The art of changes: bell-ringing, anagrams, and the culture of combination in seventeenth-century England
"Suppose we a number of things exposed, different from each other, as $a, b, c, d, e, \& c ., "$ proposed the mathematician and cryptographer John Wallis in 1685. "The Question is; how many ways the order of these may be varied? As, for instance, how many changes may be Rung upon a certain Number of Bells; or, how many ways (by way of Anagram) a certain Number of (different) Letters, may be differently ordered?". ${ }^{1}$ Wallis's question, written in an appendix to his history of algebra, is an appeal to and reflection of the early modern culture of combination. His calculation determines the number of permutations: a subset of combination in which each unit must be present every time. By establishing the number of ways of ordering the units Wallis describes the limits of the space within which variation can take place. The "Number of Alternations thus calculated", he continued, "will proceed to a vast Multitude beyond what at first one would expect": the space was large, but it was also knowable. ${ }^{2}$ The proposition that the world was made up of a fixed number of units, which could be recombined and reordered in a finite number of
ways, provided the foundation and the structure for many manifestations of the medieval and early modern ars combinatoria, from alchemy to language planning, from the kabbalah and the ars magna of Ramon Lull to the development of a mathesis universalis. The calculation Wallis describes underpins these practices.

This essay explores early modern permutational systems by examining Wallis's calculation and the examples he uses to explain it: letters and bells. In the century before he was writing both had been used in England to exemplify rigorous permutation: letters, in the craze for anagrams; bells, in the new and wildly popular practice of changeringing. Neither was part of the mainstream of the republic of letters but despite, or maybe even because of, their intellectual triviality, both show the pervasiveness of the culture of combination over the course of the seventeenth century. Attending to the form of both highlights the strategies of meaning-making in these contemporary arts of variation.

Wallis's choice of letters to explain permutations draws on a long history: the link between alphabets and combinations can be traced at least as far back as Lucretius for whom letters, like atoms, were elements to be
reordered to describe, or create, the world. ${ }^{3}$ An anagram was, as William Camden explained, "a dissolution of a Name truely written into his Letters, [...] and a new connexion of it by artificiall transposition, [...] making some perfect sence applyable to the person named." ${ }^{4}$ The anagrammatist was able, by moving around the individual letters that made up a word (often a proper name) and choosing a meaningful new order for them, to expose new meanings, even prophecies, from what was originally there. Often derided as a trifle ("some of the sowre sort", wrote Camden, "will say it is nothing but a troublous toy"), the principles of the anagram-variation within a given space, and skilful selection from it-are nevertheless shared with more complicated early modern language systems. ${ }^{5}$ The anagram remained persistently popular from the late sixteenth through the seventeenth centuries, shared in letters and copied out into commonplace books; they were fashionable in the 1630s, particularly at the court of Charles and Henrietta Maria; in the 1640s and 1650s, they had particular resonance for those who felt the world had been turned upside down. ${ }^{6}$

At the same time as English men and women were creating new meaning by moving around the letters of their names,
the sounds of their bells were beginning to ring according to an anagrammatical system. Change-ringing, which still sounds from English church towers today, uses all the bells in a tower, ringing them in rounds. ${ }^{7}$ Every bell must be rung in every round, and the order in which they are rung must never be repeated. The aim is, in theory at least, to work through all the possible orders in which the bells can be rung, without ever repeating a round, and with strict rules about which bells swap places and how. It is a system based on absolute permutation, whose mathematical foundations Wallis described.

Early writers on change-ringing emphasised its exceptional nature. In the opening lines of Tintinnalogia (1668), the first published work on the practice, Richard Duckworth ${ }^{8}$ describes "the Beginning of Changes":

It is an ancient Proverb with us in England (That Rome was not built in a day) by which expression is declared, That difficult things are not immediately done, or in a short time accomplished: But for the Art of Ringing, it is admirable to conceive in how short a time it hath increased, that the very depth of its intricacy is found out; for within these Fifty or Sixty years last past [since the beginning of the
century], Changes were not known, or thought possible to be Rang. ${ }^{9}$

The recently-invented practice was one whose "intricacy" ran swift and deep in seventeenth-century England, but the speed with which change-ringing developed has obscured in the record the motivations behind its creation. ${ }^{10}$ Following Wallis's lead to link together anagrams and bells helps to illuminate this lost ground, and suggests some reasons for the invention of change-ringing. In this essay, I examine change-ringing's form and aesthetics through its entanglements with anagrams and other manifestations of the ars combinatoria, including mathematics and music. Changeringing borrowed from combinatorial systems in order to create new meanings for old bells. But its particular way of adapting these systems went further than such related practices as anagrams, in which the principle of selection drives meaning. In change-ringing, the lack of selection is precisely where meaning is located: it was key to the practice that all the bells hanging in the tower had to be used, with no discrimination between them. By tapping into the contemporary ars combinatoria, but eschewing the principle of selection that was at its heart, change-
ringing forged at once a new strand of the culture of permutation and a new use for pre-Reformation church bells.

Change-ringing and its system

In his travel journal, the merchant and diarist Peter Mundy (c.1596-1667) suggests something of change-ringing's novelty and its distinctiveness. On his return to England in the 1650s, after decades travelling through Europe and Asia, Mundy noted a new sound that he heard at home. He describes, in London, "the sweet Ringing of our tuneable bells, especially in changes". This "deserves notice", he writes, "first, for the Art therein to bee observed, (2) their melody, and (3) the singularity of it. Not the like, nor nothing Near, to bee heard in the whole World beeside". ${ }^{11}$ Mundy's description of this rare and strange ringing appears alongside notes of other things he found intriguing on and after his travels, including thoughts about the motion of the planets, remarks on the Taj Mahal (which he saw under construction), and sketches of flying fish. This ringing was intriguing enough for Mundy to document: it was a practice, he wrote, done "no where out of England". ${ }^{12}$

Change-ringing was England's singular response to the crisis of meaning brought about by the Reformation, which saw many of bells' uses banned or made obsolete. ${ }^{13}$ In the pre-Reformation church, bells had been used to call people to church; announce the transformation of the host; commemorate births, deaths, victories, and coronations; mark curfew, and the canonical hours; warn about fires; or drive away demons and thunder that hovered in the air around the steeple. Bells' sound created a whole language of signals, the subtleties and dialects of which their hearers would have known and understood. Ringing was done for sacred purposes and for secular; most of all, for communal uses in which the sacred and the secular were impossible to distinguish from each other. Bells came under official attack during the sixteenth century, most directly in the 1547 Injunctions of Edward VI which ordered churches (which often had several bells) to reduce their number to just "one bell [...] to be rung or knelled before the sermon". ${ }^{14}$ This injunction was never strictly carried out, though, and many bells remained hanging in church towers. ${ }^{15}$ Although some of their pre-Reformation functions (particularly those related to transubstantiation) were now outlawed, others were transformed to conform to the new
religion and many simply continued. The same bells made the same sounds, though in subtly different patterns and to somewhat different ends: the existing campanological language was not made extinct, but there was an adjustment to its grammar. Although English church bells should have been consigned to disrepair or made subject to iconoclastic fury, then, in fact the utility, and familiarity, of these objects helped them to survive.

The number of bells in English churches actually grew over the course of the seventeenth century, with increasing numbers of rings of 8 bells or more. ${ }^{16}$ This was at least in part to do with change-ringing, which grew to be an extremely popular recreation. The earliest hints we have of something like it come from foreigners like Paul Hentzner, a visitor to England around 1600, who reported that "it is common for a number of them [ringers], that have got a glass in their heads, to go up into the belfry, and ring the bells for hours together for the sake of exercise."17 These long feats of endurance ringing for hours at a time developed into the rigorous, mathematical practice of change-ringing. Societies were formed from the 1630s onwards to further the activity; it continued throughout the 1640 s and 1650 s and became extremely popular in the

Restoration, practised by a very wide range of people and across large parts of the south-east, the Midlands, and East Anglia; in the eighteenth century it was yet more widely done, throughout much of the country. Change-ringing could mark church services but was also done simply for pleasure, a recreation that was practised and performed in leisure time, for fun. Other Protestant countries developed new aesthetically-driven ways of ringing old Catholic bells: the Low Countries, for instance, favored the carillon in which bells were played in tunes by a musician using a keyboard. ${ }^{18}$ Ringing "in changes" was England's solution, connected not principally to music but to combinatorics.

Change-ringing's combinatorial system was made clear in early publications on the practice. Its notation uses numbers, each one of which stands for a bell; each row of numbers represents a round. [See Fig. 1: [Richard Duckworth], Tintinnalogia (London 1668), pp. 30-31. Reproduction of original in Henry E. Huntington Library and Art Gallery, via Early English Books Online [image file submitted in separate document].] Each row of the notation contains every number, 12345, because each bell must be rung in every round. The system relies on constant
variation, in that no round may be repeated; it is constant change, but change of a very orderly kind, its goal being to exhaust all possible variations, all anagrams of those five characters, neatly completing the circuit. The permutations start and end at the same point, 12345. The number of possible rounds, or changes, is determined by the factorial of the number of bells in a tower. ${ }^{19}$ This is the calculation that Wallis described: how many ways a given number of units can be ordered, when all units must be present every time. The factorial, now represented in modern mathematical notation by an exclamation mark, is shown in Table 1. Factorials of numbers 1-12 [see separate file].

For five bells, the number we saw in the notation in Tintinnalogia, there are 5! possible changes, that is, 5 x $4 \times 3 \mathrm{x} 2 \mathrm{x} 1=120$. These numbers escalate so that for twelve bells, there are $479,001,600$ possible orders in which the bells can be rung. Finding paths through all possible changes, avoiding the forbidden repetition of a round, was a difficult task that preoccupied change-ringers and composers of the peals (known as methods) that they rang. Rules governed the ways in which the orders of the bells were allowed to change. Some of these were based on
physical constraints: if a bell finished a row last, for example, it could not then become the first bell to ring in the next round, because the weight and the wheel of the bell prohibits any bell to be rung twice in such quick succession. Most rules, however, had no basis in practice: they were arbitrary, though strict, constraints. Coursing through all possible variations of the order of a number of bells involves patterns, symmetries and, in exhausting all orders, a kind of completeness that seals the circuit.

Ringing in the early seventeenth century was rather limited: the method known as "Cambridge Forty-Eight", consisting of just 48 permutations of the 120 possible on five bells, was for a long time thought "the greatest Peal that was rang or invented". But the practice soon developed, and a method known as Plain Changes allowed ringers to work through all the permutations without ever repeating an order. By the time Duckworth was writing he could boast that "now, neither Forty-eight, nor a Hundred, nor Seven-hundred and twenty, nor any Number can confine us; for we can Ring Changes, Ad infinitum". ${ }^{20}$ The way in which change-ringing both produced and corralled its calculations is described by Duckworth as a kind of victory over number:

Although Philosophers say, No Number is infinite, because it can be numbred; for infinite is a quantity that cannot be taken or assigned, but there is (infinitum quoad nos) as they term it, that is infinite in respect of our apprehension: Therefore a Ringers knowledge may seem infinite to dive so infinitely into such an infinite Subject. ${ }^{21}$

For Duckworth the vastness of number in change-ringing orders the infinite into something that can be known and numbered-not in an abstract way, as "the Philosophers say", but in the heavy business of ringing bells, or by making material the bulk of their calculations. On a folded-out sheet on his manuscript, Mundy wrote out all 720 changes on six bells, working through all the changes and laboriously proving his command over their workings. Like Wallis later in the century, Mundy found "the strange operation \& mistery of numbers" in the system of change-ringing to be remarkable: that so few units could, when permutated in this way, "amount unto such a prodigious summ". ${ }^{22}$ For both Mundy and Duckworth, infinity retains its wonder but is satisfyingly tamed into something that is infinite in respect of "us": although the prodigious numbers may have
seemed infinitely large they can no longer "confine us" and, no matter how big, can be comprehended by the system.

For all its complexity and neatness such order remained something theoretical rather than, in most cases, actual. In fact, Duckworth and the other ringers couldn't ring changes ad infinitum. One reason for this was straightforwardly material: very few churches in England had more than five or six bells. Five bells could ring 120 changes: this is not a huge number, nor a particularly challenging set of rounds to work through, and could be rung in a few minutes. A much more difficult challenge was to find potential ways through the mass of changes possible on twelve bells, for example-and yet the first ringable set of twelve bells in England dates from the very end of the seventeenth century. Even if there had been rings of twelve bells-as there were in the following century-ringing all the 479,001,600 possible changes on these bells would have been an almost impossible feat. "If twelve men should attempt to ring all those Changes on twelve Bells", Fabian Stedman wrote in Campanalogia (1677), the second book to be published on change-ringing, "they could not effect it in less than seventy five years, twelve Lunar Months, one week, and three days, notwithstanding they ring without
intermission, and after the proportion of 720 Changes every hour". ${ }^{23}$ Coursing through all those changes becomes something superhuman, overtaking the lives of twelve men in the pursuit of campanological totality.

There was, therefore, a gap between the speculative and the practical side of ringing: the former was well developed, but the latter struggled to catch up. Early change-ringers were clear that its speculative aspects had connections to contemporary intellectual currents: Stedman declared, at the opening of Campanalogia, that "These clear dayes of Knowledge, that have ransackt the dark corners of most Arts and Sciences, and freed their hidden mysteries from the bonds of obscurity, have also registred this of Ringing, in the Catalogue of their Improvements". The "Practick" and also, crucially, the "Speculative" part of change-ringing have now, he continued, "become perfect, and worthy the knowledge of the most ingenious". ${ }^{24}$ Changeringing had uncovered patterns and permutations that had lain quietly inert within the metal of English bells for centuries, and which forged a new identity and a new purpose for these old objects.

Connections to mathematics and music

The "Invention" of the "Art of Changes", wrote Stedman, was "Mathematical, and [it] produceth incredible effects". ${ }^{25}$ Its mathematical basis in the factorial linked it to 1500 years of combinatorial thinking, both mathematical and philosophical, Asian and European, for which this calculation was the basis. ${ }^{26}$ Although the factorial connects change-ringing to the early modern ars combinatoria, early modern change-ringers' links to contemporary mathematical work were limited. Some English mathematicians used bells to explain the factorial: Wallis, of course, and Newton, who in a college notebook of c.1664-5, performed calculations including determining "how many changes 6 Bells [sic]". ${ }^{27}$ For the most part, though, change-ringing operated in parallel to contemporary combinatorics, and had little contact with the European work I will discuss below. Change-ringing methods are now explained by the kind of algebra known as group theory, concerned with the symmetrical behaviour of groups, which was properly codified only in the nineteenth century; change-ringing's contribution to mathematics was not recognized until very recently. ${ }^{28}$ In the seventeenth century, however, changeringing developed in response not to the mainstream of the
republic of letters but to the new fashion for book-based popular mathematical recreations.

This trend was initiated by the Récréations mathématiques, published in France in $1624 .{ }^{29}$ The exercises contained in this book include conjuring tricks and sleights of hand as well as mathematical questions, many derived from a long tradition of recreational problems. ${ }^{30}$ The Récréations mathématiques passed down to a more popular readership the problems of permutation and combination that were being discussed in more learned mathematical circles at the time and its English translation is, I propose, where early change-ringers learned their mathematics.

In Mathematicall Recreations (1633), the Englished Récréations mathématiques, bells are included within a section on "prodigious progression and multiplication". ${ }^{31}$ Ringers aren't given much credit here for their mathematical prowess: "It is often debated amongst our common Ringers", the author writes, "what number of Changes there might be made in $5,6,7,8$, or more Bells". Ringers, he continues, "spend much time to answere their owne doubts, entering often into a Labyrinth in the serch thereof", and are implied to be among those "that are not versed in Arithmeticke", for whom numbers present "a world
of confusion and difficultie" and who don't even know how to calculate the simple factorial. ${ }^{32}$ The date of this publication, in the 1630s, places it in those times in which, if we believe Duckworth's chronology, change-ringers were in the process of developing their permutations. Even at this early date, however, the association of bells with the factorial seems to have been unique to England: the references to bells in the Mathematicall Recreations do not appear in the original French edition. In the later part of the century, change-ringers could answer back to charges of bad mathematics-because they had perhaps read this very book (or one of the many imitations it inspired). Duckworth and Stedman's examples to qualify and explain the mass that the factorial quickly escalated to produce show their indebtedness to the recreational tradition. In Campanalogia, for instance, Stedman describes the workings of this calculation using reams of paper, diners seated at a table, amount of money paid in rent, and problems in which one party uses the factorial to outwit another-all similar to those found in the Mathematicall Recreations. ${ }^{33}$ One application of the factorial in the Mathematicall Recreations which is, surprisingly, largely absent from change-ringing literature is music. In the former, the
factorial is used first to describe variation on bells, then the human voice and "stringed Instruments, and the Gamat". ${ }^{34}$ But neither Stedman nor Duckworth explain the factorial using musical notes. Elsewhere, musical composition was closely tied to early modern combinatorics, particularly in the work of Marin Mersenne. In his Harmonie Universelle (1636), in the service of "composing the best melody of all those that can be imagined", ${ }^{35}$ Mersenne wrote out all 720 orders of the six notes ut, re, mi, fa, sol, la in both letters and musical notation; elsewhere, he prepared a manuscript in which he wrote out all 40,320 permutations of the full octave, which ran to 672 folio pages. ${ }^{36}$ Athanasius Kircher, heavily indebted to Mersenne, invented in the Musurgia universalis (1650) a device for composing music according to combinatorial principles. Although both Mersenne and Kircher were elsewhere interested in the metal, tuning, and resonance of bells, and their combinatorial compositions are tantalisingly close to change-ringing, neither knew about the practice or mention the way in which it brought together the two aspects of their musical-permutational thinking. Likewise, this continental intellectual current does not seem to have directly influenced early change-ringers. None of the early
writers on the practice mention Mersenne, for example, although it is possible that Mundy was familiar with his work. That ringers had been thinking about permutations before the publication of the Harmonie Universelle is clear from the mention they are given in the Mathematical Recreations, and suggests that change-ringing developed separately from these other combinatorial endeavors.

This is not to say that music has nothing to do with change-ringing. Bells are, of course, musical instruments, a fact mentioned by both Duckworth and Stedman, who also discuss the importance of tuning; Mundy also noted the "melody" of the bells that he heard. Indeed, change-ringing has always favored the most aesthetically-pleasing orders in which the bells can be rung, but as a preference rather than the priority. Writing on Tintinnalogia in his General History of Music (1789), Charles Burney found it "extraordinary, that melody has not been consulted in the choice of changes: there seems a mechanical order and succession in them all, without the least idea of selecting such as are most melodious and agreeable". ${ }^{37}$ Burney's scorn is unfair-there is in change-ringing much more than the "least idea" of melody, and Stedman wrote that good changeringing methods would place the notes of the bells 'that
their Musick may be rendred much more pleasant'. ${ }^{38}$ However, permutational correctness and adherence to change-ringing's rules were, in its early texts, the principal compositional principles; melody, though important, was secondary. Unlike the musical carillon, change-ringing composed sounds on bells with deliberate restrictions. It employed permutation not to find the most agreeable orders but rather as a way to quantify, and revel in, the many orders that existed.

## Change-ringing and language

The most common way in which early writers on changeringing understood their vast calculations was by forging the connection not to music but to letters, writing, and speech. I discuss in some detail two attempts to explain change-ringing in terms of language, by Peter Mundy and Fabian Stedman, before making some links to anagrams and other contemporary combinatorial ideas of language.

Peter Mundy turns to letters to explain his permutational calculations by, first, linking directly the 24 letters of the early modern alphabet with the 24 possible changes available on four bells. He gives a table with the factorials of the numbers 1-24, including the
instructions to "imagine 1234 to bee a or 1 and 2134 to bee b or 22314 to bee c or 3: \& soe of all the rest": he alphabetises the 24 changes. ${ }^{39}$ Mundy does not explain exactly what he is getting at in this calculation, but it might be that he is attempting to permute the permutations: he recognised that there were different ways of exhausting all 24 possible orders, and attempted by assigning a letter to each row to work out just how many there were. One possible way through the 24 changes is represented by the letters of the alphabet in order, a-z; the letters then permute, just as the numbers do in change-ringing notation. His figures are unfortunately slightly wrong: he made a miscalculation of 23! and 24!, so that all his further calculations were based on the wrong numbers. He realised his mistake, and corrected the factorials, but too late to alter the lengthy further workings out.

Mundy's application of his calculations is more important than their accuracy. After he has calculated the factorial of 24 , albeit incorrectly, he imagines how many pages, how many books would be required to contain the workings out of all these numbers. He reckons one book to contain 500 leaves, so a thousand pages, and each page to contain two columns of 24 changes in each. He estimates the
average dimensions of a book, and the area the books would cover. He then calculates the surface area of the earth. When laid down, he concludes, the said number of books wold not be conteyned in 754 such worlds as these [ie like ours] if they were laid one by one, butt if so bee they were to be heaped all uppon one world close packed then would the heap bee 188 1/2 foote round about or 754000 bookes one uppon another over the whole world imagine'd to be dry land. ${ }^{40}$

Mundy's calculations not only cover the earth, they cover it 754 times, or would cover 754 worlds: smothering land and sea with pages and pages of numbers, or words. The conceit of turning the factorial into written or printed words on a page is not unique to Mundy. It appears in the Mathematical Recreations, for example, in which the author imagines all the 24! possible orders of the alphabet printed into books (a slightly different calculation), then laid out over the surface of the world, covering it twelve times over; Mersenne estimated that if printed on paper the songs that can be composed from 22 notes, never repeating a note, would reach from the earth to the firmament. ${ }^{41}$ Mundy used this trope to frame his calculation in terms of a
vastness he could (begin to) comprehend: the book in which these notes are written also contain his diary entries written while he was travelling to the other side of the world, himself covering the surface of the globe.

Mundy's unpublished manuscript establishes an early link between change-ringing and language but, of all the early writers on the practice, Fabian Stedman went most deeply into its theoretical aspects and linguistic implications, often veering off from strict permutation into combination more generally. In the introductory section to Campanalogia Stedman describes the amazing number of variations available from increasingly large numbers of bells, comparing this to words. "If we consider the multitude of different words, wherewith we express our selves in Speech", he writes, "it may be thought almost impossible that such numbers should arise out of twenty four Letters; yet this Art of variation will produce much more incredible effects". He goes on to calculate "the numbers of every quantity of Letters from two to twelve, that may be produced out of the Alphabet": that is, the number of two-letter, three-letter, up to twelve-letter words that can be found in the 24 -letter alphabet, without any letter being present more than once per word. ${ }^{42}$ (Think
of this as a bag of Scrabble letters, or perhaps printing sorts, from which you take a given number to form a word.) Stedman is using combination here: choosing a certain number of items from the whole. He uses the binomial coefficient correctly (choosing $\underline{k}$ items from a total of $\underline{n}$ items-choosing 4 letters from a total of 24 ), and then multiplies this by 4! (24), because each selection of 4 letters can be ordered in 24 ways. He concludes that there are, first, 255,024 possible four-letter words. Repeating this procedure for all the other sets of words (words of 5 letters, and so on) then adding the totals together, "the whole will amount to 1402645824276320 , wherein there are not two alike, nor two letters of one sort in any one of them". ${ }^{43}$

Like Mundy, Stedman extrapolates these numbers into writing-and printing. He imagines all these combinations of letters "being written or printed on large Paper in folio"; allowing 5000 characters to a sheet, he estimates that "all the Houses in the City and Liberties of London" would not be able to contain the reams of paper thus produced. Whereas Mundy the traveller covered the whole world with his linguistic calculations, Stedman stays closer to home. His background as a printer and bookseller is evident in
the applications of his calculations: he guesses that the printed sheets he imagined would be in excess of "all the Books that ever were printed in the world, reckoning only one of each Impression". ${ }^{44}$

Stedman imagines the calculation he has performed being extended, so that all twenty-four letters of the alphabet are used, and each letter can be used more than once per word-a calculation also described in the Mathematicall Recreations, although less thoroughly examined there. ${ }^{45}$ Stedman wonders how long it would take people to say each word out loud, guessing that: the infinite numbers of them [words] would not permit a Million of men to effect it in some thousand of years: it would be evident, that there is no word or syllable in any language or speech in the world, which can be exprest with the character of our Alphabet, but might be found literatim and entire therein, and more by many thousands of Millions than can be pronounced, or that ever were made use of in any language. ${ }^{46}$ He begins by imagining sheer quantity, expressed in terms of language and time together: the duration of words spoken aloud by the labour of millions. There is excess in this not-quite-infinite verbal mass, but this provides
opportunities for linguistic development, beyond "any language" that is currently known. Indeed, these numbers of words "would not permit" the many men to speak them: these languages have a life and agency that goes beyond that of the limited humans that might try to say them out loud. Here Stedman exceeds the quantifying impulse perhaps learned from the books of mathematical recreation. In this passage the numbers he has calculated point towards a knowledge of the world that we do not yet possess.

Stedman and Mundy weren't alone in invoking the combinations and permutations of all the letters of the alphabet. Take Kircher's arca stenographia, for example, a kind of experimental writing machine described in his Polygraphia nova et universalis (1663). This machine comprised tables, each bearing a letter, which could be moved around at will. Kircher, who gives 2585201673888497666640000 as the number of combinations of the 24 letters of the alphabet, declares that "there is no conceivable sentence occurring in any language which cannot be represented on the tablets; thus this narrow box and the letters enclosed therein surpass all the libraries of the whole world." ${ }^{47}$ Kircher claims his machine to be totalising, accommodating everything that could be or has ever been
said, but in a way that is retrospective rather than generative-and therefore different from Stedman's system. The exhaustion Kircher describes points towards the limits of language rather than its potential, as Haun Saussy explains:

The lesson of the machine is that no matter how marvelous, it is still not miraculous; no matter how many combinations a finite set of elements can produce, its number still falls infinitely short of infinity; and thus the triumphant display of large numbers stands for the exhaustion of language as much as for its fecundity. ${ }^{48}$

The large but known numbers created by what Saussy calls "brute-force permutation" close down meaning-making even as they describe the vast space in which it operates. Other linguistic projects that attempted to use the fullness of permutation rarely did so with quite the gleeful optimism that Stedman displays. For Kircher-as for other writers who invoked the combination of all 24 letters of the alphabet, including Leibniz, Alsted, and the German baroque poets Quirinus Kuhlmann and Georg Philipp Harsdörffer-a machine that contains everything ever written is something that captures, archives, and anticipates human linguistic
development. ${ }^{49}$ For Stedman, it is an engine with which to explore. Crucial to this is the principle in change-ringing that no unit, no arrangement of units, is better than any other. Freed from the imperative to select, but happy to remain bounded by the limits that permutation provided, change-ringers saw only potential in the variation they had found. This wasn't infinity but it was, as Duckworth wrote, "infinitum quoad nos", the infinite "in respect of our apprehension". Change-ringers were excited to understand the limits of the possible space and then, as Duckworth wrote, "dive so infinitely into such an infinite Subject". ${ }^{50}$

For most epistemological projects the bulk of the factorial obscured rather than produced knowledge, and when total factorial plenitude was invoked in combinatorial systems it was often more to bewilder than to clarify. In $\underline{A}$ New Method of Cryptography (1666), for example, Samuel Morland revels in the potential for cryptography that, in a simple code of nine letters or units, there are potentially 362,880 transpositions (that is, 9!). He gives a table of the factorial, and even invokes the familiar transformation of these numbers into writing-sheets of paper, or volumes in print-to show the impenetrability of these profuse calculations. ${ }^{51}$ Cipher, as Katherine Ellison argues,
"perhaps more visibly than any other kind of communication, is a language one can get lost in, a language out of which readers must try to find a way." ${ }^{52}$ Bacon recognised that bells could be used to carry secret messages, using them in the De Augmentis scientarium (1623) to describe the bilateral cipher, but change-ringing's cryptographic potential was not exploited (at least, not until Dorothy L. Sayers's 1934 mystery The Nine Tailors). In part this was because change-ringing theorists wanted not to bury a message in its permutational profusion but rather to "get lost" in it-to "dive in", as Duckworth wrote, to its labyrinth. In cryptography, as the example from Morland shows, it is only informed selection that allows explorers to find their way out-quite different to change-ringing's joyful being-within.

Language and the principle of selection

I want to return to Wallis's original comparison between bells and anagrams to interrogate what kind of meaning might be made by change-ringing's lack of selection. In the activity of creating anagrams there was, as George Puttenham wrote, no "great gayne nor any great losse
unlesse it be of idle time": no net gain or loss in the making of anagrams, and (as Juliet Fleming points out) no gain or loss in the reordering of the letters, either. ${ }^{53}$ The anagram must remain "bounded by the letters", as a 1638 book of anagrams asserted: all must be present every time but, in the careful reordering of letters and selection of anagrams, something new was added-revealed-that was hidden, but immanent, in the original. ${ }^{54}$

Unlike the change-ringer, the anagrammatist selects. Each anagram has to contain every letter, but not all of the new orders of the letters has to be used. We see this in Wallis's Treatise, when he proposes moving around the letters of the word ROMA, a common anagram. Of the twentyfour words that these four letters can create, Wallis writes, "these seven are only useful; Roma, ramo, oram, mora, maro, armo, amor". "The other forms are useless", he writes, "as affording no (Latin) Word of known signification". ${ }^{55}$
[Table 2: Anagrams of the word ROMA (viable words shown in italic). From Wallis, Treatise, 117. See separate file]

Wallis lays bare the waste words of his endeavours, showing all the useless words from which the meaningful ones are to be chosen. More commonly, anagrammatists hid away the useless or contradictory reshufflings of the names on which they worked.

Makers of anagrams explained in great detail the selection they undertook. George Puttenham gives the example of Elizabeth's name, translating the English "z" to a Latin double "s". ${ }^{56}$ He is pleased with two of the results, Multa regnabis ense gloria, and Multa regnabis sene gloria. But here the principle of selection comes in, because these are the only two anagrams that will do. "Af[ter] the first search", Puttenham writes, "the same letters being by me tossed \& tranlaced fiue hundreth times, I could neuer make any other, at least of some sence \& conformitie to her Maiesties estate and the case". ${ }^{57}$ Puttenham is clear: it's not that he hasn't found any more anagrams, but rather that he hasn't found any more that can responsibly be applied to the Queen. Unpoliced anagrams could unleash unsavory or even dangerous reflections on the original word. "In a good Mans Name ye shall find some Evil, and in an evil Man's Good, according to the Searcher", ${ }^{58}$ wrote William Drummond of Hawthornden: careful selection of correct, appropriate
anagrams was paramount, because each name contained within it its own subversive potential.

Rules dictated the methods by which this activity was to be performed but there was skill involved in the interpretation of the anagrams, too. Many seventeenthcentury anagrams are accompanied by acrostics or other verses that dilate and explain it, particularly when the anagrammatist struggled to find a new word that worked. Some are "more obscure Enigma's", admits John Coysh in his poem on Francis Lenton's anagrams, and the poems help to clarify the new meaning of the name:

His lines licke into forme (like a Shee Beare)
Who newly hath produc't a shapelesse whelpe,
Makes it a perfect creature by her helpe ${ }^{59}$
Coysh draws on Pliny's description of the newborn bear cub, only given shape by the actions of its mother, to describe how the anagram-maker can create "perfect" form from something "shapelesse" and inchoate. The anagrammatist's role was both creative, licking into form, and revelatory, uncovering what was already latently there. The meaning of the anagram hovered between something made and something found, and its prognostic capabilities always lurked beneath its surface.

If anagrammatical endeavors attempted to uncover new meaning, another form of linguistic play-with a direct link to change-ringing-was concerned in a rather narrower way to preserve what Puttenham called "sence \& conformitie". In the anagram, the letter is the unit to be moved around; in what Julius Caesar Scaliger named "Proteus verse", the unit is the word. ${ }^{60}$ The most puzzled-over early modern example, which occupied figures from Vossius to Leibniz, Wallis to Browne to Bernouilli, is the verse Tot tibi sunt dotes, virgo, quot sidera caelo ("Thou hast as many virtues, O Virgin, as there are stars in heaven"). The line has eight words so it can, as Wallis remarked, be "turned absolutely" 40,320 ways (that is, 8!) but fewer ways "so as to preserve the verse". ${ }^{61}$ The skill here is not in finding meaning outside the text, but just preserving both the sense and the metre as the words are "turned". ${ }^{62}$

Proteus verse which eschews any epistemological or divinatory purpose, but just delights in the retelling of the same thing in different ways, is more like change-ringing-and it has a direct link to the practice in Roger Tisdale's poem of 1623, Pax Vobis, or Wits Changes, tuned in a Latine Hexameter of Peace. Tisdale, now best known for addressing Donne in a small volume on the practice of the
law, ${ }^{63}$ dedicated Pax Vobis to James I and the peace he had brought about, and offers hope for Charles's success in the Spanish Match. Tisdale's hexameter, "Pax tibi Rex Salem, per te Gens florida regnat", is also a chronogram: it contains all the letters that, in Roman numerals, make up the year 1623. Tisdale finds that this line can be moved around up to $9!$ ways, but can do so while retaining its sense exactly 1623 ways-neatly matching the year of its composition. Tisdale borrows from change-ringing in the title of his list of the possible orders of the verse: "The Changes rung out'. ${ }^{64}$ The words move through the sentence, as the numbers moved through the row in change-ringing notation, although Tisdale abides not by the laws of change-ringing but by the metrical rules of Latin hexameter (indeed, he claims in the prefatory poem to go "beyond" Scaliger in this, and his line is very similar, metrically, to the famous Proteus verse-though perhaps a little more inelegant). ${ }^{65}$ Tisdale claims to have selected the 1623 orders that make sense, but he fails to write them all out: "Tedious it were to reade, and more to write,/ One thing so often: therefore I recite/ Only a few for demonstration sake". ${ }^{66}$ Whether this is because he hadn't really found exactly that number of turns, or because he got bored, is
unclear. Regardless, Tisdale's experiment shows that, by 1623, there was awareness of change-ringing's system, even though he, like the other linguistic experimenters I have discussed, differs from it by discarding some of his many permutations.

What marks change-ringing out from other combinatorial systems, then, and the innovation that it makes to the ars combinatoria, is that there is no selection and no redundancy, either. Nothing is waste; all units, and all arrangements of them, are important-equally so. Most of these systems did have a purpose, using combinations to generate knowledge, find hidden meanings, to create beauty, or to reflect the real world. But although they invoke all the possibilities, rarely do they actually exhaust them: such systems always discard a remnant that they do not use, because cutting away-as in other forms of early modern writing-is what creates meaning. ${ }^{67}$ Systems that give equal preference to all combinations can go wrong, like Puttenham's unlicensed anagrams which could potentially be made to produce treacherous statements, the nonsense words generated by Wallis's permutation of the letters ROMA, or the obfuscation of Morland's codes. Not all units are equal; not all orders of words make sense; not all letters
are all used equally often. Some possibilities must be excluded or removed in order for any others to make sense; meaning, like beauty, is created by the interplay of expansiveness and constraint. Change-ringing is very like some of these combinatorial systems, but its value comes precisely from invoking the plenitude of the factorial with no selection. It is in this sense an extreme, even a pure, version of the ars combinatoria: it is total anagram.

Exhaustion of meaning, exhaustion as meaning

Where selection is the engine of meaning in these other combinatorial systems, in the exhaustive system of changeringing, meaning is created by not selecting. The distinctiveness of change-ringing's aesthetics come to light when compared with a parallel set of systems far away in time and type: some modernist works, which also employ exhaustive permutations to create a new field of signification. Serialist musical compositions by Schoenberg, Webern, and others, for example, display a commitment to using permutations of all the twelve notes of the chromatic scale, analogous to change-ringing, although these composers do select on aesthetic grounds. ${ }^{68}$ Beckett's
permutations-Murphy's "total permutability" of the order in which he can eat his five biscuits, for example, or the description, in Watt of the best way in which to organise a committee of five people to glance at each other-are more strictly concerned with exhausting all the possibilities. ${ }^{69}$ At the same time, they offer something beyond the banality of obsessive completion. Here exhaustion itself becomes something almost transcendent, and accrues meaning aside from the terms and the units that have performed the exhaustive practice. It is the "strange amalgam of shock and boredom" in modernist cultural production that Sianne Ngai has called "stuplimity". 70 In Beckett's works, Ngai argues (using How It Is in particular), "large but finite numbers take the place of the infinity associated with Kant's mathematical sublime, yet the effect of these enumerations is [...] to call attention to representational or conceptual fatigue, if not collapse". ${ }^{71}$ Extrapolating and anachronising Ngai's argument, might change-ringing's intricacy replace, or at least cover up, representational fatigue (if not collapse) in the previous meanings and uses of church bells?

In the system of change-ringing the set of variables, not biscuits but bells, are made equal and without
preference; while the older meanings of church bells still have a lingering aural presence in the layers of their sound, signification is renounced in favor of the obsessive, but active, drive towards the elimination of all the orders in which they can be rung. Change-ringing is altogether more optimistic than Beckett's futile permutations, though: if it overwrites previous meanings, it aims to create something that is joyful in its plenitude.

Change-ringing and church bells in the seventeenth century

The lack of preference and selection in the system of change-ringing had material consequences. To discuss this, we must return to the aspects of ringing that I've neglected: the groups of people who did the practice, the noisy and unavoidable sound that it made, and the context into which it emerged. The principle of the lack of selection had material, social, and aural consequences that help to explain why change-ringing came about, and became so popular, in the seventeenth century.

England had always been proud of its claim to be the "ringing isle" and parishioners felt attached to and
responsible for the bells in their parish church, which were often under the custody of the churchwardens rather than the diocese. The communal ringing of bells, done always by men and often by drunk men, had been going on for centuries, and it's this rather disorganized practice that coalesced into the strict rules and forms of changeringing. In a recent essay, Christopher Marsh argues persuasively that continued recreational ringing in the seventeenth century (which includes change-ringing, and less organized practice as well) was "a new outlet for the expression of certain deeply traditional socio-religious instincts that had been endangered by Reformation beliefs and sensibilities", and indeed that such ringing "provid[ed] a fresh language through which to explore the ancient combination of recreation and religion on holy ground", now that other sports and games were excluded from the churchyard. ${ }^{72}$ Marsh highlights the social power of this delight in ringing bells, a power that allowed the practice and the material objects to continue and remain, and for some of the older associations of the activity to be carried forward somehow too.

The obsessive plenitude of change-ringing is a related, parallel power to the social impetus that Marsh has
identified. It fits into this story of communities who struggled to find official justification for having their bells, but who nevertheless very much wanted to keep them. Change-ringing was the expression of a desire to use and keep church bells-all the bells. Using the permutational exhaustion borrowed from the ars combinatoria, taken to extremes with its lack of selection in a way that has later parallels in modernist exhaustion, change-ringing created a need for all the bells to be rung at once. Change-ringing's indifference to the sounds of individual bells, and the different ways and occasions of ringing them, formed a kind of flattening out of England's rich ringing soundscape. It formed a different system, not straightforwardly semantic or aesthetic, which joined with these other kinds of ringing as a new dialect of campanalogical language. Change-ringing, like the anagram, found something new in something old.

The combinatorial, anagrammatical extrapolations of change-ringing were particularly strong in the late seventeenth century, when that gap between theory and practice was at its greatest. The technologies of ringing were catching up with its system, but were not quite there yet. By the end of the century, when larger rings
(including one containing twelve bells) were ready to be used, and when change-ringing had been securely entrenched as a national practice, we see a marked decrease in changeringers' interest in the intricacies of abstract permutation. The authors of Campanalogia Improved (1702), a successor to Stedman's book, confided that "it will be very improper, and not in the least to our purpose to stuff this Treatise full of unnecessary and useless Examples of the Variation of Numbers". ${ }^{73}$ By this time, of course, selection was possible in the practice of change-ringing: once you have twelve bells to ring, you don't have to commit to spend seventy-five years ringing all the millions of changes possible on them. Change-ringers spent less time thinking about finitely infinite possibilities and more time devising methods, selecting melodic patterns of changes from the many available, that they would be able to ring and that sounded particularly nice. At the same time, the fad for anagrams had definitively passed: in Mac Flecknoe (composed 1676), Dryden anticipated this development by banishing Shadwell to "acrostic land", where he could use his paltry poetic talent just for anagrams, "tortur[ing] one poor word ten thousand ways"; in The

Spectator, Addison located a "regiment of Anagrams" in the "Region of false wit'. ${ }^{74}$

By the beginning of the eighteenth century, then, the theory of change-ringing had done its work. By harnessing the contemporary interest in combinations early writers on change-ringing had developed what Stedman called "the Art of Changes", which "produceth incredible effects". ${ }^{75}$ The 'meaning' of change-ringing was, and remains, difficult to define. But it was the "incredible effects" of this noisy, beautiful version of the "art of variation" that explains why the English air, still now, rings with anagrams.

I am grateful to audiences at the University of East Anglia, the University of Oxford, UCL, and the EMPHASIS seminar at the Institute of Historical Research in London for helpful feedback on versions of this paper. My particular thanks for their careful reading of it in draft form goes to the two anonymous reviewers and to Steven Connor, Dennis Duncan, Nick Gaskill, and Vanessa Harding. ${ }^{1}$ John Wallis, A Treatise of Algebra, Both Historical and Practical (London, 1685), Additional Treatise 4, Chapter II, 115.
${ }^{2}$ Wallis, Treatise, 116. On Wallis's approach to the history of algebra see Jacqueline Stedall, A Discourse Concerning Algebra: English Algebra to 1685 (Oxford: Oxford University Press, 2003).
${ }^{3}$ Lucretius, De rerum natura, 2.1013-22.
${ }^{4}$ William Camden, Remaines of a greater worke, concerning Britaine [...] (London, 1605), 150.
${ }^{5}$ Camden, Remaines, 151.
${ }^{6}$ See Lois Potter, Secret Rites and Secret Writing: Royalist Literature 1641-1660 (Cambridge: Cambridge University Press, 1989), 50-51

7 The Oxford English Dictionary gives 1872 as the first instance of "change ringing", but an earlier appearance is in the Kentish Post of 24 January 1756 ; see Order and Disorder in the Eighteenth Century: Newspaper Extracts about Church Bells and Bellringing, ed. Cyril Wratten and John Eisel ([Great Britain]: The Central Council of Bell Ringers, 2010), 100. I am grateful to John Eisel for this reference. I use "change-ringing" throughout for convenience, although seventeenth-century writers used "ringing in changes" or "the art of changes".
${ }^{8}$ Duckworth, senior bursar of Brasenose College, Oxford, and sometime rector of Steeple Aston, Oxfordshire, has been
confidently identified as the author of this anonymouslypublished work. See John C. Eisel, "Duckworth, Richard," Oxford Dictionary of National Biography, Oxford University Press, 2004; online edn, May 2014 [http://ezproxyprd.bodleian.ox.ac.uk:2167/view/article/8138, accessed 26 Feb 2017]

9 [Richard Duckworth], Tintinnalogia, or, The art of ringing (London, 1668), 1-2. 10 There are many accomplished histories of the practice, often written by change-ringers, but no firm evidence survives to explain why change-ringing takes its permutational form. Some that are particularly accessible to the non-ringer include Ron Johnston, Bell-Ringing: The English Art of Change-Ringing (Harmondsworth: Penguin, 1986), Ernest Morris, The History and Art of Change Ringing (London: Chapman and Hall, 1931); Jean Sanderson, ed., Change Ringing: The History of an English Art, 3 vols (Cheltenham: Central Council of Change Bell Ringers, 19871994); and regular pieces in Ringing World, the weekly change-ringer's journal.
${ }^{11}$ Bodl. Rawl. A.315: f.215v. The five pages Mundy devotes to change-ringing in this manuscript are undated, but observations a few pages later are dated 9 September 1654,
so perhaps he wrote them some time shortly before that. I am grateful to Natalya Din-Kariuki for sharing with me her forthcoming essay on Mundy.
${ }^{12}$ Bodl. Rawl. MS A315: f.215v.
${ }^{13}$ See David Cressy, Bonfires and Bells (London: Weidenfeld and Nicolson, 1989), Christopher Marsh, Music and Society in Early Modern England (Cambridge University Press, 2010), 454-504, and Peter Marshall, Beliefs and the Dead in Reformation England (Oxford: Oxford University Press, 2002), particularly 128-132.
${ }^{14}$ Tudor Royal Proclamations. Vol. 1: The Early Tudors (1485-1553), ed. Paul L. and James F. Larkin (New Haven, CT: Yale University Press, 1964), 399.
${ }^{15}$ On the fate of bells in English Reformation iconoclasm, see Margaret Aston, Broken Idols of the English Reformation (Cambridge: Cambridge University Press, 2015), 445-488. ${ }^{16}$ Numbers of bells are notoriously difficult to calculate, in part because the frequent recasting of bells obscures the history of their predecessors. Cyril Wratten estimated there to have been 17 rings of 8 bells in England in 1668, 49 in 1699, and 371 in 1799: part of a general increase in numbers of bells and size of rings. See Wratten, "Bells in the Eighteenth Century", in Sanderson, ed., Change Ringing,
vol. II, The Eighteenth Century: An Overview (1992) ed. John Eisel and Cyril Wratten, 40; see also Wratten, "The Growth of Change Ringing" in the same volume, 50-59.
${ }^{17}$ Paul Hentzner, Travels in England during the Reign of Queen Elizabeth, trans. Richard Bentley (London, 1797), 64
${ }^{18}$ Luc Rombouts. "Carillon." Grove Music Online. Oxford Music Online. Oxford University Press, accessed January 25, 2017,
http://www.oxfordmusiconline.com/subscriber/article/grove/m usic/04929.

19 Although the term "factorial" wasn't in use until the nineteenth century, the function itself was well known, discussed by mathematicians and amateurs, including the early writers on change-ringing.

20 [Duckworth], Tintinnalogia, 2.
${ }^{21}$ [Duckworth], Tintinnalogia, 2.
22 Bodl. Rawl. MS A315: f.215v
${ }^{23}$ Fabian Stedman, Campanalogia: or the Art of Ringing
Improved. With plain and easie rules to guide the
practitioner in the ringing all kinds of changes. To which is added, great variety of new peals (London, 1677), 4. Stedman was apprenticed as a printer, and worked as a bookseller. Campanalogia is the only known book he wrote,
though he was involved with the publication of Tintinnalogia.
${ }^{24}$ Stedman, Campanalogia, 1-2.
${ }^{25}$ Stedman, Campanalogia, 2.
${ }^{26}$ The factorial is first known in writing, in the Jewish mystical text the Sefer Yetzirah (ca. 400 CE ), and in Indian mathematics of around the same time. On permutation and the uses of the factorial see Donald Knuth, The Art of Computer Programming, Vol. 3: Sorting and Searching (2nd edition) (Reading, Mass.: Addison-Wesley, 1998), 23-24; The Art of Computer Programming, Vol. 4, Fascicle 4: Generating all Trees: History of Combinatorial Generation, esp. 52-53 (Upper Saddle River, NJ: Addison Wesley, 2006); and "Two Thousand Years of Combinatorics", in Combinatorics: Ancient and Modern ed. Robin Wilson and John J. Watkins (Oxford: Oxford University Press, 2013), 3-38. On the ars combinatoria see Paolo Rossi, Logic and the Art of Memory, trans. Stephen Clucas (Chicago: Chicago University Press, 2000) •

27 Isaac Newton, college notebook, c.1664-5, Cambridge University Library Add. MS 4000, f.12r
${ }^{28}$ On the prescient mathematics of early change-ringing see Arthur T. White, "Fabian Stedman: The First Group

Theorist?," American Mathematical Monthly 103:9 (1996): 771-778. Donald Knuth discusses the algorithm that explains the method of ringing known as Plain Changes, which he traces to Peter Mundy's manuscript, in The Art of Computer Programming, Vol. 4, Fascicle 2: Generating all Tuples and Permutations, 42-44 (Upper Saddle River, NJ: Addison Wesley, 2005), section 7.2.1.2, Algorithm P. 29 The French original has been variously attributed. Formerly ascribed to Jean Leurechon, a Jesuit priest and mathematician, the authorship has more recently been attributed to Jean Appier Hanzelet (writing under the pseudonym Henrik van Etten), a fireworks master and engraver from Lorraine. See Albrecht Heeffer, "Récréations mathématiques: A study on its authorship, sources and influence", Gibeciere 1 (2006): 79-167, esp. 11-12; for an earlier discussion see Trevor Henry Hall, "Mathematical Recreations": An Exercise in Seventeenth-Century

Bibliography (Leeds: Leeds University School of English, 1969) .
${ }^{30}$ For a detailed investigation of the sources of the problems collected in the book, see Heeffer, "Recreations", 12ff.
${ }^{31}$ [Jean Appier?], Mathematicall Recreations (London, 1633), 178. The English translation has been ascribed to the mathematician William Oughtred, whose name appears on the titlepage of the 1653 edition, but it's now thought unlikely that he was responsible. See Heeffer, "Recreations".
${ }^{32}$ Mathematicall Recreations, 185
${ }^{33}$ Stedman, Campanalogia, 16-18; Mathematicall Recreations, 188-9.
${ }^{34}$ Mathematicall Recreations, 186.
${ }^{35}$ Quoted in Floris Cohen, Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580-1650 (Dordrecht: Springer, 1984), 112.
${ }^{36}$ Marin Mersenne, Harmonie universelle, contenant la théorie et la pratique de la musique, vol. 2, Traitez de la Voix et des Chants (Paris, 1636), 111-15, 117-28. The MS is Bibliothèque Nationale de France, Fonds Français, no. 24256. See Knuth, "Two Thousand Years of Combinatorics", 10 and Cohen, Quantifying Music, 112-14.
${ }^{37}$ Charles Burney, General History of Music (London, 1789), 413
${ }^{38}$ Stedman, Campanalogia, 20.
${ }^{39}$ Bodl. Rawl. MS A315: f.216r
${ }^{40}$ Bodl. Rawl. MS A315: f.216v
41 Anon., Mathematicall Recreations, 188; Mersenne,
Harmonie, 108
42 Stedman, Campanalogia, 13.
${ }^{43}$ Stedman, Campanalogia, 14.
${ }^{44}$ Stedman, Campanalogia, 14.
${ }^{45}$ Mathematicall Recreations, 188.
${ }^{46}$ Stedman, Campanalogia, 16.
${ }^{47}$ Athanasius Kircher, Polygraphia nova et universalis ex
combinatoria arte detecta (Rome, 1663), 141. I use the
translation given in Haun Saussy, "Magnetic Language: AK and Communication" in Athanasius Kircher: The Last Man who Knew Everything ed. Paula Findlen (London: Routledge, 2004), 272 .
${ }^{48}$ Saussy, "Magnetic Language", 276-7.
${ }^{49}$ Saussy, "Magnetic Language", 276; see also Jan C. Westerhoff, "Poeta Calculans: Harsdörffer, Leibniz, and the mathesis universalis", Journal of the History of Ideas 60.3 (1999), 449-467.
${ }^{50}$ [Duckworth], Tintinnalogia, 2.
${ }^{51}$ Samuel Morland, A New Method of Cryptography (London, 1660), 4 and 7.
${ }^{52}$ Katherine Ellison, "Cryptogrammatophoria: The Romance and Novelty of Losing Readers in Code", Eighteenth-Century Fiction 20.3 (2008), 293.
${ }^{53}$ Juliet Fleming, Graffiti and the Writing Arts of Early Modern England (London: Reaktion, 2001), 123.
${ }^{54}$ Anon., in Francis Lenton, The Innes of Court
Anagrammatist: Or, the Masquers masqued in Anagrammes (London, 1634), sig. B1r.

55 Wallis, Treatise on Algebra, 117.
56 English anagrammatists allowed themselves some liberties in the choice of letters: "z" to double "s"; "w" to two "v"s, and so on. These rules weren't meant as a cheat, but as a way to allow language, and letters, to be supple enough to work with.
${ }^{57}$ George Puttenham, The Arte of English Poesie (London, 1589), 84.
${ }^{58}$ William Drummond, "The Character of a Perfect Anagram", in The Works of William Drummond, of Hawthornden
(Edinburgh, 1711), 231. On this see also William H. Sherman, "Of Anagrammatology", English Language Notes 47.2 (2009), 139-148.
${ }^{59}$ Coysh in Lenton, The Innes of Court Anagrammatist, A3vA4r.
${ }^{60}$ Julius Caesar Scaliger, Poetices Libri Septem (Lyon, 1561), Book 2, Ch. 30, 73. On the mathematical implications of Proteus verse see Donald Knuth, "Two Thousand Years of Combinatorics", in Combinatorics ed. Wilson and Watkins, 21-24; on Proteus verse as baroque poetry, see Westerhoff, "Poeta calculans".
${ }^{61}$ Wallis, Treatise on Algebra, 118.
62 There is subversive potential here, too. We might think of similar endeavors, such as cento poems created from fragments-words from Virgil, for example, rearranged to have a Christian meaning (on which see Scott McGill, Virgil Recomposed, Oxford: Oxford University Press, 2005), or the artist Kabe Wilson's reworking of Virginia Woolf's A Room of One's Own. Wilson reordered all the words of Woolf's essay to make a new text, Of One Woman Or So by Olivia N'Gowfri (2014), the story of a young black woman who goes to study at Cambridge; disgusted by the racism and sexism she encounters there, she burns down the university library.
${ }^{63}$ Roger Tisdale, The Lawyers Philosophy (London, 1622), A3r-A5v.
${ }^{64}$ Roger Tisdale, Pax Vobis, or Wits Changes, tuned in a Latine Hexameter of Peace (London, 1623), 4.

65 I am grateful to Anthony Ossa-Richardson for his advice on this verse.
${ }^{66}$ Tisdale, Pax Vobis, 6.
${ }^{67}$ On cutting as writing see Adam Smyth, ""Shreds of holinesse': George Herbert, Little Gidding, and Cutting Up Texts in Early Modern England", English Literary Renaissance 42.3 (2012): 452-481, and the special issue ed. Juliet Fleming, Bill Sherman, and Adam Smyth on "The Renaissance Collage" of The Journal of Medieval and Early Modern Studies 45.3 (2015). ${ }^{68}$ See, for instance, Arnold Schoenberg, Style and Idea: Selected Writings of Arnold Schoenberg, ed. Leo Stein, trans. Leo Black (London: Faber and Faber, 1974), 219; more generally, Arnold Whittall, The Cambridge Introduction to Serialism (Cambridge: Cambridge University Press, 2008). ${ }^{69}$ Samuel Beckett, Murphy (1938), in Works (New York: Grove Press, 2006) I, 60.The many combinatorial works of the Oulipo share structural similarities with change-ringing too, but don't aim for exhaustion in quite the same way. 70 Sianne Ngai, Ugly Feelings (Cambridge, Mass.: Harvard University Press, 2005), 2.
${ }^{71}$ Ngai, Ugly Feelings, 274.

72 Christopher Marsh, "‘At it ding dong': recreation and religion in the English belfry, 1580-1640", in Worship and the parish church in early modern Britain. ed. Alec Ryrie and Natalie Mears (Farnham: Ashgate, 2013), 151-172, at 168 and 169.

73 [John Doleman] and C.M., Campanalogia Improved, or, the Art of Ringing made easie (London, 1702), 14.
${ }^{74}$ John Dryden, 'Mac Flecknoe', lines 206, 208, in Dryden:
Selected Poems, ed. Paul Hammond and David Hopkins (Harlow: Longman Pearson, 2007), 148-9; Joseph Addison, The Spectator 63 (12 May 1711), 271.
${ }^{75}$ Stedman, Campanalogia, 2

