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Modeling skewness in portfolio choice

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Abstract

We seek the best skewness models for portfolio choice decisions. To this end, we compare the predictive ability and portfolio performance of several prominent skewness models in a sample of 10 international equity market indices. Overall, models that employ information from the option markets outperform models that only rely on stock returns. We propose an option-based skewness estimator that accounts for the skewness risk premium. This estimator offers the most informative forecasts of future skewness, the lowest prediction errors, and the best portfolio performance in most of our tests.

KEYWORDS

portfolio choice, skewness modeling, skewness risk premium

1 | INTRODUCTION

Mean–variance portfolio theory has been leading academic research in investment management since Markowitz (1952). However, starting with Arditti (1967), several studies highlight the importance of skewness for investors.[1] In response to this evidence, a growing body of research advocates embedding skewness in the portfolio selection process.[2] Still, skewness is challenging to predict since conventional skewness estimates are not persistent (Singleton & Wingender, 1986), are sensitive to outliers (Kim & White, 2004), and are affected by the frequency of the returns underlying the estimation (Neuberger, 2012). While several alternative skewness models have been developed to address these issues, it remains an open question which models have merits for predicting skewness in the context of investment decision-making.

We contribute to the literature by identifying the best skewness models for portfolio choice decisions.[3] By carrying out a broad comparison of the predictive ability and portfolio performance of several skewness models, we find that models that use information from the option markets outperform models based only on historical returns. We further propose a new option-implied estimator that accounts for the skewness risk premium and outperforms the rest of the

[1]A number of subsequent studies explore the link between skewness and asset prices, including Amaya et al. (2015), Barberis and Huang (2008), Boyer et al. (2010), Brunnermeier et al. (2007), Chen (2021), Conrad et al. (2013, 2014), Harvey and Siddique (2000), Hong and Stein (2003), Kraus and Litzenberger (1976), and Mitton and Vorkink (2007).

[2]Patton (2004), Jondeau and Rockinger (2006), Guidolin and Timmermann (2008), Harvey et al. (2010), DeMiguel et al. (2013b), and Ghysels et al. (2016) consider the use of skewness in investment decisions. Bali et al. (2008), Engle (2011), Gilbert et al. (2006), Kostika and Markellos (2013), and Lien (2010) account for skewness in risk management applications.

[3]In relation to this work, Aretz and Arisoy (2022) undertake a comparative in-sample analysis of skewness models using a sample of individual US stocks. The focus of their paper is on empirical asset pricing, while here we concentrate on out-of-sample predictive ability and portfolio choice using skewness forecasts.

skewness models in most of our tests. Our analysis is comprehensive in that we account for: (i) six skewness models; (ii) 10 international indices; (iii) three time horizons (30, 60, and 90 calendar days); (iv) two methods for assessing the information content of each model; (v) an out-of-sample forecasting horserace under two loss functions and two ranking tests; and (vi) a portfolio-based evaluation of skewness models under five portfolio performance metrics. We also perform several robustness tests.

We proxy true skewness using the *realized skewness* estimator of Neuberger (2012) in line with Kozhan et al. (2013) that use realized skewness as a measure of physical skewness. We choose this proxy as, similar to realized volatility, it can be computed for any horizon using high-frequency returns in contrast to most other skewness estimators. Six skewness models with distinct characteristics compete to predict realized skewness. The first forms predictions using a one-period lag of realized skewness. The next two employ conditional skewness estimates from the GJR-GARCH model of Glosten et al. (1993) assuming time-invariant shape parameters (Engle, 2011) and time-varying shape parameters (Bali et al., 2008). GJR-GARCH has been widely studied in the empirical literature and has been shown to be among the best performers in the family of historical conditional volatility models. The fourth model is based on the estimation of conditional quantiles of returns via a Mixed Data Sampling (QMIDAS) approach (Ghysels et al., 2016). Two attractive features of this model are that it is less sensitive to outliers and that it can be computed using returns of higher frequency than the horizon of interest. The fifth model is an option-implied skewness estimator, similar to that proposed by Conrad et al. (2013). This is a forward-looking estimator, which is computed from option prices and does not rely on historical asset returns. In addition to the five models from the literature, we propose a new option-based skewness model that accounts for a *skewness risk premium*.

We find that option-implied models are generally more informative of future skewness compared with skewness models that only use past asset returns. The best overall predictive performance is offered by the option-based skewness model we propose in this work. In most considered cases, it leads to the highest R^2's in our predictive regressions and to the lowest forecasting errors. Notably, it helps explain up to 35% of the variation in the future realized skewness of the Standard and Poor's (S&P) 500 index. Generalized autoregressive conditional heteroskedasticity (GARCH) models tend to outperform option-based models in less developed option markets.

We finally compare the skewness models in the context of portfolio performance. We use the parametric approach of Brandt et al. (2009) to construct a portfolio strategy for each skewness model. The option-implied skewness model that accounts for the skewness risk premium delivers the best portfolio performance in terms of mean return, variance, and Sharpe ratio. In some of our tests, it is the only model that leads to positive portfolio skewness. The second best alternative is the vanilla option-implied model. These results extend the findings of DeMiguel et al. (2013b) and Kourtis et al. (2016) and support the use of option-implied information and risk premia in portfolio selection.

The rest of the paper is organized as follows. Section 2 presents our data, the realized skewness proxy and the skewness models under consideration. Section 3 investigates the information content of the models while Section 4 evaluates their out-of-sample performance. We present the construction and performance of skewness-based portfolio strategies in Section 5. Our robustness tests are covered in Section 6, while Section 7 concludes the paper.

2 | DATA AND MODELS

2.1 | Data

Our sample consists of 10 equity indices corresponding to 7 international regions.[4] This wide selection of indices allows us to stress-test the robustness of our results across markets with different characteristics and levels of development. It also enables us to carry out an international diversification exercise to assess the value of the considered skewness models for portfolio selection.

We adopt two main sets of data for our analysis. First, we collect daily dividend-adjusted levels from Thomson-Reuters Datastream. For each index, we also employ a time series of the London Interbank Offer Rate (LIBOR), quoted in the same currency, to proxy the corresponding risk-free rate. LIBOR data are collected from the FRED database of the Federal Reserve Bank of St. Louis. Our second data set consists of market prices of European vanilla options written on the indices considered. We obtain this data from IVolatility. We apply several standard filters from the literature to this data set (e.g., see, Conrad

[4]Table 1 presents details of these indices and the time periods we consider. These can differ across indices according to the availability of option data

TABLE 1 Data.

Equity index	Region	Time period
AEX	The Netherlands	11/2006–5/2019
DAX	Germany	06/2002–5/2019
DJIA	The United States	06/2001–5/2019
STOXX 50	Europe	01/2003–5/2019
FTSE 100	The United Kingdom	12/2006–5/2019
HANGSENG	China	11/2008–5/2019
KOSPI	South Korea	11/2008–5/2019
NASDAQ 100	The United States	11/2000–5/2019
RUSSELL 2000	The United States	09/2003–5/2019
S&P 500	The United States	01/2000–5/2019

Note: This table lists the indices we employ in this work along with the corresponding region. It also reports the time period we consider for each index.
Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; S&P 500, Standard and Poor's 500.

et al., 2013; Stilger et al., 2016). First, we exclude all options with zero bid prices, zero open interest, and price smaller than \$3/8. Second, we only consider out-of-the-money (OTM) calls/puts with a moneyness level between 0.8 and 1.2 and maturity ranging from 7 to 270 days.[5] Third, we discard all options that violate theoretical arbitrage bounds from Merton (1973).[6] Finally, at each day, we only account for maturities that have at least two OTM calls and two OTM puts.

2.2 | Realized skewness

To assess the predictive performance of any skewness model, we need a proxy for the true skewness of asset returns for different time horizons. In the volatility modeling literature, a common proxy of the true volatility is realized volatility computed from high-frequency data as the sum of squared returns sampled at equal intervals (see Andersen et al., 2001; Barndorff-Nielsen & Shephard, 2002). However, estimating skewness, especially for lengthier horizons, is not that straightforward. As Neuberger (2012) shows, one cannot simply use the sum of high-frequency cubic returns to accurately proxy long-term skewness. This is because the latter is also driven by a leverage effect, given by the correlation between innovations in volatility and asset returns. While we could instead rely on estimators based on nonoverlapping long-term returns to estimate long-term skewness, skewness estimates would be subject to significant sampling errors due to a likely small number of observations. To resolve these issues, Neuberger (2012) proposes a skewness estimator of logarithmic returns, coined as "realized skewness." Realized skewness uses option data to capture the leverage effect and enables the use of data of frequency higher than the horizon of interest, similar to realized volatility. This motivates us to adopt realized skewness as a proxy of the true skewness of index returns in this work, similar to Kozhan et al. (2013).

Realized skewness can be computed as follows. We denote the asset price at the end of day t with S_t. If T is the time horizon, the log return from day t to day $t+T$ is $r_{t,t+T} = \ln(S_{t+T}/S_t)$.[7] Under the assumption that the asset price S_t follows a Martingale process, Neuberger (2012) shows that the realized third (central) moment of the return $r_{t,t+T}$ can be expressed as

$$\mathrm{RTM}(r_{t,t+T}) = \sum_{i \in M_{t,t+T}} \left(F(r_i) + 3\left(\Delta v^E_{i,t+T}\right)(e^{r_i} - 1) \right). \tag{1}$$

[5]Similar to Conrad et al. (2013), we define option moneyness as the ratio of strike price to spot price.
[6]We apply the following arbitrage bounds for the recorded option prices: $\max(S - Ke^{-rT}, 0) \leq C \leq S$ and $\max(Ke^{-rT} - S, 0) \leq P \leq Ke^{-rT}$, where C (P) is the call (put) option price, S is the index level, K is the strike price, r is the risk-free rate, and T is the time-to-maturity of the option contract.
[7]The return on day i is simply denoted with $r_i = r_{i-1,i}$.

TABLE 2 Descriptive statistics of realized moments.

Index	Panel A: 30 days Variance	Third moment	Skewness	Panel B: 60 days Variance	Third moment	Skewness	Panel C: 90 days Variance	Third moment	Skewness
AEX	0.358	−0.306	−1.225	0.653	−0.87	−1.401	0.957	−1.666	−1.586
	(0.58)	(0.83)	(0.52)	(0.86)	(2.18)	(0.47)	(1.26)	(3.67)	(0.49)
DAX	0.367	−0.28	−1.162	0.693	−0.764	−1.282	1.015	−1.491	−1.408
	(0.49)	(0.61)	(0.53)	(0.81)	(1.27)	(0.49)	(1.00)	(1.99)	(0.47)
DJIA	0.443	−0.406	−0.983	0.647	−0.876	−1.17	1.028	−1.869	−1.288
	(0.70)	(1.21)	(0.55)	(0.96)	(2.83)	(0.58)	(1.46)	(5.10)	(0.58)
STOXX 50	0.377	−0.325	−1.251	0.681	−0.897	−1.378	0.981	−1.662	−1.519
	(0.54)	(0.73)	(0.64)	(0.75)	(1.80)	(0.54)	(0.99)	(2.84)	(0.50)
FTSE 100	0.274	−0.244	−1.35	0.469	−0.617	−1.494	0.664	−0.995	−1.612
	(0.42)	(0.62)	(0.59)	(0.51)	(1.37)	(0.44)	(0.65)	(1.50)	(0.43)
HANGSENG	0.235	−0.123	−0.801	0.472	−0.364	−0.856	0.701	−0.633	−0.88
	(0.17)	(0.21)	(0.54)	(0.33)	(0.57)	(0.52)	(0.47)	(0.87)	(0.49)
KOSPI	0.2	−0.113	−0.933	0.371	−0.269	−0.976	0.556	−0.483	−1.057
	(0.23)	(0.23)	(0.69)	(0.39)	(0.46)	(0.84)	(0.56)	(0.72)	(1.00)
NASDAQ 100	0.43	−0.335	−1.065	0.744	−1.004	−1.286	1.082	−2.049	−1.509
	(0.64)	(0.88)	(0.57)	(1.04)	(2.63)	(0.54)	(1.52)	(4.72)	(0.53)
RUSSELL 2000	0.465	−0.403	−1.06	0.926	−1.371	−1.302	1.457	−2.867	−1.447
	(0.77)	(1.10)	(0.46)	(1.34)	(3.25)	(0.42)	(1.87)	(5.62)	(0.41)
S&P 500	0.288	−0.281	−1.466	0.536	−0.827	−1.738	0.737	−1.446	−1.966
	(0.53)	(0.95)	(0.76)	(0.78)	(2.45)	(0.74)	(0.94)	(3.76)	(0.76)

Note: This table reports the average realized variance, third moment, and skewness coefficients of the index log returns. The variance is annualized and the third moment is multiplied by 1000 for improved presentation. Standard errors for each statistic are presented in parentheses. Panels A, B, and C correspond to horizons of 30, 60, and 90 calendar days, respectively.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; S&P 500, Standard and Poor's 500.

In the above equation, $F(r) = 6(re^r - 2e^r + r + 2)$, $M_{t,t+T}$ is the set of trading days in the period $\{t, t+T\}$ and $\Delta v^E_{i,t+T}$ is the change of the index entropy variance from the end of day $i-1$ to the end of day i.[8] Hence, the third moment of long-horizon returns is the sum of the third moment of daily returns ($F(r)$) and the correlation between volatility innovations and returns (i.e., the leverage effect) captured by the daily change in the entropy variance. Neuberger (2012) finds that the second term becomes more important in generating skewness as the time horizon increases.

The *realized skewness* $\mathrm{RS}(r_{t,t+T})$ of the index return $r_{t,t+T}$ is then given by

$$\mathrm{RS}(r_{t,t+T}) = \frac{\mathrm{RTM}(r_{t,t+T})}{(\mathrm{RV}(r_{t,t+T}))^{3/2}}, \tag{2}$$

[8]The entropy variance is the option-implied variance of a contract that pays $S_{t+T} \ln S_{t+T}$ at day $t+T$. In Section A.1 in the appendix, we present the process we follow to approximate it.

where $\mathrm{RV}(r_{t,t+T}) = \sum_{i \in M_{t,t+T}} r_i^2$ is the realized variance of $r_{t,t+T}$.[9]

$\mathrm{RS}(r_{t,t+T})$ stands for the proxy of the true skewness of the index return over the period $\{t, t+T\}$. In essence, $\mathrm{RS}(r_{t,t+T})$ is the quantity that the considered skewness models compete to forecast. All comparisons are undertaken under three horizons of $T = 30$, 60, or 90 calendar days. For each horizon, Table 2 reports the mean and standard deviation of the realized variance, third moment, and skewness of each index over the whole sample. Similar to Neuberger (2012), we find that the realized third moment and skewness are always negative and become more negative as the horizon increases.

2.3 | Predictive models of skewness

2.3.1 | Lagged realized skewness (LaggedRealizedSkew)

In this work, we examine the predictive ability and economic significance of six skewness forecasting models. These models have distinct features which allow them to capture different information about the future realized skewness $\mathrm{RS}(r_{t,t+T})$. We adopt a standard convention in the time-series asset pricing literature and estimate all forecasting models using logarithmic returns.[10]

The first model is simply the lagged realized skewness (LaggedRealizedSkew), given by $\mathrm{RS}(r_{t-T,t})$. It may seem odd we include LaggedRealizedSkew as one of the potential predictors of realized skewness in light of previous work indicating that historical skewness of simple returns is not persistent over time (e.g., see Singleton & Wingender, 1986). However, realized skewness differs from conventional skewness measures in that it embeds a "leverage-effect" component which may generate a level of persistence over time. In line with this reasoning, Neuberger (2012) provides evidence that LaggedRealizedSkew can predict the realized skewness of S&P 500 at the quarterly horizon. An attractive feature of LaggedRealizedSkew is that it does not rely on a particular distributional assumption, while it has a hybrid nature, utilizing information from both the equity and the option markets.

2.3.2 | Skewness from a GARCH model with time-invariant parameters (GARCH-1)

GARCH models are a popular choice in the literature for modeling the conditional skewness of asset returns (see, e.g., Engle, 2011; Feunou et al., 2016; Harvey & Siddique, 1999; Jondeau & Rockinger, 2003). Here, we consider two approaches that yield skewness forecasts from the GJR-GARCH model of Glosten et al. (1993). We choose this model among other GARCH alternatives as it allows asymmetries in the conditional variance and can produce skewness in the conditional distribution of multiperiod returns.

To specify the GJR-GARCH model, we assume that daily index returns follow a Skewed Generalized Error (SGE) distribution, as this is proposed by Theodossiou (2015).[11] The GJR-GARCH model under the conditional SGE distribution is then specified as

$$r_t = \mu_t + \sigma_t z_t, \tag{3}$$

$$\sigma_t^2 = b_0 + b_1 \varepsilon_{t-1}^2 + b_2 I_{z_{t-1} \le 0} \varepsilon_{t-1}^2 + b_3 \sigma_{t-1}^2, \tag{4}$$

[9]If there are not enough option prices available for a given maturity T, we compute the third moment and the realized skewness using option data for the nearest maturities T (say T_1 and T_2) for which we have the necessary data, so that $T_1 < T < T_2$. We then apply a simple linear interpolation to compute the two realized moments, as in Chang et al. (2013) and Kozhan et al. (2013).

[10]While GARCH and MIDAS models are often estimated using arithmetic returns, we use logarithmic returns in line with P. R. Hansen and Lunde (2005), Kourtis et al. (2016), and Le (2020), among others. This convention allows us to fairly compare the different models against our proxy of true skewness, as the skewness of logarithmic returns can significantly differ from that of arithmetic returns (e.g., as illustrated in Table 1 in Neuberger & Payne, 2021).

[11]The SGE distribution has been widely used in various financial applications (see, e.g., Anatolyev & Petukhov, 2016; Feunou et al., 2016), mainly due to the flexibility it offers for modeling financial data. In addition, Feunou et al. (2016) show that SGE results in superior parametric models for capturing the daily conditional skewness compared with other common distributions. We provide more details of this distribution in Section A.2 in the appendix.

where μ_t is the conditional expected return, σ_t is the conditional variance, $I_{z_{t-1}\leq 0}$ takes the value of 1 if $z_{t-1} \leq 0$ and zero otherwise, and z_t follows a standardized SGE distribution with zero mean and unit variance and time-invariant shape parameters λ and κ. We note that b_2 captures the leverage effect imposed on the conditional variance process.

To compute a GJR-GARCH-based skewness estimate for a specific time horizon, we first estimate the GJR-GARCH model above using the available sample. We then apply a Monte-Carlo simulation approach, similar to Engle (2011) and Lönnbark (2016), to compute an empirical return distribution for the horizon of interest. At each day t, for a given simulation path i, we simulate the next day return using the estimated coefficients and a random standardized innovation z_t, drawn from SGE$(0, 1, \hat{\lambda}_t, \hat{\kappa}_t)$, where $\hat{\lambda}_t$ and $\hat{\kappa}_t$ are the estimates of λ and κ at day t.[12] We iterate this process to obtain a time series of daily returns for the trading days available in the next T calendar days, where $T = 30$, 60, or 90, as before. In this manner we end up with the T-horizon simulated return $\tilde{r}^i_{t,t+T}$. We repeat this procedure 10,000 times to obtain the empirical distribution of the T-horizon return. The GJR-GARCH estimate of the skewness is then the skewness of the empirical distribution. We coin this skewness estimator GARCH-1.

2.3.3 | Skewness from a GARCH model with autoregressive conditional parameters (GARCH-2)

The next skewness estimator we consider comes from a richer variation of the GJR-GARCH model which allows some of the parameters to vary over time, similar to B. E. Hansen (1994). In particular, we enable the shape parameters in the distribution of z_t in (3) to depend on past information, assuming an autoregressive structure. Bali et al. (2008) show that such a model leads to a more accurate representation of the conditional return distribution for several US stock indices.

We follow Bali et al. (2008) to specify the dynamics of the shape parameters as

$$\widetilde{\lambda}_t = \lambda_0 + \lambda_1 z_{t-1} + \lambda_2 \widetilde{\lambda}_{t-1}, \tag{5}$$

$$\widetilde{\kappa}_t = \kappa_0 + \kappa_1 z_{t-1} + \kappa_2 \widetilde{\kappa}_{t-1}, \tag{6}$$

where $\widetilde{\lambda}_t$ and $\widetilde{\kappa}_t$ are the unrestricted estimates of λ and κ. We apply a transformation to these estimates so that they are bounded in line with the assumptions of the SGE distribution ($|\lambda_t| < 1$ and $\kappa_t > 0$):

$$\lambda_t = -0.99 + \frac{1.98}{1 + e^{-\widetilde{\lambda}_t}}, \tag{7}$$

$$\kappa_t = 2 + e^{\widetilde{\kappa}_t}. \tag{8}$$

In this case, the distribution of the innovation z_t in (3) is SGE$(0, 1, \lambda_t, \kappa_t)$. Empirically, the shape parameters are again estimated by maximizing the sample log-likelihood function before estimating the GJR-GARCH model. We then follow the same simulation approach as before to compute the skewness estimates. In the rest of the paper, we refer to this skewness estimator as GARCH-2.[13]

2.3.4 | Quantile-based skewness under a mixed data sampling approach

This skewness measure relies on the estimation of the conditional quantiles of the return distribution. A benefit from following such an approach is that quantile-based skewness estimators are generally more robust to outliers than moment-based estimators (Kim & White, 2004). To estimate conditional quantiles, we follow the Mixed Data Sampling approach (QMIDAS) of Ghysels et al. (2016). The main advantage of QMIDAS is that it can directly yield

[12] λ and κ are estimated by maximizing the sample log-likelihood function for z_t.

[13] In a robustness test, we change the dynamics of the shape parameters in (5) and (6) to allow for asymmetric responses in the return innovations as in Feunou et al. (2016). We find that this change leads to qualitatively similar results in terms of out-of-sample performance.

conditional quantiles of returns at any horizon while still exploiting information in higher-frequency data. Furthermore, it does not rely on any specific distributional assumption for the return process. The QMIDAS model is described by the following equations:

$$q_\alpha(r_{t,t+T}) = \beta^0_{\alpha,T} + \beta^1_{\alpha,T} Z_{t-1}(\kappa_{\alpha,T}), \tag{9}$$

$$Z_{t-1}(\kappa_{\alpha,T}) = \sum_{d=0}^{D} \varphi_d(\kappa_{\alpha,T}) |r_{t-1-d}|, \tag{10}$$

where $q_\alpha(r_{t,t+T})$ is the α-quantile of the T-horizon return. The known conditioning variable $Z_{t-1}(\kappa_{\alpha,T})$ is a sum of weighted absolute returns as in Ghysels et al. (2016).[14] Each weight in (10) is determined by a lag polynomial function $\varphi(\cdot)$ of a low-dimensional parameter vector $\kappa_{\alpha,T}$. We specify $\varphi(\cdot)$ as discussed in the Internet Appendix IV of Ghysels et al. (2016).[15] In our main results, we choose the maximum lag $D = 250$ to account for potential long-memory effects in the return process.[16] We then run the MIDAS quantile regression for each quantile $\alpha \in (0.01, 0.025, 0.05, 0.1, 0.15, 0.2, 0.25, 0.5, 0.75, 0.8, 0.85, 0.9, 0.95, 0.975, 0.99)$. We finally employ the approach of Aretz and Arisoy (2022) to extract the moment-based skewness from the quantiles. We provide more details of this approach in Section A.3 of the appendix.

2.3.5 | Implied skewness (ImpliedSkew)

Conrad et al. (2013) propose the use of information from the option markets to construct forward-looking estimators of the ex ante skewness. Implied skewness can then be defined as the ratio of the implied third central moment to the cube of the implied volatility:

$$\mathrm{IS}_t(r_{t,t+T}) = \frac{\mathrm{ITM}_t(r_{t,t+T})}{(\mathrm{IV}_t(r_{t,t+T}))^{3/2}} = \frac{3\left(v_t^E(r_{t,t+T}) - \mathrm{IV}_t(r_{t,t+T})\right)}{\mathrm{IV}_t(r_{t,t+T})^{3/2}}, \tag{11}$$

where $\mathrm{ITM}_t(r_{t,t+T}) = 3\left(v_t^E(r_{t,t+T}) - \mathrm{IV}_t(r_{t,t+T})\right)$ is the implied third moment and $\mathrm{IV}_t(r_{t,t+T})$ is the implied variance. The latter can be calculated as

$$\mathrm{IV}_t(r_{t,t+T}) = \frac{2}{B_{t,t+T}}\left[\int_0^{S_t} \frac{P_{t,t+T}(K)}{K^2} dK + \int_{S_t}^{\infty} \frac{C_{t,t+T}(K)}{K^2} dK\right]. \tag{12}$$

To compute the implied moments, we approximate the entropy variance as in Section 2.2. We further estimate the implied variance following the same approach as the entropy variance. We input these approximations in (11) to derive the option-implied skewness.[17]

Implied skewness (ImpliedSkew) has two notable features. First, it is forward-looking, not relying on past returns as it only reflects expectations from the option markets. Second, in contrast to GARCH-based skewness and other historical models, it is model-free as it does not assume a specific distribution for the asset returns.

[14]We model conditional quantiles as a function of past returns to be consistent with Ghysels et al. (2016), however further predictive variables could be considered. For example, Aretz and Arisoy (2022) predict the quantiles of US stock returns using several of their fundamentals. We leave extensions of the QMIDAS model that consider alternative exogenous variables at the index level for future research.

[15]We would like to thank Eric Ghysels for making available the code for this estimation at https://www.mathworks.com/matlabcentral/fileexchange/45150-midas-matlab-toolbox.

[16]If we alternatively assume lag lengths of 200 or 300 days, the forecasting losses tend be higher for $D = 200$ compared with $D = 250$ and similar for $D = 300$. These results are available upon request.

[17]To estimate the implied skewness for a day t where we do not have enough options with a maturity matching the horizon T, we again use interpolation as in the case of realized skewness.

2.3.6 | Implied skewness adjusted for the skewness risk premium (AdjustedImpliedSkew)

A potential drawback of implied skewness as a predictor of the true physical skewness is that it assumes a risk-neutral probability measure. As such, implied skewness may be a biased estimator of the physical skewness. In this context, Kozhan et al. (2013) explore the difference between the nonstandardized implied and the realized third moments for the S&P 500 index using a "skew risk premium" given by

$$xs_{t,t+T} = \frac{\mathrm{ITM}_t(r_{t,t+T})}{\mathrm{RTM}(r_{t,t+T})} - 1. \tag{13}$$

$xs_{t,t+T}$ represents the excess return from investing in the fixed leg of a swap where the fixed leg is the implied third moment and the floating leg is the realized third moment. Kozhan et al. (2013) provide evidence of the existence of an economically significant third-moment risk premium for the S&P 500 index. In this setting, they show that the third-moment risk premium is highly correlated with the variance risk premium. We study whether these findings extend to the international indices in our sample. We estimate relative variance (VRP), third-moment (TRP), and skewness (SRP) risk premia from time t to $t + T$ for each of the indices as

$$\mathrm{VRP}_{t,t+T} = \frac{\mathrm{IV}_t(r_{t,t+T})}{\mathrm{RV}(r_{t,t+T})} = \mathrm{xv}_{t,t+T} + 1, \tag{14}$$

$$\mathrm{TMRP}_{t,t+T} = \frac{\mathrm{ITM}_t(r_{t,t+T})}{\mathrm{RTM}(r_{t,t+T})} = \mathrm{xs}_{t,t+T} + 1, \tag{15}$$

$$\mathrm{SRP}_{t,t+T} = \frac{\mathrm{IS}_t(r_{t,t+T})}{\mathrm{RS}(r_{t,t+T})}, \tag{16}$$

where $\mathrm{xv}_{t,t+T}$ is the variance risk premium as defined in Kozhan et al. (2013).[18]

Table 3 reports the averages and standard errors of the above premia for each index and horizon under consideration. We observe that the implied moments are on average higher than the realized moments in absolute terms in all cases, apart from Korea Composite Stock Price Index (KOSPI), where only the absolute implied skewness is lower than the absolute realized skewness. In general, the magnitude of the skewness risk premium is slightly higher than the variance risk premium while the third-moment risk premium is considerably higher than both. In line with Kozhan et al. (2013), we find that the variance risk premium and third-moment risk premium are highly positively correlated for the S&P 500 index, with correlations ranging from 0.772 to 0.875. These correlations are of similar magnitude across indices, but they are slightly lower for Hang Seng Index (HANGSENG) and KOSPI. We also observe a negative and weaker relationship between the skewness risk premium and the variance risk premium in most cases. Finally, we find that the variability of the third-moment risk premium and the skewness risk premium is considerably higher for HANGSENG and KOSPI as the corresponding standard errors show.

The evidence on the existence of a skewness risk premium for almost all indices in our sample indicates that implied skewness is a biased estimator of realized skewness. In this context, we propose to reduce the bias by correcting the implied skewness for the skewness risk premium. To this end, we divide the implied skewness by a historical average skewness risk premium, under the assumption that the historical premium captures the premium for the period $(t, t + T)$. This adjustment is similar to the correction that DeMiguel et al. (2013b) and Prokopczuk (2014) apply to improve the forecasting performance of implied volatility. We estimate a time-horizon-dependent

[18]We follow DeMiguel et al. (2013b), Prokopczuk (2014), and Kourtis et al. (2016) to employ relative risk premia in our analysis given by the ratio of the implied to the realized moment. This allows us to directly apply the skewness risk premium to adjust the implied skewness estimator in the same way as DeMiguel et al. (2013b) and Prokopczuk (2014) use a relative variance risk premium to correct the bias in implied volatility estimates. At the same time, since $\mathrm{VRP}_{t,t+T}$ and $\mathrm{TMRP}_{t,t+T}$ are simple transformations of the corresponding risk premia $\mathrm{xv}_{t,t+T}$ and $\mathrm{xs}_{t,t+T}$ in Kozhan et al. (2013), our empirical analysis of the relative risk premia can be easily linked to their findings. For example, it is simple to confirm that $\mathrm{corr}(\mathrm{VRP}_{t,t+T}, \mathrm{TMRP}_{t,t+T}) = \mathrm{corr}(\mathrm{xv}_{t,t+T}, \mathrm{xs}_{t,t+T})$.

TABLE 3 Moment risk premia.

Index	Variance risk premium	TM risk premium	Skewness risk premium	Correlation with VRP TMRP	Correlation with VRP SRP
Panel A: 30 days					
AEX	1.577 (0.781)	2.502 (1.900)	1.279 (0.634)	0.741	−0.268
DAX	1.610 (0.868)	3.068 (2.439)	1.620 (1.111)	0.710	−0.297
DJIA	1.440 (0.763)	2.478 (3.456)	1.461 (1.634)	0.352	−0.110
STOXX 50	1.717 (0.907)	3.901 (3.614)	1.799 (1.217)	0.602	−0.233
FTSE 100	1.689 (0.816)	3.907 (2.907)	1.845 (1.094)	0.657	−0.258
HANGSENG	1.639 (0.765)	4.800 (8.790)	2.106 (4.085)	0.291	0.026
KOSPI	1.659 (0.824)	1.949 (3.398)	0.924 (1.386)	0.308	−0.080
NASDAQ 100	1.562 (0.811)	3.244 (2.719)	1.674 (1.031)	0.674	−0.186
RUSSELL 2000	1.558 (0.717)	2.982 (2.076)	1.578 (0.810)	0.707	−0.285
S&P 500	1.864 (1.039)	4.021 (3.274)	1.586 (0.771)	0.772	−0.247
Panel B: 60 days					
AEX	1.679 (0.823)	2.502 (1.687)	1.134 (0.389)	0.863	−0.271
DAX	1.592 (0.776)	2.348 (1.728)	1.189 (0.575)	0.774	−0.282
DJIA	1.521 (0.791)	2.215 (1.917)	1.148 (0.630)	0.739	−0.149
STOXX 50	1.733 (0.827)	3.011 (2.008)	1.340 (0.536)	0.811	−0.329
FTSE 100	1.719 (0.781)	3.093 (2.039)	1.358 (0.529)	0.801	−0.227
HANGSENG	1.636 (0.705)	4.216 (5.378)	1.772 (2.088)	0.476	0.110
KOSPI	1.840 (1.018)	2.176 (5.462)	0.882 (1.203)	0.230	−0.081
NASDAQ 100	1.665 (0.809)	2.782 (1.933)	1.281 (0.507)	0.805	−0.227
RUSSELL 2000	1.681 (0.717)	2.728 (1.659)	1.255 (0.447)	0.804	−0.293
S&P 500	1.943 (1.004)	3.380 (2.564)	1.222 (0.441)	0.843	−0.252
Panel C: 90 days					
AEX	1.725 (0.903)	2.591 (1.854)	1.111 (0.356)	0.909	−0.273
DAX	1.633 (0.784)	2.332 (1.682)	1.107 (0.463)	0.839	−0.250
DJIA	1.550 (0.749)	2.264 (1.909)	1.093 (0.547)	0.737	−0.026
STOXX 50	1.752 (0.848)	2.876 (1.888)	1.233 (0.416)	0.873	−0.309
FTSE 100	1.675 (0.825)	2.621 (1.880)	1.179 (0.467)	0.858	−0.206
HANGSENG	1.598 (0.736)	3.954 (4.489)	1.756 (1.829)	0.517	0.077
KOSPI	2.086 (1.327)	2.146 (4.801)	0.719 (1.016)	0.339	−0.092
NASDAQ 100	1.711 (0.885)	2.563 (1.953)	1.118 (0.474)	0.827	−0.194
RUSSELL 2000	1.698 (0.763)	2.450 (1.527)	1.099 (0.351)	0.875	−0.312
S&P 500	2.028 (1.050)	3.131 (2.390)	1.056 (0.347)	0.875	−0.272

Note: This table reports the average values for the variance, third moment (TM), and skewness risk premium for each index in our data set and for horizons of 30, 60, and 90 calendar days. *Variance Risk Premium* (VRP) is defined as the ratio of implied to realized variance. *TM Risk Premium* (TMRP) is defined as the ratio of implied to realized third moment. *Skewness Risk Premium* (SRP) is defined as the ratio of implied to realized skewness. Standard deviations are reported in parenthesis. The last two columns, respectively, report the correlation between the variance and the third-moment risk premia and between the variance and the skewness risk premia. Panels A, B, and C show these statistics for horizons of 30, 60, and 90 days, respectively.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; S&P 500, Standard and Poor's 500.

historical skewness premium for each day t and horizon T using the skewness risk premia over the previous 252-T trading days as[19]

$$\bar{\mathrm{SRP}}_{t,t+T} = \frac{1}{252 - T} \sum_{i=t-251}^{t-T} \mathrm{SRP}_{i,i+T}. \tag{17}$$

The *adjusted implied skewness estimator* (AdjustedImpliedSkew) is then

$$A - \mathrm{IS}_t(r_{t,t+T}) = \frac{\mathrm{IS}_t(r_{t,t+T})}{\bar{\mathrm{SRP}}_{t,t+T}}. \tag{18}$$

In the absence of a skewness risk premium, the adjusted implied skewness (AdjustedImpliedSkew) is identical to the vanilla implied skewness. Overall, the set of competing models consists of five skewness models from the literature along with the adjusted implied skewness model we propose here. Three of the models rely on information from past returns only (GARCH-1, GARCH-2, and QMIDAS), while the rest of the models also use information from the options market (LaggedRealizedSkew, ImpliedSkew, and AdjustedImpliedSkew). The rest of the paper aims to identify the best models in terms of predicting future realized skewness and in terms of investment performance.

3 | INFORMATION CONTENT OF SKEWNESS MODELS

We launch the comparisons between the skewness models presented in Section 2 with an in-sample evaluation of the information content of each model. To provide some initial evidence of how informative each model is with regard to realized skewness, we present a plot of the latter versus the skewness estimates produced by the GARCH-2, QMIDAS, and adjusted-implied skewness models for S&P 500, STOXX 50, and HANGSENG in Figure 1.[20] GARCH-2 and QMIDAS are estimated using the full sample. The forecasting horizon is 30 days. We observe that the option-based skewness model better captures the dynamics of the realized skewness of S&P 500 and STOXX 50, which are associated with more developed option markets than HANGSENG. QMIDAS appears to perform better than the GARCH model for the S&P 500 index while all models appear to have limited information content for the HANGSENG index.

To identify which models produce the most informative forecasts of future realized skewness, we estimate Mincer–Zarnowitz regressions (Mincer & Zarnowitz, 1969), that is, we regress the T-day realized skewness on each model's corresponding skewness forecasts for each equity index using the available sample:

$$\mathrm{RS}_{t,t+T} = \alpha + \beta \widehat{F}^{m}_{t,t+T} + \beta^{\rho}\rho_t + e_{t,t+T}, \tag{19}$$

where $\widehat{F}^{m}_{t,t+T}$ is the skewness forecast at day t for a forecasting horizon of T calendar days as produced by model m. In our regressions, the two GARCH variations along with the QMIDAS model are estimated using the whole sample before extracting the corresponding skewness predictors. We also control for the correlation ρ_t between index returns and the corresponding variance risk premium over the prior 12 months to account for potential bias from violation of the martingale assumption in the derivation of realized skewness.[21]

Tables 4–6 report the regression results for horizons of 30, 60, and 90 days, respectively. We present the β coefficient from (19) as well as the adjusted R^2's. The significance of the coefficients was assessed using the heteroscedasticity and autocorrelation consistent (HAC) standard errors of Newey and West (1987) with T lags. Finally, we present the average values of all the coefficients and adjusted R^2 for each model, across indices.

[19]We also consider alternative averaging periods of 18 and 24 months and we obtain similar results.
[20]We did not include GARCH-1 and ImpliedSkew in these plots, as they can be considered special cases of GARCH-2 and AdjustedImpliedSkew, respectively, and including them would make the plots less interpretable.
[21]This adjustment was proposed in an earlier draft of Aretz and Arisoy (2022), motivated by the observation that realized skewness is associated with positive (negative) bias when the asset price and the variance risk premium are positively (negatively) correlated. We note that this control is not included in our out-of-sample tests and in the portfolio performance analysis. Results from an in-sample analysis without this control lead to similar conclusions about the comparative in-sample ranking of the models and are available from the authors.

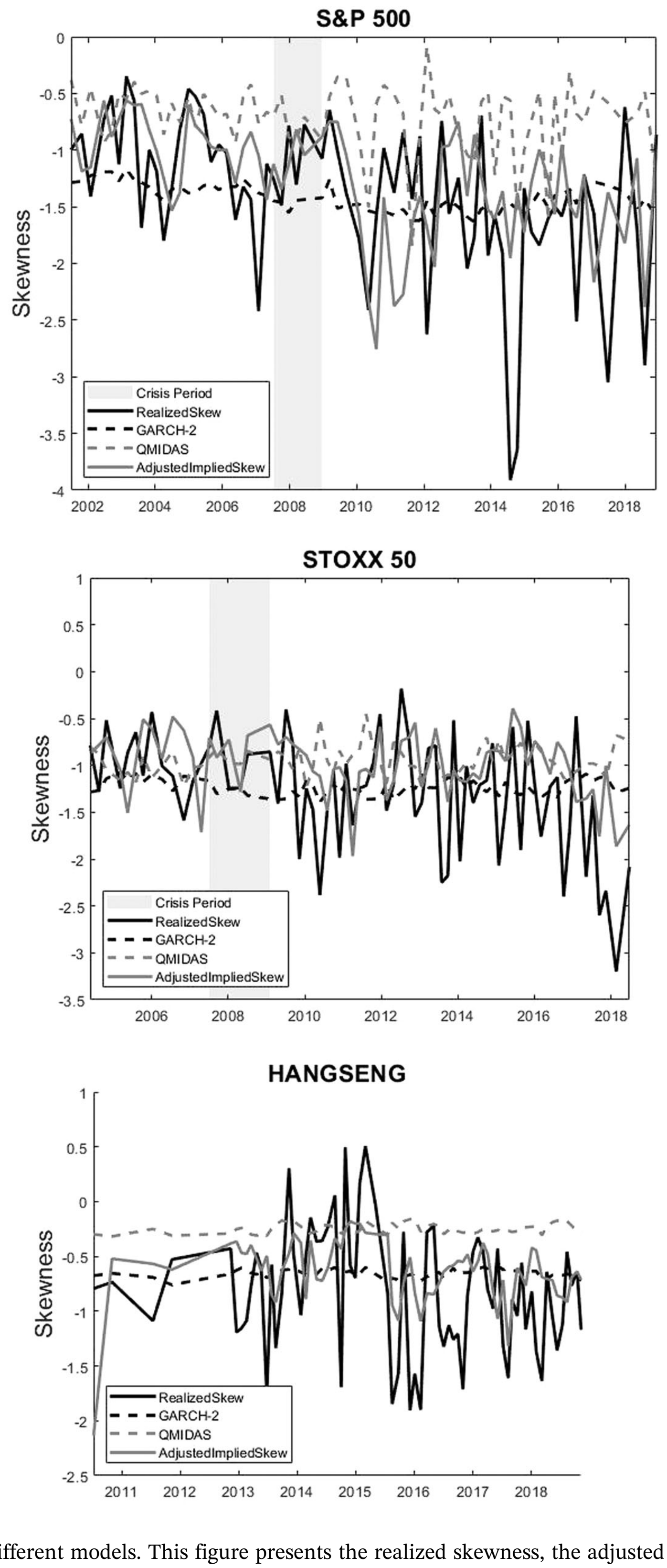

FIGURE 1 Skewness from different models. This figure presents the realized skewness, the adjusted implied skewness, the GARCH-2 skewness, and the QMIDAS-based skewness for three indices (S&P 500, STOXX 50, and HANGSENG). The horizon is 30 days. The gray area corresponds to the period from August 1, 2007 to December 31, 2008.

TABLE 4 Information content of skewness models (30 days).

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
AEX						
β	**0.207**	−0.497	−0.705	−0.169	**0.294**	**0.258**
$\bar{R}^2$ (%)	12.61	10.16	10.44	9.98	**13.19**	12.48
DAX						
β	**0.253**	−0.571	−0.910	0.483	**0.238**	**0.304**
$\bar{R}^2$ (%)	11.55	7.69	7.94	8.12	11.49	**11.57**
DJIA						
β	**0.364**	0.400	**1.948**	0.572	**0.429**	**0.443**
$\bar{R}^2$ (%)	8.55	4.87	13.33	5.07	10.59	**15.63**
STOXX 50						
β	**0.341**	−1.164	−1.451	0.005	**0.424**	**0.547**
$\bar{R}^2$ (%)	10.62	1.68	1.97	0.50	10.29	**11.96**
FTSE 100						
β	**0.114**	0.492	0.532	0.443	**0.127**	**0.282**
$\bar{R}^2$ (%)	8.02	7.21	7.18	7.97	8.63	**9.16**
HANGSENG						
β	**0.250**	**7.031**	**6.583**	−1.993	**0.220**	0.258
$\bar{R}^2$ (%)	10.86	**14.80**	11.87	8.26	7.36	7.33
KOSPI						
β	**0.252**	1.835	1.191	−0.009	**0.355**	**0.233**
$\bar{R}^2$ (%)	**9.49**	5.06	4.16	3.82	8.54	7.45
NASDAQ 100						
β	**0.436**	**−2.573**	**−2.563**	**0.629**	**0.514**	**0.675**
$\bar{R}^2$ (%)	19.98	6.15	6.00	4.24	19.95	**20.96**
RUSSELL 2000						
β	**0.184**	−1.508	**−2.309**	0.068	**0.478**	**0.556**
$\bar{R}^2$ (%)	3.93	1.77	3.40	0.63	**10.71**	8.55
S&P 500						
β	**0.296**	−1.129	**1.167**	**4.028**	**0.514**	**0.790**
$\bar{R}^2$ (%)	15.53	8.65	9.83	12.41	**25.85**	25.52
Average Results						
Average α	−0.785	−1.263	−0.941	−0.741	−0.577	−0.668
Average β	0.270	0.232	0.348	0.406	0.359	0.435
Average β^{ρ}	−0.847	−0.993	−0.794	−0.940	−0.650	−0.471
Average $\bar{R}^2$ (%)	11.12	6.80	7.61	6.10	12.66	**13.06**

Note: This table reports the results from our Mincer–Zarnowitz regressions. We regress the realized skewness of each index in Table 1 on the forecasts generated from each skewness model. The forecasting horizon is 30 calendar days. The GARCH and QMIDAS models are estimated using the whole sample. In our regressions, we control for the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months to account for potential bias in the realized skewness estimates. α and β, respectively, denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, $\overline{R}^2$ is the adjusted R^2 coefficient. Significant coefficients at the 5% level and the maximum $\overline{R}^2$ across models are highlighted in bold. The bottom part of the table contains the average values of α, β, β^{ρ}, and $\overline{R}^2$ across indices.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

TABLE 5 Information content of skewness models (60 days).

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
AEX						
β	0.114	−0.260	−0.343	−0.256	**0.324**	**0.331**
$\bar{R}^2$ (%)	11.14	10.35	10.52	12.18	13.86	**15.91**
DAX						
β	**0.190**	−0.421	−0.592	0.336	**0.326**	**0.346**
$\bar{R}^2$ (%)	8.26	5.34	5.62	6.97	10.94	**12.95**
DJIA						
β	**0.342**	0.011	**2.193**	−0.025	**0.564**	**0.453**
$\bar{R}^2$ (%)	19.53	9.22	**20.51**	9.23	17.09	17.58
STOXX 50						
β	**0.348**	−0.451	−0.555	0.044	**0.455**	**0.528**
$\bar{R}^2$ (%)	13.56	1.92	2.13	1.38	12.05	**19.61**
FTSE 100						
β	−0.055	0.255	0.191	−0.139	0.095	**0.192**
$\bar{R}^2$ (%)	2.32	2.49	2.26	2.19	3.12	**4.65**
HANGSENG						
β	**0.211**	**4.848**	**4.436**	0.068	0.198	**0.407**
$\bar{R}^2$ (%)	4.37	**9.97**	8.80	−0.06	2.16	7.75
KOSPI						
β	0.000	1.795	1.962	0.160	0.187	0.134
$\bar{R}^2$ (%)	0.48	1.97	1.99	0.23	1.74	**2.06**
NASDAQ 100						
β	**0.503**	**−1.232**	**−1.534**	0.477	**0.613**	**0.679**
$\bar{R}^2$ (%)	25.18	3.78	4.69	3.87	28.03	**31.06**
RUSSELL 2000						
β	0.122	**−1.767**	**−1.908**	**−0.186**	**0.390**	**0.332**
$\bar{R}^2$ (%)	2.59	7.54	8.31	3.31	**8.51**	5.20
S&P 500						
β	**0.358**	−0.651	**1.307**	−0.044	**0.634**	**0.685**
$\bar{R}^2$ (%)	23.18	12.68	15.44	11.84	31.85	**32.05**
Average Results						
Average α	−0.959	−1.371	−0.844	−1.215	−0.701	−0.765
Average β	0.213	0.213	0.516	0.043	0.378	0.409
Average β^{ρ}	−0.820	−0.969	−0.794	−0.880	−0.593	−0.472
Average $\bar{R}^2$ (%)	11.06	6.53	8.03	5.11	12.94	**14.88**

Note: This table reports the results from our Mincer–Zarnowitz regressions. We regress the realized skewness of each index in Table 1 on the forecasts generated from each skewness model. The forecasting horizon is 60 calendar days. The GARCH and QMIDAS models are estimated using the whole sample. In our regressions, we control for the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months to account for potential bias in the realized skewness estimates. α and β, respectively, denote the intercept and the coefficient of the forecast in the regression. In addition, β^{ρ} is the coefficient of ρ_t, and $\overline{R}^2$ is the adjusted R^2 coefficient. Significant coefficients at the 5% level and the maximum $\overline{R}^2$ across models are highlighted in bold. The bottom part of the table contains the average values of α, β, β^{ρ}, and $\overline{R}^2$ across indices.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

TABLE 6 Information content of skewness models (90 days).

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
AEX						
β	**0.401**	−0.385	−0.467	−0.415	**0.465**	**0.459**
$\bar{R}^2$ (%)	17.41	4.69	5.30	7.39	11.79	**19.92**
DAX						
β	**0.332**	−0.253	−0.314	−0.273	**0.353**	**0.367**
$\bar{R}^2$ (%)	12.08	2.27	2.39	2.24	9.54	**15.37**
DJIA						
β	**0.332**	−0.362	**1.292**	**−0.567**	**0.478**	**0.520**
$\bar{R}^2$ (%)	15.40	6.78	13.61	9.51	12.12	**16.05**
STOXX 50						
β	**0.478**	−0.142	−0.177	−0.427	**0.419**	**0.551**
$\bar{R}^2$ (%)	22.69	1.12	1.18	3.49	10.52	**22.86**
FTSE 100						
β	0.041	0.186	0.187	−0.016	0.095	0.132
$\bar{R}^2$ (%)	1.12	1.67	1.57	0.97	2.04	**2.84**
HANGSENG						
β	0.139	**3.344**	**3.177**	**−1.472**	0.045	**0.273**
$\bar{R}^2$ (%)	1.86	13.44	**13.74**	10.42	0.13	11.81
KOSPI						
β	0.057	0.693	0.781	0.166	0.071	0.052
$\bar{R}^2$ (%)	1.24	1.54	**1.60**	1.20	1.15	1.22
NASDAQ 100						
β	**0.492**	−0.441	−0.600	0.601	**0.487**	**0.504**
$\bar{R}^2$ (%)	**27.40**	3.69	4.08	3.72	20.16	26.85
RUSSELL 2000						
β	**0.275**	**−1.243**	**−1.282**	−0.459	**0.439**	**0.356**
$\bar{R}^2$ (%)	7.62	9.45	9.83	1.73	**12.50**	10.14
S&P 500						
β	**0.381**	−0.370	**0.809**	0.178	**0.690**	**0.640**
$\bar{R}^2$ (%)	24.33	12.41	14.69	11.98	33.91	**34.53**
Average Results						
Average α	−0.969	−1.605	−1.106	−1.568	−0.849	−0.870
Average β	0.293	0.103	0.341	−0.269	0.354	0.385
Average β^ρ	−0.497	−0.683	−0.651	−0.663	−0.518	−0.307
Average $\bar{R}^2$ (%)	13.11	5.70	6.80	5.26	11.39	**16.16**

Note: This table reports the results from our Mincer–Zarnowitz regressions. We regress the realized skewness of each index in Table 1 on the forecasts generated from each skewness model. The forecasting horizon is 90 calendar days. The GARCH and QMIDAS models are estimated using the whole sample. In our regressions, we control for the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months to account for potential bias in the realized skewness estimates. α and β, respectively, denote the intercept and the coefficient of the forecast in the regression. In addition, β^ρ is the coefficient of ρ_t, and $\overline{R}^2$ is the adjusted R^2 coefficient. Significant coefficients at the 5% level and the maximum $\overline{R}^2$ across models are highlighted in bold. The bottom part of the table contains the average values of α, β, β^ρ, and $\overline{R}^2$ across indices.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

We find that the skewness models can predict up to 35% of the variation in the future realized skewness. The most informative forecasts are produced by models that use forward-looking information from option markets, namely, the lagged realized and the two option-implied estimators of skewness. These models lead to the highest average R^2 across indices for all three time horizons. Indicatively for the S&P 500, at the quarterly horizon, the LaggedRealizedSkew, ImpliedSkew, and AdjustedImpliedSkew models yield an adjusted R^2 of 24.33%, 33.91%, and 34.53%, respectively. In the same setting, GARCH-1 and QMIDAS-based forecasts do not bear any significant information about the realized skewness while GARCH-2 leads to an adjusted R^2 of 14.69%. The latter model leads to the highest average R^2's among the models that do not use option data. The superior predictive ability of LaggedRealizedSkew, ImpliedSkew, and AdjustedImpliedSkew is consistent with similar findings from the volatility forecasting literature, where realized or option-implied volatilities are known to better predict future realized volatility compared with GARCH-based estimators (e.g., see Kourtis et al., 2016).

The best performance comes from the new implied skewness estimator that corrects for the skewness risk premium (AdjustedImpliedSkew). This model yields the highest average adjusted R^2 across all horizons. We find that the correction of implied skewness improves the information content of the estimator for 23 out of 30 index/horizon pairs. The improvement is greater for the quarterly horizon. For example, accounting for the skewness risk premium in the estimation of implied skewness increases the adjusted R^2 from 10.52% to 22.86% in the case of the STOXX 50 index. The original option-implied skewness estimator (ImpliedSkew) performs similarly to the lagged realized skewness model (LaggedRealizedSkew) with one model being slightly superior to the other for some indices/horizons/metrics and vice versa.

An interesting finding is that GARCH and QMIDAS forecasts fail to have any explanatory power on future realized skewness in the majority of horizon-index pairs in our analysis. In addition, in several cases, their coefficients are either negative or much higher than one. This finding is consistent with the graphical evidence in Figure 1. It appears that when QMIDAS and GARCH models are estimated using the full sample, they are not able to capture the time-varying nature of the leverage effect (Bandi & Reno, 2012; Kalnina & Xiu, 2017), which is the main driver of the dynamics in the realized skewness, as indicated by Neuberger (2012). However, for KOSPI and HANGSENG, GARCH models beat the implied skewness (adjusted implied skewness) in 5 (4) out of 6 cases. We attribute this result to the limited availability of reliable option data in these two markets. In Section 4, we alternatively estimate these models using a rolling window approach and investigate whether their comparative performance improves in an out-of-sample setting.

We further investigate whether one or more forecasting models subsume the information contained in the rest of the models using encompassing regressions. To avoid potential multicollinearity issues, we exclude GARCH-1 and ImpliedSkew from this test as GARCH-2 and AdjustedImpliedSkew are richer specifications, respectively. The resulting encompassing model is as follows:

$$\mathrm{RS}_{t,t+T} = \alpha + \beta_{\mathrm{LaggedRealizedSkew}} \hat{F}_{t,t+T}^{\mathrm{LaggedRealizedSkew}} + \beta_{\mathrm{GARCH\text{-}2}} \hat{F}_{t,t+T}^{\mathrm{GARCH\text{-}2}} \tag{20}$$

$$+\beta_{\mathrm{QMIDAS}} \hat{F}_{t,t+T}^{\mathrm{QMIDAS}} + \beta_{\mathrm{AdjustedImpliedSkew}} \hat{F}_{t,t+T}^{\mathrm{AdjustedImpliedSkew}} + \beta^{\rho} \rho_t + e_{t,t+T}. \tag{21}$$

Table 7 presents the regression coefficients and the adjusted R^2's from our encompassing regressions. We find that AdjustedImpliedSkew appears as a significant variable in the regressions in more cases than the rest of the models. The AdjustedImpliedSkew, GARCH, and QMIDAS models tend to capture different information about the future realized skewness, as the corresponding forecasts that are significant in our previous Mincer–Zarnowitz tests are in most cases significant in the multivariate setting too.[22] As a result, adjusted R^2's for the encompassing regression can be considerably larger to their analogues in the single-forecast regressions. Indicatively, in the case of the STOXX 50 index, the highest adjusted R^2 that a single model can yield on its own at the quarterly horizon is 22.86% while the encompassing regression results in an adjusted R^2 of 29.42%. The lagged realized skewness forecasts remain informative at the monthly horizon. However, they do not enter the regression at the bimonthly and quarterly horizons in most cases. This implies that AdjustedImpliedSkew captures most of the material information in LaggedRealizedSkew for longer horizons.

[22]In some cases, GARCH or QMIDAS models are associated with higher coefficients than AdjustedImpliedSkew. We note however that we cannot draw comparisons about the information content of forecasts that are significant by comparing their coefficients as these are not indicative of their contribution to the adjusted R^2.

TABLE 7 Encompassing regressions.

	AEX	DAX	DJIA	STOXX 50	FTSE 100	HANGSENG	KOSPI	NASDAQ 100	RUSSELL 2000	S&P 500
Panel A: 30 days										
α	−1.498	0.689	0.968	**−3.002**	−0.480	1.644	−0.343	**−2.498**	**−2.787**	**2.530**
β^{ρ}	**−1.373**	**−1.216**	0.019	0.123	**−1.332**	**−1.160**	1.013	0.527	0.371	0.702
$\beta_{\text{LaggedRealizedSkew}}$	**0.175**	**0.207**	**0.261**	**0.233**	0.093	**0.167**	**0.207**	**0.260**	0.089	**0.107**
$\beta_{\text{GARCH-2}}$	−0.881	1.057	0.916	−1.559	−0.071	**4.218**	0.345	**−2.159**	**−2.816**	0.586
β_{QMIDAS}	0.421	0.353	0.701	**−1.411**	**0.709**	−0.928	−0.054	−0.248	**0.424**	**3.476**
$\beta_{\text{AdjustedImpliedSkew}}$	**0.203**	**0.213**	**0.203**	**0.387**	**0.329**	0.113	0.165	**0.457**	**0.468**	**0.603**
$\bar{R}^2$ (%)	15.33	15.12	24.04	16.99	12.41	15.76	10.91	28.40	12.68	29.50
Panel B: 60 days										
α	−0.316	1.775	**1.544**	**−1.844**	−0.590	1.714	**3.845**	**−1.183**	**−3.590**	−0.147
β^{ρ}	−0.289	−0.570	−0.200	0.005	−0.502	−0.345	−0.961	**0.991**	0.382	−0.751
$\beta_{\text{LaggedRealizedSkew}}$	0.008	0.068	**0.206**	**0.155**	−0.097	0.086	−0.112	**0.218**	−0.028	0.116
$\beta_{\text{GARCH-2}}$	0.600	**1.557**	**1.907**	−0.588	0.542	**3.677**	**5.112**	−0.680	**−2.502**	0.306
β_{QMIDAS}	−0.368	**0.549**	−0.324	**−0.530**	−0.296	−0.538	**−0.771**	0.296	0.177	−0.013
$\beta_{\text{AdjustedImpliedSkew}}$	**0.317**	**0.302**	0.110	**0.416**	**0.268**	0.262	**0.174**	**0.460**	0.229	**0.563**
$\bar{R}^2$ (%)	17.80	14.54	27.18	21.62	6.98	13.82	6.47	35.97	10.80	32.90
Panel C: 90 days										
α	−0.627	−0.545	1.041	0.117	−0.747	2.227	1.072	0.114	**−2.321**	−0.561
β^{ρ}	0.297	−0.288	0.738	−0.048	−0.382	−0.522	−1.397	**1.251**	0.601	−0.809
$\beta_{\text{LaggedRealizedSkew}}$	0.193	0.183	**0.151**	**0.300**	0.036	−0.009	0.055	**0.306**	0.061	0.111
$\beta_{\text{GARCH-2}}$	0.255	0.015	**1.565**	**0.574**	0.326	**3.188**	1.725	0.258	−0.915	0.012
β_{QMIDAS}	−0.356	0.209	**−0.861**	−0.357	−0.120	0.415	−0.388	0.532	0.035	0.040
$\beta_{\text{AdjustedImpliedSkew}}$	**0.329**	**0.286**	**0.163**	**0.342**	0.139	**0.227**	0.050	**0.322**	**0.240**	**0.550**
$\bar{R}^2$ (%)	24.58	17.55	30.86	29.42	4.01	20.33	2.07	34.23	15.34	35.13

Note: This table reports the results from regressing the realized skewness on forecasts generated from the LaggedRealizedSkew, GARCH-2, QMIDAS, and AdjustedImpliedSkew models, within the same regression, for each index in Table 1. The GARCH-2 and QMIDAS models are estimated using the whole sample. In our regressions, we control for the empirical correlation (ρ_t) between daily index returns and the index variance risk premium over the prior 12 months to account for potential bias in the realized skewness estimates. α and β_m, respectively, denote the intercept and the coefficient of the forecast of model m in the regression. In addition, β^{ρ} is the coefficient of ρ_t and $\overline{R}^2$ is the adjusted R^2 coefficient. Significant coefficients at the 5% level are highlighted in bold. Panels A, B, and C, respectively, present results for a forecasting horizon of 30, 60, and 90 calendar days.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

4 | OUT-OF-SAMPLE ANALYSIS

To better assess the value of each model for decision-making purposes, we carry out an out-of-sample comparative analysis of its predictive ability. To this end, we compute the root mean squared error (RMSE) for each model/index/horizon triplet to measure forecasting losses:

$$\text{RMSE} = \sqrt{\frac{1}{K}\sum_{t=1}^{K}(\text{RS}(r_{t,t+T}) - \hat{F}_{t,t+T})^2}, \tag{22}$$

where $\widehat{F}_{t,t+T}$ is the skewness forecast and K is the total number of out-of-sample skewness forecasts.[23] To compute the forecasts from the GARCH and QMIDAS models, we use a rolling window of 1250 daily observations.[24]

We present the average losses produced by each model and for each index in Table 8 for the monthly, bimonthly, and quarterly horizon. The model with the lowest forecast errors is highlighted in bold. We also run Diebold–Mariano predictive accuracy tests (Diebold & Mariano, 1995) using HAC standard errors (Newey & West, 1987) to test whether the difference of the loss function produced by a model is significantly larger than that of the best model. Models that produce significantly less accurate forecasts than the best model at the 5% (10%) significance level are marked with two (one) asterisks.

In line with the in-sample results, the adjusted implied skewness estimator offers superior predictive ability to the rest of the models. AdjustedImpliedSkew leads to the lowest average loss in 7 out of 10 indices at the monthly and bimonthly horizons, with the differences in the loss function between AdjustedImpliedSkew and each other model being statistically significant in most cases. At the quarterly horizon, the unadjusted implied skewness produces the smallest losses in most scenarios, however, the losses produced by AdjustedImpliedSkew are statistically similar to the best model in all cases, apart from the KOSPI index. In contrast to our in-sample analysis, the GARCH models outperform the vanilla implied skewness and the lagged realized skewness models at the monthly horizon. They still underperform the adjusted implied skewness model for almost all indices. This result further highlights the importance of accounting for the skewness risk premium when modeling skewness.

How do the option-based models perform in settings with less developed option markets? To answer this question, we focus our analysis on HANGSENG and KOSPI which are associated with less liquid option markets. For HANGSENG, at the monthly and bimonthly horizon, GARCH models are only outperformed by AdjustedImpliedSkew and they lead to lower losses than both ImpliedSkew and LaggedRealizedSkew. At the quarterly horizon, GARCH-1, -2, and QMIDAS lead to the best performance likely because options are less liquid for longer maturities. For KOSPI, GARCH models offer the best performance for all three horizons while QMIDAS has similar performance with LaggedRealizedSKew and AdjustedImpliedSkew. These results indicate that GARCH and QMIDAS models can outperform option-based models when option liquidity is relatively low.

We conclude our out-of-sample analysis by using the nonparametric approach of P. R. Hansen et al. (2011), known as model confidence set (MCS), to identify a collection of models that outperform the rest of the models under a given loss function and a specific level of confidence. In our implementation of the MCS test, we use the range statistic from P. R. Hansen et al. (2011) to test the null hypothesis that two models lead to the same loss at a specific time. To compute the MCS, we use a block-bootstrap process with a block of two observations and 10,000 replications.[25]

We present the results from our MCS tests in Table 9 assuming a significance level of 5%. We find that the adjusted implied skewness (AdjustedImpliedSkew) enters the MCS in almost all considered cases, ranked generally first at the monthly and bimonthly horizons. Notably, it is the only model in the MCS for some indices (e.g., the S&P 500 for $T = 30$). Out of the two GARCH models, the simpler and more parsimonious specification is ranked higher than the more dynamic model in most cases. This result is likely due to the higher sensitivity of the second model to estimation errors that comes from the larger number of free parameters. GARCH models again perform better (worse) to LaggedRealizedSkew and ImpliedSkew in this test at the monthly (quarterly) horizon. We finally observe that the QMIDAS model never enters the MCS.

5 | PORTFOLIO PERFORMANCE

Several recent studies incorporate skewness measures in portfolio choice models and provide evidence that it improves outcomes for investors.[26] We contribute to this literature by identifying skewness models that lead to good portfolio performance. Our portfolio exercise aims to answer two questions. First, which of the competing skewness forecasting

[23]We have also assessed out-of-sample performance using the mean absolute error (MAE) loss function given by $\mathrm{MAE} = \frac{1}{K}\sum_{t=1}^{K}|\mathrm{RS}(r_{t,t+T}) - \hat{F}_{t,t+T}|$. We present the results from these tests in Section A.4 of the appendix. In a nutshell, the results are qualitatively similar to using RMSE-based losses.

[24]To better understand the effect of a different sample size in forecasting skewness, we also consider windows of 1000 and 1500 observations. We find that these alternatives do not alter the conclusions we draw in the main part of the paper about the comparative performance of the models. These results are available from the authors.

[25]We have considered alternative block lengths with similar results.

[26]For example, see Patton (2004), Jondeau and Rockinger (2006), Guidolin and Timmermann (2008), Harvey et al. (2010), DeMiguel et al. (2013b), and Ghysels et al. (2016)

TABLE 8 Out-of-sample forecasting performance.

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
Panel A: 30-day horizon						
AEX	0.627**	**0.543**	0.564	0.775**	0.552	0.561
DAX	0.633*	0.583	0.618	0.840**	0.707**	**0.581**
DJIA	0.634	0.807**	0.701**	0.858**	**0.588**	0.603
STOXX 50	0.724**	0.668	0.759**	1.107**	0.842**	**0.654**
FTSE 100	0.820**	0.734	0.707	1.011**	1.267**	**0.671**
HANGSENG	0.679*	0.650*	0.635	0.899**	0.734**	**0.608**
KOSPI	1.053**	**0.887**	0.890	1.088**	0.921	1.001
NASDAQ 100	0.638**	0.677**	0.647**	0.822**	0.707**	**0.581**
RUSSELL 2000	0.632**	0.522	0.551**	0.770**	0.601**	**0.508**
S&P 500	0.948**	0.887**	0.965**	1.244**	0.934**	**0.745**
Panel B: 60-day horizon						
AEX	0.569**	0.525	0.549**	1.071**	**0.476**	0.497
DAX	0.605**	0.594	0.629**	1.081**	0.544	**0.539**
DJIA	0.707**	0.761**	0.754**	1.415**	**0.630**	0.666
STOXX 50	0.635**	0.642	0.752**	1.132**	0.629**	**0.551**
FTSE 100	0.677**	0.601	0.573	1.002**	0.830**	**0.561**
HANGSENG	0.711**	0.655**	0.626*	0.848**	0.694*	**0.602**
KOSPI	1.490**	1.106	**1.101**	1.406**	1.237**	1.460**
NASDAQ 100	0.557**	0.620**	0.692**	0.923**	0.543*	**0.500**
RUSSELL 2000	0.569**	0.533*	0.578**	0.746**	0.509*	**0.460**
S&P 500	0.875**	0.880**	1.041**	1.462**	0.715	**0.702**
Panel C: 90-day horizon						
AEX	0.522	0.733**	0.751**	1.201**	**0.499**	0.509
DAX	0.523	0.810**	0.711**	1.205**	**0.508**	0.525
DJIA	0.654	0.716	0.650	1.430**	**0.604**	0.617
STOXX 50	0.511	0.815**	0.883**	1.221**	0.597**	**0.497**
FTSE 100	0.609	0.673*	**0.561**	1.061**	0.659*	0.580
HANGSENG	0.700**	0.572**	**0.541**	0.573*	0.746**	0.708
KOSPI	1.779**	**1.497**	1.502	1.838**	1.566*	1.780**
NASDAQ 100	**0.530**	0.728**	0.750**	1.158**	0.531	0.536
RUSSELL 2000	0.504*	0.561**	0.579*	0.836**	**0.440**	0.465
S&P 500	0.837**	0.853**	1.041**	1.611**	**0.668**	0.684

Note: For each index in Table 1, this table reports out-of-sample forecasting losses for each skewness model we consider. The forecasting horizon is 30, 60, or 90 calendar days. The GARCH and QMIDAS models are estimated using a rolling window of 1250 observations. We report root mean squared errors (RMSE) using the realized skewness of Neuberger (2012) as a proxy for the true skewness. The model with the lowest forecasting loss is highlighted in bold. One (two) asterisk(s) shows that the corresponding model is inferior to the best model at the 10% (5%) significance level, in the context of a Diebold–Mariano test.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

TABLE 9 Model confidence set.

	AEX		DAX		DJIA		STOXX 50		FTSE 100		HANGSENG		KOSPI		NASDAQ 100		RUSSELL 2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LaggedRealizedSkew	5*	0.064	4*	0.081	3*	0.445	3	0.01	4	0.01	4	0.00	5*	0.104	2	0.01	5	0.00	4	0.00
GARCH-1	1*	1.000	2*	0.919	5	0.00	2*	0.502	3*	0.188	3	0.00	1*	1.000	4	0.00	2*	0.282	2	0.00
GARCH-2	4*	0.191	3*	0.081	4	0.00	4	0.00	2*	0.317	2*	0.129	2*	0.725	3	0.00	3	0.00	5	0.00
QMIDAS	6	0.00	6	0.00	6	0.00	6	0.00	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00
ImpliedSkew	2*	0.765	5	0.00	1*	1.000	5	0.00	6	0.00	5	0.00	3*	0.725	5	0.00	4	0.00	3	0.00
AdjustedImpliedSkew	3*	0.765	1*	1.000	2*	0.445	1*	1.000	1*	1.000	1*	1.000	4*	0.104	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LaggedRealizedSkew	5	0.01	4	0.00	3	0.05	3	0.00	4	0.00	5	0.00	6	0.00	3	0.01	4	0.00	3	0.00
GARCH-1	3*	0.155	3	0.03	5	0.00	4	0.00	3*	0.399	3	0.00	2*	0.307	4	0.00	3	0.00	4	0.00
GARCH-2	4	0.01	5	0.00	4	0.00	5	0.00	2*	0.667	2*	0.504	1*	1.000	5	0.00	5	0.00	5	0.00
QMIDAS	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	4	0.00	6	0.00	6	0.00	6	0.00
ImpliedSkew	1*	1.000	2*	0.730	1*	1.000	2	0.00	5	0.00	4	0.00	3	0.03	2	0.01	2	0.03	2*	0.610
AdjustedImpliedSkew	2*	0.155	1*	1.000	2*	0.099	1*	1.000	1*	1.000	1*	1.000	5	0.00	1*	1.000	1*	1.000	1*	1.000
Panel C: 90-day horizon																				
LaggedRealizedSkew	3*	0.702	2*	0.496	4*	0.306	2*	0.514	3*	0.366	4	0.00	4	0.03	1*	1.000	3	0.00	3	0.00
GARCH-1	4	0.00	5	0.00	5	0.04	4	0.00	5	0.00	2*	0.085	1*	1.000	4	0.00	4	0.00	4	0.00
GARCH-2	5	0.00	4	0.00	3*	0.306	5	0.00	1*	1.000	1*	1.000	2*	0.611	5	0.00	5	0.00	5	0.00
QMIDAS	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3	0.00	6	0.00	6	0.00	6	0.00	6	0.00
ImpliedSkew	1*	1.000	1*	1.000	1*	1.000	3	0.00	4	0.00	6	0.00	3*	0.095	2*	0.937	1*	1.000	1*	1.000
AdjustedImpliedSkew	2*	0.702	3*	0.296	2*	0.366	1*	1.000	2*	0.598	5	0.00	5	0.01	3*	0.863	2*	0.06	2*	0.246

Note: This table reports our results for the model confidence set (MCS) test using the root mean squared error (RMSE) as a loss function and a 5% significance level. Panels A, B, and C report results for the 30-, 60-, and 90-day forecasting horizons, respectively. The columns labeled "*Rank*" present the ranking of a model in the MCS while the "*pval*" columns show the p values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

methods is more beneficial for an international investor? Second, can any of the skewness models lead to portfolios that produce higher risk-adjusted returns than naïve diversification?[27] To this end, we construct parametric portfolios that are based on each skewness model using the approach of Brandt et al. (2009). This framework allows us to directly assess the value of a skewness model for portfolio choice without having to rely on any specific distributional assumptions for the asset returns. This approach has also been adopted by DeMiguel et al. (2013b), who show that a portfolio that exploits option-implied skewness (ImpliedSkew) leads to a higher Sharpe ratio compared with $1/N$ in a sample of US stocks.

We consider an investor that at time t uses a skewness forecast to select a portfolio of N indices. The investor's portfolio weight on the j index is a linear function of the skewness forecast:

$$w_{j,t}^{m} = w_{j,t}^{1/N} + \theta_t^m \frac{1}{N}\hat{f}_{j,t}^{m}. \tag{23}$$

In the above, $w_{j,t}^{1/N} = 1/N$ is the weight of the $1/N$ portfolio which stands for the benchmark in the parametric portfolio framework as in DeMiguel et al. (2013b).[28] $\hat{f}_{j,t}^{m}$ is the 30-day ahead forecast of the skewness of the index j, generated by the model m and standardized so that the cross-sectional mean and variance at time t are 0 and 1 respectively.[29] The parameter θ_t^m is the loading on each standardized forecast and is the same across indices.

Each of the six forecasting models augments the benchmark portfolio $w_t^{1/N}$ with a zero-cost portfolio that is determined by the skewness forecasts generated by the model. As such, each model yields a unique portfolio strategy defined by the weights w_t^m. By studying the out-of-sample performance of each strategy, we can then assess the value of the corresponding skewness model for portfolio decisions. We assess the performance of the six strategies as follows. At each day t and for each model m, we compute the parameter θ_t^m which leads to the minimum variance of the daily portfolio returns over the previous 252 days, under the constraint of positive portfolio weights.[30]

We input this value in (23) to derive the portfolio weights at day t for model m. We then compute the corresponding portfolio return $r_{t,t+\tau}^m$ under three rebalancing frequencies, that is, daily ($\tau = 1$), weekly ($\tau = 5$), and biweekly ($\tau = 10$), as in DeMiguel et al. (2013b). We repeat this process to yield a series of $M - \tau$ portfolio returns/weight vectors for each skewness forecasting model and for the $1/N$ portfolio. Then, for each portfolio model m, including the $1/N$ portfolio, and rebalancing frequency τ, we compute the out-of-sample portfolio mean return, volatility, Sharpe ratio, and skewness as[31]

$$\hat{\mu}_\tau^m = \frac{1}{M-\tau}\sum_{t=1}^{M-\tau} r_{t,t+\tau}^m, \tag{24}$$

$$\left(\hat{\sigma}_\tau^m\right)^2 = \frac{1}{M-\tau-1}\sum_{t=1}^{M-\tau}\left(r_{t,t+\tau}^m - \hat{\mu}_\tau^m\right)^2, \tag{25}$$

$$\widehat{\mathrm{SR}}_\tau^m = \frac{\hat{\mu}_\tau^m}{\hat{\sigma}_\tau^m}, \tag{26}$$

$$\widehat{S}_\tau^m = \frac{Q_3(r_{t,t+\tau}) + Q_1(r_{t,t+\tau}) - 2Q_2(r_{t,t+\tau})}{Q_3(r_{t,t+\tau}) - Q_1(r_{t,t+\tau})}, \tag{27}$$

[27]DeMiguel et al. (2009) show that naïve diversification, also known as $1/N$, performs consistently better than many popular theory-based portfolio choice methods as the latter are subject to sampling errors that deteriorate portfolio performance significantly.
[28]We consider an alternative benchmark portfolio in our robustness tests and end up with similar results, as we discuss in Section 6.
[29]In our portfolio choice tests, we only employ monthly skewness forecasts as all the portfolio rebalancing intervals we account for are less than 30 calendar days.
[30]In our main analysis, we choose this calibration criterion for θ_t^m based on empirical evidence provided by DeMiguel et al. (2013a). They find that optimizing linear combinations of portfolio strategies under minimum variance leads to higher risk-adjusted returns in most of their tests than utility- or Sharpe-ratio-based optimization. Nevertheless, in Section 6, we also present results for skewness-based portfolio calibration.
[31]We use the robust skewness estimator from Bowley (1920) as a performance measure as it is less sensitive to outliers compared with the third-moment-based skewness (e.g., see Kim & White, 2004).

where $Q_i(r_{t,t+\tau})$ stands for the ith quartile of $r_{t,t+\tau}$.

We further test for the following null hypotheses:

$$H^*_{0,v} : \left(\hat{\sigma}^m_\tau\right)^2 - \left(\hat{\sigma}^*_\tau\right)^2 = 0, \tag{28}$$

$$H^*_{0,SR} : \left(\hat{\mathrm{SR}}^m_\tau\right)^2 - \left(\hat{\mathrm{SR}}^*_\tau\right)^2 = 0, \tag{29}$$

$$H^{1/N}_{0,v} : \left(\hat{\sigma}^m_\tau\right)^2 - \left(\hat{\sigma}^{1/N}_\tau\right)^2 = 0, \tag{30}$$

$$H^{1/N}_{0,SR} : \left(\hat{\mathrm{SR}}^m_\tau\right)^2 - \left(\hat{\mathrm{SR}}^{1/N}_\tau\right)^2 = 0, \tag{31}$$

where $\left(\hat{\sigma}^*_\tau\right)^2$ and $\left(\hat{\mathrm{SR}}^*_\tau\right)$ are the out-of-sample variance and Sharpe ratio of the skewness-based portfolio that leads to the lowest risk and highest Sharpe ratio, respectively. The first two tests help us to identify if one of the skewness models is superior to the rest of the models in terms of portfolio performance in a statistically significant manner. Testing for the third and fourth hypotheses can reveal if any of the skewness-based models can significantly outperform $1/N$ in terms of risk and risk-adjusted returns, respectively. To accommodate overlapping weekly and biweekly returns, we estimate p values for these tests using the nonparametric bootstrap framework of Ledoit and Wolf (2011), assuming an average block size of 5 and 5000 trials. We finally compute the average portfolio turnover for each portfolio strategy, which assesses the stability of the strategy over time as

$$\mathrm{TO}^m = \frac{1}{M-\tau-1}\frac{1}{\tau}\sum_{t=1}^{M-\tau-1}\left\| w^m_{t+\tau} - \widetilde{w}^m_{t+}\right\|_1, \tag{32}$$

where $\widetilde{w}^m_{t+}$ stands for the portfolio weights at the beginning of the period $\{t, t+\tau\}$, before rebalancing takes place, while $\|\cdot\|_1$ stands for the 1-norm.

In Table 10, we report the above metrics for portfolios of the indices in our data set, excluding Dow Jones Industrial Average (DJIA), National Association of Securities Dealers Automated Quotations (NASDAQ) 100, and RUSSELL 2000 to reduce the bias of the portfolio towards the US market.[32] In Panels A, B, and C, we report results for daily, weekly, and biweekly rebalancing, respectively.

We observe that the lowest variance and highest Sharpe ratio are always offered by the portfolio that is based on the adjusted implied skewness estimator (AdjustedImpliedSkew). This portfolio outperforms all other skewness-based portfolios and $1/N$ in terms of risk at the 5% significance level for every considered rebalancing interval. Under daily rebalancing, the AdjustedImpliedSkew-based portfolio gives a significantly larger Sharpe ratio at the 5% level than $1/N$ and all skewness-based strategies apart from the portfolio based on the simple implied skewness model. For example, the AdjustedImpliedSkew-based portfolio leads to an annualized variance of 0.013 and an out-of-sample Sharpe ratio of 0.466. In comparison, $1/N$ yields an average annualized variance of 0.015 and a Sharpe ratio of 0.287. Notably, the AdjustedImpliedSkew strategy is the only strategy that results in positive skewness under daily rebalancing. For the two other rebalancing frequencies, AdjustedImpliedSkew again offers a higher Sharpe ratio but the difference with other strategies is significant only for LaggedRealizedSkew and QMIDAS. It also leads to the third (second) highest skewness under weekly (biweekly) rebalancing among the skewness-based portfolios. We note that in almost all considered cases all strategies are associated with negative skewness.

The second best alternative out of the skewness-based portfolios is the one that relies on the vanilla option-implied estimator (ImpliedSkew). This portfolio produces higher average return and Sharpe ratio than the GARCH- and QMIDAS-based portfolios. It also generates the highest skewness under weekly rebalancing. The superior performance

[32]As a robustness check, we have also included the DJIA, NASDAQ 100, and RUSSELL 2000 in the portfolios and rerun our analysis. Our qualitative results remain the same.

TABLE 10 Portfolio performance.

	Mean	Var	p_v^{AIS}	$p_v^{1/N}$	SR	p_{SR}^{AIS}	$p_{SR}^{1/N}$	Skew	TO
Panel A: Daily rebalancing									
LaggedRealizedSkew	0.0236	0.0150	(0.00)	(0.51)	0.1926	(0.00)	(0.04)	−0.1070	0.0533
GARCH-1	0.0285	0.0150	(0.00)	(0.64)	0.2324	(0.03)	(0.30)	−0.0478	0.0142
GARCH-2	0.0290	0.0149	(0.00)	(0.35)	0.2374	(0.04)	(0.50)	−0.0578	0.0254
QMIDAS	0.0167	0.0145	(0.01)	(0.11)	0.1388	(0.00)	(0.03)	−0.0865	0.0450
ImpliedSkew	0.0417	0.0148	(0.00)	(0.29)	0.3422	(0.24)	(0.47)	−0.0435	0.0996
AdjustedImpliedSkew	**0.0532**	**0.0130**	(1.00)	(0.00)	**0.4658**	(1.00)	(0.05)	**0.0019**	0.1118
1/*N*	0.0353	0.0151	(0.00)	(1.00)	0.2869	(0.05)	(1.00)	−0.0994	**0.0045**
Panel B: Weekly rebalancing									
LaggedRealizedSkew	0.0237	0.0175	(0.00)	(0.46)	0.1794	(0.01)	(0.03)	−0.1218	0.0296
GARCH-1	0.0319	0.0176	(0.00)	(0.93)	0.2401	(0.22)	(0.34)	−0.0995	0.0071
GARCH-2	0.0303	0.0176	(0.00)	(0.72)	0.2289	(0.23)	(0.43)	−0.1761	0.0133
QMIDAS	0.0188	0.0171	(0.01)	(0.07)	0.1437	(0.04)	(0.03)	−0.1203	0.0243
ImpliedSkew	0.0378	0.0176	(0.00)	(0.96)	0.2849	(0.43)	(0.95)	**−0.0719**	0.0376
AdjustedImpliedSkew	**0.0448**	**0.0157**	(1.00)	(0.00)	**0.3574**	(1.00)	(0.37)	−0.1111	0.0421
1/*N*	0.0385	0.0176	(0.00)	(1.00)	0.2895	(0.37)	(1.00)	−0.0775	**0.0025**
Panel C: Biweekly rebalancing									
LaggedRealizedSkew	0.0240	0.0153	(0.01)	(0.29)	0.1937	(0.00)	(0.03)	**−0.0885**	0.0210
GARCH-1	0.0333	0.0158	(0.00)	(0.25)	0.2649	(0.13)	(0.30)	−0.1407	0.0051
GARCH-2	0.0323	0.0157	(0.01)	(0.57)	0.2578	(0.16)	(0.40)	−0.1490	0.0097
QMIDAS	0.0214	0.0153	(0.03)	(0.43)	0.1731	(0.03)	(0.06)	−0.1249	0.0177
ImpliedSkew	0.0409	0.0157	(0.01)	(0.50)	0.3261	(0.35)	(0.98)	−0.1288	0.0221
AdjustedImpliedSkew	**0.0499**	**0.0143**	(1.00)	(0.00)	**0.4173**	(1.00)	(0.21)	−0.1165	0.0259
1/*N*	0.0394	0.0155	(0.00)	(1.00)	0.3158	(0.21)	(1.00)	−0.1002	**0.0020**

Note: This table presents the out-of-sample performance of the equally-weighted portfolio ($1/N$) and of the parametric portfolios that use the skewness estimates from each model considered in the paper. The portfolios include as assets the following indices: AEX, DAX, STOXX 50, FTSE 100, HANGSENG, KOSPI, and S&P 500. The table reports the annualized out-of-sample average return (Mean), variance of returns (Var), and Sharpe ratio (SR) for each portfolio strategy as well as the skewness (Skew) and the average turnover (TO). It presents p values from testing the hypothesis that the variance (p_v^{AIS}) or the Sharpe ratio ($p_{\text{SR}}^{\text{AIS}}$) between a portfolio strategy and the portfolio based on the adjusted implied skewness (AdjustedImpliedSkew) are equal. It also reports p values from testing the hypothesis that the variance ($p_v^{1/N}$) or the Sharpe ratio ($p_{\text{SR}}^{1/N}$) between a portfolio strategy and $1/N$ are equal. The p values are computed using the block-bootstrap approach of Ledoit and Wolf (2011), assuming an average block size of 5 and 5000 replications. Panels A, B, and C, respectively, present results assuming daily, weekly, and biweekly rebalancing. The best performance for each metric is highlighted in bold.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

of both option-implied skewness-based portfolios is accompanied with a higher turnover compared with the rest of the portfolios. The portfolios that result from the GARCH-based skewness tend to yield the lowest levels of turnover while they outperform both the LaggedRealizedSkew- and QMIDAS-based portfolios in terms of out-of-sample Sharpe ratio. The QMIDAS-driven strategy offers lower risk than all strategies, except for AdjustedImpliedSkew. Overall, our results advocate the use of option-implied skewness in portfolio choice.

6 | ROBUSTNESS CHECKS

6.1 | Performance across different market conditions

Some of the time series in our data set are long enough to allow us to perform a subsample analysis of the forecasting performance of each skewness model. Apart from enhancing the robustness of our conclusions, such an exercise can provide evidence on whether some models perform better in different market conditions. In this context, we examine the predictive ability of the competing models in the crisis of 2007-08 and compare it to the rest of the available sample for DAX, DJIA, NASDAQ, and S&P 500.[33] We set the crisis period between 1 August 2007 and 31 December 2008. Table A3 in the appendix presents the results from our subsample analysis. We observe that the results in the subsample that excludes the crisis period are similar to our main results with the adjusted implied skewness offering the highest forecasting performance in most considered cases. Focusing on the crisis period, we observe that the adjusted implied skewness model is associated with better predictive ability in two out of four considered indices at the monthly horizon. For the longer forecasting windows, the vanilla implied skewness leads to the lowest losses, but in several cases these are statistically similar to the losses produced by AdjustedImpliedSkew.

6.2 | Alternative proxy for the true skewness

In this work, we rely on the realized skewness estimator of Neuberger (2012) as the proxy of true physical skewness in line with Kozhan et al. (2013). Similar to realized volatility which is established as a proxy of the true volatility, realized skewness can exploit the distribution of high-frequency returns to estimate skewness at any horizon. However, in contrast to realized volatility, the computation of realized skewness requires the use of option data for the calculation of the leverage effect. This creates two potential challenges in using realized skewness as a measure of the true (unobserved) skewness. First, option data may be limited in some settings, such as emerging markets. While we use interpolation to address this issue, the resulting skewness measure may still be subject to estimation errors. Second, in the literature there is evidence of mispricing between index options and the underlying index levels. For example, Constantinides et al. (2011) identify mispricing in the S&P 500 index option market. In the same fashion, realized skewness estimates may be subject to measurement errors. While potential measurement and estimation errors in realized skewness would not affect our portfolio performance results, there is a question on whether the superior forecasting performance of option-based estimators is an outcome of using an option-based skewness measure.

To address these issues, we repeat our forecasting analysis using an alternative proxy for realized skewness introduced by Neuberger and Payne (2021), which is solely based on return data. We present the results in Tables A4 and A5 in the appendix. We find that the adjusted-implied skewness estimator again leads to superior out-of-sample performance in most cases considered. It is only significantly outperformed in terms of the RMSE loss function for the KOSPI index in the monthly and bimonthly horizons. AdjustedImpliedSkew enters the MCS for 24 out of 30 index-horizon pairs. GARCH models tend to perform better than lagged realized skewness and implied skewness for the 30-day horizon, but their relative performance deteriorates as the horizon increases, similar to what we observe in our main analysis.

6.3 | Alternative specifications for the portfolio choice analysis

We consider three modifications of the portfolio exercise we carry out in Section 6.2. First, we consider an alternative portfolio as our benchmark in constructing skewness-based portfolios and assessing their performance. This is a volatility-timing (VT) strategy proposed by Kirby and Ostdiek (2012). The portfolio weights at time t are given by

$$w_{j,t}^{\mathrm{VT}} = \frac{1/\sigma_{j,t}^2}{\sum_{j=1}^{N}\left(1/\sigma_{j,t}^2\right)}, \tag{33}$$

[33]We cannot perform these tests for the rest of the indices in our sample as the corresponding data start at a later date.

where $\sigma_{j,t}^2$ is the variance of the daily returns on index j over the previous 252 days. VT shares some of the favorable properties of $1/N$, such as positive weights and low portfolio turnover. At the same time, it represents the minimum variance portfolio under zero correlations. As such, it is expected to lead to lower portfolio variance than $1/N$ and pose an additional challenge for the skewness-based portfolios. Using VT as our benchmark portfolio, we perform the same portfolio performance tests as in our main analysis and present our findings in Table A6 in the appendix for the two asset universes we consider. In general, the results are qualitatively similar to the results with $1/N$ as the benchmark. We again find that the AdjustedImpliedSkew-based strategy offers lower risk and higher risk-adjusted returns compared with VT and the rest of the skewness-based portfolios. It also yields higher skewness against most alternatives.

Second, the results from our encompassing regressions motivate us to consider if combining forecasts from two skewness models contributes to further improvements in portfolio performance. To this end, we examine the performance of parametric portfolios that exploit two pairs of forecasts. In this case, the parametric portfolio weight on the j index is given by

$$w_{j,t}^{m,l} = w_{j,t}^{1/N} + \theta_t^m \frac{1}{N}\hat{f}_{j,t}^m + \theta_t^l \frac{1}{N}\hat{f}_{j,t}^l. , \tag{34}$$

where $\hat{f}_{j,t}^m$ and $\hat{f}_{j,t}^l$ stand for the normalized forecasts from the skewness models m and l, respectively. Similar to our encompassing regressions, we consider pairs from four models, namely, LaggedRealizedSkew, GARCH-2, QMIDAS and AdjustedImpliedSkew. We calibrate the corresponding loadings θ_t^m and θ_t^l to minimize the portfolio variance over the previous year and compute the out-of-sample performance metrics as in Section 6.2. We report these metrics in Table A7 of the appendix. Looking first at portfolio risk, we find that portfolios that exploit AdjustedImpliedSkew in addition to another model lead to lower variance out-of-sample. The lowest risk is offered by the portfolio that combines forecasts from QMIDAS and AdjustedImpliedSkew. This also outperforms the portfolio based only on AdjustedImpliedSkew for the same metric. However, the AdjustedImpliedSkew-based portfolio we examine in Section 6.2 outperforms strategies based on forecast combinations in terms of mean return in all cases and Sharpe ratio in most cases. We conclude that, in our setting, exploiting combination of skewness models promotes the reduction of risk at the cost of lower risk-adjusted returns.

Finally, in our main results we observe that almost all portfolio strategies lead to negative skewness and may be less attractive to investors that seek positive portfolio skewness. This outcome could potentially be from calibrating the strategies to yield minimum risk over the previous year. To explore if we can generate higher levels of skewness for the considered strategies, we rerun our portfolio analysis using instead the value of θ_t^m that maximizes portfolio skewness over the previous year under the constraint that portfolio variance is less than the variance of $1/N$ which allows us to control portfolio risk. The corresponding results are included in Table A8 in the appendix. We again find that portfolio skewness is negative and of a similar magnitude compared with the portfolios calibrated to minimize variance. This finding is indicative of the lack of persistence in portfolio skewness. Among the competing skewness models, AdjustedImpliedSkew gives the highest skewness under daily and weekly rebalancing while LaggedRealizedSkew produces higher skewness under biweekly rebalancing. With regard to risk and risk-adjusted returns, the ranking of the models is largely consistent with our main results.

7 | CONCLUSION

We carry out a comprehensive comparison of the predictive ability and portfolio performance of several skewness models. We also propose an option-implied skewness estimator that takes into account the skewness risk premium. In our analysis we consider 10 international indices, three forecasting horizons, two ways to assess the information content of each model and two out-of-sample comparisons. We also compare the competing models in a portfolio choice framework to infer the skewness model that leads to the best out-of-sample portfolio performance under five measures. We support our results with a battery of robustness checks.

Our findings support the use of option-implied skewness for portfolio decisions. The adjusted option-based skewness model produces the most informative forecasts of future skewness while it leads to the lowest prediction errors in most of our out-of-sample tests. Portfolios based on this model outperform portfolios based on the rest of the skewness models as well as the $1/N$ portfolio in most of our tests.

In this work, we mostly focus on indices from developed international markets as we are constrained by the availability of option data. Potential extensions of our work would be to consider more indices from developing markets or to perform our tests at the stock level. As option markets are less liquid in developing markets or for individual stocks, the performance of option-based models could deteriorate. However, this could be, respectively, mitigated by using additional country-level economic variables (similar to Ghysels et al., 2016) or stock characteristics (e.g., as in Boyer et al., 2010) as additional predictors of future skewness. We leave such extensions of our work for future research.

DATA AVAILABILITY STATEMENT

The data that support the results of this study are obtained from Thomson-Reuters Datastream and IVOLATILITY. The data are not publicly available due to commercial restrictions.

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APPENDIX A

A.1 | Computation of the entropy variance

As we explain in Section 2.2, the realized skewness estimator of Neuberger (2012) is a function of the entropy variance. The latter is defined as the implied variance of a contract that pays $S_{t+T} \ln S_{t+T}$ at day $t + T$ given by

$$v^{E}_{i,t+T} = 2\mathbb{E}^{\mathbb{Q}}_{i,t+T}\left[\frac{S_{t+T}}{S_i}\ln\left(\frac{S_{t+T}}{S_i}\right) - \frac{S_{t+T}}{S_i} + 1\right], \tag{A1}$$

where the expectation is taken under the option-implied probability measure conditional on the information available on day i. The entropy variance can be calculated following Bakshi and Madan (2000). Their work shows that the payoff of an asset can be replicated using a portfolio of a risk-free zero-coupon bond and a continuum of OTM calls and puts written on the asset with varying strike prices. Let $B_{i,t+T} = e^{-r^f(t+T-i)}$ be the price of the bond, where r^f is the risk-free rate and $t + T - i$ is the time-to-maturity. Using the spanning rule of Bakshi and Madan (2000), we can compute the entropy variance by

$$v^{E}_{i,t+T} = \frac{2}{B_{i,t+T}}\left[\int_0^{S_i}\frac{P_{i,t+T}(K)}{KS_i}dK + \int_{S_i}^{\infty}\frac{C_{i,t+T}(K)}{KS_i}dK\right], \tag{A2}$$

where $C_{i,t+T}(K)$ and $P_{i,t+T}(K)$ are, respectively, the prices of an OTM call and put with strike price K and $t + T - i$ time-to-maturity. To compute the above integral, we use the approximation of Kozhan et al. (2013).[34] For a given day i, suppose that we have $N + 1$ available calls and puts with increasing strike prices $K_0, K_1, \ldots, K_N$ and maturity on day $t + T$. We define the strike price differences as

$$\Delta K_j = \begin{cases} K_1 - K_0, & j = 0, \\ \dfrac{K_{j+1} - K_{j-1}}{2}, & 0 < j < N, \\ K_N - K_{N-1}, & j = N. \end{cases} \tag{A3}$$

We use the available OTM calls and puts in our data set, $C_{i,t+T}(K_j)$ and $P_{i,t+T}(K_j)$, to approximate $v^{E}_{i,t+T}$ as

$$v^{E}_{i,t+T} \approx \frac{2}{B_{i,t+T}}\left[\sum_{K_j<S_i}\frac{P_{i,t+T}(K_j)}{S_iK_j}\Delta K_j + \sum_{S_i<K_j}\frac{C_{i,t+T}(K_j)}{S_iK_j}\Delta K_j\right]. \tag{A4}$$

We finally apply the above approximation to estimate $\Delta v^{E}_{i,t+T}$ for each trading day i in $M_{t,t+T}$ and, in turn, compute the realized third moment as in (1).

A.2 | SGE distribution

The probability density function of the SGE distribution is given by

$$f(y|\mu,\sigma,\lambda,\kappa) = \frac{C}{\sigma}\exp\left(-\frac{|y-\mu+\delta\sigma|^{\kappa}}{(1+\mathrm{sign}(y-\mu+\delta\sigma)\lambda)^{\kappa}\theta^{\kappa}\sigma^{\kappa}}\right) \tag{A5}$$

with

$$C = \frac{\kappa}{2\theta\Gamma(1/\kappa)}\delta = 2\lambda AS(\lambda)^{-1}, \quad \theta = \Gamma(1/\kappa)^{1/2}\Gamma(3/\kappa)^{-1/2}S(\lambda)^{-1}, \tag{A6}$$

[34]We would like to thank Kevin Aretz for providing us with the computer code to perform this estimation.

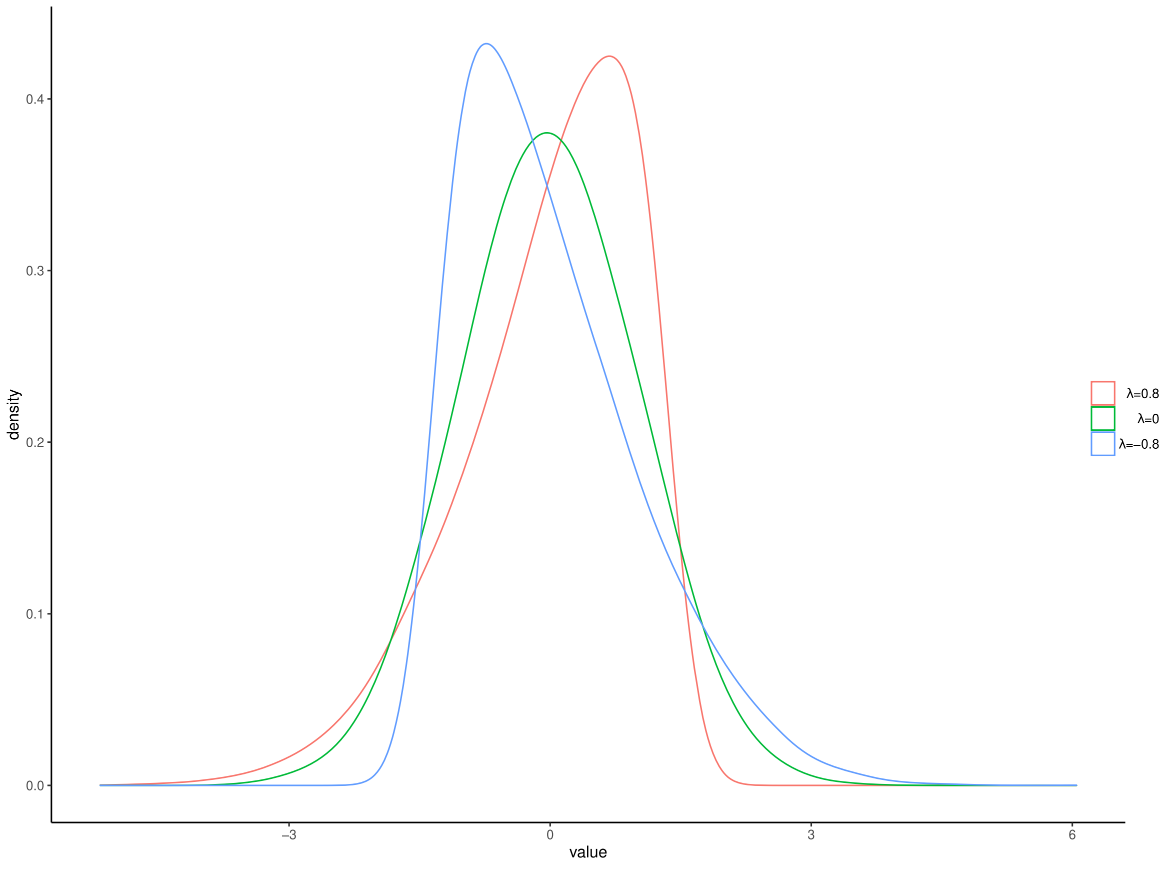

FIGURE A1 Skewed generalized error (SGE) distribution. This figure presents the probability distribution function of the SGE distribution as defined in Section A.2. Three values for the skew parameter λ are assumed (−0.8, 0, and 0.8). The shape parameter $\varkappa$ is equal to 2. [Color figure can be viewed at wileyonlinelibrary.com]

$$S(\lambda) = \sqrt{1 + 3\lambda^2 - 4A^2\lambda^2}, \quad A = \Gamma(2/\varkappa)\Gamma(1/\varkappa)^{-1/2}\Gamma(3/\varkappa)^{-1/2}, \tag{A7}$$

where $\Gamma(\cdot)$ is the Gamma function and μ, σ, λ, and $\varkappa$ are, respectively, the mean, standard deviation, asymmetry (skewness parameter), and tail-thickness (kurtosis parameter) of the distribution. The shape parameters λ and $\varkappa$ satisfy the conditions $-1 < \lambda < 1$ and $\varkappa > 0$. The distribution skews to the left (right) when $\lambda < 0$ (>0) and is symmetric when $\lambda = 0$. It exhibits fatter (thinner) tails than the normal distribution when $\varkappa < 2$ $(\varkappa > 2)$. Figure A1 presents the probability distribution function of SGE for different values of λ.

A.3 | Estimation of QMIDAS-based skewness

We present the method of Aretz and Arisoy (2022) for extracting the skewness given a set of quantiles of the returns $r_{t,t+T}$. This method offers the advantage that it allows the direct and simultaneous estimation of all distribution moments using the law of total probability. The first three conditional moments of the return distribution within two consecutive quantiles α_{j-1} and α_j are approximated by

$$\hat{\mathbb{E}}\left[r_{t,t+T}|q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] = \frac{q_{\alpha_{j-1}} + q_{\alpha_j}}{2}, \tag{A8}$$

$$\hat{\mathbb{E}}\left[r^2_{t,t+T}|q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] = \frac{q^2_{\alpha_{j-1}} + q_{\alpha_{j-1}} \times q_{\alpha_j} + q^2_{\alpha_j}}{3}, \tag{A9}$$

$$\hat{\mathbb{E}}\left[r^3_{t,t+T}|q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] = \frac{q^3_{\alpha_{j-1}} + q^2_{\alpha_{j-1}} \times q_{\alpha_j} + q_{\alpha_{j-1}} \times q^2_{\alpha_j} + q^3_{\alpha_j}}{4}. \tag{A10}$$

If J is the number of conditional quantiles, the conditional moments of the return density are given by the law of total probability:

$$\widehat{\mathbb{E}}\left[r_{t,t+T}^{m}\right] = \sum_{j=2}^{J} \frac{\alpha_j - \alpha_{j-1}}{\alpha_J - \alpha_1} \widehat{\mathbb{E}}\left[r_{t,t+T}^{m} | q_{\alpha_{j-1}} < r_{t,t+T} < q_{\alpha_j}\right] \tag{A11}$$

for $m = 1, 2, 3$. Using the latter, the skewness is then given by

$$\mathrm{QS}_t(r_{t,t+T}) = \frac{\widehat{\mathbb{E}}\left[r_{t,t+T}^{3}\right] - 3\widehat{\mathbb{E}}\left[r_{t,t+T}\right]\left(\widehat{\mathbb{E}}\left[r_{t,t+T}^{2}\right] - \widehat{\mathbb{E}}\left[r_{t,t+T}\right]^2\right) - \widehat{\mathbb{E}}\left[r_{t,t+T}\right]^3}{\left(\widehat{\mathbb{E}}\left[r_{t,t+T}^{2}\right] - \widehat{\mathbb{E}}\left[r_{t,t+T}\right]^2\right)^{3/2}}. \tag{A12}$$

A.4 | Results from out-of-sample tests using MAEs

See Tables A1 and A2.

TABLE A1 Out-of-sample forecasting performance using mean absolute errors.

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
Panel A: 30-day horizon						
AEX	0.498*	**0.438**	0.459	0.614**	0.454	0.449
DAX	0.484*	0.459	0.462	0.672**	0.549**	**0.448**
DJIA	0.479	0.627**	0.542**	0.684**	**0.451**	0.458
STOXX 50	0.543*	0.518	0.556*	0.827**	0.696**	**0.499**
FTSE 100	0.632**	0.573**	0.523	0.803**	1.085**	**0.501**
HANGSENG	0.535	0.506	0.495	0.744**	0.585	**0.493**
KOSPI	0.811**	0.675	**0.673**	0.811**	0.693	0.737
NASDAQ 100	0.492**	0.536**	0.495**	0.648**	0.582**	**0.435**
RUSSELL 2000	0.477**	0.415*	0.421**	0.610**	0.504**	**0.386**
S&P 500	0.702**	0.690**	0.706**	0.993**	0.747**	**0.552**
Panel B: 60-day horizon						
AEX	0.460**	0.419	0.441	0.815**	**0.381**	0.390
DAX	0.460**	0.490*	0.478*	0.866**	0.413	**0.412**
DJIA	0.544**	0.594**	0.565**	1.209**	**0.464**	0.504
STOXX 50	0.485*	0.510	0.578**	0.855**	0.514**	**0.429**
FTSE 100	0.538**	0.483	0.441	0.847**	0.696**	**0.436**
HANGSENG	0.554**	0.496*	0.470	0.710**	0.561*	**0.469**
KOSPI	1.041**	0.799	**0.793**	1.060**	0.900**	1.029**
NASDAQ 100	0.431**	0.498**	0.550**	0.788**	0.437**	**0.386**
RUSSELL 2000	0.441**	0.414**	0.443**	0.571**	0.413*	**0.363**
S&P 500	0.665**	0.685**	0.795**	1.236**	0.553	**0.540**
Panel C: 90-day horizon						
AEX	0.404	0.600**	0.623**	0.966**	**0.393**	0.407
DAX	0.407	0.689**	0.590**	0.978**	**0.386**	0.411

(Continues)

TABLE A1 (Continued)

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
DJIA	0.568	0.503	1.283**	**0.474**	0.474	
STOXX 50	**0.379**	0.647**	0.699**	0.916**	0.480**	0.387
FTSE 100	0.481	0.537**	**0.437**	0.964**	0.538**	0.458
HANGSENG	0.522**	0.429**	**0.398**	0.431	0.587**	0.472
KOSPI	1.291**	**1.019**	1.028	1.363**	1.056	1.201*
NASDAQ 100	0.419	0.591**	0.614**	1.030**	**0.414**	0.421
RUSSELL 2000	0.396**	0.444**	0.454**	0.662**	**0.349**	0.369
S&P 500	0.644**	0.667**	0.807**	1.395**	**0.519**	0.541

Note: For each index in Table 1, this table reports out-of-sample forecasting losses for each skewness model we consider. The forecasting horizon is 30, 60, or 90 calendar days. The GARCH and QMIDAS models are estimated using a rolling window of 1250 observations. We report mean absolute errors (MAE) using the realized skewness of Neuberger (2012) as a proxy for the true skewness. The model with the lowest forecasting loss is highlighted in bold. One (two) asterisk(s) shows that the corresponding model is inferior to the best model at the 10% (5%) significance level, in the context of a Diebold–Mariano test.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

TABLE A2 Model confidence set using mean absolute errors.

	AEX		DAX		DJIA		STOXX 50		FTSE 100		HANGSENG		KOSPI		NASDAQ 100		RUSSELL 2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LaggedRealizedSkew	5*	0.145	4*	0.218	3*	0.576	3*	0.087	4	0.00	4	0.00	5*	0.080	2	0.00	4	0.00	3	0.00
GARCH-1	1*	1.000	2*	0.704	5	0.00	2*	0.314	3	0.00	3	0.00	2*	0.802	4	0.00	2	0.04	2	0.00
GARCH-2	4*	0.145	3*	0.704	4	0.00	4	0.03	2*	0.430	2*	0.900	1*	1.000	3	0.00	3	0.01	4	0.00
QMIDAS	6	0.00	6	0.00	6	0.00	6	0.00	5	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00
ImpliedSkew	3*	0.682	5	0.00	1*	1.000	5	0.00	6	0.00	5	0.00	3*	0.802	5	0.00	5	0.00	5	0.00
AdjustedImpliedSkew	2*	0.682	1*	1.000	2*	0.661	1*	1.000	1*	1.000	1*	1.000	4*	0.180	1*	1.000	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LaggedRealizedSkew	5	0.01	3	0.02	3	0.01	2	0.00	4	0.00	4	0.00	5	0.00	2	0.00	4	0.00	3	0.00
GARCH-1	3*	0.283	5	0.00	5	0.00	3	0.00	3	0.04	3	0.00	2*	0.299	4	0.00	3	0.00	4	0.00
GARCH-2	4	0.02	4	0.00	4	0.00	5	0.00	2*	0.880	2*	0.964	1*	1.000	5	0.00	5	0.00	5	0.00
QMIDAS	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00
ImpliedSkew	1*	1.000	2*	0.920	1*	1.000	4	0.00	5	0.00	5	0.00	3	0.03	3	0.00	2	0.01	2*	0.441
AdjustedImpliedSkew	2*	0.527	1*	1.000	2	0.02	1*	1.000	1*	1.000	1*	1.000	4	0.00	1*	1.000	1*	1.000	1*	1.000
Panel C: 90-day horizon																				
LaggedRealizedSkew	2*	0.638	2*	0.238	4*	0.337	1*	1.000	3*	0.233	5	0.00	5	0.00	2*	0.720	3	0.00	3	0.00
GARCH-1	4	0.00	5	0.00	5	0.05	4	0.00	4	0.00	2	0.03	1*	1.000	4	0.00	4	0.00	4	0.00
GARCH-2	5	0.00	4	0.00	3*	0.518	5	0.00	1*	1.000	1*	1.000	2*	0.385	5	0.00	5	0.00	5	0.00
QMIDAS	6	0.00	6	0.00	6	0.00	6	0.00	6	0.00	3	0.00	6	0.00	6	0.00	6	0.00	6	0.00
ImpliedSkew	1*	1.000	1*	1.000	1*	1.000	3	0.00	5	0.00	6	0.00	3*	0.385	1*	1.000	1*	1.000	1*	1.000
AdjustedImpliedSkew	3*	0.638	3*	0.060	2*	0.956	2*	0.632	2*	0.420	4	0.00	4	0.01	3*	0.655	2*	0.056	2*	0.059

Note: This table reports our results for the model confidence set (MCS) test using the mean absolute error (MAE) as a loss function and a 5% significance level. Panels A, B, and C report results for the 30-, 60-, and 90-day forecasting horizons, respectively. The columns labeled "*Rank*" present the ranking of a model in the MCS while the "*pval*" columns show the *p* values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

A.5 | Results from a subsample analysis

See Table A3.

TABLE A3 Out-of-sample forecasting performance—subsample analysis.

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
30-Day horizon						
Crisis period						
DAX	0.618**	0.739**	0.751**	1.015**	**0.544**	0.703**
DJIA	0.502*	0.427	**0.422**	0.743**	0.435	0.446
NASDAQ	0.377**	0.448**	0.531**	0.573**	0.505**	**0.321**
S&P 500	0.467	0.499**	0.593**	1.021**	0.552*	**0.415**
Full sample excluding crisis						
DAX	0.650**	**0.566**	0.611	0.822**	0.742**	0.571
DJIA	0.666	0.980**	0.810**	0.789*	**0.650**	0.657
NASDAQ	0.670**	0.699**	0.647	0.850**	0.738**	**0.603**
S&P 500	1.006**	0.926**	1.002**	1.265**	0.984**	**0.776**
60-Day horizon						
Crisis period						
DAX	0.705**	0.759**	0.754**	0.901**	**0.584**	0.694**
DJIA	0.542**	0.529*	0.523*	1.556**	**0.469**	0.523**
NASDAQ	0.438**	0.567**	0.682**	1.149**	**0.375**	0.389
S&P 500	0.506	0.530**	0.678**	1.473**	**0.410**	0.442
Full sample excluding crisis						
DAX	0.610**	0.570	0.618*	1.143**	0.546	**0.520**
DJIA	0.766**	0.823**	0.818**	1.362**	**0.657**	0.707
NASDAQ	0.574**	0.623**	0.684**	0.925**	0.566*	**0.510**
S&P 500	0.924**	0.914**	1.080**	1.473**	0.750	**0.729**
90-day horizon						
Crisis period						
DAX	0.916	0.819	0.806**	1.136**	**0.616**	0.709**
DJIA	0.498	0.427	0.338**	1.139**	**0.459**	0.509**
NASDAQ	0.360**	0.437**	0.572**	1.264**	**0.318**	0.415**
S&P 500	0.575**	0.440**	0.520**	1.333**	**0.415**	0.431
Full sample excluding crisis						
DAX	**0.477**	0.798**	0.680**	1.255**	0.504	0.510
DJIA	0.665	0.704	0.713	1.592**	**0.587**	0.590
NASDAQ	0.544	0.765**	0.767**	1.178**	0.548	**0.544**
S&P 500	0.861**	0.875**	1.069**	1.634**	**0.681**	0.701

Note: This table reports out-of-sample forecasting losses in two subsamples for each skewness model for four indices from Table 1. The first subsample is coined "crisis period" and is from August 1, 2007 to December 31, 2008. The second subsample consists of the full sample excluding the crisis period. The GARCH and QMIDAS models are estimated using a rolling window of 1250 observations. We report root mean squared errors (RMSE) using the realized skewness of Neuberger (2012) as a proxy for the true skewness. The model with the lowest forecasting loss is highlighted in bold. One (two) asterisk(s) shows that the corresponding model is inferior to the best model at the 10% (5%) significance level, in the context of a Diebold–Mariano test. The forecasting horizon is 30, 60, or 90 calendar days.

Abbreviations: DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; GARCH, generalized autoregressive conditional heteroskedasticity; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

A.6 | Results for an alternative proxy for the true skewness

See Tables A4 and A5.

TABLE A4 Out-of-sample forecasting performance—alternative realized skewness.

	LaggedRealizedSkew	GARCH-1	GARCH-2	QMIDAS	ImpliedSkew	AdjustedImpliedSkew
Panel A: 30-day horizon						
AEX	0.636**	0.394**	**0.355**	0.665**	0.527**	0.389
DAX	0.717**	0.508**	**0.412**	0.645**	0.826**	0.423
DJIA	0.648**	**0.465**	0.512	0.874**	0.493	0.466
STOXX 50	0.791**	0.468*	0.438	0.663**	0.969**	**0.415**
FTSE 100	0.743**	0.532**	0.431**	0.607**	1.375**	**0.327**
HANGSENG	0.562**	0.491**	0.478**	0.846**	0.483**	**0.285**
KOSPI	0.795**	0.329**	**0.254**	0.403**	0.592**	0.469**
NASDAQ	0.743**	**0.438**	0.676**	0.925**	0.658**	0.509
RUSSELL 2000	0.618**	0.445	0.411	0.564**	0.730**	**0.394**
S&P 500	0.835**	0.439	0.495**	0.876**	0.953**	**0.410**
Panel B: 60-day horizon						
AEX	0.683**	0.595**	0.574**	0.850**	0.594**	**0.383**
DAX	0.728**	0.656**	**0.501**	0.914**	0.707**	0.527
DJIA	0.623**	**0.428**	0.499*	1.070**	0.553**	0.475
STOXX 50	0.833**	0.703**	0.643**	0.939**	0.855**	**0.437**
FTSE 100	0.731**	0.691**	0.554**	0.522**	1.112**	**0.298**
HANGSENG	0.583*	0.592	0.568	0.877**	**0.474**	0.531
KOSPI	0.890**	0.506*	**0.433**	0.580*	0.739**	0.576**
NASDAQ	0.728**	**0.616**	0.816**	1.051**	0.642	1.798
RUSSELL 2000	0.733**	0.545*	0.510	0.597**	0.831**	**0.395**
S&P 500	0.894**	0.567**	0.520**	0.890**	0.877**	**0.389**
Panel C: 90-day horizon						
AEX	0.743**	1.062**	1.049**	0.814**	0.714**	**0.381**
DAX	0.715**	1.027**	0.711**	0.842**	0.705**	**0.465**
DJIA	0.709**	**0.476**	0.496*	0.887**	0.631**	0.662
STOXX 50	0.889**	1.104**	0.963**	0.965**	0.953**	**0.469**
FTSE 100	0.814**	1.074**	0.837**	0.466**	1.006**	**0.328**
HANGSENG	0.616	0.538**	**0.506**	0.677**	0.573	0.563
KOSPI	0.929	0.871	0.774	0.782	0.724	**0.697**
NASDAQ	0.849**	0.812*	0.982**	1.390**	0.802**	**0.655**
RUSSELL 2000	0.823**	0.774**	0.690**	0.487*	0.828**	**0.389**
S&P 500	1.008**	0.869**	0.630**	0.968**	0.870**	**0.409**

Note: For each index in Table 1, this table reports out-of-sample forecasting losses for each skewness model we consider. The forecasting horizon is 30, 60, or 90 calendar days. The GARCH and QMIDAS models are estimated using a rolling window of 1250 observations. We report root mean squared errors (RMSE) using the realized skewness of Neuberger and Payne (2021) as a proxy for the true skewness. The model with the lowest forecasting loss is highlighted in bold. One (two) asterisk(s) shows that the corresponding model is inferior to the best model at the 10% (5%) significance level, in the context of a Diebold–Mariano test.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

TABLE A5 Model confidence set using root mean squared errors—alternative realized skewness.

	AEX		DAX		DJIA		STOXX 50		FTSE 100		HANGSENG		KOSPI		NASDAQ 100		RUSSELL 2000		S&P 500	
	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval	Rank	pval
Panel A: 30-day horizon																				
LaggedRealizedSkew	5	0.00	5	0.00	5	0.00	5	0.00	5	0.00	5	0.00	6	0.00	5	0.00	5	0.00	4	0.00
GARCH-1	3	0.00	3	0.00	1*	1.000	3	0.00	3	0.00	4	0.00	2	0.00	1*	1.000	3	0.00	2*	0.146
GARCH-2	1*	1.000	1*	1.000	4*	0.115	2*	0.263	2	0.00	2	0.00	1*	1.000	4	0.00	2*	0.331	3	0.00
QMIDAS	6	0.00	4	0.00	6	0.00	4	0.00	4	0.00	6	0.00	3	0.00	6	0.00	4	0.00	5	0.00
ImpliedSkew	4	0.00	6	0.00	3*	0.115	6	0.00	6	0.00	3	0.00	5	0.00	3	0.00	6	0.00	6	0.00
AdjustedImpliedSkew	2*	0.075	2*	0.588	2*	0.965	1*	1.000	1*	1.000	1*	1.000	4	0.00	2*	0.111	1*	1.000	1*	1.000
Panel B: 60-day horizon																				
LaggedRealizedSkew	5	0.00	5	0.00	5	0.00	4	0.00	5	0.00	4	0.03	6	0.00	3	0.00	5	0.00	6	0.00
GARCH-1	4	0.00	3	0.00	1*	1.000	3	0.00	4	0.00	5	0.00	2	0.00	1*	1.000	3	0.00	3	0.00
GARCH-2	2	0.00	1*	1.000	3	0.00	2	0.00	3	0.00	3	0.03	1*	1.000	4	0.00	2	0.00	2	0.00
QMIDAS	6	0.00	6	0.00	6	0.00	6	0.00	2	0.00	6	0.00	4	0.00	5	0.00	4	0.00	5	0.00
ImpliedSkew	3	0.00	4	0.00	4	0.00	5	0.00	6	0.00	1*	1.000	5	0.00	2*	0.193	6	0.00	4	0.00
AdjustedImpliedSkew	1*	1.000	2*	0.477	2	0.04	1*	1.000	1*	1.000	2*	0.323	3	0.00	6	0.00	1*	1.000	1*	1.000
Panel C: 90-day horizon																				
LaggedRealizedSkew	3	0.00	4	0.00	5	0.00	2	0.00	3	0.00	5	0.00	6	0.00	4	0.00	5	0.00	6	0.00
GARCH-1	6	0.00	6	0.00	1*	1.000	6	0.00	6	0.00	2	0.00	5	0.03	3	0.00	4	0.00	3	0.00
GARCH-2	5	0.00	3	0.00	2	0.02	4	0.00	4	0.00	1*	1.000	3*	0.180	5	0.00	3	0.00	2	0.00
QMIDAS	4	0.00	5	0.00	6	0.00	5	0.00	2	0.00	6	0.00	4*	0.180	6	0.00	2	0.00	5	0.00
ImpliedSkew	2	0.00	2	0.00	3	0.00	3	0.00	5	0.00	4	0.00	2*	0.356	2	0.00	6	0.00	4	0.00
AdjustedImpliedSkew	1*	1.000	1*	1.000	4	0.00	1*	1.000	1*	1.000	3	0.00	1*	1.000	1*	1.000	1*	1.000	1*	1.000

Note: This table reports our results for the model confidence set (MCS) test using the root mean squared error (RMSE) as a loss function and a 5% significance level. We use the realized skewness of Neuberger and Payne (2021) as a proxy for the true skewness. Panels A, B, and C report results for the 30-, 60-, and 90-day forecasting horizons, respectively. The columns labeled "*Rank*" present the ranking of a model in the MCS while the "*pval*" columns show the *p* values of the test. An asterisk (*) shows that the corresponding model is included in the MCS.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

A.7 | Results for alternative specifications in the portfolio choice analysis

See Tables A6–A8.

TABLE A6 Portfolio performance using a volatility-timing strategy as a benchmark.

	Mean	Var	p_v^{AIS}	p_v^{VT}	SR	p_{SR}^{AIS}	p_{SR}^{VT}	Skew	TO
Panel A: Daily rebalancing									
LaggedRealizedSkew	0.0274	0.0123	(0.00)	(0.73)	0.2469	(0.01)	(0.13)	−0.0728	0.0539
GARCH-1	0.0306	0.0123	(0.00)	(0.90)	0.2757	(0.05)	(0.23)	−0.0533	0.0135
GARCH-2	0.0301	0.0123	(0.00)	(0.83)	0.2713	(0.05)	(0.23)	−0.0595	0.0200
QMIDAS	0.0211	0.0121	(0.03)	(0.21)	0.1925	(0.01)	(0.01)	−0.0462	0.0359
ImpliedSkew	0.0397	0.0122	(0.00)	(0.27)	0.3596	(0.31)	(0.65)	0.0088	0.0825
AdjustedImpliedSkew	**0.0474**	**0.0113**	(1.00)	(0.00)	**0.4458**	(1.00)	(0.12)	**0.0092**	0.0893
VT	0.0364	0.0123	(0.00)	(1.00)	0.3280	(0.12)	(1.00)	−0.0547	**0.0068**
Panel B: Weekly rebalancing									
LaggedRealizedSkew	0.0288	0.0149	(0.00)	(0.56)	0.2357	(0.02)	(0.09)	−0.1216	0.0294
GARCH-1	0.0344	0.0151	(0.00)	(0.61)	0.2805	(0.24)	(0.24)	−0.1328	0.0069
GARCH-2	0.0328	0.0150	(0.00)	(0.96)	0.2678	(0.22)	(0.21)	−0.1576	0.0104
QMIDAS	0.0245	0.0146	(0.02)	(0.08)	0.2021	(0.04)	(0.02)	−0.1490	0.0195
ImpliedSkew	0.0384	0.0150	(0.00)	(0.95)	0.3135	(0.44)	(0.80)	**−0.0932**	0.0315
AdjustedImpliedSkew	**0.0437**	**0.0139**	(1.00)	(0.00)	**0.3703**	(1.00)	(0.52)	−0.1086	0.0341
VT	0.0404	0.0150	(0.00)	(1.00)	0.3299	(0.50)	(1.00)	−0.1309	**0.0041**
Panel C: Biweekly rebalancing									
LaggedRealizedSkew	0.0290	0.0133	(0.06)	(0.20)	0.2520	(0.01)	(0.06)	**−0.1006**	0.0208
GARCH-1	0.0359	0.0137	(0.01)	(0.12)	0.3069	(0.19)	(0.18)	−0.1547	0.0052
GARCH-2	0.0345	0.0136	(0.01)	(0.42)	0.2962	(0.16)	(0.15)	−0.1730	0.0079
QMIDAS	0.0271	0.0133	(0.08)	(0.38)	0.2350	(0.03)	(0.02)	−0.1521	0.0139
ImpliedSkew	0.0411	0.0136	(0.02)	(0.50)	0.3528	(0.39)	(0.96)	−0.1464	0.0189
AdjustedImpliedSkew	**0.0471**	**0.0127**	(1.00)	(0.01)	**0.4176**	(1.00)	(0.35)	−0.1283	0.0212
VT	0.0415	0.0134	(0.01)	(1.00)	0.3579	(0.36)	(1.00)	−0.1318	**0.0034**

Note: This table presents the out-of-sample performance of a volatility-timing strategy (VT) as defined in Section 6.3 and of the parametric portfolios that use the skewness estimates from each model considered in the paper. The portfolios include as assets the following indices: AEX, DAX, STOXX 50, FTSE 100, HANGSENG, KOSPI, and S&P 500. The table reports the annualized out-of-sample average return (Mean), variance of returns (Var), and Sharpe ratio (SR) for each portfolio strategy as well as the skewness (Skew) and the average turnover (TO). It presents p values from testing the hypothesis that the variance (p_v^{AIS}) or the Sharpe ratio ($p_{\mathrm{SR}}^{\mathrm{AIS}}$) between a portfolio strategy and the portfolio based on the adjusted implied skewness (AdjustedImpliedSkew) are equal. It also reports p values from testing the hypothesis that the variance (p_v^{VT}) or the Sharpe ratio (p_{SR}^{VT}) between a portfolio strategy and VT are equal. The p values are computed using the block-bootstrap approach of Ledoit and Wolf (2011), assuming an average block size of 5 and 5000 replications. Panels A, B, and C, respectively, present results assuming daily, weekly, and biweekly rebalancing. The best performance for each metric is highlighted in bold.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

TABLE A7 Portfolio performance for pairs of skewness models.

	Mean	Var	$p_v^{QMIDAS+AIS}$	$p_v^{1/N}$	SR	$p_{SR}^{QMIDAS+AIS}$	$p_{SR}^{1/N}$	Skew	TO
Panel A: Daily rebalancing									
LaggedRealizedSkew + GARCH-2	0.0212	0.0145	(0.00)	(0.04)	0.1765	(0.45)	(0.16)	−0.0622	0.0809
LaggedRealizedSkew + QMIDAS	0.0122	0.0140	(0.00)	(0.00)	0.1033	(0.14)	(0.02)	−0.0705	0.0903
LaggedRealizedSkew + AdjustedImpliedSkew	**0.0482**	0.0131	(0.01)	(0.00)	**0.4207**	(0.06)	(0.21)	−0.0078	0.1473
GARCH-2 + QMIDAS	0.0185	0.0145	(0.00)	(0.07)	0.1536	(0.35)	(0.10)	−0.0804	0.0586
GARCH-2 + AdjustedImpliedSkew	0.0404	0.0125	(0.11)	(0.00)	0.3611	(0.16)	(0.56)	−0.0041	0.1663
QMIDAS + AdjustedImpliedSkew	0.0294	**0.0122**	(1.00)	(0.00)	0.2660	(1.00)	(0.86)	**0.0092**	0.1682
$1/N$	0.0353	0.0151	(0.00)	(1.00)	0.2869	(0.87)	(1.00)	−0.0994	**0.0045**
Panel B: Weekly rebalancing									
LaggedRealizedSkew + GARCH-2	0.0198	0.0170	(0.00)	(0.03)	0.1516	(0.92)	(0.07)	−0.1600	0.0436
LaggedRealizedSkew + QMIDAS	0.0101	0.0167	(0.00)	(0.01)	0.0781	(0.51)	(0.00)	−0.1535	0.0477
LaggedRealizedSkew + AdjustedImpliedSkew	**0.0428**	0.0157	(0.00)	(0.00)	**0.3412**	(0.02)	(0.54)	**−0.0980**	0.0598
GARCH-2 + QMIDAS	0.0235	0.0171	(0.00)	(0.10)	0.1801	(0.98)	(0.17)	−0.1284	0.0304
GARCH-2 + AdjustedImpliedSkew	0.0332	0.0152	(0.11)	(0.00)	0.2687	(0.14)	(0.88)	−0.1330	0.0618
QMIDAS + AdjustedImpliedSkew	0.0222	**0.0149**	(1.00)	(0.00)	0.1820	(1.00)	(0.22)	−0.1311	0.0663
$1/N$	0.0385	0.0176	(0.00)	(1.00)	0.2895	(0.22)	(1.00)	−0.0775	**0.0025**
Panel C: Biweekly rebalancing									
LaggedRealizedSkew + GARCH-2	0.0199	0.0152	(0.01)	(0.29)	0.1617	(0.40)	(0.09)	-0.1211	0.0302
LaggedRealizedSkew + QMIDAS	0.0109	0.0149	(0.01)	(0.07)	0.0890	(0.07)	(0.00)	−0.1250	0.0332
LaggedRealizedSkew + AdjustedImpliedSkew	**0.0482**	0.0142	(0.14)	(0.00)	**0.4050**	(0.03)	(0.34)	−0.1293	0.0371
GARCH-2 + QMIDAS	0.0258	0.0154	(0.00)	(0.69)	0.2082	(0.65)	(0.28)	−0.1342	0.0215
GARCH-2 + AdjustedImpliedSkew	0.0386	0.0141	(0.15)	(0.00)	0.3251	(0.29)	(0.92)	−0.1403	0.0382
QMIDAS + AdjustedImpliedSkew	0.0299	**0.0138**	(1.00)	(0.00)	0.2546	(1.00)	(0.52)	−0.1510	0.0417
$1/N$	0.0394	0.0155	(0.00)	(1.00)	0.3158	(0.51)	(1.00)	**−0.1002**	**0.0020**

Note: This table presents the out-of-sample performance of the equally-weighted portfolio ($1/N$) and of the parametric portfolios that use estimates from two skewness models as described in Section 6.3. The portfolios include as assets the following indices: AEX, DAX, STOXX 50, FTSE 100, HANGSENG, KOSPI, and S&P 500. The table reports the annualized out-of-sample average return (Mean), variance of returns (Var), and Sharpe ratio (SR) for each portfolio strategy as well as the skewness (Skew) and the average turnover (TO). It presents p values from testing the hypothesis that the variance ($p_v^{\mathrm{QMIDAS+AIS}}$) or the Sharpe ratio ($p_{\mathrm{SR}}^{\mathrm{QMIDAS+AIS}}$) between a portfolio strategy and the portfolio that uses QMIDAS and adjusted implied skewness (AdjustedImpliedSkew) forecasts. It also reports p values from testing the hypothesis that the variance ($p_v^{1/N}$) or the Sharpe ratio ($p_{\mathrm{SR}}^{1/N}$) between a portfolio strategy and $1/N$ are equal. The p values are computed using the block-bootstrap approach of Ledoit and Wolf (2011), assuming an average block size of 5 and 5000 replications. Panels A, B, and C, respectively, present results assuming daily, weekly, and biweekly rebalancing. The best performance for each metric is highlighted in bold.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.

TABLE A8 Portfolio performance (alternative calibration).

	Mean	Var	p_v^{AIS}	$p_v^{1/N}$	SR	p_{SR}^{AIS}	$p_{SR}^{1/N}$	Skew	TO
Panel A: Daily rebalancing									
LaggedRealizedSkew	0.0284	0.0148	(0.92)	(0.05)	0.2339	(0.13)	(0.17)	−0.0659	0.0334
GARCH-1	0.0333	0.0151	(0.25)	(0.71)	0.2715	(0.23)	(0.71)	−0.0843	0.0152
GARCH-2	0.0320	0.0152	(0.08)	(0.18)	0.2596	(0.25)	(0.54)	−0.0961	0.0216
QMIDAS	0.0279	0.0153	(0.03)	(0.04)	0.2250	(0.07)	(0.11)	−0.1001	0.0224
ImpliedSkew	0.0415	0.0150	(0.45)	(0.72)	0.3390	(0.80)	(0.47)	−0.0702	0.0925
AdjustedImpliedSkew	**0.0436**	**0.0147**	(1.00)	(0.15)	**0.3595**	(1.00)	(0.32)	**−0.0587**	0.0700
1/*N*	0.0353	0.0151	(0.16)	(1.00)	0.2869	(0.31)	(1.00)	−0.0994	**0.0045**
Panel B: Weekly rebalancing									
LaggedRealizedSkew	0.0330	0.0176	(0.63)	(0.61)	0.2492	(0.31)	(0.31)	−0.0889	0.0184
GARCH-1	0.0326	0.0176	(0.45)	(0.66)	0.2457	(0.32)	(0.14)	−0.1035	0.0082
GARCH-2	0.0362	0.0175	(0.59)	(0.35)	0.2729	(0.84)	(0.63)	−0.1558	0.0104
QMIDAS	0.0343	0.0178	(0.08)	(0.20)	0.2570	(0.86)	(0.49)	−0.1034	0.0114
ImpliedSkew	0.0299	0.0175	(0.85)	(0.49)	0.2259	(0.49)	(0.48)	−0.1031	0.0343
AdjustedImpliedSkew	**0.0410**	**0.0174**	(1.00)	(0.27)	**0.3108**	(1.00)	(0.61)	−0.0888	0.0290
1/*N*	0.0385	0.0176	(0.27)	(1.00)	0.2895	(0.61)	(1.00)	**−0.0775**	**0.0025**
Panel C: Biweekly rebalancing									
LaggedRealizedSkew	0.0348	0.0156	(0.85)	(0.42)	0.2782	(0.35)	(0.34)	**−0.0869**	0.0129
GARCH-1	0.0339	0.0156	(0.90)	(0.55)	0.2710	(0.13)	(0.06)	−0.1210	0.0053
GARCH-2	0.0357	0.0155	(0.83)	(0.95)	0.2868	(0.28)	(0.48)	−0.0978	0.0077
QMIDAS	0.0358	0.0157	(0.68)	(0.33)	0.2858	(0.26)	(0.90)	−0.1297	0.0078
ImpliedSkew	0.0349	0.0156	(0.89)	(0.63)	0.2792	(0.29)	(0.57)	−0.1426	0.0203
AdjustedImpliedSkew	**0.0425**	**0.0156**	(1.00)	(0.82)	**0.3409**	(1.00)	(0.63)	−0.1303	0.0185
1/*N*	0.0394	0.0155	(0.83)	(1.00)	0.3158	(0.63)	(1.00)	−0.1002	**0.0020**

Note: This table presents the out-of-sample performance of the equally-weighted portfolio ($1/N$) and of the parametric portfolios that use the skewness estimates from each model considered in the paper. The parametric portfolios are calibrated to lead to maximum skewness over the previous year under the constraint that the portfolio variance is lower than the variance of $1/N$. The portfolios include as assets the following indices: AEX, DAX, STOXX 50, FTSE 100, HANGSENG, KOSPI, and S&P 500. The table reports the annualized out-of-sample average return (Mean), variance of returns (Var), and Sharpe ratio (SR) for each portfolio strategy as well as the skewness (Skew) and the average turnover (TO). It presents p values from testing the hypothesis that the variance (p_v^{AIS}) or the Sharpe ratio ($p_{\mathrm{SR}}^{\mathrm{AIS}}$) between a portfolio strategy and the portfolio based on the adjusted implied skewness (AdjustedImpliedSkew) are equal. It also reports p values from testing the hypothesis that the variance ($p_v^{1/N}$) or the Sharpe ratio ($p_{\mathrm{SR}}^{1/N}$) between a portfolio strategy and $1/N$ are equal. The p values are computed using the block-bootstrap approach of Ledoit and Wolf (2011), assuming an average block size of 5 and 5000 replications. Panels A, B, and C, respectively, present results assuming daily, weekly, and biweekly rebalancing. The best performance for each metric is highlighted in bold.

Abbreviations: AEX, Amsterdam Exchange index; DAX, Deutscher Aktien Index; DJIA, Dow Jones Industrial Average; FTSE, Financial Times Stock Exchange; GARCH, generalized autoregressive conditional heteroskedasticity; HANGSENG, Hang Seng Index; KOSPI, Korea Composite Stock Price Index; NASDAQ, National Association of Securities Dealers Automated Quotations; QMIDAS, quantiles of returns via a Mixed Data Sampling; S&P 500, Standard and Poor's 500.