MODELLING HIGHER MOMENTS AND TAIL RISK IN FINANCE

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For my family, my wife, Trang, and my son, Nam ...

Declaration

I hereby declare that this thesis is my own work except where specific reference is made to the work of others. The use of all materials from other sources has been fully and properly acknowledged. All errors remain my responsibility. Replication files and codes for all results are available upon request.

The total number of words is **65,834**.

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Abstract

This thesis aims to address three critical issues in modelling higher moments and tail risk of financial returns. First, I propose applying the Mixed Data Sampling (MIDAS) framework to forecast Value at Risk (VaR) and Expected Shortfall (ES) under a semiparametric approach. The new models exploit the serial dependence in short-horizon returns to directly forecast the tail dynamics at the desired horizon. I examine the predictive power of the new models by an extensive comparison of out-of-sample VaR and ES forecasts with the established models for a wide range of financial assets and backtests. The MIDAS-based models significantly outperform traditional GARCH-based forecasts and alternative conditional quantile specifications, especially at the multi-day forecast horizons. My analysis advocates models featuring asymmetric conditional quantile and the use of Asymmetric Laplace density to jointly estimate VaR and ES.

Second, I carry out a comprehensive comparison of the forecasting ability and economic importance of several prominent skewness models. My empirical analysis advocates the use of information from option prices to forecast skewness. Option-implied skewness and a realized skewness model, which also uses information from options, outperform two GARCH models and skewness forecasts derived from conditional quantiles. I further propose a new skewness estimator that corrects the option-implied skewness for skewness risk premium. The new estimator has the highest information content on future skewness while it consistently leads to the lowest out-of-sample forecast errors. A portfolio strategy that employs this

estimator is superior to the "*1/N*" portfolio and to the strategies based on the rest of the skewness models considered.

Third, I investigate the role of conditional higher moments, up to the fourth level, in an international portfolio allocation framework. The conditional moments of return distribution are simultaneously approximated by a set of quantile estimates using the law of total probability. My results reveal significant economic gains to an international investor by jointly incorporating conditional higher moments in the information set. The portfolio that employs both conditional skewness and kurtosis outperforms the benchmark portfolio based on mean-variance predictors and portfolio based on information up to only the third conditional moment.

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List of Abbreviations

ASharpe . . . . . . Adjusted Sharpe ratio

AL . . . . Asymmetric Laplace

As . . . . Asymmetric Slope

AR . . . . Autoregressive

CAPM . . . . Capital Asset Pricing Model

CAViaR . . . . . . Conditional Autoregressive Value at Risk

CES . . . . Conditional Expected Shortfall test

CE . . . . Certainty Equivalent

CIS . . . . Implied Skewness corrected for the SRP

CoSkew . . . . . . Co-Skewness

CoKurt . . . . . . Co-Kurtosis

CPI . . . . Consumer Pricing Index

CRRA . . . . Constant Relative Risk Aversion

DM . . . . Developed Markets

DivY . . . . Dividend Yields

DQ . . . . Dynamic Quantile Test

EM . . . . Emerging Markets

Eq. . . . . Equation

ES . . . . Expected Shortfall

ETF . . . . Exchange-Traded Fund

EVT . . . . Extreme Value Theory

Fhs . . . . Filtered Historical Simulation

FRED . . . . Federal Research Economic Data

GARCH . . . . . . Generalized Autoregressive Conditional Heteroskedasticity

GJR . . . . Glosten-Jagannathan-Runkle

GJRGARCH . . . Glosten-Jagannathan-Runkle GARCH

GPD . . . . Generalized Pareto Distribution

GPV . . . . Ghysels-Plazzi-Valkanov

HM . . . . Higher Moments

IPG . . . . Industrial Production Growth

ITM . . . . Implied Third Moment

IV . . . . Implied Variance

IS . . . . Implied Skewness

Ku . . . . Kurtosis

LRS . . . . Lagged Realized Skewness

Log-L . . . . Logarithmic Likelihood

LIBOR . . . . London Interbank Offer Rate

MAE . . . . Mean Absolute Error

MCS . . . . Model Confidence Set

MDS . . . . Martingale Difference Sequence

MIDAS . . . . . . Mixed Data Sampling

MN . . . . Multinominal Distribution

Mom . . . . Momentum

MV . . . . Mean-Variance

OKurt . . . . Orthogonalized Kurtosis

OOS . . . . Out-of-Sample

OTM . . . . Out-of-the-money

PIT . . . . Probability Integral Transform

QMIDAS . . . . . Quantile-based Skewness Forecasts with MIDAS

RMSE . . . . Root Mean Squared Error

Sav . . . . Symmetric Absolute Value

SGE . . . . Skewed Generalized Error

Sk . . . . Skewness

SkewToKurt . . . Skewness-to-Kurtosis ratio

SRP . . . . Skewness Risk Premium

SR . . . . Sharpe ratio

Std . . . . Standard Deviation

TMRP . . . . Third Moment Risk Premium

UC . . . . Unconditional Coverage Test

UES1 . . . . Unconditional Expected Shortfall - Zero Mean Discrepancy

UES2 . . . . Unconditional Expected Shortfall - PIT

U.S. . . . . United States

VaR . . . . Value at Risk

VRP . . . . Variance Risk Premium

VW . . . . Value-Weighted

Chapter 1

Introduction

Normality of asset return is a central assumption in traditional finance theories, such as the Modern Portfolio Theory of Markowitz (1952), the Efficiency Market Hypothesis of Fama (1969) and the Option Pricing Theory of Black and Scholes (1973). However, there is a plethora of empirical evidence documenting the departure of asset returns from normality, driven by significant asymmetry and tail-fatness of the return distribution.[1] Motivated by this well-documented fact, my thesis contributes to a rapidly growing stream of the literature investigating the implications of non-normality of asset returns in the context of financial modelling and applications. More specifically, I perform three empirical studies on modelling higher moments and tail risk in financial returns.

The contribution of the thesis is threefold. First, I develop a novel method to improve tail forecasts of return distribution at any horizon. The new models exploit the rich information in higher frequency returns to directly model the tail dynamics of returns measured at a lower frequency. In my empirical application, I use the new models to forecast two popular tail risk measures, namely Value at Risk (VaR) and Expected Shortfall (ES). I find that the newly proposed models outperform several established models in a wide range of financial assets and backtesting

[1]The evidence of non-normality in financial return distribution was explored as early as Mandelbrot (1963). See also Cont (2001) and the references therein for other stylised facts of financial returns.

procedures. Second, I perform the first empirical study in the literature, which explicitly seeks for the best model to forecast return skewness. Furthermore, I develop a new skewness estimator that corrects for the skewness risk premium in option-implied skewness. This new estimator consistently generates the lowest out-of-sample forecast errors and leads to a superior investing strategy compared to the popular "1/N" portfolio and to strategies based on alternative skewness models. Third, I examine the practical benefits of incorporating conditional higher moments in an international portfolio allocation framework. The robust measures of return moments are jointly estimated from a set of conditional quantiles, while portfolio optimisation is conducted on an expanded utility function of a risk-averse investor. The portfolio results confirm the investor preferences towards the higher moments of return distribution as predicted by previous theoretical studies. Moreover, my empirical analysis also contributes to the international diversification literature by documenting the heterogeneity in conditional higher moments across countries.

Accurate risk models are essential to financial institutions for their risk management decisions. However, many risk models, especially those focusing on the tail risk such as VaR and ES, fail to correctly assess the risks of financial positions and are often cited as the underlying cause leading to the 2007-2009 global financial crisis (Brownlees et al., 2011). One of the most important issues is that the calculation of these risk measures is mainly based on the 1-day ahead forecasts. This practice is clearly insufficient to provide timely warnings to investors and financial institutions to liquidate their positions, especially during periods of financial turmoil (Engle, 2011). To measure the risk for longer horizons, banks typically scale up the 1-day VaR and ES forecasts using the square-root-of-time rule (Berkowitz et al., 2011; Pérignon and Smith, 2010). This rule is based on an over-simplified assumption that financial returns are i.i.d normally distributed and that the risk is constant over the scaling period. As a result, such an assumption often leads to the poor performance of risk models in practice since financial return are not Gaussian and the shape of their distribution varies significantly over time (Engle, 2011; Hansen, 1994).

Another challenge of accurately forecasting tail risk, particularly at multi-day horizons, is the significant dependence of lower frequency returns on the higher frequency return process. For instance, Engle (2011) and Neuberger (2012) show that the non-normality of returns at longer horizons is crucially driven by the so-called leverage effects in the short-horizon returns. Moreover, a correct risk model should able to capture the multi-components nature of return volatility, which reflect information embedded in different time horizons (see, e.g., Corsi, 2009; Engle et al., 2013) or different economic conditions (see, e.g., Cenesizoglu and Timmermann, 2008; Lima and Meng, 2017).

The first study addresses these issues by applying the Mixed Data Sampling (MIDAS) method introduced by Ghysels et al. (2004) to forecast VaR and ES. The semiparametric specification of the proposed models allows the direct modelling of VaR and ES at the desired horizon, whereas the serial dependence and multi-component nature of higher frequency returns are captured by a flexible data-driven polynomial. To the best of my knowledge, I am the first to incorporate MIDAS in forecasting ES. This is important since ES has recently gained substantial attention among academics and practitioners. For instance, the "*Minimum Capital Requirements for Market Risk*" of Basel Committe on Banking Supervision (2019) has moved towards using ES, as a complement of VaR, to calculate regulatory capital requirements. This regulatory agreement is expected to be fully implemented on January 1, 2022.

More specifically, I extend the MIDAS quantile regression of Ghysels et al. (2016) to estimate VaR and ES at the desired forecast horizon using the daily return process. The conditional quantile of the return distribution is specified as a mixture of lagged higher frequency returns, which differs for each probability level and forecast horizon. I also extend the symmetric specification of Ghysels et al. (2016) to allow for asymmetric effects between negative and positive lagged returns, which subsequently yields better out-of-sample forecasts than the symmetric counterpart in most cases. I propose two approaches to forecast VaR and ES using MIDAS-based conditional quantiles. First, I adopt the recently proposed model of Taylor (2019) which estimates VaR and ES via the maximum likelihood of an Asymmetric Laplace (AL) density. Second, I follow Manganelli and Engle (2004) to combine MIDAS quantile regression with the extreme value theory (EVT). In both approaches, VaR and ES are jointly estimated and sidestep from the so-call "*non-elicitablity*" of individual ES estimation (Gneiting, 2011). To examine the accuracy of the new methods, I perform a comprehensive comparison of out-of-sample VaR and ES forecasts with established models in the literature. The empirical analysis involves an extensive dataset of 43 international equity indices; three forecast horizons (i.e., 1-day, 5-day and 10-day, respectively); twelve forecasting models; six statistical backtests on both VaR and ES; and two loss functions to compare out-of-sample forecast errors.

My benchmark models consist of two alternative semiparametric approaches to forecast VaR and ES. First, I consider the traditional GARCH-based approach of Barone-Adesi et al. (1999, 2002) and Giannopoulos and Tunaru (2005) to estimate VaR and ES at the desired horizon using filtered historical simulations. Several studies find that this approach outperforms the simple historical simulation and an analytical approximation to estimate conditional VaR and ES (Kuester et al., 2006; Lönnbark, 2016). Moreover, I also consider a simulation method proposed by McNeil and Frey (2000) that focuses on tail events using EVT. Novales and Garcia-Jorcano (2018) recently find that this method provides better ES forecasts than non EVT-based models. Second, I replace the MIDAS-based specifications of conditional quantile with an analogue from the conditional autoregressive VaR (CAViaR) model of Engle and Manganelli (2004). This model has the appealing autoregressive structure to capture impacts of volatility clustering and autocorrelation in the return process (see, e.g., Meng and Taylor, 2018; Taylor, 2019; Žikeš and Baruník, 2016).

My empirical results reveal superior performance for the new models. The benefits of the MIDAS framework becomes more pronounced at longer forecast horizons. The MIDAS-based models have the lowest number of rejections for both VaR and ES backtests at the 5- and 10-day forecast horizons. Moreover, the asymmetric MIDAS-based models generate the lowest forecast errors for both loss functions considered. The GARCH-based models provide acceptable results based on binary violation sequences, but often underestimate the risks and generate higher losses at longer forecast horizons. This is in line with the results of Slim et al. (2017) and Degiannakis and Potamia (2017) who suggest that the performance of the filter historical simulation approach depends largely on the parametric specification of the filtering model, the targeting return series and the length of the simulation period. I find that the naive aggregation of higher frequency return series to match the forecasting horizon leads to substantial loss in the conditioning information. In particular, the CAViaR-based models are inferior to all other models in multi-day forecast horizons. When the model confidence set method of Hansen et al. (2011) is employed to further compare the forecast accuracy between competing models, the CAViaR-based models are often excluded from the set of superior models, whereas the MIDAS-based models are included in most cases. This finding provides supports for the results of Engle (2011) and Neuberger (2012), who highlight the importance of accounting for serial dependence in the short-horizon return process for long-horizon return modelling. Finally, I find evidence supporting the approach of Taylor (2019) that use AL likelihood to jointly forecast VaR and ES, while the EVT-based alternatives are sensitive to the length of the in-sample estimation window.

To examine the robustness of the proposed models, I repeat the analysis on different market regimes, different country groups, alternative financial assets and estimation windows. Not surprisingly, all models produce higher forecast errors during periods of financial distress or in emerging stock markets. Moreover, the performance of the EVT-based models is substantially worse when the models are calibrated using a shorter estimation window. Nevertheless, the model ranking remains robust and is in favour of the new models. Overall, I conclude that the MIDAS framework successfully exploits the dynamics of the higher frequency return process in forecasting the tail risk of the multi-period return distribution. More specifically, I argue that it is important to directly forecast VaR and ES and account for the impact of serial dependence in higher frequency process to the tail dynamics of lower frequency return distribution. This is important for the risk management of financial institutions since inaccurate risk measurements would lead to higher probability of financial institutions to be under- or over-capitalization. In either case, over-capitalization would lead to under-investment and low probability of financial institutions, whereas under-capitalization increases the default risk in their activities.

Another stream of the literature relates the non-normality in the return distribution to its higher moments, especially skewness and kurtosis. The non-smooth variation induced by jumps in the return process and changes in market regimes result in significant asymmetry and tail-fatness in the return distribution (Bekaert et al., 1998; Guidolin and Timmermann, 2008; Huang and Tauchen, 2005). The recent financial crisis characterised by extreme returns provides a good example of the nontrivial existence of such higher moments. In his seminal work, Hansen (1994) shows that these higher moments are also time-variation (see, e.g., Brooks et al., 2005; Harvey and Siddique, 1999; Jondeau and Rockinger, 2003, , for further empirical evidence). Consequently, several empirical studies further highlight the importance of accounting for higher moments in asset pricing (e.g., Dittmar, 2002; Harvey and Siddique, 2000), portfolio allocation (e.g., Harvey et al., 2010; Jondeau and Rockinger, 2012) and risk management (e.g., Bali et al., 2008; Kostika and Markellos, 2013).

In contrast to volatility, however, there are not established proxies for the true physical values of higher moments in the literature. Previous studies rely on different models and econometric techniques to estimate and forecast higher moments of asset returns. As a result, the predictability and the practical value of return skewness and kurtosis remain largely ambiguous. While the search for a reliable estimate of ex-post kurtosis has not reached a consensus, the recent *realised skewness* estimator of Neuberger (2012) offers a reliable proxy to investigate the best method to model and forecast return skewness. Thus, in the second study, I attempt to answer this question by performing the first ever horserace between several skewness models. Furthermore, I examine the economic value of competing models based on an out-of-sample investing strategy. My empirical analysis involves 10 international equity indices; six forecasting models that utilise different information and forecasting methods; three forecast horizons (i.e. 30, 60 and 90 calendar days); two statistcial tests for assessing the information content of each forecasting model and an out-of-sample forecasting comparison under two loss functions.

The realised skewness is comprised of two components: the skewness of short-horizon returns estimated using daily returns; and the leverage effect between returns and volatility is captured by option prices. To forecast this ex-post measure, I consider five prominent models in the literature that use different approaches to estimate return skewness. The first model is based on predictive power in lagged realised skewness documented in Neuberger (2012) and Kozhan et al. (2013). Thus, I use the realised skewness, lagged by one period to forecast future return asymmetry. The next two forecasts are drawn from simulated distribution using the popular GJR-GARCH model of Glosten et al. (1993). In the first approach, the conditional skewness is generated from the asymmetry-in-volatility process as suggested by Engle (2011). In the second approach, the conditional skewness also comes from time-varying skewness in conditional return density with time-variant shape parameters. The fourth skewness forecasting model is conditional skewness approximated by a set of quantile estimates recently advocated by Ghysels et al. (2016). Next, I rely on information from option prices to estimate the forward-looking skewness estimator similar to Conrad et al. (2013). This method does not require long time series of historical returns and the implied skewness can be computed directly from daily option prices. Finally, I develop a new option-implied skewness, which adjusts for the skewness risk premium documented in Kozhan et al. (2013) and Broll (2016). This new estimator is in parallel with the evidence of improvement in the predictive power of option-implied volatility by accounting for the variance risk premium (see, e.g., Kourtis et al., 2016; Prokopczuk and Wese Simen, 2014).

My empirical results advocate the use of information from option markets to forecast skewness. The information contents of all skewness models are on average higher at longer forecast horizons. Interestingly, the encompassing regression suggests that each model appears to capture different pieces of information about future skewness. Nevertheless, the option-implied skewness is the most informative of future skewness across indices and forecast horizons. The models incorporating information from option markets provide better out-of-sample forecast performance than those only employ historical returns. The best overall performance is offered by the new implied skewness which is adjusted for the skewness risk premium. This estimator outperforms all alternative skewness forecasts across most of the comparisons. This finding is robust to various model specifications and estimation methods. Overall, my results are in line with the volatility forecasting literature: Realised and option-implied estimators are superior to GARCH-based estimators while adjusting for the relevant risk premium improves forecasting accuracy (see,e.g., DeMiguel et al., 2013; Kourtis et al., 2016; Prokopczuk and Wese Simen, 2014).

To assess the economic value of skewness forecasts, I form several investing strategies for a constant relative risk-averse investor. The optimal portfolio weight is a linear function of conditional skewness from a competing model. I use the popular $1/N$ portfolio as a benchmark investment strategy in my analysis. DeMiguel et al. (2009) show that this simple strategy consistently outperforms many popular theory-based portfolios as the former avoids estimation errors in optimisation. I find that portfolios employing the two option-implied skewness estimators outperform the rest of the skewness-based portfolios. While all skewness-based portfolios lead to lower volatility compared to the $1/N$, only the portfolio based on the corrected implied skewness estimator leads to higher average return and Sharpe ratio in all cases considered. Thus, this result extends the evidence of DeMiguel et al. (2013) and Kourtis et al. (2016) that the use of option-implied information can enhance portfolio performance. Moreover, since this is the first study assessing the economic significance of different skewness forecasts, the best method in my empirical analysis can serve as a benchmark for the future researches in the literature.

Apart from predictability, a natural question is whether market participants can exploit the time-variation in higher moments of the return distribution to improve their portfolio allocation. To this end, I attempt to answer the following two questions in the third study. Can the investor benefit from incorporating conditional higher moments in her information set? If so, does information from conditional kurtosis add value to the portfolio allocation beyond that offered by conditional skewness?

The first question is motivated by strong evidence of non-normality in the return distribution, which violates the core assumption of the classical mean-variance criteria pioneered by Markowitz (1952). The theory assumes that asset returns are normally distributed, thus the utility of an investor is only a function of expected returns and variance. However, early theoretical studies suggest that a risk-averse investor has preference to the higher moments of return distribution (Dittmar, 2002; Scott and Horvath, 1980). In particular, they are willing to give up diversification benefits to hold assets with higher skewness and lower kurtosis (Bali et al., 2019; Mitton and Vorkink, 2007). Recent empirical evidence of predictable patterns in higher moments dynamics further hints at the possibility of exploiting these characteristic to improve portfolio performance (Conrad et al., 2013; Neumann and Skiadopoulos, 2013).

The second question is based on the evidence of time-variation and heterogeneity in the tail-fatness of asset returns. For example, Ibragimov et al. (2013) and Gu and Ibragimov (2018) document that financial returns, especially those from emerging markets, are characterised by higher likelihood of extreme observations and more heavy-tailed distributions. However, the existing literature in portfolio allocation mainly focuses on the role of return skewness. Some notable examples are Chunhachinda et al. (1997); Harvey et al. (2010); Patton (2004) and Ghysels et al. (2016). Jondeau and Rockinger (2006) argue that in the case of large departure from normality, it is necessary to expand the utility function and consider expected higher moments of the return distribution up the fourth level (i.e. kurtosis). Given that there is no clear reason a priori to exclude kurtosis, it is of practical interest to investigate the marginal benefit of conditional kurtosis over the information already embedded in conditional skewness.

I address these research questions by incorporating conditional skewness and kurtosis into an international dynamic portfolio framework. The empirical analysis is conducted on stock returns of 42 international indices for the period from January 1996 to December 2017. The conditional moments of returns are estimated from a set of quantile estimates spanning over the return density. This approach has several advantages compared to previous studies. First, the quantile-based conditional higher moments are robust to the outliers. Kim and White (2004) show that return higher moments are very sensitive to outliers, which is more pronounced in the kurtosis. Thus, using quantile-based conditional higher moments ensures that the potential benefits of introducing conditional kurtosis are not driven by the extreme observations. Second, all moments of the conditional return distribution are jointly estimated using the law of total probability. This method reduces the estimation errors of forming separate forecasts for each distributional moment as typically done in the literature (see, e.g., Ghysels et al., 2016). Third, I consider the *total* skewness and kurtosis while the limited number of studies on higher moment portfolio focus on the *comoments* between asset returns and the market portfolio (see, e.g., Gao and Nardari, 2018; Jondeau and Rockinger, 2012; Martellini and Ziemann, 2010). Recent studies show that the *idiosyncratic* skewness and kurtosis also contain predictive information about future asset returns (Bali et al., 2019; Boyer et al., 2010). Therefore, my analysis provides a broader picture on the effects of higher moments on the investment decisions.

My empirical analysis reveals significant time-variation and heterogeneity in the conditional skewness and kurtosis in all equity indices. In line with Ghysels et al. (2016), I find that emerging stock markets (EMs) have less negative skewness than those of developed markets (DMs). The conditional kurtosis also exhibits a similar pattern with lower kurtosis in the EMs, although the discrepancy is less pronounced. Interestingly, after controlling for the impact of conditional skewness, the orthogonalised kurtosis of EMs is notably higher than those of DMs. This finding indicates that although EMs are favourable in terms of the conditional asymmetry, their conditional distributions are more sensitive to extreme events.

A practical challenge of employing higher return moments in portfolio allocation is the so-called "*curse of dimensionality*", which refers to the dramatic increase of the dimension in multivariate distribution modelling. To deal with this issue, I employ the parametric portfolio policy of Brandt et al. (2009) to optimise the international dynamic portfolio. More specifically, the optimal weight allocated to each market is estimated as a linear function of the distributional characteristics of its equity returns. The portfolio results suggest that the investor should allocate more weight to countries with positive (or less negative) conditional skewness and less weight the countries with higher kurtosis. This strategy is consistent with the general preference towards higher moments suggested by the theoretical works of Scott and Horvath (1980), Kimball (1993) and Dittmar (2002). The portfolio based on both conditional skewness and kurtosis also provides sizeable economic gains compared to the mean-variance portfolio and the portfolio with conditioning information only up to the third moment. More importantly, a large fraction of the improvement can be attributable to the joint dynamics of conditional higher moments. This finding is in line with results of Jondeau and Rockinger (2006) and Jondeau and Rockinger (2012), who highlight the importance of incorporating information from *both* skewness and kurtosis in the portfolio allocation.

The portfolio structure in my analysis also sheds further light on the current debate about international diversification benefits. Similar to Ghysels et al. (2016), I find that incorporating conditional skewness tilts the optimal portfolio towards EMs due to their favourable conditional asymmetry. Nevertheless, since EMs are more exposed to the extreme returns, introducing conditional kurtosis to the information set considerably reduces the portfolio weights on EMs. Notably, the EM-skewness effect disappears in the most recent period as the investor significantly increases her holding of stocks from DMs (about 30%) by shorting equity from EMs. This reaction can partly be explained by the recent rise of protectionist policies, which have slowed down the globalisation and impose negative impacts on capital inflows and prospects of emerging markets (Bekaert et al., 2016).

Finally, I perform several additional checks to further evaluate the robustness of the main findings. In particular, I investigate the benefits of conditional higher moments for a real-time investor in two ways. First, I perform an out-of-sample analysis in which the investor recursively estimates her optimal portfolio weights based on the out-of-sample forecasts of higher moments. The portfolio policy continues to exhibit positive (negative) preference of the investor to countries with higher skewness (lower kurtosis). The strategy based on both higher moments remains superior with highest average returns and lowest volatility. Second, I examine the potential impact of transaction costs using different trading cost scenarios. Again, the economic gains in the portfolio with conditional higher moments remains significant. Finally, the main results are robust to several robustness checks including alternative levels of risk aversion and inclusion of additional quantile levels for the approximation of conditional return distribution.

Chapter 2

Forecasting VaR and ES with Mixed Data Sampling

2.1 Introduction

The recent 2007-2009 financial crisis has challenged the accuracy of risk measurement models, especially those focusing on the tail risk. Yet, two important issues remain largely unexplored. First, a voluminous literature studies tail risk based on Value at Risk (VaR) estimates.[1] Although VaR plays a dominant role in the internal risk management of financial institutions and regulators, this measure fails to meet the requirements of a coherent risk metric as defined by Artzner et al. (1999). Among the alternatives, expected shortfall (ES) has recently gained more attention. Unlike VaR, ES is a coherent risk measure and offers information about the expected loss based on the tail of return distribution (Acerbi and Tasche, 2002). More specifically, the Basel Committee of Banking Supervision has incorporated ES as the main risk measure to calculate the capital requirement of

[1] Previous papers examine the predictive power of risk models in producing VaR forecasts, either explicitly (see, e.g, Berkowitz et al., 2011; Boucher et al., 2014; Brownlees and Gallo, 2010; Chen and Gerlach, 2013; Halbleib and Pohlmeier, 2012; Nieto and Ruiz, 2016) or implicitly via volatility forecasting (see, e.g, Bams et al., 2017; Berger and Missong, 2014; Brownlees et al., 2011; Slim et al., 2017)

market risk for financial institutions in its latest regulatory agreement (Basel III). Despite its importance, there is little empirical studies focusing on ES forecasts.[2] This is mainly due to the difficulty in ES estimation and backtesting procedures (Gneiting, 2011). Second, the large extant literature focuses on the 1-day ahead risk forecasts, which is clearly insufficient to warn the investors and financial institutions to liquidate their positions, especially during the financial turmoil. As emphasised by Engle (2011), p. 438, "*the financial crisis was predicable one day ahead*", and as such, the key failure of risk modelling in financial crisis lies on their deteriorations in multi-day ahead forecasts.

This study addresses these gaps by extending the novel quantile regression based on the Mixed Data Sampling (MIDAS) of Ghysels et al. (2016) to forecast VaR and ES. The new methods allow for direct forecasts of VaR and ES at the desired horizon, while the use of semiparametric specifications avoids making a restrictive assumption about the conditional return distribution. To the best of my knowledge, this is the first study in the literature that applies MIDAS to obtain ES forecasts. I perform a comprehensive analysis of the forecasting accuracy of the proposed method. The main analysis involves: 43 international indices; three forecast horizons (i.e., 1-day, 5-day and 10-day, respectively); twelve forecasting models; six statistical backtests on both VaR and ES; and an out-of-sample forecast comparison with two loss functions. I also investigate model performance under different market regimes, countries groups, alternative assets and several model specifications.

My investigation draws on two streams of the literature. First, it is well-established that financial return distribution is not normal and this fact is more pronounced at the multi-day horizon. Engle (2011) and Neuberger (2012) find that the asymmetry of return distribution increases with horizon and converges very slowly to the normality. Moreover, the long-horizon return distribution depends

[2] Some notably studies are, among others, Cai and Wang (2008), Taylor (2008), Novales and Garcia-Jorcano (2018), Taylor (2019) and Patton et al. (2019).

crucially on serial dependence and dynamic in short-horizon return process. As the return horizon is lengthier, the shape of return distribution, i.e., asymmetry (tail-fatness) is mainly driven by the covariance between lagged (squared) returns and innovations in variance (Neuberger and Payne, 2019). Fama and French (2018) recently apply bootstrapping simulations and observe that the nonnormality remains significant even at the 20- and 30-years returns. Consequently, a good forecasting model at the short horizons, such as 1-day ahead, does not necessary yield accurate forecasts at the multi-day horizons. Each quantile in a nonnormal distribution may also evolve in different dynamics and depends on different sets of information. For example, Cenesizoglu and Timmermann (2008) and Lima and Meng (2017) document the asymmetric effects of economic variables on different parts of the return distribution and time-variation in their explanatory powers. These observations suggest that a tail risk model may benefit from the direct estimation of tail areas such as quantile regression, rather than the traditional approach using conditional return distribution models, such as the GARCH family.

Second, several studies document that the dynamics of return volatility are characterised by multiple components capturing information at different time horizons. Some notable examples are Engle and Lee (1999), Chernov et al. (2003), Corsi (2009) and Engle et al. (2013). Given the strong correlation between volatility and return quantiles, it is natural to calibrate a model that could capture different components of information in modelling tail dynamics. Engle (2011) and Neuberger (2012) highlight that long-horizon return distribution depends crucially on the dynamics in short-horizon return process. Therefore, one needs to consider the serial dependence in short-horizon return when forecasting VaR and ES at the multi-horizon-ahead.

Altogether, I propose to extend the novel MIDAS quantile regression of Ghysels et al. (2016) to directly forecast VaR and ES at the desired horizon. The MIDAS framework introduced by Ghysels et al. (2004) provides an efficient method to link variables sampled at different frequencies. The use of flexible and parsimonious lag polynomials allows MIDAS to directly forecast lower frequency variables by exploiting the data-rich environment of higher frequencies variables. Andreou et al. (2012) locate the MIDAS approach in the middle of the '*direct*' and '*iterate*' methods in the forecasting literature (Marcellino et al., 2006). A number of studies document the advantage of applying MIDAS in various financial forecasts, including Andreou et al. (2013) and Kuzin et al. (2013) for macroeconomic predictions; Pettenuzzo et al. (2016) for return density; Ghysels et al. (2006) and Ghysels et al. (2019) for volatility. Moreover, MIDAS provides a suitable framework to capture different components in the tail dynamics by data-driven weighting scheme with flexible shapes. Thus, we may expect the use of MIDAS framework can enhance the estimation and predictive power of VaR and ES forecasts.

To estimate ES, however, one needs to address the central problem of "*non-elicitability*". A measure is considered as "*elicitable*" if it is the correct minimiser of at least one loss function (Gneiting, 2011). For example, VaR is elicitable since VaR can be estimated by minimizing the '*tick loss*' function, which is the main ingredient in the quantile regression of Koenker and Bassett (1978). Early empirical studies sidestep this issue in several ways. A popular approach is to estimate ES as the mean of many VaR estimates corresponding to a tail area. In particular, one can employ simulations on innovations of a location-scale model to produce an empirical density from which VaR and ES can be obtained (see,e.g, Lönnbark, 2016; Novales and Garcia-Jorcano, 2018, for recent applications). However, this method relies heavily on the distributional assumption, which I try to avoid due to the nonnormality issues mentioned above. Instead, I follow two alternative semiparametric approaches to directly model VaR and ES and allow their dynamics to vary with each quantile level.

I start from the premise that it is important to account for the serial dependence of higher frequency (i.e. daily) return process in modelling the conditional density at the desired horizon (Neuberger, 2012). For this purpose, I develop the proposed models on the MIDAS-based quantile regression of Ghysels et al. (2016). In particular, the conditional quantile is based on a mixture of lagged higher frequency returns, which is driven by the data environment and flexibly differs for each quantile level and forecast horizon. Moreover, I also develop an asymmetric specification, which provides better out-of-sample forecast performance than its symmetric counterparts of Ghysels et al. (2016) in most cases.

In the first approach, I adopt the semiparametric model of Taylor (2019) based on the Asymmetric Laplace (AL) density. The author explores the fact that although ES is not individually elicitable, it is jointly elicitable with VaR under a set of suitable scoring functions (Fissler and Ziegel, 2016). Since the AL log-likelihood is a member of this set, VaR and ES can be jointly estimated via maximum likelihood of an AL density. In the second approach, I follow Manganelli and Engle (2004) to combine quantile regression and extreme value theory (EVT). The conditional VaR and ES are estimated by fitting a Generalised Pareto Distribution (GPD) to the extreme observations that exceeded a threshold level.

In the empirical analysis, I employ two alternative semiparametric approaches in the literature as the benchmark methods. First, I consider the filtered historical simulation approach introduced by Barone-Adesi et al. (1999, 2002) and Giannopoulos and Tunaru (2005). I use two GARCH models to prefilter the data, namely the GARCH(1,1) model of Bollerslev (1987) and its asymmetric version, i.e. the GJR-GARCH(1,1) model of Glosten et al. (1993). VaR and ES forecasts are then obtained from the empirical distribution approximated from simulated paths of returns at the desired horizon using bootstrapping methods. Second, I replace MIDAS-based quantile specifications by the conditional autoregressive

VaR (CAViaR) specifications of Engle and Manganelli (2004). The CAViaR-based dynamics have attractive autoregressive structure, yet one needs to form a single-horizon return series that matches the forecast horizon in the model estimation (see, e.g., Jeon and Taylor, 2013; Taylor, 2019, for applications in VaR and ES forecasts).

I employ a battery of statistical tests to compare the out-of-sample VaR and ES forecasts between competing models. In the first stage, I analyse their absolute performance based on the desired properties of VaR and ES as risk metrics, such as the correct tail coverage and interdependent exceedances. The backtests include the unconditional coverage test of Kupiec (1995), the dynamic quantile test of Engle and Manganelli (2004), the unconditional ES test on violation residuals of McNeil and Frey (2000), the unconditional and conditional ES test using probability-integral-transform (PIT) of Du and Escanciano (2017) and the multinomial VaR test of Kratz et al. (2018). In the second stage, I investigate the relative performance of competing models in term of minimizing two loss functions. To this end, I form a set of superior models using the Model Confidence Set (MCS) technique of Hansen et al. (2011).

In summary, I obtain strong evidence in favor of the new models across quantile levels and forecasting horizons. Although my focus is to improve the multi-day horizon VaR and ES forecasts, the MIDAS-based models provide competitive performance to the benchmarks at the 1-day horizon as well. In fact, the asymmetric MIDAS-based VaR and ES forecasts are often at par with asymmetric GARCH-based models in term of absolute performance, but the former generally yields lower forecast errors. Similar to Engle and Manganelli (2004) and Taylor (2019), I find that the asymmetric specifications provide superior VaR and ES forecasts at the 1-day horizon.

The benefits of MIDAS framework are more pronounced at multi-day forecast horizons. The MIDAS-based models lead to the lowest number of test rejections for both VaR and ES forecasts at the 5- and 10-day horizons. The asymmetric MIDAS-based models also generate the lowest forecast errors and are often included in the set of superior models. In contrast, the performance of CAViaR-based models always inferior to all other models in multi-day forecasting horizons. Thus, the naive aggregation to single-horizon return series leads to substantial loss in forecasting information. This observation is in line with the findings of Neuberger (2012) who highlights the importance of accounting for the serial dependence in higher frequency returns when modelling the dynamics of lower frequency return distribution. Finally, I find evidence supporting the joint model of Taylor (2019) that uses the AL likelihood in forecasting VaR and ES. The EVT-based alternatives provide similar performance to the AL-based forecasts in the main analysis. However, their predictive powers are sensitive to the estimation sample and deteriorate significantly when the estimation window is shorter.

I perform a series of robustness tests on the main results. First, I investigate model performance across different market regimes, before, during and after the recent great financial crisis. Not surprisingly, I observe considerably higher forecast errors during the crisis period. Nevertheless, the relative ranking between competing models is generally unchanged with the proposed methods always belonging to the best performing models. Second, the main results are robust when I repeat the analysis for individual stocks, alternative asset classes and separately for developed versus emerging stock markets.

The remainder of this chapter is structured as follows: Section 2.2 presents the methodology, in which I provide details on the proposed models for VaR and ES, the benchmark methods and the backtesting procedures. Section 2.3 presents the empirical studies on the out-of-sample forecast comparison. Section 2.4 presents several robustness checks on model performance spanning different market regimes, alternative assets and alternative length of estimation windows.

Section 2.5 identifies potential limitations. Finally, section 2.6 concludes the chapter.

2.2 Methodology

2.2.1 New Models for VaR and ES Forecasts

Let $\{r_t\} = ln(P_t/P_{t-1})$ be the daily continuously compounded return series where P_t is the closing price of trading day t. The h-day horizon return is defined as $r_{t,h} = \sum_{i=1}^{h} r_{t+i}$. The h-day VaR of an asset or portfolio returns at the $(1-\alpha)\%$ confidence level is simply the conditional quantile at the probability level α, $Q_{\alpha,t-1}(r_{t,h})$.[3]

2.2.1.1 The MIDAS-based Conditional Quantile Specifications

The main ingredient of the proposed models is the MIDAS-based conditional quantile specification introduced by Ghysels et al. (2016). The conditional quantile of returns at any horizon is specified as a linear function of conditioning variables, which can be sampled at different frequencies:

$$Q_{\alpha,t-1}(r_{t,h}) = \beta^0_{\alpha,h} + \beta^1_{\alpha,h} \sum_{d=1}^{D} \varphi_d(\kappa_{\alpha,h})|r_{t-d,1}| \tag{2.1}$$

where the absolute daily return $|r_{t-d,1}|$ is the conditioning variable with a lag length of D days. $\varphi(.)$ is the polynomial function that linearly filters the conditioning variable and projects to the conditional quantile. $\kappa_{\alpha,h}$ is a low-dimensional parameter vector that parsimoniously defines the shape of the weighting function. The vector of estimated parameters $\theta_{\alpha,h} = (\beta^0_{\alpha,h}, \beta^1_{\alpha,h}, \kappa_{\alpha,h})$ is quantile-specific at the considered horizon.

[3] Throughout the chapter, I use the terms "VaR" and "conditional quantile" at the α quantile level interchangeably to imply the conditional VaR at the $(1-\alpha)$ confidence level. To simplify the notation, I drop the horizon subscript h whenever it does not cause confusions, keeping in mind that the series refers to the h-day horizon from day t to $t+h$.

Previous studies document superior performance of accounting for the asymmetric effects in forecasting volatility (see, e.g., Brownlees et al., 2011), or VaR and ES (see, e.g., Ener et al., 2012; Engle and Manganelli, 2004; Lönnbark, 2016; Taylor, 2019). Thus, a natural extension of Eq. (2.1) is to capture the potential asymmetric effects of positive and negative returns, which can be specified as follow:

$$
\begin{aligned}
&Q_{\alpha,t-1}(r_{t,h}) \\
&= \beta^{0}_{\alpha,h} + \beta^{1-}_{\alpha,h} \sum_{d=1}^{D} \varphi_d(\kappa_{\alpha,h}) I_{(r_{t-d,1}<0)} |r_{t-d,1}| + \beta^{1+}_{\alpha,h} \sum_{d=1}^{D} \varphi_d(\kappa_{\alpha,h}) I_{(r_{t-d}\geq 0)} |r_{t-d,1}|
\end{aligned}
\tag{2.2}
$$

where $I_{(.)}$ is the indicator function. To retain the parsimonious advantage of the MIDAS framework, I apply one polynomial $\varphi_d(\kappa_{\alpha,h})$ but allowing for different slope coefficients for negative and positive lagged returns. I follow Ghysels et al. (2016) to specify $\varphi(\kappa_\alpha)$ as the "Beta" function with two parameters, $\varphi(\kappa_1, \kappa_2)$, given that it provides highly flexible shapes (see, Ghysels et al., 2007, for technical discussions and alternative polynomial functions). The Beta polynomial is expressed as follows:

$$
\varphi(\kappa_1, \kappa_2) = \frac{f(\frac{d}{D}, \kappa_1, \kappa_2)}{\sum_{d=1}^{D} f(\frac{d}{D}, \kappa_1, \kappa_2)}
$$

where:

$$
\begin{aligned}
f(x, a, b) &= \frac{x^{a-1}(1-x)^{b-1}\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \\
\Gamma(a) &= \int_0^\infty e^{-x} x^{a-1} dx.
\end{aligned}
$$

In my application, I restrict $\kappa_1 = 1$ and $\kappa_2 > 1$ in order to have decaying weights on the conditioning variable.[4] The lag length is set at $D = 100$ days for all forecasting horizons. This choice is based on the observation of Ghysels et al.

[4]Similar Ghysels et al. (2016), I find that optimising both two parameters can marginally improve the goodness-of-fit in quantile estimate. However, the optimisation comes at significant computational cost and a lower convergence rate.

(2006) that using lags longer than 50 days has little effect on volatility forecasts up to 20-days horizon. Moreover, Ghysels and Qian (2019) argue that the only issue of choosing a large lag length is the loss of higher frequency data at the beginning of estimation sample. In fact, it is more problematic in choosing a too short lag length.[5]

The conditional quantile could be directly estimated using a quantile regression model pioneered by Koenker and Bassett (1978) and first applied to the financial data by Engle and Manganelli (2004). The conditional quantile is obtained by minimizing the following *tick loss* function:

$$\widehat{\theta}_{\alpha,h} = \underset{\theta_{\alpha,h}}{\operatorname{argmin}} \quad T^{-1}\sum_{t=1}^{T}\left[r_{t,h} - Q_{\alpha,t-1}(r_{t,h})\right]\left[\alpha - I_{(r_{t,h} \leq Q_{\alpha,t-1}(r_{t,h}))}\right] \tag{2.3}$$

where $\theta_{\alpha,h}$ is the unknown vector of parameters to be estimated for each quantile level and forecast horizon. The conditional ES is defined as the expected loss given a VaR violation occurred and can be expressed as:

$$ES_{\alpha,t-1}(r_{t,h}) = E\left[r_{t,h} | r_{t,h} \leq Q_{\alpha,t-1}(r_{t,h})\right] \tag{2.4}$$

However, unlike VaR, there is no loss function such that ES is the unique minimiser (Gneiting, 2011). Therefore, I employ to two alternative approaches to estimate ES based on the above MIDAS-based conditional VaR specifications.

2.2.1.2 Forecast VaR and ES with Asymmetric Laplace Distribution

In the first approach, I jointly estimate VaR and ES using Asymmetric Laplace (AL) density as proposed by Taylor (2019). This model is motivated by the work of Koenker and Machado (1999), who link the minimisation of the '*tick loss*' function in Eq. (2.3) to the maximum likelihood of an AL density specified as

[5] To check the sensitivity of this choice, I also repeat the main analysis with $D = (80, 120)$ days and observe similar results

follows:

$$f(r_t) = \frac{\alpha(1-\alpha)}{\sigma} exp\left(-\frac{(r_t - Q_\alpha(r_t))(\alpha - I(r_t \leq Q_\alpha(r_t))}{\sigma}\right) \tag{2.5}$$

where, for this density, $Q_\alpha(r_t)$ is the time-varying location, while $\sigma > 0$ and $0 < \alpha < 1$ are the scale and skew parameters, respectively. Note that the return process is not assumed to follow AL distribution since the skew parameter α is chosen corresponding to the quantile level of interest. Taylor (2019) argues that if the scale parameter σ varies over time, its maximum likelihood estimation can be interpreted as the time-varying expectation of the '*tick loss*' function:

$$\sigma_t = E_{t-1}\left[(r_t - Q_\alpha(r_t))\left(\alpha - I\left(r_t - Q_\alpha(r_t)\right)\right)\right] \tag{2.6}$$

Given that Bassett (2004) links the conditional ES to quantile regression by:

$$ES_{\alpha,t-1}(r_t) = E_{t-1}(r_t) - \frac{1}{\alpha} E_{t-1}\left[(r_t - Q_\alpha(r_t))\left(\alpha - I\left(r_t - Q_\alpha(r_t)\right)\right)\right]$$

then Eq. (2.6) can be rewritten in term of conditional ES and conditional mean $\mu_t = E_{t-1}(r_t)$ as follow:

$$\sigma_t = \alpha(\mu_t - ES_{\alpha,t-1}(r_t))$$

Thus, for given specifications of the conditional mean, conditional VaR and ES, the AL density in Eq. (2.5) can be rewritten in conditional terms as:

$$f(r_t) = \frac{1-\alpha}{\mu_t - ES_{\alpha,t-1}(r_t)} exp\left(-\frac{(r_t - Q_{\alpha,t-1}(r_t))(\alpha - I(r_t \leq Q_{\alpha,t-1}(r_t))}{\alpha(\mu_t - ES_{\alpha,t-1}(r_t))}\right) \tag{2.7}$$

Without the loss of generality, I specify the conditional mean return as an AR(1) formulation where $\mu_t = a_0 + a_1 r_{t-1}$ to account for possible autocorrelation in the return process. I follow Taylor (2019) to specify conditional ES as an exponential function of the conditional quantile to prevent possible crossovers between conditional VaR and ES:

$$ES_{\alpha,t-1}(r_t) = [1 + \exp(\gamma)]\, Q_{\alpha,t-1}(r_t) \tag{2.8}$$

where γ controls the joint dynamics of VaR and ES. The $Q_{\alpha,t-1}(r_t)$ can follow either the MIDAS-based specifications in Eq. (2.1) or (2.2). Finally, I follow the optimisation procedure of Taylor (2019) to jointly estimate VaR and ES. To assist the optimisation, I separately estimate the coefficients in the conditional mean using maximum likelihood and the conditional quantile using the MIDAS quantile regression.[6] Next, these optimised values are combined with 10^4 randomly sampled candidates for the γ coefficient in the ES formulation to form the vectors of starting parameters. The optimisation is then performed on the negative of the sample log-likelihood of Eq. (2.7). I term the models which define $Q_{\alpha,t-1}(r_t)$ in Eq. (2.1) and (2.2) as '*Midas-AL*', and '*MidasAs-AL*', respectively.

2.2.1.3 Forecast VaR and ES with Extreme Value Theory

In the second approach, I adopt the two-step estimation procedure suggested by Manganelli and Engle (2004). First, the MIDAS quantile regression of Ghysels et al. (2016) is estimated at a threshold level which is not as extreme as the quantile level of interest. The standardised quantile residuals, Z_{α_u}, are then obtained as follows:

$$Z_{\alpha_u} = \frac{r_t}{Q_{\alpha_u,t-1}(r_t)} - 1 \tag{2.9}$$

where $Q_{\alpha_u,t-1}(r_t)$ is the conditional quantile at threshold level α_u. Similar to Manganelli and Engle (2004), I choose the threshold level at $\alpha_u = 7.5\%$. Second,

[6]The estimation is based on an R code created by the author following the Matlab toolbox provided by Eric Ghysels

I fit the Generalised Pareto Distribution (GPD) to the standardised quantile residuals of threshold violations, i.e, $Z_{\alpha_u}^{exceed} = Z_{\alpha_u} | Z_{\alpha_u} > 0 \sim GPD(\widehat{\xi}, \widehat{\varsigma})$, where $\widehat{\xi} < 1$ is the shape parameter and $\widehat{\varsigma}$ is the scale parameter. Conditional VaR and ES at any quantile level $\alpha < \alpha_u$ then can be computed using the results of McNeil and Frey (2000):

$$
\begin{aligned}
&Q_{\alpha,t-1}(Z_{\alpha_u}) = \frac{\widehat{\varsigma}}{\widehat{\xi}}\left[\left(\frac{\alpha T}{T_u}\right)^{-\widehat{\xi}} - 1\right] \\
&ES_{\alpha,t-1}(Z_{\alpha_u}) = Q_{\alpha,t-1}(Z_{\alpha_u})\left(\frac{1}{1-\widehat{\xi}} + \frac{\widehat{\varsigma}}{(1-\widehat{\xi})Q_{\alpha,t-1}(Z_{\alpha_u})}\right) \\
&Q_{\alpha,t-1}(r_t) = Q_{\alpha_u,t-1}\left[1 + Q_{\alpha,t-1}(Z_{\alpha_u})\right] \\
&ES_{\alpha,t-1}(r_t) = Q_{\alpha_u,t-1}\left[1 + ES_{\alpha,t-1}(Z_{\alpha_u})\right]
\end{aligned}
$$

where T_u is the number of exceedances beyond the conditional threshold. In this approach, I denote the model that uses the Eq. (2.1) specification in quantile regression as '*Midas-Evt*', whereas I use the term '*MidasAs-Evt*' when specification in Eq. (2.2) is used.

2.2.2 Benchmark Models

In this section, I present a set of benchmark models to examine the predictive power of new methods on out-of-sample VaR and ES forecasts. The details of forecasting models are presented in Table A.1 in the Appendix.

2.2.2.1 Filtered Historical Simulation

The first benchmark method is the Filtered Historical Simulation (Fhs) introduced by Barone-Adesi et al. (1999, 2002) for VaR and extended to ES by Giannopoulos and Tunaru (2005). Kuester et al. (2006) and Lönnbark (2016) find that this approach outperforms the simple historical simulation as well as the analytical approximation in VaR forecasts. The Fhs algorithm involves a bootstrap procedure using return innovations of a conditional location-scale model to simulate return paths at the considered horizon. Given a reasonably large number of trials, these paths can form an empirical distribution, from which conditional VaR and ES can be obtained.

I consider two GARCH models to prefilter the data, namely the GARCH(1,1) model of Bollerslev (1987) and its asymmetric version, i.e., GJR-GARCH(1,1) model of Glosten et al. (1993). Brownlees et al. (2011) document that the latter provides better volatility forecasting performance relative to alternative GARCH-type models. To be consistent with the MIDAS-based models, I model conditional mean as an AR(1) process:

$$r_t = a_0 + a_1 r_{t-1} + \sigma_t z_t \tag{2.10}$$

while the conditional variance process is defined as:

$$\text{GARCH:} \quad \sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{2.11}$$

$$\text{GJR-GARCH:} \quad \sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 I_{(\varepsilon_{t-1}<0)} \varepsilon_{t-1}^2 + \beta_3 \sigma_{t-1}^2 \tag{2.12}$$

where $\varepsilon_t = \sigma_t z_t$ is the residuals from the mean equation and $z_t \sim f_z(.)$ is the series of standardised resdiuals. A common agreement in the literature is the need to account for fat tails and asymmetries in the conditional distribution of z_t, i.e., $f_z(.)$ (see, e.g., Brooks et al., 2005; Giot and Laurent, 2003; Slim et al., 2017). In my empirical application, I utilise the standardised Skewed Generalised Error (SGE) distribution of Theodossiou (2015), i.e., $z_t \sim SGE(0, 1, \lambda, \eta)$. This distribution allows for nonnormality in the return distribution, where the shape parameters $-1 < \lambda < 1$ and $\eta > 0$ control the asymmetry and tail thickness, respectively. The distributional density is symmetric when $\lambda = 0$ and skews to the left (right) when $\lambda < 0$ ($\lambda > 0$). When $\lambda = 0$ and $\eta = 2$, it gives the standardised normal distribution (see, e.g., Anatolyev and Petukhov, 2016; Feunou et al., 2016, for application of SGE distribution to financial data).

To estimate conditional VaR and ES at h-day horizon, I perform the following algorithm:

1. On day t, I randomly sample $\{z^*_{t+1}, z^*_{t+2}, .., z^*_{t+h}\}$ with replacement from the set of standardised residuals.

2. The sampled residuals are plugged into the conditional mean and variance equations (i.e., Eq. (2.10) - (2.12)) to generate a simulated path of returns $\{r^*_{t+1}, r^*_{t+2}, ..., r^*_{t+h}\}$. The bootstrapped h-day return is then constructed as $r^*_{t,h} = \sum_{i=1}^{h} r^*_{t+i}$.

3. I repeat the above steps $B = 10,000$ times to form an empirical return distribution at the h-day horizon, $\{r^b_{t,h}\} = \{r^1_{t,h}, r^2_{t,h}, ..., r^B_{t,h}\}$.

The conditional VaR is obtained as the α^{th} percentile of the simulated return distribution:

$$Q^B_\alpha(r_{t,h}) = \{r^b_{t,h}\}_{B\alpha} \tag{2.13}$$

and the corresponding conditional ES is:

$$ES^B_\alpha(r_{t,h}) = \frac{1}{B\alpha}\sum_{b=1}^{B} r^b_{t,h} I_{(r^b_{t,h} < Q^B_\alpha(r_{t,h}))} \tag{2.14}$$

where $I_{(r^b_{t,h} < Q^B_\alpha(r_{t,h}))}$ is an indicator function. I term the VaR and ES forecasts from this method as "*GARCH-Evt*" when the conditional variance in Eq. (2.11) is used and "*GJR-Fhs*" when Eq. (2.12) is used.

2.2.2.2 EVT-based Filter Historical Simulation

An alternative simulation approach is to combine FHS with EVT as proposed by McNeil and Frey (2000). To this end, I fit a GPD to those standardised residuals that exceeded the threshold, which correspond to the 7.5% percentile of the standardised residuals. Next, I follow McNeil and Frey (2000) to simulate the conditional return distribution at the h-day horizon using the following algorithm:

1. Similar to the FHS, I randomly sample $\{z^*_{t+1}, z^*_{t+2}, .., z^*_{t+h}\}$ with replacement from the set of standardised residuals.

2. If the bootstrapped z^* is lower than the threshold level, I replace it with a simulated value from a GPD ($\widehat{\xi}$,$\widehat{\eta}$), where $\widehat{\xi}$ and $\widehat{\eta}$ are the estimated GPD parameters from in-sample standardised residuals. Otherwise, the sampled standardised residuals is used.

3. Steps (3) and (4) of the FHS algorithm are then applied to form an empirical return distribution at the h-day horizon using $B = 10,000$ trails, $\{r^b_{t,h}\} = \{r^1_{t,h}, r^2_{t,h}, ..., r^B_{t,h}\}$.

The conditional VaR and ES are then obtained by Eq. (2.13) and (2.14) as above. I term the VaR and ES forecasts from this approach "*GARCH-Evt*" and "*GJR-Evt*", depending on whether the filtering model is GARCH(1,1) and GJR-GARCH(1,1), respectively.

2.2.2.3 CAViaR-based Models

In the next benchmark method, I replace the MIDAS-based specifications with two analogues drawing from the CAViaR model of Engle and Manganelli (2004):
Symmetric Absolute Value:

$$Q_{\alpha,t-1}(r_t) = \beta_0 + \beta_1 Q_{\alpha,t-2}(r_{t-1}) + \beta_2 |r_{t-1}| \tag{2.15}$$

Asymmetric Slope:

$$Q_{\alpha,t-1}(r_t) = \beta_0 + \beta_1 Q_{\alpha,t-2}(r_{t-1}) + \beta_2^- I_{(r_{t-1}<0)} |r_{t-1}| + \beta_2^+ I_{(r_{t-1}\geq 0)} |r_{t-1}| \tag{2.16}$$

The conditional VaR and ES are estimated using either AL density or EVT as described earlier. A contrasting difference between this benchmark method and MIDAS-based models is the treatment of higher frequency observations. The CAViaR model works exclusively on single-horizon setting. This means that one needs to aggregate higher frequency returns to match the target forecasting horizon to perform model estimation. I term the forecasting models "*Sav-AL*" and "*Sav-Evt*" when symmetric absolute value specification is utilised. Alternatively, I refer the models as "*As-AL*" and "*As-Evt*" when the asymmetric slope specification is employed.

2.2.3 Evaluation Methods of VaR and ES forecasts

I employ two alternative ways to evaluate the accuracy of out-of-sample VaR and ES forecasts. First, I assess the absolute performance of VaR and ES forecasts corresponding to their usages as risk measures. Second, I evaluate the relative performance of competing models using two loss functions.

2.2.3.1 Absolute Performance Evaluation

VaR backtests

I employ two popular tests to investigate the accuracy of VaR forecasts, including the unconditional coverage (UC) test of Kupiec (1995) and the dynamic quantile (DQ) test of Engle and Manganelli (2004). Under the null hypothesis of the UC test, the number of VaR violations is not statistically different from the chosen quantile level. The test can be performed using the log-likelihood ratio (LR) statistic:

$$LR = 2[T_u ln(T_u/(\alpha T)) + (T - T_u) ln((T - T_u)/(T - \alpha T))]$$

where T is the number of observations, α is the probability level and T_u is the number of VaR exceedances. The LR test statistic follows a $\chi^2(1)$ distribution. Apart from unconditional coverage, DQ further examines the dependence between VaR violations. The test statistic involves a transformation of VaR series to a hit sequence, $Hit_t = I_{(r_t < Q_{\alpha,t-1}(r_t))} - \alpha$. Under the null hypothesis of correct VaR forecasts, Hit_t should have a zero unconditional and conditional expectation given the information set available at time $t-1$. The test can be performed using a linear regression $Hit_t = X\beta + \varepsilon_t$, where X is a set of potential explanatory variables. Similar to Bali et al. (2008), I include a constant, the current level of VaR and five lags of Hit_t. The test statistic is specified as:

$$DQ = \frac{\widehat{b}' X' X \widehat{b}'}{\alpha(1-\alpha)}$$

where $\widehat{b}$ are the estimated coefficients of the linear regression and the DQ test statistic follows $\chi^2(7)$ distribution, where 7 is the column dimension of X.

ES backtests

I consider three backtesting procedures for ES forecasts. First, I employ the discrepancy test of McNeil and Frey (2000). After standardizing by corresponding VaR estimates, the standardised discrepancies between VaR violations and ES forecasts should have unconditional mean of zero under the correct risk model. This null hypothesis can be tested using bootstrap method with 10,000 trials as documented in McNeil and Frey (2000).

Second, I adopt the unconditional and conditional ES tests of Du and Escanciano (2017) due to their analogy with the VaR backtests. Instead of explicitly employing ES estimates, they implicitly examine the accuracy of the risk model in tail coverage. These tests are based on the observation that VaR violations should form a class of martingale difference sequence (MDS), indexed by the considered quantile level. Du and Escanciano (2017) argue that the cumulative violations also form MDS and provide meaningful information about the conditional tail when a violation occurs to backtest ES. The cumulative violation process is defined as the integral of VaR violations:

$$H_t(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} h_t(u)\mathrm{d}u$$

where $h_t(u) = I_{(r_t < Q_{u,t-1}(r_t))}$ is the hit indicator at quantile level, u, at time t. If the risk model is correctly specified, $h_t(u)$ has mean u. Similar to Du and Escanciano (2017), I define $u_t = F(r_t|\widehat{\theta}_\alpha, \Omega_{t-1})$ for computational purposes, where $F(.|\Omega_{t-1})$ is the conditional cumulative return distribution given the estimated parameters of the risk model, $\widehat{\theta}_\alpha$. Then, $h_t(u) = I_{(r_t < Q_{u,t-1}(r_t))} = I_{(u_t<u)}$ and the cumulative violations process can be written as:

$$H_t(\alpha, \widehat{\theta}_\alpha) = \frac{1}{\alpha}\int_0^{\alpha} I_{(u_t<u)}\mathrm{d}u = \frac{1}{\alpha}(\alpha - \widehat{u}_t)I_{(\widehat{u}_t<\alpha)}$$

The unconditional ES test can be conducted by testing the null hypothesis $H_0 : E\left[H_t(\alpha, \widehat{\theta}_\alpha)\right] = \alpha/2$ using a standard t-test:

$$U_{ES} = \frac{\sqrt{T}\left(\overline{H}(\alpha) - \alpha/2\right)}{var(H_t(\alpha))} \sim N(0,1) \tag{2.17}$$

where T is the number of forecasts and $var(H_t(\alpha)) = \sqrt{\alpha(1/3 - \alpha/4)}$, and $\overline{H}(\alpha)$ is the sample mean of $\{\widehat{H}(\alpha)\}_{t=1}^{T}$. Finally the conditional ES test can be obtained by checking whether $\{H_t(\alpha, \widehat{\theta}_\alpha) - \alpha/2\}_{t=1}^{\infty}$ are uncorrelated with the null hypothesis being $H_0 : E\left[H_t(\alpha, \widehat{\theta}_\alpha) - \alpha/2|\Omega_{t-1}\right] = 0$. I define the lag-$j$ autovariance, $\gamma_{T,j}$, and autocorrelation, $\rho_{T,j}$, of $\{H_t(\alpha)\}_{t=1}^{T}$ for $j \geq 0$ as:

$$\gamma_{T,j} = \frac{1}{T-j}\sum_{t=j+1}^{T}\left[H_t(\alpha) - \alpha/2\right]\left[H_{t-j}(\alpha) - \alpha/2\right] \quad \text{and} \quad \rho_{T,j} = \frac{\gamma_{T,j}}{\gamma_{T,0}}$$

To be consistent with the DQ test, I chose a lag order $m = 5$. The test can then be conducted using a simple Box-Pierce test statistic.

$$C_{ES}(m) = N \sum_{j=1}^{T} \hat{\rho}^2_{T,j} \sim \chi^2_m \tag{2.18}$$

Finally, I employ the multinomial VaR (MultiVaR) test of Kratz et al. (2018) to evaluate the accuracy in tail coverage by simultaneously testing VaR estimates at multiple quantile levels. From a practical viewpoint, this test has the advantage of not having to store predictive distribution $F(.|\Omega_{t-1})$ from the risk model at each forecast. For a given starting quantile level of interest, α, I consider a series of VaR forecasts at levels $\alpha_1, .., \alpha_N$ given by:

$$\alpha_j = \alpha + \frac{j-1}{N}(1-\alpha), \quad j = 1, .., N$$

I choose the starting quantile level $\alpha = 0.025$, which is equivalent to backtesting ES forecasts at the 2.5% quantile level. This choice is motivated by the requirement of Basel Committe on Banking Supervision (2019) for ES forecasts.[7] I define $I_{t,j} = I_{(r_t < Q_{\alpha_j, t-1})}$ as the violation indicator. The sequence $X_t = \sum_{j=1}^{N} I_{t,j}$ counts the number of VaR estimates being violated at each time t. Similar to individual VaR forecasts, for the sequence (X_t) to have a correct tail coverage, it should satisfy the following two properties:

1. The unconditional coverage: $P(X_t \leq j) = \alpha_{j+1}, j = 0, ..., N$ for all t.

2. The conditional coverage: X_t is independent of X_s for all $s \neq t$.

Kratz et al. (2018) show that the two above conditions can be tested using multinomial distribution of the sequence. Let $\text{MN}(T, (p_0, ..., P_N))$ be the multinomial distribution where T is the number of trials. At each trial, there are $N + 1$

[7]The use of quantile regressions cannot guard against the possibility of the well-known "*quantile crossing*". On any day when the issue is observed, I apply the recently developed method of monotonically rearrangement of Chernozhukov et al. (2010) to correct the problem.

outcomes $(0, 1, ..., N)$ depending on how many VaR levels are breached, together with corresponding probabilities $p_0, ..., p_N$. The observed cell count is defined as:

$$O_j = \sum_{t=1}^{T} I_{X_t = j}, \quad j = 0, 1, ..., N$$

then, the random vector $(O_0, O_1, ..., O_T)$ should follow the multinomial distribution if the two conditions are satisfied, $(O_0, O_1, ..., O_T) \sim \text{MN}(T, (\alpha_1 - \alpha_0, ..., \alpha_{N+1} - \alpha_N))$. The formal hypotheses are given by:

H_0: $\phi_j = \alpha_j, \quad$ for $j = 1, ..., N$

H_1: $\phi_j \neq \alpha_j, \quad$ for at least one $j \in \{1, ..., N\}$

where $0 = \phi_0 < \phi_1 < ... < \phi_N < \phi_{N+1} = 1$ is an arbitrary sequence of parameters. Kratz et al. (2018) propose several test statistics to examine the hypotheses. In my application, I choose the Nass test (Nass, 1959) with $N = 4$ as this test exhibits a good compromise between size and power of the test (for technical details, refer to Kratz et al., 2018).

2.2.3.2 Relative Performance Evaluation

To evaluate the relative accuracy and facilitate decision making between different forecasting methods, it is necessary to employ loss functions. Models that generate lower expected loss are arguably preferred over those with higher loss values. To simplify the notation in this subsection, let $\widehat{Q}_t = Q_{\alpha,t-1}(r_t)$ be the conditional VaR and $\widehat{ES}_t = ES_{\alpha,t-1}(r_t)$ be the conditional ES. Since VaR is elicitable using Eq. (2.3), Giacomini and Komunjer (2005) argue that this function is a natural choice to compare VaR forecasts:

$$L_Q(\widehat{Q}_t) = (r_t - \widehat{Q}_t)\left[\alpha - I_{(r_t \leq \widehat{Q}_t)}\right] \tag{2.19}$$

Fissler and Ziegel (2016) suggest a family of strictly consistent loss functions in which VaR and ES forecasts are jointly elicitable. I adopt a member of this family defined in Fissler et al. (2015) to jointly compare the forecast errors of VaR and ES estimates from competing models as follows:

$$\begin{aligned} L_{FZG}(\widehat{Q}_t, \widehat{ES}_t) = & (I_{(r_t<\widehat{Q}_t)} - \alpha)\widehat{Q}_t - I_{(r_t<\widehat{Q}_t)} r_t \\ & + \frac{\exp(\widehat{ES}_t)}{1+\exp(\widehat{ES}_t)} \left(\widehat{ES}_t - \widehat{Q}_t + \frac{1}{\alpha} I_{(r_t<\widehat{Q}_t)} (\widehat{Q}_t - r_t) \right) \\ & + ln \left(\frac{2}{1+\exp(\widehat{ES}_t)} \right) \end{aligned} \tag{2.20}$$

Using these loss functions, I apply the model confidence set (MCS) method of Hansen et al. (2011) to form a set of superior models. The MCS procedure starts with the initial set of forecasting models, M_0, to deliver the superior set of models $M^*_{1-\alpha^*}$, which contains smaller number of models, $m^* < M_0$, for a given significant level α^*.[8] In the main analysis, I use $\alpha^* = 5\%$ to construct the 5% MCS.[9] The test applies an elimination rule where at each step, a significance test is conducted to eliminate the worst performing model based on an equivalence test, $\delta_{\mathbb{M}}$, and an elimination rule $e_{\mathbb{M}}$, as follows:

$$\begin{aligned} H_{0,\mathbb{M}} &: E(\Delta L_{i,j,t}) = 0, \quad \text{for all} \quad i,j \in \mathbb{M} \\ H_{A,\mathbb{M}} &: E(\Delta L_{i,j,t}) \neq 0, \quad \text{for some} \quad i,j \in \mathbb{M} \end{aligned}$$

where $\mathbb{M} \subset M_0$ is the set of remaining models at each step and $\Delta L_{i,j,t}$ is the loss difference between model i and j at time t. If the null hypothesis $H_{0,M}$ is not rejected by the equivalence test $\delta_{\mathbb{M}}$, the MCS is defined as $M^*_{1-\alpha^*} = \mathbb{M}$. Otherwise, the worst performing model is eliminated using the elimination rule $e_{\mathbb{M}}$. I employ

[8] Note that I use α^* to differentiate the significant level of MCS analysis to the quantile level, α, in VaR and ES forecasts.

[9] The 10% MCS is presented in Table A.10 and provides similar result.

the equivalence test based on the range statistic in Hansen et al. (2011):[10]

$$T_{\mathbb{M}} = \max_{i \in \mathbb{M}} |t_{i,j}| \tag{2.21}$$

where

$$t_{i,j} = \frac{\overline{\Delta L}_{i,j}}{\sqrt{\widehat{Var}(\overline{\Delta L}_{i,j})}} \quad ; \quad \overline{\Delta L}_{i,j} = T^{-1} \sum_{t=1}^{T} \Delta L_{i,j,t}$$

where $\overline{\Delta L}_{i,j}$ is the average sample loss difference between models i and j, $\widehat{Var}(\overline{\Delta L}_{i,j})$ is estimate of the asymptotic variance of $\overline{\Delta L}_{i,j}$, computed using a block-bootstrap with $10,000$ trials and a block size set at $l = 4$ observations.[11]

The elimination rule is then specified as:

$$e_{\mathbb{M}} = \underset{i \in M}{arg\,max} \sup_{j \in \mathbb{M}} t_{i,j} \tag{2.22}$$

where the model with the highest value of $t_{i,j}$ is eliminated if the null hypothesis is rejected. The test is sequentially repeated until the MCS is reached at a given confidence level.

2.3 Empirical Analysis

2.3.1 Data and Descriptive Statistics

I employ daily U.S. dollar-denominated returns for 42 international indices and the MSCI world index. The sample period is from January 2, 1996 to December 31, 2017 for most of the markets with a total of 5740 days.[12] The full list of countries

[10] I also employ the alternative test statistic in Hansen et al. (2011), which is the semi-quadratic statistic. The results are presented in Table A.11 and yields similar results.

[11] The MCS results with alternative block sizes (2 and 6) or the use stationary bootstrapping in A.12 give similar results

[12] The only two exceptions are Portugal ,which starts on May 04, 1998 and Russia, which starts on April 02, 1997.

Table 2.1 List of International Indices

	Country	Source
World	World Portfolio	MSCI
Developed Markets	Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, The Netherlands, Hongkong, Ireland, Israel, Italia, Japan, South Korea, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, United States	FTSE
Emerging Markets	Brazil, Chile, China, Czech Republic, Hungary, India, Indonesia, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey	S&P/IFCI

is provided in Table 2.1. I obtain total return indices for 24 developed markets from the FTSE, and for the 18 emerging markets indices from the S&P/IFCI database. The series correspond to highly liquid and investable indices, which track real returns for a foreign investor investing on each country in the equity market. Furthermore, they are often used in the literature of international portfolio diversification literature (see Christoffersen et al., 2012; Ghysels et al., 2016).

Table 2.2 reports the descriptive statistics for the index return series. Panel A displays information about the 1-day return horizon, while Panels B and C present the metrics for the 5- and 10-day horizons, respectively. The columns provide the mean and quantiles for the cross-sectional distribution of the statistics presented in rows, including the annualised mean, annualised standard deviation, skewness, kurtosis and the Jarque-Bera statistic. With the only exception of Portugal, all markets have positive mean returns over the sample in all the three horizons. The return series have, on average, negatively skewed and leptokurtic empirical distributions. Notably, average skewness increases in absolute value with horizons, which is in line with the findings of Neuberger (2012) and Ghysels et al. (2016).

Table 2.2 Descriptive Statistics of International Indices

This table reports the descriptive statistics for the cross-section of index returns. The columns show the mean and quantiles from the distribution of cross-sectional statistics presented in the rows. Panel A reports the statistics for the 1-day horizon, while Panels B and C show the corresponding statistics for the 5- and 10-day horizon, respectively. The last row in each panel reports the Jaque-Bera test statistics under the null hypothesis of normally distributed in the return series.

	Mean	5%	25%	Median	75%	95%
			Panel A: 1-day			
Mean	0.070	0.024	0.053	0.076	0.086	0.114
Std dev	0.262	0.186	0.222	0.248	0.292	0.410
Skewness	-0.206	-0.736	-0.355	-0.192	-0.069	0.300
Kurtosis	12.305	7.291	9.238	10.848	13.230	22.826
Jarque-Bera	30544.96	4426.63	9074.24	14904.09	25029.66	106849.23
			Panel B: 5-day horizon			
Mean	0.350	0.119	0.265	0.380	0.431	0.569
Std dev	0.631	0.421	0.515	0.598	0.710	1.065
Skewness	-0.472	-0.945	-0.695	-0.566	-0.281	0.141
Kurtosis	9.162	5.383	6.228	7.917	10.363	18.562
Jarque-Bera	3083.68	306.03	526.61	1163.91	2698.50	11688.43
			Panel C: 10-day horizon			
Mean	0.700	0.238	0.530	0.759	0.863	1.137
Std dev	0.855	0.562	0.691	0.810	0.971	1.435
Skewness	-0.521	-1.181	-0.718	-0.517	-0.277	0.078
Kurtosis	7.749	4.433	5.342	6.557	8.915	16.746
Jarque-Bera	909.35	57.40	147.94	316.02	930.40	4735.75

Indeed, the Jarque-Bera statistics strongly reject the null hypothesis of normality for all indices and horizons.

2.3.2 Estimates of MIDAS-based Models

In this chapter, I am interested in the VaR and ES forecasts of two commonly used quantiles in the literature, at $\alpha = (0.01, 0.05)$ probability levels, respectively. I consider three forecast horizons: 1-day, 5-day and 10-day. The choice of 1-day horizon allows for direct comparison of my results to the established methods in the literature, which mainly focus on 1-day ahead forecasts. The choice of 10-day horizon is motivated by the baseline horizon used for the capital requirements under the Basel III regulatory agreement.

The main focus of this study is to improve the out-of-sample performance of VaR and ES forecasts using MIDAS-based models. However, the estimation results of the proposed models provide some worthy observations. For this purpose, I present the estimated parameters of the MIDAS-based models using the first estimation window of 2500 daily returns. I start by the estimation results for the MSCI world index at $\alpha = 0.05$.[13] Next, I further examine the variations in parameter estimates across countries.

Table 2.3 presents results for the AL-based models described in section 2.2.1. Columns (1) are the results for the *Midas-AL* model, while columns (2) are the results for the *MidasAs-AL* model. The row "*Log-L*" provides the maximised log-likelihood value of AL density presented in Eq. (2.7), while "*Hit*" is the empirical violation rate of the estimation sample.

I observe strong time-variation in the conditional VaR as the slope coefficients $\beta^1_{\alpha,h}$ ($\beta^{1-}_{\alpha,h}$,$\beta^{1+}_{\alpha,h}$) are statistically significant at conventional levels. The γ coefficient governing the dynamics of conditional ES is also always significant across models and horizons. Not surprisingly, the negative and positive returns have different impacts on the quantile dynamics, although the asymmetry is less pronounced at longer horizons. For example, the $\beta^{1+}_{\alpha,h}$ estimate at the 1-day horizon is -0.354 and not statistically significant, whereas its value at the 10-day horizon is 9.548 and highly significant with the magnitude almost equal to that of $\beta^{1-}_{\alpha,h}$ (-10.842). The *MidasAs-AL* model provides better goodness-of-fit than the symmetric counterpart as shown by the "*Log-L*" values. Finally, the percentages of VaR exceedances are always close to 5%, signalling good tail coverages for both models over the estimation period.

Table 2.4 reports the estimated parameters for the EVT-based models. Columns (1) correspond to the *Midas-Evt* model, while columns (2) refer to the *MidasAs-Evt* model. I also report the likelihood value of Eq. (2.7) using estimated VaR and

[13] The estimation results for $\alpha = 0.01$ provide similar conclusions.

Table 2.3 Estimation of AL-based Models at the 5% quantile for the MSCI World Index

This table provides estimated parameters of two AL-based models under the MIDAS framework for the 5% quantile level for the MSCI World index. The results are presented for 1-, 5- and 10-day return horizons. The parameters are estimated using the first moving window with 2500 observations. Columns (1) are the results for the *Midas-AL* model, while Columns (2) are the results for the *MidasAs-AL* model, which specify the conditional quantile as in Eq. (2.1) and (2.2), respectively. The numbers in parentheses below the estimated parameters are p-values, based on bootstrapped standard errors with 1,000 replications. For parameter κ_2, the null hypothesis is $\kappa_2 = 1$. The row *Log-L* reports the maximised log-likelihood value of AL distribution described in Eq. (2.7), while the row Hit (%) denotes the percentage of times the VaR is exceeded.

	1-day horizon		5-day horizon		10-day horizon	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.003	-0.004	-0.012	-0.016	-0.036	-0.045
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\beta^1_{\alpha,h}$: $\beta^{1-}_{\alpha,h}$	-1.743	-2.706	-4.265	-7.321	-1.865	-10.842
	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
$\beta^1_{\alpha,h}$: $\beta^{1+}_{\alpha,h}$		-0.354		0.966		9.548
		(0.069)		(0.088)		(0.000)
κ_2	8.523	7.147	4.968	3.060	20.039	2.613
	(0.000)	(0.000)	(0.000)	(0.011)	(0.034)	(0.000)
γ	-1.064	-1.162	-1.228	-0.959	-0.878	-1.081
	(0.000)	(0.000)	(0.000)	(0.000)	(0.013)	(0.000)
Log-L	7092.82	7179.12	931.39	945.11	388.72	406.47
Hit(%)	4.833	4.750	5.000	5.000	5.000	4.583

ES for comparison purposes, although the estimation of EVT-based models does not involve AL density maximisation. The estimation results are generally in line with those reported in Table 2.3. The asymmetric effects of lagged returns become less pronounced at longer horizon. Both models have Hit percentages close to 5%. Finally, the likelihood values are only slightly lower than their counterparts in Table 2.3, which directly maximise the AL likelihood.

Tables 2.5 and 2.6 provide a summary of the cross-sectional parameter estimates for the newly proposed models. Some observations are worth noting. First, the coefficients of negative lagged returns ($\beta^{1-}_{\alpha,h}$) have greater magnitude on average than those of lagged positive returns ($\beta^{1+}_{\alpha,h}$). This finding provides evidence of asymmetric effects of lagged returns across countries and forecast horizons. Second, the cross-sectional standard deviation of parameter κ_2 is relatively more

Table 2.4 Estimation of EVT-based Models at the 5% quantile for the MSCI World Index

This table provides estimated parameters of two EVT-based models under the MIDAS framework for the 5% quantile level for the MSCI World index. The results are presented for 1-, 5- and 10-day return horizons. The parameters are estimated using the first moving window with 2500 observations. Columns (1) are the results for the *Midas-Evt* model, while Columns (2) are the results for the *MidasAs-Evt* model, which specify the conditional quantile in Eq. (2.1) and (2.2), respectively. The numbers in parentheses below the estimated parameters are p-values, based on bootstrapped standard errors. For parameter κ_2, the null hypothesis is $\kappa_2 = 1$. The row *Log-L* reports the maximised log-likelihood value of AL distribution described in Eq. (2.7), while the row Hit (%) denotes the percentage of times the VaR is exceeded.

	1-day horizon		5-day horizon		10-day horizon	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.002	-0.004	-0.011	-0.016	-0.031	-0.033
	(0.001)	(0.000)	(0.003)	(0.000)	(0.003)	(0.000)
$\beta^1_{\alpha,h}$ $\beta^{1-}_{\alpha,h}$	-1.625	-2.726	-3.201	-7.124	-1.375	-9.564
	(0.000)	(0.000)	(0.000)	(0.000)	(0.077)	(0.000)
$\beta^1_{\alpha,h}$ $\beta^{1+}_{\alpha,h}$		0.035		2.116		6.545
		(0.160)		(0.031)		(0.000)
κ_2	8.608	6.073	5.230	2.777	18.960	2.557
	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.027)
ξ	0.085	0.185	-0.156	-0.227	0.064	0.053
β	0.349	0.294	0.467	0.520	0.585	0.380
Log-L	7000.13	7166.88	931.06	943.69	386.45	404.36
Hit(%)	5.125	5.250	5.000	5.000	5.417	4.583

pronounced than those of other parameters, particularly at the multi-day horizons. Although κ_2 does not have a direct economic interpretation, this coefficient provides information about the shape of the weighting function applied to the lagged conditioning variable. Since I apply the same lag length in all estimations, this observation highlights the flexibility of the MIDAS framework in capturing significant heterogeneity in tail dynamics across market indices and forecasting horizons (see, e.g., Gu and Ibragimov, 2018, for similar evidence of heterogeneity in the tail of international index return using the "Cubic law").

2.3.3 Out-of-Sample Forecast Evaluation

I now focus on the out-of-sample (OOS) VaR and ES forecasts from the MIDAS-based models and the benchmark models presented in Section 2.2.2. To this end, I employ a rolling window approach with a fixed length of 2500 daily

Table 2.5 Cross-sectional Estimates of AL-based Models at the 5% quantile

This table provides the average of estimated parameters across countries of the AL-based models at the 5% quantile level. Results are reported at 1-day, 5-day and 10-day return horizons, respectively. The parameters are estimated using the first moving window of 2500 observations. Columns (1) are the results for the *Midas-AL* model, while Columns (2) are the results for the *MidasAs-AL* model, which specify the conditional quantile in Eq. (2.1) and (2.2), respectively. The numbers in parentheses display cross-sectional standard deviation of the above parameters.

	1-day		5-day		10-day	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.006	-0.008	-0.018	-0.021	-0.031	-0.034
	(0.003)	(0.004)	(0.018)	(0.019)	(0.038)	(0.032)
$\beta^1_{\alpha,h}$ $\beta^{1-}_{\alpha,h}$	-1.674	-2.296	-3.908	-5.705	-5.089	-9.298
	(0.278)	(0.380)	(1.555)	(2.541)	(3.232)	(5.675)
$\beta^{1+}_{\alpha,h}$		-0.660		-1.678		-0.325
		(0.424)		(2.144)		(6.731)
κ_2	12.606	14.234	21.294	14.901	32.993	12.892
	(7.252)	(7.097)	(52.672)	(24.515)	(65.669)	(24.757)
γ	-0.914	-0.984	-0.959	-0.961	-1.094	-1.196
	(0.185)	(0.182)	(0.228)	(0.218)	(0.356)	(0.377)

Table 2.6 Cross-sectional Estimates of EVT-based Models at the 5% quantile

This table provides the average of estimated parameters across countries of the Evt-based models at the 5% quantile level. Results are reported at 1-day, 5-day and 10-day return horizons, respectively. The parameters are estimated using the first moving window of 2500 observations. Columns (1) are the results for the *Midas-Evt* model, while Columns (2) are the results for the *MidasAs-Evt* model, which specify the conditional quantile in Eq. (2.1) and (2.2), respectively. The numbers in parentheses display cross-sectional standard deviation of the above parameters.

	1-day		5-day		10-day	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.005	-0.006	-0.014	-0.018	-0.021	-0.035
	(0.004)	(0.004)	(0.017)	(0.016)	(0.032)	(0.032)
$\beta^1_{\alpha,h}$ $\beta^{1-}_{\alpha,h}$	-1.436	-2.157	-3.432	-4.708	-4.445	-6.723
	(0.288)	(0.422)	(1.367)	(2.114)	(2.526)	(4.324)
$\beta^{1+}_{\alpha,h}$		-0.450		-1.415		-0.327
		(0.397)		(1.887)		(3.800)
κ_2	11.826	12.823	17.905	19.390	17.246	22.111
	(7.527)	(7.474)	(53.094)	(40.973)	(49.149)	(56.400)
ξ	0.067	0.074	-0.007	0.011	-0.043	0.066
	(0.123)	(0.109)	(0.256)	(0.208)	(0.362)	(0.397)
ς	0.434	0.407	0.497	0.476	0.512	0.423
	(0.057)	(0.048)	(0.172)	(0.138)	(0.197)	(0.208)

observations. I estimate the parameters for each model using the most recent 2500 daily observations and obtain VaR and ES forecasts for all quantile levels and for 1-, 5- and 10-day ahead. Then, I move the estimation window 10 days forward and iterate this procedure until I reach the end of the sample. Thus, this procedure yields a total of 324 OOS forecasts, spanning the period from August 2, 2005 to December 30, 2017.

2.3.3.1 Absolute Forecasting Performance

The results for VaR forecasts of competing models at the 1% and 5% quantile levels are presented in Table 2.7. Panel A shows the results for the 1-day horizon, whereas Panels B and C display the results for the 5- and 10-day forecast horizons, respectively. The first two columns present the empirical hit percentage over the OOS period. For each test in the next columns, I report the number of rejections across the countries. Finally, column "*Total*" is the sum of rejections across quantile levels for each test. For example, the value of 3 for the *GARCH-Fhs* model at the 1% quantile in the UC column of Panel A indicates that the 1% VaR forecasts of this model at the 1-day horizon are rejected by UC test in 3 out of 43 indices. Thus, for each forecasting horizon, the best model has the lowest value in each column.

The MIDAS-based models has competitive results to the benchmark models at the 1-day horizon, but provide superior results at the 5- and 10-day horizons. All models perform reasonably well in the UC test at 1-day horizons and the levels of hit percentage are close to the quantile level. At longer forecast horizons, however, all benchmark models significantly underestimate the risk as the hit percentage often exceeds the quantile level. In contrast, the MIDAS-based models continue to produce violation rates that are close to the quantile levels. In particular, the two MIDAS-based models with AL density provide the best performance since they are not rejected in any market at both quantile levels at the 10-day horizon.

The results from the DQ test offer three additional insights. First, the asymmetric models often provide smaller number of test rejections than the symmetric alternatives, especially at the 1-day horizon. However, this effect is considerably weaker at the 10-day horizon, which is in line with the in-sample estimates of the previous subsection. Second, the performance of CAViaR-based models deteriorate significantly at the 5- and 10-day forecast horizons. For instance, the 5% VaR forecast of the *As-AL* model is rejected in only 3 out of 43 indices at the 1-day horizon. This number changes remarkably at the 10-day horizon, indicating that the *As-AL* model is rejected in 33 out of 43 markets. Third, the MIDAS-based models consistently provide competitive performance in all three forecasting horizons. In fact, the *Midas-Evt* model has the lowest number of rejections in both the 5- and 10-day forecast horizons. The contrasting performance between MIDAS-based and CAViaR-based models at the multi-day horizon highlights the deficiency of temporal aggregation to match target horizon in VaR forecasts and consistent with the simulation study in Ghysels et al. (2016).

Next, I focus on the result for ES forecasts in Table 2.8. In the columns, I present evaluation results for the four ES backtests described earlier in Section 2.2.3. These tests include the discrepancy test of McNeil and Frey (2000) (denoted UES1), the unconditional (UES2) and conditional (CES) tests of Du and Escanciano (2017) and the multi-VaR test of Kratz et al. (2018). Again, for each test, I report the number of rejections across countries, while column '*Total*' is the sum of this number across quantile levels. Lower number in each column indicates superiority.

The results are generally in line with those in Table 2.7. First, all models provide acceptable results in two unconditional ES tests with no clear superiority of one model over another. Second, similar to VaR forecasts, the models with asymmetric specification in conditional quantile yield smaller numbers of test rejection. This observation, however, is less pronounced at the 5- and 10-day forecast horizons. Finally, the CAViaR-based models are clearly the worst performing models, whereas the MIDAS-based models are superior at multi-day forecasting horizons. Particularly in the multi-VaR test, all benchmark models are inferior to the new models at 5-day and 10-day horizons.[14] This finding further highlights the benefit of MIDAS framework in exploiting the richness of daily returns to forecast the tail dynamics at multi-day return horizons.

2.3.3.2 Relative Forecasting Performance

While the absolute performance evaluation is useful to validate the competing models, it provides little insight about their relative performance. Next, I investigate the relative performance of forecasting models based on the two loss functions presented in the previous section. Table 2.9 reports the average OOS forecast losses for all models under consideration. Panel A shows results for the 1-day horizon, while Panels B and C report results for the 5- and 10-day forecast horizons, respectively. In each panel, I compute the cross-sectional average of the mean forecast losses across the 43 indices. The L_Q and L_{FZG} loss functions are reported separately for the 1% and 5% quantile levels. In each column, I highlight the cell corresponding to the best method in rows.

The most accurate methods often appear in the final two rows, which are the asymmetric MIDAS-based models. The *MidasAs-AL* model yields the most accurate forecasts at the 1% quantile, while the *MidasAs-Evt* is often the best model at 5% quantile. The only exception is the 1-day horizon, for which the *GJR-Fhs* model achieves the best performance. The CAViaR-based models also perform well at the 1-day horizon, but their average losses rise significantly at multi-day forecast horizons.

Table 2.10 presents the MCS results for the L_Q and L_{FZG} loss functions separately for each quantile level and forecast horizons. The entry in each column presents the number of times (out of 43 indices), that the model in row is excluded

[14]The only exception is the *MidasAs-AL* model at 5-day horizon

Table 2.7 Results of Out-of-Sample VaR Absolute Forecasting Performance

This table summarises the performance of out-of-sample VaR forecasts across 43 international equity indices. Forecasts are based on rolling window of 2500 observations. Panel A provides the results for the 1-day horizon, while Panels B and C reports the results for the 5- and 10-day forecast horizons, respectively. The columns labelled Hit(%) report the percentage of times the VaR estimates are exceeded. The next six columns display the absolute performance of VaR forecasts, based on the unconditional coverage test (UC) of Kupiec (1995) and the dynamic quantile test (DQ) of Engle and Manganelli (2004). For each test in column, I report the number of test rejections out of 43 indices at 5% significant level. Lower number implies superior performance.

	Hit(%)		UC			DQ		
Models	1%	5%	1%	5%	Total	1%	5%	Total
Panel A: 1-day horizon								
GARCH-Fhs	1.065	5.021	3	1	4	15	20	35
GARCH-Evt	0.997	5.064	0	0	0	14	19	33
GJR-Fhs	1.063	4.966	3	5	8	11	7	18
GJR-Evt	0.996	5.054	2	4	6	4	5	9
Sav-AL	1.074	4.994	3	1	4	21	21	42
Sav-Evt	1.042	5.188	1	1	2	21	24	45
As-AL	1.069	4.908	5	4	9	12	3	15
As-Evt	1.020	5.024	0	3	3	14	5	19
Midas-AL	1.030	4.950	2	1	3	14	21	35
Midas-Evt	1.021	5.119	1	0	1	19	24	43
MidasAs-AL	1.042	4.897	4	4	8	10	4	14
MidasAs-Evt	0.998	4.975	2	4	6	11	5	16
Panel B: 5-day horizon								
GARCH-Fhs	1.529	5.951	7	6	13	15	11	26
GARCH-Evt	1.513	5.920	7	10	17	13	10	23
GJR-Fhs	1.389	5.628	4	4	8	9	7	16
GJR-Evt	1.342	5.520	4	3	7	8	4	12
Sav-AL	1.195	5.030	0	0	0	22	24	46
Sav-Evt	1.267	5.104	0	0	0	16	17	33
As-AL	1.237	4.946	2	4	6	27	15	42
As-Evt	1.269	5.075	1	0	1	19	12	31
Midas-AL	1.023	4.706	1	1	2	11	5	16
Midas-Evt	1.012	4.808	0	1	1	7	4	11
MidasAs-AL	0.924	4.675	1	2	3	12	4	16
MidasAs-Evt	1.015	4.782	0	1	1	8	5	13
Panel C: 10-day horizon								
GARCH-Fhs	1.514	5.692	3	2	5	11	5	16
GARCH-Evt	1.514	5.641	2	2	4	13	3	16
GJR-Fhs	1.181	5.307	0	3	3	5	5	10
GJR-Evt	1.188	5.276	1	3	4	6	5	11
Sav-AL	1.261	6.265	3	3	6	21	34	55
Sav-Evt	1.557	5.579	5	1	6	23	17	40
As-AL	1.329	6.530	3	10	13	15	33	48
As-Evt	1.659	5.548	8	1	9	20	14	34
Midas-AL	1.061	4.811	0	0	0	8	1	9
Midas-Evt	1.079	4.427	0	1	1	7	1	8
MidasAs-AL	0.918	4.913	0	0	0	8	2	10
MidasAs-Evt	1.188	4.676	1	1	2	13	2	15

Table 2.8 Results of Out-of-Sample ES Absolute Forecasting Performance

This table summarises the performance of out-of-sample ES forecasts across 43 international equity indices. Forecasts are based on rolling window of 2500 observations. Panel A provides results for the 1-day horizon, while Panels B and C reports results for the 5- and 10-day forecast horizons, respectively. The next six columns display the absolute performance of ES forecasts, based on the unconditional ES test of zero discrepancy (UES1) of McNeil and Frey (2000), the unconditional (UES2) and conditional ES (CES) tests of Du and Escanciano (2017), the multi-VaR test of Kratz et al. (2018). For each test in column, I report the number of test rejections out of 43 indices at 5% significant level. Lower number implies superior performance.

	UES1			UES2			CES			
Models	1%	5%	Total	1%	5%	Total	1%	5%	Total	MultiVaR
Panel A: 1-day horizon										
GARCH-Fhs	1	1	2	2	1	3	16	35	51	1
GARCH-Evt	0	0	0	1	1	2	16	33	49	2
GJR-Fhs	1	0	1	1	3	4	7	12	19	4
GJR-Evt	2	1	3	3	2	5	8	12	20	4
Sav-AL	0	1	1	4	0	4	22	42	64	3
Sav-Evt	1	1	2	4	1	5	23	41	64	5
As-AL	1	2	3	8	3	11	11	15	26	4
As-Evt	1	2	3	6	4	10	11	8	19	4
Midas-AL	0	1	1	5	0	5	25	40	65	3
Midas-Evt	1	0	1	4	0	4	20	41	61	3
MidasAs-AL	0	2	2	4	2	6	11	10	21	1
MidasAs-Evt	1	2	3	3	2	5	10	10	20	3
Panel B: 5-day horizon										
GARCH-Fhs	1	1	2	3	4	7	1	4	5	8
GARCH-Evt	1	0	1	4	5	9	0	4	4	9
GJR-Fhs	3	2	5	3	3	6	1	4	5	7
GJR-Evt	2	2	4	0	2	2	1	2	3	5
Sav-AL	4	1	5	2	0	2	22	34	56	2
Sav-Evt	0	0	0	0	0	0	7	28	35	2
As-AL	3	3	6	13	0	13	15	13	28	3
As-Evt	2	0	2	3	0	3	11	14	25	4
Midas-AL	0	0	0	1	0	1	5	8	13	1
Midas-Evt	0	0	0	0	0	0	4	4	8	1
MidasAs-AL	1	0	1	2	1	3	4	5	9	3
MidasAs-Evt	1	2	3	2	1	3	3	4	7	1
Panel C: 10-day horizon										
GARCH-Fhs	1	0	1	0	3	3	2	2	4	6
GARCH-Evt	1	0	1	0	3	3	3	2	5	4
GJR-Fhs	1	0	1	1	1	2	2	4	6	3
GJR-Evt	0	0	0	2	1	3	2	3	5	3
Sav-AL	3	2	5	9	0	9	16	16	32	7
Sav-Evt	0	1	1	3	2	5	12	13	25	6
As-AL	4	6	10	17	2	19	11	13	24	8
As-Evt	0	1	1	3	2	5	9	10	19	8
Midas-AL	0	1	1	1	0	1	4	4	8	1
Midas-Evt	0	0	0	4	1	5	2	2	4	1
MidasAs-AL	0	2	2	0	1	1	3	2	5	1
MidasAs-Evt	0	3	3	6	2	8	3	3	6	1

Table 2.9 Summary of Out-of-Sample Forecast Losses

This table provides the average out-of-sample forecast losses at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of Eq. (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in Eq. (2.20). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	Panel A: 1-day				Panel B: 5-day				Panel C: 10-day			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	0.489	2.447	1.684	1.821	1.285	6.278	4.138	4.387	1.701	8.147	5.735	5.988
GARCH-Evt	0.490	2.448	1.684	1.821	1.274	6.223	4.126	4.373	1.698	8.132	5.735	5.988
GJR-Fhs	**0.483**	**2.415**	1.668	1.804	1.268	6.192	4.144	4.391	1.717	8.213	5.778	6.031
GJR-Evt	0.484	2.416	1.667	1.803	1.252	6.115	4.121	4.367	1.715	8.201	5.782	6.036
Sav-AL	0.500	2.498	1.702	1.840	1.469	7.146	4.473	4.743	2.240	10.708	6.420	6.733
Sav-Evt	0.500	2.499	1.703	1.842	1.437	6.999	4.418	4.684	2.146	10.862	6.203	6.661
As-AL	0.489	2.445	1.670	1.806	1.394	6.790	4.424	4.692	2.184	10.424	6.492	6.813
As-Evt	0.484	2.420	1.668	1.804	1.369	6.682	4.336	4.597	2.071	10.662	6.118	6.570
Midas-AL	0.497	2.487	1.701	1.839	1.275	6.215	4.187	4.438	1.736	8.311	5.884	6.144
Midas-Evt	0.498	2.489	1.700	1.839	1.284	6.282	4.141	4.390	1.763	8.635	5.691	5.985
MidasAs-AL	0.485	2.426	1.670	1.806	**1.230**	**5.992**	4.143	4.391	**1.621**	**7.759**	5.708	**5.963**
MidasAs-Evt	0.485	2.424	**1.667**	**1.803**	1.263	6.215	**4.087**	**4.336**	1.757	8.665	**5.677**	5.979

Table 2.10 Model Confidence Set Results

This table reports the results of the 5% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of Eq. (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in Eq. (2.20). The range statistic in Eq. (2.21) is used to the equivalence test of the MCS. Lower values corresponds to superior performance. Bold numbers indicate best methods in each column.

	1-day				5-day				10-day			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	7	7	15	13	3	3	1	**0**	3	4	2	2
GARCH-Evt	7	6	13	13	3	3	1	**0**	3	4	1	1
GJR-Fhs	2	2	2	2	2	2	1	**0**	6	7	1	1
GJR-Evt	2	2	2	2	2	2	1	**0**	6	7	1	1
Sav-AL	8	8	19	18	9	8	7	7	19	19	6	7
Sav-Evt	11	9	21	20	9	9	7	5	9	16	5	6
As-AL	5	5	**0**	**0**	7	7	5	4	17	18	7	7
As-Evt	4	4	1	1	6	6	2	2	8	17	1	3
Midas-AL	7	7	18	18	4	4	4	3	7	7	5	4
Midas-Evt	10	9	18	17	5	5	1	1	5	8	0	1
MidasAs-AL	**0**	**0**	2	2	**1**	**1**	1	1	**0**	**0**	**0**	**0**
MidasAs-Evt	4	4	3	3	2	2	**0**	**0**	3	6	**0**	**0**

from the 5% MCS. For example, the entry for L_Q function of the *GARCH-Fhs* model at the 1% quantile level and 1-day horizon is 7. This number indicates that the model is excluded from the MCS in 7 out 43 cases. Therefore, a smaller number indicates superior performance cross-sectionally.

The main findings from the MCS results are threefold. First, in line with the absolute performance evaluation, there is significant benefit of using asymmetric models at 1-day horizon, but the impact is less pronounced as the forecast horizon gets longer. Second, the *MidasAs-AL* model provides the best overall performance and often be included in the set of superior models in most cases. For example, this model is always included in the MCS in all indices at both quantile levels at the 10-day forecast horizon. The GARCH-based models also perform well but are often inferior to the asymmetric MIDAS-based models. Third, the CAViaR-based models perform worst at the multi-day forecast horizon and are often excluded from MCS, especially at the 1% quantile level.

Overall, I obtain promising results for the MIDAS-based models for VaR and ES forecasts. The proposed models consistently belong to the best performing models with low number of rejections across backtests in all quantile levels and forecasting horizons. The new methods also yield the lowest forecast errors and are often included in the set of superior models, especially at forecasting horizons longer than 1-day ahead. In contrast, the alternative models that rely on a single-horizon returns are always inferior to all other models at multi-day forecast horizons. This finding suggests significant benefits of accounting for serial dependence in short-horizon return process to predict the tail dynamics of long-horizon return distribution. Finally, I also find evidence supporting the asymmetric specification in conditional quantile. In terms of ES forecasting method, the jointly model using AL density generally provide better forecasts than the EVT-based alternative.

2.4 Robustness Checks

2.4.1 Model Performance and Market Regimes

I first evaluate model performance across different market regimes. Especially, I separate the out-of-sample forecasts into three subsamples: (i) the pre-crisis period from August 2, 2000 to July 31, 2007; (ii) the crisis period from August 1, 2007 to December 31, 2009; (iii) the post-crisis period from January 1, 2010 to December 31, 2017.

Tables A.2 and A.3 in the Appendix report the average OOS forecast losses and MCS results for the competing models for each forecasting horizon, quantile level and sub-period. Not surprisingly, the forecast losses increase significantly during the crisis period in all cases. This finding is in line with the recent result of Kourtis et al. (2016) and Symitsi et al. (2018) in the context of volatility and covariance forecasting, respectively. The MIDAS-based models generate slightly higher forecast losses than GARCH-based models during crisis at 1-day and 5-day horizon, but outperform the latter at 10-day horizon. During the pre-crisis and post-crisis sub-samples, the MIDAS-based models yield the best performance compared to all other competing models. Consistent with results of the full-sample results, the CAViaR-based forecasts often belong to the worst performing models in all sub-samples and particularly at the multi-day horizons. Finally, the *MidasAs-AL* model is often included in the superior set across three sub-samples, where the superiority is more pronounced at multi-day forecasting horizons.

2.4.2 Alternative Assets

My main results focus on the international equity indices. To further investigate the predictive power of the new models, I repeat the main analysis with alternative assets. To this end, I obtain stock prices of 20 largest companies globally listed the "*Global Top 100 companies by market capitalisation*" report by the

PricewaterhouseCoopers (PwC) on March 3, 2018. The companies are: Apple, Microsoft, Amazon.com, Tencent, Berkshire Hathaway, JPMorgan Chase, Johnson & Johnson, Exxon Mobile, Bank of America, Royal Dutch Shell, Walmart, Wells Fargo, Intel, Anheuser-Busch InBev, Taiwan Semiconductor, AT&T, Chevron, PetroChina, Novartis. The data is collected from DataStream with the maximum available sample period from January 3, 1997 to December 31, 2017.[15] I also consider two alternative asset classes, including: the Barclays U.S. Aggregate Bond Index from September 29, 2003 to December 31, 2017 as a proxy for the bond class. I also consider the S&P Goldman Sachs Commodity Total Return Index (GSCI) from January 1, 2003 to December 31, 2017 as a proxy for the commodity class. These two indices are investable and track the return of an investor from a fully collateralised portfolio of bonds and commodities. For these two indices, I collect data from the CapitalIQ database.

Table A.4 reports the average OOS forecast losses across the considered assets. In line with the main analysis, the MIDAS-based models provide clearly the best VaR and ES forecasts. The asymmetric models yield slightly lower forecast losses than the symmetric counterparts. This observation is generally in line with the model confidence set results in Table A.5. An interesting observation is that the performance of CAViaR-based models with AL density are not considerably inferior to the GARCH-based models compared to the analysis involving only stock indices.

2.4.3 Alternative Rolling Window Length

The OOS forecasts in the main analysis are conducted using rolling window of 2500 observations. This choice is largely driven by the convergence rates of the CAViaR-based models. The single-horizon setting leads to substantial loss of observations for the model estimation. For example, the CAViaR-based models are optimized

[15]Some stocks have shorter historical length but the first observation is no later than January 1, 2005

using only 250 non-overlapping return observations at the 10-day forecast horizon. Nevertheless, one may concern that using long estimation windows may give unfair advantage to the MIDAS-based methods, for example, compared to the GARCH-based models. To explore this issue, I repeat my analysis using rolling window of 1,500 and 2,000 observations, respectively. In the former case, I exclude the CAViaR-based models due to their low rates of convergence. Tables A.6 and A.7 in the Appendix show that my main conclusions are robust to the length of rolling windows. Notably, the performance of EVT-based models deteriorates remarkably in shorter estimation windows. This observation is not surprising since the numbers of extreme exceptions in these cases are lower, which thereby increases estimation errors and reduces the goodness-of-fit in the GPD estimation.

2.4.4 Performance for Different Country Groups

The return distributions in developed and emerging markets are typically characterised by distinct features. For example, Bekaert et al. (1998) show that return process in emerging countries significantly departs from normality due to frequent market jumps resulting from the non-smooth transition periods in the globalisation process. In the third chapter, I also document that the conditional distribution of emerging markets is less negatively skewed and has higher kurtosis than those of developed countries. Therefore, it is of interest to compare the model performance between the two groups of country.

Table A.8 provides the average OOS forecast losses separately for each country group. The forecast losses are substantially higher for the emerging countries in all cases. This observation may result from relatively more noisy data of the emerging stock markets. Nevertheless, the relative performance between competing models is consistent with the main results. The lowest forecast losses are often recorded in the final two rows, which correspond to the asymmetric MIDAS-based models. The MCS results in Table A.9 indicate that the asymmetric MIDAS-based model with AL density provides the best overall performance in both country groups. Therefore, I conclude that the performance of the new models is robust to different characteristics in the return process.

2.5 Suggestions for Future Research

My main analysis in this chapter focuses on VaR and ES forecast, given their practical importance to financial institutions and regulations. Thus, my result provides additional evidence on the benefits of MIDAS framework in forecasting different characteristics of return distribution (see, e.g., Ghysels et al., 2019, for recent evidence on the superiority of MIDAS in volatility forecasting). An interesting question for future research is whether the MIDAS framework can also improve return density forecast or equity risk premium, for example, by the combination of quantile forecasts (see, e.g, Lima and Meng, 2017; Wilhelmsson, 2013, for similar application).

Another extension of the current study could include the macroeconomic and financial variables in the conditioning variables. Several studies document significant explanatory power of economic variables on conditional return distribution features such as mean (Campbell and Diebold, 2009), volatility (Engle et al., 2013), correlation (Colacito et al., 2011) or different parts of return density (Cenesizoglu and Timmermann, 2008). Thus, additional information from macroeconomic conditions can further improve the forecast of tail dynamics in conditional return distribution. The MIDAS framework provides suitable setting to incorporating such variables, which typically sampled at different frequencies.

Finally, as I focus on the univariate VaR and ES forecast, a direct extension can explore multivariate VaR and ES forecasts (see, e.g Polanski and Stoja, 2017) or investigate potential benefits of MIDAS-based forecasts in a portfolio allocation (see, e.g, Dias, 2016, for the value of controlling for tail risks in portfolio selection).

2.6 Conclusion

Using the MIDAS framework, I propose new models to directly forecast VaR and ES at the desired horizon and quantile level. The semiparametric approach allows flexible dynamics in different quantile levels and avoid making distributional assumptions. In addition, the MIDAS framework utilises the data-rich environment of higher frequency return process to improve the forecast of the tail dynamics in longer horizon. Using a large cross-section of international stock indices, I examine the predictive performance of the proposed models relative to several popular forecasting models at various quantile levels and forecast horizons. Using a battery of backtesting procedures, I obtain strong evidence in favor of the proposed models, which consistently belong to the best performing methods. The MIDAS framework significantly outperforms the GARCH-based models and the alternative semiparametric models which rely on single-period quantile regression. Finally, models that incorporate asymmetry in the quantile dynamics, and use of the AL density to jointly estimate VaR and ES, generally provide the best forecasts across quantile levels and return horizons. This result is robust to different market regimes, alternative assets and forecast specifications.

Chapter 3

Forecasting Skewness

3.1 Introduction

A vast body of theoretical and empirical works showcases that investors tend to prefer assets with positive return skewness and, as such, the latter may be priced in capital markets. For example, Rubinstein (1973), Kraus and Litzenberger (1976) and Harvey and Siddique (2000) use classical capital asset pricing model and argue that return *coskewness* with the aggregate market portfolio is an important pricing factor. Drawing on behavioural models, Hong and Stein (2003), Brunnermeier et al. (2007), Mitton and Vorkink (2007) and Barberis et al. (2008) show that asset *idiosyncratic* skewness also provide significant role in pricing securities. Recent empirical studies of Conrad et al. (2013), Bali and Murray (2013) and Amaya et al. (2015) provide strong evidence on the importance of return skewness in explaining the cross-sectional heterogeneity of stock returns.

Motivated by strong evidence of return asymmetry, the literature takes a step further to promote the use of skewness forecasts to enhance financial decision-making. For instance, Patton (2004), Jondeau and Rockinger (2006), Guidolin and Timmermann (2008), Harvey et al. (2010), DeMiguel et al. (2013) and Ghysels et al. (2016) investigate the economic gains from incorporating various measures of skewness in investment decisions. Bali et al. (2008), Engle (2011) and Kostika and Markellos (2013) explore the use of skewness metrics in risk management applications. However, different skewness models tend to be employed across studies. As a result, there is a lack of consensus on how to best model and forecast skewness, which may hinder skewness-driven decisions.

To address this gap in the literature, I carry out a broad comparison of the forecasting ability and economic importance of several prominent skewness models. I also develop a new option-implied skewness estimator that outperforms the rest of the models in most of the statistical tests. My analysis is comprehensive in that I consider: (i) 10 international indices; (ii) six forecasting models that utilise different information; (iii) three forecasting horizons (i.e, 30, 60 and 90 calendar days respectively); (iv) two tests for assessing the information content of each forecasting model; (v) an out-of-sample forecasting horserace under two loss functions and (vi) an investment evaluation of skewness forecasts under four portfolio performance metrics. I also perform a series of robustness checks.

To the best of my knowledge, this is the first study in the literature that explicitly aims to identify good skewness forecasting models in financial markets. The only study that contains a test for information contents of a set of direct and indirect skewness models is concurrent work of Aretz and Arısoy (2019). There are several important differences between their work and this study. First, the focus of their paper is on empirical asset pricing, while I concentrate on forecasting and portfolio optimisation. Second, they focus on U.S. stocks, while I consider market indices, similar to Ghysels et al. (2016). Third, the forecasting models in their work are different than the ones I employ. For instance, they study models that account for specific stock characteristics, which are not available at the index level. Fourth, their forecasting experiment consists of Mincer-Zarnowitz regressions, while I include several additional tests to provide a broader analysis of skewness forecasting.

In my analysis, I draw parallels to the volatility forecasting literature. It may appear surprising that there is a lack of studies on the relative predictive ability of skewness models, given the rich literature around assessing and comparing volatility forecasts (see, Kourtis et al., 2016, and the references therein). However, in contrast to volatility, there is not an established proxy in the literature for the true physical skewness. Also, third-moment forecasting is notoriously more challenging than volatility forecasting for several reasons. First, historical skewness is generally known not to be persistent, while historical volatility is (Singleton and Wingender, 1986). Second, although volatility forecasts can be easily scaled to any horizon of interest, skewness estimates are heavily affected by the frequency of the returns used to perform the estimation (Neuberger, 2012; Neuberger and Payne, 2019). Equivalently, one cannot simply use high-frequency returns to forecast skewness of lower frequency returns. Third, skewness is considerably more sensitive to outliers compared to volatility (Kim and White, 2004).

I proxy true skewness using the *realised skewness* estimator of Neuberger (2012), also used by Aretz and Arısoy (2019). The realised skewness is comprised of two components, that are the realised skewness of higher frequency returns and the leverage effects between returns and innovations in volatility. Neuberger and Payne (2019) shows that the latter plays dominant role in driving skewness dynamics. Similar to realised volatility, realised skewness has the advantage that it can be computed for any horizon using high-frequency returns, even though extra information from the option markets is required to capture the leverage effect.

Using the ex-post realised skewness, I compare the information content and out-of-sample forecasting performance of five existing skewness forecasting models with distinct features. The first competing model I consider is realised skewness, lagged by one period. This choice is motivated by the finding of Neuberger (2012) that lagged realised skewness has some explanatory power on future realised skewness.

I further employ two conditional skewness estimators based on the popular GJR-GARCH model of Glosten et al. (1993) which can produce conditional return skewness in multiple horizons via an asymmetry-in-volatility process. The first GARCH-based skewness estimator comes from the assumption of time-invariant shape parameters in the specification of the GJR-GARCH model, as considered by Engle (2011). The second comes from Bali et al. (2008) and assumes time-varying shape parameters in the spirit of the autoregressive conditional density model of Hansen (1994). The fourth skewness forecasting model in my analysis is based on the estimation of conditional quantiles via a Mixed Data Sampling (QMIDAS) approach, as proposed by Ghysels et al. (2016). Two features of QMIDAS-based skewness are that it is less sensitive to outliers and that it can be computed using returns of higher frequency than the horizon of interest. The fifth model I include in my analysis is an option-implied skewness estimator, similar to Conrad et al. (2013). This is a forward-looking estimator, since it is solely computed by option prices and does not rely on historical asset returns.

I complement my set of skewness forecasting models with a new model that also relies on option market information. This is motivated by evidence in Kozhan et al. (2013) about the existence of a *skewness risk premium* in the S&P 500 index, which can be quantified by the difference between option-implied and realised skewness.[1] I confirm that a skewness risk premium exists for almost all indices in my dataset. Based on this finding, I propose an adjustment of the standard option-implied skewness that corrects it for the skewness risk premium. The new estimator is analogue to the volatility estimator proposed by DeMiguel et al. (2013) that employs the variance risk premium to improve the predictive performance of implied volatility.

[1]Broll (2016) also provides evidence on the skewness risk premium in the currency markets. More recently, Lin et al. (2019) show that skewness risk premium in the S&P 500 index is economically meaningful and has predictive power on future excess returns

In my empirical analysis, I find that the information content about future realised skewness increases on average with the forecasting horizon for all models. Each of my main models also appears to encapsulate different type of information about future skewness, as my encompassing regression results show. Models that use information from the option markets are more informative of future skewness and perform better out-of-sample compared to skewness models that are only computed by asset returns. The lagged realised skewness and the option-implied estimators result have higher explanatory power and lead to lower forecasting errors than the GARCH and QMIDAS models in most cases considered. The best overall forecasting performance is offered by the implied skewness estimator which accounts for the skewness risk premium. This estimator is superior to the rest of the skewness models across most of my comparisons. Notably, it explains up to 32.34% of the variation in the future realised skewness of the S&P 500. Overall, my results are consistent with parallel results in the volatility forecasting literature: Realised and option-implied estimators are superior to GARCH-based estimators while accounting for the relevant risk premium improves forecasting accuracy (see, e.g., DeMiguel et al., 2013; Kourtis et al., 2016; Prokopczuk and Wese Simen, 2014). The findings are also robust to a set of different model specifications and estimation methods I assume in my supplementary tests.

Finally, I examine the economic significance of each skewness model in the context of an international diversification setting, using the "$1/N$" portfolio as a benchmark investment strategy.[2] To this end, I use the parametric approach of Brandt et al. (2009) to construct a portfolio strategy for each skewness model, where the portfolio weights are a linear function of the corresponding skewness forecast. I find that portfolios employing the two option-implied skewness estimators outperform the rest of the skewness-based portfolios with regards to mean returns and Sharpe ratios. This result extends the evidence of DeMiguel et al.

[2]The choice of the benchmark is based on the work of DeMiguel et al. (2009) who find that $1/N$ is superior out-of-sample compared to several sample-based portfolio strategies.

(2013) and Kourtis et al. (2016) that the use of option-implied information can enhance portfolio performance. While all skewness-based portfolios lead to lower risk compared to $1/N$, only the portfolio based on the corrected implied skewness estimator leads to higher mean return and Sharpe ratio in all cases considered. In terms of economic gains, the new skewness estimator I develop in this work is superior to all other models across all time-periods and asset universes I consider, under all performance metrics, apart from portfolio turnover.

The rest of the chapter is organised as follows. Section 3.2 presents my data, the proxy for the *true* return skewness and the competing forecasting models I employ. In section 3.3, I examine the information content and out-of-sample performance of the models. I also discuss the portfolio performance of the skewness-based strategies. The robustness tests are covered in section 3.4, while I draw some limitations and future research directions in section 3.5. The last section concludes the chapter.

3.2 Methodology

3.2.1 Data

In my analysis, I adopt two main sets of data. First, I collect daily dividend-adjusted levels for 10 equity indices corresponding to 7 international regions (see Table 3.1 for details of these indices and the time periods). I source this data from Thomson-Reuters Datastream. For each index, I also employ a time-series of the London Interbank Offer Rate (LIBOR), quoted in the same currency, in order to proxy the corresponding risk-free rate. LIBOR data are collected from the FRED database of the Federal Reserve Bank of St. Louis.

The second dataset consists of market prices for all European vanilla options written on the considered indices. I obtain this data from IVolatility. I apply several standard filters from the literature to this dataset (see, e.g., Conrad et al.,

Table 3.1 List of Indices

This table lists the indices I employ in this work along with the region where each index is listed. It also reports the time period I consider for each index.

Equity Index	Region	Time Period
AEX	Netherlands	01/2006 - 12/2015
DAX	Germany	06/2001 - 12/2015
DJIA	United States	01/2000 - 12/2015
STOXX 50	Europe	02/2002 - 12/2015
FTSE 100	United Kingdom	01/2006 - 12/2015
HANGSENG	Hong Kong	12/2007 - 12/2015
KOSPI 200	South Korea	12/2007 - 12/2015
NASDAQ 100	United States	01/2000 - 12/2015
RUSSELL 2000	United States	10/2002 - 12/2015
S&P500	United States	01/2000 - 12/2015

2013; Stilger et al., 2017). First, I exclude all options with zero bid prices, zero open interest and those with prices smaller than 3/8$. Second, I only consider out-of-the-money (OTM) calls/puts with moneyness level between 0.8 and 1.2 and maturity ranging from 7 to 270 days to ensure the respective option contracts are liquid enough. Third, I discard all options that violate the theoretical arbitrage bounds in the model of Merton (1973). Finally, at each day, I only account for maturities that have at least two OTM calls and two OTM puts.

3.2.2 Realised Skewness

Skewness of asset returns is a latent variable, such as volatility. For the latter, there is a widely-used and theoretically justified proxy, i.e. realised volatility computed from high-frequency data as the sum of squared returns sampled at equal intervals (see Andersen et al., 2001; Barndorff-Nielsen and Shephard, 2002). However, estimating skewness, especially for lengthier horizons, is not as straight-forward. As Neuberger (2012) shows, one cannot simply use the sum of high-frequency cubic returns to accurately proxy long-term skewness. This is because the latter is also driven by a leverage effect, given by the correlation between innovations in volatility and asset returns. While one could instead rely on non-overlapping long-term returns to estimate skewness, the resulting estimator would be subject to significant sampling errors due to a likely small number of observations. To resolve these issues, the recent work of Neuberger (2012) proposes a new skewness estimator, coined as "*realised skewness*", which uses option data to capture the leverage effect and the use of data at higher frequency to estimate the skewness attributed to higher frequency return process. In this work, I adopt realised skewness to proxy the true skewness of index returns.[3]

I denote the asset price at the end of day t with S_t and its logarithmic value with $s_t = lnS_t$. The log return from day t to day $t+T$ is then $r_{t,t+T} = ln(S_{t+T}/S_t)$.[4] Under the assumption that the asset price S_t follows a Martingale process, Neuberger (2012) shows that the realised third (central) moment of the return $r_{t,t+T}$ can be expressed as:

$$RTM(r_{t,t+T}) = \sum_{i \in M_{t,t+T}} 3\left(\left(\Delta v^E_{i,t+T}\right)(e^{r_i} - 1) + F(r_i)\right) \tag{3.1}$$

In the above equation, $F(r) = 6(re^r - 2e^r + r + 2)$, $M_{t,t+T}$ is the set of trading days in the period $t, t+T$ and $\Delta v^E_{i,t+T}$ is the change of the index's entropy variance from the end of day $i-1$ to the end of day i. As in Neuberger (2012), the entropy variance is the option-implied variance of a contract that pays $S_{t+T} \ln S_{t+T}$ at day $t+T$:

$$v^E_{i,t+T} = 2\mathbb{E}^{\mathbb{Q}}_{i,t+T}\left[\frac{S_{t+T}}{S_i} ln\left(\frac{S_{t+T}}{S_i}\right) - \frac{S_{t+T}}{S_i} + 1\right], \tag{3.2}$$

where the expectation is taken under the option-implied probability measure conditional on the information on day i.

The entropy variance can be calculated following Bakshi and Madan (2000). The latter work shows that any payoff of an asset can be replicated using a portfolio of a risk-free zero-coupon bond and a continuum of OTM calls and puts written on the asset with varying strike prices. Let $B_{i,t+T} = e^{-r^f(t+T-i))}$ be the price of the bond, where r^f is the risk-free rate and $t+T-i$ is the time to

[3] Aretz and Arısoy (2019) also use this estimator as a proxy of the true skewness.
[4] Then the return on day t is simply $r_t = r_{t-1,t}$.

maturity. Using the spanning rule of Bakshi and Madan (2000), I can calculate the value of the fixed leg of the entropy by:

$$v^E_{i,t+T} = \frac{2}{B_{i,t+T}} \left[\int_0^{S_i} \frac{P_{i,t+T}(K)}{KS_i} dK + \int_{S_i}^{\infty} \frac{C_{i,t+T}(K)}{KS_i} dK \right], \tag{3.3}$$

where $C_{i,t+T}(K)$ and $P_{i,t+T}(K)$ are respectively the prices of an OTM call and put with strike price K and $t+T-i$ time to maturity. To compute the above integrals, I use the approximation of Kozhan et al. (2013). In particular, for a given day i, suppose that I have $N+1$ available calls and puts with increasing strike prices K_0, K_1,...,K_N and maturity $t+T$. I define the strike price differences by:

$$\Delta K_j = \begin{cases} K_1 - K_0, & j = 0 \\ \frac{K_{j+1} - K_{j-1}}{2}, & 0 < j < N \\ K_N - K_{N-1}, & j = N \end{cases} \tag{3.4}$$

I then use the available OTM calls and puts in my dataset, $C_{i,t+T}(K_j)$ and $P_{i,t+T}(K_j)$, to approximate $v^E_{i,t+T}$ as

$$v^E_{i,t+T} \approx \frac{2}{B_{i,t+T}} \left[\sum_{K_j < S_i} \frac{P_{i,t+T}(K_j)}{S_i K_j} \Delta K_j + \sum_{S_i < K_j} \frac{C_{i,t+T}(K_j)}{S_i K_j} \Delta K_j \right]. \tag{3.5}$$

I use this approximation to estimate the quantities $\Delta v^E_{i,t+T}$ for each trading day i in $M_{t,t+T}$ and, in turn, compute the realised third moment as in Eq. (3.1). The *realised skewness* $RS(r_{t,t+T})$ of the index return $r_{t,t+T}$ is then given by:

$$RS(r_{t,t+T}) = \frac{RTM(r_{t,t+T})}{(RV(r_{t,t+T}))^{3/2}}, \tag{3.6}$$

where $RV(r_{t,t+T})$ is the realised variance of $r_{t,t+T}$, calculated using the generalised variance measure of Neuberger (2012).

$$RV(r_{t,T}) = \sum_{i \in M_{t,t+T}} 2\left(e^{r_i} - 1 - r_i\right) \tag{3.7}$$

If there is not enough option prices available for a exact maturity T on a given date, I compute the third moment and the realised variance using option data for the nearest maturities T (say T_1 and T_2) for which I have the necessary data, so that $T_1 < T < T_2$. I then apply a simple linear interpolation to compute the two realised central moments, as in Chang et al. (2013) and Kozhan et al. (2013). The realised skewness coefficient is then computed using Eq. (3.6). $RS(r_{t,t+T})$ is the quantity the forecasting models compete to predict throughout this work. I consider three T of 30, 60 and 90 calendar days.

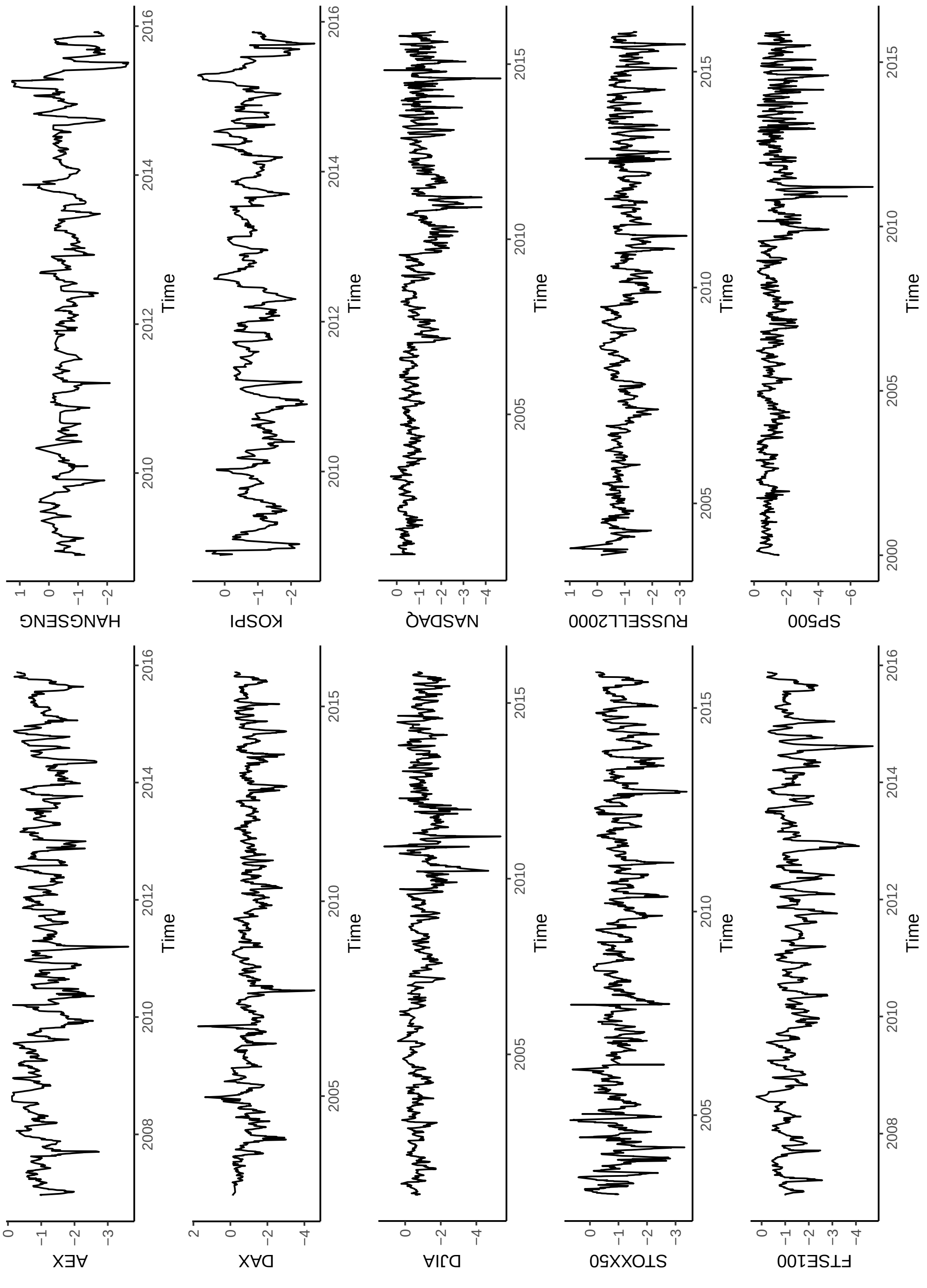
Given the importance of realised skewness in this work, I present its main features in Table 3.2. I report the mean and standard deviation of the realised variance, third moment and skewness of each index over the whole sample. Similar to Neuberger (2012), I find that the realised third moment is always negative and increases faster with the horizon than the second moment for all indices. For example, the realised variance of the S&P 500 rises from 0.332 at 30 days to 0.996 at 90 days, while the realised third moment increases from -0.321 to -2.039 for the same increase in the horizon. In the same fashion, the realised skewness is negative and increases with the horizon for all indices, apart from KOSPI. To also illustrate the evolution of realised skewness in my data, Figures 3.1, 3.2 and 3.3 plot the realised skewness of the returns of 30, 60 and 90 calendar days, respectively, for each index considered.

Table 3.2 Descriptive Statistics of Realised Moments

This table reports the average realised variance, third moment and skewness coefficients of the index log returns. The variance is annualised and the third moment is multiplied by 1,000 for better exposition. Standard errors for each statistic are presented in parentheses. Panel A, B and C correspond to horizons of 30, 60 and 90 calendar days respectively.

	Panel A: 30 days			*Panel B: 60 days*			*Panel C: 90 days*		
Index	Variance	Third Moment	Skewness	Variance	Third Moment	Skewness	Variance	Third Moment	Skewness
AEX	0.458	-0.393	-1.103	0.901	-1.237	-1.264	1.357	-2.529	-1.389
	(0.77)	(1.01)	(0.50)	(1.37)	(3.14)	(0.45)	(1.89)	(5.74)	(0.42)
DAX	0.465	-0.342	-1.060	0.911	-1.022	-1.177	1.327	-2.142	-1.29
	(0.60)	(0.67)	(0.57)	(1.10)	(1.90)	(0.52)	(1.49)	(4.19)	(0.47)
DJIA	0.359	-0.322	-1.004	0.649	-0.913	-1.195	1.030	-1.885	-1.273
	(0.60)	(1.00)	(0.65)	(1.03)	(2.90)	(0.63)	(1.50)	(5.14)	(0.59)
STOXX 50	0.450	-0.374	-1.092	0.853	-1.118	-1.207	1.274	-2.177	-1.327
	(0.61)	(0.84)	(0.56)	(1.04)	(2.32)	(0.43)	(1.43)	(3.82)	(0.40)
FTSE 100	0.357	-0.355	-1.296	0.702	-1.074	-1.43	0.994	-1.621	-1.484
	(0.58)	(1.01)	(0.61)	(1.02)	(2.85)	(0.49)	(1.22)	(3.36)	(0.48)
HANGSENG	0.334	-0.170	-0.621	0.583	-0.404	-0.694	0.841	-0.721	-0.762
	(0.32)	(0.38)	(0.55)	(0.44)	(0.72)	(0.53)	(0.53)	(0.98)	(0.54)
KOSPI	0.263	-0.151	-0.893	0.505	-0.414	-0.906	0.754	-0.718	-0.847
	(0.28)	(0.29)	(0.54)	(0.52)	(0.69)	(0.57)	(0.72)	(1.03)	(0.58)
NASDAQ 100	0.548	-0.400	-0.974	1.053	-1.240	-1.120	1.480	-2.376	-1.280
	(0.81)	(0.95)	(0.59)	(1.46)	(2.94)	(0.57)	(1.97)	(5.14)	(0.59)
RUSSELL 2000	0.516	-0.483	-1.014	1.131	-1.729	-1.230	1.765	-3.588	-1.381
	(0.85)	(1.32)	(0.49)	(1.65)	(3.88)	(0.42)	(2.40)	(6.93)	(0.41)
S&P 500	0.332	-0.321	-1.338	0.668	-1.049	-1.548	0.996	-2.039	-1.732
	(0.59)	(1.02)	(0.77)	(1.09)	(3.17)	(0.69)	(1.52)	(5.52)	(0.66)

Fig. 3.1 Realised Skewness of 30-day Index Returns



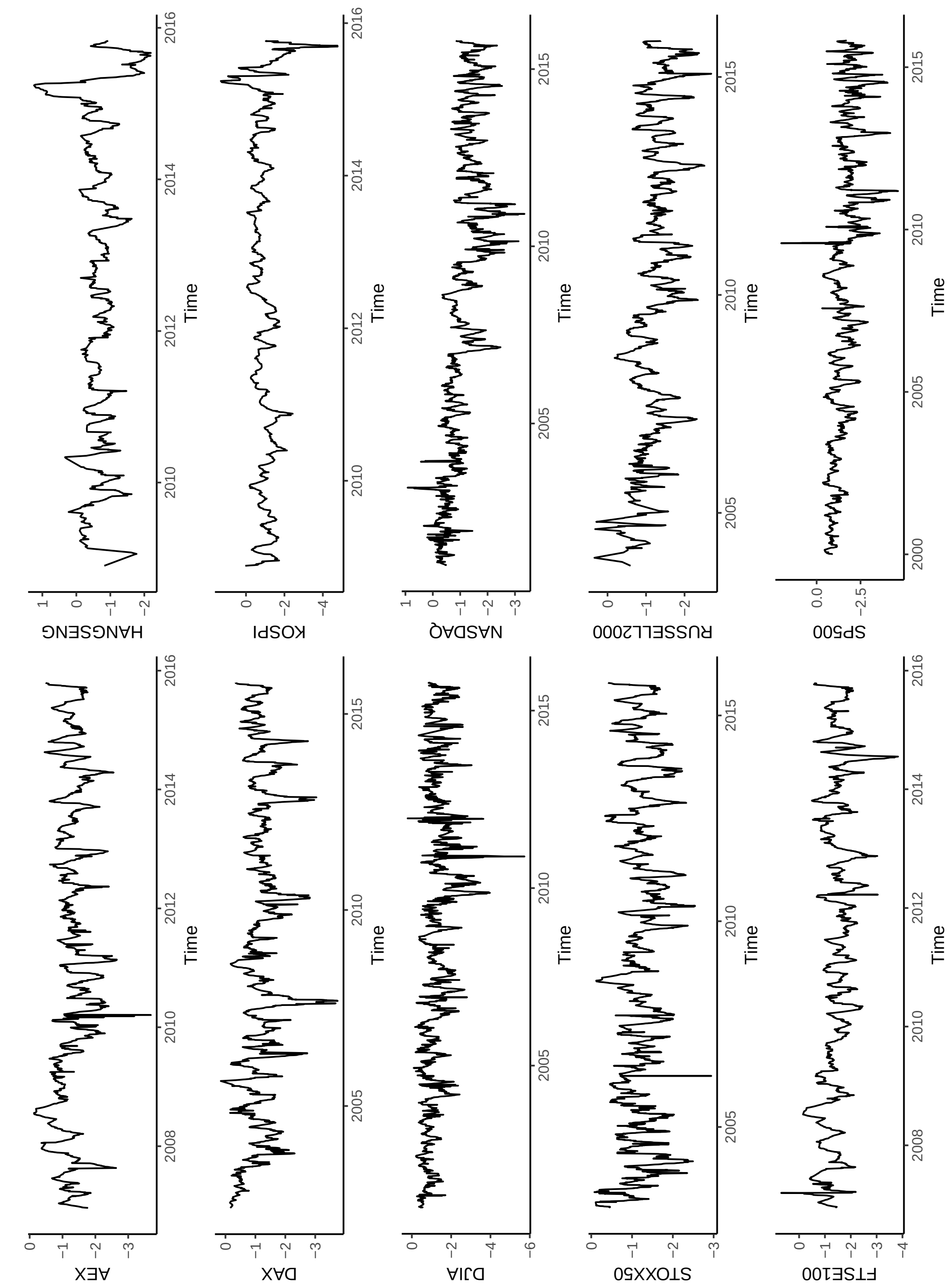


Fig. 3.2 Realised Skewness of 60-day Index Returns

Fig. 3.3 Realised Skewness of 90-day Index Returns

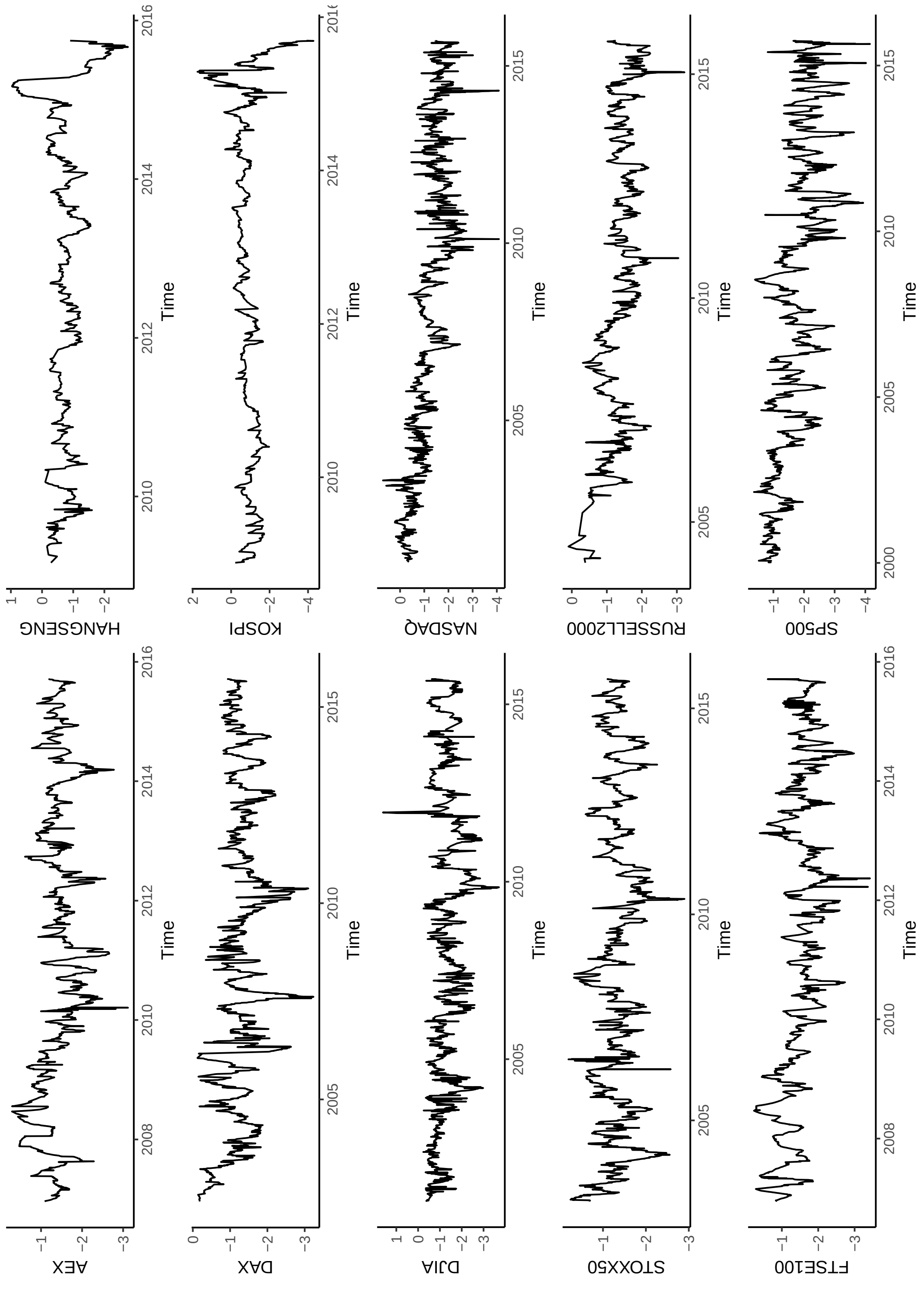


Table 3.3 List of Skewness Forecasting Models

This table summarises the competing skewness forecasting models under consideration.

Abbreviation	Description
LRS	Lagged Realised Skewness, defined as in Neuberger (2012)
GARCH-1	Skewness extracted from the GJR-GARCH model of Glosten et al. (1993), assuming a Skewed Generalised Error distribution (Theodossiou, 2015) for the daily index returns with time-invariant shape parameters
GARCH-2	Skewness extracted from the GJR-GARCH model of Glosten et al. (1993), assuming a Skewed Generalised Error distribution (Theodossiou, 2015) for the daily index returns with time-variant shape parameters
QMIDAS	Skewnss extracted from a Quantile regression with MIxed Data Sampling model, similar to Ghysels et al. (2016)
IS	Option-implied skewness, defined as in Conrad et al. (2013) and computed following the method of Kozhan et al. (2013)
CIS	A new option-implied skewness estimator that corrects for the skewness risk premium

3.2.3 Skewness Forecasting Models

3.2.3.1 Lagged realised skewness (LRS)

In this work, I examine the predictive ability and economic significance of six skewness forecasting models which I summarise in Table 3.3. These models have distinct features and may capture different information about future realised skewness. The first predictor of the future realised skewness ($RS(r_{t,t+T})$) simply is the lagged realised skewness (LRS), given by $RS(r_{t-T,t})$. Even though skewness is known not to be persistent over time (see, e.g., Singleton and Wingender, 1986), Neuberger (2012) provides evidence that LRS has some forecasting ability on the realised skewness of the S&P 500 for horizons of up to 3 months. An advantage of this estimator is that it does not rely on a particular distributional assumption while it has a hybrid nature utilising information from both the equity and the option markets.

3.2.3.2 GARCH forecasts with time-invariant parameters (GARCH-1)

GARCH models and their extensions are popular choices in the literature for modelling the conditional skewness of asset returns (see, e.g., Engle, 2011; Feunou et al., 2016; Harvey and Siddique, 1999; Jondeau and Rockinger, 2003). In this context, I consider two approaches that yield skewness forecasts from the GJR-GARCH model of Glosten et al. (1993). I choose this model among other GARCH alternatives as it allows asymmetries in the conditional variance, so that this can increase differently following a negative shock to the daily returns series than following a positive shock of the same magnitude. As such, it can produce significant skewness in the conditional distribution of multi-period returns (Engle, 2011).

To specify the GJR-GARCH model, I assume that daily index returns follow a Skewed Generalised Error distribution (SGE), as this is proposed by Theodossiou (2015). The distribution has been widely used in various financial applications (see, e.g., Anatolyev and Petukhov, 2016; Feunou et al., 2016), mainly due to the flexibility it offers for modelling financial data. In addition, Feunou et al. (2016) show that SGE results in superior parametric models for capturing the daily conditional skewness compared to other common distributions. The probability density function of the SGE distribution is given by:

$$f(y|\mu,\sigma,\lambda,\kappa) = \frac{C}{\sigma}exp\left(-\frac{|y-\mu+\delta\sigma|^{\kappa}}{(1+sign(y-\mu+\delta\sigma)\lambda)^{\kappa}\theta^{\kappa}\sigma^{\kappa}}\right), \tag{3.8}$$

with

$$C = \frac{\kappa}{2\theta\Gamma(1/\kappa))}; \quad \delta = 2\lambda AS(\lambda)^{-1}; \quad \theta = \Gamma(1/\kappa)^{1/2}\Gamma(3/\kappa)^{-1/2}S(\lambda)^{-1}$$
$$S(\lambda) = \sqrt{1+3\lambda^2-4A^2\lambda^2}; \quad A = \Gamma(2/\kappa)\Gamma(1/\kappa)^{-1/2}\Gamma(3/\kappa)^{-1/2},$$

where $\Gamma(.)$ is the Gamma function and μ, σ, λ, and κ are respectively the mean, standard deviation, asymmetry (skewness parameter) and tail-thickness (kurtosis parameter) of the distribution. The shape parameters, λ and κ satisfy the conditions $-1 < \lambda < 1$ and $\kappa > 0$. The distribution skews to the left (right) when $\lambda < 0 (> 0)$ and is symmetric when $\lambda = 0$. It exhibits fatter (thinner) tails than the normal distribution when $\kappa < 2$ ($\kappa > 2.$).

The GJR-GARCH model under the conditional SGE distribution is specified as:

$$r_t = \mu_t + \sigma_t z_t \tag{3.9}$$

$$\sigma_t^2 = b_0 + b_1 \varepsilon_{t-1}^2 + b_2 I_{z_{t-1} \leq 0} \varepsilon_{t-1}^2 + b_3 \sigma_{t-1}^2 \tag{3.10}$$

$$z_t \sim SGE\left(0, 1, \lambda, \kappa\right) \tag{3.11}$$

where b_2 captures the leverage effect imposed in the conditional variance process and z_t follows a standardised SGE distribution with zero mean and unit variance and time-invariant shape parameters λ and κ. The latter parameters are estimated by maximizing the sample log-likelihood function for z_t.

To compute a GJR-GARCH-based skewness estimate for a specific time horizon, I first estimate the GJR-GARCH model above using the available sample. I then apply a Monte-Carlo simulation approach, similar to Engle (2011) and Lönnbark (2016), in order to compute an empirical return distribution for the forecasting horizons. In particular, at each day t, for a given simulation path i, I simulate the next day return using the estimated coefficients and a random standardised innovation z_t, drawn from $SGE(0, 1, \hat{\lambda}_t, \hat{\kappa}_t)$, where $\hat{\lambda}_t$ and $\hat{\kappa}_t$ are the estimates of λ and κ at day t. I iterate this process to obtain a time series of daily returns for the trading days available in the next $T^{trading}$ trading days, where $T^{trading} = 22$, 44 or 66, for the forecast horizons of 30, 60 and 90 calendar days, respectively. The simulated path is then summed up to obtain the T-horizon simulated return

$\widetilde{r}^i_{t,t+T}$. I repeat this procedure 10,000 times to obtain the empirical distribution of T-horizon returns. The GJR-GARCH estimate of the skewness is then the sample skewness of the set of the simulated T-horizon returns. I coin this forecasting model with GARCH-1.

3.2.3.3 GARCH forecasts with autoregressive conditional parameters (GARCH-2)

The next set of skewness forecasts comes from a richer variation of the GJR-GARCH model discussed above. In particular, I allow the shape parameters governing return asymmetry to be time-variant, in the spirit of Hansen (1994). The dynamics in shape parameters (λ, κ) in the distribution of z_t in Eq. (3.9) depend on past information in an autoregressive structure. Bali et al. (2008) show that such a model allows for a more accurate representation of the conditional return distribution for several U.S. indices.

I follow Bali et al. (2008) to specify the dynamics of the shape parameters as:

$$\widetilde{\lambda}_t = \lambda_0 + \lambda_1 z_{t-1} + \lambda_2 \widetilde{\lambda}_{t-1} \tag{3.12}$$

$$\widetilde{\kappa}_t = \kappa_0 + \kappa_1 z_{t-1} + \kappa_2 \widetilde{\kappa}_{t-1}, \tag{3.13}$$

where $\widetilde{\lambda}_t$ and $\widetilde{\kappa}_t$ are the unrestricted estimates of λ and κ. The unrestricted estimates are then logistically transformed to be bounded as in the definition of SGE distribution ($|\lambda_t| < 1$ and $\kappa_t > 0$):

$$\lambda_t = -0.99 + \frac{1.98}{1 + e^{-\widetilde{\lambda}_t}} \tag{3.14}$$

$$\kappa_t = 2 + e^{\widetilde{\kappa}_t} \tag{3.15}$$

The innovation in Eq. (3.9) is then distributed as $z_t \sim SGE\,(0, 1, \lambda_t, \kappa_t)$. Empirically, the shape parameters are again estimated by maximizing the sample log-likelihood function before estimating the GJR-GARCH model. I then follow the same simulation approach as for the first version of the GJR-GARCH model to compute the skewness estimates in this case. In the rest of the paper, I refer to the time-varying version of the GARCH-based skewness forecasts with GARCH-2.

3.2.3.4 Quantile-based skewness forecasts with MIxed Data Sampling (QMIDAS)

The next forecasting model I employ relies on the estimation of the conditional quantiles of the return distribution to estimate skewness, similar to Ghysels et al. (2016). This is motivated by Kim and White (2004) who suggest that quantile-based skewness estimators are more robust to outliers than moment-based estimators. To estimate conditional quantiles, I follow the Mixed Data Sampling approach (QMIDAS) of Ghysels et al. (2016). The main advantage of the QMIDAS model is that it can directly estimate conditional quantiles of returns at any horizon while still exploiting information in higher-frequency data. Furthermore, it does not rely on any specific distributional assumption for the return process. As shown in the first study, the use of MIDAS framework provides reliable conditional quantiles by accounting for the impacts of short-horizon return process to long-horizon distribution. The QMIDAS model is described by the following equations:

$$q_\alpha(r_{t,t+T}) = \beta^0_{\alpha,T} + \beta^1_{\alpha,T} Z_{t-1}(\kappa_{\alpha,T}) \tag{3.16}$$

$$Z_{t-1}(\kappa_{\alpha,T}) = \sum_{d=0}^{D} \varphi_d(\kappa_{\alpha,T}) |r_{t-1-d}|, \tag{3.17}$$

where $q_\alpha(r_{t,t+T}|\Omega_{t-1})$ is the α-quantile of the T-horizon return. The known conditioning variable $Z_{t-1}(\kappa_{\alpha,T})$ is a sum of weighted absolute returns as in Ghysels et al. (2016) and reflects the high-frequency information. Each weight in Eq. (3.17) is determined parsimoniously by a lag polynomial function $\varphi(.)$ of a low-dimensional parameter vector $\kappa_{\alpha,T}$. I specify $\varphi(.)$ and estimate the quantiles as discussed in the Internet Appendix IV of Ghysels et al. (2016).[5] In my main results, I choose $D = 250$ to account for potential long-memory effects in the return process. I have also assumed alternative lag lengths of 200 or 300 days with similar results, as I discuss in section 3.4.3.

After estimating the conditional quantiles from the QMIDAS model, I employ the approach of Aretz and Arısoy (2019) to extract the moment-based skewness from the quantiles. First, the first three conditional moments of the return distribution between two consecutive quantiles α_{j-1} and α_j are approximated by

$$\widehat{\mathbb{E}}\left[r_{t,t+T}|q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] = \frac{q_{\alpha_{j-1}} + q_{\alpha_j}}{2} \tag{3.18}$$

$$\widehat{\mathbb{E}}\left[r_{t,t+T}^2|q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] = \frac{q_{\alpha_{j-1}}^2 + q_{\alpha_{j-1}} \times q_{\alpha_j} + q_{\alpha_j}^2}{3} \tag{3.19}$$

$$\widehat{\mathbb{E}}\left[r_{t,t+T}^3|q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] = \frac{q_{\alpha_{j-1}}^3 + q_{\alpha_{j-1}}^2 \times q_{\alpha_j} + q_{\alpha_{j-1}} \times q_{\alpha_j}^2 + q_{\alpha_j}^3}{4}, \tag{3.20}$$

Let J be the number of conditional quantiles, I then approximate the true conditional moments of the return density using the law of total probability by:[6]

$$\widehat{\mathbb{E}}\left[r_{t,t+T}^m\right] = \sum_{j=2}^{J} \frac{\alpha_j - \alpha_{j-1}}{\alpha_J - \alpha_1} \widehat{\mathbb{E}}\left[r_{t,t+T}^m|q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] \tag{3.21}$$

for $m = 1, 2, 3$. Finally, I compute the QMIDAS-based skewness as

$$QS_t(r_{t,t+T}) = \frac{\widehat{\mathbb{E}}\left[r_{t,t+T}^3\right] - 3\widehat{\mathbb{E}}\left[r_{t,t+T}\right](\widehat{\mathbb{E}}\left[r_{t,t+T}^2\right] - \widehat{\mathbb{E}}\left[r_{t,t+T}\right]^2) - \widehat{\mathbb{E}}\left[r_{t,t+T}\right]^3}{(\widehat{\mathbb{E}}\left[r_{t,t+T}^2\right] - \widehat{\mathbb{E}}\left[r_{t,t+T}\right]^2)^{3/2}}. \tag{3.22}$$

[5] I would like to thank Eric Ghysels for making available the code for this estimation at https://www.mathworks.com/matlabcentral/fileexchange/45150-midas-matlab-toolbox.

[6] In this application, I use 15 quantile levels spanning over the return density, i.e., $\alpha \in \{0.01, 0.025, 0.05, 0.1, 0.15, 0.2, 0.25, 0.5, 0.75, 0.8, 0.85, 0.9, 0.95, 0.975, 0.99\}$

3.2.3.5 Implied skewness (IS)

Conrad et al. (2013) advocates the use of information of the option markets to construct forward-looking estimator of the ex-ante skewness. In this context, implied skewness is defined as the ratio of the implied third central moment to the cube of the implied volatility:

$$IS_t\left(r_{t,t+T}\right) = \frac{ITM_t\left(r_{t,t+T}\right)}{\left(IV_t\left(r_{t,t+T}\right)\right)^{3/2}} = \frac{3(v_t^E(r_{t,t+T}) - IV_t\left(r_{t,t+T}\right))}{IV_t\left(r_{t,t+T}\right)^{3/2}}, \tag{3.23}$$

where $ITM_t\left(r_{t,t+T}\right) = 3\left(v_t^E\left(r_{t,t+T}\right) - IV_t\left(r_{t,t+T}\right)\right)$ is the implied third moment and $IV_t\left(r_{t,t+T}\right)$ is the implied variance. The latter can be calculated as

$$IV_t\left(r_{t,t+T}\right) = \frac{2}{B_{t,t+T}}\left[\int_0^{S_t} \frac{P_{t,t+T}(K)}{K^2}dK + \int_{S_t}^{\infty} \frac{C_{t,t+T}(K)}{K^2}dK\right]. \tag{3.24}$$

To compute the implied third moment and skewness, I approximate the entropy variance as in section 3.2.2 and the implied variance following the same approach as for the entropy variance. I then input these approximations in Eq. (3.23) to derive option implied skewness forecasts.[7] I note that apart from being forward-looking, implied skewness is also model-free by construction. I denote skewness forecasts from this method as IS.

3.2.3.6 Implied skewness corrected for the skewness risk premium (CIS)

It has been recently documented that correcting implied volatilities for the variance risk premium improves their predictive ability (DeMiguel et al., 2013; Kourtis et al., 2016; Prokopczuk and Wese Simen, 2014). Motivated by this evidence, I first examine whether a skewness risk premium exists in the equity markets under

[7] To estimate the implied skewness for a day t that does not have enough options with a maturity matching the horizon k, I again use interpolation as in the case of realised skewness.

study. I then introduce a new option-implied estimator of the true skewness that corrects for the skewness risk premium.

Kozhan et al. (2013) provide evidence of the existence of an economically significant third-moment risk premium for the S&P 500 index, reflected in the discrepancy between the implied and the realised skewness of the index. I choose to employ relative risk premia in my analysis instead of using simply the difference between realised and implied skewness (or third moment), given the evidence of a positive relationship between the level of the third moment and the third moment risk premium provide by Kozhan et al. (2013). The same type of connection between the level of the variance and the variance risk premium has also motivated DeMiguel et al. (2013), Prokopczuk and Wese Simen (2014) and Kourtis et al. (2016) to employ a relative variance risk premium in their respective work. To study whether their findings extend to other indices, I estimate (relative) third-moment and skewness risk premium as well as the variance risk premium from time t to $t+T$ for each of the indices as:

$$VRP_{t,t+T} = \frac{IV_{t,t+T}}{RV_{t,t+T}} \tag{3.25}$$

$$TMRP_{t,t+T} = \frac{ITM_{t,t+T}}{RTM_{t,t+T}} \tag{3.26}$$

$$SRP_{t,t+T} = \frac{IS_{t,t+T}}{RS_{t,t+T}} \tag{3.27}$$

Table 3.4 reports the averages and standard errors of the above premium for each index and horizon under consideration. The implied moments are on average higher than the realised moments in absolute terms in all cases, apart from KOSPI, where only the absolute implied skewness is lower than the absolute realised skewness. In general, the magnitude of the skewness risk premium is slightly higher than the variance risk premium while the third-moment risk premium is considerably higher than both. In line with Kozhan et al. (2013), I find that the variance risk premium and third-moment risk premium are highly positively correlated, with

Table 3.4 Moment Risk Premia

This table reports the average values for the variance, third-moment (TM) and skewness risk premium for each index in the dataset and for horizons of 30, 60 and 90 days. *Variance Risk Premium* is defined as the ratio of implied over realised variance. *TM Risk Premium* is defined as the ratio of implied over realised third moment. *Skewness Risk Premium* is defined as the ratio of implied over realised skewness. Standard deviations are reported in parenthesis. The last two columns respectively report the correlation between the variance and the third-moment risk premium and between the variance and the skewness risk premium. Panel A, B and C show these statistics for horizons of 30, 60 and 90 days respectively.

				Correlation with VRP	
Index	Variance Risk Premium	TM Risk Premium	Skewness Risk Premium	Third-Moment	Skewness
		Panel A: 30 days			
AEX	1.509 (0.777)	2.602 (2.564)	1.439 (1.013)	0.599	-0.231
DAX	1.587 (0.868)	3.236 (3.405)	1.773 (1.410)	0.510	-0.277
DJIA	1.372 (0.795)	2.326 (3.404)	1.417 (1.300)	0.342	-0.081
STOXX 50	1.647 (0.846)	3.824 (4.336)	1.884 (1.368)	0.495	-0.233
FTSE 100	1.697 (0.870)	3.848 (3.009)	1.853 (1.218)	0.661	-0.302
HANGSENG	1.661 (0.750)	4.703 (5.897)	2.051 (2.105)	0.423	0.022
KOSPI	1.682 (0.816)	2.033 (2.434)	0.953 (1.029)	0.422	-0.118
NASDAQ 100	1.521 (0.772)	3.150 (2.807)	1.723 (1.215)	0.582	-0.190
RUSSELL 2000	1.552 (0.726)	2.763 (2.149)	1.503 (0.851)	0.593	-0.305
S&P 500	1.782 (1.069)	3.807 (3.625)	1.643 (0.966)	0.634	-0.233
		Panel B: 60 days			
AEX	1.549 (0.802)	2.361 (1.754)	1.250 (0.701)	0.847	-0.295
DAX	1.545 (0.767)	2.484 (2.480)	1.314 (0.877)	0.567	-0.206
DJIA	1.485 (0.787)	2.292 (2.865)	1.168 (0.737)	0.623	-0.033
STOXX 50	1.619 (0.776)	2.936 (2.194)	1.461 (0.811)	0.716	-0.269
FTSE 100	1.674 (0.817)	2.988 (2.311)	1.414 (0.794)	0.729	-0.295
HANGSENG	1.610 (0.656)	4.445 (5.062)	1.865 (1.671)	0.554	0.216
KOSPI	1.644 (0.671)	2.000 (2.203)	0.955 (0.981)	0.369	-0.092
NASDAQ 100	1.594 (0.750)	2.659 (2.082)	1.327 (0.816)	0.675	-0.176
RUSSELL 2000	1.629 (0.743)	2.475 (1.546)	1.223 (0.555)	0.817	-0.339
S&P 500	1.810 (0.948)	2.993 (2.127)	1.238 (0.506)	0.820	-0.306
		Panel C: 90 days			
AEX	1.577 (0.842)	2.439 (1.799)	1.217 (0.517)	0.887	-0.279
DAX	1.566 (0.746)	2.449 (2.302)	1.232 (0.803)	0.647	-0.151
DJIA	1.535 (0.795)	2.467 (2.648)	1.153 (0.655)	0.646	0.072
STOXX 50	1.617 (0.754)	2.727 (1.797)	1.321 (0.563)	0.827	-0.256
FTSE 100	1.623 (0.855)	2.534 (1.965)	1.263 (0.729)	0.820	-0.312
HANGSENG	1.654 (0.689)	4.417 (4.694)	1.777 (1.660)	0.585	0.202
KOSPI	1.586 (0.658)	1.770 (2.252)	0.855 (0.938)	0.364	-0.025
NASDAQ 100	1.636 (0.755)	2.597 (2.433)	1.184 (0.877)	0.630	-0.046
RUSSELL 2000	1.666 (0.794)	2.305 (1.512)	1.073 (0.399)	0.873	-0.335
S&P 500	1.854 (0.947)	2.711 (1.860)	1.072 (0.374)	0.878	-0.344

correlations ranging from 0.342 to 0.887. I however observe a negative relation between the skewness risk premium and the variance risk premium in most cases, stemming from the inverse relationship between variance and skewness.

Given the existence of a skewness risk premium for almost all indices, the forecasting ability of the implied skewness estimator Eq. (3.23) could be enhanced if I incorporate the premium in the estimation procedure. To this end, I propose to adjust the implied skewness by dividing it to a historical average relative skewness risk premium, under the assumption of the historical premium captures well the premium for the forecasting period of interest. This adjustment is similar to the correction that DeMiguel et al. (2013) and Prokopczuk and Wese Simen (2014) apply to improve the forecasting performance of implied volatility. I estimate the historical skewness premium for each day t using the available skewness risk premium over the previous 252 trading days:[8]

$$\overline{SRP}_{t,t+T} = \frac{1}{252 - T} \sum_{i=t-252+1}^{t-T} SRP_{i,i+T}. \tag{3.28}$$

my *corrected implied skewness estimator* (CIS) is then defined as:

$$CIS_{t,t+T} = \frac{IS_{t,t+T}}{\overline{SRP}_{t,t+T}}. \tag{3.29}$$

3.3 Empirical Analysis

3.3.1 Information Content of Skewness Forecasts

I launch my empirical analysis with an in-sample evaluation of the information content of the forecasts produced by each skewness model. I estimate Mincer-Zarnowitz regressions (Mincer and Zarnowitz, 1969), i.e., I regress the T-day realised skewness on each model's corresponding skewness forecasts for each equity

[8]I also consider alternative averaging periods of about 18 and 24 months and I obtain similar results.

index using the whole sample available:

$$RS_{t,t+k} = \alpha + \beta \widehat{F}^{i}_{t,t+T} + \beta^{\rho} \rho_t + e_{t,t+T}, \tag{3.30}$$

where $\widehat{F}^{i}_{t,t+T}$ is the skewness forecast at day t produced by model i with a forecasting horizon of T calendar days. The two GARCH variations I assume along with the QMIDAS model are estimated using the whole sample before extracting the corresponding skewness forecasts. In my regressions, I control for the correlation ρ_t between index returns and the corresponding variance risk premium over the prior 12 months, as proposed by Aretz and Arısoy (2019). The latter work shows that such a control can help address the bias in $RS_{t,t+T}$ that results from a possible violation in the assumption in Neuberger (2012) that prices follow a martingale.

Tables 3.5, 3.6 and 3.7 report the regression results for forecasting horizons of 30, 60 and 90 days, respectively. I report for each index, the values of the intercept and the coefficients in Eq. (3.30) as well as the adjusted R^2's. I also report in parentheses t-statistics for the intercept and the coefficients, computed using the heteroscedasticity and autocorrelation consistent standard errors of Newey and West (1987) with T lags. Finally, I present the average values of the coefficients and adjusted R^2 for each model, across all indices.

I find that skewness forecasts become more informative as the horizon increases, as indicated by the average adjusted R^2's. For example, the average adjusted R^2 for the lagged realised skewness forecasts increases from 10.88% at the monthly horizon to 11.14% at the bimonthly horizon and to 12.98% at the quarterly horizon. The most informative forecasts are offered by models that use forward-looking information from option markets, namely the lagged realised and option-implied estimators of skewness. Indicatively for the S&P 500, at the quarterly horizon, the LRS, IS and CIS models yield an adjusted R^2 of 25.55%, 27.64% and 32.34%, respectively. In the same setting, GARCH- and QMIDAS-based forecasts do not

Table 3.5 Information Content of Skewness Forecasts (30 days)

This table reports the results from Mincer-Zarnowitz regressions. I regress the realised skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 30 calendar days. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realised skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.932**	-0.875	-0.851	**-0.715**	**-0.836**	**-0.908**
	(-11.42)	(-1.31)	(-1.30)	(-3.55)	(-6.68)	(-7.57)
β	**0.165**	0.201	0.222	**0.432**	**0.224**	**0.218**
	(2.39)	(0.36)	(0.41)	(2.03)	(2.61)	(2.01)
β^ρ	**-1.776**	**-2.072**	**-2.080**	**-1.979**	**-1.863**	**-1.599**
	(-4.00)	(-4.13)	(-4.14)	(-3.96)	(-4.08)	(-3.12)
$\overline{R}^2$	10.00%	7.65%	7.67%	9.17%	9.99%	**10.05%**
DAX						
α	**-0.811**	**-0.998**	**-0.997**	**-1.198**	**-0.941**	**-0.951**
	(-11.68)	(-11.10)	(-10.81)	(-6.46)	(-10.55)	(-11.29)
β	**0.221**	0.055	0.055	-0.210	0.072	0.107
	(3.77)	(0.70)	(0.68)	(-0.80)	(1.23)	(1.25)
β^ρ	**-1.196**	**-1.611**	**-1.610**	**-1.692**	**-1.407**	**-1.350**
	(-3.22)	(-3.42)	(-3.41)	(-3.21)	(-3.36)	(-3.17)
$\overline{R}^2$	**9.86%**	6.71%	6.70%	6.78%	7.05%	7.09%
DJIA						
α	**-0.621**	**-0.716**	**-0.373**	-0.224	**-0.299**	**-0.467**
	(-8.32)	(-4.22)	(-4.72)	(-0.38)	(-2.97)	(-5.26)
β	**0.375**	0.213	**0.606**	1.176	**0.666**	**0.676**
	(5.30)	(1.80)	(8.01)	(1.30)	(7.33)	(5.98)
β^ρ	**-0.760**	**-1.429**	**1.021**	**-1.134**	**-0.825**	-0.359
	(-2.22)	(-3.09)	(2.52)	(-2.62)	(-2.15)	(-1.00)
$\overline{R}^2$	17.06%	4.64%	**22.10%**	3.82%	13.53%	18.67%
STOXX 50						
α	**-0.845**	**-1.296**	**-1.413**	-0.690	**-0.672**	**-0.768**
	(-11.24)	(-2.89)	(-4.71)	(-1.73)	(-5.86)	(-6.62)
β	**0.224**	-0.173	-0.292	0.519	**0.246**	**0.341**
	(4.02)	(-0.46)	(-1.10)	(1.01)	(3.82)	(3.09)
β^ρ	0.414	0.525	0.574	0.655	0.650	0.630
	(1.17)	(1.31)	(1.45)	(1.54)	(1.80)	(1.71)
$\overline{R}^2$	5.29%	0.61%	0.90%	1.01%	**5.31%**	4.62%
FTSE 100						
α	**-1.193**	**-0.842**	**-0.858**	**-0.958**	**-1.030**	**-0.984**
	(-11.87)	(-2.21)	(-2.35)	(-4.25)	(-7.72)	(-5.95)
β	0.081	0.305	0.314	0.510	**0.139**	**0.288**
	(1.23)	(1.23)	(1.23)	(1.56)	(2.13)	(2.15)
β^ρ	**-1.632**	**-2.025**	**-2.039**	**-1.688**	**-1.444**	-1.170
	(-2.60)	(-3.07)	(-3.06)	(-2.53)	(-2.34)	(-1.79)
$\overline{R}^2$	6.61%	6.84%	6.83%	7.09%	**8.39%**	8.23%

Table 3.5 Information Content of Skewness Forecasts (30 days) *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.479**	0.665	**1.758**	**-0.811**	-0.237	-0.239
	(-4.16)	(1.55)	(2.32)	(-4.55)	(-1.18)	(-1.70)
β	**0.258**	**2.167**	**3.332**	-0.892	**0.335**	**0.630**
	(2.45)	(3.27)	(3.27)	(-1.29)	(2.56)	(3.85)
β^ρ	0.196	-0.053	0.020	0.132	-0.141	-0.818
	(0.24)	(-0.07)	(0.03)	(0.15)	(-0.17)	(-0.99)
$\overline{R}^2$	6.55%	**18.01%**	16.79%	0.90%	6.89%	15.35%
KOSPI						
α	**-0.524**	0.873	1.782	**-1.061**	**-0.520**	**-0.626**
	(-6.61)	(0.63)	(1.12)	(-8.85)	(-3.71)	(-5.49)
β	**0.406**	1.725	2.417	-0.377	**0.515**	**0.349**
	(5.16)	(1.27)	(1.67)	(-1.44)	(3.04)	(2.75)
β^ρ	0.039	0.715	0.662	0.623	0.619	0.760
	(0.06)	(0.85)	(0.81)	(0.65)	(0.83)	(0.99)
$\overline{R}^2$	**16.87%**	3.28%	4.66%	2.67%	10.99%	8.48%
NASDAQ 100						
α	**-0.556**	**-1.059**	**-1.177**	**-0.523**	**-0.306**	**-0.438**
	(-7.74)	(-5.78)	(-6.84)	(-1.96)	(-3.24)	(-5.09)
β	**0.430**	-0.124	-0.308	0.862	**0.497**	**0.673**
	(6.42)	(-0.59)	(-1.43)	(1.62)	(6.73)	(6.46)
β^ρ	-0.486	-0.984	-0.761	-0.803	-0.465	-0.086
	(-1.21)	(-1.63)	(-1.25)	(-1.45)	(-1.07)	(-0.19)
$\overline{R}^2$	**19.87%**	2.46%	3.40%	4.29%	17.56%	19.67%
RUSSELL 2000						
α	**-0.837**	**-1.108**	**-1.125**	**-1.016**	**-0.269**	**-0.515**
	(-10.02)	(-16.19)	(-16.68)	(-15.58)	(-2.31)	(-4.80)
β	**0.186**	-0.108	-0.132	0.024	**0.571**	**0.587**
	(2.81)	(-1.44)	(-1.83)	(0.22)	(6.59)	(5.22)
β^ρ	-0.502	-0.399	-0.341	-0.587	-0.435	0.011
	(-1.20)	(-0.81)	(-0.70)	(-1.23)	(-1.09)	(0.03)
$\overline{R}^2$	4.14%	1.81%	2.47%	0.78%	**12.43%**	9.03%
S&P500						
α	**-0.872**	**-1.081**	**-1.207**	-0.044	**-0.440**	**-0.589**
	(-9.26)	(-4.96)	(-5.75)	(-0.10)	(-2.94)	(-5.21)
β	**0.266**	0.073	-0.016	**1.609**	**0.459**	**0.608**
	(3.75)	(0.50)	(-0.11)	(2.65)	(5.11)	(5.86)
β^ρ	**-1.723**	**-2.573**	**-2.360**	**-2.178**	**-1.517**	**-1.206**
	(-3.38)	(-3.59)	(-3.28)	(-3.84)	(-2.91)	(-2.50)
$\overline{R}^2$	12.58%	6.69%	6.60%	8.26%	14.70%	**15.82%**
Aggregated results						
Average α	-0.767	-0.644	-0.446	-0.724	-0.555	-0.649
Average β	0.261	0.433	0.620	0.366	0.372	0.448
Average β^ρ	-0.743	-0.991	-0.691	-0.865	-0.683	-0.519
Average $\overline{R}^2$	10.88%	5.87%	7.81%	4.48%	10.68%	**11.70%**

Table 3.6 Information Content of Skewness Forecasts (60 days)

This table reports the results from Mincer-Zarnowitz regressions. I regress the realised skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 60 calendar days. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realised skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^{ρ} across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-1.156**	-0.817	-0.825	**-1.058**	**-0.856**	**-0.886**
	(-9.54)	(-1.55)	(-1.60)	(-5.89)	(-5.61)	(-8.69)
β	0.002	0.192	0.188	0.085	**0.229**	**0.253**
	(0.02)	(0.64)	(0.64)	(0.59)	(2.14)	(3.06)
β^{ρ}	**-2.669**	**-2.826**	**-2.828**	**-2.852**	**-2.422**	**-2.135**
	(-3.92)	(-3.92)	(-3.91)	(-3.73)	(-3.94)	(-3.38)
$\overline{R}^2$	19.70%	19.92%	19.92%	19.94%	22.01%	**23.64%**
DAX						
α	**-1.057**	**-1.065**	**-1.061**	**-1.324**	**-1.132**	**-1.119**
	(-11.20)	(-11.57)	(-11.32)	(-9.73)	(-13.41)	(-14.27)
β	0.107	0.080	0.083	-0.151	0.039	0.061
	(1.79)	(1.26)	(1.28)	(-1.28)	(0.83)	(1.03)
β^{ρ}	-0.765	**-1.155**	**-1.162**	**-1.302**	-0.867	-0.829
	(-1.61)	(-1.98)	(-1.98)	(-2.00)	(-1.74)	(-1.64)
$\overline{R}^2$	**3.57%**	3.39%	3.42%	3.51%	2.71%	2.91%
DJIA						
α	**-0.789**	**-0.906**	**-0.678**	**-0.953**	**-0.826**	**-0.806**
	(-7.85)	(-4.09)	(-7.58)	(-5.72)	(-6.25)	(-6.90)
β	**0.309**	0.113	**0.420**	0.257	**0.293**	**0.357**
	(3.66)	(1.09)	(5.65)	(1.13)	(2.72)	(2.94)
β^{ρ}	**-1.169**	**-1.962**	0.560	**-2.049**	**-1.414**	**-1.083**
	(-2.39)	(-3.25)	(0.97)	(-3.08)	(-2.47)	(-2.10)
$\overline{R}^2$	13.79%	6.15%	**21.60%**	6.51%	8.14%	11.80%
STOXX 50						
α	**-1.078**	**-1.017**	**-1.086**	**-1.517**	**-1.030**	**-1.097**
	(-9.97)	(-2.73)	(-3.04)	(-4.22)	(-7.26)	(-9.95)
β	0.113	0.115	0.084	-0.317	0.124	0.111
	(1.62)	(0.54)	(0.37)	(-0.84)	(1.55)	(1.21)
β^{ρ}	-0.126	-0.210	-0.182	-0.063	-0.015	-0.040
	(-0.18)	(-0.28)	(-0.24)	(-0.09)	(-0.02)	(-0.05)
$\overline{R}^2$	1.23%	0.10%	-0.00%	0.16%	**1.29%**	0.74%
FTSE 100						
α	**-1.319**	**-1.223**	**-1.304**	**-1.529**	**-1.140**	**-1.088**
	(-8.39)	(-3.16)	(-3.55)	(-8.66)	(-7.23)	(-8.20)
β	0.049	0.075	0.042	-0.170	0.151	**0.240**
	(0.56)	(0.42)	(0.22)	(-0.99)	(1.73)	(2.52)
β^{ρ}	**-1.654**	**-1.799**	**-1.759**	-1.183	-1.177	-1.161
	(-2.12)	(-2.39)	(-2.33)	(-1.26)	(-1.60)	(-1.72)
$\overline{R}^2$	5.70%	5.57%	5.50%	6.22%	8.23%	**10.03%**

Table 3.6 Information Content of Skewness Forecasts (60 days) *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.533**	0.649	1.774	-0.112	**-0.515**	**-0.382**
	(-3.78)	(0.98)	(1.43)	(-0.32)	(-2.44)	(-2.43)
β	0.207	1.577	2.457	2.241	0.147	**0.462**
	(1.44)	(1.96)	(1.95)	(1.56)	(1.32)	(2.69)
β^{ρ}	-0.156	-0.330	-0.305	-0.299	-0.120	-0.477
	(-0.25)	(-0.72)	(-0.65)	(-0.60)	(-0.18)	(-0.85)
$\overline{R}^2$	3.74%	16.71%	**16.92%**	8.65%	1.41%	8.04%
KOSPI						
α	**-0.620**	1.405	2.093	**-0.948**	**-0.694**	**-0.686**
	(-8.37)	(1.07)	(1.34)	(-7.30)	(-9.33)	(-7.99)
β	**0.322**	1.667	2.035	-0.131	**0.298**	**0.290**
	(2.99)	(1.71)	(1.87)	(-0.50)	(2.50)	(2.31)
β^{ρ}	-0.870	-1.608	-1.688	-1.342	-1.020	-0.918
	(-0.68)	(-1.21)	(-1.29)	(-1.02)	(-0.74)	(-0.69)
$\overline{R}^2$	**8.10%**	5.09%	6.20%	1.34%	4.64%	6.24%
NASDAQ 100						
α	**-0.547**	**-1.330**	**-1.337**	**-0.811**	**-0.290**	**-0.438**
	(-5.49)	(-5.62)	(-6.15)	(-4.11)	(-2.79)	(-4.24)
β	**0.520**	-0.186	-0.232	0.573	**0.657**	**0.701**
	(7.42)	(-1.01)	(-1.16)	(1.74)	(9.00)	(7.69)
β^{ρ}	-0.316	-0.821	-0.744	-0.557	-0.024	0.313
	(-0.68)	(-1.16)	(-1.04)	(-0.73)	(-0.06)	(0.73)
$\overline{R}^2$	28.79%	3.53%	3.93%	5.58%	28.76%	**32.66%**
RUSSELL 2000						
α	**-0.978**	**-1.575**	**-1.568**	**-1.455**	**-0.579**	**-0.762**
	(-6.51)	(-18.23)	(-18.77)	(-18.50)	(-2.64)	(-4.41)
β	**0.225**	**-0.270**	**-0.270**	**-0.227**	**0.502**	**0.447**
	(2.20)	(-3.95)	(-4.01)	(-3.66)	(3.38)	(3.28)
β^{ρ}	-0.120	0.345	0.364	-0.055	0.258	0.327
	(-0.26)	(0.76)	(0.80)	(-0.11)	(0.55)	(0.76)
$\overline{R}^2$	5.18%	11.85%	**12.38%**	8.15%	10.63%	9.24%
S&P500						
α	**-0.913**	**-1.555**	**-1.580**	**-1.437**	**-0.406**	**-0.654**
	(-7.83)	(-5.07)	(-5.55)	(-11.97)	(-2.56)	(-5.34)
β	**0.344**	-0.084	-0.107	-0.078	**0.633**	**0.584**
	(4.95)	(-0.60)	(-0.73)	(-0.63)	(6.53)	(6.69)
β^{ρ}	**-1.570**	**-2.302**	**-2.237**	**-2.430**	**-1.098**	-0.967
	(-2.95)	(-3.42)	(-3.28)	(-4.03)	(-2.08)	(-1.89)
$\overline{R}^2$	21.61%	11.47%	11.57%	11.44%	25.89%	**26.45%**
Aggregated results						
Average α	-0.899	-0.743	-0.557	-1.114	-0.747	-0.792
Average β	0.220	0.328	0.470	0.208	0.307	0.351
Average β^{ρ}	-0.942	-1.267	-0.998	-1.213	-0.790	-0.697
Average $\overline{R}^2$	11.14%	8.38%	10.14%	7.15%	11.37%	**13.17%**

Table 3.7 Information Content of Skewness Forecasts (90 days)

This table reports the results from Mincer-Zarnowitz regressions. I regress the realised skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 90 calendar days. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realised skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.944**	**-1.240**	**-1.250**	**-1.263**	**-0.839**	**-0.836**
	(-8.06)	(-2.50)	(-2.54)	(-6.51)	(-5.94)	(-8.90)
β	**0.293**	0.031	0.026	0.034	**0.315**	**0.379**
	(3.48)	(0.15)	(0.13)	(0.28)	(3.34)	(5.89)
β^ρ	-1.060	**-1.910**	**-1.904**	**-1.975**	**-1.562**	**-1.240**
	(-1.67)	(-2.43)	(-2.43)	(-2.35)	(-2.49)	(-2.37)
$\overline{R}^2$	18.90%	11.83%	11.82%	11.89%	18.21%	**25.79%**
DAX						
α	**-1.146**	**-1.114**	**-1.094**	**-1.575**	**-1.129**	**-1.096**
	(-6.51)	(-9.06)	(-8.42)	(-5.89)	(-13.21)	(-13.98)
β	0.131	0.104	0.112	-0.290	**0.134**	**0.178**
	(0.99)	(1.63)	(1.70)	(-1.04)	(2.24)	(2.91)
β^ρ	-0.124	-0.520	-0.535	-0.586	-0.207	-0.177
	(-0.31)	(-1.01)	(-1.03)	(-1.00)	(-0.46)	(-0.41)
$\overline{R}^2$	1.86%	2.50%	2.69%	1.59%	2.47%	**4.74%**
DJIA						
α	**-0.885**	**-1.595**	**-0.846**	**-1.452**	**-0.840**	**-0.908**
	(-6.75)	(-6.40)	(-10.70)	(-10.85)	(-7.03)	(-7.44)
β	**0.310**	-0.121	**0.324**	-0.225	**0.365**	**0.329**
	(3.47)	(-1.37)	(4.62)	(-1.69)	(4.45)	(3.92)
β^ρ	-0.049	-0.081	**0.970**	0.103	-0.029	-0.114
	(-0.14)	(-0.17)	(1.97)	(0.22)	(-0.07)	(-0.31)
$\overline{R}^2$	10.22%	1.47%	**21.94%**	2.72%	5.75%	7.39%
STOXX 50						
α	**-1.024**	**-1.455**	**-1.469**	**-0.961**	**-1.033**	**-1.071**
	(-7.47)	(-4.27)	(-4.06)	(-2.99)	(-7.23)	(-10.39)
β	**0.249**	-0.045	-0.057	0.449	**0.200**	**0.227**
	(2.67)	(-0.29)	(-0.31)	(1.19)	(2.40)	(2.49)
β^ρ	0.246	0.239	0.247	0.234	0.285	0.371
	(0.41)	(0.34)	(0.35)	(0.34)	(0.42)	(0.56)
$\overline{R}^2$	**6.31%**	0.12%	0.14%	1.15%	4.48%	5.57%
FTSE 100						
α	**-1.111**	**-1.988**	**-1.984**	**-1.899**	**-1.134**	**-1.149**
	(-7.24)	(-4.72)	(-4.74)	(-9.83)	(-7.97)	(-8.81)
β	**0.270**	-0.172	-0.185	**-0.500**	**0.246**	**0.256**
	(3.78)	(-1.08)	(-1.09)	(-2.44)	(3.37)	(3.42)
β^ρ	-0.423	-0.483	-0.465	0.582	0.061	-0.359
	(-0.57)	(-0.59)	(-0.57)	(0.60)	(0.07)	(-0.47)
$\overline{R}^2$	8.24%	2.29%	2.33%	6.05%	9.10%	**11.11%**

Table 3.7 Information Content of Skewness Forecasts (90 days) *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.766**	1.074	2.249	**-1.713**	**-0.753**	**-0.682**
	(-4.94)	(1.51)	(1.91)	(-2.96)	(-3.91)	(-6.51)
β	0.046	**1.816**	**2.490**	-1.686	0.035	**0.106**
	(0.24)	(2.60)	(2.57)	(-1.63)	(0.38)	(5.00)
β^{ρ}	1.777	**1.357**	**1.364**	1.631	1.768	1.093
	(1.54)	(2.08)	(2.07)	(1.56)	(1.52)	(1.32)
$\overline{R}^2$	4.10%	**30.95%**	30.35%	13.64%	4.06%	19.66%
KOSPI						
α	**-0.813**	1.856	2.553	**-0.630**	**-0.664**	**-0.567**
	(-6.22)	(1.57)	(1.86)	(-4.34)	(-5.21)	(-4.11)
β	-0.007	**1.623**	**1.874**	**0.320**	**0.216**	**0.297**
	(-0.03)	(2.27)	(2.46)	(1.97)	(2.61)	(3.23)
β^{ρ}	-0.039	-0.526	-0.557	-0.283	0.178	0.604
	(-0.03)	(-0.51)	(-0.56)	(-0.24)	(0.13)	(0.47)
$\overline{R}^2$	-0.19%	8.01%	**9.28%**	4.08%	1.66%	7.00%
NASDAQ 100						
α	**-0.660**	**-1.648**	**-1.652**	-0.381	**-0.694**	**-0.781**
	(-5.09)	(-6.79)	(-7.25)	(-0.91)	(-4.35)	(-5.83)
β	**0.576**	-0.137	-0.166	**1.403**	**0.531**	**0.520**
	(8.27)	(-0.97)	(-1.08)	(2.69)	(7.05)	(7.42)
β^{ρ}	0.668	0.802	0.828	1.158	0.774	0.861
	(1.20)	(1.07)	(1.10)	(1.63)	(1.40)	(1.57)
$\overline{R}^2$	**34.58%**	1.91%	2.18%	8.37%	22.50%	28.96%
RUSSELL 2000						
α	**-0.813**	**-1.896**	**-1.890**	**-1.798**	**-0.726**	**-0.817**
	(-5.02)	(-17.17)	(-16.93)	(-21.20)	(-3.82)	(-5.03)
β	**0.454**	**-0.301**	**-0.300**	**-0.334**	**0.507**	**0.475**
	(4.48)	(-3.91)	(-3.81)	(-4.51)	(4.47)	(4.59)
β^{ρ}	0.465	0.995	0.986	1.030	0.515	0.727
	(0.92)	(1.79)	(1.76)	(1.77)	(1.05)	(1.57)
$\overline{R}^2$	**20.28%**	17.48%	17.23%	17.58%	18.83%	19.95%
S&P500						
α	**-0.906**	**-2.178**	**-2.176**	**-1.834**	**-0.568**	**-0.750**
	(-6.28)	(-6.30)	(-6.37)	(-9.99)	(-3.27)	(-5.91)
β	**0.447**	-0.214	-0.238	-0.251	**0.655**	**0.580**
	(5.66)	(-1.77)	(-1.78)	(-1.69)	(6.36)	(7.67)
β^{ρ}	-0.959	-1.540	-1.505	**-1.832**	-0.911	-0.498
	(-1.45)	(-1.69)	(-1.65)	(-2.16)	(-1.34)	(-0.80)
$\overline{R}^2$	25.55%	9.73%	9.80%	9.44%	27.64%	**32.34%**
Aggregated results						
Average α	-0.907	-1.018	-0.756	-1.351	-0.838	-0.866
Average β	0.277	0.258	0.388	-0.108	0.320	0.335
Average β^{ρ}	0.050	-0.167	-0.057	0.006	0.087	0.127
Average $\overline{R}^2$	12.98%	8.63%	10.78%	7.65%	11.47%	**16.25%**

bear any significant information about the realised skewness. I observe a similar pattern across most considered cases with the GARCH and QMIDAS forecasts failing to have any explanatory power on future realised skewness in the majority of horizon-index pairs in my analysis. The superiority of the lagged realised and option-implied skewness estimates is consistent with similar findings from the volatility forecasting literature, where realised or option-implied volatilities are known to better predict future realised volatility compared to GARCH-based estimators (see, e.g., Kourtis et al., 2016). The poor performance of the GARCH/QMIDAS models in this test can be partly due to the inability of these models to capture the time-varying nature of the leverage effect, documented by Bandi and Renò (2012), Wang and Mykland (2014) and Kalnina and Xiu (2017). While the leverage effect is either explicitly estimated by the coefficient b_2 in the GJR-GARCH model or implicitly captured by the slope coefficient β^1 in the MIDAS model, these coefficients are estimated using a long sample, which makes them less representative of the forecasting horizon.

Among the models that use option-based information, the simple option-implied skewness (IS) performs similarly to the lagged realised skewness model (LRS) with one model being slightly superior to the other for some indices/horizons/metrics and vice versa. For instance, at the quarterly horizon, LRS forecasts subsume significant information about the future realised skewness for 7 out of 10 indices, while IS contains significant relevant information for 9 out of 10 indices. For the same horizon, LRS corresponds to a higher average adjusted R^2 across indices (12.98% vs 11.47%). Both of these models, however, are inferior to the implied skewness estimator that corrects for the skewness risk premium (CIS) in the majority of the scenarios I consider. Focusing again on the quarterly horizon, CIS predicts the realised skewness in a statistically significant manner for all indices in my sample. At the same time, it offers the highest average adjusted R^2 equal to 16.25%. CIS is generally superior to the remaining models on average across all horizons, highlighting the important of accounting for the skewness risk premium when forecasting skewness. This result is again analogue to findings in the volatility forecasting literature that promote the use of the variance risk premium in volatility forecasting using implied volatilities.

I can reach two additional conclusions from the results in Tables 3.5-3.7. First, I observe that the intercept is statistically significant in almost all scenarios I consider, indicating that the forecasts are generally biased estimates of the future realised skewness. I explore the bias in skewness forecasts for models with significant slope coefficients more formally by jointly testing the hypothesis $H_0 : \alpha = 0; \beta = 1$ with a standard Wald-type test. The hypothesis is indeed rejected for most of the markets and forecast horizons in my analysis. Second, I observe that the control for the bias in the realised skewness I include in the Mincer-Zarnowitz regressions has significant explanatory power in the majority of cases considered while it enters the regression with a negative coefficient. This result shows that the martingale assumption in Neuberger (2012) is violated in most cases and the realised skewness is generally a biased estimator of the true skewness.

I further investigate the relative importance of each forecasting model in the context of encompassing regressions. Encompassing regressions also allow us to identify whether a forecasting model subsumes the information contained in the rest of the models. The encompassing regression extends the Mincer-Zarnowitz regressions Eq. (3.30) as follows:

$$\begin{aligned} RS_{t,t+k} = \alpha &+ \beta_{LRS}\widehat{F}^{LRS}_{t,t+T} + \beta_{GARCH-2}\widehat{F}^{GARCH-2}_{t,t+T} \\ &+ \beta_{QMIDAS}\widehat{F}^{QMIDAS}_{t,t+T} + \beta_{CIS}\widehat{F}^{CIS}_{t,t+T} + \beta^{\rho}\rho_t + e_{t,t+T}. \end{aligned} \tag{3.31}$$

I exclude the GARCH-1 and IS models from the regressions as they can be considered as the restricted versions of the GARCH-2 and CIS models, respectively.

Table 3.8 presents the regression coefficients and the adjusted R^2's from my encompassing regressions for each index and horizon I consider. As before, I find

Table 3.8 Encompassing Regressions

This table reports the results from regressing the realised skewness on forecasts generated from the LRS, GARCH-2, QMIDAS and CIS models, within the same regression, for each index in Table 3.1. The GARCH-2 and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realised skewness estimates. α and β_i respectively denote the intercept and the coefficient of the forecast of model i in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. Panel A, B and C respectively present results for a forecasting horizon of 30, 60 and 90 calendar days. (1) - (10) are the ten international indices of order listed in Table 3.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Panel A: 30 days					
α	-0.113	**-0.664**	**0.868**	-0.876	0.041	1.309	1.557	0.140	**-0.506**	**0.854**
	(-0.18)	(-2.26)	(2.12)	(-1.48)	(0.12)	(1.79)	(1.34)	(0.32)	(-4.29)	(2.30)
β^ρ	**-1.443**	**-1.371**	**0.741**	0.549	-1.081	-0.574	-0.107	0.111	0.313	-0.410
	(-2.88)	(-2.94)	(2.20)	(1.39)	(-1.58)	(-0.72)	(-0.18)	(0.25)	(0.70)	(-0.68)
β_{LRS}	**0.160**	**0.220**	**0.188**	**0.170**	0.069	0.038	**0.325**	**0.279**	0.083	**0.184**
	(2.08)	(3.78)	(2.50)	(2.86)	(1.14)	(0.35)	(4.55)	(3.90)	(1.21)	(2.51)
$\beta_{GARCH-2}$	0.132	0.113	**0.386**	-0.108	0.346	**2.296**	1.942	0.113	**-0.170**	-0.124
	(0.24)	(1.21)	(5.63)	(-0.47)	(1.31)	(2.46)	(1.82)	(0.51)	(-2.12)	(-0.94)
β_{QMIDAS}	**0.528**	-0.057	**1.545**	-0.101	0.514	0.177	-0.291	0.838	**0.255**	**2.149**
	(2.41)	(-0.19)	(2.61)	(-0.18)	(1.45)	(0.27)	(-1.52)	(1.42)	(2.29)	(4.32)
β_{CIS}	0.183	0.096	**0.333**	**0.235**	**0.385**	**0.375**	0.185	**0.429**	**0.485**	**0.474**
	(1.72)	(1.21)	(3.69)	(2.04)	(3.07)	(2.49)	(1.82)	(4.20)	(4.28)	(4.49)
$\overline{R}^2$	13.82%	12.68%	31.07%	6.87%	11.38%	21.10%	21.73%	26.29%	11.31%	20.08%
					Panel B: 60 days					
α	-0.477	**-0.912**	**-0.476**	**-1.043**	-0.536	**2.584**	**2.721**	-0.243	**-1.205**	**-0.676**
	(-0.65)	(-3.82)	(-3.06)	(-2.36)	(-1.12)	(2.16)	(2.09)	(-1.40)	(-7.14)	(-2.13)
β^ρ	**-2.510**	**-1.093**	0.392	-0.283	-0.905	-0.399	-1.694	**0.929**	0.713	-0.715
	(-3.66)	(-2.08)	(0.65)	(-0.46)	(-1.12)	(-0.71)	(-1.84)	(2.18)	(1.74)	(-1.26)
β_{LRS}	-0.080	0.125	**0.169**	0.108	-0.010	-0.005	**0.220**	**0.270**	-0.043	**0.170**
	(-0.92)	(1.82)	(2.41)	(1.47)	(-0.10)	(-0.03)	(2.02)	(3.67)	(-0.47)	(2.78)
$\beta_{GARCH-2}$	0.207	0.089	**0.336**	0.429	0.297	**3.723**	**2.301**	-0.169	**-0.254**	-0.090
	(0.45)	(1.09)	(4.81)	(1.57)	(1.65)	(2.63)	(2.57)	(-1.43)	(-2.19)	(-0.47)
β_{QMIDAS}	0.062	-0.067	0.079	-0.747	-0.158	**-2.415**	-0.179	**0.611**	0.035	0.100
	(0.35)	(-0.46)	(0.44)	(-1.59)	(-1.15)	(-2.01)	(-0.74)	(3.03)	(0.35)	(0.65)
β_{CIS}	**0.311**	0.057	0.053	0.075	**0.317**	0.251	0.184	**0.455**	**0.333**	**0.456**
	(3.40)	(0.97)	(0.70)	(0.81)	(2.73)	(1.58)	(1.58)	(5.62)	(3.06)	(5.78)
$\overline{R}^2$	24.76%	5.32%	24.03%	2.51%	12.13%	20.15%	15.46%	39.63%	16.01%	28.17%
					Panel C: 90 days					
α	-0.138	-0.549	**-0.931**	-0.421	**-0.821**	1.295	**4.329**	0.076	**-1.076**	**-0.677**
	(-0.35)	(-1.46)	(-5.94)	(-0.77)	(-2.14)	(1.09)	(2.23)	(0.32)	(-5.92)	(-2.16)
β^ρ	**-1.513**	-0.381	**1.307**	0.220	0.359	1.033	0.184	**1.028**	**0.994**	-0.349
	(-2.89)	(-0.87)	(2.81)	(0.46)	(0.53)	(1.81)	(0.18)	(2.58)	(2.56)	(-0.61)
β_{LRS}	0.120	0.077	0.124	**0.183**	0.163	-0.154	-0.143	**0.387**	**0.182**	**0.189**
	(1.33)	(0.82)	(1.68)	(2.03)	(1.75)	(-1.16)	(-1.12)	(6.05)	(2.13)	(2.47)
$\beta_{GARCH-2}$	0.197	**0.170**	**0.294**	0.147	0.171	**1.838**	**2.845**	-0.030	-0.102	-0.013
	(1.14)	(2.17)	(4.94)	(0.89)	(1.40)	(2.18)	(2.48)	(-0.36)	(-1.10)	(-0.09)
β_{QMIDAS}	0.078	0.117	**-0.210**	0.246	**-0.331**	-0.258	-0.263	**0.913**	-0.071	0.014
	(0.84)	(0.43)	(-2.27)	(0.64)	(-2.23)	(-0.39)	(-1.22)	(3.20)	(-0.70)	(0.08)
β_{CIS}	**0.359**	**0.184**	-0.027	0.144	**0.190**	**0.086**	**0.353**	**0.269**	**0.262**	**0.448**
	(4.15)	(3.87)	(-0.46)	(1.73)	(2.55)	(3.05)	(3.58)	(4.55)	(3.91)	(7.11)
$\overline{R}^2$	28.11%	9.04%	25.63%	8.42%	14.80%	36.87%	19.08%	42.73%	28.50%	34.15%

that the option-based models tend to be capture more information about the future realised skewness compared to the models that only use information from past returns. The CIS model again produces the most informative estimates, based on the number it appears as a significant predictor in the regressions. An interesting observation here is that each model tends to capture different information about the realised skewness, as forecasts that are significant in my Mincer-Zarnowitz tests are in most cases significant in the multivariate setting too.[9] For example, at the monthly horizon and for the DJIA index, all forecasts are significant at the 5% level either in isolation or collectively as it can be respectively seen in Table 3.5 and in Panel A of Table 3.8. As a result, adjusted R^2's for the encompassing regression can be considerably larger to their analogues in the single-forecast regressions. Indicatively, if I consider again the DJIA index, the highest adjusted R^2 that a single model can yield on its own at the monthly horizon is 22.10% while the encompassing regression results in an adjusted R^2 of 31.07%.

3.3.2 Out-of-Sample Analysis

I evaluate the predictive ability of each of the six skewness models in an out-of-sample empirical analysis. To this end, I compute two popular loss functions for each model/index/horizon triplet, i.e., the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE), defined respectively as:

$$RMSE = \sqrt{\frac{1}{M}\sum_{t=1}^{M}\left(RS_{t,t+T} - \widehat{F}_{t,t+T}\right)^2} \tag{3.32}$$

$$MAE = \frac{1}{M}\sum_{t=1}^{M}|RS_{t,t+T} - \widehat{F}_{t,t+T}| \tag{3.33}$$

[9]There are only a handful of cases where one of the models appears to be uninformative in the multivariate framework while it produces significant results on its own.

where $\widehat{F}_{t,t+T}$ is the skewness forecast and M is the total number of out-of-sample skewness forecasts. To compute the forecasts from the GARCH and QMIDAS models, I use a rolling window of 1250 daily observations.

The results loss functions are presented in Tables 3.9, 3.10 to Table 3.11 for the monthly, bi-monthly and quarterly horizons, respectively. I report RMSE's in Panel A and MAE's in Panel B of each table. The models with the lowest forecast errors are highlighted in bold. I also run Diebold-Mariano predictive accuracy tests (Diebold and Mariano, 1995) using heteroscedasticity and autocorrelation consistent standard errors (Newey and West, 1987) to test whether the difference of the loss function produced by a model is significantly larger than that of the best model. Models that produce significantly less accurate forecasts than the best model at the 5% (10%) significance level are marked with two (one) asterisks.

In line with the in-sample results, I observe that the option-based models offer significantly superior predictive ability compared to the models that only rely on index returns. The GARCH and QMIDAS models lead to inferior out-of-sample performance in all cases, apart from the HANGSENG and KOSPI indices. The relative forecasting performance of the LRS, IS and CIS models are boosted out-of-sample by the use of forward-looking information and the lower sensitivity to estimation errors. On the contrary, the rest of the models involve the estimation of a relatively large number of parameters which makes them more sensitive to estimation errors and deteriorates their out-of-sample performance.

At the monthly and bi-monthly horizon implied skewness adjusted for the skewness risk premium (CIS) produces the lowest forecasting errors. For example, according to the MAE criterion, it is the superior model for predicting the skewness of 8 out 10 indices. The MAE's produced by CIS are significantly lower at the 5% level than the rest of the forecasts in most cases. At the quarterly horizon, the results are mixed with LRS, IS or CIS producing the best forecast in different

Table 3.9 Out-of-sample Forecasting Performance (30 days)

For each index in Table 1, this table reports out-of-sample forecasting losses for each skewness estimator generated from the models in Table 3. The forecasting horizon is 30 calendar days. The GARCH and QMIDAS models are estimated using a rolling window of 1250 observations. I report Root Mean Squared Errors (RMSE) in Panel A and Mean Absolute Errors (MAE) in Panel B using the realised skewness of Neuberger (2012) as a proxy for the true skewness. The model with the lowest forecasting loss is highlighted in bold. One (two) asterisk(s) shows that the corresponding model is inferior to the best model at the 10% (5%) significance level, in the context of a Diebold-Mariano test.

	Panel A: RMSE						Panel B: MAE					
	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX	0.648**	0.785**	0.870**	0.624**	0.553	**0.549**	0.499**	0.630**	0.696**	0.484*	0.446	**0.429**
DAX	0.653	1.130**	1.024**	0.800**	0.868**	**0.637**	0.482	0.863**	0.797**	0.630**	0.635**	**0.472**
DJIA	0.770**	1.104**	1.018**	0.964**	**0.661**	0.686	0.565**	0.836**	0.790**	0.747**	**0.479**	0.499
STOXX 50	0.661**	1.089**	0.815**	0.694**	0.865**	**0.587**	0.512**	0.863**	0.635**	0.544**	0.716**	**0.440**
FTSE 100	0.914**	0.903**	0.994**	0.963**	1.273**	**0.728**	0.686**	0.681**	0.768**	0.722**	1.050**	**0.534**
HANGSENG	0.794**	0.668	0.643	0.709	0.853**	**0.621**	0.625**	0.504	0.479	0.510	0.681**	**0.451**
KOSPI	0.632	**0.593**	1.095	0.735	0.595	0.636	0.511*	0.463	0.722*	0.563**	**0.438**	0.472**
NASDAQ 100	0.666*	1.000**	1.183**	0.814**	0.733**	**0.607**	0.491**	0.788**	0.949**	0.625**	0.594**	**0.443**
RUSSELL 2000	0.667**	0.841**	0.850**	0.714**	0.581*	**0.525**	0.496**	0.644**	0.665**	0.535**	0.479**	**0.389**
S&P500	1.004**	1.316**	1.451**	1.259**	0.934*	**0.850**	0.732**	0.999**	1.109**	0.983**	0.721**	**0.596**

Table 3.10 Out-of-sample Forecasting Performance (60 days)

For each index in Table 1, this table reports out-of-sample forecasting losses for each skewness estimator generated from the models in Table 3. The forecasting horizon is 60 calendar days. The GARCH and QMIDAS models are estimated using a rolling window of 1250 observations. I report Root Mean Squared Errors (RMSE) in Panel A and Mean Absolute Errors (MAE) in Panel B using the realised skewness of Neuberger (2012) as a proxy for the true skewness. The model with the lowest forecasting loss is highlighted in bold. One (two) asterisk(s) shows that the corresponding model is inferior to the best model at the 10% (5%) significance level, in the context of a Diebold-Mariano test.

	Panel A: RMSE						Panel B: MAE					
	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX	0.570**	1.423**	1.610**	0.642**	**0.466**	0.469	0.457*	1.137**	1.243**	0.509**	0.381	**0.374**
DAX	**0.665**	2.138**	1.580**	0.927**	0.764	0.722	0.493	1.628**	1.198**	0.724**	0.502	**0.479**
DJIA	0.780	1.880**	1.175**	1.344**	**0.740**	0.767	0.587**	1.373**	0.880**	1.097**	**0.528**	0.566*
STOXX 50	0.551	2.301**	1.236**	0.734**	0.639**	**0.542**	0.441	1.799**	0.979**	0.595**	0.511*	**0.433**
FTSE 100	0.703*	1.200**	0.849**	0.906**	0.863**	**0.604**	0.542*	0.935**	0.636**	0.739**	0.701**	**0.466**
HANGSENG	0.809	0.720	0.666	0.772	0.839**	**0.662**	0.547	0.547	0.521	0.549	0.686**	**0.501**
KOSPI	0.752	0.826	2.079**	1.069**	0.776*	**0.729**	0.526	0.633	1.416**	0.878**	0.511	**0.500**
NASDAQ 100	0.548**	1.517**	1.266**	1.052**	0.524	**0.494**	0.423**	1.107**	1.066**	0.896**	0.425**	**0.377**
RUSSELL 2000	0.518**	1.346**	0.837**	0.768**	0.446	**0.437**	0.418**	0.968**	0.657**	0.608**	0.362	**0.345**
S&P500	0.828**	2.133**	1.447**	1.429**	**0.677**	0.695	0.629**	1.502**	1.151**	1.236**	**0.516**	0.521

Table 3.11 Out-of-sample Forecasting Performance (90 days)

For each index in Table 1, this table reports out-of-sample forecasting losses for each skewness estimator generated from the models in Table 3. The forecasting horizon is 90 calendar days. The GARCH and QMIDAS models are estimated using a rolling window of 1250 observations. I report Root Mean Squared Errors (RMSE) in Panel A and Mean Absolute Errors (MAE) in Panel B using the realised skewness of Neuberger (2012) as a proxy for the true skewness. The model with the lowest forecasting loss is highlighted in bold. One (two) asterisk(s) shows that the corresponding model is inferior to the best model at the 10% (5%) significance level, in the context of a Diebold-Mariano test.

	Panel A: RMSE						Panel B: MAE					
	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX	0.487	2.249**	2.575**	0.608**	0.463	**0.444**	0.378	1.824**	1.995**	0.481*	**0.367**	0.372
DAX	**0.569**	3.687**	2.588**	0.897**	0.646	0.635	**0.425**	2.682**	1.816**	0.741**	0.465	0.472
DJIA	0.695	3.087**	1.420**	1.319**	**0.646**	0.697	0.549	2.100**	1.034**	1.141**	**0.507**	0.535
STOXX 50	**0.450**	4.066**	2.256**	0.769**	0.593**	0.500*	**0.355**	3.143**	1.637**	0.614**	0.470**	0.395*
FTSE 100	**0.571**	1.934**	0.965**	1.015**	0.671*	0.614	**0.456**	1.525**	0.663**	0.899**	0.526*	0.482
HANGSENG	0.918	0.785	**0.710**	0.794	0.969**	1.196	0.684	0.625	**0.568**	0.643	0.787*	0.671
KOSPI	1.085**	1.299**	3.500**	1.006**	0.853**	**0.782**	0.704**	1.095**	2.484**	0.781**	0.541**	**0.498**
NASDAQ 100	**0.509**	2.732**	1.616**	1.172**	0.538	0.556	**0.396**	1.829**	1.277**	1.052**	0.397	0.408
RUSSELL 2000	0.406	2.171**	1.043**	0.930**	**0.398**	0.414	0.334	1.617**	0.790**	0.686**	**0.306**	0.329*
S&P500	0.696*	3.553**	1.552**	1.349**	**0.617**	0.633	0.544**	2.374**	1.203**	1.166**	**0.475**	0.504

cases, depending on the loss function or the index assumed.[10] Nevertheless, CIS is never outperformed at the 5% significance level by any of the remaining models.

I conclude my out-of-sample analysis using the nonparametric approach of Hansen et al. (2011), known as Model Condidence Set (MCS), to identify a collection of models that outperform the rest of the models under a given loss function at specific level of confidence. Similar to the second chapter, I use the range statistic in Eq. (2.21) to test the null hypothesis that two models lead to the same loss at a specific time. To compute the MCS, I use a block bootstrap process with a block of 2 observation and 10,000 replications.[11]

I present the results from my MCS tests in Tables 3.12 and 3.13 for the RMSE and MAE losses, respectively, assuming a significance level of 5%. Three main observations from these results are noteworthy. First, corrected implied skewness (CIS) enters the MCS in almost all considered cases, ranked first in the majority of them, except for some indices at quarterly horizon. Second, IS and, to a lesser extent, LRS are included in the MCS in several cases highlighting again the importance of option-implied information when forecasting skewness. Third, the GARCH- and QMIDAS-based skewness estimators do not enter the MCS in almost all scenarios. For example, under the RMSE criterion, the GARCH and QMIDAS model are favoured by the MCS test only for the HANGSENG index, where all three are included in the MCS, and for KOSPI, where only the GARCH-1 model is included. Overall, the findings from my out-of-sample analysis are consistent with the in-sample results from the previous section.

3.3.3 Economic Value of Skewness Forecasts

I assess the economic significance of the skewness models in an international diversification setting. I aim to identify which of the six skewness forecasting

[10] An exception is the GARCH-2 that leads to the lowest losses for the HANGSENG index.
[11] I have considered alternative block lengths with similar results.

Table 3.12 Model Confidence Set under Root Mean Squared Errors

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.02	2*	0.562	3	0.00	2	0.00	3	0.00	5	0.02	3*	0.677	2	0.03	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	3*	0.276	1*	1.000	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.492	6	0.03	6	0.00	6	0.00	6	0.00
QMIDAS	3	0.02	3	0.00	4	0.00	3	0.00	4	0.00	4*	0.073	5	0.03	4	0.00	4	0.00	4	0.00
IS	2*	0.833	4	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	2*	0.968	3	0.00	2	0.02	2	0.01
CIS	1*	1.000	1*	1.000	2*	0.091	1*	1.000	1*	1.000	1*	1.000	4*	0.090	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	1*	1.000	3*	0.181	2*	0.583	2	0.02	5*	0.066	2*	0.535	3	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.138	4*	0.059	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.876	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	4*	0.066	5	0.00	4	0.00	4	0.00	4	0.00
IS	1*	1.000	3*	0.082	1*	1.000	3	0.00	4	0.00	6	0.00	3*	0.059	2	0.04	2*	0.596	1*	1.000
CIS	2*	0.886	2*	0.327	2*	0.181	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.215
Panel C: 90-day horizon																				
LRS	3*	0.136	1*	1.000	2*	0.068	1*	1.000	1*	1.000	4	0.02	4	0.00	1*	1.000	2*	0.597	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	2*	0.051	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	4	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	5	0.00	3	0.02	3	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.224	3	0.03	1*	1.000	3	0.00	3	0.00	5	0.00	2	0.00	2*	0.179	1*	1.000	1*	1.000
CIS	1*	1.000	2	0.03	3*	0.68	2	0.00	2*	0.172	6	0.00	1*	1.000	3*	0.176	3*	0.166	2*	0.141

Table 3.13 Model Confidence Set under Mean Absolute Errors

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4*	0.052	2*	0.630	3	0.00	2	0.00	3	0.00	5	0.00	4*	0.120	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	3*	0.287	2*	0.370	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.365	6	0.00	6	0.00	6	0.00	6	0.00
QMIDAS	3*	0.052	3	0.00	4	0.00	3	0.00	4	0.00	4*	0.287	5	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.341	4	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	1*	1.000	3	0.00	2	0.00	2	0.00
CIS	1*	1.000	1*	1.000	2*	0.220	1*	1.000	1*	1.000	1*	1.000	3*	0.120	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.01	2*	0.522	3	0.01	2*	0.657	2	0.02	3*	0.592	3*	0.459	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	4*	0.592	4*	0.054	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.592	6	0.00	5	0.00	5	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	5*	0.556	5	0.00	4	0.00	4	0.00	5	0.00
IS	2*	0.653	3*	0.269	1*	1.000	3	0.00	4	0.00	6	0.00	2*	0.459	3	0.00	2*	0.229	1*	1.000
CIS	1*	1.000	1*	1.000	2	0.01	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.740
Panel C: 90-day horizon																				
LRS	3*	0.862	1*	1.000	3*	0.117	1*	1.000	1*	1.000	5	0.04	3	0.00	1*	1.000	3	0.01	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	2	0.04	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	3	0.04	4	0.00	4	0.00	4	0.00	4	0.00
IS	1*	1.000	2*	0.050	1*	1.000	3	0.00	3	0.04	6	0.00	2	0.00	2*	0.979	1*	1.000	1*	1.000
CIS	2*	0.862	3	0.04	2*	0.117	2	0.00	2*	0.303	4	0.04	1*	1.000	3*	0.436	2	0.01	2	0.00

methods is more beneficial for an international investor and whether any of them can lead to portfolios that outperform naïve diversification, i.e., my benchmark in this exercise. DeMiguel et al. (2009) show that the naïve diversification, also known as the "$1/N$" portfolio, performs consistently better than many popular theory-based portfolio choice methods as the latter are subject to sampling errors that deteriorate portfolio performance significantly. In particular, I construct parametric portfolios that are based on each skewness forecasting model using the approach of Brandt et al. (2009). This framework allows us to directly assess the impact of the skewness forecasts on portfolio performance without having to rely on any specific distributional assumptions about the index returns. The same approach has been also adopted by DeMiguel et al. (2013), who show that a portfolio that exploits option-implied skewness leads to higher Sharpe ratio compared to $1/N$ in a sample of U.S. stocks.

I consider an investor that at time t uses the information on a skewness forecast to select a portfolio of N indices. The investor's portfolio weight on the i index is a linear function of the skewness forecast as

$$w_{i,t}^{m} = w_{i,t}^{1/N} + \theta_t^m \frac{1}{N} \tilde{f}_{i,t}^{m}. \tag{3.34}$$

In the above, $w_{i,t}^{1/N} = 1/N$ is the weight of the $1/N$ portfolio which stands for the benchmark in the parametric portfolio framework. $\tilde{f}_{i,t}^{m}$ is the 30-day ahead forecast of the skewness of the index i, generated by the model m and standardised so that the cross-sectional mean and variance at time t are 0 and 1 respectively. The parameter θ_t^m is the loading on each standardised forecast and is the same across indices. When θ_t^m is positive (negative), the investor will assign a larger weight on her portfolio on assets with a higher (lower) skewness prediction.

Each of the six forecasting models augments the benchmark portfolio $w_t^{1/N}$ by a zero-cost portfolio that is determined by the forecasts generated by the model. As such each forecasting model yields a unique portfolio strategy defined by the weights w_t^m. By studying the out-of-sample performance of each strategy, I can then assess the economic value of the corresponding forecasting model. In this fashion, I compute several popular performance metrics from daily portfolio returns. At each day t and for each model m, I compute the parameter θ_t^m which leads to the minimum variance of the daily portfolio returns over the previous 252 days. I input this value in Eq. (3.34) to derive the portfolio weights at day t for model m and the corresponding portfolio return r_t^m. I repeat this process until the last day of my sample to yield a series of M portfolio returns/weight vectors for each skewness forecasting model and for the $1/N$ portfolio. Then, for each portfolio model m, including the $1/N$ portfolio, I compute the out-of-sample portfolio mean return, volatility and Sharpe ratio as:

$$\widehat{\mu}^m = \frac{1}{M}\sum_{t=1}^{M} r_t^m \tag{3.35}$$

$$(\widehat{\sigma}^m)^2 = \frac{1}{M-1}\sum_{t=1}^{M}(r_t^m - \widehat{\mu}^m)^2 \tag{3.36}$$

$$\widehat{SR}^m = \frac{\widehat{\mu}^m}{\widehat{\sigma}^m} \tag{3.37}$$

I also explore the significance of the difference of the variance between a portfolio strategy and $1/N$ by testing the null hypothesis: $H_0 : (\widehat{\sigma}^m)^2 - \left(\widehat{\sigma}^{1/N}\right)^2 = 0$. I estimate p-values for this test using the non-parametric bootstrap framework of Ledoit and Wolf (2011), assuming an average block size of 10 and 5,000 trials. I also compute the average daily portfolio turnover for each portfolio strategy, which assesses the sensitivity of a portfolio strategy to transaction costs (see Kourtis (2015) as:

$$\widehat{\tau}^m = \frac{1}{M-1}\sum_{t=1}^{M-1} \|\widetilde{w}_{t+1}^{(m)} - \widetilde{w}_{t+}^{(m)}\|_1, \tag{3.38}$$

where $\widetilde{w}_{t+}^{(m)}$ stands for the portfolio weights at the beginning of the period $t+1$, before rebalancing takes place, while $\|\cdot\|_1$ stands for the 1-norm.

In Table 3.14, I report the above metrics for portfolios of the indices in my dataset, excluding the HANGSENG and KOSPI indices for which the resulting out-of-sample period would be considerably small to yield any reliable conclusions. I also exclude the DJIA, NASDAQ and RUSSELL 2000 indices in order to reduce the bias of the portfolio towards the U.S. market. As a robustness check, I have also included the DJIA, NASDAQ and RUSSELL 2000 and rerun my analysis. My qualitative results remain similar to the ones presented in this section, as discussed in the next section. I consider two time-periods. In Panel A, I report results for the period 01/2011-12/2015, i.e. the maximum period for which out-of-sample forecasts are available for all models. In Panel B, I report results for the period 03/2008-12/2015, i.e. the maximum period for which out-of-sample forecasts are available for the LRS, IS and CIS models. As such, I do not include results for the GARCH and the QMIDAS models in panel B.

Starting with the period 01/2011-12/2015, I observe that all skewness-based portfolios offer a lower risk than $1/N$ with the difference between the variances being statistical significance at the 5% level in most cases, apart from the LRS and QMIDAS portfolios. The lowest variance is offered by the portfolio that is based on the implied skewness estimator that corrects for the skewness risk premium (CIS). This portfolio is the only one that outperforms $1/N$ in terms of either average return or Sharpe ratio. In particular, the CIS-based portfolio leads to an annualised mean return of 12.61% and an out-of-sample Sharpe ratio of 0.7199. In comparison, $1/N$ yields an average annualised return of 9.24% and a Sharpe ratio of 0.5004.

The second best alternative out of the skewness-based portfolios is the one that relies on the vanilla option-implied estimator (IS). This portfolio produces higher average return and Sharpe ratio compared to all other skewness-based portfolios,

Table 3.14 Out-of-sample Performance of Skewness-based Portfolios

This table presents the out-of-sample performance of the equally-weighted portfolio ($1/N$) and of the parametric portfolios that use the skewness forecasts from each model considered in the paper. The portfolios include as assets the following indices: AEX, DAX, STOXX 50, FTSE100 and S&P 500. The table reports the annualised out-of-sample average daily return (MEAN), variance of daily returns (VAR) and Sharpe ratio (SR) for each portfolio strategy as well as the average daily turnover (TRN). It also reports p-values from testing the hypothesis that the variances between a portfolio strategy and $1/N$ are equal. The p-values are computed using the block-bootstrap approach of Ledoit and Wolf (2011), assuming an average block size of 5 and 5,000 replications. Panel A and B respectively present results for the periods 01/2011-12/2015 and 03/2008-12/2015.

	MEAN	*VAR*	*p-value*	*SR*	*TRN*
		Panel A: 01/2011-12/2015			
LRS	0.0761	0.0323	0.07	0.4233	0.1649
GARCH-1	0.0872	0.0331	0.01	0.4792	0.0674
GARCH-2	0.0796	0.0332	0.00	0.4371	0.0728
QMIDAS	0.0798	0.0338	0.52	0.4342	0.0876
IS	0.0877	0.0332	0.00	0.4816	0.1850
CIS	**0.1261**	**0.0307**	0.00	**0.7199**	0.3277
1/N	0.0924	0.0341	1.00	0.5004	0.0039
		Panel B: 03/2008-12/2015			
LRS	0.0552	0.0191	0.07	0.3986	0.2379
IS	0.0633	0.0175	0.00	0.4793	0.4095
CIS	**0.0914**	**0.0159**	0.00	**0.7252**	0.4219
1/N	0.0630	0.0220	1.00	0.4249	0.0032

apart from CIS. The superior performance of both option-implied skewness-based portfolios is however accompanied by a higher turnover while the portfolios that result from the GARCH-based skewness tend to yield lower levels of turnover. Finally, LRS underperforms all portfolios with regards to mean and risk-adjusted returns, even though it results in lower variance compared to all portfolios, apart from CIS.

Focusing on the second time period under consideration (03/2008-12/2015), which is longer than the first, I observe that option-implied skewness leads to even better relative portfolio performance. In particular, both the IS and CIS strategies outperform both $1/N$ and LRS across all performance measures, except turnover. The CIS strategy is again superior than the rest of the strategies, yielding an average return of 9.14%, a variance of 0.0159 and a Sharpe ratio of 0.7252. It again outperforms $1/N$ which in turn leads to an average return of 6.3%, a variance of 0.0220 and a Sharpe ratio of 0.4249. Finally, LRS leads to the worse overall performance out of the 4 strategies considered in this setting.

Overall, I show that the use of option-implied skewness offers significant benefits in terms of out-of-sample portfolio performance. Especially, accounting for the skewness risk premium when forecasting skewness leads to superior portfolios compared to $1/N$ or to portfolios constructed using other methods for estimating skewness. These results extend recent studies that advocate the use of option-implied information in portfolio selection (DeMiguel et al., 2013; Kourtis et al., 2016; Prokopczuk and Wese Simen, 2014).

3.4 Robustness Checks

3.4.1 Alternative Method to compute Implied and Realised Skewness

I carry out a series of checks to confirm the robustness of the main conclusions from my empirical analysis. I first examine whether my results are sensitive to the methodology I use to extract information for option markets. In particular, instead of the trapezoidal approximation of the integrals in Eq. (3.3) and (3.24) discussed in section 2, I adopt the interpolation/extrapolation method discussed in p. 1818 of DeMiguel et al. (2013) to extract option prices from implied volatilities and, subsequently, compute implied and realised skewness. Tables B.1 to table B.6 in the Appendix repeat the in-sample and out-of-sample analyses using this alternative framework to estimate all realised/implied skewness values. The results are consistent with the main findings.

3.4.2 Alternative Specification of the GARCH-2 Model

In my main analysis, when deriving the GARCH-2 model, I specify the dynamics of the shape parameters in a GARCH-type structure similar to Bali et al. (2008). Feunou et al. (2016) finds that introducing an asymmetric specification in the shape parameters dynamics improves the in-sample fit of the GARCH-2 model to the S&P 500 index. To investigate this result in an international setting, I change the dynamics of the shape parameters in Eq. (3.12) and (3.13) to allow for asymmetric responses in the return innovations as in Feunou et al. (2016):

$$\widetilde{\lambda}_t = \lambda_0 + \lambda_1^- I_{(z_{t-1} \leq 0)} z_{t-1} + \lambda_1^+ I_{(z_{t-1} > 0)} z_{t-1} + \lambda_2 \widetilde{\lambda}_{t-1} \tag{3.39}$$

$$\widetilde{\kappa}_t = \kappa_0 + \kappa_1^- I_{(z_{t-1} \leq 0)} z_{t-1} + \kappa_1^+ I_{(z_{t-1} > 0)} z_{t-1} + \kappa_2 \widetilde{\kappa}_{t-1} \tag{3.40}$$

Table B.7 to table B.12 in the appendix report the results of my analysis using the alternative specification of the GARCH-2 model described above. I find that the predictive ability of the GARCH-2 model under the new specification is similar in-sample to the original specification. However, thee out-of-sample performance of the more sophisticated model is worse as it leads to higher losses. This result is due to the increased impact of the estimation errors on out-of-sample forecasting performance, driven by an increase in the number of unknown parameters in the specification of the GARCH-2 model proposed by Feunou et al. (2016).

3.4.3 Alternative Specification of the QMIDAS Model

For the estimation of equation Eq. (3.17) in the specification of the QMIDAS modle, I have chosen a lag length D of 250 days in my main analysis. I study the sensitivity of the forecasting performance of QMIDAS-based skewness to D by considering two alternative lags, namely $D = 200$ and $D = 300$ days. In Tables B.13 to table B.24 I report empirical results for $D = 200$ and in Tables B.19-B.24 for $D = 300$. Overall, my results indicate that my main findings for the forecasting performance of the QMIDAS skewness estimator are robust to these alternative values for D.

3.4.4 Alternative Estimation Windows for the GARCH and QMIDAS Models

My out-of-sample results rely on a rolling window of 1250 observations for the estimation of two GARCH and the QMIDAS model. In this robustness test, I also consider windows of 1000 and 1500 observations. I present the corresponding results in Table B.25 to table B.28 in the Appendix. I find that these alternatives do not alter the conclusions I draw in the main part of the paper.

3.4.5 Alternative Asset Universe for the Portfolio Analysis

In my final robustness test, I examine the out-of-sample performance of the skewness-based portfolios presented in section 5 for an alternative asset universe. In particular, I assume that the assets available to the investor further include all U.S. indices, excluding only HANGSENG and KOSPI from the original list of indices. The relevant results are presented in Table B.29 in the appendix and are similar to the corresponding results in section 3.3.3.

3.5 Suggestions for future research

In my analysis, I mainly focus on indices from developed international markets and on forecasting horizons of up to three months, as option data are more reliable in these settings and longer forecasting horizons are less important for financial decision-making than the ones I assume. An interesting extension of my work is then to consider more indices from developing markets or to perform my tests at the stock level. As option markets are less liquid in developing markets or for individual stocks, the performance of option-based models could decrease in such settings. However, this decrease could be mitigated by using stock characteristics (see, e.g., Aretz and Arısoy, 2019; Boyer et al., 2010) or additional country-level economic variables (similar to Ghysels et al., 2016) as additional predictors of future skewness. Such predictors can also help with supporting the performance of skewness estimators for horizons of more than three months. I leave such extensions for future research.

Another possible extension is to combine skewness forecasts from alternative methods, given the findings in encompassing regression. Elliott and Timmermann (2016) argue that combinations of different forecasting model typically produce improved forecasts. Future research can also explore the predictability of return kurtosis. As will be shown in the next chapter, incorporating information from kurtosis forecasts significantly improves portfolio performance. The main obstacle is the lack of proxy for the true physical kurtosis. One possible solution is the new estimators recently introduced by Neuberger and Payne (2019) to estimate realised skewness and kurtosis employing *only* historical daily data.

3.6 Conclusion

I carry out a comprehensive comparison of the forecasting performance and economic significance of several skewness models from the literature. I also develop a new skewness estimator that corrects option-implied skewness for the skewness risk premium. In my analysis I consider ten international indices, three forecasting horizons, two in-sample regression tests and two out-of-sample comparisons under two loss functions. I also compare the competing models in an international diversification framework to infer the skewness model that leads to the best out-of-sample portfolio performance under four measures. I support my empirical analysis with a battery of robustness checks.

My empirical results support the use of option-implied information when forecasting skewness. Two forward-looking option-implied skewness models and a realised skewness model that also employs information from options outperform the rest of the models that only rely on historical returns, namely, two GARCH models and a skewness estimator computed from conditional quantiles (QMIDAS). The corrected option-implied skewness estimator I propose has the highest information content on future skewness while it consistently leads to the lowest forecast errors in my out-of-sample tests. A portfolio strategy based on this new estimator also outperforms strategies based on the rest of the skewness models and the $1/N$ portfolio, in terms of out-of-sample mean returns, variance and Sharpe ratio.

Chapter 4

International Portfolio Allocation: The Role of Conditional Higher Moments

4.1 Introduction

The mean-variance theory of Markowitz (1952) has laid the foundations for modern portfolio literature. The core assumption of the theory is that asset returns are normally distributed, thus the utility of an investor is only a function of expected return and variance. However, the strong evidence on the nonnormality of financial asset returns casts doubts on the efficiency of the classical mean-variance investment strategy (see, e.g., Cont, 2001, for a survey on the stylised facts of financial returns and reference therein). A direct implication of nonnormal return distribution is the potential impact of higher moments on portfolio allocation. Early theoretical models show that a risk-averse investor has positive preferences for skewness and negative preferences for kurtosis (see Kimball, 1993; Scott and Horvath, 1980). Recent empirical studies also document significant time-variation in the higher moments of asset returns, that are marginally predictable (see, e.g., Brooks et al., 2005; Conrad et al., 2013; Hansen, 1994; Neumann and Skiadopoulos, 2013). As a result, a natural question that arises is whether an investor can exploit higher moments in the distribution of returns to improve her portfolio performance.

I address this question by incorporating conditional skewness and kurtosis in an asset allocation strategy across 42 international equity indices. The moments of the conditional return distribution are jointly estimated from a set of conditional quantiles to reduce their sensitivities to outliers. The weights of each country in the optimal portfolio is then specified as a linear parameterisation of return distributional characteristics. Moreover, I apply several decompositions to isolate the components of portfolio gains driven by additional information from conditional skewness and kurtosis. An out-of-sample (OOS) allocation strategy and two alternative transaction costs scenarios are further examined to investigate the benefits of return higher moments to a real-time investor. Finally, I perform a series of robustness checks on the model specifications.

The contribution of this study is twofold. First, I incorporate *both* conditional skewness and kurtosis in the portfolio allocation, while the existing literature focuses primarily on the role of return skewness. For example, Chunhachinda et al. (1997) Patton (2004) and Harvey et al. (2010) show that incorporating skewness leads to sizeable changes in the optimal asset allocation and results in economically significant gains for the investor. More recently, DeMiguel et al. (2013) incorporate implied skewness from option prices and document similar results. There is no clear reason a priori, however, to exclude return kurtosis from the portfolio allocation. Dittmar (2002) argues that apart from the utility-based preference, kurtosis provides meaningful information about the return distribution that is distinguished from the variance and skewness. More specifically, the fourth moment captures the probability of extreme outcomes to both tails of the distribution, which is related to the tail risk of asset returns. Moreover, Jondeau and Rockinger (2006) argue that in the case of large departures from normality, a portfolio optimisation strategy up to return kurtosis is necessary to provide a good approximation of the expected utility. Thus, my analysis provides a broader picture on the effects of nonnormality of asset returns on investment decisions.

Second, I consider the *total* skewness and kurtosis, while the limited number of studies on higher moment portfolio have focused on the *comoments* between the asset returns and the market portfolio. For example, Jondeau and Rockinger (2012) and Gao and Nardari (2018) develop a distribution timing strategy based on forecasts of comoments up to the fourth order. Using this strategy, they observe significant economic gains in the distributional portfolio over the classical mean-variation portfolio. Introducing new estimators of coskewness and cokurtosis, Martellini and Ziemann (2010) document similar improvements in economic value and stability of portfolios of large dimensions. Recent studies shed light on the roles of *own* skewness and kurtosis of asset returns in explaining the cross-sectional of stock returns (see, e.g., Boyer et al., 2010; Conrad et al., 2013; Stilger et al., 2017). These results are drawn from the theoretical studies showing that the investor is willing to trade diversification for stocks with positive skewness (see, e.g., Barberis et al., 2008; Hong and Stein, 2003; Mitton and Vorkink, 2007) and those with lower exposures to the tail-related events (Dittmar, 2002). Bali et al. (2019), in particular, recently show that only the idiosyncratic components of higher moments are related to expected returns. Indeed, I also include the coskewness and kurtosis in the conditioning information and find that the systematic components of higher moments do not have significant impacts on the optimal weights of an international portfolio.

The most directly related study to mine is the recent work of Ghysels, Plazzi and Valkanov (2016, hereafter GPV). My study differs from theirs in two important ways. First, GPV only investigate the role of conditional skewness for international diversification, while I extend their work to further incorporate conditional kurtosis. Second, although I also estimate conditional higher moments from quantile estimates, I rely on an alternative approximation method. GPV draw on the third-order Cornish-Fisher expansion to scale their quantile-based skewness to the central third moment. Such truncation implicitly assumes time-invariant and zero excess-kurtosis in the return distribution. However, this assumption can be restrictive given the recent empirical evidence of Gu and Ibragimov (2018) that international indices exhibit heavy tailed distributions. Instead, I adapt the more recent method proposed by Aretz and Arısoy (2019) to approximate return moments using the law of total probability. This approach allows for simultaneously estimating all moments of the conditional return distribution, yet it retains the robust feature of quantile-based higher moments. While the latter is practically important in the case of potentially noisy data from emerging markets, the former reduces the estimation errors from making separate forecasts for each distributional moment.

My empirical analysis reveals significant time-variation and heterogeneity in conditional skewness and kurtosis at the monthly horizon in all equity indices. Emerging stock markets (EMs) have less negative skewness than those of developed stock markets (DMs). The value-weighted average conditional skewness of EMs is equal to -0.154, almost three times lower in magnitude than those of DMs, which equals to -0.446. A similar pattern is obtained in the case of conditional kurtosis, although the discrepancy is less pronounced. Interestingly, after controlling for the dynamics of skewness, the orthogonalised kurtosis is notably higher in EMs than in DMs, averaging at 1.171 for EMs and 0.426 for DMs. Thus, although EMs have less negative skewness, their conditional distributions are more exposed to extreme returns. Motivated by this observation, I incorporate both conditional skewness and orthogonalised kurtosis into the international dynamic portfolio using the parametric portfolio policy of Brandt et al. (2009). By doing so, I sidestep from the need for a high-dimensional multivariate distribution modelling, while I able to investigate the marginal impacts of conditional higher moments on the optimal portfolio.

The whole-sample portfolio results suggest that the investor should allocate more weights to the countries with positive (or less negative) conditional skewness and less weights to the countries with higher kurtosis. This portfolio policy remains robust when the investor recursively estimates her optimal portfolio weights based on the OOS forecasts of return moments or when different scenarios of transaction costs are imposed to the portfolio optimisation. In economic terms, the portfolio based on both conditional skewness and kurtosis increases the annual return by 3.4% and the certainty-equivalent rate by 3.3% compared to the mean-variance portfolio. Overall, this strategy is consistent with the general preference towards return moments suggested by the theoretical works of Scott and Horvath (1980), Kimball (1993) and Dittmar (2002). More importantly, I find that a large fraction of the economic gains is generated from the join dynamics between conditional skewness and kurtosis. When the conditional kurtosis in introduced to the information set, the investor earns an additional 1.8% annualised return, which leads to a certainty-equivalent gain of 2.1% compared to the skewness portfolio. This improvement is in line with results of Jondeau and Rockinger (2006) and Jondeau and Rockinger (2012), who highlight the importance of incorporating information from *both* skewness and kurtosis in the portfolio allocation.

My study is also related to the international diversification literature. Since the international indices are characterised by significant non-normality and heavy tails, the prospects of international diversification might go beyond the first two moments of the return distribution (Ang and Bekaert, 2002; Bekaert et al., 1998). Empirical evidence on the benefits of higher moments nevertheless remains inconclusive. For example, Christoffersen et al. (2012) argue that EMs provide significant diversification benefits due to their low tail dependence with DMs. Moreover, Pukthuanthong and Roll (2014) document weakly correlated jumps in the equity markets, which promote the international diversification associated with cross-sectional heterogeneity in the return higher moments. In contrast, Guidolin and Timmermann (2008) explore that international diversification benefits reduce substantially after accounting for time-variations in higher-order comoments of the country stock index and global market portfolio. Several recent studies further document significant risk transmission and long-run cointegration in the extreme tails of the return distribution (Chen et al., 2018; Shen, 2018). Since tail risk can be directly related to higher moments via Cornish-Fisher approximation, this evidence limits the benefits of higher moments in an international portfolio.

The current study sheds further light on this debate, using the Taylor expansion truncated at the fourth order to generalise the expected utility function of an international investor (see, e.g., Dittmar, 2002; Gao and Nardari, 2018; Guidolin and Timmermann, 2008, for similar generalisation). This generalisation allows me to investigate the international diversification benefits in the context of heterogeneity in distributional characteristics of stock returns in each country. Similar to Ghysels et al. (2016), I find that a large proportion (38.3% of the total weight) of the skewness-specific portfolio is invested in EMs, thanks to their favourable conditional asymmetry. However, since EMs also have higher orthogonalised kurtosis, the exposure of EMs reduces significantly to only 22.8% in the skewness-kurtosis-specific portfolio. Notably, the EM-skewness effect disappears in the most recent period, leading to a substantial reduction of EMs in the optimal weights. More specifically, the investor increases the holding of DMs by about 30% by shorting EMs by the end of 2017. This reaction can partly be explained by the recent rise in protectionist policies induced by the 2007-2009 global financial crisis (GFC). Globalisation has slowed down or even been reversed, with negatively impacts on capital inflows and the prospects of emerging markets (Bekaert et al., 2016).

The reminder of this chapter is structured as follows. Section 4.2 presents the methodology. Section 4.3 describes the data and discusses the estimates of conditional higher moments as well as the main portfolio results. Section 4.4 reports robustness checks, whereas Section 4.5 identifies directions for future research. Finally, Section 4.5 concludes the chapter.

4.2 Methodology

4.2.1 Expected Utility and Higher Moments of Asset Returns

Let r_t be the daily geometric return. Then the h-period geometric return is defined as $r_{t,h} = \Pi_{i=0}^{h-1}(1 + r_{t+i}) - 1$. Suppose that at time t, there are N_t countries in the investable universe based on their stock indices. The investor's portfolio allocation problem is to choose the portfolio weight to invest in country i, $w_{i,t-1}$, based on the information set available at time $t-1$ in order to maximise the expected utility function $\mathbb{E}_{t-1}[U(r_{t,h}^p)]$, where $r_{t,h}^p$ is the h-period portfolio return:

$$\begin{cases} \max\limits_{(w_{i,t-1})_{i=1}^{N_t}} E_{t-1}\left[U\left(r_{t,h}^p\right)\right] = \max\limits_{(w_{i,t-1})_{i=1}^{N_t}} E_{t-1}\left[U\left(\sum_{i=1}^{N_t}(w_{i,t-1}r_{i,t})\right)\right] \\ s.t. \sum_{i=1}^{N_t} w_{i,t-1} = 1 \end{cases} \tag{4.1}$$

The expected utility function can be approximated by a fourth-order Taylor expansion series around the expected mean return, $\mu_{t,h}$:[1]

$$\begin{aligned} \mathbb{E}[U(r_{t,h})] \approx U(\mu_{t,h}) + \frac{1}{2!}U^2(\mu_{t,h})\mathbb{E}[(r_{t,h} - \mu_{t,h})^2] + \frac{1}{3!}U^{(3)}(\mu_{t,h})\mathbb{E}[(r_{t,h} - \mu_{t,h})^3] \\ + \frac{1}{4!}U^{(4)}(\mu_{t,h})\mathbb{E}[(r_{t,h} - \mu_{t,h})^4] + O(r_{t,h}^4) \end{aligned} \tag{4.2}$$

[1] To simplify the notation, I drop the $t-1$ subscript while keeping in mind that the portfolio allocation problem is conditioning on the information set available at time $t-1$

where $U^{(i)}(\mu_{t,h})$ is the i^{th} derivative of the utility function and $O(r_{t,h}^4)$ is the Taylor remainder. Eq. (4.2) implies that the investor's expected utility depends on the first four moments of the portfolio return distribution. Although the Taylor series expansion can be extended to an infinite order, truncation at the fourth level directly relates the investor's expected utility to her preference towards higher-order moments. Under the general assumption of positive marginal utility, Scott and Horvath (1980) suggest that a risk-averse investor has a positive preference for the odd moments (mean, skewness), i.e, $U^{(1)} > 0$ and $U^{(3)} > 0$, and a negative preference for even moments (variance, kurtosis), i.e, $U^{(2)} < 0$ and $U^{(4)} < 0$. Therefore, adding information embedded in the higher moments generally provides a better approximation for the expected utility function (Ederington, 1995; Guidolin and Timmermann, 2008). Jondeau and Rockinger (2006) also argue that a portfolio optimisation strategy up to return kurtosis is necessary in the case of large departures from normality.

Empirically, the maximisation of expected utility in Eq. (4.2) requires measures of the conditional moments at the h-day horizon. Early empirical research relies on the information from the option market to estimate return higher moments under risk-neutral density.[2] The appealing feature of this approach is that option-implied moments reflect forward-looking information and require one day of option price data. Indeed, the third chapter in this thesis shows that forecasting models with information from the option market provide the best skewness forecasts among several alternative models. For this specific application, however, options data is not available for the majority of countries in the sample. I therefore rely on a set of conditional quantiles of return distribution to estimate its moments. This approach is motivated by the fact that higher moments of asset returns are

[2]Methodology to extract risk-neutral return moments from option prices is proposed in Britten-Jones and Neuberger (2000) and Bakshi et al. (2003). See, e.g., Dennis and Mayhew (2002), Bali and Murray (2013), Stilger et al. (2017) for the use of option-implied higher moments for asset pricing models and Rehman and Vilkov (2012) and DeMiguel et al. (2013) for the use of option-implied skewness for portfolio allocation.

very sensitive to outliers (Kim and White, 2004). The issue is arguably more pronounced in emerging markets due to more frequent jumps, resulting partly from the non-smooth integration of globalisation (Bekaert et al., 1998, 2016). Thus, using conditional quantile can greatly reduce the impacts of outliers in the estimates of return higher moments.[3]

4.2.2 Higher Moments Estimates

Let $F(r_{t,h}) = P(r_{t,h} < r)$ be the unconditional cumulative distribution function of $r_{t,h}$ and its conditional counterpart be $F_{t-1}(r_{t,h})$. Both functions are strictly increasing. The conditional quantile at probability level, α, is defined as $Q_\alpha(r_{t,h}) = F_{t-1}^{-1}(\alpha, r_{t,h})$. The robust estimators of skewness and kurtosis by quantiles have been proposed by Pearson (1895), Crow and Siddiqui (1967) and Groeneveld and Glen Meeden (1984). In my application, I rely on the new method in Aretz and Arısoy (2019) to approximate return higher moments from the estimated quantiles of the return distribution using the law of total probability.

The approximation is conducted on a set of J conditional quantiles sorted in an increasing order $\alpha \in \{\alpha_1 < \alpha_2 < .. < \alpha_{J-1} < \alpha_J\}$. In the main analysis, I employ nine quantile levels, that are $\alpha \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99\}$. Assume that the return distribution between two consecutive conditional quantiles follows a uniform distribution, the expected value of the first four (raw) moments of returns in each interval are:

$$\begin{aligned}
\mathbb{E}_{t-1}\left(r_{t,h}|q_{\alpha_{j-1}} < r_{t,h} < q_{\alpha_j}\right) &= \frac{q_{\alpha_{j-1}} + q_{\alpha_j}}{2} \\
\mathbb{E}_{t-1}\left(r^2_{t,h}|q_{\alpha_{j-1}} < r_{t,h} < q_{\alpha_j}\right) &= \frac{q^2_{\alpha_{j-1}} + q_{\alpha_{j-1}} \times q_{\alpha_j} + q^2_{\alpha_j}}{3} \\
\mathbb{E}_{t-1}\left(r^3_{t,h}|q_{\alpha_{j-1}} < r_{t,h} < q_{\alpha_j}\right) &= \frac{q^3_{\alpha_{j-1}} + q^2_{\alpha_{j-1}} \times q_{\alpha_j} + q_{\alpha_{j-1}} \times q^2_{\alpha_j} + q^3_{\alpha_j}}{4} \\
\mathbb{E}_{t-1}\left(r^4_{t,h}|q_{\alpha_{j-1}} < r_{t,h} < q_{\alpha_j}\right) &= \frac{q^4_{\alpha_{j-1}} + q^3_{\alpha_{j-1}} \times q_{\alpha_j} + q^2_{\alpha_{j-1}} \times q^2_{\alpha_j} + q_{\alpha_{j-1}} \times q^3_{\alpha_j} + q^4_{\alpha_j}}{5}
\end{aligned}$$

[3]A comprehensive discussion on the robust estimations of return skewness and kurtosis based on quantiles can be found in Kim and White (2004).

Ignoring the probability mass below α_1 and above α_J, the conditional (central) moments of return distribution can be approximated as follows:[4]

$$\mathbb{E}_{t-1}\left(r_{t,h}^m\right) = \sum_{j=2}^{J} \frac{\alpha_j - \alpha_{j-1}}{\alpha_J - \alpha_1} \mathbb{E}_{t-1}\left(r_{t,h}^m | q_{\alpha_{j-1}} < r_{t,h} < q_{\alpha_j}\right) \tag{4.3}$$

where $m = 1, 2, 3, 4$. Finally, the conditional volatility, skewness and kurtosis can be estimated using the conventional moment-based skewness and kurtosis formulas:

$$Vol_{t-1}(r_{t,h}) = \left[\mathbb{E}_{t-1}\left(r_{t,h}^2\right) - \mathbb{E}_{t-1}\left(r_{t,h}\right)^2\right]^{1/2} \tag{4.4}$$

$$Sk_{t-1}(r_{t,h}) = \frac{\mathbb{E}_{t-1}\left(r_{t,h}^3\right) - 3\mathbb{E}_{t-1}\left(r_{t,h}\right)\left(\mathbb{E}_{t-1}\left(r_{t,h}^2\right) - \mathbb{E}_{t-1}\left(r_{t,h}\right)^2\right) - \mathbb{E}_{t-1}\left(r_{t,h}\right)^3}{\left[\mathbb{E}_{t-1}\left(r_{t,h}^2\right) - \mathbb{E}_{t-1}\left(r_{t,h}\right)^2\right]^{3/2}} \tag{4.5}$$

$$\begin{aligned} &Ku_{t-1}(r_{t,h}) \\ &= \frac{\mathbb{E}_{t-1}\left(r_{t,h}^4\right) - 4\mathbb{E}_{t-1}\left(r_{t,h}^3\right)\mathbb{E}_{t-1}\left(r_{t,h}\right) + 6\mathbb{E}_{t-1}\left(r_{t,h2}^2\right)\mathbb{E}_{t-1}\left(r_{t,h}\right)^2 - 3\mathbb{E}_{t-1}\left(r_{t,h}\right)^4}{\left[\mathbb{E}_{t-1}\left(r_{t,h}^2\right) - \mathbb{E}_{t-1}\left(r_{t,h}\right)^2\right][^2} \end{aligned} \tag{4.6}$$

A crucial requirement for the above approximation is accurate estimates of return quantiles. I follow GPV to estimate the conditional quantiles at the h-day horizon using the MIDAS quantile regression. As documented in Chapter 2, MIDAS-based forecasts outperform alternative models in modelling return quantiles at the multi-day horizon by accounting for serial dependence in the daily return process. The conditional MIDAS-based quantile can be expressed as:

$$Q_\alpha(r_{t,h}) = \beta_{\alpha,h}^0 + \beta_{\alpha,h}^1 Z_{t-1}(\kappa_{\alpha,h}) \tag{4.7}$$

$$Z_{t-1}(\kappa_{\alpha,h}) = \sum_{d=0}^{D} \varphi_d(\kappa_{\alpha,h})|r_{t-1-d}| \tag{4.8}$$

[4] The main findings are similar when I include more extreme quantiles in the robustness check in Section 4.4.4.

In the current application, I focus the portfolio analysis on monthly holding-period returns, i.e., $h = 22$. Whereas rebalancing at shorter horizons (i.e., daily or weekly) is costly in EMs, using a longer holding-period might increase the possibility of estimation error in my skewness and kurtosis proxies. Neuberger and Payne (2019) show that as the horizon lengthens, the return skewness (kurtosis) is mainly driven by the covariance between the innovations in variance and the lagged returns (lagged squared returns). Such components can only be captured by a long series of historical returns or information from option prices.[5] The use of monthly return may therefore attempt to strike a balance between the estimation errors in the higher moment proxies and the impacts of trading costs on portfolio allocation.

The latent conditioning variable $Z_{t-1}(\kappa_{\alpha,h})$, known at time $t-1$, is a linear projection of higher-frequency information. I follow GPV to choose daily lagged absolute returns $|r_{t-1-d}|$ of $D = 250$ days as the higher frequency information. The projection weight is parsimoniously determined by the low-dimensional polynomial function $\varphi(.)$ of Eq. (4.8). Similar to Chapter 2, I choose the "Beta" polynomial, which is by far the most popular function used in the MIDAS literature. Again, I constrain the first polynomial parameter to unity and estimate the second polynomial parameter. This specification ensures monotonically declining weights to the high-frequency variables as it moves further into the past.

4.3 Empirical Analysis

4.3.1 Data and Descriptive Statistics

In my analysis, I adopt daily total return data for 42 countries, including 24 DM indices collected from the FTSE and 18 EM indices collected from the S&P /IFCI database. For each index, I use the U.S. dollar-denominated values to avoid any

[5]Neuberger and Payne (2019) show that one needs to use at least 50 years of data to produce reliable skewness and kurtosis estimates at monthly returns using historical return data

potential effects of exchange rates. All indices are investable at low trading costs and closely track the returns of a foreign investor on each country in the stock market. The data spans the period from January 1, 1996 to December 31, 2017.[6]

Table 4.1 summarises statistics of the unconditional distributions of all indices. To facilitate the comparison, I divide the sample into two groups, DMs and EMs, respectively. For each group, I sort the indices by market capitalisation as of the end of December 2017. In the first two columns, I report the annualised mean and volatility. The next two columns report the sample skewness ($Sk_{t,22}$) and its robust quantile-based measure ($Sk^Q_{t,22}$) using Eq. (4.5). Similarly, the last two columns show the sample kurtosis ($Ku_{t,22}$) and its robust quantile-based version ($Ku_{t,22}$) using Eq. (4.6). The value-weighed (VW) average of all statistics across countries within each group is displayed in the last row of each panel.

The mean returns of DMs are lower than those of EMs (9.2% and 11.5%, respectively), whereas the average volatility of the latter is significantly higher than that of the former (18.1% and 32.6%, respectively). The VW skewness ($Sk_{t,22}$) for DM is -0.618 with negative estimates in all but four DM. In contrast, the VW skewness for EM is almost zero, with 6 out of 18 countries having positive asymmetry. Interestingly, although return distributions in DM are more negatively skewed, their VW kurtosis ($Ku_{t,22}$) is lower than that of EM (4.601 vs. 5.186, respectively). The robust estimates, $Sk^Q_{t,22}$ and $Ku^Q_{t,22}$, clearly demonstrate the impact of outliers on higher moment estimates. A notable example can be found in the case of South Korea. The sample skewness (kurtosis) of South Korea decreases by approximately 70% from 0.987 (9.002) to 0.312 (3.952) in the robust version. Similar observations can be found in the EM group, for example, Philippines has $Sk_{t,22}$ switching the sign from a positive estimate of 0.189 to a negative value of -0.302 in $Sk^Q_{t,22}$. The sample distributions of EM are still nevertheless characterised by less negative skewness and higher kurtosis.

[6] The only exceptions being Portugal and Russia, which start from May 4, 1998 and February 4, 1997, respectively.

Table 4.1 Statistics of Unconditional Return Distributions

This table shows statistics of unconditional return distributions at the monthly horizon for all indices. The table displays the annualised mean, annualised standard deviation, skewness and kurtosis. The robust quantile-based versions are approximated from return quantiles as in Eq. (4.5) and (4.6). Developed and emerging markets are reported separately in Panels A and B, respectively. The last row in each panel presents the value-weighted average of each statistic using the market capitalisation of December 31, 2017.

	Mean	*Std*	$Sk_{t,22}$	$Sk^Q_{t,22}$	$Ku_{t,22}$	$Ku^Q_{t,22}$
Panel A: Developed Markets						
United States	0.099	0.157	-0.834	-0.520	4.515	3.110
Japan	0.045	0.184	0.051	0.047	3.302	2.765
United Kingdom	0.065	0.170	-0.552	-0.522	5.005	3.604
France	0.090	0.211	-0.559	-0.364	4.060	3.132
Germany	0.095	0.236	-0.487	-0.481	4.471	3.411
Canada	0.102	0.210	-0.570	-0.350	5.681	3.300
Switzerland	0.088	0.172	-0.693	-0.643	4.520	3.686
Australia	0.108	0.214	-0.612	-0.454	4.457	3.321
South Korea	0.190	0.387	0.987	0.312	9.002	3.952
Hong Kong	0.090	0.256	0.018	0.100	5.908	3.659
The Netherlands	0.079	0.222	-0.856	-0.684	5.512	3.848
Spain	0.096	0.249	-0.304	-0.267	4.171	3.229
Sweden	0.116	0.257	-0.220	-0.128	4.541	3.490
Italy	0.068	0.250	-0.286	-0.240	3.778	3.029
Denmark	0.136	0.204	-0.838	-0.509	5.808	3.881
Singapore	0.073	0.287	0.142	0.083	7.124	5.081
Belgium	0.095	0.214	-1.235	-0.609	8.350	3.950
Finland	0.117	0.318	-0.009	0.000	4.483	3.846
Norway	0.105	0.273	-0.798	-0.771	5.755	4.537
Israel	0.109	0.215	-0.524	-0.401	3.935	3.382
Austria	0.092	0.269	-0.998	-0.468	6.989	3.558
Ireland	0.047	0.239	-1.119	-0.857	5.682	4.061
New Zealand	0.077	0.218	-0.462	-0.441	3.939	3.281
Portugal	0.027	0.229	-0.478	-0.283	4.103	2.816
VW.Average	*0.092*	*0.181*	*-0.618*	*-0.418*	*4.601*	*3.214*
Panel B: Emerging Markets						
China	0.108	0.331	0.312	0.301	5.421	4.281
Taiwan	0.061	0.279	-0.038	-0.008	4.022	3.202
India	0.145	0.301	-0.105	-0.139	4.844	2.432
Brazil	0.167	0.382	-0.115	-0.053	3.851	2.999
South Africa	0.133	0.278	-0.568	-0.412	4.265	3.040
Russia	0.176	0.459	-0.216	-0.179	7.062	4.483
Mexico	0.130	0.267	-0.769	-0.155	5.678	2.908
Thailand	0.103	0.364	0.005	-0.124	5.607	4.536
Malaysia	0.050	0.296	0.521	0.365	9.758	6.683
Indonesia	0.121	0.445	0.070	-0.112	6.094	4.514
Philippines	0.060	0.287	0.189	-0.302	6.601	3.936
Poland	0.097	0.333	-0.490	-0.086	4.773	2.998
Chile	0.097	0.235	-0.399	-0.173	4.888	3.396
Turkey	0.175	0.491	0.695	0.173	6.542	3.589
Peru	0.171	0.276	-0.372	0.122	5.913	3.079
Pakistan	0.151	0.375	-0.221	0.119	7.963	4.724
Hungary	0.144	0.347	-0.578	-0.335	5.067	3.489
Czech Republic	0.125	0.284	-0.383	-0.120	4.260	2.878
VW.Average	*0.115*	*0.326*	*0.009*	*0.031*	*5.186*	*3.678*

Table 4.2 MIDAS Conditional Quantile Estimates

This table reports the MIDAS-based estimates for the 1^{st}, 5^{th}, 25^{th}, 50^{th}, 75^{th}, 95^{th} and 99^{th} conditional quantiles of the monthly returns of the U.S., China and the average across developed and emerging markets. The numbers in parentheses display p-values using bootstrapped standard errors. For κ_α, the null hypothesis is $\kappa_\alpha = 1$. The rows *Coverage* present the empirical coverage, as a percentage. The numbers in parentheses report the p-values of the Kupiec (1995) test under the null hypothesis that the empirical coverage is not statistically different from the considered quantile level.

α	0.01	0.05	0.025	0.5	0.75	0.95	0.99
				United States			
β^0	-0.031	-0.011	0.004	0.011	0.018	0.020	0.017
	(0.00)	(0.00)	(0.08)	(0.00)	(0.00)	(0.00)	(0.00)
β^1	-11.025	-7.415	-2.814	0.367	2.592	6.282	9.581
	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)
κ_α	60.849	7.619	4.997	171.437	43.488	33.182	20.597
	(0.00)	(0.00)	(0.06)	(0.89)	(0.00)	(0.00)	(0.00)
Coverage	1.008	4.984	25.009	50.092	75.101	95.108	98.974
	(0.95)	(0.96)	(0.99)	(0.89)	(0.86)	(0.71)	(0.85)
				China			
β^0	-0.021	0.011	0.031	0.039	0.038	0.056	0.122
	(0.04)	(0.04)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β^1	-13.591	-11.445	-6.011	-2.380	2.288	7.972	10.849
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
κ_α	8.602	4.255	1.662	1.242	28.839	17.081	4.235
	(0.00)	(0.00)	(0.01)	(0.04)	(0.00)	(0.00)	(0.06)
Coverage	1.008	4.965	24.991	50.073	74.973	95.071	98.992
	(0.95)	(0.91)	(0.99)	(0.91)	(0.96)	(0.81)	(0.95)
				Developed Markets			
β^0	-0.063	-0.034	0.001	0.010	0.019	0.036	0.047
	(0.04)	(0.05)	(0.26)	(0.16)	(0.00)	(0.02)	(0.03)
β^1	-9.773	-6.449	-2.961	0.236	2.997	6.622	9.365
	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)	(0.00)
κ_α	34.644	25.518	24.190	26.573	11.219	8.101	11.009
	(0.05)	(0.03)	(0.03)	(0.37)	(0.06)	(0.01)	(0.03)
Coverage	1.000	4.947	25.006	50.039	75.060	95.071	99.014
	(0.93)	(0.86)	(0.97)	(0.92)	(0.90)	(0.81)	(0.86)
				Emerging Markets			
β^0	-0.059	-0.042	-0.014	0.006	0.015	0.021	0.032
	(0.05)	(0.10)	(0.04)	(0.16)	(0.14)	(0.06)	(0.14)
β^1	-12.696	-7.680	-2.408	0.587	4.221	10.328	14.577
	(0.00)	(0.00)	(0.02)	(0.05)	(0.00)	(0.00)	(0.00)
κ_α	16.905	13.219	14.245	19.845	7.122	4.055	3.354
	(0.03)	(0.07)	(0.15)	(0.28)	(0.08)	(0.01)	(0.06)
Coverage	1.002	4.966	25.014	50.082	75.109	95.108	99.022
	(0.93)	(0.91)	(0.97)	(0.90)	(0.83)	(0.74)	(0.86)

4.3.2 Conditional Higher Moments

4.3.2.1 Conditional Quantiles

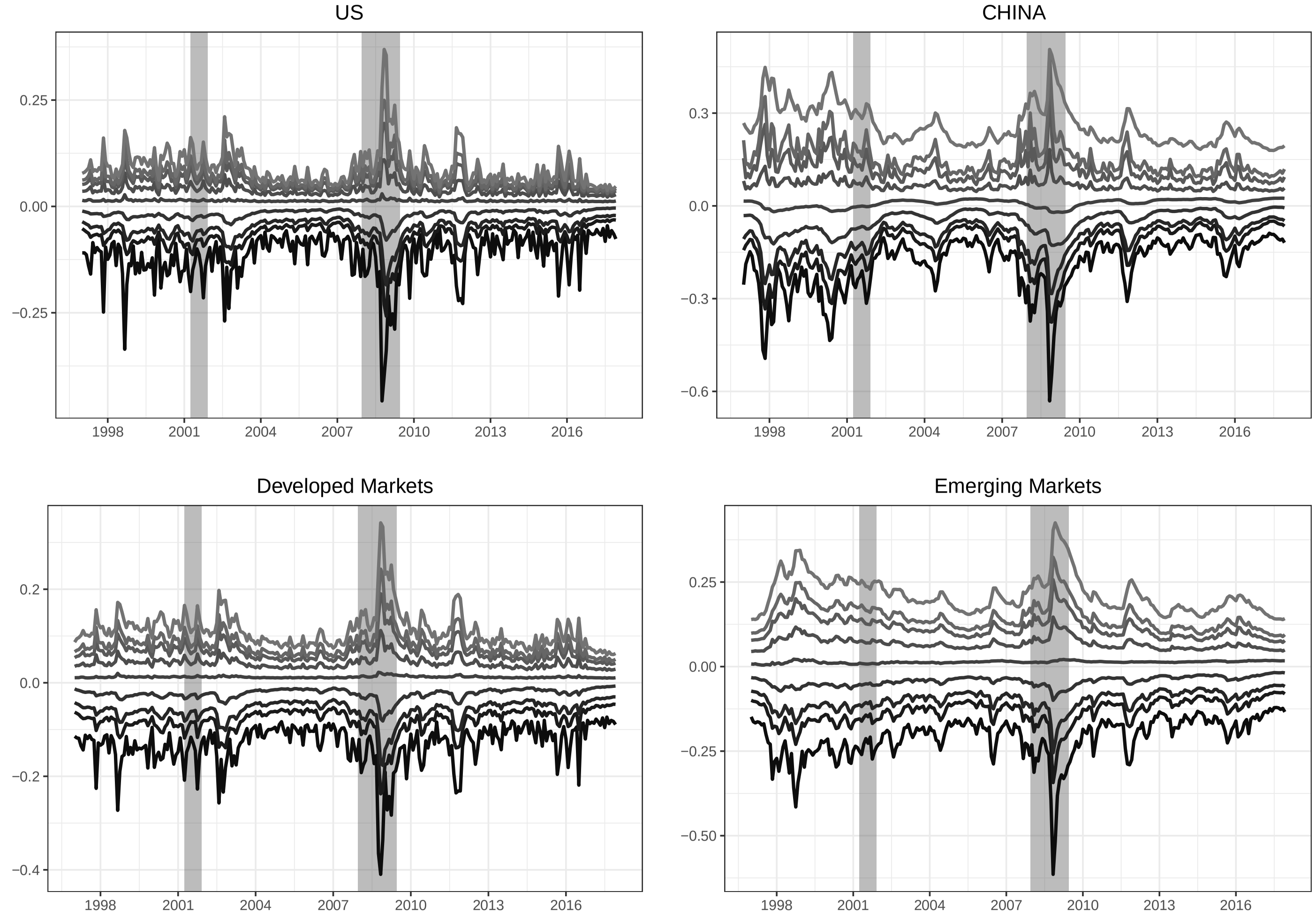
I start with an evaluation on the accuracy of probability coverage in quantile estimates. Table 4.2 reports the results from the MIDAS quantile regressions described in Eq. (4.7) - (4.8) for the average across DMs and EMs, as well as the U.S. and China, which are the two largest economies in each group. Due to the limit of space, I only report results for seven quantile levels, at $\alpha = \{0.01, 0.05, 0.25, 0.5, 0.75, 0.95, 0.99\}$. In the "*Coverage*" row, I present the proportion of realised returns that exceeded each estimated quantile. If a conditional quantile is correctly measured, I should expect this empirical coverage to be close to the value of the quantile level. The numbers in parentheses report the p-values of the Kupiec (1995) test under the null hypothesis that the empirical coverage is not statistically different from the considered quantile level.

The slope coefficients, β^1, are statistically significant in all quantile levels across the markets, implying strong time-variation in conditional quantiles. Similar to GPV, the absolute values of β^1 for the left-tail quantiles, i.e,. $\alpha \in (0.01, 0.05, 0.25)$, are relatively different from those of the right-tail quantiles, i.e., $\alpha \in (0.75, 0.95, 0.99)$, implying notable asymmetry in the conditional return distribution. The empirical coverage observed in the *Coverage* rows are very close to the corresponding quantile levels across markets. More importantly, the results of Kupiec (1995) test show that none of the conditional quantiles in all markets are rejected at the 5% significance level.

Figure 4.1 displays the conditional quantiles of monthly returns from the results in Table 4.2. The top plots present conditional quantiles of the U.S. and China, while the bottom plots present the value-weighted average of DMs and

Fig. 4.1 Conditional Quantiles of Monthly Returns in Selected Markets

This figure present the conditional quantiles from MIDAS quantile regression for U.S., China and the average across developed and emerging markets. The shaded areas correspond to recession periods defined by the National Bureau of Economic Research.



EMs.[7] In each plot, the shaded areas correspond to the recession periods defined by the National Bureau of Economic Research.

The 2007-2009 GFC is notably highlighted by spiking volatility and a widening range of conditional quantiles across markets. For China and EMs, the inter-range between quantiles also increased in the early part of the sample, which corresponds to the 1997-1998 Asian financial crisis and the increased capital flows due to financial liberalisation in these countries in the early 2000s. The magnitude of conditional quantiles in the left tails is typically higher than that in the right tails, particularly for the U.S. and DMs. This pattern signals the existence of asymmetry in the conditional return distribution. Since all plots have similar scales, China and EMs are featured with higher possibilities of extreme returns in both tails of the conditional distributions than those of the U.S. and DMs. This is in line with Ibragimov et al. (2013) and Gu and Ibragimov (2018), who document heavier tails in the emerging countries, with significantly lower tail indexes than those of developed markets.

Overall, the MIDAS-based quantiles provide good coverage of the conditional return density and are highly informative about changes in market conditions. In the next section, I plug these conditional quantiles into Eq. (4.5) - (4.6) to construct conditional skewness and kurtosis.

4.3.2.2 Approximate to Conditional Higher Moments

Table 4.3 presents the summary statistics of conditional higher moments for each country in my sample. The first four columns report the mean, standard deviation, minimum and maximum of conditional skewness while the respective statistics for conditional kurtosis are reported in the next four columns. The final column

[7] Since I perform separate quantile regressions for each quantile level, the possibility of a "*quantile crossing*" issue cannot be guarded against. In the current application, this issue is recorded at 0.98% of the full-sample estimation for any pairs of estimated quantiles in the same stock index. Whenever I observe the quantile crossing, I apply the recently developed method of monotonic rearrangement of Chernozhukov et al. (2010) to address the problem.

Table 4.3 Summary Statistics of Conditional Skewness and Kurtosis Estimates

This table summarises the conditional skewness and kurtosis for each country in the sample. The conditional skewness and kurtosis are constructed from the MIDAS-based conditional quantiles using Eq. (4.5) and (4.6). For each country, this table displays the mean, standard deviation, minimum and maximum values. The *Correlation* presents the correlation between the time series of conditional higher moments. ^{a}, ^{b} and ^{c} denote statistical significance at the 1%, 5% and 10% levels for the correlation coefficient using the stationary bootstrap with 10,000 replications.

	Conditional Skewness				Conditional Kurtosis				
Country	Mean	Std	Min	Max	Mean	Std	Min	Max	Correlation
Panel A: Developed Markets									
United States	-0.561	0.126	-1.008	-0.170	3.030	0.465	2.244	5.518	-0.618^a
Japan	-0.125	0.213	-0.842	0.479	2.895	0.197	2.654	3.939	-0.884^a
United Kingdom	-0.436	0.185	-1.224	0.317	2.985	0.317	2.400	5.069	-0.943^a
France	-0.402	0.094	-0.627	-0.122	3.062	0.551	1.844	4.565	-0.962^a
Germany	-0.466	0.135	-0.806	-0.087	3.032	0.487	1.933	4.389	-0.971^a
Canada	-0.418	0.117	-0.694	0.120	2.949	0.197	2.487	3.490	-0.944^a
Switzerland	-0.322	0.047	-0.432	-0.192	2.950	0.180	2.382	3.461	0.415^a
Australia	-0.466	0.202	-0.946	0.404	3.125	0.336	2.221	4.378	-0.828^a
South Korea	-0.150	0.362	-0.862	0.868	2.899	0.206	2.360	3.675	-0.103
Hong Kong	-0.193	0.140	-0.865	0.171	2.880	0.463	2.288	5.893	-0.564^a
The Netherlands	-0.519	0.186	-0.999	-0.056	3.329	0.285	2.663	4.236	-0.753^a
Spain	-0.133	0.229	-0.850	0.289	2.743	0.213	2.305	3.662	-0.859^a
Sweden	-0.285	0.093	-0.612	-0.057	2.866	0.347	1.971	4.011	-0.705^a
Italy	-0.136	0.081	-0.449	0.059	2.897	0.375	2.230	3.917	-0.815^a
Denmark	-0.365	0.222	-1.316	0.117	3.072	0.392	2.557	5.728	-0.773^a
Singapore	-0.352	0.351	-1.209	0.480	3.208	0.335	2.798	4.580	-0.832^a
Belgium	-0.447	0.215	-1.079	0.060	3.180	0.332	2.433	4.231	-0.920^a
Finland	0.008	0.096	-0.289	0.181	2.781	0.304	2.188	3.615	-0.489^a
Norway	-0.383	0.118	-0.685	0.135	2.937	0.172	2.371	3.318	-0.643^a
Israel	-0.336	0.028	-0.396	-0.232	3.155	0.259	2.604	3.794	-0.452^a
Austria	-0.344	0.228	-0.983	0.350	2.852	0.250	2.416	3.933	-0.896^a
Ireland	-0.451	0.102	-0.642	-0.141	3.308	0.277	2.423	3.924	-0.891^a

Table 4.3 *(continued)*

Country	Mean	Std	Min	Max	Mean	Std	Min	Max	Correlation
New Zealand	-0.236	0.168	-0.547	0.314	2.818	0.116	2.463	3.094	-0.783^a
Portugal	-0.281	0.092	-0.480	-0.033	2.811	0.132	2.552	3.639	-0.510^a
VW Average	*-0.446*	*0.107*	*-0.772*	*-0.115*	*3.002*	*0.314*	*2.456*	*4.394*	*-0.785*
Panel B: Emerging Markets									
China	0.196	0.096	-0.051	0.475	3.148	0.322	2.517	4.036	0.312^a
Taiwan	-0.098	0.226	-0.591	0.447	2.619	0.071	2.504	2.909	0.581^a
India	-0.220	0.067	-0.412	-0.059	2.654	0.031	2.592	2.741	0.850^a
Brazil	-0.269	0.139	-1.091	-0.050	2.894	0.373	2.350	5.404	-0.861^a
South Africa	-0.321	0.175	-1.146	0.036	2.727	0.321	2.237	5.163	-0.959^a
Russia	-0.162	0.255	-0.707	0.768	2.989	0.180	2.524	3.706	0.465^a
Mexico	-0.436	0.227	-1.392	0.243	3.150	0.387	2.666	6.016	-0.568^a
Thailand	-0.031	0.461	-2.150	0.655	3.192	0.708	2.831	8.415	-0.791^a
Malaysia	-0.222	0.425	-1.343	0.725	3.141	0.537	2.284	5.253	-0.879^a
Indonesia	-0.270	0.424	-1.626	0.693	3.055	0.520	2.431	5.588	-0.907^a
Philippines	-0.182	0.264	-0.850	0.420	3.015	0.212	2.669	3.738	-0.800^a
Poland	-0.194	0.096	-0.502	0.006	2.615	0.256	2.119	3.741	-0.734^a
Chile	-0.261	0.151	-1.061	-0.026	2.930	0.460	2.352	5.994	-0.985^a
Turkey	-0.028	0.337	-0.859	0.676	2.866	0.144	2.642	3.631	0.077
Peru	-0.103	0.210	-0.926	0.345	2.763	0.443	2.238	4.799	-0.845^a
Pakistan	0.023	0.221	-0.635	0.469	3.475	0.277	2.457	4.699	-0.300^b
Hungary	-0.157	0.182	-0.814	0.440	2.805	0.136	2.610	3.561	-0.803^a
Czech Republic	-0.069	0.300	-1.515	0.498	2.669	0.276	2.260	5.065	-0.825^a
VW Average	*-0.154*	*0.101*	*-0.483*	*0.074*	*2.914*	*0.138*	*2.712*	*3.572*	*-0.634*

presents the pairwise correlations between conditional skewness and kurtosis in each country.

Almost all markets exhibit negative conditional third moments with a few exceptions such as Finland, China and Pakistan. In line with GPV, I observe distinct features in the conditional asymmetry between DMs and EMs, where the former has more negative skewness on average than the latter (-0.446 vs. -0.154). For instance, the U.S. has the most negative asymmetry, at -0.561, and its conditional skewness never turns positive throughout the sample. In contrast, China has the most positive asymmetry, averaging at 0.196.

The results for conditional kurtosis offer two noteworthy points. First, the robust estimator largely reduces the impact of outliers on conditional kurtosis. The discrepancy in conditional kurtosis across countries is less pronounced than that of the third conditional moment. Their time series, however, still fluctuate considerably in most of the markets, which can be inferred from the standard deviation, minimum and maximum values. Second, the majority of countries have negative correlations between skewness and kurtosis as expected, averaging at -0.785 and -0.634 for DMs and EMs, respectively. However, the correlation statistics vary remarkably between countries, ranging from -0.985 in Chile to +0.850 in India. Several markets have significant positive correlations, mostly in the emerging stock markets. This observation indicates significant heterogeneity between the joint dynamics of higher moments between the international markets under consideration.

To further investigate the dynamics in conditional higher moments between the markets, I calculate the cross-sectional correlation across all pairs of conditional skewness and kurtosis. The average correlations between 861 pairs are relatively low and equal to 0.172 for conditional skewness and 0.110 for conditional kurtosis. A large proportion of skewness correlations, at 245 (28.5%) pairs, are negative. The corresponding number for kurtosis is even higher, with 346 (40.2%) pairs, and most of the correlations are statistically significant. The large fraction of negative correlations suggests that conditional higher moments do not exhibit a significant common pattern in their cross-country dynamics, especially in the case of conditional kurtosis.

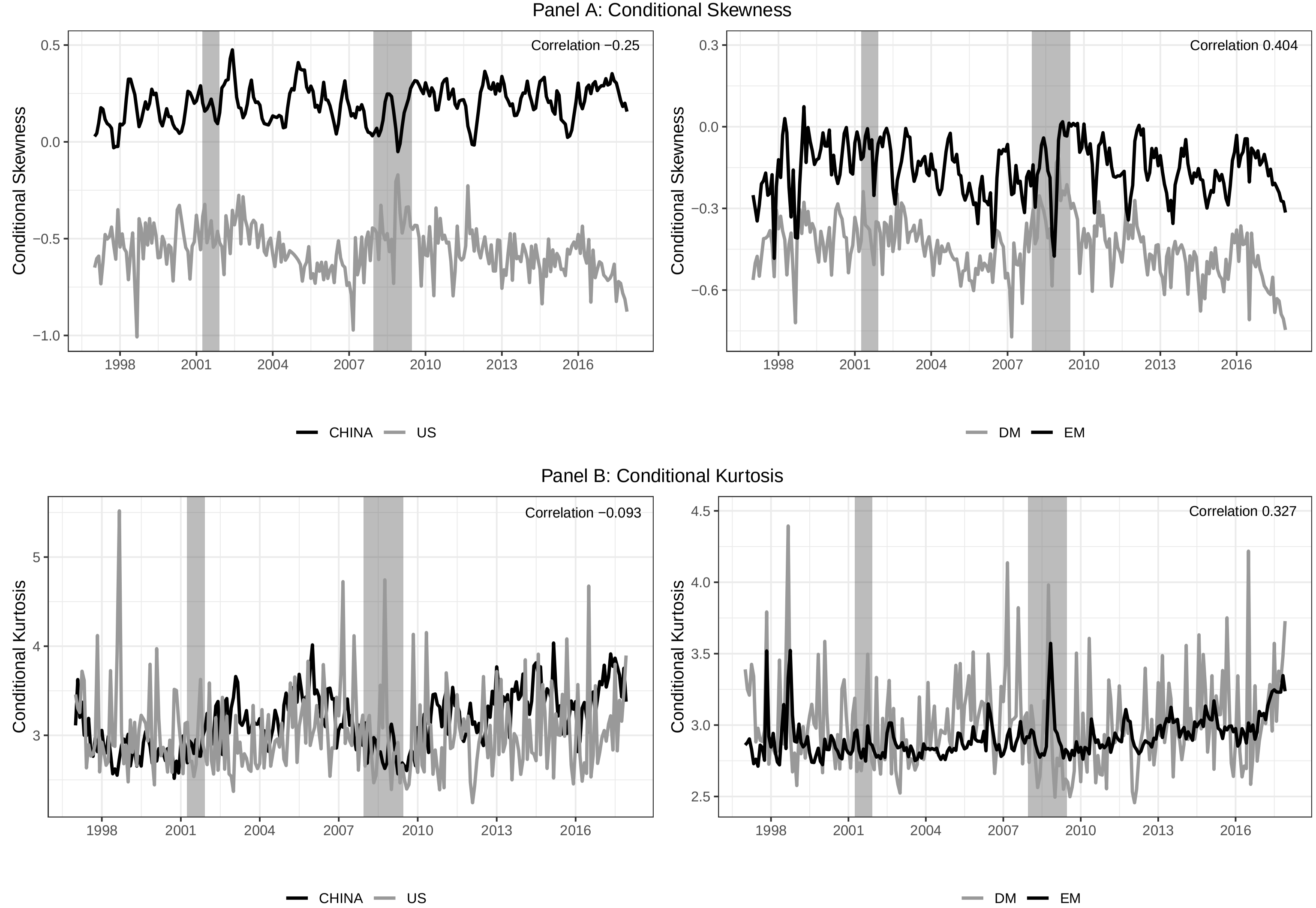
Figure 4.2 displays the time series of monthly conditional higher moments for the U.S., China and the average across DMs and EMs. Panel A shows conditional skewness while Panel B presents conditional kurtosis. On the upper right corner of each plot, I report the sample correlation between the two time series.

The conditional skewness of the U.S. is always negative over the entire sample. The most negative value occurred in September 1998 when the Russian financial crisis hit global markets. In contrast, the conditional asymmetry of China is almost always positive and has negative correlation (-0.25) with the U.S. series. In China, the most negative estimates are recorded during the 1997-1998 Asian financial crisis and the 2007-2009 GFC. The correlation turn to positive in the average series of DMs and EMs, equals to 0.404. Nevertheless, the average conditional skewness of EMs is consistently above that of DMs, hinting at higher downside risks in the latter.

The conditional kurtosis of the U.S. is weakly correlated with that of China. Moreover, the time series of conditional kurtosis of U.S. is more volatile and exhibits spikes in the periods with most negative skewness. In contrast, the conditional kurtosis of China is smoother and does not display sharp increases during the 2007-2009 GFC. In fact, the conditional kurtosis of China is much higher in 2015 after the bursting of the stock market bubble and in the most recent period due to the slowdown of the Chinese economy. The correlation between the average kurtosis of DMs and EMs is slightly lower than that of skewness, equals to 0.327 in-sample. Finally, these plots show that the quantile-based conditional higher moments are highly informative about the changes in return distribution in different market conditions. Some of the global crashes can be visually illustrated

Fig. 4.2 Conditional Skewness and Kurtosis of Monthly Returns

This figure presents the time series of monthly conditional higher moments for the U.S., China and the average across DMs and EMs.The higher moment estimates are constructed from MIDAS-based conditional quantiles presented in 4.1.



in the right-panel plots. For example, the Asian financial crisis in 1997, the Russian crisis in 1998 and the 2007-2009 GFC result in significant drops (spikes) in the conditional skewness (kurtosis) in all the markets.

4.3.2.3 Decomposing Conditional Kurtosis

I further apply several decompositions to examine the features of conditional kurtosis. First, I decompose the conditional kurtosis into its systematic and idiosyncratic components. To this end, I collect the MSCI World portfolio total returns from Datastream and estimates its conditional kurtosis as described above. Next, I run a standard CAPM-type regression:

$$Ku_{t-1}(r_{i,t}) = a^{sys} + b^{sys} Ku_{t-1}(r_{w,t}) + \epsilon_{i,t}^{sys} \tag{4.9}$$

where the dependent variable is the country-specific conditional kurtosis and the right-hand side variable is the conditional kurtosis of the world portfolio.

The result of this decomposition is presented in Panel A of Table 4.4, where I report the average slope coefficients, their cross-sectional standard deviation, the average R^2 and their standard deviation. The average of the slope estimates across all markets is relatively low, equal to 0.194. The corresponding average for DMs and EMs is 0.266 and 0.100, respectively. The global conditional kurtosis can only explain on average 17% of the time-variation in an individual country's conditional kurtosis. Again, the average fit in DMs regressions is about 0.265, higher than that of only 0.044 in EMs. This finding is similar to the decomposition of conditional skewness documented in GPV and indicates that a large part of the conditional kurtosis is country-specific. Moreover, the idiosyncratic kurtosis component is relatively more pronounced in emerging stock markets.

In the second decomposition, I separate the pure kurtosis from the component driven by conditional skewness in each country. To do so, I follow Chang et al.

Table 4.4 Kurtosis Decomposition

This table presents two decompositions of the conditional kurtosis. In Panel A, the conditional kurtosis is decomposed into its systematic and idiosyncratic components as in equation Eq. (4.9). In Panel B, the conditional kurtosis is decomposed into its skewness-related variation and pure kurtosis variation.

	A: Systematic and Idiosyncratic			B: Orthogonalised Kurtosis		
	All	DMs	EMs	All	DMs	EMs
Slope Coeffient	0.194	0.266	0.100	-5.932	-6.990	-4.522
Std	0.227	0.254	0.145	4.454	3.103	5.580
Adj R-Squared	0.170	0.265	0.044	0.640	0.766	0.471
Std	0.200	0.218	0.047	0.343	0.288	0.345

(2013) to orthogonalise conditional kurtosis by regressing its dynamics on the contemporaneous skewness estimates as follows:

$$Ku_{t-1}(r_{i,t}) = a^{sk} + b^{sk} Sk_{t-1}(r_{i,t}) + \epsilon_{i,t}^{sk} \tag{4.10}$$

I report the summary statistics of this decomposition in Panel B of Table 4.4. The average of the slope coefficients, as expected, is largely negative in all markets. The average R^2 is high, approximately 76.6% and 47.1% for DMs and EMs, respectively. However, the standard deviations of slope coefficients and R^2 are also large for both DMs and EMs. This observation indicates that a nontrival part of the fluctuations in conditional kurtosis is driven by distinct dynamics beyond those related to conditional skewness, especially for emerging countries. Therefore, I use the residuals from this decomposition to characterise the pure conditional kurtosis of each country in the portfolio analysis that follows. This approach allows my analysis to accurately examine the marginal benefits of conditional kurtosis beyond that offered by skewness.

Table 4.5 provides the summary statistics of pure kurtosis for all countries in the sample. As previously, I report the mean, standard deviation, minimum and maximum statistics. Several observations are worth highlighting. First, controlling for the impacts of skewness greatly reduces conditional kurtosis, particularly in DMs. An indicative example is the case of the U.S., where the conditional kurtosis is reduced significantly from 3.030 to only 0.083 in the orthogonalised kurtosis.

Note that the U.S. has the most negative skewness, indicating that a large part of the higher moment risk in this country comes from the return asymmetry. The orthogonalisation of kurtosis in EMs displays a less pronounced pattern. Taking Malaysia as an example, I observe that the orthogonalised kurtosis slightly reduces to 2.273 from 3.141 in the conditional kurtosis. Since their conditional skewness is also higher than average, at -0.222, this observation means that this country is exposed to both return asymmetry and kurtosis risks. Finally, the average of orthogonalised kurtosis of EMs is significantly higher than that in DMs (1.171 vs 0.426). This finding indicates that the former exhibits higher exposure to the extreme returns than the latter.

Second, the orthogonalised kurtosis exhibits significant time-variation and heterogeneity across countries. The mean statistics are significantly different from zero in almost all markets and have a large standard deviation. The average of cross-sectional correlations is 0.128 and about 32.2% (277) of the 861 kurtosis pairs between indices is negative. These observations suggest that incorporating information from the orthogonalised kurtosis may provide potential benefits for the international portfolio beyond that provided by conditional skewness.

Finally, I plot the time series of average orthogonalised kurtosis across DMs and EMs in Figure 4.3. The two series are positively correlated at 0.416 as seen in the top-right corner. The 2007-2009 GFC is marked with significant comovement in orthogonalised kurtosis between DMs and EMs, indicating increases in the tail dependence across countries. After accounting for the impact of conditional skewness, the pure kurtosis for EMs is relatively more volatile and stays at a higher level than compared to that of DMs for the large part of the sample period. Therefore, I conclude from this section that EMs have less negatively skewed returns on average, but their conditional distributions are characterised by heavier tails than those of DMs.

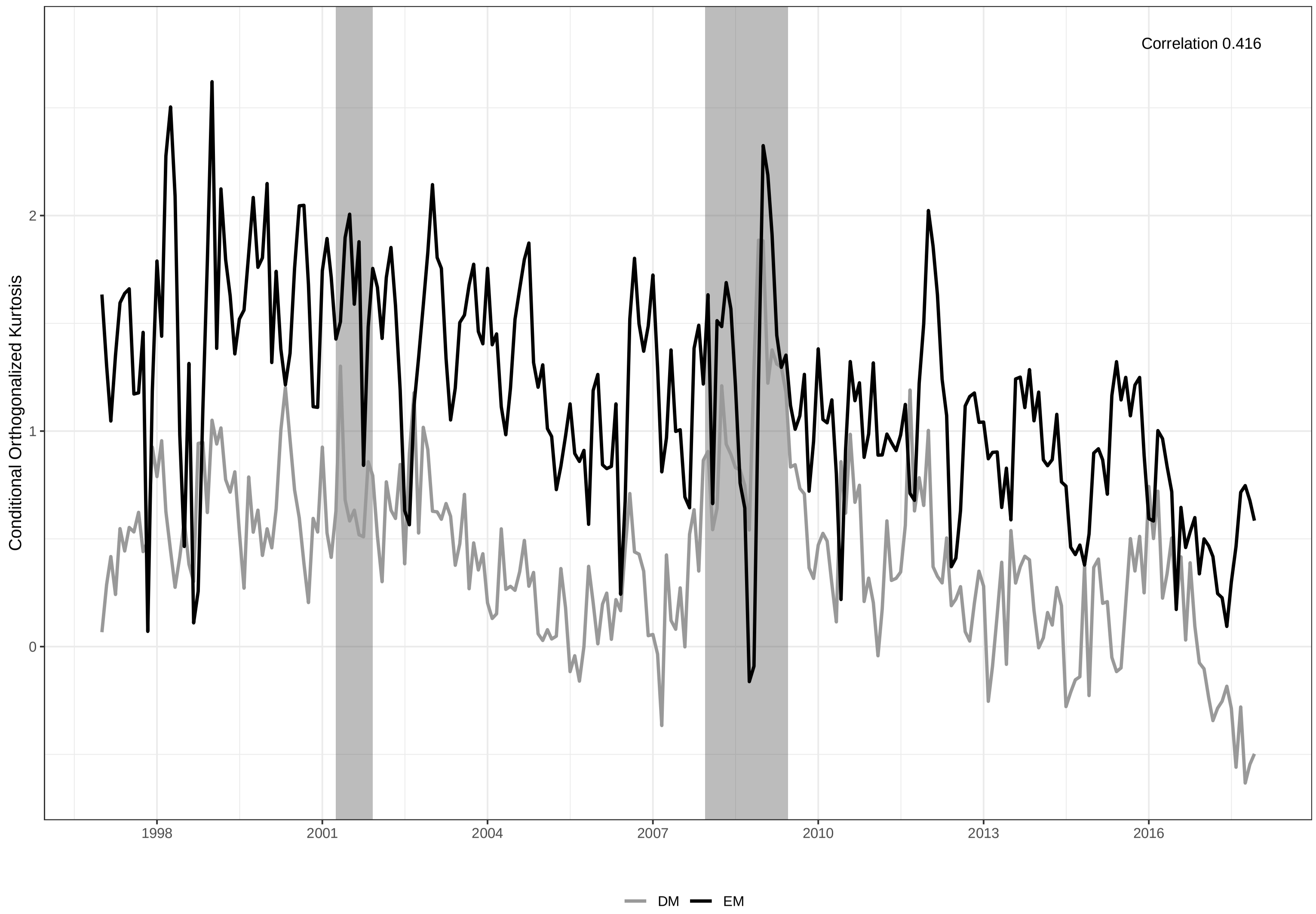
Table 4.5 Summary Statistics of Orthogonalised Kurtosis

This table presents the summary statistics for the pure kurtosis of all the countries in the sample. The orthogonalised kurtosis is the residuals from the orthogonalisation in Eq. (4.10). For each country, this table displays the mean, standard deviation, minimum and maximum values. ^{a}, ^{b} and ^{c} denote statistical significance at 1%, 5% and 10% levels for the statistics using the stationary bootstrap with 10,000 replications.

Country	Mean	Std	Min	Max
Panel A: Developed Markets				
United States	0.083^b	0.522	-0.963	2.071
Japan	2.075^a	1.226	-1.576	6.042
United Kingdom	0.348^a	0.829	-2.339	4.505
France	0.041^a	0.231	-0.419	0.928
Germany	0.108^a	0.391	-0.783	1.387
Canada	0.166^a	0.597	-1.193	3.338
Switzerland	0.072^b	0.520	-1.041	1.568
Australia	0.390^a	0.924	-1.523	5.142
South Korea	2.465^a	1.046	0.885	5.937
Hong Kong	0.869^a	1.260	-3.111	4.515
The Netherlands	0.309^a	0.886	-1.941	2.637
Spain	1.969^a	1.154	-1.272	4.246
Sweden	0.201^a	0.669	-1.716	1.827
Italy	0.621^a	1.071	-3.733	3.288
Denmark	0.691^a	1.170	-2.862	4.063
Singapore	1.458^a	1.479	-1.436	5.673
Belgium	0.477^a	1.004	-2.297	2.951
Finland	2.774^a	0.350	2.089	3.738
Norway	0.223^a	0.737	-1.546	3.449
Israel	0.012	0.271	-0.786	0.499
Austria	0.767^a	1.165	-2.032	4.576
Ireland	0.107^a	0.492	-0.820	1.705
New Zealand	0.902^a	1.275	-1.414	5.344
Portugal	0.251^a	0.779	-1.441	2.480
VW Average	*0.426*	*0.399*	*-0.633*	*1.886*
Panel B: Emerging Markets				
China	0.567^a	1.202	-2.888	3.696
Taiwan	2.221^a	0.965	0.277	4.731
India	0.234^a	0.768	-1.927	2.058
Brazil	0.480^a	0.949	-4.390	2.277
South Africa	0.495^a	0.915	-2.803	2.487
Russia	2.160^a	1.392	-0.728	7.624
Mexico	0.582^a	1.164	-2.333	4.979
Thailand	3.140^a	0.482	2.421	4.822
Malaysia	2.273^a	1.218	-0.025	5.852
Indonesia	1.959^a	1.271	-1.025	5.834
Philippines	1.964^a	1.364	-1.175	5.321
Poland	0.439^a	0.907	-2.455	2.311
Chile	0.536^a	0.934	-3.749	2.110
Turkey	2.847^a	0.279	2.391	3.834
Peru	2.079^a	1.049	-1.441	5.044
Pakistan	3.445^a	0.452	2.418	5.514
Hungary	1.553^a	1.346	-2.939	6.476
Czech Republic	2.482^a	0.598	0.991	4.160
VW Average	*1.171*	*0.503*	*-0.162*	*2.621*

Fig. 4.3 Conditional Orthogonalised Kurtosis of Monthly Returns

This figure presents the time series of orthogonalised kurtosis for the average of DMs and EMs. The orthogonalised kurtosis is the residuals from the regression in Eq. (4.10).



4.3.3 International Portfolio Allocation

4.3.3.1 Parametric Portfolio Policy

The return higher moments can enter the optimisation strategy in Eq. (4.1) and Eq. (4.2) in terms of individual moments or comoments between indices return. A traditional approach is to model the joint distribution of all assets in the portfolio. However, there are two main issues with this approach. First, since the joint distribution is mostly unknown, the investor needs to characterise the joint distribution. One method is to use a multivariate GARCH, such as the dynamic conditional correlation model of Engle (2002) to model the covariance of a pair of asset returns and derive closed-form solutions for the moments of portfolio returns. Examples of this method include Jondeau and Rockinger (2012), Boudt et al. (2015) and Gao and Nardari (2018). Alternatively, a dynamic copula model can be used to measure the interdependence between asset returns. The resulting joint distribution can be fully parametric or semi-parametric depending on the specification of the marginal distributions. Applications of this method can be found in Patton (2004) and Cerrato et al. (2017). In either case, the parametric specifications on conditional return distribution or the copula dynamics raise the risks of model misspecification errors.

Another issue is the so-called "*curse of dimensionality*". Joint distribution modelling involves the computation of a covariance, coskewness and cokurtosis matrix for all assets. The dimensionality of these matrices increases substantially with the number of assets in the portfolio. For example, for a portfolio of $n = 42$ assets as in current application, the number of elements that I have to compute is $n(n+1)/2 = 903$ for the covariance matrix , $n(n+1)(n+2)/6 = 13,244$ for the coskewness matrix and $n(n+1)(n+2)(n+3)/24 = 148,995$ for the cokurtosis matrix. Therefore, it is practically impossible to directly compute the joint distribution for such a large number of assets in the portfolio and to trace the marginal contribution of each return distribution characteristic.[8]

In this study, I avoid the need for modelling the joint distribution of asset returns using the parametric portfolio policy of Brandt et al. (2009). The general idea is to directly optimise the portfolio weights with regard to the investor's expected utility using individual asset characteristics. The optimal weight of country i in Eq. (4.1), $w_{i,t-1}$, can be specified as follows:

$$w_{i,t-1} = \overline{w}_{i,t-1} + \lambda_X^\top \frac{1}{N_t} \widetilde{X}_{t-1,i} + \lambda_{Hm}^\top \frac{1}{N_t} \widetilde{Hm}_{t-1}(r_{i,t,h}) \tag{4.11}$$

where $\overline{w}_{i,t-1}$ is the weight of index i in a value-weighted portfolio. $\widetilde{Hm}_{t-1}(r_{i,t,h})$ is the information on conditional higher moments, i.e., conditional skewness and kurtosis. $\widetilde{X}_{t-1,i}$ is a vector of country-specific characteristics, which generally capture information related to the conditional mean and variance of equity index returns. Each characteristic is standardised cross-sectionally to have zero mean and unit standard deviation. By doing so, the portfolio policy coefficients in $(\lambda_X^\top; \lambda_{Hm}^\top)$ indicate deviations from the value-weighted portfolio driven by the cross-sectional differences between countries. A positive optimal loading $\lambda_j \in (\lambda_X^\top; \lambda_{Hm}^\top)$ of asset characteristic j means that the market with a higher value of this characteristic than the cross-sectional average is preferable to the investor, and thereby leads to higher weight in the optimal portfolio.

The formulation of Eq. (4.11) indicates that the portfolio weights can be decomposed into three components. The first component, $\overline{w}_{i,t-1}$, is the weights in a value-weighted portfolio, in which the investor simply allocates her wealth across countries according to their market capitalisation. The second component, $\lambda_X^\top \frac{1}{N_t} \widetilde{X}_{t-1,i}$, measures the deviation from the naive weighting strategy using

[8] Several remedies for the curse of dimensionality are proposed in the literature. Some notable studies include the shrinkage the covariance matrix (see, e.g., Kourtis et al., 2012; Ledoit and Wolf, 2004a,b), constrained portfolio weights (Jagannathan and Ma, 2003) or imposing zero correlation between asset returns (Elton et al., 2006). However, these methods have been mainly applied to the covariance matrix, while incorporating higher moments to the optimisation strategy would require substantial computational resources.

conditioning information related to the conditional mean and volatility of the return distribution. The third component, $\lambda_{Hm}^{\top}\frac{1}{N_t}\widetilde{Hm}_{t-1}$, explicitly captures the marginal benefits of accounting for conditional higher moments. The sum of the first two component serves as the benchmark in this study.

In addition to the conditional moments of return distribution, I also exploit several standard financial and macroeconomic predictors commonly used to forecast the expected mean and volatility of stock returns. For the financial indicators, I employ the dividend yield of the corresponding index, $DivY_{i,t-1}$, and a time-series momentum, $Mom_{i,t-1}$, computed as the 12-month past returns. The use of the dividend yield as a predictor of stock returns has long history in the asset pricing literature.[9] Meanwhile, momentum is one of the most robust anomalies in the asset pricing literature, first documented by Jegadeesh and Titman (1993).[10] For macroeconomic variables, I use the growth rate of industrial production, $IPG_{i,t-1}$, and the inflation rate based on the consumer price index, $INF_{i,t-1}$. These two macroeconomic variables have been widely used in the literature to predict expected stock returns and volatility. [11]

The policy parameters are estimated by maximising the sample expected utility of Eq. (4.1). I follow GPV to specify the objective utility function for a power investor:

$$\max_{(\lambda_X,\lambda_{Hm})} \frac{1}{T}\sum_{t=1}^{T-1}\frac{(1+r_{t,p})^{1-\gamma}}{1-\gamma} \tag{4.12}$$

where γ is the relative risk aversion coefficient. In the main result, I set $\gamma = 5$ and perform a robustness check in subsection 4.4.3 using alternative levels of risk aversion coefficients.

[9] Some notable studies are Campbell and Shiller (1988), Fama and French (1989), Stambaugh (1999), Valkanov (2003), Campbell and Yogo (2006), Ang et al. (2006), Welch and Goyal (2008) and Campbell and Diebold (2009).

[10] Several researchers document the existence of momentum in the international indices, such as Rouwenhorst (1998), Asness et al. (2013) and Barroso and Santa-Clara (2015).

[11] See, e.g., Flannery and Protopapadakis (2002) Engle and Rangel (2008), and Engle et al. (2013).

4.3.3.2 Portfolio Decompositions

I apply several decompositions to further investigate the marginal benefits of conditional higher moments. Motivated by GPV, I first decompose the portfolio returns by country groups as follows:

$$r_{t,P} = r_{t,P}^{DM} + r_{t,P}^{EM} \tag{4.13}$$

where

$$r_{t,P}^{DM} = \sum_{t=1}^{T-1} I_i^{DM} w_{t-1,i} r_{t,i} \quad \text{and} \quad r_{t,P}^{EM} = \sum_{t=1}^{T-1} I_i^{EM} w_{t-1,i} r_{t,i}$$

where I_i^{DM} (I_i^{EM}) is the indicator function which takes value one if country i is developed (emerging) and zero otherwise. This decomposition helps to track the component of portfolio returns that is attributable to developed and emerging stock markets. To examine the marginal benefits of conditional higher moments, I focus my attention on the second decomposition that isolates the component of portfolio returns driven by information from these moments of the return distribution:

$$r_{t,P} = r_{t,P}^{bench} + r_{t,P}^{Hm} \tag{4.14}$$

where

$$r_{t,P}^{bench} = \sum_{t=1}^{T-1} w_{t-1,i}^{bench} r_{t,i} \quad \text{and} \quad r_{t,P}^{Hm} = \sum_{t=1}^{T-1} w_{t-1,i}^{Sk} r_{t,i} + \sum_{t=1}^{T-1} w_{t-1,i}^{KU} r_{t,i}$$

where $w_{t,i}^{Sk}$ and $w_{t,i}^{Ku}$ are the skewness- and kurtosis-induced weights. Thus, the $r_{t,P}^{Hm}$ denotes the component of portfolio returns obtained by actively managing conditional skewness and kurtosis in the information set. I term $r_{t,P}^{Hm}$ as the return on the skewness-kurtosis-specific portfolio. When only conditional skewness is considered, i.e., $w_{t,1}^{KU} = 0$, I term this component as the skewness-specific portfolio. The marginal impact of conditional kurtosis beyond that of skewness can be directly referred from the comparison between these two actively managed portfolios.

4.3.3.3 Portfolio Allocation Results

Table 4.6 displays the main results for five specifications of international portfolios with different investing strategies. Panel A reports the optimal loadings on asset characteristics. Panel B displays several statistics of the portfolio returns, which are the annualised average return and volatility, the $SkewToKurt$ ratio, the adjusted-Sharpe ratio, $ASharpe$, and the annualised certainty-equivalent returns, $CE(r_P)$. The $SkewToKurt$ is proposed by Watanabe (2006) a portfolio performance evaluation to explicitly account for nonnormality in portfolio returns, where a higher value of the ratio is preferable. The $ASharpe$ ratio is defined by Pezier and White (2008), which penalise the Share ratio of portfolio returns for negative skewness and excess kurtosis as follows:

$$ASharpe = SR \times \left[1 + \left(\frac{Skew}{6}\right) \times SR - \left(\frac{Kurt - 3}{24}\right) \times SR^2\right]$$

where SR is the conventional Sharpe ratio. The certainty-equivalent return is defined as $CE(r_P) = [u(r_P)(1-\gamma)]^{1/(1-\gamma)} - 1$, which represents the risk-free rate that an investor is willing to accept rather than holding the optimised portfolio. To examine the structure of portfolio allocation, I also report the overall average weight on EMs, $\overline{w}_{EM}$, the average weight due to conditional skewness, $\overline{w}_{EM}^{Sk}$, the average weight due to conditional kurtosis, $\overline{w}_{EM}^{Ku}$, and the average weight due to *both* conditional skewness and kurtosis, $\overline{w}_{EM}^{Hm} = \overline{w}_{EM}^{Sk} + \overline{w}_{EM}^{Ku}$. Finally, I present the results for portfolio decompositions documented above in Panel C.

In the first column of Table 4.6, I present the properties of value-weighted (VW) portfolio, in which the investor does not incorporate any conditioning information. The portfolio has positive average returns of 8.3% with an annualised volatility of 16.3%. Although the market capitalisation of EMs has been trending upward, the investor only allocates 7.5% of her wealth, on average, to emerging countries. The adjusted Sharpe ratio equals to 0.463 and the $SkewToKurt$ ratio suggests negative skewness in portfolio returns.

In the next two columns, the investor starts incorporating information related to the first two moments of the conditional return distribution. In the column titled "MV", the investor has a quadratic utility function. In the "*bench*" portfolio, she has the power utility function as in Eq. (4.12) with a relative risk aversion of 5. There are a few observations to make about the above two portfolios. The conditional volatility has a negative loading in both specifications, although not statistically significant at the 10% level. This loading implies that the investor should tilt her portfolio toward countries with lower cross-sectional conditional volatility. Among the financial and macroeconomic variables, only Mom has an insignificant loading whereas $DivY$, IPG and INF are highly significant and have the expected signs. The optimised portfolios have moderate gains compared to that of *VW*. The average returns increase remarkably to 23.6% in the MV and 25.5% in the *bench* portfolio compared to only 8.3% in the VW portfolio. However, both portfolios also exhibit significant increases in volatility, resulting in only slight improvements to the certainty-equivalent returns (0.103 in MW and 0.109 in *bench* vs 0.075 in *VW*, respectively). Finally, the optimal weight structure changes significantly . The investor allocates 30.5% (24.3%) of her wealth to EMs in the *bench* (MV) portfolio compared to only 7.5% in VW portfolio.

In the next column labelled "*Sk*", the investor includes the quantile-based conditional skewness to the information set. Several studies, including Kraus and Litzenberger (1976) and Harvey and Siddique (2000), document significant risk premia for assets with higher coskewness with the market portfolio. Therefore, I follow GPV to also consider the coskewness of a country's stock index with the MSCI World portfolio. To define the coskewness, I follow Harvey and Siddique (2000) to regress the the lagged 250-day returns of each market on the contemporaneous returns and squared returns of the MSCI World portfolio. I then use the coefficient of the squared returns as the coskewness estimate.

Overall, I confirm the main findings of GPV. The conditional skewness enters the portfolio policy with a positive loading, although it is not statically significant (p-value of 12.8%). Coskewness also has a positive sign but is insignificant. Interestingly, the interaction between conditional skewness and volatility induces notable changes in the portfolio policy. The magnitude of the loading of conditional volatility increases and becomes highly significant at the 5% level once information on skewness is included. The introduction of conditional skewness also leads to reasonable economic gains as the certainty-equivalent increases by 1.2% from the *bench* portfolio (0.121 vs. 0.109). Finally, the investor allocates higher weights to EMs in the optimal portfolio. The proportion of EMs in the skewness-specific portfolio, $\overline{w}_{EM}^{Sk}$, is 38.3% and further confirms that the tilt of optimal portfolio toward EMs results from their favourable asymmetry as documented in Table 4.3.

In the final column, labelled "*SkKurt*", the investor further adds conditional higher moments to her information set. To proxy for the pure kurtosis, she uses the orthogonalised kurtosis reported in Table 4.5. Thus, this portfolio directly extends the investing strategy of GPV to the fourth conditional moment. Given the reasoning in Dittmar (2002) and Gao and Nardari (2018), I also include the cokurtosis between each country and the global portfolio in the set of predictors. In line with Bali et al. (2019), I define the cokurtosis as the slope coefficient on the cubed returns of regressing the lagged 250-day country returns on the contemporaneous returns, squared returns and cubed returns of the MSCI World portfolio.

The conditional kurtosis has a negative optimal loading and is statistically significant at the 10% level. The -1.637 loading means that if a country has $OKurt_{t-1}(r_{t,i})$ that is one standard deviation higher than the cross-sectional average, the investor should allocate 1.637% less weight than that in the *bench* portfolio. This finding is consistent negative preference towards kurtosis documented in the theoretical studies of Scott and Horvath (1980) and Dittmar (2002). The cokurtosis variable also has a negative loading, but is largely insignificant. More importantly, the joint dynamics between the return moments again have a significant impact on the optimal portfolio policy. When the fourth conditional moment enters the information set, it induces notable changes in the magnitude and significance of the loadings of conditional skewness and conditional volatility. For example, the portfolio coefficient of conditional skewness increases from 2.275 to 3.028 and is now highly significant at the 5% level.

The portfolio properties in Panel B mark a significant improvements in *SkKurt* compared to that of the *Sk* portfolio. The average return increases from 27.1% to 28.9% and the adjusted Sharpe ratio rises from 1.014 to 1.068. In particular, the increase in the certainty-equivalent is significant, at 2.1% (from 0.121 to 0.142). In the portfolio weights, the investor slightly tilts her portfolio away from EMs. In the skewness-kurtosis-specific portfolio, the investor invests only 28.2% to EMs, which is a reduction of approximately 10% relative to the skewness-specific portfolio documented above.

The portfolio decompositions in Panel C provide some additional insights on the impact of conditional information. The first decomposition in Eq. (4.13) indicates remarkable changes in the structure of portfolio returns when the investor starts utilising conditioning variables. In the VW portfolio, only 0.6% of total portfolio returns can be attributable to EMs, compared to a large 7.7% of DMs. The returns originated from EMs then increase substantially to 20.4% out of

28.9% total returns in the *SkKurt* portfolio. In the case of DMs, their contribution only increases when the investor adds information from the conditional kurtosis.

Next, I focus on the marginal information content of kurtosis beyond that of skewness using the decomposition in Eq. (4.14). The last column in Panel C indicates that the investor earns approximately 1% higher average returns by exploiting $OKurt_{t-1}(r_{t,i})$ (from 3.4% to 4.3% in r_p^{Hm}). Notably, controlling for the exposure of extreme returns reduces the volatility of the skewness-kurtosis-specific portfolio from 12.7% to 9.6%. Consequently, the corresponding increase in the certainty-equivalent is sizeable, equals to 2.9% (from -0.8% to 2.1%). Altogether, this decomposition clearly indicates the superior performance of the $SkKurt$ portfolio coming from additional information embedded in the conditional kurtosis.

Finally, I display the time series of actively-managed weight of EMs, $w_{t-1,EM}^{Hm}$ for the skewness-kurtosis-specific portfolio in Figure 4.4. More specifically, I separately present the weight due to conditional skewness ($w_{t-1,EM}^{Sk}$) in the top plot and the weight due conditional kurtosis ($w_{t-1,EM}^{Ku}$) in the middle plot, using the decomposition of $w_{t-1,EM}^{Hm} = w_{t-1,EM}^{Sk} + w_{t-1,EM}^{Ku}$. Note that since all the characteristics are standardised cross-sectionally, a positive weight on EMs is obtained by selling DMs by the same amount, i.e., $w_{t-1,EM}^{Hm} + w_{t-1,DM}^{Hm} = 0$. Conditional skewness has more pronounced effects on the skewness-kurtosis-specific portfolio compared to kurtosis due to considerably higher optimal loadings on the former (3.028 vs. -1.637). The skewness-induced weights of EMs are mainly positive, except for the 2007-2009 GFC period. In contrast, the kurtosis-induced weights on EMs are always negative throughout the sample period due to their relatively higher kurtosis documented in Table 4.5. When I combine these effects in the overall weight, there are several periods when the investor tilts her portfolio away from EMs in addition to the GFC as documented in GPV. A notable example is the most recent period in 2017 when the EM-skewness effect disappears and is

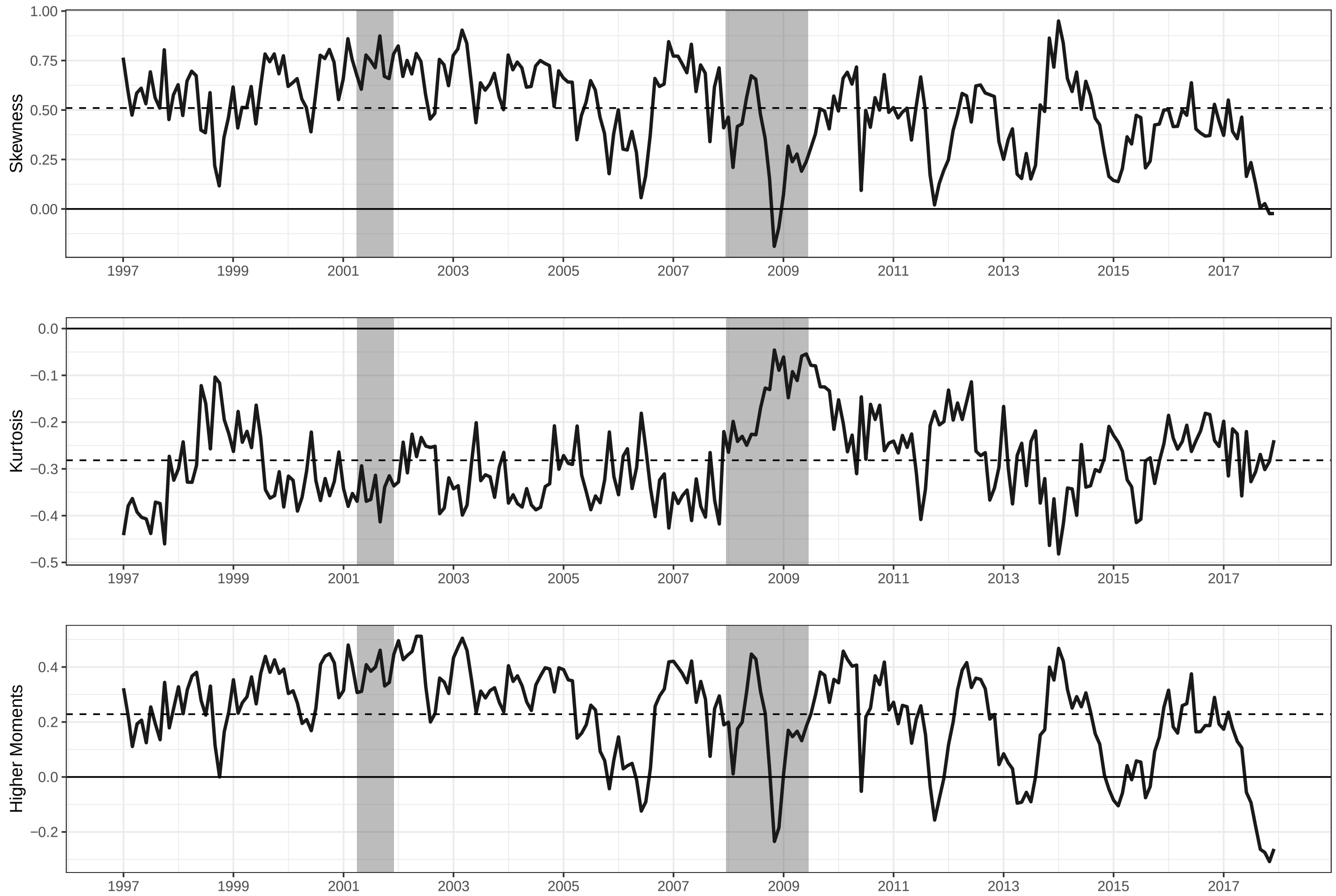
Table 4.6 International Dynamic Portfolio Allocation

This table presents the portfolio results with monthly rebalancing for an international investor. VW denotes the value-weighted portfolio. In the MV and *bench* portfolios, the investor constructs her optimal portfolio based on conditioning information related to the first two moments of return distribution. MV indicates an investor with quadratic utility, while *bench* refers to an investor with power utility and a relative risk aversion of $\gamma = 5$. In the portfolio presented in the last two columns, the investor adds conditional higher moments of returns to her information set. The $Sk_{t-1}(r_{i,t})$ is the conditional skewness constructed from Eq. (4.5). The $OKurt_{t-1}(r_{t,i})$ is the proxy for pure kurtosis, which is the residuals of orthogonal regression in Eq. (4.10). $CoSkew$ and $CoKurt$ are the coskewness and cokurtosis coefficients of country return with the MSCI World portfolio returns. $DivY$ and Mom are the dividend yields and momentum computed as the last 12-month returns, respectively. IPG and INF are the industrial production growth and inflation rate, respectively. Panel A reports the optimal portfolio coefficients, while the numbers in parentheses are their p-values. Panel B reports the portfolio return properties, including the annualised average return for the optimal portfolio, its annualised volatility, *SkewToKurt* ratio, adjusted Sharpe ratio and the annualised certainty-equivalent return. The last four rows report the overall average weight of EMs, $\overline{w}_{EM}$, and the weight of EMs in the skewness-kurtosis-specific portfolio due to skewness, $\overline{w}_{EM}^{Sk}$, kurtosis, $\overline{w}_{EM}^{Ku}$, and the sum of both higher moments, $\overline{w}_{EM}^{Hm}$. Panel C reports the decompositions to portfolio returns into country groups and separately for the skewness-kurtosis-specific portfolio.

	VW	MW	bench	Sk	SkKurt
Panel A: Optimal Loadings					
$OKurt_{t-1}(r_{t,i})$					-1.637
					(0.089)
$CoKurt_{t-1}$					-0.210
					(0.865)
$Sk_{t-1}(r_{t,i})$				2.275	3.028
				(0.128)	(0.047)
$CoSkew_{t-1}$				1.100	0.830
				(0.343)	(0.474)
$Vol_{t-1}(r_{t,i})$		-1.405	-1.416	-2.678	-2.118
		(0.177)	(0.175)	(0.034)	(0.093)
$DivY_{t-1}$		2.664	3.088	2.299	2.936
		(0.044)	(0.026)	(0.099)	(0.045)
Mom_{t-1}		0.962	1.164	1.136	1.275
		(0.313)	(0.235)	(0.261)	(0.223)
IPG_{t-1}		2.708	2.792	2.652	2.749
		(0.054)	(0.047)	(0.068)	(0.068)
INF_{t-1}		2.419	2.726	3.040	2.902
		(0.037)	(0.021)	(0.019)	(0.037)
Panel B: Portfolio Properties					
r_P	0.083	0.236	0.255	0.271	0.289
$\sigma(r_P)$	0.163	0.223	0.241	0.251	0.265
$SkewToKurt(r_P)$	-0.175	0.075	0.095	0.125	0.149
$ASharpe(r_P)$	0.463	1.009	1.019	1.014	1.068
$CE(r_P)$	0.098	0.103	0.109	0.121	0.142
$\overline{w}_{EM}$	0.075	0.243	0.305	0.352	0.348
$\overline{w}_{EM}^{Sk}$				0.383	0.510
$\overline{w}_{EM}^{Ku}$					-0.282
$\overline{w}_{EM}^{Hm}$				0.383	0.228
Panel C: Return Decompositions					
r_P^{DM}	0.077	0.076	0.070	0.077	0.085
r_P^{EM}	0.006	0.160	0.186	0.194	0.204
r_P^{Hm}				0.034	0.043
$\sigma(r_P^{Hm})$				0.127	0.096
$SkewToKurt(r_P^{Hm})$				0.009	0.110
$CE(r_P^{Hm})$				-0.008	0.021

Fig. 4.4 International Portfolio Weights

This figure presents the actively-managed weight of EMs in the skewness-kurtosis-specific portfolio in the bottom plot. The time series is decomposed to the weight due to conditional skewness and the weight due conditional kurtosis in the bottom plot.



dominated by the kurtosis effect. By the end of the sample, the investor increases her holding of DMs by approximately 30% by short-selling equity in EMs (see the bottom plot of Figure 4.4). This reverse action can partly be explained by the recently rise in protectionist policies, which have slowed down globalisation and impose negative impacts on capital inflows and the prospects of emerging markets (Bekaert et al., 2016).

Overall, the *SkKurt* portfolio successfully exploits the cross-sectional dynamics of conditional return moments up to the fourth level. More importantly, a large part of the economic gains in the optimal portfolio is attributable to the joint dynamics of conditional moments in return distribution. Indeed, this finding is in line with the conclusions of Jondeau and Rockinger (2012) who highlight the importance of jointly accounting for skewness and kurtosis in their distribution timing strategy.

4.4 Robustness Checks

I conduct several checks to assess the robustness of my main result. First, I perform an OOS analysis to examine the benefits of conditional higher moments to a real-time investor. Second, I examine how the transaction costs may affect the economic value of optimal portfolio with conditional higher moments. Third, I consider the sensitivity of the portfolio allocation with different levels of relative risk aversion. Finally, I investigate whether the main results hold when I employ more extreme quantile levels in the conditional return distribution.

4.4.1 Out-of-Sample Analysis

In real time, the investor faces estimation errors and time-variation in portfolio policy coefficients. In the current application, this concern can be important since the conditional moments are constructed from a set of separate quantile regressions. To examine this issue, I perform an OOS investing strategy as follow.

I start with the first in-sample period ending in December 2016, i.e. 10 years of data. The last observation in the MIDAS quantile regression is the conditional quantile (i.e., $Q_\alpha(r_{t,h})$ in Eq. (4.7)) of the December 2006 using index return data until the end of November 2017 (i.e., $r_{t-1,d}$ in Eq. (4.8)). I estimate the in-sample conditional volatility, skewness, coskewness, orthogonalised kurtosis and cokurtosis measures based on conditional quantiles for all countries as described above. Then, I collect all financial and macroeconomic factors observable as of November 2016 (i.e., X_{t-1} in Eq. (4.11)) for all countries. These series are combined with with conditional moments of return distribution to estimate the dynamic portfolio allocation for the first estimation window. Note that the portfolio policy uses conditioning information as of November 2016 but the optimisation requires realised returns (i.e., $r_{i,t}$ in Eq. (4.1)) until December 2016. Next, I use the in-sample MIDAS quantile coefficients together with the last 250 daily returns ending December 2006 to produce OOS quantile estimates for January 2007. Similarly, I construct the OOS estimates for conditional volatility, skewness and orthogonalised kurtosis. Finally, I use the estimated portfolio loadings (i.e., $\lambda_X^\top$ and $\lambda_{Hm}^\top$), OOS conditional moments, financial/macroeconomic variables and market capitalisation as of December 2006 to compute optimal weights for January 2007. The corresponding OOS realised portfolio return is then stored for ex-post evaluation. For the next iteration, I expand the estimation window by one month to the end of January 2007 and obtain the OOS realised portfolio return for February 2007. Iteratively, this procedure produces 132 monthly portfolio returns from January 2007 to December 2017 for the investor in "real-time". To estimate the marginal impacts of conditional higher moments, I build three optimal portfolios, namely the *bench*, Sk and $SkKurt$, which are defined above in the main analysis.

Table 4.7 presents the OOS results for the international dynamic portfolio. In Panel A, I report the average of portfolio coefficients across in-sample estimations for *bench*, *Sk* and *SkKurt* portfolios. The OOS analysis shows that the portfolio coefficients are largely similar to the whole-sample estimates in Table 4.6. The optimal loading for conditional skewness is on average positive, whereas the orthogonalised kurtosis continues to enter the optimal weighting function with negative coefficients. Among other predictors, $DivY$ continues to be provide the strongest predictor, followed by the industrial production growth. This finding confirms that, in practice, the investor should tilt her portfolio toward the markets with higher conditional skewness and away from the markets with higher conditional kurtosis.

The return properties in Panel B shows that the *SkKurt* portfolio continues to earn the highest average return, at 6.7%, compared to that of 5.4% in *bench* and 5.9% in *Sk*. This portfolio also generates the lowest volatility at 21.4%, and thereby has the highest ASharpe ratio at 0.209. The *SkewToKurt* ratio is always negative, implying negative skewness in all portfolios. Interestingly, when the investor only considers conditional skewness, the *SkewToKurt* ratio is more negative than that of the *bench*. In contrast, this ratio becomes less negative when she incorporates both conditional skewness and kurtosis. This finding further highlights the importance of accounting for the joint dynamic between conditional higher moments. The negative value of the certainty-equivalent in all portfolios is reasonable since the OOS period fully covers the 2007-2009 GFC. Nevertheless, this number is much less negative in the $SkKurt$ portfolio compared to that of the *bench* and Sk portfolios. Overall, I conclude that accounting for both conditional skewness and kurtosis yields sizeable economic gains in real-time investing strategy.

Table 4.7 International Portfolio Allocation - Out-of-sample Analysis

This table presents results of the OOS international portfolio allocation. The OOS conditional moments of returns are constructed from one-month-ahead quantile estimates by expanding the estimation sample. The first in-sample period ends at December 2006. I use the estimated portfolio coefficients using the in-sample conditioning information to form OOS optimal weights for the following month and track the ex-post realised portfolio return. This procedure generates 132 OOS portfolio returns from January 2007 to December 2017. The *bench* column refers to the benchmark portfolio of an investor with relative risk aversion of $\gamma = 5$. In the column labelled *Sk* and *SkKurt*, the investor adds conditional skewness and both conditional higher moments to her information set. The $Sk_{t-1}(r_{i,t})$ is the conditional skewness based on quantile estimates as in equation Eq. (4.5). The $Sk_{t-1}(r_{i,t})$ is the conditional skewness constructed from Eq. (4.5). The $OKurt_{t-1}(r_{t,i})$ is the proxy for pure kurtosis, which is the residuals of orthogonal regression in Eq. (4.10). $CoSkew$ and $CoKurt$ are the coskewness and cokurtosis coefficients of country return with the MSCI World portfolio returns. $DivY$ and Mom are the dividend yields and momentum computed as the last 12-month returns, respectively. IPG and INF are the industrial production growth and inflation rate, respectively. Panel A reports the average of the portfolio coefficients in 4.11 across the estimation samples. Panel B reports the portfolio return properties, including the annualised average return for the optimal portfolio, its annualised volatility, $SkewToKurt$ ratio, adjusted Sharpe ratio and the annualised certainty-equivalent.

	bench	Sk	SkKurt
Panel A: Average portfolio policy estimates			
$OKurt_{t-1}(r_{t,i})$			-1.059
$CoKurt_{t-1}$			0.140
$Sk_{t-1}(r_{t,i})$		1.522	1.967
$CoSkew_{t-1}$		-0.690	-0.963
$Vol_{t-1}(r_{t,i})$	-1.566	-2.511	-2.345
$DivY_{t-1}$	5.809	5.538	5.318
Mom_{t-1}	1.620	1.702	1.615
IPG_{t-1}	3.161	3.086	2.580
INF_{t-1}	2.305	2.266	2.061
Panel B: Portfolio Properties			
r_P	0.054	0.059	0.067
$\sigma(r_P)$	0.235	0.217	0.214
$SkewToKurt(r_P)$	-0.078	-0.085	-0.065
$ASharpe(r_P)$	0.113	0.165	0.209
$CE(r_P)$	-0.089	-0.064	-0.052

4.4.2 Transaction Costs

Another possible concern is the relatively higher transaction costs in EMs due to the lack of liquidity. Although the ETFs indices in my main analysis are relatively liquid, this issue can still lead to return distortions for the investor in practice.

As in Brandt et al. (2009), the trading cost of going long or short in a country can be expressed as a proportion of the turnover:

$$C_{t-1,i} = c_{t-1,i}|w_{t-1,i} - w_{t-2,i}| \tag{4.15}$$

where $C_{t-1,i}$ is the total cost of trading on country i, which depends on the relative trading cost $c_{t-1,i}$ and the absolute change in optimal portfolio weights formed on time $t-2$ and $t-1$. The net return of optimal portfolio is defined as:

$$r_{t,P} = \sum_{i=1}^{N_t} (w_{t-1,i}r_{t,i} - C_{t-1,i}) \tag{4.16}$$

I follow GPV to model $c_{t-1,i}$ in two alternative approaches. First, I assume constant trading costs on all countries at 0.25%, i.e., $c_{t-1,i} = 0.0025$. Thus, the total transaction cost depends solely on the overall turnover in the optimal portfolio. Second, I impose the trading cost on each country based on its relative market capitalisation to reflect relatively higher trading costs on smaller markets. To this end, the proportional transaction cost is expressed as, $c_{t-1,i} = 0.004 - 0.003 \times me_{t-1,i}$, where $me_{t-1,i}$ is the standardised market size of country i cross-sectionally.

Table 4.8 reports the optimal portfolio policy with the two alternative transaction cost modellings. The portfolio policy coefficients in Panel A provide two notable observations. First, all predictors have significant lower loadings than the main results, except only for the dividend yield. Nevertheless, the coefficients on return moments remain significant and have the expected signs. Among other variables, the two macroeconomic variables, *INF* and *IPG*, have the largest reduction in the loadings and become statistically insignificant. Second, the summary of portfolio return properties in Panel B exhibits clear impacts of imposing transaction costs on the optimal portfolio. Not surprisingly, the proportion of portfolio weights on EMs reduces remarkably after accounting for transaction costs. For example, the total weight of EMs on the *SkKurt* portfolio reduces from 34.8% to

Table 4.8 International Portfolio Allocation - With Transaction Costs

This table reports results for the international portfolio allocation accounting for transaction costs. The transaction costs are modelled in two alternative approaches. In the first specification, the transaction cost is expressed as constant at 0.25%. In the second specification, the transaction cost is relatively related to the market capitalisation with smaller markets having higher trading cost, $c_{t-1,i} = 0.004 - 0.003 \times me_{t-1}$. The *bench* column refers to the benchmark portfolio of an investor with relative risk aversion of $\gamma = 5$. In the columns labelled Sk and $SkKurt$, the investor adds conditional skewness and both conditional higher moments to her information set. Panel A reports the optimal portfolio coefficients, while the numbers in parentheses are their p-values. Panel B reports the portfolio return properties, including the annualised average return for the optimal portfolio, its annualised volatility, $SkewToKurt$ ratio, adjusted Sharpe ratio and the annualised certainty-equivalent return. The last four rows report the overall average weight of EMs, $\overline{w}_{EM}$, and the weight of EMs in the skewness-kurtosis-specific portfolio due to skewness, $\overline{w}^{Sk}_{EM}$, kurtosis, $\overline{w}^{Ku}_{EM}$, and the sum of both higher moments, $\overline{w}^{Hm}_{EM}$.

	Constant Transaction Cost			Relative Transaction Cost		
	bench	Sk	SkKurt	bench	Sk	SkKurt
Panel A: Optimal Loadings						
$OKurt_{t-1}(r_{t,i})$			-0.982			-0.888
			(0.086)			(0.089)
$CoKurt_{t-1}$			0.574			0.448
			(0.246)			(0.078)
$Sk_{t-1}(r_{t,i})$		1.107	1.993		0.673	1.335
		(0.112)	(0.073)		(0.089)	(0.020)
$CoSkew_{t-1}$		-0.369	-0.676		-0.302	-0.369
		(0.368)	(0.144)		(0.114)	(0.078)
$Vol_{t-1}(r_{t,i})$	-0.772	-1.271	-1.247	-0.631	-0.880	-0.825
	(0.167)	(0.058)	(0.083)	(0.073)	(0.051)	(0.088)
$DivY_{t-1}$	3.363	3.320	3.330	2.851	2.903	2.993
	(0.001)	(0.002)	(0.000)	(0.001)	(0.000)	(0.000)
Mom_{t-1}	1.193	1.286	1.198	0.907	1.053	1.106
	(0.017)	(0.022)	(0.023)	(0.010)	(0.011)	(0.002)
IPG_{t-1}	0.231	0.276	0.165	0.074	0.097	0.110
	(0.625)	(0.548)	(0.751)	(0.507)	(0.455)	(0.418)
INF_{t-1}	0.658	0.647	0.326	0.289	0.315	0.318
	(0.284)	(0.272)	(0.603)	(0.252)	(0.284)	(0.286)
Panel B: Portfolio Properties						
r_P	0.150	0.158	0.163	0.125	0.130	0.138
$\sigma(r_P)$	0.185	0.186	0.191	0.171	0.172	0.181
$SkewToKurt(r_P)$	0.077	0.130	0.130	0.027	0.081	0.076
$ASharpe(r_P)$	0.780	0.845	0.846	0.679	0.720	0.727
$CE(r_P)$	0.067	0.077	0.082	0.053	0.058	0.062
$\overline{w}_{EM}$	0.077	0.163	0.124	0.032	0.111	0.104
$\overline{w}^{Sk}_{EM}$		0.186	0.336		0.113	0.225
$\overline{w}^{Ku}_{EM}$			-0.169			-0.153
$\overline{w}^{Hm}_{EM}$		0.186	0.167		0.113	0.072

only 12.4% and 10.4% in the two cost scenarios, respectively. The average returns in all three portfolios decrease significantly compared to those reported in Table 4.6. Nevertheless, the *SkKurt* portfolio continues to yield the best performance, although the superiority is less pronounced. The increases in certainty-equivalent are 1.5% and 0.9% compared to the *bench* portfolio in constant and relative transaction cost scenarios, respectively.

4.4.3 Different Degree of Relative Risk Aversion

My main results assume a representative investor with a relative risk aversion $\gamma = 5$. Since investing in EMs is arguably riskier than investing in DMs, the level of risk aversion might affect the results of my analysis. I investigate this possibility by considering the relative risk aversion coefficients of $\gamma = 3$ and $\gamma = 10$, respectively, and reestimate the optimal portfolio allocation. The corresponding portfolios are reported in Table 4.9. The results reveal that the benefit of accounting for conditional higher moments is more pronounced for less risk-averse investors. The portfolio coefficients on conditional skewness and kurtosis are of greater magnitudes and highly significant for $\gamma = 3$, whereas conditional kurtosis is only significant at the 13% confidence level when $\gamma = 10$. In both cases, however, the *SkKurt* portfolio always yields the best performance. The certainty-equivalent increases by 1.7% (1.6%) compared to the *bench* portfolio when $\gamma = 3$ ($\gamma = 10$). This is in line with the results of GPV, indicating that those with low risk-aversion are better able to exploit information for conditional higher moments of return distribution.

4.4.4 Additional Conditional Quantiles

The conditional moments in my main results are approximated from conditional quantiles using the law of total probability. Therefore, it might be of interest to investigate whether employing more conditional quantiles, especially those in the

Table 4.9 International Portfolio Allocation - With Alternative Risk Aversions

This table reports results for the international portfolio allocation with alternative levels of relative risk aversion. The *bench* column refers to the benchmark portfolio of an investor with relative risk aversion of $\gamma = 5$. In the column labelled Sk and $SkKurt$, the investor adds conditional skewness and both conditional higher moments to her information set. Panel A reports the optimal portfolio coefficients, while the numbers in parentheses are their p-values. Panel B reports the portfolio return properties, including the annualised average return for the optimal portfolio, its annualised volatility, $SkewToKurt$ ratio, adjusted Sharpe ratio and the annualised certainty-equivalent return. The last four rows report the overall average weight of EMs, $\overline{w}_{EM}$, and the weight of EMs in the skewness-kurtosis-specific portfolio due to skewness, $\overline{w}_{EM}^{Sk}$, kurtosis, $\overline{w}_{EM}^{Ku}$, and the sum of both higher moments, $\overline{w}_{EM}^{Hm}$.

	$\gamma = 3$			$\gamma = 10$		
	bench	Sk	SkKurt	bench	Sk	SkKurt
Panel A: Optimal Loadings						
$OKurt_{t-1}(r_{t,i})$			-1.743			-0.814
			(0.085)			(0.124)
$CoKurt_{t-1}$			1.386			0.585
			(0.438)			(0.367)
$Sk_{t-1}(r_{t,i})$		2.503	4.304		1.792	2.869
		(0.135)	(0.072)		(0.084)	(0.081)
$CoSkew_{t-1}$		-0.718	-0.392		-0.569	-0.114
		(0.679)	(0.834)		(0.356)	(0.865)
$Vol_{t-1}(r_{t,i})$	-1.550	-2.716	-2.240	-1.484	-2.300	-2.139
	(0.162)	(0.077)	(0.051)	(0.019)	(0.063)	(0.062)
$DivY_{t-1}$	7.256	7.654	7.326	4.634	3.766	4.169
	(0.001)	(0.000)	(0.003)	(0.000)	(0.000)	(0.000)
Mom_{t-1}	2.383	2.357	2.433	1.473	1.225	1.477
	(0.152)	(0.157)	(0.147)	(0.013)	(0.042)	(0.013)
IPG_{t-1}	4.932	4.817	4.603	2.776	3.055	2.830
	(0.044)	(0.046)	(0.068)	(0.001)	(0.000)	(0.002)
INF_{t-1}	3.509	2.180	2.617	2.282	2.571	1.946
	(0.079)	(0.069)	(0.053)	(0.001)	(0.002)	(0.012)
Panel B: Portfolio Properties						
r_P	0.393	0.403	0.441	0.269	0.281	0.297
$\sigma(r_P)$	0.378	0.384	0.402	0.251	0.253	0.260
$SkewToKurt(r_P)$	0.172	0.181	0.182	0.148	0.159	0.166
$ASharpe(r_P)$	1.050	1.067	1.139	1.097	1.150	1.194
$CE(r_P)$	0.136	0.141	0.153	0.061	0.071	0.077
$\overline{w}_{EM}$	0.533	0.486	0.695	0.242	0.408	0.385
$\overline{w}_{EM}^{Sk}$		0.422	0.725		0.302	0.483
$\overline{w}_{EM}^{Ku}$			-0.300			-0.220
$\overline{w}_{EM}^{Hm}$		0.422	0.425		0.302	0.263

Table 4.10 International Portfolio Allocation - With Additional Quantiles

This table reports results for the international portfolio allocation with additional quantile levels. The *bench* column refers to the benchmark portfolio of an investor with a relative risk aversion of $\gamma = 5$. In the columns labelled *Sk* and *SkKurt*, the investor adds conditional skewness and both conditional higher moments to her information set. Panel A reports the optimal portfolio coefficients, while the numbers in parentheses are their p-values. Panel B reports the portfolio return properties, including the annualised average return for the optimal portfolio, its annualised volatility, *SkewToKurt* ratio, adjusted Sharpe ratio and the annualised certainty-equivalent return. The last four rows report the overall average weight of EMs, $\overline{w}_{EM}$, and the weight of EMs in the skewness-kurtosis-specific portfolio due to skewness, $\overline{w}_{EM}^{Sk}$, kurtosis, $\overline{w}_{EM}^{Ku}$, and the sum of both higher moments, $\overline{w}_{EM}^{Hm}$.

	bench	Sk	SkKurt
Panel A: Optimal Loadings			
$OKurt_{t-1}(r_{t,i})$			-2.152
			(0.082)
$CoKurt_{t-1}$			1.880
			(0.121)
$Sk_{t-1}(r_{t,i})$		1.483	4.361
		(0.125)	(0.023)
$CoSkew_{t-1}$		-0.706	-0.572
		(0.531)	(0.622)
$Vol_{t-1}(r_{t,i})$	-1.619	-2.218	-1.821
	(0.131)	(0.072)	(0.092)
$DivY_{t-1}$	4.832	4.636	3.846
	(0.000)	(0.002)	(0.009)
Mom_{t-1}	1.596	1.584	1.518
	(0.107)	(0.136)	(0.149)
IPG_{t-1}	3.065	2.907	1.399
	(0.033)	(0.046)	(0.093)
INF_{t-1}	2.091	1.835	1.025
	(0.073)	(0.132)	(0.432)
Panel B: Portfolio Properties			
r_P	0.217	0.226	0.259
$\sigma(r_P)$	0.215	0.215	0.245
$SkewToKurt(r_P)$	0.162	0.176	0.190
$ASharpe(r_P)$	1.026	1.078	1.101
$CE(r_P)$	0.114	0.124	0.130
$\overline{w}_{EM}$	0.144	0.215	0.291
$\overline{w}_{EM}^{Sk}$		0.185	0.545
$\overline{w}_{EM}^{Ku}$			-0.258
$\overline{w}_{EM}^{Hm}$		0.185	0.286

extreme tails, could provide better approximations, and thereby affects portfolio allocation. To answer this question, I also estimate conditional quantiles at more extreme levels, i.e. $\alpha = (0.1\%, 0.5\%, 2.5\%, 97.5\%, 99.5\%, 99.9\%)$ and recalculate the conditional moments accordingly. The portfolio results displayed in Table 4.10 show slightly stronger impacts of conditional higher moments on optimal loadings. The portfolio returns, however, are largely similar to the main results. Therefore, adding more extreme quantiles marginally highlight the role of conditional higher moments in international portfolio allocation. This benefit, however, comes at a cost of higher probability of estimation errors and uncertainty to the investor in real-time strategy.

4.5 Suggestions for Future Research

My analysis focuses on the impacts of conditional higher moments on the international diversification, given strong evidence of nonnormality in the aggregate returns. However, Albuquerque (2012) show that the shape of return distributions in individual stocks are different to those of the index returns. More specifically, the former is typically characterised with positive skewness and less pronounced kurtosis than the latter. Furthermore, Gao and Nardari (2018) recently advocate the inclusion of commodity to risky portfolio using an OOS strategy with forward-looking return higher moments. Thus, a direct extension of the current work is to examine the benefits of return higher moments on a wider range of asset asset universe, including individual stocks and alternative asset classes.

One limitation in this chapter is the rebalancing period, which I only consider monthly horizon. As shown by Neuberger and Payne (2019), return skewness (kurtosis) of longer horizon is mainly driven by the covariance between innovation in variance and lagged returns (lagged squared returns). Such components are better captured by information from option markets, which are not available in the majority of countries under consideration in the current study. Therefore, future research may narrow the sample to a subset of the indices in which option data is publicly available, at the cost of the generalisation in portfolio results. The conditional skewness and kurtosis are then can be estimated under risk neutral density as in Bakshi et al. (2003).

4.6 Conclusion

I investigate the benefit of incorporating conditional higher moments of the return distribution to international portfolio allocation. I use the MIDAS quantile regression to estimate various conditional quantiles and approximate the conditional higher moments using the law of total probability. By doing so, the conditional higher moments are robust to the outliers and can be directly used in the optimisation of expected utility function. Specifically, I regress the conditional kurtosis on contemporaneous skewness to measure the pure kurtosis orthogonalised by impacts from skewness.

My empirical results reveal significant time variations and heterogeneity in conditional skewness and kurtosis across countries. Using the parametric portfolio policy approach, I observe sizeable economic gains for the international investor in the optimal portfolio incorporating conditional higher moments. More importantly, I find that a large proportion of the economic gains is attributable to the joint dynamics of conditional skewness and kurtosis. The economic value of conditional higher moments remains robust in OOS analysis as well as accounting for transaction costs or alternative estimation specifications.

Chapter 5

Conclusion

5.1 Summary

This thesis consists of three studies on modelling higher moments and tail risk of financial returns. In the first study, I propose applying the MIDAS framework to improve tail forecasts at any horizon. In particular, I focus on VaR and ES forecasts, given their crucial roles in the risk management process of financial institutions and regulators. The MIDAS framework offers a direct estimation of VaR and ES at the desired forecast horizons, yet exploiting the data-rich environment of returns sampled at different frequencies and avoiding making restrictive distributional assumptions. A rigorous analysis including 43 international stock indices, 8 established benchmark models, 5 statistical backtests and 2 loss functions is conducted to examine the predictive power of the newly proposed models.

The out-of-sample analysis reveals that MIDAS-based models provide the best VaR and ES forecasts, both in their absolute performance as tail risk measures and their relative performance in terms of minimizing two loss functions. Moreover, the proposed methods are included in the set of superior models in most cases across quantile levels and forecast horizons. The GARCH-based models perform well in the absolute performance backtests, which are based on binary sequences of violations. However, the corresponding VaR and ES forecasts generally underestimate the risks, and as such, this leads to higher forecasting errors. The CAViaR-based models, which apply prior temporal aggregation of return series to match the forecast horizon, are inferior to all other methods, specially at the multi-day forecast horizons. Finally, the model ranking is robust to different market regimes, alternative assets and estimation specifications.

In the second study, I aim at identifying the best method to model and forecast return skewness. In particular, I perform a comprehensive horserace between five prominent skewness models with distinct features on forecasting methods in 10 international indices from 7 regions at three forecast horizons. To proxy for the "true" skewness in equity returns, I employ the recently introduced estimator, which utilises information from both historical daily returns and option prices. Under this ex-post measure, the competing forecasts are compared in both information content analysis and out-of-sample forecast evaluation. More importantly, I introduce a new skewness forecast based on option-implied skewness and adjusting for the skewness risk premium between physical and risk-neutral return distribution. The economic value of skewness forecasts is then examined in an application on the international portfolio allocation.

The empirical analysis reveals that the best overall forecasting performance is offered by the new implied skewness estimator which accounts for the average difference between realised skewness and implied skewness. The new estimator also generates superior portfolio performance compared to alternative skewness forecasts. More specifically, only the portfolio based on the corrected implied skewness produces higher average returns and Sharpe ratio than the benchmark portfolio in all cases considered. My findings are robust to a set of different model specifications and estimation methods.

The third study investigates the potential benefits of incorporating conditional higher moments of return distribution to the international portfolio allocation.

Early theoretical models suggest that a risk-averse investor has positive preference to skewness and negative preference to kurtosis. To examine this prediction, I consider the portfolio allocation problem for an investor who utilises the forecasts of return higher moments to determine her optimal portfolio weight in an investable universe involving equity returns from 42 international indices. Specially, all the conditional moments of the return distribution, namely volatility, skewness and kurtosis, are simultaneously estimated from a set of conditional quantiles using the law of total probability. In addition to conditional return moments, I also consider several financial and macroeconomic variables in the information set of the investor.

I find that the conditional higher moments exhibit significant time-variation and heterogeneity between countries in the sample. The portfolio results suggest strong evidence on the benefits of incorporating higher moment forecasts to the international portfolio allocation. Interestingly, I find that a large part of the economic gains is attributable to the joint dynamics between return higher moments. The portfolio strategy that exploits both skewness and kurtosis significantly outperforms other strategies based on predictors up to only the third moment, both in-sample and out-of-sample. The main finding is robust to the inclusion of two different scenarios on transaction costs and alternative levels of risk aversion.

5.2 Non-academic Implications

The superiority of new methods in my first study offer two main implications for the risk management process in financial institutions. First, I argue that financial institutions should use a risk model that directly forecasts VaR and ES and avoids heavy assumptions on return distributions. The Basel III and Solvency II agreements rely their capital requirements on VaR and ES forecasts at 10-day horizon. Yet, the large extant of literature focuses on 1-day ahead forecasts and scale up to multi-period estimates by assuming i.i.d normal distribution in asset returns. Such a restrictive assumption is often associated with inaccurate risk measurements and makes financial institutions sensitive to the probability of under-(over-) capitalization. Second, the risk model should account for the impacts of serial dependence in higher frequency process to the tail dynamics of lower frequency return distribution. The simple aggregation of return series to the forecast horizon results to significant loss of information in higher frequency data. My empirical analysis shows that the flexibility of the lag polynomials in MIDAS framework provides a good solution for this issue.

The results in the second study imply that the information embedded in option prices is useful in forecasting financial return characteristics. Previous studies point out the role of option-implied estimate in forecasting return volatility. My study complements these findings to the skewness forecasts and further highlights the importance of accounting for the discrepancy between physical and risk-neutral return distribution. Thus, the dynamics of these risk premiums should be taken into account when the investor develops a strategy involving option contracts. Moreover, my new skewness estimator also provides an efficient tool for practitioners to enhance their financial decision-making. Some notable examples is the use of skewness forecast in investment and hedging decisions.

The third study sheds light on the current debate of international diversification literature. Previous studies provide mixed results about the role of emerging countries in the international portfolio allocation as a separate asset class with distinct features. I argue that the diversification benefits of these countries reduces significantly once we take into account their conditional kurtosis. Although the emerging stock markets have less negative skewness on average, their return distribution is relatively more exposed to the extreme observations. Thus, the investors should pay more attention on the tail risks of assets from the emerging markets in their portfolios. This issue is relatively more critical given the recent rising of protectionist policies in the global economy, which undoubtedly increase the uncertainty of capital flows and future prospects of emerging markets.

5.3 Future Research Directions

There are several possible extensions and areas for future researches from the three studies in this thesis. The first study highlights the importance of accounting for the serial dependence of higher frequency return process in modelling the lower frequency return distribution. I limit my attention on the tail dynamics in terms of VaR and ES estimates, but future research can also explore this phenomenon in terms of return density or equity risk premium forecasts. Some possible methods are the LASSO quantile regression method of Belloni and Chernozhukov (2011) or the three-pass quantile regression filter recently proposed by Kelly and Pruitt (2015). Another possible extension is to further exploit information from financial and economic variables. Several studies, including Engle and Rangel (2008), Colacito et al. (2011), Engle et al. (2013), document the explanatory power of economic variables to return volatility and correlations. Thus, incorporating such variables to the information set may further improve the tail forecast. Finally, it would be also of interest to investigate the benefits of improved tail forecasts in a multivariate setting and asset allocation (see, e.g. Dias, 2016, for the value of controlling tail risks in portfolio selection).

The second study advocates the use of option-implied information in forecasting skewness. One possible concern is the limited number of international markets under consideration. The main reason for this limitation is the lack of reliable option data that is necessary to compute the realised skewness estimator. Neuberger and Payne (2019) recently introduce new estimators for return higher moments employing only historical daily returns. Thus, a possible extension could involve more indices from emerging markets and alternative asset classes. Another interesting question is the skewness forecasting for individual stocks. Albuquerque (2012) shows that whereas aggregated stock returns display negative skewness, firm-level stock returns are typically positively skewed due to the cross-sectional heterogeneity in the announcement of company events. Finally, since each skewness forecasting model in my study contains part of information on the realised skewness, a combination of skewness forecasts may provide better performance (Elliott and Timmermann, 2016).

The third study mainly focuses on international portfolio allocation, given strong evidence of heterogeneity in time-varying higher moments between countries. However, investing in the international markets might be costly and of interest to a limited number of investors. Future research might seek for implications of skewness and kurtosis forecasts in alternative asset universe, such as individual stocks or different asset classes. Another extension may also consider longer rebalancing horizons, although this may require information from option markets to obtain reliable proxies for return higher moments. A possible solution is to consider a subset of the current sample in which the option price data is publicly available. However, I note that this extension comes at the cost of reducing the generalisation of the results. Finally, since the conditional higher moments exhibit significant time variations, it would be of interest to examine the potential spillovers in higher moments across countries. For example, one can apply the popular method of Diebold and Yilmaz (2012) and Diebold and Yilmaz (2014) using the network analysis to construct the higher moment connectedness index. By doing so, we can have a more complete picture on how the global economy is connected in term of the tail dependence, which in turn provides some insights on the global systemic risks.

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Appendix A

Table A.1 List of VaR and ES Forecasting Models

This table summarizes the competing forecasting models for VaR and ES under consideration.

Abbreviation	Description
Benchmark Models	
GARCH-Fhs	VaR and ES are extracted from the GARCH model of Bollerslev (1987), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated using filter historical simulation with 10,000 trials.
GARCH-Evt	VaR and ES are extracted from the GARCH model of Bollerslev (1987), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated by combining filter historical simulation and EVT with 10,000 trials.
GJR-Fhs	VaR and ES are extracted from the GJR-GARCH model of Glosten et al. (1993), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated using filter historical simulation with 10,000 trials.
GJR-Evt	VaR and ES are extracted from the GJR-GARCH model of Glosten et al. (1993), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated by combining filter historical simulation and EVT with 10,000 trials.
Sav-AL	VaR and ES are jointly estimated using maximum likelihood of AL density in (2.7). VaR follows symmetric absolute value specification in (2.15), while ES dynamic follows specification in (2.8).
Sav-Evt	Conditional quantile at threshold level of 7.5% is estimated using CAViaR model with symmetric absolute value specification in (2.15). VaR and ES are jointly computed using the results of McNeil and Frey (2000).
As-AL	VaR and ES are jointly estimated using maximum likelihood of AL density in (2.7). VaR follows asymmetric slope specification in (2.16), while ES dynamic follows specification in (2.8).
As-Evt	Conditional quantile at threshold level of 7.5% is estimated using CAViaR model with asymmetric slope specification in (2.16). VaR and ES are jointly computed using the results of McNeil and Frey (2000).
New Models	
Midas-AL	VaR and ES are jointly estimated using maximum likelihood of AL density in (2.7). VaR follows MIDAS-based symmetric absolute value specification in (2.1), while ES dynamic follows specification in (2.8).
Midas-Evt	Conditional quantile at threshold level of 7.5% is estimated using MIDAS quantile regression with symmetric absolute value specification in (2.1). VaR and ES are jointly computed using the results of McNeil and Frey (2000).
MidasAs-AL	VaR and ES are jointly estimated using maximum likelihood of AL density in (2.7). VaR follows MIDAS-based asymmetric slope specification in (2.2), while ES dynamic follows specification in (2.8).
MidasAs-Evt	Conditional quantile at threshold level of 7.5% is estimated using MIDAS quantile regression with asymmetric slope specification in (2.2). VaR and ES are jointly computed using the results of McNeil and Frey (2000).

Table A.2 Out-of-sample Forecast Losses - Different Market Regimes

This table provides the average out-of-sample forecast losses for different market regimes at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: 02/08/2000 - 31/07/2007 (Pre-crisis Subsample)												
GARCH-Fhs	0.426	2.095	1.431	1.507	1.260	5.993	4.049	4.102	1.760	8.101	5.404	5.274
GARCH-Evt	0.425	2.092	1.430	1.505	1.237	5.882	3.985	4.033	1.765	8.126	5.410	5.281
GJR-Fhs	**0.416**	**2.047**	1.418	1.492	1.248	5.932	4.069	4.123	1.793	8.255	5.491	5.366
GJR-Evt	0.417	2.051	**1.417**	**1.491**	1.221	5.801	3.988	4.036	1.794	8.266	5.489	5.365
Sav-AL	0.436	2.147	1.449	1.527	1.252	5.950	4.119	4.178	1.824	8.385	5.714	5.605
Sav-Evt	0.437	2.152	1.452	1.529	1.273	6.068	4.071	4.126	1.840	9.019	5.564	5.524
As-AL	0.422	2.074	1.425	1.500	1.254	5.962	4.140	4.202	2.027	9.401	6.054	5.979
As-Evt	0.421	2.071	1.423	1.498	1.273	6.065	4.095	4.152	2.170	10.823	5.904	6.022
Midas-AL	0.433	2.131	1.448	1.525	1.137	5.384	3.989	4.036	1.602	7.353	5.608	5.485
Midas-Evt	0.432	2.127	1.447	1.524	1.224	5.814	3.976	4.023	1.666	7.818	**5.394**	**5.319**
MidasAs-AL	0.417	2.051	1.425	1.500	**1.120**	**5.302**	4.001	4.049	**1.559**	**7.137**	5.638	5.528
MidasAs-Evt	0.421	2.070	1.422	1.497	1.194	5.671	**3.938**	**3.982**	1.789	8.377	5.452	5.349
Panel B: 01/08/2007 - 31/12/2009 (Crisis Subsample)												
GARCH-Fhs	0.721	3.612	2.553	2.784	1.873	9.195	6.079	6.539	2.317	11.128	8.310	8.840
GARCH-Evt	0.720	3.609	2.555	2.786	1.855	9.105	**6.053**	**6.511**	**2.300**	**11.050**	8.307	8.836
GJR-Fhs	0.711	3.563	**2.530**	**2.759**	1.839	9.119	6.131	6.590	2.421	11.580	8.541	9.079
GJR-Evt	0.710	3.558	2.532	2.761	**1.806**	**8.869**	6.081	6.537	2.413	11.536	8.535	9.071
Sav-AL	0.749	3.755	2.591	2.825	2.585	12.525	7.441	7.984	4.016	19.246	11.042	11.801

Table A.2 *(continued)*

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Sav-Evt	0.751	3.761	2.597	2.832	2.512	12.187	7.244	7.771	3.969	19.472	10.367	11.106
As-AL	0.729	3.653	2.545	2.776	2.283	11.110	7.178	7.713	3.679	17.588	10.980	11.765
As-Evt	**0.709**	**3.555**	2.538	2.768	2.235	10.884	6.855	7.356	3.478	17.141	9.706	10.420
Midas-AL	0.744	3.727	2.588	2.822	1.897	9.409	6.316	6.786	2.563	12.262	8.808	9.369
Midas-Evt	0.745	3.733	2.590	2.825	1.923	9.406	6.200	6.663	2.679	13.119	8.506	9.090
MidasAs-AL	0.724	3.627	2.542	2.773	1.830	8.983	6.093	6.603	2.412	11.506	**8.288**	**8.794**
MidasAs-Evt	0.712	3.570	2.537	2.766	1.897	9.312	6.069	6.529	2.652	12.938	8.524	9.099
Panel C: 01/01/2010 - 31/12/2017 (Post-crisis Subsample)												
GARCH-Fhs	0.435	2.179	1.481	1.605	1.113	5.465	3.575	3.806	1.496	7.236	5.027	5.289
GARCH-Evt	0.435	2.183	1.481	1.605	1.107	5.434	3.579	3.811	1.495	7.231	5.028	5.290
GJR-Fhs	0.430	2.156	1.467	1.589	1.104	5.413	3.563	3.794	1.484	7.172	5.003	5.263
GJR-Evt	0.430	2.157	1.465	1.587	1.093	5.361	3.563	3.793	1.482	7.165	5.012	5.272
Sav-AL	0.439	2.203	1.493	1.617	1.185	5.810	3.659	3.897	1.804	8.694	5.189	5.469
Sav-Evt	0.439	2.201	1.493	1.617	1.151	5.654	3.647	3.885	1.666	8.698	5.092	5.586
As-AL	0.433	2.170	1.464	1.586	1.160	5.689	3.660	3.897	1.771	8.514	5.235	5.512
As-Evt	0.431	2.162	1.463	1.585	1.131	5.565	3.635	3.871	1.619	8.659	5.075	5.529
Midas-AL	0.438	2.198	1.492	1.617	1.111	5.452	3.591	3.824	1.517	7.340	5.057	5.320
Midas-Evt	0.439	2.200	1.491	1.616	1.106	5.457	3.561	3.794	1.508	7.469	4.904	5.199
MidasAs-AL	**0.430**	**2.154**	1.465	1.587	**1.064**	**5.218**	3.573	3.804	**1.396**	**6.754**	4.907	**5.159**
MidasAs-Evt	0.431	2.163	**1.462**	**1.584**	1.089	5.414	**3.524**	**3.758**	1.476	7.432	**4.859**	5.177

Table A.3 Model Confidence Set - Different Market Regimes

This table reports the results of the 5% Model Confidence Set (MCS) for different market regimes at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The range statistic in (2.21) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: 02/08/2000 - 31/07/2008 (Pre-crisis Subsample)												
GARCH-Fhs	6	6	6	6	6	7	2	2	20	18	4	5
GARCH-Evt	7	6	3	3	3	4	1	1	20	18	4	5
GJR-Fhs	1	1	0	0	11	10	3	3	32	30	9	11
GJR-Evt	2	2	0	0	10	10	3	3	33	31	10	11
Sav-AL	14	14	10	10	17	17	4	4	32	32	11	12
Sav-Evt	15	15	9	9	13	14	3	4	22	21	7	8
As-AL	5	5	3	3	14	14	5	5	33	32	10	12
As-Evt	3	4	2	2	13	13	4	4	31	30	8	12
Midas-AL	13	13	9	9	11	12	6	7	18	18	8	9
Midas-Evt	13	13	7	6	14	15	1	1	24	23	6	7
MidasAs-AL	7	7	5	5	5	6	3	3	13	13	5	6
MidasAs-Evt	4	4	1	1	7	7	1	1	24	25	4	4
Panel B: 01/08/2008 - 31/12/2009 (Crisis Subsample)												
GARCH-Fhs	3	3	2	2	5	5	1	1	10	11	2	2
GARCH-Evt	4	4	3	4	5	5	1	1	9	9	2	2
GJR-Fhs	2	2	1	1	4	4	2	1	12	13	2	2
GJR-Evt	2	3	1	1	4	4	2	2	12	13	2	2
Sav-AL	5	5	2	2	20	19	6	6	15	16	4	4
Sav-Evt	5	5	4	4	19	18	8	8	11	12	4	4
As-AL	2	3	2	2	11	11	6	6	15	17	4	5
As-Evt	4	5	1	1	8	7	6	6	10	12	2	2
Midas-AL	3	3	3	3	11	12	1	1	11	11	4	4
Midas-Evt	4	4	3	4	13	13	4	4	11	12	2	2
MidasAs-AL	2	2	1	1	3	3	2	2	3	3	2	2
MidasAs-Evt	4	5	3	3	7	7	1	1	8	9	2	2
Panel C: 01/01/2010 - 31/12/2017 (Crisis Subsample)												
GARCH-Fhs	2	3	11	11	7	7	2	2	5	8	3	1
GARCH-Evt	2	2	12	13	7	7	2	2	5	6	3	1
GJR-Fhs	0	0	2	3	5	5	3	3	15	15	3	1
GJR-Evt	3	3	4	4	5	5	3	3	15	15	3	1
Sav-AL	7	7	19	19	18	17	5	6	34	33	6	6
Sav-Evt	4	4	17	17	11	11	4	4	21	28	6	4
As-AL	4	4	2	3	17	16	7	7	33	33	8	6
As-Evt	2	2	5	5	11	11	3	3	24	31	4	4
Midas-AL	6	6	18	18	10	10	5	6	15	15	7	5
Midas-Evt	6	6	16	16	11	11	1	1	15	14	0	0
MidasAs-AL	3	3	2	3	1	2	2	2	2	2	2	0
MidasAs-Evt	1	2	5	5	9	9	1	1	11	17	1	0

Table A.4 Out-of-sample Forecast Losses - Alternative Assets

This table provides the average out-of-sample forecast losses for alternative assets at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. The alternative assets are defined in Section 2.4.2. L_Q denotes the quantile loss function of 2.19 and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	Panel A: 1-day				Panel B: 5-day				Panel C: 10-day			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	0.522	2.594	1.701	1.827	1.073	5.172	3.736	3.904	1.442	6.763	5.127	5.235
GARCH-Evt	0.524	2.605	1.702	1.828	1.078	5.195	3.722	3.890	1.464	6.887	5.106	5.212
GJR-Fhs	0.523	2.601	1.691	1.816	1.096	5.274	3.734	3.902	1.478	6.960	5.128	5.236
GJR-Evt	0.517	2.568	1.691	1.817	1.098	5.294	3.730	3.898	1.473	6.931	5.126	5.237
Sav-AL	0.517	2.558	1.672	1.796	1.151	5.538	3.740	3.908	1.645	7.742	5.267	5.387
Sav-Evt	0.521	2.589	1.695	1.821	1.139	5.496	3.842	4.018	1.760	8.611	5.538	5.726
As-AL	0.515	2.541	**1.648**	**1.770**	1.082	5.221	3.706	3.873	1.561	7.376	5.145	5.255
As-Evt	0.518	2.575	1.684	1.809	1.112	5.390	3.830	4.008	1.626	8.095	5.307	5.538
Midas-AL	0.513	2.548	1.686	1.811	1.044	5.037	3.597	3.756	1.376	6.494	4.917	5.021
Midas-Evt	0.519	2.580	1.693	1.818	1.067	5.144	3.656	3.818	1.436	6.980	5.011	5.139
MidasAs-AL	**0.508**	**2.523**	1.668	1.792	**1.035**	**5.004**	**3.565**	**3.721**	**1.324**	**6.272**	**4.836**	**4.937**
MidasAs-Evt	0.517	2.568	1.680	1.804	1.042	5.032	3.630	3.790	1.432	6.987	5.013	5.176

Table A.5 Model Confidence Set Result - Alternative Assets

This table reports the results of the 5% Model Confidence Set (MCS) for alternative assets at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The alternative assets are defined in Section 2.4.2. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The range statistic in (2.21) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	Panel A: 1-day				Panel B: 5-day				Panel C: 10-day			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	4	4	15	15	3	5	3	3	9	10	3	3
GARCH-Evt	4	4	15	15	3	4	3	3	11	11	3	3
GJR-Fhs	2	2	13	13	3	5	4	3	13	12	3	3
GJR-Evt	2	2	15	15	5	6	3	3	13	13	3	3
Sav-AL	2	2	6	6	9	9	1	1	10	8	3	3
Sav-Evt	4	4	14	14	9	10	7	7	17	17	7	8
As-AL	1	1	4	4	3	5	2	2	9	7	2	2
As-Evt	2	2	14	14	5	7	4	3	13	12	3	3
Midas-AL	1	1	12	12	3	3	0	0	8	7	1	1
Midas-Evt	3	3	14	15	4	4	2	2	14	13	1	2
MidasAs-AL	0	0	7	8	2	2	1	1	7	5	0	0
MidasAs-Evt	1	1	12	12	4	6	1	1	7	8	1	1

Table A.6 Out-of-sample Forecast Losses - Alternative Estimation Windows

This table provides the average out-of-sample forecast losses for alternative rolling window length at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of 2.19 and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: 1500-observtion Estimation Window												
GARCH-Fhs	0.482	2.406	1.646	1.772	1.253	6.089	4.090	4.297	1.686	8.011	5.721	5.894
GARCH-Evt	0.481	2.400	1.645	1.772	1.248	6.063	4.080	4.286	1.674	7.959	**5.706**	**5.879**
GJR-Fhs	0.512	2.555	1.712	1.844	1.328	6.446	4.242	4.459	1.844	8.772	5.888	6.076
GJR-Evt	0.474	2.364	1.630	1.754	**1.229**	**5.967**	4.071	4.276	1.712	8.127	5.761	5.935
Midas-AL	0.481	2.400	1.647	1.774	1.267	6.146	4.051	4.255	1.702	8.103	5.771	5.950
Midas-Evt	0.498	2.482	1.671	1.800	1.329	6.482	4.232	4.456	1.903	9.578	6.027	6.367
MidasAs-AL	**0.467**	**2.329**	**1.616**	**1.740**	1.245	6.040	**4.022**	**4.225**	**1.664**	**7.909**	5.721	5.887
MidasAs-Evt	0.483	2.411	1.642	1.767	1.351	6.640	4.262	4.499	1.995	10.187	6.077	6.437
Panel B: 2000-observation Estimation Window												
GARCH-Fhs	0.482	2.405	1.642	1.770	1.255	6.107	4.056	4.271	1.658	7.899	**5.622**	**5.811**
GARCH-Evt	0.480	2.396	1.642	1.770	1.254	6.099	4.048	4.262	1.680	7.999	5.629	5.820
GJR-Fhs	0.508	2.536	1.702	1.835	1.321	6.422	4.206	4.431	1.820	8.668	5.813	6.017
GJR-Evt	0.474	2.364	1.626	1.752	1.237	**6.012**	4.051	4.265	1.695	8.065	5.682	5.874
Sav-AL	0.484	2.401	1.888	2.039	1.404	6.805	4.453	4.697	2.003	9.555	6.011	6.230
Sav-Evt	0.481	2.398	1.628	1.755	1.348	6.544	4.177	4.399	1.933	9.809	5.754	6.087
As-AL	0.472	2.342	1.896	2.049	1.283	6.221	4.512	4.763	1.894	9.037	6.020	6.242
As-Evt	0.468	2.334	**1.598**	**1.722**	1.275	6.205	4.055	4.271	1.855	9.614	5.623	6.067
Midas-AL	0.481	2.399	1.646	1.775	1.276	6.197	4.035	4.249	1.672	7.980	5.720	5.916
Midas-Evt	0.499	2.488	1.671	1.802	1.336	6.525	4.222	4.456	1.884	9.528	5.960	6.327
MidasAs-AL	**0.467**	**2.331**	1.616	1.741	**1.251**	6.076	**4.009**	**4.221**	**1.646**	**7.848**	5.676	5.868
MidasAs-Evt	0.485	2.418	1.641	1.769	1.360	6.697	4.260	4.510	1.987	10.201	6.048	6.440

Table A.7 Model Confidence Set - Alternative Estimation Windows

This table reports the results of the 5% Model Confidence Set (MCS) for alternative lengths of rolling window at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The range statistic in (2.21) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: 1500-observtion Estimation Window												
GARCH-Fhs	10	10	17	18	0	0	0	0	3	4	1	1
GARCH-Evt	8	7	15	15	1	1	1	1	3	4	0	0
GJR-Fhs	17	17	27	27	3	3	8	8	8	9	0	1
GJR-Evt	5	5	7	7	0	0	0	0	8	9	1	1
Midas-AL	8	8	17	17	2	2	0	0	5	6	0	0
Midas-Evt	28	27	32	33	11	13	17	17	13	21	10	15
MidasAs-AL	0	0	0	0	0	0	0	0	1	1	2	1
MidasAs-Evt	13	12	20	20	14	16	20	20	12	28	9	17
Panel B: 2000-observation Estimation Window												
GARCH-Fhs	6	4	28	27	2	2	3	1	2	2	0	0
GARCH-Evt	7	7	28	29	1	1	2	1	3	2	0	0
GJR-Fhs	18	17	30	30	4	4	9	9	5	4	2	2
GJR-Evt	2	2	19	19	1	1	1	3	2	2	1	1
Sav-AL	8	7	43	43	8	9	13	15	17	15	2	4
Sav-Evt	9	9	17	17	5	5	8	8	9	17	2	2
As-AL	3	2	43	43	1	1	14	14	9	7	4	4
As-Evt	2	2	5	5	1	1	1	1	7	16	3	2
Midas-AL	5	5	30	30	1	2	0	0	2	2	1	1
Midas-Evt	25	24	40	40	9	13	14	16	12	19	10	13
MidasAs-AL	0	0	11	10	0	0	1	1	1	1	0	0
MidasAs-Evt	9	10	30	30	12	14	19	21	12	26	10	14

Table A.8 Out-of-sample Forecast Losses - Different Country Groups

This table provides the average out-of-sample forecast losses for different country groups at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of 2.19 and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: Developed Stock Markets												
GARCH-Fhs	0.465	2.330	1.625	1.760	1.218	5.972	3.878	4.124	1.573	7.580	5.415	5.677
GARCH-Evt	0.465	2.329	1.625	1.760	1.203	5.896	3.874	4.119	1.574	7.584	5.419	5.682
GJR-Fhs	0.463	2.318	1.610	1.743	1.214	5.947	3.891	4.137	1.608	7.730	5.485	5.750
GJR-Evt	0.463	2.317	1.610	1.743	1.191	5.834	3.869	4.114	1.602	7.702	5.487	5.752
Sav-AL	0.472	2.362	1.635	1.771	1.378	6.726	4.157	4.421	2.146	10.291	5.972	6.287
Sav-Evt	0.473	2.370	1.636	1.772	1.344	6.567	4.114	4.375	1.990	10.146	5.780	6.261
As-AL	0.467	2.337	1.608	1.741	1.310	6.401	4.136	4.402	2.082	9.993	6.146	6.479
As-Evt	**0.462**	**2.314**	1.606	1.739	1.292	6.314	4.057	4.314	1.952	9.935	5.716	6.132
Midas-AL	0.470	2.352	1.634	1.770	1.199	5.864	3.927	4.175	1.582	7.613	5.527	5.795
Midas-Evt	0.471	2.359	1.633	1.769	1.212	5.941	3.885	4.131	1.627	7.902	5.324	5.599
MidasAs-AL	0.465	2.328	1.608	1.741	**1.160**	**5.676**	3.872	4.117	**1.461**	**7.043**	**5.323**	**5.582**
MidasAs-Evt	0.462	2.315	**1.605**	**1.738**	1.202	5.912	**3.833**	**4.078**	1.612	7.920	5.341	5.636
Panel B: Emerging Stock Markets												
GARCH-Fhs	0.523	2.611	1.767	1.907	1.382	6.722	4.492	4.743	1.838	8.753	6.126	6.363
GARCH-Evt	0.524	2.615	1.767	1.908	1.374	6.686	4.465	4.715	1.829	8.711	6.120	6.358
GJR-Fhs	**0.512**	**2.554**	1.751	1.890	1.348	6.559	4.493	4.742	1.835	8.736	6.135	6.372
GJR-Evt	0.512	2.556	**1.750**	**1.888**	1.341	6.524	4.464	4.711	1.839	8.754	6.145	6.382
Sav-AL	0.537	2.682	1.793	1.936	1.589	7.694	4.894	5.172	2.345	11.170	6.960	7.263
Sav-Evt	0.536	2.673	1.796	1.939	1.559	7.567	4.827	5.100	2.327	11.674	6.719	7.142
As-AL	0.520	2.593	1.759	1.898	1.504	7.292	4.811	5.082	2.256	10.712	6.873	7.173
As-Evt	0.514	2.567	1.756	1.895	1.470	7.167	4.706	4.970	2.208	11.563	6.599	7.097
Midas-AL	0.535	2.670	1.792	1.935	1.366	6.636	4.529	4.781	1.895	9.014	6.290	6.536
Midas-Evt	0.534	2.664	1.793	1.936	1.376	6.725	4.484	4.736	1.895	9.325	6.138	6.449
MidasAs-AL	0.513	2.560	1.758	1.897	**1.312**	**6.368**	4.502	4.752	**1.790**	**8.515**	6.171	6.416
MidasAs-Evt	0.516	2.574	1.755	1.894	1.346	6.624	**4.431**	**4.684**	1.906	9.410	**6.074**	**6.372**

Table A.9 Model Confidence Set - Different Country Group

This table reports the results of the 5% Model Confidence Set (MCS) for different country groups at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The range statistic in (2.21) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: Developed Stock Markets												
GARCH-Fhs	1	1	7	6	2	2	0	0	3	4	2	2
GARCH-Evt	2	1	6	6	1	1	0	0	3	4	1	1
GJR-Fhs	0	0	1	1	1	1	0	0	4	5	1	1
GJR-Evt	0	0	1	1	1	1	0	0	4	5	1	1
Sav-AL	1	1	10	10	6	5	2	2	15	16	5	5
Sav-Evt	3	2	11	11	6	6	1	1	8	11	4	4
As-AL	1	1	0	0	5	5	2	2	12	13	4	4
As-Evt	0	0	0	0	3	3	1	1	6	10	1	2
Midas-AL	1	1	9	10	2	2	2	2	6	6	2	2
Midas-Evt	2	2	8	8	4	4	1	1	5	5	0	0
MidasAs-AL	0	0	0	0	0	0	1	1	0	0	0	0
MidasAs-Evt	0	0	1	1	1	1	0	0	2	3	0	0
Panel B: Emerging Stock Markets												
GARCH-Fhs	6	6	8	7	1	1	1	0	0	0	0	0
GARCH-Evt	5	5	7	7	2	2	1	0	0	0	0	0
GJR-Fhs	2	2	1	1	1	1	1	0	2	2	0	0
GJR-Evt	2	2	1	1	1	1	1	0	2	2	0	0
Sav-AL	7	7	9	8	3	3	5	5	4	3	1	2
Sav-Evt	8	7	10	9	3	3	5	4	1	5	1	2
As-AL	4	4	0	0	2	2	3	2	4	4	3	3
As-Evt	4	4	1	1	3	3	1	1	2	6	0	1
Midas-AL	6	6	9	8	2	2	2	1	1	1	3	2
Midas-Evt	8	7	10	9	1	1	0	0	0	3	0	1
MidasAs-AL	0	0	2	2	1	1	0	0	0	0	0	0
MidasAs-Evt	4	4	2	2	1	1	0	0	1	4	0	0

Table A.10 Model Confidence Set Results - 10% MCS

This table reports the results of the 10% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The range statistic in (2.21) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day				5-day				10-day			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	11	11	17	18	5	5	1	0	4	5	2	3
GARCH-Evt	11	11	17	18	4	4	1	0	4	4	1	2
GJR-Fhs	4	4	3	4	2	2	1	0	9	8	2	3
GJR-Evt	4	4	3	4	2	2	1	0	10	9	3	3
Sav-AL	13	13	26	27	13	12	12	10	28	27	12	12
Sav-Evt	14	14	27	29	11	10	11	10	20	26	7	13
As-AL	10	9	2	2	10	11	9	7	27	27	13	15
As-Evt	6	6	2	2	7	7	7	6	18	28	5	10
Midas-AL	13	13	24	25	4	4	5	3	10	11	3	4
Midas-Evt	13	14	23	23	6	6	1	1	9	12	1	2
MidasAs-AL	2	1	4	4	1	1	2	1	0	0	0	0
MidasAs-Evt	7	7	7	7	3	3	0	0	7	8	0	1

Table A.11 Model Confidence Set - Alternative Elimination Rule

This table reports the results of the 5% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The using the semi-quadratic statistic in Hansen et al. (2011) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	Panel A: 1-day				Panel B: 5-day				Panel C: 10-day			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	6	6	3	3	1	1	0	0	2	2	1	1
GARCH-Evt	5	5	4	4	1	1	0	0	2	2	1	1
GJR-Fhs	1	1	1	1	1	1	0	0	2	3	1	1
GJR-Evt	3	3	1	1	1	1	0	0	2	3	1	1
Sav-AL	9	8	14	15	3	3	4	4	8	10	5	5
Sav-Evt	9	8	13	13	3	3	2	3	3	8	1	4
As-AL	3	3	1	1	2	3	2	2	11	11	3	4
As-Evt	3	3	0	0	2	2	2	0	3	11	1	4
Midas-AL	6	6	14	15	1	1	0	0	2	3	0	0
Midas-Evt	7	7	14	14	1	1	0	0	1	3	0	0
MidasAs-AL	1	1	0	0	0	0	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	1	1	0	0	1	3	0	0

Table A.12 Model Confidence Set - Alternative Bootstrapping Methods

This table reports the results of the 5% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (2.19) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (2.20). The range statistic in (2.21) is used to the equivalence test of the MCS. The test statistic is constructed using alternative bootstrapping methods. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: Use of stationary bootstrapping												
GARCH-Fhs	7	6	3	3	0	0	0	0	2	3	1	1
GARCH-Evt	5	5	4	4	0	0	0	0	2	3	1	1
GJR-Fhs	2	2	1	1	1	1	0	0	3	4	1	1
GJR-Evt	3	3	1	1	1	1	0	0	3	4	1	1
Sav-AL	10	10	16	19	6	6	5	5	10	13	5	8
Sav-Evt	8	8	13	14	5	5	3	3	3	11	3	5
As-AL	3	3	1	1	3	3	4	4	10	10	4	5
As-Evt	3	3	0	0	1	1	2	2	4	13	1	6
Midas-AL	6	7	15	15	0	0	1	1	3	4	0	0
Midas-Evt	8	8	14	15	1	1	0	0	1	5	0	0
MidasAs-AL	1	1	0	0	0	1	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	0	0	0	0	2	6	0	0
Panel B: Block bootstrapping of length 2												
GARCH-Fhs	6	6	3	3	0	1	0	0	2	2	1	1
GARCH-Evt	5	5	4	4	0	1	0	0	2	2	1	1
GJR-Fhs	1	1	1	1	1	1	0	0	3	3	1	1
GJR-Evt	3	3	1	1	1	1	0	0	3	3	1	1
Sav-AL	9	10	18	18	5	3	5	4	11	12	6	4
Sav-Evt	8	8	13	13	3	3	2	3	3	7	3	7
As-AL	3	3	1	1	2	3	3	3	11	10	4	5
As-Evt	3	3	0	0	1	2	2	2	5	12	1	6
Midas-AL	5	6	16	16	0	1	1	1	3	3	0	0
Midas-Evt	8	8	14	15	1	1	0	0	1	4	0	0
MidasAs-AL	1	1	0	0	0	0	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	0	1	0	0	2	4	0	0
Panel C: Block bootstrapping of length 6												
GARCH-Fhs	6	6	3	3	1	0	0	0	2	2	1	1
GARCH-Evt	5	5	4	4	1	0	0	0	2	2	1	1
GJR-Fhs	1	1	1	1	2	1	0	0	2	2	1	1
GJR-Evt	3	3	1	1	2	1	0	0	2	2	1	1
Sav-AL	7	7	14	14	3	3	4	3	11	11	4	5
Sav-Evt	8	7	13	14	3	3	1	1	4	9	1	4
As-AL	3	3	1	1	2	1	1	1	10	11	3	3
As-Evt	3	3	0	0	1	1	0	0	3	11	1	4
Midas-AL	6	6	12	13	1	0	0	0	2	2	0	0
Midas-Evt	7	6	12	12	2	1	0	0	1	3	0	0
MidasAs-AL	1	1	0	0	0	0	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	1	0	0	0	1	3	0	0

Appendix B

Table B.1 Information Content of Skewness Forecasts (30 days) - Alternative Implied-Skewness

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 30 calendar days. Implied and realized skewness are estimated as described in section 3.4.1. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^{ρ} across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.955**	-0.967	-0.947	**-0.708**	**-0.846**	**-0.948**
	(-12.27)	(-1.43)	(-1.43)	(-3.70)	(-5.46)	(-6.55)
β	**0.151**	0.133	0.149	**0.449**	**0.211**	0.183
	(2.37)	(0.24)	(0.27)	(2.26)	(2.03)	(1.38)
β_ρ	**-1.706**	**-1.943**	**-1.949**	**-1.869**	**-1.796**	**-1.585**
	(-3.77)	(-3.75)	(-3.76)	(-3.64)	(-3.78)	(-2.90)
$\overline{R}^2$	**8.50%**	6.50%	6.51%	8.28%	8.34%	7.85%
DAX						
α	**-0.840**	**-1.029**	**-1.028**	**-1.260**	**-0.888**	**-0.917**
	(-11.38)	(-10.81)	(-10.52)	(-5.82)	(-9.70)	(-10.74)
β	**0.219**	0.056	0.056	-0.251	**0.124**	**0.176**
	(3.65)	(0.66)	(0.64)	(-0.83)	(2.21)	(2.11)
β_ρ	**-1.018**	**-1.410**	**-1.409**	**-1.531**	**-1.155**	**-1.091**
	(-2.88)	(-3.22)	(-3.20)	(-3.01)	(-3.02)	(-2.79)
$\overline{R}^2$	**9.86%**	5.34%	5.33%	5.50%	7.35%	7.13%
DJIA						
α	**-0.624**	**-0.751**	**-0.490**	0.021	-0.209	**-0.430**
	(-7.89)	(-4.12)	(-5.82)	(0.04)	(-1.57)	(-4.27)
β	**0.401**	0.227	**0.580**	1.632	**0.737**	**0.733**
	(5.57)	(1.85)	(7.25)	(1.78)	(6.85)	(6.35)
β_ρ	**-0.882**	**-1.611**	0.685	**-1.328**	**-0.940**	-0.396
	(-2.31)	(-3.17)	(1.39)	(-2.82)	(-2.11)	(-0.95)
$\overline{R}^2$	19.77%	5.62%	**20.72%**	5.39%	14.93%	20.45%
STOXX 50						
α	**-0.858**	**-1.165**	**-1.329**	**-0.792**	**-0.636**	**-0.750**
	(-11.11)	(-2.60)	(-4.44)	(-1.96)	(-5.28)	(-6.27)
β	**0.216**	-0.055	-0.210	0.396	**0.264**	**0.364**
	(3.83)	(-0.15)	(-0.79)	(0.76)	(4.13)	(3.27)
β_ρ	0.531	0.663	0.725	0.792	**0.778**	**0.764**
	(1.40)	(1.55)	(1.71)	(1.77)	(2.08)	(2.02)
$\overline{R}^2$	5.35%	1.02%	1.22%	1.31%	**6.61%**	5.67%
FTSE 100						
α	**-1.215**	**-0.767**	**-0.777**	**-0.902**	**-1.042**	**-1.016**
	(-12.16)	(-2.01)	(-2.13)	(-4.12)	(-7.61)	(-6.01)
β	0.074	0.365	0.382	0.615	**0.135**	**0.267**
	(1.12)	(1.47)	(1.51)	(1.95)	(2.07)	(1.97)
β_ρ	**-1.705**	**-2.133**	**-2.157**	**-1.728**	**-1.523**	**-1.269**
	(-2.77)	(-3.42)	(-3.45)	(-2.69)	(-2.53)	(-1.99)
$\overline{R}^2$	6.48%	7.21%	7.23%	7.57%	**8.26%**	7.83%

Table B.1 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.480**	0.677	**1.774**	**-0.801**	-0.223	-0.240
	(-4.05)	(1.55)	(2.30)	(-4.50)	(-1.13)	(-1.81)
β	**0.259**	**2.188**	**3.355**	-0.824	**0.344**	**0.619**
	(2.40)	(3.25)	(3.24)	(-1.20)	(2.71)	(4.29)
β_ρ	0.270	0.044	0.122	0.261	-0.076	-0.693
	(0.32)	(0.06)	(0.16)	(0.28)	(-0.09)	(-0.84)
$\overline{R}^2$	6.78%	**17.86%**	16.62%	0.84%	7.61%	16.15%
KOSPI						
α	**-0.650**	**3.395**	**4.514**	**-1.094**	**-0.626**	**-0.688**
	(-6.63)	(2.66)	(3.06)	(-9.00)	(-3.95)	(-5.32)
β	**0.357**	**4.349**	**5.033**	-0.127	**0.485**	**0.385**
	(4.86)	(3.47)	(3.75)	(-0.46)	(3.09)	(3.25)
β_ρ	0.017	1.340	1.284	0.959	0.778	0.767
	(0.02)	(1.60)	(1.58)	(0.86)	(0.85)	(0.82)
$\overline{R}^2$	13.41%	13.34%	**14.30%**	1.10%	9.43%	10.69%
NASDAQ 100						
α	**-0.550**	**-1.069**	**-1.196**	**-0.532**	**-0.208**	**-0.409**
	(-7.54)	(-5.96)	(-7.16)	(-2.02)	(-2.22)	(-4.61)
β	**0.441**	-0.124	-0.320	0.866	**0.564**	**0.715**
	(6.70)	(-0.60)	(-1.53)	(1.65)	(7.66)	(6.69)
β_ρ	-0.466	-0.895	-0.665	-0.702	-0.388	-0.014
	(-1.16)	(-1.49)	(-1.11)	(-1.27)	(-0.90)	(-0.03)
$\overline{R}^2$	20.45%	2.08%	3.20%	4.03%	19.85%	**21.60%**
RUSSELL 2000						
α	**-0.857**	**-1.135**	**-1.155**	**-1.027**	-0.187	**-0.494**
	(-9.62)	(-14.81)	(-15.35)	(-14.91)	(-1.42)	(-4.34)
β	**0.180**	-0.120	-0.147	0.034	**0.615**	**0.629**
	(2.64)	(-1.52)	(-1.95)	(0.32)	(6.50)	(5.18)
β_ρ	-0.498	-0.361	-0.291	-0.579	-0.398	0.025
	(-1.12)	(-0.69)	(-0.57)	(-1.15)	(-0.97)	(0.06)
$\overline{R}^2$	3.80%	1.88%	2.68%	0.68%	**13.34%**	9.60%
S&P 500						
α	**-0.869**	**-1.042**	**-1.170**	-0.020	**-0.413**	**-0.576**
	(-8.96)	(-4.74)	(-5.55)	(-0.05)	(-2.73)	(-5.07)
β	**0.267**	0.099	0.010	**1.642**	**0.461**	**0.616**
	(3.83)	(0.67)	(0.07)	(2.71)	(5.26)	(6.04)
β_ρ	**-1.813**	**-2.736**	**-2.526**	**-2.279**	**-1.551**	**-1.270**
	(-3.53)	(-3.71)	(-3.41)	(-3.89)	(-2.86)	(-2.57)
$\overline{R}^2$	12.79%	6.96%	6.79%	8.54%	15.15%	**16.46%**
Aggregated Results						
Average α	-0.790	-0.385	-0.180	-0.712	-0.528	-0.647
Average β	0.256	0.712	0.889	0.443	0.394	0.469
Average β_ρ	-0.727	-0.904	-0.618	-0.801	-0.627	-0.476
Average $\overline{R}^2$	10.72%	6.78%	8.46%	4.32%	11.09%	**12.34%**

Table B.2 Information Content of Skewness Forecasts (60 days) - Alternative Implied Skewness

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 60 calendar days. Implied and realized skewness are estimated as described in section 3.4.1. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-1.302**	-0.963	-0.980	**-1.156**	**-1.032**	**-1.033**
	(-10.61)	(-1.87)	(-1.93)	(-6.59)	(-5.87)	(-8.34)
β	-0.096	0.126	0.117	0.027	0.110	0.139
	(-1.20)	(0.44)	(0.42)	(0.21)	(0.99)	(1.46)
β_ρ	**-2.589**	**-2.480**	**-2.476**	**-2.433**	**-2.286**	**-2.135**
	(-3.77)	(-3.47)	(-3.46)	(-3.15)	(-3.61)	(-3.28)
$\overline{R}^2$	14.30%	13.59%	13.58%	13.51%	13.97%	**14.56%**
DAX						
α	**-1.080**	**-1.005**	**-1.002**	**-1.426**	**-1.126**	**-1.127**
	(-11.27)	(-9.04)	(-8.86)	(-10.34)	(-11.11)	(-11.56)
β	0.095	0.132	0.135	-0.239	0.051	0.063
	(1.65)	(1.92)	(1.91)	(-1.76)	(0.89)	(0.84)
β_ρ	-0.867	**-1.271**	**-1.273**	**-1.455**	-0.910	-0.869
	(-1.84)	(-2.16)	(-2.15)	(-2.16)	(-1.88)	(-1.74)
$\overline{R}^2$	3.71%	5.21%	5.21%	**5.47%**	3.08%	3.14%
DJIA						
α	**-0.794**	**-1.057**	**-0.748**	**-1.110**	**-0.561**	**-0.679**
	(-6.97)	(-4.11)	(-8.49)	(-5.86)	(-3.05)	(-5.68)
β	**0.364**	0.085	**0.452**	0.168	**0.589**	**0.535**
	(4.12)	(0.73)	(6.07)	(0.69)	(4.13)	(5.30)
β_ρ	-0.790	**-1.558**	0.859	**-1.588**	-0.690	-0.375
	(-1.79)	(-2.72)	(1.54)	(-2.60)	(-1.29)	(-0.87)
$\overline{R}^2$	16.70%	4.90%	**25.91%**	4.97%	12.29%	18.42%
STOXX 50						
α	**-1.084**	-0.745	**-0.866**	**-1.379**	**-1.034**	**-1.105**
	(-9.67)	(-1.90)	(-2.30)	(-3.45)	(-6.13)	(-8.94)
β	0.128	0.287	0.243	-0.141	0.134	0.128
	(1.74)	(1.29)	(1.03)	(-0.34)	(1.39)	(1.18)
β_ρ	-0.076	-0.303	-0.257	-0.025	0.010	0.046
	(-0.10)	(-0.37)	(-0.31)	(-0.03)	(0.01)	(0.06)
$\overline{R}^2$	**1.53%**	0.89%	0.49%	-0.07%	1.19%	0.88%
FTSE 100						
α	**-1.370**	**-0.923**	**-1.030**	**-1.594**	**-1.206**	**-1.162**
	(-8.41)	(-2.10)	(-2.48)	(-8.72)	(-5.97)	(-6.52)
β	0.032	0.221	0.190	-0.209	0.123	0.202
	(0.37)	(1.10)	(0.91)	(-1.22)	(1.17)	(1.64)
β_ρ	-1.515	**-1.821**	**-1.790**	-0.950	-1.175	-1.103
	(-1.79)	(-2.22)	(-2.18)	(-0.98)	(-1.47)	(-1.45)
$\overline{R}^2$	3.94%	4.75%	4.50%	5.14%	5.48%	**6.68%**

Table B.2 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.574**	0.634	1.763	-0.120	**-0.514**	**-0.377**
	(-4.32)	(0.99)	(1.46)	(-0.35)	(-2.59)	(-2.99)
β	0.178	**1.584**	**2.466**	2.293	0.162	**0.473**
	(1.28)	(2.03)	(2.01)	(1.64)	(1.58)	(3.49)
β_ρ	0.107	-0.148	-0.132	-0.137	0.077	-0.443
	(0.15)	(-0.32)	(-0.28)	(-0.27)	(0.11)	(-0.81)
$\overline{R}^2$	2.91%	17.00%	**17.20%**	8.95%	1.90%	10.47%
KOSPI 200						
α	**-0.927**	**3.215**	**3.853**	**-0.879**	**-0.782**	**-0.656**
	(-8.25)	(2.69)	(2.68)	(-5.19)	(-6.32)	(-6.33)
β	0.083	**3.044**	**3.295**	0.305	**0.291**	**0.405**
	(0.98)	(3.45)	(3.31)	(0.92)	(2.08)	(4.19)
β_ρ	-1.510	**-2.176**	**-2.138**	-1.517	-1.469	-0.410
	(-1.23)	(-2.02)	(-2.00)	(-1.15)	(-1.21)	(-0.36)
$\overline{R}^2$	2.43%	11.21%	**11.22%**	3.20%	3.68%	10.12%
NASDAQ 100						
α	**-0.522**	**-1.378**	**-1.388**	**-0.777**	-0.137	**-0.376**
	(-5.09)	(-5.82)	(-6.39)	(-4.08)	(-1.33)	(-3.88)
β	**0.547**	-0.220	-0.275	**0.656**	**0.777**	**0.792**
	(7.41)	(-1.19)	(-1.38)	(2.03)	(9.89)	(8.66)
β_ρ	-0.245	-0.687	-0.587	-0.401	0.240	0.666
	(-0.54)	(-0.98)	(-0.84)	(-0.50)	(0.62)	(1.68)
$\overline{R}^2$	31.66%	3.38%	3.96%	6.10%	36.72%	**40.46%**
RUSSELL 2000						
α	**-1.021**	**-1.602**	**-1.597**	**-1.471**	**-0.530**	**-0.768**
	(-6.75)	(-17.72)	(-18.22)	(-16.95)	(-2.24)	(-4.32)
β	**0.200**	**-0.269**	**-0.270**	**-0.219**	**0.527**	**0.450**
	(2.03)	(-3.98)	(-4.07)	(-3.54)	(3.39)	(3.26)
β_ρ	-0.159	0.410	0.437	-0.019	0.205	0.266
	(-0.33)	(0.88)	(0.94)	(-0.04)	(0.44)	(0.62)
$\overline{R}^2$	4.15%	11.36%	**12.03%**	7.41%	10.93%	8.81%
S&P 500						
α	**-0.910**	**-1.578**	**-1.595**	**-1.474**	-0.267	**-0.604**
	(-7.20)	(-5.06)	(-5.52)	(-11.66)	(-1.51)	(-4.45)
β	**0.357**	-0.086	-0.105	-0.103	**0.725**	**0.645**
	(4.79)	(-0.61)	(-0.72)	(-0.81)	(6.80)	(6.69)
β_ρ	**-1.436**	**-2.199**	**-2.144**	**-2.309**	-0.750	-0.644
	(-2.56)	(-3.07)	(-2.97)	(-3.56)	(-1.36)	(-1.18)
$\overline{R}^2$	21.30%	10.30%	10.39%	10.38%	27.26%	**27.41%**
Aggregated Results						
Average α	-0.958	-0.540	-0.359	-1.138	-0.719	-0.789
Average β	0.189	0.490	0.625	0.254	0.349	0.383
Average β_ρ	-0.908	-1.223	-0.950	-1.083	-0.675	-0.500
Average $\overline{R}^2$	10.26%	8.26%	10.45%	6.51%	11.65%	**14.09%**

Table B.3 Information Content of Skewness Forecasts (90 days) - Alternative Implied-Skewness

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 90 calendar days. Implied and realized skewness are estimated as described in section 3.4.1. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^{ρ} across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-1.209**	**-1.174**	**-1.187**	**-1.345**	**-1.028**	**-1.016**
	(-9.83)	(-2.98)	(-3.03)	(-8.53)	(-6.86)	(-10.84)
β	0.114	0.080	0.073	0.009	**0.209**	**0.260**
	(1.23)	(0.48)	(0.45)	(0.09)	(2.12)	(3.56)
β_{ρ}	**-1.388**	**-1.792**	**-1.786**	**-1.699**	**-1.535**	**-1.319**
	(-2.07)	(-2.71)	(-2.71)	(-2.52)	(-2.67)	(-2.45)
$\overline{R}^2$	10.76%	9.83%	9.81%	9.69%	12.27%	**15.96%**
DAX						
α	**-0.996**	**-1.071**	**-1.051**	**-1.768**	**-1.142**	**-1.035**
	(-6.38)	(-8.04)	(-7.47)	(-7.35)	(-10.89)	(-10.54)
β	**0.258**	**0.134**	**0.142**	-0.487	0.132	**0.229**
	(2.20)	(2.14)	(2.16)	(-1.86)	(1.79)	(3.05)
β_{ρ}	-0.051	-0.564	-0.572	-0.758	-0.194	-0.191
	(-0.12)	(-0.98)	(-0.99)	(-1.17)	(-0.39)	(-0.41)
$\overline{R}^2$	**6.70%**	3.96%	4.09%	4.11%	2.08%	6.69%
DJIA						
α	**-0.918**	**-1.663**	**-0.952**	**-1.527**	**-0.594**	**-0.731**
	(-6.55)	(-5.49)	(-12.88)	(-9.63)	(-2.52)	(-5.58)
β	**0.327**	-0.113	**0.318**	-0.209	**0.621**	**0.506**
	(3.54)	(-1.07)	(5.73)	(-1.28)	(3.45)	(5.10)
β_{ρ}	-0.120	-0.128	**0.847**	0.018	0.213	-0.076
	(-0.35)	(-0.29)	(2.38)	(0.04)	(0.56)	(-0.26)
$\overline{R}^2$	11.53%	1.77%	**25.42%**	3.09%	10.62%	17.50%
STOXX 50						
α	**-1.005**	**-1.116**	**-1.129**	**-1.226**	**-1.008**	**-1.031**
	(-8.65)	(-3.06)	(-2.93)	(-3.56)	(-5.86)	(-8.45)
β	**0.276**	0.118	0.124	0.174	**0.221**	**0.275**
	(3.27)	(0.72)	(0.65)	(0.43)	(2.18)	(2.50)
β_{ρ}	0.262	0.037	0.043	0.215	0.252	0.432
	(0.41)	(0.05)	(0.05)	(0.28)	(0.34)	(0.60)
$\overline{R}^2$	**7.75%**	0.47%	0.40%	0.20%	4.21%	7.20%
FTSE 100						
α	**-1.325**	**-1.524**	**-1.537**	**-1.792**	**-1.289**	**-1.262**
	(-7.21)	(-3.51)	(-3.55)	(-9.12)	(-6.45)	(-8.08)
β	0.168	0.021	0.017	-0.263	0.176	**0.226**
	(1.85)	(0.13)	(0.10)	(-1.22)	(1.75)	(2.49)
β_{ρ}	-0.250	-0.480	-0.472	0.177	-0.083	-0.012
	(-0.31)	(-0.60)	(-0.59)	(0.18)	(-0.10)	(-0.02)
$\overline{R}^2$	2.84%	0.22%	0.22%	2.06%	3.54%	**6.47%**

Table B.3 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.790**	0.929	2.006	**-1.709**	**-0.770**	**-0.604**
	(-5.89)	(1.45)	(1.92)	(-3.43)	(-3.99)	(-5.02)
β	-0.009	**1.655**	**2.275**	-1.702	0.011	**0.204**
	(-0.06)	(2.71)	(2.69)	(-1.86)	(0.12)	(5.36)
β_ρ	2.370	1.574	1.589	2.065	2.362	1.553
	(1.58)	(1.77)	(1.77)	(1.54)	(1.55)	(1.26)
$\overline{R}^2$	7.54%	**29.26%**	28.91%	17.33%	7.55%	18.40%
KOSPI 200						
α	**-0.961**	**3.215**	**3.977**	**-0.710**	**-1.018**	**-0.810**
	(-4.99)	(3.21)	(3.33)	(-4.72)	(-5.91)	(-4.78)
β	0.104	**2.583**	**2.790**	**0.590**	0.061	**0.290**
	(0.57)	(4.17)	(4.12)	(4.34)	(0.38)	(2.45)
β_ρ	-0.528	-1.481	-1.411	-1.311	-0.691	0.403
	(-0.48)	(-1.56)	(-1.48)	(-1.29)	(-0.63)	(0.41)
$\overline{R}^2$	1.04%	15.69%	**15.89%**	11.00%	0.34%	5.41%
NASDAQ 100						
α	**-0.528**	**-1.751**	**-1.762**	-0.352	-0.248	**-0.534**
	(-3.86)	(-7.13)	(-7.60)	(-0.81)	(-1.51)	(-3.94)
β	**0.678**	-0.199	-0.243	**1.464**	**0.859**	**0.703**
	(9.26)	(-1.30)	(-1.46)	(2.73)	(9.09)	(9.37)
β_ρ	0.656	0.748	0.805	1.126	0.741	**0.877**
	(1.42)	(1.03)	(1.13)	(1.65)	(1.85)	(2.19)
$\overline{R}^2$	47.50%	2.77%	3.46%	10.04%	44.39%	**49.21%**
RUSSELL 2000						
α	**-0.916**	**-1.920**	**-1.915**	**-1.836**	-0.486	**-0.837**
	(-5.56)	(-16.94)	(-16.79)	(-19.83)	(-1.89)	(-4.10)
β	**0.404**	**-0.281**	**-0.281**	**-0.318**	**0.676**	**0.483**
	(4.05)	(-3.78)	(-3.69)	(-4.37)	(4.28)	(3.63)
β_ρ	0.671	**1.262**	**1.253**	**1.327**	0.743	0.916
	(1.20)	(2.19)	(2.17)	(2.19)	(1.52)	(1.78)
$\overline{R}^2$	17.19%	17.20%	17.02%	17.84%	**20.82%**	16.81%
S&P 500						
α	**-0.902**	**-2.254**	**-2.257**	**-1.910**	-0.337	**-0.689**
	(-6.02)	(-6.53)	(-6.63)	(-10.50)	(-1.75)	(-5.09)
β	**0.459**	-0.231	-0.258	**-0.294**	**0.806**	**0.632**
	(5.76)	(-1.91)	(-1.94)	(-2.00)	(7.09)	(7.91)
β_ρ	-0.785	-1.237	-1.195	-1.505	-0.456	-0.268
	(-1.14)	(-1.33)	(-1.28)	(-1.72)	(-0.70)	(-0.41)
$\overline{R}^2$	25.45%	8.41%	8.55%	8.40%	30.86%	**32.76%**
Aggregated Results						
Average α	-0.955	-0.833	-0.581	-1.418	-0.792	-0.855
Average β	0.278	0.377	0.496	-0.104	0.377	0.381
Average β_ρ	0.084	-0.206	-0.090	-0.034	0.135	0.232
Average $\overline{R}^2$	13.83%	8.96%	11.38%	8.38%	13.67%	**17.64%**

Table B.4 Encompassing Regressions - Alternative Implied-Skewness

This table reports the results from regressing the realized skewness on forecasts generated from the LRS, GARCH-2, QMIDAS and CIS models, within the same regression, for each index in Table 3.1. Implied and realized skewness are estimated as described in section 3.4.1. The GARCH-2 and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β_i respectively denote the intercept and the coefficient of the forecast of model i in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. Panel A, B and C respectively present results for a forecasting horizon of 30, 60 and 90 calendar days. (1) - (10) are the ten international indices of order listed in Table 3.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Panel A: 30 days					
α	-0.272	**-0.666**	**1.095**	-0.836	0.087	1.288	**3.330**	0.083	**-0.507**	**0.881**
	(-0.44)	(-2.10)	(2.67)	(-1.36)	(0.25)	(1.72)	(2.58)	(0.19)	(-3.96)	(2.37)
β_{ρ}	**-1.393**	**-1.231**	0.396	0.601	-1.172	-0.447	0.554	0.178	0.418	-0.559
	(-2.61)	(-2.68)	(1.08)	(1.51)	(-1.85)	(-0.55)	(0.83)	(0.41)	(0.88)	(-0.90)
β_{LRS}	**0.164**	**0.204**	**0.216**	**0.156**	0.066	0.038	**0.193**	**0.268**	0.062	**0.177**
	(2.32)	(3.65)	(2.72)	(2.67)	(1.10)	(0.34)	(2.74)	(3.80)	(0.83)	(2.50)
$\beta_{GARCH-2}$	-0.013	0.128	**0.327**	-0.037	0.370	**2.238**	**3.636**	0.062	**-0.203**	-0.097
	(-0.02)	(1.32)	(4.91)	(-0.16)	(1.40)	(2.35)	(3.03)	(0.29)	(-2.37)	(-0.72)
β_{QMIDAS}	**0.579**	-0.113	**1.969**	-0.183	0.569	0.252	-0.084	0.763	**0.283**	**2.128**
	(2.93)	(-0.33)	(3.26)	(-0.31)	(1.66)	(0.39)	(-0.37)	(1.30)	(2.53)	(4.19)
β_{CIS}	0.149	**0.172**	**0.352**	**0.282**	**0.374**	**0.377**	**0.205**	**0.470**	**0.540**	**0.483**
	(1.15)	(2.26)	(3.39)	(2.41)	(2.93)	(2.83)	(1.98)	(4.46)	(4.22)	(4.66)
$\overline{R}^2$	11.91%	12.29%	31.89%	7.48%	11.54%	21.22%	22.32%	27.69%	12.30%	20.54%
					Panel B: 60 days					
α	-0.958	**-0.971**	**-0.544**	-0.775	-0.455	**2.438**	2.722	-0.258	**-1.249**	-0.636
	(-1.26)	(-3.47)	(-3.47)	(-1.59)	(-0.94)	(2.03)	(1.79)	(-1.50)	(-7.15)	(-1.93)
β_{ρ}	**-2.408**	**-1.247**	0.814	-0.393	-0.930	-0.311	-0.798	**1.163**	0.790	-0.463
	(-3.35)	(-2.23)	(1.44)	(-0.58)	(-1.14)	(-0.57)	(-0.67)	(2.67)	(1.88)	(-0.77)
β_{LRS}	-0.134	0.112	**0.175**	0.135	0.001	-0.071	0.018	**0.200**	-0.075	**0.150**
	(-1.67)	(1.74)	(2.28)	(1.73)	(0.02)	(-0.40)	(0.32)	(2.28)	(-0.82)	(2.22)
$\beta_{GARCH-2}$	0.106	0.103	**0.339**	**0.637**	**0.373**	**3.579**	**2.278**	-0.162	**-0.279**	-0.059
	(0.23)	(1.17)	(4.78)	(2.18)	(2.00)	(2.51)	(2.07)	(-1.39)	(-2.32)	(-0.30)
β_{QMIDAS}	-0.013	-0.124	-0.008	-0.834	-0.180	-2.383	0.195	**0.563**	0.053	0.039
	(-0.07)	(-0.72)	(-0.05)	(-1.60)	(-1.34)	(-1.90)	(0.62)	(2.87)	(0.51)	(0.24)
β_{CIS}	0.194	0.062	0.118	0.089	**0.279**	**0.322**	**0.322**	**0.566**	**0.362**	**0.520**
	(1.85)	(0.87)	(1.56)	(0.82)	(2.03)	(2.37)	(3.09)	(5.68)	(3.11)	(5.94)
$\overline{R}^2$	15.98%	7.12%	29.30%	3.98%	9.94%	21.12%	15.98%	44.72%	15.93%	28.72%
					Panel C: 90 days					
α	-0.518	**-0.902**	**-0.973**	-0.211	-0.577	0.740	4.080	0.015	**-1.231**	**-0.685**
	(-1.49)	(-2.51)	(-5.74)	(-0.35)	(-1.46)	(0.65)	(1.91)	(0.08)	(-6.25)	(-2.17)
β_{ρ}	**-1.622**	-0.610	**1.032**	0.072	0.154	1.146	-0.913	**1.089**	**1.234**	-0.145
	(-2.98)	(-1.44)	(2.77)	(0.14)	(0.21)	(1.60)	(-0.95)	(3.48)	(2.93)	(-0.25)
β_{LRS}	0.003	0.174	0.071	**0.218**	0.096	-0.220	-0.063	**0.352**	0.129	**0.171**
	(0.03)	(1.94)	(0.95)	(2.85)	(0.94)	(-1.70)	(-0.50)	(4.43)	(1.32)	(2.19)
$\beta_{GARCH-2}$	0.188	0.105	**0.266**	0.282	**0.293**	1.501	**2.835**	-0.059	-0.103	-0.004
	(1.21)	(1.36)	(5.38)	(1.61)	(2.17)	(1.86)	(2.20)	(-0.68)	(-1.07)	(-0.02)
β_{QMIDAS}	0.014	-0.280	-0.189	0.079	-0.272	-0.574	-0.072	**0.756**	-0.095	-0.044
	(0.17)	(-1.05)	(-1.87)	(0.19)	(-1.82)	(-0.90)	(-0.29)	(3.08)	(-0.90)	(-0.25)
β_{CIS}	**0.286**	**0.189**	0.087	**0.185**	**0.219**	**0.167**	0.145	**0.385**	**0.248**	**0.494**
	(3.42)	(2.94)	(1.04)	(1.96)	(2.43)	(3.05)	(1.32)	(5.40)	(2.22)	(7.22)
$\overline{R}^2$	16.78%	15.05%	29.02%	11.71%	9.71%	33.69%	16.68%	56.70%	24.84%	34.40%

Table B.5 Model Confidence Set under Root Mean Squared Errors - Alternative Implied-Skewness

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. Implied and realized skewness are estimated as described in section 3.4.1. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.01	2*	0.668	3	0.01	2	0.00	3	0.00	5	0.01	3*	0.475	2	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	3*	0.324	1*	1.000	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.445	6	0.04	6	0.00	6	0.00	6	0.00
QMIDAS	3	0.01	3	0.00	4	0.00	3	0.00	4	0.00	4*	0.085	5	0.04	4	0.00	4	0.00	4	0.00
IS	2*	0.218	4	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	2*	0.475	3	0.00	2	0.00	2	0.00
CIS	1*	1.000	1*	1.000	2*	0.578	1*	1.000	1*	1.000	1*	1.000	4	0.04	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	2*	0.559	3	0.04	2*	0.140	2*	0.052	5*	0.071	3*	0.710	3	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.071	2*	0.710	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.528	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	4*	0.071	5	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.649	3*	0.087	1*	1.000	3	0.00	4	0.00	6	0.00	4*	0.088	2	0.00	2*	0.202	1*	1.000
CIS	1*	1.000	1*	1.000	2*	0.285	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.248
Panel C: 90-day horizon																				
LRS	3*	0.149	1*	1.000	3	0.05	1*	1.000	1*	1.000	5*	0.051	4	0.01	3*	0.163	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.593	5	0.01	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	4	0.00	1*	1.000	6	0.01	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	5	0.00	2*	0.593	3	0.01	4	0.00	4	0.00	4	0.00
IS	2*	0.151	3*	0.099	1*	1.000	3	0.00	3	0.00	6	0.00	2	0.01	1*	1.000	1*	1.000	1*	1.000
CIS	1*	1.000	2*	0.114	2*	0.449	2*	0.085	2*	0.761	4*	0.126	1*	1.000	2*	0.163	2	0.00	2	0.01

Table B.6 Model Confidence Set under Mean Absolute Errors - Alternative Implied-Skewness

This table reports the results for the Model Confidence Set (MCS) test using the Mean Absolute Error (MAE) as a loss function and a 5% significance level. Implied and realized skewness are estimated as described in section 3.4.1. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.02	2*	0.262	3	0.00	2	0.00	3	0.00	5	0.00	4*	0.095	2	0.01	2	0.00	2	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	3*	0.413	2*	0.948	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.413	6	0.02	6	0.00	6	0.00	6	0.00
QMIDAS	3	0.02	3	0.00	4	0.00	3	0.00	4	0.00	4*	0.413	5	0.02	4	0.00	4	0.00	4	0.00
IS	2	0.03	4	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	1*	1.000	3	0.00	3	0.00	3	0.00
CIS	1*	1.000	1*	1.000	2*	0.693	1*	1.000	1*	1.000	1*	1.000	3*	0.095	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	3	0.02	3	0.00	2*	0.223	2	0.01	5*	0.412	3*	0.723	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.468	4*	0.723	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.468	6	0.00	5	0.00	5	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	4*	0.468	5	0.00	4	0.00	4	0.00	5	0.00
IS	2*	0.070	2*	0.063	1*	1.000	3	0.00	4	0.00	6	0.00	2*	0.916	3	0.00	2	0.04	2*	0.935
CIS	1*	1.000	1*	1.000	2	0.03	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000
Panel C: 90-day horizon																				
LRS	3*	0.947	1*	1.000	3	0.01	1*	1.000	2*	0.612	5	0.04	3	0.02	3*	0.113	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.278	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	4	0.04	4	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.947	2*	0.548	1*	1.000	3	0.00	3	0.03	6	0.00	2	0.02	1*	1.000	1*	1.000	1*	1.000
CIS	1*	1.000	3*	0.548	2*	0.339	2*	0.292	1*	1.000	2*	0.797	1*	1.000	2*	0.160	2	0.00	2	0.00

Table B.7 Information Content of Skewness Forecasts (30 days) - Alternative GARCH-2 Specification

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 30 calendar days. The GARCH-2 model is estimated as described in section 3.4.2. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^{ρ} across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.932**	-0.875	-0.810	**-0.715**	**-0.836**	**-0.908**
	(-11.42)	(-1.31)	(-1.25)	(-3.55)	(-6.68)	(-7.57)
β	**0.165**	0.201	0.240	**0.432**	**0.224**	**0.218**
	(2.39)	(0.36)	(0.47)	(2.03)	(2.61)	(2.01)
β_{ρ}	**-1.776**	**-2.072**	**-2.083**	**-1.979**	**-1.863**	**-1.599**
	(-4.00)	(-4.13)	(-4.17)	(-3.96)	(-4.08)	(-3.12)
$\overline{R}^2$	10.00%	7.65%	7.70%	9.17%	9.99%	**10.05%**
DAX						
α	**-0.811**	**-0.998**	**-0.983**	**-1.198**	**-0.941**	**-0.951**
	(-11.68)	(-11.10)	(-8.48)	(-6.46)	(-10.55)	(-11.29)
β	**0.221**	0.055	0.051	-0.210	0.072	0.107
	(3.77)	(0.70)	(0.65)	(-0.80)	(1.23)	(1.25)
β_{ρ}	**-1.196**	**-1.611**	**-1.594**	**-1.692**	**-1.407**	**-1.350**
	(-3.22)	(-3.42)	(-3.44)	(-3.21)	(-3.36)	(-3.17)
$\overline{R}^2$	**9.86%**	6.71%	6.67%	6.78%	7.05%	7.09%
DJIA						
α	**-0.621**	**-0.716**	**-0.757**	-0.224	**-0.299**	**-0.467**
	(-8.32)	(-4.22)	(-4.54)	(-0.38)	(-2.97)	(-5.26)
β	**0.375**	0.213	0.169	1.176	**0.666**	**0.676**
	(5.30)	(1.80)	(1.57)	(1.30)	(7.33)	(5.98)
β_{ρ}	**-0.760**	**-1.429**	**-1.378**	**-1.134**	**-0.825**	-0.359
	(-2.22)	(-3.09)	(-3.03)	(-2.62)	(-2.15)	(-1.00)
$\overline{R}^2$	17.06%	4.64%	4.18%	3.82%	13.53%	**18.67%**
STOXX 50						
α	**-0.845**	**-1.296**	**-1.489**	-0.690	**-0.672**	**-0.768**
	(-11.24)	(-2.89)	(-5.30)	(-1.73)	(-5.86)	(-6.62)
β	**0.224**	-0.173	-0.277	0.519	**0.246**	**0.341**
	(4.02)	(-0.46)	(-1.47)	(1.01)	(3.82)	(3.09)
β_{ρ}	0.414	0.525	0.569	0.655	0.650	0.630
	(1.17)	(1.31)	(1.46)	(1.54)	(1.80)	(1.71)
$\overline{R}^2$	5.29%	0.61%	1.07%	1.01%	**5.31%**	4.62%
FTSE 100						
α	**-1.193**	**-0.842**	**-0.835**	**-0.958**	**-1.030**	**-0.984**
	(-11.87)	(-2.21)	(-2.05)	(-4.25)	(-7.72)	(-5.95)
β	0.081	0.305	0.269	0.510	**0.139**	**0.288**
	(1.23)	(1.23)	(1.16)	(1.56)	(2.13)	(2.15)
β_{ρ}	**-1.632**	**-2.025**	**-1.993**	**-1.688**	**-1.444**	-1.170
	(-2.60)	(-3.07)	(-3.05)	(-2.53)	(-2.34)	(-1.79)
$\overline{R}^2$	6.61%	6.84%	6.74%	7.09%	**8.39%**	8.23%

Table B.7 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.479**	0.665	0.471	**-0.811**	-0.237	-0.239
	(-4.16)	(1.55)	(1.36)	(-4.55)	(-1.18)	(-1.70)
β	**0.258**	**2.167**	**2.197**	-0.892	**0.335**	**0.630**
	(2.45)	(3.27)	(3.46)	(-1.29)	(2.56)	(3.85)
β_ρ	0.196	-0.053	-0.062	0.132	-0.141	-0.818
	(0.24)	(-0.07)	(-0.08)	(0.15)	(-0.17)	(-0.99)
$\overline{R}^2$	6.55%	18.01%	**18.13%**	0.90%	6.89%	15.35%
KOSPI 200						
α	**-0.524**	0.873	0.577	**-1.061**	**-0.520**	**-0.626**
	(-6.61)	(0.63)	(0.44)	(-8.85)	(-3.71)	(-5.49)
β	**0.406**	1.725	1.346	-0.377	**0.515**	**0.349**
	(5.16)	(1.27)	(1.11)	(-1.44)	(3.04)	(2.75)
β_ρ	0.039	0.715	0.732	0.623	0.619	0.760
	(0.06)	(0.85)	(0.84)	(0.65)	(0.83)	(0.99)
$\overline{R}^2$	**16.87%**	3.28%	2.30%	2.67%	10.99%	8.48%
NASDAQ 100						
α	**-0.556**	**-1.059**	**-1.162**	**-0.523**	**-0.306**	**-0.438**
	(-7.74)	(-5.78)	(-7.85)	(-1.96)	(-3.24)	(-5.09)
β	**0.430**	-0.124	-0.281	0.862	**0.497**	**0.673**
	(6.42)	(-0.59)	(-1.63)	(1.62)	(6.73)	(6.46)
β_ρ	-0.486	-0.984	-0.773	-0.803	-0.465	-0.086
	(-1.21)	(-1.63)	(-1.31)	(-1.45)	(-1.07)	(-0.19)
$\overline{R}^2$	**19.87%**	2.46%	3.50%	4.29%	17.56%	19.67%
RUSSELL 2000						
α	**-0.837**	**-1.108**	**-1.012**	**-1.016**	**-0.269**	**-0.515**
	(-10.02)	(-16.19)	(-14.41)	(-15.58)	(-2.31)	(-4.80)
β	**0.186**	-0.108	0.020	0.024	**0.571**	**0.587**
	(2.81)	(-1.44)	(0.34)	(0.22)	(6.59)	(5.22)
β_ρ	-0.502	-0.399	-0.515	-0.587	-0.435	0.011
	(-1.20)	(-0.81)	(-1.03)	(-1.23)	(-1.09)	(0.03)
$\overline{R}^2$	4.14%	1.81%	0.85%	0.78%	**12.43%**	9.03%
SP500						
α	**-0.872**	**-1.081**	**-1.195**	-0.044	**-0.440**	**-0.589**
	(-9.26)	(-4.96)	(-6.00)	(-0.10)	(-2.94)	(-5.21)
β	**0.266**	0.073	-0.006	**1.609**	**0.459**	**0.608**
	(3.75)	(0.50)	(-0.05)	(2.65)	(5.11)	(5.86)
β_ρ	**-1.723**	**-2.573**	**-2.383**	**-2.178**	**-1.517**	**-1.206**
	(-3.38)	(-3.59)	(-3.39)	(-3.84)	(-2.91)	(-2.50)
$\overline{R}^2$	12.58%	6.69%	6.60%	8.26%	14.70%	**15.82%**
Aggregated Results						
Average α	-0.767	-0.644	-0.719	-0.724	-0.555	-0.649
Average β	0.261	0.433	0.373	0.366	0.372	0.448
Average β_ρ	-0.743	-0.991	-0.948	-0.865	-0.683	-0.519
Average $\overline{R}^2$	10.88%	5.87%	5.77%	4.48%	10.68%	**11.70%**

Table B.8 Information Content of Skewness Forecasts (60 days) - Alternative GARCH-2 Specification

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 60 calendar days. The GARCH-2 model is estimated as described in section 3.4.2. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-1.156**	-0.817	-0.784	**-1.058**	**-0.856**	**-0.886**
	(-9.54)	(-1.55)	(-1.50)	(-5.89)	(-5.61)	(-8.69)
β	0.002	0.192	0.185	0.085	**0.229**	**0.253**
	(0.02)	(0.64)	(0.71)	(0.59)	(2.14)	(3.06)
β_ρ	**-2.669**	**-2.826**	**-2.837**	**-2.852**	**-2.422**	**-2.135**
	(-3.92)	(-3.92)	(-3.95)	(-3.73)	(-3.94)	(-3.38)
$\overline{R}^2$	19.70%	19.92%	19.96%	19.94%	22.01%	**23.64%**
DAX						
α	**-1.057**	**-1.065**	**-0.999**	**-1.324**	**-1.132**	**-1.119**
	(-11.20)	(-11.57)	(-8.08)	(-9.73)	(-13.41)	(-14.27)
β	0.107	0.080	0.092	-0.151	0.039	0.061
	(1.79)	(1.26)	(1.42)	(-1.28)	(0.83)	(1.03)
β_ρ	-0.765	**-1.155**	**-1.169**	**-1.302**	-0.867	-0.829
	(-1.61)	(-1.98)	(-2.00)	(-2.00)	(-1.74)	(-1.64)
$\overline{R}^2$	**3.57%**	3.39%	3.55%	3.51%	2.71%	2.91%
DJIA						
α	**-0.789**	**-0.906**	**-0.910**	**-0.953**	**-0.826**	**-0.806**
	(-7.85)	(-4.09)	(-4.12)	(-5.72)	(-5)	(-6.90)
β	**0.309**	0.113	0.106	0.257	**0.293**	**0.357**
	(3.66)	(1.09)	(1.07)	(1.13)	(2.72)	(2.94)
β_ρ	**-1.169**	**-1.962**	**-1.958**	**-2.049**	**-1.414**	**-1.083**
	(-2.39)	(-3.25)	(-3.22)	(-3.08)	(-2.47)	(-2.10)
$\overline{R}^2$	**13.79%**	6.15%	6.14%	6.51%	8.14%	11.80%
STOXX 50						
α	**-1.078**	**-1.017**	**-1.103**	**-1.517**	**-1.030**	**-1.097**
	(-9.97)	(-2.73)	(-2.80)	(-4.22)	(-7.26)	(-9.95)
β	0.113	0.115	0.055	-0.317	0.124	0.111
	(1.62)	(0.54)	(0.29)	(-0.84)	(1.55)	(1.21)
β_ρ	-0.126	-0.210	-0.163	-0.063	-0.015	-0.040
	(-0.18)	(-0.28)	(-0.22)	(-0.09)	(-0.02)	(-0.05)
$\overline{R}^2$	1.23%	0.10%	-0.03%	0.16%	**1.29%**	0.74%
FTSE 100						
α	**-1.319**	**-1.223**	**-1.255**	**-1.529**	**-1.140**	**-1.088**
	(-8.39)	(-3.16)	(-2.98)	(-8.66)	(-7.23)	(-8.20)
β	0.049	0.075	0.053	-0.170	0.151	**0.240**
	(0.56)	(0.42)	(0.31)	(-0.99)	(1.73)	(2.52)
β_ρ	**-1.654**	**-1.799**	**-1.774**	-1.183	-1.177	-1.161
	(-2.12)	(-2.39)	(-2.35)	(-1.26)	(-1.60)	(-1.72)
$\overline{R}^2$	5.70%	5.57%	5.52%	6.22%	8.23%	**10.03%**

Table B.8 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.533**	0.649	0.562	-0.112	**-0.515**	**-0.382**
	(-3.78)	(0.98)	(0.92)	(-0.32)	(-2.44)	(-2.43)
β	0.207	1.577	**1.636**	2.241	0.147	**0.462**
	(1.44)	(1.96)	(1.97)	(1.56)	(1.32)	(2.69)
β_ρ	-0.156	-0.330	-0.341	-0.299	-0.120	-0.477
	(-0.25)	(-0.72)	(-0.74)	(-0.60)	(-0.18)	(-0.85)
$\overline{R}^2$	3.74%	16.71%	**17.33%**	8.65%	1.41%	8.04%
KOSPI 200						
α	**-0.620**	1.405	1.787	**-0.948**	**-0.694**	**-0.686**
	(-8.37)	(1.07)	(1.23)	(-7.30)	(-9.33)	(-7.99)
β	**0.322**	1.667	1.836	-0.131	**0.298**	**0.290**
	(2.99)	(1.71)	(1.80)	(-0.50)	(2.50)	(2.31)
β_ρ	-0.870	-1.608	-1.634	-1.342	-1.020	-0.918
	(-0.68)	(-1.21)	(-1.23)	(-1.02)	(-0.74)	(-0.69)
$\overline{R}^2$	**8.10%**	5.09%	5.71%	1.34%	4.64%	6.24%
NASDAQ 100						
α	**-0.547**	**-1.330**	**-1.341**	**-0.811**	**-0.290**	**-0.438**
	(-5.49)	(-5.62)	(-6.43)	(-4.11)	(-2.79)	(-4.24)
β	**0.520**	-0.186	-0.225	0.573	**0.657**	**0.701**
	(7.42)	(-1.01)	(-1.25)	(1.74)	(9.00)	(7.69)
β_ρ	-0.316	-0.821	-0.736	-0.557	-0.024	0.313
	(-0.68)	(-1.16)	(-1.04)	(-0.73)	(-0.06)	(0.73)
$\overline{R}^2$	28.79%	3.53%	4.02%	5.58%	28.76%	**32.66%**
RUSSELL 2000						
α	**-0.978**	**-1.575**	**-1.228**	**-1.455**	**-0.579**	**-0.762**
	(-6.51)	(-18.23)	(-12.35)	(-18.50)	(-2.64)	(-4.41)
β	**0.225**	**-0.270**	0.032	**-0.227**	**0.502**	**0.447**
	(2.20)	(-3.95)	(0.49)	(-3.66)	(3.38)	(3.28)
β_ρ	-0.120	0.345	-0.120	-0.055	0.258	0.327
	(-0.26)	(0.76)	(-0.24)	(-0.11)	(0.55)	(0.76)
$\overline{R}^2$	5.18%	**11.85%**	0.66%	8.15%	10.63%	9.24%
SP500						
α	**-0.913**	**-1.555**	**-1.580**	**-1.437**	**-0.406**	**-0.654**
	(-7.83)	(-5.07)	(-5.64)	(-11.97)	(-2.56)	(-5.34)
β	**0.344**	-0.084	-0.097	-0.078	**0.633**	**0.584**
	(4.95)	(-0.60)	(-0.74)	(-0.63)	(6.53)	(6.69)
β_ρ	**-1.570**	**-2.302**	**-2.251**	**-2.430**	**-1.098**	-0.967
	(-2.95)	(-3.42)	(-3.36)	(-4.03)	(-2.08)	(-1.89)
$\overline{R}^2$	21.61%	11.47%	11.57%	11.44%	25.89%	**26.45%**
Aggregated Results						
Average α	-0.899	-0.743	-0.685	-1.114	-0.747	-0.792
Average β	0.220	0.328	0.367	0.208	0.307	0.351
Average β_ρ	-0.942	-1.267	-1.298	-1.213	-0.790	-0.697
Average $\overline{R}^2$	11.14%	8.38%	7.44%	7.15%	11.37%	**13.17%**

Table B.9 Information Content of Skewness Forecasts (90 days) - Alternative GARCH-2 Specification

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 90 calendar days. The GARCH-2 model is estimated as described in section 3.4.2. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.944**	**-1.240**	**-1.240**	**-1.263**	**-0.839**	**-0.836**
	(-8.06)	(-2.50)	(-2.70)	(-6.51)	(-5.94)	(-8.90)
β	**0.293**	0.031	0.028	0.034	**0.315**	**0.379**
	(3.48)	(0.15)	(0.16)	(0.28)	(3.34)	(5.89)
β_ρ	-1.060	**-1.910**	**-1.912**	**-1.975**	**-1.562**	**-1.240**
	(-1.67)	(-2.43)	(-2.44)	(-2.35)	(-2.49)	(-2.37)
$\overline{R}^2$	18.90%	11.83%	11.83%	11.89%	18.21%	**25.79%**
DAX						
α	**-1.146**	**-1.114**	**-0.985**	**-1.575**	**-1.129**	**-1.096**
	(-6.51)	(-9.06)	(-5.64)	(-5.89)	(-13.21)	(-13.98)
β	0.131	0.104	0.124	-0.290	**0.134**	**0.178**
	(0.99)	(1.63)	(1.86)	(-1.04)	(2.24)	(2.91)
β_ρ	-0.124	-0.520	-0.527	-0.586	-0.207	-0.177
	(-0.31)	(-1.01)	(-1.03)	(-1.00)	(-0.46)	(-0.41)
$\overline{R}^2$	1.86%	2.50%	2.89%	1.59%	2.47%	**4.74%**
DJIA						
α	**-0.885**	**-1.595**	**-1.628**	**-1.452**	**-0.840**	**-0.908**
	(-6.75)	(-6.40)	(-6.32)	(-10.85)	(-7.03)	(-7.44)
β	**0.310**	-0.121	-0.128	-0.225	**0.365**	**0.329**
	(3.47)	(-1.37)	(-1.44)	(-1.69)	(4.45)	(3.92)
β_ρ	-0.049	-0.081	-0.080	0.103	-0.029	-0.114
	(-0.14)	(-0.17)	(-0.17)	(0.22)	(-0.07)	(-0.31)
$\overline{R}^2$	**10.22%**	1.47%	1.55%	2.72%	5.75%	7.39%
STOXX 50						
α	**-1.024**	**-1.455**	**-1.463**	**-0.961**	**-1.033**	**-1.071**
	(-7.47)	(-4.27)	(-3.54)	(-2.99)	(-7.23)	(-10.39)
β	**0.249**	-0.045	-0.042	0.449	**0.200**	**0.227**
	(2.67)	(-0.29)	(-0.26)	(1.19)	(2.40)	(2.49)
β_ρ	0.246	0.239	0.234	0.234	0.285	0.371
	(0.41)	(0.34)	(0.34)	(0.34)	(0.42)	(0.56)
$\overline{R}^2$	**6.31%**	0.12%	0.11%	1.15%	4.48%	5.57%
FTSE 100						
α	**-1.111**	**-1.988**	**-2.043**	**-1.899**	**-1.134**	**-1.149**
	(-7.24)	(-4.72)	(-4.19)	(-9.83)	(-7.97)	(-8.81)
β	**0.270**	-0.172	-0.166	**-0.500**	**0.246**	**0.256**
	(3.78)	(-1.08)	(-1.05)	(-2.44)	(3.37)	(3.42)
β_ρ	-0.423	-0.483	-0.487	0.582	0.061	-0.359
	(-0.57)	(-0.59)	(-0.60)	(0.60)	(0.07)	(-0.47)
$\overline{R}^2$	8.24%	2.29%	2.21%	6.05%	9.10%	**11.11%**

Table B.9 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.766**	1.074	0.872	**-1.713**	**-0.753**	**-0.682**
	(-4.94)	(1.51)	(1.37)	(-2.96)	(-3.91)	(-6.51)
β	0.046	**1.816**	**1.770**	-1.686	0.035	**0.106**
	(0.24)	(2.60)	(2.59)	(-1.63)	(0.38)	(5.00)
β_ρ	1.777	**1.357**	**1.343**	1.631	1.768	1.093
	(1.54)	(2.08)	(2.03)	(1.56)	(1.52)	(1.32)
$\overline{R}^2$	4.10%	**30.95%**	30.11%	13.64%	4.06%	19.66%
KOSPI 200						
α	**-0.813**	1.856	2.357	**-0.630**	**-0.664**	**-0.567**
	(-6.22)	(1.57)	(1.78)	(-4.34)	(-5.21)	(-4.11)
β	-0.007	**1.623**	**1.806**	**0.320**	**0.216**	**0.297**
	(-0.03)	(2.27)	(2.41)	(1.97)	(2.61)	(3.23)
β_ρ	-0.039	-0.526	-0.528	-0.283	0.178	0.604
	(-0.03)	(-0.51)	(-0.52)	(-0.24)	(0.13)	(0.47)
$\overline{R}^2$	-0.19%	8.01%	**8.66%**	4.08%	1.66%	7.00%
NASDAQ 100						
α	**-0.660**	**-1.648**	**-1.671**	-0.381	**-0.694**	**-0.781**
	(-5.09)	(-6.79)	(-7.40)	(-0.91)	(-4.35)	(-5.83)
β	**0.576**	-0.137	-0.177	**1.403**	**0.531**	**0.520**
	(8.27)	(-0.97)	(-1.19)	(2.69)	(7.05)	(7.42)
β_ρ	0.668	0.802	0.838	1.158	0.774	0.861
	(1.20)	(1.07)	(1.12)	(1.63)	(1.40)	(1.57)
$\overline{R}^2$	**34.58%**	1.91%	2.38%	8.37%	22.50%	28.96%
RUSSELL 2000						
α	**-0.813**	**-1.896**	**-1.397**	**-1.798**	**-0.726**	**-0.817**
	(-5.02)	(-17.17)	(-14.17)	(-21.20)	(-3.82)	(-5.03)
β	**0.454**	**-0.301**	0.056	**-0.334**	**0.507**	**0.475**
	(4.48)	(-3.91)	(0.94)	(-4.51)	(4.47)	(4.59)
β_ρ	0.465	0.995	0.599	1.030	0.515	0.727
	(0.92)	(1.79)	(0.90)	(1.77)	(1.05)	(1.57)
$\overline{R}^2$	**20.28%**	17.48%	2.52%	17.58%	18.83%	19.95%
SP500						
α	**-0.906**	**-2.178**	**-2.186**	**-1.834**	**-0.568**	**-0.750**
	(-6.28)	(-6.30)	(-6.38)	(-9.99)	(-3.27)	(-5.91)
β	**0.447**	-0.214	-0.222	-0.251	**0.655**	**0.580**
	(5.66)	(-1.77)	(-1.80)	(-1.69)	(6.36)	(7.67)
β_ρ	-0.959	-1.540	-1.532	**-1.832**	-0.911	-0.498
	(-1.45)	(-1.69)	(-1.69)	(-2.16)	(-1.34)	(-0.80)
$\overline{R}^2$	25.55%	9.73%	9.74%	9.44%	27.64%	**32.34%**
Aggregated Results						
Average α	-0.907	-1.018	-0.938	-1.351	-0.838	-0.866
Average β	0.277	0.258	0.305	-0.108	0.320	0.335
Average β_ρ	0.050	-0.167	-0.205	0.006	0.087	0.127
Average $\overline{R}^2$	12.98%	8.63%	7.20%	7.65%	11.47%	**16.25%**

Table B.10 Encompassing Regressions - Alternative GARCH-2 Specification

This table reports the results from regressing the realized skewness on forecasts generated from the LRS, GARCH-2, QMIDAS and CIS models, within the same regression, for each index in Table 3.1. The GARCH-2 model is estimated as described in section 3.4.2. The GARCH-2 and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β_i respectively denote the intercept and the coefficient of the forecast of model i in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. Panel A, B and C respectively present results for a forecasting horizon of 30, 60 and 90 calendar days. (1) - (10) are the ten international indices of order listed in Table 3.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Panel A: 30 days					
α	-0.186	**-0.654**	0.566	**-0.979**	-0.004	0.411	0.671	-0.062	**-0.446**	**0.900**
	(-0.30)	(-2.44)	(1.39)	(-2.05)	(-0.01)	(1.09)	(0.70)	(-0.18)	(-3.77)	(2.47)
β_ρ	**-1.424**	**-1.370**	-0.391	0.549	-1.015	-0.595	-0.095	0.179	-0.110	-0.367
	(-2.85)	(-2.94)	(-1.02)	(1.39)	(-1.50)	(-0.75)	(-0.16)	(0.41)	(-0.25)	(-0.61)
β_{LRS}	**0.160**	**0.222**	**0.264**	**0.168**	0.069	0.021	**0.334**	**0.276**	0.090	**0.182**
	(2.07)	(3.82)	(3.48)	(2.84)	(1.15)	(0.19)	(4.59)	(3.83)	(1.37)	(2.48)
$\beta_{GARCH-2}$	0.060	0.107	0.021	-0.136	0.255	**1.551**	1.170	-0.014	-0.036	-0.131
	(0.11)	(1.33)	(0.21)	(-0.96)	(0.97)	(2.50)	(1.29)	(-0.09)	(-0.65)	(-1.09)
β_{QMIDAS}	**0.540**	-0.087	**1.337**	-0.132	0.537	0.141	-0.332	0.615	0.086	**2.247**
	(2.34)	(-0.32)	(2.03)	(-0.26)	(1.38)	(0.22)	(-1.67)	(1.23)	(0.96)	(4.27)
β_{CIS}	0.181	0.094	**0.495**	**0.234**	**0.372**	**0.356**	0.178	**0.434**	**0.541**	**0.475**
	(1.69)	(1.21)	(4.80)	(2.03)	(3.00)	(2.38)	(1.73)	(4.24)	(4.46)	(4.50)
$\overline{R}^2$	13.79%	12.70%	25.47%	6.93%	11.12%	21.44%	20.35%	26.25%	9.76%	20.15%
					Panel B: 60 days					
α	-0.958	**-0.971**	**-0.544**	-0.775	-0.455	**2.438**	2.722	-0.258	**-1.249**	-0.636
	(-1.26)	(-3.47)	(-3.47)	(-1.59)	(-0.94)	(2.03)	(1.79)	(-1.50)	(-7.15)	(-1.93)
β_ρ	**-2.408**	**-1.247**	0.814	-0.393	-0.930	-0.311	-0.798	**1.163**	0.790	-0.463
	(-3.35)	(-2.23)	(1.44)	(-0.58)	(-1.14)	(-0.57)	(-0.67)	(2.67)	(1.88)	(-0.77)
β_{LRS}	-0.134	0.112	**0.175**	0.135	0.001	-0.071	0.018	**0.200**	-0.075	**0.150**
	(-1.67)	(1.74)	(2.28)	(1.73)	(0.02)	(-0.40)	(0.32)	(2.28)	(-0.82)	(2.22)
$\beta_{GARCH-2}$	0.106	0.103	**0.339**	**0.637**	**0.373**	**3.579**	**2.278**	-0.162	**-0.279**	-0.059
	(0.23)	(1.17)	(4.78)	(2.18)	(2.00)	(2.51)	(2.07)	(-1.39)	(-2.32)	(-0.30)
β_{QMIDAS}	-0.013	-0.124	-0.008	-0.834	-0.180	-2.383	0.195	**0.563**	0.053	0.039
	(-0.07)	(-0.72)	(-0.05)	(-1.60)	(-1.34)	(-1.90)	(0.62)	(2.87)	(0.51)	(0.24)
β_{CIS}	0.194	0.062	0.118	0.089	**0.279**	**0.322**	**0.322**	**0.566**	**0.362**	**0.520**
	(1.85)	(0.87)	(1.56)	(0.82)	(2.03)	(2.37)	(3.09)	(5.68)	(3.11)	(5.94)
$\overline{R}^2$	15.98%	7.12%	29.30%	3.98%	9.94%	21.12%	15.98%	44.72%	15.93%	28.72%
					Panel C: 90 days					
α	-0.518	**-0.902**	**-0.973**	-0.211	-0.577	0.740	4.080	0.015	**-1.231**	**-0.685**
	(-1.49)	(-2.51)	(-5.74)	(-0.35)	(-1.46)	(0.65)	(1.91)	(0.08)	(-6.25)	(-2.17)
β_ρ	**-1.622**	-0.610	**1.032**	0.072	0.154	1.146	-0.913	**1.089**	**1.234**	-0.145
	(-2.98)	(-1.44)	(2.77)	(0.14)	(0.21)	(1.60)	(-0.95)	(3.48)	(2.93)	(-0.25)
β_{LRS}	0.003	0.174	0.071	**0.218**	0.096	-0.220	-0.063	**0.352**	0.129	**0.171**
	(0.03)	(1.94)	(0.95)	(2.85)	(0.94)	(-1.70)	(-0.50)	(4.43)	(1.32)	(2.19)
$\beta_{GARCH-2}$	0.188	0.105	**0.266**	0.282	**0.293**	1.501	**2.835**	-0.059	-0.103	-0.004
	(1.21)	(1.36)	(5.38)	(1.61)	(2.17)	(1.86)	(2.20)	(-0.68)	(-1.07)	(-0.02)
β_{QMIDAS}	0.014	-0.280	-0.189	0.079	-0.272	-0.574	-0.072	**0.756**	-0.095	-0.044
	(0.17)	(-1.05)	(-1.87)	(0.19)	(-1.82)	(-0.90)	(-0.29)	(3.08)	(-0.90)	(-0.25)
β_{CIS}	**0.286**	**0.189**	0.087	**0.185**	**0.219**	**0.167**	0.145	**0.385**	**0.248**	**0.494**
	(3.42)	(2.94)	(1.04)	(1.96)	(2.43)	(3.05)	(1.32)	(5.40)	(2.22)	(7.22)
$\overline{R}^2$	15.98%	7.12%	29.30%	3.98%	9.94%	21.12%	15.98%	44.72%	15.93%	28.72%

Table B.11 Model Confidence Set under Root Mean Squared Errors - Alternative GARCH-2 Specification

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. The GARCH-2 model is estimated as described in section 3.4.2. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.01	2*	0.579	3	0.00	2	0.00	3	0.00	5	0.01	3*	0.672	2	0.03	3	0.00	3	0.00
GARCH-1	5	0.00	5	0.00	6	0.00	6	0.00	2	0.00	2*	0.366	1*	1.000	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	6	0.00	5	0.00	5	0.00	5	0.00	3*	0.366	5*	0.146	6	0.00	6	0.00	6	0.00
QMIDAS	3	0.02	3	0.00	4	0.00	3	0.00	4	0.00	4*	0.161	6	0.03	4	0.00	4	0.00	4	0.00
IS	2*	0.819	4	0.00	1*	1.000	4	0.00	6	0.00	6	0.00	2*	0.965	3	0.00	2	0.01	2	0.01
CIS	1*	1.000	1*	1.000	2*	0.091	1*	1.000	1*	1.000	1*	1.000	4*	0.146	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	1*	1.000	3*	0.181	2*	0.603	2	0.01	5*	0.080	2*	0.534	3	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	5	0.00	6	0.00	6	0.00	5	0.00	3*	0.127	4*	0.051	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	6	0.00	5	0.00	5	0.00	6	0.00	2*	0.377	5	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	4	0.00	4*	0.080	6	0.00	4	0.00	4	0.00	4	0.00
IS	1*	1.000	3*	0.090	1*	1.000	3	0.00	3	0.00	6	0.00	3*	0.051	2	0.04	2*	0.579	1*	1.000
CIS	2*	0.854	2*	0.317	2*	0.181	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.211
Panel C: 90-day horizon																				
LRS	3*	0.136	1*	1.000	2*	0.072	1*	1.000	1*	1.000	4*	0.105	4	0.00	1*	1.000	2*	0.617	3	0.00
GARCH-1	5	0.00	5	0.00	6	0.00	6	0.00	5	0.00	2*	0.970	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	6	0.00	5	0.00	5	0.00	6	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	4	0.00	3*	0.970	3	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.227	3	0.03	1*	1.000	3	0.00	3	0.01	5	0.03	2	0.00	2*	0.176	1*	1.000	1*	1.000
CIS	1*	1.000	2	0.03	3*	0.072	2	0.00	2*	0.151	6	0.03	1*	1.000	3*	0.156	3*	0.124	2*	0.157

Table B.12 Model Confidence Set under Mean Absolute Errors - Alternative GARCH-2 Specification

This table reports the results for the Model Confidence Set (MCS) test using the Mean Absolute Errors (MAE) as a loss function and a 5% significance level. The GARCH-2 model is estimated as described in section 3.4.2. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.04	2*	0.606	3	0.00	2	0.00	3	0.00	5	0.00	4*	0.134	2	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	5	0.00	5	0.00	6	0.00	2	0.00	2*	0.296	2*	0.356	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	6	0.00	6	0.00	5	0.00	5	0.00	4*	0.296	5*	0.109	6	0.00	6	0.00	6	0.00
QMIDAS	3	0.04	3	0.00	4	0.00	3	0.00	4	0.00	3*	0.296	6	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.342	4	0.00	1*	1.000	4	0.00	6	0.00	6	0.00	1*	1.000	3	0.00	2	0.00	2	0.00
CIS	1*	1.000	1*	1.000	2*	0.215	1*	1.000	1*	1.000	1*	1.000	3*	0.134	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.01	2*	0.543	3	0.01	2*	0.659	2	0.01	2*	0.577	3*	0.454	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	5	0.00	6	0.00	6	0.00	5	0.00	3*	0.577	4*	0.051	5	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	6	0.00	5	0.00	5	0.00	6	0.00	5*	0.577	5	0.00	6	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	4	0.00	4*	0.577	6	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.644	3*	0.270	1*	1.000	3	0.00	3	0.00	6	0.00	2*	0.454	3	0.00	2*	0.243	1*	1.000
CIS	1*	1.000	1*	1.000	2	0.01	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.739
Panel C: 90-day horizon																				
LRS	3*	0.869	1*	1.000	3*	0.095	1*	1.000	1*	1.000	5*	0.620	3	0.00	1*	1.000	3	0.01	3	0.00
GARCH-1	5	0.00	5	0.00	6	0.00	6	0.00	5	0.00	2*	0.914	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	6	0.00	5	0.00	5	0.00	6	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	4	0.00	3*	0.914	4	0.00	4	0.00	4	0.00	4	0.00
IS	1*	1.000	2*	0.051	1*	1.000	3	0.00	3	0.03	6	0.04	2	0.00	2*	0.974	1*	1.000	1*	1.000
CIS	2*	0.869	3	0.03	2*	0.095	2	0.00	2*	0.294	4*	0.914	1*	1.000	3*	0.372	2	0.01	2	0.00

Table B.13 Information Content of Skewness Forecasts (30 days) - MIDAS with 200 lagged days

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 30 calendar days. The conditioning variable in the QMIDAS model is a function of the previous 200 daily returns. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.932**	-0.875	-0.851	-0.849	**-0.836**	**-0.908**
	(-11.42)	(-1.31)	(-1.30)	(-1.94)	(-6.68)	(-7.57)
β	**0.165**	0.201	0.222	0.287	**0.224**	**0.218**
	(2.39)	(0.36)	(0.41)	(0.62)	(2.61)	(2.01)
β_ρ	**-1.776**	**-2.072**	**-2.080**	**-2.057**	**-1.863**	**-1.599**
	(-4.00)	(-4.13)	(-4.14)	(-4.14)	(-4.08)	(-3.12)
$\overline{R}^2$	10.00%	7.65%	7.67%	7.78%	9.99%	**10.05%**
DAX						
α	**-0.811**	**-0.998**	**-0.997**	**-1.201**	**-0.941**	**-0.951**
	(-11.68)	(-11.10)	(-10.81)	(-6.56)	(-10.55)	(-11.29)
β	**0.221**	0.055	0.055	-0.213	0.072	0.107
	(3.77)	(0.70)	(0.68)	(-0.83)	(1.23)	(1.25)
β_ρ	**-1.196**	**-1.611**	**-1.610**	**-1.700**	**-1.407**	**-1.350**
	(-3.22)	(-3.42)	(-3.41)	(-3.22)	(-3.36)	(-3.17)
$\overline{R}^2$	**9.86%**	6.71%	6.70%	6.80%	7.05%	7.09%
DJIA						
α	**-0.621**	**-0.716**	**-0.373**	-0.507	**-0.299**	**-0.467**
	(-8.32)	(-4.22)	(-4.72)	(-1.38)	(-2.97)	(-5.26)
β	**0.375**	0.213	**0.606**	0.755	**0.666**	**0.676**
	(5.30)	(1.80)	(8.01)	(1.34)	(7.33)	(5.98)
β_ρ	**-0.760**	**-1.429**	**1.021**	**-1.013**	**-0.825**	-0.359
	(-2.22)	(-3.09)	(2.52)	(-2.38)	(-2.15)	(-1.00)
$\overline{R}^2$	17.06%	4.64%	**22.10%**	3.40%	13.53%	18.67%
STOXX 50						
α	**-0.845**	**-1.296**	**-1.413**	**-0.944**	**-0.672**	**-0.768**
	(-11.24)	(-2.89)	(-4.71)	(-4.44)	(-5.86)	(-6.62)
β	**0.224**	-0.173	-0.292	0.203	**0.246**	**0.341**
	(4.02)	(-0.46)	(-1.10)	(0.71)	(3.82)	(3.09)
β_ρ	0.414	0.525	0.574	0.601	0.650	0.630
	(1.17)	(1.31)	(1.45)	(1.41)	(1.80)	(1.71)
$\overline{R}^2$	5.29%	0.61%	0.90%	0.74%	**5.31%**	4.62%
FTSE 100						
α	**-1.193**	**-0.842**	**-0.858**	**-0.911**	**-1.030**	**-0.984**
	(-11.87)	(-2.21)	(-2.35)	(-3.61)	(-7.72)	(-5.95)
β	0.081	0.305	0.314	0.556	**0.139**	**0.288**
	(1.23)	(1.23)	(1.23)	(1.57)	(2.13)	(2.15)
β_ρ	**-1.632**	**-2.025**	**-2.039**	**-1.662**	**-1.444**	-1.170
	(-2.60)	(-3.07)	(-3.06)	(-2.48)	(-2.34)	(-1.79)
$\overline{R}^2$	6.61%	6.84%	6.83%	7.15%	**8.39%**	8.23%

Table B.13 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.479**	0.665	**1.758**	-0.112	-0.237	-0.239
	(-4.16)	(1.55)	(2.32)	(-0.33)	(-1.18)	(-1.70)
β	**0.258**	**2.167**	**3.332**	2.487	**0.335**	**0.630**
	(2.45)	(3.27)	(3.27)	(1.75)	(2.56)	(3.85)
β_ρ	0.196	-0.053	0.020	0.491	-0.141	-0.818
	(0.24)	(-0.07)	(0.03)	(0.55)	(-0.17)	(-0.99)
$\overline{R}^2$	6.55%	**18.01%**	16.79%	4.96%	6.89%	15.35%
KOSPI 200						
α	**-0.524**	0.873	1.782	**-0.998**	**-0.520**	**-0.626**
	(-6.61)	(0.63)	(1.12)	(-6.00)	(-3.71)	(-5.49)
β	**0.406**	1.725	2.417	-0.235	**0.515**	**0.349**
	(5.16)	(1.27)	(1.67)	(-0.65)	(3.04)	(2.75)
β_ρ	0.039	0.715	0.662	0.752	0.619	0.760
	(0.06)	(0.85)	(0.81)	(0.77)	(0.83)	(0.99)
$\overline{R}^2$	**16.87%**	3.28%	4.66%	1.18%	10.99%	8.48%
NASDAQ 100						
α	**-0.556**	**-1.059**	**-1.177**	**-0.601**	**-0.306**	**-0.438**
	(-7.74)	(-5.78)	(-6.84)	(-2.63)	(-3.24)	(-5.09)
β	**0.430**	-0.124	-0.308	0.728	**0.497**	**0.673**
	(6.42)	(-0.59)	(-1.43)	(1.56)	(6.73)	(6.46)
β_ρ	-0.486	-0.984	-0.761	-0.810	-0.465	-0.086
	(-1.21)	(-1.63)	(-1.25)	(-1.45)	(-1.07)	(-0.19)
$\overline{R}^2$	**19.87%**	2.46%	3.40%	4.10%	17.56%	19.67%
RUSSELL 2000						
α	**-0.837**	**-1.108**	**-1.125**	**-1.012**	**-0.269**	**-0.515**
	(-10.02)	(-16.19)	(-16.68)	(-15.56)	(-2.31)	(-4.80)
β	**0.186**	-0.108	-0.132	0.030	**0.571**	**0.587**
	(2.81)	(-1.44)	(-1.83)	(0.28)	(6.59)	(5.22)
β_ρ	-0.502	-0.399	-0.341	-0.587	-0.435	0.011
	(-1.20)	(-0.81)	(-0.70)	(-1.23)	(-1.09)	(0.03)
$\overline{R}^2$	4.14%	1.81%	2.47%	0.79%	**12.43%**	9.03%
SP500						
α	**-0.872**	**-1.081**	**-1.207**	0.674	**-0.440**	**-0.589**
	(-9.26)	(-4.96)	(-5.75)	(1.10)	(-2.94)	(-5.21)
β	**0.266**	0.073	-0.016	**2.605**	**0.459**	**0.608**
	(3.75)	(0.50)	(-0.11)	(2.95)	(5.11)	(5.86)
β_ρ	**-1.723**	**-2.573**	**-2.360**	**-2.729**	**-1.517**	**-1.206**
	(-3.38)	(-3.59)	(-3.28)	(-4.59)	(-2.91)	(-2.50)
$\overline{R}^2$	12.58%	6.69%	6.60%	9.41%	14.70%	**15.82%**
Aggregated Results						
Average α	-0.767	-0.644	-0.446	-0.646	-0.555	-0.649
Average β	0.261	0.433	0.620	0.721	0.372	0.448
Average β_ρ	-0.743	-0.991	-0.691	-0.871	-0.683	-0.519
Average $\overline{R}^2$	10.88%	5.87%	7.81%	4.63%	10.68%	**11.70%**

Table B.14 Information Content of Skewness Forecasts (60 days) - MIDAS with 200 lagged days

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 60 calendar days. The conditioning variable in the QMIDAS model is a function of the previous 200 daily returns. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-1.156**	-0.817	-0.825	**-1.094**	**-0.856**	**-0.886**
	(-9.54)	(-1.55)	(-1.60)	(-5.01)	(-5.61)	(-8.69)
β	0.002	0.192	0.188	0.057	**0.229**	**0.253**
	(0.02)	(0.64)	(0.64)	(0.32)	(2.14)	(3.06)
β_ρ	**-2.669**	**-2.826**	**-2.828**	**-2.765**	**-2.422**	**-2.135**
	(-3.92)	(-3.92)	(-3.91)	(-3.53)	(-3.94)	(-3.38)
$\overline{R}^2$	19.70%	19.92%	19.92%	19.76%	22.01%	**23.64%**
DAX						
α	**-1.057**	**-1.065**	**-1.061**	**-1.310**	**-1.132**	**-1.119**
	(-11.20)	(-11.57)	(-11.32)	(-9.84)	(-13.41)	(-14.27)
β	0.107	0.080	0.083	-0.122	0.039	0.061
	(1.79)	(1.26)	(1.28)	(-1.17)	(0.83)	(1.03)
β_ρ	-0.765	**-1.155**	**-1.162**	-1.267	-0.867	-0.829
	(-1.61)	(-1.98)	(-1.98)	(-1.95)	(-1.74)	(-1.64)
$\overline{R}^2$	**3.57%**	3.39%	3.42%	3.34%	2.71%	2.91%
DJIA						
α	**-0.789**	**-0.906**	**-0.678**	**-0.962**	**-0.826**	**-0.806**
	(-7.85)	(-4.09)	(-7.58)	(-5.34)	(-6.25)	(-6.90)
β	**0.309**	0.113	**0.420**	0.262	**0.293**	**0.357**
	(3.66)	(1.09)	(5.65)	(1.03)	(2.72)	(2.94)
β_ρ	**-1.169**	**-1.962**	0.560	**-1.909**	**-1.414**	**-1.083**
	(-2.39)	(-3.25)	(0.97)	(-3.28)	(-2.47)	(-2.10)
$\overline{R}^2$	13.79%	6.15%	**21.60%**	6.08%	8.14%	11.80%
STOXX 50						
α	**-1.078**	**-1.017**	**-1.086**	**-1.765**	**-1.030**	**-1.097**
	(-9.97)	(-2.73)	(-3.04)	(-4.53)	(-7.26)	(-9.95)
β	0.113	0.115	0.084	-0.618	0.124	0.111
	(1.62)	(0.54)	(0.37)	(-1.41)	(1.55)	(1.21)
β_ρ	-0.126	-0.210	-0.182	-0.282	-0.015	-0.040
	(-0.18)	(-0.28)	(-0.24)	(-0.38)	(-0.02)	(-0.05)
$\overline{R}^2$	1.23%	0.10%	-0.00%	0.83%	**1.29%**	0.74%
FTSE 100						
α	**-1.319**	**-1.223**	**-1.304**	**-1.552**	**-1.140**	**-1.088**
	(-8.39)	(-3.16)	(-3.55)	(-6.72)	(-7.23)	(-8.20)
β	0.049	0.075	0.042	-0.190	0.151	**0.240**
	(0.56)	(0.42)	(0.22)	(-0.79)	(1.73)	(2.52)
β_ρ	**-1.654**	**-1.799**	**-1.759**	-1.203	-1.177	-1.161
	(-2.12)	(-2.39)	(-2.33)	(-1.28)	(-1.60)	(-1.72)
$\overline{R}^2$	5.70%	5.57%	5.50%	5.99%	8.23%	**10.03%**

Table B.14 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.533**	0.649	1.774	-0.585	**-0.515**	**-0.382**
	(-3.78)	(0.98)	(1.43)	(-1.11)	(-2.44)	(-2.43)
β	0.207	1.577	2.457	0.386	0.147	**0.462**
	(1.44)	(1.96)	(1.95)	(0.22)	(1.32)	(2.69)
β_ρ	-0.156	-0.330	-0.305	-0.038	-0.120	-0.477
	(-0.25)	(-0.72)	(-0.65)	(-0.06)	(-0.18)	(-0.85)
$\overline{R}^2$	3.74%	16.71%	**16.92%**	-0.28%	1.41%	8.04%
KOSPI 200						
α	**-0.620**	1.405	2.093	**-0.764**	**-0.694**	**-0.686**
	(-8.37)	(1.07)	(1.34)	(-7.72)	(-9.33)	(-7.99)
β	**0.322**	1.667	2.035	0.429	**0.298**	**0.290**
	(2.99)	(1.71)	(1.87)	(1.58)	(2.50)	(2.31)
β_ρ	-0.870	-1.608	-1.688	-1.689	-1.020	-0.918
	(-0.68)	(-1.21)	(-1.29)	(-1.23)	(-0.74)	(-0.69)
$\overline{R}^2$	**8.10%**	5.09%	6.20%	4.17%	4.64%	6.24%
NASDAQ 100						
α	**-0.547**	**-1.330**	**-1.337**	**-0.891**	**-0.290**	**-0.438**
	(-5.49)	(-5.62)	(-6.15)	(-4.25)	(-2.79)	(-4.24)
β	**0.520**	-0.186	-0.232	0.461	**0.657**	**0.701**
	(7.42)	(-1.01)	(-1.16)	(1.19)	(9.00)	(7.69)
β_ρ	-0.316	-0.821	-0.744	-0.572	-0.024	0.313
	(-0.68)	(-1.16)	(-1.04)	(-0.70)	(-0.06)	(0.73)
$\overline{R}^2$	28.79%	3.53%	3.93%	4.72%	28.76%	**32.66%**
RUSSELL 2000						
α	**-0.978**	**-1.575**	**-1.568**	**-1.458**	**-0.579**	**-0.762**
	(-6.51)	(-18.23)	(-18.77)	(-18.23)	(-2.64)	(-4.41)
β	**0.225**	**-0.270**	**-0.270**	**-0.226**	**0.502**	**0.447**
	(2.20)	(-3.95)	(-4.01)	(-3.58)	(3.38)	(3.28)
β_ρ	-0.120	0.345	0.364	-0.026	0.258	0.327
	(-0.26)	(0.76)	(0.80)	(-0.05)	(0.55)	(0.76)
$\overline{R}^2$	5.18%	11.85%	**12.38%**	7.43%	10.63%	9.24%
SP500						
α	**-0.913**	**-1.555**	**-1.580**	**-1.432**	**-0.406**	**-0.654**
	(-7.83)	(-5.07)	(-5.55)	(-12.14)	(-2.56)	(-5.34)
β	**0.344**	-0.084	-0.107	-0.074	**0.633**	**0.584**
	(4.95)	(-0.60)	(-0.73)	(-0.60)	(6.53)	(6.69)
β_ρ	**-1.570**	**-2.302**	**-2.237**	**-2.440**	**-1.098**	-0.967
	(-2.95)	(-3.42)	(-3.28)	(-4.05)	(-2.08)	(-1.89)
$\overline{R}^2$	21.61%	11.47%	11.57%	11.42%	25.89%	**26.45%**
Aggregated Results						
Average α	-0.899	-0.743	-0.557	-1.181	-0.747	-0.792
Average β	0.220	0.328	0.470	0.037	0.307	0.351
Average β_ρ	-0.942	-1.267	-0.998	-1.219	-0.790	-0.697
Average $\overline{R}^2$	11.14%	8.38%	10.14%	6.35%	11.37%	**13.17%**

Table B.15 Information Content of Skewness Forecasts (90 days) - MIDAS with 200 lagged days

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 90 calendar days. The conditioning variable in the QMIDAS model is a function of the previous 200 daily returns. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^{ρ} across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.944**	**-1.240**	**-1.250**	**-1.254**	**-0.839**	**-0.836**
	(-8.06)	(-2.50)	(-2.54)	(-6.05)	(-5.94)	(-8.90)
β	**0.293**	0.031	0.026	0.039	**0.315**	**0.379**
	(3.48)	(0.15)	(0.13)	(0.30)	(3.34)	(5.89)
β_{ρ}	-1.060	**-1.910**	**-1.904**	**-1.988**	**-1.562**	**-1.240**
	(-1.67)	(-2.43)	(-2.43)	(-2.39)	(-2.49)	(-2.37)
$\overline{R}^2$	18.90%	11.83%	11.82%	11.91%	18.21%	**25.79%**
DAX						
α	**-1.146**	**-1.114**	**-1.094**	**-1.654**	**-1.129**	**-1.096**
	(-6.51)	(-9.06)	(-8.42)	(-5.13)	(-13.21)	(-13.98)
β	0.131	0.104	0.112	-0.340	**0.134**	**0.178**
	(0.99)	(1.63)	(1.70)	(-1.11)	(2.24)	(2.91)
β_{ρ}	-0.124	-0.520	-0.535	-0.674	-0.207	-0.177
	(-0.31)	(-1.01)	(-1.03)	(-1.08)	(-0.46)	(-0.41)
$\overline{R}^2$	1.86%	2.50%	2.69%	1.89%	2.47%	**4.74%**
DJIA						
α	**-0.885**	**-1.595**	**-0.846**	**-1.588**	**-0.840**	**-0.908**
	(-6.75)	(-6.40)	(-10.70)	(-9.45)	(-7.03)	(-7.44)
β	**0.310**	-0.121	**0.324**	**-0.421**	**0.365**	**0.329**
	(3.47)	(-1.37)	(4.62)	(-2.28)	(4.45)	(3.92)
β_{ρ}	-0.049	-0.081	**0.970**	0.278	-0.029	-0.114
	(-0.14)	(-0.17)	(1.97)	(0.63)	(-0.07)	(-0.31)
$\overline{R}^2$	10.22%	1.47%	**21.94%**	5.04%	5.75%	7.39%
STOXX 50						
α	**-1.024**	**-1.455**	**-1.469**	**-1.323**	**-1.033**	**-1.071**
	(-7.47)	(-4.27)	(-4.06)	(-6.10)	(-7.23)	(-10.39)
β	**0.249**	-0.045	-0.057	0.040	**0.200**	**0.227**
	(2.67)	(-0.29)	(-0.31)	(0.16)	(2.40)	(2.49)
β_{ρ}	0.246	0.239	0.247	0.231	0.285	0.371
	(0.41)	(0.34)	(0.35)	(0.31)	(0.42)	(0.56)
$\overline{R}^2$	**6.31%**	0.12%	0.14%	0.08%	4.48%	5.57%
FTSE 100						
α	**-1.111**	**-1.988**	**-1.984**	**-1.913**	**-1.134**	**-1.149**
	(-7.24)	(-4.72)	(-4.74)	(-8.98)	(-7.97)	(-8.81)
β	**0.270**	-0.172	-0.185	**-0.498**	**0.246**	**0.256**
	(3.78)	(-1.08)	(-1.09)	(-2.21)	(3.37)	(3.42)
β_{ρ}	-0.423	-0.483	-0.465	0.623	0.061	-0.359
	(-0.57)	(-0.59)	(-0.57)	(0.64)	(0.07)	(-0.47)
$\overline{R}^2$	8.24%	2.29%	2.33%	5.95%	9.10%	**11.11%**

Table B.15 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.766**	1.074	2.249	**-1.798**	**-0.753**	**-0.682**
	(-4.94)	(1.51)	(1.91)	(-2.84)	(-3.91)	(-6.51)
β	0.046	**1.816**	**2.490**	-2.116	0.035	**0.106**
	(0.24)	(2.60)	(2.57)	(-1.69)	(0.38)	(5.00)
β_ρ	1.777	**1.357**	**1.364**	1.199	1.768	1.093
	(1.54)	(2.08)	(2.07)	(1.44)	(1.52)	(1.32)
$\overline{R}^2$	4.10%	**30.95%**	30.35%	14.78%	4.06%	19.66%
KOSPI 200						
α	**-0.813**	1.856	2.553	-0.210	**-0.664**	**-0.567**
	(-6.22)	(1.57)	(1.86)	(-0.64)	(-5.21)	(-4.11)
β	-0.007	**1.623**	**1.874**	**1.171**	**0.216**	**0.297**
	(-0.03)	(2.27)	(2.46)	(2.10)	(2.61)	(3.23)
β_ρ	-0.039	-0.526	-0.557	-0.590	0.178	0.604
	(-0.03)	(-0.51)	(-0.56)	(-0.52)	(0.13)	(0.47)
$\overline{R}^2$	-0.19%	8.01%	**9.28%**	3.93%	1.66%	7.00%
NASDAQ 100						
α	**-0.660**	**-1.648**	**-1.652**	-0.402	**-0.694**	**-0.781**
	(-5.09)	(-6.79)	(-7.25)	(-1.21)	(-4.35)	(-5.83)
β	**0.576**	-0.137	-0.166	**1.301**	**0.531**	**0.520**
	(8.27)	(-0.97)	(-1.08)	(3.14)	(7.05)	(7.42)
β_ρ	0.668	0.802	0.828	1.177	0.774	0.861
	(1.20)	(1.07)	(1.10)	(1.70)	(1.40)	(1.57)
$\overline{R}^2$	**34.58%**	1.91%	2.18%	12.47%	22.50%	28.96%
RUSSELL 2000						
α	**-0.813**	**-1.896**	**-1.890**	**-1.794**	**-0.726**	**-0.817**
	(-5.02)	(-17.17)	(-16.93)	(-21.44)	(-3.82)	(-5.03)
β	**0.454**	**-0.301**	**-0.300**	**-0.328**	**0.507**	**0.475**
	(4.48)	(-3.91)	(-3.81)	(-4.54)	(4.47)	(4.59)
β_ρ	0.465	0.995	0.986	1.063	0.515	0.727
	(0.92)	(1.79)	(1.76)	(1.82)	(1.05)	(1.57)
$\overline{R}^2$	**20.28%**	17.48%	17.23%	17.88%	18.83%	19.95%
SP500						
α	**-0.906**	**-2.178**	**-2.176**	**-1.828**	**-0.568**	**-0.750**
	(-6.28)	(-6.30)	(-6.37)	(-9.48)	(-3.27)	(-5.91)
β	**0.447**	-0.214	-0.238	-0.245	**0.655**	**0.580**
	(5.66)	(-1.77)	(-1.78)	(-1.57)	(6.36)	(7.67)
β_ρ	-0.959	-1.540	-1.505	**-1.792**	-0.911	-0.498
	(-1.45)	(-1.69)	(-1.65)	(-2.09)	(-1.34)	(-0.80)
$\overline{R}^2$	25.55%	9.73%	9.80%	9.34%	27.64%	**32.34%**
Aggregated Results						
Average α	-0.907	-1.018	-0.756	-1.376	-0.838	-0.866
Average β	0.277	0.258	0.388	-0.140	0.320	0.335
Average β_ρ	0.050	-0.167	-0.057	-0.047	0.087	0.127
Average $\overline{R}^2$	12.98%	8.63%	10.78%	8.33%	11.47%	**16.25%**

Table B.16 Encompassing Regressions - MIDAS with 200 lagged days

This table reports the results from regressing the realized skewness on forecasts generated from the LRS, GARCH-2, QMIDAS and CIS models, within the same regression, for each index in Table 3.1. The conditioning variable in the QMIDAS model is a function of the previous 200 daily returns. The GARCH-2 and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β_i respectively denote the intercept and the coefficient of the forecast of model i in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. Panel A, B and C respectively present results for a forecasting horizon of 30, 60 and 90 calendar days. (1) - (10) are the ten international indices of order listed in Table 3.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Panel A: 30 days					
α	0.109	**-0.676**	0.309	**-1.253**	0.097	1.252	1.802	-0.013	**-0.502**	**1.601**
	(0.16)	(-2.31)	(1.09)	(-2.72)	(0.29)	(1.96)	(1.51)	(-0.03)	(-4.24)	(2.82)
β_ρ	**-1.605**	**-1.378**	**0.790**	0.456	-1.054	-0.627	-0.108	0.127	0.316	-0.965
	(-3.29)	(-2.94)	(2.37)	(1.14)	(-1.55)	(-0.80)	(-0.18)	(0.28)	(0.71)	(-1.65)
β_{LRS}	**0.154**	**0.220**	**0.191**	**0.173**	0.069	0.041	**0.331**	**0.280**	0.084	**0.182**
	(1.98)	(3.78)	(2.49)	(2.92)	(1.16)	(0.38)	(4.77)	(3.87)	(1.21)	(2.62)
$\beta_{GARCH-2}$	0.312	0.110	**0.358**	-0.326	0.350	**2.290**	**2.226**	0.063	**-0.174**	-0.199
	(0.59)	(1.17)	(5.19)	(-1.27)	(1.35)	(2.73)	(2.01)	(0.28)	(-2.18)	(-1.58)
β_{QMIDAS}	0.525	-0.069	0.725	-0.315	0.564	-0.091	-0.449	0.623	**0.267**	**3.336**
	(1.17)	(-0.23)	(1.66)	(-0.92)	(1.48)	(-0.11)	(-1.54)	(1.16)	(2.39)	(4.28)
β_{CIS}	0.199	0.095	**0.349**	**0.247**	**0.385**	**0.371**	0.187	**0.429**	**0.484**	**0.466**
	(1.85)	(1.21)	(3.87)	(2.16)	(3.09)	(2.39)	(1.86)	(4.19)	(4.27)	(4.37)
$\overline{R}^2$	12.42%	12.69%	29.92%	7.10%	11.47%	21.06%	21.60%	26.10%	11.42%	21.34%
					Panel B: 60 days					
α	-0.388	**-0.846**	**-0.491**	**-1.490**	-0.468	1.713	4.837	-0.331	**-1.210**	**-0.673**
	(-0.57)	(-3.40)	(-2.99)	(-2.72)	(-0.99)	(1.45)	(1.76)	(-1.83)	(-7.20)	(-2.13)
β_ρ	**-2.503**	**-1.047**	0.471	-0.336	-0.998	-0.408	-1.059	0.723	0.751	-0.717
	(-3.59)	(-1.99)	(0.83)	(-0.54)	(-1.26)	(-0.74)	(-1.02)	(1.63)	(1.87)	(-1.26)
β_{LRS}	-0.081	0.126	**0.168**	0.115	-0.012	0.010	**0.245**	**0.256**	-0.040	**0.170**
	(-0.93)	(1.83)	(2.40)	(1.54)	(-0.12)	(0.05)	(2.54)	(3.44)	(-0.43)	(2.77)
$\beta_{GARCH-2}$	0.268	0.108	**0.338**	0.090	0.340	2.077	3.762	-0.112	**-0.306**	-0.084
	(0.65)	(1.23)	(4.69)	(0.41)	(1.87)	(1.90)	(1.95)	(-0.94)	(-2.79)	(-0.45)
β_{QMIDAS}	0.046	-0.025	0.054	-0.713	-0.170	0.694	-0.547	0.362	0.100	0.095
	(0.24)	(-0.19)	(0.24)	(-1.71)	(-0.91)	(0.44)	(-1.33)	(1.69)	(1.06)	(0.63)
β_{CIS}	**0.313**	0.058	0.057	0.074	**0.317**	0.213	**0.237**	**0.470**	**0.326**	**0.455**
	(3.33)	(0.98)	(0.74)	(0.80)	(2.72)	(1.18)	(2.21)	(5.69)	(2.99)	(5.77)
$\overline{R}^2$	24.73%	5.27%	23.96%	2.75%	11.93%	18.01%	16.06%	37.77%	16.29%	28.16%
					Panel C: 90 days					
α	-0.135	**-0.725**	**-1.029**	-1.034	**-0.836**	2.100	**3.899**	0.389	**-1.077**	**-0.674**
	(-0.34)	(-2.01)	(-6.06)	(-1.53)	(-2.13)	(1.87)	(2.47)	(1.49)	(-5.92)	(-2.20)
β_ρ	**-1.530**	-0.452	**1.371**	0.203	0.317	**1.120**	0.445	**0.984**	**1.001**	-0.354
	(-2.95)	(-0.97)	(3.07)	(0.38)	(0.46)	(1.99)	(0.42)	(2.52)	(2.57)	(-0.63)
β_{LRS}	0.122	0.079	0.127	**0.193**	0.165	-0.134	-0.123	**0.375**	**0.180**	**0.188**
	(1.35)	(0.85)	(1.78)	(2.14)	(1.75)	(-1.01)	(-0.93)	(5.87)	(2.11)	(2.46)
$\beta_{GARCH-2}$	0.188	**0.140**	**0.283**	-0.003	0.161	**2.201**	**2.780**	**0.176**	-0.101	-0.010
	(1.04)	(2.12)	(4.88)	(-0.01)	(1.31)	(2.76)	(2.58)	(2.11)	(-1.07)	(-0.07)
β_{QMIDAS}	0.093	-0.012	**-0.349**	-0.145	-0.300	0.441	-0.933	**0.954**	-0.072	0.009
	(0.87)	(-0.05)	(-2.68)	(-0.45)	(-1.82)	(0.73)	(-1.12)	(4.09)	(-0.72)	(0.06)
β_{CIS}	**0.358**	**0.182**	-0.022	0.149	**0.187**	**0.086**	**0.357**	**0.264**	**0.262**	**0.448**
	(4.14)	(3.80)	(-0.38)	(1.78)	(2.53)	(3.18)	(3.57)	(4.54)	(3.89)	(7.11)
$\overline{R}^2$	28.15%	8.96%	26.74%	8.31%	14.42%	36.97%	19.06%	44.16%	28.52%	34.15%

Table B.17 Model Confidence Set under Root Mean Squared Errors - MIDAS with 200 lagged days

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. The conditioning variable in the QMIDAS model is a function of the previous 200 daily returns. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labelled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.00	2*	0.562	3	0.00	2	0.00	3	0.00	5	0.01	4*	0.814	2	0.03	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	3*	0.256	1*	1.000	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.484	6*	0.078	6	0.00	6	0.00	6	0.00
QMIDAS	1*	1.000	3	0.00	4	0.00	3	0.00	4	0.00	4*	0.150	3*	0.814	4	0.00	4	0.00	4	0.00
IS	3*	0.966	4	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	2*	0.958	3	0.00	2	0.01	2	0.00
CIS	2*	0.966	1*	1.000	2*	0.091	1*	1.000	1*	1.000	1*	1.000	5*	0.125	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	1*	1.000	3*	0.187	2*	0.611	2	0.01	5	0.04	2*	0.530	3	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.141	4*	0.054	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.866	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	4*	0.141	5	0.00	4	0.00	4	0.00	4	0.00
IS	1*	1.000	3*	0.083	1*	1.000	3	0.00	4	0.00	6	0.00	3*	0.054	2	0.04	2*	0.581	1*	1.000
CIS	2*	0.860	2*	0.308	2*	0.187	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.217
Panel C: 90-day horizon																				
LRS	3*	0.132	1*	1.000	2*	0.064	1*	1.000	1*	1.000	4	0.02	4	0.00	1*	1.000	2*	0.593	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	2	0.02	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	4	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	5	0.00	3	0.02	3	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.225	3	0.03	1*	1.000	3	0.00	3	0.00	5	0.00	2	0.00	2*	0.172	1*	1.000	1*	1.000
CIS	1*	1.000	2	0.03	3*	0.064	2	0.00	2*	0.178	6	0.00	1*	1.000	3*	0.172	3*	0.168	2*	0.147

Table B.18 Model Confidence Set under Mean Absolute Error - MIDAS with 200 lagged days

This table reports the results for the Model Confidence Set (MCS) test using the Mean Absolute Error (MAE) as a loss function and a 5% significance level. The conditioning variable in the QMIDAS model is a function of the previous 200 daily returns. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.01	2*	0.623	3	0.00	3	0.00	3	0.00	5	0.00	5*	0.160	2	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	4*	0.272	2*	0.378	5	0.00	5	0.00	5	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.524	6	0.02	6	0.00	6	0.00	6	0.00
QMIDAS	1*	1.000	3	0.00	4	0.00	2	0.00	4	0.00	3*	0.524	4*	0.160	4	0.00	4	0.00	4	0.00
IS	3*	0.612	4	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	1*	1.000	3	0.00	2	0.00	2	0.00
CIS	2*	0.828	1*	1.000	2*	0.227	1*	1.000	1*	1.000	1*	1.000	3*	0.160	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	2*	0.543	3	0.02	2*	0.680	2	0.01	3*	0.576	3*	0.471	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	4*	0.433	4*	0.052	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.576	6	0.00	5	0.00	5	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	5*	0.353	5	0.00	4	0.00	4	0.00	5	0.00
IS	2*	0.634	3*	0.239	1*	1.000	3	0.00	4	0.00	6	0.00	2*	0.471	3	0.00	2*	0.263	1*	1.000
CIS	1*	1.000	1*	1.000	2	0.02	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.740
Panel C: 90-day horizon																				
LRS	3*	0.884	1*	1.000	3*	0.091	1*	1.000	1*	1.000	5	0.04	3	0.00	1*	1.000	3	0.01	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	2	0.04	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	3	0.04	4	0.00	4	0.00	4	0.00	4	0.00
IS	1*	1.000	2	0.04	1*	1.000	3	0.00	3	0.03	6	0.00	2	0.00	2*	0.971	1*	1.000	1*	1.000
CIS	2*	0.884	3	0.03	2*	0.091	2	0.00	2*	0.295	4	0.04	1*	1.000	3*	0.417	2	0.01	2	0.00

Table B.19 Information Content of Skewness Forecasts (30 days) - MIDAS with 300 lagged days

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 30 calendar days. The conditioning variable in the QMIDAS model is a function of the previous 300 daily returns. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^{ρ} across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.932**	-0.875	-0.851	**-0.641**	**-0.836**	**-0.908**
	(-11.42)	(-1.31)	(-1.30)	(-3.01)	(-6.68)	(-7.57)
β	**0.165**	0.201	0.222	**0.496**	**0.224**	**0.218**
	(2.39)	(0.36)	(0.41)	(2.25)	(2.61)	(2.01)
β_ρ	**-1.776**	**-2.072**	**-2.080**	**-1.872**	**-1.863**	**-1.599**
	(-4.00)	(-4.13)	(-4.14)	(-3.73)	(-4.08)	(-3.12)
$\overline{R}^2$	10.00%	7.65%	7.67%	9.62%	9.99%	**10.05%**
DAX						
α	**-0.811**	**-0.998**	**-0.997**	**-1.198**	**-0.941**	**-0.951**
	(-11.68)	(-11.10)	(-10.81)	(-7.30)	(-10.55)	(-11.29)
β	**0.221**	0.055	0.055	-0.205	0.072	0.107
	(3.77)	(0.70)	(0.68)	(-0.92)	(1.23)	(1.25)
β_ρ	**-1.196**	**-1.611**	**-1.610**	**-1.707**	**-1.407**	**-1.350**
	(-3.22)	(-3.42)	(-3.41)	(-3.26)	(-3.36)	(-3.17)
$\overline{R}^2$	**9.86%**	6.71%	6.70%	6.83%	7.05%	7.09%
DJIA						
α	**-0.621**	**-0.716**	**-0.373**	**-0.785**	**-0.299**	**-0.467**
	(-8.32)	(-4.22)	(-4.72)	(-2.68)	(-2.97)	(-5.26)
β	**0.375**	0.213	**0.606**	0.322	**0.666**	**0.676**
	(5.30)	(1.80)	(8.01)	(0.74)	(7.33)	(5.98)
β_ρ	**-0.760**	**-1.429**	**1.021**	**-1.060**	**-0.825**	-0.359
	(-2.22)	(-3.09)	(2.52)	(-2.48)	(-2.15)	(-1.00)
$\overline{R}^2$	17.06%	4.64%	**22.10%**	3.12%	13.53%	18.67%
STOXX 50						
α	**-0.845**	**-1.296**	**-1.413**	-0.634	**-0.672**	**-0.768**
	(-11.24)	(-2.89)	(-4.71)	(-1.93)	(-5.86)	(-6.62)
β	**0.224**	-0.173	-0.292	0.609	**0.246**	**0.341**
	(4.02)	(-0.46)	(-1.10)	(1.38)	(3.82)	(3.09)
β_ρ	0.414	0.525	0.574	0.720	0.650	0.630
	(1.17)	(1.31)	(1.45)	(1.74)	(1.80)	(1.71)
$\overline{R}^2$	5.29%	0.61%	0.90%	1.50%	**5.31%**	4.62%
FTSE 100						
α	**-1.193**	**-0.842**	**-0.858**	**-0.956**	**-1.030**	**-0.984**
	(-11.87)	(-2.21)	(-2.35)	(-4.16)	(-7.72)	(-5.95)
β	0.081	0.305	0.314	0.508	**0.139**	**0.288**
	(1.23)	(1.23)	(1.23)	(1.54)	(2.13)	(2.15)
β_ρ	**-1.632**	**-2.025**	**-2.039**	**-1.687**	**-1.444**	-1.170
	(-2.60)	(-3.07)	(-3.06)	(-2.53)	(-2.34)	(-1.79)
$\overline{R}^2$	6.61%	6.84%	6.83%	7.03%	**8.39%**	8.23%

Table B.19 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.479**	0.665	**1.758**	-0.232	-0.237	-0.239
	(-4.16)	(1.55)	(2.32)	(-0.55)	(-1.18)	(-1.70)
β	**0.258**	**2.167**	**3.332**	1.434	**0.335**	**0.630**
	(2.45)	(3.27)	(3.27)	(0.92)	(2.56)	(3.85)
β_ρ	0.196	-0.053	0.020	0.289	-0.141	-0.818
	(0.24)	(-0.07)	(0.03)	(0.33)	(-0.17)	(-0.99)
$\overline{R}^2$	6.55%	**18.01%**	16.79%	0.66%	6.89%	15.35%
KOSPI 200						
α	**-0.524**	0.873	1.782	**-1.044**	**-0.520**	**-0.626**
	(-6.61)	(0.63)	(1.12)	(-7.05)	(-3.71)	(-5.49)
β	**0.406**	1.725	2.417	-0.316	**0.515**	**0.349**
	(5.16)	(1.27)	(1.67)	(-1.13)	(3.04)	(2.75)
β_ρ	0.039	0.715	0.662	0.742	0.619	0.760
	(0.06)	(0.85)	(0.81)	(0.77)	(0.83)	(0.99)
$\overline{R}^2$	**16.87%**	3.28%	4.66%	1.77%	10.99%	8.48%
NASDAQ 100						
α	**-0.556**	**-1.059**	**-1.177**	**-0.653**	**-0.306**	**-0.438**
	(-7.74)	(-5.78)	(-6.84)	(-3.04)	(-3.24)	(-5.09)
β	**0.430**	-0.124	-0.308	0.601	**0.497**	**0.673**
	(6.42)	(-0.59)	(-1.43)	(1.42)	(6.73)	(6.46)
β_ρ	-0.486	-0.984	-0.761	-0.930	-0.465	-0.086
	(-1.21)	(-1.63)	(-1.25)	(-1.72)	(-1.07)	(-0.19)
$\overline{R}^2$	**19.87%**	2.46%	3.40%	3.85%	17.56%	19.67%
RUSSELL 2000						
α	**-0.837**	**-1.108**	**-1.125**	**-1.014**	**-0.269**	**-0.515**
	(-10.02)	(-16.19)	(-16.68)	(-15.59)	(-2.31)	(-4.80)
β	**0.186**	-0.108	-0.132	0.027	**0.571**	**0.587**
	(2.81)	(-1.44)	(-1.83)	(0.25)	(6.59)	(5.22)
β_ρ	-0.502	-0.399	-0.341	-0.587	-0.435	0.011
	(-1.20)	(-0.81)	(-0.70)	(-1.23)	(-1.09)	(0.03)
$\overline{R}^2$	4.14%	1.81%	2.47%	0.79%	**12.43%**	9.03%
SP500						
α	**-0.872**	**-1.081**	**-1.207**	-0.352	**-0.440**	**-0.589**
	(-9.26)	(-4.96)	(-5.75)	(-1.28)	(-2.94)	(-5.21)
β	**0.266**	0.073	-0.016	**1.230**	**0.459**	**0.608**
	(3.75)	(0.50)	(-0.11)	(2.90)	(5.11)	(5.86)
β_ρ	**-1.723**	**-2.573**	**-2.360**	**-2.216**	**-1.517**	**-1.206**
	(-3.38)	(-3.59)	(-3.28)	(-3.93)	(-2.91)	(-2.50)
$\overline{R}^2$	12.58%	6.69%	6.60%	8.73%	14.70%	**15.82%**
Aggregated Results						
Average α	-0.767	-0.644	-0.446	-0.751	-0.555	-0.649
Average β	0.261	0.433	0.620	0.471	0.372	0.448
Average β_ρ	-0.743	-0.991	-0.691	-0.831	-0.683	-0.519
Average $\overline{R}^2$	10.88%	5.87%	7.81%	4.39%	10.68%	**11.70%**

Table B.20 Information Content of Skewness Forecasts (60 days) - MIDAS with 300 lagged days

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 60 calendar days. The conditioning variable in the QMIDAS model is a function of the previous 300 daily returns. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-1.156**	-0.817	-0.825	**-1.030**	**-0.856**	**-0.886**
	(-9.54)	(-1.55)	(-1.60)	(-5.27)	(-5.61)	(-8.69)
β	0.002	0.192	0.188	0.109	**0.229**	**0.253**
	(0.02)	(0.64)	(0.64)	(0.68)	(2.14)	(3.06)
β_ρ	**-2.669**	**-2.826**	**-2.828**	**-2.866**	**-2.422**	**-2.135**
	(-3.92)	(-3.92)	(-3.91)	(-3.83)	(-3.94)	(-3.38)
$\overline{R}^2$	19.70%	19.92%	19.92%	20.02%	22.01%	**23.64%**
DAX						
α	**-1.057**	**-1.065**	**-1.061**	**-1.342**	**-1.132**	**-1.119**
	(-11.20)	(-11.57)	(-11.32)	(-8.24)	(-13.41)	(-14.27)
β	0.107	0.080	0.083	-0.162	0.039	0.061
	(1.79)	(1.26)	(1.28)	(-1.14)	(0.83)	(1.03)
β_ρ	-0.765	**-1.155**	**-1.162**	-1.251	-0.867	-0.829
	(-1.61)	(-1.98)	(-1.98)	(-1.94)	(-1.74)	(-1.64)
$\overline{R}^2$	**3.57%**	3.39%	3.42%	3.29%	2.71%	2.91%
DJIA						
α	**-0.789**	**-0.906**	**-0.678**	**-0.951**	**-0.826**	**-0.806**
	(-7.85)	(-4.09)	(-7.58)	(-6.33)	(-6.25)	(-6.90)
β	**0.309**	0.113	**0.420**	0.250	**0.293**	**0.357**
	(3.66)	(1.09)	(5.65)	(1.28)	(2.72)	(2.94)
β_ρ	**-1.169**	**-1.962**	0.560	**-2.058**	**-1.414**	**-1.083**
	(-2.39)	(-3.25)	(0.97)	(-3.16)	(-2.47)	(-2.10)
$\overline{R}^2$	13.79%	6.15%	**21.60%**	6.72%	8.14%	11.80%
STOXX 50						
α	**-1.078**	**-1.017**	**-1.086**	**-1.715**	**-1.030**	**-1.097**
	(-9.97)	(-2.73)	(-3.04)	(-6.49)	(-7.26)	(-9.95)
β	0.113	0.115	0.084	**-0.545**	0.124	0.111
	(1.62)	(0.54)	(0.37)	(-1.97)	(1.55)	(1.21)
β_ρ	-0.126	-0.210	-0.182	-0.028	-0.015	-0.040
	(-0.18)	(-0.28)	(-0.24)	(-0.04)	(-0.02)	(-0.05)
$\overline{R}^2$	1.23%	0.10%	-0.00%	0.68%	**1.29%**	0.74%
FTSE 100						
α	**-1.319**	**-1.223**	**-1.304**	**-1.332**	**-1.140**	**-1.088**
	(-8.39)	(-3.16)	(-3.55)	(-8.77)	(-7.23)	(-8.20)
β	0.049	0.075	0.042	0.071	0.151	**0.240**
	(0.56)	(0.42)	(0.22)	(0.49)	(1.73)	(2.52)
β_ρ	**-1.654**	**-1.799**	**-1.759**	**-1.840**	-1.177	-1.161
	(-2.12)	(-2.39)	(-2.33)	(-2.17)	(-1.60)	(-1.72)
$\overline{R}^2$	5.70%	5.57%	5.50%	5.62%	8.23%	**10.03%**

Table B.20 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.533**	0.649	1.774	**-1.009**	**-0.515**	**-0.382**
	(-3.78)	(0.98)	(1.43)	(-4.88)	(-2.44)	(-2.43)
β	0.207	1.577	2.457	-0.807	0.147	**0.462**
	(1.44)	(1.96)	(1.95)	(-1.23)	(1.32)	(2.69)
β_ρ	-0.156	-0.330	-0.305	0.170	-0.120	-0.477
	(-0.25)	(-0.72)	(-0.65)	(0.27)	(-0.18)	(-0.85)
$\overline{R}^2$	3.74%	16.71%	**16.92%**	1.58%	1.41%	8.04%
KOSPI 200						
α	**-0.620**	1.405	2.093	**-0.842**	**-0.694**	**-0.686**
	(-8.37)	(1.07)	(1.34)	(-8.82)	(-9.33)	(-7.99)
β	**0.322**	1.667	2.035	0.140	**0.298**	**0.290**
	(2.99)	(1.71)	(1.87)	(0.87)	(2.50)	(2.31)
β_ρ	-0.870	-1.608	-1.688	-1.227	-1.020	-0.918
	(-0.68)	(-1.21)	(-1.29)	(-0.85)	(-0.74)	(-0.69)
$\overline{R}^2$	**8.10%**	5.09%	6.20%	1.85%	4.64%	6.24%
NASDAQ 100						
α	**-0.547**	**-1.330**	**-1.337**	**-0.938**	**-0.290**	**-0.438**
	(-5.49)	(-5.62)	(-6.15)	(-7.17)	(-2.79)	(-4.24)
β	**0.520**	-0.186	-0.232	**0.306**	**0.657**	**0.701**
	(7.42)	(-1.01)	(-1.16)	(2.01)	(9.00)	(7.69)
β_ρ	-0.316	-0.821	-0.744	-0.664	-0.024	0.313
	(-0.68)	(-1.16)	(-1.04)	(-0.92)	(-0.06)	(0.73)
$\overline{R}^2$	28.79%	3.53%	3.93%	5.39%	28.76%	**32.66%**
RUSSELL 2000						
α	**-0.978**	**-1.575**	**-1.568**	**-1.462**	**-0.579**	**-0.762**
	(-6.51)	(-18.23)	(-18.77)	(-19.03)	(-2.64)	(-4.41)
β	**0.225**	**-0.270**	**-0.270**	**-0.238**	**0.502**	**0.447**
	(2.20)	(-3.95)	(-4.01)	(-3.91)	(3.38)	(3.28)
β_ρ	-0.120	0.345	0.364	-0.072	0.258	0.327
	(-0.26)	(0.76)	(0.80)	(-0.15)	(0.55)	(0.76)
$\overline{R}^2$	5.18%	11.85%	**12.38%**	10.02%	10.63%	9.24%
SP500						
α	**-0.913**	**-1.555**	**-1.580**	**-1.432**	**-0.406**	**-0.654**
	(-7.83)	(-5.07)	(-5.55)	(-12.46)	(-2.56)	(-5.34)
β	**0.344**	-0.084	-0.107	-0.058	**0.633**	**0.584**
	(4.95)	(-0.60)	(-0.73)	(-0.50)	(6.53)	(6.69)
β_ρ	**-1.570**	**-2.302**	**-2.237**	**-2.444**	**-1.098**	-0.967
	(-2.95)	(-3.42)	(-3.28)	(-4.01)	(-2.08)	(-1.89)
$\overline{R}^2$	21.61%	11.47%	11.57%	11.42%	25.89%	**26.45%**
Aggregated Results						
Average α	-0.899	-0.743	-0.557	-1.205	-0.747	-0.792
Average β	0.220	0.328	0.470	-0.093	0.307	0.351
Average β_ρ	-0.942	-1.267	-0.998	-1.228	-0.790	-0.697
Average $\overline{R}^2$	11.14%	8.38%	10.14%	6.66%	11.37%	**13.17%**

Table B.21 Information Content of Skewness Forecasts (90 days) - MIDAS with 300 lagged days

This table reports the results from Mincer-Zarnowitz regressions. I regress the realized skewness of each index in Table 3.1 on the forecasts generated from each model in Table 3.3. The forecasting horizon is 90 calendar days. The conditioning variable in the QMIDAS model is a function of the previous 300 daily returns. The GARCH and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β respectively denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. The bottom panel of the table contains the average values of α, β and β^ρ across indices.

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
AEX						
α	**-0.944**	**-1.240**	**-1.250**	**-1.263**	**-0.839**	**-0.836**
	(-8.06)	(-2.50)	(-2.54)	(-9.06)	(-5.94)	(-8.90)
β	**0.293**	0.031	0.026	0.033	**0.315**	**0.379**
	(3.48)	(0.15)	(0.13)	(0.44)	(3.34)	(5.89)
β_ρ	-1.060	**-1.910**	**-1.904**	**-1.976**	**-1.562**	**-1.240**
	(-1.67)	(-2.43)	(-2.43)	(-2.54)	(-2.49)	(-2.37)
$\overline{R}^2$	18.90%	11.83%	11.82%	11.98%	18.21%	**25.79%**
DAX						
α	**-1.146**	**-1.114**	**-1.094**	**-1.442**	**-1.129**	**-1.096**
	(-6.51)	(-9.06)	(-8.42)	(-8.84)	(-13.21)	(-13.98)
β	0.131	0.104	0.112	-0.139	**0.134**	**0.178**
	(0.99)	(1.63)	(1.70)	(-0.88)	(2.24)	(2.91)
β_ρ	-0.124	-0.520	-0.535	-0.450	-0.207	-0.177
	(-0.31)	(-1.01)	(-1.03)	(-0.84)	(-0.46)	(-0.41)
$\overline{R}^2$	1.86%	2.50%	2.69%	0.99%	2.47%	**4.74%**
DJIA						
α	**-0.885**	**-1.595**	**-0.846**	**-1.609**	**-0.840**	**-0.908**
	(-6.75)	(-6.40)	(-10.70)	(-9.43)	(-7.03)	(-7.44)
β	**0.310**	-0.121	**0.324**	**-0.452**	**0.365**	**0.329**
	(3.47)	(-1.37)	(4.62)	(-2.39)	(4.45)	(3.92)
β_ρ	-0.049	-0.081	**0.970**	0.301	-0.029	-0.114
	(-0.14)	(-0.17)	(1.97)	(0.68)	(-0.07)	(-0.31)
$\overline{R}^2$	10.22%	1.47%	**21.94%**	5.32%	5.75%	7.39%
STOXX 50						
α	**-1.024**	**-1.455**	**-1.469**	**-1.283**	**-1.033**	**-1.071**
	(-7.47)	(-4.27)	(-4.06)	(-4.65)	(-7.23)	(-10.39)
β	**0.249**	-0.045	-0.057	0.088	**0.200**	**0.227**
	(2.67)	(-0.29)	(-0.31)	(0.27)	(2.40)	(2.49)
β_ρ	0.246	0.239	0.247	0.263	0.285	0.371
	(0.41)	(0.34)	(0.35)	(0.36)	(0.42)	(0.56)
$\overline{R}^2$	**6.31%**	0.12%	0.14%	0.14%	4.48%	5.57%
FTSE 100						
α	**-1.111**	**-1.988**	**-1.984**	**-1.854**	**-1.134**	**-1.149**
	(-7.24)	(-4.72)	(-4.74)	(-9.48)	(-7.97)	(-8.81)
β	**0.270**	-0.172	-0.185	**-0.455**	**0.246**	**0.256**
	(3.78)	(-1.08)	(-1.09)	(-2.16)	(3.37)	(3.42)
β_ρ	-0.423	-0.483	-0.465	0.407	0.061	-0.359
	(-0.57)	(-0.59)	(-0.57)	(0.41)	(0.07)	(-0.47)
$\overline{R}^2$	8.24%	2.29%	2.33%	5.21%	9.10%	**11.11%**

Table B.21 *(continued)*

	LRS	GARCH-1	GARCH-2	QMIDAS	IS	CIS
HANGSENG						
α	**-0.766**	1.074	2.249	**-1.712**	**-0.753**	**-0.682**
	(-4.94)	(1.51)	(1.91)	(-3.28)	(-3.91)	(-6.51)
β	0.046	**1.816**	**2.490**	-1.684	0.035	**0.106**
	(0.24)	(2.60)	(2.57)	(-1.88)	(0.38)	(5.00)
β_ρ	1.777	**1.357**	**1.364**	1.155	1.768	1.093
	(1.54)	(2.08)	(2.07)	(1.44)	(1.52)	(1.32)
$\overline{R}^2$	4.10%	**30.95%**	30.35%	18.91%	4.06%	19.66%
KOSPI 200						
α	**-0.813**	1.856	2.553	**-0.697**	**-0.664**	**-0.567**
	(-6.22)	(1.57)	(1.86)	(-5.79)	(-5.21)	(-4.11)
β	-0.007	**1.623**	**1.874**	**0.277**	**0.216**	**0.297**
	(-0.03)	(2.27)	(2.46)	(2.24)	(2.61)	(3.23)
β_ρ	-0.039	-0.526	-0.557	-0.426	0.178	0.604
	(-0.03)	(-0.51)	(-0.56)	(-0.39)	(0.13)	(0.47)
$\overline{R}^2$	-0.19%	8.01%	**9.28%**	6.23%	1.66%	7.00%
NASDAQ 100						
α	**-0.660**	**-1.648**	**-1.652**	**-1.432**	**-0.694**	**-0.781**
	(-5.09)	(-6.79)	(-7.25)	(-5.11)	(-4.35)	(-5.83)
β	**0.576**	-0.137	-0.166	0.011	**0.531**	**0.520**
	(8.27)	(-0.97)	(-1.08)	(0.04)	(7.05)	(7.42)
β_ρ	0.668	0.802	0.828	0.758	0.774	0.861
	(1.20)	(1.07)	(1.10)	(1.01)	(1.40)	(1.57)
$\overline{R}^2$	**34.58%**	1.91%	2.18%	1.19%	22.50%	28.96%
RUSSELL 2000						
α	**-0.813**	**-1.896**	**-1.890**	**-1.776**	**-0.726**	**-0.817**
	(-5.02)	(-17.17)	(-16.93)	(-21.57)	(-3.82)	(-5.03)
β	**0.454**	**-0.301**	**-0.300**	**-0.320**	**0.507**	**0.475**
	(4.48)	(-3.91)	(-3.81)	(-4.41)	(4.47)	(4.59)
β_ρ	0.465	0.995	0.986	0.992	0.515	0.727
	(0.92)	(1.79)	(1.76)	(1.71)	(1.05)	(1.57)
$\overline{R}^2$	**20.28%**	17.48%	17.23%	17.05%	18.83%	19.95%
SP500						
α	**-0.906**	**-2.178**	**-2.176**	**-1.851**	**-0.568**	**-0.750**
	(-6.28)	(-6.30)	(-6.37)	(-9.97)	(-3.27)	(-5.91)
β	**0.447**	-0.214	-0.238	-0.233	**0.655**	**0.580**
	(5.66)	(-1.77)	(-1.78)	(-1.70)	(6.36)	(7.67)
β_ρ	-0.959	-1.540	-1.505	**-1.705**	-0.911	-0.498
	(-1.45)	(-1.69)	(-1.65)	(-1.97)	(-1.34)	(-0.80)
$\overline{R}^2$	25.55%	9.73%	9.80%	9.36%	27.64%	**32.34%**
Aggregated Results						
Average α	-0.907	-1.018	-0.756	-1.492	-0.838	-0.866
Average β	0.277	0.258	0.388	-0.288	0.320	0.335
Average β_ρ	0.050	-0.167	-0.057	-0.068	0.087	0.127
Average $\overline{R}^2$	12.98%	8.63%	10.78%	7.64%	11.47%	**16.25%**

Table B.22 Encompassing Regressions - MIDAS with 300 lagged days

This table reports the results from regressing the realized skewness on forecasts generated from the LRS, GARCH-2, QMIDAS and CIS models, within the same regression, for each index in Table 3.1. The conditioning variable in the QMIDAS model is a function of the previous 300 daily returns. The GARCH-2 and QMIDAS models are estimated using the whole sample. In the regressions, I control for the the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months in order to account for any bias in the realized skewness estimates. α and β_i respectively denote the intercept and the coefficient of the forecast of model i in the regression. In addition, β^ρ is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient while the numbers in parentheses denote t-statistics, estimated using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Significant coefficients at the 5% level are highlighted in bold. Panel A, B and C respectively present results for a forecasting horizon of 30, 60 and 90 calendar days. (1) - (10) are the ten international indices of order listed in Table 3.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Panel A: 30 days					
α	0.099	**-0.669**	-0.004	-0.556	0.056	1.102	2.340	-0.068	**-0.504**	0.561
	(0.16)	(-3.06)	(-0.02)	(-0.87)	(0.17)	(1.73)	(1.85)	(-0.20)	(-4.28)	(1.74)
β_ρ	**-1.365**	**-1.382**	**0.745**	0.596	-1.099	-0.561	-0.122	0.077	0.319	-0.453
	(-2.71)	(-2.99)	(2.22)	(1.56)	(-1.61)	(-0.72)	(-0.21)	(0.17)	(0.72)	(-0.79)
β_{LRS}	**0.161**	**0.219**	**0.188**	**0.167**	0.069	0.043	**0.317**	**0.284**	0.084	**0.195**
	(2.07)	(3.79)	(2.47)	(2.87)	(1.14)	(0.40)	(4.40)	(3.95)	(1.21)	(2.74)
$\beta_{GARCH-2}$	0.268	0.113	**0.361**	0.020	0.364	**2.397**	**2.782**	0.026	**-0.174**	-0.098
	(0.52)	(1.41)	(5.12)	(0.07)	(1.40)	(2.46)	(2.32)	(0.13)	(-2.17)	(-0.79)
β_{QMIDAS}	**0.561**	-0.062	0.240	0.149	0.498	-0.866	**-0.548**	0.536	**0.268**	**1.723**
	(2.56)	(-0.29)	(0.73)	(0.26)	(1.41)	(-0.85)	(-2.37)	(1.27)	(2.39)	(4.52)
β_{CIS}	0.184	0.095	**0.348**	0.227	**0.383**	**0.369**	0.183	**0.428**	**0.485**	**0.489**
	(1.73)	(1.21)	(3.81)	(1.89)	(3.06)	(2.31)	(1.83)	(4.17)	(4.27)	(4.61)
$\overline{R}^2$	14.27%	12.69%	29.58%	6.88%	11.32%	21.32%	22.79%	26.24%	11.38%	21.33%
					Panel B: 60 days					
α	-0.566	**-0.835**	**-0.467**	**-1.187**	-0.297	1.455	**4.509**	**-0.378**	**-1.189**	**-0.697**
	(-0.75)	(-3.43)	(-3.21)	(-2.61)	(-0.63)	(1.37)	(2.23)	(-2.34)	(-7.03)	(-2.18)
β_ρ	**-2.512**	**-1.037**	0.358	-0.189	**-1.494**	-0.361	-1.565	**0.876**	0.606	-0.726
	(-3.74)	(-2.00)	(0.60)	(-0.32)	(-2.11)	(-0.66)	(-1.55)	(2.15)	(1.46)	(-1.24)
β_{LRS}	-0.079	0.126	**0.170**	0.123	-0.004	0.015	**0.226**	**0.291**	-0.056	**0.172**
	(-0.92)	(1.84)	(2.43)	(1.68)	(-0.04)	(0.08)	(2.29)	(4.04)	(-0.62)	(2.82)
$\beta_{GARCH-2}$	0.121	0.113	**0.334**	0.287	0.336	2.110	**3.532**	-0.190	-0.159	-0.095
	(0.24)	(1.45)	(4.75)	(1.27)	(1.72)	(1.87)	(2.49)	(-1.54)	(-1.29)	(-0.50)
β_{QMIDAS}	0.115	-0.023	0.091	**-0.700**	0.030	-0.224	-0.307	**0.366**	-0.077	0.118
	(0.50)	(-0.15)	(0.58)	(-2.06)	(0.23)	(-0.49)	(-1.58)	(3.80)	(-0.69)	(0.80)
β_{CIS}	**0.311**	0.058	0.053	0.068	**0.316**	0.169	0.193	**0.433**	**0.351**	**0.440**
	(3.40)	(0.98)	(0.69)	(0.74)	(2.59)	(1.28)	(1.89)	(5.39)	(3.21)	(5.52)
$\overline{R}^2$	24.81%	5.27%	24.09%	2.91%	11.53%	17.92%	16.30%	40.02%	16.27%	27.18%
					Panel C: 90 days					
α	-0.104	-0.327	**-1.047**	-0.873	**-0.772**	1.858	**5.259**	**-0.559**	**-1.069**	**-0.674**
	(-0.27)	(-1.18)	(-6.05)	(-0.92)	(-2.01)	(1.68)	(2.05)	(-3.26)	(-5.94)	(-2.12)
β_ρ	**-1.490**	-0.359	**1.385**	0.241	0.210	1.074	0.258	0.738	**0.983**	-0.389
	(-2.94)	(-0.89)	(3.11)	(0.45)	(0.31)	(1.88)	(0.25)	(1.74)	(2.54)	(-0.69)
β_{LRS}	0.118	0.073	0.125	**0.191**	0.165	-0.140	-0.130	**0.407**	**0.185**	**0.184**
	(1.31)	(0.77)	(1.75)	(2.13)	(1.76)	(-1.06)	(-1.02)	(6.22)	(2.17)	(2.38)
$\beta_{GARCH-2}$	0.225	**0.234**	**0.283**	0.050	0.171	**2.109**	**3.329**	-0.001	-0.113	0.008
	(1.49)	(2.86)	(4.88)	(0.17)	(1.37)	(2.61)	(2.25)	(-0.01)	(-1.24)	(0.05)
β_{QMIDAS}	0.056	0.231	**-0.372**	-0.071	-0.285	0.155	-0.268	-0.034	-0.055	-0.010
	(1.01)	(1.45)	(-2.78)	(-0.14)	(-1.86)	(0.35)	(-1.20)	(-0.19)	(-0.57)	(-0.06)
β_{CIS}	**0.361**	**0.181**	-0.022	0.149	**0.194**	**0.085**	**0.370**	**0.283**	**0.262**	**0.439**
	(4.17)	(3.84)	(-0.39)	(1.79)	(2.60)	(3.12)	(3.63)	(4.93)	(3.91)	(6.95)
$\overline{R}^2$	28.22%	9.55%	26.88%	8.22%	14.30%	36.78%	19.23%	39.78%	28.45%	32.87%

Table B.23 Model Confidence Set under Root Mean Squared Errors - MIDAS with 300 lagged days

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. The conditioning variable in the QMIDAS model is a function of the previous 300 daily returns. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4	0.01	2*	0.586	3	0.00	2	0.00	3	0.00	5	0.01	3*	0.653	2	0.02	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	3*	0.255	1*	1.000	5	0.00	4	0.00	5	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.486	6*	0.072	6	0.00	6	0.00	6	0.00
QMIDAS	3	0.03	3	0.00	4	0.00	3	0.00	4	0.00	4*	0.064	5*	0.072	4	0.00	5	0.00	4	0.00
IS	2*	0.831	4	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	2*	0.963	3	0.00	2	0.01	2	0.00
CIS	1*	1.000	1*	1.000	2*	0.095	1*	1.000	1*	1.000	1*	1.000	4*	0.095	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	1*	1.000	3*	0.206	2*	0.612	2	0.02	4*	0.144	2*	0.534	3	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.144	4	0.05	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.880	6	0.00	5	0.00	4	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	5	0.03	5	0.00	4	0.00	5	0.00	4	0.00
IS	1*	1.000	3*	0.077	1*	1.000	3	0.00	4	0.00	6	0.00	3	0.05	2	0.03	2*	0.557	1*	1.000
CIS	2*	0.876	2*	0.322	2*	0.206	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.207
Panel C: 90-day horizon																				
LRS	3*	0.164	1*	1.000	2*	0.064	1*	1.000	1*	1.000	4	0.03	4	0.00	1*	1.000	2*	0.611	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	2	0.03	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	5	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	4	0.00	3	0.03	3	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.235	3	0.03	1*	1.000	3	0.00	3	0.00	5	0.00	2	0.00	2*	0.186	1*	1.000	1*	1.000
CIS	1*	1.000	2	0.03	3*	0.064	2	0.00	2*	0.157	6	0.00	1*	1.000	3*	0.175	3*	0.135	2*	0.149

Table B.24 Model Confidence Set under Mean Absolute Error - MIDAS with 300 lagged days

This table reports the results for the Model Confidence Set (MCS) test using the Mean Absolute Error (MAE) as a loss function and a 5% significance level. The conditioning variable in the QMIDAS model is a function of the previous 300 daily returns. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	4*	0.065	2*	0.628	3	0.00	2	0.00	3	0.00	5	0.00	4*	0.118	2	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	2	0.00	3*	0.215	2*	0.360	5	0.00	5	0.00	4	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	4	0.00	5	0.00	2*	0.358	6	0.00	6	0.00	6	0.00	6	0.00
QMIDAS	3*	0.085	4	0.00	4	0.00	3	0.00	4	0.00	4*	0.215	5	0.00	4	0.00	4	0.00	5	0.00
IS	2*	0.360	3	0.00	1*	1.000	5	0.00	6	0.00	6	0.00	1*	1.000	3	0.00	2	0.00	2	0.00
CIS	1*	1.000	1*	1.000	2*	0.203	1*	1.000	1*	1.000	1*	1.000	3*	0.118	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.01	2*	0.519	3	0.01	2*	0.672	2	0.02	3*	0.569	3*	0.470	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	4*	0.458	4	0.04	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	3	0.00	2*	0.569	6	0.00	5	0.00	4	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	5*	0.157	5	0.00	4	0.00	5	0.00	5	0.00
IS	2*	0.643	3*	0.250	1*	1.000	3	0.00	4	0.00	6	0.00	2*	0.470	3	0.00	2*	0.253	1*	1.000
CIS	1*	1.000	1*	1.000	2	0.01	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.731
Panel C: 90-day horizon																				
LRS	3*	0.862	1*	1.000	3*	0.111	1*	1.000	1*	1.000	5	0.03	3	0.00	1*	1.000	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	2	0.03	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	3	0.03	4	0.00	4	0.00	4	0.00	5	0.00
IS	1*	1.000	2*	0.056	1*	1.000	3	0.00	3	0.04	6	0.00	2	0.00	2*	0.984	1*	1.000	1*	1.000
CIS	2*	0.862	3	0.03	2*	0.111	2	0.00	2*	0.315	4	0.03	1*	1.000	3*	0.402	2	0.00	2	0.00

Table B.25 Model Confidence Set under Root Mean Squared Errors - 1000-observations Window

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. The GARCH and QMIDAS models are estimated using a rolling window of 1,000 observations. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	3	0.02	2*	0.528	3	0.00	2	0.00	2	0.00	3	0.00	3	0.03	2	0.02	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	3	0.00	5	0.00	4	0.03	5	0.00	5	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	5	0.00	4	0.00	6	0.01	6	0.00	6	0.00	5	0.00
QMIDAS	4	0.01	3	0.00	4	0.00	3	0.00	4	0.00	2	0.00	5	0.03	4	0.00	4	0.00	4	0.00
IS	2*	0.878	4	0.00	1*	1.000	4	0.00	6	0.00	6	0.00	1*	1.000	3	0.00	2	0.02	2	0.01
CIS	1*	1.000	1*	1.000	2*	0.063	1*	1.000	1*	1.000	1*	1.000	2*	0.287	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	1*	1.000	3*	0.103	1*	1.000	2	0.00	3*	0.140	3*	0.321	3	0.02	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	4	0.00	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	4	0.00	5	0.00	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	5	0.00	2*	0.582	4	0.00	4	0.00	4	0.00	4	0.00
IS	1*	1.000	3*	0.067	1*	1.000	3	0.00	3	0.00	6	0.00	2*	0.321	2	0.02	2*	0.884	1*	1.000
CIS	2*	0.943	2*	0.347	2*	0.103	2*	0.779	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.086
Panel C: 90-day horizon																				
LRS	3*	0.452	1*	1.000	3	0.01	1*	1.000	1*	1.000	1*	1.000	4	0.00	1*	1.000	2*	0.386	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.618	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	5	0.00	4*	0.618	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	4	0.00	2*	0.714	3	0.02	4	0.00	4	0.00	4	0.00
IS	2*	0.452	3*	0.316	1*	1.000	3	0.00	3	0.00	5*	0.221	2	0.02	2*	0.143	1*	1.000	1*	1.000
CIS	1*	1.000	2*	0.316	2	0.01	2	0.00	2	0.04	6*	0.221	1*	1.000	3*	0.143	3	0.04	2	0.03

Table B.26 Model Confidence Set under Mean Absolute Error - 1000-observations Window

This table reports the results for the Model Confidence Set (MCS) test using the Mean Absolute Error (MAE) as a loss function and a 5% significance level. The GARCH and QMIDAS models are estimated using a rolling window of 1,000 observations. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	3	0.02	2*	0.583	3	0.00	2	0.00	2	0.00	5	0.00	3	0.02	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	3	0.00	3	0.00	4	0.02	5	0.00	5	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	4	0.00	4	0.00	6	0.00	6	0.00	6	0.00	5	0.00
QMIDAS	4	0.00	3	0.00	4	0.00	3	0.00	5	0.00	2	0.00	5	0.00	4	0.00	4	0.00	4	0.00
IS	2*	0.510	4	0.00	1*	1.000	4	0.00	6	0.00	6	0.00	1*	1.000	3	0.00	2	0.00	2	0.00
CIS	1*	1.000	1*	1.000	2*	0.152	1*	1.000	1*	1.000	1*	1.000	2	0.02	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3	0.00	2*	0.474	3	0.00	1*	1.000	2	0.00	3*	0.518	3*	0.506	2	0.00	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	4	0.00	4	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	5	0.00	6	0.00	5	0.00	5	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	2*	0.861	5	0.00	4	0.00	4	0.00	5	0.00
IS	2*	0.619	3*	0.296	1*	1.000	3	0.00	3	0.00	6	0.00	2*	0.685	3	0.00	2*	0.616	1*	1.000
CIS	1*	1.000	1*	1.000	2	0.01	2*	0.547	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.460
Panel C: 90-day horizon																				
LRS	2*	0.850	1*	1.000	3	0.03	1*	1.000	1*	1.000	1*	1.000	4	0.00	1*	1.000	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3	0.02	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	5	0.00	5	0.00	4	0.00	4	0.02	6	0.00	5	0.00	5	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	4	0.00	4	0.00	5	0.00	2	0.02	3	0.00	4	0.00	4	0.00	5	0.00
IS	1*	1.000	2*	0.331	1*	1.000	3	0.00	3	0.01	5	0.02	2	0.04	2*	0.659	1*	1.000	1*	1.000
CIS	3*	0.850	3*	0.295	2	0.03	2	0.00	2*	0.107	6	0.00	1*	1.000	3*	0.622	2	0.00	2	0.00

Table B.27 Model Confidence Set under Root Mean Squared Errors - 1500-observations Window

This table reports the results for the Model Confidence Set (MCS) test using the Root Mean Squared Error (RMSE) as a loss function and a 5% significance level. The GARCH and QMIDAS models are estimated using a rolling window of 1,500 observations. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	3*	0.184	2*	0.714	3	0.00	3	0.00	4	0.00	6	0.00	3*	0.052	2	0.04	4	0.00	3	0.00
GARCH-1	5*	0.110	6	0.00	4	0.00	6	0.00	2	0.01	3*	0.377	1*	1.000	5	0.00	5	0.00	4	0.00
GARCH-2	6	0.00	5	0.00	6	0.00	4	0.00	5	0.00	2*	0.791	6	0.00	6	0.00	6	0.00	6	0.00
QMIDAS	4*	0.165	3	0.00	5	0.00	2	0.00	3	0.01	4*	0.077	4	0.00	4	0.00	3	0.00	5	0.00
IS	2*	0.998	4	0.00	1*	1.000	5	0.00	6	0.00	5*	0.077	2*	0.052	3	0.00	2*	0.059	2*	0.069
CIS	1*	1.000	1*	1.000	2*	0.053	1*	1.000	1*	1.000	1*	1.000	5	0.00	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3*	0.114	1*	1.000	2*	0.390	2*	0.218	2	0.01	6	0.00	3*	0.522	3	0.03	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.078	1*	1.000	6	0.00	6	0.00	4	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.01	2*	0.078	6	0.00	5	0.00	5	0.00	5	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	3	0.03	3	0.01	5	0.00	5	0.00	4	0.00	4	0.00	6	0.00
IS	1*	1.000	3*	0.056	1*	1.000	4	0.00	5	0.00	4*	0.078	4	0.03	2*	0.235	2*	0.880	1*	1.000
CIS	2*	0.824	2*	0.262	3*	0.212	1*	1.000	1*	1.000	1*	1.000	2*	0.522	1*	1.000	1*	1.000	2*	0.107
Panel C: 90-day horizon																				
LRS	2*	0.427	1*	1.000	2*	0.060	1*	1.000	1*	1.000	5*	0.059	5	0.00	1*	1.000	2*	0.564	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.138	4	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	1*	1.000	6	0.00	5	0.00	5	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	2*	0.447	3	0.00	4	0.00	4	0.00	5	0.00
IS	3*	0.312	3	0.02	1*	1.000	3	0.00	3	0.00	4*	0.096	2	0.00	2*	0.358	1*	1.000	1*	1.000
CIS	1*	1.000	2	0.05	3*	0.060	2	0.03	2*	0.609	6*	0.059	1*	1.000	3*	0.146	3	0.04	2*	0.387

Table B.28 Model Confidence Set under Mean Absolute Error - 1500-observations Window

This table reports the results for the Model Confidence Set (MCS) test using the Mean Absolute Error (MAE) as a loss function and a 5% significance level. The GARCH and QMIDAS models are estimated using a rolling window of 1,500 observations. Panel A, B and C report results for the 30-, 60- and 90-day forecasting horizons, respectively. The columns labeled '*Rank*' present the ranking of a model in the MCS while the '*pval*' columns show the p-values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

	AEX		DAX		DJIA		STOXX50		FTSE100		HANGSENG		KOSPI		NASDAQ		RUSSELL2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LRS	3*	0.291	2*	0.652	3	0.00	3	0.00	4	0.00	6	0.00	5	0.00	2	0.03	4	0.00	3	0.00
GARCH-1	5*	0.211	6	0.00	4	0.00	5	0.00	2	0.00	3*	0.816	1*	1.000	5	0.00	5	0.00	4	0.00
GARCH-2	6	0.00	5	0.00	6	0.00	4	0.00	5	0.00	2*	0.816	6	0.00	6	0.00	6	0.00	6	0.00
QMIDAS	4*	0.211	3	0.00	5	0.00	2	0.00	3	0.00	4*	0.816	4	0.00	4	0.00	3	0.00	5	0.00
IS	2*	0.628	4	0.00	1*	1.000	6	0.00	6	0.00	5	0.00	2	0.00	3	0.00	2	0.00	2	0.00
CIS	1*	1.000	1*	1.000	2*	0.115	1*	1.000	1*	1.000	1*	1.000	3	0.00	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LRS	3*	0.121	2*	0.598	3	0.01	2*	0.154	2	0.01	2*	0.341	2*	0.877	3	0.01	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	5	0.00	4*	0.089	3*	0.877	5	0.00	6	0.00	4	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	5	0.03	6	0.00	6	0.00	5	0.00	5	0.00
QMIDAS	4	0.01	4	0.00	5	0.00	3*	0.075	3	0.00	3*	0.331	5	0.00	4	0.00	4	0.00	6	0.00
IS	2*	0.476	3*	0.153	1*	1.000	4	0.00	6	0.00	6	0.00	4	0.04	2	0.01	2*	0.471	1*	1.000
CIS	1*	1.000	1*	1.000	2	0.01	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	1*	1.000	2*	0.286
Panel C: 90-day horizon																				
LRS	1*	1.000	1*	1.000	2*	0.079	1*	1.000	2*	0.771	4*	0.252	4	0.00	2*	0.607	3	0.00	3	0.00
GARCH-1	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3*	0.252	5	0.00	6	0.00	6	0.00	6	0.00
GARCH-2	6	0.00	5	0.00	4	0.00	5	0.00	4	0.00	2*	0.252	6	0.00	5	0.00	4	0.00	4	0.00
QMIDAS	4	0.00	4	0.00	5	0.00	4	0.00	5	0.00	1*	1.000	3	0.00	4	0.00	5	0.00	5	0.00
IS	3*	0.767	3*	0.066	1*	1.000	3	0.00	3	0.00	5*	0.252	2	0.00	1*	1.000	1*	1.000	1*	1.000
CIS	2*	0.767	2*	0.066	3*	0.079	2	0.03	1*	1.000	6*	0.100	1*	1.000	3*	0.285	2	0.00	2	0.01

Table B.29 Out-of-sample performance of skewness-based portfolios - Alternative Asset Universe

This table presents the out-of-sample performance of the equally-weighted portfolio ($1/N$) and of the parametric portfolios that use the skewness forecasts from each model considered in the paper. The portfolios include as assets all indices from Table 3.34, except from HANGSENG and KOSPI 200. The table reports the annualised out-of-sample average daily return (MEAN), variance of daily returns (VAR) and Sharpe ratio (SR) for each portfolio strategy as well as the average daily turnover (TRN). It also reports p-values from testing the hypothesis that the variances between a portfolio strategy and $1/N$ are equal. The p-values are computed using the block-bootstrap approach of Ledoit and Wolf (2011), assuming an average block size of 5 and 5,000 replications. Panel A and B respectively present results for the periods 01/2011-12/2015 and 03/2008-12/2015.

	MEAN	*VAR*	*p-value*	*SR*	*TRN*
		Panel A: 01/2011-12/2015			
LRS	0.0623	0.0173	0.26	0.4747	0.1800
GARCH-1	0.0650	0.0178	0.49	0.4866	0.1080
GARCH-2	0.0859	0.0175	0.06	0.6498	0.0886
QMIDAS	0.0821	0.0180	0.96	0.6119	0.0713
IS	0.0999	0.0177	0.55	0.7500	0.2405
CIS	**0.1086**	**0.0162**	0.01	**0.8525**	0.3556
1/N	0.0831	0.0180	1.00	0.6191	0.0039
		Panel B: 03/2008-12/2015			
LRS	0.0970	0.0269	0.02	0.5913	0.1678
IS	0.1098	0.0281	0.52	0.6550	0.1850
CIS	**0.1429**	**0.0264**	0.00	**0.8796**	0.3291
1/N	0.1110	0.0284	1.00	0.6592	0.0042