University of East Anglia

FINANCE, INVESTMENT AND STRUCTURAL CHANGE

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by

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Abstract

How have advanced economies developed over the last few decades? Some common trends are clear even to the casual observer: a move away from manufacturing and towards services; an increasing reliance on intangible capital, such as intellectual property or organisational capital; and an increase in the size and importance of the financial sector. This thesis asks whether the first of these phenomena can be partly explained by the second and third.

Investment in intangible capital is one kind of irreversible investment: such capital can only be resold at a discount to its purchase price, if it can be resold at all. Chapter 1 develops a partial equilibrium model of an individual firm's optimal investment programme across its life cycle, when investment is partially irreversible. This model is consistent with many stylised facts found in the empirical literature: young firms are reliant on external financing to grow, but eventually reach maturity and are self-financing, while investment irreversibility is positively associated with aggregate corporate savings and negatively associated with aggregate corporate borrowing.

Firms with these characteristics are embedded in a two-sector general equilibrium model in Chapter 2. The degree of investment irreversibility is found to affect the relative sizes of the two sectors – which we label 'manufacturing' and 'services' – but the direction of this effect depends on consumer preferences. Government subsidies to capital liquidation are found often to be welfare-enhancing, but these subsidies increase the size of the manufacturing sector relative to the services sector, so may be misinterpreted as a deliberate boost to manufacturing.

In Chapter 3, the relationship between finance and structural change is examined both theoretically and empirically. A general equilibrium model is developed, predicting that financial development will accelerate structural change towards services and away from manufacturing. This prediction is supported by empirical evidence that structural change accelerated following bank branching deregulation in the United States, which happened in a staggered fashion from the 1970s to the 1990s, where the principal estimation strategy is a pooled ridge augmented synthetic controls study.

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Introduction

The structure of advanced economies is changing. In former ages the backbone of the United Kingdom's economy was industrial: textile manufacturing, or ship building, or coal mining. Today, most British workers are employed in the services sector – healthcare workers, baristas, finance professionals, shop attendants – while manufacturing accounts for a relatively small portion of the economy. Nor is this unique to the United Kingdom. Across the rich world, services form an increasingly large share of national economies.

On the one hand, the decline in manufacturing has been argued to have contributed to growing inequality (Novta and Pugacheva, 2019), to Brexit (Becker et al., 2017) and to the election of Donald Trump (Bonvillian, 2016) and other populist leaders; moreover, it seems clear that the changing locus of global manufacturing has exacerbated tensions between China and the West. On the other hand, the move towards services has been coincident with growing GDP per capita, and some would argue that a transition towards the jobs of the future should actively be encouraged. This thesis is agnostic about whether structural change away from manufacturing and towards services is 'a good thing'. Nonetheless, regardless of whether policymakers might wish to accelerate or inhibit such change, understanding its causes is an indispensable first step.

In particular, this thesis seeks to understand whether other broad trends in advanced economies have contributed to structural change. The rise of intangible capital – software rather than factories, for instance – particularly among fast-growing high-technology firms is likely to have substantial macroeconomic impacts. While there are various ways of thinking about the distinction between traditional capital and intangible capital, we focus on the difficulty in reselling such capital. Chapter 1 describes the life cycle of a firm that undertakes partially irreversible investment; that is, investment in capital which can only be resold at a discount to its purchase price. Despite the lack of explicit adjustment costs, we find that firms accumulate capital gradually, and we find that irreversibility is positively associated with net corporate savings, both of which are consistent with stylised empirical facts. In Chapter 2 we embed these firms in a two-sector general equilibrium model, and study the effect of investment irreversibility on the structural composition of the economy. We find that there is such an effect, but its direction depends on whether households consider services and manufactured goods to be complements or substitutes. In either case, increased irreversibility is associated with reduced output.

The irreversibility of investment affects the collateral value of capital, which helps determine the risk taken by lenders – and hence the interest rate charged – when extending credit to firms. In turn, this affects firms' accumulation decisions. When studying irreversible investment, therefore, the financial sector plays a crucial role, but we can abstract from irreversibility and study the effect of finance as the channeller of savings into investment. The increasing financialisation of economies such as the United States and the United Kingdom is widely appreciated even among lay audiences. There is a general consensus among economists that financial development causes output growth to accelerate, but there is much less work explicitly linking finance to structural change, either theoretically or empirically. This relationship is explored in Chapter 3. We predict that financial development should cause structural change to accelerate – a better functioning financial sector channels savings more effectively into investment, increasing the pace of capital accumulation and productivity growth, the latter of which is the ultimate engine of structural change in our model – and we confirm this prediction empirically.

This thesis examines the links between structural change, investment irreversibility and financial development. This work is novel: there is relatively little existing literature studying the effect of finance on structural change, and even less concerning the effect of investment irreversibility on the structure of the economy. Before turning to the substance of the research, we will briefly describe some of these important concepts.

Structural change

As economies develop, the shares of output, employment and consumption accounted for by particular industries do not remain static. At the highest level, economic activity can be categorised into four broad sectors: agriculture, manufacturing, services and government. Figure 1 shows clearly some stylised facts of structural change in currently advanced economies. As incomes rise, considering each market sector as a share of either aggregate employment or aggregate output (value added): agriculture declines; services increase; and manufacturing increases up to a middle income level, before declining as economies become more advanced.

Comparable trends exist among output measures in lower-income countries, and although the pattern is a little less neat, consumption dynamics also follow a reasonably similar pattern. Herrendorf et al. (2014) present a comprehensive discussion of theory and evidence concerning growth and structural change. Of particular relevance to this thesis, once countries reach middle income level, manufacturing appears to decline monotonically and services increase monotonically as a share of the economy, with agriculture accounting for only a very small share of workers or output.

There are at least two existing theories which purport to explain the mechanisms underlying structural change[1]. First, Kongsamut et al. (2001) suggest that structural change is a consequence of the form of household preferences, with consumers shifting into manufac-

[1]Another obvious potential contributor to the decline in manufacturing in advanced economies is increased trade competition from middle-income economies. In particular, Autor et al. (2013) find that increased Chinese import competition exacerbated job losses in U.S. manufacturing between 1990 and 2007. However, they find that this explains only a quarter of the decline in aggregate U.S. manufacturing employment. Trade therefore does not seem to explain the entire phenomenon of strcutrual change, but is rather an accelerant of the process.

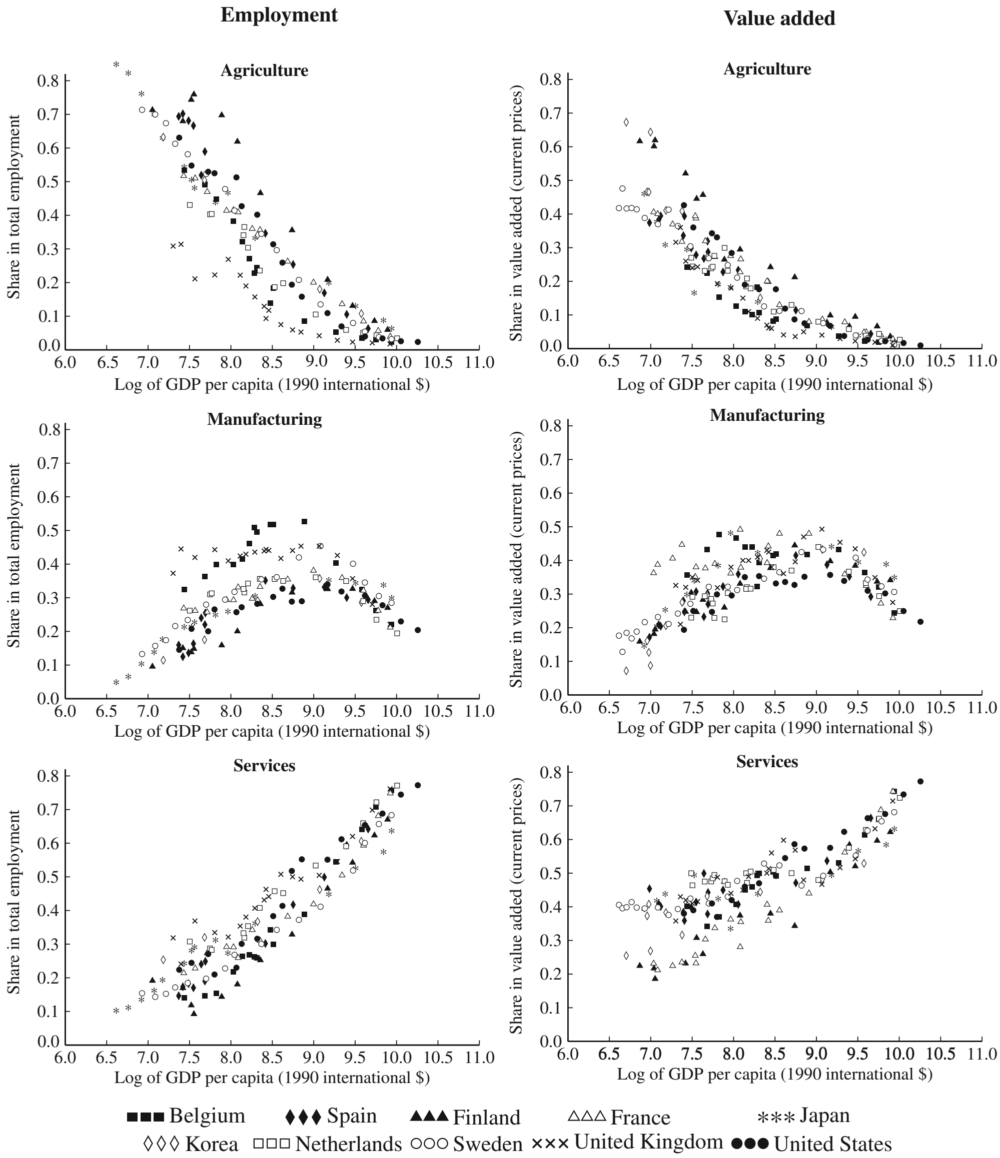

Figure 1: Structural change in currently rich countries

Structural composition of currently rich countries as incomes rise. As a share of either aggregate employment or aggregate output (value added): agriculture declines; services increase; and manufacturing increases up to a middle income level, before declining as economies become more advanced.

Source: Figure 6.1 in Herrendorf et al. (2014, p. 861) tured goods and then services as incomes rise. In particular, such a story is able to generate the inverted 'U' shape of the share of manufacturing. Second, Ngai and Pissarides (2007) assume different rates of technological progress in different sectors, with labour flowing to the slow-growing sector to satisfy consumers, who consider goods from different sectors to be complementary. This is consistent with observed structural change in advanced economies, and with observed differences in total factor productivity (TFP) growth rates (Jorgenson and Stiroh, 2000). In Chapter 3, we take the latter, supply-side story as a starting point and include a financial sector in the general equilibrium model.

Before stating the research questions of this thesis explicitly, we will discuss financial development and irreversible investment.

Intangible capital and irreversible investment

A important phenomenon in the last few decades of advanced economies' development has been the rise of intangible capital. Haskel and Westlake (2018) describe intangible capital as having four distinctive features: spillovers, scalability, synergies and sunkenness. Intangible capital such as software can generate spillovers, since the ideas around which that software is crafted can be hard to protect. By a similar token, such capital is often scalable; software, once written, can be deployed more widely at essentially zero marginal cost. Intangible capital can often exhibit synergies as ideas complement each other, such as the synergy between GPS data and mapping software.

We focus particularly on the final characteristic of intangible capital, sunkenness. Such capital can take the form of investments in process improvements, for example, which can increase the productivity of a firm but which have no resale value. Thus investment in intangible capital can be thought of as being partly or completely irreversible. Figure 2 shows the increasing share of intangible capital in U.S. firms; if this capital is harder to liquidate, then investment is becoming commensurately more irreversible. A substantial portion of this thesis is concerned with the effects of irreversible investment on macroeconomic outcomes, where the precise degree of investment irreversibility can be parametrised.

Figure 2 shows that decreasing leverage has been coincident with the rise of intangible capital, and Falato et al. (2013) suggest that the two are causally linked. Indeed, the models advanced in Chapters 1 and 2 of this thesis also have the characteristic that the corporate sector becomes less leveraged as investment becomes more irreversible. When capital cannot be liquidated at a high price, its collateral value is reduced and its effective cost to the firm is increased. Thus less capital is employed when irreversibility is high, and less debt finance is extended to firms in the growth phase.

There is a large body of literature showing that investment irreversibility can affect the firm's investment programme (e.g. Bertola and Caballero, 1994; Abel and Eberly, 1996, 1999; Cui and Shibata, 2017) and can affect – to a greater or lesser extent – a general equilibrium economy (e.g. Bernanke, 1983; Faig, 2001; Kogan, 2001; Veracierto, 2002; Hugonnier et al., 2005; Sim, 2007). Debt finance is an important channel through which investment irre-

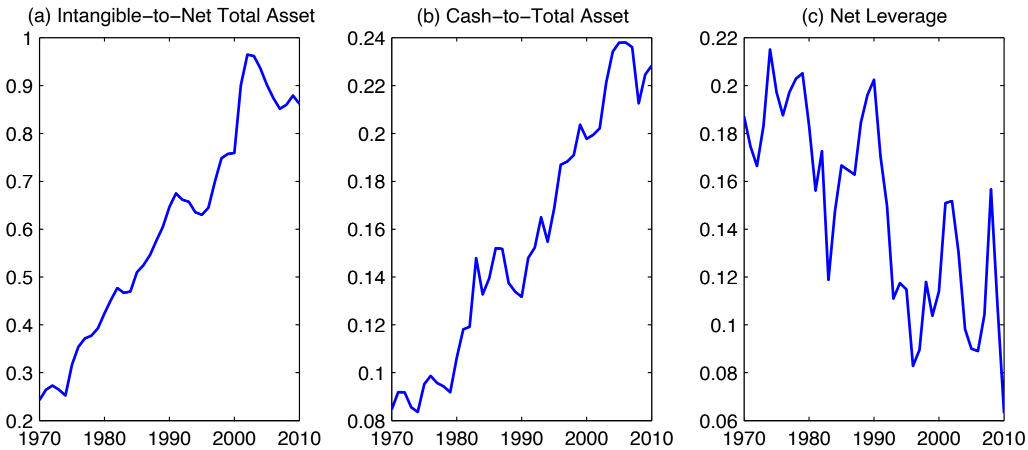

Figure 2: The rise of intangible capital in the United States

Rising intangible capital and cash, and declining leverage among firms in the United States. "The sample includes all Compustat firm-year observations from 1970 to 2010 with positive values for the book value of total assets and sales revenue for firms incorporated in the United States." (Falato et al., 2013, p. 41).
Source: Figure 1 in Falato et al. (2013, p. 41)

versibility can affect economic outcomes, but the financial sector is important regardless of the presence of irreversibility. We therefore turn to a short discussion of financial development and its effects.

Financial development

Economists have long considered that finance might be important for economic growth. The financial sector has been argued to play a number of important roles in the economy: the pooling of savings; the assessment of risk and project screening; the diversification of risk; 'grease in the wheels' of commerce and trading, and so on. While this idea has at times been contested, with some economists arguing that finance follows growth and not vice versa (see e.g. Levine, 2005, p. 867), there is now a general consensus that financial development – the better functioning and greater reach of the financial sector – has a causal effect on increased economic growth (see e.g. King and Levine, 1993; Levine et al., 2000; Rioja and Valev, 2004b,a).

It is well known that the financial sectors of countries like the United Kingdom and the United States are rather large, and have grown over time. This is true more generally: the richer a country is, the larger its financial sector tends to be. Figure 3 shows various measures of financial depth (liquid liabilities of the financial sector as a fraction of GDP, private money as a fraction of total money, and commercial credit as a fraction of GDP) for countries grouped into quartiles by GDP per capita. Clearly, the richer the country, the larger the financial sector.

Often in the empirical literature size measures such as these are taken as proximate indices of financial development. However, recent evidence suggests that financial sectors can grow inefficiently large (Arcand et al., 2015), so financial depth is an imperfect measure

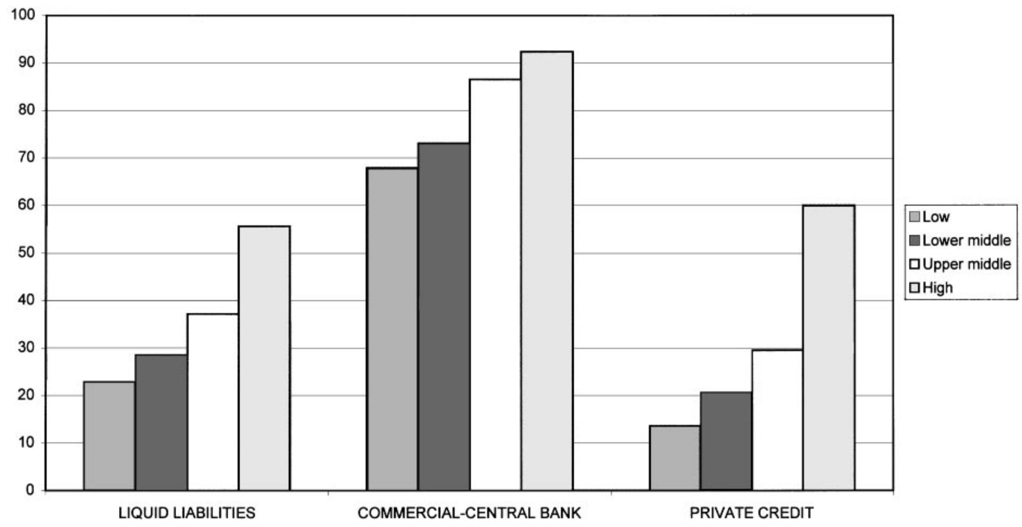

Figure 3: Financial depth and national income

Financial depth by quartile of national GDP per capita, average 1960-1995. See Table 9 in Levine et al. (2000, p. 67) for a list of countries included in the sample. "LIQUID LIABILITIES = liquid liabilities of the financial system (currency plus demand and interest-bearing liabilities of banks and nonbank financial intermediaries) divided by GDP, times 100. COMMERCIAL-CENTRAL BANK = assets of deposit money banks divided by assets of deposit money banks plus central bank assets, times 100. PRIVATE CREDIT = credit by deposit money banks and other financial institutions to the private sector divided by GDP, times 100." (Levine et al., 2000, p. 66)
Source: Figure 1 in Levine et al. (2000, p. 40)

of the quality of the financial sector. When assessing the effects of financial development, an alternative approach is to estimate the impact of a shock to the financial sector, such as bank branching deregulation in the United States. From the 1970s to the 1990s, individual U.S. states deregulated their banking markets to allow multi-branch banks[2]. Jayaratne and Strahan (1998) find that this episode caused 'financial development' in the sense we wish to isolate: banks' operating costs and loan losses reduced, and these savings were substantially passed on to borrowers, who faced a lower cost of credit post-deregulation. A reduction in losses suggests that banks were better at screening or monitoring the projects they funded, while lower overheads led to a reduced spread between savers' rate of return and borrowers' cost of credit. It seems plausible that this should have macroeconomic consequences, and in fact bank branching deregulation has been found to accelerate output growth (Jayaratne and Strahan, 1996), supporting the general proposition that finance encourages growth.

Indeed, the preponderance of the evidence suggests that finance causally affects growth. Structural change is known to go hand in hand with growth, but there is comparatively little work directly considering the link between financial development and structural change. This link is considered explicitly in Chapter 3, where we build a general equilibrium model of finance and structural change and estimate the effect of bank branching deregulation on the structure of U.S. state economies.

[2]See Section 3.2.2 for a more detailed description of bank branching deregulation in the United States.

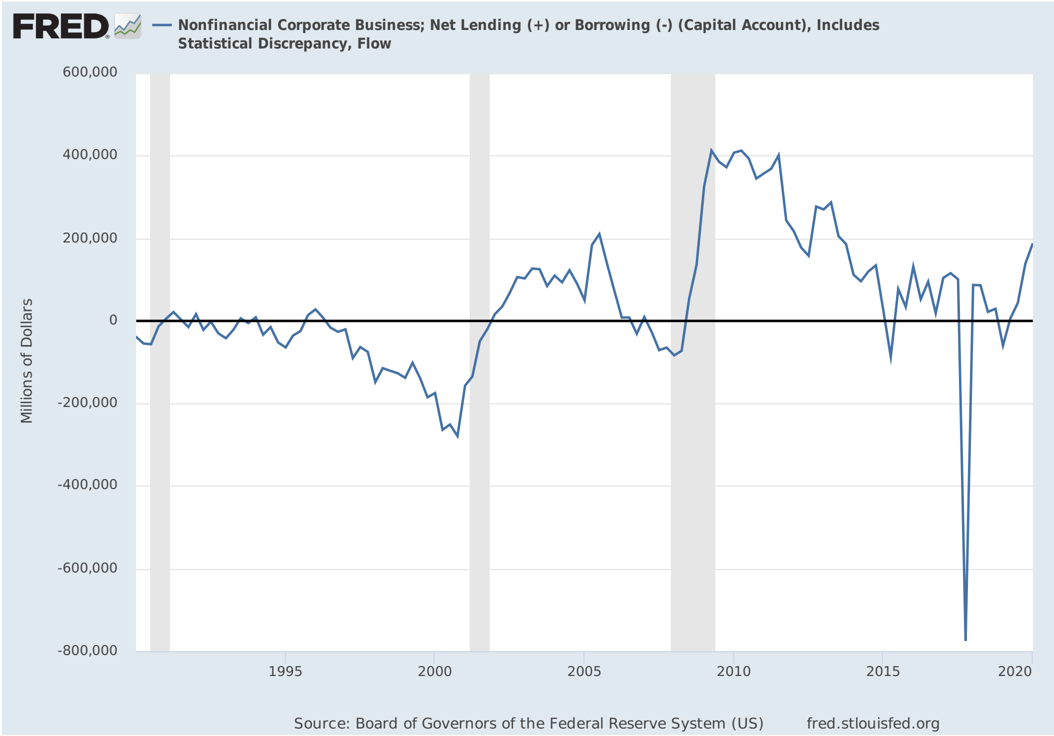

Figure 4: Net lending by U.S. nonfinancial corporations

Net lending by U.S. nonfinancial corporations between 1990 and 2020.
Source: `https://fred.stlouisfed.org/series/NCBLACQ027S`

Research questions

Put very simply, this thesis asks the following question: do finance and investment irreversibility affect structural change in advanced economies? The rise of intangible capital, the increasing sizes of financial sectors, and structural change away from manufacturing and towards services have all been happening in rich countries over recent decades, but there is relatively little theory or evidence linking them. Specifically, we seek to answer the following questions:

1. **Can simple financial frictions generate a realistic life cycle for firms in the absence of adjustment costs?** This thesis focuses on the effects of finance and investment irreversibility. In order for the financial sector to have an effect on firms' investment decisions, at some point in their lives firms must rely on credit from banks (we focus on credit rather than equity markets in this thesis). However, as shown by Figure 4, U.S. nonfinancial corporations have saved more than they have borrowed for most of the last two decades, meaning some individual firms must be net lenders. One way for these two characteristics to coexist is for firms to be borrowers when young and lenders when mature. This implies a life cycle for firms. Convex adjustment costs can artificially induce gradual capital accumulation, but a life cycle that arises from more fundamental frictions would throw more light on the process of firm growth.

2. **Does investment irreversibility affect such firms' life cycle?** Since we embed

microfounded firms into a general equilibrium model in order to study the effect of investment irreversibility on the aggregate economy, we require that irreversibility must affect the firms themselves. Empirical evidence suggests that an increased reliance on intangible capital leads to increased corporate savings; if irreversibility changes firms' life cycle, size and revenue in order that the average firm saves more across its lifetime, this will aggregate to give rise to the observed macroeconomic effects.

3. **Does investment irreversibility affect long-run macroeconomic outcomes, including the structural composition of the economy?** As previously noted, greater use of intangible capital is a significant trend in the recent development of advanced economies, as is structural change away from manufacturing and towards services. There is no existing literature considering whether the former affects the latter, as far as we are aware. To the extent that structural change is an important contributor to the changing nature of society, understanding its catalysts and retardants is useful for policymakers, with potential policy implications.

4. **Does finance have a role to play in structural change?** There is a large literature assessing the relationship between finance and growth, both theoretically and empirically. While much attention has been paid to the effect of finance on the *size* of the economy, comparatively little work has considered the link between finance and the *structure* of the economy. An appreciation of the structural change implications of finance is important when determining financial regulation, for example, or when setting industrial policy, particularly in light of the increasing financialisation of many advanced economies.

All of these questions are addressed theoretically, and the final question is also addressed empirically. In order not to bury the lede, the answers are yes, yes, somewhat[3], and yes respectively. Chapter 1 considers the first two of these questions, while Chapter 2 considers the third and Chapter 3 the fourth.

Contributions

In this thesis, we make a number of original research contributions.

- **Chapter 1** develops a model of the firm's life cycle in the presence of financial and liquidity constraints. Other work considers elements of this problem, but as far as we are aware, this is the only model to consider a firm's optimal investment programme, across its entire life cycle, with a parameter that directly encodes the degree of investment irreversibility. This parameter is found to have a significant effect on corporate leverage and firm dynamics: increased irreversibility leads to decreased leverage, and to smaller firms that reach maturity (that is, reach their terminal size) at a younger age.

[3] The answer to the third question is 'somewhat' because we predict that increasing irreversibility should have a large effect on long-run output, wages and consumption, but a modest effect on the long-run structural composition of the economy.

- **Chapter 2** advances a general equilibrium model of structural change in response to changing investment reversibility. While not the first model to consider irreversibility in a general equilibrium setting, it is unique in studying the structural change implications of irreversibility. We predict that when consumers consider services and manufactured goods to be complements, increased irreversibility is associated with a larger manufacturing sector and a smaller services sector, in proportional terms, and it leads to decreased output, wages and consumption. We also consider the policy implications of the model, concluding that subsidies to capital liquidation are often welfare-enhancing, particularly when investment is substantially irreversible.

- **Chapter 3** considers the link between finance and structural change, both empirically and theoretically. We make several contributions in this chapter: we contribute to a rather small literature in modelling the link between the financial sector and structural change; we add to an ever-expanding body of work on the synthetic control method and its extensions, and in particular we propose a simple statistical test for the validity of pooling multiple case studies; and we use bank branching deregulation in the United States to show empirically that financial development does indeed accelerate structural change.

The thesis concludes by summarising our findings and considering potential avenues for future research.

Chapter 1

Investment Irreversibility, Finance, and the Firm's Life Cycle

Abstract

We develop a model of a financially constrained firm's optimal investment programme when investment is partially irreversible, which generates many of the stylised facts of a firm's life cycle: gradual capital accumulation, growth when young followed by maturity, and retention of liquid wealth alongside reliance on borrowing during the growth phase. Investment is subject to a cash-in-advance constraint. Firms with little internal liquid wealth must therefore borrow from banks, which extend one-period loans, in order to finance investment. The risk of firm failure makes lending risky, so banks charge higher interest rates when firms do not have sufficient collateral. A reduction in the reversibility of investment – for example, a reduction in the tangibility of firms' capital – increases aggregate corporate cash holdings as a multiple (share) of output, while reducing aggregate corporate borrowing as a multiple (share) of output, consistent with empirical evidence.

1.1 Introduction

Capital functions as a productive input for firms, but also as collateral for loans, and as an asset with potential resale value. This multiplicity of uses hints at a rich role for capital to play in investment decisions. In this chapter, we develop a partial equilibrium model of an individual firm's life cycle, when investment is partially reversible and subject to a cash-in-advance constraint. There are several stylised facts identified in the empirical literature which we wish to capture. First, growing firms tend to be young, and young firms tend to experience credit constraints which inhibit growth (Binks and Ennew, 1996; Beck et al., 2005). Second, and on a related note, small firms' growth is inhibited by a lack of internal funds, and a typical small firm retains all its revenue (Carpenter and Petersen, 2002; Oliveira and Fortunato, 2006). Third, on a macroeconomic level, an increased reliance on intangible capital – which can be thought of, among other things, as a reduction in the reversibility of investment – leads to increased corporate cash balances and reduced debt capacity (Falato

*This chapter is the result of joint work with Mich Tvede and Simone Valente.

et al., 2013). The model developed below is consistent with all of these stylised facts, which arise from simple financial frictions and the three uses of capital we have identified.

The study of investment theory in the neoclassical framework stretches back at least to Jorgenson (1963), who formalised a model in which firms wish to maximise their net worth and undertake an optimal programme of investment to that end. As with our model, much subsequent theory has taken the same essential premise and enriched the setting in some respect.

The present model considers firms which have access to debt financing, so relates to the literature on investment in the presence of financial frictions. Hubbard (1997) surveys the literature to that date and develops a model in which finance plays a role in determining business investment, where markets are subject to frictions such as information asymmetry and principal-agent problems. This gives rise to the proposition, familiar from the study of corporate finance, that firms fund themselves hierarchically: preferably using internal funds, then using external financing if necessary. Many other models posit real effects of finance. Bernanke et al. (1999), for example, envisage a setting in which financial frictions exacerbate real shocks. They include a costly-state-verification problem which is similar in spirit to our hypothesis, described in more detail in Section 1.2, that banks can only liquidate capital at a discount to firms.

Cash-in-advance constraints are another way to generate the non-neutrality of finance or monetary policy (Abel, 1985; Lucas and Stokey, 1985; Chu and Cozzi, 2014). Firms in our model face a cash-in-advance constraint, so must borrow to finance investment when they do not have sufficient internal wealth. Firms face an exogenous death risk in any given period, so lending to firms is risky. Banks therefore request collateral from firms.

There is a rich literature concerning the role of collateral in financial markets. Demanding collateral from borrowers limits the risk to lenders, so collateral is most often associated with riskier borrowers and riskier loans (Berger and Udell, 1990). Indeed, theory has long predicted that increased collateral lowers borrowing costs (Barro, 1976), and that collateral is used as a means to cope with adverse selection and moral hazard issues (Bester, 1987). Collateral can therefore have real effects. Bernanke and Gertler (1989) model the relationship between borrowers' net worth and the business cycle: in an upturn, net worth is increased, which lowers borrowing costs and encourages investment, exacerbating the boom. Clementi and Hopenhayn (2006) examine optimal contracting when lenders cannot monitor project outcomes, and consider the effects on investment behaviour and firm dynamics. As in our model, collateral plays an important role; a lack of suitable collateral can constrain firms' borrowing. Similarly, in Rampini and Viswanathan (2010) enforcement constraints give rise to a need for collateral, which is important therefore in the firm's investment and financing decisions. In richer settings, the quantity of collateral required – equivalently, borrowers' leverage – is endogenously determined. Fostel and Geneakoplos (2014) survey the theory of endogenous leverage and collateral determination in models with incomplete markets.

As well as the quantity, the *quality* of collateral is important. Gorton and Ordoñez (2014) suppose that is is costly for borrowers to learn about the quality of collateral, so when there is a credit boom, lenders are willing to lend without incurring this cost. This leads credit to be extended to poor quality borrowers. When a shock causes lenders to have an incentive to produce information about the quality of collateral, a crisis occurs, and credit is (potentially severely) restricted. In an empirical study of the U.S. airline industry, Benmelech and Bergman (2009) confirm that collateral quality matters: borrowing costs are lower when debt is secured by more 'redeployable' capital. Redeployability is closely related to the notion of investment reversibility – capital is likely to resell at a price closer to its purchase cost if it is useful to other producers – and in the model we develop, reversibility has a substantial effect on borrowing.

The reversibility, or irreversibility, of investment has garnered much attention in recent decades. Bertola and Caballero (1994) generate smooth investment in the presence of shocks by relying on irreversible investment: once a firm has made an investment, the resale value of installed capital is zero. Abel and Eberly (1999) examine the joint effects of irreversibility and uncertainty on the long-run accumulation of capital. Abel and Eberly (1996) enrich this essential premise by allowing *partial* reversibility of investment, which is the approach we take below. Cui and Shibata (2017) examine a similar problem when there is informational asymmetry between the firm owner and manager.

A particular kind of irreversible investment is investment in *intangible* capital. Haskel and Westlake (2018) describe intangible capital as having four defining characteristics: scalability, sunkenness, spillovers and synergies. For our purposes in particular, sunkenness is important. Intangible capital such as improved production processes or knowledge capital is very hard to liquidate, so the expenditure on such capital is – to a greater or lesser extent, which we parametrise in our model – sunk, with important economic consequences. Falato et al. (2013) find that U.S. businesses have held steadily increasing cash balances in recent decades, and attribute much of this change to firms' growing reliance on intangible capital, a trend which is increasingly well-documented (Corrado et al., 2009; Corrado and Hulten, 2010). This reduction in firms' indebtedness is confirmed by D'Mello et al. (2018), who find that the leverage ratio of the median non-regulated firm in the United States declined by almost 50% between 1980 and 2010. Rampini and Viswanathan (2013) construct a model in which the tangibility of capital affects firms' leverage: higher tangibility allows higher leverage and increases investment. These stylised facts are also consistent with our model.

Once we understand the firm's optimal investment programme, we can describe the entire life cycle of a firm that is born with zero liquid wealth and zero capital. Mueller (1972) described two competing theories of the firm's life cycle, shareholder value maximisation and growth maximisation. Either way, the youngest and smallest firms are likely to be the fastest growing. This is consistent with the empirical evidence (Evans, 1987a,b), and is generated by financial market frictions in the model by Cooley and Quadrini (2001). In broad terms, our model generates a similar life cycle for the firm. Convex adjustment costs are sometimes invoked as a means of generating smooth investment and other empirical regularities (Cooper, 2006; Cooper and Haltiwanger, 2006); by contrast, our model does not rely on any adjustment costs (except for the partial irreversibility of investment), but still generates gradual capital accumulation. Young firms retain all their revenue, rely on borrowing to finance investment, and grow quickly; mature firms can be entirely self-financing, and may choose to pay dividends, which young firms never do.

While most of the theoretical papers cited above rely on stochastic productivity terms, incomplete enforcement, principal-agent problems, asymmetric information, or some combination, essentially the only constraints in our model are that surviving firms cannot be insolvent, and banks only extend one-period loans. The risk of firm failure makes lending risky, so banks charge higher interest rates when firms do not have sufficient collateral. The importance of collateral gives rise to a key role for investment reversibility: when capital resells at a higher price, less of it is needed in order to be able to repay a loan in the event of firm failure.

There are a handful of theoretical studies at the nexus of finance, irreversibility, and the firm's life cycle. Holt (2003) is similar to our study in many respects, but firms are not able to borrow and investment is fully irreversible. Caggese (2007) allows firms to undertake two kinds of investment – fully reversible and fully irreversible – so there is no 'index of reversibility' whose effect can be studied. Shibata and Nishihara (2018) develop a model that is closely related to ours, but do not focus on the firm's life cycle, and place an upper limit on debt issuance. This chapter, as far as we are aware, presents the only model that considers the firm's life cycle under financial and liquidity constraints, with a parameter encoding investment reversibility whose economic effect can be studied both for an individual firm and in the aggregate.

The rest of the chapter is arranged as follows. Section 1.2 sets up the firm's optimal investment problem, while Section 1.3 solves it. Numerical modelling of the firm's life cycle and aggregate outcomes, such as the relationship between investment reversibility and corporate cash balances, is undertaken in Section 1.4. Section 1.5 concludes.

1.2 Model setup

We consider an individual firm's investment decision. The firm invests in project-specific capital using internal liquid wealth – comprising retained profits – or loans from the banking sector, or some combination of the two. Our distinction between *capital* and *liquid funds* relies on imperfect substitutability between man-made productive inputs and financial assets. We assume that liquid wealth can be converted costlessly into physical capital, but the reverse is not true, and the discount at which capital can be liquidated is our notion of the *reversibility* of investment. Besides the behaviour of the firm, we also consider the behaviour of a risk-neutral banking sector in setting the interest rate charged on loans when the firm finances investment wholly or partly with borrowed funds.

1.2.1 The firm

Time is discrete and indexed by t. The firm produces y_t units of output at date t by means of k_t units of project-specific physical capital, according to

$$y_t = \varphi_t f(k_t)$$

where $\varphi_t > 0$ is a productivity parameter and $f : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a C^2-class production function satisfying

$$f'(k_t) \equiv \partial f(k_t)/\partial k_t > 0 \qquad \text{and} \qquad f''(k_t) \equiv \partial^2 f(k_t)/\partial k_t^2 < 0.$$

The strict concavity of the production function implies strictly positive operative profits when the firm compensates physical capital at its marginal productivity. The firm purchases Δ_t units of the final consumption good and converts them into Δ_t units of physical capital to be used in its own production process. Once installed, the capital of the firm is generally not re-convertible one-for-one into the final good or into other firms' capital: re-conversion may be performed only at a strictly positive cost. The productivity parameter φ_t is stochastic: the firm is essentially a *project* affected by a death shock such that

$$\varphi_t = \begin{cases} \bar{\varphi} \geq 1 & \text{with probability} \quad 1-\delta \\ 0 & \text{with probability} \quad \delta \end{cases}$$

where $\delta \in (0,1)$ is an exogenous probability of project failure that may hit the firm at any date. After capital has been installed and used for production, if the 'bad state' which occurrs with probability δ is realised at date t, the firm's output is and will remain zero from date t onwards. The firm has perfect information about the probability of project failure: prior to date t, the units of output that the firm expects to sell at date t are given by

$$\mathbb{E}_t(y_t) = (1-\delta)\,\bar{\varphi} f(k_t)\,.$$

Investment Δ_t is made immediately prior to date $t \geq 1$ so that the net capital stock available for production at date t is determined by the *capital accumulation constraint*

$$k_t = (1-\mu)\,k_{t-1} + \Delta_t, \tag{1.1}$$

where k_{t-1} is the installed capital used in production at date $t-1$, and the exogenous constant $\mu \in [0,1]$ represents depreciation, that is, the fraction of the capital stock that is destroyed or becomes obsolete during the production process. Immediately after any date $t \geq 1$, the firm has $(1-\mu)\,k_t$ units of residual capital in any state – that is, the firm has some residual capital regardless of whether it is suffers project failure at date t.

The firm is subject to a cash-in-advance constraint with respect to investment expenditure. Since Δ_t must be paid up front, the firm must either borrow on the credit market or have sufficient internal liquid wealth to cover investment before production begins. Immediately prior to each date t, the firm has $a_{t-1} \geq 0$ units of liquid wealth on its balance sheet. At date zero, the firm is born with zero liquid wealth and zero capital, $a_0 = k_0 = 0$. At all subsequent dates, $t \geq 1$, liquid wealth represents cumulative retained profits from the previous dates. At date t, the firm can use a_{t-1} units of liquid wealth to finance part of its internal capital investment or to purchase financial assets yielding the *risk-free interest rate* r, which we assume to be constant and which the firm takes as given. Formally,

$$a_{t-1} = \underbrace{a_t^F}_{\text{financial asset purchases}} + \underbrace{s_t}_{\text{self-financed capital investment}} \tag{1.2}$$

where s_t is the *self-financed* part of the firm's total capital investment Δ_t. Since the firm may finance part of its physical capital investment by borrowing b_t from banks, we have

$$\Delta_t = \underbrace{s_t}_{\text{self-financed}} + \underbrace{b_t}_{\text{debt-financed}} = a_{t-1} - a_t^F + b_t. \tag{1.3}$$

Note that the liquidity constraint is well-specified only if a_{t-1}, a_{t-1}^F and s_t are all non-negative. Therefore, self-financed investment cannot exceed liquid wealth,

$$a_{t-1} \geq s_t. \tag{1.4}$$

We assume that debt-financed investment is subject to the repayment of the loan rate r_t^b, which is firm-specific and depends on how banks evaluate the risk of obtaining repayment.

The objective of the firm is to maximize the present discounted value of the dividends paid to shareholders, including the liquidation of any residual capital in case the firm dies. Considering a given date t at which the firm survives, the ex-post dividend that the firm would pay to shareholders is

$$\begin{aligned} \bar{\psi}_t^S = {} & \underbrace{a_{t-1}}_{\text{initial liquid wealth}} - \underbrace{a_t^F}_{\text{financial asset purchases}} + \underbrace{(1+r)\, a_t^F}_{\text{financial asset returns}} + \underbrace{p\bar{\varphi} f(k_t)}_{\text{revenues from output sales}} \\ & - \underbrace{\Delta_t}_{\text{investment}} + \underbrace{b_t - \left(1 + r_t^b\right) b_t}_{\text{loan credit minus repayment}} - \underbrace{a_t}_{\text{retained wealth}} \end{aligned} \tag{1.5}$$

$$= (1+r)\, a_t^F + p\bar{\varphi} f(k_t) - \left(1 + r_t^b\right) b_t - a_t, \tag{1.6}$$

where p is the price at which the firm can sell output – like the safe interest rate r, we assume that this is constant and taken by the firm as given – and where the last term is obtained by substituting $\Delta_t = a_{t-1} - a_t^F + b_t$ from equation (1.3). Equation (1.6) says that, absent technological failure, the dividend paid to shareholders is the residual of the revenues from the firm's financial assets and output sales after subtracting the repayment of the loan and the amount of retained liquid wealth kept within the firm for the next period. We will henceforth assume that the firm always repays its debt in case of survival, so that $\bar{\psi}_t^S \geq 0$ holds even in the case of zero retained wealth, $a_t = 0$. This implies that loan repayments are subject to the upper bound

$$(1+r)\,a_t^F + p\bar{\varphi} f(k_t) \geq \left(1+r_t^b\right) b_t. \tag{1.7}$$

Consider now a given date t at which the firm experiences technological failure: the firm shuts down, earns no revenues from output sales but can still re-convert the $(1-\mu)\,k_t$ residual units of physical capital into homogeneous final output and sell it on the market, obtaining $\theta^F(1-\mu)\,k_t$ with $0 < \theta^F < 1$. Since the purchase price of new capital to be installed is unity, the hypothesis $\theta^F < 1$ captures the *partial irreversibility* of investment: if capital can be uninstalled, resold and redeployed at very little cost, or easily converted to the consumption good, so θ^F is nearly unity, then investment is highly reversible. If instead capital is difficult to uninstall, or difficult to resell, or very specific to a particular project, so that θ^F is almost zero, then investment is almost completely irreversible.

Differently from the case of survival, technological failure may cause insolvency towards banks: the market value of all the firm's liquid assets at the end of the period might be insufficient to repay the outstanding loan. Whether the firm is solvent or (partially) insolvent crucially depends on the resale value of installed capital: full solvency requires

$$(1+r)\,a_t^F + \theta^F(1-\mu)\,k_t \geq \left(1+r_t^b\right) b_t. \tag{1.8}$$

If the solvency condition in equation (1.8) is (not) satisfied, the firm will (not) be able to pay dividends to shareholders in the event of failure. Formally, the ex-post dividend in case of technological failure reads

$$\bar{\psi}_t^F = \max\left\{(1+r)\,a_t^F + \theta^F(1-\mu)\,k_t - \left(1+r_t^b\right) b_t, 0\right\}. \tag{1.9}$$

From equations (1.6) and (1.9), the ex-ante dividend that the firm expects immediately prior to date t is given by $(1-\delta)\,\bar{\psi}_t^S + \delta\bar{\psi}_t^F$. Given the firm begins life with zero liquid wealth and capital, the firm chooses the time paths of the choice variables $\left\{a_t, a_t^F, b_t, k_t\right\}_{t=1}^{\infty}$ that maximizes the present-discounted value of expected dividends,

$$\begin{aligned}\Psi_0 &\equiv \frac{(1-\delta)\,\bar{\psi}_1^S + \delta\bar{\psi}_1^F}{1+r} + \frac{1-\delta}{1+r}\cdot\frac{(1-\delta)\,\bar{\psi}_2^S + \delta\bar{\psi}_2^F}{1+r} + ... \\ &= \frac{(1-\delta)\,\bar{\psi}_1^S + \delta\bar{\psi}_1^F}{1+r} + \sum_{t=2}^{\infty}\left(\frac{1-\delta}{1+r}\right)^{t-1}\left[\frac{(1-\delta)\,\bar{\psi}_t^S + \delta\bar{\psi}_t^F}{1+r}\right],\end{aligned} \tag{1.10}$$

subject to the accumulation constraint in equation (1.1) and the non-negativity constraints implied by equations (1.3) and (1.4). The firm has perfect foresight with respect to the loan rate r_t^b charged by banks.

1.2.2 The banking sector

The banking sector is perfectly competitive and risk-neutral, and faces a cost of funds equal to the risk-free rate r. It makes loans to firms, such that each loan has an expected return of

r. In the event of a debtor firm failing and being unable to repay its debt by liquidating its own capital, the bank seizes the firm's capital and undertakes the liquidation itself. Capital thus acts as collateral, but the bank is at a disadvantage relative to the firm in liquidating capital; it is able to liquidate capital at unit value θ^B with

$$0 < \theta^B < \theta^F.$$

The difference between the firm's own liquidation value – that is, investment reversibility θ^F – and the capital resale price θ^B faced by banks captures costs that arise in the event of bankruptcy, and is very similar in spirit to the assumption made in costly state verification models. The closer θ^B is to θ^F, the better the financial sector is at handling firm failure and deploying capital somewhere else that is useful, so θ^B is essentially an index of *financial development*.

The bank can be thought of as offering a 'menu' of loan sizes and rates from which the firm may select. The interest rate faced by the firm depends on the amount it borrows, b_t. Recalling our previous discussion of the solvency conditions, the payoffs for the bank are characterized as follows. If the firm survives date t, by assumption the bank is repaid in full. If the firm dies, the bank is repaid in full if the solvency condition in equation (1.8) holds, and receives the liquidation value otherwise. Formally, the ex-post earnings for the bank in the various cases read

$$\begin{cases} \left(1+r_t^b\right) b_t & \text{if} \quad (1+r)\, a_t^F + p\bar{\varphi} f\,(k_t) \geq \left(1+r_t^b\right) b_t & \text{survival} \\ \left(1+r_t^b\right) b_t & \text{if} \quad (1+r)\, a_t^F + \theta^F (1-\mu)\, k_t \geq \left(1+r_t^b\right) b_t & \text{death with solvency} \\ (1+r)\, a_t^F + \theta^B (1-\mu)\, k_t & \text{if} \quad (1+r)\, a_t^F + \theta^F (1-\mu)\, k_t < \left(1+r_t^b\right) b_t & \text{death with insolvency} \end{cases}$$

Since the banking sector is perfectly competitive, the lending rate will always be such that the expected repayment from the firm is $(1+r)\, b_t$. We can thus distinguish two scenarios. First, if the firm is able to repay its debt in the event of death – that is, if inequality (1.8) holds – then the bank charges the safe rate, $r_t^b = r$, and the supply of loans is characterized by

$$1 + r_t^b = 1 + r \quad \text{for} \quad b_t \leq a_t^F + \frac{\theta^F (1-\mu)\, k_t}{1+r}, \qquad \text{“small loan scenario”}. \tag{1.11}$$

We label this case as the "small loan scenario" because it arises only when b_t is sufficiently small to be covered by the residual assets of the firm in case it shuts down. In the second scenario, instead, the firm is not able to repay its debt in the event of death. In this case, the loan rate charged by the bank is determined by the no-arbitrage condition over the bank's expected returns,

$$(1+r)\, b_t = (1-\delta)\left(1+r_t^b\right) b_t + \delta\left[(1+r)\, a_t^F + \theta^B (1-\mu)\, k_t\right],$$

which yields

$$1+r_t^b=\frac{1}{1-\delta}\left(1+r-\delta\frac{(1+r)\,a_t^F+\theta^B\,(1-\mu)\,k_t}{b_t}\right),\qquad \text{“large loan scenario”},\tag{1.12}$$

and holds when inequality (1.8) is violated. We can interpret the right hand side of equation (1.12) as two components: the safe rate augmented by the death factor, $(1+r)\,/\,(1-\delta)$, minus the marginal partial compensation offered by the firm's residual assets in case of death. As is intuitive, the loan rate charged by the bank is increasing in the size of the loan b_t. In particular, for given a_t^F and k_t, we have

$$\lim_{b_t\to\infty}1+r_t^b=\frac{1+r}{1-\delta},$$

which defines the upper bound for the loan rate charged by banks on large loans. The lower bound is determined by the maximum loan size in the “small loan scenario”, denoted by $\bar{b}_t$, determined by the solvency condition in equation (1.8) as follows,

$$\bar{b}_t\equiv a_t^F+\frac{\theta^F\,(1-\mu)\,k_t}{1+r}\tag{1.13}$$

$$\lim_{b_t\to\bar{b}_t^+}1+r_t^b=\frac{1+r}{1-\delta}\left(1-\delta\frac{(1+r)\,a_t^F+\theta^B\,(1-\mu)\,k_t}{(1+r)\,a_t^F+\theta^F\,(1-\mu)\,k_t}\right)>1+r,\tag{1.14}$$

where the strict inequality in equation (1.14) holds as long as $\theta^F>\theta^B$. Therefore, for given a_t^F and k_t, the supply of loans is as shown in Figure 1.1.

1.2.3 A recursive formulation of the firm's problem

Equation (1.10) represents the firm's expected net present value, but hints at a natural recursive formulation of the firm's problem. Let $V_t\,(a_{t-1},k_{t-1})$ be the firm's value function: that is, at time t, given capital and retained liquid wealth at time $t-1$, $V_t\,(a_{t-1},k_{t-1})$ takes the maximal possible present value of the firm's lifetime future expected dividend stream. Relating this to the infinite-horizon problem,

$$V_1\,(a_0,k_0)=\max_{\left\{a_t^F,s_t,b_t,a_t,k_t\right\}_{t=1}^{\infty}}\Psi_0.$$

The firm's Bellman equation writes this problem recursively, and takes the form

$$V_t\,(a_{t-1},k_{t-1})=\max_{\left(a_t^F,s_t,b_t,a_t,k_t\right)}\left\{\frac{1-\delta}{1+r}\cdot\bar{\psi}_t^S+\frac{\delta}{1+r}\cdot\bar{\psi}_t^F+\frac{1-\delta}{1+r}V_{t+1}\,(a_t,k_t)\right\},$$

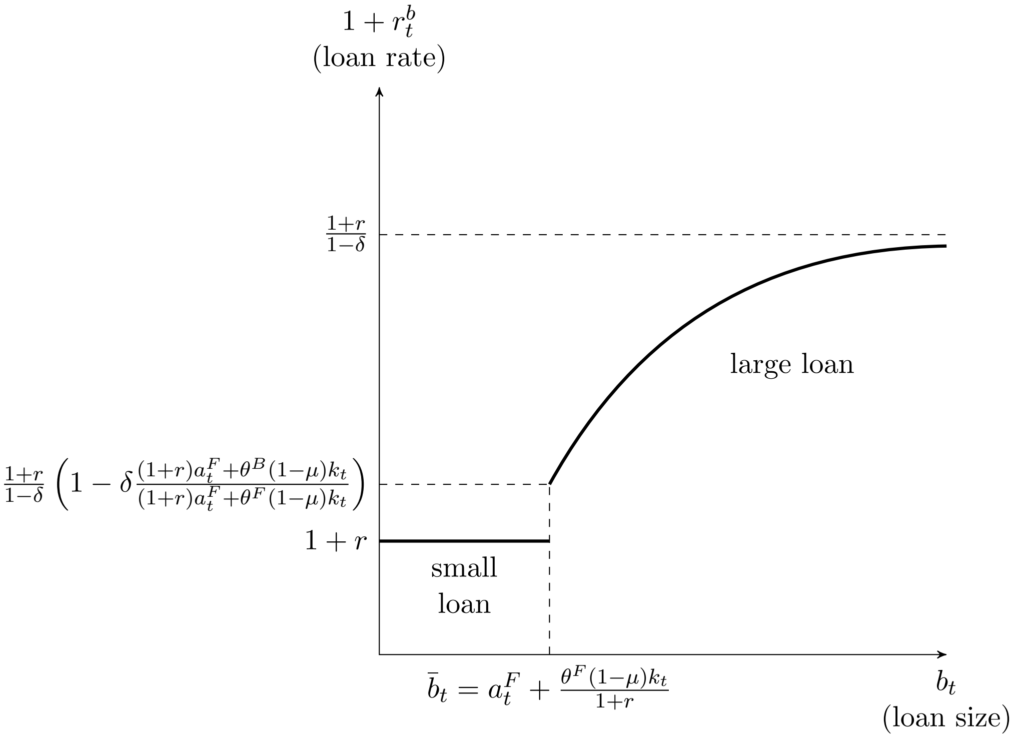

Figure 1.1: Interest rate discontinuity

where the survival dividend $\bar{\psi}_t^S$ and the failure dividend $\bar{\psi}_t^F$ are given by equations (1.6) and (1.9). Explicitly, the firm's Bellman equation is

$$
\begin{aligned}
V_t\left(a_{t-1},k_{t-1}\right) = \max_{\left(a_t^F,s_t,b_t,a_t,k_t\right)} \Bigg\{ & (1-\delta)\,\frac{(1+r)\,a_t^F + p\bar{\varphi}f\left(k_t\right) - \left(1+r_t^b\right)b_t - a_t}{1+r} \\
& + \delta\frac{\max\left\{(1+r)\,a_t^F + \theta^F\left(1-\mu\right)k_t - \left(1+r_t^b\right)b_t, 0\right\}}{1+r} \\
& + (1-\delta)\,\frac{V_{t+1}\left(a_t,k_t\right)}{1+r}\Bigg\},
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
k_t &= (1-\mu)\,k_{t-1} + s_t + b_t \\
a_t^F + s_t &= a_{t-1} \\
1+r_t^b &= \begin{cases} 1+r & \text{for } b_t \le a_t^F + \theta^F\frac{1-\mu}{1+r}k_t \\ \frac{1}{1-\delta_t}\left(1+r-\delta_t\frac{(1+r)a_t^F+\theta^B(1-\mu)k_t}{b_t}\right) & \text{for } b_t > a_t^F + \theta^F\frac{1-\mu}{1+r}k_t, \end{cases}
\end{aligned}
$$

from equations (1.1), (1.2), (1.11) and (1.12). The maximum small loan, given by equation (1.13), can be expressed as

$$\begin{aligned}\bar{b}_t &= a_t^F + \frac{\theta_F (1-\mu) k_t}{1+r} \\ &= a_t^F + \frac{a_{t-1} + (1-\mu) k_{t-1}}{(1+r)/\theta^F (1-\mu) - 1},\end{aligned}$$

where the second line follows from substituting in the constraints. Thus a small loan is a loan that satisfies $b_t \leq \bar{b}_t$, while a large loan is a loan that satisfies $b_t > \bar{b}_t$. It is convenient to define net borrowing B_t, which is the difference between borrowing and financial investment. The threshold level of net borrowing is therefore given by

$$\begin{aligned}\bar{B}_t &\equiv \bar{b}_t - a_t^F \\ &= \frac{a_{t-1} + (1-\mu) k_{t-1}}{(1+r)/\theta^F (1-\mu) - 1}.\end{aligned}$$

Given $\bar{B}_t$, we can define $\bar{k}_t (a_{t-1}, k_{t-1})$, the maximum possible capital that can be employed in period t by taking out only a small loan,

$$\begin{aligned}\bar{k}_t (a_{t-1}, k_{t-1}) &\equiv a_{t-1} + (1-\mu) k_{t-1} + \bar{B}_t \\ &= a_{t-1} + (1-\mu) k_{t-1} + \frac{a_{t-1} + (1-\mu) k_{t-1}}{(1+r)/\theta^F (1-\mu) - 1} \\ &= \frac{a_{t-1} + (1-\mu) k_{t-1}}{1 - \theta^F (1-\mu)/(1+r)}. \end{aligned} \tag{1.15}$$

The quantity $\bar{k}_t (a_{t-1}, k_{t-1})$ will be useful later. The constraints allow two control variables to be eliminated from the Bellman equation, which becomes

$$\begin{aligned}V_t (a_{t-1}, k_{t-1}) = &\max_{(b_t, a_t, k_t)} \\ &\left\{ (1-\delta) \frac{(1+r)(a_{t-1} + b_t + (1-\mu) k_{t-1} - k_t) + p\bar{\varphi} f(k_t) - \left(1 + r_t^b\right) b_t - a_t}{1+r} \right. \\ &+ \delta \frac{\max \left\{ (1+r)(a_{t-1} + b_t + (1-\mu) k_{t-1} - k_t) + \theta^F (1-\mu) k_t - \left(1 + r_t^b\right) b_t, 0 \right\}}{1+r} \\ &\left. + (1-\delta) \frac{V_{t+1} (a_t, k_t)}{1 + r_t^s} \right\}.\end{aligned}$$

Note that by substituting the constraints into equation (1.7), it follows that in order for the firm to be able to repay its loan fully if it does not fail, the loan must be subject to the bound

$$\begin{aligned} & (1 + r_t) a_t^F + p\bar{\varphi} f(k_t) \geq \left(1 + r_t^b\right) b_t \\ \Rightarrow \quad & (1+r)(a_{t-1} + b_t + (1-\mu) k_{t-1} - k_t) + p\bar{\varphi} f(k_t) \geq \left(1 + r_t^b\right) b_t. \end{aligned}$$

If the firm chooses to retain maximal liquid wealth each period that it survives, so it always chooses $\bar{\psi}_t^S = 0$, then comparing this expression to the Bellman equation makes it clear that this is equivalent to

$$a_t \geq 0.$$

That is, when a_t is maximal, the condition that the firm must be able to repay its loan fully if it does not fail is equivalent to the condition that the firm must at all times have non-negative liquid wealth. We will see later that it is never sub-optimal for the firm to choose to retain maximal liquid wealth, and therefore the loan repayment constraint can be expressed as $a_t \geq 0$.

1.3 Solving the firm's problem

1.3.1 Best capital stocks and constrained capital stocks

Before solving the firm's problem, it will be useful to give some formal definitions.

Definition 1.1 *The first-best capital stock k^* is the capital stock whose marginal product, in expected terms, equals the risk-free return, when the firm rather than the bank undertakes capital liquidation in the event of firm failure (this is the first-best since $\theta^F > \theta^B$). This is the capital stock that satisfies*

$$(1-\delta)\, p\bar{\varphi} f'(k^*) + (1-\delta)(1-\mu) + \delta\theta^F (1-\mu) = 1 + r, \tag{1.16}$$

where the three terms on the LHS represent the expected marginal product of capital, the expected value of undepreciated capital, and the expected liquidation value of capital.

Definition 1.2 *The boundary-best capital stock is the maximum possible capital stock $\bar{k}_t(a_{t-1}, k_{t-1})$ that can be employed in period t by taking out only a small loan, given by equation* (1.15)*.*

Definition 1.3 *The small-loan-constrained capital stock $\bar{k}_t^S$ is the maximum capital stock that can be employed at time t given that the firm takes out a small loan and obeys the condition $a_t \geq 0$. Explicitly,*

$$\bar{k}_t^S (a_{t-1}, k_{t-1}) \equiv \max\left\{k_t : p\bar{\varphi} f(k_t) + (1+r)(a_{t-1} + (1-\mu) k_{t-1} - k_t) \geq 0\right\}. \tag{1.17}$$

While $\bar{k}_t$ is the maximum capital stock that ensures the firm is solvent if it *fails*, $\bar{k}_t^S$ is the maximum capital stock that ensures the firm is solvent if it *survives*, supposing that it faces the risk-free borrowing cost r. It could be the case that $\bar{k}_t^S < \bar{k}$, so it convenient to define it as a separate quantity.

Definition 1.4 *The large-loan-best capital stock k^{**} is the capital stock whose marginal product, in expected terms, equals the risk-free return, when the bank undertakes capital liquidation* *in the event of firm failure (which occurs when the firm takes out a large loan). This is the capital stock that satisfies*

$$(1-\delta)\,p\bar{\varphi}f'\left(k^{**}\right)+(1-\delta)\,(1-\mu)+\delta\theta^{B}\,(1-\mu)=1+r. \tag{1.18}$$

Note that equation (1.18) is the same as the expression defining k^*, except that θ^B has replaced θ^F. Therefore $f'(k^{**}) > f'(k^*)$, so $k^{**} < k^*$. We require one final definition.

Definition 1.5 *The large-loan-constrained capital stock $\bar{k}_t^L$ is the maximum capital stock that can be employed at time t given that the firm takes out a large loan and obeys the condition $a_t \geq 0$. Explicitly,*

$$\bar{k}_t^L\,(a_{t-1},k_{t-1}) \equiv \max \left\{k_t\,:\,p\bar{\varphi}f\,(k_t)+\frac{1+r}{1-\delta}\,(a_{t-1}+(1-\mu)\,k_{t-1}-k_t)+\frac{\delta\theta^{B}\,(1-\mu)\,k_t}{1-\delta}\geq 0\right\}. \tag{1.19}$$

Thus $\bar{k}_t^L$ is the maximum capital stock that ensures the firm is solvent if it *survives*, supposing that it takes a large loan. Below we will show that the firm always chooses one of these five capital stocks, so long as it doesn't enter a given period with 'too much' capital (this will be made precise later).

1.3.2 First order conditions

Much of the difficulty in solving the firm's problem comes from the discontinuity between the small loan and the large loan regimes. Therefore we will begin by considering each of these regimes in isolation. First we will restrict our attention to small loans. When the firm is in the small loan regime, $r_t^b = r$, so the Bellman equation becomes

$$\begin{aligned} V_t\,(a_{t-1},k_{t-1}) = \max_{(a_t,k_t)} \Bigg\{ & (1-\delta)\,\frac{(1+r)\,(a_{t-1}+(1-\mu)\,k_{t-1}-k_t)+p\bar{\varphi}f\,(k_t)-a_t}{1+r} \\ & +\delta\frac{(1+r)\,(a_{t-1}+(1-\mu)\,k_{t-1}-k_t)+\theta^{F}\,(1-\mu)\,k_t}{1+r} \\ & +(1-\delta)\,\frac{V_{t+1}\,(a_t,k_t)}{1+r}\Bigg\}. \end{aligned} \tag{1.20}$$

Note that under these circumstances, b disappears from the problem as the borrowing rate is the same as the safe rate. We know by Benveniste and Scheinkman (1979) that the value function is differentiable[1], so long as there is some optimal solution to the firm's investment

[1]Differentiability requires convexity of the technology set defining feasible combinations of (a_t, k_t) given (a_{t-1}, k_{t-1}), which can be demonstrated in the interior of each of the small loan regime and large loan regime. Note that the value function does not satisfy the requirements for differentiability in Benveniste and Scheinkman (1979) at the boundary between taking a small loan and a large loan – inspection of Figure 1.1 suggests this intuitively, since there is a discontinuity in the interest rate at this point.

problem. Then the first order conditions for V are

$$\frac{\partial V_{t+1}(a_t, k_t)}{\partial a_t} = 1 \tag{1.21}$$

$$\frac{\partial V_{t+1}(a_t, k_t)}{\partial k_t} = \frac{1+r}{1-\delta} - p\bar{\varphi}f'(k_t) - \frac{\delta\theta^F(1-\mu)}{1-\delta}. \tag{1.22}$$

So long as both of these first order conditions hold, it is possible to characterise the capital stock chosen by the firm.

Lemma 1.6 *When the firm takes out a small loan, if the first order conditions (equations* (1.21) *and* (1.22)*) on the Bellman equation hold at time t and $t+1$, the firm employs the first-best capital stock k^* at time t.*

Proof. Equation (1.21) can be interpreted as indifference between retaining wealth and paying a dividend. We can be still more specific about equation (1.22), by finding an explicit value for $\partial V_{t+1}(a_t, k_t)/\partial k_t$. Suppose that there are differentiable optimal policy functions[2] at the point of interest,

$$\begin{aligned} a_{t+1} &= h(a_t, k_t) \\ k_{t+1} &= g(a_t, k_t). \end{aligned}$$

Then we can update the Bellman equation by one period and rewrite it as

$$\begin{aligned} V_{t+1}(a_t, k_t) = &(1-\delta)\frac{(1+r)(a_t + (1-\mu)k_t - g(a_t,k_t)) + p\bar{\varphi}f(g(a_t,k_t)) - h(a_t,k_t)}{1+r} \\ &+ \delta\frac{(1+r)(a_t + (1-\mu)k_t - g(a_t,k_t)) + \theta^F(1-\mu)g(a_t,k_t)}{1+r} \\ &+ (1-\delta)\frac{V_{t+2}(h(a_t,k_t), g(a_t,k_t))}{1+r}. \end{aligned}$$

Totally differentiating with respect to k_t yields the envelope condition

$$\begin{aligned} \frac{\partial V_{t+1}(a_t, k_t)}{\partial k_t} = &(1-\mu) - \frac{\partial g(\cdot)}{\partial k_t} + \frac{1-\delta}{1+r}\left[p\bar{\varphi}f'(g(\cdot))\frac{\partial g(\cdot)}{\partial k_t} - \frac{\partial h(\cdot)}{\partial k_t}\right] \\ &+ \frac{\delta\theta_F(1-\mu)}{1+r}\frac{\partial g(\cdot)}{\partial k_t} + \frac{1-\delta}{1+r}\left[\frac{\partial V_{t+2}(\cdot)}{\partial h(\cdot)}\frac{\partial h(\cdot)}{\partial k_t} + \frac{\partial V_{t+2}(\cdot)}{\partial g(\cdot)}\frac{\partial g(\cdot)}{\partial k_t}\right] \\ = &(1-\mu) + \frac{1-\delta}{1+r}\frac{\partial h(\cdot)}{\partial k_t}\left[\frac{\partial V_{t+2}(\cdot)}{\partial h(\cdot)} - 1\right] \\ &+ \frac{1-\delta}{1+r}\left[\frac{\partial V_{t+2}(\cdot)}{\partial g(\cdot)} + p\bar{\varphi}f'(g(\cdot)) + \frac{\delta\theta^F(1-\mu)}{1-\delta} - \frac{1+r}{1-\delta}\right]\frac{\partial g(\cdot)}{\partial k_t} \\ = &1-\mu, \end{aligned} \tag{1.23}$$

since by the first order conditions both terms in the square brackets in equation (1.23) are

[2] Araujo (1991), among others, gives some conditions under which the optimal policy functions are differentiable. The setting studied here does not satisfy these requirements, but Lemma 1.8 – which does not depend on Lemmas 1.6 or 1.7 – guarantees that a_t is a differentiable function of a_{t-1} and k_{t-1}. We simply assume that the same is true for capital k_t at the point of interest, which seems at least plausible.

equal to zero. We can substitute this back into the first order condition to get the optimal capital stock, implicitly defined by

$$(1-\delta)\, p\bar{\varphi} f'(k_t) + (1-\delta)(1-\mu) + \delta\theta^F (1-\mu) = 1+r.$$

Note that this is precisely the first-best capital stock, $k_t = k^*$, as defined in equation (1.16). ■

Next we will restrict our attention to the large loan regime. Then the Bellman equation becomes

$$\begin{aligned}
V(a_{t-1}, k_{t-1}) = \max_{(b_t, a_t, k_t)} \Bigg\{ & (1-\delta) \frac{(1+r)(a_{t-1} + b_t + (1-\mu) k_{t-1} - k_t) + p\bar{\varphi} f(k_t) - a_t}{1+r} \\
& - (1-\delta) \frac{\frac{1}{1-\delta}\left(1 + r - \delta \frac{(1+r) a_t^F + \theta^B (1-\mu) k_t}{b_t}\right) b_t}{1+r} \\
& + (1-\delta) \frac{V_{t+1}(a_t, k_t)}{1+r} \Bigg\} \\
= \max_{(a_t, k_t)} \Bigg\{ & (1-\delta) \frac{(1+r)(a_{t-1} + (1-\mu) k_{t-1} - k_t) + p\bar{\varphi} f(k_t) - a_t}{1+r} \\
& + \delta \frac{(1+r)(a_{t-1} + (1-\mu) k_{t-1} - k_t) + \theta^B (1-\mu) k_t}{1+r} \\
& + (1-\delta) \frac{V_{t+1}(a_t, k_t)}{1+r} \Bigg\}.
\end{aligned}$$

This is exactly the same as the Bellman equation in the case of small loans, except the bank's liquidation value of capital θ^B has replaced the firm's liquidation value (i.e. reversibility value) θ^F. Thus we immediately have the following result.

Lemma 1.7 *When the firm takes out a large loan, if the first order conditions (equations* (1.21) *and* (1.22)*, replacing* θ^F *with* θ^B*) on the Bellman equation hold at time* t *and* $t+1$*, the firm employs the first-best capital stock* k^{**} *at time* t*.*

Note that Lemmas 1.6 and 1.7 only apply when the first order conditions hold. We will proceed to characterise the situations when they do and do not hold, and the firm's decision when they do not hold. First we derive some useful results about the value function.

1.3.3 Properties of the value function and other intermediate results

First, we present and prove the result that the firm always weakly prefers to retain maximal liquid wealth. Thus we can always set the firm's retained liquid wealth a_t to be the maximal possible level, and the firm's problem reduces to the one-dimensional choice of k_t at time t.

Lemma 1.8 *The firm never has a strict incentive to pay a dividend in any given period. In fact, in some cases, the firm has a strict incentive to retain liquid wealth.*

Proof. The firm wishes to maximise the present value of its expected lifetime dividend stream. Given the time invariance of exogenous parameters, by equation (1.10) this is given by

$$\begin{aligned}\Psi_0 &\equiv \sum_{t=1}^{\infty}\left(\frac{1-\delta}{1+r}\right)^{t-1}\left[\frac{(1-\delta)\,\bar{\psi}_t^S+\delta\bar{\psi}_t^F}{1+r}\right]\\ &=\sum_{t=1}^{\infty}\left(\frac{1-\delta}{1+r}\right)^{t-1}\left[\frac{(1-\delta)\left(p\bar{\varphi}f\left(k_t\right)+(1+r)\,a_t^F-\left(1+r_t^b\right)b_t-a_t\right)}{1+r}\right.\\ &\qquad\left.+\frac{\delta\max\left\{(1+r)\,a_t^F+\theta^F(1-\mu)\,k_t-\left(1+r_t^b\right)b_t,0\right\}}{1+r}\right].\end{aligned}$$

Suppose the firm has selected some (not necessarily optimal) sequence of choice variables $\{a_t,k_t,b_t\}_{t=1}^{\infty}$. Suppose further that at date T the firm decides to increase a_T – or equivalently decrease its dividend at date T, recalling that the dividend must still be non-negative – by a quantity α_T, which it invests in financial assets. In the event of survival, or if there is positive wealth to pay out in the event of death, the firm pays out an additional $(1+r)\,\alpha_T$ in its dividend at date $T+1$. Suppose that the capital stock and loan amount chosen by the firm are unchanged at all dates, and retained liquid wealth is unchanged at all dates except T. Call the new present value of its expected lifetime dividend stream $\Psi_0^{\alpha_T}$. Then

$$\begin{aligned}\Psi_0^{\alpha_T}-\Psi_0=\left(\frac{1-\delta}{1+r}\right)^{T}\Bigg[&\underbrace{-\alpha_T}_{\text{decrease in }\bar{\psi}_T^S}\\ &+\underbrace{\frac{(1-\delta)\left[(1+r)\,\alpha_T+\left(r_{T+1}^b-r_{T+1}^{b,\alpha_T}\right)b_{T+1}\right]}{1+r}}_{\text{increase in expected survival dividend, }(1-\delta)\left(\bar{\psi}_{T+1}^{S,\alpha_T}-\bar{\psi}_{T+1}^S\right)}\\ &+\underbrace{\frac{\delta\max\left\{(1+r)\left(a_{T+1}^F+\alpha_T\right)+\theta^F(1-\mu)\,k_{T+1}-\left(1+r_{T+1}^{b,\alpha_T}\right)b_{T+1},0\right\}}{1+r}}_{\text{new expected failure dividend, }\delta\bar{\psi}_{T+1}^{F,\alpha_T}}\\ &-\underbrace{\frac{\delta\max\left\{(1+r)\,a_{T+1}^F+\theta^F(1-\mu)\,k_{T+1}-\left(1+r_{T+1}^b\right)b_{T+1},0\right\}}{1+r}}_{\text{old expected failure dividend, }\delta\bar{\psi}_{T+1}^F}\Bigg],\end{aligned}$$

where the notation r_{T+1}^{b,α_T} reflects that the firm's borrowing rate depends on its collateral, of which financial wealth is a component, when it is in the large loan regime. That is, in the large loan regime,

$$\begin{aligned}1+r_{T+1}^b&=\frac{1}{1-\delta}\left[1+r-\frac{(1+r)\,a_{T+1}^F+\theta^B(1-\mu)\,k_{T+1}}{b_{T+1}}\right]\\ 1+r_{T+1}^{b,\alpha_T}&=\frac{1}{1-\delta}\left[1+r-\frac{(1+r)\left(a_{T+1}^F+\alpha_T\right)+\theta^B(1-\mu)\,k_{T+1}}{b_{T+1}}\right].\end{aligned}$$

Given that k_t and b_t are unchanged at all dates, there are three possible situations:

(i) *The quantity b_{T+1} is a small loan, whether or not the additional liquid wealth α_T is retained at date T.* In this case, $r^b_{T+1} = r^{b,\alpha_T}_{T+1} = r$, and the failure dividends $\bar{\psi}^F_{T+1}$ and $\bar{\psi}^{F,\alpha_T}_{T+1}$ are both (potentially) strictly positive. Thus

$$\begin{aligned}
\Psi_0^{\alpha_T} - \Psi_0 &= \left(\frac{1-\delta}{1+r}\right)^T \left[-\alpha_T + (1-\delta)\,\alpha_T + \delta\left(\bar{\psi}^{F,\alpha_T}_{T+1} - \bar{\psi}^F_{T+1}\right)\right] \\
&= \left(\frac{1-\delta}{1+r}\right)^T \left[-\alpha_T + (1-\delta)\,\alpha_T + \delta\alpha_T\right] \\
&= 0.
\end{aligned}$$

(ii) *The quantity b_{T+1} is a large loan, whether or not the additional liquid wealth α_T is retained at date T.* In this case, $\bar{\psi}^F_{T+1} = \bar{\psi}^{F,\alpha_T}_{T+1} = 0$, so

$$\begin{aligned}
\Psi_0^{\alpha_T} - \Psi_0 &= \left(\frac{1-\delta}{1+r}\right)^T \left[-\alpha_T + (1-\delta)\,\alpha_T + \frac{(1-\delta)\left(r^b_{T+1} - r^{b,\alpha_T}_{T+1}\right) b_{T+1}}{1+r}\right] \\
&= \left(\frac{1-\delta}{1+r}\right)^T \left[-\alpha_T + (1-\delta)\,\alpha_T + \delta\alpha_T\right] \\
&= 0.
\end{aligned}$$

(iii) *The quantity b_{T+1} is a large loan when the firm does not retain the additional liquid wealth α_T at date T, and a small loan when it does.* In this case $r^{b,\alpha_T}_{T+1} = r$, while $\bar{\psi}^F_{T+1} = 0$. Therefore

$$\begin{aligned}
\Psi_0^{\alpha_T} - \Psi_0 &= \left(\frac{1-\delta}{1+r}\right)^T \Bigg[-\alpha_T + (1-\delta)\,\alpha_T + \frac{(1-\delta)\left(r^b_{T+1} - r\right) b_{T+1}}{1+r} \\
&\qquad + \delta\left\{(1+r)\left(a^F_{T+1} + \alpha_T\right) + \theta^F(1-\mu)\,k_{T+1} - (1+r)\,b_{T+1}\right\}\Bigg] \\
&= \left(\frac{1-\delta}{1+r}\right)^T \Bigg[-\alpha_T + (1-\delta)\,\alpha_T \\
&\qquad + (1+r)\,b_{T+1} - \delta\left\{(1+r)\,a^F_{T+1} + \theta^B(1-\mu)\,k_{T+1}\right\} - (1-\delta)\,b_{T+1} \\
&\qquad + \delta\left\{(1+r)\left(a^F_{T+1} + \alpha_T\right) + \theta^F(1-\mu)\,k_{T+1} - (1+r)\,b_{T+1}\right\}\Bigg],
\end{aligned}$$

and so

$$\begin{aligned}
\Psi_0^{\alpha_T} - \Psi_0 > & \left(\frac{1-\delta}{1+r}\right)^T \Big[-\alpha_T + (1-\delta)\,\alpha_T + \delta\alpha_T \\
& \quad - \delta \left\{ (1+r)\, a_{T+1}^F + \theta^B (1-\mu)\, k_{T+1} - (1+r)\, b_{T+1} \right\} \\
& \quad + \delta \left\{ (1+r)\, a_{T+1}^F + \theta^F (1-\mu)\, k_{T+1} - (1+r)\, b_{T+1} \right\} \Big] \\
= & \left(\frac{1-\delta}{1+r}\right)^T \delta \left(\theta^F - \theta^B\right)(1-\mu)\, k_{T+1} \\
> & 0.
\end{aligned}$$

In all three cases, therefore, $\Psi_0^{\alpha_T} - \Psi_0 \geq 0$, and in some cases this inequality is strict. Therefore the firm always weakly prefers to retain additional (and therefore maximal) liquid wealth from period T to period $T+1$, and hence it never has a strict incentive to pay a dividend in period T. As T was arbitrary, it follows that the firm never has a strict incentive to pay a dividend in any given period. Indeed, we have also seen that the firm sometimes strictly prefers to retain liquid wealth.

Note moreover that the preceding analysis simply examines adjusting a_T, while keeping k_t and b_t the same at all dates. It's possible that retaining liquid wealth allows the firm to do even better than this, by expanding its opportunity set (that is, making new and preferable combinations of k_t and b_t possible). ■

Corollary 1.9 *The value function has the property that for all $\alpha > 0$, and all $a_t, k_t \geq 0$,*

$$V_{t+1}(a_t + \alpha, k_t) - V_{t+1}(a_t, k_t) \geq \alpha.$$

Proof. Fix a date T. Suppose there were some combination of a_T, k_T and $\alpha > 0$ for which

$$V_{T+1}(a_T + \alpha, k_T) < V_{T+1}(a_T, k_T) + \alpha.$$

Then the firm would strictly prefer to pay the quantity α as a dividend at date T rather than retain it until date $T+1$. This contradicts Lemma 1.8, hence it's always true that

$$V_{t+1}(a_t + \alpha, k_t) - V_{t+1}(a_t, k_t) \geq \alpha$$

as required. ■

Next we show that the first-best capital stock as defined in equation (1.16) is indeed the best choice for firms that are not liquidity constrained.

Lemma 1.10 *Firms that are not liquidity constrained – that is, firms that are indifferent between paying a dividend or retaining liquid wealth, and firms that can attain either k^* or k^{**} – will choose to take a small loan and employ the first-best capital stock k^*. Moreover,* *firms that are not able to employ k^* with a small loan at date t have an incentive (possibly strict) to retain wealth from date $t-1$ to date t in order to achieve k^* in the future.*

Proof. For a firm that is indifferent between retaining liquid wealth and paying a dividend, equation (1.21) (the first order condition on a) holds. By Lemma 1.6, if equation (1.21) holds, the optimal capital stock in the small loan regime is characterised by k^*. By Lemma 1.7, if equation (1.21) holds, the optimal capital stock in the large loan regime is characterised by k^{**}. Since these are the only candidate maxima, the global maximum (that is, the optimal choice for a firm that is indifferent between paying a dividend and retaining wealth) must be one of the two. We can compute the expected lifetime dividend stream associated with being permanently at k^* in the small loan regime and at k^{**} in the large loan regime, assuming zero starting wealth, and assuming the firm can borrow as much as it likes at the risk free rate r,

$$
\begin{aligned}
\Psi_0^S =& \frac{1-\delta}{1+r}\left[p\bar{\varphi}f\left(k^*\right) - \frac{\left[1+r-\delta\theta^F\left(1-\mu\right)\right]k^*}{1-\delta}\right] \\
&+\sum_{t=2}^{\infty}\left(\frac{1-\delta}{1+r}\right)^t\left\{p\bar{\varphi}f\left(k^*\right)+\frac{\left[\delta\theta^F\left(1-\mu\right)-\mu\right]k^*}{1-\delta}\right\} \\
=&-\left(1-\frac{\mu}{1+r}\right)k^*+\sum_{t=1}^{\infty}\left(\frac{1-\delta}{1+r}\right)^t\left\{p\bar{\varphi}f\left(k^*\right)+\frac{\left[\delta\theta^F\left(1-\mu\right)-\mu\right]k^*}{1-\delta}\right\} \\
=&\frac{\mu k^*}{1+r}+\frac{1-\delta}{r+\delta}\left\{p\bar{\varphi}f\left(k^*\right)+\frac{\left[\delta\theta^F\left(1-\mu\right)-\mu-r-\delta\right]k^*}{1-\delta}\right\} \\
=&\frac{\mu k^*}{1+r}-\frac{\delta\mu k^*}{r+\delta}+\frac{1-\delta}{r+\delta}\left\{p\bar{\varphi}f\left(k^*\right)-\frac{\left[1+r-\left(1-\delta\right)\left(1-\mu\right)-\delta\theta^F\left(1-\mu\right)\right]k^*}{1-\delta}\right\} \\
=&\left(\frac{1}{1+r}-\frac{\delta}{r+\delta}\right)\mu k^*+\frac{1-\delta}{r+\delta}\left\{p\bar{\varphi}f\left(k^*\right)-p\bar{\varphi}f'\left(k^*\right)k^*\right\},
\end{aligned}
$$

where the substitution follows from equation (1.16), and analogously

$$
\Psi_0^L=\left(\frac{1}{1+r}-\frac{\delta}{r+\delta}\right)\mu k^{**}+\frac{1-\delta}{r+\delta}\left\{p\bar{\varphi}f\left(k^{**}\right)-p\bar{\varphi}f'\left(k^{**}\right)k^{**}\right\},
$$

where Ψ_0^S corresponds to permanent adherence to the small loan regime and Ψ_0^L corresponds to permanent adherence to the large loan regime. We can calculate

$$
\begin{aligned}
\Psi_0^S-\Psi_0^L=&\left(\frac{1-\delta}{r+\delta}\right)p\bar{\varphi}\left\{\left[f\left(k^*\right)-f'\left(k^*\right)k^*\right]-\left[f\left(k^{**}\right)-f'\left(k^{**}\right)k^{**}\right]\right\} \\
&+\left(\frac{1}{1+r}-\frac{\delta}{r+\delta}\right)\mu\left(k^*-k^{**}\right).
\end{aligned}
$$

Note that

$$\begin{aligned}\frac{d}{dk_t}\left[f\left(k_t\right)-f'\left(k_t\right)k_t\right] &= f'\left(k_t\right)-f'\left(k_t\right)-f''\left(k_t\right)k_t \\ &= -f''\left(k_t\right)k_t \\ &> 0,\end{aligned}$$

by assumption of the properties of $f\left(k_t\right)$. Therefore

$$f\left(k^*\right)-f'\left(k^*\right)k^* > f\left(k^{**}\right)-f'\left(k^{**}\right)k^{**},$$

and since

$$\frac{1}{1+r} > \frac{\delta}{r+\delta}$$

it follows that

$$\Psi_0^S - \Psi_0^L > 0.$$

Clearly therefore k^* in the small loan regime is preferable to k^{**} in the large loan regime. Therefore, when k^* is not feasible with a small loan, the firm may have a strict incentive to retain liquid wealth in order to be able to attain k^* with a small loan in the future.

Finally, in order to show that k^* really is a maximum, we must show that when the firm employs the first-best capital stock, the first order condition on a – equation (1.21) – really does hold. Fix some date T, and assume that the firm takes a small loan and chooses $k_T = k^*$. It follows from the proof of Lemma 1.8 that if retaining wealth does not change the firm's choice of loan regime, and does not change the choice of future capital stock, then the firm is indifferent between retaining wealth and paying a dividend. It is shown below, in Theorem 1.19, that when the firm chooses $k_T = k^*$, then k^* is more than feasible with a small loan in period $T+1$ also. Therefore the marginal unit of retained liquid wealth does not affect the firm's decision problem next period, so the first order condition on a holds and k^* is truly the optimal choice for a firm that is not liquidity constrained.

Note that by Lemma 1.8, this does not present the firm with any kind of decision problem: the firm weakly prefers to retain liquid wealth under any circumstances, so retaining wealth up to the point that k^* is feasible with a small loan is 'costless' for the firm. ■

Corollary 1.11 *When the firm takes a small loan at time t, the firm chooses the minimum of the first-best, boundary-best and small-loan-constrained capital stocks,* $k_t = \min\left\{k^*, \bar{k}_t, \bar{k}_t^S\right\}$.

Proof. Suppose the firm takes a small loan. By Lemma 1.10, the first-best capital stock k^* is the optimal choice for a firm that is not liquidity constrained. If k^* is not feasible, but the firm has still chosen to take a small loan, it will get as close to this capital stock as possible, while obeying the constraint that retained liquid wealth must be non-negative, $a_t \geq 0$. That is, it will choose the boundary-best capital stock $\bar{k}_t$ if this does not violate the non-negativity of liquid wealth. If it does violate this constraint, the firm will choose the maximum capital stock that does not violate the constraint, $\bar{k}_t^S$, in order to get as close as possible to the first-best capital stock k^*. ■

The preceding analysis tells us how the firm behaves when it's optimal for it to take a small loan. By Lemma 1.10 we know that firms in the large loan regime may have a strict incentive to retain liquid wealth in order to make k^* feasible in the small loan regime at some point in the future, so we cannot rely on Lemma 1.7 to characterise the firm's behaviour when it takes a large loan as the first order condition on a may not hold. In order to show how the firm does choose to behave when it takes a large loan, we first derive another intermediate result about the value function.

Lemma 1.12 *The value function has the property that for all $a_t, k_t \geq 0$, and for $k_t + \kappa \geq 0$,*

$$V_{t+1}(a_t, k_t + \kappa) = V_{t+1}(a_t + (1-\mu)\kappa, k_t)$$

so long as $(1-\mu)k_t \leq k^*$ and $(1-\mu)(k_t+\kappa) \leq k^*$.

Proof. Fix a date T, and suppose for a contradiction that there is some combination of a_T, k_T and κ for which $V_{T+1}(a_T, k_T+\kappa) \neq V_{T+1}(a_T + (1-\mu)\kappa, k_T)$. In particular, this means that $k_{T+1}(a_T, k_T+\kappa) \neq k_{T+1}(a_T + (1-\mu)\kappa, k_T)$, otherwise capital and retained liquid wealth would both be identical at time $T+1$ and hence the value functions would be equal.

Suppose without loss of generality that $\kappa > 0$. Any combination of capital investment and financial investment at time $T+1$ that is feasible given $(a_T, k_T+\kappa)$ is also feasible given $(a_T + (1-\mu)\kappa, k_T)$, but the reverse is not true (that is, liquid wealth can be converted costlessly into capital but not vice versa). Specifically, for some given amount of borrowing b_{T+1}, the range of possible capital stocks at date $T+1$ without liquidating any capital is given by

$$\begin{cases} k_{T+1}(a_T, k_T+\kappa) & \in [(1-\mu)(k_T+\kappa), a_T + (1-\mu)(k_T+\kappa) + b_{T+1}] \\ k_{T+1}(a_T + (1-\mu)\kappa, k_T) & \in [(1-\mu)k_T, a_T + (1-\mu)(k_T+\kappa) + b_{T+1}]. \end{cases}$$

Since $k_{T+1}(a_T, k_T+\kappa) \neq k_{T+1}(a_T + (1-\mu)\kappa, k_T)$, and since the firm's ability to borrow at a given interest rate is identical in both situations when installed capital is identical, it must therefore be the case that

$$\begin{aligned} & k_{T+1}(a_T, k_T+\kappa) > k_{T+1}(a_T + (1-\mu)\kappa, k_T) \\ \Rightarrow \quad & k_{T+1}(a_T, k_T+\kappa) \leq (1-\mu)(k_T+\kappa). \end{aligned} \tag{1.24}$$

Essentially, given $(a_T, k_T+\kappa)$, the firm arrives at date $T+1$ with 'too much' capital, and therefore chooses either the smallest possible capital stock at date $T+1$ without liquidating capital, or indeed liquidates some capital and chooses a capital stock strictly smaller than the stock it inherits. Since we have assumed $(1-\mu)(k_T+\kappa) \leq k^*$, the firm cannot be in the small loan regime at date $T+1$: the only way the firm could have 'too much' capital in the small loan regime is to arrive at date $T+1$ with more than the first best capital stock, $(1-\mu)(k_T+\kappa) > k^*$, which we are explicitly ruling out. Therefore given $(a_T, k_T+\kappa)$, the firm will take a large loan in period $T+1$. By equation (1.15), taking a large loan means that

$$\begin{aligned} k_{T+1}(a_T, k_T+\kappa) &> \frac{a_T + (1-\mu)(k_T+\kappa)}{1-\theta^F(1-\mu)/(1+r)} \\ &> a_T + (1-\mu)(k_T+\kappa) \\ &\geq (1-\mu)(k_T+\kappa), \end{aligned}$$

so the optimally chosen capital stock is strictly greater than the capital stock the firm inherits from date T. This contradicts equation (1.24); the firm cannot have arrived at date $T+1$ with 'too much' capital. It follows that the original assumption that $V_{T+1}(a_T, k_T+\kappa) \neq V_{T+1}(a_T + (1-\mu)\kappa, k_T)$ cannot be true, and hence

$$V_{T+1}(a_T, k_T+\kappa) = V_{T+1}(a_T + (1-\mu)\kappa, k_T).$$

■

Corollary 1.13 *The value function has the property that for all $a_t, k_t \geq 0$, and for $k_t+\kappa \geq 0$, $a_t+\alpha \geq 0$ and $\alpha + (1-\mu)\kappa > 0$,*

$$V_{t+1}(a_t+\alpha, k_t+\kappa) - V_{t+1}(a_t, k_t) \geq \alpha + (1-\mu)\kappa$$

so long as $(1-\mu)k_t \leq k^*$ and $(1-\mu)(k_t+\kappa) \leq k^*$. In particular, when $(1-\mu)k_t \leq k^*$, the value function $V_{t+1}(a_t, k_t)$ is strictly and monotonically increasing in $a_t + (1-\mu)k_t$.

Proof. By Lemma 1.12, since $(1-\mu)k_t \leq k^*$ and $(1-\mu)(k_t+\kappa) \leq k^*$,

$$V_{t+1}(a_t+\alpha, k_t+\kappa) = V_{t+1}(a_t+\alpha+(1-\mu)\kappa, k_t).$$

By Corollary 1.9, therefore, since $\alpha + (1-\mu)\kappa > 0$,

$$\begin{aligned} V_{t+1}(a_t+\alpha, k_t+\kappa) - V_{t+1}(a_t, k_t) &= V_{t+1}(a_t+\alpha+(1-\mu)\kappa, k_t) - V_{t+1}(a_t, k_t) \\ &\geq \alpha + (1-\mu)\kappa. \end{aligned}$$

It follows immediately that the value function $V_{t+1}(a_t, k_t)$ is strictly and monotonically increasing in $a_t + (1-\mu)k_t$. ■

By this result, when the firm does not inherit 'too much' capital, so $(1-\mu)k_t \leq k^*$, the value function can be expressed as $V_{t+1}(a_t, k_t) = V_{t+1}(\Omega_t)$, a function of *inherited wealth*, defined as follows.

Definition 1.14 *Inherited wealth that the firm carries from date t to date $t+1$ is defined as*

$$\Omega_t \equiv a_t + (1-\mu)k_t. \tag{1.25}$$

By Corollary 1.13, so long as $(1-\mu)\,k_t \le k^*$, inherited wealth Ω_t is the one-dimensional state variable describing the firm's problem.

We are now in a position to characterise the capital stock chosen by the firm in the large loan regime even when the first order conditions do not hold.

Lemma 1.15 *In the large loan regime, the firm chooses the minimum of the large-loan-best and the large-loan-constrained capital stocks, $k_t = \min\left\{k^{**}, \bar{k}_t^L\right\}$.*

Proof. Fix some date T, and suppose the firm takes a large loan at date T. By Lemma 1.10, this means that k^* is not feasible with a small loan at time T, otherwise this is what the firm would have chosen. Therefore $(1-\mu)\,k_{T-1} < \bar{k}_T < k^*$. By Lemma 1.8, the firm always weakly prefers to retain maximal liquid wealth – hence the firm will not pay a dividend if it survives date T – and since we are considering the large loan regime, the firm will not pay a dividend if it fails. Thus

$$\begin{aligned} V_T\left(\Omega_{T-1}\right) &= \max_{\{\Omega_T\}}\left\{\frac{1-\delta}{1+r}\cdot\bar{\psi}_T^S + \frac{\delta}{1+r}\cdot\bar{\psi}_T^F + \frac{1-\delta}{1+r}V_{T+1}\left(\Omega_T\right)\right\} \\ &= \max_{\{\Omega_T\}}\left\{\frac{1-\delta}{1+r}V_{T+1}\left(\Omega_T\right)\right\}. \end{aligned}$$

By Corollary 1.13 it follows that $V_{T+1}\left(\Omega_T\right)$ is maximised when Ω_T is maximised. Setting a_T equal to maximum retained wealth, we have

$$\begin{aligned} \Omega_T &= a_T + (1-\mu)\,k_T \\ &= p\bar{\varphi}f\left(k_T\right) + \frac{1+r}{1-\delta}\left[\Omega_{T-1} - k_T\right] + \frac{\delta\theta^B\left(1-\mu\right)k_T}{1-\delta} + (1-\mu)\,k_T \\ \Rightarrow \qquad \frac{\partial\Omega_T}{\partial k_T} &= p\bar{\varphi}f'\left(k_T\right) - \frac{1+r}{1-\delta} + \frac{\delta\theta^B\left(1-\mu\right)}{1-\delta} + (1-\mu)\,. \end{aligned}$$

Setting this equal to zero yields

$$(1-\delta)\,p\bar{\varphi}f'\left(k_T\right) + (1-\delta)\,(1-\mu) + \delta\theta^B\,(1-\mu) = 1+r.$$

Note that this is precisely the large-loan-best capital stock, $k_T = k^{**}$, as defined in equation (1.18). If the firm takes a large loan and has sufficient wealth, therefore, it will choose $k_T = k^{**}$. However, in order for the non-negative wealth constraint to be satisfied, $a_T \ge 0$, we know that $k_T \le \bar{k}_T^L\left(a_{T-1}, k_{T-1}\right)$. Therefore the firm chooses $k_T = \min\left\{k^{**}, \bar{k}_T^L\right\}$. ■

Finally, it only remains to determine when the firm chooses a large loan and when it chooses a small loan. In order to characterise the firm's decision, given (a_{t-1}, k_{t-1}), we will define

$$\begin{aligned} k_t^S &\equiv \min\left\{k^*, \bar{k}_t, \bar{k}_t^S\right\} \\ k_t^L &\equiv \min\left\{k^{**}, \bar{k}_t^L\right\}, \end{aligned}$$

so that k_t^S is the capital stock that the firm will choose if it takes a small loan, and k_t^L is the capital stock that the firm will choose if it takes a large loan. Note that it's possible for k_t^L to be too small to necessitate a large loan, in which case the firm will certainly take a small loan whatever choice of capital it makes, and will therefore certainly choose $k_t = k_t^S$. By definition, this happens when $k_t^L \leq \bar{k}_t$.

We will define the decision quantity D_t as

$$\begin{aligned} D_t \equiv & p\bar{\varphi}\left[f\left(k_t^S\right) - f\left(k_t^L\right)\right] - \frac{\left[1 + r - (1-\delta)(1-\mu) - \delta\theta^F(1-\mu)\right]k_t^S}{1-\delta} \\ & + \frac{\left[1 + r - (1-\delta)(1-\mu) - \delta\theta^B(1-\mu)\right]k_t^L}{1-\delta}. \end{aligned} \tag{1.26}$$

We are now able to characterise the firm's choice between a small loan and a large loan.

Lemma 1.16 *Suppose the first-best capital stock k^* is not feasible at date t with a small loan.*

(i) Suppose that $\bar{k}_t \leq \bar{k}_t^S$. If $D_t > 0$, the firm chooses a small loan and employs the capital stock k_t^S at date t. If $D_t < 0$, the firm chooses a large loan and employs the capital stock k_t^L at date t. If $D_t = 0$, the firm is indifferent between employing k_t^S with a small loan and employing k_t^L with a large loan at date t.

(ii) Now suppose that $\bar{k}_t^S < \bar{k}_t$. The firm chooses a small loan and employs the capital stock $\bar{k}_t^S$ at date t.

Proof. Fix some date T. Take Ω_{T-1} as given, and assume that k^* is not feasible with a small loan at date T. If the firm takes a large loan at date T, the failure dividend will be equal to zero and we can assume that the firm will choose to retain maximum liquid wealth in the event of survival. If the firm takes a small loan at date T, by Corollary 1.11, the firm will employ the minimum of the boundary-best capital stock and the small-loan-constrained capital stock, $k_T = \min\left\{\bar{k}_T, \bar{k}_T^S\right\}$.

(i) Suppose first that $\bar{k}_T \leq \bar{k}_T^S$, so that the firm chooses the boundary-best capital stock $\bar{k}_T$ if it takes a small loan. In this case, the failure dividend in the small loan regime will also be equal to zero and we can also assume that the firm will choose to retain maximum liquid wealth in the event of survival. Whether the firm takes a small loan or a large loan, therefore, it follows that

$$V_T\left(\Omega_{T-1}\right) = \max_{\Omega_T} \frac{1-\delta}{1+r} V_{T+1}\left(\Omega_T\right).$$

By Corollary 1.13, $V_{T+1}\left(\Omega_T\right)$ is maximised when Ω_T is maximised. Suppose the firm takes a small loan and chooses $k_T = \bar{k}_T$, and denote by $\bar{a}_T$ the corresponding maximal retained liquid wealth. Let $\bar{\Omega}_T$ denote the value of Ω_T associated with this policy. We note by equation (1.15) that

$$\Omega_{T-1} = \left[1 - \frac{\theta^F (1-\mu)}{1+r}\right] \bar{k}_T.$$

Then

$$\begin{aligned}
\bar{\Omega}_T &= \bar{a}_T + (1-\mu)\bar{k}_T \\
&= p\bar{\varphi} f\left(\bar{k}_T\right) + (1+r)\left[\Omega_{T-1} - \bar{k}_T\right] + (1-\mu)\bar{k}_T \\
&= p\bar{\varphi} f\left(\bar{k}_T\right) + \left[1+r-\theta^F(1-\mu)\right]\bar{k}_T - \left[1+r-(1-\mu)\right]\bar{k}_T \\
&= p\bar{\varphi} f\left(\bar{k}_T\right) + \left(1-\theta^F\right)(1-\mu)\bar{k}_T.
\end{aligned}$$

Now suppose the firm takes a large loan and chooses $k_T = k_T^L$, and denote by a_T^L the corresponding maximal retained liquid wealth. Let Ω_T^L denote the value of Ω_T associated with this policy. Then

$$\begin{aligned}
\Omega_T^L =& a_T^L + (1-\mu)k_T^L \\
=& p\bar{\varphi} f\left(k_T^L\right) + \frac{1+r}{1-\delta}\left[\Omega_{T-1} - k_T^L\right] + \frac{\delta\theta^B(1-\mu)k_T^L}{1-\delta} + (1-\mu)k_T^L \\
=& p\bar{\varphi} f\left(k_T^L\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]k_T^L}{1-\delta} + \frac{1+r}{1-\delta}\Omega_{T-1} \\
=& p\bar{\varphi} f\left(k_T^L\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]k_T^L}{1-\delta} \\
&+ \frac{\left[1+r-\theta^F(1-\mu)\right]\bar{k}_T}{1-\delta}.
\end{aligned} \tag{1.27}$$

Note that

$$\begin{aligned}
\bar{\Omega}_T - \Omega_T^L =& p\bar{\varphi}\left[f\left(\bar{k}_T\right) - f\left(k_T^L\right)\right] - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)\right]\bar{k}_T}{1-\delta} \\
&+ \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]k_T^L}{1-\delta} \\
=& D_T,
\end{aligned}$$

where D_T is defined by equation (1.26). Then it follows that choosing $\bar{k}_T$ with a small loan is weakly (strictly) preferable to choosing k_T^L with a large loan when D_T is weakly (strictly) greater than zero, and the reverse is true when D_T is weakly (strictly) less than zero, so long as $\bar{k}_t \leq \bar{k}_T^S$.

(ii) Suppose instead that that $\bar{k}_T^S < \bar{k}_T$, so that the firm chooses the small-loan-constrained capital stock if it takes a small loan at date T. Note that in this case, $D_T > 0$ is a sufficient but not a necessary condition for the firm to take a small loan at date T. $D_T > 0$ guarantees that the future value of the firm is greater in the small loan regime than in the large loan regime, but it understates the true benefit to the firm of taking a small loan, because when $\bar{k}_T^S < \bar{k}_T$ the firm will pay a strictly positive failure dividend in the event of death.

For convenience, we will define the functions

$$F(k_T) \equiv p\bar{\varphi} f(k_T) + (1+r)(\Omega_{T-1} - k_T) \tag{1.28}$$

$$\begin{aligned} G(k_T) &\equiv p\bar{\varphi} f(k_T) + \frac{1+r}{1-\delta}(\Omega_{T-1} - k_T) + \frac{\delta\theta^B(1-\mu)k_T}{1-\delta}, \\ &= p\bar{\varphi} f(k_T) + \frac{1+r}{1-\delta}\Omega_{T-1} - \frac{\left[1+r-\delta\theta^B(1-\mu)\right]k_T}{1-\delta} \end{aligned} \tag{1.29}$$

so that $F\left(\bar{k}_T^S\right) = 0$ by equation (1.17), and $G\left(\bar{k}_T^L\right) = 0$ by equation (1.19). Then we can write

$$\begin{aligned} G(k_T) &= p\bar{\varphi} f(k_T) + \frac{\left[1+r-\theta^F(1-\mu)\right]\bar{k}_T}{1-\delta} - \frac{\left[1+r-\delta\theta^B(1-\mu)\right]k_T}{1-\delta} \\ &= F(k_T) + \frac{\delta}{1-\delta}\left\{\left[1+r-\theta^F(1-\mu)\right]\bar{k}_T - \left[1+r-\theta^B(1-\mu)\right]k_T\right\}. \end{aligned}$$

Since we are assuming that the firm chooses $\bar{k}_T^S$ in the small loan regime, if this quantity is larger than the large-loan-constrained capital stock $\bar{k}_T^L$, then $k_T^S > k_T^L$ which clearly implies that $D_T > 0$. As previously argued, this is a sufficient condition for the firm to take a small loan at date T.

Therefore suppose $\bar{k}_T^S < \bar{k}_T^L$. By definition, this means that $F\left(\bar{k}_T^L\right) < 0$. Also by definition, $G\left(\bar{k}_T^L\right) = 0$. Therefore

$$\begin{aligned} F\left(\bar{k}_T^L\right) &= G\left(\bar{k}_T^L\right) + \frac{\delta}{1-\delta}\left\{\left[1+r-\theta^B(1-\mu)\right]\bar{k}_T^L - \left[1+r-\theta^F(1-\mu)\right]\bar{k}_T\right\} \\ &= \frac{\delta}{1-\delta}\left\{\left[1+r-\theta^B(1-\mu)\right]\bar{k}_T^L - \left[1+r-\theta^F(1-\mu)\right]\bar{k}_T\right\} \\ &< 0, \end{aligned}$$

which implies that

$$\begin{aligned} \left[1+r-\theta^B(1-\mu)\right]\bar{k}_T^L &< \left[1+r-\theta^F(1-\mu)\right]\bar{k}_T \\ \Rightarrow \qquad \bar{k}_T^L &< \bar{k}_T, \end{aligned}$$

and so by definition of $\bar{k}_T$ the firm cannot be in the large loan regime if it chooses the large-loan-constrained capital stock $\bar{k}_T^L$. In reality, $\bar{k}_T^L$ is either not feasible or suboptimal in the small loan regime. Therefore the firm chooses the small-loan-constrained capital stock, $k_T = k_T^S = \bar{k}_T^S$.

■

1.3.4 'Too much' capital

So far the discussion has assumed that the firm does not enter any given period with 'too much' capital – that is, with more than the first-best capital stock. Indeed, the firm would never have an incentive to do this; when the exogenous parameters are time-invariant, this would only happen if the firm is born with excessive capital. Nonetheless, this situation is analysed for the sake of completeness.

If the firm inherits too much capital at time t, so $(1-\mu)\,k_{t-1} > k^*$, it is clear from Lemma 1.10 that the firm will not choose to install any more capital for use at time t. This means that the firm is certainly in the small loan regime: any money that is borrowed will simply be invested in financial assets, and the firm will be solvent in the case of death. If the firm decides to choose $k_t \neq (1-\mu)\,k_{t-1}$, therefore, this can only be because the firm has *liquidated* capital and reduced its capital stock. Since the firm is only able to liquidate capital at a value of $\theta^F < 1$, its decision problem is no longer described by the Bellman equation (1.20). Instead the Bellman equation becomes

$$\begin{aligned} V_t\left(a_{t-1}, k_{t-1}\right) = \max_{(a_t,k_t)} \Bigg\{ & (1-\delta)\,\frac{(1+r)\left(a_{t-1} + \theta^F\left[(1-\mu)\,k_{t-1} - k_t\right]\right) + p\bar{\varphi}f\left(k_t\right) - a_t}{1+r} \\ & + \delta\frac{(1+r)\left(a_{t-1} + \theta^F\left[(1-\mu)\,k_{t-1} - k_t\right]\right) + \theta^F\left(1-\mu\right)k_t}{1+r} \\ & + (1-\delta)\,\frac{V_{t+1}\left(a_t, k_t\right)}{1+r} \Bigg\}, \end{aligned} \tag{1.30}$$

where the first two appearances of θ^F reflect the fact that when *reducing* its capital stock, the firm can only trade capital for liquid wealth at a discount. The first order conditions on the Bellman equation then become

$$\frac{\partial V_{t+1}\left(a_t, k_t\right)}{\partial a_t} = 1 \tag{1.31}$$

$$\frac{\partial V_{t+1}\left(a_t, k_t\right)}{\partial k_t} = \frac{\theta^F\left(1+r\right)}{1-\delta} - p\bar{\varphi}f'\left(k_t\right) - \frac{\delta\theta^F\left(1-\mu\right)}{1-\delta}. \tag{1.32}$$

We define the quantity $k^\dagger$ of capital as the quantity that satisfies

$$\begin{aligned} \sum_{t=1}^{t^\dagger}\left[\frac{(1-\delta)\,(1-\mu)}{1+r}\right]^t & \left[p\bar{\varphi}f'\left[(1-\mu)^{t-1}\,k^\dagger\right] + \frac{\delta\theta^F\,(1-\mu)}{1-\delta}\right] \\ & = \theta^F\,(1-\mu) - \left[\frac{(1-\delta)\,(1-\mu)}{1+r}\right]^{t^\dagger}, \end{aligned} \tag{1.33}$$

where

$$t^\dagger \equiv \left\lfloor \log_{(1-\mu)}\left(\frac{k^*}{k^\dagger}\right)\right\rfloor + 1.$$

Given this definition, we can characterise the firm's decision when it has excessive capital.

Lemma 1.17 *Take* (a_{t-1}, k_{t-1}) *as given, and assume that the firm arrives at date* t *with 'too much' capital, so* $(1-\mu) k_{t-1} > k^*$*. Then the firm chooses* $k_t = \min\{(1-\mu) k_{t-1}, k^\dagger\}$*, where* $k^\dagger$ *is defined as in equation* (1.33)*. In all subsequent periods* $\tau > t$*, the firm chooses* $k_\tau = \max\{(1-\mu) k_{\tau-1}, k^*\}$*.*

Proof. Suppose without loss of generality that $(1-\mu) k_0 > k^*$, and we are considering the firm's decision from date 1 onwards. Let k_1 be the optimal choice of capital at date 1. There are two possible situations we can consider. First, suppose the optimal choice of capital at date 1 is less than the inherited capital from date 0, so $k_1 < (1-\mu) k_0$. In this case, if k_0 is increased, the firm will simply choose to liquidate the extra undepreciated capital at a value of θ^F when it arrives at date 1. It follows that

$$\frac{\partial V_1(a_0, k_0)}{\partial k_0} = \theta^F (1-\mu),$$

and the firm will choose k_1 to be the maximum level of capital such that $\partial V_1(a_0, k_0)/\partial k_1 = \theta^F (1-\mu)$. That is, the firm will liquidate capital when it arrives at date 1, up until the point that further liquidation would be suboptimal. This quantity will be made explicit below.

Second, suppose the optimal choice of capital at date 1 is simply the inherited capital from date 0, so $k_1 = (1-\mu) k_0$. If the firm chooses not to liquidate any capital when it arrives at date 1, it must be because the marginal benefit of doing so is no greater than the marginal cost of doing so. Suppose that in all subsequent periods τ the firm chooses $k_\tau = \max\{(1-\mu) k_{\tau-1}, k^*\}$ (it will be shown later that this is optimal). Then

$$t_0 = \left\lfloor \log_{(1-\mu)} \left(\frac{k^*}{k_0}\right) \right\rfloor.$$

is the number of periods for which the firm will employ the capital stock inherited from the previous period – that is, the number of periods for which the inherited capital stock is weakly more than the first best capital stock k^*. Since the purpose of retaining liquid wealth is to make k^* feasible with a small loan we can set retained liquid wealth equal to zero at each date τ that satisfies $(1-\mu) k_\tau > k^*$. In fact, the firm will only have the potential need to retain some liquid wealth at date t_0 in order to make k^* feasible at date $t_0 + 1$, given that $(1-\mu) k_{t_0} < k^*$. Denote by $\bar{V}$ the expected lifetime dividend stream associated with this choice of policy (as distinct from the true value function, since we have not yet demonstrated that this policy is optimal). Expanding the Bellman equation over t_0 periods, $\bar{V}$ is therefore given by

$$\begin{aligned}
\bar{V}_1(a_0, k_0) =& a_0 + \bar{V}_1(0, k_0) \\
=& a_0 + \sum_{t=1}^{t_0-1} \left(\frac{1-\delta}{1+r}\right)^{t-1} \left[\frac{(1-\delta)\Psi_t^S + \delta \Psi_t^F}{1+r}\right] \\
&+ \left(\frac{1-\delta}{1+r}\right)^{t_0-1} \bar{V}_{t_0}\left(0, (1-\mu)^{t_0-1} k_0\right),
\end{aligned}$$

where

$$\Psi_t^S = p\bar{\varphi} f\left[(1-\mu)^t k_0\right]$$
$$\Psi_t^F = \theta^F (1-\mu)^{t+1} k_0$$

and

$$\bar{V}_{t_0}\left(0, (1-\mu)^{t_0-1} k_0\right) = \frac{(1-\delta)\left\{f\left[(1-\mu)^{t_0} k_0\right] - \left[k^* - (1-\mu)^{t_0+1} k_0\right]\right\}}{1+r} + \frac{\delta\theta^F (1-\mu)^{t_0+1} k_0}{1+r} + \bar{V}_{t_0+1}(a^*, k^*),$$

where a^* is the profit associated with k^*. We assume the firm has no liquid wealth carried over from the previous period, since it is weakly optimal for the firm to retain just enough liquid wealth at time t_0 in order to make k^* exactly feasible at time t_0+1 with zero borrowing, so

$$a_{t_0} = k^* - (1-\mu)^{t_0+1} k_0.$$

Therefore the derivatives of these quantities are

$$\frac{\partial \Psi_t^S}{\partial k_0} = (1-\mu)^t p\bar{\varphi} f'\left[(1-\mu)^t k_0\right]$$
$$\frac{\partial \Psi_t^F}{\partial k_0} = \theta^F (1-\mu)^{t+1}$$
$$\frac{\partial \bar{V}_{t_0}\left(0, (1-\mu)^{t_0-1} k_0\right)}{\partial k_0} = \frac{(1-\delta)\left\{(1-\mu)^{t_0} f'\left[(1-\mu)^{t_0} k_0\right] + (1-\mu)^{t_0+1}\right\}}{1+r} + \frac{\delta\theta^F (1-\mu)^{t_0+1}}{1+r}.$$

Thus it follows that

$$\begin{aligned} \frac{\partial \bar{V}_1(a_0, k_0)}{\partial k_0} &= \sum_{t=1}^{t_0} \left(\frac{1-\delta}{1+r}\right)^{t-1} \left[\frac{(1-\delta)(1-\mu)^t p\bar{\varphi} f'\left[(1-\mu)^t k_0\right] + \delta\theta^F (1-\mu)^{t+1}}{1+r}\right] \\ &\quad + \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t_0} \\ &= \sum_{t=1}^{t_0} \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^t \left[p\bar{\varphi} f'\left[(1-\mu)^t k_0\right] + \frac{\delta\theta^F (1-\mu)}{1-\delta}\right] \\ &\quad + \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t_0}. \end{aligned} \tag{1.34}$$

Given that the firm has optimally chosen $k_1 = (1-\mu) k_0$, if it is optimal for the firm to choose $k_\tau = \max\{(1-\mu) k_{\tau-1}, k^*\}$ at all dates $\tau > 1$, $\bar{V}$ is the true value function. In the case of general k_0, therefore, we find the firm's optimal capital choice as $k_1 = \min\left\{(1-\mu) k_0, k^\dagger\right\}$

by defining $k^{\dagger}$ as the solution to

$$\left.\frac{\partial \bar{V}_2(0,k)}{\partial k}\right|_{(1-\mu)k=k^{\dagger}} = \theta^F(1-\mu).$$

Next, therefore, we must show that allowing capital to depreciate naturally until the firm reaches k^* is indeed optimal behaviour, given that $k_1 = (1-\mu)k_0$. Clearly, by Lemma 1.10, if k^* is larger than $(1-\mu)k_{\tau-1}$, then k^* is what the firm would prefer. Suppose therefore $(1-\mu)k_{\tau-1} > k^*$. The firm will certainly not choose $k_\tau > (1-\mu)k_{\tau-1}$, so it is only left to determine whether the firm will choose to liquidate any capital and employ $k_\tau < (1-\mu)k_{\tau-1}$. The firm will certainly not choose to liquidate capital if

$$\frac{\partial V_\tau(0,k_{\tau-1})}{\partial k_{\tau-1}} > \theta^F(1-\mu), \tag{1.35}$$

where V is the true value function, since the cost of liquidating capital in this situation is greater than the benefit. If therefore we can show that equation (1.35) holds at all dates $\tau > 1$, we have shown that the firm will not choose to undertake any liquidation at any of these dates, and it will simply allow its capital to depreciate naturally until it returns to employing the first-best capital stock k^*. Suppose $\tau = 2$; if we can show that equation (1.35) holds for $\tau = 2$, the the firm will choose $k_2 = (1-\mu)k_1$ and faces a very similar decision at time 3, so inductively it must be true for all $\tau \in [1, t_0]$. We have assumed that $k_1 = (1-\mu)k_0$. By equation (1.34),

$$\begin{aligned}
\frac{\partial \bar{V}_2(0,k_1)}{\partial k_1} - \frac{\partial \bar{V}_1(0,k_0)}{\partial k_0} =& \frac{\partial \bar{V}_2(0,k_1)}{\partial k_0}\frac{\partial k_0}{\partial k_1} - \frac{\partial \bar{V}_1(0,k_0)}{\partial k_0} \\
=& \frac{1}{1-\mu}\frac{\partial \bar{V}_2(0,k_1)}{\partial k_0} - \frac{\partial \bar{V}_1(0,k_0)}{\partial k_0} \\
=& \sum_{t=2}^{t_0}\left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t-1}\left[p\bar{\varphi}f'\left[(1-\mu)^t k_0\right] + \frac{\delta\theta^F(1-\mu)}{1-\delta}\right] \\
&+ \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t_0-1} \\
&- \sum_{t=1}^{t_0}\left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t}\left[p\bar{\varphi}f'\left[(1-\mu)^t k_0\right] + \frac{\delta\theta^F(1-\mu)}{1-\delta}\right] \\
&- \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t_0} \\
=& \left[1 - \frac{(1-\delta)(1-\mu)}{1+r}\right]\Bigg\{ \\
&\sum_{t=2}^{t_0}\left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t-1}\left[p\bar{\varphi}f'\left[(1-\mu)^t k_0\right] + \frac{\delta\theta^F(1-\mu)}{1-\delta}\right] \\
&+ \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t_0-1}\Bigg\} \\
&- \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right] + \frac{\delta\theta^F(1-\mu)}{1-\delta}\right\}.
\end{aligned}$$

Let

$$Q \equiv \frac{(1-\delta)(1-\mu)}{1+r}$$
$$R \equiv \frac{\delta\theta^F(1-\mu)}{1-\delta}.$$

Then

$$\begin{aligned}\frac{\partial \bar{V}_2(0,k_1)}{\partial k_1} - \frac{\partial \bar{V}_1(0,k_0)}{\partial k_0} &= (1-Q)\left[\sum_{t=2}^{t_0} Q^{t-1}\left\{p\bar{\varphi}f'\left[(1-\mu)^t k_0\right]+R\right\}+Q^{t_0-1}\right] \\ &\quad - Q\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\} \\ &> (1-Q)\left[\sum_{t=2}^{t_0} Q^{t-1}\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\}+Q^{t_0-1}\right], \\ &\quad - Q\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\}\end{aligned}$$

since $f'' < 0$. Thus

$$\begin{aligned}\frac{\partial \bar{V}_2(0,k_1)}{\partial k_1} - \frac{\partial \bar{V}_1(0,k_0)}{\partial k_0} &> (1-Q)\left[\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\}\sum_{t=2}^{t_0} Q^{t-1}+Q^{t_0-1}\right] \\ &\quad - Q\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\} \\ &= (1-Q)\left[\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\}\frac{Q-Q^{t_0}}{1-Q}+Q^{t_0-1}\right] \\ &\quad - Q\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\} \\ &= \left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\}\left[Q-Q^{t_0}-Q\right]+(1-Q)Q^{t_0-1} \\ &= Q^{t_0-1}\left[1-Q-Q\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\}\right].\end{aligned}$$

Since $(1-\mu)k_0 > k^*$, it follows that $f'\left[(1-\mu)k_0\right] < f'(k^*)$. By equation (1.16), therefore,

$$\begin{aligned}f'\left[(1-\mu)k_0\right]+R &< f'(k^*)+R \\ &= \frac{1+r}{1-\delta}-(1-\mu)-\frac{\delta\theta^F(1-\mu)}{1-\delta}+\frac{\delta\theta^F(1-\mu)}{1-\delta} \\ &= \frac{1+r}{1-\delta}-(1-\mu).\end{aligned}$$

Therefore

$$\begin{aligned}1-Q-Q\left\{p\bar{\varphi}f'\left[(1-\mu)k_0\right]+R\right\} &> 1-Q-Q\left[\frac{1+r}{1-\delta}-(1-\mu)\right] \\ &= 1-Q-\left[(1-\mu)-Q(1-\mu)\right] \\ &= (1-Q)\left[1-(1-\mu)\right] \\ &= \mu(1-Q) \\ &> 0.\end{aligned}$$

Therefore it follows that

$$\frac{\partial \bar{V}_2(0, k_1)}{\partial k_1} > \frac{\partial \bar{V}_1(0, k_0)}{\partial k_0}.$$

Since we have assumed that $k_1 = (1-\mu) k_0$, the firm's choice of k_1 coincides for $\bar{V}$ and the true value function V. Therefore

$$\begin{aligned}
V_1(a_0, k_0) &= a_0 + \frac{(1-\delta)\Psi_1^S + \delta\Psi_1^F}{1+r} + \frac{1-\delta}{1+r} V_2(0, k_1) \\
\bar{V}_1(a_0, k_0) &= a_0 + \frac{(1-\delta)\Psi_1^S + \delta\Psi_1^F}{1+r} + \frac{1-\delta}{1+r} \bar{V}_2(0, k_1) \qquad (1.36) \\
\Rightarrow \qquad V_1(\cdot) - \bar{V}_1(\cdot) &= \frac{1-\delta}{1+r}\left[V_2(\cdot) - \bar{V}_2(\cdot)\right] \\
\Rightarrow \qquad V_2(\cdot) &= \bar{V}_2 + \frac{1+r}{1-\delta}\left[V_1(\cdot) - \bar{V}_1(\cdot)\right]
\end{aligned}$$

which implies that

$$\begin{aligned}
\frac{\partial V_2(\cdot)}{\partial k_1} &= \frac{\partial \bar{V}_2(\cdot)}{\partial k_1} + \frac{1+r}{1-\delta}\left[\frac{\partial V_1(\cdot)}{\partial k_0} - \frac{\partial \bar{V}_1(\cdot)}{\partial k_0}\right]\frac{\partial k_0}{\partial k_1} \\
&= \frac{\partial \bar{V}_2(\cdot)}{\partial k_1} + \frac{1+r}{(1-\delta)(1-\mu)}\left[\frac{\partial V_1(\cdot)}{\partial k_0} - \frac{\partial \bar{V}_1(\cdot)}{\partial k_0}\right] \\
&> \frac{\partial \bar{V}_1(\cdot)}{\partial k_0} + \frac{1+r}{(1-\delta)(1-\mu)}\left[\frac{\partial V_1(\cdot)}{\partial k_0} - \frac{\partial \bar{V}_1(\cdot)}{\partial k_0}\right] \\
&= (1-Q)\frac{\partial \bar{V}_1(\cdot)}{\partial k_0} + Q\frac{\partial V_1(\cdot)}{\partial k_0}, \qquad (1.37)
\end{aligned}$$

since we have shown that $\partial \bar{V}_2(\cdot)/\partial k_1 > \partial \bar{V}_1(\cdot)/\partial k_0$. Clearly the true value function has the property

$$\frac{\partial V_1(\cdot)}{\partial k_0} \geq \theta^F (1-\mu),$$

since the firm can always liquidate undepreciated capital. By equation (1.36),

$$\begin{aligned}
\frac{\partial \bar{V}_1(a_0, k_0)}{\partial k_0} &= \frac{\partial \bar{V}_1(a_0, k_0)}{\partial k_1}\frac{\partial k_1}{\partial k_0} \\
&= (1-\mu)\frac{\partial \bar{V}_1(a_0, k_0)}{\partial k_1} \\
&= (1-\mu)\frac{1-\delta}{1+r}\left[\frac{\partial \Psi_1^S}{\partial k_1} + \frac{\delta}{1-\delta}\frac{\partial \Psi_1^F}{\partial k_1} + \frac{\partial \bar{V}_2(0, k_1)}{\partial k_1}\right] \\
&= (1-\mu)\frac{1-\delta}{1+r}\left[p\bar{\varphi}f'(k_1) + \frac{\delta\theta^F(1-\mu)}{1-\delta} + \frac{\partial \bar{V}_2(0, k_1)}{\partial k_1}\right].
\end{aligned}$$

The first order condition in equation (1.32) gives the optimal capital stock when the firm is above the first-best capital stock and can only liquidate capital (that is, equation (1.32) characterises the capital stock the firm would choose if it inherits even more capital than this). Therefore k_1 must be less than or equal to the capital stock that satisfies this equation,

and so

$$p\bar{\varphi}f'(k_1) \geq \frac{\theta^F(1+r)}{1-\delta} - \frac{\delta\theta^F(1-\mu)}{1-\delta} - \frac{\partial \bar{V}_2(0,k_1)}{\partial k_1}, \tag{1.38}$$

and it follows that

$$\begin{aligned}\frac{\partial \bar{V}_1(a_0,k_0)}{\partial k_0} &\geq (1-\mu)\frac{1-\delta}{1+r}\left[\frac{\theta^F(1+r)}{1-\delta} - \frac{\delta\theta^F(1-\mu)}{1-\delta} - \frac{\partial \bar{V}_2(0,k_1)}{\partial k_1}\right.\\ &\qquad\left. + \frac{\delta\theta^F(1-\mu)}{1-\delta} + \frac{\partial \bar{V}_2(0,k_1)}{\partial k_1}\right]\\ &= (1-\mu)\frac{1-\delta}{1+r}\frac{\theta^F(1+r)}{1-\delta}\\ &= \theta^F(1-\mu).\end{aligned}$$

Substituting this back into equation (1.37) yields

$$\begin{aligned}\frac{\partial V_2(\cdot)}{\partial k_1} &> (1-Q)\frac{\partial \bar{V}_1(\cdot)}{\partial k_0} + Q\frac{\partial V_1(\cdot)}{\partial k_0}\\ &\geq (1-Q)\theta^F(1-\mu) + Q\theta^F(1-\mu)\\ &= \theta^F(1-\mu),\end{aligned}$$

so the cost of liquidating capital at time 2 strictly exceeds the benefit of doing so, and the firm will choose $k_2 = (1-\mu)k_1$. It follows therefore that $\bar{V}_t$ coincides with the true value function V_t at all times t such that the firm inherits 'too much' capital, $(1-\mu)k_{t-1} > k^*$, and such that the derivative of $\bar{V}$ is at least as great as the benefit of liquidating undepreciated capital, $\partial \bar{V}_t(\cdot)/\partial k_{t-1} \geq \theta^F(1-\mu)$.

Fix some date T, take (a_{T-1}, k_{T-1}) as given, and suppose $(1-\mu)k_{T-1} > k^*$. Let $k^\dagger$ satisfy

$$\left.\frac{\partial \bar{V}_T(0,k)}{\partial k}\right|_{(1-\mu)k=k^\dagger} = \theta^F(1-\mu).$$

If $(1-\mu)k_{T-1} > k^\dagger$, then the firm will liquidate capital at time T and choose $k_T = k^\dagger$. Otherwise the firm will simply choose the inherited capital stock $k_T = (1-\mu)k_{T-1}$.

By equation (1.34), $k^\dagger$ is the quantity that satisfies

$$\begin{aligned}\theta^F(1-\mu) = \sum_{t=1}^{t^\dagger}\left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^t\left[p\bar{\varphi}f'\left[(1-\mu)^{t-1}k^\dagger\right] + \frac{\delta\theta^F(1-\mu)}{1-\delta}\right]\\ + \left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t^\dagger},\end{aligned}$$

or equivalently

$$\sum_{t=1}^{t^\dagger}\left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^t\left[p\bar{\varphi}f'\left[(1-\mu)^{t-1}k^\dagger\right]+\frac{\delta\theta^F(1-\mu)}{1-\delta}\right]$$
$$=\theta^F(1-\mu)-\left[\frac{(1-\delta)(1-\mu)}{1+r}\right]^{t^\dagger},$$

where

$$\begin{aligned}t^\dagger &= \left\lfloor \log_{(1-\mu)}\left(\frac{(1-\mu)k^*}{k^\dagger}\right)\right\rfloor \\ &= \left\lfloor \log_{(1-\mu)}\left(\frac{k^*}{k^\dagger}\right)\right\rfloor + 1.\end{aligned}$$

■

1.3.5 The firm's behaviour and dynamics

We now have all the necessary results to describe the firm's behaviour.

Theorem 1.18 (The firm's behaviour) *Take (a_{t-1}, k_{t-1}) as given. Suppose that the firm does not inherit more than the first-best capital stock, so $(1-\mu)k_{t-1} \leq k^*$.*

(i) If $\min\{\bar{k}_t, \bar{k}_t^S\} \geq k^*$ – that is, if the first-best capital stock is feasible with a small loan – the firm takes a small loan and chooses $k_t = k^*$.

(ii) If $\min\{\bar{k}_t, \bar{k}_t^S\} < k^*$ – that is, if the first-best capital stock is not feasible with a small loan – then:

(a) If $\bar{k}_t(\Omega_{t-1}) \leq \bar{k}_t^S(\Omega_{t-1})$:

(1) If $D_t > 0$, where D_t is defined as in equation (1.26), the firm still takes a small loan and chooses the boundary-best capital stock $k_t = \bar{k}_t(\Omega_{t-1})$.

(2) If $D_t < 0$, the firm takes a large loan and chooses the optimal large loan capital stock $k_t = k_t^L = \min\{k^{**}, \bar{k}_t^L\}$.

(3) If $D_t = 0$, the firm is indifferent between choosing $k_t = \bar{k}_t(\Omega_{t-1})$ with a small loan and choosing $k_t = k_t^L = \min\{k^{**}, \bar{k}_t^L\}$ with a large loan.

(b) If $\bar{k}_t^S(\Omega_{t-1}) < \bar{k}_t(\Omega_{t-1})$, then the firm takes a small loan and chooses the small-loan-constrained capital stock $k_t = \bar{k}_t^S(\Omega_{t-1})$.

Now suppose $(1-\mu)k_{t-1} > k^*$.

(iii) The firm takes a small loan and chooses $k_t = \min\{(1-\mu)k_{t-1}, k^\dagger\}$.

Moreover, the firm always weakly prefers to retain maximal liquid wealth from date t to date $t+1$.

Proof. Suppose $(1-\mu)k_{t-1} \leq k^*$.

(i) When $\min\left\{\bar{k}_t, \bar{k}_t^S\right\} \geq k^*$, by Lemma 1.10, the firm takes a small loan and chooses the first-best capital stock $k_t = k^*$.

(ii) If $\min\left\{\bar{k}_t, \bar{k}_t^S\right\} \geq k^*$, then:

(a) If $\bar{k}_t \leq \bar{k}_t^S$:

1) If $D_t > 0$, by Lemma 1.16, the firm chooses a small loan and the boundary-best capital stock $k_t = \bar{k}_t\left(\Omega_{t-1}\right)$.

2) If $D_t < 0$, by Lemma 1.16, the firm takes a large loan and chooses the capital stock $k_t = k_t^L = \min\left\{k^{**}, \bar{k}_t^L\right\}$.

3) If $D_t = 0$, by Lemma 1.16, the firm is indifferent between choosing $k_t = \bar{k}_t\left(\Omega_{t-1}\right)$ with a small loan and choosing $k_t = k_t^L = \min\left\{k^{**}, \bar{k}_t^L\right\}$ with a large loan.

(b) If $\bar{k}_t^S\left(\Omega_{t-1}\right) < \bar{k}_t\left(\Omega_{t-1}\right)$, by Lemma 1.16 the firm takes a small loan and chooses the small-loan-constrained capital stock $k_t = \bar{k}_t^S\left(\Omega_{t-1}\right)$.

Now suppose $(1-\mu)\, k_{t-1} > k^*$.

(iii) By Lemma 1.17, the firm takes a small loan and chooses $k_t = \min\left\{(1-\mu)\, k_{t-1}, k^\dagger\right\}$.

By Lemma 1.8, the firm always weakly prefers to retain maximal liquid wealth from date t to date $t+1$. ■

Next we have a result concerning the dynamics of the firm's capital-loan choice.

Theorem 1.19 *Suppose $(1-\mu)\, k_{t-1} \leq k^*$. If the firm survives from date t to date $t+1$, then*

$$\Omega_t > (1+r)\, \Omega_{t-1}.$$

It follows that, so long as the firm survives and there is a strictly positive interest rate $r > 0$, it will in finite time end up taking a small loan and employing the first-best capital stock k^*.

Proof. Fix a time T, and take Ω_{T-1} as given. Suppose $(1-\mu)\, k_{T-1} \leq k^*$. We can consider two situations:

(i) *It is optimal for the firm to take a large loan at time T, and therefore the firm chooses* $k_T = k_T^L = \min\left\{k^{**}, \bar{k}_T^L\right\}$. In this case, we first note that

$$\begin{aligned}
&\frac{d}{dk}\left\{p\bar{\varphi} f(k) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]k}{1-\delta}\right\}\Bigg|_{k=k_T^L} \\
&= p\bar{\varphi} f'\left(k_T^L\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]}{1-\delta} \\
&\geq p\bar{\varphi} f'\left(k^{**}\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]}{1-\delta} \\
&= 0.
\end{aligned}$$

By the properties of $f(k_T)$, it follows that

$$p\bar{\varphi}f\left(k_T^L\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]k_T^L}{1-\delta} > 0.$$

By equation (1.27), therefore,

$$\begin{aligned}\Omega_T &= p\bar{\varphi}f\left(k_T^L\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^B(1-\mu)\right]k_T^L}{1-\delta} + \frac{1+r}{1-\delta}\Omega_{T-1}\\ &> \frac{1+r}{1-\delta}\Omega_{T-1}\\ &> (1+r)\,\Omega_{T-1}.\end{aligned}$$

(ii) *It is optimal for the firm to take a small loan at time T, and therefore the firm chooses* $k_T = k_T^S = \min\left\{k^*, \bar{k}_T, \bar{k}_T^S\right\}$. We note again that in this situation that

$$\begin{aligned}\frac{d}{dk}&\left\{p\bar{\varphi}f(k) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)\right]k}{1-\delta}\right\}\Bigg|_{k=k_T^S}\\ &= p\bar{\varphi}f'\left(\bar{k}_T\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)\right]}{1-\delta}\\ &\geq p\bar{\varphi}f'(k^*) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)\right]}{1-\delta}\\ &= 0.\end{aligned}$$

Again, by the properties of $f(k_T)$, it follows that

$$p\bar{\varphi}f\left(k_T^S\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)\right]k_T^S}{1-\delta} > 0.$$

Letting a_T^S be maximal retained wealth associated with employing k_T^S at time T,

$$\begin{aligned}\Omega_T =& a_T^S + (1-\mu)\,k_T^S\\ =& p\bar{\varphi}f\left(k_T^S\right) + (1+r)\left(\Omega_{T-1} - k_T^S\right) + (1-\mu)\,k_T^S\\ =& p\bar{\varphi}f\left(k_T^S\right) - \frac{\left[(1-\delta)(1+r)-(1-\delta)(1-\mu)\right]k_T^S}{1-\delta} + (1+r)\,\Omega_{T-1} \qquad (1.39)\\ >& p\bar{\varphi}f\left(k_T^S\right) - \frac{\left[1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)\right]k_T^S}{1-\delta} + (1+r)\,\Omega_{T-1}\\ >& (1+r)\,\Omega_{T-1}.\end{aligned}$$

In each case, $\Omega_T > (1+r)\,\Omega_{T-1}$. So long as the firm survives from date T to date $T+1$, therefore, inherited wealth Ω grows by more than the interest rate. With positive growth, bounded below at a value strictly greater than unity (so long as there is a strictly positive interest rate), this means Ω will be large enough in finite time for the firm to be able to employ the first-best capital stock k^* with a small loan. ■

1.3.6 The transversality condition

In order to ensure that Theorem 1.18 does indeed describe an optimal policy for the firm, it is sufficient to show that this policy obeys a transversality condition (see e.g. Theorem 4.15 in Stokey et al., 1989, p. 98). Since the firm's problem is stochastic, the appropriate transversality condition is

$$\lim_{t\to\infty} (1+r)^{-t}\,\mathbb{E}_0\left[\frac{d\psi_{t+1}/d\Omega_t}{1+r}\Omega_t\right] = 0, \tag{1.40}$$

where $\psi_{t+1}/(1+r)$ is the value at date t of the dividend that the firm pays at date $t+1$ (that is, its instantaneous utility function) and, by Corollary 1.13 and equation (1.25), Ω_t is the state variable[3]. The expectation is taken at time 0; that is, the expectation is taken without knowing the date at which the firm will fail, and in particular is not conditional on the firm having survived to time $t-1$. Equation (1.40) guarantees that the firm does not expect to 'leave money on the table': in the limit as $t\to\infty$, the additional dividend that the firm is able to pay is, in expected discounted terms, zero.

Lemma 1.20 *The policy path described in Theorem 1.18 satisfies the transversality condition in equation* (1.40)*, and thus describes an optimal policy path.*

Proof. First, note that the firm does not pay a dividend if it survives, so $d\psi^S_{t+1}/d\Omega_t = 0$. Thus we need only consider the failure dividend,

$$\frac{d\psi^F_{t+1}}{d\Omega_t} = \frac{d}{d\Omega_t}\max\left\{(1+r)\,\Omega_t + \left[\theta^F(1-\mu) - (1+r)\right]k_t, 0\right\}.$$

Since we are considering this derivative as $t\to\infty$, by Theorem 1.19 – except for an initial finite number of periods – we are considering the situation when $k_t = k^*$. In particular, this also means that the firm is in a small loan environment, so will pay a strictly positive dividend if it fails. Thus, as $t\to\infty$,

$$\begin{aligned}\frac{d\psi^F_{t+1}}{d\Omega_t} &= \frac{d}{d\Omega_t}\left\{(1+r)\,\Omega_t + \left[\theta^F(1-\mu) - (1+r)\right]k^*\right\}\\ &= 1+r.\end{aligned}$$

The expectation at time 0 is therefore

$$\mathbb{E}_0\left[\frac{d\psi_{t+1}/d\Omega_t}{1+r}\Omega_t\right] = \delta\,(1-\delta)^t\,\Omega_t,$$

since the firm survives (with probability $1-\delta$) for t periods and fails (with probability δ)

[3] Note that $\Omega_t = a_t + (1-\mu)\,k_t$ is the state variable for the firm's problem only if the firm has not overaccumulated capital, which it will not unless it is born with more than the first-best capital stock. Even in this case, the capital stock will depreciate and will settle at the first-best capital stock in finite time, so in the limit as $t\to\infty$ the state variable is guaranteed to be the one-dimensional variable Ω_t whatever the firm's initial capital stock.

Table 1.1: Comparative statics for an individual firm

	Parameter						
Capital Stock	$\boldsymbol{\theta^F}$	$\boldsymbol{\theta^B}$	$\boldsymbol{\delta}$	$\boldsymbol{\mu}$	$\bar{\varphi}$	$\boldsymbol{p}$	$\boldsymbol{r}$
k^*	$+$	$\cdot$	$-$	$-$	$+$	$+$	$-$
$\bar{k}_t$	$+$	$\cdot$	$\cdot$	$-$	$\cdot$	$\cdot$	$-$
$\bar{k}_t^S$	$\cdot$	$\cdot$	$\cdot$	$-$	$+$	$+$	$-$
k^{**}	$\cdot$	$+$	$-$	$-$	$+$	$+$	$-$
$\bar{k}_t^L$	$\cdot$	$+$	$-$	$-$	$+$	$+$	$-$

Classifying the effect of the exogenous parameters on the firm's decision quantities, for given Ω_{t-1}. The symbol $+$ means that the capital stock moves with the parameter, $-$ means that the capital stock moves against the parameter, while $\cdot$ means that the capital stock is not affected by the parameter. The effects on $\bar{k}_t^L$ hold when $\bar{k}_t^L > \bar{k}_t$ – that is, when the employment of $\bar{k}_t^L$ necessitates a large loan.

immediately thereafter. The transversality condition now reads

$$\lim_{t\to\infty} \delta \left(\frac{1-\delta}{1+r}\right)^t \Omega_t = 0.$$

This condition is satisfied so long as the growth rate of Ω_t tends to something strictly less than $(1+r)/(1-\delta)$ as $t \to \infty$. By equation (1.39), when $k = k^*$,

$$\begin{aligned}
\Omega_{t+1} &= p\bar{\varphi} f(k^*) - \frac{[(1-\delta)(1+r) - (1-\delta)(1-\mu)]\, k^*}{1-\delta} + (1+r)\,\Omega_t \\
&= \text{constant} + (1+r)\,\Omega_t \\
\Rightarrow \qquad \frac{\Omega_{t+1}}{\Omega_t} &= \frac{\text{constant}}{\Omega_t} + 1 + r \\
\Rightarrow \qquad \lim_{t\to\infty} \frac{\Omega_{t+1}}{\Omega_t} &= 1 + r < \frac{1+r}{1-\delta},
\end{aligned}$$

since Theorem 1.19 shows that Ω_t is unbounded as t grows. It follows therefore that the firm obeys the transversality condition in equation (1.40), and Theorem 1.18 does indeed describe an optimal policy for the firm. ■

1.3.7 Comparative statics for an individual firm

The parameters specifying the environment faced by the firm, and therefore potential capital stocks, are

$$\text{parameters: } \theta^F, \theta^B, \delta, \mu, \bar{\varphi}, p, r,$$

while the potential choices of capital are

$$\text{capital stocks: } k^*, \bar{k}_t(\Omega_{t-1}), \bar{k}_t^S(\Omega_{t-1}), k^{**}, \bar{k}_t^L(\Omega_{t-1})$$

Table 1.2: Modelling choices

Parameter	Value	Description
β	0.7	output elasticity of capital
$\bar{\varphi}$	10	productivity
p	1	price of output
μ	0.05	per-period depreciation
δ	0.1	per-period death risk
r	0.02	interest rate
θ^B	0.1	bank's liquidation value of capital

Modelling choices for numerical simulation. As it will be allowed to vary in the following simulations, no value has been specified for the resale value of capital θ^F.

assuming that the firm does not inherit more than the first-best capital stock at date t, so $(1-\mu)\,k_{t-1} \leq k^*$. Table 1.1 classifies the effect that each parameter has on each capital stock, taking inherited wealth Ω_{t-1} as given. The results are mostly straightforward to show, but the derivation for the effect of δ on the large-loan-constrained capital stock $\bar{k}_t^L\,(\Omega_{t-1})$ is provided in Appendix A.1. Inspection of Table 1.1 makes clear that increases in θ^F, θ^B, p and φ are 'good' for the firm – that is, they allow the firm to employ more capital given (a_{t-1}, k_{t-1}) – while δ, μ and r are 'bad' for the firm. This makes intuitive sense: it seems reasonable that increasing the resale value of capital, the price of output, and productivity should all be positive for the firm, while increasing the death risk, depreciation and the interest rate will all increase costs for the firm.

1.4 Numerical modelling of the firm's life cycle and comparative statics

Theorem 1.18 gives us a complete description of the firm's behaviour over its life cycle. Therefore we can turn to numerical modelling to understand how shocks to parameters affect the equilibrium character of the firm. Table 1.2 details the parameter and function choices made for numerical simulation. In particular, the production function is a Cobb-Douglas function with capital as the only input, $f\,(k) = k^{\beta}$, and an elasticity parameter of $\beta = 0.7$.

Simulations of the firm's life cycle were conducted with the values $\theta^F = 0.2$ and $\theta^F = 0.8$, to compare a situation in which investment is relatively reversible with a situation in which investment is relatively irreversible, and the results are presented in Figure 1.2. Intuitively, when θ^F is smaller, firms grow more slowly and reach a smaller size at maturity, in terms both of accumulated capital and of revenue. When $\theta^F = 0.8$ firms reach maturity after around 24 periods of operation; at this point, they have reached their terminal size, that is, the quantity of capital employed and revenue are constant from this point until the firm fails. At maturity, the firm starts to carry positive balances of liquid wealth because it is no longer liquidity constrained, and does not need to spend all liquid wealth on further capital accumulation. When $\theta^F = 0.2$, maturation only takes around 14 periods, and the size of the

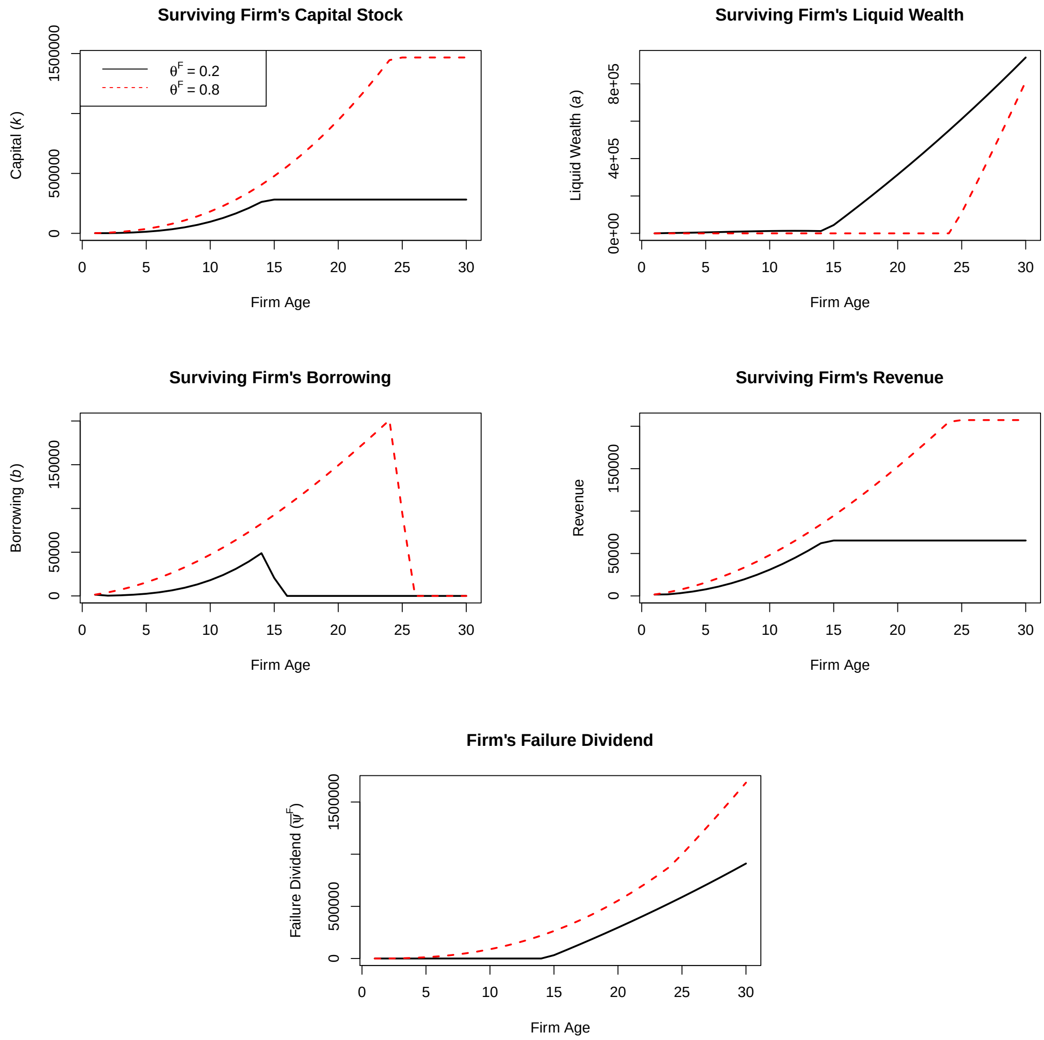

Figure 1.2: The firm's life cycle with two different resale values of capital

Simulations of the firm's life cycle with $\theta^F = 0.2$ (the solid black line) and $\theta^F = 0.8$ (the dotted red line). When the resale value of capital is lower, growth in capital and revenue are slower, firms achieve a lower terminal size – that is, mature firms are smaller when θ^F is smaller – and firms of any age are worth less when they fail. Moreover, when θ^F is smaller, while long-established firms carry smaller balances of liquid wealth, medium-age firms carry larger balances of liquid wealth.

firm when it reaches maturity is rather smaller. Clearly, when θ^F is smaller, a given firm's failure dividend (the value of the firm when it fails) is less at any age than when θ^F is larger.

Perhaps most interestingly, when θ^F is smaller, firms reach maturity and start accumulating liquid wealth at a younger age. Thus medium-age firms actually carry *larger* cash balances when the resale value of capital is *lower*. It's clear from Figure 1.2 that the very oldest firms would indeed carry larger cash balances when θ^F is larger – if both lines in the top right panel of Figure 1.2 were extended, they would clearly cross at some point – but this suggests that the aggregate effect of θ^F on firms' cash balances economy-wide is ambiguous.

Figure 1.3 presents average results across all firms. Due to the exogenous death risk, there must be $(1-\delta)$ times as many firms that survive to be $\tau+1$ periods old as there are firms that survive to be τ periods old. This eventual tapering off among old firms allows calculations

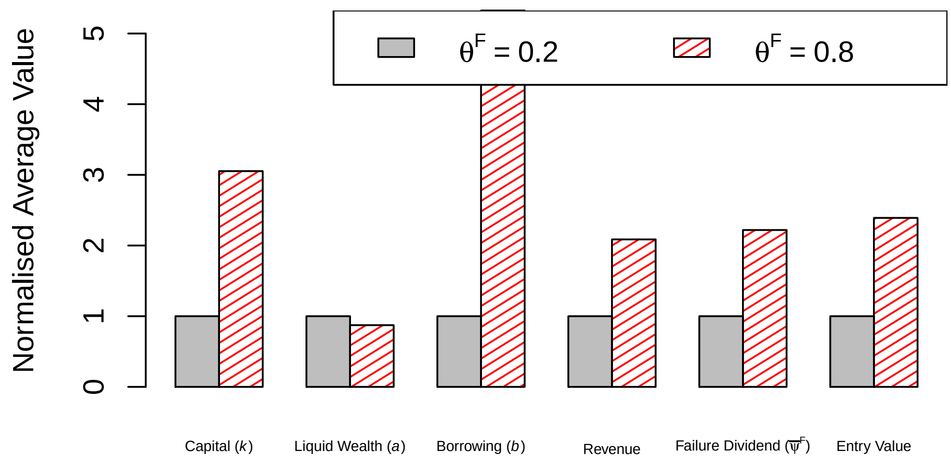

Figure 1.3: The average firm with two different resale values of capital

Simulations of average firm characteristics with $\theta^F = 0.2$ (the solid grey bars) and $\theta^F = 0.8$ (the hatched red bars), with the values for $\theta^F = 0.2$ normalised to unity. Most of the quantities shown are greater for a higher value of θ^F, but not uniformly greater. A smaller value of θ^F leads to increased cash stocks and reduced borrowing, particularly relative to revenues and capital stocks.

to be made of average characteristics at a snapshot in time[4]. Suppose we freeze time at date T, and suppose we assume there is a continuum of firms of unit measure: there must be δ firms that are in their first period of production, $\delta(1-\delta)$ in their second, $\delta(1-\delta)^2$ in their third, and so on. Summing across all firms' capital stocks, given the different densities of firms of different ages, gives the average capital stock. We also calculate average liquid wealth balances, average revenues and average failure dividends (bearing in mind that only the proportion δ of firms of any given age will fail). Finally, we also calculate the entry value of a firm (that is, the expected discounted lifetime dividend stream of a firm that is born the following period with no liquid wealth or capital, $V_0(0)$).

These quantities are normalised to unity for the case when $\theta^F = 0.2$, and presented as relative quantities for $\theta^F = 0.8$. Note that four of the five values are higher for the higher value of θ^F, but they are not uniformly greater. Note also that average liquid wealth is marginally lower for $\theta^F = 0.8$ than for $\theta^F = 0.2$. More particularly, a reduction in θ^F implies that firms' cash balances will increase significantly relative to revenue, and increase even more relative to capital stocks.

Sensitivity analysis is conducted in Appendix A.2, using the same approach as in this section but varying each of the other parameters one at a time.

The model developed in this chapter is a partial equilibrium model, so does not take

[4]For mature firms, liquid wealth grows at a rate that tends towards $1+r$ per period, and there are $1-\delta$ as many firms surviving each subsequent period. Therefore in order for the average characteristics to be well-defined (finite), we require $(1+r)(1-\delta) < 1$.

account of price effects, much less productivity growth. We must therefore be sceptical when interpreting the absolute difference in, for example, capital or liquid wealth for the two values of θ^F shown in Figure 1.3. We cannot conclude with confidence that capital employed really does increase in absolute terms when θ^F increases. Nonetheless, if we take the ratio of capital, liquid wealth, failure dividends and entry values to revenue – which is the same as nominal output, given we have a continuum of firms of unit measure – then this gives us values as a share of the economy, which are amenable to more meaningful interpretation and comparison[5]. Therefore we consider the relationship between the resale value of capital θ^F and average capital, liquid wealth, failure dividends and the firm's entry value, each expressed as a share (multiple) of revenue.

Figure 1.4 shows the results of these simulations. Capital per revenue and borrowing per revenue are clearly increasing with θ^F, while liquid wealth per revenue is clearly decreasing with θ^F. This suggests that a decline in the resale value of capital ought to lead to greater aggregate liquid wealth on firms' balance sheets, along with less capital and hence lower investment, and also less borrowing, relative to GDP. This is consistent with the stylised facts discussed earlier.

Firms' entry value and dividends paid relative to GDP both increase with θ^F in most of the range considered, but at low levels of investment reversibility, a further decrease in reversibility causes these ratios to increase. The effect on entry values and dividends are therefore ambiguous. Moreover, the sensitivity analysis shown in Figures A.9, A.10, A.11, A.12 and A.13 show that the effect of θ^F on failure dividends, in particular, is very sensitive to parameter values, not just in magnitude but in direction. Therefore we cannot be confident in predicting any particular effect of investment reversibility on dividends.

1.5 Conclusions

We developed a partial equilibrium model of a representative firm's optimal investment programme across its life cycle, when there is a positive, exogenous risk of firm failure in any given period, when firms face a cash-in-advance constraint on investment expenditure, and when banks can only extend one-period loans. We find that surviving firms strictly prefer to retain all liquid wealth when young and growing, and accumulate capital gradually, even though they do not face convex adjustment costs. Young firms rely on credit markets, but mature firms are able to self-finance all desired investment.

By conducting numerical analysis, we find that aggregated across a continuum of firms, investment reversibility is positively associated with capital stocks and borrowing, and negatively associated with liquid wealth retained by firms, expressed as a share (multiple) of revenue. Thus, in particular, an increasing reliance on intangible capital leads to greater

[5]The sensitivity analysis conducted in Appendix A.2 shows that doubling the firm's productivity from $\varphi = 10$ to $\varphi = 20$ does not make much difference at all to any of the panels in Figure 1.4. Since price p only occurs as a multiplier on the production function, as does productivity φ, the same would hold for price changes. Therefore even though we have developed a partial equilibrium model, when taking quantities as a share (multiple) of revenue, the effects are similar to those we would find in a general equilibrium analysis.

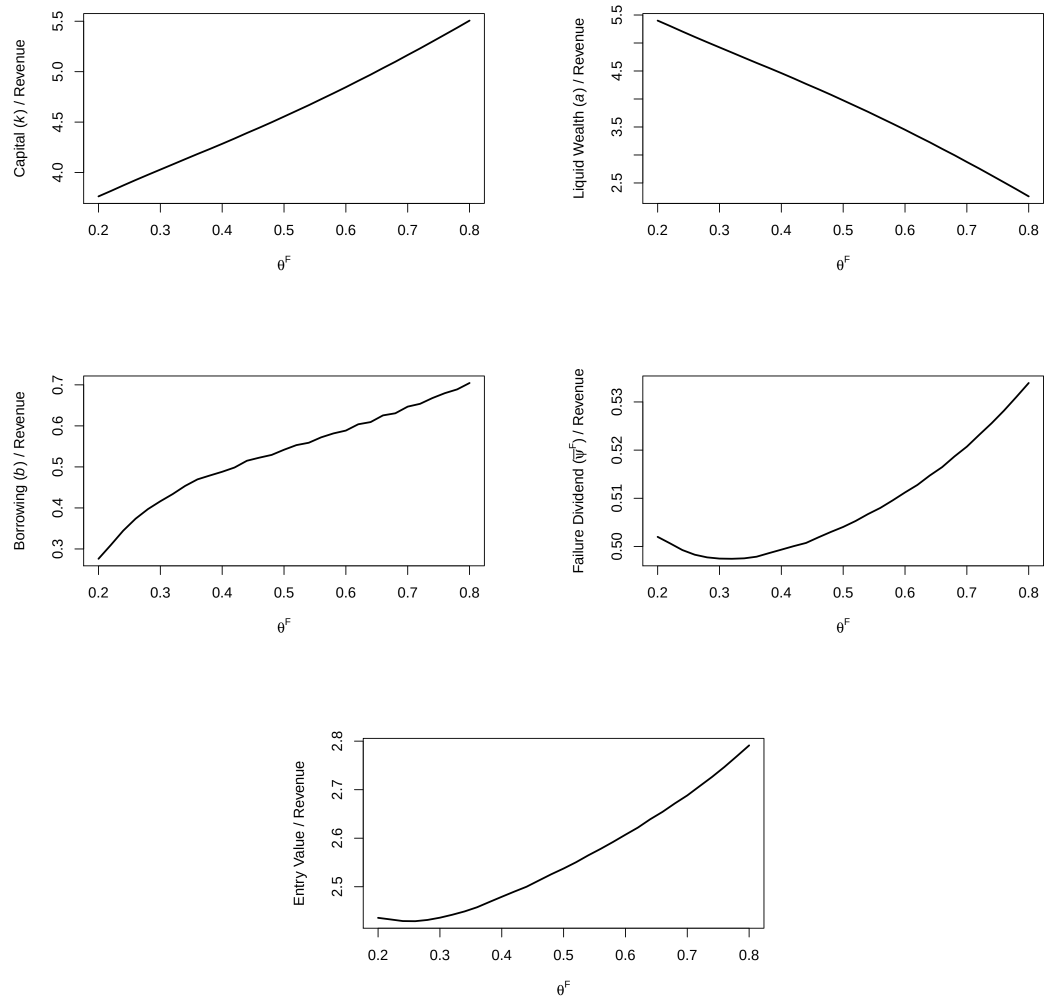

Figure 1.4: Average firm characteristics divided by revenue, plotted against θ^F

Simulations of average firm characteristics divided by average firm revenue, plotted against the resale value of capital θ^F. Capital k and borrowing b increase uniformly as a share (multiple) of revenue with θ^F, while liquid wealth a decreases uniformly. The average failure dividend and the firm's entry value both increase as a share (multiple) of revenue with θ^F in most of the range shown, but appear to have a quadratic form and are minimised around $\theta^F = 0.33$ and $\theta^F = 0.25$ respectively.

cash stocks on corporate balance sheets and reduced debt, as found by Falato et al. (2013). Sensitivity analysis confirms these results hold for a wide array of parameter choices.

Chapter 2

Investment Irreversibility and Structural Change in General Equilibrium

Abstract

This chapter develops a general equilibrium model of an economy in which the degree of investment irreversibility affects the relative sizes of the manufacturing and services sectors. In a benchmark one-sector model, output per worker and the wage rate are increasing in investment reversibility, and simulations show that taxing households to subsidise capital liquidation – thereby increasing the reversibility of investment from firms' perspective – can be welfare-enhancing. In the two-sector model investment reversibility affects the structural composition of the economy. The direction of this effect depends on consumer preferences: when consumers consider manufactured goods and services to be 'more complements than substitutes', the share of services in the economy is increasing with investment reversibility. Regardless of consumer preferences, subsidising capital liquidation increases the manufacturing shares of labour and output. Such subsidies, even when they are welfare-enhancing, may therefore be perceived as a *de facto* fillip to the manufacturing sector.

2.1 Introduction

There is an extensive body of work studying the effects of investment irreversibility on individual firms' capital formation decisions and on aggregate economic performance, but comparatively little attention has been paid to the relationship between irreversibility and the structural composition of the economy. In advanced economies like the United States, two striking trends in recent decades have been an increasing reliance on intangible capital (Corrado et al., 2009; Corrado and Hulten, 2010) and a secular shift away from manufacturing towards services[1] (Herrendorf et al., 2014). Is there any relationship between the two?

In this chapter, we develop a general equilibrium model in which individual firms face a degree of irreversibility in their investments; that is, capital can only be liquidated at a

[1]See Chapter 3 for a more empirically-founded discussion of this trend in the United States.

discount to its purchase price, where we parametrise the precise level of reversibility and study its effect on competitive equilibrium outcomes. The model suggests that in the long run increased reversibility causes increased output, wages and consumption. Decreasing reversibility leads to decreased corporate borrowing and increased corporate saving, consistent with empirical evidence (Falato et al., 2013). It also causes a reduction in the number of firms in the market, consistent with an observed reduction in the number of listed firms[2] in the United States (Doidge et al., 2018). The effect of reversibility on structural composition – the balance between services and manufacturing – depends on households' preferences. We also consider government subsidies of capital liquidation, finding that at low levels of underlying investment reversibility, such subsidies can be welfare-enhancing. However, these subsidies cause an increase in the size of the manufacturing sector, so may be perceived as a boondoggle for manufacturers even when they are welfare-enhancing.

While early examinations of a firm's optimal investment programme in the neoclassical framework, such as Jorgenson (1963), assumed perfect reversibility of investment – that is, capital can be liquidated at its purchase price, so the installation of capital does not constitute a sunk cost – a later body of work considered the more realistic situation in which investment is either fully irreversible or only reversible at a cost[3] (e.g. Bertola and Caballero, 1994; Abel and Eberly, 1996, 1999). A number of studies have embedded such micro-level irreversibility into a general equilibrium model, as we do in this chapter. Bernanke (1983) suggests that irreversibility can give rise to investment cycles; for example, at the beginnings of recessions whose duration and severity are not yet know, investors may hold off making investments against a time when the future is clearer. This model emphasises the 'option value' of not investing: when the economic environment is uncertain, taking irreversible action precludes the possibility of taking some other action at a later date, so there is an option value associated with *not* taking that irreversible action. In a later contribution, Veracierto (2002) advances a model in which, despite having significant implications for individual plants or firms, irreversibility has no major effect on the aggregate business cycle. This helps justify our simple stochastic environment in which there are only firm-level shocks, rather than aggregate shocks, since we are primarily interested in long-run outcomes rather than the business cycle.

Further emphasising the importance of general equilibrium modelling, Faig (2001) highlights the difference between the effects of investment irreversibility on individual markets and economy-wide, and suggests that irreversibility – despite effectvely being an additional cost to capital accumulation – can under certain circumstances spur capital creation. Hugonnier et al. (2005) consider irreversible investment in the framework of the aforementioned option value of waiting to invest. When risk aversion is introduced, and this framework is extended to general equilibrium with endogenous consumption, the option value of waiting is diminished, with implications for the pattern of investment.

Focussing on asset prices and investment thresholds, Kogan (2001) models a two-sector

[2]This prediction must be intepreted with some caution. Doidge et al. (2018) suggest that the fall in the number of listed firms is because public markets are not well-suited to financing small firms with a large proportion of intangible capital, not necessarily that such firms don't exist.

[3]See Chapter 1 for a deeper discussion of this literature.

economy – the 'sector of interest' and 'everything else' – in the presence of irreversibility. Notably, "[u]nlike in standard partial-equilibrium models of irreversible investment, the link between aggregate uncertainty and investment in general equilibrium is ambiguous and depends on households preferences" (Kogan, 2001, p. 227-228). This prefigures the importance in our model of household preferences in determining the response of structural composition to investment reversibility.

Sim (2007) develops a general equilibrium model in which the magnitude of the 'hangover effect' associated with investment irreversibility (not being able to liquidate over-accumulated capital, which would tend to increase capital stocks) and the 'user cost effect' (the strictly positive option value of waiting to invest, which would tend to decrease capital stocks) are compared, finding that for a wide variety of parameters, the user cost effect dominates the hangover effect. This lends support to our simple setup in which there is no uncertainty about the aggregate economic environment – the only stochastic element of our model is the exogenous probability each period of any individual firm permanently failing – and so there is no real hangover effect. A firm only ever has too much capital if it fails and thus requires zero capital, and in this case capital is liquidated and the firm wound up.

Firms in our model, while identical technologically, are heterogeneous in terms of size, age, wealth, collateral and so on. This heterogeneity is important for some of our results about the effect of reversibility on corporate savings[4]. There is an extensive literature concerned with heterogeneous firms in the field of international economics, attempting to explain observed differences between exporting and non-exporting firms, and the consequent effects on the macroeconomy (see e.g. Helpman et al., 2004; Ghironi and Melitz, 2005; Bernard et al., 2007; Melitz and Redding, 2014). In an interesting paper, Manova (2013) considers the interaction between credit constraints and firm heterogeneity. In some respects, this is similar to our setup: firms face upfront costs associated with exporting, just as in our model firms face upfront costs associated with investment, and thus rely on credit markets. In common with most of the literature however, firms in Manova (2013) differ in terms of their productivity, which is not the case in our model.

A handful of models have considered investment irreversibility in the presence of technologically heterogeneous firms. Gala (2007) employs irreversible investment in the presence of productivity shocks and heterogeneous firms to generate a realistic cross-section of stock returns, and assesses the performance of asset pricing models at explaining data thus generated. In contrast to our own findings, Jamet (2004) constructs a general equilibrium model in which irreversibility increases the long-run aggregate capital stock in the economy. Firm heterogeneity generates a non-spike distribution of firms in the steady state when there is investment irreversibility, even without firm entry and exit, since the model admits historical dependence. By inducing entry and exit, we are able to generate a non-spike distribution of firm sizes in equilibrium even with technologically homogeneous firms.

In the two-sector version of our model, we show that investment reversibility modestly affects the structural composition of the economy. Herrendorf et al. (2014) present some

[4]See Section 1.4 for more discussion of firm-level effects of reversibility on corporate savings.

stylised facts associated with structural change, notably that in advanced economies, manufacturing is secularly decreasing and services secularly increasing over time as a share of output, consumption and employment. At least two theoretical paradigms exist to explain this phenomenon. Kongsamut et al. (2001) embed consumer preferences in a Stone-Geary utility function, and thus increasing wealth leads to a different desired mix of products. Conversely, Ngai and Pissarides (2007) show that under certain assumptions, labour will flow towards sectors with relatively slow productivity growth, and relative price changes will be such that nominal output and consumption shares will increase in the backward sector. Even within the supply-side story, there has been very little consideration of the effect of investment irreversibility on the structural composition of the economy. We find that decreasing investment reversibility – due, for example, to an increasing reliance on intangible capital – in fact *decreases* the share of services in the economy when consumer preferences are relatively inelastic. Nonetheless, this effect is rather small, so it may be happening concurrently with unequal technological progress in different sectors, and the technological progress dominates. The effect of investment irreversibility on aggregate output is rather greater: if these changes were happening concurrently, therefore, the shift to intangible capital could help explain secularly low growth rates in the advanced world in recent decades.

We also consider the welfare and structural composition implications of subsidies for capital liquidation, which would increase the reversibility of investment from firms' perspective. De Long and Summers (1991) suggest that the social return to equipment installation is greater than the private return, since equipment investment is associated with productivity growth, implying that equipment subsidies are a desirable policy. While productivity is a fixed, separate parameter in our model – so if the productivity spillovers story is correct, we are underestimating the benefit of subsidies – we nonetheless find that capital liquidation subsidies can be welfare-enhancing.

A number of empirical studies provide evidence of the effects of capital subsidies consistent with our model. Harris (1991) examines the effect of factor subsidies in Northern Irish industry, finding that capital subsidies caused manufacturing output to increase, as we find. This paper also suggests that capital subsidies caused manufacturing employment to decrease, which is the opposite of the prediction we make: however, unlike our model, Harris (1991) relies on a partial equilibrium model in which capital subsidies cause manufacturers to substitute away from labour to capital. Our general equilibrium setting, in which labour is supplied inelastically, precludes the possibility of *all* sectors substituting away from labour, and we find that in fact capital subsidies cause labour to flow to manufacturing from services.

Bergström (2000) examines the effect of capital subsidies in Sweden on the performance of firms, concluding that subsidies can accelerate firm growth, but have little effect on firm productivity. Similarly, considering the effect of regional capital subsidies in Greece on firm performance, Tzelepis and Skuras (2004) find that subsidies are positively associated with firm growth. Harris and Trainor (2005) study the effect of capital subsidies for manufacturing firms in Northern Ireland and conclude that real gross manufacturing output would have been 7-10% lower in the absence of grants. All of these findings are broadly consistent with our model. Additionally, Harris and Trainor (2005) find some evidence for the proposition that capital subsidies increase total factor productivity; while this effect is not present in our model, if true, this only strengthens our assertion that capital subsidies can sometimes be welfare-enhancing.

The rest of the chapter is organised as follows. Section 2.2 develops a baseline one-sector general equilibrium model, studies the effects of investment reversibility on this economy, and undertakes welfare and policy analysis. Section 2.3 does the same for the two-sector model, additionally considering the effect of investment reversibility on the balance between services and manufacturing in the economy. Section 2.4 concludes.

2.2 A one-sector general equilibrium model

2.2.1 Model setup

In this model, we consider the firms introduced in Chapter 1 to be intermediate firms, producing an intermediate good that is used alongside labour in the production of the final consumption good. The model operates as follows. Time is discrete, and in each period a homogeneous intermediate good is produced using project-specific capital; then a homogeneous final good is produced using labour and the intermediate good. Final output can be consumed, saved as liquid wealth, or converted costlessly into project-specific capital, but capital can only be liquidated at a discount. We take the final good as the numeraire in this economy and fix its price to unity.

There is a continuum of identical households of fixed measure L, each of which is endowed with a unit of labour at each period in time. Households supply labour inelastically to the final sector, which is perfectly competitive and exhibits constant returns to scale, so that inputs are compensated according to their marginal product. At date t, the final sector employs all labour and all intermediate output produced at time t.

There is an infinite supply of potential intermediate firms, each of which comes into existence upon the payment of a fixed entry cost. Intermediate firms operate exactly as described in Chapter 1, taking project-specific capital as their only input, and relying in some situations on credit from the banking sector; the intermediate sector is also competitive, but firms exhibit decreasing returns to scale, so are able to make profits. The equilibrium condition for firm entry is that the firm's expected discounted lifetime dividend stream – that is, the firm's value function on entry – must equal the entry cost.

The banking sector takes deposits from households and lends to intermediate firms to fund capital purchases when the firms do not have sufficient liquid wealth to fund themselves, and to fund startup costs for firms. Once a bank has funded a firm's startup costs, it owns all shares in the firm. The banking sector is risk-neutral, and therefore requires an expected return of the risk-free rate r_t on any asset it holds at time t. A simple assumption that households face transaction costs not faced by banks gives rise to financial intermediaries. Assets held by banks are loans to the intermediate sector and shares in intermediate firms, while liabilities are deposits held by households and firms.

Each period is divided into three subperiods, and the timing is as follows:

Subperiod 1. The banking sector uses deposits from households and firms to finance startup costs for new intermediate firms, and to make loans to intermediate firms in order to fund capital purchases

Subperiod 2. Production takes place in the intermediate sector

Subperiod 3. Simultaneously, the following happen: the final sector purchases all intermediate output, produces final output, and pays a wage to households; intermediate firms pay back their loans to the financial sector, where they are able; failed intermediate firms' capital is liquidated, and failure dividends are paid to their shareholders (the banking sector); interest is paid on households' and firms' deposits with the banking sector; households consume their chosen quantity of final output; households' savings and firms' retained liquid wealth are deposited in the banking sector.

We turn now to a more detailed consideration of each sector.

2.2.2 The banking sector

The perfectly competitive banking sector takes deposits from households and firms with excess liquid wealth, and uses them to fund the entry cost of startups and to make loans to young firms so that they can install capital. Banks fund startups and own firms directly; banks therefore act as a risk-pooling intermediary for savers in the economy, so that households and individual firms can access the safe rate of return, while bearing no idiosyncratic risk associated with individual holdings in their portfolios. This could be justified by assuming that banks have informational advantages in funding firms, or can exploit economies of scale, or by assuming that individual savers are risk-averse. We note that there is no possibility for arbitrage: as described in Chapter 1 banks only make loans at time t that have an expected return of r_t, which is the prevailing safe rate at time t. It is clear to see that, by construction, the expected return on holding a share in a firm is also r_t, since the value function of any firm is given by

$$V_t\left(\Omega_{t-1}\right) = \frac{\left(1-\delta\right)\psi_{t+1}^{S} + \delta\psi_{t+1}^{F}}{1+r_t} + \frac{1-\delta}{1+r_t}V_{t+1}\left(\Omega_t\right),$$

where ψ^S and ψ^F are the survival and failure dividends respectively. Thus $\left(1+r_t\right)V_t$ is exactly equal to the expected dividend plus the new expected share value (that is, the principal plus the expected capital gain).

The banking sector has both assets and liabilities. Assets comprise the loan portfolio and ownership of firms, while liabilities are deposits from households and from firms with retained liquid wealth. In any period t, the banking sector's net worth is therefore V_t^B, where

$$V_t^B \equiv \underbrace{N_t\bar{b}_t}_{\text{loan portfolio}} + \underbrace{N_t\bar{V}_t}_{\text{firm ownership}} - \underbrace{N_t\bar{a}_t}_{\text{firms' deposits}} - \underbrace{L\phi_t}_{\text{households' deposits}},$$

where N_t is the number of intermediate firms at time t, barred quantities are averages across all intermediate firms, L is the (fixed) number of households, and ϕ_t is the quantity of deposits held in the banking sector at time t by each household. Since we assume that the banking sector makes zero profits, imposing the condition that it came into existence with zero net worth, this implies that $V_t^B = 0$ at all times t. Thus

$$L\phi_t = N_t\left(\bar{b}_t + \bar{V}_t - \bar{a}_t\right). \tag{2.1}$$

We also note the law of motion for the value of the banking sector's firm ownership,

$$\begin{aligned}\underbrace{N_{t+1}\bar{V}_{t+1}}_{\text{aggregate firm value at time } t+1} &= \underbrace{(1+r_t)\,N_t\bar{V}_t}_{\text{return on firm ownership at time } t} - \underbrace{N_t\bar{\psi}^F_{t+1}}_{\text{aggregate failure dividends}} \\ &\quad + \underbrace{(N_{t+1} - (1-\delta)\,N_t)\,e}_{\text{value of new firms}},\end{aligned}$$

where e is the entry cost for new firms[5] and $\bar{\psi}^F$ is the average failure dividend. Thus in the steady state[6], where quantities are constant and so not time-dependent,

$$\begin{aligned} N\bar{V} &= (1+r)\,N\bar{V} - N\bar{\psi}^F + \delta Ne \\ \Rightarrow \qquad \bar{V} &= \frac{1}{r}\left(\bar{\psi}^F - \delta e\right).\end{aligned}$$

Combining this with equation (2.1) yields the steady-state relationship

$$L\phi = N\left(\bar{b} - \bar{a}\right) + \frac{N}{r}\left(\bar{\psi}^F - \delta e\right), \tag{2.2}$$

which will be useful later, since all of the quantities on the RHS of equation (2.2) are quantities that we can calculate numerically in the steady state. Next, we consider the final sector.

2.2.3 The final production sector

The final output sector is perfectly competitive, and produces output Y_t at time t by use of labour L_t and intermediate output I_t. Aggregate final output is given by

$$Y_t \equiv I_t^{\gamma} L_t^{1-\gamma}, \tag{2.3}$$

where L_t is aggregate labour and $\gamma \in (0,1)$ is an elasticity parameter. Many of the results derived below would flow through for any competitive CRS production technology in the final

[5] As discussed further in Section 2.2.4.1, the equilibrium entry condition is that firms' value on entry is equal to the entry cost.

[6] For a precise definition of equilibrium and the steady state in this economy, see Section 2.2.6.

sector, but we assume a concrete Cobb-Douglas functional form for convenience. The price of intermediate goods and the wage rate respectively are given by

$$\begin{aligned} p_t &= \gamma \Gamma_t^{\gamma-1} \\ w_t &= (1-\gamma)\,\Gamma_t^{\gamma}, \end{aligned}$$

where Γ is the quantity of intermediate good per worker,

$$\Gamma_t \equiv I_t / L_t.$$

Thus the price of intermediate output is clearly decreasing in Γ, while the wage rate is increasing in Γ. Rearrangement of these equations gives

$$\Gamma_t = \left(\frac{\gamma}{p_t}\right)^{\frac{1}{1-\gamma}} \tag{2.4}$$

$$w_t = (1-\gamma)\left(\frac{\gamma}{p_t}\right)^{\frac{\gamma}{1-\gamma}}. \tag{2.5}$$

Thus we immediately have the following result.

Lemma 2.1 *Intermediate output per worker Γ, final output per worker Y/L, and the wage rate w are all in one-to-one correspondence with – and decreasing in – the price of intermediate output p.*

Proof. The result follows immediately from equations (2.3), (2.4) and (2.5). ■

Lemma 2.1 will be useful to us, because when characterising the steady state, the price of intermediate output will be determined by considering an individual firm's problem, and we can then use this result to specify Γ, Y/L and w. Indeed, we now proceed to derive some useful results about the intermediate sector.

2.2.4 The intermediate production sector

Intermediate firms are modelled exactly as firms in Chapter 1. We have fully characterised those firms' behaviour when the parameters that are exogenous to the firm, such as the interest rate and price of intermediate output, are fixed – which is indeed the case in the steady state – so we can lift these results wholesale when characterising the steady state of this economy. Intermediate firms take final output, convert it costlessly into project-specific capital, and use this capital to produce the intermediate good, which is used in the production of the final good. Intermediate firms are all technologically identical, and they produce a homogeneous intermediate good, but they differ in terms of age and size: some firms die and new firms are born at every date. More mature firms have built up more capital and internal liquid wealth, produce more output, and are less reliant on the financial sector to fund their operations.

In Chapter 1 we only required the intermediate firms' production function to obey the usual Inada conditions. For simplicity, and consistency with the final sector, we will here assume it takes Cobb-Douglas form,

$$f(k) = k^{\beta},$$

with $\beta \in (0,1)$, where k is project-specific capital employed by that firm. Thus an intermediate firm's output at time t is given by

$$\begin{aligned} y_t &= \varphi_t k_t^{\beta} \\ \varphi_t &= \begin{cases} \bar{\varphi} > 0 & \text{with probability } 1-\delta \\ 0 & \text{with probability } \delta. \end{cases} \end{aligned}$$

The possibility of firm failure means loans extended by the financial sector are risky, and capital functions as collateral. If the firm is able to repay its loan in the event of failure, it liquidates its own capital at the price $\theta^F \in (0,1)$. If the firm is unable to repay its loan in the event of failure, the bank seizes the firm's capital and undertakes the liquidation itself at a price $\theta^B \in \left(0, \theta^F\right)$. Thus θ^F, which will be a key parameter of interest when we turn to numerical modelling, indexes the 'reversibility of investment'. Intermediate firms produce a homogeneous intermediate good: at time t, there is a continuum of intermediate firms of measure N_t, where firms are indexed by j, and aggregate intermediate output is given by

$$I_t \equiv \int_0^{N_t} y_{j,t}\, dj,$$

where $y_{j,t}$ is firm j's output at time t.

2.2.4.1 Firm entry

At any time t, we assume that there is an infinite pool of potential startup firms which can become operational upon payment of a fixed entry cost e. Firms are born with zero liquid wealth and zero capital. In equilibrium, the cost of entry will equal the present value of the expected lifetime dividend stream. That is, the equilibrium condition for a firm entering the marketplace at time t is

$$V_0 \equiv V_t(0) = e.$$

The firm pays the cost e at time t and begins production at time $t+1$.

2.2.4.2 Determining the price of intermediate output

In order to show that the interest rate in the steady state must determine the price of intermediate output, we first derive some useful results.

Lemma 2.2 *Holding other parameters and the price of intermediate output constant, an intermediate firm's value function on entry is continuous, differentiable, strictly monotone* *and decreasing in the interest rate r, and in particular,*

$$\frac{dV_0}{dr} < -\frac{(1-\delta)\,e}{(1+r)^2}. \tag{2.6}$$

Proof. Consider an intermediate firm. We will proceed by backward induction: first we will show that equation (2.6) holds for all mature firms, and then we will show that if it holds at date $t+1$, it must also hold at date t.

Fix some date T and Ω_{T-1}, and suppose that the firm employed the first-best capital stock k^* at time $T-1$, so that in particular the firm can continue to employ k^* for as long as it survives. Then by the proof of Lemma 1.10 the firm's value function at date T is given by

$$V_T(\Omega_{T-1}) = \left(\frac{1}{1+r} - \frac{\delta}{r+\delta}\right)\mu k^* + \frac{1-\delta}{r+\delta}\left[p\bar{\varphi}f(k^*) - p\bar{\varphi}f'(k^*)\,k^*\right] + C(\Omega_{T-1}),$$

where C is some constant based on inherited wealth Ω_{T-1}. Note that k^* is implicitly defined by

$$p\bar{\varphi}f'(k^*) - \frac{1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)}{1-\delta} = 0,$$

so by the Implicit Function Theorem we can take the differential dk^*/dr. Moreover, since $f'' < 0$ by assumption, $dk^*/dr < 0$. Thus

$$\begin{aligned}
\frac{dV_T}{dr} =& -\left[\frac{1}{(1+r)^2} - \frac{\delta}{(r+\delta)^2}\right]\mu k^* - \frac{1-\delta}{(r+\delta)^2}\left[p\bar{\varphi}f(k^*) - p\bar{\varphi}f'(k^*)\,k^*\right] \\
&+ \left(\frac{1}{1+r} - \frac{\delta}{r+\delta}\right)\mu\frac{dk^*}{dr} + \frac{1-\delta}{r+\delta}\left[p\bar{\varphi}f'(k^*)\frac{dk^*}{dr} - p\bar{\varphi}f'(k^*)\frac{dk^*}{dr} - \frac{k^*}{1-\delta}\right] \\
=& -\left[\frac{1}{(1+r)^2} - \frac{\delta}{(r+\delta)^2}\right]\mu k^* - \frac{1-\delta}{(r+\delta)^2}\left[p\bar{\varphi}f(k^*) - p\bar{\varphi}f'(k^*)\,k^*\right] \\
&+ \left(\frac{1}{1+r} - \frac{\delta}{r+\delta}\right)\mu\frac{dk^*}{dr} - \frac{k^*}{r+\delta} \\
<& 0.
\end{aligned}$$

Now consider some arbitrary date t, and suppose inductively that $dV_{t+1}/dr < 0$. Suppose the firm takes a large loan at time t. Then since the firm will not pay a dividend whether it survives or fails,

$$\begin{aligned}
V_t(\Omega_{t-1}) &= \frac{1-\delta}{1+r}V_{t+1}(\Omega_t) \\
\Rightarrow \qquad \frac{dV_t}{dr} &= -\frac{1-\delta}{(1+r)^2}V_{t+1}(\Omega_t) + \frac{1-\delta}{1+r}\frac{dV_{t+1}}{dr} \\
&< -\frac{1-\delta}{(1+r)^2}V_{t+1}(\Omega_t).
\end{aligned}$$

By Corollary 1.13 the value function is strictly and monotonically increasing in inherited wealth Ω, so $V_{t+1}(\Omega_t) > V_0(0)$, and by the free entry condition, $V_0 = e$ where e is the firm's entry cost. Thus

$$\frac{dV_t}{dr} < -\frac{(1-\delta)\,e}{(1+r)^2},$$

as required. Suppose instead that the firm takes a small loan at time t. The firm will not pay a dividend if it survives, but it may pay a dividend if it fails. By equation (1.20) the value function is

$$V_t(\Omega_{t-1}) = \delta\Omega_{t-1} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]k_t}{1+r} + \frac{1-\delta}{1+r}V_{t+1}(\Omega_t).$$

Therefore

$$\begin{aligned}
\frac{dV_t}{dr} &= -\frac{\delta\theta^F(1-\mu)\,k_t}{(1+r)^2} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dr} - \frac{1-\delta}{(1+r)^2}V_{t+1}(\Omega_t) + \frac{1-\delta}{1+r}\frac{dV_{t+1}}{dr} \\
&< -\frac{1-\delta}{(1+r)^2}V_{t+1}(\Omega_t) + \left\{\frac{1-\delta}{1+r} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dV_{t+1}}\right\}\frac{dV_{t+1}}{dr} \\
&< -\frac{(1-\delta)\,e}{(1+r)^2} + \left\{\frac{1-\delta}{1+r} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dV_{t+1}}\right\}\frac{dV_{t+1}}{dr}.
\end{aligned}$$

Note that

$$\frac{dV_{t+1}}{dk_t} = \frac{dV_{t+1}}{d\Omega_t}\cdot\frac{d\Omega_t}{dk_t}.$$

By the proof of Theorem 1.19,

$$\begin{aligned}
\frac{d\Omega_t}{dk_t} &= p\bar{\varphi}f'(k_t) - \frac{(1-\delta)(1+r)-(1-\delta)(1-\mu)}{1-\delta} \\
&\geq p\bar{\varphi}f'(k^*) - \frac{1+r-(1-\delta)(1-\mu)-\delta\theta^F(1-\mu)}{1-\delta} + \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1-\delta} \\
&= \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1-\delta}.
\end{aligned}$$

Again by Corollary 1.13[7],

$$\frac{dV_{t+1}}{d\Omega_t} \geq 1.$$

Thus

$$\begin{aligned}
& & \frac{dV_{t+1}}{dk_t} &\geq \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1-\delta} \\
\Rightarrow & & \frac{1-\delta}{1+r} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dV_{t+1}} &\geq 0,
\end{aligned}$$

[7]Strictly speaking, this condition only follows from Corollary 1.13 if we can show that V is a differentiable function of Ω. This is straightforward to show so long as the policy functions are differentiable; as in Chapter 1, we simply assume this to be the case.

and so

$$\begin{aligned}\frac{dV_t}{dr} &< -\frac{(1-\delta)\,e}{(1+r)^2} + \left\{\frac{1-\delta}{1+r} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dV_{t+1}}\right\}\frac{dV_{t+1}}{dr} \\ &\leq -\frac{(1-\delta)\,e}{(1+r)^2}\end{aligned}$$

as required. ■

Next we show that V_0 is increasing in p.

Lemma 2.3 *Holding other parameters constant, an intermediate firm's value function on entry is continuous, differentiable, strictly monotone and increasing in the price of intermediate output p. Moreover, $V_0 = 0$ when $p = 0$, and $\lim_{p\to\infty} V_0 = \infty$.*

Proof. The proof follows a similar structure to the proof of Lemma 2.2. Again, we will proceed by backward induction.

Fix some date T and Ω_{T-1}, and suppose that the firm employed the first-best capital stock k^* at time $T-1$, so that in particular the firm can continue to employ k^* for as long as it survives. Then by the proof of Lemma 1.10 the firm's value function at date T is given by

$$V_T(\Omega_{T-1}) = \left(\frac{1}{1+r} - \frac{\delta}{r+\delta}\right)\mu k^* + \frac{1-\delta}{r+\delta}\left[p\bar{\varphi}f(k^*) - p\bar{\varphi}f'(k^*)\,k^*\right] + C(\Omega_{T-1}),$$

where C is some constant based on inherited wealth Ω_{T-1}. Since $dk^*/dp > 0$,

$$\begin{aligned}\frac{dV_T}{dp} =& \left(\frac{1}{1+r} - \frac{\delta}{r+\delta}\right)\mu\frac{dk^*}{dp} + \frac{1-\delta}{r+\delta}\left[\bar{\varphi}f(k^*) - \bar{\varphi}f'(k^*)\,k^*\right] \\ &+ \frac{1-\delta}{r+\delta}\left[p\bar{\varphi}f'(k^*)\frac{dk^*}{dp} - p\bar{\varphi}f'(k^*)\frac{dk^*}{dp}\right] \\ >& \frac{1-\delta}{r+\delta}\left[\bar{\varphi}f(k^*) - \bar{\varphi}f'(k^*)\,k^*\right] + \frac{1-\delta}{r+\delta}\left[p\bar{\varphi}f'(k^*)\frac{dk^*}{dp} - p\bar{\varphi}f'(k^*)\frac{dk^*}{dp}\right] \\ =& \frac{1-\delta}{r+\delta}\left[\bar{\varphi}f(k^*) - \bar{\varphi}f'(k^*)\,k^*\right] \\ >& 0.\end{aligned}$$

Now consider some arbitrary date t, and suppose inductively that $dV_{t+1}/dp > 0$. Suppose the firm takes a large loan at time t. Then since the firm will not pay a dividend whether it survives or fails,

$$\begin{aligned}V_t(\Omega_{t-1}) &= \frac{1-\delta}{1+r}V_{t+1}(\Omega_t) \\ \Rightarrow \qquad \frac{dV_t}{dp} &= \frac{1-\delta}{1+r}\frac{dV_{t+1}}{dp} \\ &> 0,\end{aligned}$$

as required. Suppose instead that the firm takes a small loan at time t. Then

$$V_t(\Omega_{t-1}) = \delta\Omega_{t-1} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]k_t}{1+r} + \frac{1-\delta}{1+r}V_{t+1}(\Omega_t)$$
$$\Rightarrow \qquad \frac{dV_t}{dp} = -\frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dp} + \frac{1-\delta}{1+r}\frac{dV_{t+1}}{dp}.$$

If $k_t = k^*$, as at time T, this implies that $dV_t/dp > 0$. Suppose therefore that $k_t < k^*$. Then

$$\begin{aligned}\frac{dV_t}{dp} &= -\frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dp} + \frac{1-\delta}{1+r}\frac{dV_{t+1}}{dp} \\ &= \left[\frac{1-\delta}{1+r} - \frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{dV_{t+1}}\right]\frac{dV_{t+1}}{dp} \\ &> 0,\end{aligned}$$

where the reasoning is the same as in the proof of Lemma 2.2, but the inequality is strict because we are supposing $k_t < k^*$. As required, again, we have $dV_t/dp > 0$. Since V_t is differentiable, it follows immediately that it is continuous.

Clearly if $p = 0$ then $V_0 = 0$, because there is no benefit to firms in producing output and optimal behaviour is not to produce anything at all[8]. Suppose therefore that $p > 0$. Since firms are born with zero capital and zero liquid wealth, they must take a large loan in their first period of operation. Now consider the firm's behaviour in its second period of operation. It has inherited some quantity of wealth Ω_1. It may or may not be optimal to do so, but the firm is certainly able to take a small loan in its second period of operation, and make its operations small enough that it is able to pay a strictly positive dividend immediately after production. Suppose the firm chooses the capital level $k_2 < \min\left\{\bar{k}_2, \bar{k}_2^S\right\}$. Then it is able to pay the dividend

$$\bar{\psi}_2^S = p\bar{\varphi}f(k_2) - (1+r)k_2 > 0.$$

Clearly as $p \to \infty$, also $\bar{\psi}_2^S \to \infty$, keeping k_2 fixed (and indeed the firm might be able to do even better than this). Thus since

$$\begin{aligned}V_0 &= \frac{1-\delta}{1+r}V_1(0) \\ &= \left(\frac{1-\delta}{1+r}\right)^2 V_2(\Omega_1) \\ &> \left(\frac{1-\delta}{1+r}\right)^2 \delta\bar{\psi}_2^S,\end{aligned}$$

it is also the case that $V_0 \to \infty$ as $p \to \infty$. ■

Lemmas 2.2 and 2.3 will be enough to guarantee a steady-state relationship between r

[8] Strictly, we also want to ensure that $\lim_{p\to 0} V_0 = 0$, to ensure that the value function is right continuous at zero. With reference to the proof of Lemma 1.10, we note that $V_0 < \Psi_0^S$, and that since $f(k^*) - f'(k^*)k^*$ is increasing in k^* which is increasing in p, it follows that that $\lim_{p\to 0}\Psi_0^S = 0$. Since V_0 is bounded below by zero, the result follows immediately.

and p, but we defer this discussion to Section 2.2.6 once we have formally defined the steady state. Before saying something about the equilibrium of this economy, however, we must finally consider households.

2.2.5 Households

There is a continuum of identical households of fixed measure L, each of which is endowed with a unit of labour at each date. Labour is supplied inelastically to the final sector, attracting the wage rate w_t at time t. At time t, each household wishes to maximise the utility function

$$U_t \equiv \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho}\right)^{\tau} \ln\left(c_\tau\right),$$

where $\rho > 0$ is the subjective discount rate and c is per-household consumption. Households deposit savings in banks, earning the return r_t on financial assets ϕ_t held at time t. The household's wealth constraint is therefore

$$\phi_{t+1} = \left(1 + r_t\right)\phi_t + w_t - c_t. \tag{2.7}$$

The Lagrangean for the household's problem is

$$\mathbb{L} \equiv \sum_{t=1}^{\infty} \left\{ \left(\frac{1}{1+\rho}\right)^{t} \ln\left(c_t\right) - \lambda_t \left[\phi_{t+1} - \left(1 + r_t\right)\phi_t - w_t + c_t\right] \right\},$$

yielding the first order conditions

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial c_t} &= \frac{1}{\left(1+\rho\right)^t c_t} - \lambda_t = 0 \\ \Rightarrow \qquad \lambda_t &= \frac{1}{\left(1+\rho\right)^t c_t} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial \phi_t} &= -\lambda_{t-1} + \left(1 + r_t\right)\lambda_t = 0 \\ \Rightarrow \qquad \lambda_{t-1} &= \left(1 + r_t\right)\lambda_t. \end{aligned}$$

Finally, the optimal consumption path must obey the transversality condition

$$\lim_{t\to\infty} \lambda_t \phi_{t+1} = \lim_{t\to\infty} \frac{\phi_{t+1}}{\left(1+\rho\right)^t c_t} = 0. \tag{2.8}$$

Combining the first order conditions yields the relationship between consumption growth and the interest rate,

$$\frac{c_{t+1}}{c_t} = \frac{1 + r_{t+1}}{1+\rho}. \tag{2.9}$$

Dynamic analysis of this economy is difficult, as discussed further in Section 2.2.9, so we limit ourselves below to steady states and comparative statics of steady states: all that is important about the household's problem therefore is that $r = \rho$ when consumption is invariant, as implied by equation (2.9). As with the final sector's production technology, we could have assumed a generic form for households' utility function, and it would only affect the transition of the economy; we simply assume the concrete functional form proposed here for convenience.

2.2.6 Characterising the steady state and comparative statics

Definition 2.4 *We define an equilibrium as a sequence of prices and quantities*

$$\{r_t, p_t, w_t, Y_t, c_t\}_{t=-\infty}^{\infty}$$

such that banks, intermediate firms and the final sector maximise profits, households maximise utility, and at each date the markets for labour, for the intermediate good and for final output clear.

Since there is only one final production sector, which exhibits constant returns to scale and is perfectly competitive, market clearing for labour and the intermediate good simply means that those inputs are compensated according to their marginal product. Market clearing for the final good means that consumption demand plus investment demand is equal to final output plus liquidated capital at each date, given the fixed unit price of the final good and the prevailing interest rate.

Definition 2.5 *We define a steady state in this economy to be an equilibrium such that prices and quantities are invariant over time.*

Note that we use 'the steady state' to refer to the nontrivial steady state – that is, we ignore the trivial steady state in which production and consumption are permanently zero. We have a straightforward first result about what the steady state looks like.

Lemma 2.6 *In the steady state, if it exists, the interest rate equals households' subjective discount rate, $r = \rho$.*

Proof. The result follows immediately from equation (2.9) when $c_{t+1} = c_t = c$ and $r_{t+1} = r$ are time-invariant. ■

We can also pin down consumption in the steady state.

Lemma 2.7 *In the steady state, if it exists, consumption is given by*

$$\begin{aligned} c &= w + r\phi \\ &= w + \left[\rho\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta e\right] N/L. \end{aligned}$$

Proof. The result follows immediately from equations (2.2) and (2.7) when $c_{t+1} = c_t = c$, $\phi_{t+1} = \phi_t = \phi$, $w_t = w$ and $r_t = r = \rho$ are time-invariant. ■

Next, we show that p and r are co-determined in the steady state. It proves convenient for the statement and proof of Lemma 2.8 to consider r to be a function of p in the appropriate domain, but we might equally consider p to be a function of r, as we do in the proof of Lemma 2.9.

Lemma 2.8 *Suppose $V_0 = e$ where $e > 0$ is some fixed entry cost. Taking all other parameters as given, the steady-state price of intermediate output p is a function of the steady-state interest rate r. Consider the inverse $r(p)$ of this function. Then $r(p)$ is continuous, differentiable, strictly monotone and increasing in p. Moreover, $\lim_{p\to 0} r(p) < 0$ and $\lim_{p\to\infty} r(p) = \infty$.*

Proof. By Lemma 2.6, the steady-state interest rate r is determined by households' discount rate. It is not strictly true therefore that r is a 'function' of the steady-state price of intermediate output p, but with some abuse of terminology we will consider this to be the case, as it will be convenient to consider the mapping $r(p)$ that is the inverse of the function $p(r)$.

Considering r as a function of p, therefore, it follows immediately from Lemmas 2.2 and 2.3 that $dr/dp > 0$, and since r is a differentiable function of p, it must also be a continuous function of p.

Now, suppose for a contradiction that $\lim_{p\to 0} r \geq 0$. This implies that

$$\lim_{p\to 0} V_0|_{r=0} \geq e,$$

since the relationship between r and p is implicitly defined by the restriction $V_0 = e$, and we know by Lemma 2.2 that V_0 is decreasing in r for any given p. However, this is clearly a contradiction: we also know by Lemma 2.3 that V_0 is a continuous function of p for any given r, and clearly when $p = 0$ the firm derives no benefit from producing output, so $V_0 = 0$ when $p = 0$. Thus in fact

$$\lim_{p\to 0} V_0|_{r=0} = 0$$

and it follows that $\lim_{p\to 0} r < 0$.

Next we wish to show that $\lim_{p\to\infty} r = \infty$. Suppose for a contradiction that $\lim_{p\to\infty} = \tilde{r} < \infty$. By Lemma 2.2, considering V_0 as a function of p and r,

$$e = V_0(p, r) \equiv V_0[p, r(p)] \geq V_0(p, \tilde{r}),$$

since r is an implicit function of p. Thus

$$e \geq \lim_{p\to\infty} V_0(p, \tilde{r}).$$

However, this contradicts Lemma 2.3, which shows that $\lim_{p\to\infty} V_0 = \infty$ for any given value of r. Thus $\lim_{p\to\infty} r = \infty$ as required. ■

Lemma 2.8 can be thought of as a kind of supply curve: although it is a relationship between two prices, rather than a price and a quantity, it codifies the relationship between those two prices that is required by the entry condition on the supply side of the economy. Thus the steady-state interest rate – which is pinned down by households' discount rate – also determines the steady-state price of intermediate output, which in turn allows the calculation of all other quantities in the steady state.

We therefore have all the ingredients needed to characterise the steady state of this economy. First however, we must show that the steady state exists.

Lemma 2.9 *The steady state exists and is uniquely determined.*

Proof. By Lemma 2.6, the interest rate in the steady state – if it exists – is determined by the households' subjective discount rate; in p-r space, therefore, Lemma 2.6 describes a flat quasi-demand curve that must be satisfied in the steady state. Lemma 2.8 describes a upward-sloping quasi-supply curve in p-r space. Moreover, we know that this quasi-demand curve implies a strictly positive price p for any $r \geq 0$, and is unbounded. These two curves must therefore intersect, and jointly characterise the only permissible nontrivial steady state. By Lemma 2.1 and equations (2.3), (2.4) and (2.5), the price of intermediate output determines the wage rate, intermediate output per worker, and final output per worker. Lemma 2.7 determines consumption in the steady state, which – since it is constant – clearly satisfies the transversality condition in equation (2.8). Finally, given an individual firm's output – which is determined by p and r, in addition to the exogenous parameters – the number of firms can be calculated as aggregate intermediate output divided by per-firm intermediate output. ■

Note that even in the steady state, intermediate firms are dying and being born each period. While individual firms are born and die, however, the distribution is invariant. Since a representative share δ of all firms fail in each period, in the steady state there must be $1 - \delta$ times as many firms in their $(T + 1)$-th period of operation as there are firms in their T-th period of operation. Thus in the steady state there are $N\delta$ firms in their first period of operation, $N\delta\,(1 - \delta)$ in their second period of operation, $N\delta\,(1 - \delta)^2$ in their third period of operation, and so on; summing across all firm ages, there is a mass N of firms overall. Thus the distribution of firms, and hence the quantity of intermediate output produced, is invariant in the steady state even though individual firms change over time.

It is difficult to derive aggregate quantities analytically, precisely because there is a continuum of firms of different ages and sizes. Nonetheless, it is possible to find these quantities numerically. Before turning to numerical methods, however, we will derive some analytical results about the steady state of this economy, and in particular the effect of investment reversibility θ^F. In order to say something about the aggregate effect of θ^F, we first examine its effect on individual intermediate firms.

Lemma 2.10 *Holding other parameters and the price of intermediate output constant, an intermediate firm's value function on entry is continuous, differentiable, strictly monotone and increasing in the firm's liquidation value of capital θ^F.*

Proof. Once again, the proof proceeds by backwards induction, like the proofs of Lemmas 2.2 and 2.3. Fix some date T and Ω_{T-1}, and suppose that the firm employed the first-best capital stock k^* at time $T-1$, so that in particular the firm can continue to employ k^* for as long as it survives. Then by the proof of Lemma 1.10 the firm's value function at date T is given by

$$V_T\left(\Omega_{T-1}\right) = \left(\frac{1}{1+r} - \frac{\delta}{r+\delta}\right)\mu k^* + \frac{1-\delta}{r+\delta}\left[p\bar{\varphi} f\left(k^*\right) - p\bar{\varphi} f'\left(k^*\right)k^*\right] + C\left(\Omega_{T-1}\right),$$

where C is some constant based on inherited wealth Ω_{T-1}. Thus

$$\begin{aligned} \frac{dV_T}{d\theta^F} &= \left(\frac{1}{1+r} - \frac{\delta}{r+\delta}\right)\mu\frac{dk^*}{d\theta^F} - p\bar{\varphi} f''\left(k^*\right)\frac{dk^*}{d\theta^F} \\ &> 0, \end{aligned}$$

since $\delta < 1$ and $f'' > 0$. Now consider some arbitrary date t, and suppose inductively that $dV_{t+1}/d\theta^F > 0$. Suppose the firm takes a large loan at time t. Then since the firm will not pay a dividend whether it survives or fails,

$$\begin{aligned} V_t\left(\Omega_{t-1}\right) &= \frac{1-\delta}{1+r}V_{t+1}\left(\Omega_t\right) \\ \Rightarrow \qquad \frac{dV_t}{d\theta^F} &= \frac{1-\delta}{1+r}\frac{dV_{t+1}}{d\theta^F} \\ &> 0, \end{aligned}$$

as required. Suppose instead that the firm takes a small loan at time t. Then

$$\begin{aligned} V_t\left(\Omega_{t-1}\right) &= \delta\Omega_{t-1} - \frac{\delta\left[1+r-\theta^F\left(1-\mu\right)\right]k_t}{1+r} + \frac{1-\delta}{1+r}V_{t+1}\left(\Omega_t\right) \\ \Rightarrow \qquad \frac{dV_t}{d\theta^F} &= -\frac{\delta\left[1+r-\theta^F\left(1-\mu\right)\right]}{1+r}\frac{dk_t}{d\theta^F} + \frac{\delta\left(1-\mu\right)k_t}{1+r} + \frac{1-\delta}{1+r}\frac{dV_{t+1}}{dp}. \end{aligned} \tag{2.10}$$

If $k_t = k^*$, then as already shown, $dV_t/d\theta^F > 0$. Suppose therefore that $k_t \neq k^*$. There are two possibilities. First suppose that $k_t = \bar{k}_t^S$. By Table 1.1, $d\bar{k}_t^S/d\theta^F = 0$, so equation (2.10) clearly shows that $dV_t/d\theta^F > 0$. Finally suppose that $k_t = \bar{k}_t$. Then

$$\begin{aligned} \bar{k}_t &= \frac{\Omega_{t-1}}{1-\theta^F\left(1-\mu\right)/\left(1+r\right)} \\ \Rightarrow \qquad \frac{d\bar{k}_t}{d\theta^F} &= \frac{\left(1-\mu\right)\Omega_{t-1}}{\left(1+r\right)\left[1-\theta^F\left(1-\mu\right)/\left(1+r\right)\right]^2} \\ &= \frac{\left(1-\mu\right)\bar{k}}{1+r-\theta^F\left(1-\mu\right)}, \end{aligned}$$

and thus equation (2.10) becomes

$$
\begin{aligned}
\frac{dV_t}{d\theta^F} &= -\frac{\delta\left[1+r-\theta^F(1-\mu)\right]}{1+r}\frac{dk_t}{d\theta^F}+\frac{\delta(1-\mu)k_t}{1+r}+\frac{1-\delta}{1+r}\frac{dV_{t+1}}{dp} \\
&= -\frac{\delta(1-\mu)k_t}{1+r}+\frac{\delta(1-\mu)k_t}{1+r}+\frac{1-\delta}{1+r}\frac{dV_{t+1}}{dp} \\
&= \frac{1-\delta}{1+r}\frac{dV_{t+1}}{dp} \\
&> 0.
\end{aligned}
$$

In any case, it follows that $dV_t/d\theta^F > 0$, and hence $dV_0/d\theta^F > 0$ as required. ■

Corollary 2.11 *Taking all other parameters as given, the steady-state price of intermediate output p is in one-to-one correspondence with – and decreasing in – investment reversibility θ^F.*

Proof. The result follows immediately from Lemmas 2.3 and 2.10. ■

Corollary 2.11 shows that an increase in investment reversibility θ^F will cause the steady state price of intermediate output p to decrease, by which we immediately have the following result about comparative statics.

Corollary 2.12 *The steady-state values of intermediate output per worker Γ, final output per worker Y/L, and the wage rate w are all in one-to-one correspondence with – and increasing in – investment reversibility θ^F.*

Proof. The result follows immediately from Lemma 2.1 and Corollary 2.11. ■

Corollary 2.12 describes qualitatively the relationship between the firm's liquidation value of capital θ^F and aggregate quantities. Intuitively, the lower is θ^F – perhaps because capital is less tangible – the lower will be output per worker and the wage rate. In order to calculate the magnitude of these effects directly, we turn now to numerical modelling.

2.2.7 Numerical modelling

We now consider the quantitative effect of θ^F on the economy by turning to numerical modelling. Table 2.1 details the parameter choices made for the following numerical modelling exercises. Figure 2.1 plots the results of the numerical modelling exercise. Consistent with Corollary 2.12, aggregate (final) output is increasing with investment reversibility θ^F, as are aggregate consumption, aggregate capital, and the number of intermediate firms. Corporate borrowing increases with θ^F, while corporate savings decrease with θ^F.

This is broadly consistent with stylised empirical evidence. Our results suggest that a decrease in investment reversibility, for example a reduction in capital tangibility, would tend to: put downward pressure on output, which could help explain slow growth in recent years; increase net corporate savings, as has been observed; and lead to fewer firms, increasing market concentration.

Table 2.1: One-sector modelling choices

Parameter	Value	Description
β	0.7	intermediate sector output elasticity of capital
$\bar{\varphi}$	10	intermediate sector productivity
μ	0.05	intermediate sector per-period depreciation
δ	0.1	intermediate sector per-period death risk
ρ	0.02	households' subjective discount rate
θ^B	0.1	banks' liquidation value of capital
e	100	intermediate firms' entry cost
γ	0.3	final sector output elasticity of intermediate goods
L	1	labour supply / number of households

Modelling choices for numerical simulation of the one-sector model. As it will be allowed to vary in the following simulations, no value has been specified for the resale value of capital θ^F.

The numerical modelling sensitivity analysis undertaken in Appendix B.1 shows that these broad effects hold for a wide variety of parameter choices.

In order to assess the relationship between investment reversibility and structural change, however, we need to model two final output sectors. This is the approach we take in Section 2.3.

2.2.8 Welfare and policy implications

Suppose now that there is a government in this economy, which is able to levy a per-period lump sum tax on households and use that money to subsidise firms' capital liquidation. That is, we now suppose that firms can liquidate capital at a value of

$$\theta^F = \bar{\theta}^F + \sigma,$$

where σ is a subsidy provided by the government and $\bar{\theta}^F$ is the underlying reversibility of investment. The government's budget constraint in period t is

$$\sigma_t \delta K_t^S \leq L\mathbb{T}_t,$$

where we define K^S as the aggregate quantity of capital that firms in the small loan regime are employing, so that δK^S is the quantity of liquidated capital that is being subsidised, and where $\mathbb{T}$ is the lump sum tax levied on each household. We allow σ and $\mathbb{T}$ to be positive or negative, so that it's possible for the government to tax capital liquidation and subsidise households, and we suppose that $\theta^F \in (0,1)$ as before[9]. Either a tax or a subsidy could be implemented by altering the tax code, for example: capital liquidation could be taxed directly, or it could be made tax deductible, or made to qualify for tax credits.

[9]If we allowed $\theta^F > 1$ there would be an arbitrage possibility for firms: they would have an incentive to purchase and immediately resell capital, which is not a situation we wish to allow. We also suppose that $\theta^F > 0$, otherwise firms would have an incentive simply to abandon unwanted capital rather than liquidate it.

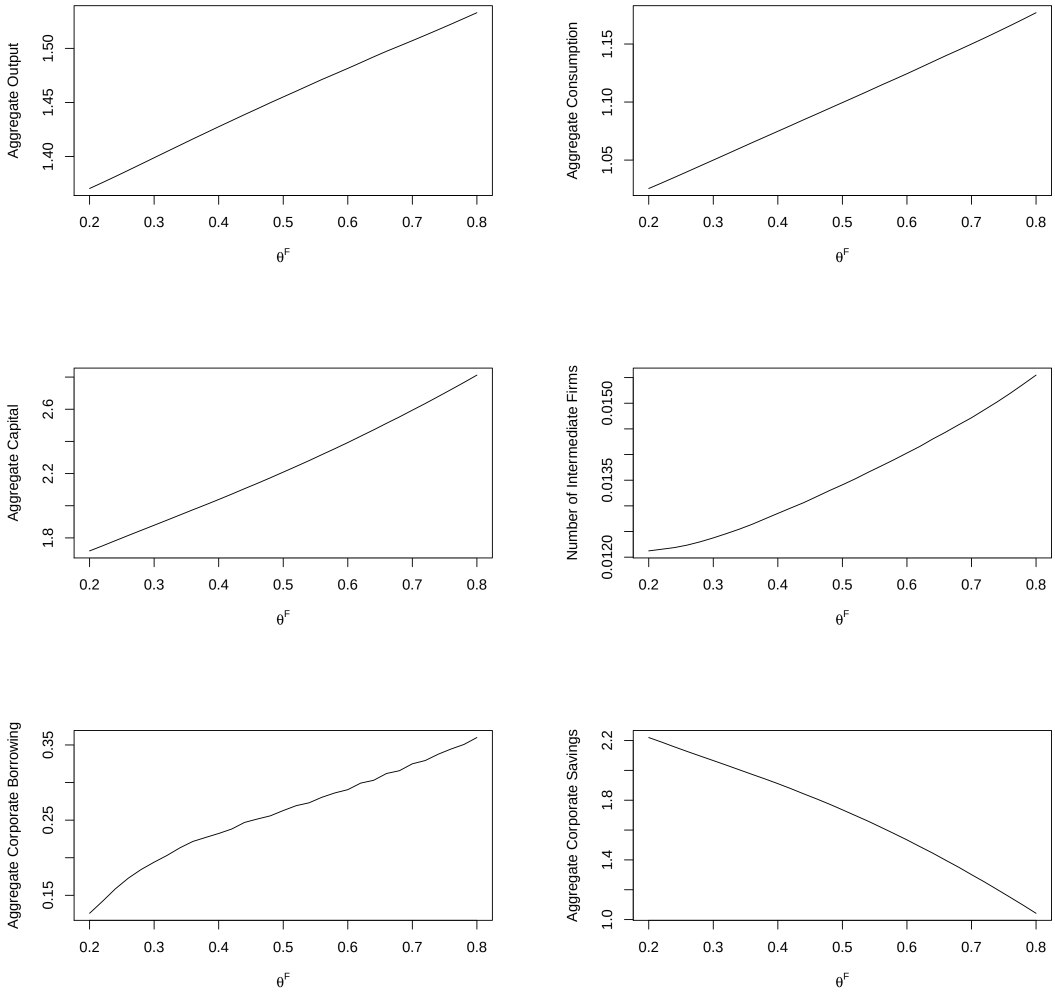

Figure 2.1: The effect of investment reversibility θ^F on one-sector general equilibrium

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. Aggregate output is increasing with θ^F, consistent with Corollary 2.12, as are aggregate consumption, aggregate capital, and the number of intermediate firms. Corporate borrowing increases with θ^F, while corporate savings decrease with θ^F.

As far as firms are concerned, the liquidation value of capital is simply θ^F, so the intermediate sector of the economy behaves exactly as previously modelled. However, modifying Lemma 2.7, in the steady state households are now consuming

$$\begin{aligned} c &= w + \rho\phi - \mathbb{T} \\ &= w + \rho\phi - \sigma\delta K^S. \end{aligned} \tag{2.11}$$

We assume that the government's budget constraint holds with equality in the steady state, since if it did not the government would be stockpiling wealth indefinitely, which is clearly not welfare-maximising. Taking θ^F as fixed, two different underlying reversibility values $\bar{\theta}_1^F$ and $\bar{\theta}_2^F$ will imply two different subsidy levels, σ_1 and σ_2. Suppose that $\bar{\theta}_1^F < \bar{\theta}_2^F$, so that $\sigma_1 > \sigma_2$. Given that θ^F determines the wage and household savings, $w + \rho\phi$ will be the same in both of these situations, but $\sigma\delta K^S$ will increase with σ. Consumption therefore increases with $\bar{\theta}^F$ and will be higher for the $\bar{\theta}_2^F$ case than for the $\bar{\theta}_1^F$ case. Thus we have the following result.

Lemma 2.13 *Taking θ^F as fixed, steady state consumption is a strictly increasing linear function of $\bar{\theta}^F$.*

Proof. The result follows immediately from equation (2.11), given that $w + \rho\phi$ and δK^S depend only on θ^F and not on σ. ■

Households' utility is a function only of consumption, and in the steady state consumption is the same in all periods, so steady state welfare is increasing in steady state consumption. Thus, by Lemma 2.13, it's always better to have higher underlying investment reversibility $\bar{\theta}^F$ than to achieve the same level of θ^F with a lower underlying level of investment reversibility that is boosted using subsidies. Nonetheless, given some value of $\bar{\theta}^F$, it may still be possible to increase welfare by subsiding (or indeed taxing) capital liquidation; we must therefore consider the effect of a marginal increase in the subsidy. By equation (2.11),

$$\begin{aligned} \frac{dc}{d\sigma} &= \frac{dw}{d\sigma} + \rho\frac{d\phi}{d\sigma} - \delta K^S - \sigma\delta\frac{dK^S}{d\sigma} \\ &= \frac{dw}{d\theta^F} + \rho\frac{d\phi}{d\theta^F} - \delta K^S - \sigma\delta\frac{dK^S}{d\theta^F}, \end{aligned}$$

since $d\theta^F/d\sigma = 1$ given fixed $\bar{\theta}^F$. The sign of this differential is ambiguous: we know by Lemma 2.12 that $dw/d\theta^F > 0$, and clearly $-\delta K^S < 0$. Note however that the only term that depends on σ and not merely θ^F is the final term; so long as K^S is increasing in θ^F, therefore – which numerical modelling suggests is the case – then for a given level of θ^F, it follows that $dc/d\sigma$ is decreasing in σ.

Numerical modelling of welfare changes in response to changes in the level of subsidy suggests that, in general, welfare forms an inverted 'U' shape when plotted against the subsidy. If $dc/d\sigma$ is decreasing in σ for a given level of θ^F, then this means in practice that the peak of the inverted 'U' shape will occur at a lower level of θ^F when $\bar{\theta}^F$ is lower. For example, if $\bar{\theta}^F = 0.3$, welfare might be maximised if the government were to offer a subsidy of $\sigma = 0.2$

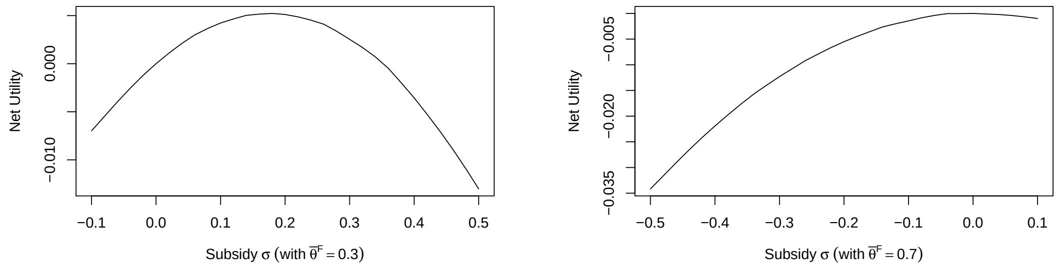

Figure 2.2: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. 'Net Utility' is calculated as the difference in the instantaneous utility function $\ln(c)$ relative to the case of zero subsidy or tax.

so that $\theta^F = 0.5$; but it does not follow that if $\bar{\theta}^F = 0.7$, then welfare would be maximised or even increase if the government imposed a tax of $\sigma = -0.2$ so that, once again, $\theta^F = 0.5$.

We cannot say much more analytically about the effects of government intervention, but Figure 2.2 shows numerical modelling of the welfare effects of subsidies and taxes when $\bar{\theta}^F = 0.3$ – so underlying reversibility is relatively low – and when $\bar{\theta}^F = 0.7$ – so underlying reversibility is relatively high – given the parameters laid out in Table 2.1. This modelling suggests that when the underlying reversibility of investment is low, the government could increase welfare by subsidising capital liquidation up to a certain point and paying for it with a tax on households. It also suggests that the government is less likely to be able to increase welfare by taxing capital liquidation and transferring the proceeds to households when the underlying reversibility of investment is high.

The sensitivity analysis conducted in Appendix B.2 suggest that this general pattern holds for a wide variety of parameter choices; indeed, even when underlying reversibility is relatively high, the government may *still* be able to increase welfare by subsidising capital liquidation[10]. The sensitivity analysis also reinforces that the precise level of subsidy or tax that maximises welfare depends importantly on the values of all parameters, including the underlying reversibility of investment $\bar{\theta}^F$.

[10]When underlying reversibility $\bar{\theta}^F$ is sufficiently high, taxing capital liquidation can in principle be welfare-enhancing. Nonetheless, this situation does not arise very often: most combinations of parameters we tested showed that capital subsidies were still more likely than taxes to be welfare-enhancing, even when underlying reversibility is relatively high.

2.2.9 Dynamics of the model

We can write down the laws of motion for consumption and capital. Consumption follows the Euler condition derived in equation (2.9), while capital changes according to

$$K_t = \underbrace{(1-\delta)(1-\mu)K_{t-1}}_{\text{undepreciated capital among surviving firms}} + \underbrace{\delta\left[\zeta_{t-1}\theta^F + (1-\zeta_{t-1})\theta^B\right](1-\mu)K_{t-1}}_{\text{liquidated capital among failed firms}} + \underbrace{Y_{t-1} - Lc_{t-1}}_{\text{final output minus aggregate consumption}},$$

where K is aggregate capital and $\zeta_t \in [0,1)$ is the share of firms at time t which take a small loan and liquidate their capital at the rate θ^F if they fail, while the remaining $1-\zeta_t$ share of firms take a large loan and the bank liquidates their capital at the rate θ^B if they fail. Thus part of the dynamical system is

$$\begin{aligned} c_t &= \frac{1+r_t}{1+\rho}c_{t-1} \\ K_t &= \left\{1-\delta+\delta\left[\zeta_{t-1}\theta^F + (1-\zeta_{t-1})\theta^B\right]\right\}(1-\mu)K_{t-1} + Y_{t-1} - Lc_{t-1}. \end{aligned}$$

In the standard neoclassical framework the evolution of these two variables would be sufficient to conduct a dynamical analysis of the system, but it is more difficult in this case. There are couple of issues: first, it is not straightforward to say anything analytically about ζ, or its endogenous evolution. Second, and more problematically, although intermediate firms are assumed all to be identical technologically, they are heterogeneous in age and size, and thus their marginal product of capital differs. Capital alone does not fully characterise the economy; the *distribution* of capital is also crucial. We cannot therefore model the intermediate sector as a representative firm, and the interest rate is not simply the marginal product of capital, because there is no single marginal product of capital in the intermediate sector.

Thus the state variable that characterises this economy must be multi-dimensional – aggregate capital alone is inadequate – and the dynamics of the model are not amenable to the usual straightforward analysis. While this is an interesting direction for potential future research, we cannot at present say much about the dynamic properties of this model, and must content ourselves with an analysis of comparative statics. The same is true of the two-sector model to which we now turn our attention.

2.3 A two-sector general equilibrium model

In order to study the effect of investment reversibility on structural change, we will extend the model from Section 2.2 to a two-sector setting. Intermediate firms and the banking sector are identical to those previously described, but we will need to revisit the household and final production sector.

2.3.1 Two final production sectors

We now assume that there are two perfectly competitive final production sectors, manufacturing and services, which we will index by M and S respectively. They both have access to Cobb-Douglas production technologies,

$$\begin{aligned} Y_{M,t} &= I_{M,t}^{\gamma_M} L_{M,t}^{1-\gamma_M} \\ Y_{S,t} &= I_{S,t}^{\gamma_S} L_{S,t}^{1-\gamma_S}, \end{aligned}$$

where $1 > \gamma_M > \gamma_S > 0$ so that manufacturing production is more capital intensive (strictly speaking, more intensive in intermediate inputs, which themselves are produced with capital) than services production. The market-clearing conditions are $I_{M,t} + I_{S,t} = I_t$, where I_t is aggregate intermediate output, and $L_{M,t} + L_{S,t} = L$. We assume that the manufactured good is the numeraire, with price fixed to unity, and that it can be converted costlessly into capital for the intermediate sector. Services can only be consumed in the period they are produced, and the price of services, denoted by p_S, is determined in the competitive market. Aggregate output, in terms of manufactured goods, is therefore

$$Y_t \equiv Y_{M,t} + p_{S,t} Y_{S,t}.$$

By Lemma 2.8, the choice of exogenous parameters determines the steady-state price of intermediate output. Thus

$$\begin{aligned} p_{I,t} &= \gamma_M I_{M,t}^{\gamma_M - 1} L_{M,t}^{1-\gamma_M} \\ &= \gamma_M \left(\frac{I_{M,t}}{L_{M,t}} \right)^{\gamma_M - 1} \\ &= \gamma_M \Gamma_{M,t}^{\gamma_M - 1}, \end{aligned}$$

where Γ_M is the intermediate input-labour ratio in the manufacturing sector, and p_I is the price of intermediate goods, as distinct from the price of services p_S. Thus p_I and Γ_M are co-determined,

$$\Gamma_{M,t} = \left(\frac{\gamma_M}{p_{I,t}} \right)^{\frac{1}{1-\gamma_M}}. \tag{2.12}$$

Similarly for services,

$$p_{I,t} = p_{S,t} \gamma_S \Gamma_{S,t}^{\gamma_S - 1}.$$

The wage rate is also equalised across both sectors, so that

$$\begin{aligned} w_t &= (1 - \gamma_M) \Gamma_{M,t}^{\gamma_M} \\ &= p_{S,t} (1 - \gamma_S) \Gamma_{S,t}^{\gamma_S}. \end{aligned}$$

Since Γ_M is in one-to-one correspondence with p_I, in turn so is w,

$$w_t = (1 - \gamma_M) \left(\frac{\gamma_M}{p_{I,t}} \right)^{\frac{\gamma_M}{1-\gamma_M}}. \tag{2.13}$$

Dividing the price equations by the wage equations yields

$$\begin{aligned} \frac{1-\gamma_S}{\gamma_S} \Gamma_{S,t} &= \frac{1-\gamma_M}{\gamma_M} \Gamma_{M,t} \\ \Rightarrow \qquad \Gamma_{S,t} &= \left(\frac{\gamma_S}{1-\gamma_S} \right) \frac{1-\gamma_M}{\gamma_M} \Gamma_{M,t} \\ &= \left(\frac{\gamma_S}{1-\gamma_S} \right) \frac{1-\gamma_M}{\gamma_M} \left(\frac{\gamma_M}{p_{I,t}} \right)^{\frac{1}{1-\gamma_M}}, \end{aligned} \tag{2.14}$$

thus Γ_S is determined. We can combine this with the price equation to derive the price of services,

$$\begin{aligned} p_{S,t} &= \frac{p_{I,t}}{\gamma_S} \Gamma_{S,t}^{1-\gamma_S} \\ &= \frac{p_{I,t}}{\gamma_S} \left[\frac{\gamma_S (1-\gamma_M)}{(1-\gamma_S) \gamma_M} \right]^{1-\gamma_S} \left(\frac{\gamma_M}{p_{I,t}} \right)^{\frac{1-\gamma_S}{1-\gamma_M}}, \end{aligned} \tag{2.15}$$

which is clearly decreasing in p_I, since $\gamma_M > \gamma_S$.

2.3.2 Households

There are now two types of consumption open to the household, of manufactured goods and of services, which must be reflected in the household's utility function. We suppose at time t, each household wishes to maximise the utility function

$$U_t \equiv \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau} \nu \left(c_{M,\tau}, c_{S,\tau} \right),$$

where c_M and c_S denote consumption of manufactured goods and services respectively, and where

$$\nu \left(c_M, c_S \right) = \ln \left[\left(\omega_M c_M^{(\epsilon-1)/\epsilon} + \omega_S c_M^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \right],$$

where $\epsilon > 0$ is an elasticity parameter and $\omega_M, \omega_S > 0$ are constant weighting parameters. Aggregate consumption c_t, in terms of manufactured goods, is given by

$$c_t = c_{M,t} + p_{S,t} c_{S,t},$$

and the household's wealth constraint is now

$$\begin{aligned} \phi_{t+1} &= (1 + r_t) \phi_t + w_t - c_{M,t} - p_{S,t} c_{S,t} \\ &= (1 + r_t) \phi_t + w_t - c_t. \end{aligned}$$

In order to solve the household's problem, we form the Lagrangean,

$$\mathbb{L} \equiv \sum_{t=1}^{\infty} \left\{ \left(\frac{1}{1+\rho} \right)^t \nu \left(c_{M,t}, c_{S,t} \right) - \lambda_t \left[\phi_{t+1} - (1+r_t) \phi_t - w_t + c_{M,t} + p_{S,t} c_{S,t} \right] \right\}.$$

We take the first order conditions

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial c_{M,t}} &= \frac{\nu_{M,t}}{(1+\rho)^t} - \lambda_t = 0 \\ \Rightarrow \qquad \lambda_t &= \frac{\nu_{M,t}}{(1+\rho)^t} \\ \frac{\partial \mathbb{L}}{\partial c_{S,t}} &= \frac{\nu_{S,t}}{(1+\rho)^t} - p_{S,t} \lambda_t = 0 \\ \Rightarrow \qquad \lambda_t &= \frac{\nu_{S,t}}{(1+\rho)^t p_{S,t}}, \end{aligned}$$

immediately yielding the static efficiency condition

$$\begin{aligned} p_{S,t} &= \frac{\nu_S}{\nu_M} = \frac{\omega_S}{\omega_M} \left(\frac{c_{M,t}}{c_{S,t}} \right)^{1/\epsilon} \\ \Rightarrow \qquad \frac{p_{S,t} c_{S,t}}{c_{M,t}} &= \left(\frac{\omega_S}{\omega_M} \right)^{\epsilon} p_{S,t}^{1-\epsilon}. \end{aligned} \tag{2.16}$$

Dynamic efficiency once again dictates that

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial \phi_t} &= -\lambda_{t-1} + (1+r_t) \lambda_t = 0 \\ \Rightarrow \qquad \lambda_{t-1} &= (1+r_t) \lambda_t. \end{aligned}$$

Again, the optimal consumption path must obey the transversality condition

$$\lim_{t \to \infty} \lambda_t \phi_{t+1} = \lim_{t \to \infty} \frac{\nu_{M,t} \phi_{t+1}}{(1+\rho)^t c_t} = 0.$$

Combining both sets of first order conditions, we see that

$$\frac{\nu_{M,t}}{\nu_{M,t+1}} = \frac{\nu_{S,t}}{\nu_{S,t+1}} = \frac{1+r_{t+1}}{1+\rho}, \tag{2.17}$$

which together with equation (2.16) implies

$$\frac{c_{t+1}}{c_t} = \frac{1+r_{t+1}}{1+\rho}, \tag{2.18}$$

so again, when consumption is invariant, $r = \rho$.

2.3.3 Characterising the steady state and comparative statics

Similarly to the one-sector model, we define an equilibrium and the steady state in this economy as follows.

Definition 2.14 *We define an equilibrium as a sequence of prices and quantities*

$$\{r_t, p_{I,t}, p_{S,t}, w_t, Y_{M,t}, Y_{S,t}, L_{M,t}, L_{S,t}, c_t\}_{t=-\infty}^{\infty}$$

such that banks, intermediate firms and both final sectors maximise profits, households maximise utility, and at each date the markets for labour, for the intermediate good and for final output clear.

Since we now have two sectors employing labour and the intermediate good, we need the additional requirement that wages and compensation for the intermediate good must be equilibrated across both final production sectors at any date. Otherwise, market clearing is defined in much the same way as in the one-sector model.

Definition 2.15 *We define a steady state in this economy to be an equilibrium such that prices and quantities are invariant over time.*

Again, we use 'the steady state' to refer to the nontrivial steady state. All that is left to do in order to characterise the steady state of this economy is to determine the labour shares in each final production sector. Individual intermediate firms are entirely characterised by the supply-side exogenous parameters – it is the number of firms that adjusts in order to ensure general equilibrium with households – so we can in particular determine average firm characteristics. Let $\bar{y}$ be the average intermediate output produced by individual intermediate firms. Then aggregate intermediate output in the steady state is given by

$$\begin{aligned} I &= I_M + I_S \\ &= \Gamma_M L_M + \Gamma_S L_S \\ &= \Gamma_M L + (\Gamma_S - \Gamma_M) L_S \end{aligned}$$

since $L_M + L_S = L$, but also $I = N\bar{y}$, so

$$N = \frac{I}{\bar{y}} = \frac{\Gamma_M L + (\Gamma_S - \Gamma_M) L_S}{\bar{y}}.$$

Thus by equation (2.2) and the household's wealth constraint, steady-state consumption is given by

$$\begin{aligned} c &= w + r\phi \\ &= w + \left[\rho\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta e\right] N/L \\ &= w + \left[\rho\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta e\right] \frac{\Gamma_M L + (\Gamma_S - \Gamma_M) L_S}{L\bar{y}}. \end{aligned} \tag{2.19}$$

The only quantity on the RHS of this expression that cannot yet be determined is L_S; we therefore require another expression relating c and L_S. By equation (2.16),

$$\begin{aligned}
c &= c_M + p_S c_S \\
&= \left[\left(\frac{\omega_M}{\omega_S}\right)^{\epsilon} p_S^{\epsilon-1} + 1\right] p_S c_S \\
&= \left[\left(\frac{\omega_M}{\omega_S}\right)^{\epsilon} p_S^{\epsilon-1} + 1\right] p_S Y_S \\
&= \left[\left(\frac{\omega_M}{\omega_S}\right)^{\epsilon} p_S^{\epsilon-1} + 1\right] p_S \Gamma_S^{\gamma_S} L_S, \qquad (2.20)
\end{aligned}$$

since all services output is consumed. Combining equations (2.19) and (2.20),

$$L_S = \frac{w + \left[r\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta e\right] \Gamma_M / \bar{y}}{\left[\left(\frac{\omega_M}{\omega_S}\right)^{\epsilon} p_S^{\epsilon-1} + 1\right] p_S \Gamma_S^{\gamma_S} - \left[r\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta e\right] \left(\Gamma_S - \Gamma_M\right) / L\bar{y}}. \qquad (2.21)$$

The steady state values of intermediate output, final output in each sector and in aggregate, and consumption can all now be calculated using L_S. By equation (2.18), it is clear that Lemma 2.6 holds in the two-sector model just as it does in the one-sector model. Existence of the steady state follows by an argument very similar to that in Lemma 2.9 combined with equation (2.21). Further, we can say something about how θ^F affects this economy in the steady state.

Corollary 2.16 *The steady-state price of services p_S is in one-to-one correspondence with – and increasing in – investment reversibility θ^F.*

Proof. The result follows immediately from Corollary 2.11 and equation (2.15). ■

Corollary 2.17 *In the steady state, the ratio of consumption expenditure on services to consumption expenditure on manufacturing is in one-to-one correspondence with θ^F. Moreover, it is increasing in θ^F if $\epsilon < 1$, while it is decreasing in θ^F if $\epsilon > 1$.*

Proof. The result follows immediately from Corollary 2.16 and equation (2.16). ■

Corollary 2.17 makes it clear that the division of consumption between services and manufacturing, and its response to changes in θ^F, depend crucially on the value of ϵ: if $\epsilon < 1$, then (price-adjusted) services consumption increases relative to manufacturing consumption as θ^F increases, but if $\epsilon > 1$ then the reverse is true. When we conduct numerical modelling exercises, therefore, we will consider values of ϵ that are both greater than and less than unity, since they have very different implications for structural change.

Note that it is possible for other indicators of structural change to behave differently. If ϵ is sufficiently close to one, and if γ_M and γ_S are sufficiently close to each other, it would be possible for the services consumption share to increase with θ^F while the services labour share decreases with θ^F, or vice versa, or for the labour or output shares to have non-monotonic relationships with θ^F.

Table 2.2: Two-sector modelling choices

Parameter	Value	Description
γ_M	0.7	manufacturing sector output elasticity of intermediate goods
γ_S	0.3	services sector output elasticity of intermediate goods
ω_M	0.5	manufacturing weight in households' utility function
ω_S	0.5	services weight in households' utility function

Modelling choices for numerical simulation of the two-sector model, where they differ from the one-sector model values detailed in Table 2.1. As it will be allowed to vary in the following simulations, no value has been specified for the resale value of capital θ^F. Similarly, no value has been specified for ϵ, as we will run simulations with different values.

2.3.4 Numerical modelling

Where relevant, parameter choices are all as they were for the one-sector model, as detailed in Table 2.1. Now there are two final production sectors, however, we need a few more parameters, which are detailed in Table 2.2. As seen in Corollary 2.17, the relationship between investment reversibility and structural change depends importantly on the value of ϵ. We therefore present two sets of numerical results: those for $\epsilon = 0.5$, and those for $\epsilon = 2$.

Figure 2.3 shows the numerical modelling results when $\epsilon = 0.5$, so services and manufacturing are 'more complements than substitutes'[11]. In this case, aggregate (final) output and aggregate consumption are increasing in θ^F, as in the one-sector model. The price of services is also increasing in θ^F, as we showed would be the case in Corollary 2.16. When $\epsilon = 0.5$, the share of labour employed in the services sector is increasing in θ^F, as are the share of output and consumption accounted for by the services sector. Thus θ^F has a significant effect not only on aggregate output and consumption, but also on the balance between services and manufacturing in the economy.

Figure 2.4 shows the numerical modelling results when $\epsilon = 2$, so services and manufacturing are 'more substitutes than complements'. Again, aggregate (final) output, aggregate consumption and the price of services are all increasing in θ^F. However, the labour, consumption and output shares accounted for by services are all now *decreasing* in θ^F. Thus consumer preferences play a crucial determining role in the relationship between investment reversibility and structural change.

Interestingly, in both cases, the effect on structural change is rather small. From the largest to the smallest modelled value of θ^F, the labour share in services only changes by a few percentage points. There appears to be a much stronger effect on aggregate output and consumption than there is on the balance of the economy between services and manufacturing. While decreasing investment reversibility due to the increasing intangibility of capital may have had an effect on structural change, therefore, these results suggest that they cannot

[11] Note that as $\epsilon \to 0$, the utility function approaches the Leontief (perfect complements) function; as $\epsilon \to 1$ the utility function approaches the Cobb-Douglas function; and as $\epsilon \to \infty$ the utility function approaches the linear, perfect substitutes function. Thus in some sense, when $\epsilon < 1$ services and manufacturing are 'more complements than substitutes', and when $\epsilon > 1$, they are 'more substitutes than complements'.

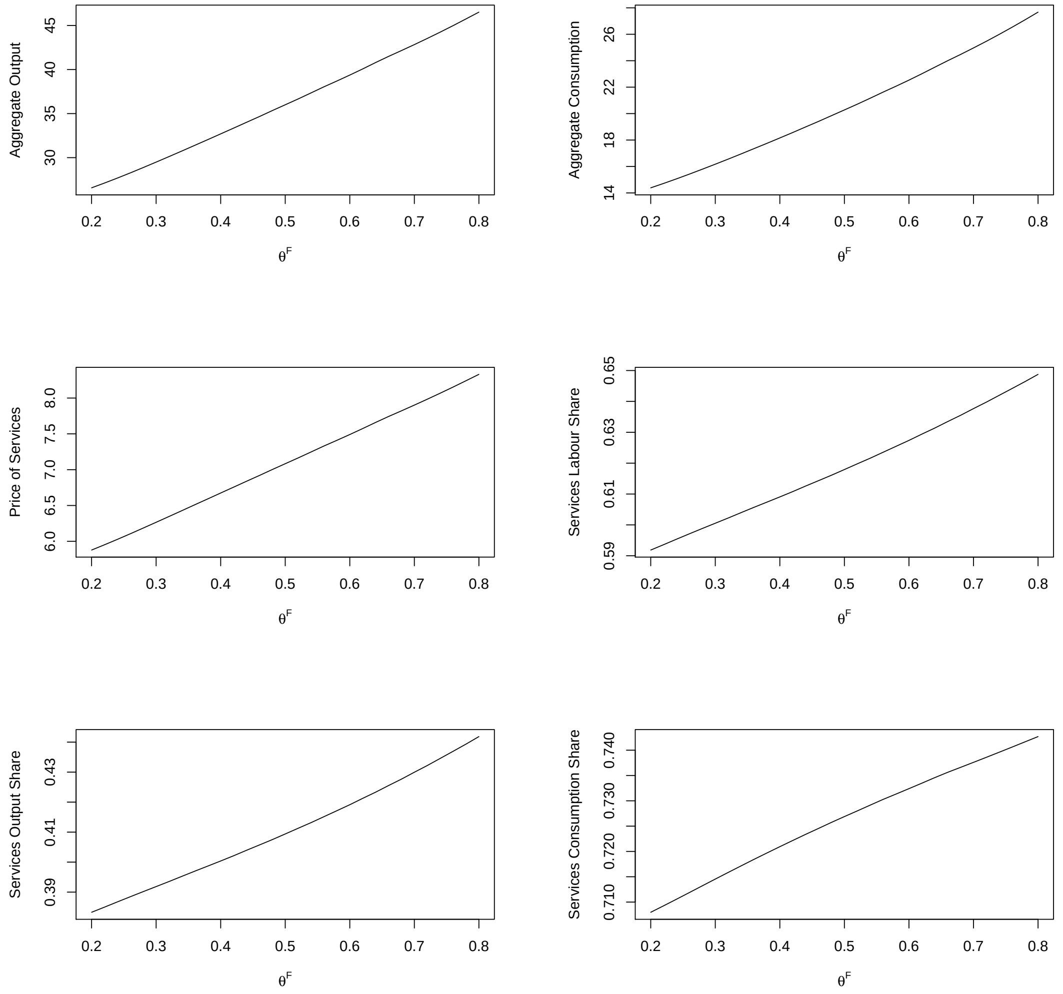

Figure 2.3: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. Aggregate output and aggregate consumption are increasing with θ^F, as is the price of services. When $\epsilon = 0.5$, the share of labour employed in the services sector is increasing with θ^F, as are the share of output and consumption accounted for by the services sector.

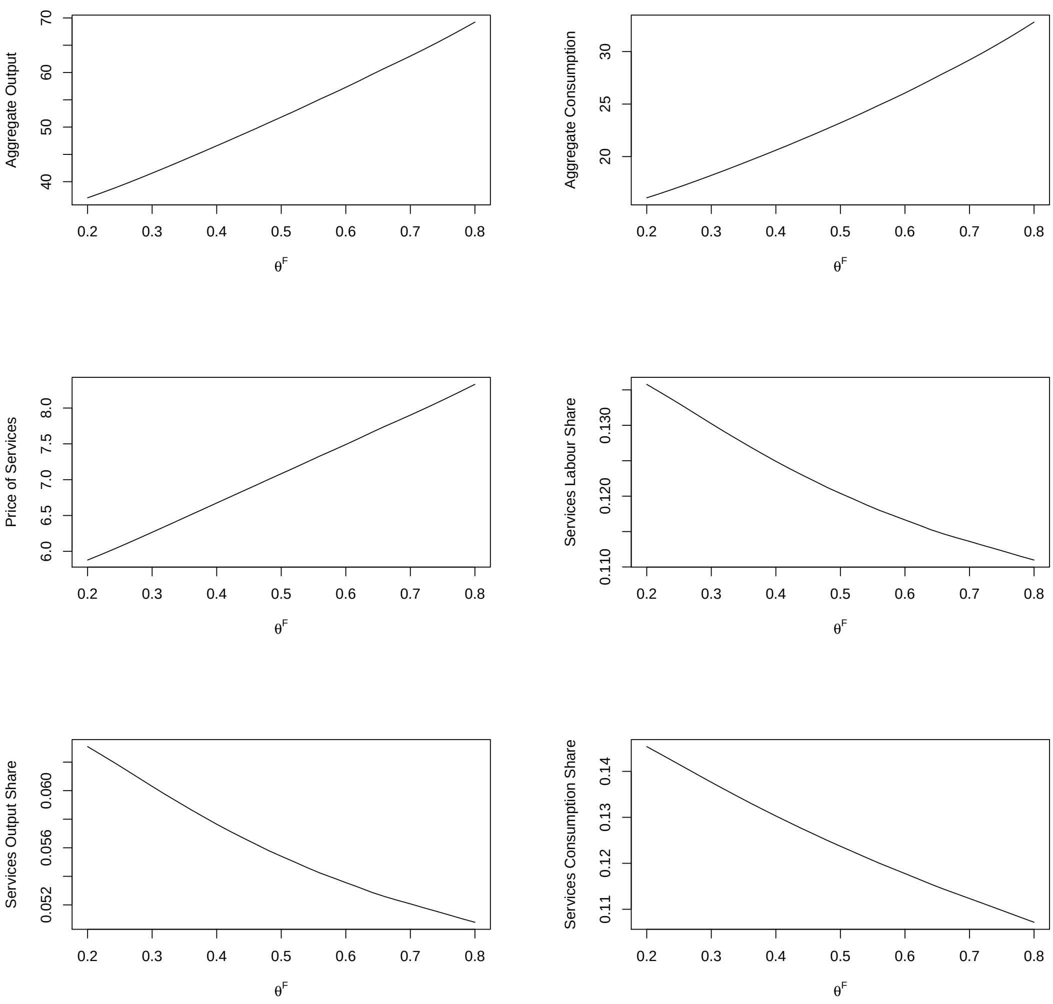

Figure 2.4: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. Aggregate output and aggregate consumption are increasing with θ^F, as is the price of services. When $\epsilon = 2$, the share of labour employed in the services sector is decreasing with θ^F, as are the share of output and consumption accounted for by the services sector.

explain the much larger structural change that has been observed in advanced economies over the last few decades.

The numerical modelling sensitivity analysis undertaken in Appendix B.3 shows that these broad effects hold for a wide variety of parameter choices.

2.3.5 Welfare and policy implications

Suppose now, as in Section 2.2.8, that there is a government, which is able to subsidise or tax firms' capital liquidation and tax or subsidise households to balance its budget. Again, we assume that

$$\theta^F = \bar{\theta}^F + \sigma,$$

where $\bar{\theta}^F$ is the underlying reversibility of investment and σ is the liquidation subsidy paid by the government. As before, we assume that $\theta^F \in (0,1)$. Individual intermediate firms simply face a liquidation value of capital equal to θ^F, but since the tax or subsidy can only be paid in the manufactured good, there is an implication for the structural composition of this economy that makes the analysis more complex than in the one-sector case. The government's budget constraint in period t is again

$$\sigma_t \delta K_t^S \leq L\mathbb{T}_t,$$

Therefore steady state consumption is now equal to

$$\begin{aligned}
c &= w + \rho\phi - \mathbb{T} \\
&= w + \rho\phi - \sigma\delta K^S \\
&= w + \left[\rho\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta e\right]\frac{N}{L} - \sigma\delta N\bar{k}^S \\
&= w + \left[\rho\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta\left(e + \sigma L\bar{k}^S\right)\right]\frac{N}{L} \\
&= w + \left[\rho\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta\left(e + \sigma L\bar{k}^S\right)\right]\frac{\Gamma_M L + \left(\Gamma_S - \Gamma_M\right)L_S}{L\bar{y}},
\end{aligned}$$

where, as before, K^S denotes aggregate capital employed in the small loan regime, and $\bar{k}^S \equiv K^S/N$. Also, as in equation (2.20),

$$c = \left[\left(\frac{\omega_M}{\omega_S}\right)^{\epsilon} p_S^{\epsilon-1} + 1\right] p_S \Gamma_S^{\gamma_S} L_S.$$

Combining these two equations gives the quantity of labour employed in the services sector,

$$L_S = \frac{w + \left[r\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta\left(e + \sigma L\bar{k}^S\right)\right]\Gamma_M/\bar{y}}{\left[\left(\frac{\omega_M}{\omega_S}\right)^{\epsilon} p_S^{\epsilon-1} + 1\right] p_S\Gamma_S^{\gamma_S} - \left[r\left(\bar{b} - \bar{a}\right) + \bar{\psi}^F - \delta\left(e + \sigma L\bar{k}^S\right)\right]\left(\Gamma_S - \Gamma_M\right)/L\bar{y}}. \tag{2.22}$$

In this equation, everything except σ is either constant or depends only on θ^F. Thus we have the following result.

Lemma 2.18 *Taking θ^F as fixed, the steady-state quantity of labour employed in the services sector is a strictly increasing function of $\bar{\theta}^F$.*

Proof. The result follows immediately from equation (2.22), since the numerator is decreasing and the denominator increasing in σ, and when θ^F is fixed $d\bar{\theta}^F/d\sigma = -1$. ∎

Lemma 2.18 says that an economy with a certain underlying level of investment reversibility and no taxes or subsidies on capital liquidation will employ more labour in the services sector than an economy with a lower underlying level of investment reversibility that 'catches up' with subsidies on capital liquidation. Thus in some sense, economies with a low level of investment reversibility might be thought to be subsidising the manufacturing sector when they are subsidising capital liquidation, since manufacturing is the more capital-intensive sector.

We can also say something about the response of consumption shares of manufacturing and services to subsidies in this economy.

Lemma 2.19 *Taking $\bar{\theta}^F$ as fixed, the steady-state ratio of consumption expenditure on services to consumption expenditure on manufacturing is in one-to-one correspondence with σ. Moreover, it is increasing in σ if $\epsilon < 1$, while it is decreasing in σ if $\epsilon > 1$.*

Proof. When $\bar{\theta}^F$ is fixed, $d\theta^F/d\sigma = 1$. Given that the firm's problem depends only on θ^F and not $\bar{\theta}^F$ or σ, the result follows immediately from Corollary 2.17. ∎

Thus we know how consumption shares will respond to increased capital liquidation subsidies, at least directionally, but it is not straightforward to say much more analytically about the effect of government intervention in the two-sector case. Nonetheless, we can undertake numerical modelling of comparative statics to assess the welfare effects of subsidies and taxes subsidies and taxes when $\bar{\theta}^F = 0.3$ – so underlying reversibility is relatively low – and when $\bar{\theta}^F = 0.7$ – so underlying reversibility is relatively high – given the parameters laid out in Tables 2.1 and 2.2. The results are presented in Figure 2.5 for the case when $\epsilon = 0.5$ and in Figure 2.6 for the case when $\epsilon = 2$. As in the one-sector case, this modelling suggests that when the underlying reversibility of investment is low, the government could increase welfare by subsidising capital liquidation and paying for it with a tax on households. It also suggests that the government is less likely to be able to increase welfare by taxing capital liquidation and transferring the proceeds to households when the underlying reversibility of investment is high. The cases for $\epsilon = 0.5$ and $\epsilon = 2$ are very similar in their policy prescriptions. The sensitivity analysis conducted in Appendix B.4 shows that the same broad conclusions hold for a wide variety of parameter values: at very low levels of underlying investment reversibility, welfare can often be increased by subsidising capital liquidation, but only rarely can welfare be increased by taxing capital liquidation when the underlying reversibility of investment is very high.

We can also examine the effect of liquidation subsidies and taxes on the steady state structural composition of the economy. Figure 2.7 shows the effect of government intervention on the balance between services and manufacturing when $\epsilon = 0.5$, and Figure 2.8 shows the

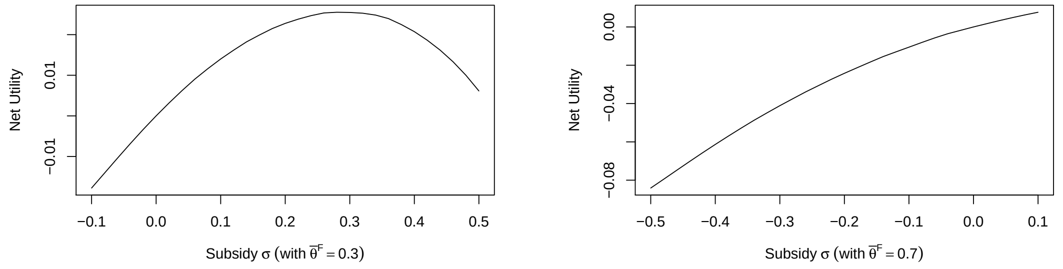

Figure 2.5: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. 'Net Utility' is calculated as the difference in the instantaneous utility function $\nu(c_M, c_S)$ relative to the case of zero subsidy or tax.

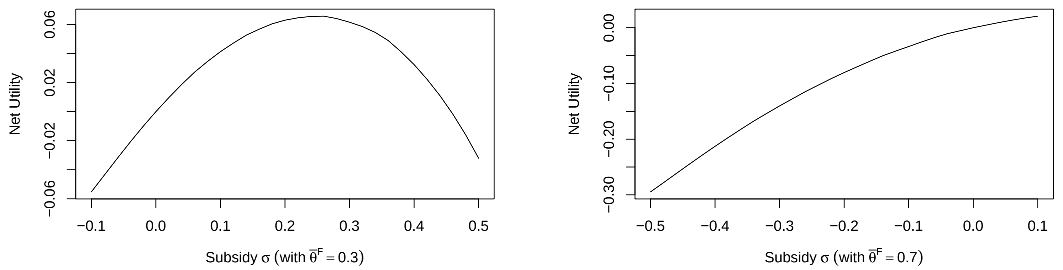

Figure 2.6: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. 'Net Utility' is calculated as the difference in the instantaneous utility function $\nu(c_M, c_S)$ relative to the case of zero subsidy or tax.

effect when $\epsilon = 2$. As with the effect of θ^F when there is are no subsidies or taxes, the response to σ of the services share of consumption depends crucially on ϵ: for $\epsilon = 0.5$ the services share of consumption is increasing with σ, while for $\epsilon = 2$ it is decreasing with σ. However, in both cases the services share of labour and of output are decreasing with σ. Thus it could be that in an economy with low levels of underlying investment reversibility, a welfare-enhancing capital liquidation subsidy may look *de facto* like a fillip for the manufacturing industry, with an increase in both manufacturing employment and the manufacturing share of output. Sensitivity analysis conducted in Appendix B.4 shows that this relationship between structural composition and capital liquidation subsidies or taxes holds for a wide array of parameter choices.

2.4 Conclusions

We developed a general equilibrium model of an economy in which individual firms face a degree of micro-level investment irreversibility. We find, using a combination of analytical and numerical methods, that a reduction in investment reversibility – perhaps due to a greater reliance on intangible capital – is in the long run associated with: reduced output, wages, consumption and aggregate capital; fewer firms in operation; and reduced corporate borrowing and increased corporate saving. In a two-sector model, we find that reduced investment reversibility reduces the price of services relative to manufactured goods, but has an ambiguous effect on the share of employment, output and consumption accounted for by each sector. The direction of these effects depends on consumer preferences.

The model predicts that capital liquidation subsidies can be welfare-enhancing. While it's more intuitive to expect this effect when the underlying reversibility of investment is low, we find that subsidies are often welfare-enhancing even when the underlying reversibility of investment is relatively high. While the effect on consumption shares of such subsidies depends on consumer preferences, the effects on output and employment shares do not: subsidies increase the manufacturing share of both output and employment. Thus such subsidies may be interpreted as a *de facto* boon to the manufacturing sector, even when they are welfare-enhancing.

Sensitivity analysis suggests that these effects hold for a wide variety of parameter choices.

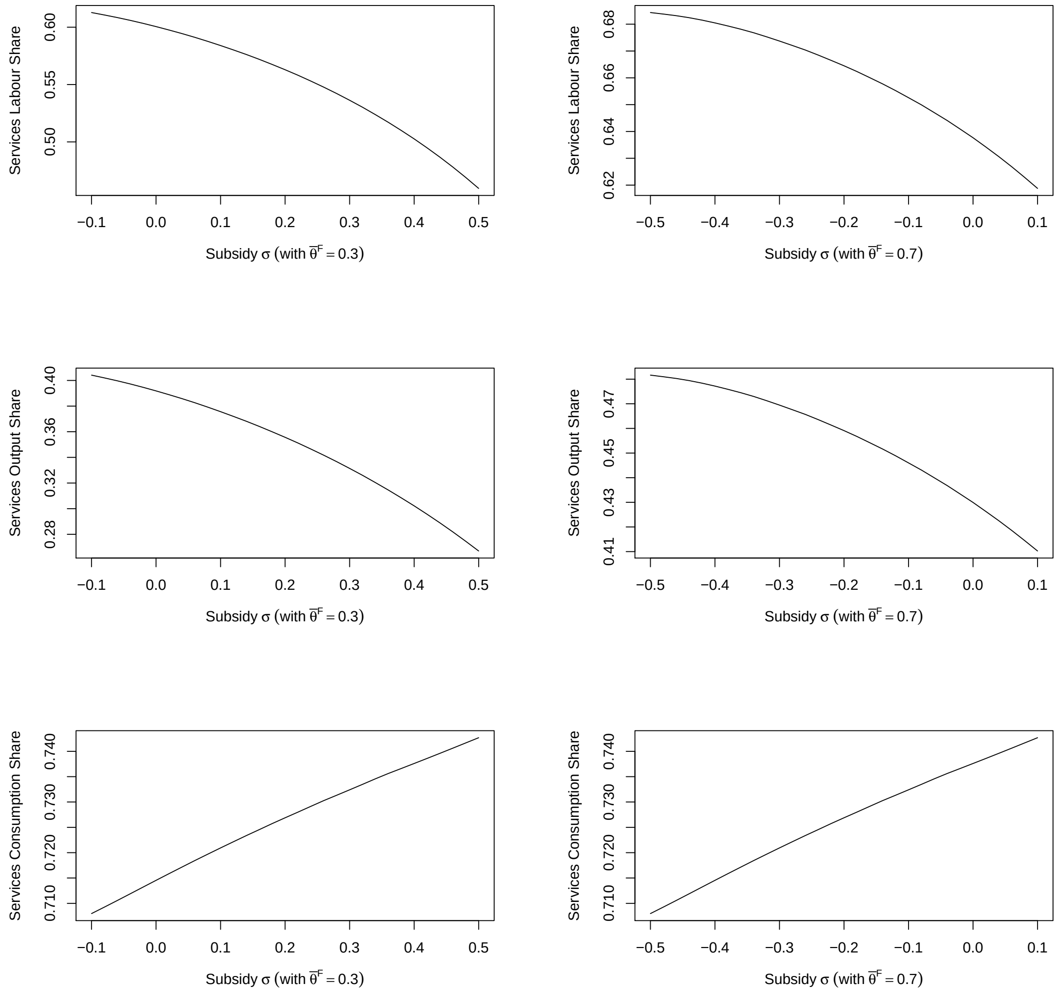

Figure 2.7: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$.

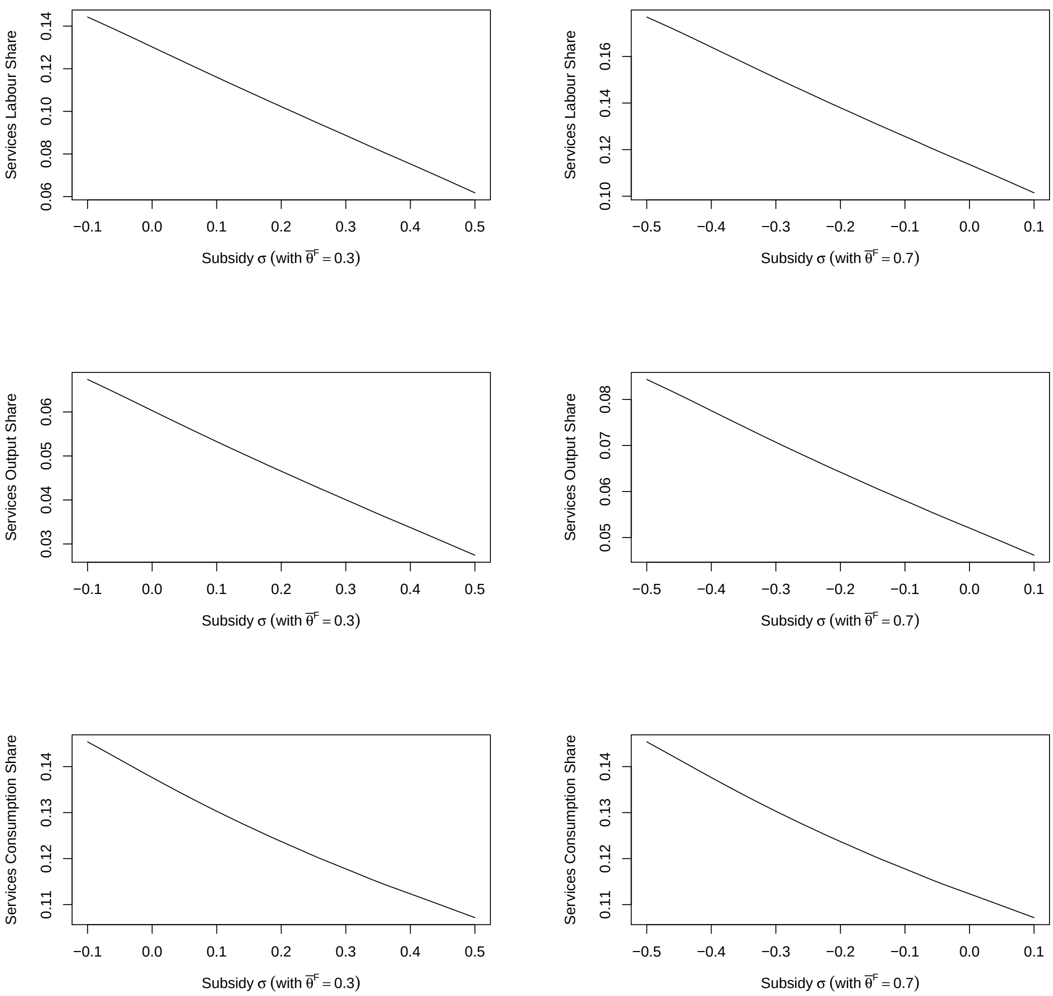

Figure 2.8: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$.

Chapter 3

Finance and Structural Change: Theory and Evidence From a Pooled Synthetic Controls Study

Abstract

We examine the effect of bank branching deregulation in the United States, which occurred state-by-state from the 1970s to the 1990s, on structural change in individual states. We find that bank branching deregulation accelerates the structural change that was already underway: services account for a greater share of the economy than they would have in the absence of deregulation. These results are consistent with the model we develop of structural change in response to a positive financial shock, which also explains existing empirical results. Our estimation strategy is a pooled ridge augmented synthetic controls study. Synthetic counterfactuals are constructed for individual deregulation events, then pooled to increase their statistical power.

3.1 Introduction

Does finance affect structural change? There is a large literature linking finance and growth, which is known to go hand-in-hand with the evolving structure of the economy – as developed economies get richer, manufacturing shrinks as a share of the economy, while services grow (Herrendorf et al., 2014) – but comparatively little attention has been paid to the direct relationship between finance and structural change.

In this chapter, we examine the effect of bank branching deregulation in the United States on the structure of individual states' economies. Prior to the 1970s, most states had unit banking restrictions that were relaxed state-by-state in a quasi-random order until the mid-1990s. Most banks in most states were limited to a single branch, until bank branching deregulation allowed for expansion, typically by merger and acquisition (M&A) in the first instance, and later by *de novo* branching. Using a pooled ridge augmented synthetic controls

*This chapter is the result of joint work with Corrado Di Maria.

study, we find that bank branching deregulation accelerates the structural change that was already underway: states that deregulate end up with a significantly higher share of services and lower share of manufacturing in the economy than they would have done in the absence of deregulation.

Our contribution is three-fold. First, we develop an infinite-horizon general equilibrium model of finance and structural change that is consistent with existing empirical evidence and predicts accelerated structural change in response to financial development. Second, we add to the nascent field of pooled synthetic controls studies and ridge augmented synthetic controls studies for policy evaluation, and in particular we propose a simple statistical test that is suggestive of the overall pre-treatment goodness of fit in such a pooled study. Third, we show empirically that bank branching deregulation did in fact cause faster structural change towards services and away from manufacturing than would have occurred without deregulation, drawing a direct line from finance to structural change, consistent with our theory.

There is a long history of thought linking finance to the real economy, stretching at least as far back as Schumpeter (1911), who argued that finance had a vital role to play in promoting economic growth by allocating society's savings, screening projects and monitoring managers. Since then a large theoretical and empirical literature has arisen on the finance-growth nexus[1]. Over the past three decades, in particular, the evidence that finance causally affects growth has become much stronger, and our results build on this edifice.

In an early contribution, King and Levine (1993) show using cross-country panel data that financial development is strongly associated with – and predictive of – growth in real per capita output. Levine et al. (2000) improve upon this work by using instrumental variables and generalised method-of-moments (GMM) dynamic panel estimators to demonstrate that output growth appears to be caused by financial development. In the long run, Beck et al. (2000) find that this growth effect is primarily a result of total factor productivity (TFP) growth, rather than capital accumulation. However, later work suggests that this may vary by the stage of economic development, with finance primarily promoting capital accumulation in less developed economies and TFP growth in more developed economies (Rioja and Valev, 2004b). Indeed, Rioja and Valev (2004a) find that the effect of finance on growth may also vary based on the level of financial development in an economy.

Since we are examining the relationship between finance and structural change, we are interested in the effect of financial development on different sectors of the economy and on different kinds of firms. Carreira and Silva (2010) present some stylised facts pertaining to firms' financial constraints: among them, that smaller and younger firms are more financially constrained; and that financial liberalisation appears to ease these constraints. In a famous paper, Rajan and Zingales (1998) show that industries with greater reliance on external finance grow faster in more financially developed economies. However, for reasons of data availability, they restrict their analysis to industries within the manufacturing sector. Chakraborty and Mallick (2012) find using U.S. survey data that manufacturing firms have a

[1] For a more comprehensive survey of the theoretical and empirical literature to that date, see Levine (2005).

larger proportional gap between actual debt and desired debt than services firms, suggesting manufacturing firms are more credit constrained. This is perhaps intuitive, since manufacturing firms will typically require far more physical capital and therefore have higher fixed startup costs than services firms.

Causal identification of the effect of financial development is difficult. One approach is to examine the effects of a plausibly exogenous shock to the financial sector. To this end, several papers have studied the effects of bank branching deregulation, as we do in this study. Krozsner and Strahan (1999) find that the timing of bank branching deregulation can be explained by a combination of new technologies in banking, such as ATMs and telephone banking, and the private interests of banks in any given state, which suggests that deregulation is othorgonal to many outcome variables of interest. Jayaratne and Strahan (1996) find that deregulation leads to significantly higher output growth, although employing a different methodology, Huang (2008) casts doubt on these results. Using the same episode, Beck et al. (2010) find that bank branching deregulation reduces income inequality. Jerzmanowski (2017) shows that bank branching deregulation affects economic growth through a mixture of physical capital accumulation and TFP growth, while finding no effect on human capital. In particular, the effect of deregulation on manufacturing works by increasing TFP growth rather than accelerating capital accumulation.

By contrast, there has been very little work on the effect of bank branching deregulation – or indeed financial development more broadly – on structural change. Herrendorf et al. (2014) present some stylised facts of structural change. Expressed as a share of the economy, whether in terms of employment or value-added, for both currently rich and currently poor countries, the following stylised facts appear to hold: the richer an economy in terms of GDP per capita, the smaller the share of agriculture; the richer an economy, the greater the share of services; and the richer an economy, the greater the share of manufacturing, up to a certain point – thereafter, manufacturing decreases as a country becomes richer, forming an inverted 'U' shape.

There are two broad theories of the drivers of structural change. The first suggests that structural change is a largely demand-driven phenomenon. This is exemplified by Kongsamut et al. (2001), who embed the machinery required to generate structural change in a Stone-Geary utility function. Intuitively, there is a subsistence level of agricultural goods required in the economy, but as the economy develops, consumers do not derive as much utility from endlessly more food, and preferences shift into consuming more manufactured goods and then services.

The second theory, as described by Ngai and Pissarides (2007), is a supply-driven shift. In this model, there are exogenous and varying rates of TFP growth across different sectors. If consumers consider goods produced in different sectors to be complements, then labour must flow to the slow-growing sectors in order to maximise utility. Supposing, for example, that TFP growth in services has been slow, while in manufacturing it has been fast – and the estimates in Jorgenson and Stiroh (2000) suggest that productivity growth in services between 1958 and 1996 may actually have been *negative*, while TFP growth in the industrial machinery and equipment sector, for example, was rather high – then we would expect to see a smaller share of labour in manufacturing over time, and a larger share in services. This is consistent with the broad sweep of the data. We develop a model based on Ngai and Pissarides (2007) in Section 3.3, adding a role for financial intermediation, in order to study the effect of finance on structural change.

Among the few papers to theorise an explicit link between finance and structural change, Buera et al. (2011) model a two-sector economy which is subject to financial frictions. Manufacturing firms are large in scale and have relatively large financing needs, while services firms are small in scale and have relatively small financing needs. Thus financial frictions disproportionately disadvantage manufacturing firms, leading to relatively low TFP growth in manufacturing and relatively high prices for manufactured goods relative to services, both of which are empirical regularities.

Acharya et al. (2011) examine a very similar question to the question posed in this chapter, also studying the effect of bank branching deregulation in the United States, but approach it by way of portfolio analysis. They use industry-level average growth rates and covariances to construct a benchmark portfolio that an investor restricted to any given U.S. state would wish to hold, then show that post-deregulation, convergence of the aggregate economy towards that benchmark portfolio accelerates. Implicit in this approach, however, is the idea that converging towards the tangency portfolio cannot change the tangency portfolio; everything is reallocation, and nothing is technology. One contribution of our study is that our causal inference is valid even without making this assumption. Manganelli and Popov (2015) conduct a similar portfolio reallocation-type investigation on an international scale.

Examining an economy at a much earlier stage of development, Heblich and Trew (2019) show that greater access to banks caused a faster rate of industrialisation in England and Wales over the period 1817-1881. This is a useful point of comparison to our results: while we find that finance particularly stimulates services, they find that it promotes industrial sectors. This is further evidence that the role finance has to play depends crucially on the state of economic development, as found by Rioja and Valev (2004a). While more work is needed to establish this definitively, it appears that finance may 'grease the wheels' of structural change, and accelerate whatever shift predominates in any given economy at any given time.

Finally, this chapter relates to the literature on the synthetic control method (SCM), which is at the heart of our main estimation strategy. The SCM was introduced by Abadie and Gardeazabal (2003) to study the effect of terrorism on the Basque Country. They construct a synthetic Basque Country as the convex combination of other Spanish regions that minimises a measure of distance between actual and synthetic outcomes prior to the onset of terrorism, and see how the actual and synthetic Basque Countries diverge after the onset of terrorism. If the synthetic Basque Country is a good fit for the real Basque Country in all respects except the onset of terrorism, then any difference between the two after 'treatment' – in this case, the onset of terrorism – can be interpreted as the causal effect of that treatment. The same method has been used to assess the efficacy of a tobacco control policies in California

(Abadie et al., 2010) and the economic effect of German reintegration in 1990 (Abadie et al., 2015). Statistical inference is performed using placebo tests.

These instances of the SCM each consider a single case study. Dube and Zipperer (2015) extend this idea by using a mean percentile rank test to pool multiple synthetic control studies. This is the approach we take later in the chapter, with particular attention paid to the validity of such pooling exercises.

The synthetic control method is preferable to some more traditional policy evaluation methods in several respects. First, the SCM is specification-free. Second, the treatment need not be exogenous: we find in Section 3.5.1 that the timing of bank branching deregulation is likely to be related to the *ex ante* structural composition of the economy, which undermines the causal interpretation of the difference-in-differences regressions. However, so long as there is a 'good fit' when creating a synthetic counterpart for each treated unit, this is not a problem for the SCM. While not something that can be tested directly, we propose a simple statistical test for the overall 'goodness of fit' relative to placebo deregulations that provides some indication of the validity of pooling several studies. In order to alleviate concerns with the goodness of fit, we improve pre-treatment fit by employing the ridge augmented synthetic control method (ridge ASCM) (Ben-Michael et al., 2019).

The rest of this chapter is organised as follows. Section 3.2 lays out the stylised facts of structural change, describes bank branching deregulation in the United States, and presents suggestive evidence linking the two. Section 3.3 develops a model of structural change and its interaction with finance that is consistent with the suggestive evidence previously found. Section 3.4 outlines the data used for our empirical investigation, and Section 3.5 estimates the effects of bank branching deregulation on structural change, using first difference-in-differences regressions and then pooled synthetic controls studies, both standard and ridge augmented. Section 3.6 concludes.

3.2 Framework

3.2.1 Structural change in the United States

The United States has been secularly shifting away from manufacturing and towards services for several decades. Figure 3.1 shows the gradual increase in services employment and output, and the gradual decrease in manufacturing employment and output, with the output trajectories more volatile than the employment trajectories[2]. The grey lines represent individual U.S. states and the District of Columbia (D.C.), while the red lines represent the (unweighted) average across all states.

The United States is not alone in this: it is a stylised fact that in sufficiently rich economies, increases in GDP per capita coincide with decreases in the manufacturing share of employment and output, and with increases in the services share of employment and output (Herrendorf et al., 2014, p. 861). Figure 3.1 makes clear that the United States had, by 1970, become sufficiently rich that the manufacturing share of both employment and output were

[2]For a description of the data used to construct these graphs, see Section 3.4.

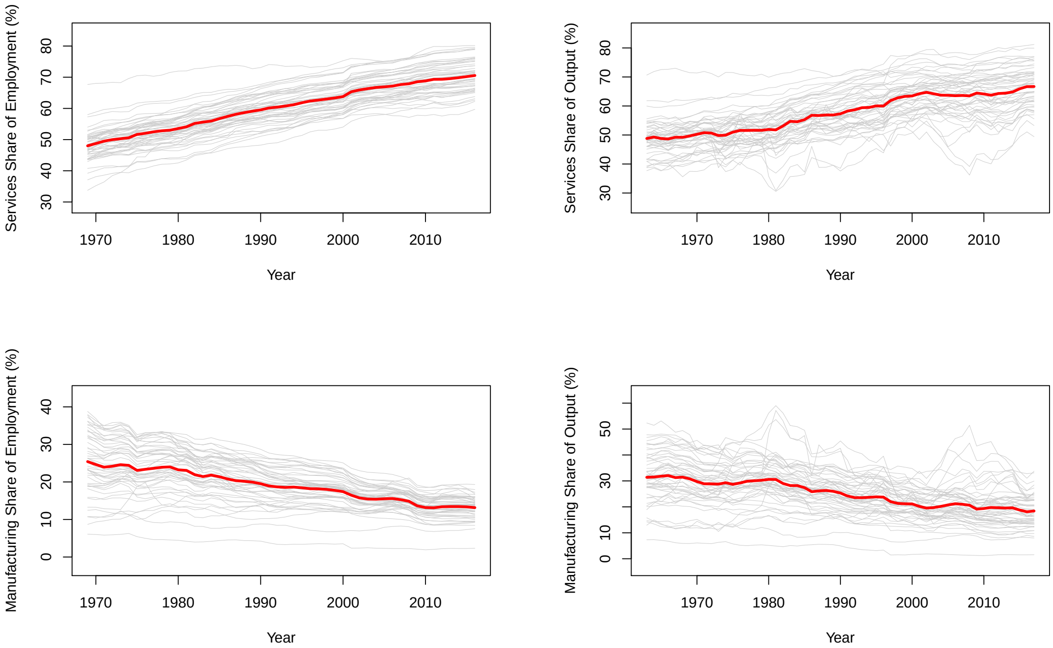

Figure 3.1: Secular shifts in structural composition

Long-term trends in the sectoral composition of U.S. states and D.C. The grey lines represent individual U.S. states and D.C., while the red lines represent the (unweighted) average across all states.

in decline. This has important implications for the external validity of our results; in this respect, the United States was already a rich country during the period under study, and so any effects of finance on structural composition that we are able to identify may not be so instructive in understanding financial development in less rich economies.

There are two major theories underlying this process: demand-driven change (Kongsamut et al., 2001) and supply-driven change (Ngai and Pissarides, 2007). There is some evidence for the supply-driven story: in the model presented by Ngai and Pissarides (2007), and the model presented below, changes in sectoral composition are a result of different growth rates in sector-level productivity. In the presence of relatively inelastic consumer preferences across different types of goods or services, sectors with slower productivity growth must compensate by employing a larger share of labour. Moreover, sectors with faster productivity growth can produce goods more cheaply, so relative prices change. This is consistent with the stylised facts in the United States: Jorgenson and Stiroh (2000) find that TFP growth in services firms in the latter part of the 20th century may actually have been negative, which might explain ever-increasing shares of employment in services.

There is a wealth of evidence suggesting that greater financial development is linked to higher GDP per capita, but very little work linking financial conditions to the structural composition of the economy. By studying a positive shock to financial development, we can assess the effects on structural composition.

3.2.2 Bank branching deregulation

At the beginning of the 1970s, most states in the U.S. had unit banking restrictions. Generally a bank consisted of a single branch, and most banks were prohibited from expansion within states by any means. Apart from a few historical exceptions, banks were further prohibited from crossing state lines by the Douglas Amendment to the Bank Holding Company Act of 1956. Banks therefore were typically small enterprises.

From the 1970s until the mid-1990s, individual states relaxed restrictions on bank branching. Most states allowed bank branching by merger and acquisition (M&A) in the first instance, then shortly afterwards allowed expansion by *de novo* branching. This process culminated in the passage of the federal Riegle-Neal Interstate Banking and Branching Efficiency Act in 1994, which effectively lifted any remaining bank branching restrictions across the United States. Following the literature, we take as the date of deregulation the year in which bank branching was first permitted by M&A[3].

Krozsner and Strahan (1999, pp. 1460-1461) identify three reasons for bank branching deregulation beginning in the 1970s in particular. First, ATMs became more prevalent and allowed some banking services to be accessed remotely; second, banking by mail and telephone increased, with checkable money market funds and the Merrill Lynch Cash Management Account; and third, general technological progress reduced the costs of transport and communication. All of these advances served to weaken the geographical link between banks and their customers. The authors also find that the timing and order of states' deregulation can be explained by a private interest model: they show "that deregulation occurs earlier in states with fewer small banks, in states where small banks are financially weaker, and in states with more small, presumably bank-dependent, firms" (Krozsner and Strahan, 1999, p. 1438). Jayaratne and Strahan (1998) find that post-deregulation, banks' operating costs and loan losses decrease, and that these cost reductions are mostly passed on to borrowers in the form of reduced interest rates. Therefore bank branching deregulation can reasonably be taken as a positive shock to financial development.

The effects of bank branching deregulation have been studied several times. Jayaratne and Strahan (1996) find that deregulation increased growth in real GDP per capita – although comparing outcomes in contiguous pairs of counties across state lines, where one state deregulated and the other did not, Huang (2008) finds less support for this result. Beck et al. (2010) find that income inequality, as measured by the Gini coefficient, is reduced after bank branching deregulation, primarily by increasing the incomes of those at the bottom of the distribution.

Jerzmanowski (2017) finds that bank branching deregulation accelerates both capital accumulation and TFP growth, while having no effect on human capital growth; moreover, increases in the rate of growth in the manufacturing sector are driven entirely by increased TFP growth, rather than increased capital accumulation. Using a more granular decomposition of the economy into smaller sectors, Acharya et al. (2011) find that shares of output in

[3]More precisely, we follow previous authors in taking the year of M&A deregulation as the year in which the process of deregulation was *completed*. See for instance Jayaratne and Strahan (1996, p. 646).

the economy converge more quickly to the optimal tangency portfolio post-deregulation than pre-deregulation.

The latter two papers suggest, implicitly or explicitly, that bank branching deregulation is related to the structural composition of the economy. We consider the broad categories of manufacturing and services, and examine this link more closely.

3.2.3 Suggestive evidence of a link

Figure 3.2 shows each state's year of bank branching deregulation against its share of services and manufacturing in employment and output in both 2016 and 1969, for states that had not undergone bank branching deregulation by 1969. On average, states that deregulated earlier have a higher services share and a lower manufacturing share of both employment and output today. In naïve regressions, all four of these relationships are significant at the 5% level. Visual inspection suggests that initial sectoral shares – that is, shares of services and manufacturing in employment and output in 1969, prior to deregulation – are not related to the year of deregulation, except for the manufacturing share of employment, where the line of best fit appears to be downward-sloping. Nonetheless, in naïve regressions, none of these relationships – not even the relationship between the year of deregulation and the manufacturing share of employment in 1969 – are significant at the 5% level.

In combination with Krozsner and Strahan (1999), who plausibly show that the timing and order of bank branching deregulation are explained by national technological trends and the private interests of banks in any given state, this evidence suggests at first glance that bank branching deregulation is essentially orthogonal to *ex ante* structural composition[4]. Moreover, it appears that the timing of bank branching deregulation *is* related to *ex post* structural composition. While this falls a long way short of being evidence of any particular causality, it does suggest that there is a correlation between bank branching deregulation and structural composition. We proceed to model this relationship formally.

3.3 A model of finance and structural change

In this section, we develop a model of structural change in response to a financial shock that is consistent with the suggestive evidence presented above, and with wider empirical evidence. In this model, structural change away from manufacturing and towards services, in terms of employment and output, accelerates in response to a positive financial shock. The model is based on Ngai and Pissarides (2007), with elements of Mehlum et al. (2016) and a learning-by-doing externality associated with capital accumulation in the spirit of Romer (1986). Parts of the solution follow the example of the Ramsey model in Barro and Sala-i-Martin (2004).

[4] We show in Section 3.5.1 that this is in fact may be untrue: the timing of bank branching deregulation does appear to be related to *ex ante* structural composition.

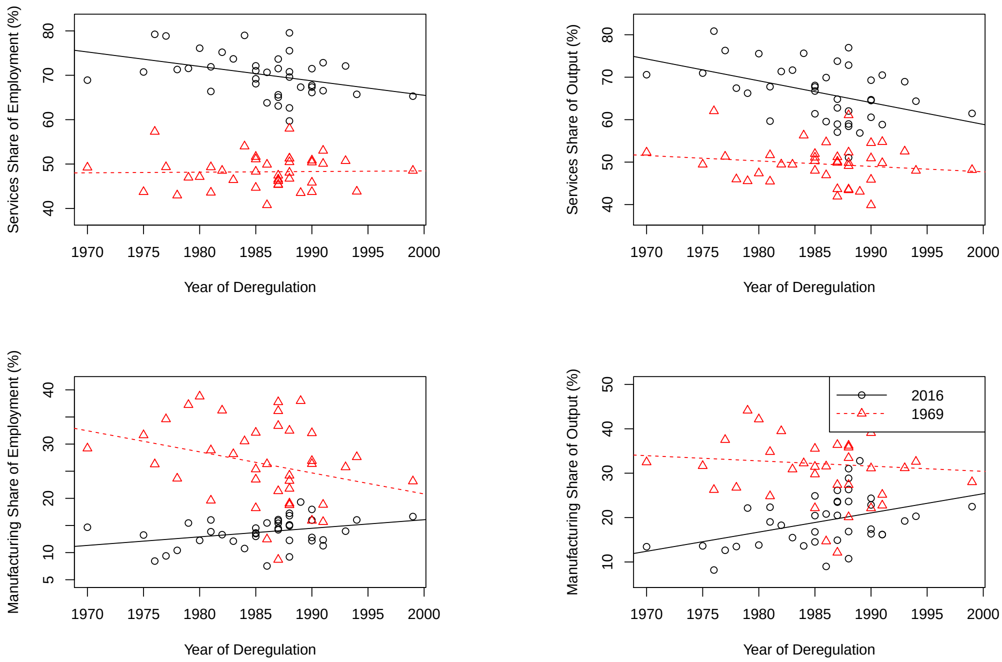

Figure 3.2: Year of deregulation and structural composition

Scatter plots of the 2016 and 1969 services and manufacturing shares of employment and output against the year of bank branching deregulation for each U.S. state and D.C. that had not deregulated by 1969, with lines of best fit. On average, states that deregulated earlier have a higher services share and a lower manufacturing share of both employment and output; in naïve regressions, the year of deregulation is a significant predictor at the 5% level of all sectoral shares in 2016 (that is, all the solid black lines of best fit have a slope significantly different from zero at the 5% level). There does not seem to be a strong relationship between the year of deregulation and sectoral shares in 1969, apart from the manufacturing share of employment; nonetheless, in naïve regressions, no sectoral share in 1969 is a significant predictor at the 5% level of the year of deregulation (that is, none of the dotted red lines of best fit have a slope significantly different from zero at the 5% level), even the manufacturing share of employment.

3.3.1 Households

Time is continuous and indexed by t. Total population in the model economy is $N(t)$ and grows at the exogenous rate g,

$$\dot{N} = gN.$$

Each household consists of an infinitely lived dynasty of unit mass at time zero, growing at the same rate as the economy-wide population growth rate. Each household is endowed with labour equal to its mass at each instant, which is supplied inelastically to the productive sectors. The optimiser at time zero maximises with perfect foresight the present value of the utility streams enjoyed by all dynasty members,

$$U = \int_0^\infty e^{-\rho t} \nu(c_M, c_S)\, dt,$$

where $c_M(t)$ is per-capita consumption of manufactured goods and $c_S(t)$ is per-capita consumption of services, where instantaneous utility ν is given by

$$\nu = \ln(\Phi(c_M, c_S))$$
$$\Phi(c_M, c_S) = \left(\omega_M c_M^{(\epsilon-1)/\epsilon} + \omega_S c_S^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)},$$

and where $\epsilon, \omega_M, \omega_S > 0$ and $\omega_M + \omega_S = 1$. Labour is divided between manufacturing and services, where $n_M(t), n_S(t)$ denote the shares of labour employed in the manufacturing and services sectors respectively, and where $n_M + n_S = 1$. We further assume that $\epsilon < 1$, so that consumer preferences are relatively inelastic.

We take the manufactured good as the numeraire and fix its price to unity, $p_M = 1$. We will allow the price of services output $p_S(t)$ to be determined in the decentralised market. Agents work, with each agent supplying inelastically one unit of labour at each instant and receiving the wage rate $w(t)$. Households also earn asset income $r_q(t)\, q(t)$, where q is the per-capita stock of financial assets and r_q the associated rate of return. Individual savings $\beta(t)$ therefore equal the difference between total current incomes $r_q q + w$ and expenditures on goods and services $c_M + p_S c_S$. The budget constraint reads

$$\beta = r_q q + w - c_M - p_S c_S. \tag{3.1}$$

In order to transform equation (3.1) into a dynamic wealth constraint we need to specify the process by which savings are transformed into physical capital.

3.3.2 Financial intermediation and capital accumulation

Capital is the productive good in the economy, and financial assets owned by all households in the economy represent ownership of physical capital. The value of the aggregate capital stock $K(t)$ thus matches the value of total financial assets,

$$K = Nq. \tag{3.2}$$

This equation is a clearing condition in the financial market, where banks operate as intermediaries between households and firms. On the one hand, banks act as financial intermediaries that collect all household savings and invest them into (shares of) a 'capital fund' representing ownership of the stock of physical capital to be rented to firms for production purposes. On the other hand, banks give final producers access to physical capital by setting up rental contracts whereby firms can use $K(t)$ units of physical capital by paying the associated rental rate $r_k(t)$ at each instant t.

We assume that there are costs associated with financial intermediation – these could be informational, or to do with the need for special expertise, or co-ordination problems, or many other kinds of costs – that are lower for banks than for households, giving rise to the need for a financial sector. We will model these costs as a wedge between rental rates and the returns on households' savings, which will be made explicit shortly.

The activities of the banking sector are represented by two basic relationships. First, the savings-investment identity

$$N\beta = \dot{K} \tag{3.3}$$

implies that banks transform new savings into additional physical capital available for production purposes – that is, banks add current savings to the aforementioned capital fund. Second, for simplicity, we model a constant marginal cost of intermediation: the total cost to banks in any given period is proportional to the amount of capital rented by final producers, θK, where θ is a constant parameter reflecting exogenous circumstances that make intermediation costly[5]. This can be thought of as a 'wealth depletion effect': in general, assuming that $\theta > 0$ implies that financial intermediation erodes the *ex post* value of the stock of assets held by households. The bank branching deregulation episode we study in the empirical section of this chapter lowered banks' operating costs and firms' borrowing costs (Jayaratne and Strahan, 1998), so can be modelled in this context as a negative shock to the intermediation wedge parameter θ. The instantaneous cash flow of the banking sector as a whole reads $r_k K - r_q Nq - \theta K$. Assuming a competitive banking sector, the zero-profit condition reads $r_k K = r_q Nq + \theta K$. Given the aggregate constraint in equation (3.2), the zero-profit condition implies

$$r_k = r_q + \theta.$$

[5]Note that, as in the previous chapters, θ is used to denote the key parameter in the effect of which we are interested. However, θ^F was used in previous chapters to denote the value at which firms could liquidate capital: thus a higher value of θ^F effectively *reduced* the cost of capital. By contrast, in this chapter, a higher value of θ effectively *increases* the cost of capital for firms by imposing a larger premium over the return earned on savings by households. Financial development is therefore modelled here as a reduction in θ.

Time-differentiating equation (3.2), and using the savings-investment identity (3.3), we obtain

$$\dot{K} = \dot{N}q + N\dot{q} = gNq + N\dot{q} = N\beta.$$

Using this to eliminate savings β from equation (3.1) yields the law of motion of financial assets per capita,

$$\dot{q} = (r_q - g)\, q + w - c_M - p_S c_S. \tag{3.4}$$

We are now able to write the law of motion for aggregate capital,

$$\begin{aligned} & \dot{K} = r_k K + Nw - [Nc_M + p_S N c_S] - \theta K \\ \Rightarrow \quad & \dot{k} = r_k k + w - [c_M + p_S c_S] - (\theta + g)\, k, \end{aligned} \tag{3.5}$$

where $k = K/N$ is capital per capita, and where the term in square brackets is consumption spending on goods and services.

3.3.3 Production sectors

The production functions in the manufacturing and services sectors, F^M and F^S respectively, are Cobb-Douglas and are identical except for a difference in productivity, with a unit continuum of identical firms in each sector, so that

$$\begin{aligned} F^M (n_M N k_M, n_M N) &= \int_0^1 A_M n_{M,j} N k_{M,j}^{\alpha}\, dj \\ &= A_M N k_M^{\alpha} \int_0^1 n_{M,j}\, dj \\ &= A_M n_M N k_M^{\alpha}, \end{aligned}$$

where $n_{M,j} = N_{M,j}/N$ and $k_{M,j} = K_{M,j}/N_{M,j}$ are the labour share and capital-labour ratio in firm j respectively. Similarly,

$$F^S (n_S N k_S, n_S N) = A_S n_S N k_S^{\alpha}.$$

We assume that A_S is constant and set it equal to unity[6], while A_M is the result of a learning-by-doing externality,

$$A_M = A_0 \left(\frac{K}{N}\right)^{\eta} = A_0 k^{\eta},$$

where $\eta > 0$, which is not internalised by manufacturing firms. In order to work with a conventional neoclassical growth model, we assume that $\alpha + \eta < 1$. We include this learning-by-doing spillover in order to generate straightforwardly the post-deregulation acceleration in

[6] This fits with the stylised facts in Jorgenson and Stiroh (2000), who find that TFP growth has been about zero in many services industries in the United States in the later 20th century, while finding that it has been positive and substantial in many manufacturing industries.

manufacturing TFP growth found by Jerzmanowski (2017), but the stylised facts of structural change are consistent with other mechanisms, such as exogenous technology gaps or different input factor intensities across sectors.

3.3.4 Solving the model

Firms wish to maximise their profits,

$$\begin{aligned}\pi_M(t) &= A_M n_M N k_M^\alpha - w n_M N - r_k n_M N k_M \\ \pi_S(t) &= p_S n_S N k_S^\alpha - w n_S N - r_k n_S N k_S,\end{aligned}$$

with respect to labour $N_i = n_i N$ and capital $K_i = n_i N k_i$ in each sector i. Taking the first order conditions, the interest rate and wage are given by the private returns to capital and labour,

$$\begin{aligned}r_k = F_K^M &= A_M n_M N \cdot \alpha k_M^{\alpha-1} \cdot \frac{\partial k_M}{\partial K_M} \\ &= \alpha A_M n_M N k_M^{\alpha-1} \cdot \frac{1}{n_M N} \\ &= \alpha A_M k_M^{\alpha-1} \\ w = F_N^M &= A_M k_M^\alpha + \alpha A_M n_M N k_M^{\alpha-1} \cdot \frac{\partial k_M}{\partial N_M} \\ &= A_M k_M^\alpha + \alpha A_M n_M N k_M^{\alpha-1} \cdot \left(-\frac{k_M}{N_M}\right) \\ &= (1-\alpha) A_M k_M^\alpha,\end{aligned} \tag{3.6}$$

and similarly

$$\begin{aligned}r_k &= p_S F_K^S = p_S \cdot \alpha k_S^{\alpha-1} \\ w &= p_S F_N^S = p_S \cdot (1-\alpha) k_S^\alpha.\end{aligned} \tag{3.7}$$

Equalising the two expressions for r_k in equations (3.6) and (3.7) yields the result

$$\frac{A_M}{p_S} = \left(\frac{k_S}{k_M}\right)^{\alpha-1},$$

while comparing the two expressions for w gives

$$\frac{A_M}{p_S} = \left(\frac{k_S}{k_M}\right)^{\alpha}.$$

This clearly implies that the capital-labour ratio is the same in each sector, $k_S = k_M = k$, and that the price of services output is given by $p_S = A_M = A_0 k^\eta$. It follows that

$$A_M = p_S = \frac{F_K^M}{F_K^S} = \frac{F_N^M}{F_N^S}.$$

In order to derive the efficiency conditions, we define the current-value Hamiltonian for households facing the wealth constraint in equation (3.4),

$$H = \nu + \lambda_q(t)\left[(r_q - g)\, q + w - c_M - p_S c_S\right],$$

with choice variables c_M and c_S, and state variable q. The first order optimality conditions for the choice variables yield

$$\begin{aligned} \frac{\partial H}{\partial c_M} = 0 \quad &\Rightarrow \quad \nu_M = \lambda_q \\ \frac{\partial H}{\partial c_S} = 0 \quad &\Rightarrow \quad \nu_S = p_S \lambda_q, \end{aligned}$$

yielding the static efficiency conditions

$$\frac{\nu_S}{\nu_M} = p_S = A_M = \frac{F_K^M}{F_K^S} = \frac{F_N^M}{F_N^S}. \tag{3.8}$$

For q, we get the dynamic efficiency condition

$$\begin{aligned} \frac{\partial H}{\partial q} &= \rho\lambda_q - \dot{\lambda}_q \\ \Rightarrow \quad -\frac{\dot{\nu}_M}{\nu_M} &= r_q - (\rho + g) \\ &= r_k - (\theta + \rho + g) \\ &= \alpha A_0 k^{\alpha+\eta-1} - (\theta + \rho + g)\,. \end{aligned} \tag{3.9}$$

Combining equation (3.8) with the utility function, we get

$$\begin{aligned} \frac{p_S c_S}{c_M} &= \left(\frac{\omega_S}{\omega_M}\right)^{\epsilon} A_M^{1-\epsilon} \\ &= \left(\frac{\omega_S}{\omega_M}\right)^{\epsilon} (A_0 k^{\eta})^{1-\epsilon} \\ &\equiv x_S, \end{aligned} \tag{3.10}$$

which is the ratio of consumption expenditure on services to consumption expenditure on manufactured goods. We will define $x_M \equiv 1$, the ratio of consumption expenditure on manufactured goods to itself, and will let

$$X \equiv x_M + x_S = 1 + x_S.$$

Further define

$$c \equiv p_S c_S + c_M,$$

which is total per capita consumption expenditure. Clearly $c = c_M X$. We can therefore express the labour share in services using equation (3.10), by noting that

$$\begin{aligned} x_S &= \frac{p_S c_S}{c_M} \\ &= \frac{p_S c_S X}{c} \\ &= \frac{A_M n_S k^\alpha X}{c} \\ &= \frac{n_S y X}{c}, \end{aligned} \tag{3.11}$$

since all services output is consumed, and where per-capita output in terms of manufactured goods is defined as

$$\begin{aligned} y &\equiv \frac{F^M + p_S F^S}{N} \\ &= A_M n_M k^\alpha + p_S n_S k^\alpha \\ &= A_M k^\alpha \\ &= A_0 k^{\alpha+\eta}, \end{aligned} \tag{3.12}$$

since $p_S = A_M$. Then by equation (3.11), the labour shares employed in each sector can be expressed as

$$n_S = \frac{x_S}{X}\left(\frac{c}{y}\right) \tag{3.13}$$

$$\begin{aligned} n_M &= 1 - n_S \\ &= 1 - \frac{x_S}{X}\left(\frac{c}{y}\right) \\ &= \frac{1}{X}\left(\frac{c}{y}\right) + \left(1 - \frac{c}{y}\right). \end{aligned} \tag{3.14}$$

The capital accumulation condition in equation (3.5) yields the condition

$$\begin{aligned} \frac{\dot{k}}{k} &= \frac{w}{k} - \frac{c_M + p_S c_S}{k} + r_k - (\theta + g) \\ &= (1-\alpha) A_0 k^{\alpha+\eta-1} - c/k + \alpha A_0 k^{\alpha+\eta-1} - (\theta + g) \\ &= A_0 k^{\alpha+\eta-1} - c/k - (\theta + g). \end{aligned} \tag{3.15}$$

It follows immediately by the definition of A_M that productivity growth in manufacturing grows according to

$$\frac{\dot{A}_M}{A_M} = \eta\frac{\dot{k}}{k} = \eta\left[A_0 k^{\alpha+\eta-1} - c/k - (\theta + g)\right]. \tag{3.16}$$

By equation (3.8), we note that

$$\frac{\nu_S}{\nu_M} = \frac{\Phi_S}{\Phi} \cdot \frac{\Phi}{\Phi_M} = \frac{\Phi_S}{\Phi_M} = p_S,$$

and it follows that since Φ is homogeneous of degree one,

$$\begin{aligned} \Phi &= \Phi_M c_M + \Phi_S c_S \\ &= \Phi_M c_M + \Phi_M p_S c_S \\ &= \Phi_M c. \end{aligned}$$

Then

$$\nu_M = \frac{\Phi_M}{\Phi} = \frac{1}{c},$$

so by the dynamic efficiency condition in equation (3.9), per-capita consumption grows at the rate

$$\begin{aligned} \frac{\dot{c}}{c} &= -\frac{\dot{\nu}_M}{\nu_M} \\ &= \alpha A_0 k^{\alpha+\eta-1} - (\theta + \rho + g) . \end{aligned} \tag{3.17}$$

This differential equation and the law of motion for capital in equation (3.15) together define the dynamics of the economy. By equation (3.12), per-capita output growth follows

$$\frac{\dot{y}}{y} = (\alpha + \eta) \frac{\dot{k}}{k} = (\alpha + \eta) \left[A_0 k^{\alpha+\eta-1} - c/k - (\theta + g) \right] . \tag{3.18}$$

We can also characterise how labour shares change over time. First note that

$$\frac{\dot{X}}{X} = \frac{\dot{x}_S}{X} = \frac{(1-\epsilon)\, x_S}{X} \cdot \frac{\dot{A}_M}{A_M}.$$

By equation (3.13),

$$\begin{aligned} \frac{\dot{n}_S}{n_S} &= \frac{\left(\dot{\overline{c/y}}\right)}{c/y} + \frac{\dot{x}_S}{x_S} - \frac{\dot{X}}{X} \\ &= \frac{\left(\dot{\overline{c/y}}\right)}{c/y} + (1-\epsilon) \left(1 - \frac{x_S}{X}\right) \frac{\dot{A}_M}{A_M} \\ &= \frac{\left(\dot{\overline{c/y}}\right)}{c/y} + \frac{(1-\epsilon)\, \eta}{X} \left[A_0 k^{\alpha+\eta-1} - c/k - (\theta + g) \right] , \end{aligned} \tag{3.19}$$

and since $n_M = 1 - n_S$, it follows that

$$\begin{aligned}
\frac{\dot{n}_M}{n_M} &= -\frac{\dot{n}_S}{n_S} \cdot \frac{n_S}{n_M} \\
&= -\frac{\dot{\left(c/y\right)}}{c/y} \frac{n_S}{n_M} - \frac{(1-\epsilon)\,\eta}{X} \left[A_0 k^{\alpha+\eta-1} - c/k - (\theta+g)\right] \frac{n_S}{n_M}.
\end{aligned} \tag{3.20}$$

Finally, it is straightforward to see that in both sectors i,

$$\frac{p_i F^i}{y} = n_i,$$

so output shares are the same as labour shares. Define the consumption-capital ratio and its growth rate as

$$\begin{aligned}
z\,(t) &\equiv \frac{c}{k} \\
\gamma_z\,(t) &\equiv \frac{\dot{z}}{z}.
\end{aligned}$$

We are now in a position to collect some results. See Appendix C.1 for proofs of all results.

Lemma 3.1 *The steady state quantities of per-capita capital , per-capita consumption and the consumption-capital ratio satisfy*

$$\begin{aligned}
k_{SS} &= \left(\frac{\alpha A_0}{\theta+\rho+g}\right)^{\frac{1}{1-(\alpha+\eta)}} \\
c_{SS} &= \left[\left(\frac{1}{\alpha}-1\right)(\theta+\rho+g)+\rho\right] k_{SS} \\
&= \left(\frac{1}{\alpha}-1\right)\left[\frac{\alpha A_0}{(\theta+\rho+g)^{(\alpha+\eta)}}\right]^{\frac{1}{1-(\alpha+\eta)}} + \rho\left(\frac{\alpha A_0}{\theta+\rho+g}\right)^{\frac{1}{1-(\alpha+\eta)}} \\
z_{SS} &= \left(\frac{1}{\alpha}-1\right)(\theta+\rho+g)+\rho.
\end{aligned}$$

Thus a positive shock to financial development, parametrised as a negative shock to the investment wedge variable θ, increases the steady-state levels of per-capita capital and per-capita consumption, and decreases the steady-state consumption-capital ratio.

We study the stability properties of this economy by considering the eigenvalues of the 2×2 system of differential equations governing the dynamics of the economy, linearised around the steady state. By equations (3.15) and (3.17) we have that

$$\begin{aligned}
\dot{k} &= A_0 k^{\alpha+\eta} - c - (\theta+g)\,k \\
\dot{c} &= \alpha A_0 k^{\alpha+\eta-1} c - (\theta+\rho+g)\,c.
\end{aligned}$$

We can define the Jacobian matrix $\mathbf{J}$ as

$$\mathbf{J} \equiv \begin{pmatrix} \partial \dot{k}/\partial k & \partial \dot{k}/\partial c \\ \partial \dot{c}/\partial k & \partial \dot{c}/\partial c \end{pmatrix}.$$

Then the eigenvalues λ of this matrix satisfy the characteristic polynomial of this matrix evaluated at the steady state. That is, they satisfy

$$\mathbf{p}\left(\mathbf{J}\right) \equiv \left|\mathbf{J} - \lambda \mathbf{I}\right|_{(k,c)=(k_{SS},c_{SS})} = 0, \tag{3.21}$$

where $\mathbf{I}$ is the 2×2 identity matrix. Considering each derivative in turn, we have

$$\begin{aligned}
\dot{k}_{k_{SS}} &= \left(\alpha + \eta\right) A_0 k_{SS}^{\alpha+\eta-1} - \left(\theta + g\right) \\
&= \frac{\eta}{\alpha}\left(\theta + \rho + g\right) + \rho \\
\dot{k}_{c_{SS}} &= -1 \\
\dot{c}_{k_{SS}} &= \alpha\left(\alpha + \eta - 1\right) A_0 k_{SS}^{\alpha+\eta-2} c_{SS} \\
&= -\left[1 - \left(\alpha + \eta\right)\right]\left(\theta + \rho + g\right)\left[\left(\frac{1}{\alpha} - 1\right)\left(\theta + \rho + g\right) + \rho\right] \\
\dot{c}_{c_{SS}} &= \alpha A_0 k_{SS}^{\alpha+\eta-1} - \left(\theta + \rho + g\right) \\
&= 0,
\end{aligned}$$

where the notation is

$$\dot{k}_{k_{SS}} \equiv \left.\frac{\partial \dot{k}}{\partial k}\right|_{(k,c)=(k_{SS},c_{SS})},$$

and so on. The characteristic polynomial in equation (3.21) is therefore

$$\begin{aligned}
\mathbf{p}\left(\mathbf{J}\right) &= \lambda^2 - \left(\dot{k}_{k_{SS}} + \dot{c}_{c_{SS}}\right)\lambda + \left(\dot{k}_{k_{SS}}\dot{c}_{c_{SS}} - \dot{k}_{c_{SS}}\dot{c}_{k_{SS}}\right) \\
&= \lambda^2 - \dot{k}_{k_{SS}}\lambda + \dot{c}_{k_{SS}}.
\end{aligned}$$

The coefficient on λ^2 is positive, while the coefficient on λ and the constant term are negative, so $\mathbf{p}\left(\mathbf{J}\right)$ has two real roots, one positive and negative. Thus the economy in this model exhibits saddle path stability. Call the negative eigenvalue λ_-. Then the eigenvector associated with λ_- is a linear approximation of the stable arm around the steady state, and satisfies

$$\begin{aligned}
\frac{c - c_{SS}}{k - k_{SS}} &= \frac{\lambda_- - \dot{k}_{k_{SS}}}{\dot{k}_{c_{SS}}} \\
&= -\left(\lambda_- - \dot{k}_{k_{SS}}\right).
\end{aligned}$$

Figure 3.3 plots this linearisation about the steady state, including the stable arm, with parameters given in Table 3.1 and financial intermediation wedge parameter $\theta = 0.05$.

Table 3.1: Modelling choices

Parameter	Value	Description
A_0	1	productivity parameter
α	0.7	capital elasticity of output
η	0.1	capital elasticity of productivity
ρ	0.01	households' discount rate
g	0.05	population growth rate

Modelling choices for numerical simulation. As it will be allowed to vary in the following simulations, no value has been given for the financial intermediation wedge parameter θ.

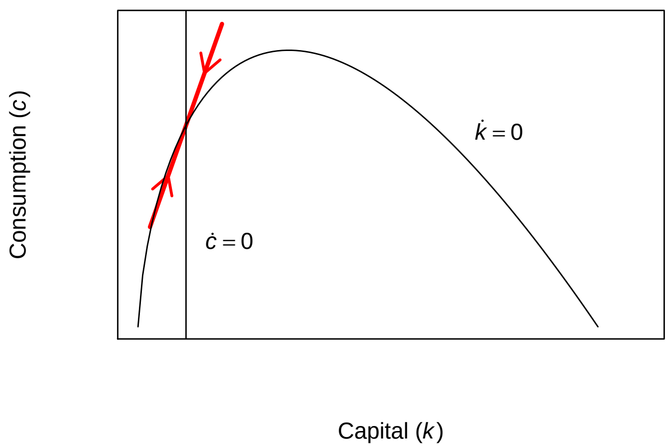

Figure 3.3: The saddle path

The stable arm, or saddle path, in the economy, linearised around the steady state, with financial intermediation wedge parameter $\theta = 0.05$. The black lines represent invariant capital and consumption, $\dot{k} = 0$ and $\dot{c} = 0$ respectively, while the red line is the linear approximation of the stable arm of this economy.

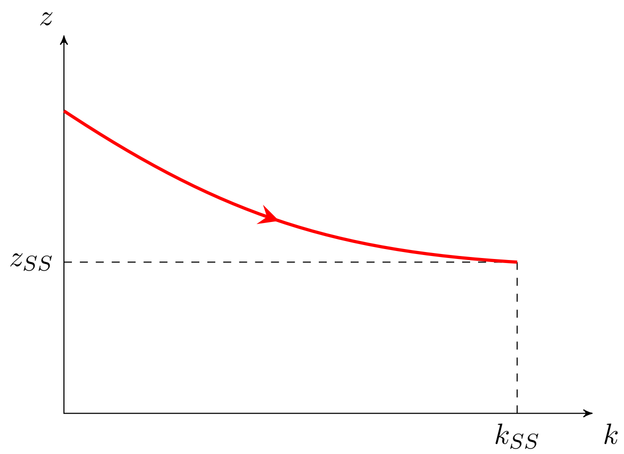

Figure 3.4: The consumption-capital ratio along the saddle path

With a little work, we can also say something about the exact stable dynamics of this economy.

Lemma 3.2 *Suppose the economy is growing, so that $k < k_{SS}$, $\dot{k} > 0$ and $\dot{c} > 0$. Along the saddle path, the consumption-capital ratio z is decreasing towards the steady state level z_{SS}. That is, while $k < k_{SS}$, $z > z_{SS}$ and $\gamma_z < 0$. Thus the consumption-capital ratio along the saddle path looks something like Figure 3.4.*

We can also say something about the effects of a shock to θ. Simulation of a modest reduction in θ, from 0.05 to 0.04, show potentially large effects on long-run consumption. Figure 3.5 shows the effects on the steady state of the economy after such a negative shock to θ, with a nearly 49% increase in long-run consumption. Moreover, it is possible to make some analytical statements about the effects of a negative shock to θ.

Lemma 3.3 *Suppose the economy is growing, so that $k < k_{SS}$, $\dot{k} > 0$ and $\dot{c} > 0$, and suppose there is a negative shock to θ. Then there is also a negative shock to per-capita consumption c. Such a shock looks something like Figure 3.6.*

Given the results we have derived so far, we can make some statements about the effect of financial development on economic growth and overall consumption.

Theorem 3.4 *Capital intensity, manufacturing productivity, per-capita output and per-capita consumption grow according to the following expressions:*

$$
\begin{aligned}
\frac{\dot{k}}{k} &= A_0 k^{\alpha+\eta-1} - c/k - (\theta+g) \\
\frac{\dot{A}_M}{A_M} &= \eta \left[A_0 k^{\alpha+\eta-1} - c/k - (\theta+g)\right] \\
\frac{\dot{y}}{y} &= (\alpha+\eta)\left[A_0 k^{\alpha+\eta-1} - c/k - (\theta+g)\right] \\
\frac{\dot{c}}{c} &= \alpha A_0 k^{\alpha+\eta-1} - (\theta+\rho+g)\,.
\end{aligned}
$$

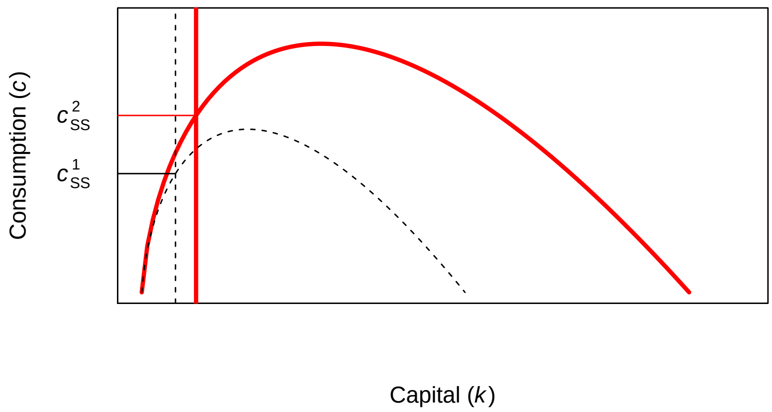

Figure 3.5: The economy given a negative shock to θ

The economy before and after a positive financial shock (a negative shock to θ). The dashed black lines show the economy when $\theta = 0.05$, and the solid red lines when $\theta = 0.04$, holding all other parameters constant. The shock causes a nearly 49% increase in long run consumption, from c^1_{SS} to c^2_{SS}.

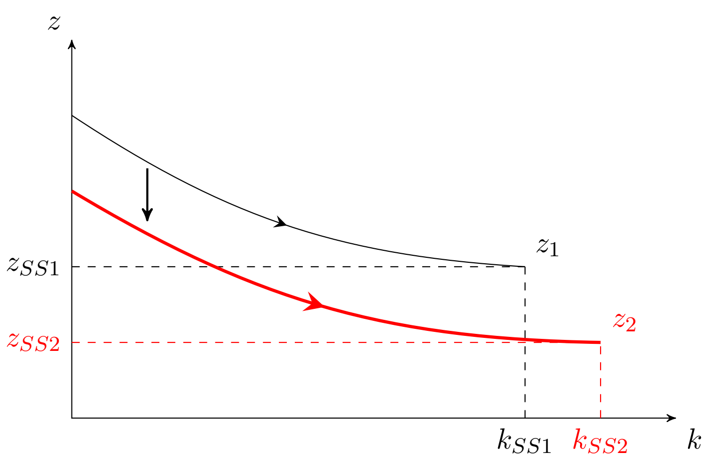

Figure 3.6: The consumption-capital ratio along the saddle path given a negative shock to θ

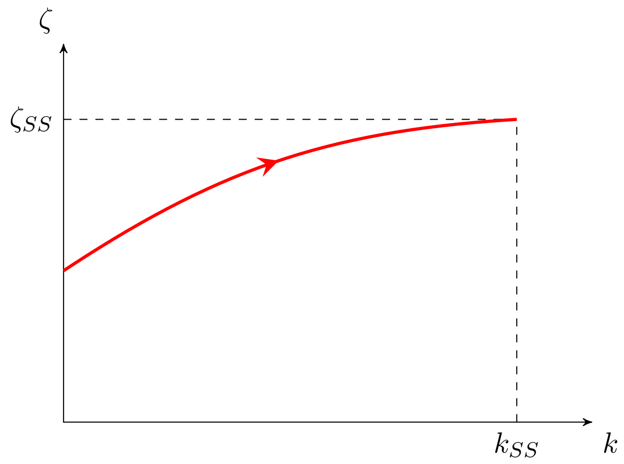

Figure 3.7: The consumption-output ratio along the saddle path

Suppose the economy is growing, so that $k < k_{SS}$, $\dot{k} > 0$ and $\dot{c} > 0$. A positive shock to financial development, parametrised as a negative shock to the investment wedge variable θ, increases all of these growth rates, since it leaves per-capita capital k unchanged and causes a negative shock to c/k, by Lemma 3.3. It also causes a negative shock to the level of per-capita consumption c.

Next we can say something about the consumption-output ratio. Define the consumption-output ratio and its growth rate as

$$\zeta(t) \equiv \frac{c}{y}$$
$$\gamma_\zeta(t) \equiv \frac{\dot{\zeta}}{\zeta}.$$

Lemma 3.5 *Suppose the economy is growing, so that $k < k_{SS}$, $\dot{k} > 0$ and $\dot{c} > 0$. Along the saddle path, consumption-output ratio ζ is increasing towards the steady state level ζ_{SS}. That is, while $k < k_{SS}$, $\zeta < \zeta_{SS}$ and $\gamma_\zeta > 0$. Thus the consumption-output ratio along the saddle path looks something like Figure 3.7.*

We require a final intermediate result before considering structural change in this economy.

Lemma 3.6 *Suppose the economy is growing, so that $k < k_{SS}$, $\dot{k} > 0$ and $\dot{c} > 0$, and suppose there is a negative shock to θ. Then there is also a negative level shock to the consumption-output ratio c/y, but a weakly positive shock to its growth rate.*

Finally, we can characterise structural change in this economy, and its interaction with financial development.

3.3.5 Model predictions: structural change in response to a financial shock

Theorem 3.7 *The labour shares n_S and n_M, in the services and manufacturing industries respectively, grow according to the following expressions:*

$$\frac{\dot{n}_S}{n_S} = \frac{\dot{\zeta}}{\zeta} + \frac{(1-\epsilon)\,\eta}{X}\left[A_0 k^{\alpha+\eta-1} - c/k - (\theta+g)\right]$$
$$\frac{\dot{n}_M}{n_M} = -\frac{\dot{\zeta}}{\zeta}\frac{n_S}{n_M} - \frac{(1-\epsilon)\,\eta}{X}\left[A_0 k^{\alpha+\eta-1} - c/k - (\theta+g)\right]\frac{n_S}{n_M},$$

where $\zeta = c/y$ is the consumption-output ratio. Suppose the economy is growing, so that $k < k_{SS}$, $\dot{k} > 0$ and $\dot{c} > 0$. This economy exhibits structural change: services employment is increasing and manufacturing employment decreasing over time.

A positive shock to financial development, parametrised as a negative shock to the investment wedge variable θ:

(i) causes aggregate per-capita output growth $\dot{y}/y$ and capital growth $\dot{k}/k$ to accelerate

(ii) causes a negative level shock to the services labour share n_S and a positive level shock to the manufacturing labour share n_M, but accelerates the growth rate $\dot{n}_S/n_S$

(iii) increases the long-run level of n_S and decreases the long-run level of n_M

(iv) increases the manufacturing productivity growth rate $\dot{A}_M/A_M$

(v) increases the growth rate of capital in the services sector, $\dot{K}_S/K_S$, but has an ambiguous effect on the growth rate of capital in the manufacturing sector, $\dot{K}_M/K_M$.

The output share is equal to the labour share in each sector, so output shares follow the same structural dynamics as labour shares and exhibit the same responses to financial shocks.

The model is therefore consistent with existing empirical evidence. Jayaratne and Strahan (1996) find that bank branching deregulation accelerated growth, as predicted by the model. Jerzmanowski (2017) finds that deregulation caused an acceleration in TFP growth in the manufacturing sector and an acceleration in aggregate capital accumulation, but that it did not cause an acceleration in capital accumulation in the manufacturing sector, all of which is also consistent with the model. Furthermore, the model predicts that bank branching deregulation will, over a sufficiently long time horizon, accelerate structural change towards services and away from manufacturing, both by affecting the transitional dynamics and by affecting the long-run shares of labour and output accounted for by each sector. We will proceed to estimate this proposition empirically.

3.4 Data

3.4.1 BEA data

Herrendorf et al. (2014) suggest three measures of structural change: changes over time in relative sectoral shares in each of employment, output (value added), and consumption. Data on sectoral shares of consumption by U.S. state and year do not appear to exist prior to the 1990s, which is too late to be of use in studying bank branching deregulation. However, the U.S. Bureau of Economic Analysis (BEA) provides industry-level employment data by U.S. state and year from 1969-2000 (aggregated using SIC code) and from 2001-2016 (aggregated using NAICS code). This allows the calculation of sectoral shares of employment. The BEA also provides industry-level output data by U.S. state and year from 1963-1996 (aggregated using SIC code) and from 1997-2017 (aggregated using NAICS code). According to the BEA's official methodology notes, "[n]o matter how a GDP by state component is estimated, it is always adjusted to be consistent with BEA's definition of value added" (U.S. Department of Commerce and Bureau of Economic Analysis, 2017, p. iii). This data therefore allows the calculation of sectoral shares of value added, as proposed by Herrendorf et al. (2014).

Where a particular state-year-industry combination has not been provided – typically in order to avoid the disclosure of confidential information – we have interpolated linearly between the nearest available entries. However, the numbers involved are typically small. For example, employment in mining in D.C. is redacted in the years 1998-2000, so in calculating aggregate employment numbers for the manufacturing sector in D.C. in those years, interpolated figures are used for mining employment. The number of employees in mining in DC in 1997 is only 309, however, so it seems reasonable to assume that this will not skew the results significantly. Indeed, where data is redacted to avoid the disclosure of confidential information, almost by definition the numbers are likely to be small.

There is a discontinuity in the data when the aggregation method switches from aggregation by SIC code to NAICS code, and the BEA cautions against combining the two datasets to form a unified dataset spanning from the 1960s to the present. However, there are several reasons why this is not likely to be a major concern for this exercise. First, we are largely interested in shares of output or employment, so changes in absolute values are not important unless they are systematically different from one sector to another[7]. Second, since we are aggregating into four major sectors, unless there are large changes in sector assignment between the two methodologies, small-scale differences in methodology will not be significant. Third, while the difference-in-differences regressions span both datasets, the synthetic controls study does not in fact use any data from after 1996 (there were no deregulations late enough, with enough states left as members of the donor pool, for this situation to arise). For completeness, we also report the difference-in-differences results obtained using only the data aggregated using SIC code in Appendix C.4.

3.4.2 IPUMS USA data

Control data was taken from the University of Minnesota's IPUMS USA database (Ruggles et al., 2019). This database collates and harmonises U.S. census microdata. Three control variables were constructed at the state-year level: the average years of education of state

[7]The SIC output data is presented in chained 1997 U.S. dollars, while the NAICS output data is presented in chained 2012 U.S. dollars. Conversion from one to the other would therefore be necessary if we were to report absolute values spanning the two datasets. However, this issue does not arise in the results presented below.

residents, the average age of state residents, and the proportion of state residents who are black. Where a particular state-year combination doesn't appear in the data, the value is constructed by linearly interpolating between the latest observation for that state prior to the year of interest, and the earliest observation for that state subsequent to the year of interest.

We have collected data from all three sources across a span of 55 years from 1963 to 2017, for 50 states and D.C., so that variables with an observation for every year have $55 \times 51 = 2,805$ observations. However, employment data in particular is only available from 1969 to 2016, so many of the results below are calculated using data that is from 1969 and more recent. Table 3.2 reports descriptive statistics for the data.

3.4.3 Date of deregulation

The year of bank branching deregulation in each state is taken from Amel (1993), Jayaratne and Strahan (1996), Krozsner and Strahan (1999), and Beck et al. (2010). Following this literature, we take the year of deregulation to be the year that a state allowed bank branching by merger and acquisition (M&A). Appendix C.3 lists the year of deregulation for each state.

3.5 Empirical results

3.5.1 Difference-in-differences regressions

A standard method of assessing the effect of a staggered treatment is to fit difference-in-differences regressions. This type of regression has been used to assess the effects of bank branching deregulation on output growth (Jayaratne and Strahan, 1996) and inequality as measured by the Gini coefficient (Beck et al., 2010), among others. Our model predicts that the effect of financial deregulation on structural change includes a growth effect, so we need to allow the effect of deregulation on sectoral shares to grow or diminish over time. We therefore estimate the following specification,

$$Y_{st} = \beta_0 + \beta_1 D_{st} + \beta_2 T_{st} + \beta_3 D_{st} T_{st} + \boldsymbol{\gamma}_1' \mathbf{X}_s + \boldsymbol{\gamma}_2' \mathbf{X}_t + \varepsilon_{st}, \tag{3.22}$$

where Y_{st} is the outcome variable of interest in state s in year t, D_{st} is a treatment dummy that is equal to 1 once a state has deregulated and equal to 0 otherwise, T_{st} is equal to the number of years since deregulation (it takes a negative value prior to deregulation), $\mathbf{X}_s$ is a vector of state dummies accounting for state fixed effects, $\mathbf{X}_t$ is a vector of time dummies accounting for time fixed effects, and ε_{st} is an error term. This specification allows us to account for unobserved, time-invariant state effects by including state dummies, and for unobserved nationwide shocks by including time dummies. The estimated value of β_1 indicates the level effect caused by deregulation, while the estimated value of β_3 indicates whether the effect of deregulation is changing over time.

We estimate the effect of deregulation on four outcome variables: services share of employment, services share of output, manufacturing share of employment, and manufacturing share of output. We do not estimate the effect of bank branching deregulation on agriculture,

Table 3.2: Descriptive statistics

Statistic	N	Mean	St. Dev.	Min	Max
Services Share of Employment (%)	2,448	60.3	8.4	33.7	80.1
Services Share of Output (%)	2,805	57.5	9.2	30.6	81.2
Manufacturing Share of Employment (%)	2,448	19.0	6.5	1.9	38.8
Manufacturing Share of Output (%)	2,805	25.1	9.3	1.2	59.0
Output per Worker ($000)	2,448	46.9	27.2	7.5	141.5
Output per Services Worker ($000)	2,448	44.8	25.4	8.2	131.6
Output per Manufacturing Worker ($000)	2,448	62.3	45.5	8.3	423.5
Population	2,499	5,138,421.0	5,754,892.0	296,000.0	39,250,017.0
Total Employment	2,448	2,774,139.0	3,120,785.0	143,816.0	23,265,312.0
Total Output ($million)	2,805	141,227.6	247,679.6	993	2,746,873
Employment Rate (%)	2,448	55.9	11.4	37.1	137.7
GDP per Capita ($000)	2,499	28.4	20.9	3.1	192.3
GDP Growth (%)	2,754	6.5	4.3	−31.0	44.2
Average Years of Education	2,805	10.1	1.0	7.3	12.5
Proportion of Black Residents (%)	2,805	10.2	11.9	0.0	70.8
Average Age of Residents	2,805	34.5	2.9	24.8	42.4

Data from BEA and IPUMS USA. Services share of employment, services share of output, manufacturing share of employment, manufacturing share of output, output per worker, output per services worker, output per manufacturing worker, employment rate, GDP per capita and GDP growth are the authors' own calculations based on BEA data. Population, total employment and total output are from BEA data. Average years of education, proportion of black residents and average age of residents are the authors' own calculations based on IPUMS USA data. Note that the maximum employment rate in the data is above 100%, but this is for D.C., which has many workers commuting from nearby states; all other states' maximum employment rate is well under 100%.

as it accounts for such a small share of most state economies over the period in question. We also do not estimate the effect of bank branching deregulation on the government share of the economy; while this may yield interesting results, almost by definition any effect cannot be argued to be a clean consequence of market forces at work in the financial sector.

Table 3.3 reports the results of the regressions[8]. As in Beck et al. (2010), standard errors are clustered at the state level (that is, at the level of the treatment unit), accounting for the fact that the error term may exhibit serial correlation within states. Consistent with Jayaratne and Strahan (1996) and Beck et al. (2010), the year of deregulation is excluded, as are Delaware and South Dakota, which have long been major centres for the credit card industry. Not all of the coefficients on the deregulation dummy are significant, suggesting that the effect of deregulation on structural composition does not operate solely – or even primarily – by having an instantaneous, permanent level effect on the outcome variables. Conversely, all the coefficients on 'Dereg.*Time' are highly significant, suggesting that deregulation causes the share of services to increase over time, in both employment and output, while having the opposite effect on manufacturing, consistent with the theory in Section 3.3.

These results are apparently very strong, and certainly suggest that structural change accelerates post-deregulation. However, the fact that the coefficients on 'Time' are significant in Table 3.3 raises concerns about the validity of a causal interpretation of the effect of bank branching deregulation. Difference-in-difference regressions rely on a number of assumptions, notably the assumption of exogenous treatment: assignment to treatment should not be correlated with the outcome variable, conditional on observables. Significant coefficients on 'Time' suggest that pre-deregulation structural composition is correlated with the number of years until deregulation, raising concerns about the exogeneity of treatment. In order to alleviate these concerns, we turn to the synthetic control method, which allows us to relax the exogeneity requirement somewhat.

3.5.2 Pooled synthetic controls

3.5.2.1 Synthetic controls

The synthetic control method (SCM) is a recent development in case study analysis. When a treatment takes place in only one or a small number of units, it is not generally possible to use traditional techniques to estimate the treatment effect. Without being able to identify a plausible counterfactual had the unit not been treated – which becomes much easier when there are large numbers of treated and untreated units that are similar in characteristics – it is impossible to identify the treatment effect. Even when only one unit is treated, however, the SCM provides a rigorous, quantitative framework for constructing a counterfactual.

For a treated unit, the SCM constructs a 'synthetic' treated unit as a convex combination of untreated control units, in order to minimise the distance between the real unit and the

[8]These regressions use the full span of BEA data available. However, as noted in Section 3.4, there is a change in methodology when the BEA switches from aggregation using SIC code to aggregation using NAICS code. For completeness, the results in Table 3.3 are replicated in Table C.4 in Appendix C.4 using only SIC-based data. None of the results are contradictory.

Table 3.3: Difference-in-differences regressions

	Dependent variable:			
	Services Share of Employment (%)	Services Share of Output (%)	Man. Share of Employment (%)	Man. Share of Output (%)
Deregulation	-0.234^{*}	0.076	0.065	-0.811^{***}
	(0.129)	(0.201)	(0.185)	(0.245)
Time	-0.110^{*}	1.294^{***}	0.769^{***}	-1.518^{***}
	(0.065)	(0.113)	(0.077)	(0.141)
Dereg.*Time	0.249^{***}	0.284^{***}	-0.360^{***}	-0.407^{***}
	(0.017)	(0.020)	(0.023)	(0.024)
Constant	40.399^{***}	66.346^{***}	40.627^{***}	8.841^{***}
	(0.935)	(2.196)	(1.217)	(2.725)
Observations	1,833	2,106	1,833	2,106
R^2	0.972	0.925	0.874	0.856
Adjusted R^2	0.971	0.922	0.868	0.849

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Difference-in-differences OLS estimates of the effect of bank branching deregulation on structural composition, accounting for year and state fixed effects. Standard errors clustered at the state level in parentheses. 'Deregulation' is a dummy that is equal to 1 when a state has deregulated, and equal to 0 otherwise. 'Time' is the time in years since deregulation (this variable takes a negative value prior to deregulation). The year of deregulation is excluded for each state. States that deregulated prior to 1963 are excluded (Alaska, Arizona, California, Delaware, the District of Columbia, Idaho, Maryland, Nevada, North Carolina, Rhode Island, South Carolina and South Dakota).

synthetic unit prior to treatment. Subject to some assumptions, divergence between the real unit and the treated unit post-treatment can be interpreted as the causal effect of treatment.

The SCM was introduced by Abadie and Gardeazabal (2003) in order to study the economic effects of terrorism in the Basque Country. Since it is impossible to observe the Basque Country in the absence of terrorism over the period of interest, a synthetic Basque Country is constructed as a convex combination of other Spanish regions. 'Treatment' in this case is the onset of violence.

When constructing the synthetic Basque Country, weights are given to each of the other Spanish regions – known collectively as the donor pool – in order to minimise the gap between the real and synthetic Basque Countries in the period prior to treatment for the outcome variable of interest, in this case GDP per capita. Formally, following the notation and exposition in Abadie and Gardeazabal (2003), let $\mathbf{W} = (w_2, ..., w_{J+1})'$ be a $(J \times 1)$ vector of weights on the J regions in the donor pool – where we reserve the index 1 for the treated region, the Basque Country – such that the weights are non-negative and sum to unity. Let $\mathbf{X}_1$ be a $(K \times 1)$ vector of predictor variables for GDP per capita in the Basque Country, and $\mathbf{X}_0$ be a $(K \times J)$ matrix containing the same K variables for each of the J regions in the donor pool. Let $\mathbf{V}$ be a diagonal $(K \times K)$ matrix with non-negative entries. Given $\mathbf{V}$, the optimal weight vector $\mathbf{W}^* (\mathbf{V})$ is chosen to minimise

$$(\mathbf{X}_1 - \mathbf{X}_0\mathbf{W})' \mathbf{V} (\mathbf{X}_1 - \mathbf{X}_0\mathbf{W}).$$

The optimal matrix $\mathbf{V}^*$ is chosen to minimise the root mean squared error (RMSE) in GDP per capita between the real and synthetic Basque Countries in the pretreatment period. $\mathbf{V}^*$ can be thought of as a weighting of the importance of the K different predictor variables.

The SCM has a number of desirable properties relative to the traditional difference-in-differences estimator. While the fixed effects model can control for time-invariant unobserved unit-specific characteristics, the synthetic controls estimator "allows the effects of confounding unobserved characteristics to vary with time" (Abadie et al., 2010, p. 495). Moreover, "treatment and control states need not follow parallel trends, conditional on observables" (Dube and Zipperer, 2015, p. 8). Importantly, the SCM does not impose a functional form on the effect of deregulation, and so avoids many of the pitfalls associated with misspecified difference-in-difference regressions. Unlike with traditional regressions, an analyst using the SCM can be completely agnostic about both the direction and the character of any treatment effect under study. This ensures that we will not infer an incorrect effect because of functional misspecification: the data speak for themselves – although this does mean that it will be harder to distinguish between the level and growth effects predicted by the model in Section 3.3. Finally, we can relax the necessity for treatment exogeneity: given a good synthetic control, the difference between the treated unit and the synthetic control is treatment alone, and the counterfactual is valid even when treatment is endogenous. Thus while the difference-in-differences estimates derived above may not be valid, in principle, estimates derived using the SCM can be.

The SCM is however subject to some assumptions. First, the treated unit is assumed to be in the convex hull of the donor pool, so that a good fit for the treated unit is assumed to exist. If this assumption holds, then even when assignment to treatment is non-random and correlated with unobservable confounders, causal inference is valid. While it's not possible to test directly the existence of a good synthetic control, we go some way towards addressing this concern with a statistical validity test that is described below. Second, validity requires "that outcomes of the untreated units are not affected by the intervention implemented in the treated unit" (Abadie et al., 2010, pp. 494-495). Since we are focussed on intra-state bank branching deregulation, it seems plausible to assume that there is very little effect of deregulation in one state on outcomes in any other. Indeed, Huang (2008) studies the effects of intra-state branching deregulation by examining pairs of contiguous counties across state lines, one of which belonged to a state that deregulated and the other of which belonged to a state that did not deregulate for at least three years subsequently. In order to test indirectly for cross-border spillover effects, deregulated counties are also compared to 'hinterland' counties that are not contiguous with any deregulated county, and no major difference is found to the contiguous case (Huang, 2008, pp. 701-702). This suggests that spillover effects are not a major concern.

We use synthetic controls to study the effect of bank branching deregulation on the same four outcome variables as we studied with difference-in-differences regressions: services share of employment, services share of output, manufacturing share of employment, and manufacturing share of output. The predictors we use to construct our $\mathbf{X}_0$ vector and $\mathbf{X}_1$ matrix are the state's population, total employment, total output, GDP per capita, GDP growth, employment rate, average years of education, proportion of residents who are black, and average age. We use these predictors because there is reason to believe that socio-demographic and economic variables such as these are good predictors of long-term growth potential (Sala-i-Martin et al., 2004), and we know that growth goes hand-in-hand with structural change. Each of these predictors is used for the full pretreatment span of time, from 1969 to the year prior to deregulation. As is common practice, we also use two observations, in 1969 and the year prior to deregulation, of each of the four outcome variables. These predictors are all calculated from BEA data, except for the average years of education, proportion of residents who are black, and average age, which are calculated from IPUMS USA data.

3.5.2.2 Statistical inference and pooling

Statistical inference when using the SCM is typically done by running placebo tests – either across time, by assigning the treatment date to some time when treatment did not actually occur, or across units, by assigning as the treated unit a member of the donor pool (Abadie et al., 2010). If the treatment does indeed have an effect, one would expect to see a greater deviation between the real and synthetic treated units after treatment in the genuine, non-placebo case than when running placebo tests. If some measure of deviation, say the post-treatment RMSE, is at the tail of the placebo distribution then we can have some confidence that the effect is statistically significant.

This method relies on having a reasonably large number of units in the donor pool, or a reasonably large time span across which to run placebo tests. In the case of bank branching deregulation, this presents a challenge. For example, there are only 9 deregulation events for which there are at least 3 other states in the donor pool, if we impose the requirements that there be at least 5 years of pre-treatment data to generate the synthetic treatment unit, and at least 10 years of post-treatment data in order to allow the treatment some time to take effect. If a deregulated state has only 3 other states in the donor pool, then even in principle a placebo test across units could only achieve a 40% significance level[9].

In order to enhance the statistical power of such placebo tests, we will pool different deregulation events and consider their joint statistical significance. Following an idea used by Dube and Zipperer (2015) to estimate the effect of minimum wage increases, we will calculate the mean percentile rank of the estimated treatment effect relative to the placebo tests. Under the null hypothesis of uniform distribution, this statistic has a distribution that can be calculated exactly even for small samples. Section 3.5.2.3 describes in detail the method used for constructing this distribution and calculating p-values.

3.5.2.3 Model selection and p-value calculation

When choosing the specification for constructing synthetic control units, there are competing imperatives. Clearly the longer the pre-treatment span of time, the better[10], as this improves the fit of the synthetic control unit. The longer the post-treatment span of time, the better, as this gives more time for the treatment to take effect and be discernible: the difference-in-differences regressions are suggestive of the fact that treatment may have an increasingly large effect over time. Finally, for each deregulation event, the more states in the donor pool, the better, as this increases the chance that a good synthetic control unit can be constructed. However, tightening these restrictions has the effect of reducing the number of deregulation events that can be used, which reduces their joint statistical power.

In order to get a good balance of all these requirements, our specification is to require that each treated state has at least 5 years of pre-treatment data, and a donor pool of at least 3 states that do not deregulate for at least 10 years after the treated state deregulates. We judge that this should balance the need for enough deregulation events to pool with the likelihood of a reasonably good fit and enough time to see gradual effects take hold. This specification gives us 9 deregulation events to study[11]. Tightening any of these requirements results in fewer deregulation events available for study, which reduces the statistical power of the pooled study.

In order to get some estimate of the size of the effect of deregulation, we will focus on the value of the outcome variable at the end of the specified post-treatment period. For instance, in our baseline specification, when assessing the effect of deregulation on the share

[9] See Section 3.5.2.3 for a detailed description of the calculation of p-values.

[10] Data for all the variables of interest are only available from 1969, as described in Section 3.4, so for example a state that deregulated in 1975 would only have 6 years of pre-treatment data available.

[11] The bank branching deregulation events captured in this specification are those in Alabama, Connecticut, Maine, New Jersey, New York, Ohio, Pennsylvania, Utah and Virginia.

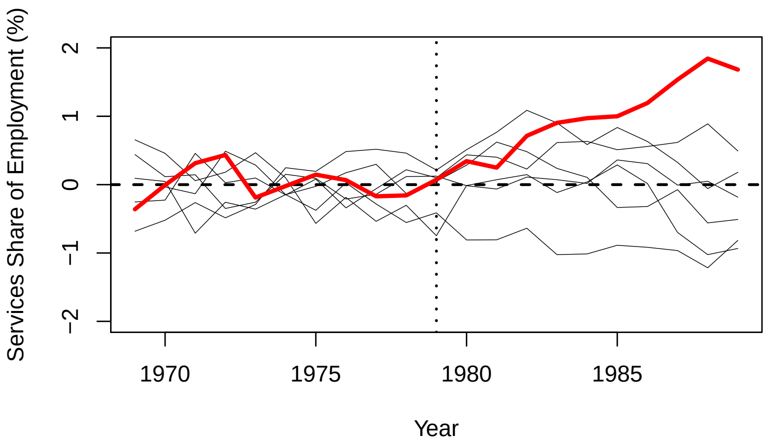

Figure 3.8: Ohio placebo test

Placebo test of the effect of bank branching deregulation on the services share of output in Ohio. The synthetic Ohio in this case is optimally calculated as 9% Kentucky, 34% Missouri and 57% Wisconsin. Deregulation took place in 1979 in Ohio. The red line represents the gap between the real Ohio and the synthetic Ohio, while the black lines represent the gaps between real but untreated states in Ohio's donor pool and their synthetic counterparts.

of services employment in the economy, we will focus on the gap between the share of services employment in the treated state and the share of services employment in the synthetic state, 10 years after deregulation occurs. We calculate two key values: first, the mean gap across all of the deregulation events; and second, a p-value associated with the mean percentile rank across all of the deregulation events.

The p-value is calculated as follows. Suppose there are N deregulation events under study and a specified post-treatment span of T years. For each $i \in [1, N]$, suppose there are $N_i - 1$ states in the donor pool. Including the treated state and the placebo studies, therefore, there are N_i values for the gap in the outcome variable (real value minus synthetic value) T years after deregulation. Any placebo deregulations that have a mean squared prediction error (MSPE) prior to deregulation greater than 5 times that of the treated state are removed from consideration, to exclude comparisons with states that do not have a well-fitting synthetic counterpart. Suppose this leaves $n_i - 1$ 'valid' placebo tests, so that for each treated state there are now n_i values for the gap in the outcome variable T years after deregulation. Suppose the treated state is at rank j_i out of those n_i values, so there are $j_i - 1$ placebo states that have a smaller gap and $n_i - j_i$ placebo states that have a larger gap. Then the percentile rank for state i is calculated as

$$\pi_i = \frac{j_i}{n_i + 1}.$$

This has the property of being symmetrical – that is, a treated state that has exactly as many placebo values above it as below it would generate a percentile rank of 0.5.

As an example, Figure 3.8 shows a placebo test of the effect of deregulation on the services share of employment in Ohio. The synthetic Ohio in this case is optimally calculated as 9% Kentucky, 34% Missouri and 57% Wisconsin. Since the effect on Ohio (the red line) is greater 10 years after deregulation than it is for any of the placebo states, and since there are 6 states in the donor pool, the percentile rank in this case would be

$$\pi_{\text{Ohio}} = \frac{j_{\text{Ohio}}}{n_{\text{Ohio}} + 1} = \frac{7}{8} = 0.875.$$

Once percentile ranks have been calculated for each individual deregulation event, the mean percentile rank across all deregulation events under study is calculated as

$$\bar{\pi} = \frac{1}{N} \sum_{i=1}^{N} \frac{j_i}{n_i + 1}. \tag{3.23}$$

Under the null hypothesis that each π_i is uniformly distributed – that is, under the null hypothesis that there is in fact no treatment effect – the exact distribution of $\bar{\pi}$ can be simulated using Monte Carlo methods[12]. For each $i \in [1, N]$, we take a random draw of $\hat{j}_i \in [1, n_i]$, then calculate

$$\hat{\pi} = \frac{1}{N} \sum_{i=1}^{N} \frac{\hat{j}_i}{n_i + 1}.$$

We repeat this five million times to construct an exact distribution for $\hat{\pi}$. The distribution of $\hat{\pi}$ allows us to assign p-values to the true value $\bar{\pi}$. If, for example, $\bar{\pi}$ is at either the 2.5th or the 97.5th percentile of the distribution of $\hat{\pi}$, we assign to $\bar{\pi}$ a p-value of 5%.

Note that the p-value does not directly relate to the estimated size of the effect. Instead, it is derived from the extremity of the genuine results in relation to placebo results. Thus we cannot construct a confidence interval around the point estimates reported in Section 3.5.2.5. Instead, we report a point estimate and report the p-value associated with there being *some* non-zero effect. In that sense, the point estimates are best thought of as giving some indication of the size and direction of the effect, rather than any precise value.

3.5.2.4 Validity test

There is a danger in pooling multiple synthetic control studies that the pooling is invalid; that is, we must guard against the possibility that the apparent effect of deregulation 10 years after the event is statistically different from zero only because the SCM studies we pooled

[12] The distribution we have calculated also assumes that, under the null hypothesis, the individual percentile ranks π_i are independent. Since the states in the donor pool have some overlap – the states that deregulate latest appear in the donor pool for multiple deregulation events – this assumption may not be correct. Dube and Zipperer (2015) address this concern by randomly permuting the state assignment in the whole dataset and conducting the entire exercise again as a placebo. They iterate this one million times and derive a distribution which allows statistical inference. However, as they note, this is computationally expensive – given that the computation for Section 3.5.2.5 of this chapter takes several hours, it would not be feasible to replicate this methodology – and they find little difference from the assumption of independence, concluding that“accounting for donor overlap has little impact on the estimated critical values, justifying our use of the mean of independent uniform distributions” (Dube and Zipperer, 2015, p. 15).

were systematically bad fits. For example, if we find that deregulation has a significantly positive effect on the services share of employment, we want to ensure this is not because there was a significant gap between the real states and their synthetic counterparts even prior to deregulation.

We propose therefore a simple statistical test of the validity of the pooling. We repeat the percentile rank test described above, but instead of using the percentile ranks from estimate of the gap 10 years after deregulation, we use percentile ranks based on the RMSE between the real state and its synthetic counterpart prior to treatment. The null hypothesis of this test is that the mean percentile rank generated from the RMSE is equal to 0.5. The test will therefore detect a situation in which the treated states are systematically harder to fit with synthetic counterparts than the placebo states, and will indicate that we should not rely on the results generated from pooling those studies[13]. Note that this test does not guarantee that any given synthetic control study generates a good fit; it does however give us some confidence that if the treated and placebo states end up behaving differently after deregulation, it's not because they were already behaving differently prior to deregulation.

3.5.2.5 Pooled synthetic controls results

Table 3.4 shows the results of the pooled synthetic controls study. 10 years after deregulation, states have a significantly higher services share of both employment and output than they would have done if they did not deregulate. This accords with the suggestive evidence from the difference-in-differences regressions in Section 3.5.1, and more particularly, matches the predictions made by the model in Section 3.3. Deregulation appears to have a statistically significant negative effect on the manufacturing share of output, also as predicted, and we find a negative point estimate for the effect of deregulation on the manufacturing share of employment, which is significant just below the 5% level. However, neither of the results pertaining to manufacturing pass the validity test. Our results suggest that bank branching deregulation causes a statistically significant increase in the share of output and employment accounted for by services, but that we are not able to fit the treated states' manufacturing shares using states that have not yet deregulated, so we cannot confidently say anything about the effect of bank branching deregulation on manufacturing.

In order to deal with the poor pre-treatment fit for manufacturing shares, we turn to augmented synthetic controls.

3.5.2.6 Ridge augmented synthetic controls

Ben-Michael et al. (2019) propose the augmented synthetic control method as an extension of the SCM to settings where a good pre-treatment fit is not feasible. The traditional SCM requires the treated unit to lie within the convex hull of the donor pool; that is, it restricts the

[13] The two-sided test will *also* detect a situation in which the treated states are systematically *easier* to fit with synthetic counterparts than the placebo states. While an argument could be made for considering only the one-sided version of this test, the placebo results are also invalid if the placebo fits are poor. We therefore use the two-sided version of the test.

Table 3.4: Pooled synthetic control results

Outcome Variable	Mean Gap	Mean Percentile Rank	p-Value	Validity p-Value
Services share of employment (%)	1.958	0.698	0.009***	0.333
Services share of output (%)	1.798	0.674	0.030**	0.785
Manufacturing share of employment (%)	−1.299	0.342	0.054*	0.003***
Manufacturing share of output (%)	−2.484	0.300	0.015**	0.017**

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$
Results from a pooled synthetic controls study requiring a minimum of 3 states in the donor pool, a minimum of 5 years of pre-treatment data, and 10 years of post-treatment data. 'Mean Gap' is the mean gap between the treated state and the synthetic state 10 years after deregulation across the 9 deregulation events fitting the specification. 'Mean Percentile Rank' is calculated as in equation (3.23), 10 years after deregulation. 'p-Value' indicates how statistically different 'Mean Percentile Rank' is from 0.5. The null hypothesis is that 'Mean Percentile Rank' is equal to 0.5, and thus that deregulation has no effect. 'Validity p-Value' indicates how statistically different the mean percentile rank of the RMSE prior to deregulation is from 0.5. The null hypothesis is that the mean percentile rank of the RMSE is equal to 0.5, and thus that the treated states have synthetic counterparts that are just as good as those of the placebo states, in which case the pooling is valid.

weight vector $\mathbf{W}$ to non-negative entries. The augmented SCM allows extrapolation outside this convex hull by permitting negative weights on donor units.

The following exposition closely mirrors the basic outline presented in Ben-Michael et al. (2019). Formally, as before, let $\mathbf{W} = (w_2, ..., w_{J+1})'$ be a $(J \times 1)$ vector of weights on the J units in the donor pool, such that the weights are still non-negative and sum to unity. We will index by 1 the treated unit, while the units in the donor pool are indexed from 2 to $J+1$. Let $\mathbf{X}_1$ be a $(T_0 \times 1)$ vector of observations of the outcome variable for the treated unit in the T_0 periods prior to treatment[14], and $\mathbf{X}_0$ be the $(T_0 \times J)$ matrix containing the outcome variable for the J donor units in the same T_0 periods. Then the optimal SCM weight vector $\mathbf{W}^{\text{SCM}}$ is chosen to minimise

$$(\mathbf{X}_1 - \mathbf{X}_0\mathbf{W})'(\mathbf{X}_1 - \mathbf{X}_0\mathbf{W}) + \sum_{i=2}^{J+1} \xi(w_i),$$

where ξ is a function that penalises the dispersion of the weights w_i. However, when the weights are constrained to be non-negative, as they are here, "the particular choice of dispersion penalty does not play a central role" (Ben-Michael et al., 2019, p. 5), so we do not dwell on the choice of ξ.

The weights $\mathbf{W}^{\text{SCM}}$ are the baseline SCM weights, essentially chosen by optimising pre-treatment fit and restricting weights to be non-negative, that we will augment by fitting an outcome model. Suppose for simplicity of exposition that there is only one post-treatment period, and let Y_i be the outcome in unit i in the period after unit 1 is treated, in the absence of treatment (this is the observed outcome for all units in the donor pool, and the untreated counterfactual we wish to estimate for unit 1). Let $Y_i = m(\mathbf{X}_i) + \varepsilon_i$ be a working outcome

[14]Note that, in the traditional SCM, the fit is based on K predictor variables (which may or may not include pre-treatment observations of the outcome variable). Here, the fit is solely based on minimising the distance between the real and synthetic treated units prior to treatment.

model, where m is some function, $\mathbf{X}_i$ is the $(T_0 \times 1)$ vector of pre-treatment outcomes in unit i, and ε_i is an independent error term with zero expectation. Then we can write down the bias of the SCM estimator based on the weights $\mathbf{W}^{\text{SCM}}$,

$$\text{bias} = Y_1 - \sum_{i=2}^{J+2} w_i^{\text{SCM}} Y_i = m\left(\mathbf{X}_1\right) - \sum_{i=2}^{J+2} w_i^{\text{SCM}} m\left(\mathbf{X}_i\right) + \mathbb{E}\left[\varepsilon_1 - \sum_{i=2}^{J+2} w_i^{\text{SCM}} \varepsilon_i\right].$$

Given an estimator $\hat{m}$ for m, we therefore have an estimator for the bias of the SCM estimator,

$$\widehat{\text{bias}} = \hat{m}\left(\mathbf{X}_1\right) - \sum_{i=2}^{J+2} w_i^{\text{SCM}} \hat{m}\left(\mathbf{X}_i\right),$$

which yields a bias-corrected SCM estimator for Y_1,

$$\begin{aligned} \hat{Y}_1^{\text{aug}} &= \sum_{i=2}^{J+2} w_i^{\text{SCM}} Y_i + \left(\hat{m}\left(\mathbf{X}_1\right) - \sum_{i=2}^{J+2} w_i^{\text{SCM}} \hat{m}\left(\mathbf{X}_i\right)\right) \\ &= \hat{m}\left(\mathbf{X}_1\right) + \sum_{i=2}^{J+2} w_i^{\text{SCM}} \left(Y_i - \hat{m}\left(\mathbf{X}_i\right)\right). \end{aligned}$$

All that remains to fit such an estimator therefore is to choose a function to employ as the estimator $\hat{m}$. Ben-Michael et al. (2019) focus primarily on a ridge-regularised linear model, $\hat{m}\left(\mathbf{X}_i\right) = \hat{v}_0^r + \mathbf{X}_i' \hat{\mathbf{v}}^r$, where

$$\left\{\hat{v}_0^r, \hat{\mathbf{v}}^r\right\} = \arg\min_{v_0, \mathbf{v}} \frac{1}{2} \sum_{i=2}^{J+1} \left(Y_i - \left(v_0 + \mathbf{X}_i' \mathbf{v}\right)\right)^2 + \chi^r \mathbf{v}' \mathbf{v},$$

given a penalty hyper-parameter χ^r that determines how sensitive we are to deviations from the initial SCM estimates. This leads to the ridge augmented synthetic control estimator

$$\hat{Y}_1^{\text{aug}} = \sum_{i=2}^{J+2} w_i^{\text{SCM}} Y_i + \left(\mathbf{X}_1 - \sum_{i=2}^{J+2} w_i^{\text{SCM}} \mathbf{X}_i\right)' \hat{\mathbf{v}}^r.$$

The choice of χ^r is important: as $\chi^r \to \infty$, the ridge-augmented estimator converges to the original SCM estimator; as $\chi^r \to 0$, by contrast, pre-treatment fit becomes perfect but only at the expense of potentially significant extrapolation outside the convex hull of the donor pool, which could exacerbate errors in the post-treatment estimate. In practice, a 'leave one out' cross-validation approach to the selection of χ^r is proposed, in which χ^r is chosen to minimise the mean squared error between the true pre-treatment outcomes in the treated unit and estimates of those outcomes based on the ridge augmented synthetic control estimator (Ben-Michael et al., 2019, p. 13).

The ridge-augmented synthetic control estimator has a number of interesting properties, elucidated in the original paper. First, it is a weighting estimator which allows for potentially negative weights on donor units, but which penalises deviation from the baseline SCM estimator, so limits extrapolation bias and over-fitting to noise. Second, ridge augmented SCM (ridge ASCM) improves the pre-treatment fit of a synthetic control relative to traditional SCM, suggesting we may have more luck in finding good fits for pre-treatment manufacturing shares than we did in the results reported in Table 3.4. Finally, the authors show using simulation studies that there are estimation gains from using ridge ASCM relative to SCM alone. These properties make it a useful method to adopt, given the poor pre-treatment fits for the manufacturing shares of the economy found in Section 3.5.2.5, since we will get a better fit, with potentially more accurate results, while minimising unnecessary extrapolation outside the convex hull of the donor states. We therefore turn now to implementing ridge ASCM to study the effects of bank branching deregulation on structural composition.

3.5.2.7 Pooled ridge ASCM results

Given the ridge ASCM, we can conduct a pooled study based on individual synthetic controls, conducted exactly as in Section 3.5.2.5. The results of this pooled study are presented in Table 3.5. We find that bank branching deregulation caused a significantly increased services share of employment, and a significantly reduced manufacturing share of both employment and output, 10 years after deregulation. While we find a positive point estimate for the effect of deregulation on the services share of output, the mean percentile rank is not quite significant, with a p-value of around 13%. Ridge ASCM has substantially improved the pre-treatment fit relative to SCM alone: all of the validity p-values have increased, and we do not come close to rejecting the null hypothesis – that the treated states are fit as well as the placebo studies – for any of the outcome variables. The point estimates found for the effect on services are very similar to those found with traditional SCM, but the magnitude of the effects on the manufacturing shares are substantially increased with ridge ASCM. This suggests that, while we could not find good fits for the manufacturing shares using traditional synthetic controls, the bad fits were actually working to *decrease* rather than increase the apparent effect of deregulation.

We therefore find that bank branching deregulation has a significant effect on structural change, accelerating the secular shift that was already underway towards services and away from manufacturing, as our theory predicted in Section 3.3. The United States from the 1970s to the 1990s was relatively economically developed, so it is unclear whether these results would hold in the context of a less developed economy; indeed, Heblich and Trew (2019) find that finance increases manufacturing employment in an industrialising economy. Looking at the same bank branching deregulation episode as we consider in this chapter, other work has shown that manufacturing firms achieve faster TFP growth post-deregulation (Jerzmanowski, 2017), consistent with our model. This mechanism may only work in relatively industrialised economies, when manufacturing firms have accumulated sufficient capital but would like to invest more in R&D, for example. Nonetheless, our results show a significant effect of bank branching deregulation on those states in the pooled synthetic controls study.

Table 3.5: Pooled ridge ASCM results

Outcome Variable	Mean Gap	Mean Percentile Rank	p-Value	Validity p-Value
Services share of employment (%)	2.081	0.738	0.003***	0.865
Services share of output (%)	1.807	0.628	0.130	0.821
Manufacturing share of employment (%)	−2.297	0.304	0.018**	0.635
Manufacturing share of output (%)	−3.566	0.231	0.001***	0.737

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$
Results from a pooled ridge ASCM study requiring a minimum of 3 states in the donor pool, a minimum of 5 years of pre-treatment data, and 10 years of post-treatment data. 'Mean Gap' is the mean gap between the treated state and the synthetic state 10 years after deregulation across the 9 deregulation events fitting the specification. 'Mean Percentile Rank' is calculated as in equation (3.23), 10 years after deregulation. 'p-Value' indicates how statistically different 'Mean Percentile Rank' is from 0.5. The null hypothesis is that 'Mean Percentile Rank' is equal to 0.5, and thus that deregulation has no effect. 'Validity p-Value' indicates how statistically different the mean percentile rank of the RMSE prior to deregulation is from 0.5. The null hypothesis is that the mean percentile rank of the RMSE is equal to 0.5, and thus that the treated states have synthetic counterparts that are just as good as those of the placebo states, in which case the pooling is valid.

3.6 Conclusions

Existing evidence shows that bank branching deregulation accelerates output growth (Jayaratne and Strahan, 1996) and accelerates TFP growth in the manufacturing sector (Jerzmanowski, 2017). We construct an infinite-horizon general equilibrium model in which a positive financial shock causes faster output growth and faster TFP growth in manufacturing; in our model, such a shock also accelerates structural change, leading to a greater share of services and a smaller share of manufacturing in the economy.

We proceed to assess the effect on structural change of state-by-state bank branching deregulation in the United States, initially using difference-in-differences regressions. We find that deregulation increases services over time as a share of the economy, and decreases manufacturing. To alleviate concerns with endogeneity of treatment, we employ a more robust estimation strategy by exploiting the synthetic control method (SCM). The SCM is specification-free, and does not rely on exogeneity of treatment for valid inference. We follow Dube and Zipperer (2015) in pooling the results across several synthetic control case studies in order to increase their statistical power, determining statistical significance with reference to the mean percentile rank, whose distribution is known exactly under the null hypothesis of uniform distribution of individual percentile ranks. We also propose a simple statistical test of the validity of pooling multiple synthetic controls studies. To improve the pre-treatment fit of the synthetic controls, we further employ the ridge augmented synthetic control method (ridge ASCM). We find that bank branching deregulation significantly increases the share of services and decreases the share of manufacturing in the economy 10 years after deregulation, consistent with our theoretical predictions.

Conclusions

This thesis aimed to determine whether there is a link between structural change and both investment irreversibility, such as an increase in the use of intangible capital, and financial development in advanced economies. First, a partial equilibrium model was developed of a firm's life cycle and optimal investment programme. Consistent with stylised empirical facts, this model predicts that firms should rely on external finance and retain all revenue when young, before reaching maturity and becoming less reliant on external finance; that capital is accumulated gradually, despite there being no adjustment costs; and that increasing investment irreversibility is associated with less corporate borrowing and more corporate saving overall. The novel approach of studying the firm's life cycle in the presence of an explicit parameter encoding investment irreversibility allowed us to show that a greater reliance on intangible capital – which is harder to liquidate than traditional capital, and investment in which is therefore less reversible – should have not just macroeconomic but also firm-level effects. When investment irreversibility increases, firms grow more slowly and reach maturity at a younger age and a smaller size.

Once these firms and their optimal investment programme had been characterised, they were embedded in a two-sector general equilibrium model. When consumers consider services and manufactured goods to be complements, increasing investment irreversibility leads to a greater long-run share of manufacturing in the economy, both in terms of output and employment. This effect however is rather modest, with much greater effects on aggregate output, consumption and wages. We consider the welfare implications of investment irreversibility: government subsidies to capital liquidation, financed by taxes on households, are found often to increase long run consumption and therefore to be welfare-enhancing. This is true for a wide array of parameter choices, but it is particularly true when the underlying irreversibility of investment is high.

Finally, we predict using a general equilibrium model that financial development should accelerate structural change towards services and away from manufacturing. In addition to deriving novel predictions, this model also explains existing evidence that bank branching deregulation accelerated output growth (Jayaratne and Strahan, 1996), and that it accelerated manufacturing TFP growth and aggregate capital accumulation, but did not accelerate capital accumulation in the manufacturing sector (Jerzmanowski, 2017). Using a pooled ridge augmented synthetic controls study, we show empirically that bank branching deregulation in the United States did indeed accelerate structural change in the manner predicted.

What does this all add up to? At least in theory, both investment irreversibility and financial development *do* affect the structural composition of the economy, and in the case of financial development this link is confirmed empirically. The research questions posed in the introduction have therefore been answered. However, we predict in Chapter 2 that when $\epsilon < 1$ increasing irreversibility should increase the size of the manufacturing sector, not the services sector. This clearly does not accord with the broad sweep of recent experience. The predicted effect is rather small, though, and could easily be dominated by structural change driven by differential TFP growth rates, if the two were to be modelled simultaneously. Thus it may still be the case that our predictions in Chapter 2 are correct: that increasing irreversibility *does* increase the size of the manufacturing sector, *ceteris parabus*. A rigorous empirical study would be required to test this hypothesis in the real world (see below for a more detailed discussion of potential future work following on from this thesis).

In conducting this research, we sought to understand whether coincident trends in macroeconomics could have some effect on the structural composition of the economy, and to understand the channels through which any such effects might operate. On that front, we have made significant progress: increasing investment irreversibility essentially increases the cost of capital to firms, while increasing financial development essentially decreases its cost. It seems natural therefore that these two phenomena should have effects that are pushing in opposite directions, both in aggregate growth terms and in structural change terms. An obvious future stream of work would be to model both of these phenomena together, and check how closely such a model accords with real-world data.

Further work

There are a number of natural extensions to the work in this thesis. Surprisingly, the firms in Chapter 1 essentially only ever spend a single period in the large loan regime when their life cycle is simulated. This gives a limited role to the bank's liquidation value of capital θ^B, and indeed tweaking this variable changes very little in aggregate outcomes (as is also the case in Chapter 2). Although finance plays an important role by limiting the amount firms can borrow while still only paying the risk-free interest rate r, 'financial development' as parametrised by θ^B is not particularly important. While for the purposes of studying the effects of investment irreversibility it was sufficient that the *firm's* liquidation value of capital was important, a different microfounded theory of investment which places greater importance on the quality of the financial system and the *bank's* liquidation value of capital may therefore yield interesting results regarding the firm's optimal investment programme.

Chapter 2 predicts that investment irreversibility will affect the structural composition of the economy, although the direction of this effect depends on household preferences. First, an empirical study confirming the relationship between investment irreversibility and structural change would be instructive: is there actually a relationship at all? In which direction does it run? In the absence of direct estimates of households' elasticity of substitution between goods and services produced in different sectors – which may exist, but which we have been unable to find – an estimate of the direction of the effect of increasing irreversibility on structural change may also provide indirect evidence of household preferences.

The inclusion in Chapter 2 of a continuum of firms that are heterogeneous in size and age makes dynamical analysis of the economy difficult, since we cannot take the usual approach of assuming a representative firm. A model of investment irreversibility and structural change that does not rely on this variety of firms but produces similar stylised facts would be useful for studying the transition of the economy. More challengingly, but perhaps more interestingly, the ability to aggregate neatly across such a continuum would allow for dynamical analysis without sacrificing the microfoundations of investment irreversibility currently present in the model.

The theoretical model in Chapter 3 relies on learning-by-doing spillovers to capital accumulation in the manufacturing sector to generate differential productivity growth rates in different sectors, which is the ultimate engine of structural change. There are other mechanisms that would generate structural change, however: exogenous productivity gaps as in Ngai and Pissarides (2007), or different input factor intensities as in Chapter 2. The synthesis of these mechanisms with a financial sector may generate similar predictions to those in Theorem 3.7, and would be a relatively straightforward extension to the chapter.

A very natural project to consider would be to combine two of stories in this thesis, the effects of investment irreversibility and productivity growth respectively on structural change. If we assume $\epsilon < 1$ in households' utility function in Chapter 2, then decreasing investment reversibility – for example, as a result of the employment of more intangible capital – leads to a larger manufacturing sector and a smaller services sector, contrary to observed shifts over time. However, this effect is rather small. Conversely, assuming that $\epsilon < 1$ in Chapter 3 generates an acceleration in structural change of the kind that we see in the data, which lends some support to the idea that services and manufactured goods really are complements in households' utility functions. How do we square these results? Perhaps TFP growth – the engine of structural change in Chapter 3 – can only occur when firms move to new, less tangible input factors. Google can only work if it runs on software, rather than heavy machinery. Thus *both* stories might be true at once: the move to less tangible capital tends to slow structural change, but not enough to cancel it out altogether. Since the effects of investment irreversibility on aggregate output are predicted in Chapter 2 to be rather greater than the effects on structural composition, this shift might manifest as a slowing in growth. Perhaps therefore secular stagnation – persistently low growth in advanced economies over recent years despite low unemployment, and the inability of monetary policy to combat it – is a temporary, supply-side phenomenon associated with the increasing use of intangible capital in more productive technologies.

Finally, it was stated in the introduction that this thesis was not concerned with a normative analysis of structural change. There is clearly scope to take at least one step in that direction, however. While the general equilibrium models employed above have assumed wage equalisation between sectors, and homogeneous households, the most cursory glance at the real world shows this is a substantial simplification. Anecdotally, it seems that there has been a proliferation in low-status, low-wage, insecure employment in advanced economies' services industries, perhaps at the expense of manufacturing jobs that were at least perceived to be better for employees. A very naïve analysis might conclude that this is fine, so long as output is growing. A slightly less naïve analysis might suggest that output growth is the domain of economics, and 'dividing the pie' is the domain of politics, so economists shouldn't concern themselves with such questions. Even if this is true, economists have a role to play in providing politicians with options and studying the effects of different policies. There might therefore be much to learn from a model of structural change in which workers vary in their level of human capital, and different jobs demand different types of worker. Such a model might accord more closely with observed reality, and help policymakers understand the best way to chart a path into an uncertain future.

Appendix A

Chapter 1 Appendices

A.1 Derivation of comparative statics for an individual firm in Section 1.3.7

Given the definitions in equations (1.16), (1.15), (1.17) and (1.18), the effect of each parameter on k^*, $\bar{k}_t(\Omega_{t-1})$, $\bar{k}_t^S(\Omega_{t-1})$ and k^{**} is straightforward to derive, as are the effects of θ^B, μ, $\bar{\varphi}$, p and r on $\bar{k}_t^L(\Omega_{t-1})$ given the definition in equation (1.19).

Fix some date T and take Ω_{T-1} as given. In order to assess the effect of δ on the large-loan-constrained capital stock $\bar{k}_t^L(\Omega_{T-1})$, we note that $\bar{k}_T^L$ must satisfy

$$G\left(\bar{k}_T^L\right) = 0,$$

where $G(k_T)$ is defined as in equation (1.29). By the Implicit Function Theorem,

$$\frac{\partial \bar{k}_T^L}{\partial \delta} = -\left(\left.\frac{\partial G}{\partial k_T}\right|_{k_T=\bar{k}_T^L}\right)^{-1}\left(\left.\frac{\partial G}{\partial \delta}\right|_{k_T=\bar{k}_T^L}\right).$$

By the properties of $f(k_T)$, it's clear that G must be decreasing in k_T at the point where $G(k_T) = 0$ – that is, at $k_T = \bar{k}_T^L$. Thus

$$\operatorname{sgn}\left\{\frac{\partial \bar{k}_T^L}{\partial \delta}\right\} = \operatorname{sgn}\left\{\left.\frac{\partial G}{\partial \delta}\right|_{k_T=\bar{k}_T^L}\right\}.$$

We proceed therefore to consider $\partial G/\partial\delta$. We note that

$$\begin{aligned}
\left.\frac{\partial G\left(k_T\right)}{\partial\delta}\right|_{k_T=\bar{k}_T^L} &= \frac{1+r}{\left(1-\delta\right)^2}\left(\Omega_{T-1}-\bar{k}_T^L\right)+\left[\frac{1}{1-\delta}+\frac{\delta}{\left(1-\delta\right)^2}\right]\theta^B\left(1-\mu\right)\bar{k}_T^L \\
&= \frac{1}{\left(1-\delta\right)^2}\left[\left(1+r\right)\left(\Omega_{T-1}-\bar{k}_T^L\right)+\theta^B\left(1-\mu\right)\bar{k}_T^L\right] \\
&< \frac{1-\mu}{\left(1-\delta\right)^2}\left[\left(1+r\right)\Omega_{T-1}-\left(1+r-\theta^F\left(1-\mu\right)\right)\bar{k}_T^L\right] \\
&< \frac{1-\mu}{\left(1-\delta\right)^2}\left[\left(1+r\right)\Omega_{T-1}-\left(1+r-\theta^F\left(1-\mu\right)\right)\bar{k}_T\right] \\
&= \frac{1-\mu}{\left(1-\delta\right)^2}\left[\left(1+r\right)\Omega_{T-1}-\left(1+r\right)\Omega_{T-1}\right] \\
&= 0,
\end{aligned}$$

where the final inequality follows from the fact that in order to be under consideration by the firm, by definition, $\bar{k}_T^L$ must be a capital stock that necessitates a large loan, so given fixed Ω_{T-1} it must be larger than $\bar{k}_T$, which is the largest capital stock possible taking only a small loan. Therefore δ has a negative effect on $\bar{k}_T^L\left(\Omega_{T-1}\right)$.

A.2 Numerical modelling sensitivity analysis

Here we repeat the numerical simulations undertaken in Section 1.4, but varying parameters other than θ^F one at a time to assess the effect of these parameters. First, Figure A.1 shows the firm's life cycle for $\theta^F = 0.2$ and $\theta^F = 0.8$, as before, but having decreased the output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$; then Figure A.2 shows the same leaving β as it was in the baseline specification, but increasing depreciation from $\mu = 0.05$ to $\mu = 0.1$; Figure A.3 illustrates the case when the firm's death increases risk from $\delta = 0.1$ to $\delta = 0.2$ in any given period; Figure A.4 illustrates the case when interest rates increase from $r = 0.02$ to $r = 0.05$; Figure A.5 illustrates the case when productivity increases from $\varphi = 10$ to $\varphi = 20$; and finally Figure A.6 shows the firm's life cycle when the bank's liquidation value of capital increases from $\theta^B = 0.1$ to $\theta^B = 0.2$.

Figure A.7 shows average firm characteristics when varying one parameter at a time from its value in the baseline specification.

Figure A.8 shows the response of capital, liquid wealth, borrowing, failure dividends and entry value, as a share (multiple) of revenue like in Figure 1.4, but having decreased the output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$; then Figure A.9 shows the same when increasing depreciation from $\mu = 0.05$ to $\mu = 0.1$; Figure A.10 illustrates the case when the firm's death increases risk from $\delta = 0.1$ to $\delta = 0.2$ in any given period; Figure A.11 illustrates the case when interest rates increase from $r = 0.02$ to $r = 0.05$; Figure A.12 illustrates the case when productivity increases from $\varphi = 10$ to $\varphi = 20$; and finally Figure A.13 illustrates the case when the bank's liquidation value of capital increases from $\theta^B = 0.1$ to $\theta^B = 0.2$.

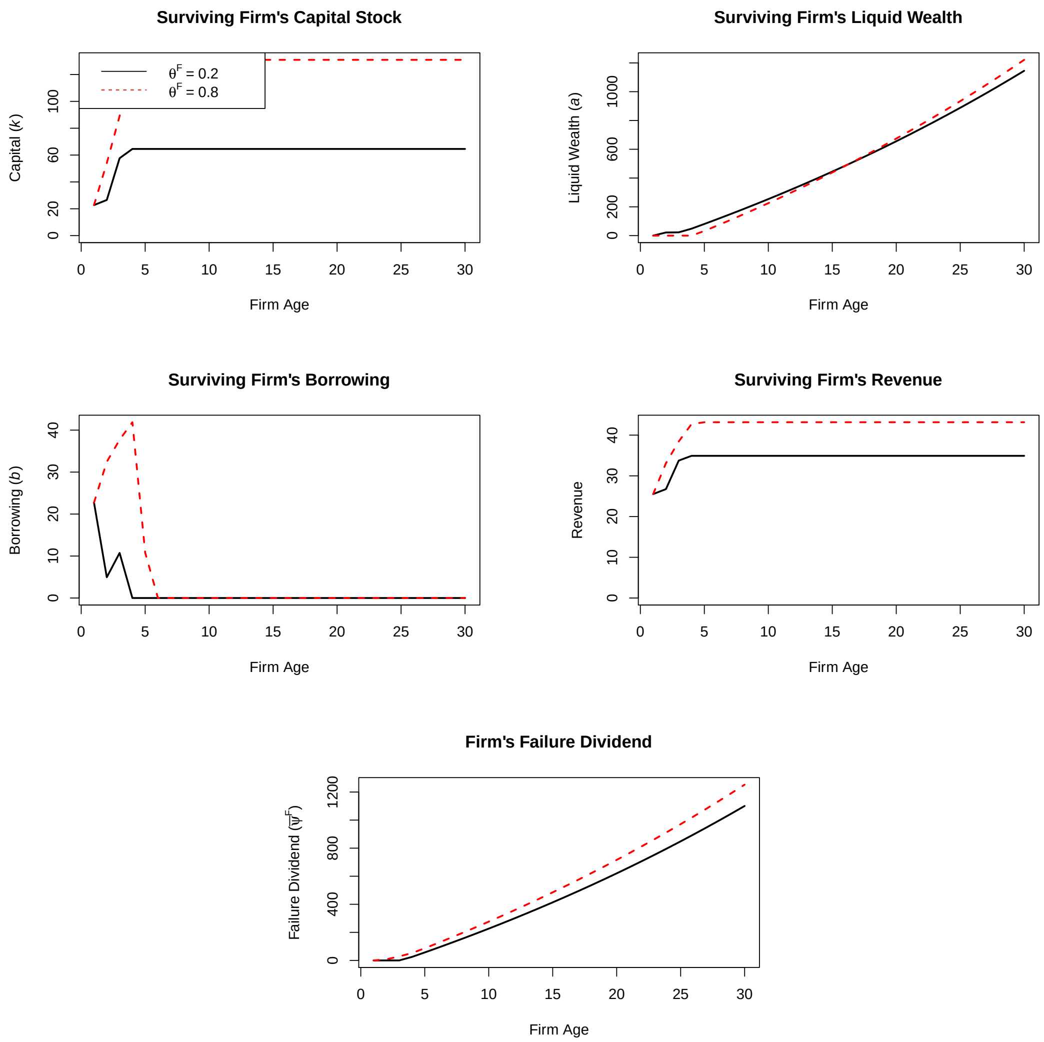

Figure A.1: The firm's life cycle: decrease in β

Simulations of the firm's life cycle with $\theta^F = 0.2$ (the solid black line) and $\theta^F = 0.8$ (the dotted red line). This is the same as Figure 1.2, but decreasing the output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

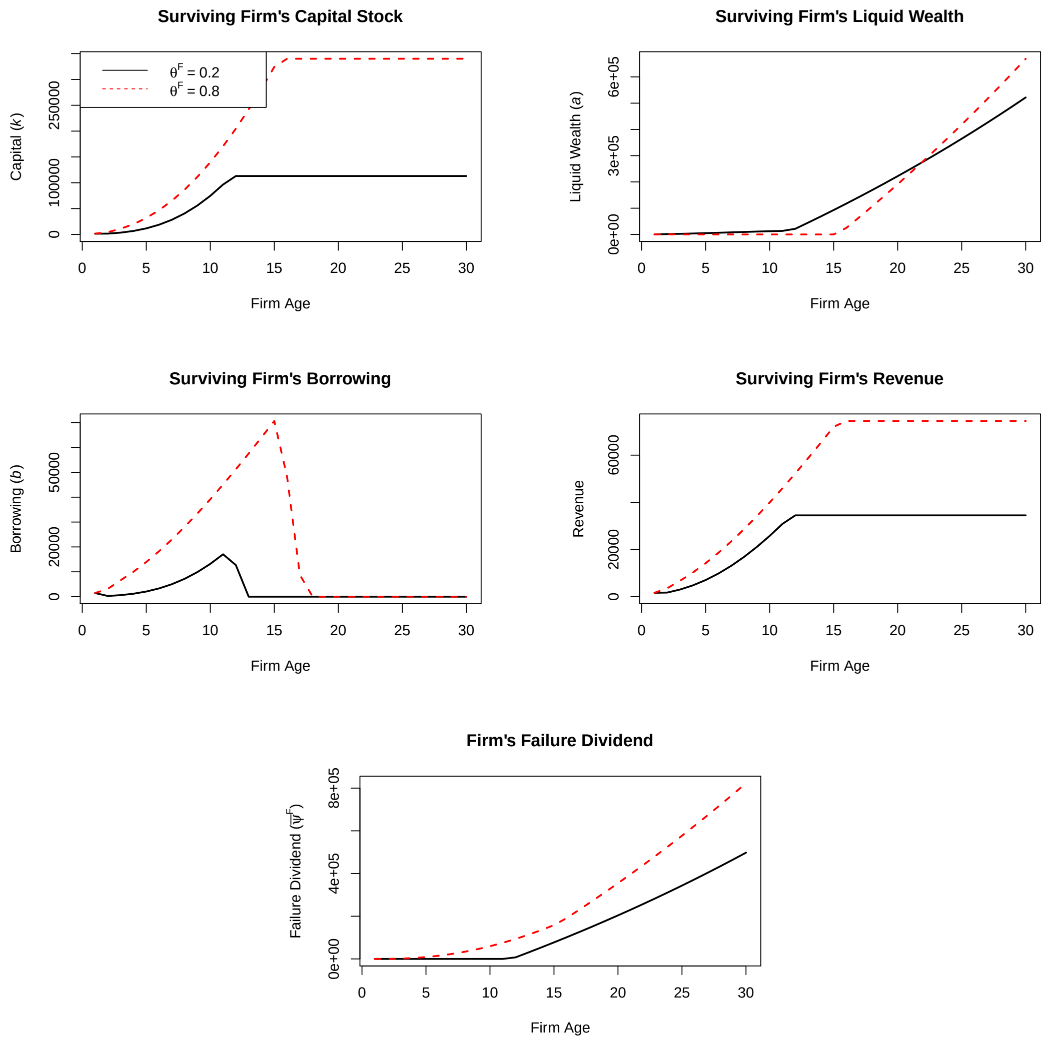

Figure A.2: The firm's life cycle: increase in μ

Simulations of the firm's life cycle with $\theta^F = 0.2$ (the solid black line) and $\theta^F = 0.8$ (the dotted red line). This is the same as Figure 1.2, but increasing depreciation from $\mu = 0.05$ to $\mu = 0.1$.

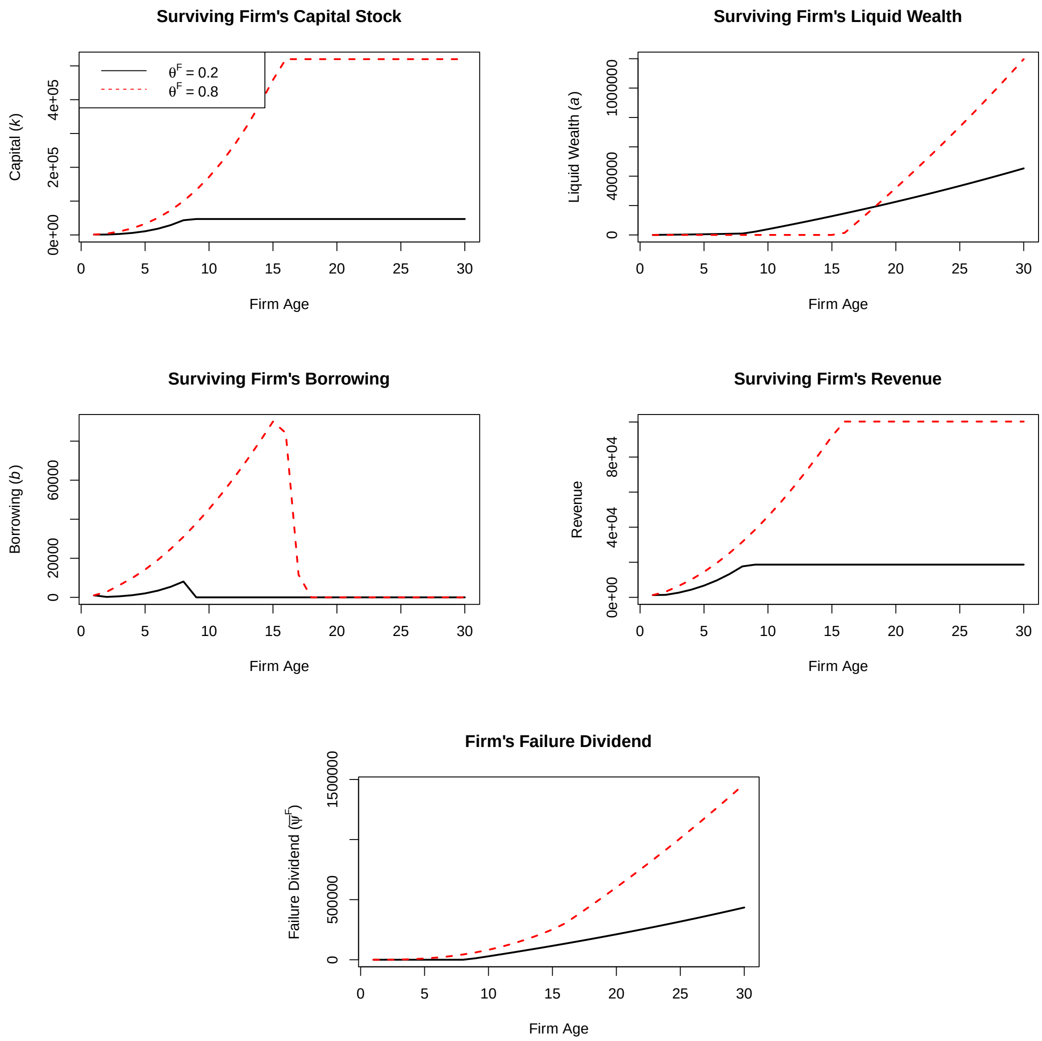

Figure A.3: The firm's life cycle: increase in δ

Simulations of the firm's life cycle with $\theta^F = 0.2$ (the solid black line) and $\theta^F = 0.8$ (the dotted red line). This is the same as Figure 1.2, but increasing the firm's per-period death risk from $\delta = 0.1$ to $\delta = 0.2$.

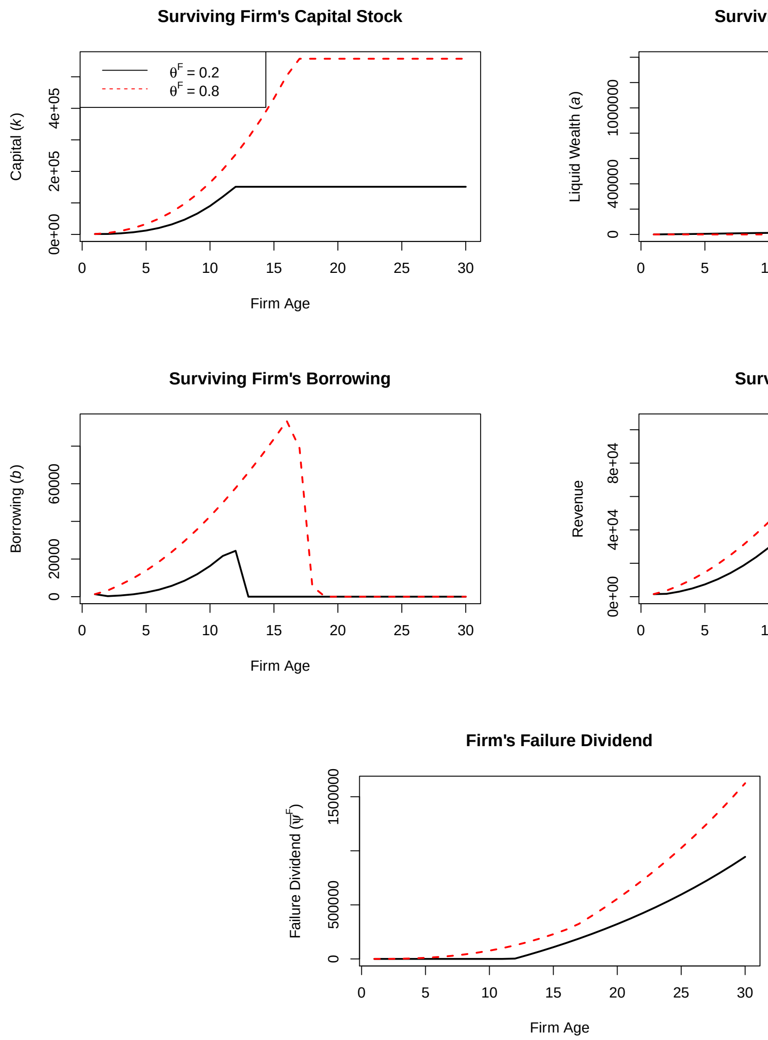

Figure A.4: The firm's life cycle: increase in r

Simulations of the firm's life cycle with $\theta^F = 0.2$ (the solid black line) and $\theta^F = 0.8$ (the dotted red line). This is the same as Figure 1.2, but increasing the interest rate from $r = 0.02$ to $r = 0.05$.

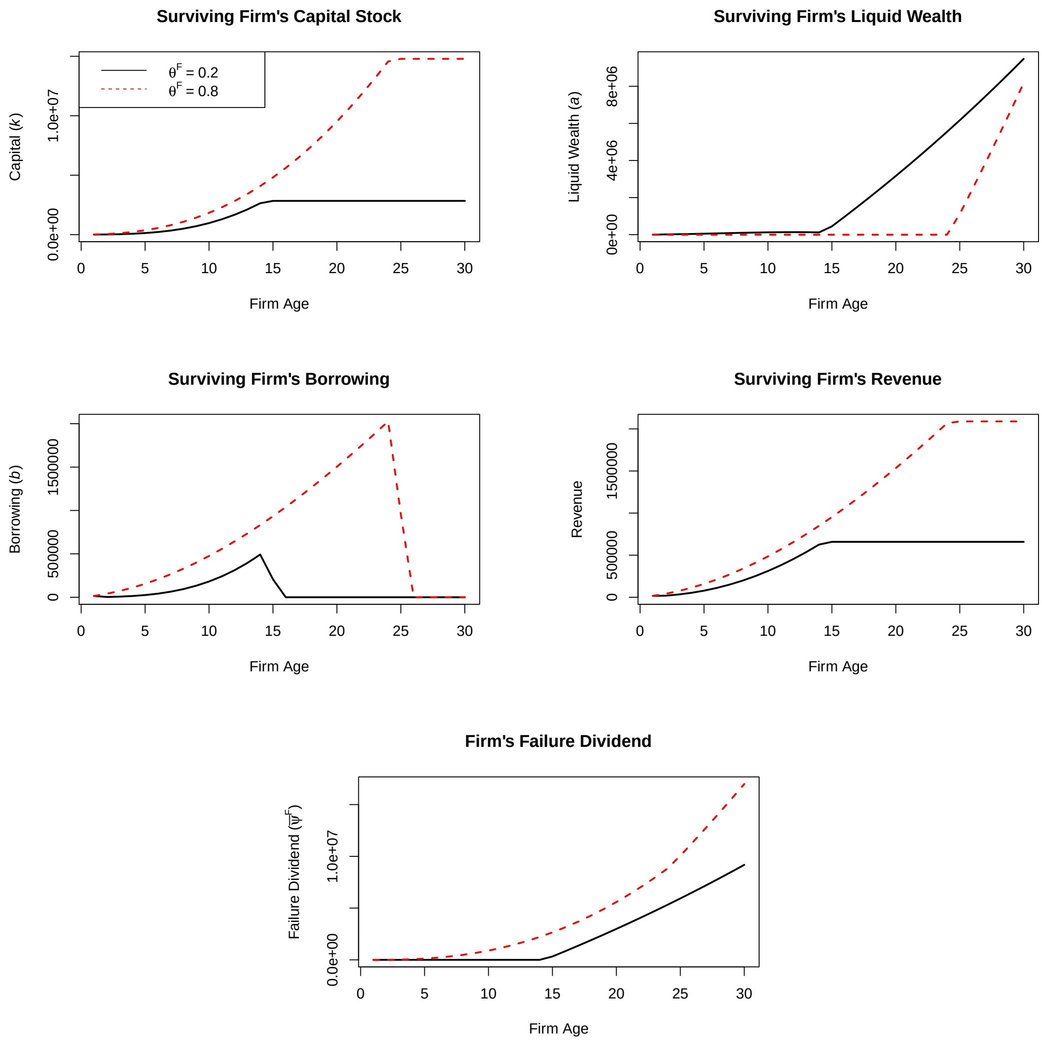

Figure A.5: The firm's life cycle: increase in φ

Simulations of the firm's life cycle with $\theta^F = 0.2$ (the solid black line) and $\theta^F = 0.8$ (the dotted red line). This is the same as Figure 1.2, but increasing the firm's productivity from $\varphi = 10$ to $\varphi = 20$.

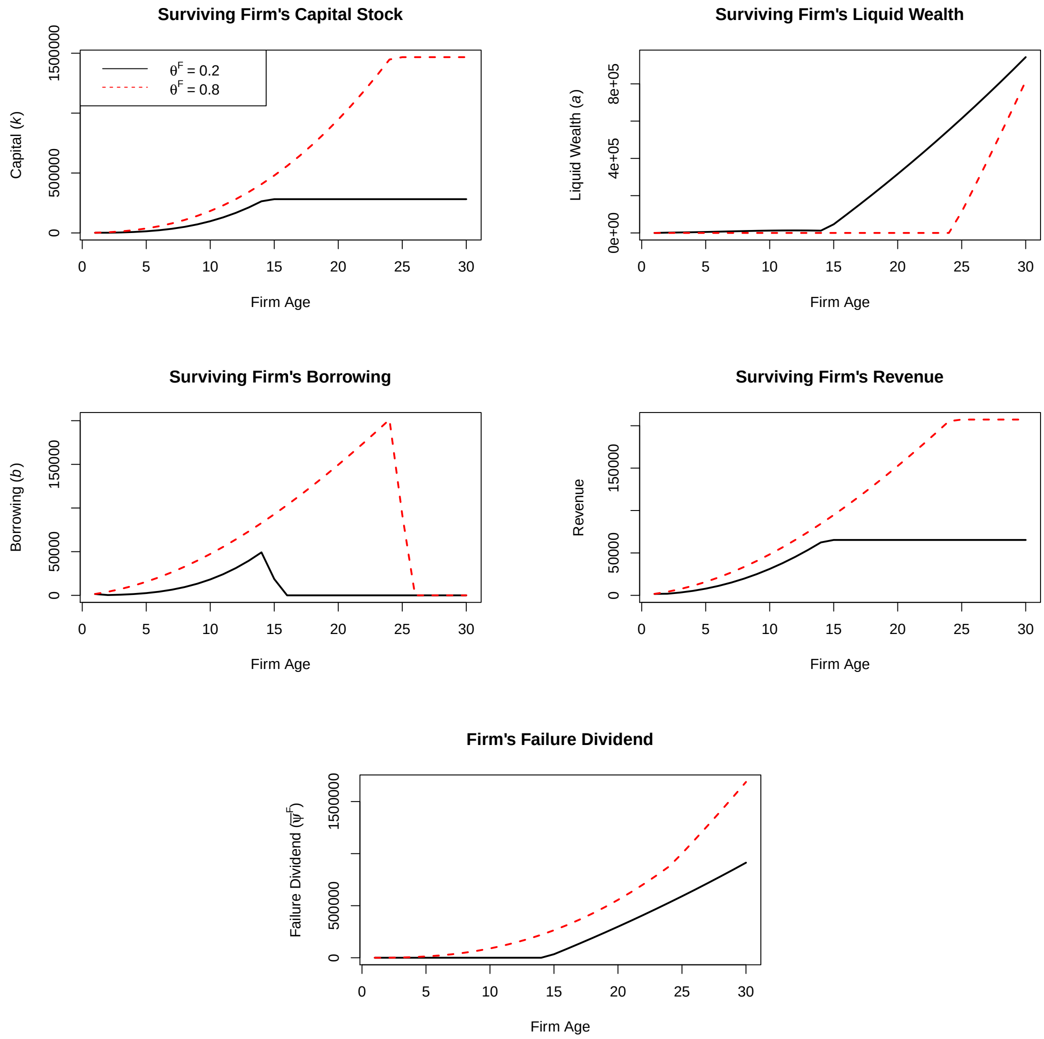

Figure A.6: The firm's life cycle: increase in θ^B

Simulations of the firm's life cycle with $\theta^F = 0.2$ (the solid black line) and $\theta^F = 0.8$ (the dotted red line). This is the same as Figure 1.2, but increasing the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

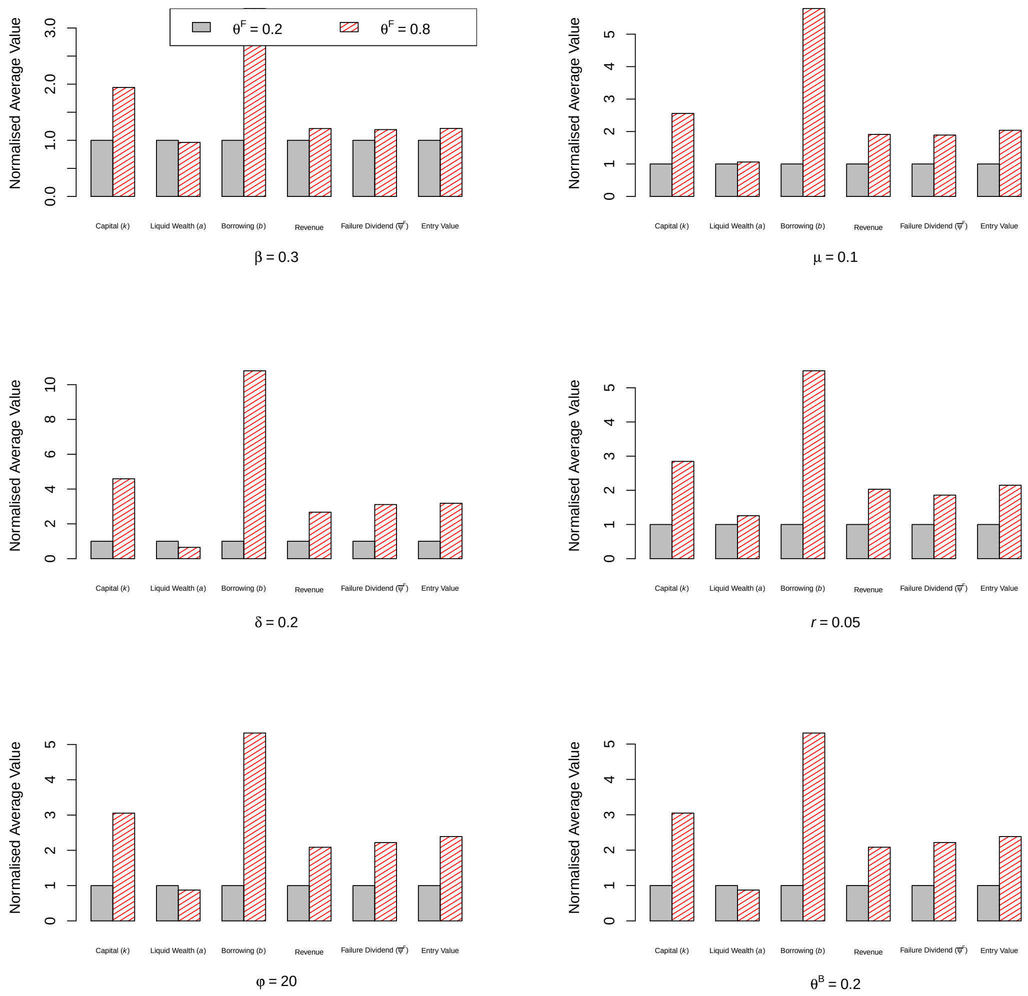

Figure A.7: The average firm: sensitivity analysis

Simulations of average firm characteristics with $\theta^F = 0.2$ (the solid grey bars) and $\theta^F = 0.8$ (the hatched red bars), with the values for $\theta^F = 0.2$ normalised to unity. Each panel shows average firm characteristics when a single parameter is increased relative to the baseline specification; the new value is specified under each chart. Varying θ^B makes almost no difference to the firm's behaviour, so the bottom right panel of this figure is almost identical to the baseline specification.

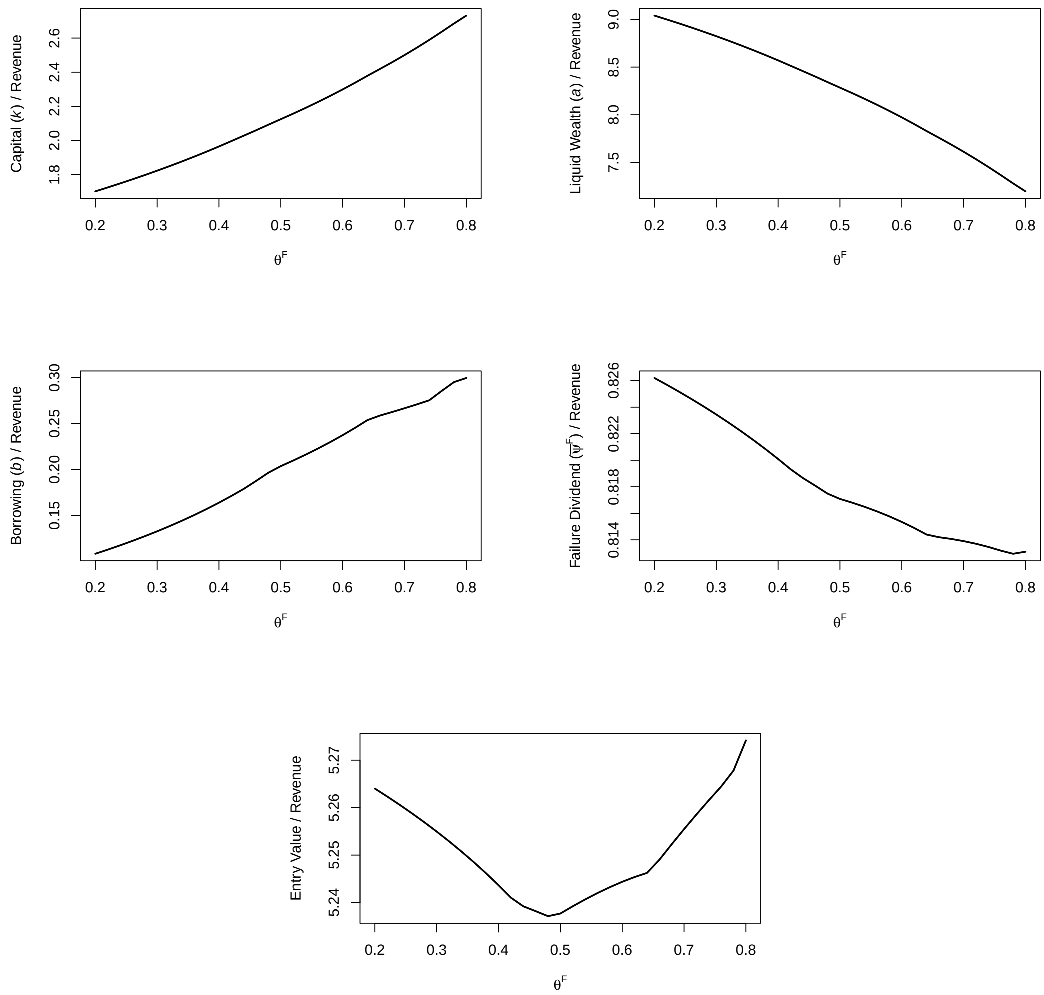

Figure A.8: Average firm characteristics divided by revenue: decrease in β

Simulations of average firm characteristics divided by average firm revenue, plotted against the resale value of capital θ^F. This is the same as Figure 1.4, but decreasing the output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

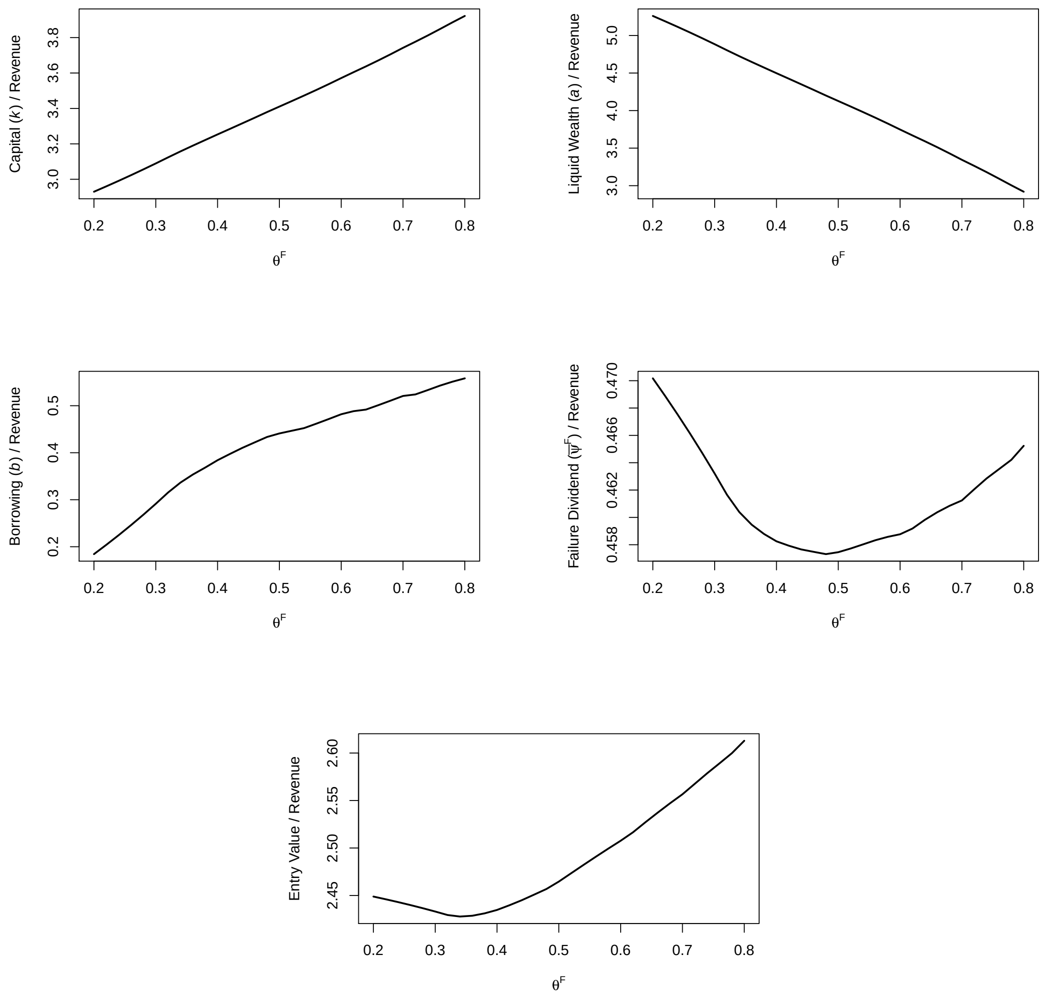

Figure A.9: Average firm characteristics divided by revenue: increase in μ

Simulations of average firm characteristics divided by average firm revenue, plotted against the resale value of capital θ^F. This is the same as Figure 1.4, but increasing depreciation from $\mu = 0.05$ to $\mu = 0.1$.

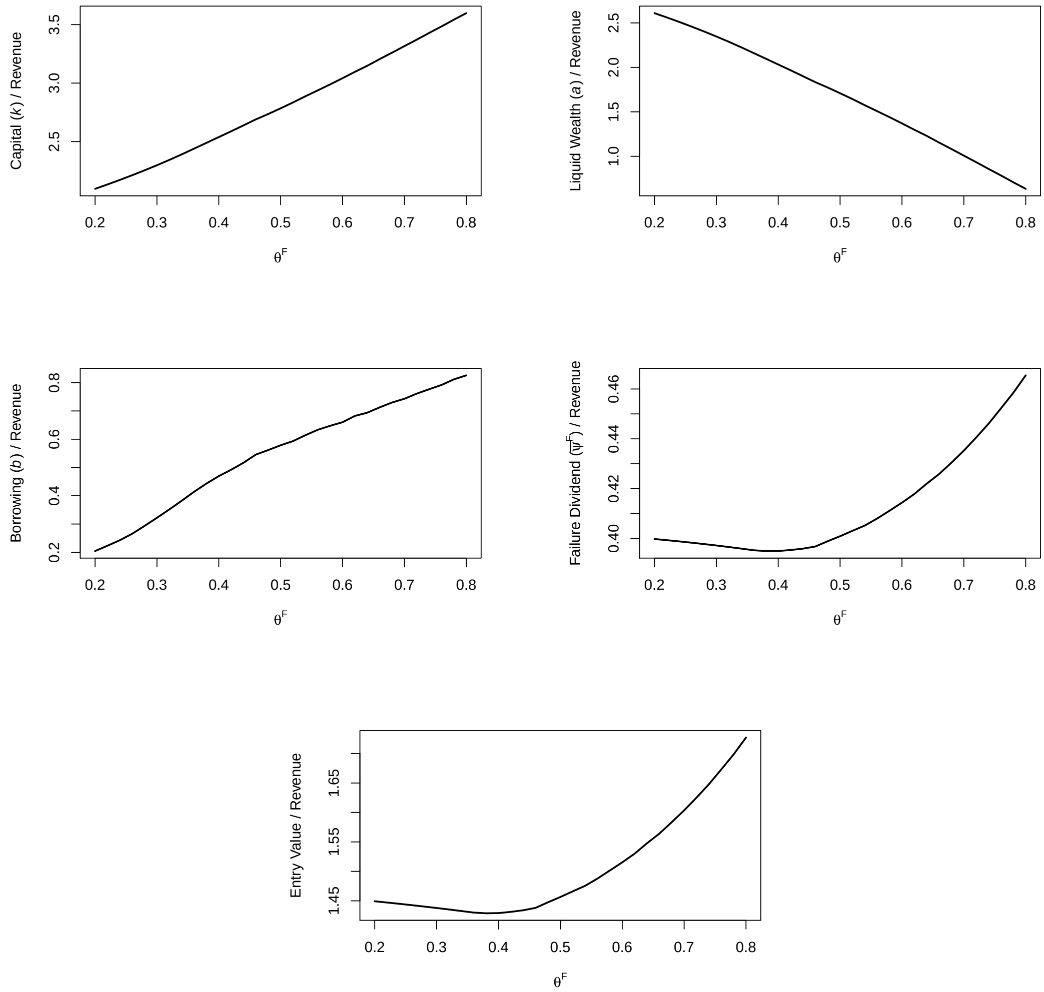

Figure A.10: Average firm characteristics divided by revenue: increase in δ

Simulations of average firm characteristics divided by average firm revenue, plotted against the resale value of capital θ^F. This is the same as Figure 1.4, but increasing the firm's per-period death risk from $\delta = 0.1$ to $\delta = 0.2$.

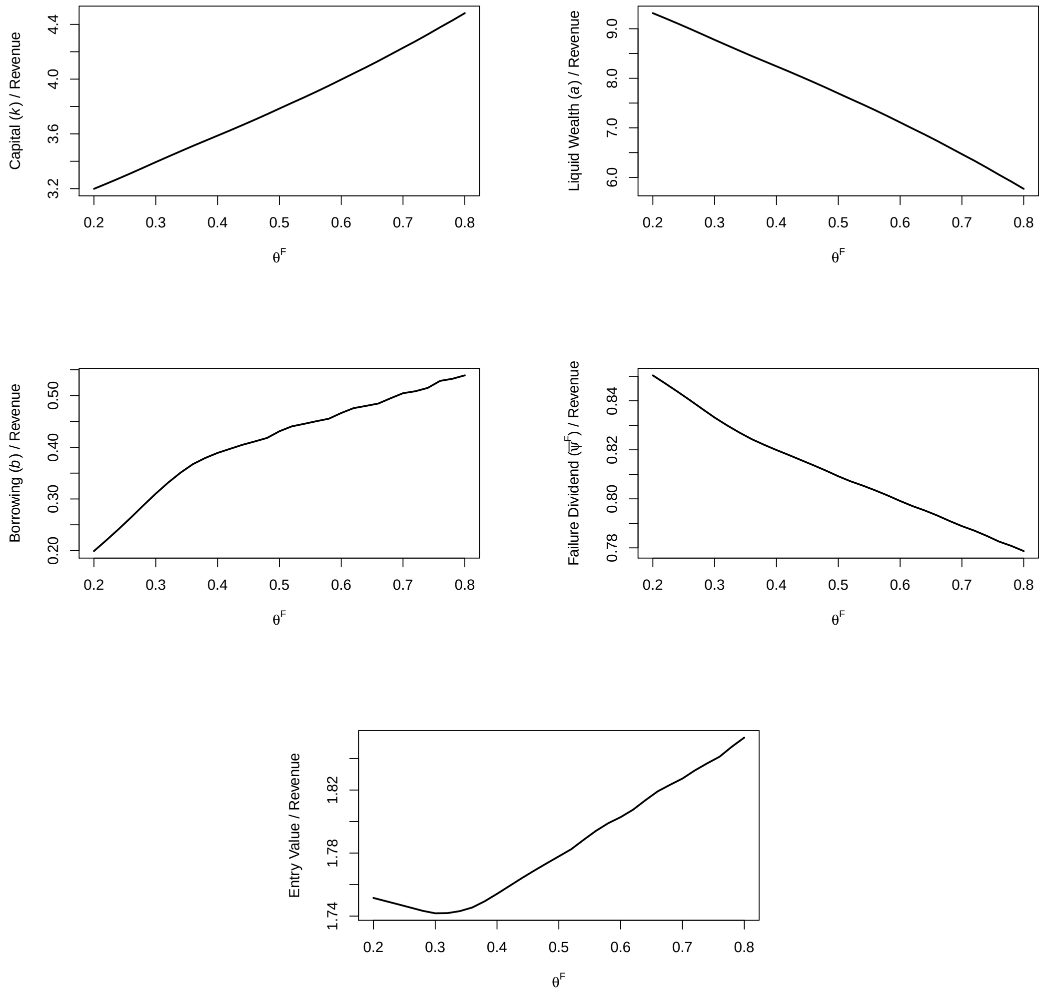

Figure A.11: Average firm characteristics divided by revenue: increase in r

Simulations of average firm characteristics divided by average firm revenue, plotted against the resale value of capital θ^F. This is the same as Figure 1.4, but increasing the interest rate from $r = 0.02$ to $r = 0.05$.

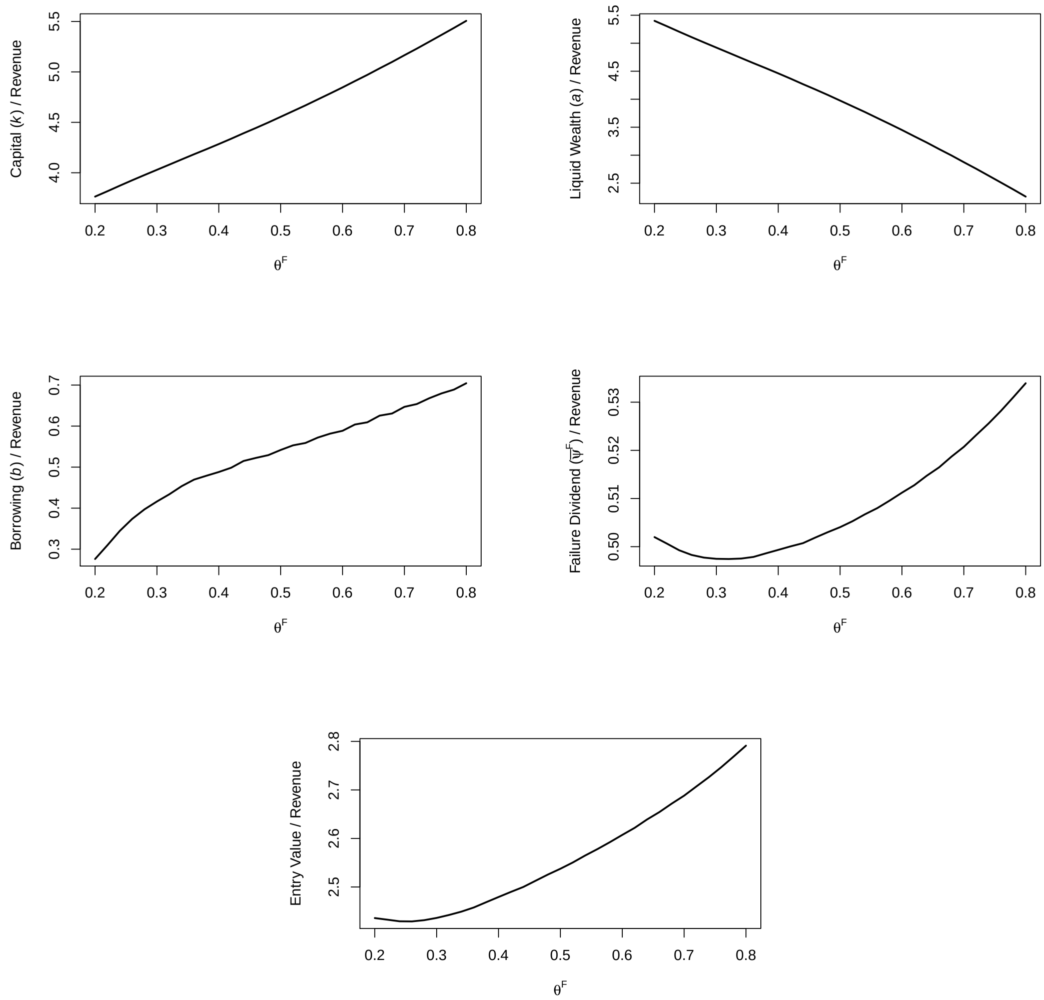

Figure A.12: Average firm characteristics divided by revenue: increase in φ

Simulations of average firm characteristics divided by average firm revenue, plotted against the resale value of capital θ^F. This is the same as Figure 1.4, but increasing the firm's productivity from $\varphi = 10$ to $\varphi = 20$.

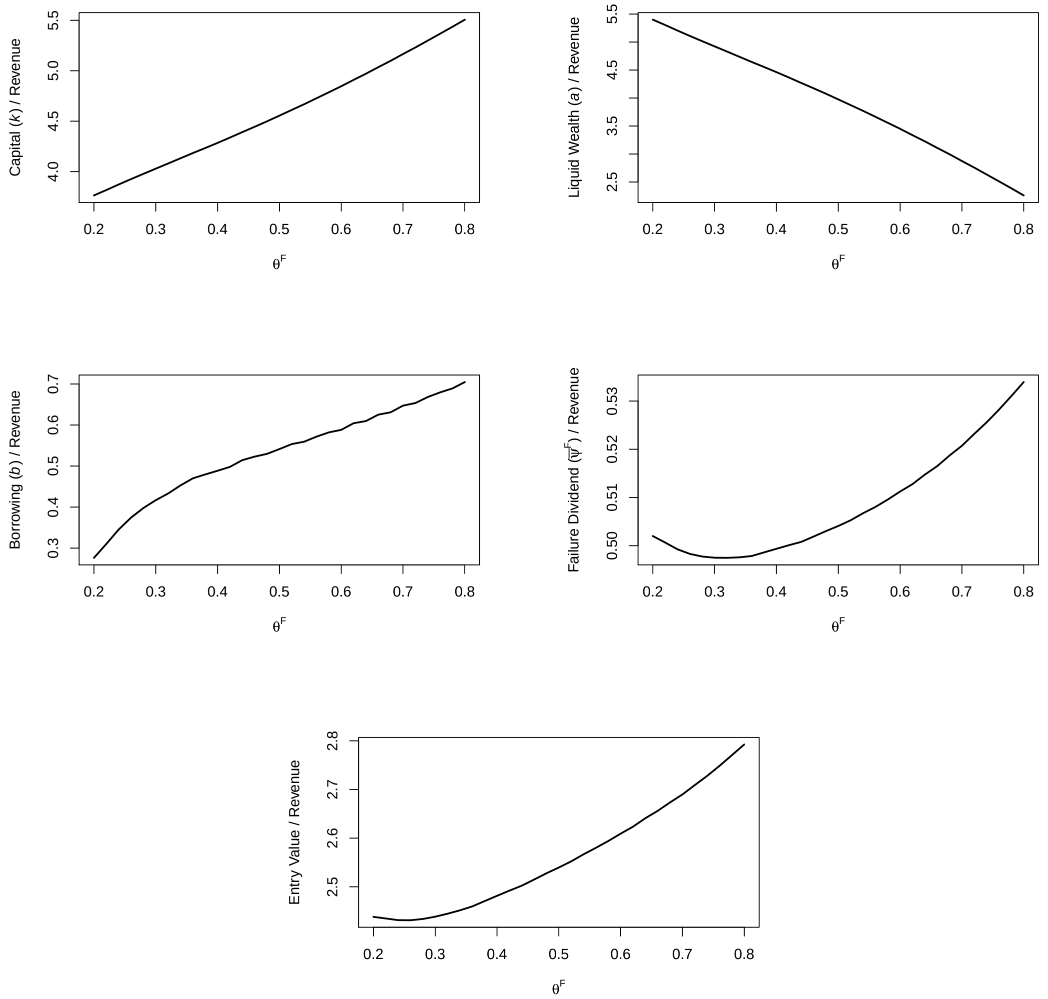

Figure A.13: Average firm characteristics divided by revenue: increase in θ^B

Simulations of average firm characteristics divided by average firm revenue, plotted against the resale value of capital θ^F. This is the same as Figure 1.4, but increasing the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

Appendix B

Chapter 2 Appendices

B.1 Numerical modelling sensitivity analysis: one-sector model

Here we repeat the numerical simulations undertaken in Section 2.2.7, but varying parameters other than θ^F one at a time to assess the effect of these parameters. These results are presented in Figures B.1 to B.8.

B.2 Welfare sensitivity analysis: one-sector model

Here we repeat the numerical simulations of the welfare effects of government intervention undertaken in Section 2.2.8, but varying parameters other than θ^F one at a time to assess the effect of these parameters. These results are presented in Figures B.9 to B.16.

B.3 Numerical modelling sensitivity analysis: two-sector model

Here we repeat the numerical simulations undertaken in Section 2.3.4, but varying parameters other than θ^F one at a time to assess the effect of these parameters. First, we conduct sensitivity analysis of the two sector economy when $\epsilon = 0.5$, so that services and manufactured goods are 'more complements than substitutes' in households' utility function. These results are presented in Figures B.17 to B.24. Then we do the same when $\epsilon = 2$, so that services and manufactured goods are 'more substitutes than complements' in households' utility function. These results are presented in Figures B.25 to B.32.

B.4 Welfare sensitivity analysis: two-sector model

Here we repeat the numerical simulations of the welfare effects of government intervention undertaken in Section 2.3.5, but varying parameters other than θ^F one at a time to assess the effect of these parameters. First, we conduct sensitivity analysis of welfare effects in

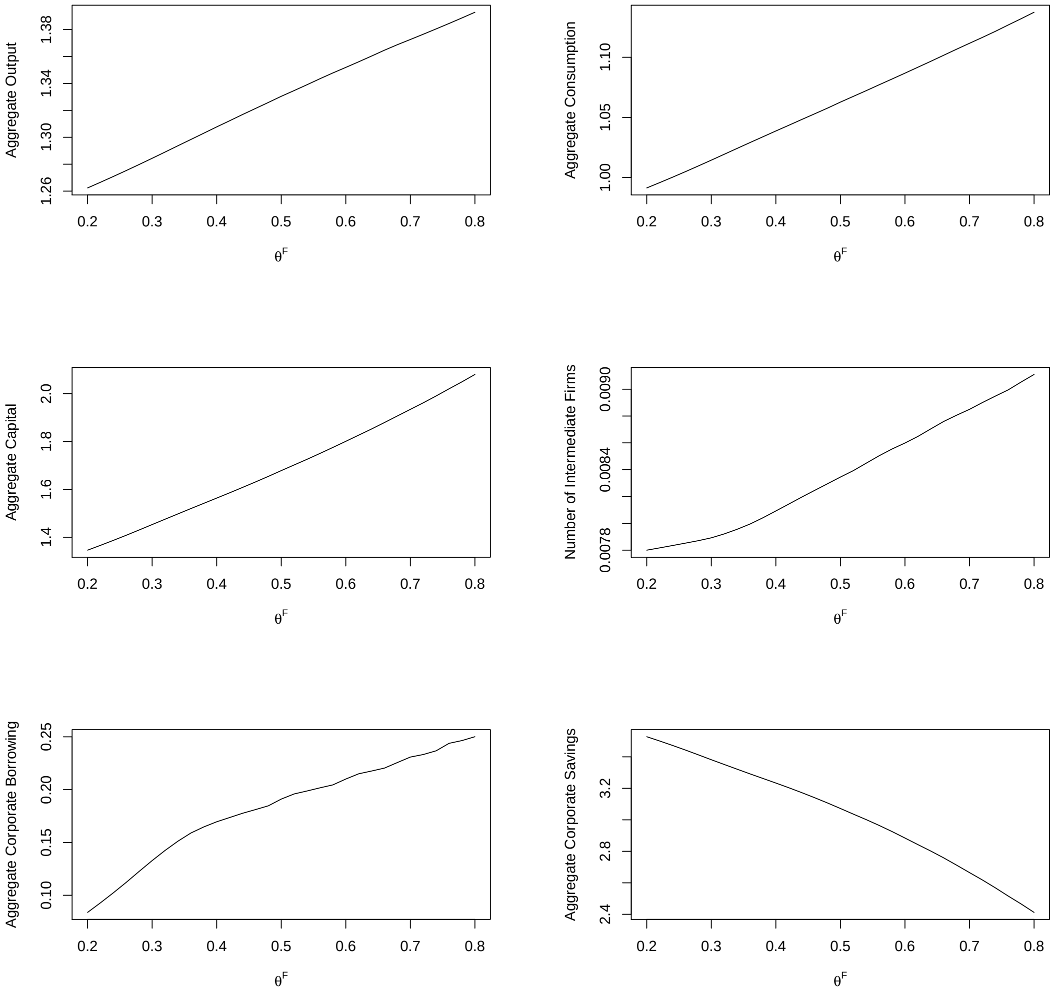

Figure B.1: The effect of investment reversibility θ^F on one-sector general equilibrium: increase in ρ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

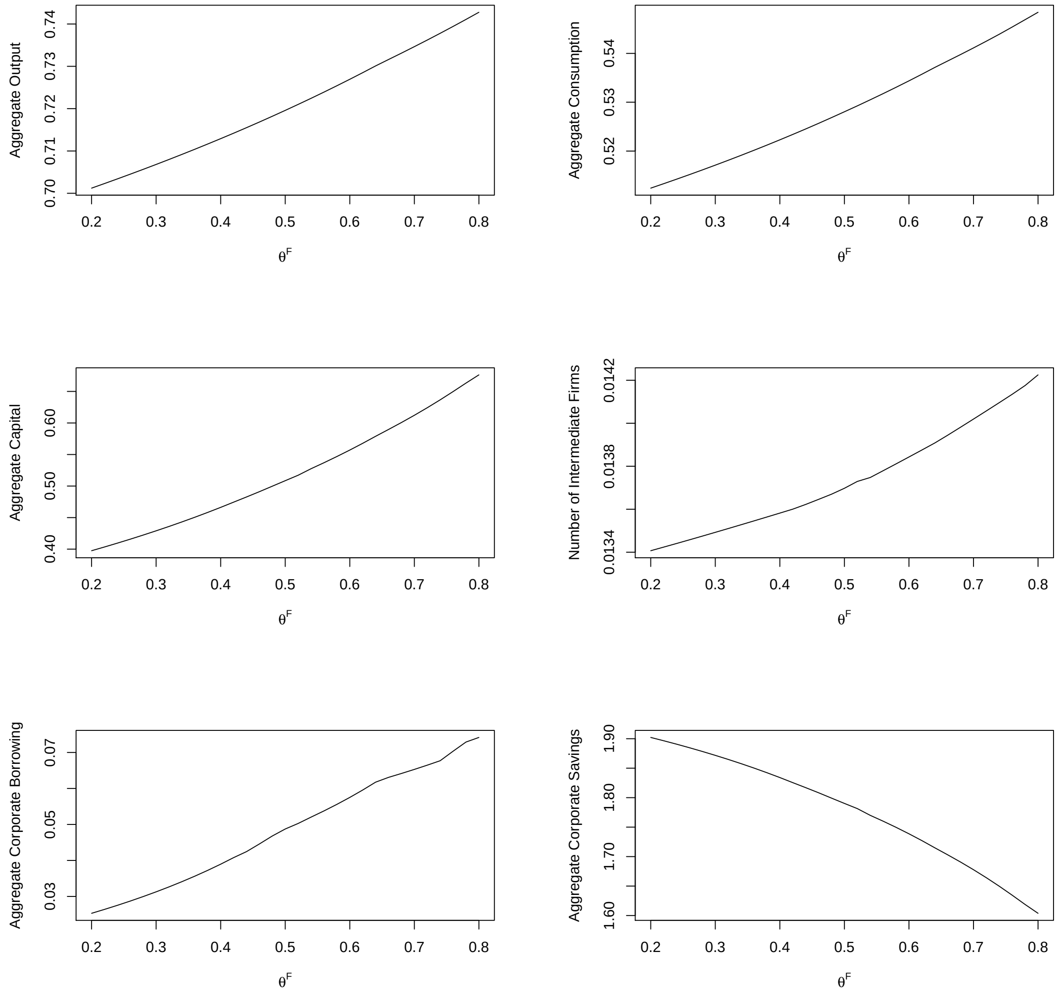

Figure B.2: The effect of investment reversibility θ^F on one-sector general equilibrium: decrease in β

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

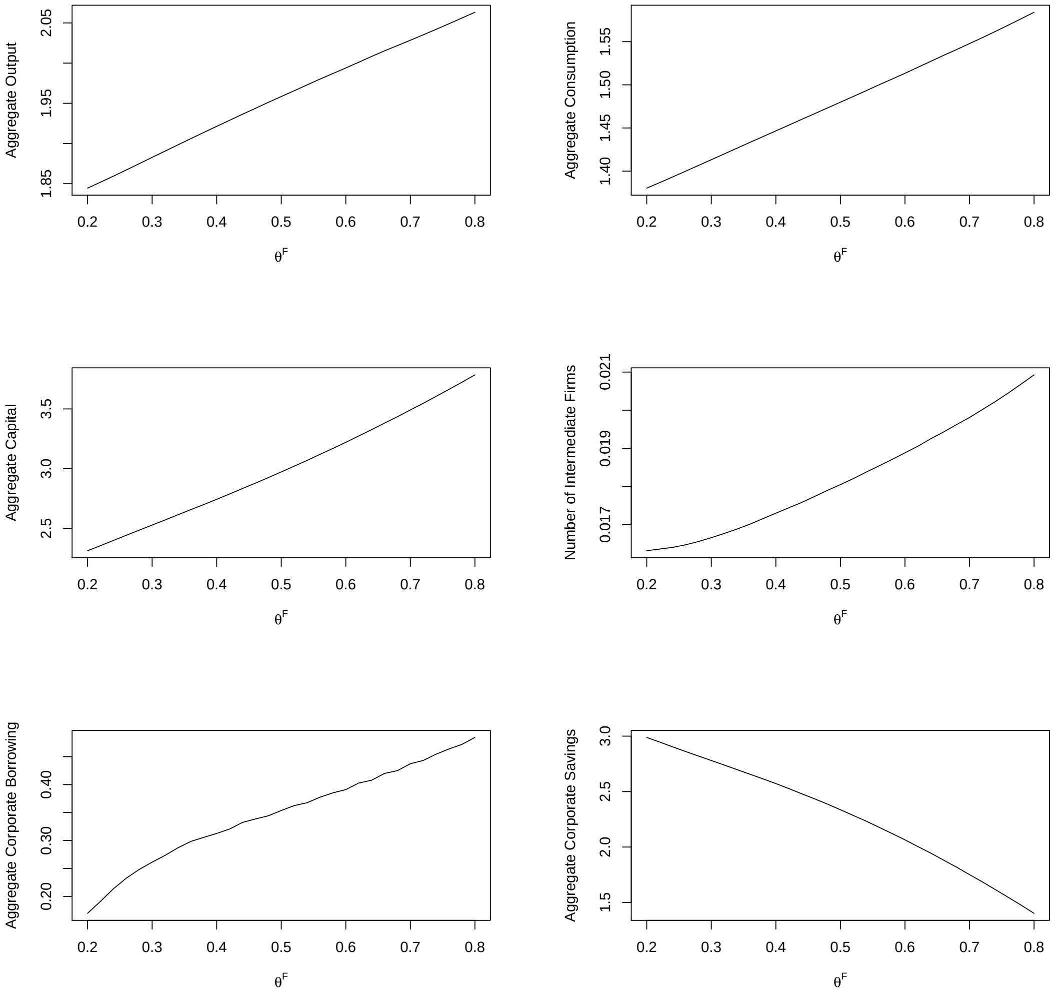

Figure B.3: The effect of investment reversibility θ^F on one-sector general equilibrium: increase in $\bar{\varphi}$

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

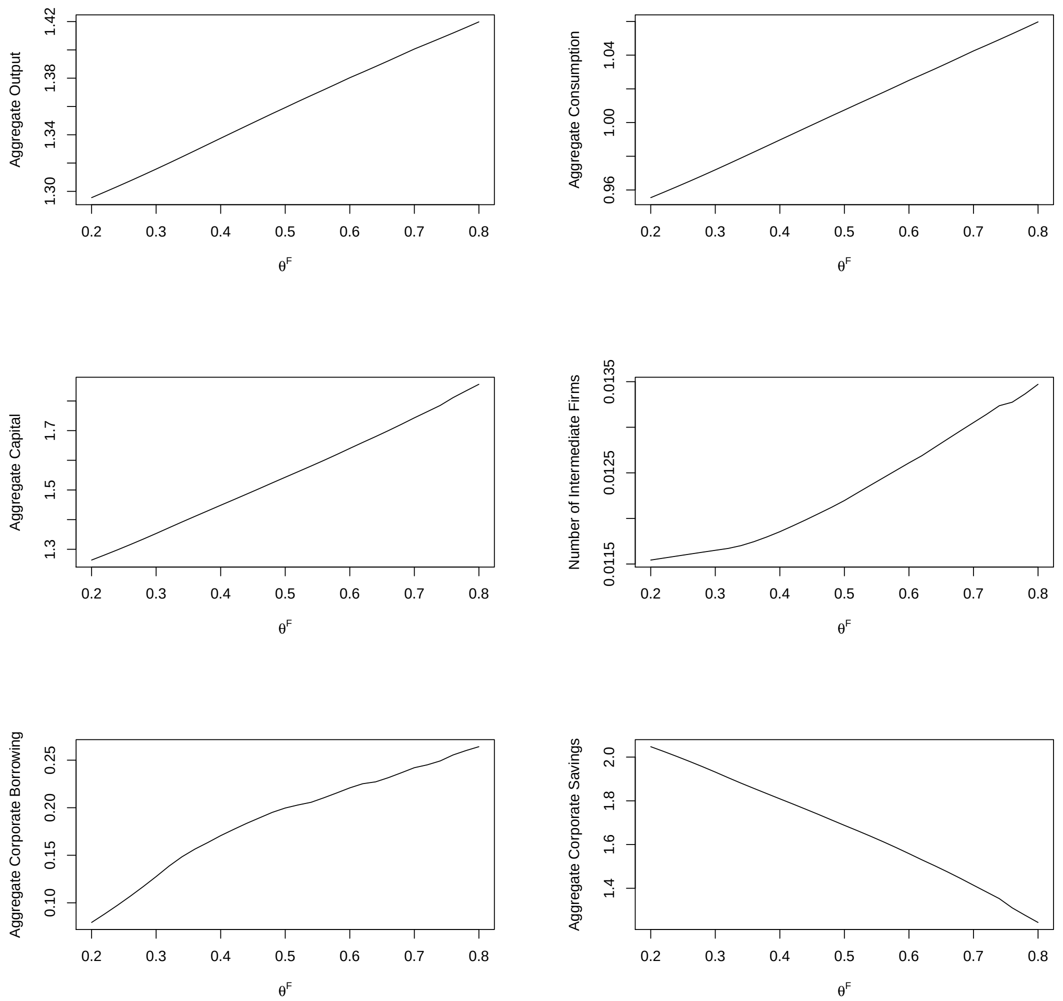

Figure B.4: The effect of investment reversibility θ^F on one-sector general equilibrium: increase in μ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

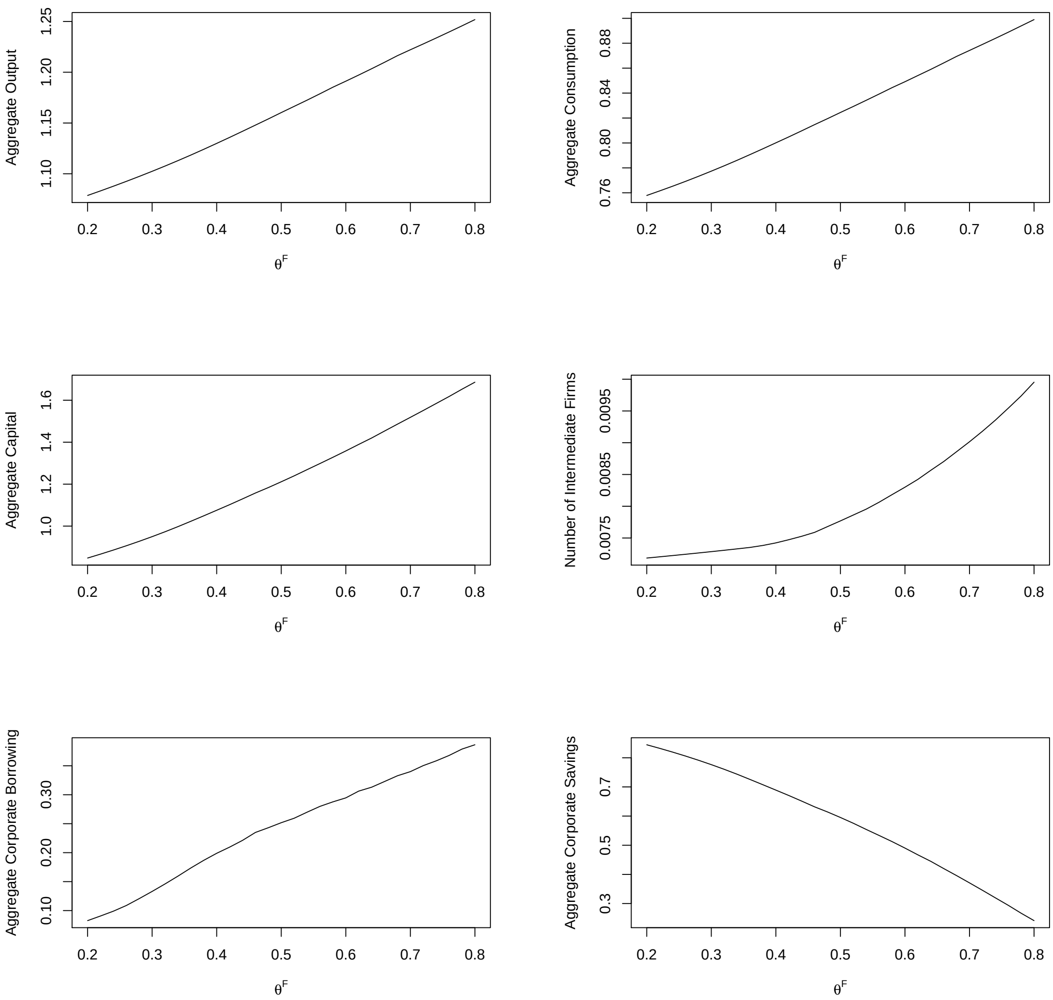

Figure B.5: The effect of investment reversibility θ^F on one-sector general equilibrium: increase in δ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

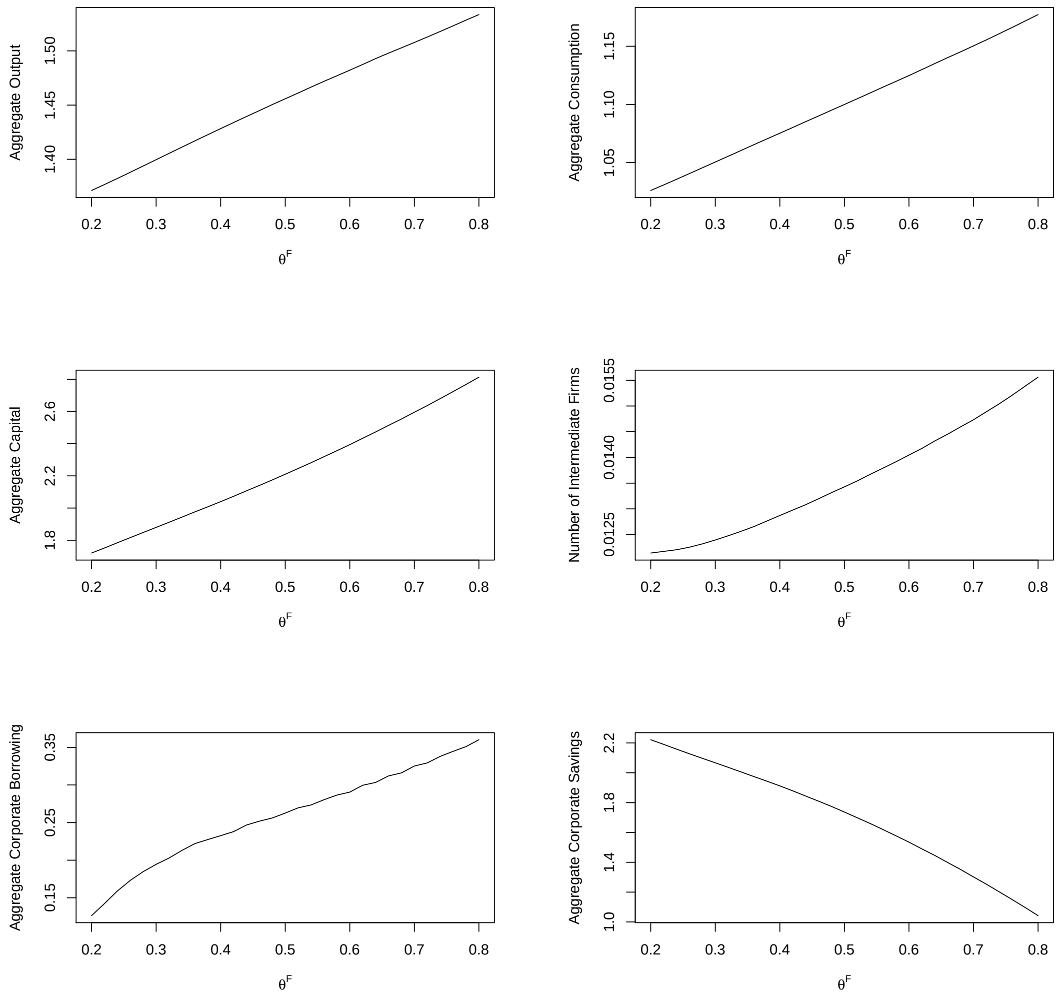

Figure B.6: The effect of investment reversibility θ^F on one-sector general equilibrium: increase in θ^B

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

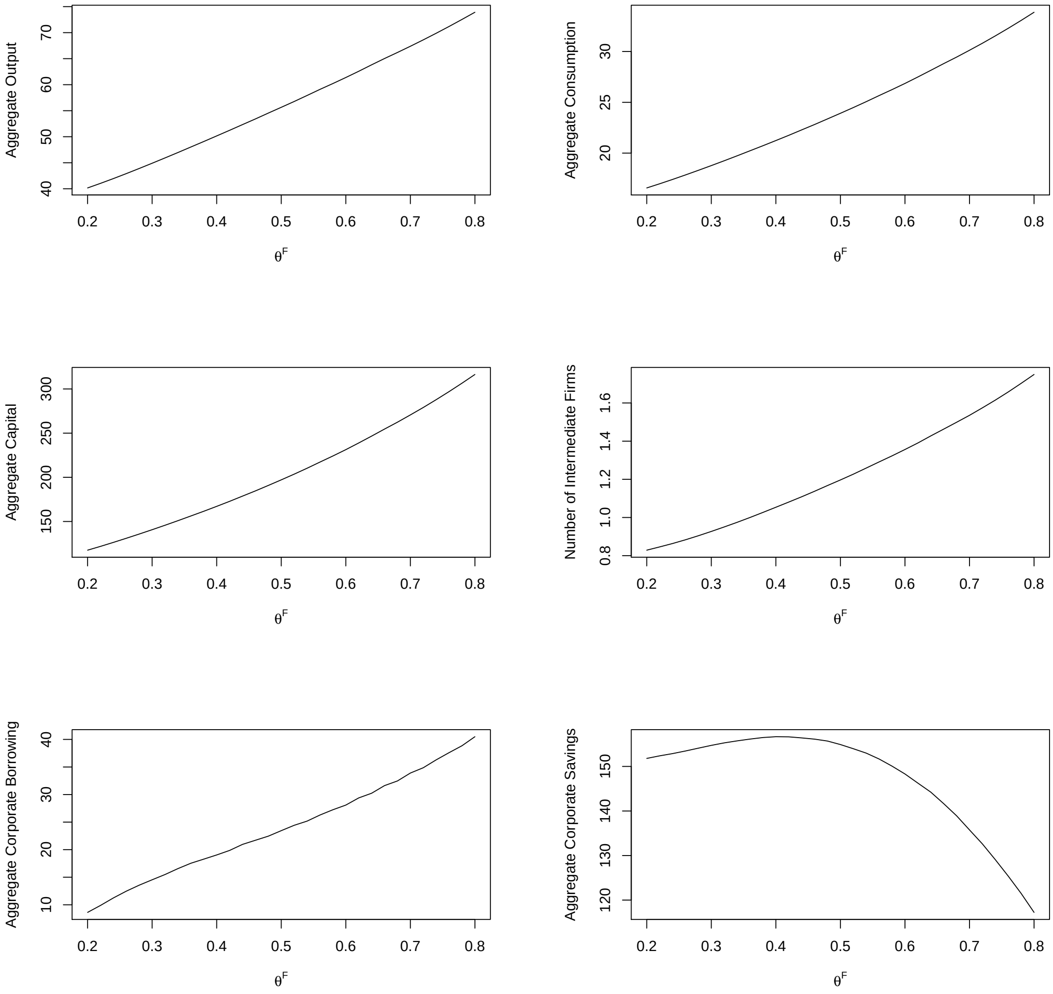

Figure B.7: The effect of investment reversibility θ^F on one-sector general equilibrium: increase in γ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with an increase in the final sector's output elasticity of intermediate goods from $\gamma = 0.3$ to $\gamma = 0.7$.

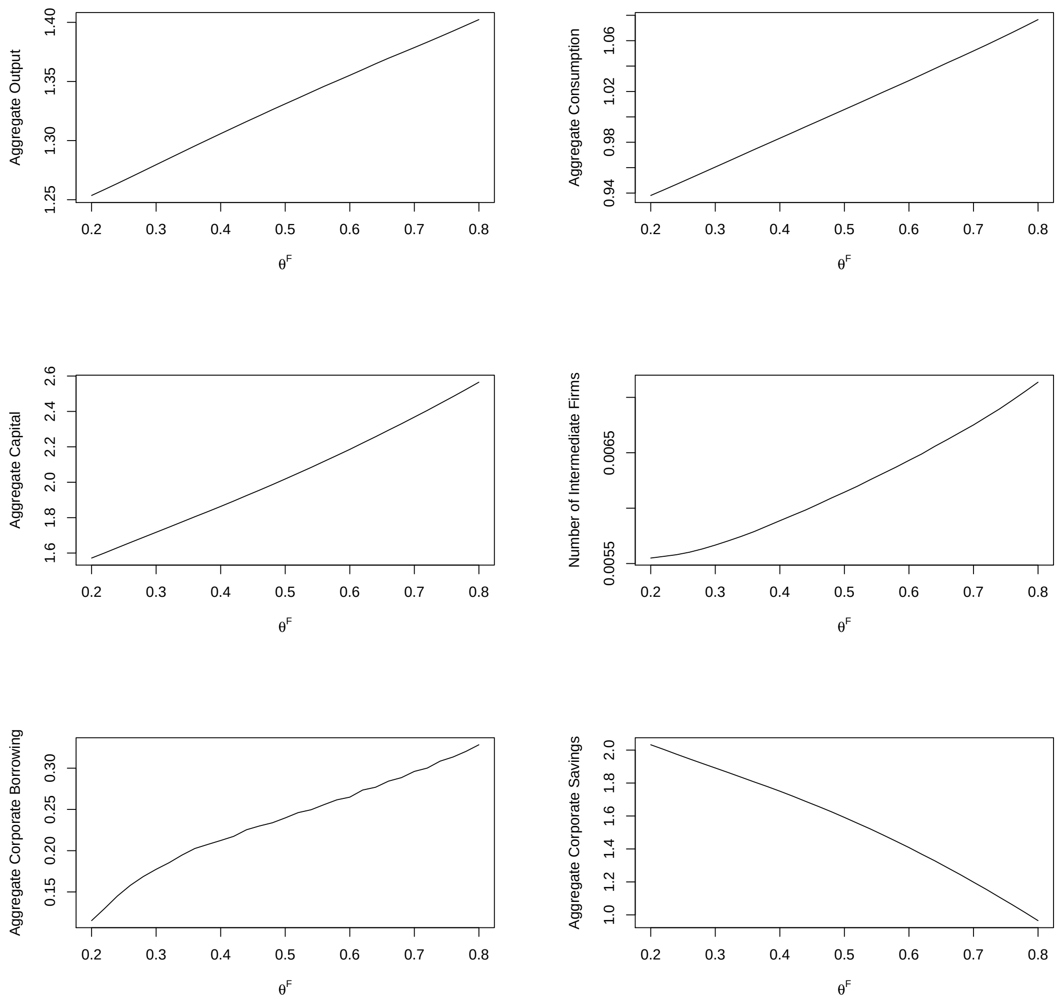

Figure B.8: The effect of investment reversibility θ^F on one-sector general equilibrium: increase in e

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.1, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

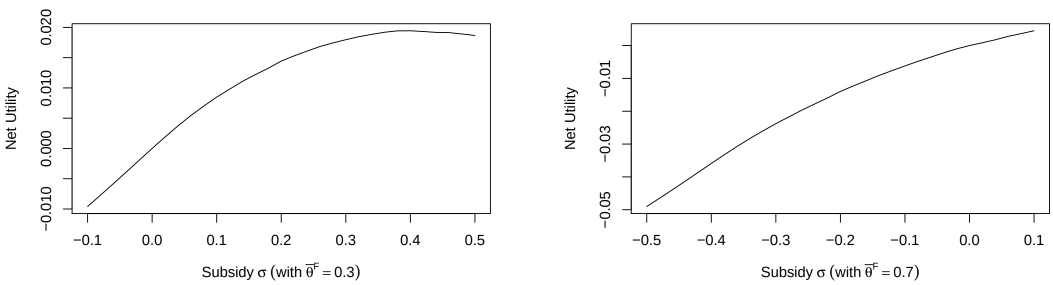

Figure B.9: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: increase in ρ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

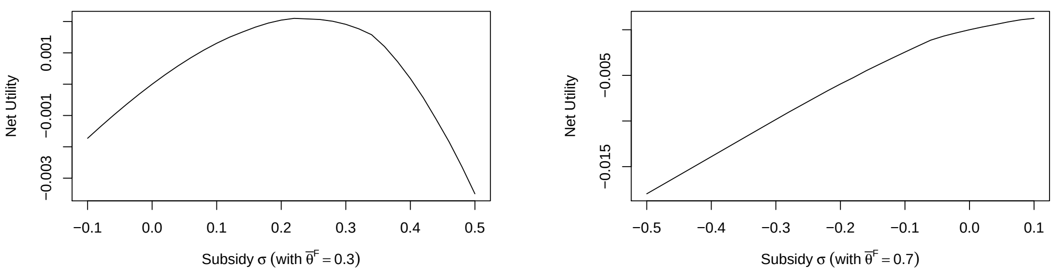

Figure B.10: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: decrease in β

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

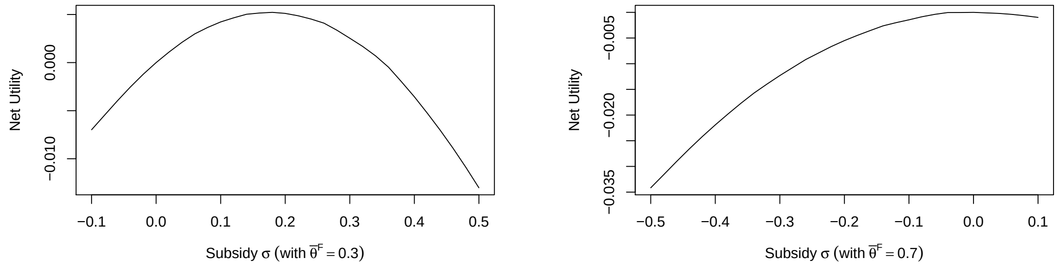

Figure B.11: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: increase in φ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

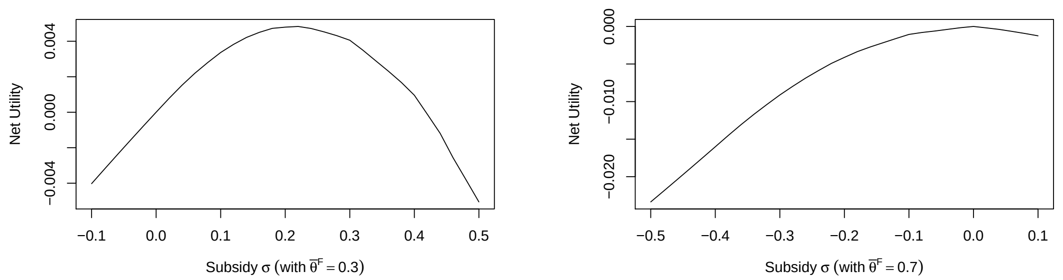

Figure B.12: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: increase in μ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

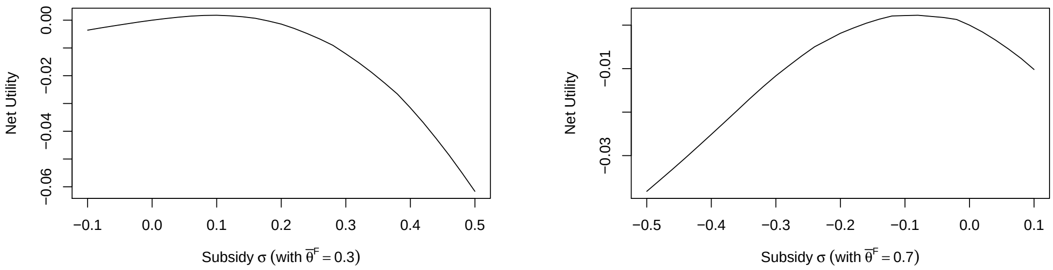

Figure B.13: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: increase in δ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

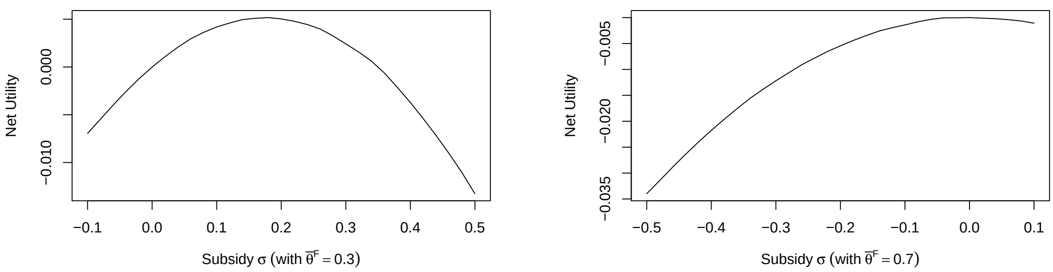

Figure B.14: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: increase in θ^B

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

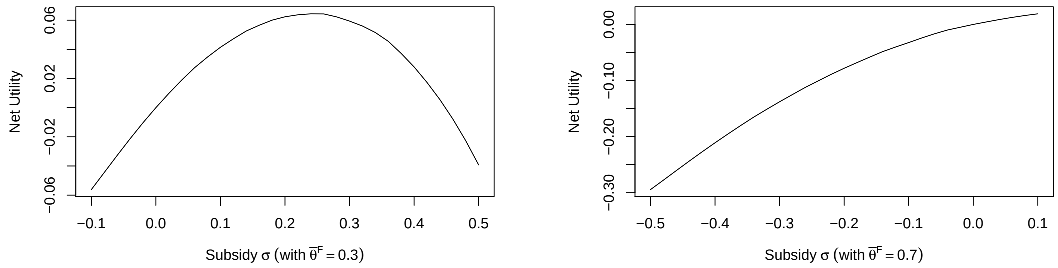

Figure B.15: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: increase in γ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with an increase in the final sector's output elasticity of intermediate goods from $\gamma = 0.3$ to $\gamma = 0.7$.

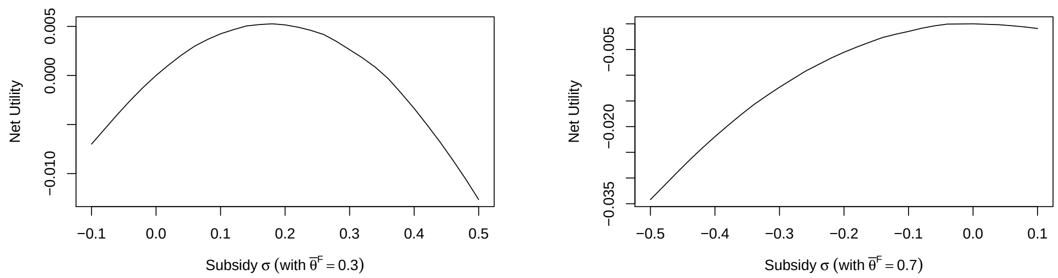

Figure B.16: Welfare changes in response to capital liquidation subsidies and taxes in the one-sector model: increase in e

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.2, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

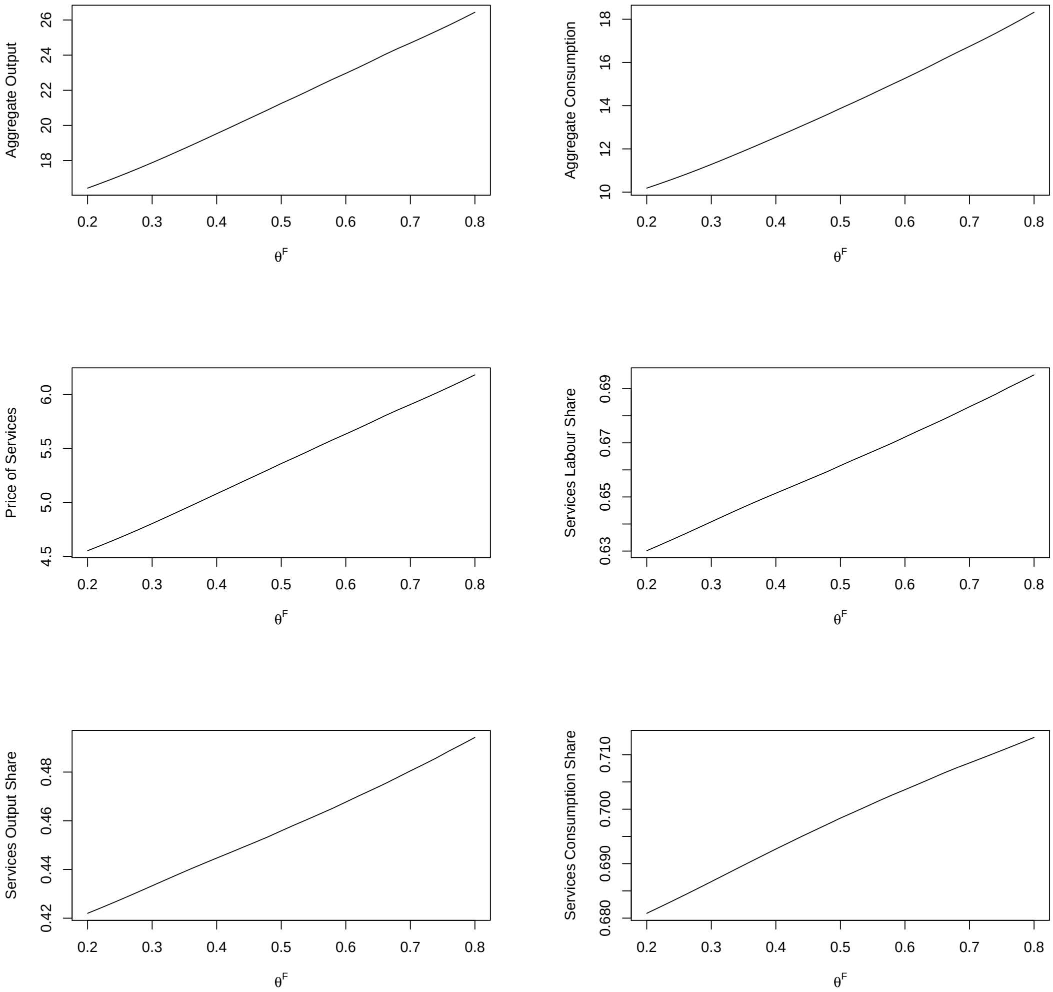

Figure B.17: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: increase in ρ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

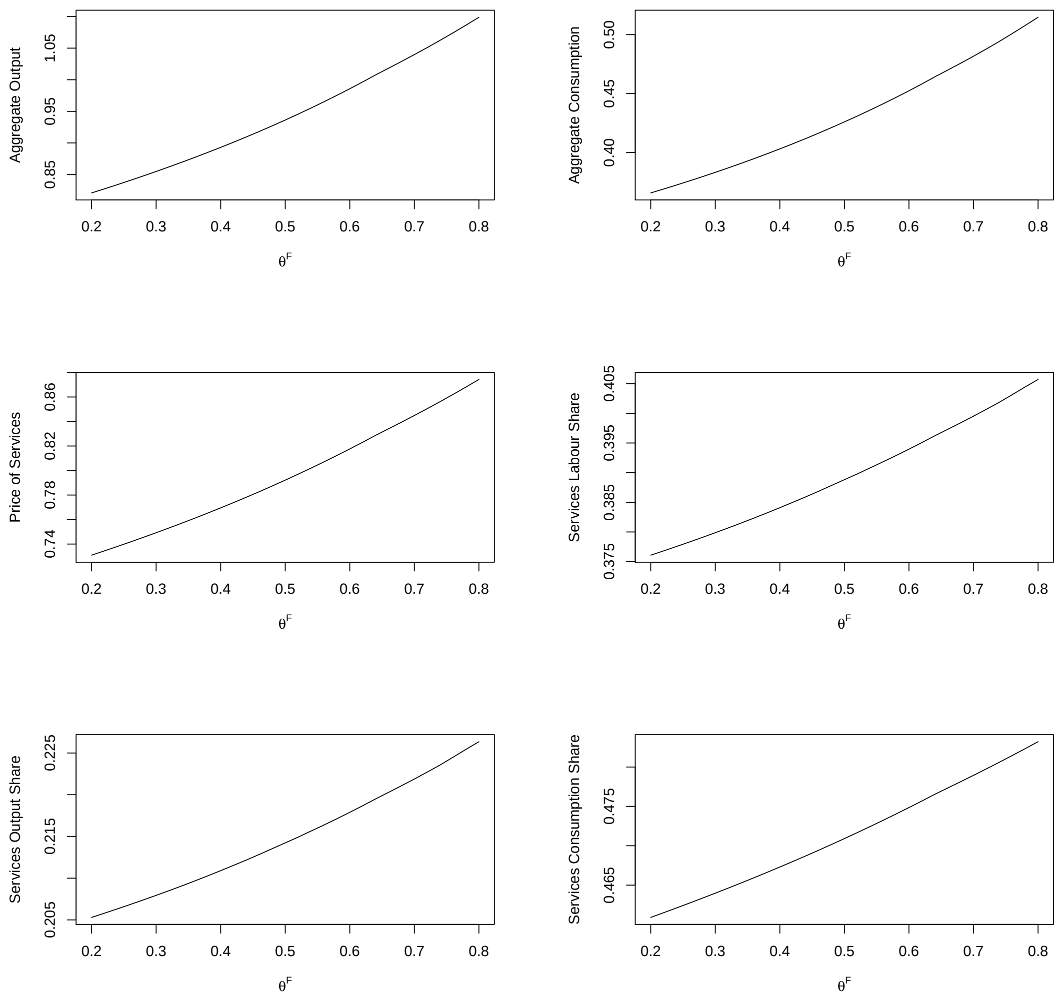

Figure B.18: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: decrease in β

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

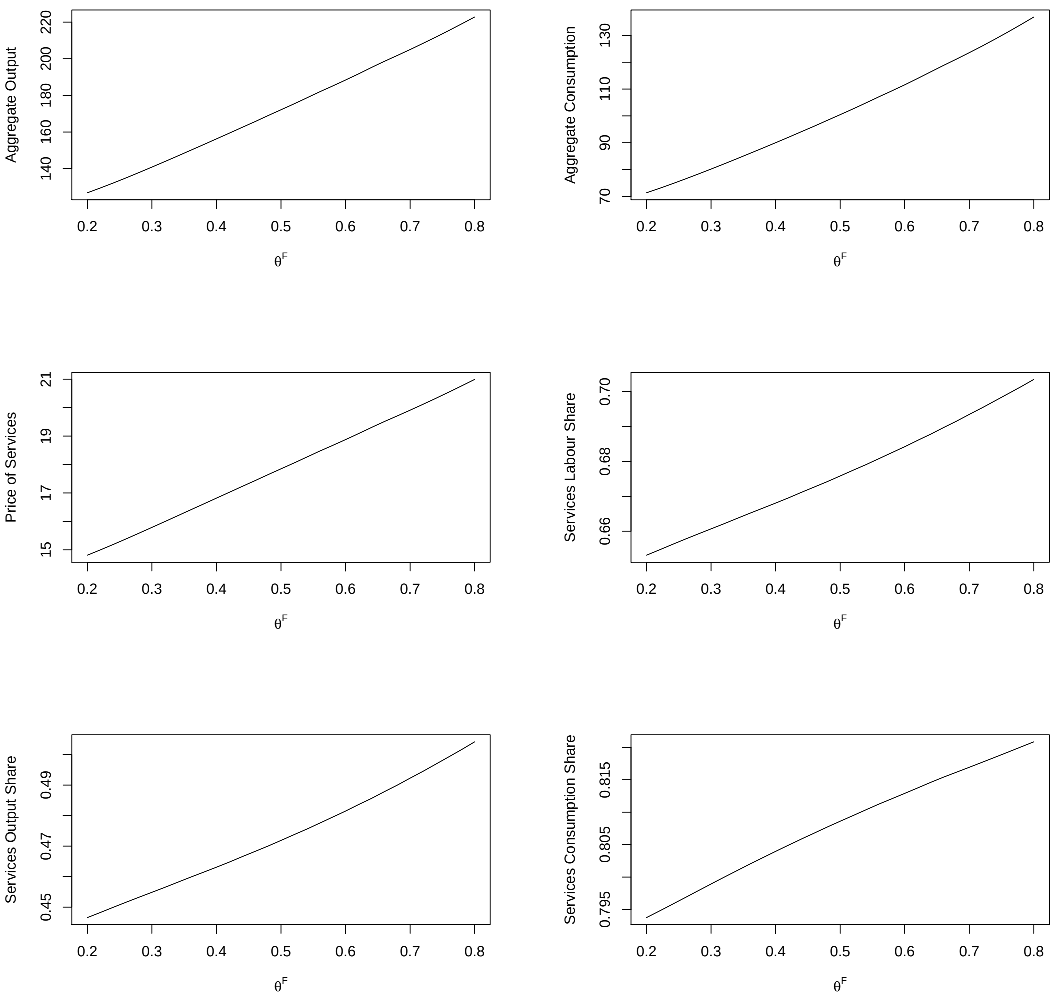

Figure B.19: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: increase in $\bar{\varphi}$

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

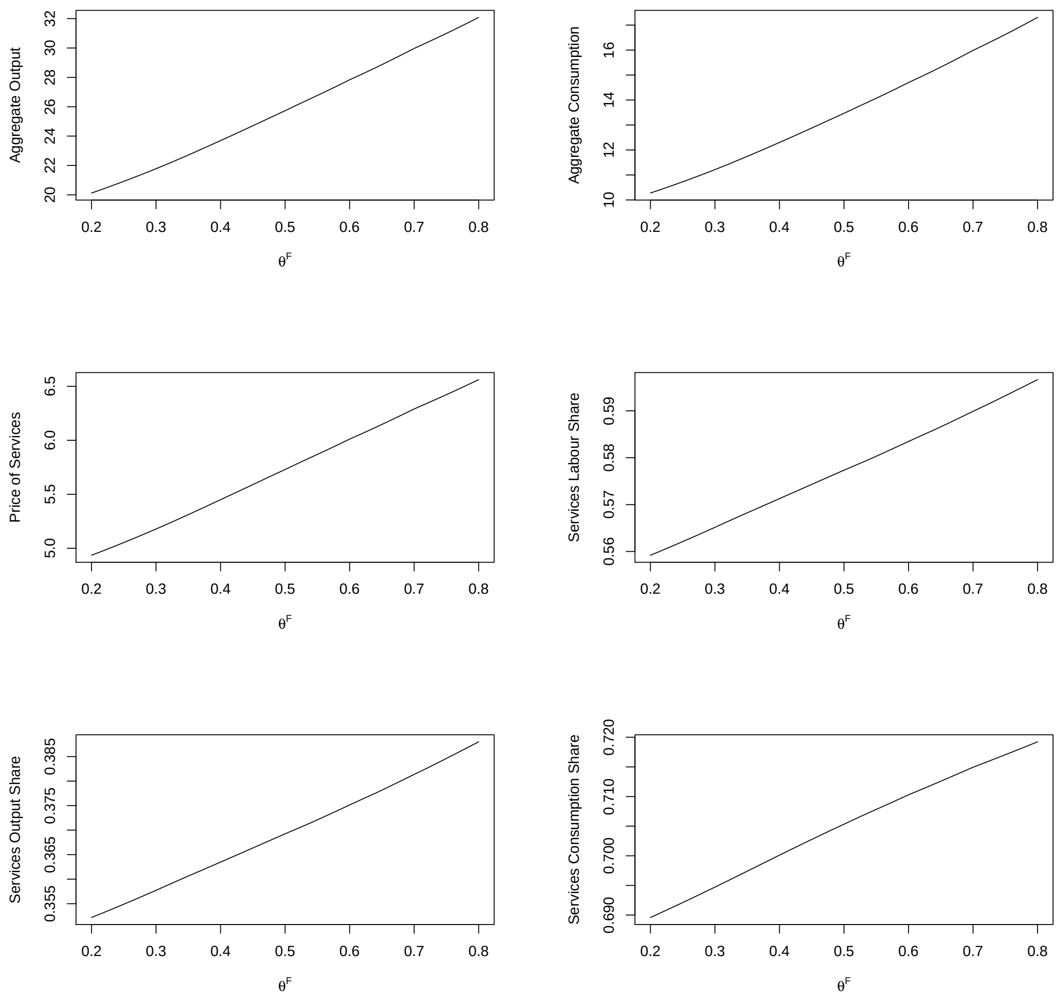

Figure B.20: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: increase in μ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

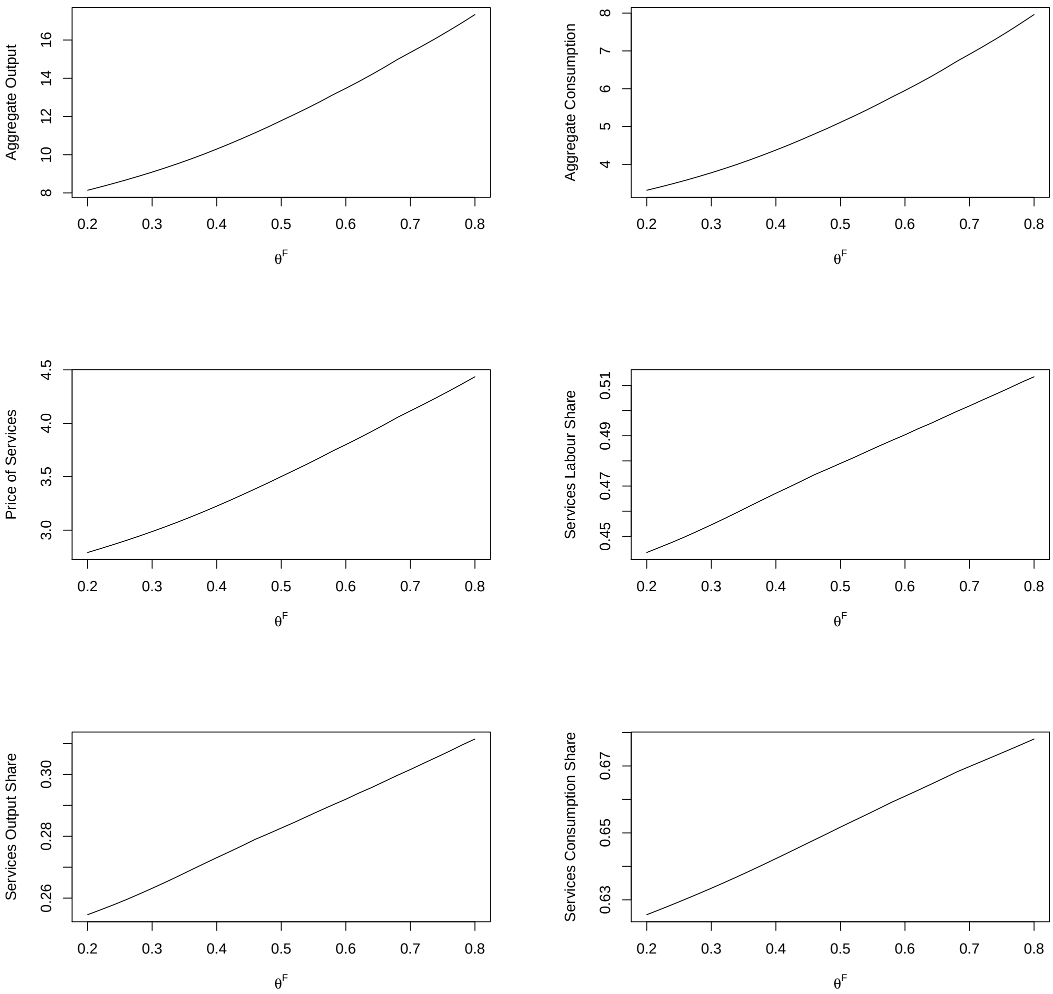

Figure B.21: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: increase in δ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

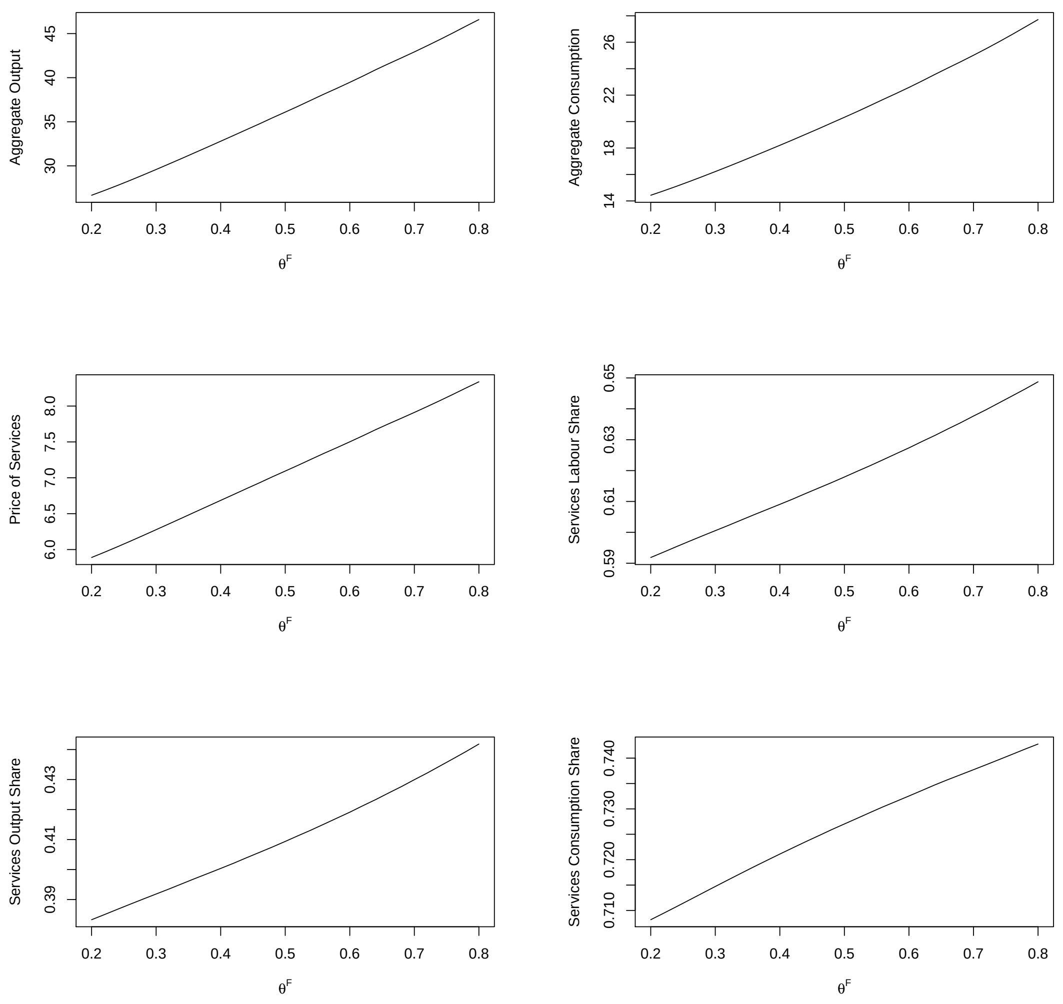

Figure B.22: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: increase in θ^B

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

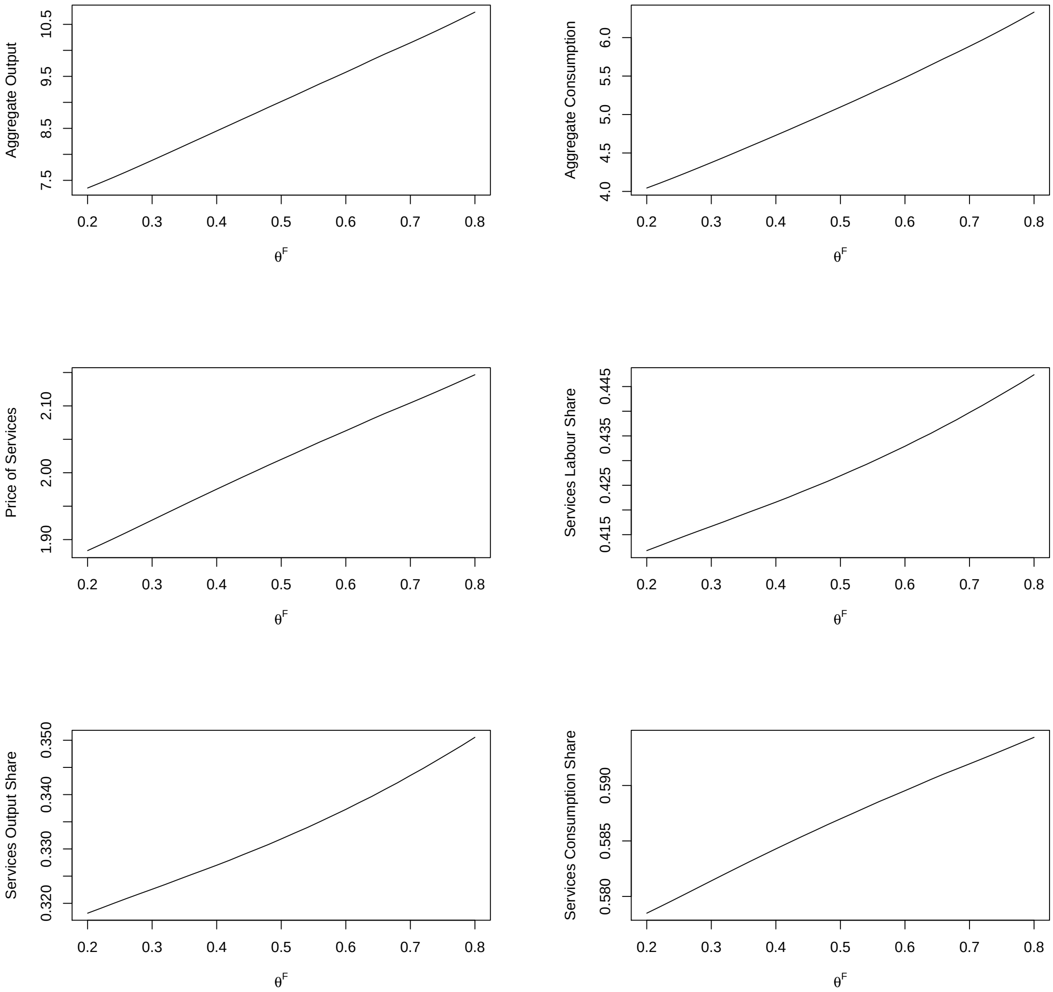

Figure B.23: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: decrease in γ_M and increase in γ_S

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with a decrease in the manufacturing sector's output elasticity of intermediate goods from $\gamma_M = 0.7$ to $\gamma = 0.6$, and an increase in the services sector's output elasticity of intermediate goods from $\gamma_S = 0.3$ to $\gamma_S = 0.4$.

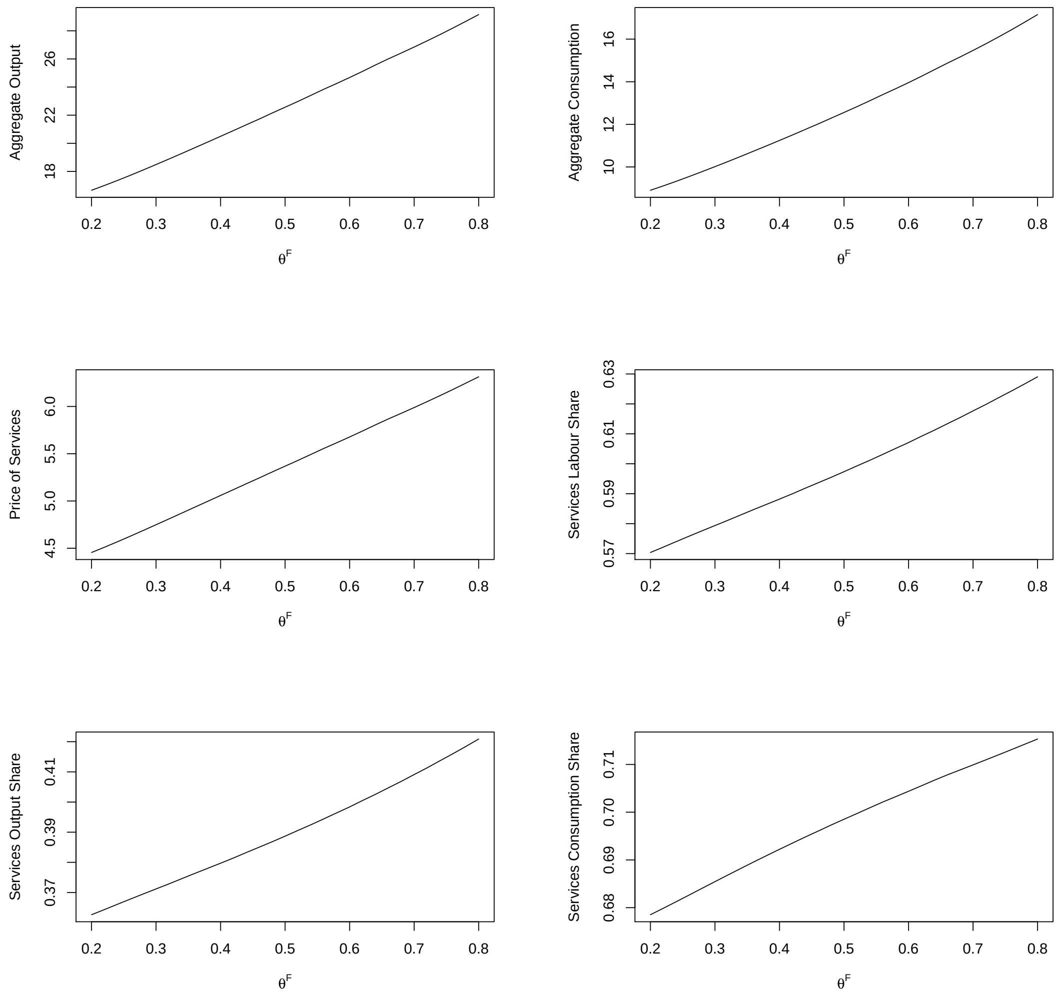

Figure B.24: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 0.5$: increase in e

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.3, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

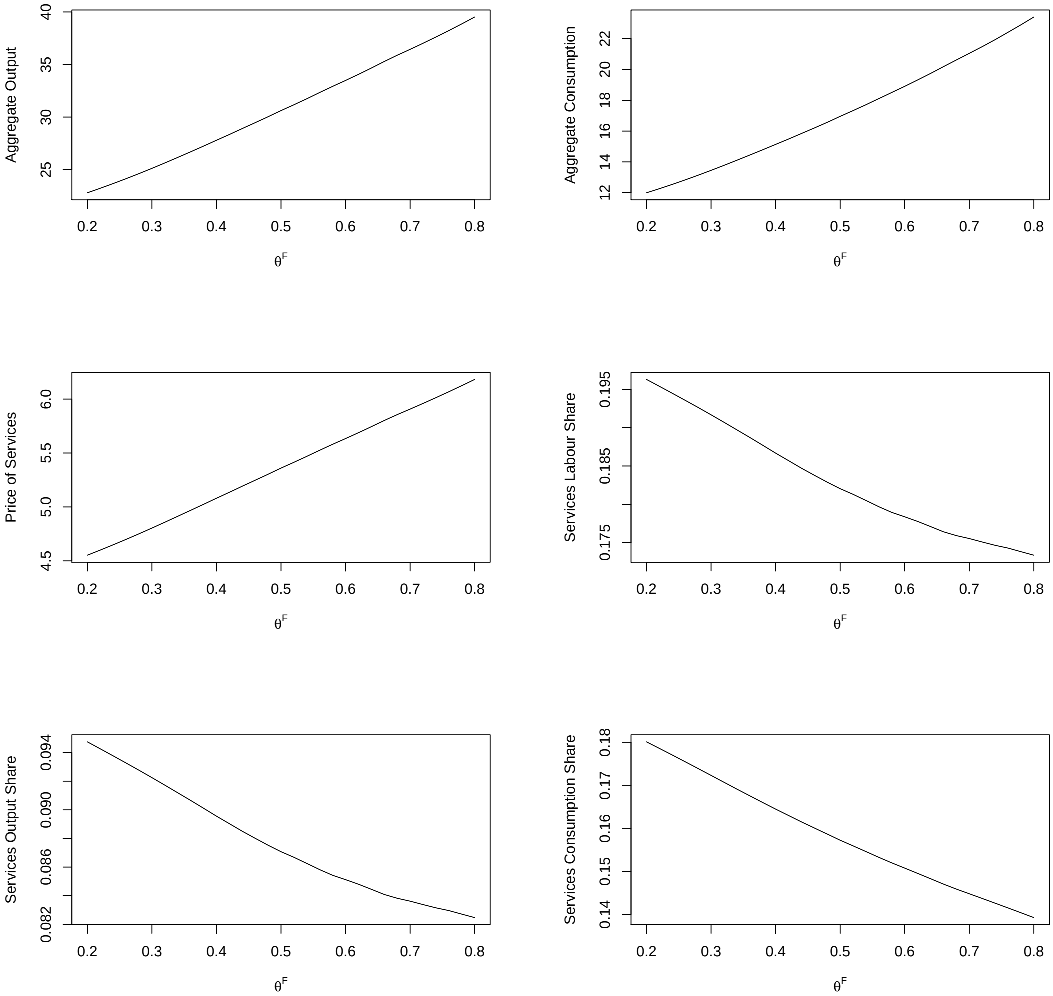

Figure B.25: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: increase in ρ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

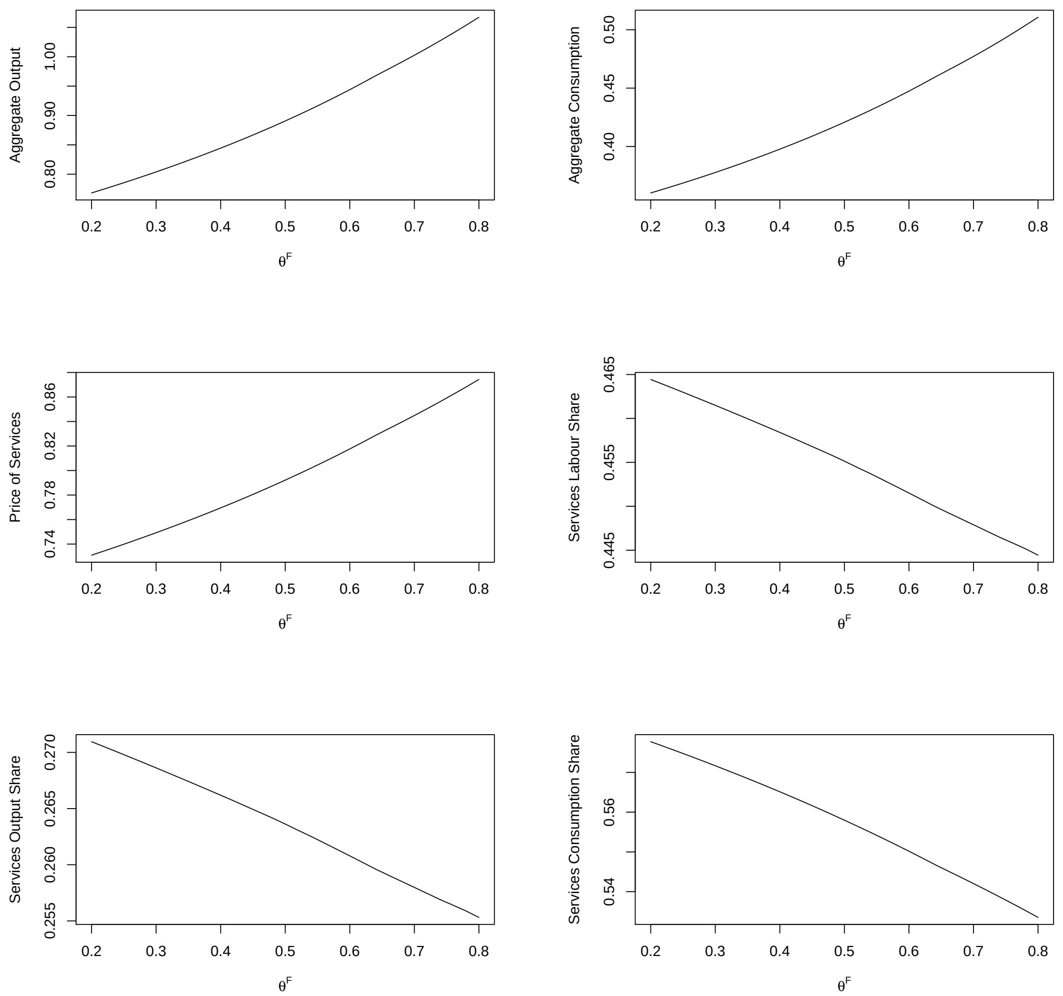

Figure B.26: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: decrease in β

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

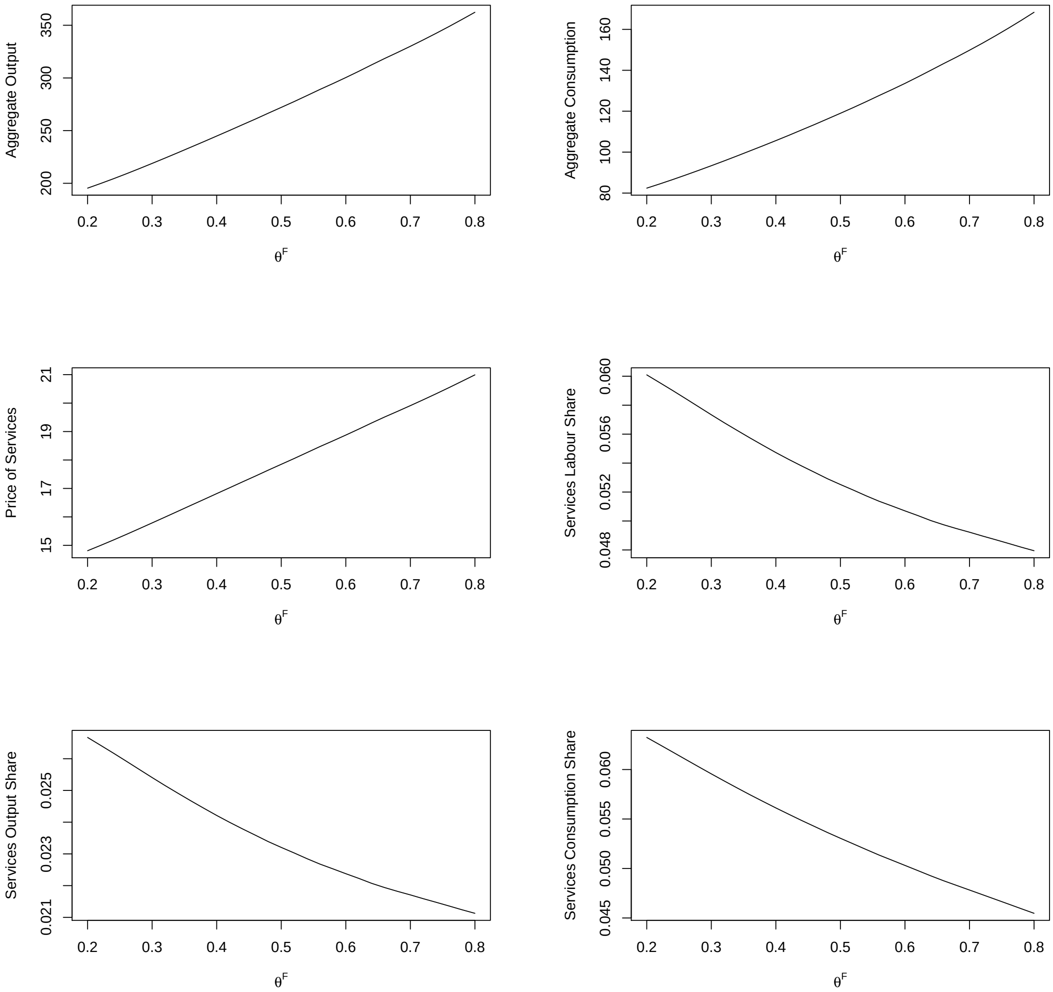

Figure B.27: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: increase in $\bar{\varphi}$

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

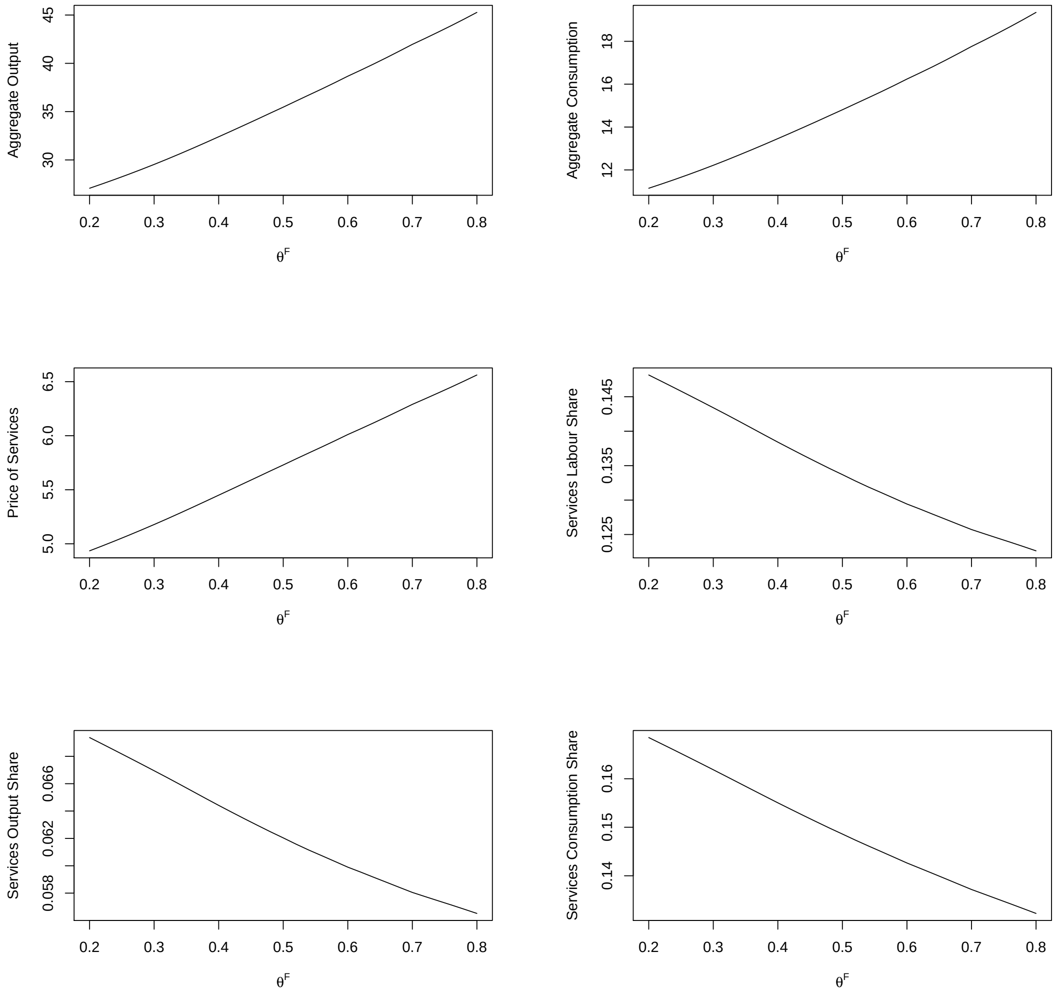

Figure B.28: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: increase in μ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

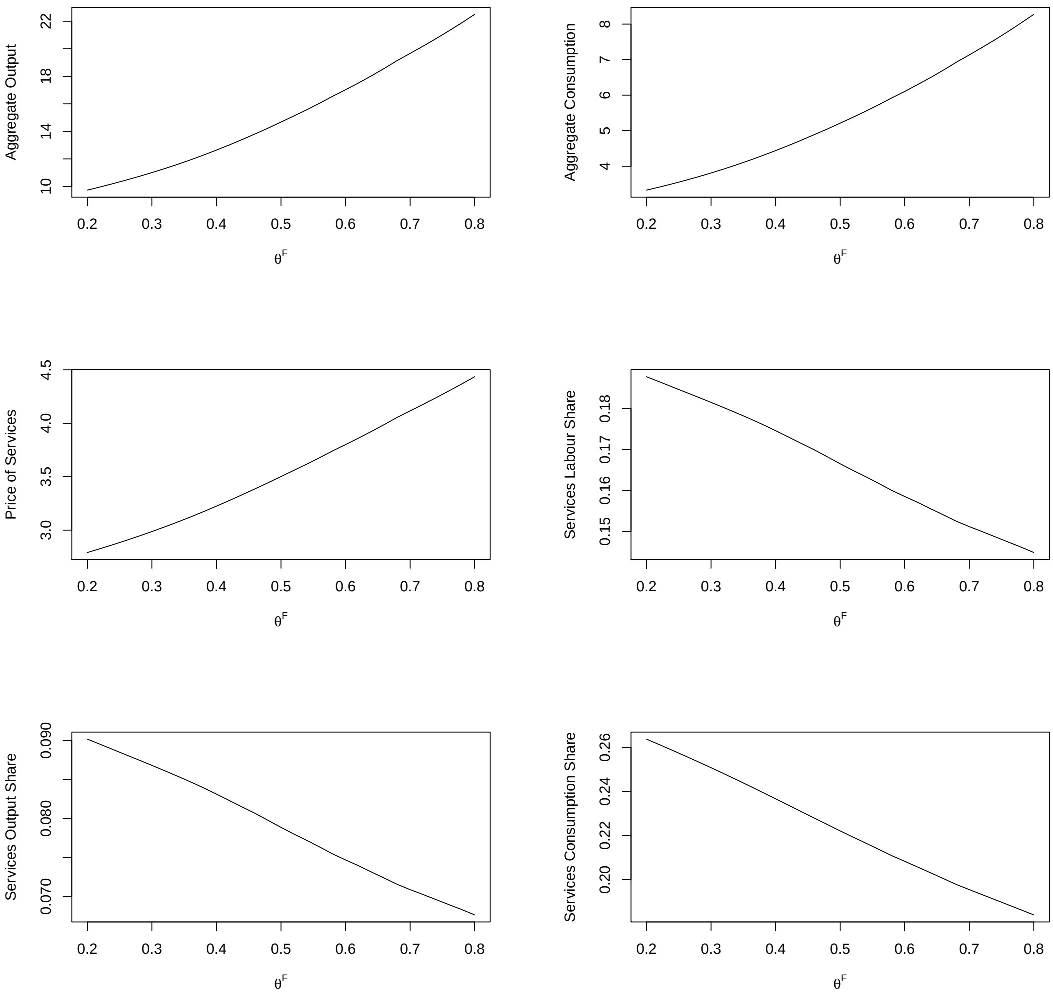

Figure B.29: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: increase in δ

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

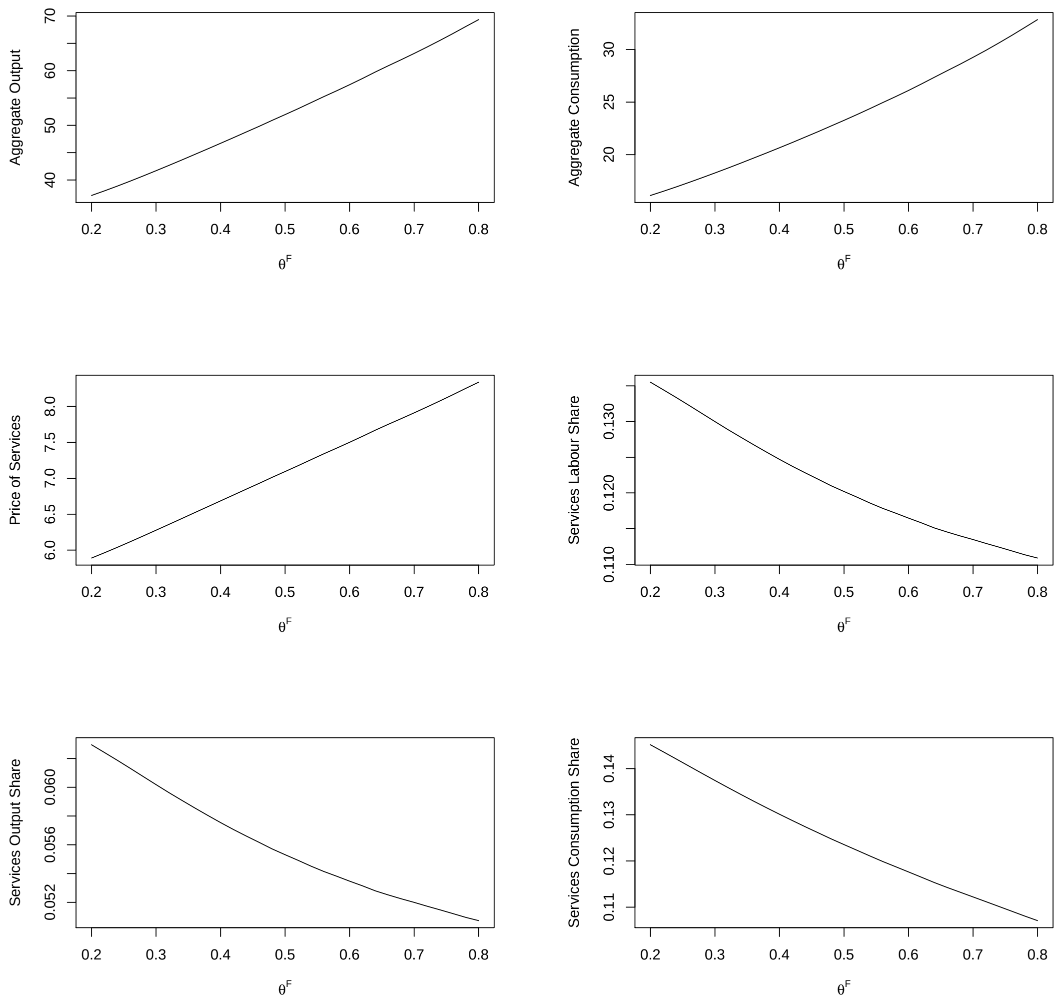

Figure B.30: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: increase in θ^B

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

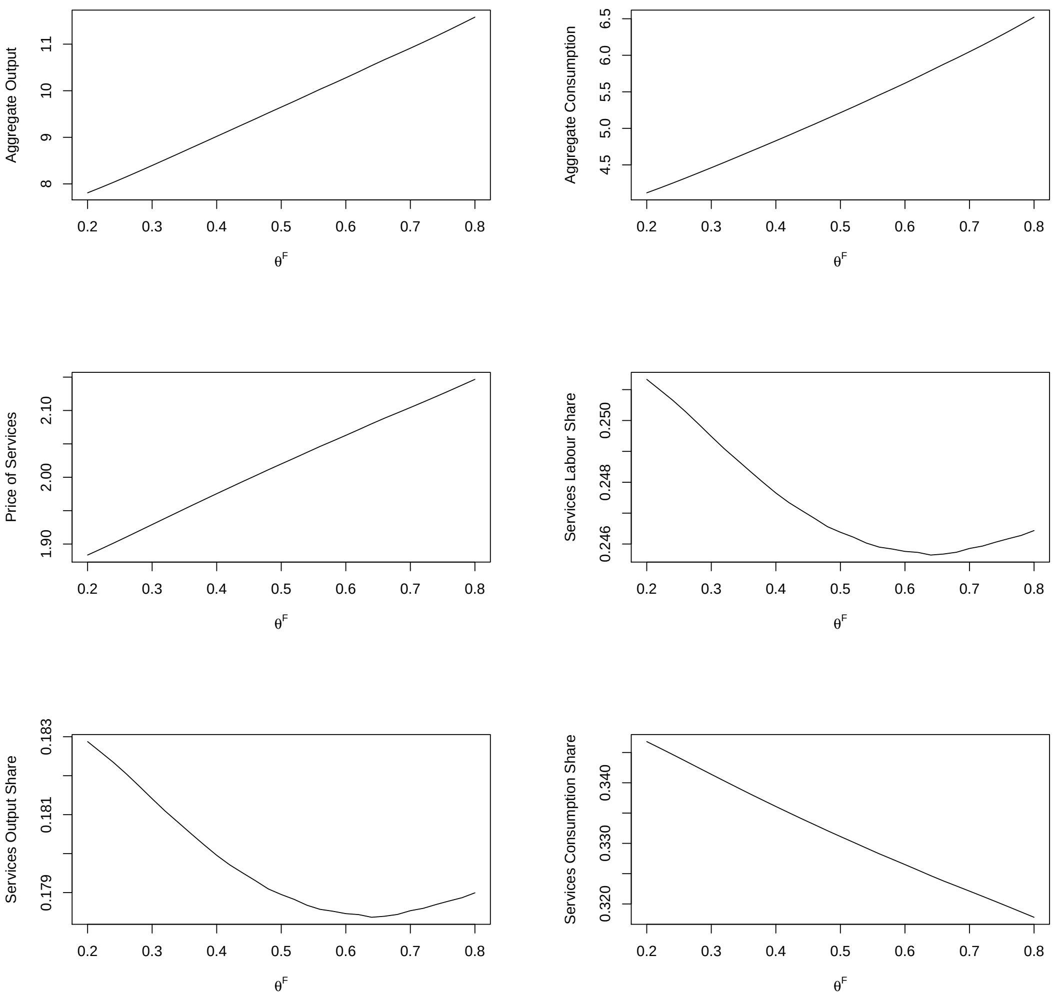

Figure B.31: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: decrease in γ_M and increase in γ_S

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with a decrease in the manufacturing sector's output elasticity of intermediate goods from $\gamma_M = 0.7$ to $\gamma = 0.6$, and an increase in the services sector's output elasticity of intermediate goods from $\gamma_S = 0.3$ to $\gamma_S = 0.4$.

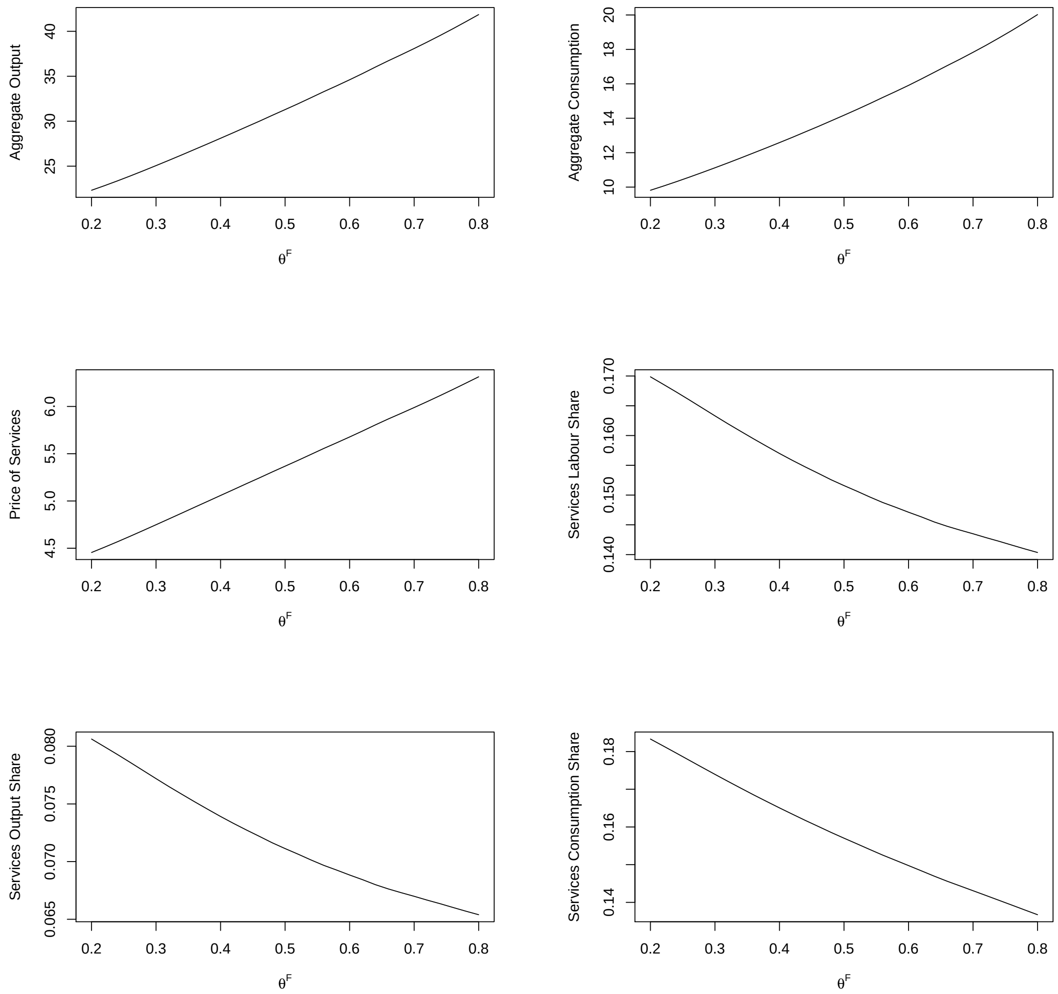

Figure B.32: The effect of investment reversibility θ^F on two-sector general equilibrium, with $\epsilon = 2$: increase in e

Numerical modelling of the economy in general equilibrium, plotted against investment reversibility θ^F. This is the same as Figure 2.4, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

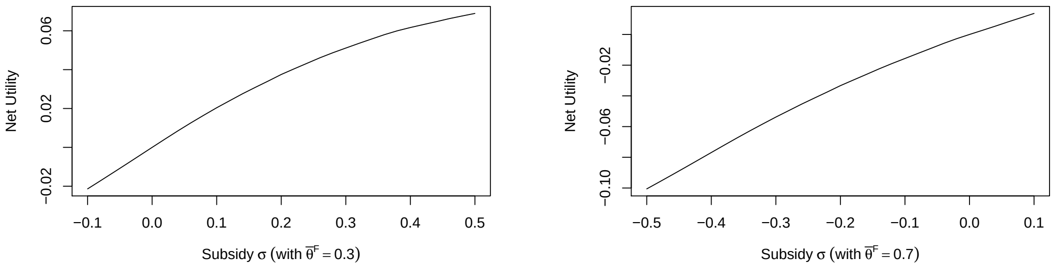

Figure B.33: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: increase in ρ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

the two sector economy when $\epsilon = 0.5$, so that services and manufactured goods are 'more complements than substitutes' in households' utility function. These results are presented in Figures B.33 to B.40. Then we do the same when $\epsilon = 2$, so that services and manufactured goods are 'more substitutes than complements' in households' utility function. These results are presented in Figures B.41 to B.48.

Next, we consider the implications of capital liquidation subsidies and taxes on the balance between services and manufacturing. First, we conduct sensitivity analysis of structural change effects in the two sector economy when $\epsilon = 0.5$. These results are presented in Figures B.49 to B.56. Then we do the same when $\epsilon = 2$. These results are presented in Figures B.57 to B.64.

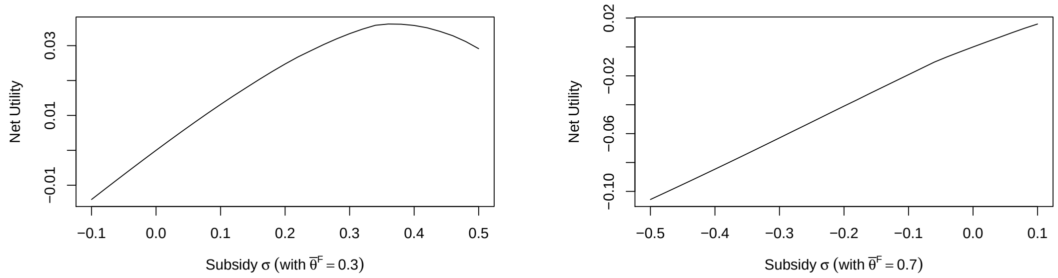

Figure B.34: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: decrease in β

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

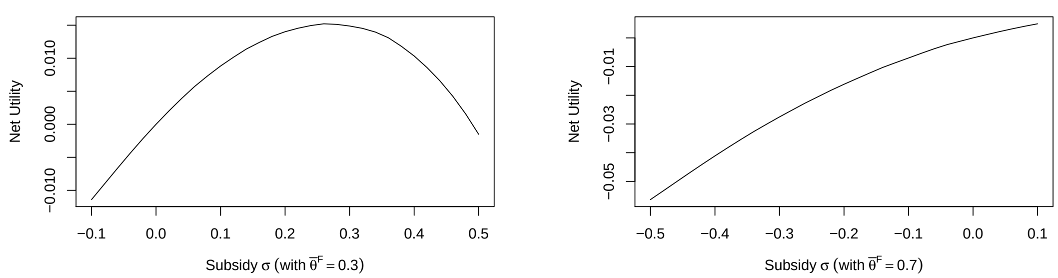

Figure B.35: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: increase in φ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

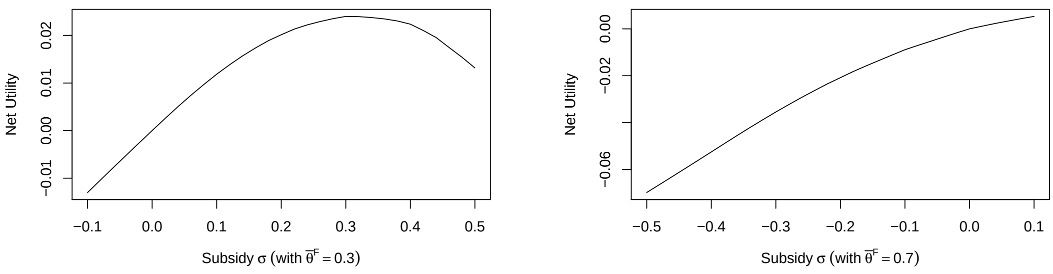

Figure B.36: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: increase in μ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

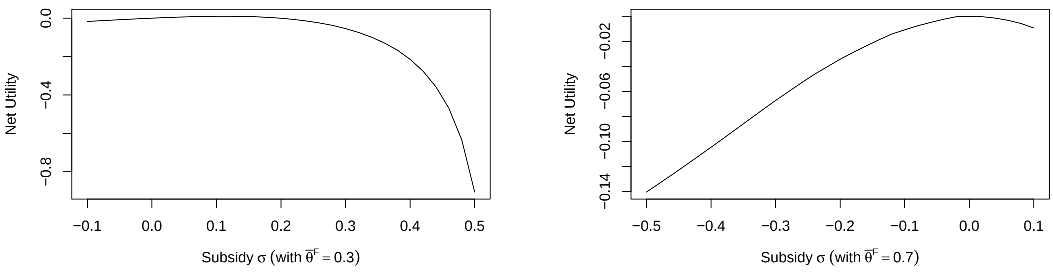

Figure B.37: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: increase in δ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

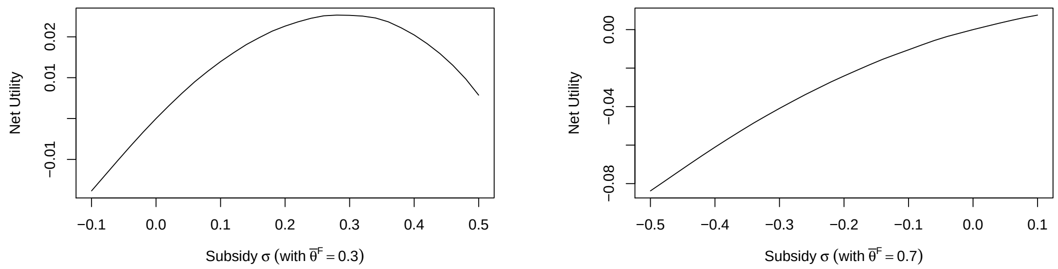

Figure B.38: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: increase in θ^B

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

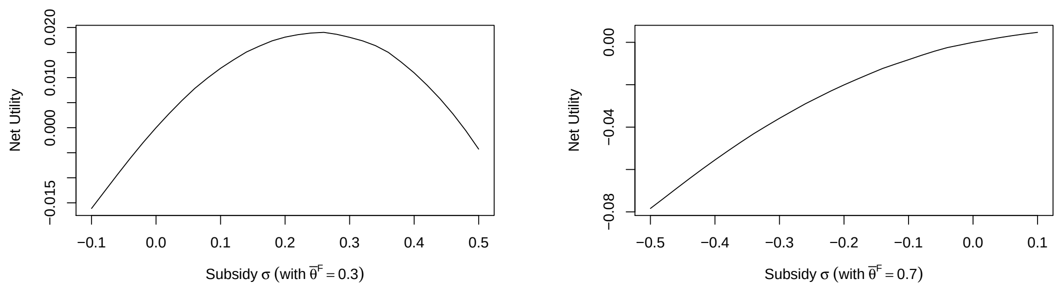

Figure B.39: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: decrease in γ_M and increase in γ_S

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with a decrease in the manufacturing sector's output elasticity of intermediate goods from $\gamma_M = 0.7$ to $\gamma = 0.6$, and an increase in the services sector's output elasticity of intermediate goods from $\gamma_S = 0.3$ to $\gamma_S = 0.4$.

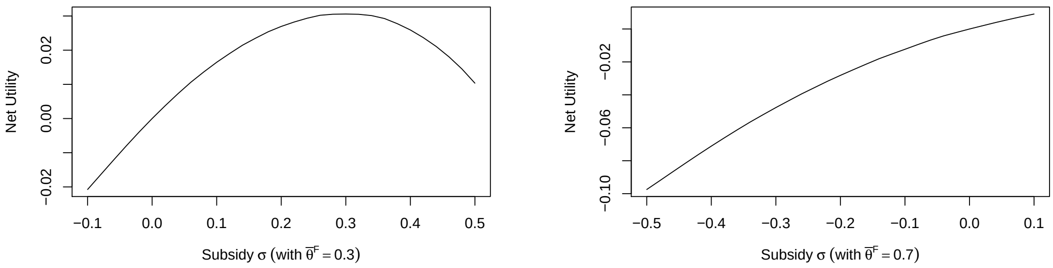

Figure B.40: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 0.5$: increase in e

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

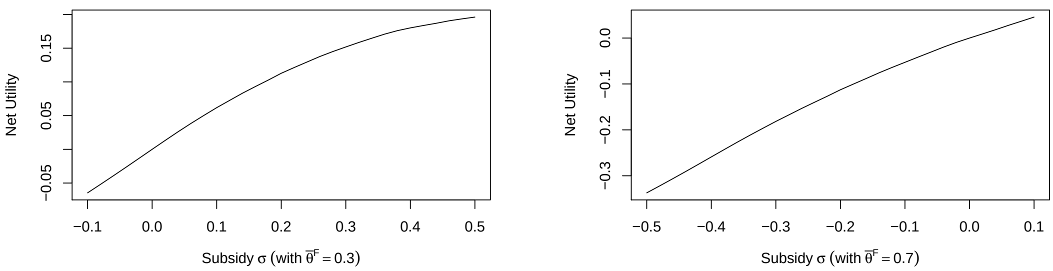

Figure B.41: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: increase in ρ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

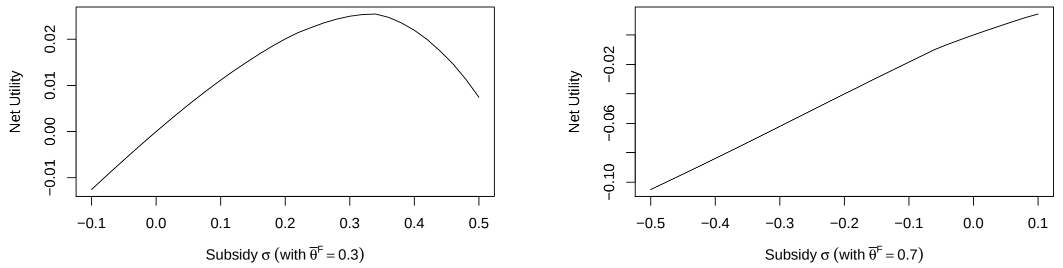

Figure B.42: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: decrease in β

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

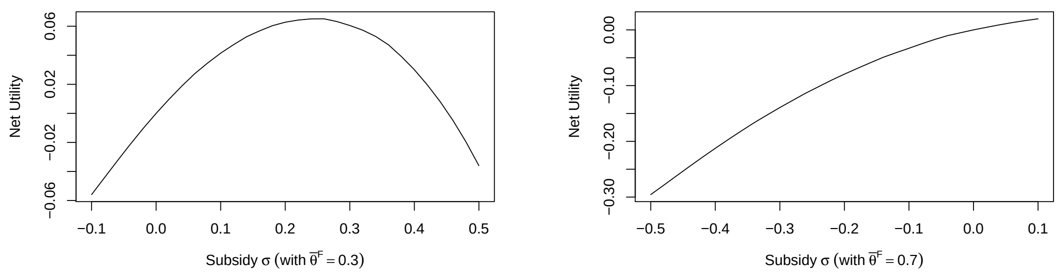

Figure B.43: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: increase in φ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

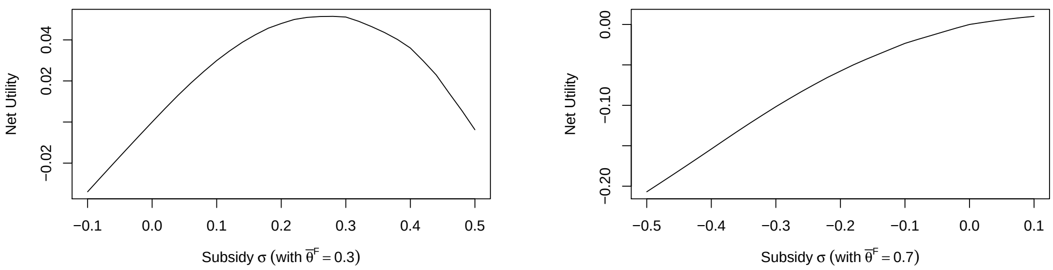

Figure B.44: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: increase in μ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

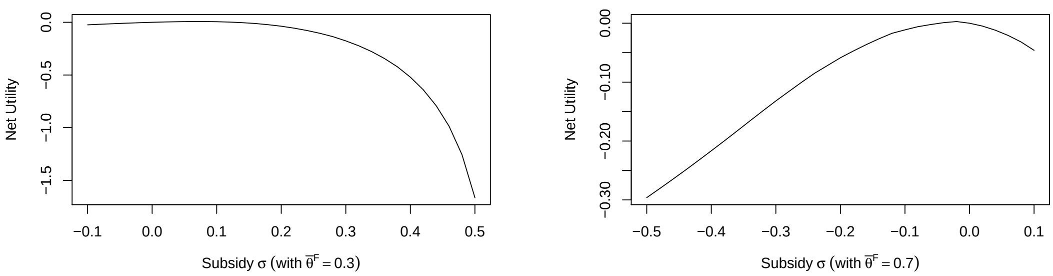

Figure B.45: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: increase in δ

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

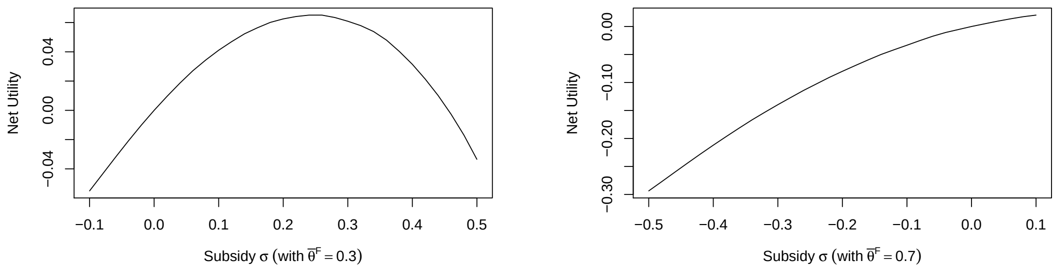

Figure B.46: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: increase in θ^B

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

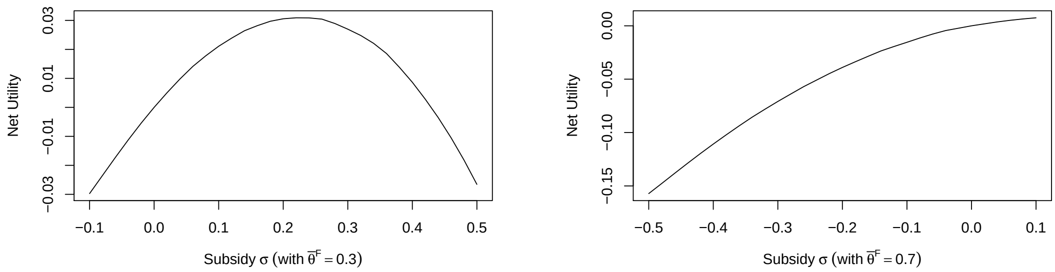

Figure B.47: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: decrease in γ_M and increase in γ_S

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with a decrease in the manufacturing sector's output elasticity of intermediate goods from $\gamma_M = 0.7$ to $\gamma = 0.6$, and an increase in the services sector's output elasticity of intermediate goods from $\gamma_S = 0.3$ to $\gamma_S = 0.4$.

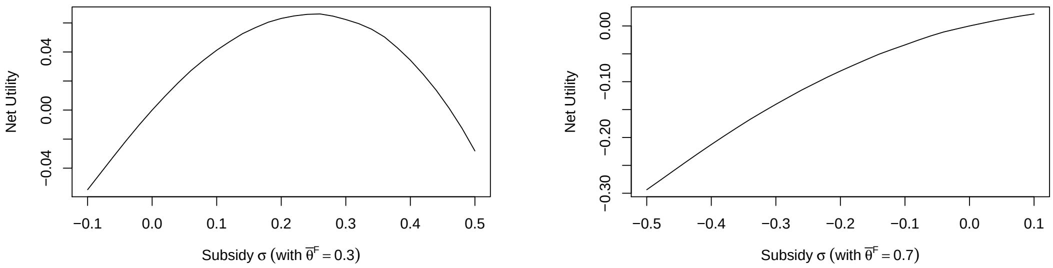

Figure B.48: Welfare changes in response to capital liquidation subsidies and taxes in the two-sector model, with $\epsilon = 2$: increase in e

Numerical modelling of welfare changes in response to capital liquidation subsidies and taxes. The left panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panel shows the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.5, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

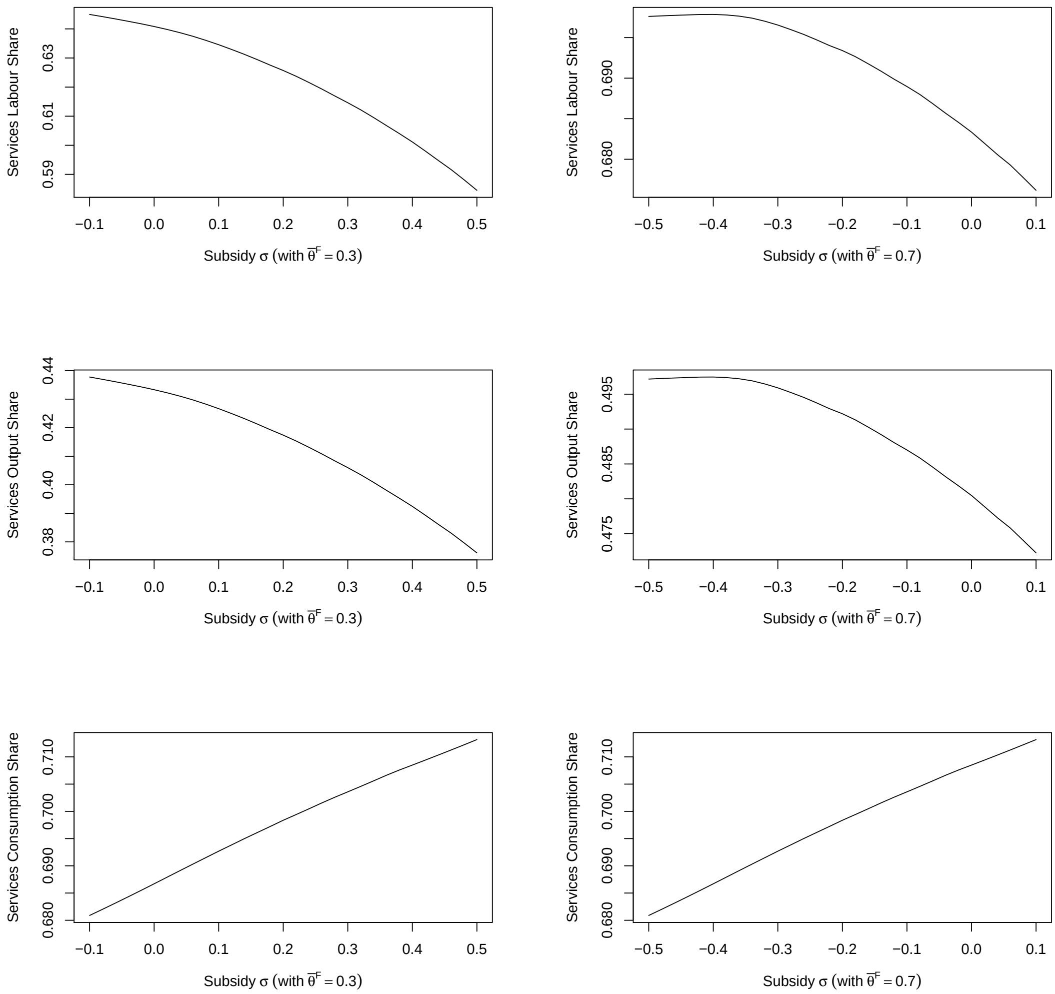

Figure B.49: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: increase in ρ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

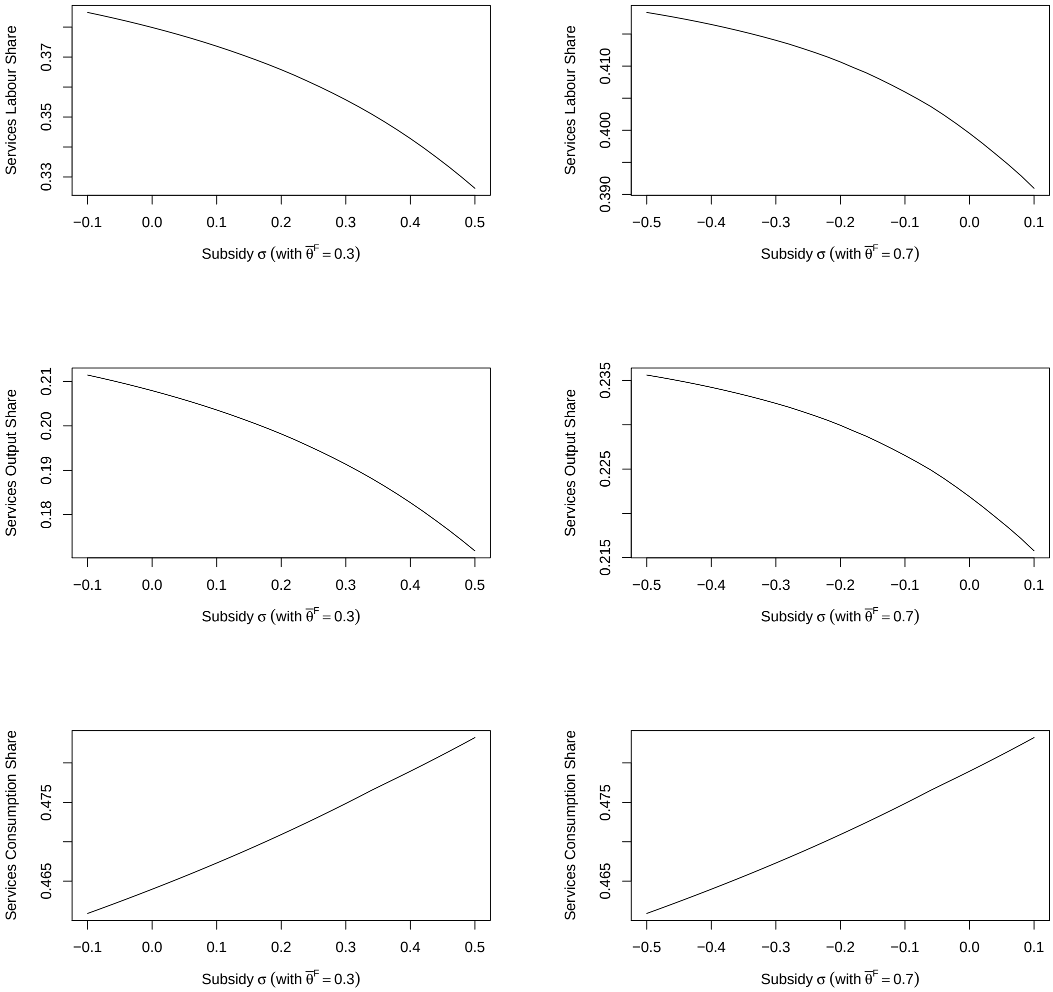

Figure B.50: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: decrease in β

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

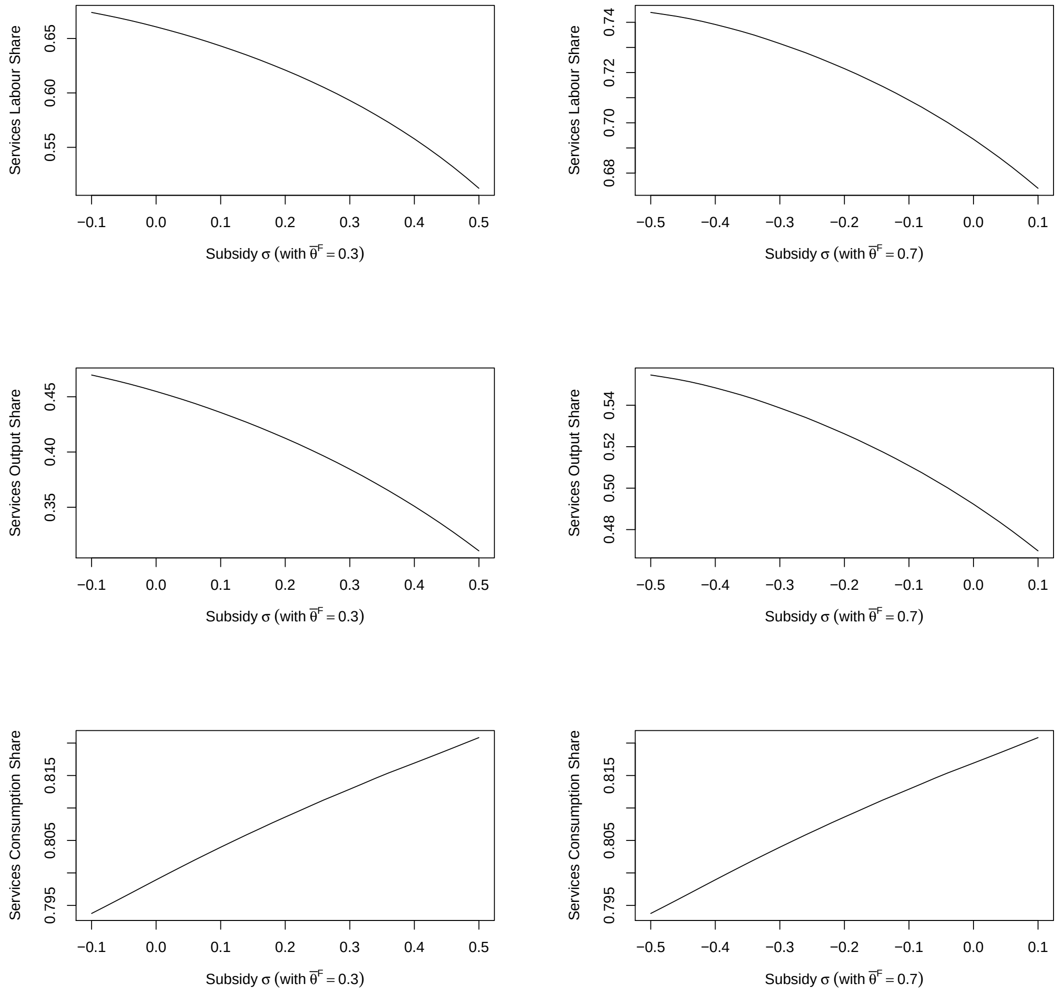

Figure B.51: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: increase in ϕ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

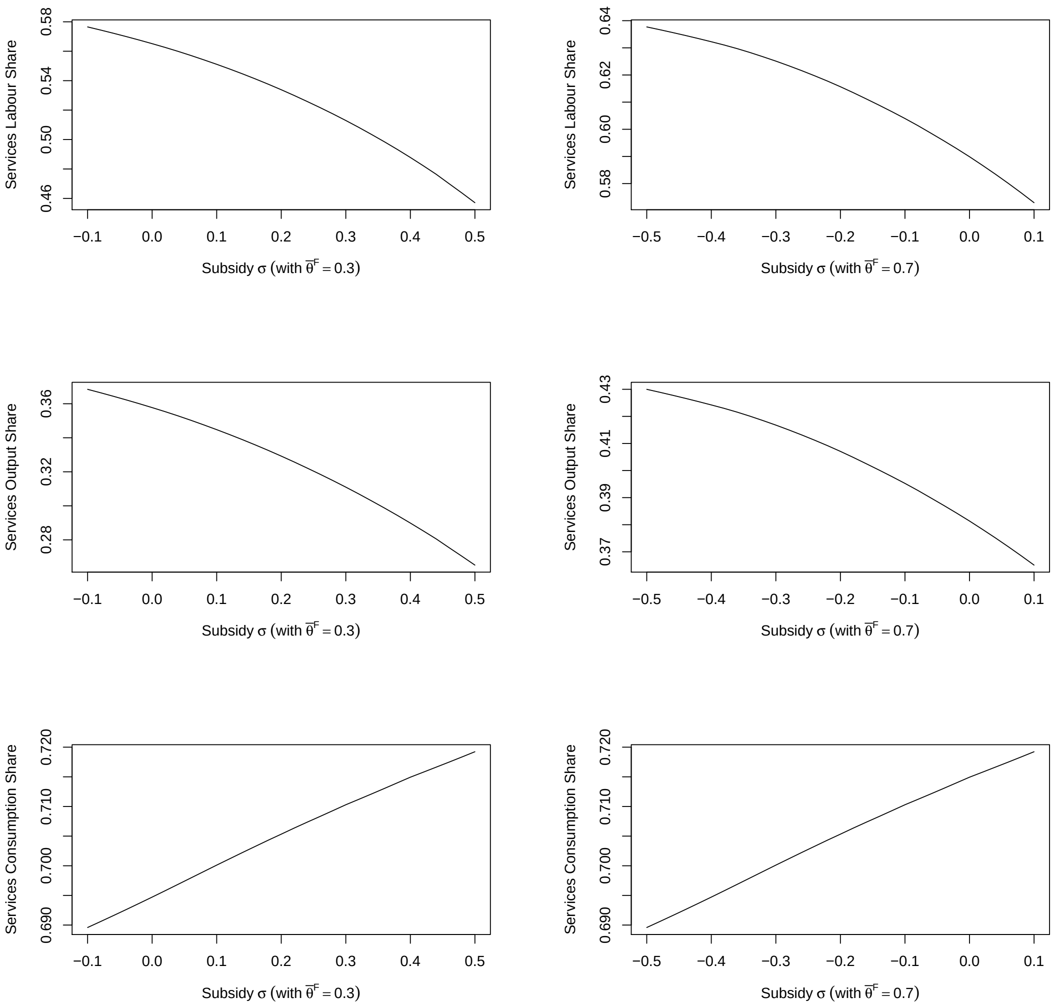

Figure B.52: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: increase in μ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

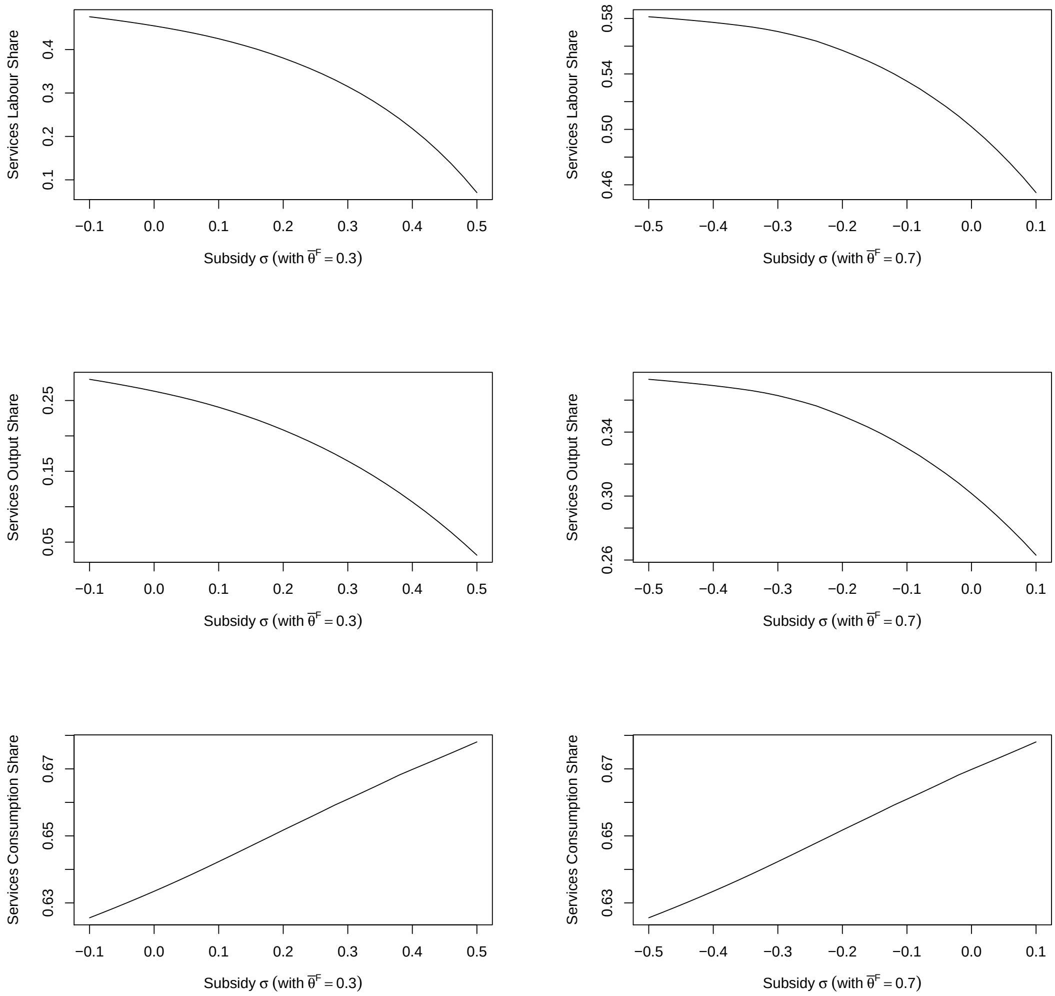

Figure B.53: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: increase in δ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

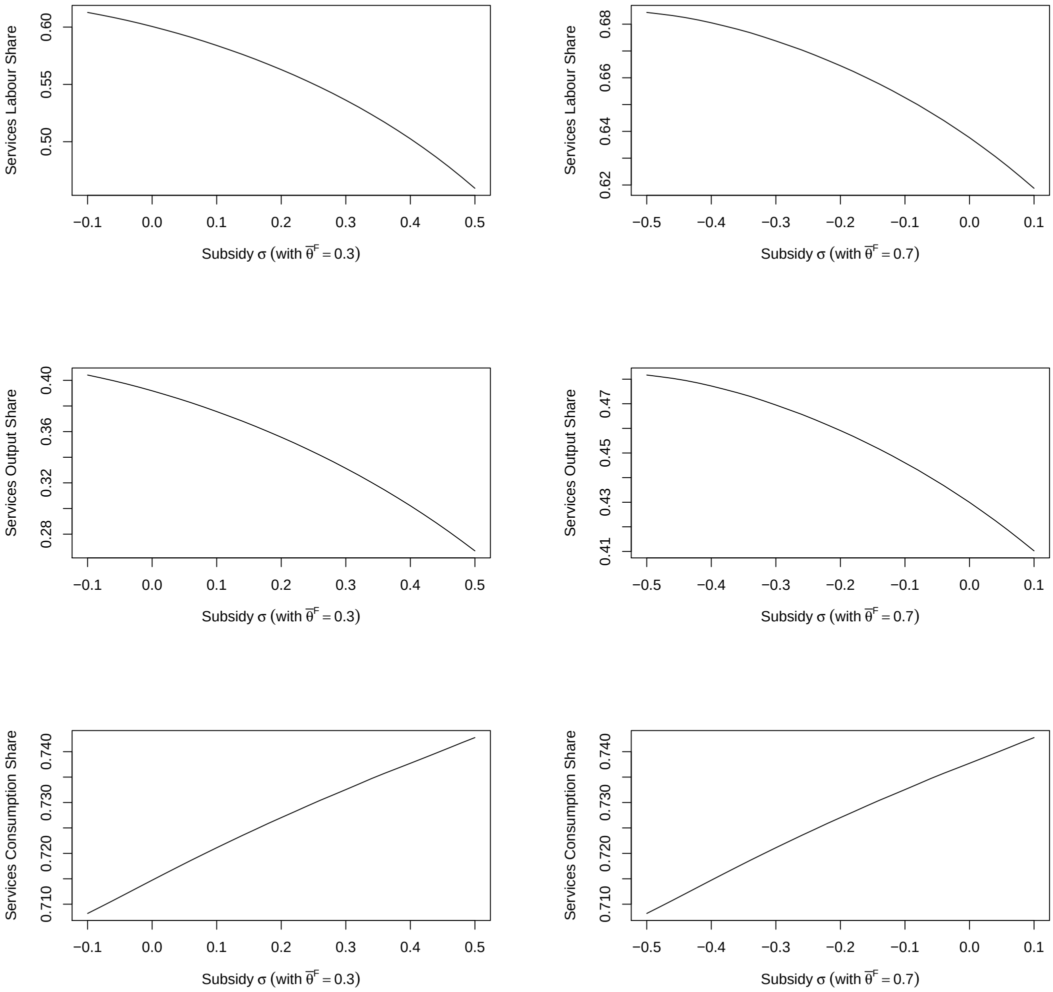

Figure B.54: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: increase in θ^B

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

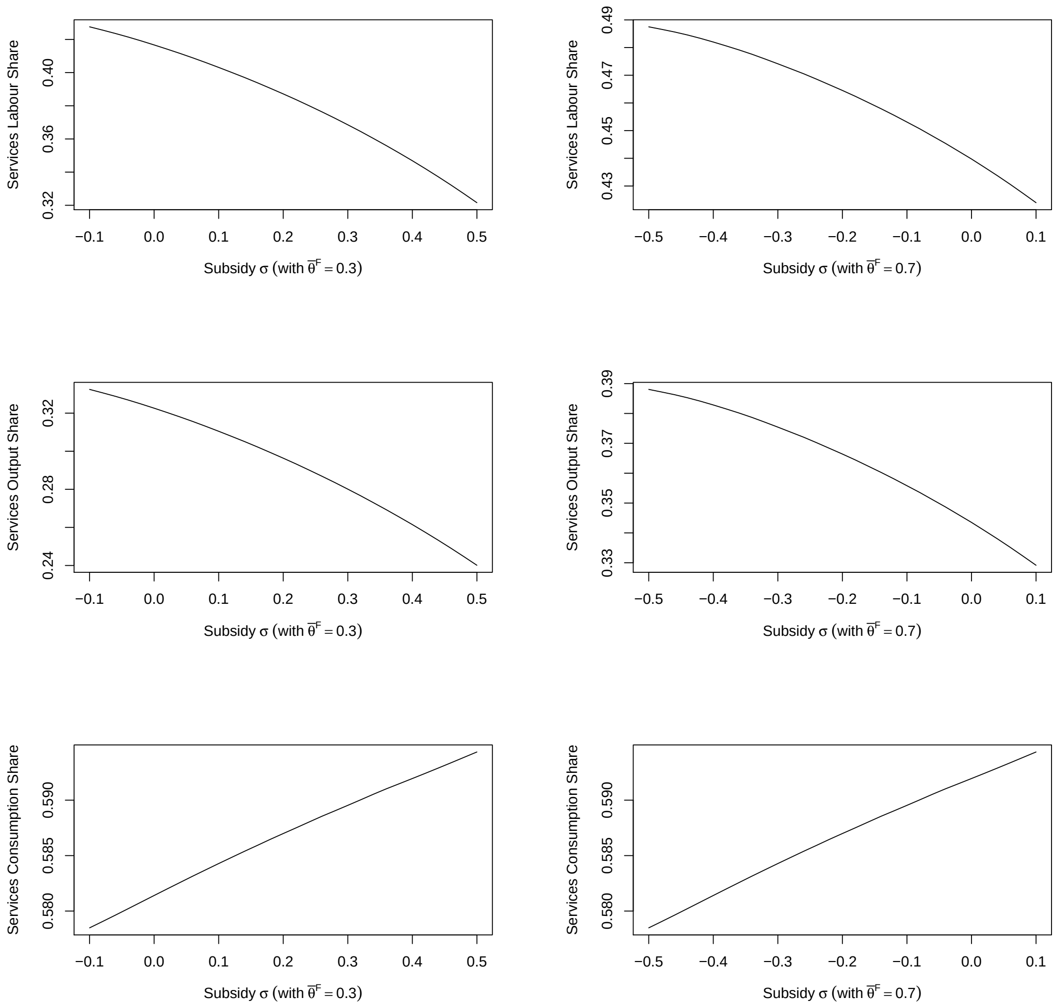

Figure B.55: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: decrease in γ_M and increase in γ_S

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with a decrease in the manufacturing sector's output elasticity of intermediate goods from $\gamma_M = 0.7$ to $\gamma = 0.6$, and an increase in the services sector's output elasticity of intermediate goods from $\gamma_S = 0.3$ to $\gamma_S = 0.4$.

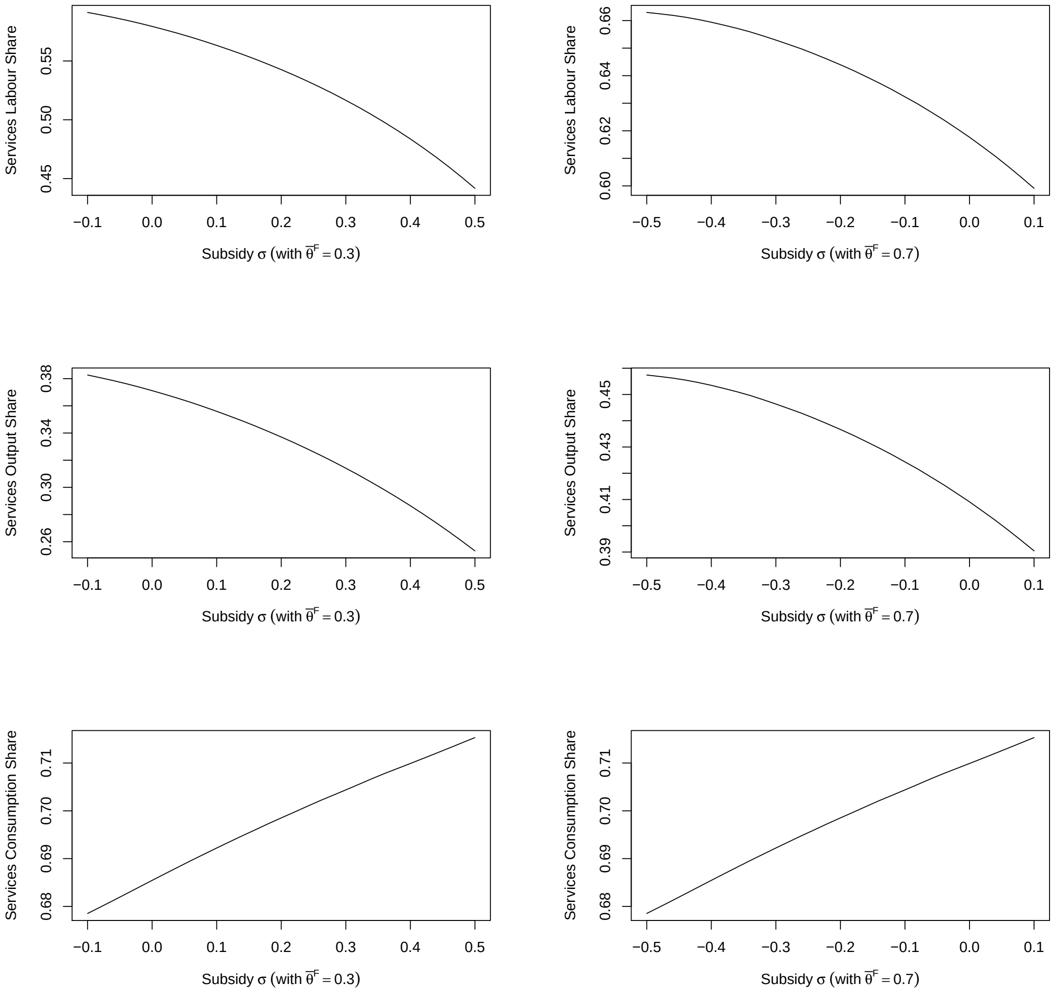

Figure B.56: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 0.5$: increase in e

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.7, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

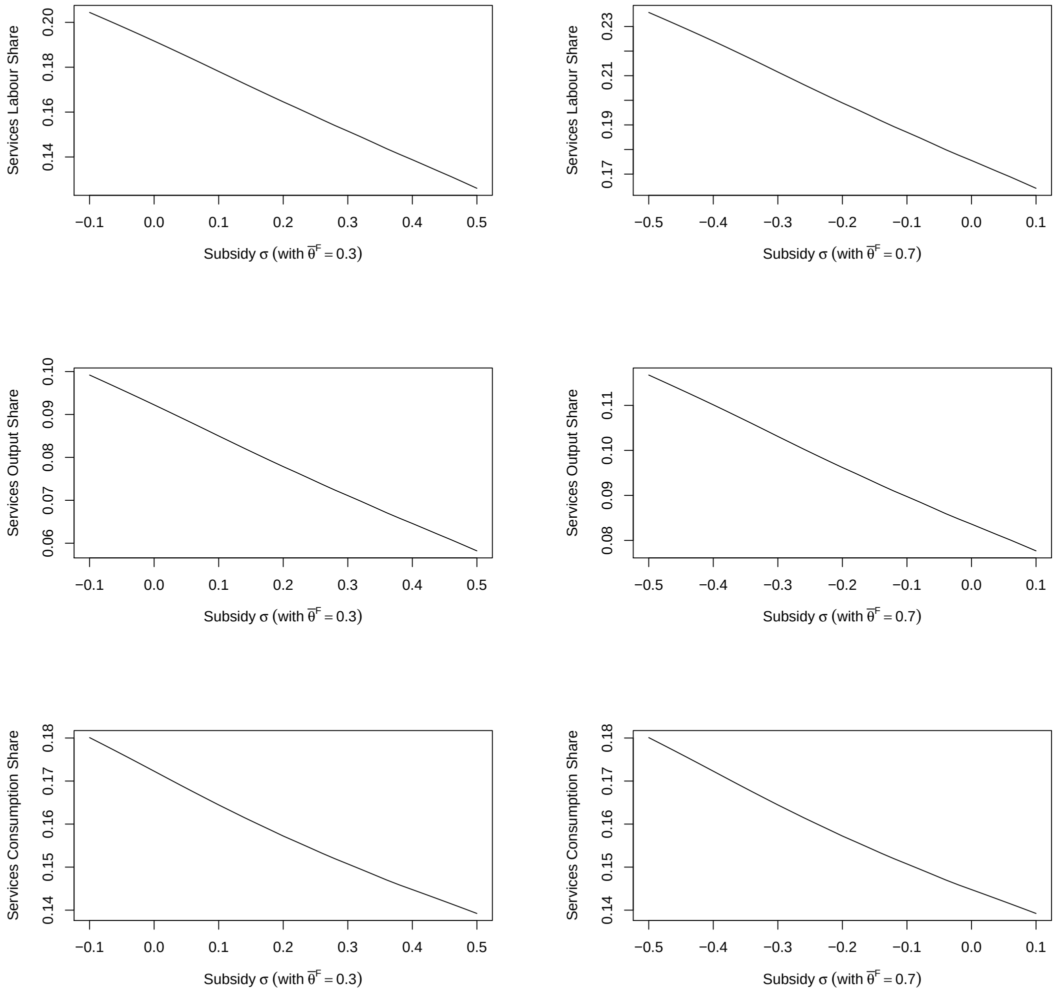

Figure B.57: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: increase in ρ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with an increase in households' subjective discount rate from $\rho = 0.02$ to $\rho = 0.05$.

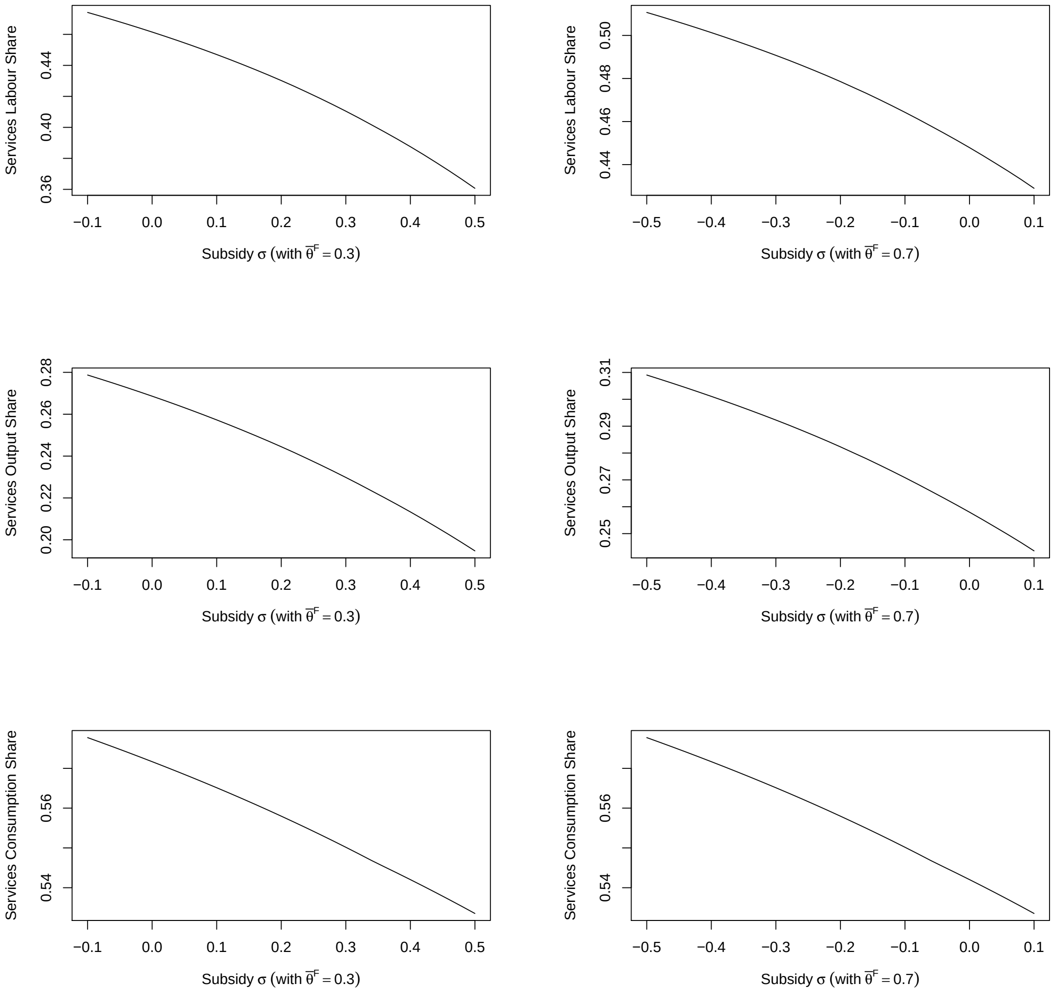

Figure B.58: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: decrease in β

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with a decrease in intermediate firms' output elasticity of capital from $\beta = 0.7$ to $\beta = 0.3$.

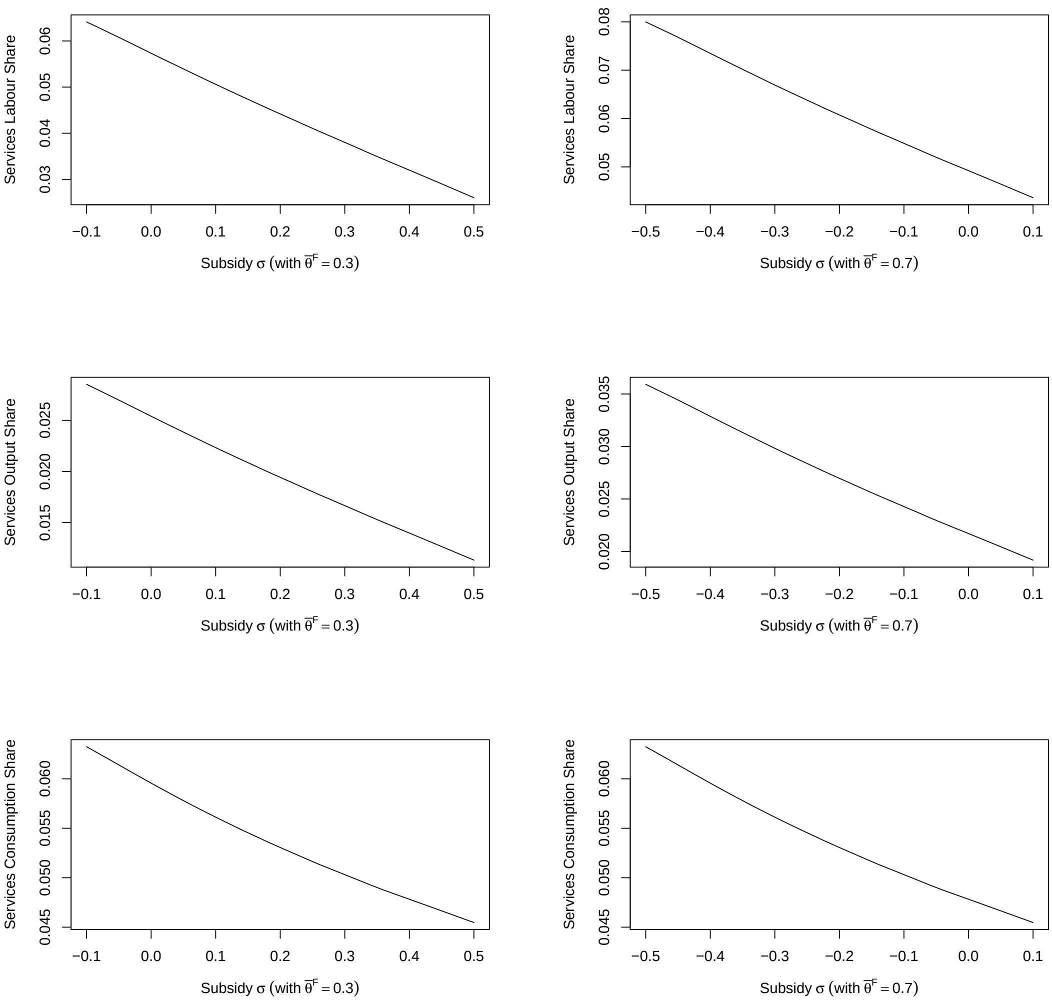

Figure B.59: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: increase in ϕ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with an increase in intermediate firms' productivity from $\bar{\varphi} = 10$ to $\bar{\varphi} = 20$.

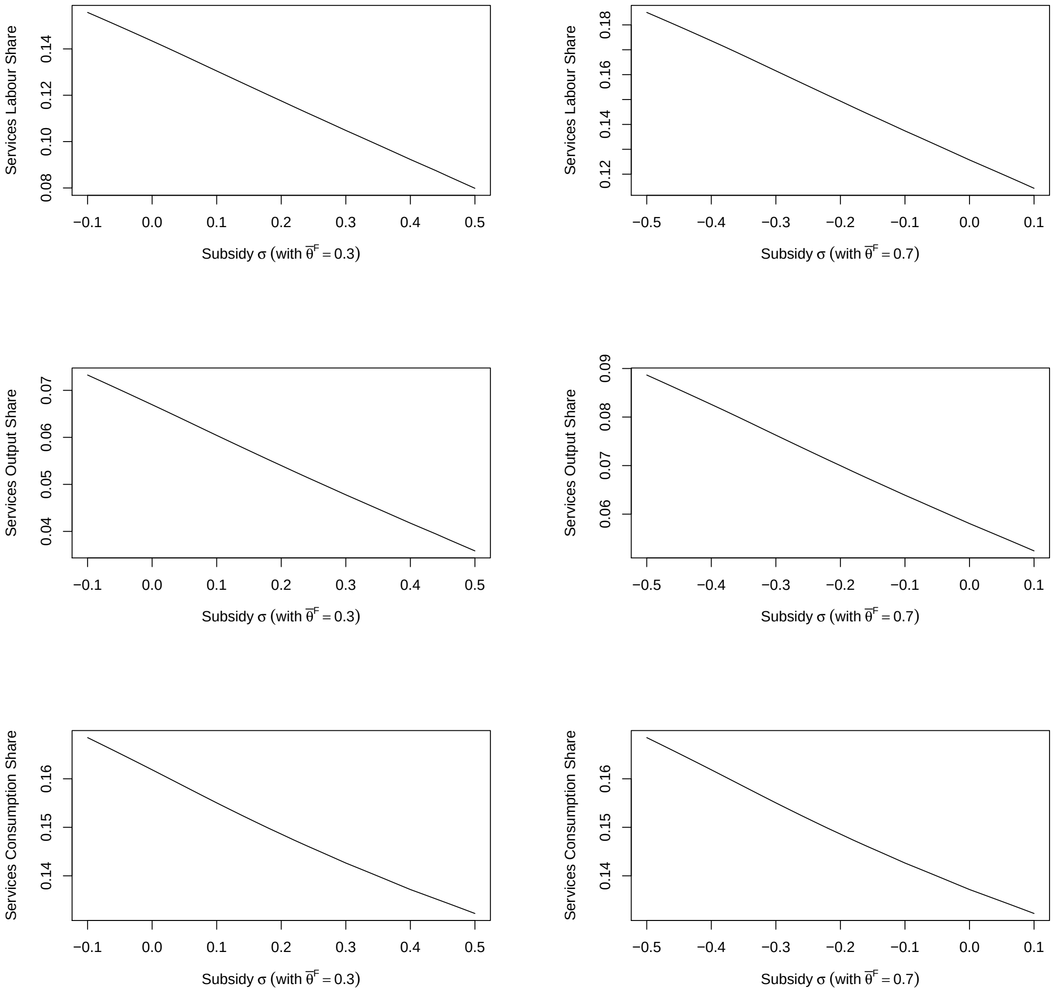

Figure B.60: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: increase in μ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with an increase in capital depreciation from $\mu = 0.05$ to $\mu = 0.1$.

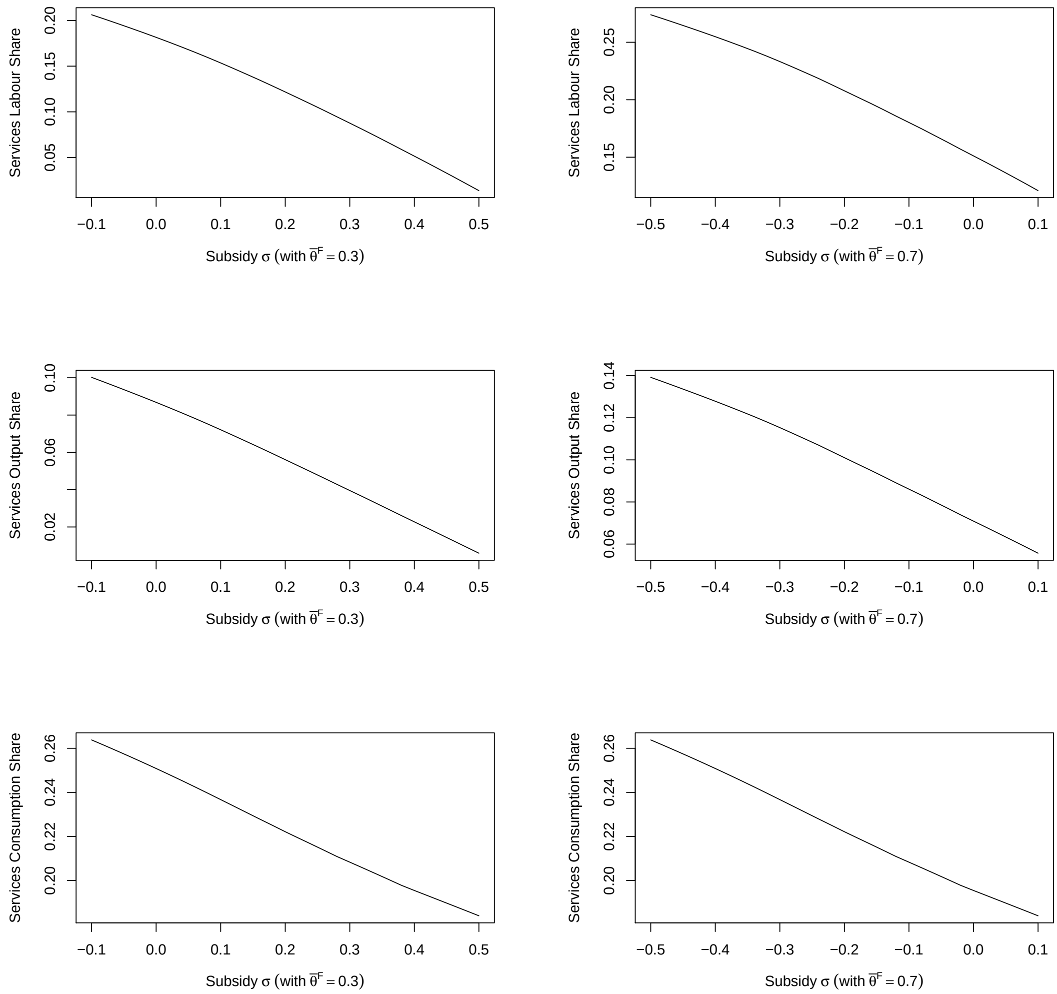

Figure B.61: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: increase in δ

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with an increase in intermediate firms' death risk from $\delta = 0.1$ to $\delta = 0.2$.

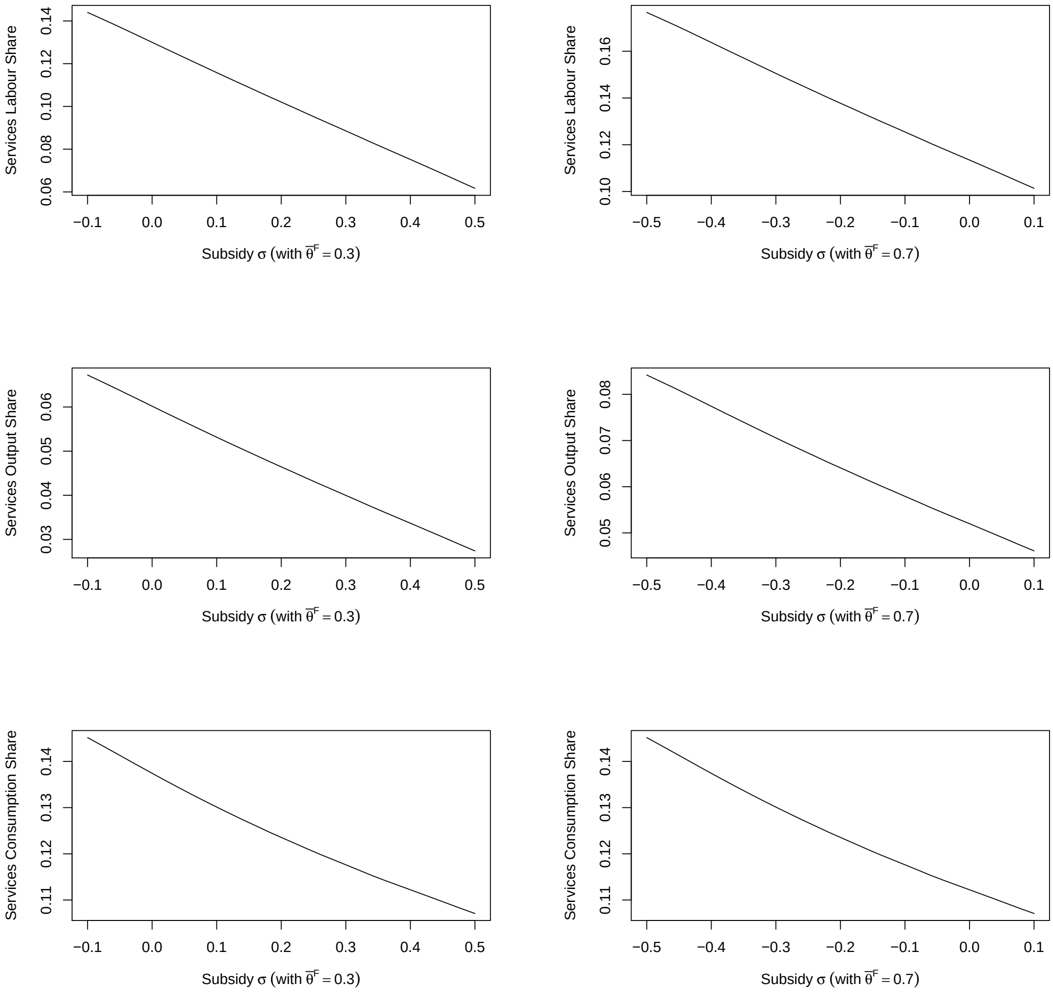

Figure B.62: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: increase in θ^B

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with an increase in the bank's liquidation value of capital from $\theta^B = 0.1$ to $\theta^B = 0.2$.

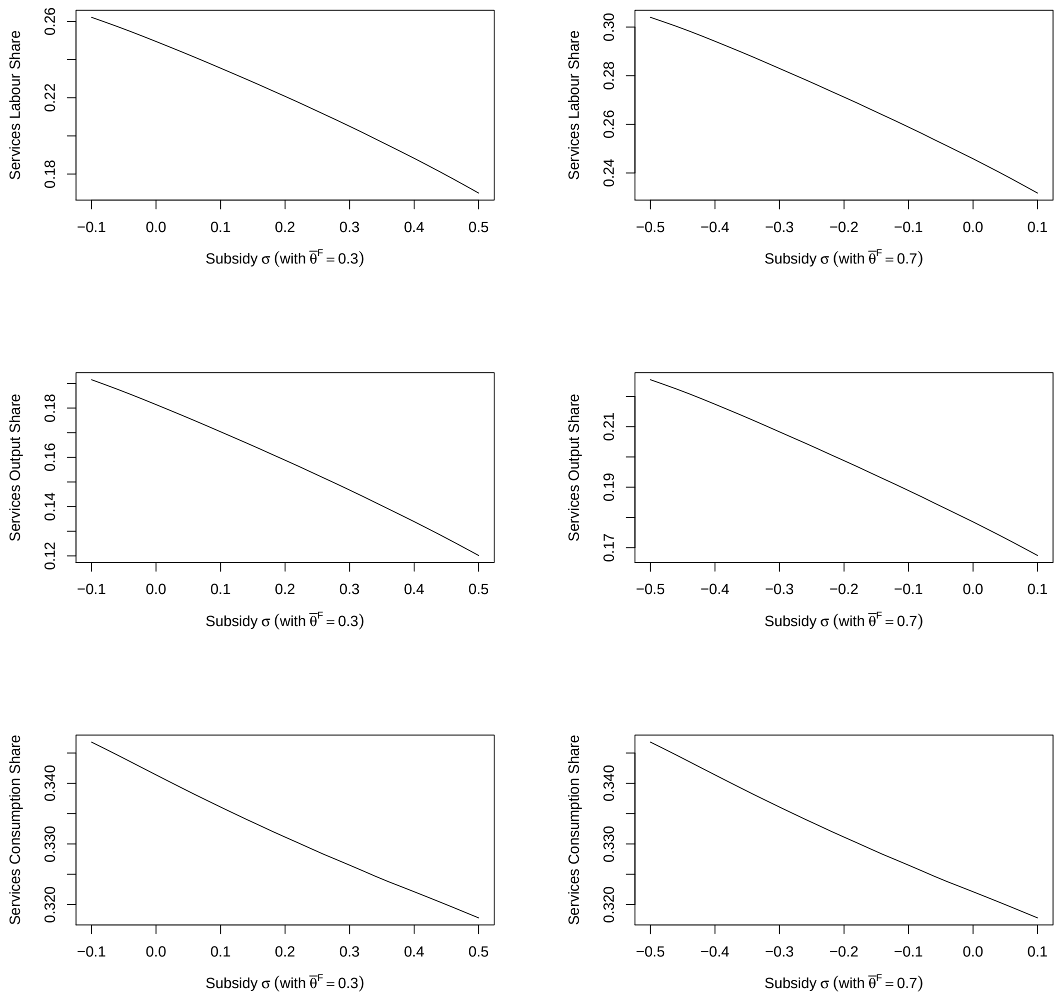

Figure B.63: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: decrease in γ_M and increase in γ_S

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with a decrease in the manufacturing sector's output elasticity of intermediate goods from $\gamma_M = 0.7$ to $\gamma = 0.6$, and an increase in the services sector's output elasticity of intermediate goods from $\gamma_S = 0.3$ to $\gamma_S = 0.4$.

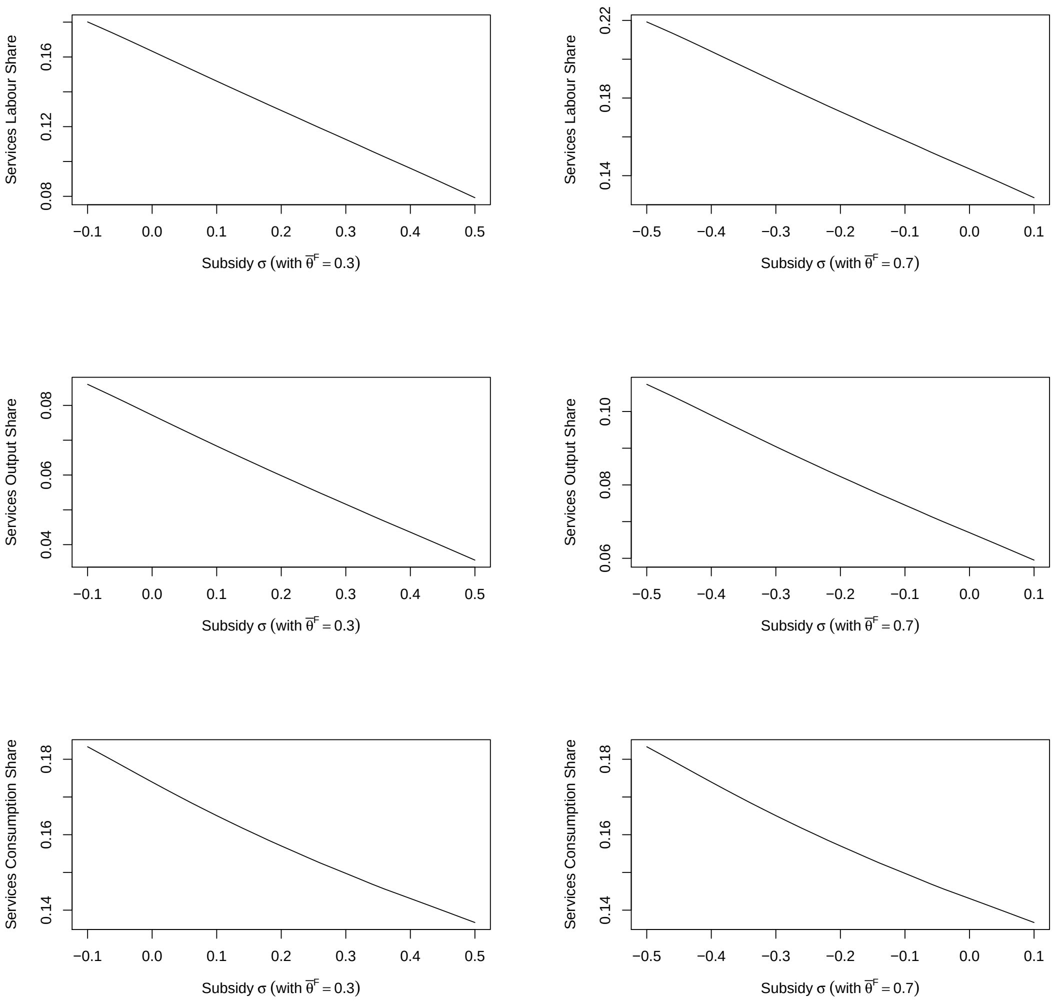

Figure B.64: Structural composition implications of capital liquidation subsidies and taxes, with $\epsilon = 2$: increase in e

Numerical modelling of the change in the balance between services and manufacturing in response to capital liquidation subsidies and taxes. The left panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.3$, and liquidation subsidies range from $\sigma = -0.1$ to $\sigma = 0.5$. The right panels show the situation where the underlying reversibility of investment is $\bar{\theta}^F = 0.7$, and liquidation taxes range from $\sigma = -0.5$ to $\sigma = 0.1$. This is the same as Figure 2.8, but with an increase in intermediate firms' entry cost from $e = 100$ to $e = 200$.

Appendix C

Chapter 3 Appendices

C.1 Proofs

Proof of Lemma 3.1. The result follows immediately from setting both the differential equations that characterise the dynamics of this economy, equations (3.15) and (3.17), equal to zero. ■

Proof of Lemma 3.2. Note that

$$\begin{aligned}\gamma_z &= \frac{\dot{c}}{c} - \frac{\dot{k}}{k} \\ &= z - (1-\alpha) A_0 k^{\alpha+\eta-1} - \rho, \end{aligned} \tag{C.1}$$

so in the steady state, $\gamma_{z_{SS}} = 0$ and $z_{SS} = (1-\alpha) A_0 k_{SS}^{\alpha+\eta-1} + \rho$. Now suppose $\dot{k} > 0$, $\dot{c} > 0$, so that $k < k_{SS}$ and the economy is growing towards the steady state. Then

$$\begin{aligned}\gamma_z &= z - (1-\alpha) A_0 k^{\alpha+\eta-1} - \rho \\ &= z - z_{SS} + (1-\alpha) A_0 \left[k_{SS}^{\alpha+\eta-1} - k^{\alpha+\eta-1}\right].\end{aligned}$$

Since $k_{SS} > k$ by assumption, and $\alpha + \eta < 1$, the term in the square brackets is negative, so $\gamma_z < z - z_{SS}$. Thus if $z \leq z_{SS}$, it follows that $\gamma_z < 0$, so z is getting further and further below its steady-state level, and the economy never reaches equilibrium.

It follows therefore that $z > z_{SS}$ along the saddle path while the economy is growing. Now consider the time derivative of γ_z,

$$\begin{aligned}\dot{\gamma}_z &= \dot{z} - (1-\alpha)(\alpha+\eta-1) A_0 k^{\alpha+\eta-2} \dot{k} \\ &= \gamma_z z - \gamma_z (1-\alpha) A_0 k^{\alpha+\eta-1} - \gamma_z \rho \\ &\qquad + \gamma_z (1-\alpha) A_0 k^{\alpha+\eta-1} + \gamma_z \rho - (1-\alpha)(\alpha+\eta-1) A_0 k^{\alpha+\eta-2} \dot{k} \\ &= \gamma_z \left[z - (1-\alpha) A_0 k^{\alpha+\eta-1}\right] + \gamma_z \rho + (1-\alpha) A_0 k^{\alpha+\eta-1} \left[\gamma_z - (\alpha+\eta-1)\frac{\dot{k}}{k}\right] \\ &= \gamma_z^2 + \gamma_z \rho + (1-\alpha) A_0 k^{\alpha+\eta-1} \left\{\gamma_z + [1-(\alpha+\eta)]\frac{\dot{k}}{k}\right\}.\end{aligned}$$

Suppose $\gamma_z \geq 0$. Since $\dot{k}/k > 0$ by assumption, and $\alpha + \eta < 1$, this would imply that $\dot{\gamma}_z > 0$. We know that $z > z_{SS}$, so if $\gamma_z \geq 0$ and $\dot{\gamma}_z > 0$, the consumption-labour ratio is accelerating away from its steady state level, and never reaches equilibrium. Thus we know that at all points along the saddle path in the growth phase of the economy, $z > z_{SS}$ and $\gamma_z < 0$. The consumption-capital ratio along the saddle path therefore looks something like Figure 3.4. ■

Proof of Lemma 3.3. We know from Lemma 3.1 that a negative shock to θ causes an increase in k_{SS}. At the moment of the shock, per-capita capital k remains unchanged, but its growth rate may change. Thus denote by θ_1, c_1, k_1, z_1 the variables of interest on the saddle path immediately prior to the shock, and let θ_2, c_2, k_2, z_2 be the variables on the new saddle path immediately after the shock, so that $\theta_2 < \theta_1$ and $k_2 = k_1$. Let k_{SS1} and k_{SS2} be the steady-state levels of per-capita capital before and after the shock respectively, where $k_{SS2} > k_{SS1}$. We will define

$$\phi(t) \equiv \frac{z_1}{z_2}.$$

Thus

$$\begin{aligned} \frac{\dot{\phi}}{\phi} &= \gamma_{z_1} - \gamma_{z_2} \\ &= z_1 - z_2, \end{aligned}$$

by equation (C.1), since $k_2 = k_1$. Suppose that $k_1 = k_{SS1}$, so that the economy is in the steady state prior to the negative shock. Then $\gamma_{z_1} = 0$, so $\dot{\phi}/\phi = -\gamma_{z_2} = z_1 - z_2$. Suppose $z_2 \geq z_1$; then $\gamma_{z_2} \geq 0$, which we know from Lemma 3.2 is not possible in the growth phase of the economy. Since the new steady state level of per-capita capital k_{SS2} is greater than the previous steady state level of per-capita capital $k_{SS1} = k_2$, then the economy is still in its growth phase immediately after the shock.

Thus, conceiving of $z(k)$ as a function of k (as in Figure 3.4), $z_2(k_{SS1}) < z_1(k_{SS1})$. Suppose for a contradiction that there were some level of capital $\bar{k} < k_{SS1}$ such that $z_2(\bar{k}) = z_1(\bar{k})$, so that also $c_2(\bar{k}) = c_1(\bar{k})$. This would imply that, evaluated at $k = \bar{k}$,

$$\begin{aligned} \frac{d\phi/dk}{\phi} &= \frac{dz_1/dk_1}{z_1} - \frac{dz_2/dk_2}{z_2} \\ &= \frac{\dot{z}_1}{z_1} \cdot \frac{dz_1/dk_1}{dz_1/dt} - \frac{\dot{z}_2}{z_2} \cdot \frac{dz_2/dk_2}{dz_2/dt} \\ &= \gamma_{z_1} \frac{dt}{dk_1} - \gamma_{z_2} \frac{dt}{dk_2} \\ &= \frac{\gamma_{z_1}}{\dot{k}_1} - \frac{\gamma_{z_2}}{\dot{k}_2}. \end{aligned}$$

Note that by equation (3.15), evaluated at $\bar{k}$ (so that $k_1 = k_2 = \bar{k}$),

$$\begin{aligned} &\dot{k}_1 - \dot{k}_2 = (\theta_2 - \theta_1)\,\bar{k} < 0 \\ \Rightarrow\quad &\dot{k}_1 < \dot{k}_2 \\ \Rightarrow\quad &\frac{1}{\dot{k}_1} > \frac{1}{\dot{k}_2}. \end{aligned} \tag{C.2}$$

Thus, since $\gamma_{z_1} < 0$ by Lemma 3.2,

$$\begin{aligned} \frac{d\phi/dk}{\phi} &= \frac{\gamma_{z_1}}{\dot{k}_1} - \frac{\gamma_{z_2}}{\dot{k}_2} \\ &< \frac{\gamma_{z_1}}{\dot{k}_2} - \frac{\gamma_{z_2}}{\dot{k}_2} \\ &= \frac{1}{\dot{k}_2}\left(\gamma_{z_1} - \gamma_{z_2}\right) \\ &= \frac{1}{\dot{k}_2}\left(z_1 - z_2\right) \\ &= 0. \end{aligned}$$

So if there is some $\bar{k}$ for which $z_1\left(\bar{k}\right) = z_2\left(\bar{k}\right)$, then z_1 is decreasing in k faster than z_2 at that point. Therefore if $z_2 \geq z_1$ anywhere to the left of k_{SS1}, it must be that $z_2 \geq z_1$ everywhere to the left of k_{SS1}. However, we just showed that $z_2\left(k_{SS1}\right) < z_1\left(k_{SS2}\right)$; assuming that both z_1 and z_2 are continuous functions of k, therefore, this is a contradiction, and $z_2 < z_1$ for all $k < k_{SS1}$. Thus z_2 looks something like Figure 3.6.

It follows that, given fixed k, a negative shock to θ causes a negative shock to z, and thus to c. ■

Proof of Lemma 3.5. By Lemma 3.1 the steady state level of ζ is

$$\begin{aligned} \zeta_{SS} &= \frac{c_{SS}}{k_{SS}} \cdot \frac{k_{SS}}{y_{SS}} \\ &= \left[\left(\frac{1}{\alpha} - 1\right)(\theta + \rho + g) + \rho\right] \cdot \frac{k_{SS}}{A_0 k_{SS}^{\alpha+\eta}} \\ &= \left[\left(\frac{1}{\alpha} - 1\right)(\theta + \rho + g) + \rho\right] \frac{k_{SS}^{1-(\alpha+\eta)}}{A_0} \\ &= \left[\left(\frac{1}{\alpha} - 1\right)(\theta + \rho + g) + \rho\right] \frac{\alpha A_0}{A_0\left(\theta + \rho + g\right)} \\ &= 1 - \alpha + \frac{\alpha\rho}{\theta + \rho + g}. \end{aligned}$$

Consider now the growth rate of ζ,

$$\begin{aligned}
\gamma_\zeta &= \frac{\dot{c}}{c} - \frac{\dot{y}}{y} \\
&= \left(\frac{\dot{c}}{c} - \frac{\dot{k}}{k}\right) + \left(\frac{\dot{k}}{k} - \frac{\dot{y}}{y}\right) \\
&= z - (1-\alpha) A_0 k^{\alpha+\eta-1} - \rho + [1-(\alpha+\eta)] \frac{\dot{k}}{k} \\
&= \frac{c}{k} - (1-\alpha) \frac{y}{k} - \rho + [1-(\alpha+\eta)] \frac{\dot{k}}{k} \\
&= \left[\zeta - (1-\alpha) - \rho \left(\frac{k}{y}\right)\right] \frac{y}{k} + [1-(\alpha+\eta)] \frac{\dot{k}}{k}.
\end{aligned} \tag{C.3}$$

When the economy is in its growth phase, the second term in the above expression is clearly positive. Suppose that $\zeta \geq 1 - \alpha + \rho\,(k/y)$. Then $\gamma_\zeta > 0$, and ζ is also clearly growing faster than k/y, so ζ is diverging from its steady state level. Thus along the saddle path, $\zeta < 1 - \alpha + \rho\,(k/y)$. Substituting in for the growth rate of k,

$$\begin{aligned}
\gamma_\zeta &= \left[\zeta - (1-\alpha) - \rho \left(\frac{k}{y}\right)\right] \frac{y}{k} + [1-(\alpha+\eta)] \left[A_0 k^{\alpha+\eta-1} - z - (\theta+\rho+g)\right] \\
&= \left[\zeta - (1-\alpha) - \rho \left(\frac{k}{y}\right)\right] \frac{y}{k} + [1-(\alpha+\eta)] \left[\frac{y}{k} - \left(\frac{c}{y} \cdot \frac{y}{k}\right) - (\theta+\rho+g)\right] \\
&= \left[(\alpha+\eta)\,\zeta - \eta - \rho \left(\frac{k}{y}\right)\right] \frac{y}{k} - [1-(\alpha+\eta)]\,(\theta+\rho+g)\,.
\end{aligned} \tag{C.4}$$

Clearly, the second term in the above expression is positive, so subtracting it gives a negative value. If

$$\zeta \leq \frac{\eta + \rho\,(k/y)}{\alpha+\eta},$$

then $\gamma_\zeta < 0$. Since k/y is growing with k, it follows that when k is growing,

$$\frac{k}{y} < \frac{k_{SS}}{y_{SS}} = \frac{\alpha}{\theta+\rho+g} < \frac{\alpha}{\rho},$$

and so $\rho\,(k/y) < \alpha$. Thus

$$\begin{aligned}
\frac{\eta + \rho\,(k/y)}{\alpha+\eta} &= 1 - \frac{\alpha}{\alpha+\eta} + \frac{\rho\,(k/y)}{\alpha+\eta} \\
&< 1 - \alpha + \rho \left(\frac{k}{y}\right) \\
&< 1 - \alpha + \frac{\alpha\rho}{\theta+\rho+g} \\
&= \zeta_{SS}.
\end{aligned}$$

So

$$\begin{aligned} \zeta &\leq \frac{\eta + \rho\,(k/y)}{\alpha + \eta} \qquad\qquad (C.5) \\ \Rightarrow \quad \zeta &< \zeta_{SS} \quad , \quad \gamma_\zeta < 0. \end{aligned}$$

Therefore if equation (C.5) holds anywhere to the left of k_{SS}, then ζ diverges from ζ_{SS} and never reaches its steady state. Thus along the saddle path of the economy, in the growth phase, we have shown that

$$\frac{\eta + \rho\,(k/y)}{\alpha + \eta} < \zeta < 1 - \alpha + \rho\left(\frac{k}{y}\right).$$

Now consider the time derivative of γ_ζ,

$$\begin{aligned} \dot{\gamma}_\zeta &= (\alpha + \eta)\, A_0 k^{\alpha+\eta-1} \dot{\zeta} + [(\alpha + \eta)\,\zeta - \eta]\,(\alpha + \eta - 1)\, A_0 k^{\alpha+\eta-2} \dot{k} \\ &= A_0 k^{\alpha+\eta-1} \left\{ (\alpha + \eta)\,\gamma_\zeta \zeta + [(\alpha + \eta)\,\zeta - \eta]\,(\alpha + \eta - 1)\,\frac{\dot{k}}{k} \right\}. \end{aligned}$$

Since $(\alpha + \eta)\,\zeta > \eta + \rho\,(k/y) > \eta$ and $\alpha + \eta < 1$, it follows that the second term in the curly brackets is negative. If γ_ζ were also weakly negative, then it would also be decreasing, since $\dot{\gamma}_\zeta < 0$, and ζ would never converge to its steady state level. Thus along the saddle path in the growth phase of the economy, $\gamma_\zeta > 0$. The consumption-output ratio therefore looks something like Figure 3.7. ■

Proof of Lemma 3.6. A negative shock to θ leaves per-capita capital k unaffected in the moment, and thus also leaves y unaffected, but by Lemma 3.3 there is a negative shock to per-capita consumption c. It follows immediately that there is a negative shock to $\zeta = c/y$. Thus, in particular, we know that $d\zeta/d\theta > 0$.

Now consider the effect of θ on $\zeta = c/y$. Denote by θ_1, k_1, ζ_1 the variables of interest on the saddle path immediately prior to the shock, and let θ_2, k_2, ζ_2 be the variables on the new saddle path immediately after the shock, so that $\theta_2 < \theta_1$ and $k_2 = k_1$. Let k_{SS1} and k_{SS2} be the steady-state levels of per-capita capital before and after the shock respectively, where $k_{SS2} > k_{SS1}$. Define

$$\psi\,(t) = \frac{\zeta_1}{\zeta_2}.$$

By equation (C.4),

$$\begin{aligned} \gamma_\zeta &= \left[(\alpha + \eta)\,\zeta - \eta - \rho\left(\frac{k}{y}\right)\right]\frac{y}{k} - [1 - (\alpha + \eta)]\,(\theta + \rho + g) \\ \Rightarrow \quad \frac{\dot{\psi}}{\psi} &= \gamma_{\zeta_1} - \gamma_{\zeta_2} \\ &= (\alpha + \eta)\,\frac{y}{k}\,(\zeta_1 - \zeta_2) - [1 - (\alpha + \eta)]\,(\theta_1 - \theta_2)\,. \end{aligned}$$

Suppose that $k_1 = k_{SS1}$, so that the economy is in the steady state prior to the negative shock. Then $\gamma_{\zeta_1} = 0$, so $\dot{\psi}/\psi = -\gamma_{\zeta_2}$. Since the new steady state level of per-capita capital k_{SS2} is greater than the previous steady state level of per-capita capital $k_{SS1} = k_2$, then the economy is still in its growth phase immediately after the shock. Thus by Lemma 3.5, $\gamma_{\zeta_2} > 0$, and $\dot{\psi}/\psi = -\gamma_{\zeta_2} < 0$. It follows that

$$\begin{aligned}
(\alpha+\eta)\frac{y(k_{SS1})}{k_{SS}}(\zeta_1-\zeta_2) - [1-(\alpha+\eta)](\theta_1-\theta_2) < 0 & \\
\Rightarrow \quad \frac{\zeta_1-\zeta_2}{\theta_1-\theta_2} &< \left[\frac{1-(\alpha+\eta)}{\alpha+\eta}\right]\frac{k_{SS1}}{y(k_{SS1})} \\
\Rightarrow \quad \left.\frac{d\zeta}{d\theta}\right|_{k=k_{SS1}} &= \lim_{\theta_2\to\theta_1}\left.\frac{\zeta_1-\zeta_2}{\theta_1-\theta_2}\right|_{k=k_{SS1}} \\
&\le \left[\frac{1-(\alpha+\eta)}{\alpha+\eta}\right]\frac{k_{SS1}}{y(k_{SS1})} \\
\Rightarrow \quad \left.\frac{d\zeta}{d\theta}\right|_{k=k_{SS}} &\le \left[\frac{1-(\alpha+\eta)}{\alpha+\eta}\right]\frac{k_{SS}}{y(k_{SS})}.
\end{aligned}$$

So we have been able to put an upper bound on $d\zeta/d\theta$ at the steady state level of per-capita capital k_{SS}. Conceiving of ζ as a function of k, as in Figure 3.7, we wish to show the same upper bound applies to this differential at all values of $k \le k_{SS}$.

Suppose for a contradiction that there exists some $\bar{k} < k_{SS}$ such that

$$\left.\frac{d\zeta}{d\theta}\right|_{k=\bar{k}} > \left[\frac{1-(\alpha+\eta)}{\alpha+\eta}\right]\frac{\bar{k}}{y(\bar{k})}. \tag{C.6}$$

Define

$$\chi(k) = \zeta(k)\frac{y(k)}{k}.$$

Then

$$\left.\frac{d\chi}{d\theta}\right|_{k=\bar{k}} > \left[\frac{1-(\alpha+\eta)}{\alpha+\eta}\right] \ge \left.\frac{d\chi}{d\theta}\right|_{k=k_{SS}}.$$

Thus $d\chi/d\theta$ is, on average, decreasing with k from its value at $\bar{k}$ towards the value it assumes at k_{SS}. By the Mean Value Theorem, there exists some $\hat{k} \in (\bar{k}, k_{SS})$ such that

$$\left.\frac{d\chi}{d\theta}\right|_{k=\hat{k}} > \left[\frac{1-(\alpha+\eta)}{\alpha+\eta}\right] \tag{C.7}$$

$$\left.\frac{d^2\chi}{d\theta\, dk}\right|_{k=\hat{k}} < 0. \tag{C.8}$$

Note that

$$\begin{aligned}
\frac{d\gamma_\zeta}{d\theta} &= \frac{d\left(\dot{\zeta}/\zeta\right)}{d\theta} \\
&= \frac{1}{\zeta}\left[\frac{d\dot{\zeta}}{d\theta} - \gamma_\zeta\frac{d\zeta}{d\theta}\right].
\end{aligned}$$

Therefore, since $d\zeta/d\theta > 0$ and $\gamma_\zeta > 0$ for all $k < k_{SS}$,,

$$\begin{aligned}\frac{d\dot{\zeta}}{d\theta} &= \zeta\frac{d\gamma_\zeta}{d\theta} + \gamma_\zeta\frac{d\zeta}{d\theta} \\ &> \zeta\frac{d\gamma_\zeta}{d\theta} \\ &= \zeta\left\{(\alpha+\eta)\left(\frac{y}{k}\right)\frac{d\zeta}{d\theta} - [1-(\alpha+\eta)]\right\} \\ &= \zeta\left\{(\alpha+\eta)\frac{d\chi}{d\theta} - [1-(\alpha+\eta)]\right\},\end{aligned}$$

so by equation (C.7)

$$\left.\frac{d\dot{\zeta}}{d\theta}\right|_{k=\hat{k}} > 0.$$

Note that

$$\begin{aligned}\frac{d^2\chi}{d\theta\,dk} &= \frac{dt}{dk}\cdot\frac{d^2\chi}{d\theta\,dt} \\ &= \frac{1}{\dot{k}}\cdot\frac{d\dot{\chi}}{d\theta},\end{aligned}$$

where

$$\begin{aligned}\dot{\chi} &= \dot{\zeta}\frac{y}{k} + \zeta\frac{d\,(y/k)}{dt} \\ &= \dot{\zeta}\frac{y}{k} - \zeta\,[1-(\alpha+\eta)]\,\frac{y}{k^2}\dot{k} \\ \Rightarrow\quad \frac{d\dot{\chi}}{d\theta} &= \left(\frac{y}{k}\right)\frac{d\dot{\zeta}}{d\theta} - \zeta\,[1-(\alpha+\eta)]\left(\frac{y}{k^2}\right)\frac{d\dot{k}}{d\theta} \\ &> \left(\frac{y}{k}\right)\frac{d\dot{\zeta}}{d\theta},\end{aligned}$$

since we know by Theorem 3.4 that $d\dot{k}/d\theta < 0$. Combining all our results, we see that

$$\begin{aligned}\left.\frac{d^2\chi}{d\theta\,dk}\right|_{k=\hat{k}} &= \frac{1}{\dot{k}}\cdot\left.\frac{d\dot{\chi}}{d\theta}\right|_{k=\hat{k}} \\ &> \frac{1}{\dot{k}}\left(\frac{y}{k}\right)\left.\frac{d\dot{\zeta}}{d\theta}\right|_{k=\hat{k}} \\ &> 0,\end{aligned}$$

which contradicts equation (C.8). Therefore our original assumption that there was some $\bar{k}$ satisfying equation (C.6) was false, and at all points $k \leq k_{SS}$,

$$\frac{d\zeta}{d\theta} \leq \left[\frac{1-(\alpha+\eta)}{\alpha+\eta}\right]\frac{k}{y}.$$

It follows immediately that

$$\begin{aligned}\frac{d\gamma_\zeta}{d\theta} &= (\alpha+\eta)\left(\frac{y}{k}\right)\frac{d\zeta}{d\theta} - [1-(\alpha+\eta)]\\ &\leq 0,\end{aligned}$$

so a negative shock to θ causes a weakly positive shock to the growth rate γ_ζ of the consumption-output ratio $\zeta = c/y$. ■

Proof of Theorem 3.7. The growth rates for n_S and n_M were derived in equations (3.19) and (3.20). If $\dot{k} > 0$, it follows by Lemma 3.5 that $\dot{\zeta} > 0$. Thus $\dot{n}_S/n_S > 0$ and $\dot{n}_M/n_M < 0$, generating structural change. Suppose there is a negative shock to θ:

(i) By Theorem 3.4, aggregate per-capita output growth $\dot{y}/y$ and capital growth $\dot{k}/k$ both accelerate in response to a negative shock to θ.

(ii) It is clear from equation (3.10) that x_S (and hence X) does not change when there is a shock to θ, so by Lemma 3.6 and equation (3.13) it follows that a negative shock to θ causes a negative level shock to n_S, and consequently a positive level shock to n_M. By Lemma 3.6, a negative shock to θ causes a weakly positive shock to $\dot{\zeta}/\zeta$, and by Theorem 3.4 it causes a strictly positive shock to $\dot{k}/k$. Thus a negative shock to θ causes a strictly positive shock to the growth rate of n_S.

(iii) Denote by $n_{S,SS}$ the steady-state share of employment in services. At any point, by equation (3.13),

$$\begin{aligned}n_S &= \frac{x_S}{X}\left(\frac{c}{y}\right)\\ &= \frac{(\omega_S/\omega_M)^\epsilon (A_0k^\eta)^{1-\epsilon}}{1+(\omega_S/\omega_M)^\epsilon (A_0k^\eta)^{1-\epsilon}}\left(\frac{c}{y}\right).\end{aligned}$$

Thus

$$\begin{aligned}n_{S,SS} &= \frac{(\omega_S/\omega_M)^\epsilon \left(A_0k_{SS}^\eta\right)^{1-\epsilon}}{1+(\omega_S/\omega_M)^\epsilon \left(A_0k_{SS}^\eta\right)^{1-\epsilon}}\left(\frac{c_{SS}}{y_{SS}}\right)\\ &= \frac{(\omega_S/\omega_M)^\epsilon \left(A_0k_{SS}^\eta\right)^{1-\epsilon}}{1+(\omega_S/\omega_M)^\epsilon \left(A_0k_{SS}^\eta\right)^{1-\epsilon}}\left(1-\alpha+\frac{\alpha\rho}{\theta+\rho+g}\right).\end{aligned}$$

By Lemma 3.1, a negative shock to θ causes a positive shock to k_{SS}. Thus a negative shock to θ causes a positive shock to $n_{S,SS}$, and consequently causes a negative shock to $n_{M,SS}$.

(iv) Productivity in the manufacturing sector is given by a learning-by-doing externality associated with capital accumulation, $A_M = A_0k^\eta$. Thus

$$\frac{\dot{A}_M}{A_M} = \eta\left(\frac{\dot{k}}{k}\right).$$

By Theorem 3.4, a negative shock to θ causes a positive shock to $\dot{k}/k$, and therefore also to $\dot{A}_M/A_M$.

(v) Capital in the services sector is given by $K_S = n_S N k$. Thus

$$\begin{aligned}\frac{\dot{K}_S}{K_S} &= \frac{\dot{n}_S}{n_S} + \frac{\dot{N}}{N} + \frac{\dot{k}}{k} \\ &= \frac{\dot{n}_S}{n_S} + g + \frac{\dot{k}}{k}.\end{aligned}$$

We have shown that $\dot{n}_S/n_S$ increases in response to a negative shock to θ, as does $\dot{k}/k$ by Theorem 3.4, and g is clearly unchanged by θ. Thus capital accumulation accelerates in the services sector in response to a positive financial shock. Similarly, in the manufacturing sector,

$$\frac{\dot{K}_M}{K_M} = \frac{\dot{n}_M}{n_M} + g + \frac{\dot{k}}{k},$$

and $g + \dot{k}/k$ increases in response to a negative shock to θ. However, the effect of such a shock on $\dot{n}_M/n_M$ is ambiguous; note that

$$\begin{aligned}\frac{\dot{n}_M}{n_M} &= -\frac{\dot{n}_S}{n_S} \cdot \frac{n_S}{n_M} \\ \Rightarrow \qquad \frac{d}{d\theta}\left(\frac{\dot{n}_M}{n_M}\right) &= -\underbrace{\frac{d}{d\theta}\left(\frac{\dot{n}_S}{n_S}\right)}_{<0}\underbrace{\frac{n_S}{n_M}}_{>0} - \underbrace{\frac{d}{d\theta}\left(\frac{n_S}{n_M}\right)}_{>0}\underbrace{\frac{\dot{n}_S}{n_S}}_{>0},\end{aligned} \tag{C.9}$$

so the sign of the equation (C.9) is ambiguous. Thus the effect of a positive financial shock on capital accumulation in the manufacturing sector is also unclear: it could be either positive or negative, depending on how the growth rate $\dot{n}_M/n_M$ responds to such a shock.

■

C.2 Sector classification

Industries are classified into four major sectors (agriculture, manufacturing, services and government) according to the methodology outlined in Appendix A of Herrendorf et al. (2014, pp. 932-933). Employment data are aggregated by the BEA according to SIC code from 1969-2000, and by NAICS code from 2001-2016. Table C.1 shows how BEA employment data is classified into sector. Output data are aggregated by the BEA according to SIC code from 1963-1996, and by NAICS code from 1997-2017. Table C.2 shows how BEA output data is classified into sector.

Table C.1: Classification of BEA employment data into sector

Sector	SIC Code (1969-2000)	NAICS Code (2001-2016)
Agriculture	Farm employment ([01-02]) Agricultural services, forestry, and fishing ([07-09])	Farm employment (111-112) Forestry, fishing, and related activities (113-115)
Manufacturing	Mining (B) Construction (C) Manufacturing (D)	Mining, quarrying, and oil and gas extraction (21) Construction (23) Manufacturing (31-33)
Services	Transportation and public utilities (E) Wholesale trade (F) Retail trade (G) Finance, insurance, and real estate (H) Services (I)	Utilities (22) Wholesale trade (42) Retail trade (43-45) Transportation and warehousing (48-49) Information (51) Finance and insurance (52) Real estate and rental and leasing (53) Professional, scientific, and technical services (54) Management of companies and enterprises (55) Administrative and support and waste management and remediation services (56) Educational services (61) Health care and social assistance (62) Arts, entertainment, and recreation (71) Accommodation and food services (72) Other services (except government and government enterprises) (81)
Government	Government and government enterprises	Government and government enterprises

Table C.2: Classification of BEA output data into sector

Sector	SIC Code (1963-1996)	NAICS Code (1997-2017)
Agriculture	Agriculture, forestry, and fishing (A)	Agriculture, forestry, fishing, and hunting (11)
Manufacturing	Mining (B)	Mining, quarrying, and oil and gas extraction (21)
	Construction (C)	Construction (23)
	Manufacturing (D)	Manufacturing (31-33)
Services		Utilities (22)
		Wholesale trade (42)
		Retail trade (44-45)
		Transportation and warehousing (48-49)
	Transportation and public utilities (E)	Information (51)
	Wholesale trade (F)	Finance, insurance, real estate, rental, and leasing (52-53)
	Retail trade (G)	Professional and business services (54-56)
	Finance, insurance, and real estate (H)	Educational services, health care, and social assistance (61-62)
	Services (I)	Arts, entertainment, recreation, accommodation, and food services (71-72)
		Other services (except government and government enterprises) (81)
Government	Government and government enterprises	Government and government enterprises

Table C.3: Timing of M&A bank branching deregulation

State	Code	Dereg. Year	State	Code	Dereg. Year
Alabama	AL	1981	Montana	MT	1990
Alaska	AK	<1963	Nebraska	NE	1985
Arizona	AZ	<1963	Nevada	NV	<1963
Arkansas	AR	1994	New Hampshire	NH	1987
California	CA	<1963	New Jersey	NJ	1977
Colorado	CO	1991	New Mexico	NM	1991
Connecticut	CT	1980	New York	NY	1976
Delaware	DE	<1963	North Carolina	NC	<1963
District of Columbia	DC	<1963	North Dakota	ND	1987
Florida	DL	1988	Ohio	OH	1979
Georgia	GA	1983	Oklahoma	OK	1988
Hawaii	HI	1986	Oregon	OR	1985
Idaho	ID	<1963	Pennsylvania	PA	1982
Illinois	IL	1988	Rhode Island	RI	<1963
Indiana	IN	1989	South Carolina	SC	<1963
Iowa	IA	1999	South Dakota	SD	<1963
Kansas	KS	1987	Tennessee	TN	1985
Kentucky	KY	1990	Texas	TX	1988
Louisiana	LA	1988	Utah	UT	1981
Maine	ME	1975	Vermont	VT	1970
Maryland	MD	<1963	Virginia	VA	1978
Massachusetts	MA	1984	Washington	WA	1985
Michigan	MI	1987	West Virginia	WV	1987
Minnesota	MN	1993	Wisconsin	WI	1990
Mississippi	MS	1986	Wyoming	WY	1988
Missouri	MO	1990			

C.3 Timing of bank branching deregulation

The year of bank branching deregulation in each state is taken from Amel (1993), Jayaratne and Strahan (1996), Krozsner and Strahan (1999), and Beck et al. (2010). Following this literature, we take the year of deregulation to be the year that a state allowed bank branching by merger and acquisition (M&A). Table C.3 lists the year of deregulation for each state.

C.4 Difference-in-differences regressions with pre-1997 data

Table C.4 replicates Table 3.3, restricted to pre-1997 data. This is to avoid the discontinuity in the data when the BEA switched from aggregating data by SIC code to aggregating data by NAICS code. None of these results directly contradict the results found in Section 3.5.1; that is, there are no coefficients that are estimated to be significantly positive using one dataset and significantly negative using the other dataset. However, some coefficients that are significant when estimated using the full dataset are not significant when estimated using the restricted dataset, and vice versa. Nonetheless, the estimates of 'Time*Dereg.' are significantly positive for services and significantly negative for manufacturing across both datasets.

Table C.4: Difference-in-differences regressions (pre-1997 dataset)

	Dependent variable:			
	Services Share of Employment (%)	Services Share of Output (%)	Man. Share of Employment (%)	Man. Share of Output (%)
Deregulation	0.116	0.620^{***}	-0.564^{***}	-1.572^{***}
	(0.100)	(0.205)	(0.151)	(0.267)
Time	-0.449^{***}	1.199^{***}	0.993^{***}	-1.474^{***}
	(0.048)	(0.135)	(0.088)	(0.179)
Dereg.*Time	0.175^{***}	0.193^{***}	-0.239^{***}	-0.278^{***}
	(0.017)	(0.027)	(0.025)	(0.032)
Constant	35.908^{***}	65.631^{***}	43.312^{***}	8.877^{***}
	(0.779)	(2.566)	(1.284)	(3.353)
Observations	1,054	1,288	1,054	1,288
R^2	0.975	0.912	0.919	0.878
Adjusted R^2	0.974	0.906	0.913	0.871

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Difference-in-differences OLS estimates of the effect of bank branching deregulation on structural composition, accounting for year and state fixed effects. Standard errors clustered at the state level in parentheses. 'Deregulation' is a dummy that is equal to 1 when a state has deregulated, and equal to 0 otherwise. 'Time' is the time in years since deregulation (this variable takes a negative value prior to deregulation). The year of deregulation is excluded for each state. States that deregulated prior to 1963 are excluded (Alaska, Arizona, California, Delaware, the District of Columbia, Idaho, Maryland, Nevada, North Carolina, Rhode Island, South Carolina and South Dakota). Data is restricted to the pre-1997 BEA dataset (aggregated by SIC code).

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