Tail Risk Interdependence

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Abstract

We present a framework focused on the interdependence of high-dimensional tail events. This framework allows us to analyze and quantify tail interdependence at different levels of extremity, decompose it into systemic and residual part and to measure the contribution of a constituent to the interdependence of a system. In particular, tail interdependence can capture simultaneous distress of the constituents of a (financial or economic) system and measure its systemic risk. We investigate systemic distress in several financial datasets confirming some known stylized facts and discovering some new findings. Further, we devise statistical tests of interdependence in the tails and outline some additional extensions.

Key words: co-exceedance, systemic distress, risk contribution, extreme risk interdependence, relative entropy

JEL: C32, G1

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1. Introduction

Tail interdependence is important in economics, finance, insurance and in many other areas of applied probability and statistics. The literature suggests a close relationship between tail dependence[1] and systemic risk (e.g., Bae et al., 2003; Poon et al., 2004; Hautsch et al., 2015; Lehkonen, 2015). Indeed, the last financial crisis was as much about multidimensional dependence and feedback loops as it was about extreme events. Although a precise definition of systemic risk seems remarkably elusive, this term often refers to a situation where many institutions fail or are distressed simultaneously (e.g., Allen et al., 2012; Patro et al., 2013). Financial stability aside, tail dependence is paramount for many other important applications in economics and finance, such as portfolio decisions, risk management and multidimensional options (e.g., Ang and Bekaert, 2002; Einmahl et al., 2009; Cherubini and Luciano, 2002), credit derivatives, collateralised debt obligations and insurance (e.g., Hull and White 2006; Kalemanova et al., 2007; Su and Spindler, 2013).

Systemic risk has both a time-series and a cross-section dimension. In the first dimension, some institutions take risks which, over time build up and may become excessive. The cross-section dimension materialises then in the transmission of risk from these institutions to others in the system. Both these dimensions are also intrinsic features of dependence. Indeed, as dependence is essential in the way shocks propagate over time and across institutions, it is the primary ingredient to understand systemic risk and financial crises. The time-series and the cross-section dependence have been identified previously in the literature as important dimensions of the systemic risk. For example, Kelly and Jiang (2014) estimate the effect of time-varying tail risk in stock returns with a panel approach by exploiting the finding that tail

[1] While we often use the terms interdependence and dependence interchangeably, we distinguish between the two concepts as follows. Dependence refers to the relationship between *two* random variables whereas interdependence refers to the relationship among *two or more* variables. Hence the latter concept nests the former.

risks of individual assets display a high degree of commonality. Hautsch et al. (2015), on the other hand, use a two-stage quantile regression approach to construct firm-specific Value-at-Risk (VaR), then compute its contribution to systemic risks and, in the process, they construct a tail risk network that captures how a stock's tail risk influences that of another stock over time. Francq and Zakoïan (2018) develop an asymptotic theory for the conditional VaR of high-dimensional dynamic portfolios with and without elliptical distributions.

In this paper, we adopt the more general concept of systemic distress which captures the tendency of constituents of any system to get into distress simultaneously. In particular, the dependence in negative tails is a practical measure of systemic distress that captures the tendency of the constituents of a multidimensional system to exceed certain critical thresholds simultaneously. The literature contains several measures of tail dependence (e.g., Li, 2009; Colangelo et al., 2005; Joe, 1997; Sibuya, 1960). Extreme dependence has been also captured by copulas (e.g., Oh and Patton, 2017; Boero et al., 2010) and multivariate extreme value theory (e.g., Chan-Lau et al., 2012; Poon et al., 2004). Heffernan (2001) provides a directory of coefficients of tail dependence which, however, either depend on distributional assumptions or are feasible only in low dimensions.

The complex nature of tail dependence, manifested in time-varying strength and shape over the support of the distribution, complicates the analysis considerably. A key reason is that dependence measured in one part of the distribution cannot be used as a substitute for the dependence in other parts. The problem is compounded further by the small number of observations in the tails. In this context, Jondeau (2016) points out that the existing techniques quickly run into scalability problems as the dimension of data goes up and argues that this often leads to poor finite sample properties of the estimators.

This paper makes the following three main contributions. First, we propose a framework to analyse the systemic distress of large systems without requiring knowledge of, or imposing assumptions on their inner workings, relying instead only on their observed behaviour as captured in a series of multidimensional observations. To address the small number of observations available in the tails, we exploit the fact that the observations which do not fall into tails have nevertheless something to say about those tails. By carefully partitioning the support of the distribution, we make use of the dependence information in some parts of the distribution to 'augment' the available information and, thereby, to estimate more reliably the dependence in parts which do not contain sufficient observations. Our non-parametric framework is applicable to high dimensional economic and financial data and provides an alternative to the existing models, which tend to be highly parameterized and suffer from the problems emphasised by Jondeau (2016) and discussed above.

Second, we propose a coefficient of tail interdependence (CTI) which can be used to define and measure dependence in different settings. CTI relies on a concept from information theory employed previously by Darbellay and Wuertz (2000) and Dionisio et al. (2004). In the bivariate case, our measure converges to the classic tail dependence coefficient of Sibuya (1960) and Joe (1997) and has similarities with the dependence measures of Poon et al. (2004), Oh and Patton (2017). We discuss these aspects in detail in Sections 2.3 and 3.3. As our framework is nonparametric, it is like other non-parametric techniques, data-intensive. However, our systemic coefficient of tail interdependence does not suffer from this drawback and can be estimated reliably even in very high dimensions from relatively small samples.

Third, we decompose CTI into systemic and residual dependence or, alternatively, into contributions of constituents (e.g., assets) to the dependence of a system (e.g., portfolio). These decompositions can, for example, help portfolio managers identify and exclude from (include into) their portfolios assets that make positive (negative) contributions to tail risk (see also Das and Uppal, 2004 for a discussion of the impact of systemic risk on portfolio allocation).

The CTI can be applied to an array of problems relating to high dimensional extreme events. In this paper, we use it to examine the distress of G7 bond and equity markets and find that dependence of G7 stock index returns is strongly asymmetric, with a rotated-J shape, i.e. higher in the negative tails, falls considerably in the central part of the distribution and then increases in the positive tails, although not to the same level as for the negative tails. This finding confirms those of previous studies (e.g. Poon et al., 2004; Longin and Solnik, 2001) and suggests that portfolio diversification and tail risk hedging would be challenging for an investor exposed to these economies due to the tendency of the markets in the developed economies to get into distress and crash simultaneously. However, G7 sovereign bonds appear to display similar dependence in both, negative and positive tails, the reasons for which require further investigation.

We also examine the dependence of the 30 constituents of the Dow Jones Industrial Average (DJ30) index as well as the Fama-French-Carhart (FFC) asset pricing factors. A similar exercise is conducted by Jondeau (2016) who investigates the tail dependence between the Fama-French portfolio returns and the market return and shows that the tail dependence is often asymmetric. However, while the difference is economically sizable, it is statistically insignificant. We, on the other hand, examine the dependence among the Fama-French-Carhart factors and the impact of these factors on the dependence among the constituents of the DJ30 index. We find, in particular, that most of the latter dependence is accounted for by the first factor – the market risk premium – while the remaining factors account for very little of the tail dependence. The inability of the FFC factors to account for the dependence of the DJ30 returns in the tails is a direct manifestation of the tail interdependence of the factors themselves. This is an important finding suggesting that FFC factors cannot be used as tail risk factor proxies.

The paper proceeds as follows. In Section 2, we illustrate the theoretical framework and define our measure of tail dependence. In Section 3, we apply the framework, conditionally and unconditionally, to financial datasets and compare it to two benchmark measures. Section 4 summarizes the paper. In the Appendix, we define formally our framework in higher dimensions, elaborate on the theoretical properties of our dependence measures and provide full results of the comparison with two benchmark measures.

2. Measurement and statistical testing of tail interdependence

2.1. Setup and Notation

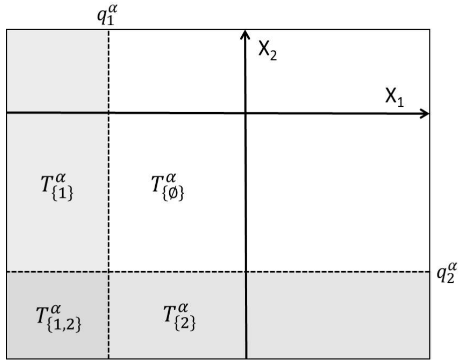
We present here our setup in the bivariate case. Suppose the portfolio of a safety-first investor contains two assets. Either of these assets may experience a large loss - a negative tail event. Therefore, on any given day, there are four possible outcomes: both assets experience a negative tail event; the first asset experiences a negative tail event but the second does not; the second asset experiences a negative tail event but the first does not; and finally, neither of the assets experiences a negative tail event.

To express this setup formally, let $\mathcal{N} = \{1, 2\}$ be the set of assets and $\mathbf{X} = (X_1, X_2)$ the vector of asset returns with a continuous joint CDF F (PDF f) and strictly increasing marginals F_1, F_2.[2] For a nominal probability level $\alpha \in (0, 1)$ and a subset $C \subseteq \mathcal{N} = \{1, 2\}$ of assets, let u_C^α be the probability under F of the *negative joint tail* T_C^α, i.e., probability that $X_i \leq F_i^{-1}(\alpha)$ for all $i \in C$ and, simultaneously, $X_s > F_s^{-1}(\alpha)$ for all $s \in \mathcal{N} \backslash C$. For the *positive joint tail* T_C^α, u_C^α is defined as the probability that $X_i \geq F_i^{-1}(1-\alpha)$ for all $i \in C$ and, simultaneously, $X_s < F_s^{-1}(1-\alpha)$ for all $s \in \mathcal{N} \backslash C$. Thus, only assets in subset C experience a tail event, while the assets in $\mathcal{N} \backslash C$ do not.

[2]The assumptions on F are for the ease of notation and are not crucial for our framework.

Therefore, the four negative joint tails (JT) that concern our safety-first investor can be expressed as follows: $T^{\alpha}_{\varnothing}$ is the JT when neither of the assets experiences a tail event and has probability $u^{\alpha}_{\{\varnothing\}}$; $T^{\alpha}_{\{1\}}$ is the JT (with probability $u^{\alpha}_{\{1\}}$) when the first asset experiences a negative tail event but the second does not; $T^{\alpha}_{\{2\}}$ is the JT (with probability $u^{\alpha}_{\{2\}}$) when the second asset experiences a negative tail event but the first does not; and finally, $T^{\alpha}_{\{1,2\}}$ is the JT (with probability $u^{\alpha}_{\{1,2\}}$) when both assets experience a negative tail event. These joint tails are illustrated in Figure 1.

Figure 1: The partition of the two-dimensional outcome space into JTs



Notes: The figure illustrates the partition of a two-dimensional outcome space into negative joint tails, where $X_i \leq F_i^{-1}(\alpha) = q_i^{\alpha}$.

In the Appendix, we generalize the above definitions of JTs and their probabilities to an arbitrary number n of assets, i.e., to the set of assets $\mathcal{N} = \{1, 2, .., n\}$. In this general case, we define the collection $\mathcal{T}^{\alpha} = (T^{\alpha}_{C})_{C \subseteq \mathcal{N}}$ of all JTs and the vector $\mathbf{u}^{\alpha} = (u^{\alpha}_{C})_{C \subseteq \mathcal{N}}$ of corresponding probabilities. Intuitively, u^{α}_{C} is the probability that all assets in the subset $C \subseteq \mathcal{N}$ exceed their α-quantiles (or $(1-\alpha)$-quantiles), while the remaining assets do not. We shall refer to $\mathbf{u}^{\alpha}$ as the *tail interdependence structure* (TIS). Clearly, $\mathbf{u}^{\alpha}$ is a (discrete) PDF as the union of all negative (positive) JTs covers the entire sample space. In our general framework defined in the Appendix, we allow for mixed joint tails, where some assets experience negative while others positive events. In this study, however, we restrict our attention to all positive and all negative JTs. In what follows, we adapt the convention that $\alpha \leq 1/2$ $(\alpha > 1/2)$

implies all negative JTs $\mathcal{T}^{\alpha}$ (all positive JTs $\mathcal{T}^{1-\alpha}$).

2.2. *Coefficient of Tail Interdependence*

The interdependence of the JTs captured by the TIS $\mathbf{u}^{\alpha}$ is fully defined by the *multi-information* (MI) (Cover and Thomas, 2006),

$$D(\mathbf{u}^{\alpha}||\boldsymbol{\pi}^{\alpha}) = \sum_{C\subseteq\mathcal{N}} u_C^{\alpha} \ln \frac{u_C^{\alpha}}{\pi_C^{\alpha}}, \tag{1}$$

where $\boldsymbol{\pi}^{\alpha} = (\pi_C^{\alpha})_{C\subseteq\mathcal{N}}$ is the corresponding TIS under tail independence, $\pi_C^{\alpha} = \alpha^{\#C}(1-\alpha)^{n-\#C}$ is the probability of the JT T_C^{α} under tail independence (computed as the product of marginal probabilities of $\#C$ exceedances and $n-\#C$ non-exceedances) and $\#C$ is the cardinality of set C. Note that $D(\mathbf{u}^{\alpha}||\boldsymbol{\pi}^{\alpha})$ is well-defined as $\pi_C^{\alpha} > 0$ for all $\alpha \in (0,1)$ and $C \subseteq \mathcal{N}$.

MI is non-negative and equals zero in case of independence only, i.e., if and only if $\mathbf{u}^{\alpha} = \boldsymbol{\pi}^{\alpha}$. In statistics, $D(\mathbf{u}^{\alpha}||\boldsymbol{\pi}^{\alpha})$ is known as the Kullback-Leibler divergence between the PDFs $\mathbf{u}^{\alpha}$ and $\boldsymbol{\pi}^{\alpha}$. MI quantifies the *total* amount of interdependence among random variables that arises from pairwise or more complex interactions. It is widely used, e.g., in physics (e.g., Schneidman et al., 2003; Chicharro and Ledberg, 2012) and biosciences (e.g., Wennekers and Ay, 2003).

We use MI (1) to measure tail interdependence. Specifically, we normalize MI to obtain the *coefficient of tail interdependence* (CTI),

$$\kappa(\mathbf{u}^{\alpha}) = \frac{D(\mathbf{u}^{\alpha}||\boldsymbol{\pi}^{\alpha})}{(1-n)\ln \alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{2}$$

where the denominator is a normalization factor derived formally in the Appendix.

2.3. *Properties of CTI*

The CTI has the following properties. It lies in the unit interval. It is scale invariant under strictly increasing transformations of the underlying variables. It is robust to outliers and invariant under the permutation of the underlying variables. The CTI is decomposable into a systemic and a residual component or into contributions of the underlying variables to interdependence. It can be used as a non-parametric statistic in statistical tests. It is computable at different levels of extremity $\alpha \in (0,1)$ and for positive, negative or mixed tails as specified by user's interest, e.g. his portfolio holdings. Finally and crucially, the CTI is efficiently computed for high-dimensional empirical distributions. We elaborate on these properties in the Appendix. Here, we briefly discuss the properties that are relevant for the empirical section.

TIS offers a wealth of information on the tail interdependence. On the other hand, its dimension grows exponentially in n, which complicates its (empirical) application for higher n. Therefore, we define *systemic TIS* as the $(n+1)-$dimensional vector $\widetilde{\mathbf{u}}^\alpha = (\widetilde{u}_k^\alpha)_{k=0}^n$ where,

$$\widetilde{u}_k^\alpha = \sum_{C \subseteq \mathcal{N}: \# C = k} u_C^\alpha,$$

is the probability of exactly $k = 0, ..., n$ exceedances. Obviously, $\widetilde{\mathbf{u}}^\alpha$ is also a (discrete) PDF. Furthermore, we compute from $\mathbf{u}^\alpha$ and $\boldsymbol{\pi}^\alpha$ the conditional probabilities $\mathbf{u}^{\alpha,k} = (u_C^\alpha / \widetilde{u}_k^\alpha)_{C \subseteq \mathcal{N}: \# C = k}$ and $\boldsymbol{\pi}^{\alpha,k} = (\pi_C^\alpha / \widetilde{\pi}_k^\alpha)_{C \subseteq \mathcal{N}: \# C = k}$, respectively, given that k exceedances have occurred.[3] Then, we show in the Appendix that the CTI can be decomposed into *systemic* and *residual CTIs* as follows,

$$\begin{aligned} \kappa(\mathbf{u}^\alpha) &= \frac{D(\widetilde{\mathbf{u}}^\alpha || \widetilde{\boldsymbol{\pi}}^\alpha)}{(1-n)\ln \alpha^\alpha (1-\alpha)^{1-\alpha}} + \sum_{k=0}^{n} \frac{D(\mathbf{u}^{\alpha,k} || \boldsymbol{\pi}^{\alpha,k})}{(1-n)\ln \alpha^\alpha (1-\alpha)^{1-\alpha}} \\ &= \widetilde{\kappa}(\mathbf{u}^\alpha) + \sum_{k=0}^{n} \widetilde{u}_k^\alpha \kappa^k(\mathbf{u}^\alpha), \quad 0 \leq \widetilde{\kappa}(\mathbf{u}^\alpha) \leq \kappa(\mathbf{u}^\alpha) \leq 1, \end{aligned} \tag{3}$$

with $\widetilde{\kappa}(\mathbf{u}^\alpha) = \kappa(\mathbf{u}^\alpha) = 0$ in the case of tail independence and $\widetilde{\kappa}(\mathbf{u}^\alpha) = \kappa(\mathbf{u}^\alpha) = 1$ for perfect dependence (i.e., when all exceedances always occur together). It is easy

[3] For example, in the bivariate case $u_{\{2\}}^{\alpha,1} = u_{\{2\}}^\alpha / \widetilde{u}_1^\alpha = u_{\{2\}}^\alpha / (u_{\{1\}}^\alpha + u_{\{2\}}^\alpha)$ is the conditional probability of X_2 exceeding when $k = 1$, i.e., when exactly one exceedance has occurred.

to show that the equality $\widetilde{\kappa}(\mathbf{u}^\alpha) = \kappa(\mathbf{u}^\alpha)$ holds for any level of dependence in the bivariate case.

The measure $\widetilde{\kappa}(\mathbf{u}^\alpha)$ quantifies the *systemic* tail interdependence by the normalized divergence between the distributions $\widetilde{\mathbf{u}}^\alpha$ and $\widetilde{\boldsymbol{\pi}}^\alpha$ of the *total* number of exceedances under $\mathbf{u}^\alpha$ and under $\boldsymbol{\pi}^\alpha$ (i.e., under tail independence), respectively. On the other hand, each $\kappa^k(\mathbf{u}^\alpha)$ quantifies the conditional interdependence among subsets of variables, given that k exceedances have occurred. We refer to it as residual interdependence. The term 'systemic dependence' reflects the fact that the first term in (3) captures dependence that is jointly generated by all constituents. The second term in (3) measures the additional or *remaining* dependence among the constituents given that k of them have exceeded their respective VaR.

Given a sufficient number of observations, the computation of the CTI $\kappa(\mathbf{u}^\alpha)$ does not suffer from the curse of dimensionality. However, if the log number of observations is smaller relative to the number of variables, $\kappa(\mathbf{u}^\alpha)$ will not be estimated reliably. In this case, we focus on the systemic component $\widetilde{\kappa}(\mathbf{u}^\alpha)$ of $\kappa(\mathbf{u}^\alpha)$ that can be reliably estimated with substantially fewer observations.

Proposition 1 below connects the systemic CTI to the literature on the multivariate extreme value theory (e.g. Asimit et al., 2016; Bücher et al., 2015).

Proposition 1. *For the n-dimensional random vector $\mathbf{X}$ with a continuous joint CDF F and strictly increasing marginal CDFs,*

$$\lim_{\alpha \to 0} \widetilde{\kappa}(\mathbf{u}^\alpha) = \frac{1}{n-1} \sum_{k=1}^{n} (k-1)\lambda_k,$$

where a unique $\lambda_k = \lim_{\alpha \to 0} \Pr\{k \text{ exceedances}\}/\alpha$ exists and is bounded for all $k = 1, ..., n$.

A special case of Proposition 1 is the bivariate CDF F. Then, as $\alpha \to 0$, both the CTI $\kappa(\mathbf{u}^\alpha)$ and the systemic CTI $\widetilde{\kappa}(\mathbf{u}^\alpha)$ converge to the classic (lower or upper) tail-dependence coefficient of Sibuya (1960) and Joe (1997). They define the tail-dependence coefficient as the limit conditional probability that one variable exceeds its threshold when the other does. In the bivariate case, this limit probability is equal to λ_2 as defined in Proposition 1. The systemic CTI can be interpreted, therefore, as a generalization of the classic, bivariate tail-dependence coefficient to high-dimensional random variables and to less extreme levels of severity.

2.4. Statistical Tests

The TIS concept can be also used in the context of statistical hypothesis testing. Our first procedure detailed in the Appendix tests whether an empirical TIS $\widehat{\mathbf{u}}^\alpha$ is compatible with a TIS $\mathbf{u}^\alpha$ derived from some hypothesized PDF f. It uses as test statistic the divergence

$$D(\widehat{\mathbf{u}}^\alpha || \mathbf{u}^\alpha) = \sum_{C \subseteq \mathcal{N}} \widehat{u}_C^\alpha \ln \frac{\widehat{u}_C^\alpha}{u_C^\alpha}. \tag{4}$$

Under the null, the divergence in (4) follows asymptotically the χ^2-distribution with $d = 2^n - n - 1$ degrees of freedom. If exceedances are mutually independent under f, this procedure boils down to a *test of tail independence*. The same null can be also tested using the empirical distribution $\widetilde{\mathbf{u}}^\alpha$ of the total number of exceedances. The corresponding test statistic (4) follows then asymptotically the χ^2-distribution with $d = n - 1$ degrees of freedom.

Another interesting question is whether two empirical TIS, computed in the negative and in the positive tails are *symmetric*. In the Appendix, we derive the corresponding test statistic that follows asymptotically the χ^2-distribution with $d = 2^n - 1$ degrees of freedom[4].

[4] For small sample sizes, one can test the null with a generalized version of the Fisher exact test (see, e.g., Mehta and Hilton, 1993)

3. Tail interdependence in financial data

We apply our tail interdependence and systemic distress framework to the daily returns of equity indices and treasury bonds of G7 countries and to the 30 constituents of Dow Jones Industrial Average (DJ30) stock index. We focus on G7 data due to their paramount importance in international capital markets. According to Bloomberg, the G7 indices, at approximately $40 trillion market capitalisation, represent about 60% of the global market whereas DJ30 index, at around $6.5 trillion market capitalisation, represents about 22% of the U.S. total market. Similarly, according to the Bank for International Settlements, the bond markets are also very large, where the U.S. bond market in particular stands out at around $31.2 trillion. Moreover, the relative transparency and regulation of these markets ensure that any dependence structure in the returns cannot be explained away by simple market inefficiencies but is probably a manifestation of deeper structural relationships.

The emphasis on market returns in our empirical studies is motivated by the aim to incorporate the most up-to-date information but, obviously, the framework can be applied to any series of observations generated by a (financial or economic) system. We use returns divided by the time-varying standard deviation and, for the DJ30 constituents, also the residuals of the regression of their returns on FFC factors. The latter factors are the dominant pricing factors in the literature but little is known about their mutual relationship or ability to account for tail risk.

In all statistical tests that we present below we adhere to the following convention. We say that the null is strongly rejected (or rejected with a high significance) if the p-value of the relevant test does not exceed 0.01. A simple rejection occurs with a p-value below 0.1.

3.1. Daily returns in G7 markets

In this subsection we investigate the daily returns of the main equity indices and 10-years treasuries for G7 countries (US, UK, Germany, Japan, Canada, France and Italy), which here we take to proxy the health of the underlying economies and sovereign risk. We compute the daily returns between 29 March 1991 and 16 July 2015. The data was obtained from Datastream and results in $T = 6,334$ synchronized observations.[5] Summary statistics are reported in Table 1. In particular, the data has mean returns close to zero and displays volatility clustering. We also observe that the returns are leptokurtic and negatively skewed.

Table 1: Summary Statistics for G7 Daily Returns

	US	UK	Germany	Japan	Canada	France	Italy
Mean B	0.023	0.029	0.026	0.017	0.029	0.028	0.035
Mean S	0.037	0.0318	0.030	0.004	0.037	0.034	0.023
SD B	0.456	0.396	0.332	0.273	0.396	0.351	0.457
SD S	1.119	1.046	1.172	1.269	0.966	1.217	1.344
Skewness B	-0.152	-0.007	-0.347	-0.605	-0.180	-0.162	0.183
Skewness S	-0.282	-0.193	-0.001	-0.217	-0.714	-0.111	-0.141
Kurtosis B	5.916	6.374	5.909	9.333	4.914	5.731	16.51
Kurtosis S	12.17	9.795	13.43	9.039	14.01	7.942	6.982

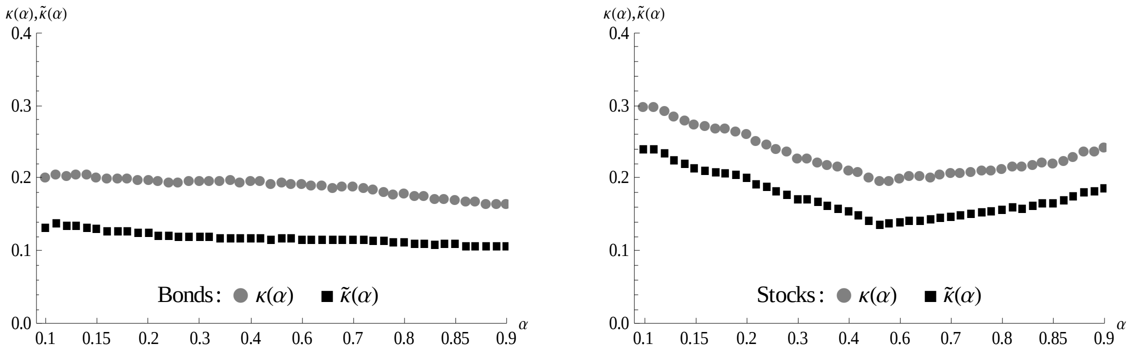
Notes: The table reports the mean, standard deviation, skewness, kurtosis for the synchronized daily log returns for G7 equity indices (S) and 10-years treasuries (B) for the sample period from 29 March 1991 to 16 July 2015. The sample was obtained from Datastream and contains 6,334 observations.

The left panel of Figure 2 shows the total ($\kappa(\alpha)$) and systemic ($\widetilde{\kappa}(\alpha)$) CTIs computed in the negative and the positive JTs for the empirical distribution of the daily

[5] While a lower frequency would account better for different opening times across G7 countries and for microstructure effects, it would result in a dramatic loss of observations.

returns of G7 treasury bonds. We follow here the convention that the CTIs for $\alpha \in[0.1, 0.5]$ correspond to the negative joint tails $\mathcal{T}^{\alpha}$ and for $\alpha \in(0.5, 0.9]$ to the positive joint tails $\mathcal{T}^{1-\alpha}$. There is a very mild asymmetry between the negative and the positive tails in the sample: the interdependence in the negative tails is slightly higher relative to the positive tails for both CTIs. However, our interdependence symmetry test cannot reject the null of the same interdependence structure, at both the total and systemic level. Therefore, treasury bond returns appear to display similar interdependence in both, negative and positive tails. Moreover, while interdependence does not vary with α, the total CTI is almost double the systemic CTI, which indicates a pronounced tail interdependence among subgroups of G7 countries, most likely Eurozone countries.

Figure 2: Tail Interdependence for G7 returns



Notes: The left (right) panel shows the total $\kappa(\alpha)$ and the systemic $\widetilde{\kappa}(\alpha)$ CTIs computed in the negative and the positive JTs for the empirical distribution of the daily log returns of 10-years treasuries (equity indices) in the G7 countries. The results for $\alpha \in [0.1, 0.5]$ correspond to the negative JTs $\mathcal{T}^{\alpha}$ and for $\alpha \in (0.5, 0.9]$ to the positive JTs $\mathcal{T}^{1-\alpha}$.

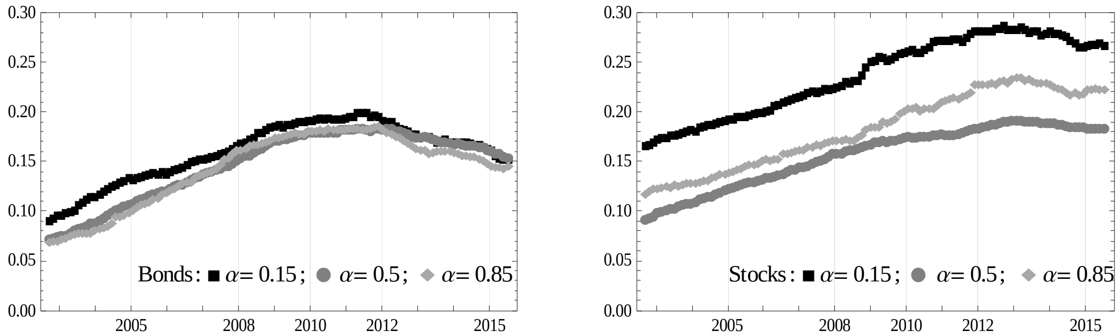
The right panel of Figure 2 shows the dependence in the negative tails $\mathcal{T}^{\alpha}$ ($0 < \alpha < 0.5$) and in the positive tails $\mathcal{T}^{1-\alpha}$ ($0.5 < \alpha < 1$) for the empirical distribution of the daily returns in G7 equity markets. For each α in the set $\{0.1, 0.15, ..., 0.5\}$, we tested the null that the interdependence in the negative tails $\mathcal{T}^{\alpha}$ is the same as in the positive tails with the same level of extremity. When $0.35 < \alpha < 0.5$, we cannot reject the null and, therefore, conclude that the dependence is symmetric. For $0 < \alpha < 0.35$, on the other hand, we strongly reject the null and conclude that the dependence is asymmetric. Therefore, negative extreme returns are indeed more closely tied together than their positive counterparts. Moreover, the total CTI is clearly larger than the systemic CTI, which indicates a pronounced tail interdependence among groups of countries, most likely the EU countries.

While the interdependence of extreme equity returns is strongly asymmetric (see also, Poon et al., 2004; Longin and Solnik, 2001), it appears that bond returns do not display such asymmetry. Although the reasons for this finding require further investigation, we conjecture that it is due to bond returns not being subject to leverage effects that affect equity returns.

3.1.1. Integration of G7 markets

In this subsection, we address questions pertaining to market integration by examining the evolution of their tail interdependence over time. We compute the systemic CTI in the windows $[t-3000, t]$ for $t = 3001, 3031, \ldots, T$ and $\alpha \in \{0.15, 0.5, 0.85\}$. The left panel of Figure 3 shows the systemic CTI $\widetilde{\kappa}(\alpha)$ computed in the negative tail ($\alpha = 0.15$), the central part of the distribution ($\alpha = 0.5$) and positive tails ($\alpha = 0.85$) for the empirical distribution of the daily returns of G7 treasury bonds. The evolution of the interdependence in the central part of the distribution and positive tails have the same pattern through time. Moreover, even if there was some asymmetry in dependence between extreme losses and extreme gains in the past, this effectively is no longer present.

Figure 3: Evolution of the CTI in G7 Returns over Time



Notes: The evolution of the systemic $\widetilde{\kappa}(\alpha)$ CTI for log returns of 10-years treasuries (left panel) and equity indices (right panel) in G7 countries from 9 March 1991 to 16 July 2015 computed in the windows $[t-3000, t]$ for $t = 3001, 3031, \ldots, T$ and $\alpha \in \{0.15, 0.5, 0.85\}$.

The right panel of Figure 3 shows the systemic CTI $\widetilde{\kappa}(\alpha)$ computed from the empirical distribution of the daily returns of G7 equity indices. Unlike the patterns in the left panel, there are strong asymmetries among the negative tail, where systemic distress materializes, the central part of the distribution which reflects small day-to-day price moves, and positive tails capturing extreme gains. In particular, the negative tail displays the strongest dependence, followed by the positive tail. This is a confirmation of the rotated-J shape dependence depicted in the right panel of Figure 2. Further, while dependence increased steadily until the Eurozone crisis, it has remained largely flat at the highest level since then. These findings coupled with those of Figure 2, have implications for portfolio diversification and hedging. In particular, they suggest that diversification has become harder over time as financial markets of developed economies have become more integrated. Further, hedging tail risk of equity returns is similarly challenging due to the tendency of the markets of the developed economies to get into distress and crash simultaneously.

3.1.2. Dynamic tail interdependence structure

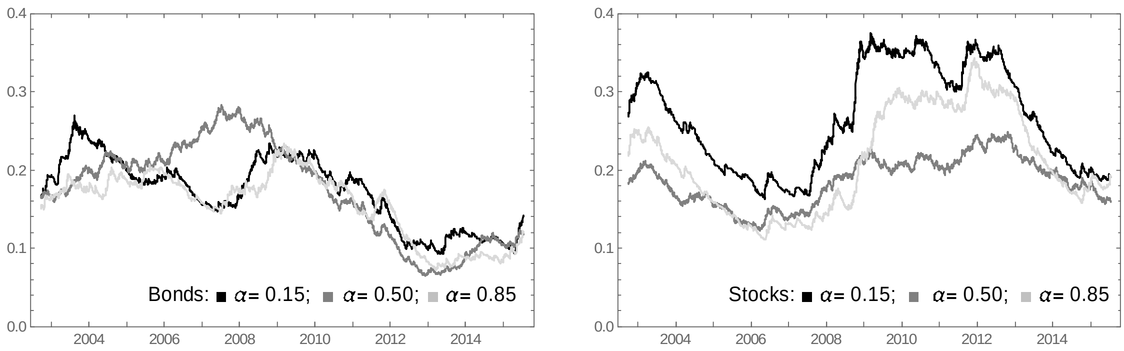
While the static estimation of the CTI is useful for highlighting the joint evolution of, and long term trends in the G7 economies, it overlooks important details regarding short term developments and reactions of these economies to events. In other words, it suffers from "ghost features" - the large impact of influential observations entering and leaving the estimation window. For example, it is difficult to detect the granular impact of either the global financial or the sovereign debt crisis. Inferences on these aspects necessitate a dynamic specification of CTI, where the dependence in the next period depends more on the current level of dependence than the dependence that prevailed a long time ago. As it is now standard in the literature, this problem can be addressed by putting a larger weight on the most recent observations which are more informative about the dependence in the subsequent periods than the observations in the past. Here, we model the dynamics of the systemic TIS $\widetilde{\mathbf{u}}^{\alpha}(t+1)$ at date $t+1$ by an exponentially weighted moving average (EWMA) process with the parameter $\gamma \in [0, 1]$,

$$\widetilde{\mathbf{u}}^{\alpha}(t+1;\gamma) = \gamma\widetilde{\mathbf{u}}^{\alpha}(t;\gamma) + (1-\gamma)\mathbf{z}(t), \tag{5}$$

where $\mathbf{z}(t)$ is a discrete $(n+1)$-dimensional PDF that puts all probability on the observed number of exceedances in period t and $\widetilde{\mathbf{u}}^{\alpha}(0)$ is the sample TIS. The EWMA model remedies the 'ghost features' problem of the static framework and, therefore, it is better suited to capturing short term dynamics in risk measures. However, while the EWMA is a relatively simple technique to the weighting of past observations it is not the only approach, and we employ it here simply as a first approximation. An alternative to EWMA would be a GARCH-like model. However, EWMA-based tail forecasts have been shown to be superior to those based on GARCH models in many cases (see, e.g., Alexander and Leigh, 1997; Boudoukh, Richardson and Whitelaw, 1997; Guermat and Harris, 2002). We estimate the EWMA model (5) for the CTI of the G7 bond and equity indices by maximum likelihood and present the results in

Figure 4.

Figure 4: Dynamic CTI in G7 Returns over Time



Notes: The systemic CTI $\widetilde{\kappa}(\alpha)$ for log returns of 10-years treasuries (left panel) and equity indices (right panel) in G7 countries from 9 March 1991 to 16 July 2015 computed for $\alpha \in \{0.15, 0.5, 0.85\}$ by the EWMA (5) with the parameter value $\gamma = 0.995$ estimated by maximum likelihood.

As in Figure 3, the left panel in Figure 4 shows the dynamic CTI for the G7 bonds estimated for the negative ($\alpha = 0.15$), central ($\alpha = 0.5$) and positive ($\alpha = 0.85$) parts of the empirical distribution. The dependence in the negative tail steadily decreases during the 'Great Moderation' period of 2002 – 2007, jumps suddenly with the onset of the global financial crisis and begins to decrease after the crisis reaching historical lows. Note however, that the dependence increases around the end of 2010 at the onset of the sovereign debt crisis but the increase is mild and relatively short-lived. The dependence in the positive tail follows a similar pattern although it is generally lower.

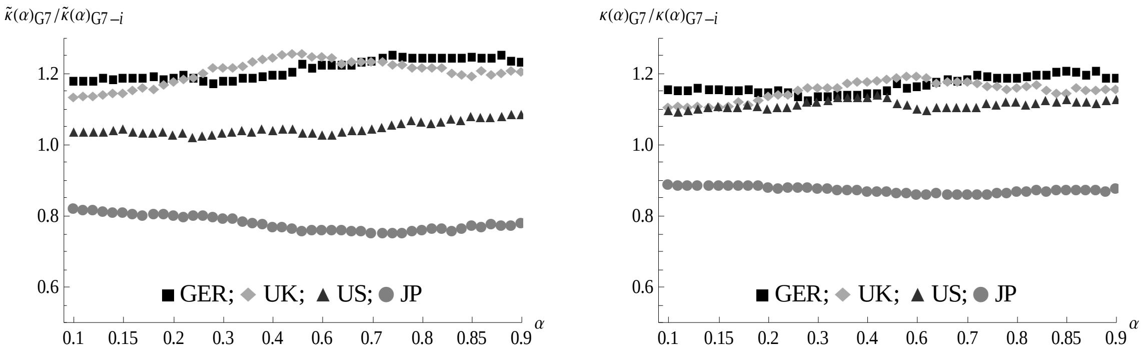
Interestingly, the dependence in the central part of the distribution increases throughout the 'Great Moderation' and its divergence from the dependence in the tails peaks around 2007 at the onset of the financial crisis. This divergence in dependence is then reversed with the onset of the financial crisis. While this convergence is partly accounted for by the increase in dependence in the negative (and positive) tail, the decrease in the dependence in the central part also plays an important role. To the best of our knowledge, the finding that the dependence of moderate G7 bond returns (i.e. in the central part of the distribution) substantially increased during the 'Great Moderation' and then decreased during the crisis is new to the literature, the reasons for which would require further investigation.

Turning to the right panel of Figure 4 showing the dependence in the G7 equity indices, despite fluctuations over time, the rotated-J shape of the dependence observed previously is clear. The dependence is considerably higher in the negative tails, followed by that of the positive tails, while the dependence of the returns in the central part is generally considerably lower. The exception is the period 2005-2007 when the dependence in the central and positive parts of the distribution look very similar. Consistent with intuition, the dependence in all three parts of the distribution significantly decreases during the 'Great Moderation' and begins to increase in 2007 with the onset of the global financial crisis. The increase is substantially larger for the dependence in the negative tail and is almost vertical during 2008. This dependence stays at historically high levels until 2011 when it decreases briefly, and then increases again at the onset of the sovereign debt crisis to similar levels through to the end of 2012. It then begins to decrease steadily until it reaches similar levels to the dependence in the other parts of the distribution, reducing the asymmetry in the process.

3.1.3. Contribution to tail dependence in G7 markets

It is important for the study of spillovers and contagion to isolate the impact or contribution of an individual component to the overall systemic distress. As the CTI can be computed for different subsets of variables, we can find the marginal contribution of each variable to the interdependence in the subsets of other variables and then, decompose the CTI into individual contributions using e.g., Shapley values as in Tarashev et al. (2016). Here, however, we simply compute the contribution of a single variable as the ratio of the CTIs that include and exclude that particular variable. Figure 5 depicts the contribution of US, UK, Germany and Japan to the (systemic) interdependence of the main equity indices computed as the ratio $CTI(\alpha)_{G7}/CTI(\alpha)_{G7\backslash i}$, where $CTI(\alpha)_{G7}$ is the (systemic) CTI for all seven countries and $CTI(\alpha)_{G7\backslash i}$ for all countries but $i \in\{US, UK, GER, JP\}$. We observe that Japan makes a negative contribution. This would suggest that the Japanese equity index may be an effective (tail) risk diversifying asset in G7 equity portfolios.

Figure 5: Interdependence Contributions



Notes: The contributions of US, UK, Germany and Japan to the (systemic) interdependence of the main equity indices in the G7 countries computed as the ratio $CTI(\alpha)_{G7}/CTI(\alpha)_{G7\backslash i}$, where $CTI(\alpha)_{G7}$ is the (systemic) CTI for all seven countries and $CTI(\alpha)_{G7\backslash i}$ for all countries but $i \in\{US, UK, GER, JP\}$. The results for $\alpha \in[0.1, 0.5]$ correspond to the negative JTs $\mathcal{T}^{\alpha}$ and for $\alpha \in(0.5, 0.9]$ to the positive JTs $\mathcal{T}^{1-\alpha}$.

3.2. Stock and factor interdependence

In this section, we focus on the interdependence among the 30 constituent stocks of Dow Jones Industrial Average index and relate this interdependence to the Fama-French-Carhart factors. The data spans the period 1 January 1990 - 21 November 2012 (5770 synchronized daily returns obtained from Datastream on 22 November 2012, while the FFC factors for the same period were obtained from Kenneth French's website). Summary statistics are reported in Table 2. For all four factors (and the

DJ30 index constituents, which are not shown) daily log returns are zero, negatively skewed and leptokurtic.

Table 2: Summary Statistics for the Fama-French-Carhart Factor Returns

	RPm	SMB	HML	MOM
Mean	0.000	0.000	0.000	0.000
SD	0.012	0.006	0.006	0.009
Skewness	-0.105	-0.268	-0.108	-0.956
Kurtosis	10.99	7.163	9.337	14.69

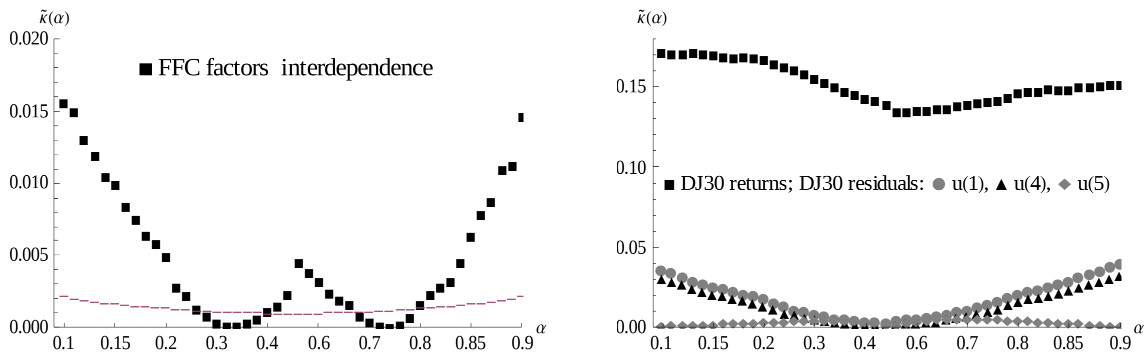
Notes: The table reports the mean, standard deviation, skewness, kurtosis for the Fama-French-Carhart factors Market Risk Premium (RPm), Small minus Big (SMB), High minus Low (HML) and Momentum (MOM). The data spans the period from 1 January 1990 through 21 November 2012 (5770 observations obtained from Kenneth French's website).

Due to the curse of dimensionality, total CTI is unreliable because of the high number of JTs (among the total of 2^{30} JTs) containing no observations. Thus, in the ensuing discussion we focus on the systemic CTI which is robust to the curse of dimensionality. The right panel of Figure 6 shows that the DJ30 returns are highly interdependent and asymmetric. While the FFC factors account for a high degree of this dependence in the central part of the distribution, the factors are unable to account for the strong systemic dependence of the DJ30 constituent returns in the tails of the distribution. Moreover, comparing the interdependence of the residuals $u(1)$ of a regression of the DJ30 index constituent returns on the first FFC factor returns (market risk premium) with the interdependence of the residuals $u(4)$ of the same dependent variables on all four FFC factor returns, it appears that most of the (systemic) interdependence is accounted for by the market risk premium. This comparison makes it clear that the remaining three FFC factors account for very little of the systemic distress of the DJ30 constituents. The inability of the FFC factors to account for the inderdependence of the DJ30 returns in the tails is a direct manifestation of the tail interdependence of the factors themselves. The systemic CTI depicted in the left panel of Figure 6 reveals that the FFC factors are significantly interdependent for $\alpha < 0.2$ and $\alpha > 0.8$. As a potential additional factor that accounts for the strong interdependence of the residuals in the tails, we explore *market dispersion* F_d. We estimate F_d by computing the standard deviation of the DJ30 constituents for every day in the sample. Then, we compute the residuals $u(5)$ by normalizing $u(4)$ with these estimates,

$$u_i(5) = u_i(4)/F_d, \quad i = 1, .., 30.$$

As the systemic CTI of $u(5)$ shows in the right panel of Figure 6, F_d accounts for a large part of the interdependence in the JTs for $\alpha \leq 0.3$ and $\alpha \geq 0.7$.

Figure 6: Interdependence of Fama-French-Carhart factors and DJ30 index constituent stocks



Notes: The left panel of this figure shows the systemic interdependence for the Fama-French-Carhart (FFC) factors. The dashed line marks the 1% critical values for the test statistic (4) in the test of independence. The right panel shows the systemic interdependence for the DJ30 index constituent returns as well as for the residuals $u(1)$ of a regression of the DJ30 returns on the first FFC factor (the market risk premium) and the residuals $u(4)$ and $u(5)$ of a regression of the DJ30 returns on all FFC factors, where the latter residuals are normalized by the market dispersion F_d.

3.3. Comparison with alternative tail dependence measures

In this section, we compare the dependence measured by CTI with two other important measures of tail dependence: the non-parametric bivariate measure of Poon et al. (2004) and the parametric high-dimensional factor copula model of Oh and Patton (2017). Both measures rely on extreme value theory. In the interest of space, we present a summary of the results - a detailed discussion on their measures and results can be found in the Appendix. Tables 4 and 5 present the dependence results of Poon et al. (2004) and Oh and Patton (2017) using the residuals of the regression for the DJ30 constituent returns on the first FFC factor. For comparison, the analogous results using CTI are also reported in both tables. We observe that all three models deliver qualitatively similar results. In particular, they show that dependence is stronger in the negative tails, not only across pairs of returns but also across industries, and that it increases when tails become more extreme, i.e. for lower α's.

As discussed in detail in the Appendix, the measure of Poon et al. (2004) also finds more occurrences of asymptotic dependence in the left tail than in the right tail. It is important to note that CTI has similarities with the Poon et al.'s (2004) tail dependence measure. Both are non-parametric and, at a fundamental level, employ the same building blocks - the log of the ratio of joint and cross-product of marginal probabilities (see their equation 3 and our equation 1). Our contribution also has similarities with the parametric factor copula model of Oh and Patton (2017). For example, their lower and upper tail dependence coefficients (their equations 5 and 6) converge to the classic lower and upper tail dependence coefficients of Sibuya (1960). Our Proposition 1 above shows that in the limit, CTI also converges to the same tail dependence coefficients. Therefore, it is not surprising that the results of all three measures are similar and show that the dependence is higher in the negative tail than positive tail.

Table 4: Tail dependence of selected pairs of DJ30 constituents and Apple with the Poon et al. (2004) and CTI measures

	Tail dependence measure χ		Tail dependence measure $\bar{\chi}$		CTI measure	
	Left Tail Quantile 5%	Right Tail Quantile 5%	Left Tail Quantile 5%	Right Tail Quantile 5%	Left Tail Quantile 5%	Right Tail Quantile 5%
AXP	0.89*	0.82*	0.35	0.33	0.11	0.07
BA	0.80*	0.63	0.34	N.A.	0.11	0.04
C	0.90*	0.76	0.37	N.A.	0.13	0.07
CAT	0.80*	0.97*	0.39	0.37	0.12	0.14
DD	0.89*	0.87*	0.39	0.37	0.15	0.11
GE	0.86*	0.68	0.39	N.A.	0.13	0.09
JPM	0.79*	0.73	0.37	N.A.	0.11	0.08
XOM	0.84*	0.73	0.34	N.A.	0.09	0.07

Notes: The table reports the tail dependence measures $\bar{\chi}$ and χ proposed by Poon et al (2004). Those pairs indicated with an asterisk * do not reject the null hypothesis of $\bar{\chi} = 1$ and hence are asymptotically dependent; their tail dependence measure is given by χ. Otherwise, the pair is considered asymptotically independent and its co-dependence is given by $\bar{\chi}$. We also report the analogous CTI measure for comparison. The results are estimated with the residuals of the regression for the DJ30 returns on the first FFC factor.

Table 5: Lower\ Upper tail dependence coefficients among six industries at 5% quantile based on Oh and Patton (2017) and CTI measure

	Oh and Patton (2017)						CTI measure					
	SIC2	SIC3	SIC4	SIC5	SIC6	SIC7	SIC2	SIC3	SIC4	SIC5	SIC6	SIC7
SIC2		0.26	0.26	0.26	0.26	0.26		0.17	0.15	0.12	0.14	0.09
SIC3	0.33		0.33	0.29	0.33	0.33	0.20		0.17	0.13	0.24	0.17
SIC4	0.33	0.41		0.29	0.36	0.36	0.16	0.17		0.13	0.17	0.11
SIC5	0.33	0.36	0.36		0.29	0.29	0.16	0.14	0.12		0.14	0.10
SIC6	0.33	0.41	0.44	0.36		0.27	0.15	0.25	0.15	0.11		0.12
SIC7	0.33	0.41	0.44	0.36	0.45		0.12	0.15	0.10	0.11	0.09	

Notes: The table shows the tail dependence between six industries implied by the Factor skew t-t model of Oh and Patton (2017) and our CTI measure. The results are estimated using the residuals of the regression for the DJ30 returns on the first FFC factor. The lower (upper) triangular entries correspond to dependence coefficients in the left (right) tail.

However, there are also differences in that Poon et al.'s measure is bivariate whereas ours is, typically, high-dimensional and therefore, they are suited to different applications. The differences with the factor-copula model of Oh and Patton (2017) arise because theirs is a computationally-intensive parametric technique requiring prior classification of the stock returns on the basis of SIC (or some other classification tool) and then produces a bivariate measure of tail dependence. Our technique, on the other hand, produces a measure of mutual dependence in a high-dimensional system. Although our tail dependence measure can be easily applied to the bivariate case, as we have done in this section, its 'natural ambience' would be a genuinely high-dimensional application.

4. Conclusion

Tail interdependence is a direct and intuitive measure of the likelihood of the constituents of a system to get into distress (or exuberance) simultaneously. However, in practice this is a challenging problem due to the small number of tail observations and the curse of dimensionality. We propose a versatile non-parametric framework to analyze and quantify tail dependence with several extensions. We apply it to the data from the G7 countries and the constituents of the DJ30 index and find some intriguing facts. For the latter dataset, we find, for example, that while the Fama-French-Carhart factors do well to account for interdependence in the central part of the distribution, they do not in the tails of the distribution because they are strongly tail dependent. In the former dataset, we find that while the interdependence of extreme equity returns is strongly asymmetric, bond returns do not display such asymmetry, perhaps because they are not subject to leverage effects. Our results imply also that diversification has become harder over time as financial markets of developed economies have become more integrated. Further, hedging tail risk of equity returns is similarly challenging due to the tendency of the markets of the developed economies to get into distress and crash simultaneously.

An interesting avenue for future research would be to investigate rigorously the optimal modelling of the dynamic properties of (systemic) CTI as well as the relationship between the CTI of main asset classes or financial institutions with the macroeconomic indicators

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