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Multi-Objective Optimization of Nonlinear Quarter Car Suspension System – PID and LQR Control

MP Nagarkar^{a,b,*}, Dr. YJ Bhalerao^c, Dr. GJ Vikhe Patil^c, RN Zaware Patil^d

^aDepartment of Mech Engg, SCSM CoE, Ahmednagar-414003, India

^bDepartment of Mech Engg, AVCoE, Sangamner, India.

^cMAE Alandi, Pune, India

^dDVVP CoE, Ahmednagar, India.

Abstract

This paper presents modeling, control and optimization of a nonlinear quarter car suspension system. A mathematical model of nonlinear quarter car along with seat and driver is developed and simulated in Matlab/Simulink® environment. Input road condition is taken as class C road and vehicle travelling at 80kmph. Active control of suspension system is achieved using PID and LQR control actions. Instead of guessing and or trial and error method to determine the PID and LQR control parameters, a GA based optimization algorithm is implemented. The optimization function is modeled as multi-objective problem comprising of frequency weighted RMS acceleration, VDV, suspension space, tyre deflection and controller force. It is observed that optimized parameters gives better control as compared to the classical parameters and passive suspension system. Further simulations are carried out on suspension system with seat and driver model. The PID controller gives better ride comfort by reducing RMS head acceleration and VDV. Results are presented in time and frequency domain.

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Keywords: Nonlinear Quarter Car; Genetic Algorithm; PID; LQR; Human model.

Nomenclature

A	Acceleration	c	Damping	f	Control Force	k	Stiffness
m	Mass	Q, R	Weight Matrices	xr	Road Profile		

* Corresponding author. Tel.:91-0241-2568383.

E-mail address: maheshnagarkar@rediffmail.com

1. Introduction

Performance requirements for a suspension system are to adequately support the vehicle weight, to provide effective ride quality by isolating the chassis against excitations mainly due to road roughness and to maintain the wheels in the appropriate position so as to have a better handling and to keep tyre in contact with the ground. The passive suspension systems are trade-off between ride comfort and performance [1].

Researchers have used various control strategies. Narayana S and Senthil S [2] used optimal control with full state information to control quarter car model. Demir O, Keskin I and Cetin S [3] presented control of nonlinear half car model using Hybrid Fuzzy PID controller with coupled rules (HFPIDCR). Rao MVC and Prahlad V [4] presented a tunable fuzzy logic control of quarter car active suspension system. Yoshimura T, Kume A, Kurimoto M and Hino J [5] presented quarter car active suspension system using the concept of sliding mode control, NN, GA and LQG control was presented by [7] with an objective to improve ride comfort and road holding.

Optimal control of nonlinear quarter car model is implemented in this paper. Optimal control method is having quadratic performance indexes. In designing LQR controller, the selection of weighting matrices, matrices Q and R, is key issue which directly affects the control action. Elmadany MM, and Al-Majed MI [8] presented the LQR problem with full state feedback for suspension system. Yahaya MS, Ruddin M, Ghani A and Ahmad N [9], Zhen L, Luo C and Hu D [10] and Darus R and Nur IE [11] presented LQR control scheme to control an actuator in an active suspension system. The values of weighting matrices were selected for simulation. Oral Ö, Çetin L and Uyar E [12] presented an approach to optimal control problem where Q and R matrices are not selected by trial and error but calculated for time domain. But, for minimum oscillations, the weighting parameters need to be adjusted. Thus becomes trial and error method in next stages. Thus for control application, LQR weight matrices parameters were arbitrarily chosen by the authors [9, 10, 11, 15] or adjusted by trial - error till desired performance is achieved [12].

Metered and Elawaf [16] had implemented particle swarm optimization (PSO) algorithm to tune the PID controller implemented on a semi-active quarter car model suspension system. Kesarkar and Selvaganesan [17] designed fractional order PID controller using artificial bee colony algorithm with objective functions such as integral absolute error, integral square error and integral time absolute error. Nui [18] had implemented GA based optimization method to tune PID parameters of active suspension system. GA based optimized PID controller improves the dynamic performance of a active suspension system and improves stability. Hamid and Hamid [19] analysed a fuzzy based PID controller for a half car active suspension system. In this analysis, suspension working space is the criterion under observation.

This paper presents mathematical modeling of nonlinear quarter car (NLQC) along with a human model. The nonlinear quarter car model consists of quadratic tyre stiffness and cubic stiffness in suspension spring as nonlinearities. Further, optimal and PID control is implemented with control objective to minimize frequency weighted RMS sprung mass acceleration (hereafter called as RMS sprung mass acceleration), RMS suspension space requirement, tyre dynamic force and road holding along with RMS optimal control force. The constraints during optimization are maximum control force, RMS sprung mass acceleration, maximum sprung mass acceleration, maximum suspension space requirement, maximum tyre deflection and maximum unsprung mass displacement. Here GA based evolutionary optimization technique is used to search the optimum parameters. Simulations are further extended to human model to analyze the effect on driver to analyze RMS acceleration and VDV at head.

2. Methodology

2.1. Mathematical Modeling – Nonlinear Quarter Car Suspension-Seat-Driver Model

The mathematical model of a quarter car suspension system consists of basic elements of the suspension system such as sprung mass m_s (representing chassis) and unsprung mass m_{us} (representing wheel assembly and axle). A suspension system of commercial vehicles generally consists of coil springs. In the present analysis, a nonlinear quarter car model having quadratic tyre stiffness and cubic stiffness in suspension spring nonlinearities is considered along with seat suspension model consists of a frame and cushion is shown in Fig.1 [20, 21, 22].

In this study a 4 DoF lumped parameter human model suggested by [23] is used in optimization study. It consists of head and neck mass (m_h), chest and upper torso mass (m_{ut}), lower torso mass (m_{lt}) and thigh and pelvis mass (m_t).

According to D’Alembert’s principle, the governing equations of motion representing nonlinear quarter car suspension-seat-human model are represented as –

$$\left. \begin{aligned}
 m_{us}\ddot{x}_{us} &= -k_t(x_{us} - x_r) + k_s(x_s - x_{us}) + c_s(\dot{x}_s - \dot{x}_{us}) + k_{tnl}(x_{us} - x_r)^2 + k_{snl}(x_s - x_{us})^3 - f \\
 m_s\ddot{x}_s &= -k_s(x_s - x_{us}) - c_s(\dot{x}_s - \dot{x}_{us}) - k_{snl}(x_s - x_{us})^3 + k_f(x_f - x_s) + c_f(\dot{x}_f - \dot{x}_s) + f \\
 m_f\ddot{x}_f &= -k_f(x_f - x_s) - c_f(\dot{x}_f - \dot{x}_s) + k_c(x_c - x_f) + c_c(\dot{x}_c - \dot{x}_f) \\
 m_c\ddot{x}_c &= -k_c(x_c - x_f) - c_c(\dot{x}_c - \dot{x}_f) + k_{tp}(x_t - x_c) + c_{tp}(\dot{x}_t - \dot{x}_c) \\
 m_t\ddot{x}_t &= -k_{tp}(x_t - x_c) - c_{tp}(\dot{x}_t - \dot{x}_c) + k_{lt}(x_{lt} - x_t) + c_{lt}(\dot{x}_{lt} - \dot{x}_t) \\
 m_{lt}\ddot{x}_{lt} &= -k_{lt}(x_{lt} - x_t) - c_{lt}(\dot{x}_{lt} - \dot{x}_t) + k_{ut}(x_{ut} - x_{lt}) + c_{ut}(\dot{x}_{ut} - \dot{x}_{lt}) \\
 m_{ut}\ddot{x}_{ut} &= -k_{ut}(x_{ut} - x_{lt}) - c_{ut}(\dot{x}_{ut} - \dot{x}_{lt}) + k_h(x_h - x_{ut}) + c_h(\dot{x}_h - \dot{x}_{ut}) \\
 m_h\ddot{x}_h &= -k_h(x_h - x_{ut}) - c_h(\dot{x}_h - \dot{x}_{ut})
 \end{aligned} \right\} \quad (1)$$

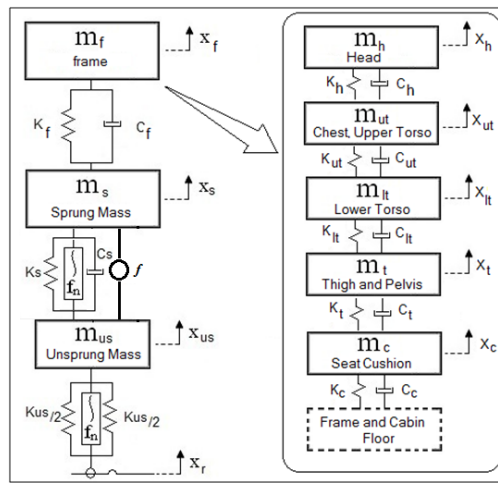


Fig. 1. Nonlinear Quarter Car System – with Seat and Human Model [26]

The nonlinear quarter car seat-suspension-driver model parameters are as follows –

- | | | | |
|-------------------|------------------|-------------------------|--------------------------|
| $m_h = 5.31;$ | $c_h = 400;$ | $k_h = 310000;$ | |
| $m_{ut} = 28.49;$ | $c_{ut} = 4750;$ | $k_{ut} = 183000;$ | |
| $m_{lt} = 8.62;$ | $c_{lt} = 4585;$ | $k_{lt} = 162800;$ | |
| $m_t = 12.78;$ | $c_t = 2064;$ | $k_t = 90000;$ | |
| $m_c = 1;$ | $c_c = 200;$ | $k_c = 18000;$ | |
| $m_f = 15;$ | $c_f = 830;$ | $k_f = 31000;$ | |
| $m_s = 290;$ | $c_s = 700;$ | $k_s = 23500;$ | $k_{snl} = 100k_s [22];$ |
| $m_{us} = 40;$ | $k_t = 190000;$ | $kt_{nl} = 1.5 kt [6].$ | |

2.2. Multi-Objective Optimization

The suspension system has to perform several conflicting objectives such as ride comfort, road holding, and suspension/rattle space requirements. Also, in this study, human model is incorporated to optimize the objective functions considering the human body responses rather than only the seat. Thus, the optimization problem becomes multi-objective in nature with conflicts. In the solution of MOO problems, Non-dominated sort GA-II (NSGA-II) [24, 25] is one of the MOEAs using GA strategy is implemented. In NSGA-II, to preserve the diversity and uniform

spread of optimal front, a crowding distance (CD) operator is used. Number of generations is used as stoppage criterion.

2.3. Objective Functions:

RMS Acceleration: As per ISO 2631-1 [27], RMS acceleration is given by

$$A_w = \left\{ \frac{1}{T} \int_0^T [a_w(t)]^2 dt \right\}^{1/2} \quad (2)$$

A major portion of the vibration experienced by the occupants of an automobile enters the body through the seat [28,29]. Hence is it necessary to measure the whole body vibrations. As per ISO 2631-1 [27], VDV is one measure for WBV. VDV is called as fourth power vibration dose and assesses the cumulative effect (dose) of vibration.

$$VDV = \left\{ \int_0^T [a_w(t)]^4 dt \right\}^{1/4} \quad (3)$$

Suspension Travel: Suspension travel is characterized by the relative travel between the sprung mass and unsprung mass. Due to random input, RMS suspension space travel is taken as one of the objective functions.

$$\text{RMS Suspension Travel} = \left\{ \frac{1}{T} \int_0^T [(x_s(t) - x_{us}(t))]^2 dt \right\}^{1/2} \quad (4)$$

Dynamic Tyre Force: Dynamic tyre force is related to tyre deflection. RMS tyre deflection is one of the objectives.

$$\text{RMS Tyre Deflection} = \left\{ \frac{1}{T} \int_0^T [(x_{us}(t) - x_r(t))]^2 dt \right\}^{1/2} \quad (5)$$

Control force is introduced as one of the objective functions so as to find optimum control force for ride comfort.

$$\text{RMS F} = \sqrt{\frac{\sum_{i=1}^N f_i^2}{N}} \quad (6)$$

According to Baual and et al. [30], at least, 125 mm of suspension travel is required and maximum seat acceleration should not increase 4.5 m/s^2 so as to avoid hitting the suspension stop. To minimize dynamic tyre forces, maximum tyre deflection should not increase 0.058m. These are the constraints of the problem.

The formulation of optimization problem is as follows –

$$\begin{aligned} \text{fobj1} &= \text{Minimize (RMS f)}, & \text{fobj2} &= \text{Minimize (VDV)}, & \text{fobj3} &= \text{Minimize (Aw)}, \\ \text{fobj4} &= \text{Minimize (RMS Suspension Travel)}, & \text{fobj5} &= \text{Minimize (RMS Tyre Deflection)} \end{aligned}$$

Constraints:

$$a_{\max, \text{seat}} \leq 4.5 \text{ m/s}^2, \text{ Max. } (x_s - x_{us}) \leq 0.125 \text{ m}, \text{ Max. } (x_{us} - x_r) \leq 0.058 \text{ m},$$

Search Space:

During optimization, the design parameters are suspension spring stiffness; suspension damping and seat cushion parameters. The search space is –

$$\text{LQR Control: } Q_{11} \in [10e10, 10e11], Q_{22} \in [10e6, 10e7], Q_{33} \in [0.01, 10], Q_{44} \in [0.01, 10], R_1 \in [0.001, 0.1]$$

$$\text{PID Control: } K_P \in [2000, 15000], K_I \in [10, 150], K_D \in [15000, 50000]$$

2.4. Control Theory:

LQR Control:

The control law is full state feedback type with disturbance rejection i.e. road unevenness rejection. Let us consider nonlinear system for control

$$\dot{x} = A x + G(x) + B_f f \quad (7)$$

The linear feedback control is given by [13-15]

$$f = [-R^{-1} B^T P] x \quad (8)$$

is optimal in order to transfer the system (6) from an initial to final state $x(t)=0$.

PID Control:

PID stands for proportional, integral and derivative. These controllers are designed to eliminate the need for continuous operator attention. In order to avoid the small variation of the output at the steady state, the PID controller is so designed that it reduces the errors by the derivative nature of the controller. A PID controller is depicted in Fig. 2 [18, 19]. The set-point is where the measurement to be. Error is defined as the difference between set-point and measurement. PID computes the control signal based on following equations:

$$u(t) = k_p e(t) + k_i \int e(t)dt + k_d [d/dt e(t)] \quad (9)$$

$$e(t) = r(t) - y(t) \quad (10)$$

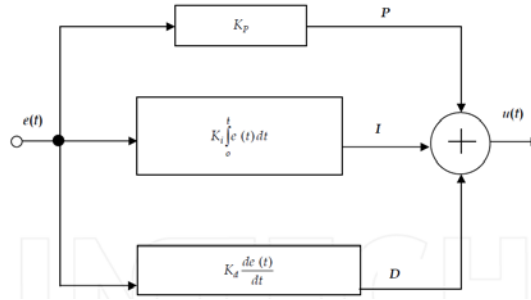


Fig. 2. PID Control.

3. Results and Discussion

Multi-objective optimization of non-linear quarter car model is simulated in Matlab/Simulink® environment using NSGA-II algorithm.

Road condition is modelled as class C road (average road) with velocity of 80 kmph. Refer Fig. 3.

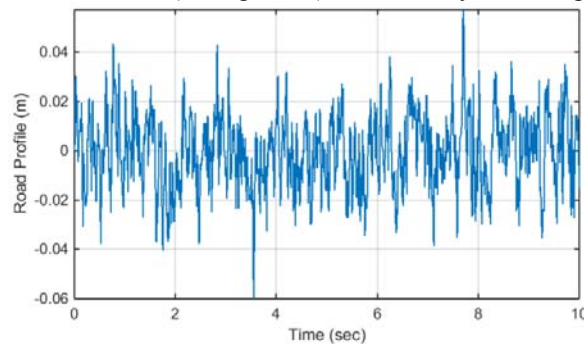


Fig. 3. Road Surface (Class C, Velocity 80 kmph).

3.1 LQR Control:

Trade off front of 100 different solutions for objective functions and satisfying the constraints is obtained after optimization. For ride comfort and health criterion, RMS acceleration and VDV are selected as optimum values. Hence from the trade-off front, minimum values of RMS acceleration and VDV are chosen along with the LQR parameters, matrices Q and R, are selected and simulated further alongwith passive suspension system.

It is observed that, RMS acceleration for GA optimized LQR controller is 0.656586 m/s^2 , which is a little uncomfortable. In LQR controlled system, RMS acceleration is reduced by 30% (passive suspension system has RMS acceleration 0.9322 m/s^2 which is uncomfortable) and VDV is reduced by 30% as compared to the passive suspension system. Also RMS suspension space and RMS tyre deflection is less as compared to the passive suspension system. Also constraints are not violated in GA optimized system. The results of RMS optimal control

force, RMS sprung mass acceleration, RMS suspension space, dynamic tyre force, road holding, maximum controller force, maximum sprung mass acceleration, are tabulated in Table 1 and refer Fig. 4(a).

3.2 PID Control

Trade off front of 100 different solutions for objective functions and satisfying the constraints is obtained after optimization. For ride comfort and health criterion, RMS acceleration and VDV are selected as optimum values.

It is observed that, RMS acceleration for GA optimized PID controller is 0.6178 m/s^2 , which is a little uncomfortable. RMS acceleration is reduced by 34% (passive suspension system has RMS acceleration 0.9322 m/s^2 which is uncomfortable) and VDV is reduced by 33% as compared to the passive suspension system. Also constraints are not violated in GA optimized system. The results are tabulated in Table 1 and refer Fig. 4(b).

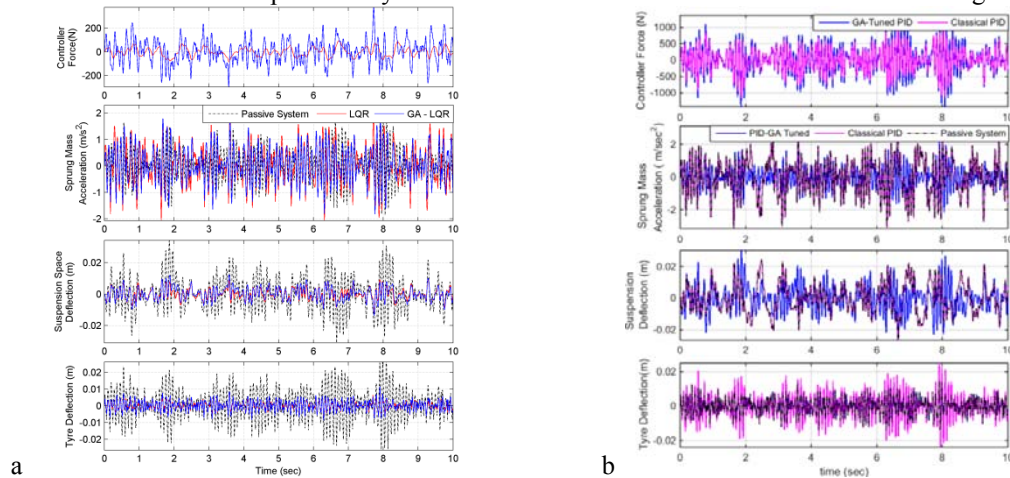


Fig. 4. Time Domain Results – (a) LQR Control; (b) PID Control

Further Frequency domain analysis is also carried out for LQR, PID and system and passive system. Results are shown in Fig. 5(a) and (b). It is observed that, PID and LQR controlled system experiences low amplitudes of vibrations as compared to the passive suspension system.

Table 1. Results – LQR Control.

Parameter	LQR Control-GA	LQR Control	PID-GA	PID Classical	Passive System
LQR Control Force (N)	100.5815	36.0332	445.4385	346.8979	--
VDV ($\text{m/s}^{2.75}$)	1.51049	1.602	1.4410	1.6174	2.1509
A_w (m/s^2)	0.656586	0.697	0.6178	0.6890	0.9322
RMS Suspension Space Deflection (m)	0.003956	0.0033	0.01007	0.008700	0.009311
RMS Tyre Deflection (m)	0.002834	0.0024	0.008864	0.007300	0.004473
Max Control Force (N)	377.0774	94.2117	1423.1810	1173.24400	--
Max Acceleration (m/s^2)	1.895686	2.3003	1.8615	2.1800	3.1672
Max Suspension Space Deflection (m)	0.013691	0.0115	0.03426	0.03050	0.0305
Max Tyre Deflection (m)	0.011538	0.0101	0.02639	0.02510	0.0251

To check the effect of vibrations on human body, the suspension system along with human model is simulated. Nonlinear quarter car along with human model is simulated using PID and compared with passive system. VDV at head, RMS Head acceleration, crest factor, seat to head transmissibility, vibrations transmitted from seat to upper torso are observed. Results are tabulated in Table 2. Refer Fig. 6(a) for time domain results and Fig. 6 (b) for frequency domain results.

Table 2. Human Model Results – PID and Passive System.

Parameter	PID	Passive System
VDV_h (m/s ^{2.75})	2.8387	4.8180
Aw_h (m/s ²)	1.0978	2.0408
CF	3.3829	3.3109
Amplitude ratio at Head [26]	1.0192	1.2033
Amplitude ratio at Upper torso [26]	1.0277	1.1969

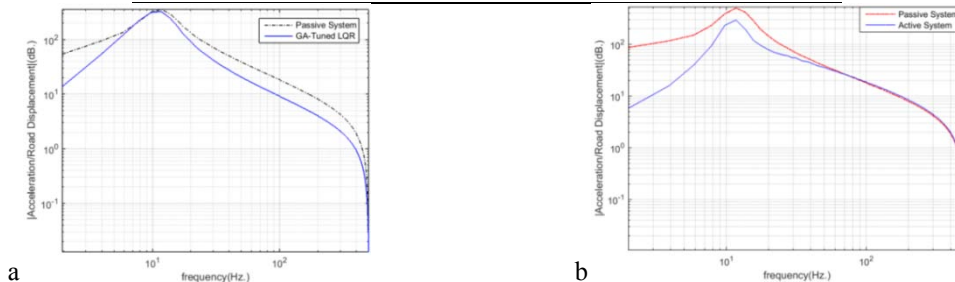


Fig. 5. (a) LQR Control Results – Frequency domain; (b) Fig. 8. PID Control Results – Frequency Domain.

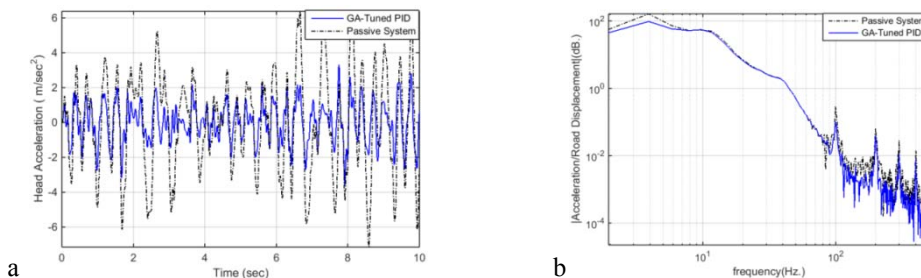


Fig. 6. Head Acceleration (a) Time Domain Results; (b) Frequency Domain Results.

Table 3. Controller Parameters.

Type	Parameters
LQR Controller	Un-optimized: Q=diag([1e+008, 1e+004, 010, 10]), R=[0.1] Optimized: Q=diag[8.139E+11, 9376, 3.5318, 6.9873], R=[0.002474]
PID	Classical: KP=3055, KI=0.7; KD=32060. Optimized: KP=200, KI=50.9245; KD=48176.4

4 Conclusion

This paper presents multi-objective optimization of Active Nonlinear Quarter Car Suspension system with driver, having quadratic nonlinearities in tyre and cubic nonlinearity in suspension spring.

LQR and PID control strategies are implemented on a nonlinear quarter car system. Controller parameters are then tuned using NSGA-II algorithm. In optimization problem comfort and health criterion consisting of VDV, RMS acceleration, along with stability criterions consisting of suspension space and tyre deflection are used as objective functions. ISO 2631-1 methodology is adopted and successfully implemented. Numerical simulations shows that active control system minimizes the frequency weighted RMS acceleration and VDV as compared to passive suspension system thus improving the ride comfort. Results are also presented in frequency domain which shows that active control system experiences less amplitude as compared to the passive one. Further results are extended on human model, which shows that active control system shows minimum acceleration at head, VDV at head, crest factor, amplitude ratio at head and upper torso, thus providing comfort with health criterion.

References

- [1] J. Y. Wong. Theory of ground vehicles. John Wiley & Sons, Inc., NY (2001).
- [2] S. Narayanan, S. Senthil. Stochastic optimal active control of a 2-DoF quarter car model with non-linear passive suspension elements. *Journal of Sound Vibrations*; 211, 3 (1998) 495-506.
- [3] D. Ozgur, K. Ilknur, C. Saban. Modeling and control of a nonlinear half-vehicle suspension system: a hybrid fuzzy logic approach. *Nonlinear Dynamics* 67(2012) 2139–2151.
- [4] MVC Rao, V. Prahlad. A tunable fuzzy logic controller for vehicle-active suspension system. *Fuzzy Sets Systems* 85 (1997) 11-21.
- [5] T. Yoshimura, A. Kume, M. Kurimoto, J. Hino. Construction of an active suspension system of a quarter car model using the concept of sliding mode control. *Journal of Sound Vibration* 239, 3 (2001) 187-199.
- [6] C. Lauwerys, J. Swevers, S. Paul. Robust linear control of an active suspension on a quarter car test-rig. *Control Eng Pract* 13 (2005) 577–586.
- [7] C. Tang, Z. Tiaoxia. The Research on Control Algorithms of Vehicle Active Suspension System. *Proceedings of IEEE International Conference on Vehicular Electronics and Safety* (2005) 320 – 325.
- [8] M.M. Elmadany, M.I. Al-Majed. Quadratic Synthesis of Active Controls for a Quarter-Car Model. *Journal of Vibration and Control* 7 (2001) 1237-1252.
- [9] M.S. Yahaya, M. Ruddin, A. Ghani, N. Ahmad. LQR Controller for Active Car Suspension. *Proceedings of TENCON* (2000) 441-444.
- [10] L. Zhen, C. Luo, D. Hu. Active Suspension Control Design Using a Combination of LQR and Backstepping. *Proceedings of the 25th Chinese Control Conference* (2006) 123-25.
- [11] R. Darus, N.I. Enzai. Modeling and Control Active Suspension System for a Quarter Car Model. *Proceedings of International Conference on Science and Social Research* (2010) 1203-06.
- [12] Ö. Oral, L. Çetin, E. Uyar. A novel method on selection of Q and R matrices in the theory of optimal control. *International Journal of System and Control*. 1, 2 (2010) 84-92.
- [13] M.P. Nagarkar, G.J. Vikhe. GA-Based Multi-Objective Optimal Control of Nonlinear Quarter Car Suspension. *Proceedings of Innovative Design and Development Practices in Aerospace and Automotive Engineering*. (2016) 481-491.
- [14] M.P. Nagarkar, G.J. Vikhe, Multi-Objective Optimization of LQR Control Quarter Car Suspension System using Genetic Algorithm. *FME Transactions*. 44 (2016) 187-196.
- [15] A.M. Tusset, M. Rafikov, J.M. Balthazar. An intelligent controller design for magneto rheological damper based quarter-car model. *Journal of Vibration and Control* 15, 12 (2009) 1907-1920.
- [16] H. Metered, A. Elswaf, T. Vampola, Z. Sika. Vibration Control of MR-Damped Vehicle Suspension System Using PID Controller Tuned by Particle Swarm Optimization. *SAE International Journal of Passenger Cars – Mechanical Systems*. 8,2 (2015) 426-435.
- [17] A.A. Kesarkar, N. Selvagesan. Tuning of optimal fractional-order PID controller using an artificial bee colony algorithm. *Systems Science & Control Engineering: An Open Access Journal*. 3 (2015) 99-105.
- [18] Niu Xiang-jie. The optimization for PID controller parameters based on Genetic Algorithm. *Applied Mechanics and Materials*, 513-517 (2014) 4102-4105.
- [19] H. Khodadadi, H. Ghadiri. Self-tuning PID controller design using fuzzy logic for half car active suspension system. *International Journal of Dynamics and Control*. doi: 10.1007/s40435-016-0291-5. 1-9
- [20] C. McGee, M. Haroon, D. Adams, Y. Luk. A Frequency Domain Technique for Characterizing Nonlinearities in a Tire-Vehicle Suspension System. *Journal of Vibrations and Acoustics* 127 (2005) 61-76.
- [21] Q. Zhu, M. Ishitobi. Chaotic vibration of a nonlinear full-vehicle model. *International Journal of Solids Structures* 43 (2005) 747–759.
- [22] J. Lixia, L. Wanxiang. Chaotic vibration of a nonlinear quarter -vehicle model. *Proceedings of IEEE Vehicle Power and Propulsion Conference, China* (2008) 1-4.
- [23] P. Boileau, S. Rakheja. Whole-body vertical biodynamic response characteristics of the seated body biodynamic response under vertical vibration. *Journal of Sound and Vibrations* 215, 4 (1998) 841-862.
- [24] K Deb, A. Pratap, S. Agarwal, T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Trans Evolutionary Computations*. 6,2 (2002) 182-197.
- [25] Song L. NGPM-A NSGA-II Program in Matlab - User Manual. Version 4.1. (2011) 1-20.
- [26] M.P Nagarkar, G.J. Vikhe, R.N. Zaware. Optimization of nonlinear quarter car suspension- seat-driver model. *Journal of Advanced Research*. 7,6 (2016) 991-1007.
- [27] ISO: 2631-1. Mechanical vibration and shock - Evaluation of human exposure to whole-body vibration. (1997).
- [28] J.L. Van Niekerka, W.J. Pielemeierb, J.A. Greenberg. The use of seat effective amplitude transmissibility (SEAT) values to predict dynamic seat comfort. *Journal of Sound and Vibrations* 260, (2003) 867–88.
- [29] M. Bovenzi. Health Effects of Mechanical Vibrations. *Giornale Italiano Di Medicina Del Lavoro Ed Ergonomia* 27,1 (2005) 58-64.
- [30] A. Baumal, J. McPhee, P. Calamai. Application of genetic algorithms to the design optimization of an active vehicle suspension system. *Computer Methods in Applied Mechanics and Engineering*. 163 (1998) 87-94.

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