# Color matching in the wild 

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#### Abstract

We present a method that, given two different views of the same scene taken by two cameras with unknown settings and internal parameters, corrects the colors of one of the images making it look as if it was captured under the other camera settings. Our method is able to deal with any standard non-linear encoded images (gamma-corrected, logarithmic-encoded, or any other) without requiring any previous knowledge of the encoding.

To this end, our method makes use of two important observations. First, the camera imaging pipeline from RAW to sRGB can be well approximated by considering just a per-pixel shading and a color transformation matrix, and second, for correcting the images we only need to estimate a single matrix -that will contain information from both of the original images- and an approximation of the shading term (that emulates the non-linearity).

Our proposed method is fast and the results have no spurious artifacts. The method outperforms the state-of-the-art when compared with other methods that do not require knowledge of the encoding used. It is also able to compete with -and even surpass in some cases- methods that consider information about image encoding.


## 1. Introduction

Color matching is a well-known problem in color imaging, in which it is necessary to map the colors of one image, called the source, to those of a second image, the reference. Color stabilization, the focus of this paper, is a particular color matching problem. In color stabilization, the source and reference images are instances of the same scene, captured by different cameras or settings and with respect to different view points.

The pipelines of the different digital cameras are proprietary to the camera makers, and different cameras perform different operations (color characterization, white-balance, etcetera) in a different order. This said it is well-accepted [1] that a good approximation for the image values of the core pixels (in fact almost all pixels save those that are very close to the border of the color gamut) is
$I_{\text {out }}=N L\left(I_{\text {lin }} A\right)$,
where $I_{\text {lin }}$ is the linear image read by the camera sensor after demosaicing, and $I_{\text {out }}$ is the output image. Here both $I_{\text {lin }}$ and $I_{\text {out }}$ denote $n \times 3$ matrices, where $n$ is the number of pixels, $A$ is a $3 \times 3$ matrix which
carries color information and white balance and the value $N L$ defines a non-linearity (for Low-Dynamic Range images a power-law-gamma correction- and for High-Dynamic Range image either a logarithmic encoded curve or some standard such as HLP or PQ are used). Of course, this is a simplified version of the pipeline, as some other processing techniques, including denoising or contrast enhancement, are also applied. Yet, Eq. (1) is a reasonable approximation and is employed and deployed in different color stabilization methods, including [2-4].

Importantly, Finlayson et al. [5] recently proposed another approximation for the camera processing pipeline based on color homographies. That approach teaches us that we can also approximate the camera processing pipeline as
$I_{\text {out }}=S I_{\text {lin }} A$,
where $S$ is a diagonal matrix of size $n \times n$ that represents a pixelwise shading adjustment, and $A, I_{\text {lin }}$ and $I_{\text {out }}$ are as previously defined. Broadly, deploying either Eqs. (1) or (5) will well describe the mapping from linear raw to the non-linear final rendered image. That is, in terms of fitting error, the two methods perform equally well.

[^0]

 (right).

In this paper, we present a method that is able to color-stabilize any two images of the same scene that were captured under any combination of color matrices and non-linearities without using any prior information on how the images were processed. Both images do not require to be registered, they just require to have some common region -for example, two views of the same scene that are close enough-. We show how to combine the knowledge given by Eqs. (1) and (5), and how to model this combination. To the best of our knowledge, ours is the first paper to tackle the color stabilization problem in the wild. Indeed the prior art $[2,6]$ has mainly focused on the case where images are simply gamma-corrected cases. Moreover, both gamma-encoded and logarithmic-encoding images have been considered [3,4], but knowing the type of encoding being used was required to use those methods. A schematic idea of our problem is shown in Fig. 1.

The paper is organized as follows. In the next section, we discuss the related work. Then, in Section 3 we present our method. After that, we present the different experiments carried out and their results. The paper ends with a short conclusion.

## 2. Related work

Matching images to make them more color consistent is an important problem encountered both in industry and academia.

In industry, color consistency is, for the most part, assured by skilled manual work; either by colorists performing color grading of movies in post-production houses or by technicians that use camera control units (CCU) [7] in live TV broadcasts. In both cases, these skilled technicians may require either having the cameras properly characterized -for example, by using the ACES framework [8]- or the inclusion of color-charts in the shots.

In academia, two related but slightly different approaches have been used for solving the problem of the color matching images. First, there is the general color transfer problem. Here, we have two images, a source and a reference, and we would like them to match in terms of their color distributions. Posed in the most abstract form the content of the images -i.e. what is in the scene being imaged- can be completely different between the image pair. As an example, we may transfer the colors from one image of a sunset scene (the reference image) to another (of somewhere shot midday, the source image) in the hope of making the second source image look like it was captured at sunset.

The color stabilization problem can be thought of as a special case of color transfer. In color stabilization, the image pair -to be color matched- is acquired for the same scene, and so the underlying content captured in each image is similar. However, in camera pipelines the rendition of a pair of images can change a surprising amount even when they are captured close to each other in time. That is small changes in the content of the images can result in large changes in the color rendition. In color stabilization we would like the color matched image (say the second source image matched to the first reference) to look similar to the first and in particular the colors of the same semantic regions of the scene should almost be the same.

Reinhard et al. [9] presented a seminal work on the color matching problem, where they proposed to first convert both source and
reference images to a decorrelated color space, and then transfer the mean and the variance from the reference image to the source one. Pitié et al. [10,11] went one step further and defined the images as probability density functions. Elegantly, in Pitié et al.'s work, the color matching problem was shown to be a particular case of optimal transport (OT), in which the goal is to minimize the cost of transferring probability density distributions of the source image into the reference one.

This idea of using optimal transport as a way of performing color matching was later also considered by Rabin et al. [12] and Ferradans et al. [13]. More recently, Grogan and Dahyot have defined a new model following L2-divergence that shares some components with OT cost function [14]. Other color transfer methods include, Kotera [15] where it was proposed to match the principal components of the two images by rotation and scaling, Pouli and Reinhard [16] in which the authors performed a multi-scale histogram matching, or Xiao and Ma first proposed in [17] to consider different color statistics, and later on in [18] proposed a method that is gradient preserving together with a new evaluation metric for color transfer.

Nguyen et al. [19] proposed to split color constancy and color transfer by first applying color constancy to the input, followed by luminance matching and a final alignment of the color gamut via a linear transformation. Hwang et al. [20] introduced a moving leastsquares approach, in which a probability measure was introduced to guarantee robustness against both noise and outliers. Gong et al. [21, 22] proposed to color transfer the colors using a projective transformation and a mean intensity mapping. Recently, some methods aiming at photorealistic color transfer based on deep learning architectures have also appeared [23-26]. Still on deep learning, there are new topics that might be applied in the future to this problem, such as distilling datasets in basis and hallucination for efficiency reasons -opening the way to color correct only the basis images [27]-, or knowledge factorization [28] aiming at specializing larger networks to specific sub-tasks.

All the methods mentioned up until now are global, but some local methods also exist. For example, Tai et al. [29] proposed to segment images into different regions, to represent their distribution using Gaussian Mixture Models (GMM) to represent color distributions, and finally match these distributions. Similarly, Xiang et al. [30] also considered first a GMM representation of color areas to later match them.

Color stabilization focuses on the case in which some regions or objects appear in both the reference and the source images, and therefore it is possible to use this extra information. HaCohen et al. [6] presented the first work on this research line, where they propose to compute dense correspondences between the images, combined with a global color mapping model. Vazquez-Corral and Bertalmío [2] proposed a color stabilization algorithm built on the observation that in digital cameras the color encoding can be approximated as a matrix multiplication followed by a power law (gamma correction). Following this observation, they show how to color match the source image to the reference, it is enough to estimate a power law ( $\gamma$ value) for each


 $g(\cdot)$ for each color channel. The final color stabilized image is then computed as $g_{k}\left(I_{s}\right) H, k=\{R, G, B\}$.
of the images, and a single $3 \times 3$ matrix. Frigo et al. [31] proposed a method focusing on color stabilizing video sequences. This method is based on estimating a non-linearity and channel-based scaling.

To the best of the authors' knowledge, there are only two color stabilization works for logarithmic images. One is the method of VazquezCorral and Bertalmío [3], which relies on finding a sufficiently large number of achromatic matches among source and reference. This need to detect achromatic matches may be a challenging limitation, in some cases. The other method [4] builds from the method in [2]. In short, it converts the logarithmic-encoded image into a power-law representation to be able to apply the [2] but also improves the quality of the estimation by allowing the matrix to be a projective $4 \times 4$ transform. A limitation of both of these methods is that they both need to know how the images are encoded (logarithmic vs. gamma) in order to obtain good results. Moreover, as images are also tone mapped for preference neither the gamma or logarithmic model will account for image encoding in all cases.

Color consistency differs from color stabilization in the sense that in color consistency there are not only two but a set of images from the same scene that need to be color matched. HaCohen et al. [32] extended their color stabilization approach [6], to the case of having more than two images. Later, Park et al. [33] proposed a model in which they estimate two different parameters for the matching: a gamma correction and a white balance constant. Finally, and related to color consistency, different color transfer methods have also been considered for image stitching (see Xu and Milligan [34] for a review). Xia et al. [35] the authors compute parametric curves for each channel under color, gradient and contrast constraints on the overlapping regions.

The method presented in this paper draws from the sources of [2,4], combining these earlier works with the Color Homographies [5]. We will make the simplifying assumption that Eqs. (1) and (5) are, more or less, equivalent -a reasonable one to make as adopting either method accounts for real camera data well [5]- and, in so doing, will use them interchangeably.

## 3. Our approach

### 3.1. General idea

Let us start by supposing that we have two rendered images, $I_{r}$ and $I_{s}$-under-script referring to reference and source-, that are broadly of the same scene content and so they have a set of $n$ points $\left(p_{1}, \ldots, p_{n}\right)$ from the scene in common. Then we can define $\mathcal{I}_{r}$ and $\mathcal{I}_{s}$ as the $n \times 3$ matrices corresponding to the RGB values in each of the images for the $n$ corresponding points.

In [2], Vazquez-Corral and Bertalmío stated -following [1]- that the relationship between the RAW image and the two rendered images for the indexed pixels can be well-approximated as:
$\mathcal{I}_{r}=N L_{r}\left(\mathcal{I}_{\text {in }} A_{r}\right)$
$\mathcal{I}_{s}=N L_{s}\left(\mathcal{I}_{i n} A_{s}\right)$,

Here, the $n \times 3$ matrix $\mathcal{I}_{\text {in }}$ represents the RAW image at the points $\left(p_{1}, \ldots, p_{n}\right)$-, $A_{i}$ are $3 \times 3$ matrices and $N L_{i}$ are non-linearities (including a power-law, a logarithmic encoding or a tone mapping applied for enhancement). Substituting, we can write:
$\mathcal{I}_{r}=N L_{r}\left(N L_{s}^{-1}\left(\mathcal{I}_{s}\right) H\right)$,
where $H=A_{s}^{-1} A_{r}$.
In previous work, the form of Eq. (4) was used to discover the unknown model parameters $\left(N L_{1}, N L_{2}\right.$ and $\left.H\right)$ and these parameters were applied to the whole $I_{s}$ image to obtain the color stabilized output. In $[2,4]$ the corresponding points in the reference and source images are found for using SIFT. Although we chose SIFT due to its simplicity, other matching methods can also be utilized. Then, Eq. (4) is recast as an objective function with the model parameters found to minimize a mean-square error. A necessary constraint in their model is that they need to know the form of the non-linearity. That is, it is assumed to be a gamma or a logarithmic encoding.

In this paper, we will instead use the Homography approximation to the camera processing pipeline given by Finlayson et al. [5] to avoid placing any constraint on the form of the camera non-linearity. Using the same notation as in the previous equations, Finlayson et al. teaches us that
$\mathcal{I}_{r}=S_{r} \mathcal{I}_{\text {in }} A_{r}$
$\mathcal{I}_{s}=S_{s} \mathcal{I}_{\text {in }} A_{s}$,
where $S_{i}$ is a diagonal matrix of size $n \times n$ that represents a pixel-wise shading. Note this model is in actuality approximate (but it is convenient for modeling purposes to assume equality). Importantly, modeling on a large set of real cameras indicates that it fits -accounts for a real camera encoding- as well as Eq. (3). In reality, both representations model real camera data with small errors.

Using Eq. (5), we can rewrite Eq. (4) as:
$\mathcal{I}_{r}=S_{r} S_{s}^{-1} \mathcal{I}_{r} A_{s}^{-1} A_{r}$.
It follows that
$\mathcal{I}_{r}=S \mathcal{I}_{s} H$,
where $H=A_{s}^{-1} A_{r}$ (as in Eq. (4)) and $S=S_{r} S_{s}^{-1}$.
When two images (or sets of pixels thereof) are related according to Eq. (7) we say that the two images are a shading homography apart (see [5] for more details about this term).

With respect to the homography model of image encoding, a rendered image is a shading homography from its linear raw counterpart. Given two images of the same scene encoded differently (the problem at hand in color stabilization), these pair of rendered images must be related by a shading homography (this is the import of Eqs. (5) through (7)).

In other words, we can see that $S$ in Eq. (7) is performing the same type of operation as the combination of the two non-linearities in Eq. (4). We can therefore use $S$-that is a diagonal, $n \times n$ matrix- to obtain the effect of the non-linearities on the images.

Inputs: The reference and the source images $I_{r}$ and $I_{s}$.
Output: The color stabilized image $I_{s t a b}$.
Obtain the $m$ corresponding points for $I_{r}$ and $I_{s}$ (e.g. using SIFT); From those corresponding points:

- Estimate $S$ and $H$ from Eq. (8).
- Estimate the global non-linearity $g(\cdot)$ from $S$ using Eq. (9).
$I_{s t a b}=g\left(I_{s}\right) H$.
Algorithm 1: Algorithm for our color stabilization approach

Let us rewrite Eq. (7) as an objective function to be minimized to find the best shading terms and the best color correction matrix:
$\min _{S, H}\left\|\mathcal{I}_{r}-S \mathcal{I}_{s} H\right\|$.
Of course, once we have related the two images (e.g. for the $n$ SIFT corresponding points) we need to apply the learned relationship to all the pixels in the source image. Thus, once we obtain $H$ and $S$, we need to be able to propagate their use to all the pixels in the image. This is trivial for the matrix $H$ as we can directly use it for all the image pixels. However, the matrix $S$ is tied to the pixels at hand (i.e. the $n$ sift points) so cannot be used directly.

Instead, we fit a global curve $g(\cdot)$ that better estimates its behavior for the set of pixels, and we apply that curve to the whole image. The estimation of this curve is performed as

$$
\begin{equation*}
\min _{g}\left\|S \mathcal{I}_{s}-g\left(\mathcal{I}_{s}\right)\right\| \tag{9}
\end{equation*}
$$

Finally, after the fitting of $g(\cdot)$, we carry our a final refinement of $H$ :
$\min _{H}\left\|\mathcal{I}_{r}-g\left(\mathcal{I}_{s}\right) H\right\|$.
A detailed schematic of our method is presented in Fig. 2, see also Algorithm 1.

### 3.2. Implementation

A minimum for Eq. (8) is found by using the (ALS) Alternating Least Squares Method [36]. The ALS method guarantees the finding of a local minima. It iteratively solves for $S$ (holding $H$ fixed) and then $H$ (holding $S$ fixed) until convergence. The ALS method has a number of desirable properties. It is easy to implement, the individual steps are least-squares optimal and the algorithm converges quickly. A detailed explanation of the ALS algorithm is presented in the appendix but also see [36].

Our minimization returns $H$ and $S$ from Eq. (7). But, although $H$ can be directly applied now to the whole image, this is not the case for $S$. In order to be able to apply $S$ to the full image, we approximate its behavior by solving for a global curve $g(\cdot)$.

The global curve $g$ is computed by fitting a polynomial of order $r-1$ to the data points estimated from $S$ using least squares. Let us denote the $j$ th row of $S I_{s},\left[S I_{s}\right]_{j}=\left[\begin{array}{lll}R_{j} & G_{j} & B_{j}\end{array}\right]$. Similarly, the corresponding point in the non-shading is denoted $\left[I_{s}\right]_{j}=\left[\begin{array}{lll}\hat{R}_{j} & \hat{G}_{j} & \hat{B}_{j}\end{array}\right]$. Let us adopt the notation $q_{k, j}$ and $\hat{q}_{k, j}$ to denote the R, G and B of the $j$ th row in respectively $S I_{s}$ and $I_{s}(k \in\{R, G, B\})$. We would like a polynomial expansion of $I_{s}$ to approximately equal $S I_{s}$ :
$q_{k, j} \approx \beta_{0}+\beta_{1} \hat{q}_{k, j}+\cdots+\beta_{r} \hat{q}_{k, j}^{r}$
for fixed coefficients $\beta_{i}, i=1,2, \ldots, r$.
Let us now consider how to find the optimal polynomial coefficients. Let the $((k-1) j+j)$ th row of an $k N \times r+1$ matrix $A$ contain the row vector $\left[\beta_{0} \beta_{1} \hat{q}_{k, j} \cdots \beta_{r} \hat{q}_{k, p}^{r}\right]$ and the corresponding $((k-1) j+j)$ th element of an $k N \times 1$ vector $\underline{b}$ equal $q_{k, j}$. We would like
$\underline{b} \approx A \underline{\beta}$,

We solve for $\underline{\beta}$ in the least-squares sense using the Moore-Penrose inverse. It is these coefficients that define the function $g()$. For our purposes, we found we could fit our data well with $r=4$ (3rd order polynomial expansion). An important detail is that the curve $g()$ should be monotonic. For our regression this was found, naturally, to be the case. But, monotonicity can also be explicitly enforced by rephrasing the regression as a constrained optimization.

We consider multiple configurations for $H$ and $S$,

1. $H$ as a $3 \times 3$ or $4 \times 4$ matrix: As it was explained in [2], for those pixels that fall inside the core of the gamut of the color space, a linear $3 \times 3$ matrix suffices for its rendering. However, as it was shown in [4] (and following [5]), the use of a projective $4 \times 4$ matrix allowed us to obtain better results; as it was able to better correct those pixels in the gamut boundary. Therefore, in this paper, we have decided to try both of these possibilities and compare their results.
2. $S$ as a 1-D or 3-1D curve: In this case, we study how to perform the approximation of $S$ by the global curve $g(\cdot)$. This global curve can be learned in what we call 1D, i.e. we learn the same curve for the three color channels. This case is similar to the one of the $3 \times 3$ color matrix: It suffices for pixels inside the gamut [2]. However, in order to better deal with pixels in the gamut boundary, that are affected by different operations such as gamut mapping or sharpening, we also introduce a $3 \times 1 \mathrm{D}$ version for the global curve approximation of $S$. In this case, we do not obtain a single curve $g(\cdot)$, but one for each of the color channels, i.e. $g_{k}(\cdot)$ with $k=R, G, B$.

## 4. Experiments and results

### 4.1. Methods considered

In this paper, we evaluate our method against those used in [4]: we consider eight state-of-the-art methods for color transfer, stabilization and consistency, Reinhard et al. [9] (Reinhard), Kotera [15] (Kotera), Xiao and Ma [17] (Xiao), Pitié et al. [11] (Pitie), Ferradans et al. [13] (Ferradans), and Park et al. [33] (Park). We also add four more recent methods, Grogan and Dahyot [14] (Grogan), and the deep-learning approaches of Chiu and Gurari [24] (Chiu), Yoo et al. [26] (Yoo), and Ho et al. [25] (Ho). All these methods are general in the sense that they share the common property that their computations -like our approach- do not require "metadata" e.g. knowledge of the form of any curve. We want to emphasize that for the Pitié et al. [11] approach, we focus only on the global part of their method and for Grogan and Dahyot [14], we selected the color transfer approach, no correspondences, using 50 colors for the $k$-means clustering. In common with the study [4], we do not include the method of Hacohen et al. because it was unable to solve all the different images.

Further, we also compare our approach with the methods of Gil Rodríguez et al. [37] (Gil18) and [4] (Gil20) that do require knowledge of the form of the non-linearity applied to the images. In this case, we perform a global comparison at the end of the section to assess how far are we from methods that input extra information.

### 4.2. Datasets

In our experiments we use the two datasets used in [4]. The first dataset consists of 35 image pairs where each pair is repeated 4 times for 4 different encodings. These are where the

1. source and reference are both gamma-corrected images.
2. source and reference are both logarithmically encoded
3. source image is logarithmically-encoded and the reference image is gamma-encoded.
4. source image is gamma-encoded and reference image is logarithmically-encoded.

Table 1
Results for the comparison of the 35 pairs in the two gamma encoded images case.

|  | $\Delta E_{00}$ |  | PSNR L |  | CPSNR |  | CID |  | RMSE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Median | Mean | Median | Mean | Median | Mean | Median | Mean | Median |
| Kotera | 11.111 | 7.686 | 21.122 | 23.877 | 19.786 | 21.040 | 0.458 | 0.394 | 0.145 | 0.089 |
| Pitie | 3.567 | 3.394 | 26.162 | 25.946 | 25.696 | 25.769 | 0.174 | 0.157 | 0.055 | 0.051 |
| Reinhard | 4.777 | 4.652 | 25.525 | 25.162 | 23.904 | 23.571 | 0.205 | 0.191 | 0.068 | 0.066 |
| Xiao | 4.377 | 4.232 | 25.940 | 26.077 | 25.183 | 25.270 | 0.196 | 0.160 | 0.059 | 0.055 |
| Ferradans | 5.522 | 5.308 | 23.715 | 23.874 | 23.028 | 22.560 | 0.260 | 0.237 | 0.078 | 0.074 |
| Park | 3.428 | 3.020 | 27.604 | 27.381 | 26.595 | 26.384 | 0.157 | 0.134 | 0.051 | 0.048 |
| Grogan | 5.632 | 4.944 | 24.174 | 24.566 | 23.694 | 24.290 | 0.255 | 0.226 | 0.074 | 0.066 |
| Chiu | 4.600 | 4.433 | 23.853 | 24.023 | 23.490 | 23.591 | 0.227 | 0.222 | 0.071 | 0.067 |
| Ho | 11.487 | 12.664 | 21.034 | 20.552 | 19.743 | 20.077 | 0.519 | 0.586 | 0.128 | 0.117 |
| Yoo | 5.466 | 5.191 | 22.846 | 22.777 | 22.392 | 22.635 | 0.277 | 0.259 | 0.080 | 0.075 |
| Proposed $3 \times 31 \mathrm{D}$ | 3.700 | 3.355 | 26.803 | 26.631 | 26.083 | 25.741 | 0.160 | 0.145 | 0.053 | 0.052 |
| Proposed $4 \times 41 \mathrm{D}$ | 3.461 | 3.236 | 26.903 | 26.876 | 26.358 | 26.093 | 0.153 | 0.137 | 0.052 | 0.051 |
| Proposed $3 \times 33 \times 1 \mathrm{D}$ | 3.219 | 2.972 | 27.000 | 26.864 | 26.587 | 26.588 | 0.140 | 0.134 | 0.050 | 0.048 |
| Proposed $4 \times 43 \times 1 \mathrm{D}$ | 3.15 | 2.993 | 26.995 | 27.000 | 26.613 | 26.598 | 0.137 | 0.13 | 0.050 | 0.047 |

In this way, we are able to evaluate our method for a full set of different non-linearities, proving its generalization. As a cautious remark, let us note here that all the images appearing in the paper have been converted to sRGB for display purposes.

The second dataset used in [4] was made by ARRI. It contains HDR videos. From the videos, linear RAW data is obtained by using ARRIRAW Converter [38]. In [4] they select three different scenes, and for each scene, they set as a reference image the raw encoded with one of the 3 different options \{PQ, HLG, Log C \}. In total, there are 9 sets of pairs of images ( 3 scenes and 3 encodings). Additionally, there is an extra pair comparing two different Log C ARRI curves. Therefore, this dataset has a total of 10 image pairs.

### 4.3. Metrics

In our experiments, we are predicting the reference image by processing the source. Given the estimated and ground-truth reference images we compute 5 different metrics that quantify, in some way, the error between these images. The metrics we consider are

- PSNR for luminance channel (PSNR L),
- color PSNR defined as CPSNR is the mean of the PSNR of the three color channels,
- root mean squared error (RMSE),
- mean $\Delta E_{00}^{*}$ [39] color difference computed in the CIELab color space $\left(\Delta E_{00}\right)$,
- CID [40] is the color extension of SSIM [41].

For each metric, we compute the mean error and the median error for the 35 image pairs. We repeated the process for the second dataset as well, but this time after applying a gamma value of 2.2 to the linear images. This helped us to ensure that we could compare the same metrics for both datasets.

### 4.4. Results

The results are subdivided into the 4 different combinations of non-linearities present in the dataset.

### 4.4.1. Gamma to gamma

Results for this case are presented in Table 1. Here, and in the tables that follow, we show in green the best results, and then in respectively blue and yellow, the second and third best results. It is evident that, overall, the best performing algorithm is $4 \times 4,3 \times 1 \mathrm{D}$ which means the color correction is a projective transform and there is an individual per channel tone curve. This algorithm has the 7 (of 10) best scores for the reported statistics (the mean and median of 5 metrics). The next best method again has per color channel non-linearities, but employs a $3 \times 3$ color matrix. For the case of PSNR L the method from Park et al. works best with our methods in 2nd and 3rd place.

Qualitative results for this case are shown in Fig. 3. In this Figure, we show from left to right the reference image, the source image, the ground-truth image, the result of processing the source with a competing method and the result of our processing. To re-emphasize, the source and reference images are of similar content to the groundtruth but some spatial detail is different (e.g. different camera position or zoom). The source image is in pixel registration to the ground truth. All the algorithms are trying to match the ground truth by learning the tone and color (transfer function) from relating the out of registration source to the reference.

In the first row of Fig. 3, notice that in the output from the method of Grogan et al. there is a clear pinkish cast. There is no cast either in the ground-truth or the output from our method. Notice also how much better we render the graffiti on the door. Regarding the images in the bottom row we see that the color on the floor and the wall are not properly restored by the method of Choi et al., in comparison, our method delivers good color stabilization. For each of these 2 examples, we also display the CID error (which is much lower for our method).

### 4.4.2. $L o g$ to $\log$

Results are presented in Table 2. We see that our proposed method using 3 different curves (one per color channel) is able to improve on all the other methods both when the color matrix $H$ is defined as $3 \times 3$ or $4 \times 4$ for all the metrics except the mean PSNR L (again Park's et al. method is best). Our best results are obtained when a $4 \times 4$ projective color mapping is used.

Qualitative results are shown in Fig. 4. From left to right: the reference, source, ground-truth images, the result of a competing method and our result. In the example in the first row of Fig. 4, we see that the output from Pitie's et al. algorithm has false colors, especially in the floor region. Regarding the bottom image, both our and Reinhard's et al. algorithm work quite well. Though, the color accuracy is significantly better for our method. Quantitatively, the CID metric numbers are consistent with our visual descriptions (our numbers are lower).

### 4.4.3. Gamma to $\log$

In the previous two cases, both non-linearities were following the same non-linearity equation but with different parameters. In this subsection, this is no longer the case, as the source image is encoded using a gamma curve and the reference image is encoded using a logarithmic curve. Results for this asymmetric case are presented in Table 4. Save for the mean Delta E metric, our approach (per channel tone mapping and a projective color map) works best overall. For the mean Delta E score, the Pitie method is slightly better but the difference is small and is likely not visually significant. Evidently, the second best method is our proposed method where we use per color tone curves and a $3 \times 3$ color transform matrix.

Qualitative results for this case are shown in Fig. 5. Regarding the comparisons in the top row, colors from the Kotera algorithm are


Fig. 3. Results for the Gamma to Gamma case. Please note the color of the wall and the color of the graffiti on the door in the top image, and the color of the floor and the wall in the bottom image. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Results for the comparison of the 35 pairs in the two logarithmic encoded images case.

|  | $\Delta E_{00}$ |  | PSNR L |  | CPSNR |  | CID |  | RMSE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Median | Mean | Median | Mean | Median | Mean | Median | Mean | Median |
| Kotera | 14.234 | 8.381 | 18.586 | 21.081 | 17.615 | 19.676 | 0.551 | 0.481 | 0.179 | 0.104 |
| Pitie | 3.978 | 4.044 | 25.797 | 25.369 | 25.119 | 25.099 | 0.207 | 0.201 | 0.059 | 0.056 |
| Reinhard | 7.878 | 7.916 | 22.656 | 22.512 | 19.899 | 19.369 | 0.364 | 0.368 | 0.107 | 0.108 |
| Xiao | 5.632 | 5.599 | 24.330 | 23.910 | 23.199 | 23.190 | 0.272 | 0.264 | 0.072 | 0.069 |
| Ferradans | 8.587 | 7.047 | 19.351 | 20.831 | 18.925 | 20.25 | 0.395 | 0.325 | 0.128 | 0.097 |
| Park | 6.768 | 4.548 | 26.217 | 26.162 | 23.961 | 24.196 | 0.296 | 0.210 | 0.083 | 0.062 |
| Grogan | 6.161 | 5.995 | 23.402 | 23.899 | 22.740 | 22.952 | 0.303 | 0.281 | 0.079 | 0.077 |
| Chiu | 5.314 | 5.397 | 22.614 | 22.605 | 22.232 | 22.068 | 0.278 | 0.284 | 0.082 | 0.081 |
| Но | 12.137 | 11.590 | 19.121 | 19.676 | 18.221 | 18.128 | 0.548 | 0.597 | 0.146 | 0.152 |
| Yoo | 14.409 | 13.553 | 14.777 | 14.535 | 14.797 | 14.388 | 0.628 | 0.648 | 0.193 | 0.192 |
| Proposed $3 \times 31 \mathrm{D}$ | 4.465 | 4.622 | 25.682 | 25.746 | 24.558 | 24.904 | 0.204 | 0.181 | 0.065 | 0.060 |
| Proposed $4 \times 41 \mathrm{D}$ | 4.041 | 4.000 | 25.396 | 25.855 | 24.946 | 25.466 | 0.185 | 0.172 | 0.060 | 0.054 |
| Proposed $3 \times 33 \times 1$ D | 3.78 | 3.512 | 25.939 | 26.576 | 25.545 | 25.887 | 0.172 | 0.155 | 0.056 | 0.052 |
| Proposed $4 \times 43 \times 1$ D | 3.760 | 3.450 | 25.825 | 26.674 | 25.455 | 25.952 | 0.172 | 0.151 | 0.057 | 0.052 |



Fig. 4. Results for the case of two logarithmic encoded images. Please, focus on the general look and the color checker in the top image. Also, please look at the pink color on the wall, the yellow color of the truck, and the color of the floor in the bottom image. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Fig. 5. Results for the case in which the source image is gamma encoded and the reference image is logarithmically encoded. Please, focus on the color of the books and the sofas in the top image, and on the gray and yellow of the walls in the bottom one. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 3
Results for the comparison of the 35 pairs in the case where the source image is gamma encoded and the reference image is logarithmically encoded.

|  | $\Delta E_{00}$ |  | PSNR L |  | CPSNR |  | CID |  | RMSE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Median | Mean | Median | Mean | Median | Mean | Median | Mean | Median |
| Kotera | 12.658 | 9.202 | 18.629 | 20.748 | 17.893 | 20.089 | 0.538 | 0.430 | 0.162 | 0.099 |
| Pitie | 3.752 | 3.903 | 25.957 | 25.538 | 25.378 | 25.217 | 0.184 | 0.173 | 0.057 | 0.055 |
| Reinhard | 6.438 | 6.246 | 22.861 | 22.642 | 21.776 | 21.666 | 0.291 | 0.291 | 0.084 | 0.083 |
| Xiao | 6.794 | 5.734 | 23.023 | 22.77 | 22.097 | 22.215 | 0.322 | 0.314 | 0.081 | 0.077 |
| Ferradans | 6.317 | 6.165 | 22.222 | 21.826 | 21.577 | 21.357 | 0.318 | 0.298 | 0.089 | 0.086 |
| Park | 12.808 | 9.620 | 20.746 | 22.593 | 18.779 | 19.351 | 0.510 | 0.454 | 0.147 | 0.108 |
| Grogan | 9.450 | 8.775 | 19.243 | 19.329 | 18.967 | 18.464 | 0.418 | 0.453 | 0.127 | 0.122 |
| Chiu | 6.345 | 6.039 | 21.083 | 20.888 | 20.800 | 20.769 | 0.326 | 0.317 | 0.096 | 0.093 |
| Нo | 10.331 | 11.233 | 20.930 | 20.889 | 20.131 | 19.418 | 0.509 | 0.543 | 0.140 | 0.139 |
| Yoo | 8.105 | 7.948 | 19.431 | 19.685 | 19.204 | 19.442 | 0.436 | 0.442 | 0.120 | 0.107 |
| Proposed $3 \times 31 \mathrm{D}$ | 7.031 | 4.804 | 22.626 | 24.081 | 22.158 | 23.737 | 0.282 | 0.239 | 0.095 | 0.065 |
| Proposed $4 \times 41 \mathrm{D}$ | 4.419 | 4.213 | 25.112 | 25.342 | 24.662 | 25.109 | 0.195 | 0.183 | 0.061 | 0.056 |
| Proposed $3 \times 33 \times 1 \mathrm{D}$ | 4.007 | 3.752 | 25.867 | 26.091 | 25.384 | 25.386 | 0.180 | 0.171 | 0.057 | 0.055 |
| Proposed $4 \times 43 \times 1 \mathrm{D}$ | 3.911 | 3.710 | 25.898 | 26.094 | 25.458 | 25.489 | 0.174 | 0.160 | 0.056 | 0.054 |

Table 4
Results for the comparison of the 35 pairs in the case where the source image is logarithmically encoded and the reference image is gamma encoded.

|  | $\Delta E_{00}$ |  | PSNR L |  | CPSNR |  | CID |  | RMSE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Median | Mean | Median | Mean | Median | Mean | Median | Mean | Median |
| Kotera | 15.704 | 12.405 | 17.017 | 18.864 | 15.970 | 16.625 | 0.631 | 0.586 | 0.199 | 0.148 |
| Pitie | 3.909 | 3.830 | 25.796 | 25.498 | 25.225 | 25.020 | 0.200 | 0.201 | 0.059 | 0.056 |
| Reinhard | 7.928 | 7.516 | 21.260 | 21.056 | 18.883 | 18.687 | 0.393 | 0.392 | 0.117 | 0.116 |
| Xiao | 7.926 | 7.554 | 21.446 | 20.539 | 20.438 | 20.059 | 0.403 | 0.416 | 0.100 | 0.099 |
| Ferradans | 8.578 | 7.954 | 19.654 | 19.163 | 19.172 | 18.518 | 0.381 | 0.369 | 0.122 | 0.119 |
| Park | 5.895 | 5.242 | 24.038 | 23.352 | 22.972 | 22.294 | 0.305 | 0.290 | 0.078 | 0.077 |
| Grogan | 9.864 | 9.217 | 18.714 | 18.901 | 18.486 | 18.663 | 0.473 | 0.466 | 0.135 | 0.123 |
| Chiu | 5.929 | 6.064 | 21.481 | 21.246 | 21.240 | 21.285 | 0.313 | 0.322 | 0.094 | 0.093 |
| Ho | 10.647 | 10.138 | 20.155 | 19.855 | 18.674 | 19.263 | 0.481 | 0.479 | 0.135 | 0.128 |
| Yoo | 5.471 | 5.207 | 22.794 | 22.726 | 22.341 | 22.307 | 0.273 | 0.256 | 0.080 | 0.077 |
| proposed $3 \times 31 \mathrm{D}$ | 7.351 | 5.882 | 21.841 | 22.723 | 21.394 | 22.296 | 0.330 | 0.296 | 0.104 | 0.079 |
| proposed $4 \times 41 \mathrm{D}$ | 4.916 | 4.763 | 24.491 | 24.462 | 24.014 | 24.082 | 0.237 | 0.221 | 0.069 | 0.065 |
| Proposed $3 \times 33 \times 1 \mathrm{D}$ | 4.244 | 3.866 | 25.099 | 24.862 | 24.709 | 24.662 | 0.198 | 0.166 | 0.062 | 0.061 |
| Proposed $4 \times 43 \times 1 \mathrm{D}$ | 4.248 | 4.106 | 24.967 | 24.886 | 24.634 | 24.886 | 0.196 | 0.168 | 0.062 | 0.062 |

everywhere a little wrong whereas our output matches the ground truth. In the second example (bottom row) focus on the window of the blinds in the shop. We correctly render these as slightly yellowish where as they are bluer in the method of Xiao et al. Again the CID scores tell the same story: our method is performing better than the state of the art.

### 4.4.4. $\log$ to gamma

Results for this case are presented in Table 3. In this case, the method of Pitie is able to overcome us in some metrics. This said we need to remember that the method of Pitie showed a quite bad performance in the cases of two gamma-encoded and two logarithmicencoded images, therefore proving that our method is much more robust to different non-linearities than that of Pitie et al.. We believe it is also interesting to point out that in this particular setting, our method with 3 different curves (one per color channel) and a $3 \times 3$ matrix seems to slightly improve over the $4 \times 4$ matrix case.

Qualitative results are shown in Fig. 6. In the top row, we compare our output to those of Park et al. for one image. Clearly, our method delivers good fidelity with the original but the Park et al. output is wrong (especially the ground in the image). The model of Ferradans et al. is considered in the second example. Here our method again delivers good results with the Ferradans method delivering an image that has an erroneous color cast. As for the other 3 qualitative examples, the CID numbers are consistent with the qualitative comparisons.

### 4.5. Comparisons against methods that require encoding information

In this subsection, we want to compare the results obtained by our agnostic method against those obtained by the methods of Gil

Table 5
Comparison between the results of our method and the results of methods inputting extra information for the 140 comparisons ( 35 comparisons per non-linearity pair). Our method is able to compete well, especially for $\Delta E_{00}$, CID, and RMSE without the need for any extra information.

|  | $\Delta E_{00}$ | PSNR L | CPSNR | CID | RMSE |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Ours | 3.767 | 25.921 | 25.540 | 0.169 | 0.056 |
| Gil18 | 3.928 | 27.186 | 25.852 | 0.179 | 0.056 |
| Gil20 | 3.371 | 27.515 | 26.442 | 0.154 | 0.050 |

Rodríguez et al. [37] (Gil18) and [4] (Gil20). We want to note that the results of these two methods can be understood as upper-bounds for our method, as they consider the information about the type of curve used in the encoding. In Table 5 we present the results as the average for all the previous four cases. In our method, we consider the case of a $4 \times 3$ matrix $H$ and three different tone-mapping curves.

As was to be expected, our method is not able to outperform these methods which use extra information. This said, we want to remark on how our method gets close to them for $\Delta E_{00}$ (only a difference of 0.39 versus Gil20, and better than Gil18), CID (only a difference of 0.15 versus Gil20, and better than Gil18), and RMSE (only a difference of 0.006 versus Gil20, and equal to Gil18)

## 4.6. $H L G / P Q / L o g C$ arri dataset

Results for this dataset are shown in Table 6, where we present the mean over the 10 image pairs. We show our results for the case of a $4 \times 3$ matrix $H$ and considering three different curves. We can see how our method outperforms all the other methods that do not require extra


 a grayish cast. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Reference (PQ)


Ground Truth


Ground Truth


Ours


Ours

 are tone mapped using [42]. Images from ARRI [43]. In the case of PQ curve, we set up the absolute luminance of the display to $1000 \mathrm{~cd} / \mathrm{m}^{2}$.

Table 6
Results show mean averages over 10 pairs, where reference and source images are encoded using HLG, PQ and logarithmic curves. In the case of PQ curve, we set up the absolute luminance of the display to $1000 \mathrm{~cd} / \mathrm{m}^{2}$.

|  |  | $\Delta \mathrm{E}_{00}$ | PSNR L | CPSNR | CID | RMSE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { O} \\ & . \quad \\ & 0 \\ & \text { 亿 } \end{aligned}$ | Kotera | 3.344 | 32.505 | 30.567 | 0.110 | 0.045 |
|  | Pitie | 1.022 | 40.047 | 40.134 | 0.035 | 0.021 |
|  | Reinhard | 1.861 | 35.311 | 35.020 | 0.062 | 0.040 |
|  | Xiao | 1.891 | 32.965 | 32.789 | 0.061 | 0.032 |
|  | Ferradans | 4.820 | 24.692 | 24.624 | 0.183 | 0.073 |
|  | Park | 1.624 | 38.250 | 36.795 | 0.044 | 0.029 |
|  | Grogan | 3.615 | 28.981 | 28.558 | 0.143 | 0.052 |
|  | Chiu | 2.559 | 27.082 | 26.765 | 0.212 | 0.050 |
|  | Ho | 5.444 | 25.135 | 24.861 | 0.254 | 0.075 |
|  | Yoo | 3.334 | 26.006 | 25.713 | 0.266 | 0.056 |
|  | Ours | 0.780 | 51.982 | 51.550 | 0.016 | 0.011 |
| O | Gil18 | 1.775 | 36.086 | 35.636 | 0.068 | 0.029 |
|  | Gil20 | 0.310 | 48.324 | 47.649 | 0.002 | 0.005 |

information and also the one of Gil 18 [37] -which makes use of the additional knowledge about the form of the non-linearity-. Also, our method is able to outperform the method of Gil Rodríguez et al. [4] -another one using extra information- for both PSNR L and CPSNR.

In Fig. 7 we show two examples of the outputs generated by our method. Our results are very close to the ground truth.

### 4.7. Time comparison

Table 7 presents the execution times for methods that do not require any learning. Processing times are computed for images of size $768 \times 1152$. We see that our method is faster than competing methods that deliver similar good performance to our method-Park and Pitie. Simpler methods that perform worse on the Tables -Reinhard and Xiaoare slightly faster. When possible, we used the code provided by the original authors. The executions were performed on a desktop computer

Table 7
Average running time (s) on 35 images of size $768 \times 1152$.

|  | Kotera | Pitie | Reinhard | Xiao | Ferradans | Park | Grogan | Ours |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Time (s) | 8.867 | 9.107 | 2.153 | 2.059 | 53.790 | 316 | 9.440 | 5.198 |

with Ubuntu 22.04.4 LTS as the operating system and the following specifications: Intel(R) Core(TM) i5-8400 CPU at 2.80 GHz processor, 16 GB RAM, and Nvidia GeForce GTX 1060 graphics card. We use Matlab version R2019a.

### 4.8. Realistic scenarios

Fig. 8 demonstrates the ability of our method to work with different cameras without knowing their characteristics. In detail, we show how we can convert between two Smartphone cameras: an iPhone 12 (Phone 1) and a Realme 8 Pro (Phone 2). Fig. 9 goes one step beyond and shows how our method is also able to work with images downloaded from the internet, where we do not know any information about the capture conditions.

### 4.9. Limitations

Our method relies on having good enough pixel-to-pixel correspondences between the source and the reference images. For this reason, it may present limitations when the viewpoints from the two images are too different to detect enough correspondences in the shared regions, when one of the images is a large zoom-in of the other, or when one of the images is captured under extreme lighting conditions such as low-light. These limitations might be reduced if replacing SIFT by some other methods such as [44-46].


Fig. 8. Examples with images captured under realistic scenarios. Our method is able to color match the images between two Smartphone cameras: an iPhone 12 (Phone 1 ) and a Realme 8 Pro (Phone 2).


Fig. 9. Examples with images captured under realistic scenarios. Our method is able to color correct images downloaded from internet, without any further information about the capture conditions.

## 5. Conclusions

In this paper, we have introduced a color matching method able to deal with any encoded images (gamma-corrected, logarithmic-encoded, or any other) that share some content but are out-of-registration. To this end, we aimed to understand the color transform from source to reference -in a known pixel specific way- so we can apply the learned transform to the source so it looks like the reference when transformed.

Our method has 4 steps. First, we find common points in the source and reference images using the SIFT methodology. Then we relate the corresponding source and reference points using a shading homography (a global color transform and a per pixel shading term). Third, we approximate the shading change using an optimized tone-curve. Fourth we apply the tone-curve and the color transform to the source. A key strength of our method is that we make no a priori assumption about the shape of the tone function or that it belongs to a particular mathematical family (e.g. gamma curves)

Extensive experiments demonstrate that our method advances the state of the art. Moreover, we also compete with or even improve over methods that employ additional information about the type of non-linearities used in encoding the images.

Further work will aim at generalizing this approach to other colorrelated tasks such as color characterization, exposure correction, image harmonization, or HDR imaging.

## CRediT authorship contribution statement

Raquel Gil Rodríguez: Methodology, Conceptualization, Writing - review \& editing. Javier Vazquez-Corral: Methodology, Conceptualization, Writing - review \& editing, Writing - original draft. Marcelo Bertalmío: Writing - review \& editing. Graham D. Finlayson: Writing - review \& editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

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## Appendix. Minimization of the alternating least squares

Our goal is to minimize:
$\min _{S, H}\left\|\mathcal{I}_{r}-S \cdot \mathcal{I}_{s} \cdot H\right\|$
where $\mathcal{I}_{r}, \mathcal{I}_{s}$ represent the RGB values of the corresponding pixels in the reference image and the source image respectively. We will minimize this equation iteratively for $H$ and $S$. To do so, three steps in each iteration need to be performed -let us consider as an example the first iteration, $\mathcal{I}^{0}=\mathcal{I}_{s}$ :

Inputs: The corresponding pixels of the reference image $\mathcal{I}_{r}$ and the corresponding pixels of the source image $\mathcal{I}_{s}$.
Output: $H^{i}, S^{\text {final }}$.
Initialize $\mathcal{I}^{0}=\mathcal{I}_{s}, i=0$, and $S^{\text {final }}=I d$;
repeat
$\min _{H^{i}}\left\|\mathcal{I}_{r}-H^{i} \mathcal{I}^{i}\right\| ;$
$\min _{S^{i}}\left\|\mathcal{I}_{r}-H^{i} \mathcal{I}^{i} S^{i}\right\| ;$
Update $S^{\text {final }} \leftarrow S^{\text {final }} \cdot S^{i}$;
Update $\mathcal{I}^{i+1}=\mathcal{I}^{i} S^{i}$;
Update $i \leftarrow i+1$.
until $\left\|\mathcal{I}^{i+1}-\mathcal{I}^{i}\right\|<\epsilon$, where $\epsilon$ is a predefined tolerance value;

## Algorithm 2: ALS algorithm for color stabilization

## 1. Minimizing for $H$ (assuming $S$ is the identity): <br> $$
\begin{equation*} \min _{H^{1}}\left\|\mathcal{I}_{r}-H^{1} \mathcal{I}^{0}\right\| \tag{A.2} \end{equation*}
$$

2. Minimizing for $S$ :

$$
\begin{equation*}
\min _{S^{1}}\left\|\mathcal{I}_{r}-H^{1} \mathcal{I}^{0} S^{1}\right\| \tag{A.3}
\end{equation*}
$$

3. Updating the image for the next iteration:

$$
\begin{equation*}
\mathcal{I}^{1}=H^{1} \mathcal{I}^{0} S^{1} \tag{A.4}
\end{equation*}
$$

We keep iterating over these operations until $\left\|\mathcal{I}^{i+1}-\mathcal{I}^{i}\right\|<\epsilon$.
Let us look in more detail into the minimization of the first two steps. The first step is a standard least squares minimization and can therefore be solved by
$H^{1}=\mathcal{I}_{r}\left(\mathcal{I}^{0}\right)^{+}$,
where ${ }^{+}$denotes the pseudo-inverse.
The second step is more intricate. We should remember that $S$ is a diagonal $n \times n$ image. Therefore, we need to force the solution of the minimization to be diagonal. We can do that in closed form as:
$S_{(j j)}^{1}=\left(V_{(j)} \cdot \mathcal{I}_{r(j)}\right) /\left\|V_{(j)}\right\|^{2}$,
where $V=H^{1} I^{0}, V_{(j)}$ and $\mathcal{I}_{r_{(j)}}$ are 1-by-3 vectors (the values of the column j in $V$ and $\mathcal{I}_{r}$ ), and $\cdot$ denotes the scalar product. An algorithm presenting this minimization is shown in Algorithm 2.

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