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Stacking sequence optimization of composite panels for blending characteristics using lamination parameters

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1 Abstract

Stacking sequence optimization of laminated composite structures to satisfy ply continuity (blending) requirements has recently attracted considerable attention. In this paper, lamination parameter-based method is examined for finding the best stacking sequence of laminated composite wing structures with blending and manufacturing constraints. The optimization procedure is to use lamination parameters and numbers of plies of the pre-defined angles (0, 90, 45 and -45 degrees) as design variables with buckling, strength and ply percentage constraints while minimizing the material volume in the top level optimization run. Based on the previous research by the authors, two new criteria, stack homogeneity index and 90° ply angle jump index, are implemented to define a single objective function. This objective function is minimized to achieve the best stacking sequence of laminate composite wing structures in the local level optimization with the consideration of blending requirements. The results of the application of this approach are compared to published results to demonstrate the potential of the developed technique.

2 Key Words: Laminated Composite, Optimization, Stacking Sequence, Blending, Lamination Parameters.

3 Introduction

Stacking sequence optimization of laminated composite structures to satisfy ply continuity (blending) requirements has recently attracted considerable attentions [1-5].

Liu *et al*. [2,6] presented a bi-level (global and local) strategy for optimization of a composite wing box structure. At the global level, continuous optimization of thicknesses of 0, 90, 45 and -45 degree plies was performed to minimize the weight of a wing box subject to strain and buckling constraints. For a given the number of plies of each orientation and in-plane loads, a permutation genetic algorithm (GA) was used at the local level to optimize the stacking sequence in order to maximize the buckling load. The optimum buckling load, which was treated as a function of the loading and the numbers of plies of 0, 90, 45 and -45 degree orientation, was evaluated by a cubic polynomial response surface approximation.

The use of lamination parameters to represent the in-plane and flexural stiffness in the optimization of laminated composites has been investigated. It was first used by Tsai *et al* [7] and later applied to the buckling optimization of orthotropic laminated plates by Fukunaga and Hirano [8]. Miki [9] and Fukunaga [10] used lamination parameters for tailoring mechanical properties of laminated composites. In a laminated composite optimization problems, lamination parameters can be used as design variables instead of layer thicknesses and ply angles in order to avoid falling into local optima. Diaconu *et al.*[11] used a variational approach to determine feasible regions in the space of lamination parameters as constraints in the optimization problem.

Herencia and Weaver [12] applied a mathematical programming technique and a GA to optimize anisotropic laminated composite panels with T-stiffeners. In the first step, weight optimization based on mathematical programming was performed where the skin and a stiffener were parameterized using lamination parameters, subject to the constraints on buckling, strength as well as practical design rules. A composite layup of a panel was determined using a GA in the second level by meeting the target values of lamination parameters coming from the top level. Herencia *et al.*[13] used the same approach for optimization of laminated composite panels with T-stiffeners, but with a different objective function at the second level. Instead of minimizing the squared distance between the target lamination parameters from the first step and the actual lamination parameters, the maximum value of the linearised design constraints was taken as the objective function. The authors' conclusion was that in the determination of the stacking sequence the minimum squared distance might not be the best objective.

Ply compatibility (also referred to as blending) between adjacent panels is a very important consideration in the design of composite structures, it has been considered by Liu and Haftka [2], Liu *et al*. [6], Soremekun *et al*. [14] and Seresta *et al*. [3]. Liu and Haftka [2] defined the composition continuity and the stacking sequence continuity measures that were used in an optimization process, also by Toropov *et al*. [15]. Soremekun *et al*.[14] and Seresta *et al.*[3] developed two blending methods, inward and outward blending, to improve the ply continuity between adjacent panels using a guide based GA. Liu and Krog [4] developed a new approach to identifying a laminate stacking sequence in individual wing panels satisfying inter-panel continuity constraints. In this method, a conventional stacking sequence identification problem was transformed into a problem of shuffling of a set of global ply layout cards. A permutation GA was applied to find an optimal card sequence, which uses the ply angle percentages and the chordwise and spanwise laminate thickness distributions as input data. The authors' conclusion was that it allowed to considerably reduce the design space and hence the solution time. Recently, two bi-level composite optimization procedures were investigated by Liu and Toropov *et al.*[5] to seek the best stacking sequence of laminated composite wing structures with blending and manufacturing constraints. Two examined approaches are: a smeared stiffness-based method, that aims to neutralize the stacking sequence effects on the buckling performance, and a lamination parameter-based method, that uses lamination parameters as design variables to formulate the membrane stiffness matrix $\mathbf{A}$ and bending stiffness matrix $\mathbf{D}$. The advantage of the smeared stiffness-based method is that it avoids a stack optimization at the local (bottom) level by performing a quicker post-processing function of ply shuffling. The advantage of the lamination parameter-based approach is that there is no need to check whether the strength or buckling constraints have been violated as long as the lamination parameters obtained after the local level optimization match the given lamination parameter values that came from the top level optimization.

In this paper, lamination parameter-based method is used for the optimization of stacking sequence of laminated composite structures. At the top level optimization, the total number of plies and the lamination parameters related to the bending stiffness matrix are treated as the design variables. Buckling and strength constraints are applied at this level and the total mass is the objective function. Next, a multi-objective function is built up for the local level optimization, which is composed of three criteria: non-dimensional lamination parameters match and two indices (stack homogeneity index and 90° ply angle jump index) as introduced in Section 7. Then, a permutation GA is used to shuffle the layers to minimize this objective function. This is embedded into a blending procedure applied at this level to achieve the global ply continuity.

4 Lamination Parameter-Based Method

Lamination parameters were first introduced by Tsai *et al.*[7]. It is known that the stiffness matrices $\mathbf{A}$ and $\mathbf{D}$ are governed by 12 lamination parameters and five material parameters. For orthotropic symmetric and balanced laminates, the number of independent lamination parameters can be reduced to eight. The elements of the membrane stiffness matrix $\mathbf{A}$ and the bending stiffness matrix $\mathbf{D}$ can be expressed as:

$$\begin{bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{16} \\ A_{26} \\ A_{66} \end{bmatrix} = h \begin{bmatrix} 1 & \xi_1^A & \xi_2^A & 0 & 0 \\ 1 & -\xi_1^A & \xi_2^A & 0 & 0 \\ 0 & 0 & -\xi_2^A & 1 & 0 \\ 0 & 0 & -\xi_1^A & 0 & 1 \\ 0 & \xi_3^A/2 & \xi_4^A & 0 & 0 \\ 0 & \xi_3^A/2 & -\xi_4^A & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix},$$

$$\begin{bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{16} \\ D_{26} \\ D_{66} \end{bmatrix} = \left(\frac{h^3}{12}\right) \begin{bmatrix} 1 & \xi_1^D & \xi_2^D & 0 & 0 \\ 1 & -\xi_1^D & \xi_2^D & 0 & 0 \\ 0 & 0 & -\xi_2^D & 1 & 0 \\ 0 & 0 & -\xi_1^D & 0 & 1 \\ 0 & \xi_3^D/2 & \xi_4^D & 0 & 0 \\ 0 & \xi_3^D/2 & -\xi_4^D & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}. \tag{1}$$

This suggests that the use of lamination parameters as design variables in the composite optimization can be very beneficial. It is known [1,11] that the relationship between the out-of-plane lamination parameters can be expressed as:

$$2(1+\xi_2^D)(\xi_3^D)^2 - 4\xi_1^D\xi_3^D\xi_4^D + (\xi_4^D)^2 \le (\xi_2^D - 2(\xi_1^D)^2 + 1)(1-\xi_2^D). \tag{2}$$

For the majority of aeronautical structures symmetric and balanced laminates with ply orientations of 0, 90, 45 and -45 degrees are used, also in this paper. Thus, $\xi_4^D = 0$ and the relationship (2) can be rewritten as:

$$(\xi_3^D)^2 \le \frac{(\xi_2^D - 2(\xi_1^D)^2 + 1)(1-\xi_2^D)}{2(1+\xi_2^D)}. \tag{3}$$

This corresponds to a constraint defining the feasible region for lamination parameters in the optimization of alaminated composite. Here, the non-dimensional lamination parameters are defined as:

$$
\begin{aligned}
V_{[0,1,2,3,4],i}^{D} &= 1+\left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} [1, \cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta] z^2 dz \\
V_{0,i}^{A} &= \left(\frac{1}{h_i}\right) \int_{-h_i/2}^{h_i/2} 1 dz = 2(n_0^i + n_{90}^i + 2n_{45}^i) = 1 \\
V_{1,i}^{A} &= \left(\frac{1}{h_i}\right) \int_{-h_i/2}^{h_i/2} \cos 2\theta dz = 2(n_0^i - n_{90}^i) = \frac{n_0^i - n_{90}^i}{n_0^i + n_{90}^i + 2n_{45}^i} \\
V_{2,i}^{A} &= \left(\frac{1}{h_i}\right) \int_{-h_i/2}^{h_i/2} \sin 2\theta dz = 0 \\
V_{3,i}^{A} &= \left(\frac{1}{h_i}\right) \int_{-h_i/2}^{h_i/2} \cos 4\theta dz = 2(n_0^i + n_{90}^i - 2n_{45}^i) = \frac{n_0^i + n_{90}^i - 2n_{45}^i}{n_0^i + n_{90}^i + 2n_{45}^i} \\
V_{4,i}^{A} &= \left(\frac{1}{h_i}\right) \int_{-h_i/2}^{h_i/2} \sin 4\theta dz = 0
\end{aligned}
\tag{4}
$$

where A indicates membrane effects,

D indicates bending effects,

i is the panel number,

n_0^i is half the number of 0° plies in the total stack of the i^{th} panel,

$n_{\pm 45}^i$ is half the number of pairs of $\pm 45^\circ$ plies in the total stack of the i^{th} panel,

n_{90}^i is half the number of 90° plies in the total stack of the i^{th} panel,

h_i is the total thickness of the panel i,

θ is the ply angle.

In the formulae above the values of $V_{0,i}^D = 1 + 2/3$, $V_{4,i}^D = 1$ can be immediately evaluated, and the following condition holds: $V_{[1,2,3],i}^D \geq 0$.

The vector form of lamination parameters related to the out-of-plane stiffness matrix can be written as:

$$
\mathbf{V}_i^D = V_{[1,2,3],i}^D = 1+\left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} [\cos 2\theta, \sin 2\theta, \cos 4\theta] z^2 dz. \tag{5}
$$

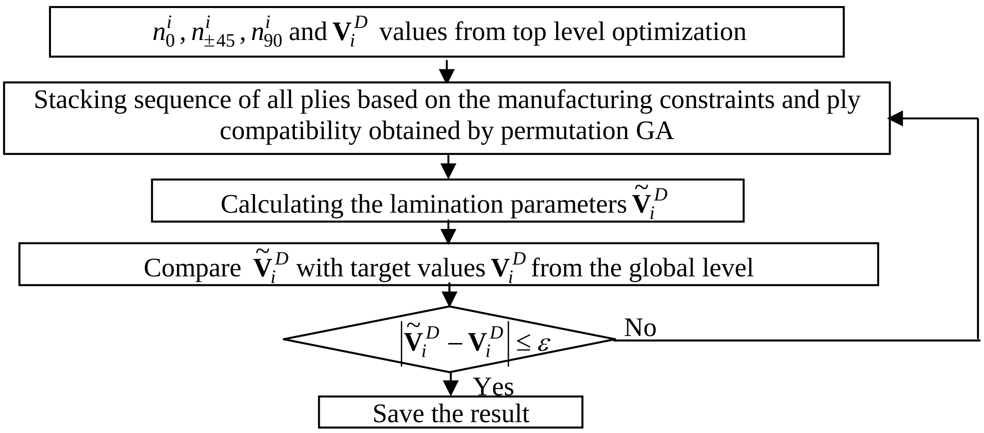

Figure 1. Flow chart of the panel stacking sequence optimization process

In this approach, the lamination parameters related to the out-of-plane stiffness matrix $\mathbf{V}_i^D$ and the numbers of plies of each orientation (n_0^i, $n_{\pm 45}^i$, n_{90}^i) are taken as the design variables in the top level. The material volume is the objective function, and the constraints are imposed on buckling, strength, percentages of the numbers of plies of each orientation as well as the feasibility of lamination parameters. Then, in the local level, a stacking sequence optimization is performed by matching the lamination parameters $\mathbf{V}_i^D$ that came from the top level optimization with the lamination parameters $\tilde{\mathbf{V}}_i^D$ computed in the local level optimization subject to satisfaction of the composite design rules and manufacturing requirements. A permutation genetic algorithm (permGA) is used for the local level optimization runs carried out iteratively in order to ensure the ply compatibility of adjacent panels as presented in Section IV. A schematic of the optimization process at this level is shown in Figure 1. The advantage of this approach is that there is no need to check whether the strength or buckling constraints have been violated as long as the lamination parameters obtained after the local level optimization match the given lamination parameter values that came from the top level optimization. In the ply compatibility optimization process it is also required to keep the values of lamination parameters in all the stacks (panels) of the whole structure matching the corresponding values that came from the top level optimization.

5 Composite Design Rules

According to aircraft industry manufacturing requirements [15,21], the laminate layup design rules applied to each panel are as follows:

1) The stack is balanced, i.e. the number of 45º and -45º plies is the same in each of the components.
2) Due to the damage tolerance requirements, the outer plies for the skin should always contain at least one set of ±45º plies.
3) The number of plies (N_{max}) in any one direction placed sequentially in the stack is limited to four.
4) A 90º change of angle between two adjacent plies is to be avoided, if possible.
5) All three ply orientations (0° , 90° and $\pm 45^\circ$) should be spread uniformly through the stack.

6 Shared Layers Blending (SLB)

In aerospace engineering, a typical wing is a multi-panel tailored composite structure. To improve structural integrity and avoid stress concentration between two adjacent panels, ply blending should be ensured. Although such requirements have been considered by several research groups [2-5], a problem of optimization of multi-panel aircraft structures with a comprehensive consideration of buckling, strength, manufacturing constraints as well as general composite design rules including ply blending still remains to be addressed to satisfaction of aircraft industry.

In this section the Shared Layers Blending (SLB) process is applied to satisfy the global blending requirement as well as the general layup design rules. Two illustrative examples are given to demonstrate this process.
First, ranking of all panels in terms of the numbers of plies of each angle is performed. Then, for each ply angle, out of all panels the minimum number of plies is selected. This set of three ply numbers defines the first set of shared layers among all panels. The thinnest panel that includes the first shared layers is identified. The first shared layers will be placed outermost in the stacks for all panels. The remaining layers in the thinnest panel are placed after the first shared layers. Next, after this first stage, for the remaining layers of all the panels, except the thinnest panel, the same procedure is applied as at the first stage. This is repeated until the last panel is considered. Finally, for the adjacent panels, the local blending between them is performed for the remaining layers in the adjacent panels. The scheme for the local blending consideration was introduced in the authors' previous paper [5]. Thus, the stacks for all the panels will become inwardly blended (outer blending), where the outer layers of all the panels are continuous. If the shared layers are placed at the position next to the mid plane instead of the outermost position, the inner blending (outwardly blended composite) will be created. In this paper, the outer blending procedure is adapted due to the damage tolerance requirements resulting in $\pm 45^\circ$ plies places on the outside of the stack.

6.1 Example 1

In the first example three panels are linked together with the numbers of plies of each orientation (as coming from the top level optimization) given in Table 1. A flowchart of this approach is shown in Figure 2.

Table 1. Numbers of plies of each orientation for a three-panel laminated structure

Panel number	panel 1	panel 2	panel 3
Number of plies ($n_0 / n_{\pm 45} / n_{90}$)	40/17/7	35/14/9	29/6/12

Using this approach, the first set of shared layers in this example will be $n_0/n_{45}/n_{90} = 29/6/7$ for all three panels and also panel 3 is selected as the first (thinnest) panel. The second shared set will be $n_0/n_{45}/n_{90} = 6/8/0$ for the panels 1 and 2 only. Now the remaining layers in the three-panel structure will be:

$n_0/n_{45}/n_{90} = 5/3/0$ for panel 1, $n_0/n_{45}/n_{90} = 0/0/2$ for panel 2, and $n_0/n_{45}/n_{90} = 0/0/5$ for panel 3. In this example, because no shared layers are available between the panels 1 and 2, the local shared set will be $n_0/n_{45}/n_{90} = 0/0/2$ for the remaining layers in the panels 2 and 3. Thus, the remaining layers in the panel 1 after the SLB procedure will be $n_0/n_{45}/n_{90} = 5/3/0$, no remaining layers left in the panel 2, and $0/0/3$ in the panel 3. The blending schematic is shown in Figure 3.

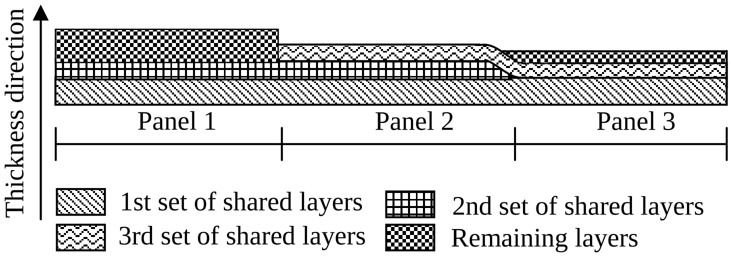

Figure 2. Flowchart for the shared layers blending scheme.

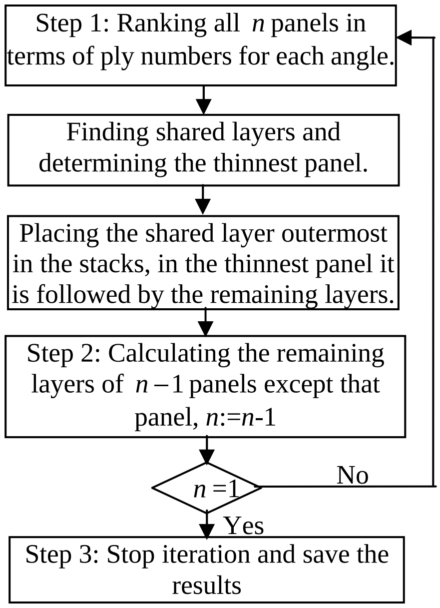

Figure 3. Illustration of shared layers blending concept for the three-panel linked structure.

6.2 Stack Repair

Two issues arise from the results in Example 1 that need to be addressed in the blending scheme. The first one is that the group of remaining layers in the panel 3 consists only of five 90 degree plies that violates the ply composition rule. The second issue is that the total number of plies in the second set of shared layers truncated between the adjacent panels 2 and 3 can be considered too large (23 plies).

For the first issue, having five 90 degree plies as remaining layers in panel 3 means that too many plies were selected for the first set of shared layers. A slightly larger number of plies including at least one 0 or ± 45 degree ply has to be included into the set of remaining layers for the panel 3. Therefore, having more than four plies of the same orientation together can be avoided in the panel 3 by reserving some layers from the first set of shared layers. In the Example 1, one ply with 0 degree orientation is reserved from the first set of shared layers to avoid five plies of the same orientation together remaining in the panel 3. Thus the first set of shared layers in Example 1 will be $n_0/n_{45}/n_{90} = 28/6/7$. The second shared set will be $n_0/n_{45}/n_{90} = 7/8/0$ for panel 1 and panel 2. Now the remaining layers in the three-panel structure will be: $n_0/n_{45}/n_{90} = 5/3/0$ for panel 1, $n_0/n_{45}/n_{90} = 0/0/2$ for panel 2, and $n_0/n_{45}/n_{90} = 1/0/5$ for panel 3. The local shared set will be $n_0/n_{45}/n_{90} = 0/0/2$ for panel 2 and panel 3. The remaining layers in the panels 3 will be $1/0/3$ and the ply with 0 degree orientation extracted from the first set of shared layers will help avoiding five plies of 90° orientation together.

For the second issue, let's assume that the number of truncated plies between the panels 2 and 3 is too large. This means that too many plies are selected as the second set of shared layers in the above blending procedure. Thus, the number of plies used as the second set of shared layers has to be re-adjusted to satisfy the requirement. Generally, for practical stack compositions that satisfy realistic constraints on ply orientation percentages, a solutions for a stack repair can always be found: If a problem happens at a certain stage, the algorithm steps back and plies are removed from the previous set of shared layers in the above blending scheme. This is repeated iteratively until the obtained stack satisfies all the design rules and constraints.

7 Optimization Using a Permutation GA

In the lamination parameter-based method, lamination parameters related to the out-of-plane stiffness matrix are obtained from the top level optimization. Given these values, a stacking sequence finding while satisfying the layup rules and the requirements of the blending scheme should be performed. A permutation GA is an ideal tool for such a composite laminate optimization problem. Each string in the coding represents a unique stacking sequence. An example of using the genetic operators with a permutation encoding is given below.

7.1 Encoding

1) Mutation - two numbers are selected and exchanged e.g. 3rd and 5th:

[1 2 **3** 4 **5**] $\Rightarrow$ [1 2 **5** 4 **3**].

2) Crossover can be done in a variety of ways, such as 'simple crossover', 'cycle crossover', 'inversion' and 'swap adjacent cells'. The 'swap adjacent cells' method, implemented in this work, is illustrated below:

[1 **2** **3** 4 5] $\Rightarrow$ [1 **3** **2** 4 5].

Also, to reflect the layup rules of composite laminate design and manufacturing requirements, substrings that represent stacks of layers such as $45/0/-45$, $45/90/-45$, $45/0_2/-45$ and $45/90_2/-45$ are implemented in the permutation GA coding in order to improve the stacking sequence design of composite laminates.

7.2 Inclusion of manufacturing requirements in the objective

For the laminated composite structure optimization, the constraints of the manufacturing requirements are applied to create a feasible design. Apart of composite design rules in section 5, two criteria, stack homogeneity index and 90° ply angle jump index, are introduced in this paper.

The stack homogeneity requirement implies that plies of all three possible orientations (0°, 90° and $\pm 45^\circ$) occur in the stack with the frequency that is as uniform as possible. In order to quantify this requirement, it is proposed to monitor the composition of the string of ply angles that characterizes the stack. The lengths of all substrings that contain only two out of three possible ply angles are calculated. A divider between such substrings can be either an occurrence of a third ply angle or one of the following five possible blocks of plies bounded by a pair of 45° and -45° plies: $45^\circ/-45^\circ$, $45^\circ/0^\circ/-45^\circ$, $45^\circ/0^\circ/0^\circ/-45^\circ$, $45^\circ/90^\circ/-45^\circ$ and $45^\circ/90^\circ/90^\circ/-45^\circ$. Also, in counting the substring length, occurrences of the same ply angle in two, three, or four sequential plies is counted as one. Thus, the maximum length of such substrings (N_h) contributes to the definition of the *stack homogeneity index*:

$$H = 2\frac{N_h t}{h} \tag{6}$$

where h is the total thickness of the panel,

t is the ply thickness.

The requirement of minimization of the number of occurrences of 90° change in the ply angle for any two consecutive plies in the stack is quantified by the 90° *ply angle jump index*:

$$A = 2\frac{N_a t}{h} \tag{7}$$

where N_a is the total number of occurrence of 90° ply angle jump in the consecutive plies in the half stack.

In order to combine stack homogeneity index (H), 90° ply angle jump index(A) and non-dimensional lamination parameters match (L) into a single-objective function, homogeneity index (H) and 90° ply angle jump index (A) can be weighted again the non-dimensional lamination parameters match (L) and formulated as:

$$f = W_1 L + W_2 A^2 + W_3 H^2$$
$$L = \sum_{i=1}^{3}\left(\frac{V_i^D - \tilde{V}_i^D}{V_i^D}\right)^2 \tag{8}$$

where f is the objective function,

W_1 is the weighting coefficient for lamination parameters,

W_2 is the weighting coefficient for 90° ply angle jump index,

W_3 is, the weighting coefficient for stack homogeneity index,

V_i^D is the i^{th} lamination parameters from the top level optimization,

$\tilde{V}_i^D$ is the i^{th} computed lamination parameters.

When these three criteria have equal importance to the objective function, the weighting coefficients can be selected as $W_1 = W_2 = W_3 = 1/3$.

7.3 Example 2

In this example, calculation of stack homogeneity index (H) and 90° ply angle jump index (A) will be demonstrated. A symmetric, balanced laminated is given as:

$[90/0/45/90/-45/90/90/0/0/0/90/0/45/0/-45/90/0/45/90/90/-45/0/90/45/0/0/45/0/0/90/0/90/45/90/90/-45/0]_s$.

The total number of occurrence of 90° ply angle jump in the consecutive plies in the half stack (N_a) of the above example can be calculated as: $A = 2\dfrac{N_a t}{h} = 2\times\dfrac{t}{h}\times 9 = \dfrac{18}{74} = 0.243$.

The maximum length of such substrings (N_h) in the above example 2 can be shown in Figure 5. The first length of substrings is 2 because the third angle orientation of block of plies $45^\circ/90^\circ/-45^\circ$ follows immediately after the first ply with 90 angle orientation and the second ply with 0 angle orientation. The second value of substring length is 4; the third value is 2; the fourth value is 2; the fifth value is 4 and the six value is 1. Thus, the maximum length of such substrings (N_h) is 4. Thus, the stack homogeneity index $H = 2\times\dfrac{t}{h}\times 4 = \dfrac{8}{74} = 0.108$.

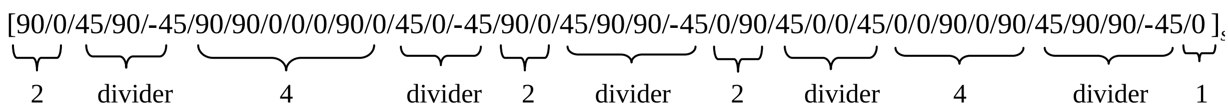

Figure 5 The illustration of substring and divider for the calculation of stack homogeneity index

8 Wing Box Example

The wing box model with material properties and loads [5] used to illustrate the methods discussed in previous sections is shown in Figures 6 and 7. Due to the aircraft industry manufacturing requirements, 0 or 90 degree plies are required to be inserted into pairs of ± 45 plies to avoid 90° change between two adjacent plies. Thus, the bending-twisting coupling terms D_{16} and D_{26} are nonzero from the contributions of off-axis layers and the distance of the positive and negative angle plies from the laminate center plane. In this work, the number of 45 degree plies in the top skin and the number of 0 and 90 degree plies in the bottom skin are rounded up to achieve the discrete optimal design. The designs of panels at the bottom skin will use the results from previous work [5]. Only stacking sequence of plies for the panels at the top skin will be considered for by GA runs in order to illustrate the effects of the weighting coefficients on the results in terms of non-dimensional lamination parameters match, 90° ply angle jump index, stack homogeneity index and buckling load factor.

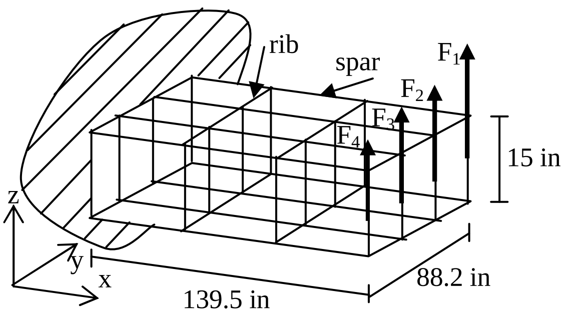

Figure 6. Geometry of the wing box

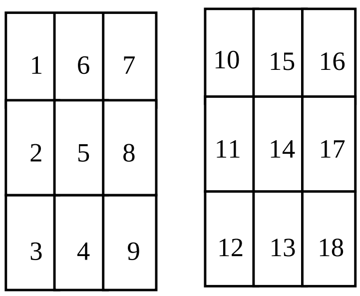

Figure 7. Bottom and top skin configurations

8.1 Problem with Two Designable Substructures

If the layup of all panels in the top skin is the same and all the bottom skin panels are also are the same, the total number of design variables for the wing box is twelve: n_0^t, $n_{\pm 45}^t$, n_{90}^t, n_0^b, $n_{\pm 45}^b$, n_{90}^b, $(V_1^D)^t$, $(V_2^D)^t$, $(V_3^D)^t$, $(V_1^D)^b$, $(V_2^D)^b$ and $(V_3^D)^b$ in the lamination parameter-based method. The results for the objective function and the violation of constraints at the top level are shown in Table 3 for the lamination parameter-based method. In contrast with the results from Liu *et al*. [6], the objective function is reduced to 180 as compared to 208. In the local level optimization, given the lamination parameters from the top level, permutation GA is used to obtain the stacking sequence for the top skin as presented in Table 4. In order to illustrate the effects of the weighting coefficients W_1, W_2, and W_3 on the results in terms of non-dimensional lamination parameters match L, stack homogeneity index H, 90° ply angle jump index A and buckling load factor, ten cases are investigated. Without consideration of stack homogeneity index H and 90° ply angle jump index A, permutation GA can fit the lamination parameters with the ones from top level quite well. When W_2 increases and W_1 keeps constant, buckling load factor decreases and the difference between computed lamination parameters and the ones from the top level optimization increases. When W_2 is equal to 1.0, the best values for 90° ply angle jump index and stack homogeneity index are obtained ($A = 1.10\text{e-}2$ and $H = 8.46\text{e-}3$), while non-dimensional lamination parameters match is quite large ($L = 7.89$) and buckling load factor is comparatively smaller. In Case 5 (in bold), the effect of W_2 on 90° ply angle jump index and stack homogeneity index is obviously important. A large value W_2 can result in the small value stack homogeneity index and 90° ply angle jump index, but the same conclusion can not be drawn for the large value W_3. In this paper, weighting coefficients W_1=0.5, W_2=0.45 and W_3=0.05 will be selected for the following research. Stacking sequence of the panel 16 for each case study run is listed in Table 5.

Table 3 Continuous and rounded optimal design with 12 variables for lamination parameter-based method

	n_0	n_{45}	n_{90}	n_0	n_{45}	n_{90}	V_1	V_2	V_3
	(Continuous)			(Rounded)					
Top skin panels	34.492	7.445	26.139	34	8	26	0.9434	1.0065	1.2108
Bottom skin panels	8.163	1.480	2.181	9	1	3	0.8944	1.0435	0.9710
Buckling load factor	1.0009			1.0183					
Total number of stacks	177.65			180					
Total number of stacks[6]	208.76			208					

Table 4 Features of designs for the panel 16 by the permutation GA runs

Case study	V_1	V_2	V_3	L	A	H	W_1	W_2	W_3	f	Buckling load factor
C1	0.9434	1.0065	1.2095	7.29e-5	1.17e-1	2.10e-2	1.0	0.0	0.0	7.29e-5	1.0210
C2	0.9434	1.0064	1.2097	2.38e-4	5.02e-2	1.10e-2	0.9	0.05	0.05	3.27e-3	1.0210
C3	0.9431	1.0063	1.2125	1.17e-3	3.88e-2	1.10e-2	0.8	0.15	0.05	7.33e-3	1.0204
C4	0.9430	1.0062	1.2211	4.51e-3	3.39e-2	1.10e-2	0.6	0.35	0.05	1.51e-2	1.0191
C5	**0.9434**	**1.0075**	**1.2430**	**4.61e-2**	**2.92e-2**	**8.46e-3**	**0.5**	**0.45**	**0.05**	**3.66e-2**	**1.0156**
C6	1.1018	1.0080	1.2304	7.89	1.10e-2	8.46e-3	0.0	1.0	0.0	1.11e-2	0.9842
C7	1.0893	1.0064	1.2894	6.79	2.92e-2	1.52e-3	0.0	0.0	1.0	1.56e-3	0.9756
C8	0.9434	1.0063	1.2130	1.00e-3	4.45e-2	1.10e-2	0.5	0.05	0.45	7.70e-3	1.0205
C9	0.9429	1.0063	1.2143	9.38e-4	3.88e-2	1.10e-2	0.6	0.05	0.35	6.39e-3	1.0201
C10	0.9434	1.0066	1.2225	3.24e-3	3.88e-2	4.33e-3	0.8	0.05	0.15	5.19e-3	1.0189

Table 5 Stacking sequence of the panel 16 for each case study run at local level optimization

Case study	stacking sequence
C1	$[(\pm 45)_2/45/90/-45/90/45/90_2/-45/90/0/45/90/-45/90_3/0_3/90/0/90_2/0_3/90_2/0_2/90/0_3/90/0_4/90/45/0/-45/0_2/90/0_3/90/0_2/90/0_2/90/0/90_2/45/0/-45/45/0/-45/90/0_2/90/0_2/90]_s$
C2	$[(\pm 45)_2/45/90_2/-45/45/90/-45/90_2/0/45/90/-45/90_4/0_4/90_2/0_3/90_2/0_4/90/0/90/0_4/45/0/-45/90/0_3/90/0_3/90_2/0_4/90_3/45/0/-45/90_2/0_4/45/0/-45/90]_s$
C3	$[(\pm 45)_2/45/90/-45/45/90_2/-45/90_2/0/45/90/-45/90_3/0_4/90_3/0_4/90_3/0_4/90/0_4/90_2/0_2/45/0/-45/0_4/90_2/0_3/90_2/0/90_4/45/0/-45/0_2/45/0/-45/0_2]_s$
C4	$[(\pm 45)_2/45/90/-45/90/45/90_2/-45/0/90_4/45/90/-45/0_4/90_3/0_4/90_3/0_4/90/0_4/90_2/0_2/45/0/-45/0_4/90_2/0_4/90_3/0_2/45/0/-45/90_2/0_2/45/0/-45/90]_s$
C5	$\mathbf{[(\pm 45)_2/0_3/45/90_2/-45/90_3/45/90_2/-45/0/90_4/45/0_2/-45/0/90_3/0_2/90_4/0/45/0/-45/0_4/90/0_4/90/0_3/90_2/0_4/45/0_2/-45/0_2/45/0_2/-45/90_3/0_2/90]_s}$
C6	$[(\pm 45)_2/0_2/45/90_2/-45/90_3/0_4/45/0_2/-45/0_4/45/90_2/-45/0_2/45/90_2/-45/0_4/90_3/0_4/45/90/-45/90_2/0_4/90_4/0_4/90_3/0_2/45/0_2/-45/90_4]_s$
C7	$[(\pm 45)_2/90/0_3/90_2/45/90/-45/0_4/90_2/0_2/45/0_2/-45/90/0_4/90_3/45/90_2/-45/0_4/90/0_4/45/90_2/-45/90_3/0_4/90/45/90_2/-45/0_2/90_4/45/0/-45/0_3/90/0]_s$
C8	$[(\pm 45)_2/45/90/-45/90_2/45/90_2/-45/0/45/90/-45/90_3/0_4/90_3/0_3/90_2/0/90/0_4/90/0_4/45/0/-45/90/0_2/90/0_4/90_2/0_4/90_4/0_2/45/0/-45/90_2/45/0/-45/0_2]_s$
C9	$[(\pm 45)_2/45/90/-45/45/90_2/-45/90_4/0/45/90/-45/90/0_4/90_3/0_4/90_3/0_4/90/0_4/90/0_2/45/0/-45/90/0_4/90_2/0_4/90_2/45/0/-45/90_4/0_2/45/0/-45/0_2]_s$
C10	$[(\pm 45)_2/45/90/-45/45/90/-45/90_2/0_2/90_4/0_3/90_4/45/0_2/-45/0_3/90_2/0_4/90_2/0_3/45/90_2/-45/90/0_2/90_3/45/0/-45/0_3/90/0_4/90/0_2/45/0/-45/0_4/90_2]_s$

8.2 Problem with Six Designable Substructures

If the top and bottom skins are divided into three parts: root, intermediate and tip part, the results are listed in Tables 6. The weight is reduced considerably as compared to the case of two designable substructures. The objective function is 464 for discrete optimal design that is the same as the result of Liu *et al*.[6]. It should be noted that when the lamination parameter-based method was used shear buckling in the bottom skin occurred (buckling load reduction by 4%) as shown in Table 7. This is due to the application of a blending procedure to a part of the structure that has a relatively few plies in which case blending caused a poor match between the target and obtained values of lamination parameters. This can be repaired by adding some layers manually. The second buckling mode corresponds to the top skin, the magnitude of the load factor is close to the value from the top level optimization. This is guaranteed by arriving at a good match with the lamination parameters from top level optimization when a local optimization is performed. With the consideration of weighing coefficients W_2 and W_3 in the objective, the stacking sequence of plies compared with the results[5] has better stack homogeneity index and 90° ply angle jump index in Table 7. Summarising, due to the limited number of plies in the bottom panel, it was difficult to shuffle the plies to match the lamination parameters from the top level while satisfying ply continuity in the bottom skin. For the top skin (that has a much greater number of plies), the lamination parameters are quite close to the ones from the top level optimization and the outer blending with the layup rules requirements did not cause any problems.

Table 6 Continuous and rounded optimal design with 36 variables for lamination parameter-based method

	n_0	n_{45}	n_{90}	n_0	n_{45}	n_{90}	V_1	V_2	V_3
	(Continuous)			(Rounded)					
Top skin panels									
Panel no.16	30.20	12.54	24.56	28	16	22	1.1268	1.0102	1.2132
Panel no.17	18.69	20.53	12.10	26	13	19	1.1610	1.0086	1.3022
Panel no.18	24.43	5.40	8.92	22	6	14	1.2398	1.0098	1.0982
Bottom skin panels									
Panel no.7	4.39	1.30	1.28	5	1	1	1.3715	1.0579	0.7382
Panel no.8	3.92	1.20	2.06	4	1	2	1.1144	1.0576	0.7906
Panel no.9	7.48	1.72	2.68	8	2	3	0.8432	1.0485	0.9308
Buckling load factor	1.0039			1.0349					
Total number of plies	456.68			464					
Total number of plies[6]	465.63			464					

Table 7 Stacking sequence and lamination parameters of the panel at local level
(W_1=0.5, W_2=0.45, W_3=0.05)

Panel no.	V_1	V_2	V_3	Buckling load factor
16	1.1594	1.0073	1.2128	1.0366 (2nd buckling factor)
17	1.1816	1.0080	1.2002	
18	1.2366	1.0120	1.1079	
7	1.2630	1.0547	0.8958	
8	1.2604	1.0547	0.8958	
9	1.2084	1.0296	1.1270	0.9615 (1st buckling factor)

Stacking sequence:

16	$[(\pm 45)_2/45/0/\text{-}45/0_4/90/0_3/45/90/\text{-}45/0_4/45/0_2/\text{-}45/90_2/0_4/90_4/45/0/\text{-}45/90_2/0_2/90_4/0/90_3/45/90_2/\text{-}45/45/0/\text{-}45/45/0_2/\text{-}45/0/(\pm 45)_4/90/45/90_2/\text{-}45/45/0_2/\text{-}45/\pm 45]_s$
17	$[(\pm 45)_2/45/0/\text{-}45/0_4/90/0_3/45/90/\text{-}45/0_4/45/0_2/\text{-}45/90_2/0_4/90_4/45/0/\text{-}45/90_2/0_2/90_4/0/90_3/45/90_2/\text{-}45/45/0/\text{-}45/45/0_2/\text{-}45/0/(\pm 45)_4]_s$
18	$[(\pm 45)_2/45/0/\text{-}45/0_4/90/0_3/45/90/\text{-}45/0_4/45/0_2/\text{-}45/90_2/0_4/90_4/45/0/\text{-}45/90_2/0_2/90_4/0]_s$
7	$[\pm 45/0_4/90/0]_s$
8	$[\pm 45/0_4/90_2]_s$
9	$[\pm 45/0_4/90_2/45/90/\text{-}45/0_4]_s$

8.3 Problem with Nine Designable Substructures

Due to the limitation on the number of design variables in ANSYS, all panels in the top skin only are considered designable in the top level optimization and the configuration of panels in the bottom skin is fixed and is the same as the discrete optimal results for the case of six designable substructures above. The results in Table 8 and 9 are obtained by the optimization with lamination parameter-based method. With the objective function to target the lamination parameters from the top level, the plies are shuffled with blending consideration. The buckling load factor has decreased 1% from 1.0213 at the top level to 1.016 at the local level. That shows that the lamination parameter-based method works well for the optimization of laminated composite structures if a small difference between the lamination parameters from top level optimization and the ones calculated in the local level can be produced. This can typically be achieved for realistic aircraft structures where the number of plies is not too small so that blending does not prevent from arriving at a good match of lamination parameters. With the consideration of stack homogeneity index and 90° ply angle jump index in the objective, better stacking sequence of plies compared with the results [5] can be obtained while satisfying blending requirements.

Table 8 Continuous and rounded optimal design with 54 variables for lamination parameter-based method

Panel no.	n_0	n_{45}	n_{90}	n_0	n_{45}	n_{90}	V_1	V_2	V_3
	(Continuous)			(Rounded)					
10	27.07	14.44	21.40	27	15	21	1.0978	1.0094	1.2446
11	25.34	12.85	19.08	25	13	19	1.1261	1.0086	1.2905
12	20.73	5.67	12.84	21	6	13	1.2319	1.0089	1.0736
13	20.70	5.66	12.84	21	6	13	1.2311	1.0087	1.0745
14	25.35	13.24	19.28	25	14	19	1.1189	1.0083	1.2596
15	27.66	15.70	22.04	28	16	22	1.0947	1.0096	1.2001
16	27.48	15.81	22.07	27	16	22	1.0987	1.0102	1.2013
17	25.56	13.49	19.36	26	14	19	1.1224	1.0082	1.2492
18	20.99	6.05	13.05	21	7	13	1.2243	1.0071	1.0460
Buckling load factor	1.0014			1.0213					
Total number of plies	1177.32			1192					

Table 9 Stacking sequence and lamination parameters of the panel at local level with 54 variables
(W_1=0.5, W_2=0.45, W_3=0.05)

Panel no.	V_1	V_2	V_3	Buckling load factor
10	1.1536	1.0077	1.1971	
11	1.1703	1.0082	1.1876	
12	1.2327	1.0129	1.0834	
13	1.2327	1.0129	1.0834	
14	1.1658	1.0081	1.1909	
15	1.1463	1.0075	1.1991	
16	1.1481	1.0075	1.1987	1.016
17	1.1636	1.0080	1.1922	
18	1.2280	1.0121	1.0982	

Stacking sequence:

10 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/(\pm 45)_2/90_3/45/90_2/-45/45/90/-45/(45/0_2/-45)_2/(\pm 45)_3/0/45/90_2/-45/0]_s$

11 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/(\pm 45)_2/90_3/45/90_2/-45/45/90/-45/(45/0_2/-45)_2/(\pm 45)_2]_s$

12 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/\pm 45]_s$

13 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/\pm 45]_s$

14 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/(\pm 45)_2/90_3/45/90_2/-45/45/90/-45/(45/0_2/-45)_2/(\pm 45)_3]_s$

15 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/(\pm 45)_2/90_3/45/90_2/-45/45/90/-45/(45/0_2/-45)_2/(\pm 45)_3/0/45/90_2/-45/0/45/90/-45/0]_s$

16 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/(\pm 45)_2/90_3/45/90_2/-45/45/90/-45/(45/0_2/-45)_2/(\pm 45)_3/0/45/90_2/-45/0/45/90/-45]_s$

17 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/(\pm 45)_2/90_3/45/90_2/-45/45/90/-45/(45/0_2/-45)_2/(\pm 45)_3/0]_s$

18 $[(\pm 45)_2/45/0/-45/0_3/45/0_2/-45/0_4/90_2/0_4/45/90/-45/(90_4/0_3)_2/0/90_2/(\pm 45)_2]_s$

9 Conclusions

A bi-level composite optimization procedure was investigated and lamination parameter-based method was examined for seeking the best stacking sequence of laminated composite wing structures with blending and manufacturing constraints. Two new criteria, stack homogeneity index and 90° ply angle jump index, are implemented to define the objective function. This objective function was minimized to achieve the best stacking sequence of laminate composite wing structures in the local level optimization with the consideration of blending constraints.

Local optimization was performed to shuffle layers while matching the lamination parameter passed from the top level and two new criteria. The stacking sequence optimization in this level can also be done efficiently using a permutation GA because it does not call any numerical simulation and only deals with calculating the objective in terms of the non-dimensional lamination parameters match, stack homogeneity index and 90° ply angle jump index by simple formulae. Once the stacking sequence is determined that satisfies the blending and manufacturing requirements, no buckling analysis needs to be performed if the target values of the lamination parameters, passed from the top level, were kept.

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