INTRODUCTION

The fourth Topic Working Group (TWG4) of the third conference of the International Network for Didactic Research in University Mathematics (INDRUM2020) was dedicated to students’ and teachers’ practices in the teaching and learning of mathematics at university level. Eleven papers and three posters were proposed and discussed in two thematic groups: digital or other resources and the use of technology and teachers’ practices and innovations. In this report, we present a synthesis of the papers and the posters in each thematic group. Also, we present a summary of our discussion on emerging issues related to the recent Covid-19 outbreak, especially in relation to the shift to online or blended modes of teaching. We conclude with a reflection on the studies presented on the TWG4 and propositions for future research.

DIGITAL OR OTHER RESOURCES AND THE USE OF TECHNOLOGY

Five papers and three posters addressed topics related to resources, specifically digital resources, and use of technology in the teaching and learning of university mathematics. Specifically, Fleischmann, Mai, and Biehler proposed the design and the evaluation frame of a four-week bridging course. The course employed a blended learning design that included face-to-face lectures incorporated together with self-regulated e-learning with multimedia learning materials outside the lectures. The paper proposed a methodological approach for the evaluation of the course that connected the teaching design with student responses to a questionnaire. Results suggest that students appreciated the integration of interactive activities to the face-to-face part of the course. Regarding assessment, Hadjerrouit discussed student engagement with a computer-based assessment that provided formative feedback. The study employed Gibson’s affordance theory (1977) on the physical properties of an object and the user-object interactions, to analyse 15 teacher students’, who engaged with the computer-based assessment, responses to a questionnaire. Findings indicate that student interactions with the assessment through the formative feedback created affordances for learning at the technological, mathematical, and assessment level.

Gueudet, Buteau, Muller, Mgombelo, and Sacristán employed the instrumental approach (Rabardel 1995) to analyse the engagement of university students with programming in the context of “authentic” mathematical investigations. The study considered programming as an artefact that develops to an instrument incorporating a structure of schemes that have mathematical only (m-schemes), programming only (p-schemes) or both mathematical and programming (p+m-schemes) goals. The structure
of schemes above was illustrated in the case of the participation of one undergraduate student. This participation is elaborated further in the Buteau, et al. poster in which the development of student’s engagement with programming is visualised in a diagram that incorporates the complex structure of m-, p-, and p+m-schemes developed by the student. The instrumental approach was used in the Topphol poster as well to analyse university students’ participation in task-based interviews, where mathematical tasks are seen as instruments in the development of mathematical competences.

The Heinrich, Hattermann, Salle, and Schumacher paper explains the relationship between the interactivity of 63 pairs of students working in different instructional media on a descriptive statistics activity and their learning gain after participating to this activity. The paper proposes a theoretical instrument based on the ICAP-framework on the engagement activities between learners or between a learner and learning materials (Chi & Menekse, 2015) to analyse students’ interactivity. In addition, learning gain through student responses to pre- and post- test was measured. Findings indicate a significant link between students’ communicational behaviour and their learning gain. Collaboration is also related to the Glassmeyer poster that analysed the affordances offered by portfolio, which use peer feedback within an online graduate course on problem solving, for mathematics teachers’ practices.

Finally, Sabra employed the documentational approach (Gueudet, 2017) to study the relation between research and teaching practices of three university teachers who are also active researchers. The study focused on the interactions between resources and teachers by analysing those teachers’ research activities and teaching practices together. Analysis of audio-recorded interviews proposed three forms of use of research resources in teaching practices: use research resource in teaching instantiation processes; research resource to scaffold the learning of a given content; and, no relation of resources.

In the discussion session of the group, we had the chance to address emerging issues from the recent Covid-19 outbreak, especially in relation to the shift to online or blended modes of teaching. Specifically, we dealt with two questions: How would the research knowledge we have been accumulating all these years in research on the use of technology and resources in the teaching and learning of mathematics at university level help us to address emerging situations due to Covid-19? and What new research can emerge from the impact of the Covid-19 outbreak on the teaching and learning of mathematics at university level?

Our discussion highlighted that studies on students’ participation and communication with peers and teachers could inform studies on students’ collaboration online. However, when online is the central medium of communication, other factors should be considered as well, such as technological affordances and availability (or lack of) and changes in the visual mediation (e.g. gestures, body language, etc.). In addition, there are potential methodological consequences. For example, studies that were possible before the pandemic (e.g., classroom face-to-face observations or activities that involve students’ physical interaction), probably will not be possible in an online
mode. For example, a study that involves classroom observations should change radically in the midst of the pandemic where remote is the dominant mode of teaching.

A concern that emerged from our discussion is that current online teaching may privilege direct instruction led by the teacher with less opportunities for student engagement. A potential approach might be alternating between short pieces of direct instruction (before learners go into cinema-mode, quiet and passive attendance) and invitations for student interaction and contributions between such short pieces of direct instruction. E-assessment might be an issue as well; studies in this area are gaining more significance in the current circumstances.

It might be too early to study and experiment on online teaching. However, one observation is that after Covid-19 outbreak there is a substantial attention to teaching. This increase of attention might be an opportunity for enhancement of the teaching provision at university level overall.

TEACHERS’ PRACTICES AND INNOVATIONS

Six papers of our group were related to teachers’ practices and innovation in university mathematics. Specifically, Gascón and Nicolás drew on the anthropological theory of the didactic (Chevallard, 1999) to analyse the transition of future teachers from the institution of tertiary mathematics (as students) to an institution of secondary education (as teachers). Their study put forward the necessity for future teachers to undertake deep changes in the institutional “teaching ends” of mathematics and to look for a missing epistemological model. Still on transition, Ghedamsi and Fattoum investigated the possibility to reduce differences in the learning expectations of calculus in the transition between high school and university by engaging high school teachers in reforming their actions and making a connection between the two levels. They deployed a collaborative method founded on guided reflection (Husu, Toom & Patrikainen, 2008) to support teacher reflection on his/her actions by taking into account transitional issues.

In a different transition, this time from mathematics to mathematics education, is the work of Biza and Nardi who presented examples of activities and their assessment frame for mathematics undergraduate students’ introduction to mathematics education research. The proposed activities follow task design principles that contextualise the use of theory and the mathematical content to specific learning situations (MathTASK). Students’ responses to these activities are assessed in relation to clarity; coherence; consistency; specificity; use of terms and constructs from mathematics education theory; and, use of terms and processes from mathematical theory. The application of these activities and the assessment frame is exemplified through the responses from one student.

Drawing on literature results about students’ difficulties and affordance for the teaching of limit notion, Chorlay and Mesnil analysed and compared three lectures by focusing on the use of definitions and examples. Post lecture interviews were used towards a further analysis of lecturers’ actions and an investigation of the possibilities
for lecturers to discuss alternatives to their actions. The results show that all three lecturers identified possibilities to consider potential changes on their own actions. Still on the teaching of calculus at university level, Karavi, Potari, and Zachariades analysed the characteristics of proof teaching in an introductory mathematical analysis lecture and lecturer’s rationale underlying this teaching. Findings show a link between the pattern of proof teaching and the development of proof image for students as well as the impact of lecturer’s experience on the building of this pattern.

Finally, Martinez, Gehrtz, Rasmussen, LaTona-Tequida, and Vroom explored what guides course coordinators’ actions towards the goal of improving students’ learning. They draw on Philipp’s (2007) review of mathematics teachers’ beliefs and affect to shape what they call “orientation toward coordination”. The analysis of interviews with coordinators resulted in the identification of two main orientations: humanistic-growth orientation and knowledge-managerial orientation. Raising awareness to such orientations provides coordinators with materials to reflect on how they can act on the available drivers for change at their institutions.

**REFLECTION AND WAYS FORWARD**

In reflection on the studies presented and discussed in the group, it would appear that teaching interventions were at the heart of our group also in relation to the use of resources and digital technology. We were introduced to design principles and evaluation approaches that can facilitate the design and assess the effectiveness of such interventions. In addition, evidence was shared on how and what type of collaborative and participatory approaches in learning university mathematics may generate substantial learning gains. Furthermore, the role of digital curriculum resources and educational technology, for example programming, in both teaching and learning at university level, was a significant part of the works presented in the group.

Transition was a recurring theme into research on studies on students’ and teachers’ practices. We discussed studies addressing issues related to the transition from secondary education to university and, also, studies related to the transition from university to school level, especially in relation to teacher preparation. The importance of double discontinuity raised by Klein (1908/1932) was highlighted as essential in research that goes beyond the description of the problem. Such research proposed interventions that can prepare students for the transition while they are at secondary education or interventions that can prepare teachers before embarking for a teaching profession. Also, we discussed the transition for mathematical to mathematics education practices, in which undergraduate mathematics students are introduced to the theory (and the practices) of mathematics education.

At a more general level, the role of theory in university mathematics research was central in our discussions with the expansion of the use of well-established theoretical perspective to address new research questions. Some examples are the use of the instrumentalational approach in the case of programming; the use of the documentational approach in the analysis of teachers’ research practices; the use of the anthropological
theory of the didactic in the transition of teachers to secondary education.

We dedicated the final session on potential open questions and research areas related to students’ and teachers’ practices that deserve more attention for the years to come. Topics that emerged from our discussion regard a range of areas. For example, inclusive mathematical experiences, was one of these areas, especially in relation to the challenges for teachers in the current (and the post-) pandemic era of serving students with special needs. Another emerging area was the equity in university mathematics education in relation to student opportunities for access to tertiary education. Furthermore, more research is needed on challenges and opportunities in e-learning and e-teaching, such as blended approaches to teaching, e-assessment or e-collaboration. In addition, we discussed the need for more opportunities for collaboration between mathematics education researchers, mathematics educators, mathematicians and mathematics teachers. Finally, we would like to investigate further new methodological and theoretical approaches with potencies in research on e-teaching and e-learning. We look forward to the next INDRUM conference and the new advances in research on teachers’ and students’ practices.

REFERENCES


