RECEPTIVITY OF A FLAT PLATE WITH ROUNDED LEADING EDGE

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Summary A generalized theory for investigating the effect of leading-edge body geometry on boundary-layer receptivity is analyzed for bodies with rounded noses which tend to a flat, finite-thickness, airfoil downstream. The theory utilizes far-downstream asymptotic expansions of the induced eigenmodes in the large Reynolds number regime, together with numerical results, in order to determine the leading edge receptivity coefficient. The generalized theory is applied to a Rankine body and it is found that the receptivity level of instability waves induced in the boundary layer decreases rapidly as a function of increasing nose radius. The decrease observed is more rapid than for a parabolic nosed body, and this appears to be due to the adverse pressure gradient generated along the majority of the Rankine body.

BACKGROUND

When an aerodynamic body is placed in a mean flow containing unsteady perturbations, the position on the body where the boundary-layer transitions from a laminar to turbulent flow is strongly influenced by the free-stream perturbation. The mechanism by which the free-stream perturbation induces instability waves in the boundary layer is known as receptivity. The unsteady perturbation typically has a long wavelength and as it interacts with the boundary layer it transfers its energy to Tollmien-Schlichting instability waves (T-S waves) in the boundary layer, which have a much shorter wavelength. This transfer of energy and wavelength shortening process is usually a result of non-parallel mean flow effects. The Receptivity Coefficient characterises the amplitude of the T-S mode which after decaying, begins to grow exponentially in magnitude downstream of the lower branch point leading eventually to transition.

Motivated by experimental results [1], in this paper we focus on receptivity in the region of the leading edge, specifically for a body consisting of a flat plate of thickness $2d$ with a rounded leading edge with radius of curvature r_n. In this case, far downstream the inviscid, free-stream slip velocity at the edge of the boundary layer takes the non-dimensional form

$$U_f(x) \sim 1 + \frac{\alpha}{x} + O\left(x^{-2}\right), \tag{1}$$

where x is a streamwise coordinate and α is a constant.

We consider the incoming flow to be parallel to the flat plate with constant magnitude U but superimposed with a small oscillation of frequency ω. It is assumed that the Reynolds number based on the acoustic length is large. Starting from the non-dimensional vorticity equation, its form within the boundary layer is rewritten in terms of coordinates tangential and normal to the surface of the body, with the additional assumption that the non-dimensional nose radius $R_n = \omega r_n/U$ is $O(1)$ or smaller. Using the assumption that the time-dependent component is small, the governing equation is decomposed into a steady equation and a linearised equation for the unsteady perturbation.

The general structure of the solution for the body considered is similar to that on a flat plate [2]. The leading-order solution of the Linearised Unsteady Boundary-Layer Equation (LUBLE) develops a two-layer structure and the scaled streamfunction consists of the sum of a Stokes solution (forced by the local unsteady slip velocity) and a set of eigensolutions, $\phi = \phi_{\mathrm{St}} + \sum_i C_i\psi_i$. The wavelength of the disturbance shortens with distance downstream, leading to terms previously ignored being significant and the appearance of a Triple-Deck structure with development of Tollmien-Schlicting (T-S) waves. The first eigensolution of the LUBLE region matches on to the T-S wave which eventually grows, and hence C_1 is termed the Receptivity Coefficient and characterises the effect of the unsteady free-stream disturbance on the point of transition. The present paper focuses on how the the Receptivity Coefficient can be obtained for the body introduced earlier, using a novel combination of analytic and numeric techniques.

FORMULATION & RESULTS FOR A RANKINE BODY

The steady and unsteady boundary layer equations are re-written in terms of scaled tangential and normal coordinates, $\xi = \int_0^x U_f(x')\,\mathrm{d}x'$ and $N = U_f y(2\xi)^{-1/2}$ and take the form

$$\phi_{1NNN} + \phi_1\phi_{1NN} = \beta(\xi)\left(\phi_{1N} - 1\right) + 2\xi\left(\phi_{1N}\phi_{1N\xi} - \phi_{1NN}\phi_{1\xi}\right), \qquad \mathcal{L}(\phi_2) = \mathrm{i}\Omega(\xi) - 2\beta(\xi), \tag{2}$$

where ϕ_1 and ϕ_2 are the steady and unsteady streamfunctions, $\mathcal{L}$ is a linear differential operator involving ϕ_1 and the slip velocity (dependent on the geometry of the body) enters through the functions $\beta = 2\xi U_f'/U_f$ and $\Omega = 2\xi/U_f^2$. In order to calculate the Receptivity Coefficient, first the steady and unsteady PDEs (**??**) must be solved numerically. The solution at $\xi = 0$ is found by solving the ODEs using a fourth-order Runge-Kutta method. This solution is then

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marched downstream using a Keller Box scheme where a variable grid is used in both the ξ and N directions. The grid points are concentrated close to the body and close to the leading edge.

Next the asymptotic form of the eigenfunctions ψ_i must be found, which depends on the form of the slip velocity given by (**??**). The large-ξ forms of ψ_i have previously been derived for a parabolic body [2]. For a parabola the slip velocity also has the form (**??**), and hence the results can be used to obtain ψ_i for the present body, though involving parameters which must be extracted from the numerical solution of ϕ_1. The results for ψ_i are not included in this extended abstract, however it should be noted that for real ξ the eigenfunctions are inversely ordered and that far downstream they are exponentially small compared with the Stokes solution.

Finally the unsteady shear stress on the body obtained from the numeric solution is compared with that from the asymptotic form $\phi = \phi_{\mathrm{St}} + \sum_i C_i\psi_i$. in order to extract the Receptivity Coefficient C_1. However, as previously noted for real ξ the eigenfunctions are exponentially small which means that the coefficients are very difficult to extract. For the case of a flat plate or a parabola where $U_f(\xi)$ is known explicitly, this problem is circumvented by extending the path of integration into the complex plane since if $\arg(\xi) \in (-\pi/6, -5\pi/6)$, the eigenfunctions grow exponentially and $\phi \sim C_1\psi_1$. For more general bodies, extending into the complex plane poses more challenges.

We now consider the flow with non-dimensional complex potential $w = z + A\log z$, where $z = x_c + \mathrm{i}y_c$. It can easily be shown that this corresponds to flow past a flat plate of half-thickness $D = A\pi$ and nose radius $R_n = \frac{3}{2}A$. Setting $\theta = y_c/A$ with $\theta \in [0, \pi]$, the slip velocity is given by

$$U_f = \left(1 + \frac{\sin^2\theta}{\theta^2} - \frac{\sin 2\theta}{\theta}\right)^{1/2} \sim 1 + \frac{A}{x} + \frac{A^2}{x^2} + O(x^{-3}),. \tag{3}$$

and so comparing this with (**??**) we find $\alpha = A$. In theory we can then extend θ into the complex plane defining a path linking $\theta = 0$ to $\theta = \pi$ in such a way that $\arg(\xi) \in (-\pi/6, -5\pi/6)$ so that the Receptivity Coefficient can be extracted. In practice, more care is required. On the real axis, the numerical solution breaksdown if the steady wall shear approaches zero. Physically this would correspond to boundary layer separation. While this does not occur for the Rankine body considered, once the path of integration is extended into the complex plane it is found that the real part of the steady wall shear does approach zero, leading to numerical problems and so the path of integration must be carefully chosen. This is accomplished by taking the path of integration along the real axis until past the position of minimum wall shear then shifting into the complex plane so that it lies in the sector $\arg(\xi) \in (-\pi/6, -5\pi/6)$ for large $|\xi|$. An important check of this technique is that choosing different integration paths (subject to the restrictions described previously) should give the same value of Receptivity Coefficient and this was shown to be true.

CONCLUSIONS

Plotting the results for the absolute value of the Receptivity Coefficient it is seen that there is a rapid decrease in $|C_1|$ as the nose radius increases, but with a small secondary local maximum at $A \approx 0.035$. This can be compare with the results for a parabolic body [2] when $|C_1|$ is maximum at a non-zero nose radius, followed by a slower decay in $|C_1|$ with increasing nose radius.

Just as stability is affected by adverse and favourable pressure gradients, it has been suggested that receptivity is similarly sensitive to mean pressure gradients [3, 4]. For the Rankine body, a favourable pressure gradient is followed by an adverse pressure gradient along the majority of the body, whereas the parabolic body has a favourable pressure gradient along the whole body. The present formulation provides the opportunity to investigate this relationship more carefully by considering other leading edge geometries.

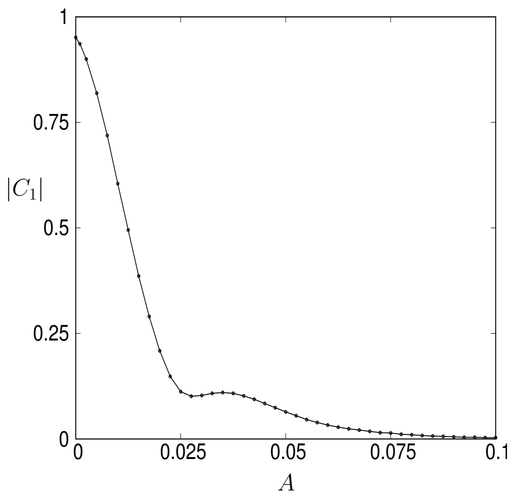

Figure 1. $|C_1|$ as a function of A

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