The proposition of an analytical tool for the evaluation of mathematics teachers' diagnostic competencies in the noticing framework

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This paper proposes an analytical tool for the evaluation of prospective mathematics teachers' (PTs) diagnostic competencies through noticing critical incidents in hypothetical classroom situations (mathtasks). Data were collected from nineteen PTs attending an undergraduate course for one semester. Data analysis highlighted variations within the four characteristics of teachers' diagnostic competencies that are described by four internal levels inspired by how teachers notice. This analysis resulted to the analytical rubric we present and exemplify in this paper. We see the potential of the proposed rubric in research and mathematics teacher professional development.

Keywords: Noticing, mathtask, prospective teachers (PTs), professional development.

Introduction

Research has highlighted the significant role of critical incidents, which, according to the Goodell (2006), are classroom incidents that have the potential to trigger teacher reflections on students' mathematical learning. Such reflections have been connected to the development of diagnostic competencies, namely their competencies to interpret students' mathematical actions by identifying the rationale behind these actions (Prediger, 2010). Very often, teachers encounter unanticipated situations in their lessons to which they are expected to respond on the spot. Especially prospective teachers, with limited teaching experience, often face difficulties to give immediate interpretations and multifaceted responses when they needed. For that reason, teachers' education on ways of analyzing students' thinking and different teaching practices is pertinent (Grossman, 2011). Such education can be supported by the analysis of critical incidents (Psycharis & Potari, 2017). Also, studies report improvement in teachers' engagement with mathematical and pedagogical terminology when they interpret students' reasoning (Grisham et al., 2002).

Recently, researchers have been using videos, pictures or texts from classroom incidents in order to support teachers' observation of students' reactions (Prediger & Zindel, 2017; Sherin & Van Es, 2003; Van Es, 2011). The ability of teachers to observe students' mathematical thinking is attributed by Van Es (2011) to their ability of *noticing*. She argues that teachers need to learn how to observe and interpret classroom interactions that affect learning. Teacher noticing is analyzed in three dimensions: the monitoring of noteworthy events; their justification; and, their interpretation in order to make an appropriate teaching plan. The key element for teachers' noticing ability is whether the substantiation of their arguments is based on the principles of teaching and learning. Van Es proposed a two-component analytical framework for teacher noticing: *What teachers notice* and *How teachers notice*. The How teachers notice component can be categorized into four levels: *Level 1*, when the analysis of a fact they notice includes general impressions, providing descriptive and evaluative comments, with little or no evidence to support it; *Level 2*, when in addition to Level

1 the analysis includes some interpretive comments referring to noteworthy events or interactions as evidence; *Level 3*, when in addition to Level 2 the analysis includes interpretive comments and the elaboration of specific noteworthy events and interactions; and, *Level 4*, when in addition to Level 3 the analysis include connections between the events and principles of teaching and learning and proposals for alternative pedagogical solutions (Van Es, 2011).

Noticing has also been connected to the diagnostic competences of PTs through their interpretation of aspects they have noticed in familiar or non-familiar instructional episodes (Prediger & Zindel, 2017). Characterisation of mathematics teachers' diagnostic competencies is related to the work of Biza et al. (2018). In their work, Biza and colleagues design hypothetical classroom situations (mathtasks[1], see Figure 1) that are inspired by mathematical and pedagogical issues likely to occur in the mathematics teaching practice, and they invite mathematics PTs and in-service teachers to reflect on these situations. The classroom situations, although hypothetical, are designed with potential critical incidents in mind (in the sense of Goodell, 2006) that PTs are invited to notice, interpret, and propose intended actions. Mathtasks were used in the study we present here towards the familiarisation of PTs with critical incidents that may arise in their classroom as we explain in the next section. In earlier work of Biza et al. (2018), the analysis of teacher responses to mathtasks proposed a typology of four characteristics of teachers' diagnostic competencies in recognising the issues in the incident described in the classroom situation and in responding to students' needs: *consistency*, how *consistent* a response is in the way it conveys the link between the respondent's stated pedagogical priorities and their intended actions; *specificity*, how *contextualized and specific* a response is to the incident under consideration; *reification of pedagogical discourse (RPD)*, how *reified*[2] the pedagogical discourse of the response is in order to describe and interpret the pedagogical and mathematical issues of the incident and to propose appropriate actions; and, *reification of mathematical discourse (RMD)*, how *reified* the *mathematical discourse* of the response is in order to describe and interpret the underpinning mathematical content of the incident and to propose appropriate actions (Biza et al., 2018). The four characteristics are attentive to teachers' both mathematical and pedagogical discourses. However, the operationalization of the typology in analysis requires more transparency on the level of sophistication within each one of the four characteristics (e.g., what does justify a high level of RPD?). Such lack of transparency is addressed by this study that examines the research question: "what levels of variation can be identified within each one of the four characteristics of PTs' diagnostic competencies as evident in their responses to hypothetical classroom situations (mathtasks)". To address this issue, we propose an analytical tool that draws on Van Es's (2011) levels of *how teachers notice* in order to describe variations within each one of the four characteristics as we exemplify through the analysis of empirical data in the next section.

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[1]Mathtasks are designed in the context of the MathTASK research and development program on mathematics teachers' pedagogical and mathematical discourses (https://www.uea.ac.uk/groups-and-centres/a-z/mathtask).

[2] Reified means that the mathematical discourse (the mathematical content and practices PTs have become familiar during their studies) and the pedagogical discourse (the theories and findings from mathematics education research PTs have become familiar during their studies) have been integrated productively into PTs' responses.

Methodology

The research took place in the context of a 14-week mathematics education undergraduate course at a Greek university with nineteen PTs. The 19 PTs who participated in the research were also those who participated in the semester course.The condition for attending the course was the completion by PTs of mathematical courses and at least four courses related to psychology, philosophy and mathematics education. During the course, PTs engaged with school based activities – such as lesson observations, lesson planning, delivering sessions, noticing of critical events from the classroom and the interpretation of these events – and university based activities – such as introduction to theories and findings from research into the teaching and learning of mathematics, engagement with mathtasks and discussion on PTs' school school based activities. One of the aims of the course was the development of PTs' pedagogical discourse also through their engagement with research in mathematics education literature.

Reasoning

In a class of maths, students are asked to solve the following problem:
"Can you make the two columns of numbers below add up to the same total by swapping just two numbers between the columns? Explain why or why not."

1	7
3	2
8	4
5	9

The following conversation between students A and B takes place:

Student A: If I add up the number in the columns, I get totals of, umm…17 and 22. So we need to make these the same.
Student B: How about we just try swapping some numbers and see what happens!
Student A: Okay, let's try the top two numbers first…If we swap 1 and 7, we get new totals of 23 and 16. That's worse than before!
Student B: Let's try some others…what about swapping 5 and 7?
Student A: No, that gives 19 and 20.
Student B: We're getting closer, though!
Student A: What about if we swap two numbers that are close together, like 2 and 3?
Student B: Ummm…that gives 16 and 23, that can't be right.
Student A: We could be here doing this forever!
Student C joins the conversation.
Student C: Maybe it can't be done and we have to show why not.
Student A: How would we do that then? We can't try every single possible swap…that would take too long!

You have just heard this exchange between students A, B and C.

Questions: **Lesson for: (specify the class)**

a. Solve this mathematical problem. What is the main goal of this problem?
b. For what reasons do you believe that this episode is important? (from mathematical and pedagogical view)
c. How you would interpret this dialogue? (refer to the literature)
d. How would you respond to Students A, B and C and to the whole class?

In this paper we draw on PTs responses to the mathtask of Figure 1 (translated from Greek), the last of the three mathtasks used in the course as instances of potential critical incidents, that was given to PTs towards the end of the semester. PTs' responses were collected electonically one week after the assignment. The hypothetical scenario of the mathtask is based on an open mathematical problem, different from what is considered as usual in the Greek mathematics school curriculum. We expected PTs to analyze the goals of the activity, to identify the flexibility of using it in different classes (and specify the class), to notice the issues in students' dialogue, to interpret these issues and to respond to them accordingly. Thus, we aimed to trigger PTs' noticing in the given incident and through this to analyze and evaluate their diagnostic competencies. All the PTs who attended the course consented to the use of their work for research purposes. The research was implemented within the framework of the qualitative research methodology.

The data were coded with the analytical tool of a rubric (Andrande, 2000) with two-dimentions (see Table 1): the typology of the four characteristics (criteria) proposed by Biza et al. (2018) in rows; and within each of the characteristics, in columns, the four levels inspired by Van Es's (2011) levels of *how teachers notice* and concern quality differences within each characteristic.

Initially, the first author analysed PTs' responses in relation to the four characteristics by looking for quality differences within each characteristic. Then, the quality differences between these levels were described with an adaptation of Van Es's (2011) levels. The rubric in Table 1 is the outcome of this phase of the analysis. Finally, PTs' responses were analysed again with the use of the rubric. The other two authors validated the analysis in each one of the three phases described above. In this paper we present the rubric together with examples from the final analysis that concerns the RPD characteristic with some reference to the other characteristics.

Results

In Table 1 we present the rubric with the four characteristics in each row and the levels in each column. Then, we exemplify the RPD levels from the responses of PTs to the mathtask of Figure 1.

Table 1: The rubric of four characteristics

	Level 1: Irrelevant	Level 2: Superficial	Level 3: Evolving	Level 4: Multidimensional
Consistency	There is no consistency in the interpretation of the incident and the proposed actions.	There is consistency in the interpretation of the incident and the proposed actions. There are general references to the incident with superficial interpretation of what is happening in it.	There is consistency in the interpretation of the incident and the proposed actions. There are specific references to the incident with interpretations and identification of connections of what is happening in it.	There is consistency in the interpretation of the incident and the proposed actions. There are targeted suggestions based on evidence from what is happening in the incident, specific and/or alternative approaches, links to principles of teaching and learning related to them.
Specificity	There is no accuracy in the descriptions of the incident.	There is accuracy in the descriptions of the incident. There is a general reference to what is happening in the incident.	There is accuracy in the description of the incident. There are specific references to the incident with interpretations and identification of connections of what is happening in it.	There is accuracy in the description of the incident. There are detailed interpretations based on evidence of what is happening in the incident, which are connected to teaching and learning principles related to them.

RPD	Wrong, irrelevant or no use of pedagogical terminology.	Limited use of pedagogical terminology. There is general reference to pedagogical issues from the incident.	Good use of pedagogical terminology. There is interpretation of interactions happening in the incident with evidence, using pedagogical terms related to principles of teaching and learning.	Very good use of pedagogical terminology. There are interpretations and identification of connections in what is happening in the incident with reference to relevant literature of principles of teaching and learning and alternative pedagogical suggestions.
RMD	Wrong or no use of mathematical terminology.	Limited use of mathematical terminology. There is general reference to pedagogical issues from the incident.	Good use of mathematical terminology. There is interpretation of interactions happening in the incident with evidence, using mathematical terms and description of suggestions based on the mathematical content.	Very good use of mathematical terminology. There are interpretations and identification of connections in what is happening in the incident with reference to the relevant literature of the basic principles of teaching and learning and alternative suggestions focused on the mathematical content.

Exemplification of RPD levels

The Levels in Table 1 are inspired by Van Es's (2011) levels and the headings in the table were named in order to attribute the quality differences from one level to another. To exemplify the levels of the RPD characteristic we present characteristic examples from four different PTs' interpretations at various levels in terms of the RPD. The levels within each one of the characteristics are interconnected in the responses of each one of the PTs. Due to space limitation, we only exemplify the rubric from PTs' responses to questions b and c of the mathtask in Figure 1.

Specifically, at the *Irrelevant level* (L1), Mania responds to question b:

> From the teaching point of view, we can see how the students manage a problem without solution and how the need of the concept of the proof emerges, as student C quote**.**

Mania seems to recognize two teaching objectives: managing a problem without solution and the emergence of a need for proving. Then, in question c, she writes:

> The concept of proof is not a standalone concept, it comes together with the sense of "legitimizing a proposition" and "theory" [her quotation marks]. … It requires a substantial transition of the student to an epistemological state: the transition from a practical state (ruled by a kind of practical logic) to a theoretical state (ruled by the physical particularity of a theory).

Her response above includes text retrieved from the internet. This is not necessarily a problem when the right reference is used, which she has not done. Also, it seems that she is using terminology without connection to elements arising from the dialogue in Figure 1. Yet it is not clear, when she refers to "theory" and "epistemological state" if she means proof and, while she refers to "a practical state", if she means the tests that students make when they try to give a solution.

Mania's responses are coded as L1 for the other three characteristics as well, which means that she does not interpret the elements from the dialogue, neither she proposes how she could manage this open-ended task in a classroom. Especially for L1 in RMD, she presents a few tests from MATLAB as solution and insisted on experimentation, which alone is not enough to help students to think of an algebraic explanation.

At the *Superficial level* (L2), we present part of Anna's response to question b in which she highlights the pedagogical interest of the episode because:

> The dialogue, the exchange of views and the collaboration between students are encouraged in this episode. Moreover, the intervention of the teacher cultivates the mathematical thinking and ability of abstraction. It is an open-ended problem that allows students to take initiatives.

Then, in question c, she writes:

> [she describes a mathematical solution] This problem was given to the class with the aim [to make] students to distinguish the meaning of even and odd [numbers].

She notes that through an open-ended activity, dialogue and different ways of resolving are favored. Nevertheless, the comments in her interpretation are general without connection to points of the episode. Moreover, she doesn't cite any reference from relevant literature. Overall, Anna focuses on the mathematical content both to her response the problem and to her interpretation of students' answers, without making any reference or connection to the principles of teaching and learning. We notice that comments such as "exchange of views and cooperation are encouraged" or "teacher cultivates mathematical thinking and abstract abilities" and "open-ended problems allow students take initiatives" are general in terms of the pedagogical content and brief that they can be applied to other cases as well. In addition, she does not support her points with evidence from the incident under consideration and with references to student interactions. The difference between Mania and Anna is the relevance of Anna's pedagogical comments to the critical incidents she identified in the dialogue. Moreover, in relation to the characteristics of Consistency and Specificity, Anna's response is at L2. However, in relation to the RMD characteristic, her response is at L3: Anna's interpretations are based on the mathematical content of the problem and she proposes specific interventions as a response to the difficulties of the students she identified in the dialogue.

At the *Evolving level* (L3), Vaso's response in question b included the following:

> The teacher lets the students discuss the exercise with each other without intervening. This practice encourages the exploration and the exchange of views. The problem is a good example of a task that sharpens students' mathematical thinking and curiosity. The students are expected to engage actively and try to think of a shorter way to solve it.

Here she points out that the teacher promotes the dialogue without guiding the students. Furthermore, she commends students' involvement as important in the demanding activity of the incident. However, she neither makes any comment on students' approaches nor she makes connections to the dialogue. Continuing with her interpretation, she comments in question c:

> …students face difficulties in relating the proof with their attempts to find a solution. The literature confirms that a large percentage of students find it difficult to acknowledge the importance and usefulness of proof. This often happens because students do not understand why the proof validates the original claim or they do not yet fully understand the meaning of proof. In this example, proof occurs as a result of a conjecture-and-test process to solve a problem which turns into an algorithmic exercise. The students seem to find it difficult to move from conjecture to proof.

Finally, we notice that Vaso refers to basic principles of teaching and learning such as tasks of high or low demand, key points of the proof (tests, conjecture, and proof), students' difficulties in understanding the meaning of a task, student habits with algorithmic solutions. This indicates that Vaso has studied the relevant literature suggested by the course although she does not refer directly to it. This approach is more detailed and focused in comparison to Anna's. Vaso's interpretation is more justified as she uses evidence from the situation to describe students' interactions through pedagogical terms. In relation to the other characteristics, Vaso's response is at L4.

At the *Multidimensional level* (L4), Anastasia's response involves pedagogical terms connected with the incident and references to relevant literature. In question b, Anastasia noticed some significant points of the dialogue like the cooperation between the students through the discussion, the investigation through tests to lead to the conjecture and the proof. Moreover, she focuses on the type of activity and the flexibility of strategies it allows to the students. She also refers to time effectiveness, number of tests and the conjecture-proof relationship. Later in question c she writes:

> According to F. Furinghetti, open problems include activities with a short formulation that does not imply the solution method, but instead stimulate the production of conjectures and encourage discovery. As M. Mariotti points out, the teacher plays a key role in helping students dealing with such problems. […] Student A observes that the solution to the problem seems time consuming. This student concern could clearly be a sign of despair or lack of willingness on his part to be further involved in the solution process. On the other hand, it may be a way by which the student expresses the need to change the way they work, or try to come up with a shorter solution.[…] It is possible, however, that student A's observation that the problem is time-consuming may have caused student C to "think cunningly" and assume that something else might have happened. [...] In any case, he seems [student C] to have realized part - if not all - of the functions of the mathematical proof (according to Bell, Hanna and de Villiers), that is, to verify the truth of a proposition, to explain it, to contribute to the discovery and exploration of new situations, concepts and properties and to "communicate" the new knowledge. The evolution of the event is expected to be critical too.

Here Anastasia describes in detail what happens in the incident by commenting on students' reactions. Her answers reveal familiarity with the curriculum and the relevant literature on teaching and learning. She refers to the open problems that motivate the production of conjectures and she comments on the role of the teacher in managing such problems in the classroom, with references to relevant literature. The pedagogical terminology she uses is constantly in connection with points of the incident. The importance she gives to the interaction is crucial in order to provoke further dialogue about the mathematical proof. Overall, she approached the incident comprehensively by interpreting the students' reactions and their interactions and offered alternative pedagogical solutions. All the above with the fact that she connects the pedagogical terms with the literature, classify her response as L4. Finally, Anastasia's response is at L4 for all the other characteristics.

Conclusion

In this paper, we address the lack of transparency on the level of sophistication of PTs diagnostic competencies and we proposed an analytical rubric that describes how teachers notice within each one of the four characteristics of PTs' diagnostic competencies (Biza et al., 2018). At the broader research, in which this rubric was used, we explore the levels of the characteristics of PTs in three consecutive mathtasks, the third one (Figure 1) is discussed in this paper, in order to study the development of PTs diagnostic competencies across the course. The analysis evidence that the typology of the four characteristics and the proposed quality differences within them provide a detailed picture of PTs' diagnostic competencies and their development as PTs moved from the first mathtask to the third one across the course. We note that such development is attributed to the course design that prioritized appropriate connections to the teaching and learning of mathematics literature and supported PTs' reflective activities. Therefore, we would say that the results agree with studies that report the benefit of teachers' interpretation of their students' mathematical thinking (e.g., Psycharis & Potari 2017; Van Es & Sherin 2010). Also these results reinforce research findings that emphasize the critical role of the kind of intervention (in our case based on the use of critical incidents in hypothetical classroom situations) on the improvement of PTs' diagnostic competence (Prediger & Zindel, 2017). The proposed rubric has affordances to map out PTs' development across the course and to identify areas for further enhancement. Following Grisham er al. (2002) observations about teachers' engagement with terminology, the co-existence of mathematical and pedagogical aspects in the MathTASK activities and the proposed rubric makes them valuable tools for PTs' education and teacher professional development as well.

In this paper, we chose to present the levels of RPD in a classroom situation related to a mathematical problem that was not familiar to PTs due to the critical role of the teacher in such situations and the variety of responses we elicited in our data. The choice of the classroom situation and its impact on PTs' response is of interest for future research. Moreover, we observed interrelations between the four characteristics. For instance, PTs who improved their RMD level across the course, they demonstrated similar improvement in the Consistency levels of their responses. In the future, it would be interesting to see in more detail the factors that influence the level change of one characteristic in comparison to the level change of another.

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