TACTILE CONSTRUCTION OF MATHEMATICAL MEANING: BENEFITS FOR VISUALLY IMPAIRED AND SIGHTED PUPILS

Angeliki Stylianidou, Elena Nardi

University of East Anglia

Tactile perception in inclusive mathematics education has been associated mostly with visually impaired (VI) pupils. We endorse an alternative perspective: that tactile perception can be of high relevance to the mathematical learning of sighted pupils too. Here, we report from a study in which we explore the impact of universally designed mathematical practices upon the mathematical learning of VI and sighted pupils in class. In the episodes we report here, we invited the VI pupil and the sighted pupils of a Year 5 (Y5) class to construct meaning of shapes through touch. We found that tactile perception led not only to better inclusion of the VI pupil but also brought benefits to sighted pupils too. We conclude with our study's aim to contribute to inclusion and challenge ableism in the mathematics classroom.

INTRODUCTION

Inclusive education has been an issue of international consideration especially since the Convention on the Rights of Persons with Disabilities (CRPD) (United Nations, 2006). Article 2 of the CRPD defines two different ways in which inclusion can be achieved: "reasonable accommodation" and "universal design". The former denotes modification and adjustments which are done to ensure that people with disabilities enjoy all human rights and fundamental freedoms on an equal basis with others, without imposing a disproportionate or undue burden. The latter denotes the design of environments, services and tools that can be used by every person, to the biggest extent possible, without the necessity for adaptation or specialised design.

In the study we draw from in this paper, we focus on the inclusion of VI pupils in mathematics lessons. We investigate how inclusion and disability are constructed in the discourses of teaching staff and pupils in mainstream mathematics classrooms; and, we examine how collaboratively designed mathematics lessons impact upon teaching staff's and pupils' (both VI and sighted) discourses on inclusion and disability. Our work resonates with studies (e.g. Nardi, Healy, Biza, & Fernandes, 2018) which challenge "ableism" in mathematics education – the "network of beliefs, processes and practices that produce a particular kind of self and body (the corporeal standard) that is projected as the perfect, species-typical and therefore essential and fully human" (Campbell, 2001, p. 44) and which objects to a perspective on disability "as a diminished state of being human" (p. 44).

A conjecture that our study explores is whether, and if so how, universally designed mathematical practices lead not only to better inclusion of VI pupils but also bring benefits to all pupils. In this paper, we discuss the impact of one tactile mathematical

2019. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 343-350). Pretoria, South Africa: PME.

task upon the mathematical learning of both VI and sighted pupils. We present evidence from one classroom task. We include evidence of a mathematical contribution, made by a sighted pupil, which involves tactile as well as visual perception of a shape that is a circle minus a circular segment. We then include evidence of a mathematical contribution, made by a VI pupil, which involves tactile perception of the same shape and of a circle. We highlight the benefits that the pupils' tactile constructions of mathematical meaning have generated and we conclude with a broader discussion of these benefits as they emerge from the study's ongoing analyses.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Vygotskii (1993) suggested that, in VI pupils, the substitution of their eyes with their hands may result in the emergence of perspectives that differ from those of sighted pupils, due to the difference in the sensory tool through which they access mathematics and construct mathematical meaning. In the context of mainstream mathematics classrooms with VI and sighted pupils, tactile perception has been associated mostly with VI pupils (e.g. Argyropoulos & Stamouli, 2006; Leuders, 2016).

This association of tactile perception with visual impairment often results in the inclusion of VI pupils through accommodations of visual materials typically used for sighted pupils. While in some cases accommodations are successful – e.g. design of tactile shapes for the VI pupil while the rest of the class used the visual shapes of the school textbook (Argyropoulos & Stamouli, 2006) – in other cases such accommodations have limitations. The limitations can be technical or affective and social. Technical limitations constitute, for example, interpretation of mathematical notations as images by a screen reading software (JAWS) as well as errors and missing elements in Braille textbooks (Bayram, Corlu, Aydın, Ortaçtepe, & Alapala, 2015). An example of affective and social limitations is found in the adaptation of a visual task with linear patterns represented with dots of two colours. The provision of counters with different textures instead of different colours may lead to processing overload for the VI pupils and also to difficulties in their communication with sighted pupils on the mathematical task (Leuders, 2016).

Acknowledging the limitations of the above adaptation, Leuders (2016) suggests the implementation of the linear pattern task through a universally designed practice: the transformation of the visual task to an auditory task, which will be the same across the entire class. Leuders' suggestion, alongside suggestions of other researchers, e.g. Healy, Fernandes and Frant's (2013) work on multimodal mathematical tasks, are starting to shift the long-established perspective on tactile perception from the VI pupils to the entire class. Healy, Fernandes and Frant (2013) argue that the multimodal nature of mathematical representations, which meets the sensory needs of every pupil in the classroom, benefits not only the pupils with sensory impairments but also the pupils with no sensory impairments in that it allows them to develop a range of ways to think mathematically (Healy, Fernandes, & Frant, 2013).

The enrichment of sighted pupils with mathematical opportunities provided through multimodal tasks, in combination with the affective and social limitations frequently caused to VI pupils through problematic accommodations, have led us towards designing tactile mathematical tasks and trialling them with the entire class.

Our study's theoretical framework is sociocultural and draws upon: Vygotskian sociocultural theory of learning (Vygotskii, 1993); Sfard's discursive perspective, known as the theory of commognition (Sfard, 2007); the social model of disability (Oliver, 2009); and, the theory of embodied cognition (Gallese & Lakoff, 2005). In this paper, we focus on the discursive activity of a sighted pupil and of a VI pupil from the same class. We use the theory of embodied cognition (Gallese & Lakoff, 2005) to analyse the two pupils' mathematical contributions which are constructed through bodily tools. We draw upon Vygotskii's theory of mediation (1993) to explore the different mathematical meaning making within each pupil.

In what follows, we present the study's context, participants and methods. We then sample from the data with two episodes which evidence tactile construction of mathematical meaning about shapes by the pupils. We conclude with highlighting the benefits of tactile meaning making upon all pupils and make the case that these benefits are a key feature of inclusive mathematics classrooms.

METHODOLOGY

The study we draw from in this paper constitutes part of the first author's doctoral thesis [see (Stylianidou & Nardi, 2018) for further information on the study]. Data collection was conducted in four UK mainstream primary mathematics classrooms (one Year 1, one Year 3 and two Year 5 classes; pupils' ages: 6-10). The VI pupils' presence and the willingness of teaching staff and pupils to participate in the study constituted our criteria for the selection of the classrooms. We collected data after securing ethical approval by our institution's Research Ethics Committee and ensuring participant anonymity, confidentiality and right to withdraw from the study.

We collected data through observations of 29 mathematics lessons (33.5 hours in total); individual interviews with 5 class teachers (6 interviews, 2 hours and 10 minutes in total); individual interviews with 4 teaching assistants (6 interviews, 2 hours and 15 minutes in total); focussed-group interviews with 35 pupils (16 interviews, 2 hours in total); 2 ten-minute individual interviews with one pupil; written transcripts of the teaching staff's contributions in the design of the three lessons that constituted an intervention phase of the study; photographs of the pupils' work in the three intervention lessons; and, pupils' evaluation forms of the intervention lesson in two classes.

During observations, written notes were kept in all lessons. 21 lessons were audio-recorded and 14 lessons were also video-recorded. All interviews were audio-recorded, except four, following interviewee requests. For these, written notes were kept instead. Data analysis is ongoing and consists of analysis of inclusion and disability discourses of teaching staff and pupils in the mathematics classroom, both before and after the design and implementation of intervention lessons. Our analysis focuses on the impact of classroom practices upon the participating pupils and teaching staff. It is with this focus that our study's conjecture – outlined in the Introduction – is explored.

The mathematical task that is the focus of the episodes we present in this paper is as follows. The teacher asks the class to close their eyes and describe two shapes (see Figure 1, hereafter we call the second shape "Shape X"), both of which were constructed with Wikki Stix. The shapes were constructed on the same white A4 paper and copies of the paper were given to the class. Wikki Stix is a flexible teaching tool made of a wax and yarn combination and can be used for VI pupils' learning. In this paper, we focus on Shape X. At some point during the pupils' engagement with the task, the teacher also gives circles of various colours and sizes to the class and asks them the difference between Shape X and these circles.

In this task, we wanted to invite the entire class to explore mathematics through touch. We saw this invitation as potentially beneficial for both the VI and the sighted pupils: it would make the VI pupil feel that he is no more the only child in class who accesses mathematics differently from his peers. It would increase the sighted pupils' familiarity with a sense which is mostly associated with VI pupils and often under-used by sighted pupils. Furthermore, being aware of the characteristics of vision and touch – vision is wholistic and touch is gradual, allowing the exploration of an object from its individual parts to its whole (Ochaita & Rosa, 1995) – we wanted to explore whether vision may generate a misinterpretation of the two shapes (Shape X, for example, at first glance may be perceived as a circle) and whether touch may generate a more accurate interpretation of the shapes. In any case, we have argued that inviting the entire class to explore mathematics through touch could possibly lead to broadening everyone's perspectives on what constitute valid mathematical practices. We use pseudonyms for the pupils.

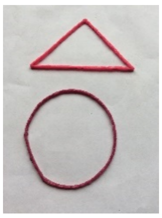

Figure 1: The two shapes made with Wikki Stix. Shape X is a circle minus a circular segment.

Figure 2: Luke holding the yellow circle.

TWO Y5 EPISODES: TACTILE CONSTRUCTION OF SHAPES

The following episodes come from a recapping lesson on mathematical topics in a Y5 class. One of these topics was shapes. Luke is the VI pupil of this class, is blind in one eye and has reduced vision in the other. The class has a general teaching assistant who supports pupils that need help at particular instances and whose role is not on supporting the VI pupil specifically. On the day of the lesson implementation, the teaching assistant was not in the classroom and the teacher was the only member of teaching staff in there. We will first present the contribution of a sighted pupil (Zak), made while in conversation with the first author, on Shape X. His contribution consists of two parts: in the first part he accesses the shape through touch and in the second part he accesses it visually. We noted that Zak's tactile contribution was common among pupils in the class, including the VI pupil, and we have selected Zak as typical. We will then present the contribution of the VI pupil (Luke), to the entire class, on the difference between Shape X and a circle (the yellow circle seen in Figure 2).

A factual account of the episodes

Shape X: Zak's contribution

During the entire lesson, the first author was sitting next to Luke. Zak was sitting in Luke's table too. At some point during the pupils' engagement with the shapes in Figure 1, Zak talks to the first author and tells her that he feels a straight line segment on Shape X when he has his eyes closed while that it is not that clear for him that there is a straight line segment when he actually opens his eyes and sees the shape.

Difference between Shape X and the circle: Luke's contribution

When the teacher invites the class to share their experiences of the two shapes in Figure 1, the conversation turns to Shape X. Luke then proposes the following comparison with the yellow circle.

> "With the normal circle like this *[he shows and grabs the yellow circle]* feels like, feels like it's gonna roll more *[he positions the yellow circle as if it is ready to roll]*. That one [*he shows Shape X]* feels like it's just gonna bob up and down."

A preliminary analytical account of the episodes

Zak's contribution

Zak constructs different meanings of Shape X through touch and through vision. While he confidently states the existence of a straight line segment when he feels the shape with his hands, he does not clearly see a straight line segment when he sees the shape with his eyes. Drawing upon Vygotskii's (1993) theory of mediation, according to which material, semiotic and sensory tools impact upon the construction of meaning, we attribute Zak's different mathematical constructions to the different sensory tools through which he accesses the shape each time: his hands and his eyes.

Drawing upon the theory of embodied cognition, according to which concepts – and therefore mathematical constructions – are embodied (Gallese & Lakoff, 2005), we relate each of Zak's mathematical constructions to the characteristics of the corresponding sensory tool. Zak clearly feels the straight line segment with his hands, possibly because tactile perception is characterised by gradual perception of an object, allowing the exploration of the object from its individual parts to its whole. So Zak had to trace Shape X gradually and successively to be able to perceive it as a whole. On the other hand, Zak does not clearly see the straight line segment with his eyes, possibly because visual perception is wholistic, namely allows the perception of the entire object at once. Since the straight line segment covered only a small part of Shape X – and the curved part dominates the shape – Zak's uncertainty regarding the existence of a straight line segment when he sees the object with his eyes is understandable. We may even surmise that Zak would not have noticed the straight line segment at all with his eyes if he had not firstly perceived the shape with his hands.

We have evidence that Zak appreciated tactile construction of mathematical meaning about shapes. In the evaluation form of the lesson, Zak wrote that he liked "the hidden facts on the shapes" (Figure 3).

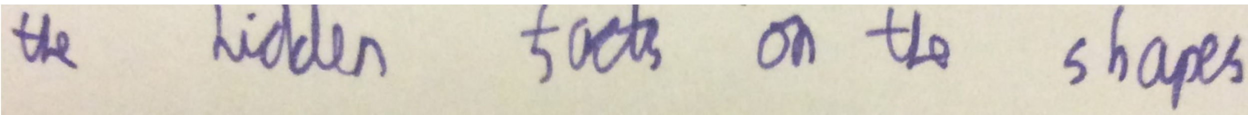

Figure 3: Zak's response to what he liked on the shape task

We see Zak's statement as evidence in favour of the conjecture our study explores. He benefited mathematically through touch and he also appreciated a form of perception that is mostly associated with VI pupils – we note that he is aware of Luke's learning Braille at school, and his own experience with tactile meaning making in mathematics may indeed help him develop a non-ableist (Campbell, ibid) perspective on the mathematical learning and capabilities of VI pupils.

Luke's contribution

Luke makes different meanings of the circle and of Shape X through touch. He feels that the circle is going to roll more – while Shape X is not; it is instead going to "bob up and down". Drawing upon Vygotskii's (1993) theory of mediation, we attribute Luke's different mathematical constructions to the different material tools which he accesses each time: the circle and Shape X. Therefore, while in Zak's case the material tool is the same in both cases and the sensory tool changes each time, in Luke's case it is the other way around: the sensory tool is the same in both cases and the material tool changes each time. We see Luke's use of the word "normal" as acknowledgement that Shape X is not a circle. We also note that Luke's description of Shape X does not come from an actual practical implementation of the rolling of the shape but from imagining the shape doing so. We see his imagining the rolling of the shape as an instance of what Gallese and Lakoff (2005) label as "embodied imagination" (p. 456).

We see Luke's meaning making about Shape X and the circle ("rolling", "bobbing up and down") as quite different from those of the other pupils who – as evidenced in the data we collected from this lesson – mostly draw upon properties of shapes and whose contributions relate to the description of shapes in school textbooks. We see Luke's contribution not only as different but also as refreshingly practical. For example, before Luke, Zak said that in a circle "you need everything to have a slight edge". We also note that Luke's contribution, especially his description of a circle, was highly appreciated by the teacher, who integrated it into his lesson and initiated its further discussion with the entire class. We see the teacher as "attuning" (Nardi et al., 2018, p. 154) his lesson to the mathematical productions of a VI learner and we argue that inviting the class to participate in tactile construction of mathematical meaning about shapes not only resulted in Luke's better inclusion – he takes centre stage in parts of the lesson – but also benefited others in class.

DISCUSSION AND CONCLUSION

In this paper, we see evidence in favour of our conjecture: tactile perception may lead not only to better inclusion of VI pupils but can also bring benefits to sighted pupils, too. We see these benefits as a key feature of inclusive mathematics classrooms. Similar benefits emerged in other mathematical topics, e.g. numbers, in which we invited the entire class to access mathematics through touch. More examples of mathematical tasks and of generated benefits will be included in the first author's doctoral thesis and subsequent papers. Based on our findings with regard to the benefits of tactile perception upon both VI and sighted pupils, we can argue that tactile perception of both VI and sighted pupils has the potency to generate multiple benefits to all pupils in inclusive mathematics classrooms. We envisage that more tactile tasks will be designed and trialled in inclusive mathematics classrooms with all pupils.

Our evidence and analyses resonate with Healy, Fernandes and Frant's (2013) argument about the impact that universally designed tasks may have upon the entire class: tactile construction of mathematical meaning benefits both the VI pupil and the sighted pupils, in that it allows them to develop a broader repertoire of ways to think mathematically. We see the invitation of the entire class to experience mathematics through the same sensory tool as a way to challenge ableism in the mathematics classroom and also as a way to create more inclusive mathematics classrooms.

ACKNOWLEDGEMENTS

We would like to thank wholeheartedly: the educational authorities in our region who have very warmly supported the study and have facilitated access to the particular classrooms; the schools' head teachers and Special Educational Needs and Disabilities Coordinators (SENDCOs) for their support and encouragement; and, the University of East Anglia for funding the study through a Social Sciences Faculty Doctoral Studentship. More importantly, we would like to thank the teaching staff and pupils, who have so eagerly embraced participation in the study.

References

Argyropoulos, V., & Stamouli, M. (2006). A collaborative action research project in an inclusive setting: Assisting a blind student. *British Journal of Visual Impairment*, *24*(3), 128-134.

Bayram, G. İ., Corlu, M. S., Aydın, E., Ortaçtepe, D., & Alapala, B. (2015). An exploratory study of visually impaired students' perceptions of inclusive mathematics education. *British Journal of Visual Impairment*, *33*(3), 212-219.

Campbell, F. (2001). Inciting legal fictions: 'Disability's' date with ontology and the ableist body of the law. *Griffith Law Review*, *10*(1), 42-62.

Gallese, V., & Lakoff, G. (2005). The brain's concepts: The role of the sensory-motor system in conceptual knowledge. *Cognitive Neuropsychology*, *22*(3-4), 455-479.

Healy, L., Fernandes, S.H.A.A., & Frant, J. B. (2013). Designing tasks for a more inclusive school mathematics. In C. Margolinas (Ed.), *Task Design in Mathematics Education: Proceedings of the 22nd International Commission on Mathematical Instruction study* (pp. 61-68). Oxford, UK: ICMI.

Leuders, J. (2016). Tactile and acoustic teaching material in inclusive mathematics classrooms. *British Journal of Visual Impairment*, *34*(1), 42-53.

Nardi, E., Healy, L., Biza, I., & Fernandes, S.H.A.A. (2018). 'Feeling' the mathematics of disabled learners. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 147-170). Sense Publishers.

Ochaita, E., & Rosa, A. (1995). Percepção, ação e conhecimento nas crianças cegas. [Perception, action and knowledge of blind children]. In C. Coll, J. Palacios, & A. Marchesi (Eds.). *Desenvolvimento psicológico e educação: Necessidades educativas especiais e aprendizagem escolar* (Vol. 3, pp. 183-197). Porto Alegre: Artes Médicas.

Oliver, M. (2009). *Understanding disability: From theory to practice* (2nd ed.). Basingstoke, UK: Palgrave Macmillan.

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The Journal of the Learning Sciences*, *16*(4), 565-613.

Stylianidou, A., & Nardi, E. (2018). Inclusion and disability in the primary mathematics classroom: Examples of teaching staff discourses on the participation of visually impaired pupils. In J. Golding, N. Bretscher, C. Crisan, E. Geraniou, J. Hodgen, & C. Morgan (Eds.), *Research Proceedings of the 9th British Congress on Mathematics Education* (pp. 167-174). UK: University of Warwick.

United Nations. (2006). *United Nations Convention on the Rights of Persons with Disabilities*. http://www.un.org/disabilities/documents/convention/convoptprot-e.pdf

Vygotskii, L.S. (1993). *The collected works of L.S. Vygotsky: The fundamentals of defectology* (R. W. Rieber & A. S. Carton, Eds.). Dordrecht: Springer.