Inference in the Context of Inquiry[1]

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Much contemporary epistemological work is affected by two interconnected problems caused by the dissociation of its analyses from the context of enquiry. First, key epistemological notions like knowledge, reasoning and inference are presented in a manner that tenaciously eludes explication and proliferates puzzles. Second, the materials of scientific enquiry prove powerless to offer insight yielding solutions to puzzles or the desired explications. In this paper I show that these difficulties vanish when epistemological work is reconstructed along the lines set out by Dewey's pragmatism, focussing in particular on a contrast between the insuperable difficulties attending a standard epistemological analysis of inference and the significant philosophical progress made possible by a pragmatist outlook on inference. In the process, I introduce a novel application of Dewey's study of inference in scientific practice to mathematical enquiry.

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1. Epistemology and Scientific Practice

Contemporary analytic epistemology has, unsurprisingly, paid sustained attention to phenomena like thought, reasoning, inference. It has often done so without including the pattern of enquiry as a necessary background to the analysis of knowledge. It has, in other words, often chosen to study knowledge apart from enquiry. One important consequence of this choice is that, once made, it allows the possession or acquisition of knowledge to be consistent with many hypothetical scenarios in which enquiry is arbitrarily curtailed, sidetracked by irrelevant considerations or suppressed altogether in favour of immediate having (e.g. in perception).

When the possibilities listed are admitted, epistemological analysis forces itself to face manifold puzzles concerning knowledge, without being able to unravel them by appealing to the structure of enquiry, which has been removed from its theoretical horizon.

One distinctive, undesirable consequence of the epistemological standpoint just described is that, instead of providing a better understanding and appreciation of knowledge, especially as it is encountered in the proceedings of the sciences, it renders knowing a rather impenetrable and puzzling affair. This result, however, does not depend on the elusive nature of subject matter, but upon the distinctive stance adopted to investigate it, which, in particular, neglects the experimental character of knowledge as a process and divorces it from the deliberate intention and methodical effort to settle problematic situations.

When knowledge is explicitly related to the pattern of enquiry, many central problems of analytic epistemology are not permitted to arise, because they more or less openly violate the logical conditions of enquiry.

The last point will be illustrated in detail with respect to a recent study of inference, but may, for clarity's sake, be exemplified with the help of one extremely familiar problem discussed in Gettier's influential paper[2] on the inadequacy of the definition of knowledge as justified true belief. While discussing this problem, Gettier introduces the fictitious case of two job applicants, say A and B, one of whom, e.g. B, is morally certain that A will be offered the job they are both applying for and who is, moreover, informed that A has ten coins in his pockets. B conjectures that the successful candidate holds ten coins in his pockets. When B is offered the job, he

2 Gettier (1963).

realises that his own pockets contain ten coins: his conjecture thus holds true. It is argued that B's statement can be regarded an instance of knowledge without thereby conforming to the definition criticised, because sufficient justification for it is missing.

The purpose of Gettier's scenario is not central here. What matters is that his conclusion is intelligible insofar as no distinction is drawn between the circumstance that a given statement is true and the circumstance that a statement is arrived at as a judgment[3], after an enquiry has been drawn to a close.

If the distinction is drawn, then B's statement can count as knowledge only in case it has been reached as a judgment. That B's statement cannot be so understood is clear from a quick inspection of Gettier's example. The unsettled situation confronting B concerns the identity of the candidate most likely to be hired between A and B. Confronted with this uncertainty, which would be plausibly responded to by selecting data that lead to a probabilistic evaluation, B contemplates a feature of A that is entirely irrelevant to the problem (even where bribing were allowed). Moreover, B does not even take care to check that the feature singled out should uniquely determine *the* applicant most likely to be hired. Carelessness here implies that B's statement may be equivalent to the significantly less informative statement that one of A or B will be hired.

It is because of this equivalence that B happens to utter a true statement. Its truth does not rectify the misguided approach taken to tackling the given problematic situation. If B is, by contrast, uttering a casual statement that happens to be true, then no enquiry is taking place and his performance is as relevant to the analysis of knowledge as two arithmetical mistakes cancelling each other may be to the definitions of certain primitive recursive functions.

What Gettier's contribution shows is that, if the structure of enquiry is to set no constraints on the definition of knowledge, then what is called knowing may be uncontrollably haphazard.

This pessimistic conclusion does not arise from the impossibility of identifying salient features of knowledge when it occurs in the context of enquiry, but from an implicit proscription of enquiry as a context that may be appealed to in order to clarify the structure of knowledge. Without relying upon this context, epistemological problems multiply without offering natural prospects of resolution,

3 As defined in John Dewey, The Collected Works of John Dewey, 1882-1953, ed. by Jo Ann Boydston (Carbondale and Edwardsville: Southern Illinois University Press, 1967-1990) LW12: 123.

as the literature deriving from Gettier's contribution has made apparent[4].

The situation has not gone wholly unnoticed in the field of epistemology. A tentative response to it has consisted in the adoption of a more explicit emphasis on the practical aspects of knowing or on knowing as a special ability or competence. Knowledge has been discussed as a cognitive ability or a quality associated with such ability (e.g. creditability[5], competence to know[6] or belief produced by intellectual ability[7]): these more recent proposals offer plausible suggestions but they remain inherently remote from a conception of knowledge that transcends the psychic sphere directly to include a process that establishes new objects and that has a formative impact on subject matter. As will be shown with reference to an illustrative example, the psychic focus distinctive of epistemological contributions keeps them at a remove from analytical effectiveness.

The purpose of this paper is not only to examine this difficulty, but also to indicate a promising way to evolve epistemological investigations beyond their traditional predicaments, by integrating into them the active and formative aspects of knowledge. The integration is made possible by reinstating the context of enquiry as a fundamental reference, since this context does not only include the subject in various psychic states, but data, operations of selection and transformation, methods, theories.

More precisely, if the unqualified stress on knowledge as competence or ability, as may be ascribed to a skilled agent, is replaced by a focus on the logical function of reflective and methodical thinking in connection with the determination of problems and their resolutions within enquiry, then epistemological analysis can greatly gain from the resulting outlook, which is certainly familiar to the pragmatist.

The outlook proposed has two central advantages over traditional epistemology: first, the more structured and richer instances of knowing, which arise in scientific practice, can be made to function as illuminating sources that clarify the generic characters of knowledge; second, conundrums produced by a relative disregard for the methodical character of enquiry no longer highlight fundamental problems because they originate from neglect of materials inconsistent with the problems' pertinence to knowing. As a result of the advantages just described, scientific practice and its epistemological analysis become fruitfully interdependent: the former can be used as a source of data for the latter, and the latter's refinement can

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4 Familiar ones appear in Chisholm (1989) and Hetherington (1998). See also Hazlett (2015).

5 Greco (2003).

6 Miracchi (2015).

7 Greco (2009).

yield a better and more profound understanding of the former.

In this paper I do not propose to reconstruct the entirety of epistemological research along the lines indicated but wish to make use of one contribution to the analysis of inference in order to provide a concrete example of the reconstruction I believe to be necessary and fruitful. The contribution in question is an article by P. Boghossian[8], which I have chosen because it focusses on a problem of obvious relevance to scientific practice, namely the nature and function of inference, and because it does so against the background of a 'practical' conception of inference as an activity in which human agents engage deliberately and for a definite purpose.

Despite this characterisation, Boghossian's analysis remains bound to the sphere of mental activities, as opposed to the broader context in which mental activity is continuous with intervention and transformation of selected materials. Deprived of empirical anchors, the analysis surrenders inference, if conjecturally, to the realm of extra-natural phenomena, which is as relevant to the procedures and techniques of scientific enquiry as these procedures and techniques are effective at casting light upon it.

The difficulties affecting Boghossian's analysis are to be studied in the next three sections. They contrastively highlight the need for a philosophical approach to inference that essentially involves the context of enquiry. The final sections of this paper rely on Dewey's work to frame this account and show its fertility when applied to mathematical practice in particular.

2. Reasoning and Its Aim

Boghossian's study of inference is based on a general conception of reasoning as a deliberate and purposive human activity. The conception is not, however, linked to the context of enquiry: without this link, a tension arises between the conception adopted by Boghossian and the character of inferential practice. To understand the tension, it suffices to reflect on what Boghossian qualifies as reasoning's aim:

> […] reasoning is something we do, not just something that happens to us. And it is something we do, not just something that is done by sub-personal bits of us. And it is something that we do with an aim – that of figuring out what follows or is supported by other things one

8 Boghossian (2014).

believes[9].

Due to its intra-psychic character, the aim of reasoning just described is immediately at odds with inferential practice in scientific enquiry: for instance, in physical enquiry the main function of reasoning is, speaking in very general terms, reflectively to control the interaction of objects under given conditions for the sake of tracking or directing their behaviour over time. Even a restriction to less specialised enquiries does not fail to evince the same leading preoccupation with handling in reflection the possible effects of action and choice. The 'intellectualist' focus on pure relationships between beliefs is meant to vindicate inference's theoretical character, but it does so to the detriment of its function.

It is noteworthy that the aim of reasoning declared above should also not be typical of Boghossian's own examples of inference. It suffices to consider only the first he provides, i.e. the formal inference from the premisses 'it rained last night' and 'if it rained last night, then the streets are wet' to the conclusion 'the streets are wet', a belief then taken to 'affect' choice of footwear[10].

An agent holding and recalling beliefs about weather conditions commonly intends to use them in connection with future actions involving exposure to or protection from those conditions. In fact, an agent holding beliefs about the weather would treat them as available information, rather than purely internal states, and would use such information for a practical purpose, but not primarily to discover what else she believes, on their basis, about the weather.

Boghossian's example, unlike his definition, suggests that the aim of reasoning is to evolve reflection to a point of continuity with action for the sake of resolving an initial state of uncertainty. If the suggestion is taken up, then it connects reasoning with the investigation of determinate problems: the context of enquiry must, in this case, be supplied as a necessary reference.

If, however, the suggestion is not taken up, what is thrown into relief as the dominant aspect of reasoning, and inference in particular, is psychic bondage. Inferential activity primarily controls which beliefs are bound to follow from others held: its function, which cannot be prospectively related to the goals of enquiry, comes to be essentially entangled with a justificatory constraint, called by Boghossian the *Taking Condition*[11]. In reasoning, and more specifically inference, certain beliefs are

9 Boghossian (2014), 5.

10 Ibid., 2.

11 Boghossian (2014), 5.

taken by an inferring agent to support her conclusions, drawn in virtue of such support.

Although it is as plausible to say that inference involves adopting certain conditions as grounds of what follows from them as it is to say that reasoning involves giving reasons, the hypostatisation of this processes as a psychic condition compresses the whole dynamic arc of enquiry into a static, subjectivistic constraint. Exclusive focus on this constraint generates two characteristic problems. First, it produces an explicit disconnection of inference, nominally referred to scientific practice, from the processes of enquiry occurring therein. Second, it substitutes the prospective analysis of inference as an activity operative in tackling problems with dialectical engagement in psychic retrogression, i.e. in the search for psychic mechanisms that underlie the Taking Condition. This search does not only widen the gap between scientific practice and epistemological analysis, but, as will be clarified, threatens to force inferential practice itself out of the domain of natural phenomena.

Section 3 discusses the first of the problems just described, i.e. the separation of inference from enquiry, while section 4 examines the problematic consequences of psychic retrogression.

3. Inference without Enquiry

While discussing the analytical fruitfulness of the Taking Condition, Boghossian introduces a puzzle erected on specific mathematical content. The treatment of this content exhibits the estranged perspective on scientific enquiry mandated by the characterisation of reasoning and inference given in Section 2. The purpose of Boghossian's example is not very significant for present purposes, but the way he frames his puzzle is of central interest:

> Consider someone who claims to infer Fermat's Last Theorem (FLT) directly from the Peano axioms, without the benefit of any intervening deductions, or knowledge of Andrew Wiles's proof of that theorem. No doubt such a person would be unjustified in performing such an inference, if he could somehow get himself to perform it. […] For the Peano Axioms to FLT transition to be a real inference, the thinker would have to be taking it that the Peano axioms support FLT' s being true. And no ordinary person could so take it, at least not in a way that's unmediated by the proof of FLT from the

Peano Axioms[12].

The problem implicit in the above quote is whether FLT follows from the first-order theory known as Peano Arithmetic (PA). This problem, as far as its inferential dimension goes, is for Boghossian the problem of determining whether it is possible to take FLT to follow from PA, i.e. whether a case of the Taking Condition occurs. It is of crucial significance that the problem is subjectivistically presented (someone puts forward a claim, but no conjectures are made or proof strategies are sketched), that it is divorced from the typical procedures of mathematical enquiry – since the claim is made without the 'benefit of any intervening deductions', i.e. without any deliberate attempt at mathematical reasoning – and that it is detached from its institution on the basis of available data, namely the proof of FLT arrived at by Wiles (the claim is supposed to be made by someone not acquainted with the proof of FLT given by Andrew Wiles[13]).

If Boghossian's mathematical scenario is to be considered intelligible, inference must be thoroughly removed from the structure of mathematical enquiry. If it were not removed, the scenario would lose intelligibility for clear mathematical reasons.

The question of whether FLT may be 'inferred' from PA is today a of definite mathematical question because of the proof given by Andrew Wiles. Wiles' proof relies, in particular, on results from cohomological number theory that can be traced back to the use of mathematical frameworks called universes, whose existence requires principles stronger than Zermelo-Fraenkel set theory with Choice (Colin McLarty has published an accessible discussion of the principles involved[14]).

Although it is known that, in practice, the results of interest may be obtained without an appeal to universes[15], an especially hard problem is to determine whether the higher-order content occurring in Wiles' proof can be further reduced to first-order content manageable in PA or, alternatively, higher-order content manageable in a conservative extension of PA (this would guarantee the existence of a proof from PA, even if one could not be exhibited).

If 'inference' has any content and purpose, an agent claiming to have inferred FLT from PA is claiming to have established something quite specific, namely a new

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12 Ibid., 6.

13 Wiles (1995).

14 McLarty (2010).

15 Ibid., 371–372.

arithmetical technology that yields a first-order unwinding of Wiles' proof. If an agent simply insists that she is taking PA to be sufficient grounds for FLT, that agent is stating a conjecture, not making an inference. If, finally, the agent insists that she can bring herself to assert the Taking Condition relative to PA and FLT, this result, insofar as it is arithmetically inconsequential, does not describe any inferential activity within mathematical enquiry.

Boghossian seems to think that the Taking Condition can account for the 'considerable feeling'[16] that the inference is unjustified. What has been noted suggests that, if inference is understood as a phase of reasoning, and reasoning takes place within scientific enquiry, rather than apart from it, then no inference is taking place, not even an unjustified one. The Taking Condition does not shed any light on the logical position of FLT as a problem whose solution is related to the data provided by PA.

It is revealing that the Taking Condition is understood as operative by Boghossian only under the explicit exclusion of available information about the proof of FLT and of any intervening deduction. To demand such explicit exclusion is to deny knowledge its transactional character and the power to modify subject matter (in this case, to reconstruct higher-order arguments), replacing it with a distinctive focus on psychic existence, i.e. the fact that an agent may or may not achieve, or perhaps adequately achieve, a mental state in which she takes certain beliefs as conditions for others. A dominant focus on mental states thus proves an obstruction to the proper assimilation of the problems, procedures and results occurring in scientific practice, commonly regarded as paradigmatic instances of reasoning and knowledge.

As the next section will show, the same psychic focus does not only create external problems affecting the relation of epistemological analysis to scientific knowledge; it also creates internal problems for the articulation of epistemological analysis.

4. Psychic Retrogression

It is Boghossian's intention to offer an explication of the 'taking' distinctive of what he calls the Taking Condition. Because the explication is of a psychic state and has been, as the mathematical scenario discussed earlier shows, severed from the

16 Boghossian (2014), 6.

procedures and activities typical of enquiry, it can only take the form of what may be described as a psychic retrogression. Boghossian must look for psychic mechanisms that underlie the Taking Condition at the basis of inference. This search, as will now be seen, faces difficulties that follow directly from of its one-sided focus on antecedent mental structures.

After considering possible explications of Taking that he is forced to dismiss as untenable, Boghossian turns to John Broome's work[17] for a more promising alternative: Broome proposes that reasoning should be conceived as the application of rules to the contents of premiss-attitudes for the sake of constructing conclusions. Although the proposal resembles and is congenial to Boghossian's, a revealing problem is shown to affect it, namely the circular interdependence of inference and the application of a rule.

If inference arises when beliefs are employed as conditions for the application of a rule, then beliefs must be recognised as conditions of application before they are so employed. For the recognition to be possible, a mental representation of the rule to be applied is required, on which inference can operate. Given a fixed rule, Boghossian takes the relevant inference to progress from the premisses: 'if input x is an A, then y must be carried out' and 'input x is an A', to the conclusion: 'y must be carried out'. The conclusion is that, if inference (here *modus ponens*) requires following a rule, then following a rule requires inference.

This problem is worthy of attention because it essentially rests on the psychic framing of the investigation. A rule that can be referred to the procedure it directs does not, as such a set of instructions, require an underlying prompt, say an inference, for its application. Consider *modus ponens*, the syntactic rule repeatedly exemplified by Boghossian.

The condition for the application of this rule is the recognition of 'X implies Y, X' as well-formed formulae and of the latter formula as an initial segment of the former. The recognition task may be carried out by means of a parsing algorithm for the language considered. Once the task is carried out, the rule itself indicates the operation to follow, namely the production of the string 'Y'. The rule does not in particular require inferring the string 'Y' from the fact that 'X implies Y, X' are the conditions of its application: it mandates the *production* of Y given a preliminary algorithmic check.

If the question of the application of a rule is however abstracted from the concrete operations the rule is designed to direct and transformed into the question

17 Broome (2013).

of what inner replica of the rule may cause an agent to resolve to apply it – apart from responding to instructions designed for use – Boghossian's argument stands. Independently of any special evaluation of the philosophical standpoint supporting the argument, it is clear that Boghossian's outlook forces the sublimation of inferential practice into a realm of self-contained psychic interactions, precisely because it cannot reach out to the concrete phases of enquiry, in which specialised activities produce and transform initially given materials, conceptual or existential.

The introduction of psychic conditions, which can only modify the internal states of an agent apart from the agent's possible worldly engagements, cannot promise an easy prospect of logical clarification with respect to activities like inference and reasoning, which are continuously intertwined with such engagements.

The culmination of Boghossian's analysis vindicates the last statement. While rules are required to admit mental representations, the problem identified with Broome's proposal now mandates such mental representations to be non-inferential: it is thus suggested that they should control thought and behaviour in a non-inferential way[18]. The psychic status of non-inferential control raises further problems: Boghossian does not wish to assimilate it to a disposition, which might reduce the Taking Condition to a form of causation (with premisses causing conclusions), and notes the possibility of a sub-personal mechanism, which is however unacceptable as the basis of deliberate reasoning.

No escape from the magic circle of psychic existence, taken apart from its interconnection with enquiry, seems possible. Because Boghossian cannot take following a rule to be engagement in regulated behaviour, he is induced to admit 'following a rule as an unanalyzable primitive' because 'we can have no expectation that we will be able to give a non-circular analysis of what following a rule of inference amounts to'[19]. It follows that the analysis of reasoning pursued 'makes it difficult to see what naturalistic process inference could consist in'[20]. The proliferation of psychic states leads to a conclusion that evokes the old philosophical predicament of psychic versus material existence.

Because this conclusion depends on the suppression of enquiry in favour of psychic retrogression, it is plausible to think that inference is to regain intelligibility once reintegrated within the context of enquiry. I will next show that this is indeed the case.

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18 Boghossian (2014), 14.

19 Ibid., 17.

20 Ibid., 18.

5. Inference in the context of enquiry

Boghossian's study of inference provides motivation for and clues to the construction of an alternative. The motivation comes from the several difficulties already highlighted. The clues come from the fact that the difficulties and conclusions are closely connected with detachment from the operations of enquiry and substitution of psychic states or psychic mechanisms for these operations.

A further consideration might be offered to depart from Boghossian's epistemological stance and turn to a closer study of inference as a reflective phase within enquiry. It is the fact that reasoning and inference as encountered in scientific practice are intimately connected with the advancement of knowledge.

The terms 'reasoning' and 'inference' are at least markers of significant transitions within scientific investigations, as a result of which control over phenomena and their interdependencies, formulations of new and sharper questions occur. The possibilities disclosed by scientific advances highlight the role of reasoning, and inference in particular, in the production of novelty.

Even the justificatory quality assignable to inference as securing certain results within the arc of enquiry may be traced back to the primary goal of advancing it: issues of correctness or validity indicate how hard it may be to advance enquiry under given circumstances. If an activity, once taken up, could proceed completely unhindered to achieve the goals that had called it forth, no interesting question about its validity would occur or at least could be easily discerned. The question of validity arises as soon as the course of enquiry is stopped in its tracks or goes patently astray relative to a set of leading goals: it then becomes necessary to institute the phases of reasoning as a problem, rather than wielding its technology as a problem-solving instrumentality.

Reasoning may thus be turned into an object of investigation, but it becomes one under the pressure of progressive enquiry. It is therefore important to understand inference essentially in connection with advancing enquiry. This understanding of inference or inferential thinking is prominent in Dewey's logical writings. It is clearly formulated in the following passage:

> Thinking in the mode of inference insists upon terminating in an intellectual advance, in a consciousness of truths hitherto escaping us. […] Thinking endeavors to compel things as they present themselves, to yield up something hitherto obscure or concealed. This advance and extension of knowledge through thinking seems to be well designated

> by the term "inference"[21].

Dewey's characterisation of inferential thinking does not only emphasise its function, but also clarify its ground. In his wording, something 'concealed' is compelled to present itself. The fact that the subject matter of enquiry, as initially given, does not exhibit or realise the totality of its connections is thus closely connected with a need for inference. Inference is therefore not simply a style of reasoning but a mode of activity imposed upon us by the structure of experience. The activity of inference arises as a distinctive need, as Dewey observes in *Experience and Nature*:

> The difference between the appearing and the unappearing is of immense practical and theoretical import, imposing upon us need for inference, which would not exist if things appeared to us in their full connections, instead of with sharply demarcated outlines due to limits of perceptibility[22].

It is important to note that the 'full connections' referred to in the passage quoted, even though they may be present yet undetected are more significantly absent yet subject to institution as soon as the conditions governing them can be isolated and regulated. Inferential practice resembles as much the activity of the astronomer discovering new celestial bodies by means of more powerful instruments of observation as it does the activity of the physicist who finds a way of improving the range and resolution of optical instruments.

Dewey's account of inference makes it unproblematic to dispense with the Taking Condition. If inferential practice is played out between the appearing and unappearing, the latter comes to be instituted as a conclusion on the basis of what appears, provided the appearing is selected as data connected with the determination of a problem. The whole process – with its conceptual instrumentalities, technique and operations – leading to the institution of a previously unappearing outcome, is the inferential nexus. No subjective Taking apart from its operations and consequences needs to be postulated to account for it.

Because of its close connection with experience presented by the contrast between appearing and unappearing, inference does not, on the picture just offered, look like an extra-natural event but firmly sits within experience, as an activity

21 MW 1: 166–7.

22 LW1: 112.

imposed upon enquiring agents by its structure and their concerns. A naturalistic context for inference does not only secure its naturalistic comprehension but also suggests the possibility of a productive interaction between the formal study of inference and the development of scientific enquiry. The latter provides nontrivial materials that support the articulation of the former, while the formal study of inference contributes to situating scientific practice within a general theory of knowledge.

The reciprocity just envisaged will now be described concretely: the next section offers a more structured account of inferential practice than has been outlined so far, and one that is sufficient for application to scientific content. The section after the next will bring the more structured account of inferential practice in contact with a representative episode from mathematical enquiry.

6. Inductive Inference and Mathematical Enquiry

As already noted, in MW1 Dewey offers a generic characterisation of thought in the inferential mode. He does not, however, provide a closer examination of the stages of inference: this is done in chapter XXI of LW12, in the context of a discussion of scientific method, in which Dewey studies the structure of inductive inference. A leading goal of Dewey's in this context is to describe the logical relationship between generalisations and the particulars that at once ground them and are reconstituted in a controlled way by them.

Dewey's analysis of this interaction provides a helpful way to understand how generalities are arrived at as solutions to problems and how they regulate novel interactions of particulars. As such it offers, once slightly modified in a way shortly to be explained, a useful analytical framework for the study of inferential practice in the context of mathematical enquiry, as opposed to empirical science, which is Dewey's primary focus.

My goal in the remainder of this paper is to show how Dewey's study of inductive inference can be reconstructed to deal with inference in mathematical enquiry and to apply the reconstructed study to the close examination of a salient episode in the history of model theory, chosen for its special significance as an illustration of the production of novelty in mathematics.

Focussing on specific mathematical content will show the perspective on inference adopted from Dewey's work capable of gaining depth and detail from the analytical assimilation of scientific practice, a result earlier recognised to lie beyond the reach of traditional epistemology. The assimilation of scientific practice by philosophical analysis will also provide a direct way of avoiding Boghossian's suggested conclusion that inference may have extra-natural traits: inference understood as a structured practice carried out by agents coping with problematic situations loses all possible extra-natural connotations.

I shall now proceed concisely to outline Dewey's conception of inductive inference, indicate its modifications needed for application to mathematical enquiry and delineate a mathematical context to which Dewey's conception, once modified, can be insightfully applied.

On Dewey's view, inductive inference is based on the interrelationship between existential particulars or data and discursive principles articulated to direct their transformation and interaction. In the initial phase of inference, data is selected that is capable of determining a problem and suggesting a mode of solution. Data works as a suggestion because it indicates a hypothesis that, in the intermediate phase of the inferential arc, can be deductively developed until it indicates operations and interventions that help tackle the problem determined at the outset. In the final phase, the operations indicated by deductive development are adopted as an instrument of control and confirmation, which can institute new data deliberately and fix conditions under which particulars are inserted into novel interactions directed by the operations outlined in the earlier phase[23].

Dewey's picture refers to existential data at the start of the inferential arc and to existential interventions directed by hypothetico-deductive developments in its final phase, in which new data is instituted that can test the generality presiding over its production.

This picture can be easily modified to apply to the materials of mathematical enquiry, which Dewey distinguishes as discursive from existential ones. It suffices to consider, in the initial phase of inductive inference, as described above, the results of earlier mathematical enquiry as the objects to be treated as particulars. Such particulars supply data or they can be associated with specific data closely related to a given problem.

The sequel of the inferential process is as Dewey describes it, except that it applies to mathematical content rather than existential. Deductive development establishes general results that do not put an end to mathematical enquiry but direct its prosecution through a reorganisation of mathematical content along new lines, which may easily be understood in terms of the interaction between general principles

23 See LW12: 423.

and particular settings or results reconstituted in light of the general approach that guides their investigation.

Instead of instituting new data that empirically test its generalities, inference in the mathematical context institutes new data that correspond as special applications to its generalities or that are posited as contents informed by their dependence upon a formative generality.

The best way of clarifying the modified picture of the inferential arc just presented is to rehearse and enrich the formal picture delineated with reference to a concrete example. The one I have chosen is Mal'tsev's introduction of a general method to prove local theorems in group theory[24].

Mal'tsev's work provides an especially striking example of novelty in mathematical enquiry, thus supplying an ideal illustration of the inferential mode of thinking as qualified in the preceding section[25]: his work on local theorems introduced a novel methodology to prove them, novel proofs of existing local theorems, proofs of novel theorems, and a novel insight into the general principles underlying such theorems. For the sake of keeping this paper relatively self-contained, I conclude this section with a brief explication of what a local theorem is in the context of group theory.

An abstract group **G** is a nonempty set equipped with the associative operation of addition $+_G$, the neutral element **0** and the one-place function '$-_G$' assigning to each element g of **G** its additive inverse $-_G g$. I shall take **G** to be associated with a first-order language L containing the function symbols +, - and the constant symbol 0. The terms of L are expressions formed by applying the given function symbols repeatedly to variables and to the symbol 0: thus, for instance, 0, $x+x$, x, $0 + (-x)$ are terms of L or L-terms. A term is evaluated on **G** once particular elements of **G** are assigned to its variables.

Consider the finite subset $\{g_1, \ldots, g_n\}$ of **G**. This set generates a subgroup **H** of **G** as follows: the elements of **H** are the evaluations of L-terms in the variables $x_1, \ldots, x_n$ on $g_1, \ldots, g_n$. The subgroup **H** is said to be a *finitely generated* subgroup of **G**.

A group **G** has a *local property* P, or, equivalently, **G** is locally P, if every finitely generated subgroup of **G** has the property P. A local theorem in group theory asserts that a group is P if, and only if, it is locally P. In other words, the property P transfers

24 Regrettably, I only have space to focus on Mal'tsev (1941). Later, fundamental contributions, which cannot be discussed here, are Mal'tsev (1956, 1957).

25 See Er'sov (2011), 939 for an assessment of Mal'tsev (1941) as an instance of mathematical discovery. Er'sov goes so far as to describe it as a 'transcendental leap'.

from **G** to all of its finitely generated substructures and vice versa. This background suffices to explore the inferential structure of Mal'tsev's theorem.

7. Inferential Structure within Mathematical Practice: An Illustration

A local theorem relates **G** to its finitely generated subgroups. A problem suggested by the existence of local theorems is whether it is possible to determine conditions that uniformly guarantee the transfer of local properties to **G**.

Groups carry data that can be expressed by first-order formulae in the language L or they can model conditions that are so expressible. In Mal'tsev work groups are no longer seen as algebraic objects but rather as model-theoretic objects associated with linguistic conditions. This transformation focusses attention on the institution of linguistic data and suggests a mode of solution to the problem of determining general conditions under which local theorems hold. The suggestion comes from the interaction between linguistic data and models allowed by the Compactness theorem for first-order logic, which provides a general connection between the local and global existence of models (a set of first-order sentences has a model iff its finite subsets do).

The suggestion is to place the local-global equivalence supplied by Compactness in the service of group-theoretic argument, by coordinating its local side with finitely generated subgroups and its global side with whole groups.

The coordination is to be carried out by way of deductive development, which Mal'tsev articulates with respect to a special type of linguistic data, namely first-order, hereditary L-sentences, i.e. ones that transfer from a group to its subgroups (an example is the sentence $\forall x \forall y(x+y = y+x)$, modelled by Abelian groups).

Given a group **G**, Mal'tsev's key idea is to construct a set S of first-order sentences whose finite parts are modelled by the finitely generated subgroups of **G** if **G** is locally P. By Compactness, S has in this case a model **K** with the property P. The set S can be made to include a complete description of **G** (in technical terms, S may include the diagram of **G**). Because P is hereditary, P then transfers from **K** to its subgroup **G** and a local theorem for **G** can be proved (Mal'tsev focusses essentially on stating P in S, but the need for the diagram of **G** as part of S is explicitly pointed out in Robinson's first communication and summary of Mal'tsev's results[26]).

26 Robinson (1960), 62.

Apart from its formal correctness, Mal'tsev's strategy is of interest if it can cover algebraically significant properties. In order to show that it does, Mal'tsev specialises his approach to the property of being a group **G** of type [$E_1, \ldots, E_k$], with $E_1, \ldots, E_k$ first-order, hereditary conditions and *k* a fixed positive integer. For **G** to be of type [$E_1, \ldots, E_k$], there has to be a chain $\mathbf{G} = \mathbf{G}_0 \supseteq \mathbf{G}_1 \supseteq \ldots \supseteq \mathbf{G}_k = \{\mathbf{0}\}$, such that $\mathbf{G}_i$ is normal in $\mathbf{G}_{i-1}$ and the quotient group $\mathbf{G}_{i-1}/\mathbf{G}_i$ has the property E_i. At this particular stage of the inferential arc, dominated by discursive development, it is important to note that institution of new data in a language that expands L guides the proof of Mal'tsev general method to obtain local theorems.

Because chains of subgroups cannot be codified in the language L, Mal'tsev ingenuously introduces a sequence of constant symbols c^i_g (i = *1*, …, *k*) for each *g* in **G**, which enable him to write, in particular, conditions of the form:

$c^1_g + c^1_h = c^1_{g+h}$, with *g*, *h* in **G**,

which guarantee the existence of a homomorphism:

$\square_1(g) = c^1_g$, with *g* in **G**

from **G** to a group $\mathbf{X}_1$ that may be built from constant symbols. Then Mal'tsev considers first-order data of the form:

$(c^1_g = c^1_e \wedge c^1_h = c^1_e) \rightarrow c^2_g + c^2_h = c^2_{g+h}$

which guarantee that the kernel of $\square_1$, call it $\mathbf{G}_1$, is in turn homomorphic to a group $\mathbf{X}_2$. Instead of interpreting a chain of subgroups into **G**, as would standardly be done today, Mal'tsev uses first-order data to identify a chain of isomorphic copies determined by suitable homomorphisms, which guarantee the normality conditions on the $\mathbf{G}_i$.

The deductive articulation just outlined leads Mal'tsev to the final phase of the inferential arc, namely the 'controlled reconstitution of particulars'[27]. In this context particulars are specific group-theoretic properties, 'reconstituted' as first-order data that directly depend on Mal'tsev's general method to establish local theorems.

Particulars are reconstituted in essentially two ways. Either they are placed, as given materials, under the direct action of Malt'sev's general method yielding local theorems, or they already exhibit an interdependence between global and local properties, which can be newly modulated and refined through the institution of new first-order data suitable for an application of Malt'sev's method.

As an instance of the first type of reconstitution, consider a group-theoretic property like solvability of length *k*. A group **G** is solvable of length *k* if it contains a

27 LW12: 435.

normal chain of length k such that the quotients $\mathbf{G}_{i-1}/\mathbf{G}_i$ ($i = 1, \dots, k$) are Abelian. It is possible to reconstitute this property in terms of first-order data by noting that, if each of $E_1, \dots, E_k$ is the sentence $\Box x \Box y(x+y = y+x)$, a group **G** of the corresponding type is solvable of length *k*. It now follows from this immediate reconstitution and Mal'tsev's general method that solvability of length *k* is equivalent to local solvability of length *k*.

The second type of reconstitution is nicely illustrated by Mal'tsev's discussion of a result obtained by Černikov to the effect that, if every finite subgroup of a locally finite torsion group has a Sylow sequence, then so does the group itself.

For present purposes, we omit the full definition of a Sylow sequence but note that it is a special chain (not necessarily finite) of normal subgroups of a given group **G**. The notion of a Sylow sequence is not first-order in the language L, even enriched with constant symbols, but Mal'tsev introduces a clever way of instituting new first-order data to reconstitute Černikov's theorem in terms of his approach, for any given group **G**. The new first-order language reconstitutes the defining features of a Sylow sequence by viewing its elements as the points of a linear order designated by the binary relation symbol '>'[28].

Unary predicates of the form A_g, with g an element of **G**, are then used to associate with each element of the given linear order the elements of a subgroup of **G**. If S is a set of first-order conditions associated with **G** and forcing the existence of a Sylow sequence, the fact that **G** has a Sylow sequence locally guarantees an application of Compactness, which provides a model of **K** including **G** as a subgroup, up to isomorphism.

The reconstitution achieved does not merely subsume data under Mal'tsev's general method to obtain local theorems but also refines Černikov's theorem, because it shows the hypothesis of local finiteness to be superfluous and handles the notion of a Sylow sequence without restricting it to torsion groups.

A final instance of the reconstitution of given material may be offered. At the close of his paper, Mal'tsev shows that a theorem on the lattice of subgroups of a group, at the time of writing proved for countable[29] and uncountable[30] groups separately, has a single, completely general proof, obtained once more by instituting the theorem's key condition as a set of hereditary first-order data. This result sheds

28 Mal'tsev (1941), 18.

29 Baer (1933).

30 Sadowskii (1941).

light on the content of existing proofs and, in particular, produces a novel standpoint that transcends their differentiation and disengages the local-global relation from cardinality constraints, thus reorganising the conceptual interconnections between distinct mathematical traits. Limits of space prevent any more than an allusion to the important role played by reconstitution of particulars in the continuum of enquiry. While established general principles enable actual reconstitutions of particulars, they also suggest wider possibilities of reconstitution, e.g., in our case, the search for a local principle that applies to structures and substructures (rather than groups and subgroups alone) relative to linguistic conditions that may be stronger than first-order. Mal'tsev obtains just one such result in his 1957 work.

Reconstitution does not only introduce a novel perspective under specified conditions, but also prepares the progressive extension of enquiry. The inferential arc has therefore both a circumstantial significance in the context directly within its purview and a programmatic significance when related to the continuum of enquiry.

8. Pragmatism and Scientific Practice

This paper opened with the detection of difficulties frequently arising in epistemological work, which were traced back to the suppression, perceived or not, of the context of enquiry. A contrast was drawn between the salient consequences of this suppression and the salient consequences of an alternative philosophical outlook informed by Dewey's conception of inference.

In the former case, two phenomena stood out, namely the difficulty of carrying analysis to fruition and a general imperviousness of analysis to insights extractible from scientific practice. By contrast, when Dewey's conception of inference was adopted as a means of philosophical orientation, an articulation of the inferential arc well suited to the fine descriptive analysis of episodes in mathematical enquiry became available.

The difference between the typical results of either philosophical outlook is so striking that it does not only recommend a reorientation of epistemological work along pragmatist lines, but also indicates that the fruitful route for epistemology is to transcend its current form and turn into the logical study of scientific practice.

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