Extracting information from option prices in the markets

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Abstract

By their nature, options markets are forward-looking. The risk-neutral densities (RND) provide information on market's view regarding the future movements of the underlying index and the perception of the risk. In Chapter 2, we use S&P 500 index option prices and the recently introduced China's 50 Exchange-Traded Fund options to extract densities and find that all methods adopted fit both option data well. However, the non-parametric method outperforms the parametric approaches on the basis of RMSE, MAE, and also the MAPE. We also investigate the dynamic behavior of the densities from smoothing the implied volatility smile in both markets, especially the impacts of higher moments on the price levels and returns of underlying assets. Chapter 3 examines the impact of macroeconomic announcements on S&P 500 option prices and 50 ETF option prices. We aim to distil information with the RND from both options data by employing the stochastic volatility inspired (SVI) method. We investigate the densities and test market efficiency based on the impact of implied moments on current returns. Furthermore, we also distinguish between types of the macroeconomic indicators and examine the reactions of RNDs. In Chapter 4, we apply the Recovery Theorem of Ross (2015) to deduce both the physical distribution and pricing kernel from option prices. The time-homogeneity and irreducibility of the Markov Chain and the path-independence in pricing kernel are two main restrictions. This study aims to test the efficiency of the Recovery Theorem with the application to the options written on Adidas AG. The interpretation of risk aversion and real-world probability distribution is provided. Chapter 5 concludes.

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Chapter 1

Introduction

This thesis consists of three main chapters. Although linked by the common theme of measurement of financial risk using options market data, each chapter has been written as a self-contained research study with focuses on different objectives of financial options research.

Chapter 2 is concerned with the risk-neutral density. Past decades have seen much attention given to implied volatility (see, Jiang and Tian, 2005; Busch, Christensen and Nielsen, 2011; DeMiguel, et al., 2013, for example). But studies by now have not limited in implied volatility, the risk neutral density has also been much attention recently, and this is our focus in Chapter 2. The risk-neutral density, which is a full probability distribution over the value of the underlying asset at option expiration, gives abundant information to academics, practitioners and policymakers. Methods to extract the risk-neutral density have been developed, such as the single lognormal approach by Jackwerth and Rubinstein (1996); the mixture of lognormals model by Bates (1996), Melick and Thomas (1997), Liu et al., (2007); the nonparametric ways by Aït-Sahalia and Lo (1998), Aït-Sahalia and Duarte (2003), Bondarenko (2003), and Bliss and Panigirtzoglou (2004). In this chapter, we use three alternative approaches: 1) Single lognomal; 2) Mixture of two lognormals; and 3) Smoothing implied volatility smile method. In the context of the pricing accuracy, we conclude that the Smoothing implied volatility smile method outperforms the other two alternative ways when adopting the two sets of data, the S&P 500 options over the period Jan 2008 - Mar 2008 and the 50 ETF options during June, 2015 - early Dec, 2015.

Chapter 3 investigates the relationship between the option market and the macroeconomy. By their nature, option markets are forward looking. The first four moments of the risk-neutral densities are the most common information to evaluate the expectation of the investors how the market will be. Theoretically, the option market should have anticipated the macroeconomic news. To address this issue, in Chapter 3, we use the S&P 500 options and China's 50 ETF options to test the impact of these movements on returns of the underlying index. We also test the weak form and semi-strong form Efficient Market Hypothesis (EMH). It is worth noting that we test the effect of lagged return, which amounts to a test of weak-form EMH, and the effect of the lagged moments that amounts to a test of semi-strong form EMH. In this study, we also investigate how the anticipated and unanticipated news has been reflected in the options market.

We begin Chapter 4 with the introducing of the recovery theorem by Ross (2015) and the relationship among the risk-neutral density, the real-world density and the risk aversion. Risk neutrality has been widely assumed in numerous studies, as well as previously in Chapters 2 and 3. However, empirical and experimental studies have found that investors are risk averse (see Thaler et al., 1997; Schubert et al., 1999; Isaac and James, 2000; Goeree, Holt and Palfrey, 2003; Lewellen, 2006; Bollerslev, Gibson and Zhou, 2011; Wilcox, 2011; Bekaert, Hoerova, and Duca, 2013; Dew-Becker et al., 2017, for example). The option prices are interpreted as reflecting to reflect the investors' risk aversion and real-world probability distribution. Studies by Aït-Sahalia and Lo (2000), Jackwerth (2000), Bliss and Panigirtzoglou (2004) and Chabi-Yo, Garcia and Renault (2007) have argued that the ratios between the risk-neutral density and the physical one result in the markets' aggregate risk aversion[1]. Intuitively, according to the previous studies, we compute the risk aversion only through finding the risk neutral density and the natural probability density. Next investigation follows the Ross (2015), the Chapter 4 intends to find the real-world density and risk aversion directly from the option prices. Two main nonparametric assumptions are given in current methodology: 1) the risk-neutral process is restricted to be a time-homogeneous and irreducible Markov Chain in a finite state space; 2) the pricing kernel is independent of asset path.

[1] Mathematically, $Risk - neutral\ probability = Physical\ probability \times Risk\ aversion$. In a no-arbitrage economy, the risk-neutral expected return is the risk-adjusted physical return, i.e., $E^{\mathbb{Q}}(r_t, \tau) = E^{\mathbb{P}}(r_t, \tau) - Risk\ premium = r_f$.

Chapter 5 concludes the thesis by summarising the chapters. We have also discussed and highlighted several ideas on which further study would be beneficial.

In the thesis, several different software packages are used, including the Stata 12 (StataCorp, 2011), the MATLAB 13a (The MathWorks, Inc., 2007), R version 3.3.2 (R Core Team, 2014), and the RStudio 1.0.136 (RStudio Team, 2015), for different purposes.

Chapter 2

Extracting Implied Risk-Neutral Densities from S&P 500 Index Option Prices and 50 ETF Options Prices

2.1 Introduction

The implied volatility is one of the most important concepts in the financial econometrics. By definition, the implied volatility is the inverse problem of option pricing. The past decades or so have seen much attention focused on it. Theoretically, the implied volatility can be regarded as a good predictor of the future volatility of the underlying asset, even its familiar pattern 'volatility smile' seems not to consistent with the Black-Scholes formula. However, extracting important but unobservable parameters from option prices in the market is not limited to implied volatility.

Nowadays, focuses have turned to risk-neutral density. The price of the options written on a given asset with different strike prices with the same time-to-maturity delivers the risk-neutral density, which has the ability to indicate the market assessment of the probability of the payoff over the series of the strike prices.

Investors revise their expectations in the light of the new information. According to the efficient market hypothesis, the prices reflect all the information in the market. Only the new or unanticipated information can influence the expectation of the market, which are all reflected in the risk neutral densities. By studying the risk neutral density, we can easily obtain the markets' beliefs. For instance, the density shows whether the market places a higher probability on an upward movement of the state of the underlying asset than a downward movement of state. In the meantime, the risk-neutral density has superior performance than implied volatility. Because implied volatility is a measure of the second moment of the distribution of the price of the underlying. The risk-neutral density embodies all the moments. Furthermore, the evolution of the risk neutral densities can reveal information on how the market's beliefs change over time.

In order to extract a well-behaved risk neutral density from a set of option prices, two problems need to be solved. Firstly, the theory calls for the option's strike prices to be continuous. As a matter of fact, take the S&P 500 index options market as an example, the market only trades with a small number of discrete strikes, with at least 5 points apart and up 25 points apart or even more in some parts of the available range of strikes. Secondly, the other problem is that we can only extract the middle portion of the density as a result of solving the first problem with interpolating and smoothing, which does not extend further to the both tails because of the small range of the strike prices.

Current chapter would like to examine how the moments of the risk-neutral densities evolve over time. Focusing on parametric and non-parametric methods, we try to extract the information content of the risk neutral densities extracted from S&P 500 index option prices and 50 ETF options prices, in particular to investigate the response of the risk neutral density to the fluctuation in the S&P 500 index and 50 ETF prices, respectively. Furthermore, the analysis of the moments of the density and what the dynamic behaviour of the densities are will also be examined. With respect to the literature review, this chapter also aims to summarize on the significant papers. To answer these question, this chapter will adopt the smoothing implied volatility smile method by Figlewski (2009) to extract the risk-neutral density and distil the information from the densities for further investigation.

The contribution here is threefold: The first contribution is that numerous papers have done a summary of the existing methodologies to find risk-neutral density, but few have done a review of the application of risk-neutral density. Secondly, this chapter finds definitively which is the best method for extracting the risk-neutral density. In other words, rare study has investigate the performance of Figlewski's smoothing implied volatility smile method by comparing it with other alternative methodologies. Thirdly, this chapter also investigates the relationship between the moments implied by risk-neutral density and the current value of the underlying.

The remainder of this chapter is organized as follows. The following section gives a review of the previous literature both the theoretical literature and the empirical literature related to this topic. Section 2.3 describes the data. Section 2.4 documents the methodologies adopted for obtaining the implied risk neutral densities from option prices. The results will be carried out in the Section 2.5. The conclusion of this chapter is provided in Section 2.6, as well as a further study.

2.2 Literature Review

In this part, I would like to review and present some existing applications by the two main model categories and also address the gap between the existing papers and current study.

The literature on extracting the risk neutral density from the option prices and the application of risk neutral density is broad. To date, several methods for extracting the implied risk-neutral density have been developed. Depending on the degrees of freedom and number of parameters needed to define the model, a general classification can be divided into two main categories: parametric methods and non-parametric methods (Jackwerth, 1999).

The *parametric methods*, which rely on particular assumptions, have been most often used related to the recovery of risk-neutral density. These methods try to select known density functions, and fit these parameters by minimizing the the squared difference between the empirical risk neutral densities and the fitted densities. This approach to estimate the risk neutral density function directly starts with the assumption that the risk neutral distribution of S_T belongs to a parametric family. One of the most classical approaches in this area is the the single lognormal approximation, which is based on the Black-Scholes model. However, this method has been criticised by Gemmill and Saflekos (2000), where the parametric approaches via double lognormal method and single lognormal are both adopted to examine the double-lognormal assumption in option pricing and the performance around crashes and British general elections. Their study finds that Double-lognormal method is much better than the one-lognormal approach at fitting observed option prices. Melick and Thomas (1997) study the risk neutral density from the Crude oil American options prices during Persian Gulf crisis to investigate market's perceptions. They find options market reflected a significant probability of a major disruption in oil prices, and distributions implied by mixture of three lognormals are better that of by using single lognormal method. Bahra (1997) and Cheng (2010) also compare the parametric methodologies using single lognormal or a mixture of lognormals in different applications.

Methods within parametric category are diverse, Bates (1991) studies the jump-diffusion approach and finds that a strong perception of downside risk in the market and the crash was predicted by the S&P 500 futures options over 1985-1987. Coutant, Jondeau and Rockinger (1999) investigate the methods with Hermite expansion and Maximum Entropy. They study show that both method fit the option prices better than that by lognormal distribution.

The *non-parametric methods*, with advantages of no distributional assumptions and no assumed functional relationship, could result in more accurate, flexible and robust models. There are several different non-parametric method have been applied in literature. Rubinstein (1994) study the risk neutral density with implied binomial trees model; Ait-Sahalia and Lo (1998) examine with the kernel estimation methods; Avellaneda (1998) studies with maximum entropy approaches.

A non-parametric approach is proposed by Shimko (1993), where the method is based on smoothing implied volatility smile method with parametric assumption of lognormal distributions on the tails of density. This model makes it possible to analyse probability movements and compare them across markets. Since then, various smoothing methods have been used in this approach. Campa, Chang and Reider (1998) study the cubic spline smoothing methods in the application of Dollar-mark, dollar-yen and key EMS cross-rates options. They find that, based on the moments of the risk-neutral density, the stronger a currency the more expectations are skewed to a further appreciation of the currency. Cooper (1999) also studies the cubic spline smoothing methods. It concludes that the pricing performance of method based on smoothing implied volatility performs better than that from the mixture lognormals.

This work, however, is more closely related to the approach in the study

Figlewski (2009). It uses the non-parametric model with a combination of implied volatility smile method and Generalized Extreme Value (GEV) distribution method to find the risk-neutral density in a 12-year daily S&P 500 index options prices over the period January 4, 1996 to February 20, 2008. It finds that densities are always left skewed and the left tail responds more than the right to the market return. Inspired by this study, Birru and Figlewski (2012) adopt the same method to investigate the behavior of the risk neutral probability density by using the intraday S&P index options data during the fall of 2008. They confirm that a strong pattern has been found in the RND shape responds to changes of stock index.

A summary of the recovery methodologies can also be found in Aparicio and Hodges (1998), Bahra (1997), Bliss and Panigirtzoglou (2002), Banz and Miller (1978), Campa *et al.* (1998), Grith and Kratchemer (2010), Jackwerth (1999, 2004).

Follow on the discussion of parametric methods and non-parametric methods, current study would like to investigate the smoothing implied volatility smile method by Figlewski (2009) for its flexibility. I will also adopt the single lognormal distribution method and a mixture of lognormal method to compare the pricing performances. The most important objective is to see how the risk-neutral moments behave and how they reflected in the stock markets.

2.3 Data

2.3.1 S&P 500 index options

This section starts with the introduction of the data source. It will also discuss the criteria for option selection and clean the data for further study. A descriptive statistics is also reported in this part.

Our input dataset includes daily call and put option data on S&P 500 index, between 02 Jan, 2008 and 19 Mar, 2008, expired on 22 Mar, 2008. Table 2.1 shows the source of each data variable.

Table 2.1: Data Sources

Variables	Sources
Option prices	OptionMetrics
Risk-free interest rate	Federal Reserve Bank of St. Louis
Dividend yield	OptionMetrics
S&P 500 Index Level	S&P Dow Jones Indices LLC

2.3.1.1 Criteria for option selection

In this chapter, we begin with bid and ask quotes rather than the transaction prices for calls and puts with a given expiration date[1].

Moreover, due to their illiquidity, in-the-money and far out-of-the-money for both call and put options will be eliminated in this study. In this case, a minimum bid price is set of 0.50 for this study.

Only at-the-money and out-of-the-money options will be used because there is more trading in these. Furthermore, taking options that are too much out of the money can lead to negative probabilities and outliers when calculating the implied volatility.

Moreover, we also eliminate the data which seems to be unreasonable, including the example of implied volatility around -99.99%[2].

2.3.1.2 Descriptive Statistics

Figure 2.1 presents the evolution of the S&P 500 index. To the naked eye, it seems that the S&P 500 index has a decreasing trend during this sample period.

[1]Transactions, in many times, take place sporadic in option market, and even worse in a single stock options. However, market quotes are continuous on a trading day. Therefore, it is better to use the bid and ask quotes in this study.

[2]As stated in the OptionMetrics (2010), in this case, the implied volatility will be set to -99.99% if any of the following conditions holds: 1. The midpoint of the bid/ask price is below intrinsic value. 2. The vega of the option is below 0.5. 3. The implied volatility calculation fails to converge.

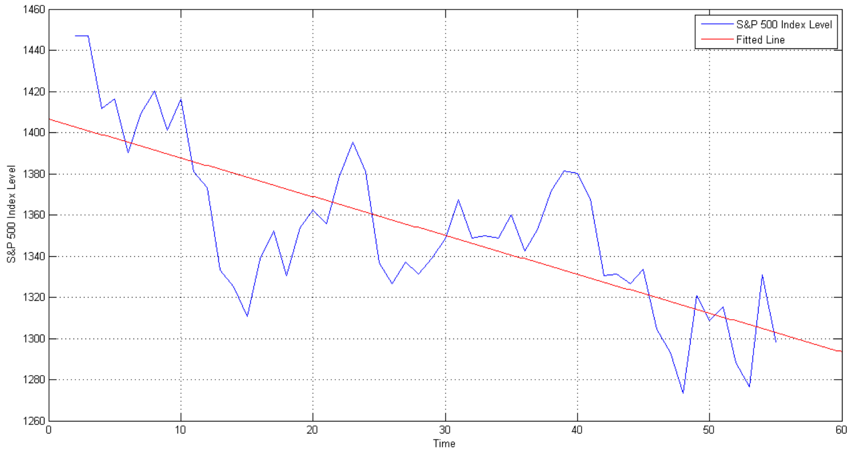


Figure 2.1: S&P 500 index during the sample period

In this sample data, 125 to 205 options were traded on each trading day, with 125 observations on 02 Jan, 2008 and 03 Jan, 2008 and 205 observations on 14 Feb, 2008. Moreover, the data sample includes option prices for 54 trading days and maturities from 3 to 80 days. Their implied volatilities averaged around 28.68%. And about 95% of them are in the range between 13.81% and 45.96%. Table 2.2 shows a summary statistics of the data which has been cleaned by the criteria.

Table 2.2: Descriptive Statistics for US market

Variable	Obs	Mean	Std. Dev.	Min	Max
Call Bid	3453	77.5304	93.7640	0.5	750.7
Call Ask	3453	79.6173	94.0965	0.6	752.7
Put Bid	3738	65.5225	79.6935	0.5	622
Put Ask	3738	67.6166	80.1659	0.55	626
Call IV	3453	0.2612	0.1056	0.1381	1.6582
Put IV	3738	0.3104	0.1590	0.1663	2.2127
S&P 500 Index	7191	1352.802	36.9483	1273.37	1447.16
Risk-Free Rate	7191	0.0332	0.0053	0.0254	0.0468
Dividend Yield	7191	0.0086	0.0043	0.0048	0.0170

2.3.2 50 ETF options

We adopt options data written on China's 50 ETF from 26 June, 2015, updated to early December, 2015. The options are standard European style, which means the option holders have the right to exercise the contracts at the strike price at the maturation. All the data variables, including the Option prices, Risk-free interest rate, Dividend yield and 50 ETF price level are collected from Wind Data-stream. The raw data also includes closing prices, option Greeks across various maturities. The contracts mature in December 2015, January 2016, March 2016 and June 2016.

Table 2.3: Descriptive Statistics for Chinese market

Variable	Obs	Mean	Std. Dev.	Min	Max
Call Price	1,886	0.4105	0.3556	0	1.4423
Put Price	1,886	0.3002	0.2003	0.0002	0.9741
Call IV	1,886	0.1042	0.0492	0	0.1992
Put IV	1,886	0.1681	0.0387	0.0854	0.2765

This dataset is quite different with the S&P 500 options as it does not contain too much information. For example, it only has the transaction data, but not the bid prices and the ask prices. Also, as we can see from the descriptive statistics, the prices for the options are quite low, so I did not use the same criteria for option selection to exclude the prices less than 0.5. I keep most of the raw data unless it has some missing data or noise data.

2.4 Theoretical Framework

2.4.1 Extract the risk neutral density from option prices and estimation with discrete data

Asset pricing theory states that the theoretical option price is equal to the discounted value of the expected payoff under the risk neutral measure. And the density under the risk neutral measure results in the risk neutral density.

$$C = e^{-r\tau} \int_{X}^{\infty} f(S_T)(S_T - X)d(S_T) \tag{2.1}$$

$$P = e^{-r\tau} \int_{0}^{X} f(S_T)(X - S_T)d(S_T) \tag{2.2}$$

In this study, C, S_T, X, r and τ all have the standard meaning of option valuation. C= call price; S_T= the index level at expiration date T; X= exercise price or strike price; r= risk-free interest rate; τ = time to expiration date. And we also use $f(S_T)$= risk neutral density; $F(S_T)$= risk neutral distribution.

I then verify the calculation of the risk neutral density for call option. Here we have:

$$
\begin{aligned}
\frac{\partial C}{\partial X} &= e^{-r\tau}\frac{\partial}{\partial X}\int_X^{\infty} f(S_T)(S_T - X)d(S_T) \\
&= -e^{-r\tau}\int_X^{\infty} f(S_T)d(S_T) \\
&= -e^{-r\tau}\left[F(\infty) - F(X)\right] \\
&= -e^{-r\tau}\left[1 - F(X)\right] \qquad (2.3)
\end{aligned}
$$

In the second line of this differentiation, we use Leibniz's Rule to differentiate with respect to an integration limit. The $F(\infty)$ in the third line is equal to 1 according to the property of probability distribution.

If we take a second differentiation, we have:

$$\frac{\partial^2 C}{\partial X^2} = e^{-r\tau} f(S_T) \qquad (2.4)$$

Therefore,

$$f(S_T) = e^{r\tau}\frac{\partial^2 C}{\partial X^2} \qquad (2.5)$$

The density for call option $f(S_T)$ is approximated as

$$f(S_T) \approx e^{r\tau}\frac{C_{n-1} - 2C_n + C_{n+1}}{(\Delta X)^2} \qquad (2.6)$$

Next followed by the calculation of the risk neutral density for put option. Taking the first and second differentiation respectively:

$$
\begin{aligned}
\frac{\partial P}{\partial X} &= e^{-r\tau}\frac{\partial}{\partial X}\int_0^{X} f(S_T)(X - S_T)d(S_T) \\
&= e^{-r\tau}\int_0^{X} f(S_T)d(S_T) \\
&= e^{-r\tau}F(X) \qquad (2.7)
\end{aligned}
$$

$$\frac{\partial^2 P}{\partial X^2} = e^{-r\tau} f(S_T) \tag{2.8}$$

Therefore,

$$f(S_T) = e^{r\tau} \frac{\partial^2 P}{\partial X^2} \tag{2.9}$$

The density for put option $f(S_T)$ is approximated as

$$f(S_T) \approx e^{r\tau} \frac{P_{n-1} - 2P_n + P_{n+1}}{(\Delta X)^2} \tag{2.10}$$

However, when extracting the risk neutral density from discrete option prices only, i.e. the squares for calls and circles for puts in Figure 2.2, we will have the result in Figure 2.3.

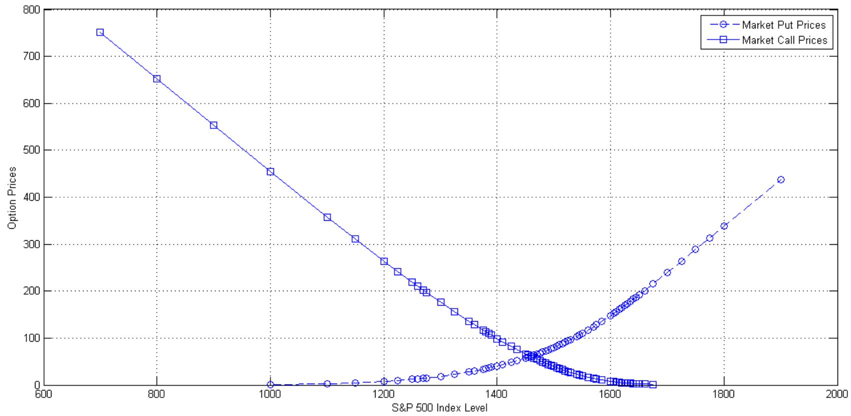


Figure 2.2: Market Option Prices

Note: The curves represented the interpolated option prices for our market calls and market puts in the figure below look highly good, without any bumps or wiggles between the market prices.

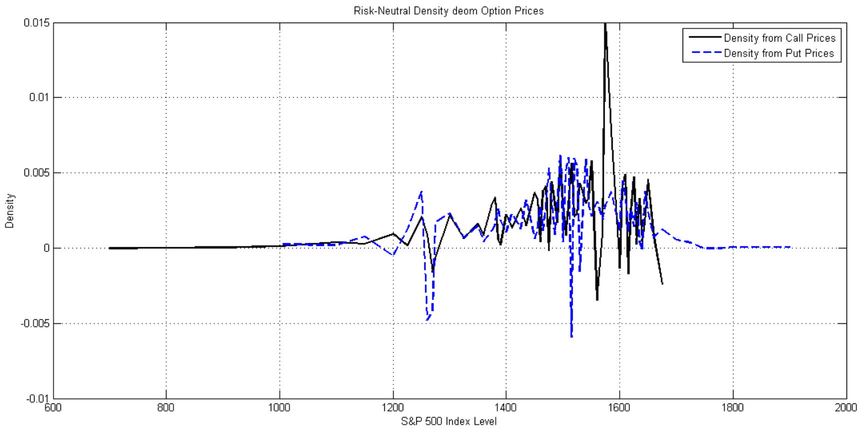


Figure 2.3: Risk Neutral Density from Raw Options Prices

Note: The unacceptable densities in the following graph result from the discreteness of the strike prices.

Firstly, to the naked eye, the curves from Figure 2.2, which shows the option prices to the calls and puts without any jumps among the prices, look nice. It seems that if we take the second derivative of the valuation function, we will get a reasonable risk-neutral density. However, as we can see from Figure 2.3, which plots the risk neutral densities from the raw options prices, seems to be clearly unacceptable as densities for both the calls and puts have some negative values. As proper distributions, risk-neutral densities must be non-negative. Moreover, the extreme fluctuations in the middle portion, as well as the shape differences between these two RNDs, violate our prior beliefs that the risk-neutral density should be smooth.

To solve this problem, in their seminal work, Breeden and Litzenberger (1978), which relates the risk-neutral density to the curvature of the option price function, state that the risk-neutral density is proportional to the second derivative of a European call price with respect to the strike price. They also show how the risk-neutral density could be extracted from the prices of options with a continuum of strikes.

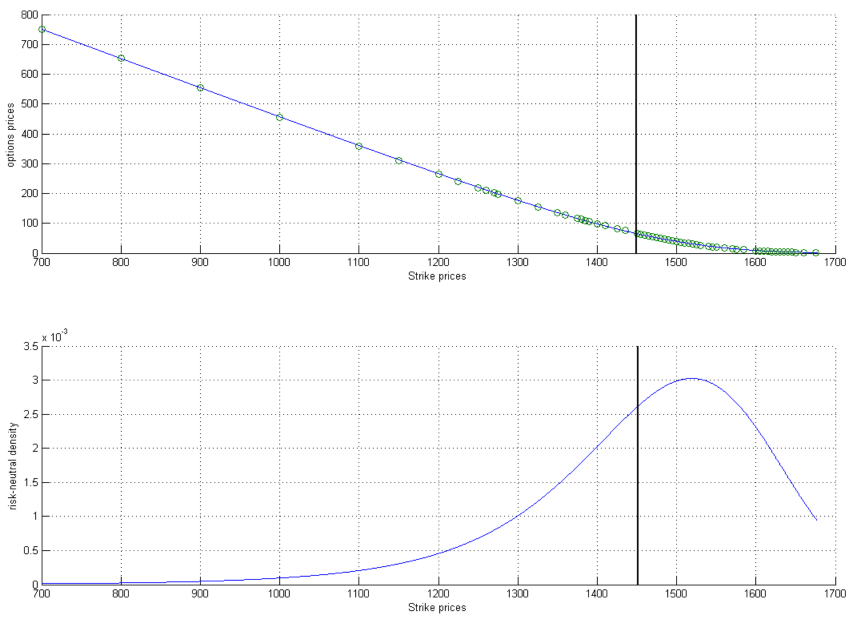
$$f(S_T) = e^{r\tau} \frac{\partial^2 C}{\partial X^2} \tag{2.11}$$

Use the same way, we yield a procedure for obtaining the risk-neutral density from put prices.

$$f(S_T) = e^{r\tau} \frac{\partial^2 P}{\partial X^2} \tag{2.12}$$

The following will show an example when adopting the terminologies from Breeden and Litzenberger (1978).

Figure 2.4: Relationship between the risk-neutral density to the curvature of the option price function.



Figures above describe the result of Breeden and Litzenberger (1978), which implies that the call's curvature in strike is the risk-neutral density. From the upper figure, its slope goes larger along with the strikes and the curvature, which measures the slope-of-the-slope, with the values go down to approximately zero to both sides and peaks at strike of near 1530. This is consistent with the risk-neutral density in the nether graph. We can also find that the mode in the density to the left of the current index level, which is represented as the vertical line.

When investigating a density, we also look at the tails of the density. In order to extract entire densities. Numerous methods have been applied for recovering implied risk-neutral densities from option prices. this section, some of these methods are described. The following subsections will introduce the single lognormal distribution (LN) method and the double lognormal (DLN) distribution method, respectively, and following with the smoothing implied volatility smile (SML) method. We will take the S&P 500 options as the example throughout this section.

2.4.2 Single lognomal

This methodology assumes the density to be single lognormal, which is consistent with that S&P 500 index follows Geometric Brownian Motion (GBM).

$$f(S_T) = \frac{1}{S_T \beta \sqrt{2\pi}} e^{-\frac{(lnS_T - \alpha)^2}{2\beta^2}} \tag{2.13}$$

$L(S_T; \alpha, \beta)$ is the lognormal distribution used in the methodology, and α , β are parameters in the lognormal distribution, $\alpha = lnS_t + (\mu - \sigma^2/2)\tau$ and $\beta = \sigma\sqrt{\tau}$. μ and σ represent the return expectation and the standard deviation, respectively.

The expected value of S&P 500 index level at its expiry is equal to S_t exp$(r\tau)$, which should have the same value of the mean of the lognormal distribution, $e^{\alpha+\frac{1}{2}\beta^2}$.

If we rewrite the Black-Scholes formula, we have:

$$C = e^{-r\tau}\left[e^{\alpha+\frac{1}{2}\beta^2}N(d_1) - XN(d_2)\right] \tag{2.14}$$

$$P = e^{-r\tau}\left[XN(-d_2) - e^{\alpha+\frac{1}{2}\beta^2}N(-d_1)\right] \tag{2.15}$$

Where:

$$d_1 = \frac{-ln(X) + \alpha + \beta^2}{\beta}$$

$$d_2 = d_1 - \beta$$

Here we have $N(\cdot)$ is the cumulative normal distribution function. This method estimates α and β by minimizing the sum of the squared deviation between the observed option prices (C^i_{mkt} and P^i_{mkt} stand for observed call prices and observed put prices, respectively) and the theoretical option prices from equation (2.14) and (2.15) for both calls and puts. In addition, we have one more condition in the minimization problem that, by exploiting the fact, in the absence of arbitrage the mean of risk-neutral function should be equal to $e^{r\tau}S_t$[3]. This is because European options on S&P 500 index futures and S&P 500 index with a particular strike and τ should be the same, and the underlyings are equal at expiry.

$$\min_{\alpha,\beta}\left\{\sum_{i=1}^{n}\left[C^i - C^i_{mkt}\right]^2 + \sum_{i=1}^{n}\left[P^i - P^i_{mkt}\right]^2 + \left[e^{\alpha+\frac{1}{2}\beta^2} - e^{r\tau}S_t\right]^2\right\} \quad (2.16)$$

With a constraint of $\beta > 0$, this minimization aims to find α and β over the observed option prices across a series of strike prices. Note that in order to extend the tails, in this method, we assume the implied volatilities are constant, which means the values of the implied volatility beyond the available strikes are equal to the implied volatility of smallest strike and largest strike in the data. Therefore, the tails assume to follow the lognormal distribution.

2.4.3 Mixture of two lognormals

An alternative way is to conduct a flexible parametric form, mixture of lognormals, on the densities and determine the parameters by maximizing the fit of the option prices from the risk neutral density to the market prices, that is, the function tries to minimize the squared differences between the prices computed by the risk neutral density and that of traded in the market.

According to asset pricing theory, shown as (2.1) and (2.2), we assume the risk-neutral density $f(S_T)$ follows a mixture of k lognormals,

$$f(S_T) = \sum_{j=1}^{k}\theta_j L(\alpha_j, \beta_j) \quad (2.17)$$

Where $L(\alpha_j, \beta_j)$ is the lognormal distribution j in the total number of k lognormals used in the methodology, θ_j is the weight of the i^{th} lognormal distribution. Moreover, the weight θ_i should satisfy the condition, $\sum_{j=1}^{k}\theta_j = 1$, $\theta_j > 0$ for every j. It is worth noting that lognormal distribution 'j' means the j^{th} lognormal distribution in (2.16), we only have one lognormal distribution. Therefore no 'j' will appear in this equation.

This chapter, particularly, uses a double lognormal distribution method, k=2.

[3]In MATLAB R2013a version, this problem can be solved by the derivative-free minimization routine *fminsearch*, a procedure for tackling complex constrained non-linear minimization problems.

This method assumes the risk neutral density follow a functional form of mixture of two lognormals, and then estimate the parameters by minimizing the squared difference between the actual call and put option prices and the option prices fitted by the assuming distribution. As suggested by Melick and Thomas (1997), the double lognormal method makes this model more flexible than the single lognormal one as it increases its ability to capture the accurate pdf and higher goodness-of-fit is generated. And also, comparing with the single lognormal method, this alternative method has higher ability to extract the contributions to the volatility smile, namely the skewness and kurtosis.

$$f(S_T) = \theta L(\alpha_1, \beta_1) + (1-\theta) L(\alpha_2, \beta_2) \tag{2.18}$$

That is,

$$C = e^{-r\tau} \int_X^{\infty} [\theta L(\alpha_1, \beta_1) + (1-\theta) L(\alpha_2, \beta_2)](S_T - X) d(S_T) \tag{2.19}$$

$$P = e^{-r\tau} \int_0^{X} [\theta L(\alpha_1, \beta_1) + (1-\theta) L(\alpha_2, \beta_2)](X - S_T) d(S_T) \tag{2.20}$$

It also can be expressed as the weighted Black-Scholes formula,

$$C = e^{-r\tau} \{ \theta \left[e^{\alpha_1 + \frac{1}{2}{\beta_1}^2} N(d_1) - XN(d_2) \right] + (1-\theta) \left[e^{\alpha_2 + \frac{1}{2}{\beta_2}^2} N(d_3) - XN(d_4) \right] \} \tag{2.21}$$

$$P = e^{-r\tau} \{ \theta \left[XN(-d_2) - e^{\alpha_1 + \frac{1}{2}{\beta_1}^2} N(-d_1) \right] + (1-\theta) \left[XN(-d_4) - e^{\alpha_2 + \frac{1}{2}{\beta_2}^2} N(-d_3) \right] \} \tag{2.22}$$

The parameters d_1 and d_2 are given by:

$$d_1 = \frac{-ln(X) + \alpha_1 + {\beta_1}^2}{\beta_1} \qquad d_3 = \frac{-ln(X) + \alpha_2 + {\beta_2}^2}{\beta_2}$$

$$d_2 = d_1 - \beta_1 \qquad d_4 = d_3 - \beta_2$$

Furthermore, with the assumption of arbitrage-free, with a special case of zero strike price, we compute the present value of the future price.

$$C(0,\tau) = e^{-r\tau}\{\theta e^{\alpha_1+\frac{1}{2}\beta_1{}^2} + (1-\theta)e^{\alpha_2+\frac{1}{2}\beta_2{}^2}\} = e^{-r\tau}f(\tau) \tag{2.23}$$

Where the $C(X,\tau)$ represents the call price with strike X and the time to maturity τ, and $f(\tau)$ is the forward price of the underlying in τ time. In the absence of arbitrage, the forward price should be equal to first moment, the mean, of the risk neutral density.

This method tries to capture the parameters, $\{\alpha_1, \beta_1, \alpha_2, \beta_2, \theta\}$, by minimizing the sum of the squared differences between the fitted option prices and the market prices for both calls and puts. In the absence of arbitrage, we have the mean of the risk-neutral function should be equal to $f(\tau)$.[4]

$$\min_{\alpha_1,\alpha_2,\beta_1,\beta_2,\theta}\{\sum_{i=1}^{n}\left[C^i - C^i_{mkt}\right]^2 + \sum_{i=1}^{n}\left[P^i - P^i_{mkt}\right]^2 + \left[\theta e^{\alpha_1+\frac{1}{2}\beta_1{}^2} + (1-\theta)e^{\alpha_2+\frac{1}{2}\beta_2{}^2} - f(\tau)\right]^2\} \tag{2.24}$$

With the restriction $\beta_1 > 0$, $\beta_2 > 0$ and $0 \leq \theta \leq 1$, for all the observations with strikes $X = X_1, X_2, X_3, \ldots, X_n$. In this method, we also assume both tails follow the lognormal distributions.

2.4.4 Smoothing implied volatility smile method

The smoothing implied volatility smile method is originally developed by Shimko (1993). This method smoothes the implied volatility space computed by the Black-Scholes formula[5]. The continuum of fitted implied volatilities are then converted back to a continuous of option prices. The implied risk-neutral densities, finally, can be obtained by applying equations (2.11) and (2.12).

This method can solve the two problems came from the nature of the option prices. On one hand, the smoothness of the density as we have discussed in the first part in the Section 2.4. On the other hand, the theory calls for options with

[4]The same as in the single lognormal approach, this problem can be also solved by MATLAB's derivative-free minimization routine *fminsearch* which works well.

[5]This method tries smooth the implied volatility/strikes space from the Black-Scholes formula instead of option price/strike leads to a better interpolation result, due to the more linearity and smoother than the option price.

a continuum of strike prices from 0 to infinity. However, that is clearly not the case in the market. Therefore, interpolation and smoothing are needed to solve the problems. The following states the procedures in detail for extracting the risk-neutral density on 02 Jan, 2008 from the available S&P 500 index option prices.

This method begins with bid and ask quotes, which have been cleaned at the beginning, for calls and puts with a given expiration date.

Secondly, we compute the implied volatilities from the call prices and put prices by using equations (2.25) and (2.26), respectively.

$$C = S_t N(d_1) - Xe^{-r\tau} N(d_2) \tag{2.25}$$

$$P = Xe^{-r\tau} N(-d_2) - S_t N(-d_1) \tag{2.26}$$

The parameters d_1 and d_2 is given by:

$$d_1 = \frac{\ln(S_t/X) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}},$$

$$\begin{aligned} d_2 &= \frac{\ln(S_t/X) + (r - \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \\ &= d_1 - \sigma\sqrt{\tau}. \end{aligned}$$

where $N(\cdot)$ is the cumulative normal distribution function, τ is time to expiration date, $T - t$. X, r, τ, σ have the standard meaning of option as indicated in the equation (2.1) and (2.2), and S_t is the current S&P 500 index level.

Previous studies adopt various smoothing functions, including cubic splines (Bates, 1991; Bu and Hadri, 2007; Monteiro, Tütüncü and Vicente, 2008), natural spline (Bliss and Panigirtzoglou, 2002, 2004; Liu, Shackleton, Taylor and Xu, 2007, 2009), quadratic polynomial (Shimko, 1993; Jackwerth and Rubinstein, 1996). Using the same smoothing methodology as Figlewski (2009), we then interpolate the implied volatility curve by using a 4th degree polynomial smoothing[6]. Before this, we blend the implied volatility for calls and puts with strike between (S_t - 50) to (S_t + 50) by the following equation,

[6]Using MATLAB, we interpolate the blended implied volatility by a 4th degree polynomial smoothing.

$$\sigma_{blend}(X) = w\sigma_{put}(X) + (1 - w)\sigma_{call}(X) \tag{2.27}$$

Where,

$$w = \frac{X_{high} - X}{X_{high} - X_{low}},$$

In this case, as we can see from the Figure 2.5, we use the blended IVs with the strikes between 1400 and 1495, the put IVs with strikes up to 1400 and the call IVs with strikes start from 1500. And we apply the 4th degree polynomial smoothing to fit the IVs, which is also showed in the figure below:

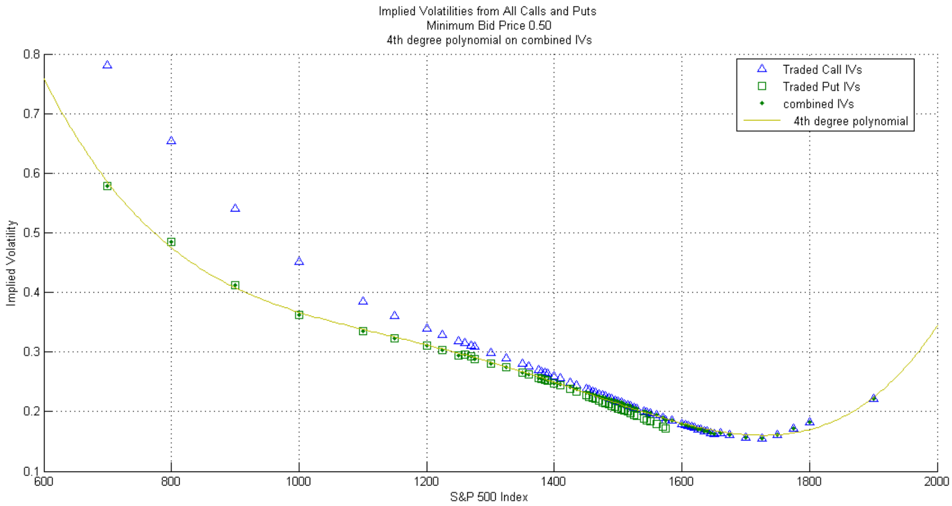


Figure 2.5: Fit the volatility smile on 02 Jan, 2008

Fourthly, we convert the interpolated implied volatility curve back to option prices space and extract the middle portion of the risk-neutral density, which shows in Figure 2.6.

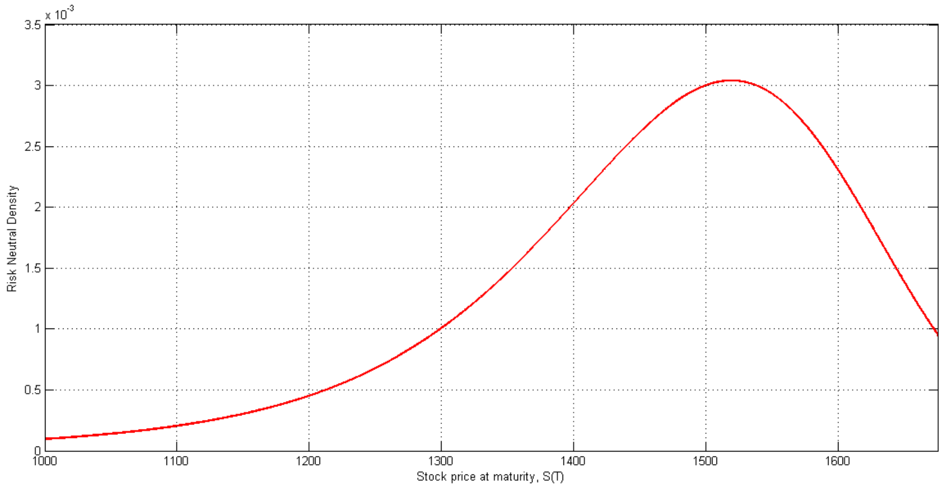


Figure 2.6: Empirical Risk-Neutral Density on 02 Jan, 2008

Finally, this method tries to append the two tails to the risk-neutral density beyond the range from X_2 to X_{n-1} via Generalized Extreme Value (GEV) distribution. The standard GEV distribution has one parameter ξ [7], which determines the shape of the both tails:

$$F(z) = exp[-(1+\xi z)^{-\frac{1}{\xi}}] \tag{2.28}$$

where

$$z = \frac{S_T - \mu_{GEV}}{\sigma_{GEV}}$$

Where, the μ_{GEV} and σ_{GEV} determine the location and the scale of the distribution, respectively. In order to extend the tails for risk-neutral density by fitting with Generalized Extreme Value Distribution. In this case, we need to follow the below conditions: First of all, the total probability should the same between the tails in the empirical RND and GEV functions. And, we also want both tails in the GEV density to have the same shapes as those in the empirical RND, where each of the two GEV densities should go through the two points in the empirical RND, respectively. The details of how to fit the Generalized Extreme Value distribution are showed in Appendix II.

[7] In the Generalized Extreme Value distribution, ξ determine which distribution the tails follow:

$$\xi = \begin{cases} < 0 & \text{Weibull distribution} \quad \text{finite tails, which do not extend to infinity} \\ =0 & \text{Gumbel distribution} \quad \text{tails similar to the normal} \\ > 0 & \text{Frchet distribution} \quad \text{fat tails relative to the normal} \end{cases}$$

Here we implement our methods to compute an example of the risk neutral density on 02 Jan, 2008. These S&P 500 index options data are traded in Chicago Board Option Exchange (CBOE) on 02 Jan, 2008, where the S&P 500 index level is 1447.16. These options have 80 days to maturity, the risk-free rate is 4.68% and the dividend yield is 1.71%.

Figure 2.7 shows the risk-neutral densities extract from various methods, including the single lognormal, a mixture of two lognormals and smoothing implied volatility smile method.

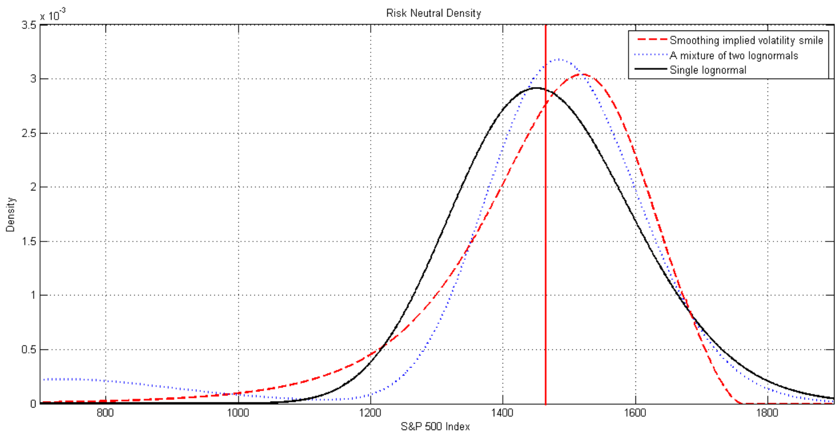


Figure 2.7: Risk-Neutral Densities extracted from various methods

As we can also see from the figure, these methods result in different densities. The density extracted by smoothing implied volatility smile seems to be quite different from the lognormal densities assumed in the Black-Scholes framework. The density is skewed to the left relative to the lognormal family. The next section, we would like to show the results using S&P 500 options and 50 ETF options.

2.5 Results

2.5.1 Implied volatility surface

The variation of the implied volatilities, which are backed out from Black-Scholes model, of European options across strike prices or moneyness and time to maturity result in the implied volatility surface (see Kamal and Gatheral, 2010).

In other words, the implied volatility surface changes over the time and also the strike prices.

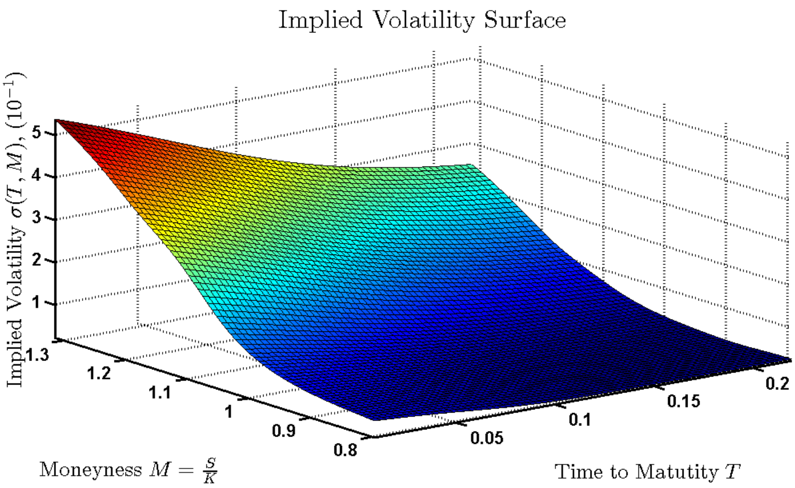


Figure 2.8: Graph of the S&P 500 Index Options implied volatility surface expiry on 22 Mar, 2008.

The implied volatility surface for S&P 500 Index Options shows in the Figure 2.8. As we can see in this figure, the height of the surface measures the value of implied volatility for each maturity and moneyness combination.

For a given expiration date, the implied volatilities decrease as the strike prices goes down, that is, the S&P 500 index options exhibit a well-known “skew”, which is also confirmed by Rubinstein (1994) and Jackwerth and Rubinstein (1996).

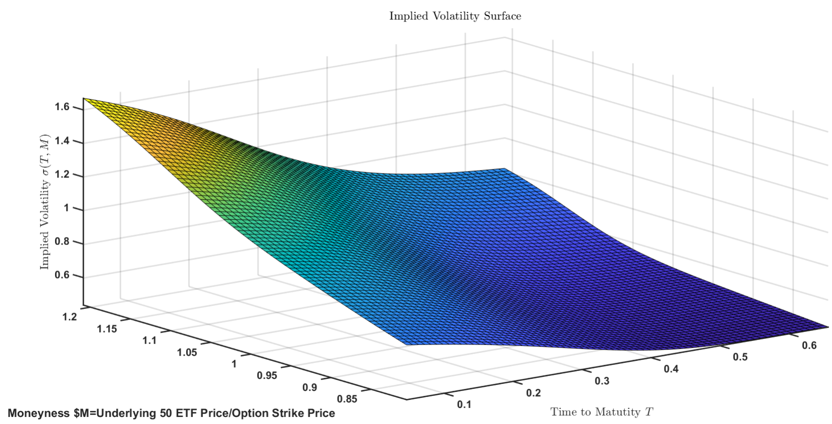


Figure 2.9: Graph of the 50 ETF Options implied volatility surface expiry on 23 Dec, 2015.

We can see the similar patterns between the S&P 500 Index Options implied volatility surface and the 50 ETF Options implied volatility surface. But the later one exhibit higher implied volatility when options are nearly expired and with higher moneyness.

2.5.2 Testing Risk Neutral Densities

In order to test the risk-neutral density, this section would like to test the goodness-of-fit among these methods.

The existence of various risk-neutral density estimation methods from option prices come out of a question of the best method to be chosen in application. In order to answer this question, we focus on the goodness-of-fit measures, which compares the market prices and the fitted option prices using the Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and the Mean Absolute Percentage Error (MAPE). The method presenting the smallest pricing errors will be considered as the best model to fit the option prices. A comparison of goodness of fit among these three methods over the sample period is presented.

Table 2.4: Summary statistics of the goodness-of-fit measures for three models for S&P 500 Options

	Single lognormal	a mixture of two lognormals	smoothing implied volatility smile
RMSE	3.4862	1.9950	0.3621
MAE	3.3140	1.5058	0.3318
MAPE	0.4054	0.1188	0.0438

Table 2.5: Summary statistics of the goodness-of-fit measures for three models for 50 ETF Options

	Single lognormal	a mixture of two lognormals	smoothing implied volatility smile
RMSE	0.0782	0.0363	0.0188
MAE	0.0616	0.0214	0.0096
MAPE	0.6908	0.1114	0.0530

Table 2.4 and 2.8 show the summary statistics of the RMSE, MAE, and MAPE for these approaches for the US and Chinese markets. In both markets,

the single lognormal method with the largest accuracy measures, means it fits the options market prices worst. Though compared with single lognormal method, the mixture of two lognormal does better. The smoothing implied volatility smile does best, only has some small values. These are consistent with the both markets.

Therefore, this part concludes that smoothing implied volatility smile performs best, at least for the S&P 500 index options and 50 ETF options during this sample period. The following section would like to apply the smoothing implied volatility smile in order to examine the information content and try to investigate the evolution of the moments.

2.5.3 The Dynamic Behavior of the S&P 500 Risk Neutral Density

The movements in the entire density could provide interesting information to daily changes in the underlying expectations. It is important to gauge changes of the moments. In this subsection, we extract the risk-neutral moments from S&P 500 index option prices from 02 Jan, 2008 to 19 Mar, 2008, expired on 22 Mar, 2008. Theoretically, the moments from the risk-neutral densities changes as the time-to-expiry changes because of a fixed expiration date for all the options in this case. The first two moments are the mean and the variance of the density, respectively. Here we define the mean of a risk-neutral density is the expected value of the index, or the forward index. Followed by the variance, which is the most common measure of dispersion of a distribution about its mean, and by definition always positive. Skewness, a measure of asymmetry, is the third moment of the density. And the fourth moment is the kurtosis of the density. A distribution with negative excess kurtosis is called platykurtic, and a distribution with positive excess kurtosis is called leptokurtic. The higher moments will not be included in this study. Details about the calculation of moments are shown in Appendix III. And we can also find the summary statistics of the moments as well.

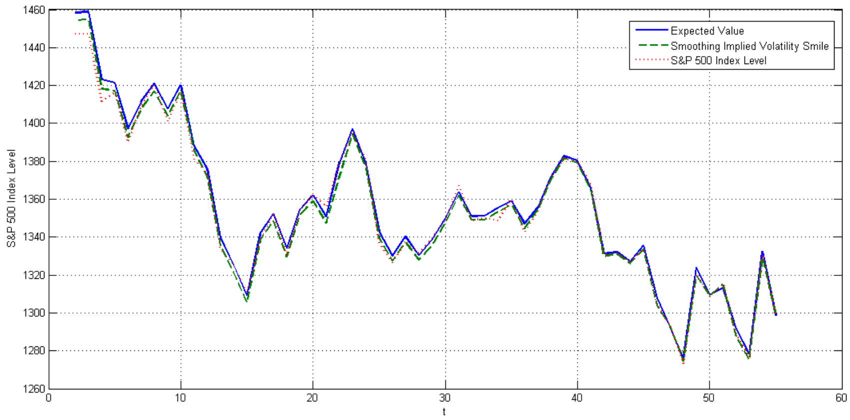


Figure 2.10: S&P 500 futures index, the means of risk-neutral densities and underlying index level over time

The Figure 2.10 shows the time varying S&P 500 expected value/futures index, the means of risk-neutral densities and underlying index level over the period from 02 Jan, 2008 to 19 Mar, 2008. It also attempts to shed some light on the accordant fluctuations among the S&P 500 index level, the S&P 500 futures prices with delivery in March 2008 and the means computed smoothing implied volatility smile. We can find that the means are moving along with the S&P 500 index. It is consistent with the theory we presented earlier in this chapter. As the time-to-maturity is small, the Figure 2.10 also indicates that the S&P 500 index is close to the futures price.

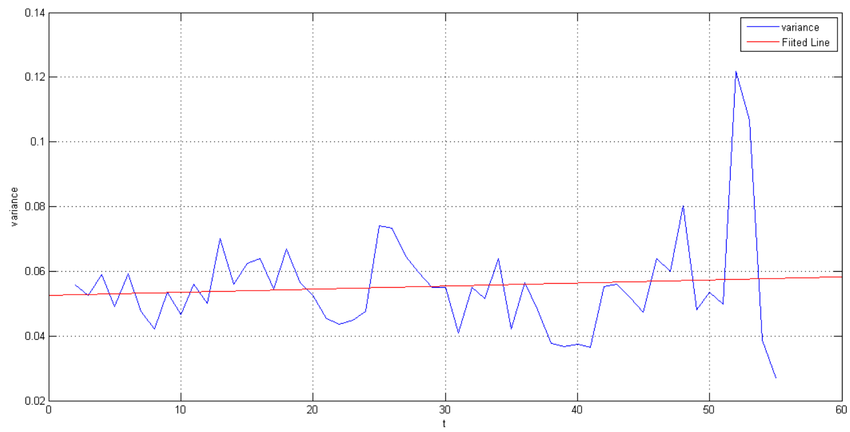


Figure 2.11: Variance of Risk-Neutral Densities

The variance of the risk neutral density represents the market's uncertainty around the expected value of S_T, given today's index level S_t and other information. The change in the risk neutral variance measures the resolution of uncertainty. For example, when the new interest target is announced today, compared with the variance of risk-neutral density yesterday, the change in the variance can judge how much information is provided by this interest rate target announcement. The variance in the figure above seems to be stationary[8] during this period. Although the slope of the fitted line is near zero (i.e. 9.5037e-05), we can see from the figure, the fluctuation of the variance become larger along the time from the minimum of 0.027 to its maximum of 0.122, with a lowest variance by the end of option lifetime. This can be also confirmed by Figlewski (2012), the risk neutral density evolves towards its maturity, the variance will collapse to zero. Moreover, a spike occurs when the variance collapses. This could be the reason that investors tend to trade more on these options when the expiration date comes, i.e., it worth noting that a large part of do not trade the options to hedge therefore they want to exercise the options before the expiration date. Hentschel (2003) also states that the options near expiration provide more extremely noisy volatility than those with mid-term and long-term options.

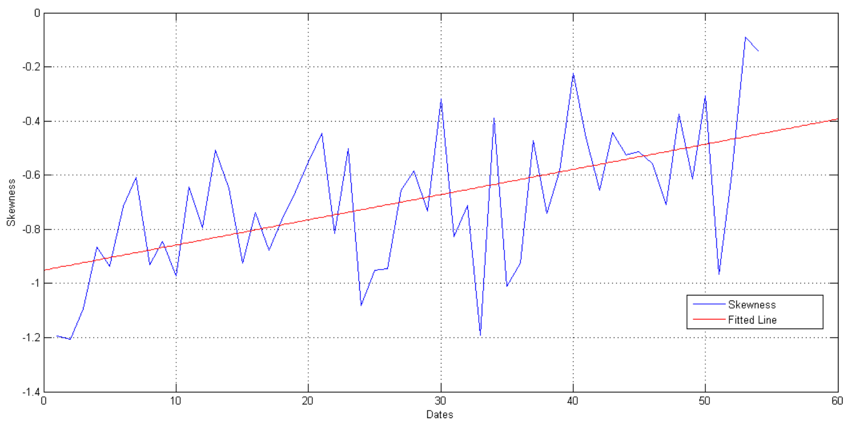


Figure 2.12: Skewness of Risk-Neutral Densities

The skewness are always negative, which means that the risk-neutral densities are asymmetric and the tails are skewed left. This suggests that a higher chance

[8] Conducting a unit root test, we find that the test statistic (-5.272) is less than the 1% Critical value (-3.634), and with the p-value of 0.0001 is much smaller than 0.01. We reject the hypothesis of a unit root. We find evidence that the variance is stationary.

of large price decrease in the S&P 500 index have been predicted in the market and reflected in the traded option prices. Therefore, the risk-neutral density gives a negative skewness with a pronounced left tail. This result can be supported by Bliss and Panigirzoglou (2002), who found that the mean of skewness extracted from the FTSE 100 options' risk-neutral densities is equal to -0.54. This is also consistent with the evidence from Corrado and Su (1997) and Figlewski (2009). They all state that the implied volatility skewness in S&P 500 index options is often negative, which seems to be a contrary result from the skewness of lognormal distribution. Moreover, it has been stated by Melick and Thomas (1997), in the case of oil prices, the densities were skewed to right, and suggested a relatively high probability for a large increase in the future oil price.

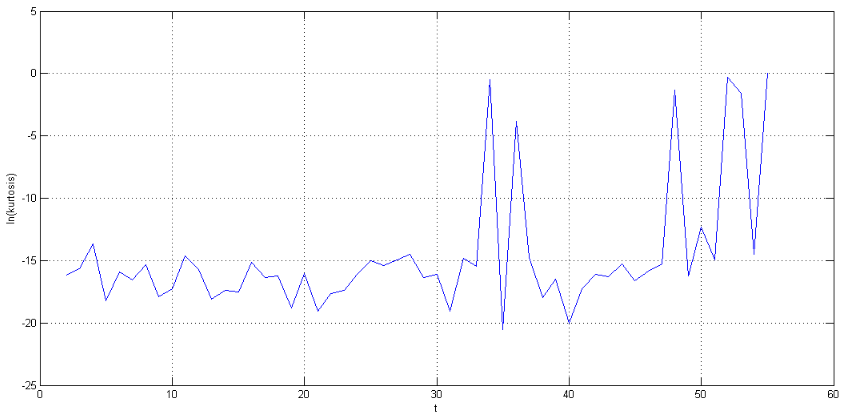


Figure 2.13: Kurtosis of Risk-Neutral Densities

The kurtosis, which measures the fatness of the tails of a probability distribution, ranges from 1.23e-09 and 1.020718 are all larger than that of Gaussian distribution. As it is in the Figure 2.13, most of the algorithms of the kurtosis are negative, with the last one is approximately equal to 0. This indicates that the RNDs are fat-tailed compared to the normal distribution, but not significant.

Table 2.6 shows the result of relationship between S&P 500 index and higher risk-neutral moments. Although this regression does not fit the S&P 500 index well, both the skewness and the kurtosis are significant. The coefficients for both higher moments are negative. This means with one unit increase in skewness and kurtosis, the S&P 500 index will decrease by 54.33 and 72.78, respectively. Hence, the higher moments are dynamically linked to the underlying.

Table 2.6: S&P 500 index and higher risk-neutral moments

Dependent Variable	S&P 500 index
skewness	-54.334 ***
	(-2.93)
kurtosis	-72.783 ***
	(-2.85)
constant	1318.92 ***
	(95.25)
obs	54
R-squared	0.2523

Table 2.6 reports the estimation results of regressions of the S&P 500 index on the corresponding higher moments. The estimated model is

$$S_t = \beta_0 + \beta_1 * skewness_t + \beta_2 * kurtosis_t + \varepsilon_t$$

Here we have the significant levels: *p <= 0.1,**p <= 0.05,***p <= 0.01.

Table 2.7: S&P 500 index returns and higher risk-neutral moments

Dependent Variable	S&P 500 index returns
skewness	0.0188 **
	(2.37)
kurtosis	-0.022 **
	(-2.10)
constant	0.0124 **
	(2.06)
obs	53
Adj R-squared	0.1643

Table 2.7 reports the estimation results of regressions of the S&P 500 index returns on the corresponding higher moments. The estimated model is

$$r_t = \beta_0 + \beta_1 * skewness_t + \beta_2 * kurtosis_t + \varepsilon_t$$

Here we have the significant levels: *p <= 0.1,**p <= 0.05,***p <= 0.01.

2.5.4 The Dynamic Behavior of the 50 ETF Risk Neutral Density

In this subsection, we extract the risk-neutral moments from 50 ETF option prices from 26 June to 06 December, 2015.

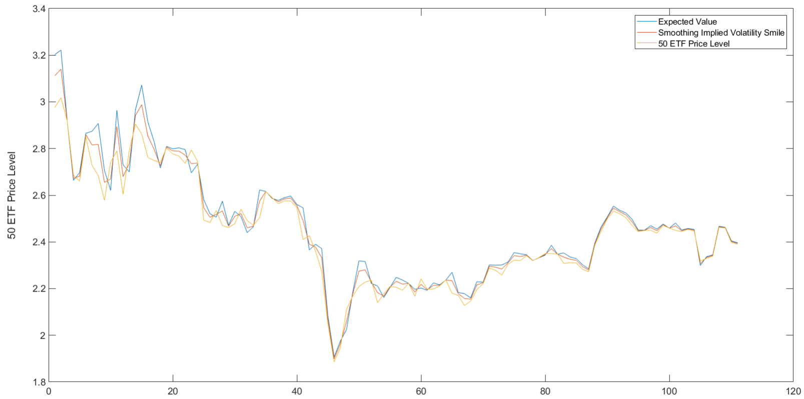


Figure 2.14: 50 ETF futures price, the means of risk-neutral densities and underlying price level over time

The Figure 2.14 shows the time varying 50 ETF expected value/futures index, the means of risk-neutral densities and underlying index level. We can also find that, as the time-to-maturity is small, the Figure 2.14 also indicates that the 50 ETF is close to the futures price. However, the different point with the case for S&P 500 options is that the series always move together over the time but in the case of 50 ETF options, more volatile among the series can be found at the beginning.

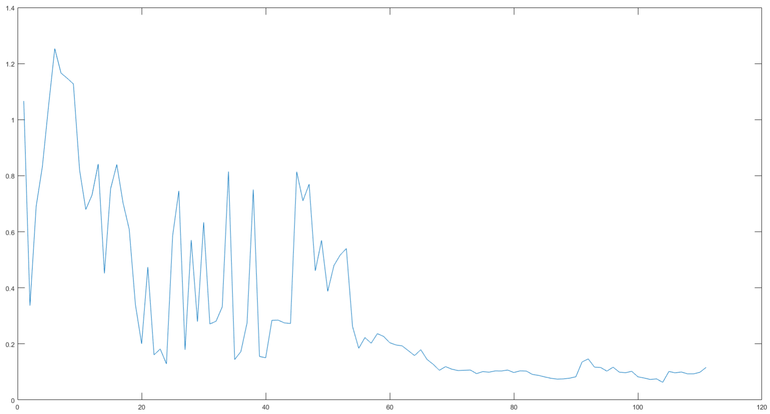


Figure 2.15: Variance of Risk-Neutral Densities

When looking at the time series of variance, it seems that the variance has experienced a decrease trend. Theoretically, the change in the risk neutral variance measures the resolution of uncertainty. As expiration approaches, the variance decreases due to the information has been released in the market. This is not consistent with the evidence from S&P 500 options, it could be the reason that the two datasets have different sample period.

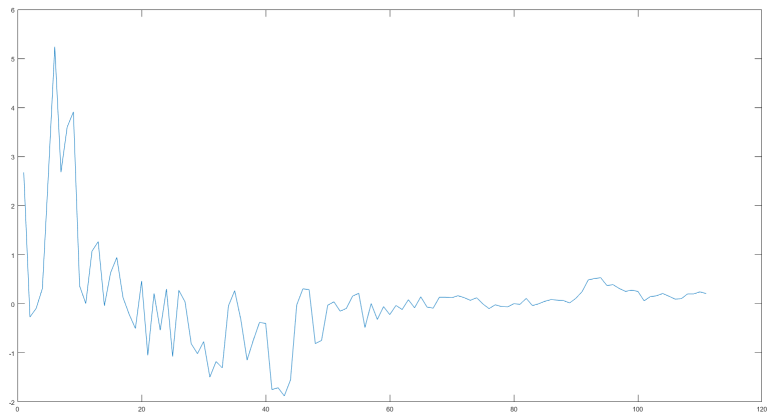


Figure 2.16: Skewness of Risk-Neutral Densities

The skewnesses seem to be more volatile in the first half time span than those in the second half. More specifically, the skewness behaved a huge positive in the second sample. The tendency to its negative value presents that the densities skewed to left, which indicated a decrease expectation in the market.

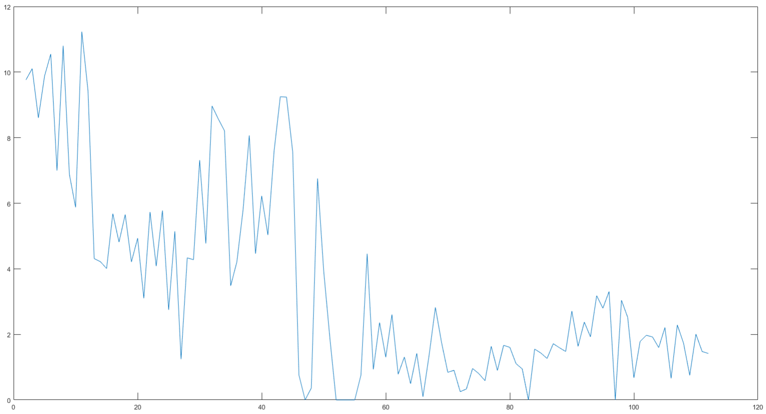


Figure 2.17: Kurtosis of Risk-Neutral Densities

In terms of the kurtosis, we can see that in the first half period, the risk neutral density exhibits the heavy tail. But in the second half, the values are below 3, which do not show the leptokurtic distributions characteristics.

Table 2.8: 50 ETF price and higher risk-neutral moments

Dependent Variable	50 ETF price
skewness	0.029 ***
	(.0079)
kurtosis	0.047 ***
	(.0052)
constant	2.268 ***
	(.0240)
obs	111
R-squared	0.471

Table 2.8 reports the estimation results of regressions of the 50 ETF price on the corresponding higher moments. The estimated model is

$$S_t = \beta_0 + \beta_1 * skewness_t + \beta_2 * kurtosis_t + \varepsilon_t$$

Here we have the significant levels: *p <= 0.1,**p <= 0.05,***p <= 0.01.

Table 2.8 shows the result of relationship between 50 ETF prices and higher risk-neutral moments. it is quite different with the one from US market. The skewness and the kurtosis are positively affect the 50 ETF price. This might due to the different sample periods. This means the different effects of the skewness and kurtosis might come from the different economic condition. Both effects are negative when it is in the crisis, but positive when it is in the normal time.

Table 2.9: 50 ETF price returns and higher risk-neutral moments

Dependent Variable	50 ETF price returns
skewness	0.0007 **
	(0.0015)
kurtosis	-0.0005 **
	(0.0009)
constant	-0.0003 **
	(0.0044)
obs	53
Adj R-squared	0.1643

Table 2.9 reports the estimation results of regressions of the 50 ETF price returns on the corresponding higher moments. The estimated model is

$$r_t = \beta_0 + \beta_1 * skewness_t + \beta_2 * kurtosis_t + \varepsilon_t$$

Here we have the significant levels: *p <= 0.1,**p <= 0.05,***p <= 0.01.

Comparing the results in S&P 500 index options market, we find that the higher moments in respective markets have the same effect to the US and Chinese underlying asset returns.

2.6 Conclusion and Directions for Further Study

In this chapter, we used three comprehensive methodologies for extracting the risk-neutral densities over the S&P 500 index options and 50 ETF options prices. Using the smoothing implied volatility smile method, we have solved two significant technique problems through smoothing and interpolation.

Moreover, we have presented the results showing a comparison among the risk-neutral densities for S&P 500 index and 50 ETF options extracted from different methodologies and find that densities by smoothing implied volatility smile is far different from that by the benchmark methodology, a mixture of two lognormals. Due to the flexibility of the non-parametric method, not surprisingly, the smoothing implied volatility smile method fits the data better than the single lognormal method and mixture of two lognormals method.

Then the moments are examined. In the case of S&P 500 index options, with respect to to the mean of the densities, it seems that the models yields the right expected value owing to the reasonably trend with the futures prices. The variances are stationary with a mean approximately equal to 0.052 with a smallest value of 0.0271 by the maturity, which is consistent as stated in Figlewski (2012).

Especially to the skewness, the densities are negatively skewed, which can be explained by a greater fear in the decreasing of S&P 500 index relative to increasing in the market. And the excess kurtosis indicates that the densities are always fat-tailed, which also means greater tail risk.

I compare the results from 50 ETF options with S&P 500 index options. It seems that they have quite different patterns in the behavior of the moments. when looking into the higher moments on the indices and the returns, they still have opposite impact. One of the most important factors might due to the economic environment. The time period for S&P 500 index options is 2008, where the financial crisis taken place. But that for 50 ETF options is much more stable.

In terms of further study, I would apply the option data from with various expiration dates, to investigate the properties of the risk neutral density before and during the Financial Crisis and to examine whether Risk Neutral Density can anticipate the Financial Crisis? Furthermore, some other methods, such as semi-parametric methods on Edgeworth expansions, Hermite polynomials and non-parametric methods on tree-based methods, Kernel regression, will be under consideration.

2.7 Appendices

Appendix I: Sample Data on 02 Jan, 2008

Table 2.10: Sample Data on 02 Jan, 2008

Interest rate=4.68063%	Underlying Index Level: 1447.16	Trading date: 02 Jan, 2008
Dividend Yield=1.706934%	Time to Expiration Date: 80 days	Expiration date: 22 Mar, 2008

	CALLS				PUTS			
Strikes	Bid	Ask	Mid-price	IVs	Bid	Ask	Mid-price	IVs
700	750.4	752.4	751.4	0.781807	0	0.5	0.25	0.578907
800	651.4	653.4	652.4	0.654094	0.05	0.5	0.275	0.484391
900	552.4	554.4	553.4	0.540375	0.3	0.5	0.4	0.412137
1000	454	456	455	0.451662	0.75	1	0.875	0.362914
1100	356.9	358.9	357.9	0.385251	2.3	3.1	2.7	0.335037
1150	309.5	311.5	310.5	0.361272	4.3	5.1	4.7	0.323571
1200	263.1	265.1	264.1	0.339179	7.3	8.3	7.8	0.311175
1225	240.4	242.4	241.4	0.328671	9.4	10.1	9.75	0.303376
1250	218.2	220.2	219.2	0.319027	11.6	12.4	12	0.294578
1260	209.4	211.4	210.4	0.314848	12.8	14.4	13.6	0.295146
1270	200.8	202.8	201.8	0.311455	14	15.6	14.8	0.292027
1275	196.4	198.4	197.4	0.308926	14.7	15.6	15.15	0.288592
1300	175.2	177.2	176.2	0.299167	18.2	19.3	18.75	0.281263
1325	154.7	156.7	155.7	0.289689	22.2	24.2	23.2	0.274539
1350	134.9	136.9	135.9	0.279926	27.2	28.7	27.95	0.265101
1360	127.2	129.2	128.2	0.275945	29.4	31.4	30.4	0.262722
1375	115.9	117.9	116.9	0.269855	32.9	34.9	33.9	0.25697
1380	112.2	114.2	113.2	0.267778	34.2	36.2	35.2	0.255275
1385	108.6	110.6	109.6	0.265949	35.5	37.5	36.5	0.253393
1390	105	107	106	0.263942	36.9	38.9	37.9	0.251744
1400	97.9	99.9	98.9	0.259801	39.7	41.7	40.7	0.247868
1410	91	93	92	0.255733	42.7	44.7	43.7	0.244027
1425	80.9	82.9	81.9	0.249189	47.5	49.5	48.5	0.237947
1435	74.5	76.5	75.5	0.245125	50.9	52.9	51.9	0.23363
1450	65.2	67.2	66.2	0.238601	56.5	58.5	57.5	0.227419
1455	62.2	64.2	63.2	0.23637	58.5	60.5	59.5	0.225394
1460	59.3	61.3	60.3	0.23429	60.5	62.5	61.5	0.223133
1465	56.4	58.4	57.4	0.23198	62.6	64.6	63.6	0.221005

1470	53.6	55.6	54.6	0.22981	64.7	66.7	65.7	0.218629
1475	50.9	52.9	51.9	0.227775	66.9	68.9	67.9	0.216375
1480	48.2	50.2	49.2	0.225494	69.2	71.2	70.2	0.214237
1485	45.6	47.6	46.6	0.223339	71.6	73.6	72.6	0.212212
1490	43.1	45.1	44.1	0.221306	74	76	75	0.209912
1495	40.6	42.6	41.6	0.219009	76.5	78.5	77.5	0.207714
1500	38.3	40.3	39.3	0.217213	79.1	81.1	80.1	0.205614
1505	36	38	37	0.215143	81.8	83.8	82.8	0.20361
1510	33.8	35.8	34.8	0.213185	84.5	86.5	85.5	0.201303
1515	31.6	33.6	32.6	0.210938	87.5	89.5	88.5	0.199892
1520	29.6	31.6	30.6	0.209201	90.2	92.2	91.2	0.196944
1525	27.6	29.6	28.6	0.207166	93.2	95.2	94.2	0.194892
1530	25.7	27.7	26.7	0.20524	96.2	98.2	97.2	0.192498
1540	22.1	24.1	23.1	0.201289	102.5	104.5	103.5	0.187929
1545	20.4	22.4	21.4	0.199258	105.8	107.8	106.8	0.18576
1550	19	20.6	19.8	0.197341	109.2	111.2	110.2	0.183676
1560	16.1	17.7	16.9	0.193887	116.1	118.1	117.1	0.178773
1570	13.4	15	14.2	0.189998	123.4	125.4	124.4	0.174187
1575	12	13	12.5	0.185538	127.1	129.1	128.1	0.17146
1585	10.3	11.3	10.8	0.184818	134.8	136.8	135.8	-99.99
1600	7.5	8.5	8	0.179699	146.9	148.9	147.9	-99.99
1605	6.7	7.7	7.2	0.178063	151	153	152	-99.99
1610	5.9	6.9	6.4	0.175984	155.2	157.2	156.2	-99.99
1615	5.3	6.3	5.8	0.175026	159.5	161.5	160.5	-99.99
1620	4.7	5.5	5.1	0.172821	163.8	165.8	164.8	-99.99
1625	4.1	4.9	4.5	0.170986	168.2	170.2	169.2	-99.99
1630	3.6	4.4	4	0.169631	172.6	174.6	173.6	-99.99
1635	3.1	3.9	3.5	0.167813	177.1	179.1	178.1	-99.99
1640	2.7	3.5	3.1	0.166615	181.6	183.6	182.6	-99.99
1645	2.3	3.1	2.7	0.164957	186.1	188.1	187.1	-99.99
1650	2.1	2.6	2.35	0.163439	190.7	192.7	191.7	-99.99
1660	1.7	2.2	1.95	0.163597	200	202	201	-99.99
1675	1.05	1.55	1.3	0.160507	214.2	216.2	215.2	-99.99
1700	-	-	-	-	238.3	240.3	239.3	-99.99
1725	-	-	-	-	262.8	264.8	263.8	-99.99
1750	-	-	-	-	287.5	289.5	288.5	-99.99
1775	-	-	-	-	312.2	314.2	313.2	-99.99
1800	-	-	-	-	336.9	338.9	337.9	-99.99
1900	-	-	-	-	436	438	437	-99.99

Appendix II: Fit Tails with Generalized Extreme Value Distribution

In order to fit the tails of risk-neutral density to Generalized Extreme Value Distribution. The following conditions need to satisfied:

$$F_{GEV}(X(\alpha_0)) = \alpha_0 \tag{2.29}$$

$$f_{GEV}(X(\alpha_0)) = f_{RND}(X(\alpha_0)) \tag{2.30}$$

$$f_{GEV}(X(\alpha_1)) = f_{RND}(X(\alpha_1)) \tag{2.31}$$

Where $F_{GEV}(.)$ and $f_{GEV}(.)$ are the GEV distribution function and GEV density function, respectively. $X(\alpha)$ is the strike price corresponding to the α-quantile of the risk-neutral density. $f_{RND}(.)$ is the estimated risk-neutral density from the available strike prices.[9]

On 02 Jan, 2008, the 1% and 2% quantiles on the left tail of the density extracted from the available strikes are 1006.72 and 1013.49, respectively, and the the 98% and 99.5% quantiles on the right tail of the density are 1663.51 and 1673.66, respectively.

Appendix III: Calculation of moments for density

The mean of the density is equal to the weighted outcomes from each strike prices with the corresponding probability in the density.

$$Mean : \mu = E(X) \tag{2.32}$$

Variance, which measures the dispersion of a density, is defined as the weighted average squared deviation from the mean.

$$Variance : \sigma = E[X - \mu]^2 \tag{2.33}$$

The Skewness is a measure of asymmetry of one density. If one density has a longer tail to the left means the density is negatively skewed. Otherwise, it is

[9] In practice, we also use Matlab's minimization routine *fminsearch* and it works well.

positive skewed.

$$Skewness : \gamma = E\left[\left(\frac{X-\mu}{\sqrt{\sigma}}\right)^3\right] \tag{2.34}$$

The Kurtosis measures to what extent one density peaked.

$$Kurtosis : \kappa = E\left[\left(\frac{X-\mu}{\sqrt{\sigma}}\right)^4\right] \tag{2.35}$$

Appendix IV: Calculation of goodness-of-fit measures

Root Mean Square Error (RMSE),

$$RMSE = \sqrt{\frac{1}{n}\Sigma_{i=1}^{n}\Big(y_i - x_i\Big)^2} \tag{2.36}$$

Mean absolute error (MAE),

$$MAE = \frac{\sum_{i=1}^{n}|y_i - x_i|}{n} = \frac{\sum_{i=1}^{n}|e_i|}{n} \tag{2.37}$$

Mean Absolute Percentage Error (MAPE),

$$MAPE = \frac{100\%}{n}\sum_{t=1}^{n}\left|\frac{y_i - x_i}{y_i}\right| \tag{2.38}$$

Appendix V: Selected computational code

1. STATA code

```
. summm
    . count if comb==6150
    . dydx midp strike if comb==6150 cp==0, gen(a0)
    . dydx a0 strike if comb==6150 cp==0 , gen(b0)
    . line b0 strike
    . dydx midp strike if comb==6150 cp==1, gen(a1)
    . dydx a1 strike if comb==6150 cp==1 , gen(b1)
    . line b1 strike
```

2. R code

```
library(RND)
    library(foreign)
    mydata=read.dta(”01080308.dta”)
    y=subset(mydata, comb==”6150”)
    a=subset(y,cp==”0”)
    b=subset(y,cp==”1”)
    x <- merge(a, b, by=c(”strike”))
    head(x)
    strikes =x[,“strike”]
    sp500.calls=(x[,“bid.x”]+ x[,“ask.x”])/2
    sp500.puts=(x[,“bid.y”]+ x[,“ask.y”])/2
    matplot(strikes, cbind(sp500.calls, sp500.puts), type=“b”, pch=19, xlab=“Strikes”,
ylab=“price”, main=“S&P 500 Calls and Puts”)
    legend(“topright”, c(“Calls”,“Puts”), col=c(“Black”,“Red”),pch=19, bty=“n”)
    te=x[1,12]
    y=x[1,13]
    r=x[1,11]
    s0=x[1,5]
    MOE(market.calls= sp500.calls, market.puts= sp500.puts, s0=s0, call.strikes=strikes,
put.strikes=strikes, te=te, r=r, y=y, file.name=“6150”, lambda=1)
```

3. MATLAB code

Information extracted from European option prices use programs written in Matlab 2014.

```
    %% Extract the implied volatilities in Single lognormal and mixture of
lognormals.
    for k=1:length(Call)
    CallIV(k) = fminsearch(@(v) BSIV(v,Call(k),S,K(k),r,q,T,‘C’), 0.15);
    PutIV(k) = fminsearch(@(v) BSIV(v,Put(k), S,K(k),r,q,T,‘P’), 0.15);
    end
    %% Parameter estimation in Single lognormal and mixture of lognormals.
    LNmeans = [2.0 3.0];
    LNstdev = [0.5 0.2];
```

```
LNweight = 0.5;
start = [LNmeans LNstdev LNweight];
options = optimset(‘MaxFunEvals’, 1e5, ‘MaxIter’, 1e5);
beta = fminsearch(@(b) FindLNparams(b,Call,Put,S,K,r,T), start, options);
%% Fit the right tail with GEV distribution in Smoothing Implied Volatility Smile Method.
a1R = 0.995;
a0R = 0.98;
Ka1R = quantile(K2, a1R);
Ka0R = quantile(K2, a0R);
fKa1R = interp1(K2(1:length(RND)), RND, Ka1R);
fKa0R = interp1(K2(1:length(RND)), RND, Ka0R);
% Find the GEV parameters for the right tail.
start = [1300 360 0];
options = optimset(‘MaxFunEvals’, 1e8, ‘MaxIter’, 1e6, ‘TolX’, 1e-10, ‘TolFun’, 1e-10);
betaR = fminsearch(@(betaR) FitGEVRightTail(betaR,a0R,a1R,Ka0R,Ka1R,fKa1R,fKa0R), start, options);
Mean = betaR(1);
sigma = betaR(2);
phi = betaR(3);
%% Fit the left tail with the GEV distribution
a1L = 0.01;
a0L = 0.02;
Ka1L = quantile(K2, a1L);
Ka0L = quantile(K2, a0L);
fKa1L = interp1(K2(1:length(RND)), RND, Ka1L);
fKa0L = interp1(K2(1:length(RND)), RND, Ka0L);
% Find the GEV parameters for the left tail
start1 = [-1300 160 -0.1];
options = optimset(‘MaxFunEvals’, 1e8, ‘MaxIter’, 1e6, ‘TolX’, 1e-10, ‘TolFun’, 1e-10);
betaL = fminsearch(@(betaL) FitGEVLeftTail(betaL,a0L,a1L,Ka0L,Ka1L,fKa1L,fKa0L), start1, options);
Mean = betaL(1);
```

```
sigma = betaL(2);
phi = betaL(3);
```

Chapter 3

The reaction of option prices to macroeconomic announcements: Evidence from S&P 500 Index Options and China's 50 ETF Options

3.1 Introduction

How do macroeconomic fundamentals reflect in the option markets? Past decades or so have been much attention focused on the implied volatility. Theoretically, the implied volatility can be regarded as a good predictor of the future volatility of the underlying asset, even its familiar pattern 'volatility smile' seems not to consistent with the Black-Scholes formula. Recent researches by Nikkinen and Sahlström (2004), Vähämaa and Äijö (2011), Gospodinov and Jamali (2012), etc., have examined the impact of macroeconomic announcements, either the scheduled macroeconomic announcements or the non-scheduled macroeconomic announcements, on the changes in implied volatility derived from the options data (also see Kim and Kim, 2003; Kearney and Lombra, 2004; Füss, Mager, Wohlenberg and Zhao, 2011; and Tanha, Dempsey and Hallahan, 2014). Moreover, investigations to examine the impacts of the good news and the bad news have also been implemented (see Fostel and Geanakoplos, 2012; Äijö, 2008;

Nofsinger and Prucyk, 2003). Ederington and Lee (1996) find an inverse relationship between the time to maturity and the effect of new information on the implied volatility, they also conclude that implied volatility only rises following price innovations due to unscheduled announcements, but declines following scheduled announcements. Fung (2007) indicates that the implied volatility provides a signal to the Hong Kong stock market crash in 1997.

However, extracting important but unobservable parameters from option prices in the market is not limited to implied volatility. Nowadays, focuses have turned to risk-neutral density. The prices of the option written on a given asset with different strike prices with the same time-to-maturity delivers the risk-neutral density, which has the ability to indicate the market assessment of the probability of the payoff over the series of the strike prices.

A main spirit of this chapter examines how the macroeconomic news reflect in the option market. Investors revise their expectations in the light of the new information. According to the efficient market hypothesis, the prices incorporate all the information in the market. Only the new or unanticipated information would influence the expectation of the market. When the announcement does not deviate from the market expectation, theoretically, the densities will not change with respect to this information. By studying the risk neutral density, we can easily obtain the markets' beliefs. For instance, the density shows whether the market places a higher probability on an upward movement of the state of the underlying asset than a downward movement of state. In the meantime, the risk-neutral density has superior performance than implied volatility. Because implied volatility is a measure of the second moment of the distribution of the price of the underlying. The risk-neutral density embodies all the moments. Furthermore, the evolution of the risk neutral densities can release the information about how the market's beliefs change over time. It might be considered to look at how market beliefs changes to either scheduled or unscheduled macroeconomic announcements, such as scheduled CPI release and the adjustment on the base rate by the central bank, respectively.

Current study offers the research on China's 50 ETF option market, which was introduced in early 2015 and is the only option in Chinese market. It is evident from the Shanghai Stock Exchange Month Market Statistics in 2015 that the trading volume of this option rose around 50% at the end of the year. In July 2017, because of the surprising interest to the investors, the trading volume has increased up to 05 million and average daily nominal trading volume around 0.25 billion RMB. It is the fact that the China's 50 ETF options market is a young market. Also, the Chinese financial market has unusual restrictions by the regulators. Will this market response to the macroeconomic news? Does the response differ to that in the mature market, e.g. the response of the S&P 500 options market to the U.S. macroeconomic news? To our best of knowledge, previous studies mostly concentrate on the financial markets in developed countries, and little research has done on the effect of macroeconomic news announcement with respect to the option market in developing countries, especially rare can be found in the China's 50 ETF options. How integrated the Chinese stock market reacts to the scheduled regularly macroeconomic news announcements and understanding how the risk neutral densities extracted from China 50 ETF options react to the macroeconomic news announcements is definitely significant to both investors and policy-makers. For the purpose of comparison, we also further examined the S&P 500 options, which has been used in the Chapter 2.

The contribution here is threefold. Firstly, while a wealth of research has investigated the response of the Chinese financial markets react to the macroeconomic news announcements, this study employ a new set of options data, which is written on China's 50 ETF and rarely been investigated by now. The use of such dataset will contribute to the existing literature by providing a newly investigation on risk-neutral density from an emerging market. We also try to examine the influence of the macroeconomic announcements on the 50 ETF options. Secondly, the tests for EMH based on the spot market and options market are included. Thirdly, this chapter also evaluates the effect of surprises (or shocks) in 18 types of macroeconomic news announcements on China 50 EFT options market and 57 types of those on the US options market. The announcements were divided into groups in order to answer the research questions. In both market, due to limitation of the time spans of the two data, thereby the limitation of the number of each macroeconomic announcement type, we didn't find any significant effect of each type of announcement on the financial option market. We investigate the option market response the good news and bad news.

The remainder of this chapter is organized as follows. The following section gives a review of the related literature. Section 3.3 describes the options data adopted to obtain the implied risk neutral densities and related macroeconomic variables. The theoretical framework for estimation for implied densities is given in Section 3.4. The econometric analysis will be carried out in Section 3.5. A conclusion of this chapter is provided in Section 3.6, followed by a further study in Section 3.7.

3.2 Literature Review

The study of the risk-neutral density is numerous. From the methodology perspective, main approaches to examine the impact of macroeconomic news announcements on financial markets including the GARCH model (see Bonser-Neal and Tanner, 1996; Connolly and Taylor, 1994; Dominguez, 2003; Nikkinen, Omran, Sahlström and Äijö, 2006; Äijö, 2008; Bekaert and Wu, 2000; Li and Engle, 1998; Kim, McKenzie and Faff, 2004; Roache and Rossi, 2010), simple OLS model (see Christie-David, Chaudhry and Koch, 2000; Beber and Brandt, 2006; Hess, Huang and Niessen, 2008; Kilian and Vega, 2008; Gospodinov, and Jamali, 2012) and others like EGARCH model (see Kim and Sheen, 2000), fractional autoregressive integrated moving average (FARIMA) model (see Onan, Salih and Yasar, 2014), Autoregress model (see De Goeij and Marquering, 2006), Vector Error Correction Model (VECM) model (see Yoshino, Taghizadeh-Hesary, Hassanzadeh and Prasetyo, 2014), etc.

Beber and Brandt (2006) compare the option-implied moments before and after the announcements and find that the announcements reduce the uncertainty on all news, which is consistent with the study Jiang, Konstantinidi and Skiadopoulos (2012). However, the changes in the higher-order moments depend on the characteristics of the news. Äijö (2008) shows that good news would decrease the implied volatility and also the skewness of the RND, while increase the kurtosis. Birru and Figlewski (2010) also show how the risk-neutral moments response to the macroeconomic news announcements. Steeley (2004) studies whether the distribution of stock prices is influenced by new public information and finds that volatility reduced after news releases as uncertainty was resolved. The results suggest that there is also higher moment sensitivity to macroeconomic surprises. Vähämaa and Äijö (2011) documented that the implied volatility is significantly affected by the Fed's monetary policy decisions. More interestingly, Brenner, Pasquariello and Subrahmanyam (2009) show that the asset returns react asymmetrically to the information content of these surprise announcements. However, Mandler (2002) does not find any effect of European Central Bank meeting on the RND. Adopting the double lognormal method to extract the RND, Gemmill and Saflekos (2000) find that RND does not help to reveal investors' sentiment during British elections. Other literature on how the risk-neutral density response to the macroeconomic announcements can be found from Mandle (2002), Castrén (2005), Galati, Melick and Micu (2005), and Hattori, Schrimpf and Sushko (2016).

With regard to the Chinese financial market, although, Baum, Kurov and Wolfe (2015) try to study how the Chinese scheduled macroeconomic announcements influenced on the global financial and commodity futures markets. Tang, et al. (2013) examine the impact of monetary policies, including the changes of interest rate and the required reserve ratio, on the Chinese stock markets, rare can be found through Chinese option market. Since the launch of the Shanghai 50 exchange-traded fund (ETF) option in China's financial markets in early 2015, China has come into the "era of option", which not only make Chinese capital market more complete but also promote and enhance Shanghai's international influence. It would be interesting to look at the Chinese option market.

3.3 Data

In this section, I will describe the data that will be using in both Chinese market and the US market.

3.3.1 Chinese data

Our data we collected allow us to address our research questions. This section describes the China 50 ETF, the China 50 ETF options and the macroeconomic news announcements data.

3.3.1.1 China 50 ETF

We obtain the China 50 ETF price levels span from 26 June updated to 06 December, 2015, from the Wind Data-stream. The 50 ETF returns we use here are the continuously compounded returns.

The continuously compounded returns are calculated as,

$$r_t = lnP_t - lnP_{t-1} \qquad t = 1, 2, ..., T, \tag{3.1}$$

Where P_t is the closing price of the China 50 ETF price level at time t.

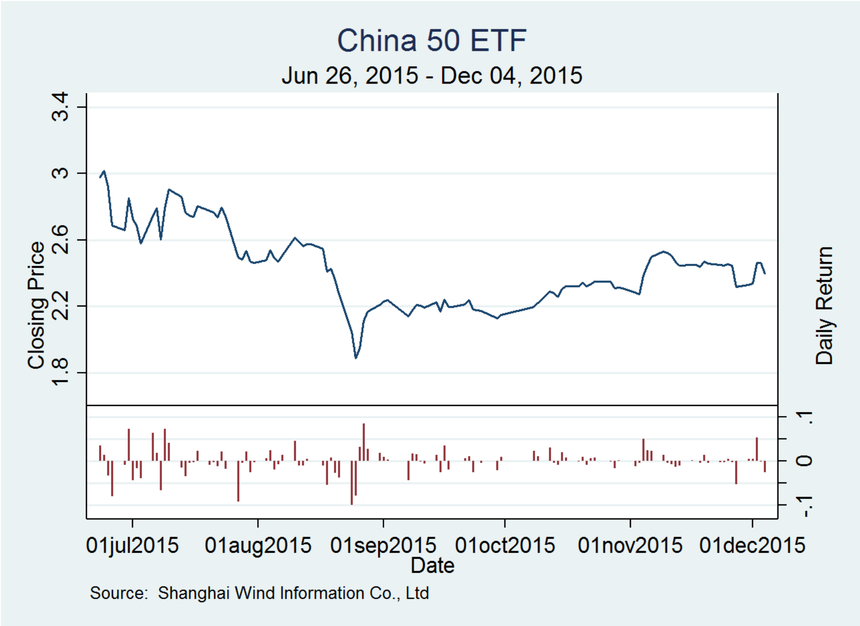


Figure 3.1: China 50 ETF and Returns Over the Sample Period.

As we can see from the figure above, the top portion of the figure shows the series of the price of the China 50 ETF and the returns are showed in the bottom portion of the figure over the sample period. It seems that the first four months has witnessed a sharply decrease and more volatile, especially fall from the maximum of 3.427 to its minimum at 1.886. The series later on seems to be more stable with a slight rise. These are consistent with the series of the returns, in other words, more spikes are showed during the first half of the time span than that over the last three months. The figure also shows that the China 50 ETF had experience a continuous fall in July and August, especially a 8.48% in Shanghai stock market and 9.98% in China 50 ETF price on 24 August, marking the largest decrease since 2007.

By 8-9 July 2015, the Shanghai stock market had fallen 30 percent over three weeks as 1,400 companies, or more than half listed, filed for a trading halt in an attempt to prevent further losses.Values of Chinese stock markets continued to drop despite efforts by the government to reduce the fall. After three stable weeks the Shanghai index fell again on 24 August by 8.48 percent, marking the largest fall since 2007.

3.3.1.2 China 50 ETF Options

We adopt options data written on China's 50 ETF from 26 June, 2015, updated to early December, 2015. The options are standard European style, which means the option holders have the right to exercise the contracts at the strike price at the maturation. The data is collected from Wind Data-stream[1]. The raw data includes closing prices, option Greeks across various maturities. The contracts mature in December 2015, January 2016, March 2016 and June 2016.

Evidence from the SSE's data[2], it seems that the China 50 ETF option sees stable operation and played a significant and positive role in Chinese financial market. Obviously, we do have limitations in current dataset. The emerging option market starts in early 2015, a short period of the option data can be collected in this study.

3.3.1.3 Macroeconomic News Announcements in China

The release of the macroeconomic news announcements are scheduled in advance, such as Consumer Price Index, Gross Domestic Product, Foreign Exchange Reserves, etc. And in this chapter, we call the release of macroeconomic news announcements are not pre-scheduled (Unscheduled) announcements, including the change of the base rate and the change of the Reserve Requirement Ratio. According to the classifications from Reuters DataStream, the classifications of the macroeconomic news announcements can be divided into Money & Finance, Government Sector, Consumer Sector, Industry Sector, National Account, Prices, External Sector, Surveys & Cyclical and Other. In this research, we adopted vast of macroeconomic indicators. Furthermore, we split all the news into good and bad news by applying the methodology of Lee (1992) and Nofsinger and Prucyk (2003), who considered a positive asset return comes from a good news and vice versa[3].

[1] Shanghai Wind Information Co., Ltd

[2] According to the trading data in SSE, the first trading day of China 50 ETF option meets the market expectation, with the volume of 18,843 contracts among which the volume of the call option contracts and put option contract are 11,320 and 7,523, respectively; the turnover of premium reaches RMB 28.7 million and the nominal value traded is RMB 431.8 million; the open interest totals 11,720 contracts.

[3] Here, for some announcements are released at weekends or on holidays, we regard the news as a good news if the return is positive in the following trading day. Moreover, we also consider a zero return as a sign of good news.

Table 3.1: Sample macroeconomic news announcements in China

Date	Classifications	Event Name	Period	Unit	Actual	Reuters Poll	Surprise	Prior
15-Jul-15	National Account	GDP - GDP QQ SA	Q2 2015	%	1.8	1.7	0.1	1.3
24-Jul-15	Surveys & Cyclical	Manuf PMI - HSBC Mfg PMI Flash	Jul. 2015	–	48.2	49.7	-1.5	49.6
01-Aug-15	Surveys & Cyclical	PMI Manuf - NBS Manufacturing PMI	Jul. 2015	–	50	50.2	-0.2	50.2
08-Aug-15	External Sector	Trade - Exports YY	Jul. 2015	%	-9.1	-1	-8.1	1.5
08-Aug-15	External Sector	Trade - Imports YY	Jul. 2015	%	-8.1	-8	-0.1	-6.1
08-Aug-15	External Sector	Trade - Trade Balance	Jul. 2015	USD B	42.05	53.25	-11.2	45.5
09-Aug-15	Prices	Inflation - PPI YY	Jul. 2015	%	-5.4	-5	-0.4	-4.8
09-Aug-15	Prices	Inflation - CPI YY	Jul. 2015	%	1.6	1.5	0.1	1.4
10-Aug-15	Government Sector	Money and lending - New Yuan Loans	Jul. 2015	CNY B	1480	738	742	1279.1
10-Aug-15	Surveys & Cyclical	Money and lending - Outstanding Loan Growth	Jul. 2015	%	15.5	13.6	1.9	13.4
12-Aug-15	National Account	Activity indicators - Urban investment (ytd)yy	Jul. 2015	%	11.2	11.5	-0.3	11.4
12-Aug-15	Industry Sector	Activity indicators - Industrial Output YY	Jul. 2015	%	6	6.6	-0.6	6.8
12-Aug-15	Consumer Sector	Activity indicators - Retail Sales YY	Jul. 2015	%	10.5	10.6	-0.1	10.6
21-Aug-15	Surveys & Cyclical	Manuf PMI - HSBC Mfg PMI Flash	Aug. 2015	–	47.1	47.7	-0.6	48.2
01-Sep-15	Surveys & Cyclical	PMI Manuf - NBS Manufacturing PMI	Aug. 2015	–	49.7	49.7	0	50
08-Sep-15	External Sector	Trade - Exports YY	Aug. 2015	%	-5.7	-6	0.3	-9.1
08-Sep-15	External Sector	Trade - Imports YY	Aug. 2015	%	-13.8	-8.2	-5.6	-8.1
08-Sep-15	External Sector	Trade - Trade Balance	Aug. 2015	USD B	59.9	48.2	11.7	42.05
23-Sep-15	Surveys & Cyclical	Manuf PMI - HSBC Mfg PMI Flash	Sep. 2015	–	47	47.5	-0.5	47.1
01-Oct-15	Surveys & Cyclical	PMI Manuf - NBS Manufacturing PMI	Sep. 2015	–	49.8	49.6	0.2	49.7

Source: Thomson Reuters Economic Data

For description, several macroeconomic announcement (see samples in Table 3.1) have been picked. Thomson Reuters Economic Data provides full information about each announcement in a neatly organized layout. For the purpose of answering the research questions, current study selects the information we need. The 'Date' indicates the date that macroeconomic news announced. Second and third columns show the classification and the indicator of a particular announcement. The column 'Period' represents the time period of the indicator. The following four columns show the unit of the indicator, the actual announcement data, the Reuters Poll data, and the surprise data, respectively. To be notice, the Reuters Poll data, unlike almost mainstream polls, the data is entirely collected via online surveys, which allows to collect much more data and more flexible than traditional (phone) research. Surprise (or shock) stands for the difference between the actual announcement and Reuters Poll data. Last column is the announcement data in previous period. Let's take the first announcement for example. The quarterly GDP Growth Rate (seasonally adjusted), which is classified in the 'National Account', for second quarter in 2015 announced in July 15, 2015 was 1.8%. However, the Reuters Poll data was 1.7%, and thus the difference between the announced data and the predicted data was 0.1%, which was the surprise (see Figure 3.3).

Following table describes the dummies used to analyse the news announced.

The announcements dummy represents dates with/-out macroeconomic announcements (1/0); surprise dummy represents dates with/-out surprises (1/0); and good/bad dummy represents the macroeconomic announcement is a good news (1) or a bad news (0). The related the information and easier understanding of the dummies can be found in the table in Appendix VI.

	good	bad	total
surprise	17	23	40
no surprise	2	4	6
no announcement			65
total	19	27	111

Table 3.2: Summary Statistics for Dummies

Figure 3.2 shows announcements days with the China 50 ETF closing prices over the study period. The dashed lines indicate all dates with macroeconomic announcements, including 'surprise' news and 'no-surprise' news. It seems that most of the news is announced around the mid next month. The dashed lines in Figure 3.3 show all dates with 'surprise' news. From the figure we can find that the majority announcements have not been anticipated by the market.

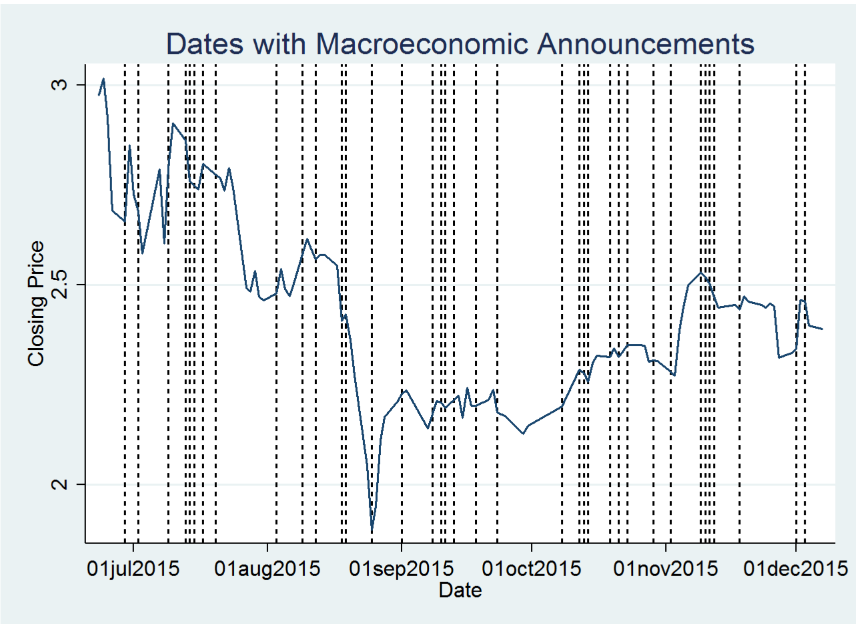


Figure 3.2: Dates with macroeconomic announcements (scheduled and unscheduled)

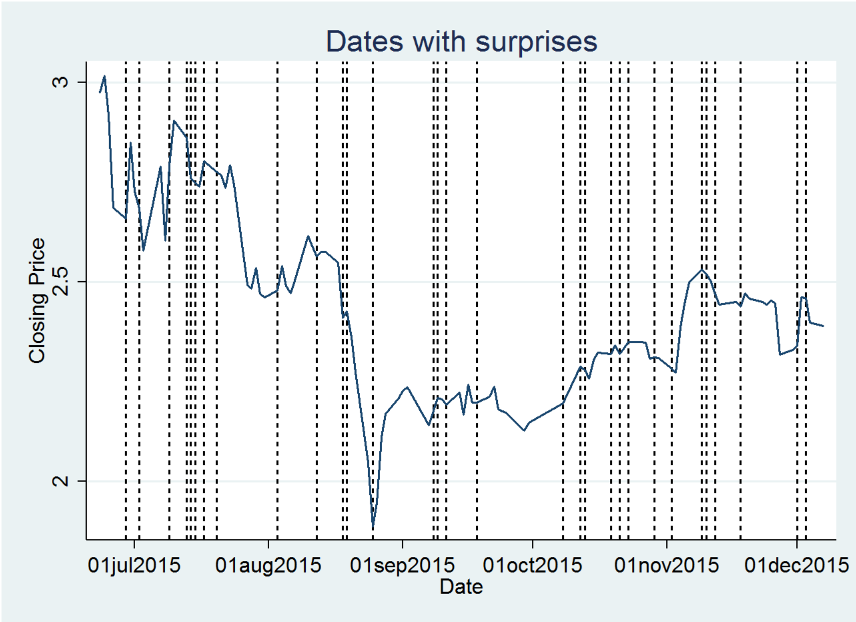


Figure 3.3: Dates with macroeconomic surprises in China

3.3.2 US data

We now move to the US data, the S&P 500 index and the corresponding options data used are the same as those in Chapter 2. We still need the macroeconomic announcements in US over the same period that options traded.

3.3.2.1 Macroeconomic News Announcements in the US

The US macroeconomic announcements data is also collected from the Thomson Reuters Economic Data. The data obtained is from 02 Jan, 2008 to 19 Mar, 2008. Following table describes the dummies used to analyse the news announced.

	good	bad	total
surprise	19	14	33
no surprise	5	4	9
no announcement			12
total	24	18	54

Table 3.3: Summary Statistics for Dummies

Figure 3.4 shows announcements days with the S&P 500 index over the study period. The dashed lines in Figure 3.5 show all dates with ‘surprise’ news. From the figure we can find that the majority announcements have not been anticipated by the market.

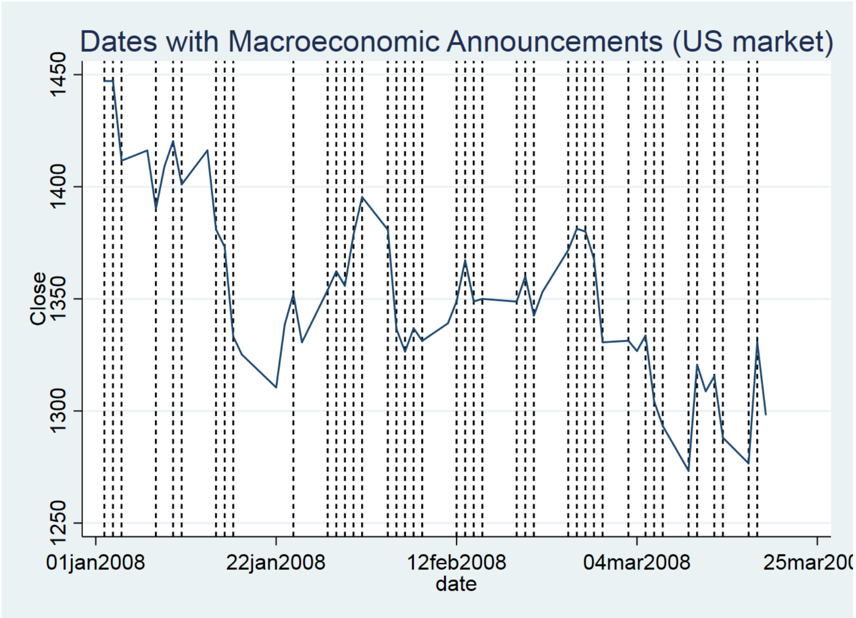


Figure 3.4: Dates with macroeconomic announcements (scheduled and unscheduled) in US

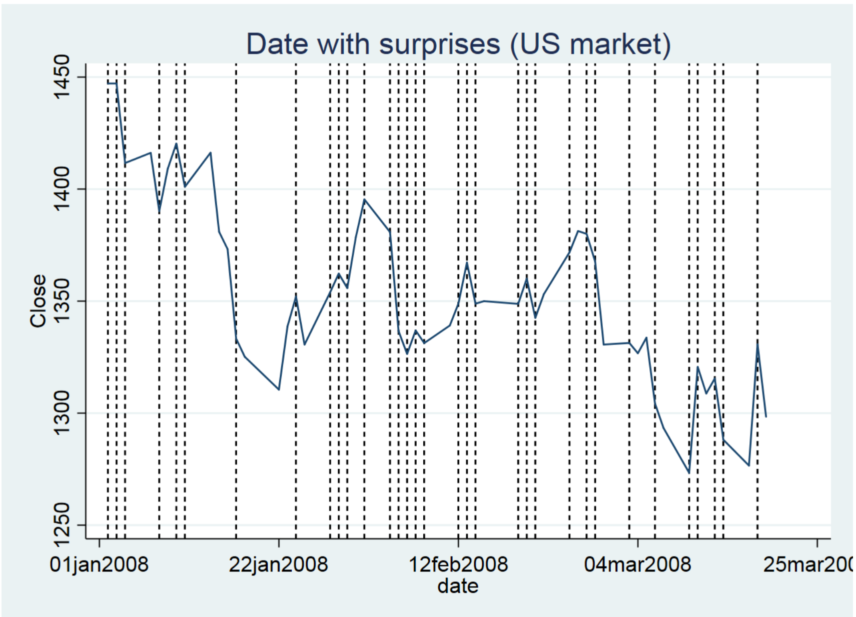


Figure 3.5: Dates with macroeconomic surprises in US

3.4 Estimation Framework

3.4.1 Estimating the risk neutral density

As stated in Bahra (1997), the theoretical option price is equal to the discounted value of the expected payoff under the risk neutral measure ($\mathcal{Q}$ measure).

$$C = e^{-r\tau} \int_X^{\infty} f(S_T)(S_T - X)d(S_T) \tag{3.2}$$

$$P = e^{-r\tau} \int_0^{X} f(S_T)(X - S_T)d(S_T) \tag{3.3}$$

Where, C, S_T, X, r and τ all have the standard meaning of option valuation; C= call price; S_T= the index level at expiration date T; X= exercise price or strike price; r= risk-free interest rate; τ = time to expiration date. And we also use $f(S_T)$= risk neutral density; $F(S_T)$= risk neutral distribution.

Breeden and Litzenberger (1978) indicate that the risk-neutral density is proportional to the second derivative of a European call price with respect to the strike price (see the mathematical proof in Chapter 2). They also show how the risk-neutral density could be extracted from the prices of options with a continuum of strikes, see equation (3.4) and (3.5) for call options and put options, respectively.

$$f(S_T) = e^{r\tau}\frac{\partial^2 C}{\partial X^2} \tag{3.4}$$

$$f(S_T) = e^{r\tau}\frac{\partial^2 P}{\partial X^2} \tag{3.5}$$

Firstly, when extracting the risk-neutral density, the theory calls for the option's strike prices to be continuous. As a matter of fact, the market only trades with a small number of discrete strikes, with at least 0.5 Chinese Yuan apart and up 1 Chinese Yuan apart or even more in some parts of the available range of strikes for 50 ETF options (see example in Table 3.14 in Appendix I). Secondly, the other problem is that we can only extract the middle portion of the density as a result of solving the first problem with stochastic volatility inspired method, which does not extend further to the both tails because of the small range of the strike prices. we convert the interpolated implied volatility curve back to option prices space and extract the middle portion of the risk-neutral density. Finally, we also extend the two tails to the risk-neutral density beyond the range from X_2 to X_{n-1}.

3.4.2 Stochastic volatility inspired (SVI) parametrization

In this section, we will discuss the stochastic volatility inspired (SVI) methodology to extract the risk-neutral density. In order to answer the research question, the moments computed from the recovered density will be used to estimate the effect of the news on the options market.

The stochastic volatility inspired parametrization originally developed by Merrill Lynch in 1999 is a parametric model for stochastic implied volatility. In the case of a call option this is a mapping, which is consistent with Gatheral and Jacquier (2014):

$$(X, \tau) \to \sigma_t^{BS}(X, \tau) \tag{3.6}$$

This is the so called implied volatility surface at date t.

$$var(x; a, b, s, \rho, m) = a + b\left\{\rho(x - m) + \sqrt{(x - m)^2 + s^2}\right\} \tag{3.7}$$

Where, x is the log(X/F) or log(Strike price/Futures price); a gives the overall level of variance; b gives the angle between the left and right asymptotes; s determines the smoothness of the vertex; ρ determines the orientation of the graph; and m translated the graph from left to right.

The left and right asymptotes are given by:

$$var_L(x; a, b, \rho, m) = a - b(1 - \rho)(x - m) \tag{3.8}$$

$$var_R(x; a, b, \rho, m) = a - b(1 - \rho)(x - m) \tag{3.9}$$

By definition, the overall level of variance is always positive; and the variance increases linearly with $|x|$.

Based on the nature of volatility, as well as the properties of the SVI equations, we have the following deductions, which are also consistent with the Gatheral

(2006),

- Increasing 'a' will lead to a vertical translation of the volatility smile in the positive direction, i.e., increases the general level of variance;
- An Increase of 'b' increases the slopes of both the put and call wings, i.e., decreases the angle between the put and call wing and thereby tightening the smile;
- Increasing 'ρ' decreases (increases) the slope of the left (right) wing, a counter-clockwise rotation of the smile;
- Increasing 'm' results in a horizontal translation of the smile in the positive direction (to the right);
- Increasing 'σ' reduces the at-the-money curvature of the smile.

Graphs in Appendix IV show how the sensitivity of the parameters to the volatility smile. Furthermore, we will follow the estimation by Gatheral (2006), which uses an objective function minimization process compared to the implied volatility given by Black-Scholes:

$$func = \underset{a,b,s,\rho,m}{\arg\min} \sum (\hat{\sigma}_{SVI} - \sigma_t^{BS})^2 \tag{3.10}$$

The estimated implied volatility curve would be converted to the option prices through Black-Scholes model.

$$\hat{C} = S_t N(\hat{d}_1) - Xe^{-r\tau} N(\hat{d}_2) \tag{3.11}$$

The parameters $\hat{d}_1$ and $\hat{d}_2$ is given by:

$$\hat{d}_1 = \frac{\ln(S_t/X) + (r + \hat{\sigma}^2_{SVI}/2)\tau}{\hat{\sigma}_{SVI}\sqrt{\tau}}, \tag{3.12}$$

$$\begin{aligned} \hat{d}_2 &= \frac{\ln(S_t/X) + (r - \hat{\sigma}^2_{SVI}/2)\tau}{\hat{\sigma}_{SVI}\sqrt{\tau}} \\ &= \hat{d}_1 - \hat{\sigma}_{SVI}\sqrt{\tau}. \end{aligned} \tag{3.13}$$

where $N(\cdot)$ is the cumulative normal distribution function, τ is time to expiration date, $T - t$. X, r, and $\hat{\sigma}_{SVI}$ are the strike price, risk-free rate and fitted volatility, respectively.

Finally, we would use the equation 3.4 to find a smoothed risk neutral density.

3.5 Empirical Results

3.5.1 Time-varying Densities

Here in this subsection, we extract the risk-neutral densities through SVI method from the China 50 ETF option markets. The figure below is the sample density from 29 June to 16 July.

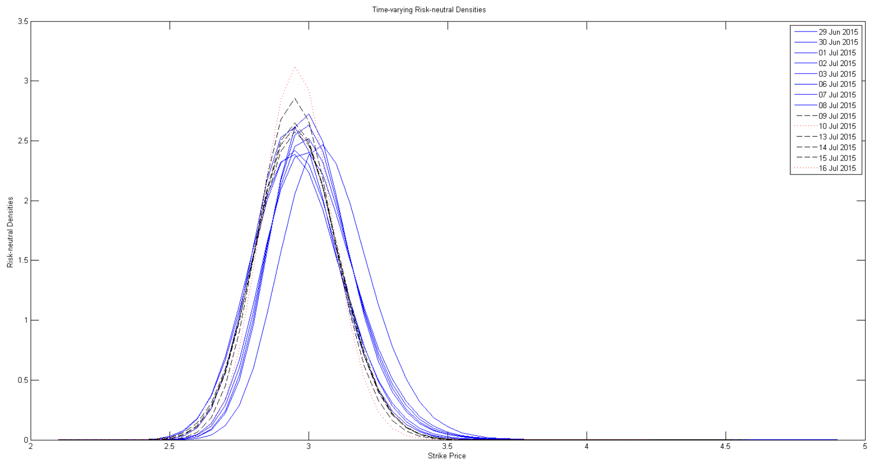


Figure 3.6: Time-Varying Risk-neutral Densities

In the figure, '- - -' represents the densities with the macroeconomic announcements on that day; '........' stands for the densities on the day after macroeconomic news announced.

According to the macroeconomic news data, on 09 July, 2015, the data on CPI and PPI were announced at 02:30 in the morning, the surprises on both CPI and PPI have produced, thereby the movement in the density. The density on 16 Jul, 2015, whose kurtosis is the greatest in the figure, shows a higher, sharper peak than those on the other days. This implies that the expectations of the value in the China 50 ETF prices on option expiration day has been moved from the shoulders of a distribution into its centre and both tails. The figure also indicates that the densities move along with the time. However, we still cannot find out whether the densities response to the macroeconomic announcements. The following part is to extract the moments of the density so as to investigate how the moments react to the news respectively.

3.5.2 Risk-neutral Moments

Applying with the Stochastic Volatility Inspired method, we can find the risk-neutral density and thus the moments of the densities. The corresponding moments for the densities can be found in the following figure.

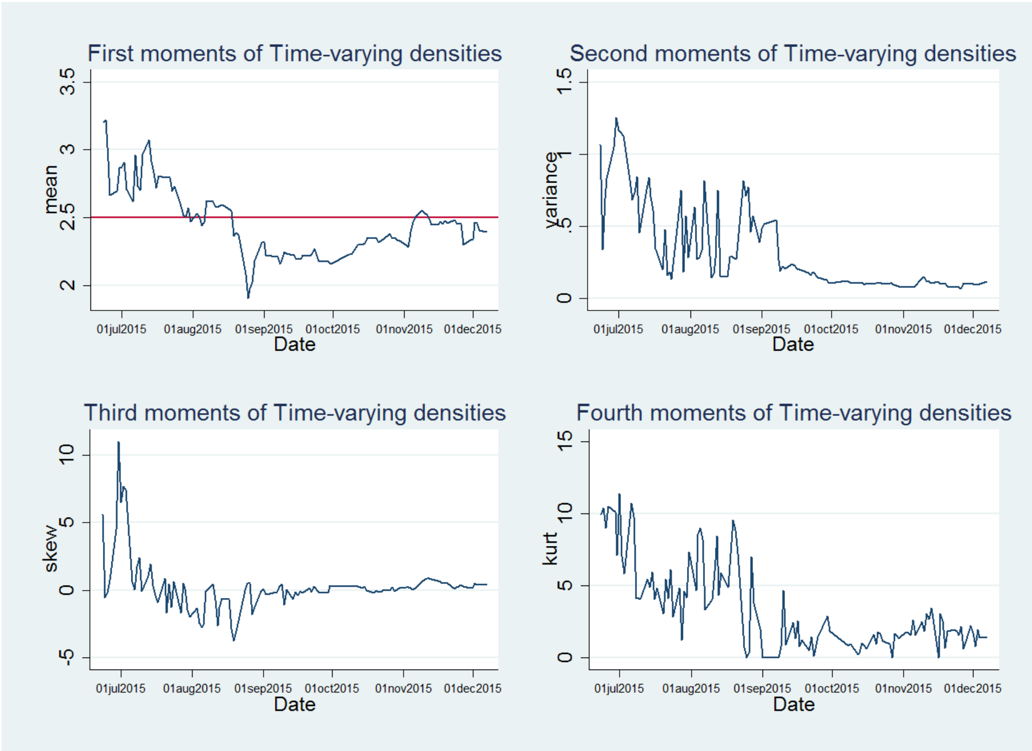


Figure 3.7: Time-Varying Moments

The movement of the means corresponds to the series of the China 50 ETF prices, which is reasonable because the means are always approximate to the expected value of the underlying price under the no-arbitrage condition. This is consistent with the findings by Figlewski (2009) and Fabozzi, Leccadito and Tunaru (2014). The horizontal line indicated the terminal value of the 50 ETF index level on the expiration date. Despite large fluctuations in August, the means move towards to terminal value of the underlying. Evidence from the time series of variance, it seems that the variance has experienced a decrease trend. Theoretically, the change in the risk neutral variance measures the resolution of uncertainty. As expiration approaches, the variance decreases due to the information has been released in the market. The skewnesses seem to be more volatile in the first half time span than those in the second half. More specifically, the skewness behaved a huge positive due to the great fall before July. The tendency to its negative value presents that the densities skewed to left, which indicated a decrease expectation in the market. The huge drop on 24 Aug corresponds to the market expectation, the skewness recover to the zero axis afterwards. This is also consistent with the variance, which demonstrated a large increase in the uncertainty from 0.27 to 0.81. In terms of the kurtosis, the 'heavy tail' phenomenon is presented in the first half period, however it does not exist thereafter. It even behaved a thinner tail than that of the normal distribution.

Table 3.4 reports the descriptive statistics and the correlations for the variables in this study. China 50 ETF price level has a mean of 2.44. Return has a mean of -0.0012, which means the underlying has experienced a fall over the sample period. Summary statistics for the moments are also reported in this table. Panel B indicates that the variables do not seem to be correlated with each other in current study.

Table 3.4: Descriptive Statistics and Correlations for the Variables (50 ETF market)

Panel A: Descriptive Statistics for Moments

Variable	Obs	Mean	Std. Dev.	Min	Max
50 ETF price	111	2.4384	0.2277	1.886	3.018
50 ETF return	111	-0.0012	0.0300	-0.0998	0.0842
mean	111	2.4671	0.2494	1.9072	3.2219
variance	111	0.3272	0.3053	0.0629	1.2536
skewness	111	0.2231	1.9954	-3.7381	10.9844
kurtosis	111	3.4719	3.0362	3.12e-10	11.4133

Panel B: Correlations among Returns and Moments

	return	mean	variance	skewness	kurtosis
return	1.0000				
mean	0.0314	1.0000			
	0.7434				
variance	-0.1195	0.4649	1.0000		
	0.2118	0.0000			
skewness	0.0799	0.3527	0.5339	1.0000	
	0.4048	0.0001	0.0000		
kurtosis	-0.0220	0.6367	0.5289	0.0666	1.0000
	0.8186	0.0000	0.0000	0.4873	

Table 3.5 reports the descriptive statistics and the correlations for the variables in the US market. Although the two datasets are with different time spans, the

correlations between the variables are consistent with low values. This also means the variables are not correlated with each other.

Table 3.5: Descriptive Statistics and Correlations for the Variables (S&P 500 market)

Panel A: Descriptive Statistics for Moments					
Variable	Obs	Mean	Std. Dev.	Min	Max
S&P 500 level	54	1352.836	40.133	1273.37	1447.16
S&P 500 return	54	0.002	0.016	-0.041	0.033
mean	54	1353.406	41.240	1275.153	1456.381
variance	54	0.237	0.028	0.1651	0.353
skewness	54	-0.695	0.262	-1.205	-0.090
kurtosis	54	0.053	0.190	0.000	1.021

Panel B: Correlations among Returns and Moments					
	return	mean	variance	skewness	kurtosis
return	1.0000				
mean	-0.1315	1.0000			
	0.3433				
variance	0.2218	-0.5483	1.0000		
	0.1069	0.0000			
skewness	-0.3151	-0.4111	-0.1770	1.0000	
	0.0203	0.0020	0.2004		
kurtosis	0.2611	-0.3384	0.1452	0.0293	1.0000
	0.0565	0.0123	0.2949	0.8335	

3.5.3 The Time-varying Behaviour of Risk-neutral Moments

Current section is to present the evolutions of Risk-neutral Moments from both Chinese market and the US market. The main results will focus on the effects of the change in the underlying prices on the change in the moments recovered from risk-neutral densities. Furthermore, I will also assess both the contemporaneous effect and lagged effect of the impact of underlying index on the change in the risk-neutral moments for both markets. Before looking at such effects, I would like to examine the stationarity for the variables in both markets.

From both tables, I have tested the stationarity for the variables in their levels and also in their differences. We can see that all the variables in both markets are stationary, I(0).

There is a substantial literature on how the Risk-neutral Moments behave

Table 3.6: Unit root test for the Variables (50 ETF market)

	in level		in difference	
variables	Test Statistic	p-value for Z(t)	Test Statistic	p-value for Z(t)
50 ETF price	-2.602	0.0926	-7.751	0.0000
50 ETF return	-8.989	0.0000	-11.169	0.0000
mean	-3.232	0.0182	-7.222	0.0000
variance	-3.539	0.0070	-13.386	0.0000
skewness	-3.075	0.0285	-16.994	0.0000
kurtosis	-3.710	0.0040	-11.105	0.0000

Table 3.7: Unit root test for the Variables (S&P 500 market)

	in level		in difference	
variables	Test Statistic	p-value for Z(t)	Test Statistic	p-value for Z(t)
S&P 500 price	-3.146	0.0233	-7.439	0.0000
S&P 500 return	-7.106	0.0000	-13.212	0.0000
mean	-3.083	0.0279	-6.802	0.0000
variance	-4.314	0.0004	-3.702	0.0041
skewness	-4.387	0.0003	-12.069	0.0000
kurtosis	-3.800	0.0029	-4.045	0.0012

when the underlying price/index level moves. Table 3.8 shows the results of regressing the changes in each risk-neutral moments on the change in the underlying index for the Chinese market, and Table 3.9 for the US market. In terms of the contemporaneous effect and lagged effect of the impact of underlying index. This study will adopt the following specifications,

Regression assessing the contemporaneous effect:

$$\Delta moment_t = \alpha + \beta \Delta close_t + \varepsilon_t \tag{3.14}$$

Regression assessing the lagged effect:

$$\Delta moment_t = \alpha_0 + \beta_0 \Delta close_t + \sum_{i=1}^{2} \beta_i \Delta close_{t-i} + \varepsilon_t \tag{3.15}$$

We would rationally assume that the risk-neutral density tend to response to the change of the underlying by equal magnitude. In other words, obviously, *ceteris paribus*, one unit change in the 50 ETF index level would move the same amount of the first moments of the density in both equations (3.14) and (3.15).

The corresponding estimation results can be found in Table 3.8. We can see that the changes in the 50 ETF closing values ($\Delta close_t$) are statistically significant different from zero in almost all cases, apart from the models with changes in skewness, i.e. model (3) and model (7). It concludes that, from regressions (1), (2) and (4), we do find the contemporaneous effects between the 50 ETF and almost risk-neutral moments. It does not exist any relationship between Skewness and the contemporaneous underlying index.

Theoretically, the coefficient β in equation (3.14) should be equal to 1 for the first moment. It is the reason that, if the market is efficient, the changes of the current index should be responded and reflected in the risk neutral density. We test the null hypothesis (β in equation 3.14 is equal to 1), which has been rejected significantly, and also the case of contain the lagged dependent variable (model 5). Therefore, when the index increases (falls), the density moves right (left) substantially less than the change in the 50 ETF index level, i.e., the markets have underreacted to the changes in the underlying asset. Underreaction to the news would always lead to misallocation of the prices. Because the coefficients are less than 1, which means the densities are not quite fully response to the changes of the change in the index. This is consistent with the study Frazzini (2006), which shows the financial stock markets always underreact to the news due to disposition effect. Related evidence can be also found from Chen, et al. (2016).

However, in terms of the lagged effect, we find that almost all the lagged independent variable do not exhibits statistical significance but the first lag of change in 50 ETF index in regression of change in mean (model 5) and change in skewness (model 7). This means the lagged change in the underlying 50 ETF index affect the changes in the mean and the skewness. The positive lagged change in underlying drives the change in mean positively, but falls the skewness.

Furthermore, I use the ‘nlcom’ and ‘testnl’[4] commands in STATA to test the nonlinear combinations of estimators and the test nonlinear hypotheses after estimation. The test for independent variables in model (1) and model (5) are statistically significant. In model 1, when testing whether coefficient of the change in closing price is equal to one. We do find evidence to reject the null hypothesis of coefficient is equal to one. The 95% confidence interval spans from 0.5670 to 0.9420, which means this model indicates the risk neutral density has

[4]The null hypothesis for testnl is H0: $\beta_0 + \beta_1 + \beta_2 = 1$ in equation (15).

underreacted to the underlying prices change.

More interestingly, in model (5), we do not find evidence to reject the null hypothesis in test nonlinear hypotheses after estimation. The 95% confidence interval is from 0.7735 to 1.3499, which means this model indicates the risk neutral density do rationally react to the underlying prices change.

When looking into the results in the US market, we find that the coefficients in Model (1) in Table 3.9 is close to 1. This means that one unit change in the S&P 500 index level would change the mean of the density and near the same amount of the change. It is even closer to 1 when adding the lagged changes in the closing prices in Model (5). The change in the closing price has the same effect in both markets. The same as the effect on the change in variance in China, the contemporary effect is also negative in Model (2) and (6). However, the effects are different in the case of changes in the higher moments. The signs turn to the opposite in the S&P 500 options market.

Table 3.8: Regressions of change in moments with respect to the change in underlying index in Chinese market

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Δmean	Δvariance	Δskew	Δkurt	Δmean	Δvariance	Δskew	Δkurt
Δclose	0.755***	-0.991***	-1.454	5.479**	0.655***	-0.777***	-0.764	4.908*
	(0.0957)	(0.239)	(1.970)	(2.632)	(0.0851)	(0.234)	(1.912)	(2.814)
Δclose_lag1					0.477***	-0.334	-3.583*	1.710
					(0.0820)	(0.225)	(1.841)	(2.710)
Δclose_lag2					-0.0704	0.328	-0.710	-0.992
					(0.0845)	(0.232)	(1.898)	(2.794)
constant	-0.00331	-0.0139	-0.0552	-0.0484	0.000775	-0.00948	-0.0220	-0.0413
	(0.00712)	(0.0178)	(0.147)	(0.196)	(0.00610)	(0.0167)	(0.137)	(0.202)
N	110	110	110	110	108	108	108	108
R-sq	0.366	0.137	0.005	0.039	0.518	0.159	0.041	0.040
adj. R-sq	0.360	0.129	-0.004	0.030	0.504	0.135	0.013	0.013
rmse	0.0744	0.186	1.533	2.048	0.0629	0.173	1.414	2.081

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3.9: Regressions of change in moments with respect to the change in underlying index in US market

	(1) Δmean	(2) Δvariance	(3) Δskew	(4) Δkurt	(5) Δmean	(6) Δvariance	(7) Δskew	(8) Δkurt
Δclose	0.952***	-0.000881***	0.00664**	-0.00394**	0.966***	-0.00102*	0.00586	-0.00226
	(0.0228)	(0.00017)	(0.00191)	(0.00144)	(0.0377)	(0.000344)	(0.00332)	(0.00236)
Δclose_lag1					0.103*	0.000264	-0.00721	0.00123
					(0.0407)	(0.00037)	(0.00358)	(0.00254)
Δclose_lag2					0.0238	-0.000136	0.00102	-0.00092
					(0.0321)	(0.000292)	(0.00283)	(0.00201)
constant	-0.105	-0.00457	0.034	0.00153	0.598	0.00243	-0.0126	0.0247
	(0.515)	(0.00383)	(0.0431)	(0.0326)	(0.768)	(0.00699)	(0.0675)	(0.048)
N	42	42	42	42	18	18	18	18
R-sq	0.978	0.402	0.232	0.157	0.979	0.416	0.357	0.097
adj. R-sq	0.977	0.387	0.213	0.136	0.975	0.291	0.219	-0.097
rmse	3.29	0.0245	0.276	0.208	2.928	0.0267	0.258	0.183

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

3.5.4 Tests of weak-form and semi-strong-form EMH

Table 3.10: Regressions with lagged effect in Chinese market

	(1) return	(2) return	(3) return	(4) return	(5) return	(6) return
L.return	0.127	0.134	0.139	0.118	0.122	0.164*
	(0.0948)	(0.0928)	(0.0954)	(0.0950)	(0.0934)	(0.0924)
L.mean		-0.0271**				-0.0313**
		(0.0112)				(0.0154)
L.variance			0.0103			0.0305**
			(0.00942)			(0.0133)
L.skew				0.00155		0.000576
				(0.00143)		(0.00183)
L.kurt					-0.00193**	-0.00192
					(0.000927)	(0.00137)
constant	-0.00139	0.0655**	-0.00475	-0.00174	0.00533	0.0724**
	(0.00285)	(0.0278)	(0.00420)	(0.00287)	(0.00428)	(0.0355)
N	110	110	110	110	110	110
R-sq	0.016	0.067	0.027	0.027	0.054	0.149
adj. R-sq	0.007	0.050	0.009	0.009	0.037	0.108
rmse	0.0299	0.0293	0.0299	0.0299	0.0295	0.0283

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Models (1) - (5) in Table 3.10 aim to regress the returns on the lag return and four lagged moments. Testing the effect of lagged return amounts to a test of weak-form EMH. It seems we cannot reject the weak-form EMH from model (1). In model (3) and (4), the lagged variance and lagged skewness also do not have any significant effect on current return. However, model (2) and (5) show negative impact on return. The model (6) also indicates the intertemporal relationship between the moments and the returns. Analogous to the contemporaneous effect.

Table 3.11: Regressions with lagged effect in US market

	(1) return	(2) return	(3) return	(4) return	(5) return	(6) return
L.return	-0.186	-0.155	-0.00743	-0.109	-0.147	0.0949
	(0.167)	(0.153)	(0.140)	(0.185)	(0.159)	(0.154)
L.mean		0.000174**				0.0000875
		(0.0000591)				(0.0000853)
L.variance			-0.436***			-0.316*
			(0.0923)			(0.131)
L.skew				0.0106		0.0169
				(0.0110)		(0.0112)
L.kurt					-0.0569*	-0.0201
					(0.0238)	(0.0223)
constant	0.00284	-0.234**	0.106***	0.0101	0.00433	-0.0292
	(0.00258)	(0.0801)	(0.0219)	(0.00800)	(0.00252)	(0.135)
N	42	42	42	42	42	42
R-sq	0.030	0.207	0.383	0.053	0.154	0.455
adj. R-sq	0.006	0.167	0.352	0.004	0.110	0.379
rmse	0.0167	0.0153	0.0135	0.0168	0.0158	0.0132

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

We refer to these lags as lag-mean-in-return effect and lag-variance-in-return effect.

One possible explanation for these effects can be provided by considering the behavior of the option traders. Testing the effect of the lagged moments amounts to a test of semi-strong form EMH, model (6) shows that the lagged mean has a negative effect on current return. Why is this? The lag-mean-in-return effect implies if the mean of the risk neutral density rises, *ceteris paribus*, the demand for call options will rise, and the demand for put options will fall. Traders who are buying call options will short-sell the underlying in order to hedge. Traders who are selling put options will sell the underlying in order to restore their hedge. Both will cause the (current) price of the underlying to fall. In terms of the lag-variance-in-return effect, one possible explanation is that if the variance of investors' expectations increases, both the prices of puts and calls increase, the purchase and the sale of the underlyings take place at the same time. However, due to the risk aversion of the investors, the purchase of the puts and underlyings would be higher than sale of the underlyings to hedge for the calls. This phenomenon will increase the price of the underlying and thus a positive return. Our result is consistent with the skew response puzzle by Constantinides and Lian (2015). In their paper, they argue that as the increase in variance, investors demand more puts as insurance and result in credit-constrained in writing puts. Theoretically, in this condition, more underlyings are needed to be hedge. Both effects might result from the behavior of the hedgers.

Turning to the US market in Table 3.11, The lagged return also does not have any effect on the return, which means we cannot reject the weak-form EMH. However, the lagged implied moments, except the lagged skewness, do have significant impact on today's return, see Model (2), (3) and (5). There are substantial differences in the effect of lagged mean and lagged variance between the two markets. One of the reason might come from the different sample in the two markets. When it is in the crisis period, as we have in S&P 500 options market, the increase in the uncertainty yesterday will decease the return today, as the results shown in Model (2) and (6).

3.5.5 Effect of announcements on densities

3.5.5.1 Effect of announcements on densities in Chinese market

Table 3.12 shows the regression of moments on the dummies. As we can see from the Figure 3.1, a huge crash was taken place on 24 Aug, 2015. Evidence from the structural break test, we find that the date on 24 Aug is a significant break point for all four moments over the sample period ($p < 0.05$). Therefore, the regression in this subsection consist of the whole period, pre-crash period and the post-crash period. What we are interested is the change of the moments.

The regression of the changes in mean, from Panel A, shows how does mean change with respect to the news over sample period and two sub-sample period. For the whole sample period, the density shifts in the case of announcement, surprise and the good news dummy. The announcement yesterday and the surprise today would decrease the changes in mean of the density for the whole period and the post-crash period. Both good news today and yesterday would increase the mean of the density. In other words, these would shift the density to the right, but the news today for the regression is not significant in pre-crash.

The good news yesterday decrease the variance today. This is not surprising because the release of the good news will decrease the overall uncertainty of the market for the whole period and the pre-crash period. But we did not find a significant effect in the post-crash period. Comparing with the effect of announcement and the surprise have opposite effect in the volatile period and post-crash period.

In terms of the regressions for skewness. We did not find any effect for the whole period and the pre-crash period. In the post-crash period, the announcement, surprise and their lags have significant effect on the skewness. With the interest on the left tails, we find that the announcement yesterday and surprise today would show an increase of the downside risk.

The kurtosis measures both the 'peakedness' of the density and the heaviness of the tails. The lagged surprise decrease the kurtosis of the density. We also find different effect of announcement and the surprise in the pre-crash and post-crash period.

Table 3.12: Effects of the news on densities

Panel A: Regressions for moments of Risk-Neutral Density Dependent variable: Changes in Mean			
	(1) Whole period	(2) pre-crash	(3) post-crash
announce	0.0441	0.0826	0.0249
	(0.0315)	(0.106)	(0.0215)
L.announce	$\boxed{-0.0511*}$	-0.151	$\boxed{-0.0369*}$
	(0.0314)	(0.146)	(0.0205)
surprise	$\boxed{-0.0596*}$	-0.0902	$\boxed{-0.0435*}$
	(0.0318)	(0.0971)	(0.0233)
L.surprise	0.00802	0.0883	0.0154
	(0.0319)	(0.137)	(0.0216)
good	$\boxed{0.0517*}$	0.0474	$\boxed{0.0382*}$
	(0.0267)	(0.0664)	(0.0213)
L.good	$\boxed{0.0804***}$	$\boxed{0.146**}$	$\boxed{0.0381*}$
	(0.0264)	(0.0648)	(0.0210)
constant	-0.00845	-0.0244	0.00418
	(0.0131)	(0.0309)	(0.00999)
N	110	44	66
R-sq	0.145	0.182	0.153
adj. R-sq	0.096	0.050	0.066
rmse	0.0885	0.127	0.0529

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Panel B: Regressions for moments of Risk-Neutral Density Dependent variable: Changes in Variance			
	(1) Whole period	(2) pre-crash	(3) post-crash
announce	-0.0826	$\boxed{-0.621**}$	$\boxed{0.0655***}$
	(0.0706)	(0.238)	(0.0245)
L.announce	0.0175	0.245	-0.0134
	(0.0704)	(0.328)	(0.0233)
surprise	0.0835	$\boxed{0.573**}$	$\boxed{-0.0673**}$
	(0.0715)	(0.219)	(0.0265)
L.surprise	0.0137	-0.233	0.0174
	(0.0715)	(0.309)	(0.0246)
good	-0.0883	-0.00575	-0.0226
	(0.0601)	(0.149)	(0.0242)
L.good	$\boxed{-0.103*}$	$\boxed{-0.262*}$	0.00830
	(0.0593)	(0.146)	(0.0238)
constant	0.0125	0.0451	-0.00973
	(0.0294)	(0.0697)	(0.0114)
N	110	44	66
R-sq	0.064	0.265	0.166
adj. R-sq	0.009	0.146	0.081
rmse	0.199	0.285	0.0601

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Panel C: Regressions for moments of Risk-Neutral Density Dependent variable: Changes in Skewness			
	(1)	(2)	(3)
	Whole period	pre-crash	post-crash
announce	0.0158	-2.240	$\boxed{0.453**}$
	(0.550)	(1.973)	(0.180)
L.announce	-0.594	-1.528	$\boxed{-0.323*}$
	(0.549)	(2.794)	(0.171)
surprise	-0.241	1.426	$\boxed{-0.453**}$
	(0.557)	(1.862)	(0.195)
L.surprise	0.208	0.325	$\boxed{0.381**}$
	(0.557)	(2.633)	(0.180)
good	-0.195	0.487	-0.189
	(0.468)	(1.272)	(0.178)
L.good	-0.164	-0.321	0.00802
	(0.462)	(1.241)	(0.175)
constant	0.238	0.670	0.0124
	(0.229)	(0.593)	(0.0834)
N	110	44	66
R-sq	0.033	0.095	0.194
adj. R-sq	-0.024	-0.052	0.113
rmse	1.548	2.425	0.441

Standard errors in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01

Panel D: Regressions for moments of Risk-Neutral Density Dependent variable: Changes in Kurtosis			
	(1)	(2)	(3)
	Whole period	pre-crash	post-crash
announce	-0.122	$\boxed{4.830**}$	$\boxed{-1.301**}$
	(0.744)	(2.235)	(0.619)
L.announce	0.567	1.668	0.578
	(0.742)	(3.084)	(0.589)
surprise	0.519	$\boxed{-3.550*}$	$\boxed{1.455**}$
	(0.754)	(2.056)	(0.671)
L.surprise	$\boxed{-1.197*}$	-1.627	$\boxed{-1.157*}$
	(0.754)	(2.907)	(0.621)
good	0.164	-1.106	0.0183
	(0.633)	(1.405)	(0.612)
L.good	0.754	1.011	0.199
	(0.625)	(1.371)	(0.602)
constant	-0.150	-0.768	0.141
	(0.310)	(0.655)	(0.287)
N	110	44	66
R-sq	0.042	0.153	0.132
adj. R-sq	-0.013	-0.015	0.043
rmse	2.093	2.677	1.521

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

3.5.5.2 Effect of announcements on densities in US market

Table 3.13: Effects of the news on densities in US market

Regressions for moments of Risk-Neutral Density for S&P 500 options market				
	(1) change in mean	(2) change in variance	(3) change in skewness	(4) change in kurtosis
announce	12.54	-0.00236	-0.000338	-0.122
	(8.467)	(0.0152)	(0.156)	(0.118)
L.announce	6.148	-0.0155	0.0399	0.0322
	(8.976)	(0.0161)	(0.166)	(0.125)
surprise	6.306	-0.00381	0.128	-0.0635
	(6.364)	(0.0114)	(0.118)	(0.0888)
L.surprise	-8.040	0.0192	-0.109	0.0487
	(7.057)	(0.0127)	(0.130)	(0.0985)
good	$\boxed{-36.27***}$	$\boxed{0.0358***}$	$\boxed{-0.258*}$	0.114
	(5.473)	(0.00982)	(0.101)	(0.0764)
L.good	5.304	-0.0145	$\boxed{0.269*}$	$\boxed{-0.172*}$
	(5.420)	(0.00972)	(0.100)	(0.0756)
constant	-3.208	-0.00731	-0.0276	0.119
	(7.779)	(0.0140)	(0.144)	(0.109)
N	42	42	42	42
R-sq	0.579	0.348	0.299	0.228
adj. R-sq	0.507	0.236	0.178	0.096
rmse	15.25	0.0274	0.282	0.213

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

We test the structural break and do not find break with the sample period for US data. Therefore, we test the effect of news on the risk-neutral moments with the whole period. Table 3.13 shows the regressions of moments in the announcement dummy, surprise dummy, good new dummy and their lags. We find that the implied moments in S&P 500 index options market only react to the good news dummy. This is not surprising and is what we expected. As a mature financial market, the S&P 500 market has anticipated almost the news and announcements before they are announced. Comparing to the 50 ETF options markets, the S&P 500 options markets have the higher information symmetry, due to much more investors are participated in the market, which results in the activeness and also accelerate the transaction of the market. Finally, these improves the efficiency of the market. From the table, we can see that the good news does decrease the change in mean and the change in skewness, but increase the change in variance. However, the good news yesterday increase the change in skewness but decrease the change in kurtosis.

3.6 Conclusions

This study investigates the impact of macroeconomic new announcements on the China's 50 Exchange-Traded Fund (50 ETF) options and the S&P 500 options. To examine this issue, we try to distil the information with the risk-neutral densities extracted from the two markets. Current study employs the stochastic volatility inspired (SVI) and find out the moments from the risk-neutral densities. We firstly investigate the information content of the density from 50 ETF options prices and the S&P 500 options prices.

In the Chinese market, for the whole sample period, the density changes in the case of the announcement, surprise and the good news. The good news yesterday falls the variance today. And the surprise makes the kurtosis lower. In the pre-crash period, the macroeconomic announcements do have some effect on the densities. Good news would increase the mean of the density. The surprise do increase the uncertainty of the market. In contrast, the announcement and the good news result in the fall of the variance. In the post-crash period, we also find that the announcement yesterday and surprise today show an increase in the downside risk. The announcement and surprise on variance and kurtosis have an opposite effect in both volatile period and post-crash period. However, there is no effect of the macroeconomic news announcement on the skewness in the whole period and the pre-crash period.

When looking at the US market, we test the structural break and do not find break. Therefore, we test the effect of news on the risk-neutral moments with the whole period. The good news contemporaneously drives down the change in mean and change in skewness, but drive up the change in variance. Moreover, the good news also has the lagged effect on the change in skewness and change in kurtosis. The good news has a positive lagged effect on the change in skewness but negative lagged effect on the change in skewness.

However, due to limitation of the time spans of the two data, thereby the limitation of the number of each macroeconomic announcement type, we did not find any significant influence on the financial option market. This comes out with an idea for the further research to have a longer time span investigation.

Moreover, we also distinguish between types (good or bad) of the macroeconomic indicators and examine how the RND responds to the variety of macroeconomic announcements.

3.7 Further study

One possible idea is to extend the dataset, constructing the fixed time to maturity densities, to have multiple expiration dates and longer period of moments so as to examine the time-varying of the preferences and expectations from the markets. This would also allow us to observe more macroeconomic announcements. We would also be able to examine the reaction of the option markets to various macroeconomic news.

The risk premium, the difference between risk-neutral density and physical probability distribution, has been widely examined. Testing the risk premium, as one of our ideas, it is of interest to me to find how the risk premium could be reflected in the option market and fundamental markets. How the risk premium vary with the information flows?

In early 2010s, the European Central bank (ECB) began paying -0.1$\%$ on deposits. Thirdly, in a Black-Scholes economy, the interest rate is assumed to be strictly positive. A negative rate implies that investing money at money market would result in a loss. However, one common but wrong solution was to set the rate simply to zero. The risk free rate comes into the formula in the form e^{-rT} in the valuation equation, in a negative interest rate environment, this portion of the equation will just add a discount, instead of a premium to the value of the option. Examining how this phenomenon affect the valuation of options as well as the risk-neutral densities would be interesting.

3.8 Appendices

Appendix I: Sample Data on 09 Jul, 2015

Table 3.14: Sample Data on 09 Jul, 2015

Interest rate=4.68063%	Underlying Index Level: 2.792	Trading date: 09 Jul, 2015
Dividend Yield=1.706934%	Time to Expiration Date: 168 days	Expiration date: 23 Dec, 2015

	CALLS		PUTS	
strike price	close price	implied volatility	close price	implied volatility
2.5	0.65	0.675	0.27	0.581
2.55	0.569	0.592	0.24	0.502
2.6	0.593	0.662	0.28	0.527
2.65	0.569	0.661	0.302	0.524
2.7	0.492	0.586	0.289	0.471
2.75	0.449	0.558	0.312	0.467
2.8	0.402	0.525	0.373	0.513
2.85	0.41	0.564	0.37	0.472
2.9	0.423	0.608	0.431	0.514
2.95	0.351	0.537	0.46	0.512
3	0.348	0.558	0.493	0.515
3.1	0.338	0.59	0.554	0.51
3.2	0.304	0.586	0.641	0.536
3.3	0.277	0.589	0.668	0.475
3.4	0.272	0.618	0.795	0.548
3.5	0.219	0.577	0.886	0.571
3.6	0.191	0.568	0.94	0.535

Appendix II: Results from Raw Option Prices on 09 Jul, 2015

Figure below represented the interpolated option prices for calls and puts.

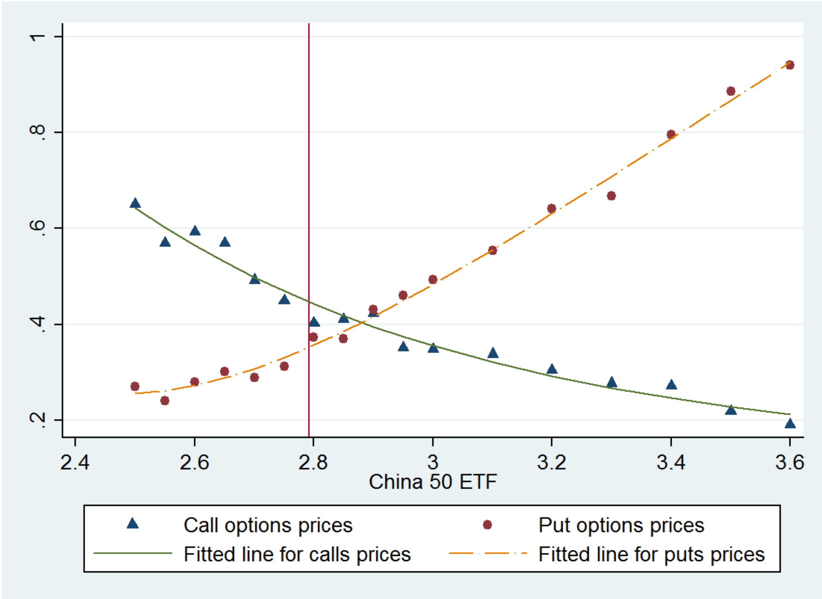


Figure 3.8: Market Option Prices

The unacceptable densities in the following graph result from the discreteness of the strike prices.

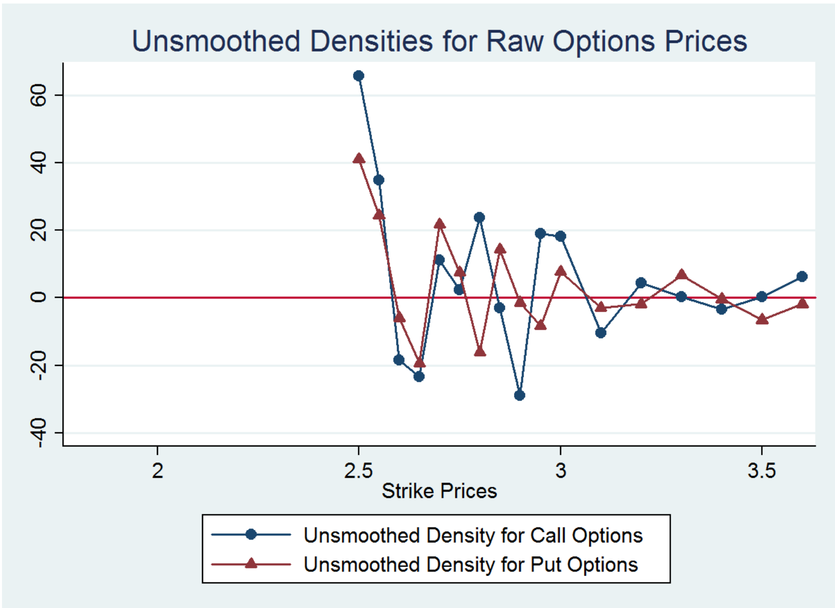


Figure 3.9: Empirical Risk-Neutral Density

Appendix III: Descriptive Statistics for announcements

Table 3.16: Descriptive Statistics for announcements in Chinese market

Variable	Obs	Mean	Std. Dev.	Min	Max
Foreign Domestic Investment					
actual	5	8.29	.4873913	7.72	8.93
reuterspoll	0				
surprise	0				
prior	5	8.676	.9100168	7.72	10.06
Foreign Exchange Reserves					
actual	4	3.5425	.105633	3.44	3.69
reuterspoll	1	3.75	.	3.75	3.75
surprise	1	-.06	.	-.06	-.06
prior	4	3.615	.1112055	3.51	3.73
M2 Money Supply					
actual	5	13	.6855654	11.8	13.5
reuterspoll	5	12.46	1.043072	11	13.2
surprise	5	.54	.680441	-.1	1.6
prior	5	12.46	1.119375	10.8	13.3
Retail Sales					
actual	5	10.76	.2073643	10.5	11
reuterspoll	5	10.6	.2738613	10.2	10.9
surprise	5	.16	.1949359	-.1	.4
prior	5	10.58	.311448	10.1	10.9
Industrial Output					
actual	5	6.04	.4722289	5.6	6.8
reuterspoll	5	6.16	.3286335	5.8	6.6
surprise	5	-.12	.5357238	-.6	.8
prior	5	6.14	.4037327	5.7	6.8
Consumer Price Index					
actual	5	1.58	.2683282	1.3	2
reuterspoll	5	1.58	.2167948	1.3	1.8
surprise	5	0	.1870829	-.2	.2
prior	5	1.56	.2966479	1.2	2
Producer Price Index					
actual	5	-5.58	.4868264	-5.9	-4.8
reuterspoll	5	-5.34	.5856621	-5.9	-4.5
surprise	5	-.24	.181659	-.4	0
prior	5	-5.32	.6058053	-5.9	-4.6
Trade Balance					
actual	5	53.902	9.348183	42.05	61.7
reuterspoll	5	53.738	7.146605	46.79	64.75
surprise	5	.164	11.82881	-11.2	13.57
prior	5	53.106	8.662482	42.05	60.36
Imports					
actual	5	-13.44	6.318465	-20.4	-6.1
reuterspoll	5	-12.44	3.983466	-16	-8
surprise	5	-1	5.970343	-5.6	8.9
prior	5	-13.22	6.100574	-20.4	-6.1
Exports					
actual	5	-4.82	4.025792	-9.1	1.5
reuterspoll	5	-3.3	2.796426	-6.3	-.2
surprise	5	-1.52	4.452191	-8.1	2.5
prior	5	-4.12	3.852532	-9.1	1.5
Gross Domestic Product					
actual	2	6.95	.0707106	6.9	7
reuterspoll	2	6.85	.0707106	6.8	6.9
surprise	2	.1	0	.1	.1
prior	2	7	0	7	7
Urban investment					
actual	5	10.8	.5338538	10.2	11.4
reuterspoll	5	10.96	.4929504	10.2	11.5
surprise	5	-.16	.2701851	-.5	.2
prior	5	11.04	.461519	10.3	11.4
Manufacturing Purchasing Managers' Index					
actual	6	48.1	.8390474	47.2	49.4
reuterspoll	2	47.9	.5656849	47.5	48.3
surprise	2	.55	.3535534	.3	.8
prior	6	48.2	.9402133	47.2	49.4
Outstanding Loan Growth					
actual	5	15.02	.9066422	13.4	15.5
reuterspoll	5	14.8	.9300537	13.6	15.6
surprise	5	.22	.9731392	-.6	1.9
prior	5	14.74	.9736529	13.4	15.5
House Prices					
actual	5	-2.34	2.02682	-4.9	.1
reuterspoll	0				
surprise	0				
prior	5	-3.5	1.939072	-5.7	-.9
PBOC Deposit Rate					
actual	1	2	.	2	2
reuterspoll	0				
surprise	0				
prior	3	2	.25	1.75	2.25
PBOC Lending Rate					
actual	1	4.85	.	4.85	4.85
reuterspoll	0				
surprise	0				
prior	3	4.85	.25	4.6	5.1
PBOC Reserve Requirement Ratio					
actual	1	18.5	.	18.5	18.5
reuterspoll	0				
surprise	0				
prior	3	18.33333	.2886751	18	18.5

Table 3.17: Descriptive Statistics for announcements in US market

Variable	Obs	Mean	Std. Dev.	Min	Max
ADP Employment Change					
actual	3	46	68.942	-18	119
reuterspoll	3	35	18.02776	15	50
surprise	3	11	59.73274	-33	79
previous	3	109.6667	68.47871	37	173
Average Hourly Earnings (MoM)					
actual	3	0.3	0.1	0.2	0.4
reuterspoll	3	0.3	0	0.3	0.3
surprise	3	0	0.1	-0.1	0.1
previous	3	0.3666667	0.1527525	0.2	0.5
Average Hourly Earnings (YoY)					
actual	3	3.7	0	3.7	3.7
reuterspoll	3	3.733333	0.1527526	3.6	3.9
surprise	3	-0.0333333	0.1527525	-0.2	0.1
previous	3	3.733333	0.057735	3.7	3.8
Average Weekly Hours					
actual	3	33.73333	0.0577341	33.7	33.8
reuterspoll	3	33.76667	0.0577341	33.7	33.8
surprise	3	-0.0333333	0.057735	-0.1	0
previous	3	33.76667	0.0577341	33.7	33.8
Building Permits (MoM)					
actual	3	1.037333	0.0488808	0.984	1.08
reuterspoll	3	1.070333	0.060136	1.023	1.138
surprise	3	-0.033	0.0284781	-0.058	-0.002
previous	3	1.069333	0.0184752	1.048	1.08
Business Inventories					
actual	3	0.6666667	0.2516611	0.4	0.9
reuterspoll	3	0.4	0	0.4	0.4
surprise	3	0.2666667	0.2516611	0	0.5
previous	3	0.4	0.3	0.1	0.7
Capacity Utilization					
actual	3	81.1	0.6928186	80.3	81.5
reuterspoll	3	81.3	0	81.3	81.3
surprise	3	-0.2	0.6928203	-1	0.2
previous	3	81.53333	0.0577341	81.5	81.6
Chicago Purchasing Managers' Index					
actual	2	48	4.949747	44.5	51.5
reuterspoll	2	51.35	2.333452	49.7	53
surprise	2	-3.35	2.616295	-5.2	-1.5
previous	2	53.95	3.464824	51.5	56.4
Construction Spending (MoM)					
actual	3	-0.7333333	0.7371115	-1.3	0.1
reuterspoll	3	-0.5333333	0.1527525	-0.7	-0.4
surprise	3	-0.2	0.6557439	-0.8	0.5
previous	3	-0.6666667	0.7094599	-1.3	0.1
Consumer Confidence					
actual	2	82.15	8.131728	76.4	87.9
reuterspoll	2	84.75	3.889087	82	87.5
surprise	2	-2.6	4.242641	-5.6	0.4
previous	2	88.25	0.4949726	87.9	88.6
Consumer Credit Change					
actual	3	10.63333	6.30661	4.5	17.1
reuterspoll	3	7.566667	0.8144528	7	8.5
surprise	3	3.066667	5.653613	-2.7	8.6
previous	3	8.766667	7.217571	4.5	17.1
Consumer Price Index (MoM)					
actual	3	0.2666667	0.2309401	0	0.4
reuterspoll	3	0.2666667	0.057735	0.2	0.3
surprise	3	-2.48E-09	0.2645751	-0.3	0.2
previous	3	0.5333333	0.2309401	0.4	0.8
Consumer Price Index (YoY)					
actual	3	4.133333	0.1527526	4	4.3
reuterspoll	3	4.166667	0.057735	4.1	4.2
surprise	3	-0.0333333	0.1527525	-0.2	0.1
previous	3	4.233333	0.1154702	4.1	4.3
Consumer Price Index ex Food & Energy (MoM)					
actual	3	0.1666667	0.1527525	0	0.3
reuterspoll	3	0.2	0	0.2	0.2
surprise	3	-0.0333333	0.1527525	-0.2	0.1
previous	3	0.2666667	0.057735	0.2	0.3
Consumer Price Index ex Food & Energy (YoY)					
actual	2	2.4	0.1414214	2.3	2.5
reuterspoll	2	2.4	0	2.4	2.4
surprise	2	0	0.1414214	-0.1	0.1
previous	2	2.45	0.0707106	2.4	2.5

Table 3.17 - Continued

Variable	Obs	Mean	Std. Dev.	Min	Max
Continuing Jobless Claims					
actual	5	2764	61.01229	2685	2828
reuterspoll	5	2751.2	44.41509	2688	2805
surprise	5	12.8	32.59908	-45	34
previous	5	2747.4	54.03055	2669	2807
Core Personal Consumption Expenditure - Price Index (MoM)					
actual	2	0.25	0.0707107	0.2	0.3
reuterspoll	2	0.2	0	0.2	0.2
surprise	2	0.05	0.0707107	0	0.1
previous	2	0.2	0	0.2	0.2
Core Personal Consumption Expenditure - Price Index (YoY)					
actual	2	2.2	0	2.2	2.2
reuterspoll	2	2.2	0	2.2	2.2
surprise	2	0	0	0	0
previous	2	2.2	0	2.2	2.2
Current Account					
actual	1	-167.2	.	-167.2	-167.2
reuterspoll	1	-184.4	.	-184.4	-184.4
surprise	1	17.2	.	17.2	17.2
previous	1	-178.5	.	-178.5	-178.5
Durable Goods Orders					
actual	2	-0.1499999	6.434672	-4.7	4.4
reuterspoll	2	-1.05	4.17193	-4	1.9
surprise	2	0.9	2.262742	-0.7	2.5
previous	2	2.45	2.757717	0.5	4.4
Durable Goods Orders ex Transportation					
actual	2	0.4999999	2.969848	-1.6	2.6
reuterspoll	2	-0.65	0.9192388	-1.3	0
surprise	2	1.15	2.05061	-0.3	2.6
previous	2	1.1	2.12132	-0.4	2.6
Existing Home Sales (MoM)					
actual	2	4.9	0.0141421	4.89	4.91
reuterspoll	2	4.88	0.0989949	4.81	4.95
surprise	2	0.02	0.0848528	-0.04	0.08
previous	2	4.955	0.0636397	4.91	5
Existing Home Sales Change (MoM)					
actual	2	-1.3	1.272792	-2.2	-0.4
reuterspoll	2	-1.4	0.5656854	-1.8	-1
surprise	2	0.1	1.838478	-1.2	1.4
previous	2	-0.9	1.838478	-2.2	0.4
Factory Orders (MoM)					
actual	3	0.4666667	2.400694	-2.3	2
reuterspoll	3	0.3	2.523886	-2.5	2.4
surprise	3	0.1666667	0.5507571	-0.4	0.7
previous	3	1.466667	0.6806859	0.7	2
Fed Interest Rate Decision					
actual	2	2.625	0.5303301	2.25	3
reuterspoll	2	2.875	0.5303301	2.5	3.25
surprise	2	-0.25	0	-0.25	-0.25
previous	2	3.25	0.3535534	3	3.5
Gross Domestic Product Annualized					
actual	2	0.6	0	0.6	0.6
reuterspoll	2	1	0.2828427	0.8	1.2
surprise	2	-0.4	0.2828427	-0.6	-0.2
previous	2	2.75	3.040559	0.6	4.9
Housing Price Index (MoM)					
actual	1	-1.1	.	-1.1	-1.1
reuterspoll	1	-1.1	.	-1.1	-1.1
surprise	1	0	.	0	0
previous	1	-0.4	.	-0.4	-0.4
Housing Starts (MoM)					
actual	3	1.05	0.0398874	1.004	1.075
reuterspoll	3	1.051667	0.0854888	0.995	1.15
surprise	3	-0.0016667	0.1253568	-0.146	0.08
previous	3	1.079	0.0793032	1.004	1.162
ISM Manufacturing PMI					
actual	3	49.13333	1.357694	48.3	50.7
reuterspoll	3	48.83333	1.527525	47.5	50.5
surprise	3	0.3	2.685144	-2.1	3.2
previous	3	49.96667	1.357693	48.4	50.8
ISM Non-Manufacturing PMI					
actual	2	49.25	6.576095	44.6	53.9
reuterspoll	2	53.25	0.3535534	53	53.5
surprise	2	-4	6.222539	-8.4	0.4
previous	2	54	0.1414192	53.9	54.1

Table 3.17 - Continued

Variable	Obs	Mean	Std. Dev.	Min	Max
Import Price Index (MoM)					
actual	3	0.5	0.8888194	-0.2	1.5
reuterspoll	3	0.4333333	0.4041452	0	0.8
surprise	3	0.0666667	0.8326664	-0.6	1
previous	3	1.333333	1.457166	-0.2	2.7
Import Price Index (YoY)					
actual	2	12.3	1.979899	10.9	13.7
reuterspoll	2	11.5	1.838478	10.2	12.8
surprise	2	0.8	0.1414213	0.7	0.9
previous	2	11.15	0.3535534	10.9	11.4
Industrial Production (MoM)					
actual	3	-0.1666667	0.4618802	-0.7	0.1
reuterspoll	3	-0.0333333	0.1154701	-0.1	0.1
surprise	3	-0.1333333	0.4163332	-0.6	0.2
previous	3	0.1666667	0.1154701	0.1	0.3
Initial Jobless Claims					
actual	10	343.8	26.36412	302	378
reuterspoll	10	341.2	13.35665	320	360
surprise	10	2.6	25.50033	-32	58
previous	10	341.8	24.14217	302	373
Michigan Consumer Sentiment Index					
actual	5	73.96	5.085568	69.6	80.5
reuterspoll	5	74.28	3.986477	70	79
surprise	5	-0.32	4.623527	-7.4	5.5
previous	5	74.96	4.713067	69.6	80.5
Monthly Budget Statement					
actual	1	17.8	.	17.8	17.8
reuterspoll	1	32	.	32	32
surprise	1	-14.2	.	-14.2	-14.2
previous	1	38.2	.	38.2	38.2
NAHB Housing Market Index					
actual	3	19.66667	0.5773503	19	20
reuterspoll	3	19.33333	0.5773503	19	20
surprise	3	0.3333333	0.5773503	0	1
previous	3	19	1	18	20
NY Empire State Manufacturing Index					
actual	3	-8.306667	15.90707	-22.23	9.03
reuterspoll	3	3.7	8.772685	-6.3	10.1
surprise	3	-12.00667	9.596616	-19.02	-1.07
previous	3	2.37	12.20837	-11.72	9.8
Net Long-Term TIC Flows					
actual	1	56.5	.	56.5	56.5
reuterspoll	1	76	.	76	76
surprise	1	-19.5	.	-19.5	-19.5
previous	1	90.9	.	90.9	90.9
New Home Sales (MoM)					
actual	2	0.603	0.0028284	0.601	0.605
reuterspoll	2	0.6225	0.0318198	0.6	0.645
surprise	2	-0.0195	0.0289914	-0.04	0.001
previous	2	0.6195	0.0205061	0.605	0.634
New Home Sales Change (MoM)					
actual	1	-1.6	.	-1.6	-1.6
reuterspoll	1	-0.7	.	-0.7	-0.7
surprise	1	-0.9	.	-0.9	-0.9
previous	1	-4.7	.	-4.7	-4.7
Nonfarm Payrolls					
actual	3	-5.333333	80.30774	-76	82
reuterspoll	3	51.66667	20.20726	30	70
surprise	3	-57	61.48984	-106	12
previous	3	58.33333	71.50058	-22	115
Nonfarm Productivity					
actual	2	1.8	0	1.8	1.8
reuterspoll	2	1.4	0.5656854	1	1.8
surprise	2	0.4	0.5656854	0	0.8
previous	2	4.05	3.181981	1.8	6.3
Pending Home Sales (MoM)					
actual	3	-1.366667	1.305118	-2.6	0
reuterspoll	3	-0.7666667	0.2516611	-1	-0.5
surprise	3	-0.6	1.452584	-2.1	0.8
previous	3	-0.1333333	3.365016	-2.6	3.7
Personal Consumption Expenditures - Price Index (MoM)					
actual	2	0.35	0.0707107	0.3	0.4
reuterspoll	2	0.15	0.0707107	0.1	0.2
surprise	2	0.2	0	0.2	0.2
previous	2	0.65	0.4949747	0.3	1

Table 3.17 - Continued

Variable	Obs	Mean	Std. Dev.	Min	Max
Personal Income (MoM)					
actual	2	0.25	0.3535534	0	0.5
reuterspoll	2	0.3	0.1414214	0.2	0.4
surprise	2	-0.05	0.212132	-0.2	0.1
previous	2	0.45	0.0707107	0.4	0.5
Philadelphia Fed Manufacturing Survey					
actual	1	-24	.	-24	-24
reuterspoll	1	-10	.	-10	-10
surprise	1	-14	.	-14	-14
previous	1	-20.9	.	-20.9	-20.9
Producer Price Index (MoM)					
actual	3	0.3333333	0.6506407	-0.3	1
reuterspoll	3	0.3	0.1	0.2	0.4
surprise	3	0.0333333	0.6110101	-0.5	0.7
previous	3	1.3	1.769181	-0.3	3.2
Producer Price Index (YoY)					
actual	3	6.7	0.6082762	6.3	7.4
reuterspoll	3	7.233333	0.3785938	6.8	7.5
surprise	3	-0.5333333	0.5131602	-1.1	-0.1
previous	3	6.966667	0.5859464	6.3	7.4
Producer Price Index ex Food & Energy (MoM)					
actual	2	0.3	0.1414214	0.2	0.4
reuterspoll	2	0.2	0	0.2	0.2
surprise	2	0.1	0.1414214	0	0.2
previous	2	0.3	0.1414214	0.2	0.4
Producer Price Index ex Food & Energy (YoY)					
actual	3	2.233333	0.2081666	2	2.4
reuterspoll	3	2.1	0.1	2	2.2
surprise	3	0.1333333	0.1527525	0	0.3
previous	3	2.1	0.1732051	2	2.3
Retail Sales (MoM)					
actual	3	-0.1333333	0.4618802	-0.4	0.4
reuterspoll	3	0.0333333	0.2081666	-0.2	0.2
surprise	3	-0.1666667	0.6658328	-0.6	0.6
previous	3	0.3333333	0.7023769	-0.4	1
Retail Sales ex Autos (MoM)					
actual	3	0.0333333	0.4163332	-0.3	0.5
reuterspoll	3	0.1666667	0.057735	0.1	0.2
surprise	3	-0.1333333	0.3785939	-0.4	0.3
previous	3	0.6666666	1.059874	-0.3	1.8
Richmond Fed Manufacturing Index					
actual	1	-5	.	-5	-5
reuterspoll	1	-7	.	-7	-7
surprise	1	2	.	2	2
previous	1	-8	.	-8	-8
Trade Balance					
actual	3	-59.99333	2.759153	-63.12	-57.9
reuterspoll	3	-60	0.9539378	-61.1	-59.4
surprise	3	0.0066667	3.490692	-3.72	3.2
previous	3	-59.61333	3.037125	-63.12	-57.82
Unemployment Rate					
actual	3	4.9	0.0999999	4.8	5
reuterspoll	3	4.933333	0.1154699	4.8	5
surprise	3	-0.0333333	0.2081666	-0.2	0.2
previous	3	4.866667	0.1527526	4.7	5
Wholesale Inventories					
actual	3	0.9	0.1732051	0.8	1.1
reuterspoll	3	0.3666667	0.057735	0.3	0.4
surprise	3	0.5333333	0.2309401	0.4	0.8
previous	3	0.6333333	0.5686241	0	1.1

Table 3.18: Lists of macroeconomic news announcements

Announcements	Abbreviations	Unit	Good/Bad	Scheduled/Unscheduled	All
Money & Finance					
Foreign Domestic Investment	FDI	% change	3/2	5/0	5
Foreign Exchange Reserves	FX	$ Tln	1/2	3/0	3
Government Sector					
M2 Money Supply	M2	% change	2/3	5/0	5
Consumer Sector					
Retail Sales	RS	% change	0/4	4/0	4
Industry Sector					
Industrial Output	IND	% change	0/4	4/0	4
Prices					
Consumer Price Index	CPI	% change	1/3	4/0	4
Producer Price Index	PPI	% change	1/3	4/0	4
External Sector					
Trade Balance	TB	$ Bln	1/2	3/0	3
Imports	IMP	% change	1/2	3/0	3
Exports	EXP	% change	1/2	3/0	3
National Account					
Gross Domestic Product	GDP	% change	0/2	2/0	2
Urban investment	UI	% change	0/4	4/0	4
Surveys & Cyclical					
Manufacturing Purchasing Managers' Index	PMI	Level	3/2	5/0	5
Outstanding Loan Growth	OLG	% change	2/3	5/0	5
Other					
House Prices	HP	% change	2/2	4/0	4
PBOC Deposit Rate	DEPR	% change	1/1	0/2	2
PBOC Lending Rate	LENR	% change	1/1	0/2	2
PBOC Reserve Requirement Ratio	REQR	% change	1/1	0/2	2

Notes: This table shows the macroeconomic news announcement we adopted in this chapter, and we also list their classifications, the reported units, the number of good and bad news, the number of scheduled and unscheduled news and the total number of the announcements.

Appendix IV: Changes of the variables in the volatility smile

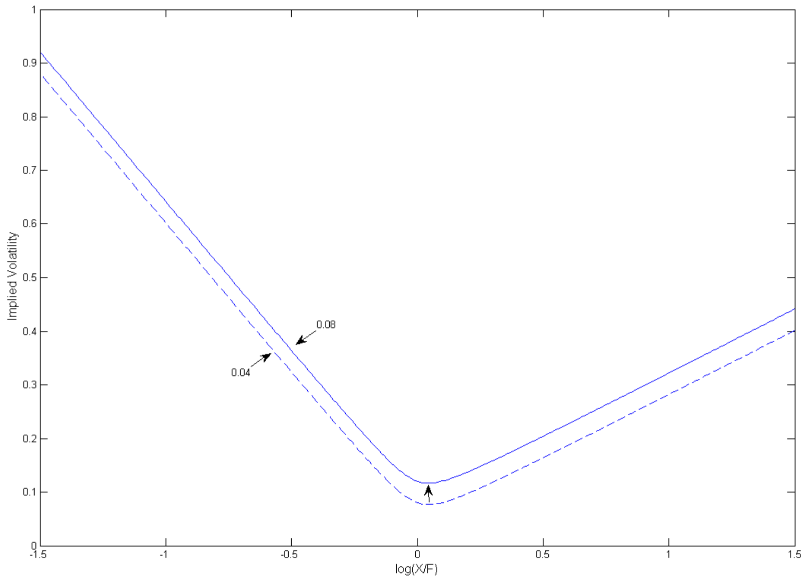


Figure 3.10: Change a

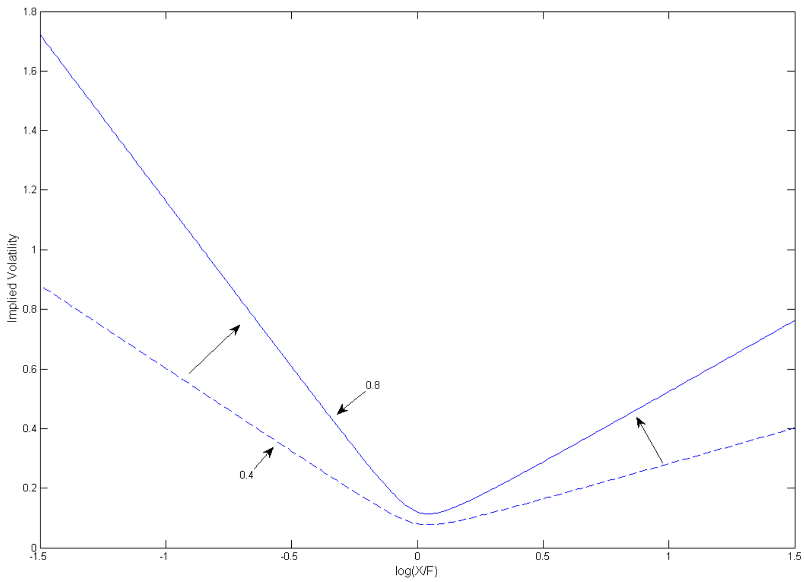


Figure 3.11: Change b

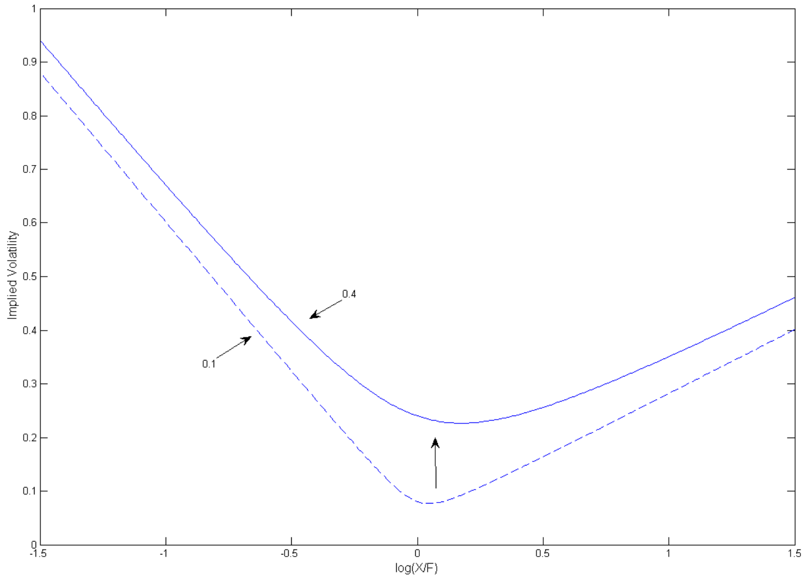


Figure 3.12: Change s

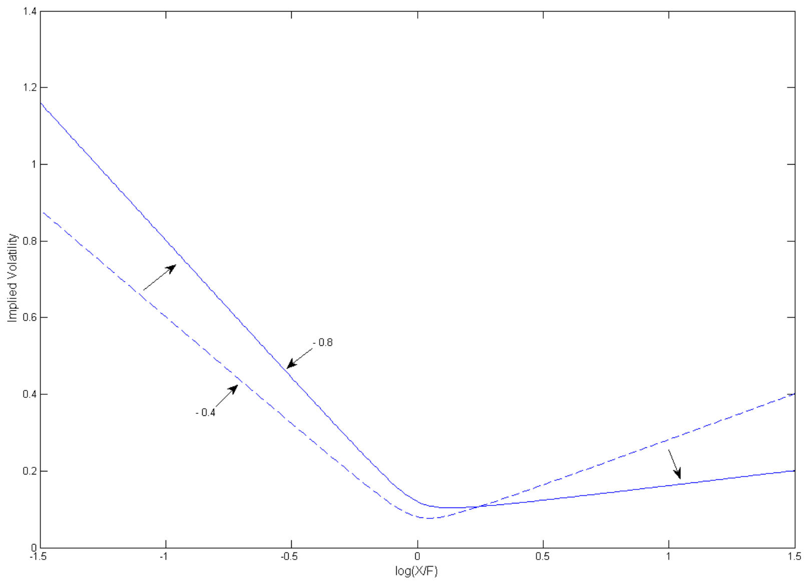


Figure 3.13: Change ρ

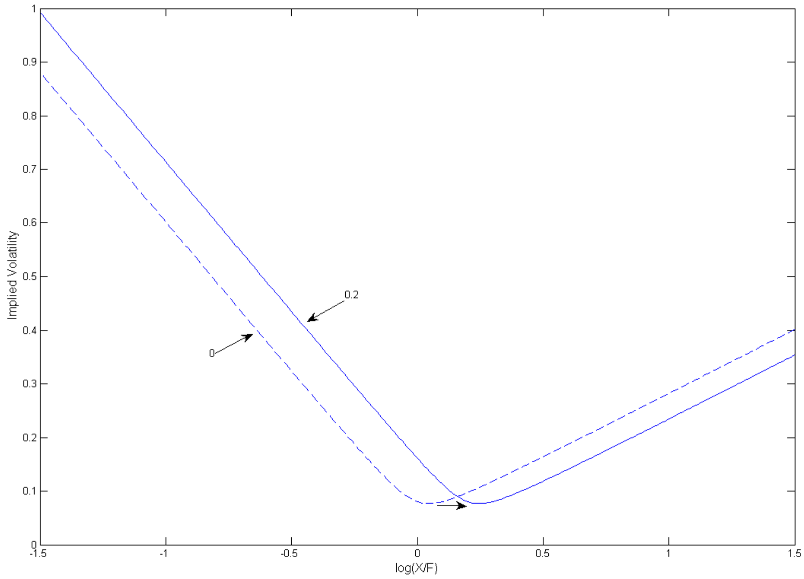


Figure 3.14: Change m

Appendix V: Surprises over the sample period

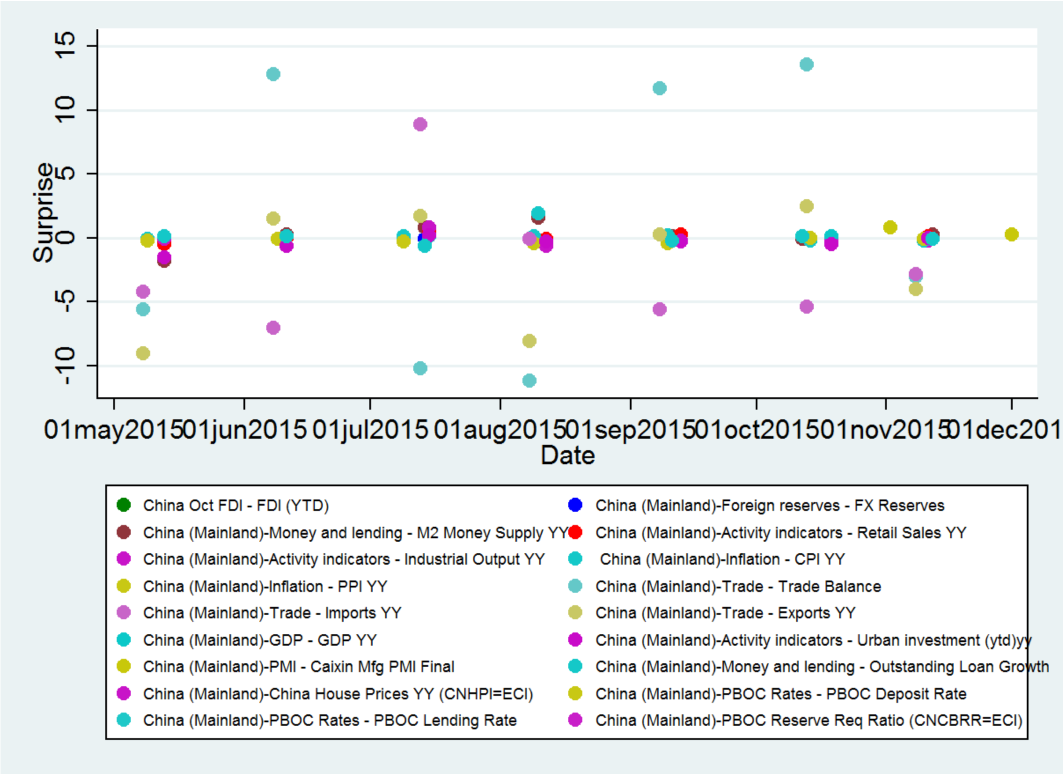


Figure 3.15: Surprises over the sample period in China

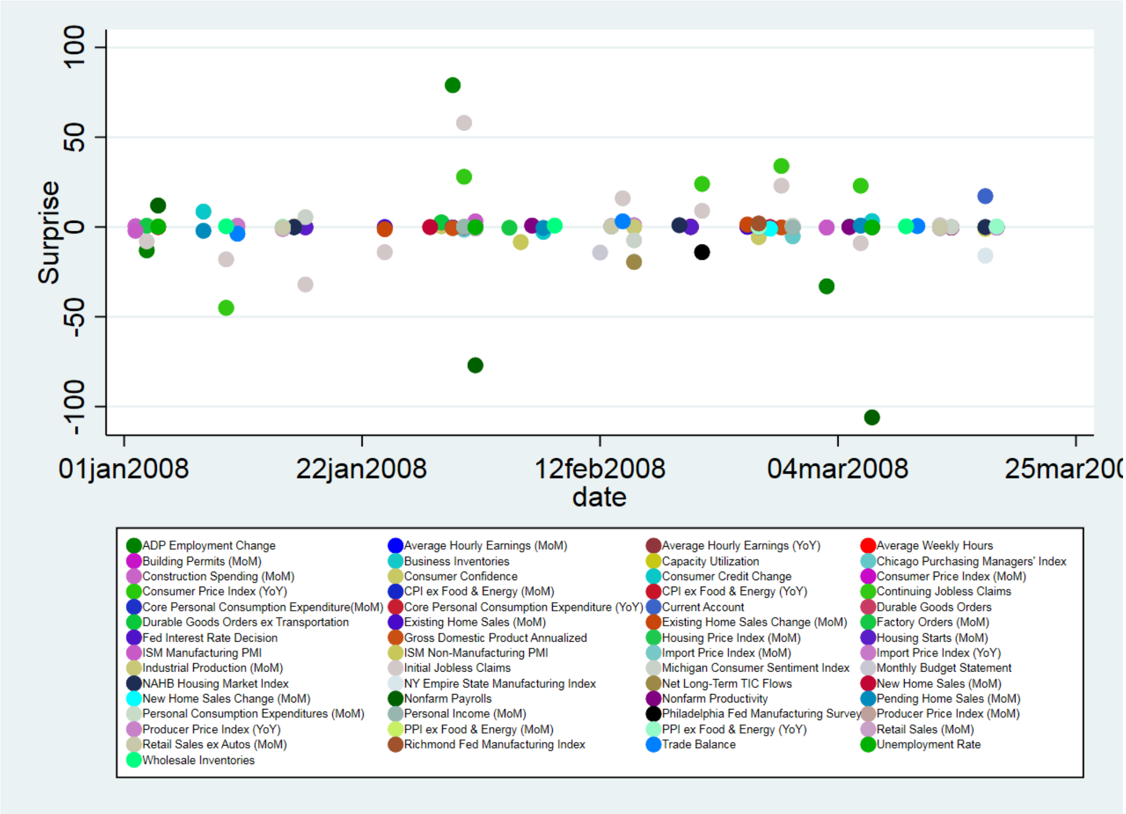


Figure 3.16: Surprises over the sample period in the US

Appendix VI: Dummies description

	0	1
announcements dummy	no announcement	have announcement
surprise dummy	no surprise	have surprise
good dummy	bad news	good news

Table 3.19: Dummy Clarification

Chapter 4

Can we recover: Evidence from stock option prices

4.1 Introduction

Options market infers information about the perception of the investors on the probability distribution of future prices and the corresponding pricing kernel (Jackwerth and Menner, 2017; Borovička, Hansen and Scheinkman, 2016).

$$Physical\ probability\ distribution = \frac{Risk\ neutral\ density}{pricing\ kernel} \tag{4.1}$$

Technically, the risk-neutral densities are the natural probabilities, which is equivalent to the physical probabilities, that have been adjusted for the risk premium. If the representative investor is risk averse, we would expect a positive risk premium. In fact, the natural probability densities, sometimes we say the real world densities, are better to incorporate all the components in the market and reflect the investors' perception of the market (see de Vincent-Humphreys and Noss, 2012). The existence of the risk aversion (of a representative agent) has also examined in literature, such as the projection of the pricing kernel under the $\mathbb{P}$ measure, which is the same as physical measure or real world measure, in Bondarenko (2014). Theoretically, under the Black-Scholes assumption, the real world density and the risk neutral density are exchangeable, due to the assumption of risk neutral for all the investors. In this case, the pricing kernel is equal to one.

Neither the pricing kernel nor the physical probability distribution is theoretically easily observed from option market prices. Since we cannot be able to compute the risk premium from the option prices as we could do from the bond/-stock markets. It does not seem that, in traditional option pricing framework, the investor's risk attitude is included as a factor in the pricing function. In other words, in the Black-Scholes formula, it does not exist one term to account for the risk attitude of the investors. There are several ways to find the physical probability distribution. One way to find the real probability distribution, Aït-Sahalia and Lo (2000), Jackwerth (2000), Constantinides, Jackwerth and Perrakis (2007) state that it approximates to the kernel density estimated from the time series of past returns. Another popular method to compute the real-world density is to use a parametric way of transforming from risk-neutral density (see Bliss and Panigirtzoglou, 2004; Liu, et al. 2007).

Recent study by Ross (2015) presents a recovery theorem, the so-called Theorem I, which allows separating the real-world probability distribution and risk aversion only using the state prices. This was not possible until his study. In the paper, two main nonparametric assumptions allow him to be able to disentangle the pricing kernel from option prices, 1) the risk-neutral process is restricted to be a time-homogeneous and irreducible Markov Chain in a finite state space; 2) the pricing kernel is path independent, i.e., independent of asset path (see Ross, 2015; Audrino, Huitema and Ludwig, 2015). Under this circumstance, the pricing kernel can be expressed as a form of a discounted rate and a positive function, and the solutions for both elements are unique. Ross attempts to find the physical probability distribution through the following steps: 1) from market prices to state price; 2) from the state price to state price transition matrix; 3) from state price transition matrix to real-world transition probability matrix.

Although the studies in the Risk-Neutral Density is abundant, little investigation has been empirically conducted to find the real-world density. Most of the studies examined the Ross recovery theorem in the context of S&P 500 index options. One contribution of current study is to adopt the recovery theorem with the stock options. Here in this study, I adopt the Adidas AG stock options in the European market. Secondly, as in Ross (2015) paper, it does not fully clarify the theory, another contribution is to clearly explain the recovery theorem.

More specifically, we investigate the implied volatility surface and the risk-neutral density surface. Evidence shows negatively implied volatility-time to expiry and implied volatility-strike relationships. Empirically, we have successfully estimated the real-world density for the Adidas AG stock options. We also discussed the evolution of the risk-neutral density and the real-world density.

The remainder of the chapter is organized as follow. Next section provides a review of related literature. Section 4.3 gives a brief presentation of the related theories would be applied in the Ross recovery theorem. Section 4.4 shows how the recovery theorem can be implemented in practice. The data used in current study is introduced in Section 4.5. The results are stated in the Section 4.6. Finally, a conclusion is provided in Section 4.7.

4.2 Literature Review

The Theorem I has been drawn much attention recently. Schneider and Trojani (2018) aim to estimate the conditional moments of the physical probability distribution through an almost model-free recovery method on the S&P 500 options data over 1990-2014. They indicate that their method is entirely forward-looking and the pricing kernel can be expressed as a polynomial function of both risk-neutral and physical moments. However, there is no unique pricing kernel without the restriction of the minimum kernel variance. Their results also show that the recovered moments do predict the future realized moments. For stock options, Massacci, Williams and Zhang (2016) adopt the Ross Unimodal approach to extract to an 11×11 transition matrix for Apple stock option data.

However, several studies find difficulties when empirically replicate the Ross recovery theorem. For example, Jackwerth and Menner (2017) implement the Ross recovery empirically and find several empirical problems. Their recovered physical probabilities in the results fail to match the empirical distribution, i.e. the historical return distribution. Aiming to test the theorem in a skeptical view through the long-term bond futures and options data, Bakshi, Chabi-Yo and Gao (2017) have challenged to conduct the Recovery theorem. Still, an agnostic result has been given in the context of the accuracy of the Recovery theorem. It has also been criticized by Borovička, Hansen and Scheinkman (2016), who document that the recovery probability distribution seems to be biased due to the unrealistic restriction in the pricing kernel.

The existence of the drawbacks of the Ross Recovery has attracted some extensions by an increasing number of studies. For example, exploring Ross' results in a continuous-time diffusion setting, Carr and Yu (2012) introduce the John Long's Numeraire Portfolio, instead of using the representative agent model. Following Carr and Yu (2012), Walden (2017) extends the results to an unbounded intervals in a success. Also, Linetzky and Qin (2016) extend the underlying process in the Ross (2015) theorem from a discrete time and irreducible finite state Markov chains framework to a continuous-time Markov process with a general state space (i.e., Borel right process), a similar study can be found in Park (2016).

Relaxing the assumption of time homogeneous in the state prices, the study Jensen, Lando and Pedersen (2018) show a closed-form solution to find both the physical probability and risk preferences only using asset prices. Adopting the standard call and put options written on S&P 500 index over the period 1996-2014, they find that the recovered physical probability might help to predict the distribution of the future return.

For studies related to the Ross recovery theorem, readers are also suggested to read Dubynskiy and Goldstein (2013), Martin and Ross (2013), Tran and Xia (2015), Flint and Mare (2016), Liu (2014), and Christensen (2017) to name a few.

4.3 Theoretical Framework

This section briefly documents several theories as a background to the recovery theorem. The following theories are serving as the fundamentals to achieve the recovery theorem.

4.3.1 Probability Measures

Theoretically, in modern financial theory, we have two basic probability measure, the risk-neutral measure, we denoted by $\mathbb{Q}$, and the physical probability measure, we denoted by $\mathbb{P}$. Intuitively, the risk-neutral measure is distinct from the physical measure, which describes the actual stochastic dynamics of markets. In risk-neutral condition, any risky asset, here we say options, has the same expected return as the money market account. In other words, the risk-neutral density is the natural probability distribution that has been 'risk-adjusted' - the risk premium has been subtracted from the density under the physical probability measure. Formally, we have following definitions,

Definition 1. *In a complete market, a* ***risk-neutral measure*** *(i.e., equivalent martingale measure) is a probability measure, in which case the current price of an asset at time t is equivalent to the discounted value of expected futures payoff of the asset at the risk-free rate, given the information $\mathcal{F}_t$ available at time t.*

Under this circumstance, the risky asset (options in this case) has the same expected return as the riskless bond. Namely, risk-neutral measure indicates that investors require no premium for bearing the risk.

Definition 2. ***Physical probability measure****, also named real-world probability measure and natural probability, has the expectation determined by the investors belief or subjective perspective on a risky assets future price.*

Broadly speaking, physical probability distribution incorporates the risk-neutral density and pricing kernel. In a specific condition, if investors were risk neutral, we say the risk-neutral probabilities coincide with physical probabilities. However, if it is not the case, investors are risk-averse, then risk-neutral probabilities would be risk-adjusted taking into account the price effect of investors' risk preference (Siu, 2008; Carr and Yu, 2012, and Ross, 2015).

4.3.2 Markov process and Markov Chain

We have a set of states, $\Omega = \{1, 2, 3, ...\}$. The process initially starts in one of these states, let's say state i, and moves from one state to another (state j). Each move is called a step.

In terms of the discrete time Markov chains, we denote p_{ij} measure the probability the process changes from state i to state j. Mathematically, we also call p_{ij} transition probabilities. Appendix II shows the diagram of a transition matrix. For one-step transition matrix,

$$P = \{p_{ij}\}. \tag{4.2}$$

Definition 3. *A Markov chain $(X(t) : t \in T^1)$ is said to be time-homogeneous if*

$$P(X(s+t) = j | X(s) = i) \tag{4.3}$$

[1] T is the set of natural number, T = {1, 2, ... }.

is independent of s. When this holds, putting s = 0 gives

$$P(X(s+t)=j|X(s)=i)=P(X(t)=j|X(0)=i), \quad \forall t\in T. \tag{4.4}$$

This indicates that, in the condition of Time Homogeneity, the probability does not depend on the time t, but only rely on the state i and j.

When extended to the two step probabilities,

$$\begin{aligned} p_{ij}^2 &= P(X_2=j|X_0=i) \\ &= \sum_{k\in\Omega} P(X_1=k|X_0=i)(X_2=j|X_1=k,X_0=i) \\ &= \sum_{k\in\Omega} p_{ik}p_{kj} \end{aligned} \tag{4.5}$$

Using the same way, we have $P^{(n)}=P^n$ for a n-step transition probability.

Definition 4. *A* ***stationary distribution of a Markov chain*** *is a probability distribution π such that,*

$$\pi \cdot P = \pi. \tag{4.6}$$

Definition 5. *A matrix M is called* ***non-negative matrix*** *if all entries of the matrix are non-negative, i.e., $m_{i,j}\geq 0$, $\forall i\ \forall j$.*

It is clearly that the state price transition matrix P is a nonnegative matrix (i.e., some entries might be zero).

Definition 6. *A Markov chain P is called* ***irreducible matrix*** *if there exists some t such that,*

$$P^t(i,j)>0, \forall i \forall j. \tag{4.7}$$

4.3.3 Perron-Frobenius theorem

The Perron-Frobenius theorem (PF in the sequel) is proposed by Oskar Perron in 1907 (see Perron, 1907) and extended by Ferdinand Georg Frobenius, which are shown in Appendix III. Their theorem provided numerous useful results for non-negative matrices.

Theorem 1. *(Perron-Frobenius Theorem) As stated in the Perron-Frobenius theorem (Meyer, 2000), let A be a non-negative $n\times n$ matrix with all the element*

a_{ij} *is strictly positive, i.e.* $a_{ij} > 0$. *Then A has a positive eigenvalue* λ *which is equal to the spectral radius of A. Eigenvalue* λ *has a unique positive eigenvector v.*

From the Perron-Frobenius Theorem, we know that a non-negative irreducible matrix have a unique positive eigenvalue λ. Refer to its proof, we recommend the readers to the studies of Borobia and Trfas (1992), MacCluer (2000) and Horn and Johnson (2012).

4.3.4 Representative Agent model

Representative agent model has been widely adopted in literature (see Geweke 1985; Eichenbaum, Hansen and Singleton, 1988; Hartley, 1996; Aït-Sahalia and Lo, 2000; Epstein and Schneider, 2008; Verdelhan, 2010; and Backus, Chernov and Zin, 2014, for example), and it is also the case in current study. Representative agent theory, which is differently with the heterogeneous agents model, reduces the heterogeneity of the behavior between the agents to a single representative agent. As it states in the representative agent model, in a competitive and frictionless market, we assume all agents act in the same manner and try to maximize her expected utility function. In such model, a representative agent's utility reflects the markets' true beliefs and also the risk aversion. Based on the Representative Agent model, several popular theories have been developed, including the Consumption-based Capital Asset Pricing Model (C-CAPM) by Breeden (1979) and arbitrage pricing theory (APT) by Ross (1976).

4.3.5 Arrow-Debreu Security

Definition 7. *An* ***Arrow-Debreu security*** *(hereafter A-D), also called state-contingent claim, that pays you £1 of the state occurs. Assuming the current state is i, the A-D security would pay £1 if the state ending at j and zero if other states.*

For example, the foreign exchange rate GBP to USD is 1.30, a state price is the price of A-D Security that pays you £1 in a year if the GBP/USD goes up to 1.40.

We denote the price of one A-D Security by $\pi_{i,j}$, state price transition matrix by $p_{i,j}$ and the pricing kernel by $\varphi_{i,j}$. We will have $\pi_{i,j} = p_{i,j} \times \varphi_{i,j}$. Theoretically, risk-neutral probability is the discounted state price at the agree exercise time, thereby, Risk-neutral density is related but not equal to the Arrow-Debreu security price. Breeden and Litzenberger (1978) have shown the relationship between Risk-neutral probability distributions and A-D prices. The probabilities of all states occurred must sum up to one, that is $\sum_{j=1}^{n} p_{i,j} = 1$. Studies in risk-neutral probability distribution can be found in previous two chapters.

4.3.6 Pricing Kernel

Definition 8. ***Pricing kernel****, which is a fundamental concept of asset pricing in finance and also named stochastic discounted factor, measures the market risk aversion over equity returns.*

Its existence results from the law of one price and the no arbitrage condition (see Ahn, Conrad, and Dittmar, 2003; Dittmar, 2002). The pricing kernel, theoretically, is positive but generally decreasing, sometimes with some increasings. This is so-called pricing kernel puzzle (Dittmar, 2002; Rosenberg and Engle, 2002) and we will not focus too detail in this study.

Empirically, two methodologies often adopted to find the Pricing Kernel, 1) one, based on the representative agent theory, is to compute the parameters by fitting maximizing some agent's utility; 2) an alternative one is to estimate based on cross-sectional option data and historical returns. We will present the estimation methodology below.

4.4 Estimation Framework

Previous literature has documented that one of the three concepts, i.e. the risk-neutral density, real-world probability distribution and the pricing kernel, can theoretically be extracted from the other two. In this section, I will show how we can estimate the real-world probability distribution and the risk aversion directly from the state price in a representative agent condition. In line with the description of estimates mentioned above, throughout the study, we have the following notations,

· State price matrix, S, where S $:= (s_{i,j})$ is a $n \times m$ matrix and normally $n \leq m$, i.e., the number columns of the tenor is not less than the number rows of the moneyness, which is equal to $\frac{S}{X}$, where X denotes the strike price for the option price;

· State price transition matrix, P, where P $:= (p_{i,j})$ is an $n \times n$ matrix, and $p_{i,j}$ is the state i price of an Arrow-Debreu security paying off in state j.;

· Natural probability transition matrix, F, where F $:= (f_{i,j})$ is an $n \times n$ matrix;

· Discount rate or discounted factor, δ;

· Pricing kernel, φ.

Several studies have empirically implemented the Ross Recovery Theorem (see, Carr and Yu, 2012; Borovička, Hansen and Scheinkman, 2016; Park, 2016; and Walden, 2017, for example). Here we consider a discrete time framework. Current study adopts the option prices written on the Adidas AG (listed in German Stock Exchange (Xetra)).

4.4.1 From Option Prices to State Prices

As it might not be able to observe a continuum strikes, we estimate the state price expressed as a tenor matrix from option prices, i.e. a matrix with rows of state prices and columns of times to maturity in years. Quite a few methodologies have been examined in previous studies, such as mixture of lognormals, Smoothed Implied Volatility Smile Method, Generalised beta distribution, Hermite Polynomials method, etc.

We collect the volatility surface, then interpolate the implied volatility, convert the smoothed implied volatility to the state price. Breeden and Litzenberger (1978) indicate that the state price density is uniquely equivalently to the second derivative of the option pricing equation concerning the strikes[2]. Details for how to estimate the state price can be found in Chapter 2 of this thesis.

[2]For European call option valuation, its price is $c = e^{-r\tau} \int_X^\infty f(S_T)(S_T - X)d(S_T)$. For European put option, $p = e^{-r\tau} \int_0^X f(S_T)(X - S_T)d(S_T)$. Therefore, the second derivative for the calls and puts are $f(S_T) = e^{r\tau} \frac{\partial^2 c}{\partial X^2}$ and $f(S_T) = e^{r\tau} \frac{\partial^2 p}{\partial X^2}$, respectively.

4.4.2 From State Prices to Markov Chain

This subsection aims to estimate the state price transition matrix P from the state price. In this stage we assume the transition matrix follows the time-homogeneous Markov Chain. In other words, The time-homogeneous transition matrix presents that, in a market, the probability from one state to another state are the same and not depend on time t. . Mathematically, we denote the first column of the state price matrix S (an $n \times m$ matrix) as $\mathbf{s}_1$, thereby, $\mathbf{s}_1$ is a vector.

$$S = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1m} \\ s_{21} & s_{22} & \dots & s_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \dots & s_{nm} \end{bmatrix}$$

$$\mathbf{s}_1 = (s_{11} s_{21} \dots s_{n1})^\intercal \tag{4.8}$$

Due to the time-homogeneous property, we have $\mathbf{s}_2^\intercal = \mathbf{s}_1^\intercal P$. Using the same way, we have $\mathbf{s}_{t+1}^\intercal = \mathbf{s}_t^\intercal P$ for each $t \in [1, m-1]$. More interestingly, you can also find the paper Jensen, Lando and Pedersen (2019), who relax the assumption of the time-homogeneity in P. Tran and Xia (2015) have shown that, in the recovery process, the optimization can be expressed as,

$$P = \underset{p_{i,j} \geq 0}{\arg\min} \|AP - B\|_2^2 \tag{4.9}$$

where,

$$A = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1,m-1} \\ s_{21} & s_{22} & \dots & s_{2,m-1} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \dots & s_{n,m-1} \end{bmatrix}^T ,$$

$$B = \begin{bmatrix} s_{12} & s_{12} & \dots & s_{1m} \\ s_{22} & s_{22} & \dots & s_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n2} & s_{n2} & \dots & s_{nm} \end{bmatrix}^T .$$

Other optimization methods included the Tikhonov method by Audrino, Huitema and Ludwig (2015) and modified Tikhonov method by Kiriu and Hibiki (2019).

4.4.3 From Markov Chain to Real-world Transition Matrix

Previous subsection has derived the state price transition matrix P. In this section, we show how to compute the Real-world Transition Matrix F from P. More specifically, F would be uniquely determined by the P under the Perron-Frobenius theorem.

The pricing kernel from state i to state j, theoretically, can be estimated by following equation,

$$\varphi_{i,j} = \frac{p_{i,j}}{f_{i,j}} \tag{4.10}$$

where, $p_{i,j}$ stands for the risk-neutral density and $f_{i,j}$ measures the physical probability distribution.

Proposition 1. *Under the condition of Time Separable Utility, we would define the pricing kernel $\varphi_{i,j}$ as:*

$$\varphi_{i,j} = \delta \frac{U'(c_j)}{U'(c_i)} \tag{4.11}$$

The assumption of the Time Separable Utility is amounted to the path independent in pricing kernel. We say, different with the empirical (stochastic) pricing kernel, if the pricing kernel is path independent, then combine (4.10) and (4.11), we have

$$\varphi_{i,j} = \frac{p_{i,j}}{f_{i,j}} = \delta \frac{u_j}{u_i} \tag{4.12}$$

where, $u_i \equiv U'(c_i)$, δ is the discounted rate and is a positive constant (theoretically, $0 < \delta \leq 1$), and u_i, u_j, which are strictly positive functions and

$u_i, u_j \in U$, represent the marginal utility at state i and state j, respectively. Several ways can be adopted to generate the transition-independent kernel, including the intertemporally additive utility function (see Ross, 2015).

Recall that,

$$P = \delta \frac{u_j}{u_i} F, \tag{4.13}$$

we have the diagonal matrix U,

$$U = \begin{bmatrix} u_1 & 0 & \dots & 0 \\ 0 & u_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u_n \end{bmatrix}$$

Where, u_j, u_i are the diagonal elements of the matrix, and should be positive values. The inverse of U is,

$$U^{-1} = \begin{bmatrix} \frac{1}{u_1} & 0 & \dots & 0 \\ 0 & \frac{1}{u_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{u_n} \end{bmatrix}$$

Theorem 2. *(Recovery theorem) In a complete and frictional market, if the pricing kernel is path independent and discounted rate is a positive constant, then given an irreducible state-price matrix P, we can uniquely recover the real-world transition matrix F.*

Proof.

$$P = \delta U^{-1} F U \tag{4.14}$$

$$F = \frac{1}{\delta} U P U^{-1} \tag{4.15}$$

Let **1** is a n $\times$ 1 vector, $(1, 1, \dots, 1)^\intercal$. With the fact that the sum of each row of the real-world transition probability matrix must be equal to 1. We have,

$$F \cdot \mathbf{1} = \mathbf{1} \tag{4.16}$$

Therefore, substituting the equation (4.15) into (4.16), we have,

$$F\mathbf{1} = \frac{1}{\delta} U P U^{-1} \mathbf{1} \tag{4.17}$$

$$\mathbf{1} = \frac{1}{\delta} U P U^{-1} \mathbf{1} \tag{4.18}$$

Rearranging this equation, we have,

$$P U^{-1} \mathbf{1} = \delta U^{-1} \mathbf{1} \tag{4.19}$$

Let $U^{-1}\mathbf{1} = x$, therefore,

$$Px = \delta x, \quad where \ x_i = \frac{1}{u_i}. \tag{4.20}$$

□

Such problem can be expressed as the problem to find the eigenvalue and the eigenvector of a matrix. Because P is a non-negative and irreducible matrix. Perron and Frobenius theorem mentioned in Section 4.2 documents that there exists an only eigenvector and it is larger and equal to zero. The theorem also documents that δ is unique and equal to the spectral radius.

Proposition 2. *Given the transition matrix in the recovery theorem, either for state price probability matrix P or real-world probability matrix F, such stochastic matrix has an eigenvalue of one*

Proof. As in the transition probability matrix P or F, the sum of its row must be 1. We have

$$P \cdot \mathbf{1} = 1 \cdot \mathbf{1}$$

and

$$F \cdot \mathbf{1} = 1 \cdot \mathbf{1}.$$

□

As also stated in previous studies, we have the absolute value of any eigenvalue for such stochastic matrix is less or equal to one. Moreover, eigenvalue one for both P and F, the corresponding eigenvector, say v, we have,

$$P \cdot v = v \tag{4.21}$$

and

$$F \cdot v = v. \tag{4.22}$$

4.5 Data

I intend to investigate a stock option. Randomly chosen from the Eurex Exchange, the options data written on Adidas AG on 15 March, 2013, was chosen to be examined. The time series of its stock price shows in Figure 4.1 below. The spot (closing) price of the stock is 79.75. As we can see from the figure, the stock behaves around 80 Euro over 2013-2015, following by a shape increase. The stock price doubled from 15 March 2013 to the end of the 2017. All the prices were recorded in Currency EUR. The same as the S&P 500 index options and the China 50 ETF options, one Adidas AG option equals rights over 100 underlying shares. The underlying firm, Adidas AG produces and markets a wide range of athletic and sports products. The Company's segments are around the whole world, include Europe, America, Asia, etc.; Each market includes wholesale, retail and e-commerce business activities.

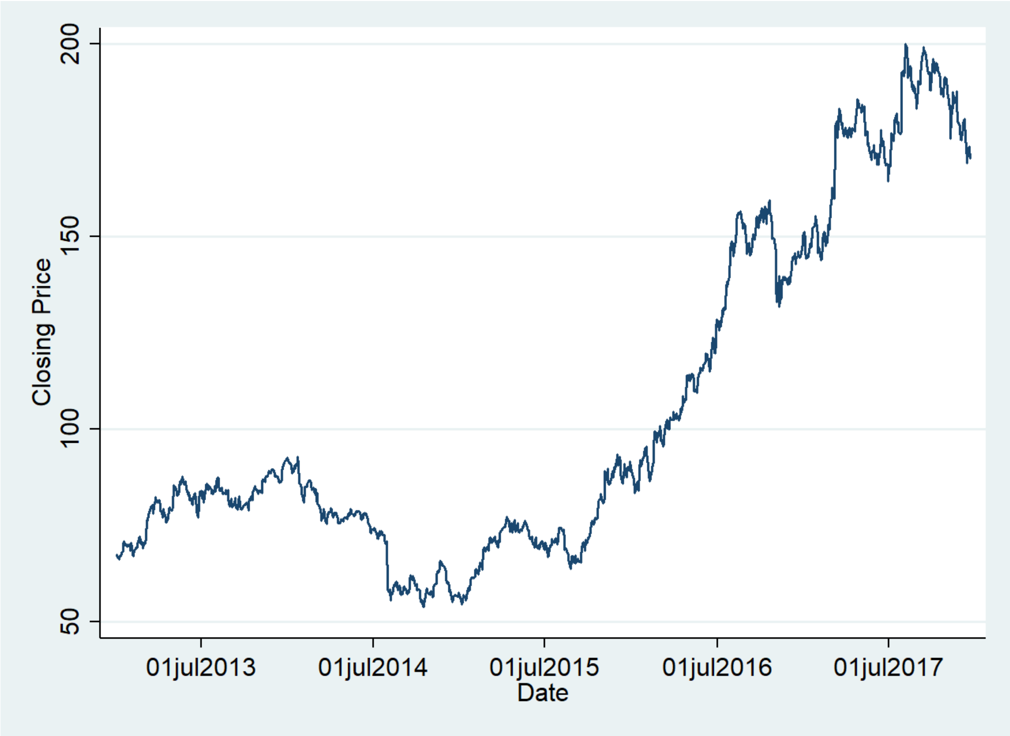


Figure 4.1: Adidas AG Closing Prices

I choose Adidas AG option prices trading on 15 March 2015, with multiple maturities[3]. This study only uses the European call options. We then construct

[3]The maturities are 19/04/2013, 17/05/2013, 21/06/2013, 20/09/2013, 20/12/2013,

an 8 $\times$ 11 matrix, see Table 4.1.

4.6 Empirical Results

Section 4.3 and Section 4.4 outlined the necessary theoretical and practical backgrounds, we try to implement the recovery theorem as follows:

1. We extract the risk-neutral densities and estimate the state price matrix S from Adidas AG option prices on 15 March, 2013 with 11 different maturities;

2. Then we aim to estimate the state transition probability matrix by finding the optimized matrix with regarding to a minimization problem with Matlab, $\arg\min_{p_{i,j}\geq 0} \|AP - B\|_2^2$. In this step, we also restrict all the elements to be nonnegative, therefore the state transition probability matrix would be a nonnegative matrix;

3. We define the matrix A as $S'_{1:8,1:T-1}$ and matrix B as $S'_{1:8,2:T}$ and find a risk-neutral transition probability matrix P;

4. Using the Ross recovery theorem, the real-world Transition Probability Matrix would be found.

4.6.1 Implied Volatility Surface and Risk-Neutral Density

20/06/2014, 19/12/2014, 19/06/2015, 18/12/2015, 16/12/2016, and 15/12/2017

Table 4.1: Matrix of option prices

Expiration	19/04/2013	17/05/2013	21/06/2013	20/09/2013	20/12/2013	20/06/2014	19/12/2014	19/06/2015	18/12/2015	16/12/2016	15/12/2017
τ	0.0959	0.1726	0.2685	0.5178	0.7671	1.2658	1.7644	2.2630	2.7616	3.7589	4.7562
Strikes											
48	31.7600	30.4600	30.4900	28.8100	30.6500	29.9300	30.3800	30.0400	30.7100	31.1600	31.7800
52	27.7600	26.4700	26.5200	26.6400	26.7900	26.2200	26.8700	26.6500	27.4300	28.1500	28.9500
60	19.7700	18.5100	18.6300	18.9600	19.4100	19.3300	20.3100	20.4800	21.4700	22.5700	23.6200
68	11.8100	10.7600	11.0900	11.9700	12.8000	13.2800	14.6900	15.1600	16.3000	17.6400	18.9600
76	4.4000	4.2600	4.9200	6.3500	7.4600	8.4900	10.1100	10.7400	11.9400	13.5000	14.9600
84	0.5300	0.8900	1.4200	2.7500	3.7600	4.9400	6.5300	7.3100	8.4600	10.1000	11.6000
92	0.0600	0.1100	0.2800	1.0000	1.6700	2.7200	4.0400	4.8100	5.8300	7.4100	8.8700
100	0.0300	0.0100	0.0100	0.5700	0.7200	1.4100	2.3900	3.0500	3.9200	5.3400	6.7000

Note: The first row indicates the expiry date, and second row shows the corresponding time to expiry (in year).

Table 4.1 shows the tensor matrix of the option prices with fixed strikes. The corresponding scatters plot can be found in Figure 4.2.

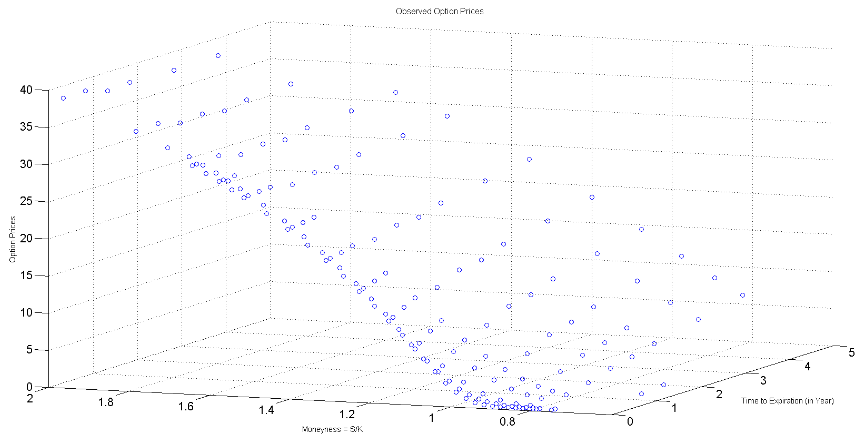


Figure 4.2: Observed Call Prices

This will be used for the following optimisations and estimations. We have eight strikes and eleven times to expiry, an 8×11 matrix. One of our aims is to find an 8 by 8 transition matrix.

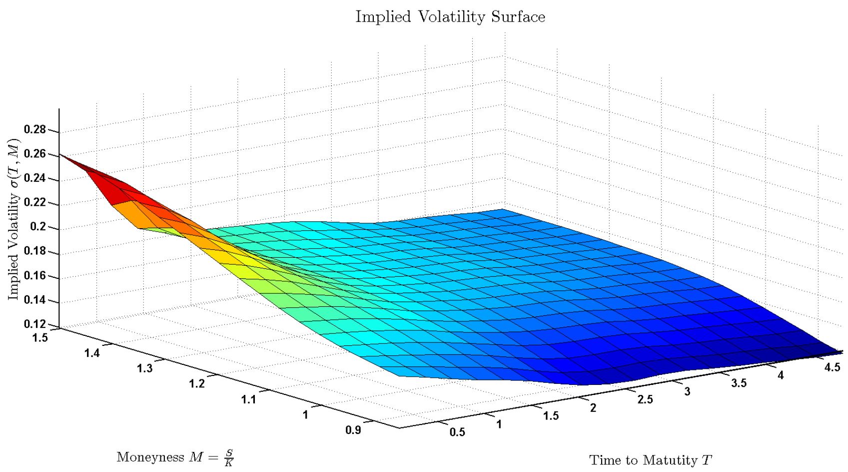


Figure 4.3: Implied Volatility Surface

Figure 4.3 presents the implied volatility surface for the call prices trading on 15 March 2013. The corresponding table of smoothed implied volatility surface can be found in Table 4.7. Evidence has shown that this is consistent with the traditional financial theory. Implied volatility for the call option goes down as the strikes increase.

Table 4.2: Average of implied volatility

Moneyness	(1.6, 1.4]	(1.4, 1.2]	(1.2, 1.1]	(1.1, 1]	(1, 0.9]	(0.9, 0.8]
τ						
(0, 1]	0.23955	0.18743	0.17816	0.17003	0.16700	0.16497
(1, 2]	0.22424	0.18594	0.17550	0.16937	0.16642	0.16437
(2, 3]	0.20452	0.18137	0.17288	0.16732	0.16437	0.16232
(3, 4]	0.18445	0.17093	0.16468	0.16051	0.15776	0.15591
(4, 5)	0.17260	0.15679	0.15269	0.15119	0.14893	0.14750

The implied volatility for shorter term option is typically higher. The implied volatility shows an inverse relation with strike price, therefore, a positive implied volatility-moneyness relationship.

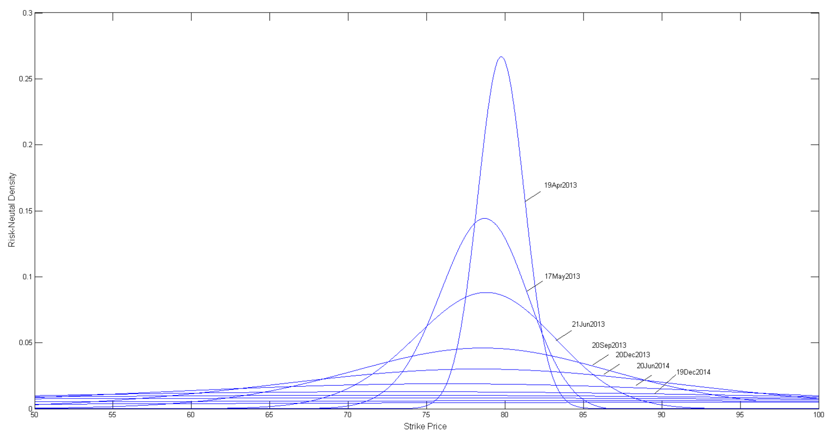


Figure 4.4: Risk-Neutral Densities on 15 March 2013 with multiple expiries

We find out the evolution of the RNDs with multiple expiries. It seems that the densities tend to a flatter curve as the increase of the expiry (see Backwell, 2015). This indicates that, given a particular trading date, the expiry of the future prices is higher when the market expects with a longer period to the prices, which is consistent with the heterogeneous belief of the investors to the future. This has been fully examined in my previous studies in this thesis. The recorded state price matrix can be seen from the Table 4.3.

Table 4.3: State price matrix

Date	19/04/2013	17/05/2013	21/06/2013	20/09/2013	20/12/2013	20/06/2014	19/12/2014	19/06/2015	18/12/2015	16/12/2016	15/12/2017
Strikes											
48	0.0002	0.0003	0.0039	0.0083	0.0014	0.0034	0.0003	0.0006	0.0006	0.0004	0.0002
52	0.0002	0.0003	0.0008	0.0095	0.0053	0.0083	0.0072	0.0095	0.0094	0.0069	0.0052
60	0.0005	0.0033	0.0055	0.0108	0.0120	0.0131	0.0147	0.0133	0.0123	0.0102	0.0105
68	0.0086	0.0195	0.0214	0.0214	0.0198	0.0197	0.0163	0.0141	0.0127	0.0123	0.0103
76	0.0553	0.0489	0.0417	0.0316	0.0256	0.0194	0.0156	0.0155	0.0138	0.0116	0.0100
84	0.0531	0.0405	0.0369	0.0289	0.0252	0.0208	0.0170	0.0145	0.0133	0.0111	0.0098
92	0.0069	0.0106	0.0136	0.0206	0.0178	0.0142	0.0131	0.0116	0.0113	0.0097	0.0088
100	0.0004	0.0019	0.0097	0.0012	0.0010	0.0077	0.0092	0.0086	0.0092	0.0083	0.0077

The corresponding risk-neutral density surface is also presented below.

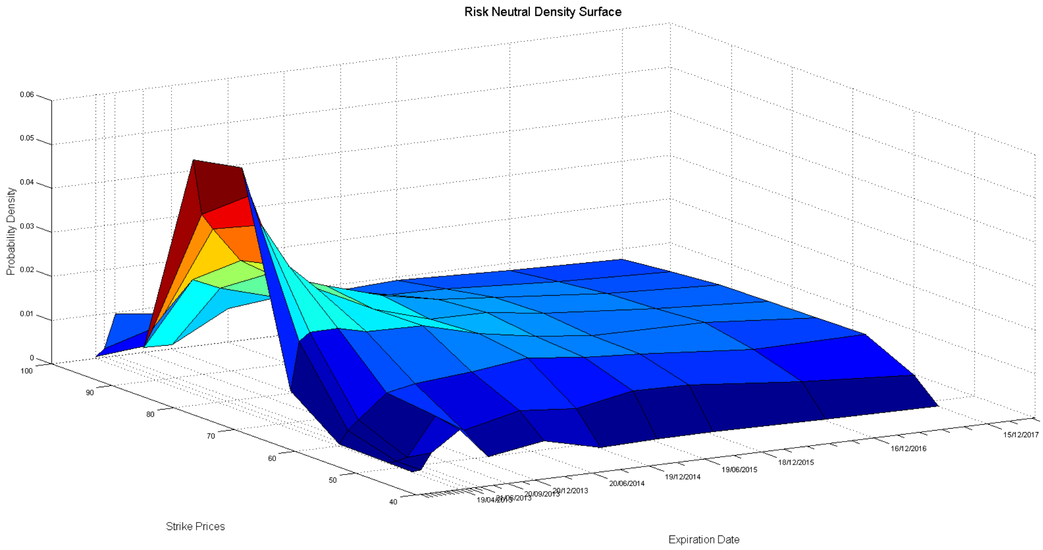


Figure 4.5: Risk-Neutral Density Surface on 15 March 2013

Each dot line of the Expiration date Axis as a single option Expiration date. More Expiration days took place for near terms options. From the figure above, the near term of options the steeper the risk-neutral density. The RNDs of longer term options behave flatter.

4.6.2 Implementation of Recovery Theorem

After estimating the Step III, the state price transition matrix is found to be as follow,

Table 4.4: The Transition matrix

	State on date t+1							
State on date t	0.110536	0.073110	0.061060	0.052145	0.024347	0.025542	0.055951	0.087848
	0.122194	0.142363	0.145421	0.077858	0.024610	0.027942	0.096004	0.157029
	0.139192	0.193440	0.222707	0.145484	0.031553	0.034612	0.169256	0.166704
	0.140768	0.134059	0.142204	0.228087	0.124296	0.143951	0.204630	0.126792
	0.109347	0.087106	0.071003	0.080853	0.382226	0.354358	0.081920	0.090423
	0.123438	0.100901	0.086211	0.123513	0.312913	0.303053	0.114674	0.096383
	0.162459	0.138017	0.139800	0.214486	0.071827	0.078252	0.197576	0.151662
	0.092066	0.131003	0.131595	0.077574	0.028228	0.032290	0.079989	0.123158

This procedure can be implemented by the ‘fmincon’[4] in Matlab. We use Matlab to find the optimized matrix with regarding to minimization of Frobenius norm. All the elements in the matrix are restricted to be nonnegative and less than 0.9999[5]. The sum of the each row for the state transition matrix is equal to a unit. The corresponding figure can be below.

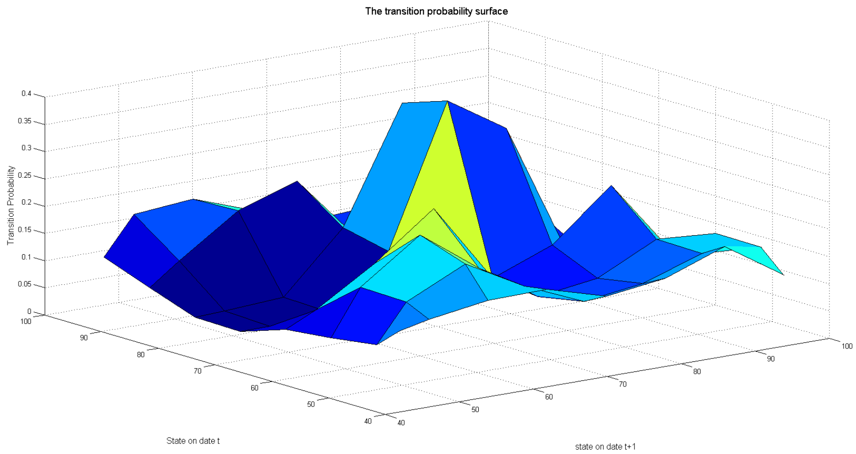


Figure 4.6: State Transition Probability Matrix

We compute the eigenvalues and the corresponding eigenvectors for the transition probability matrix. We find a maximum eigenvalue of 0.9999. Furthermore, evidence also been finding that Perron-Frobenius eigenvalue, the so-called Perron root δ, is also equal to 0.9999, whose computation is based on the Chanchana (2007).

Step IV allows us to find the Real-World Transition Probability Matrix, the result is shown as in Table 4.5.

---

[4] $x = fmincon(@(x)obj(x, S1, S2), x0, [], [], Aeq, beq, lb, ub);$

[5] The computed eigenvalues are (0.9999, 0.4948, 0.1481, 0.05222, -0.0073, 0.0069 + 0.0013i, 0.006967 - 0.0013i, 0.007972). The corresponding eigenvector for eigenvalue 0.9999 is $\begin{bmatrix} 0.134733 \\ 0.218966 \\ 0.320902 \\ 0.421010 \\ 0.500297 \\ 0.481735 \\ 0.362796 \\ 0.199209 \end{bmatrix}$.

Table 4.5: Real-World Transition Probability Matrix

	State on date t+1							
State on date t	0.110536	0.118817	0.145430	0.162941	0.090406	0.091324	0.150659	0.129887
	0.075189	0.142363	0.213120	0.149698	0.056230	0.061473	0.159065	0.142861
	0.058441	0.131993	0.222707	0.190869	0.049192	0.051959	0.191353	0.103487
	0.045049	0.069724	0.108391	0.228087	0.147705	0.164715	0.176335	0.059994
	0.029448	0.038124	0.045543	0.068039	0.382226	0.341210	0.059405	0.036005
	0.034524	0.045863	0.057428	0.107943	0.324970	0.303053	0.086361	0.039857
	0.060333	0.083300	0.123656	0.248902	0.099049	0.103906	0.197576	0.083277
	0.062268	0.143995	0.211982	0.163946	0.070892	0.078084	0.145674	0.123158

The estimation is based on the Theorem 2. We also confirm that the sum of the probabilities for one state to other possible states is equal to one. This would allow us to find the recovered density for the Adidas AG options.

4.6.3 Recovered density

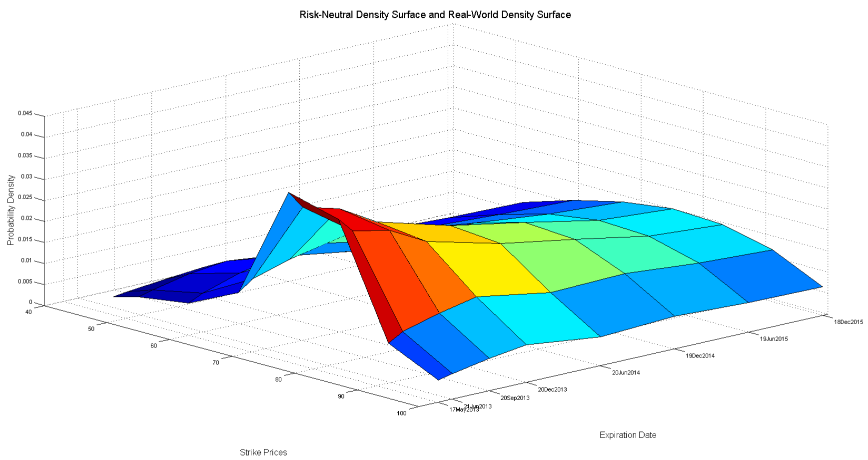


Figure 4.7: Real-World Density Surface

Figure 4.7 plots the Real-World Density for each Expiation date.

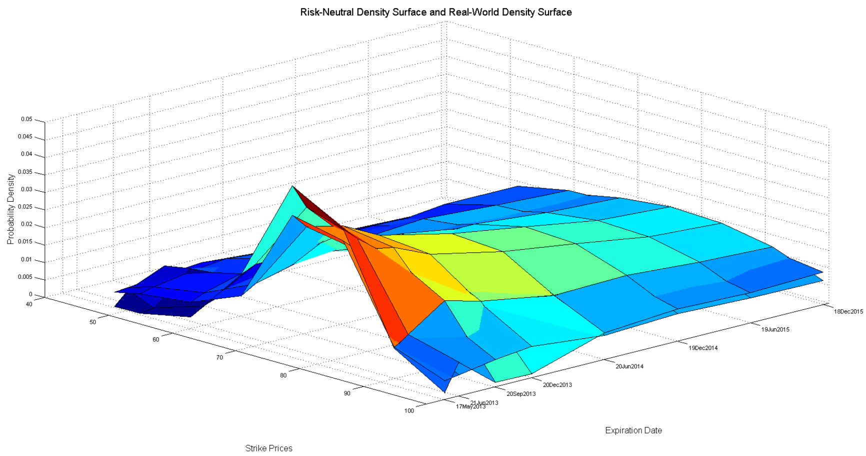


Figure 4.8: Risk-Neutral Density and Real-World Density Surface

A comparison between risk-neutral density surface and real-world density surface is shown in Figure 4.8. Readers are also suggested to see the Figure 4.10 and 4.11 for the plots of each density. It seems that the Risk-Neutral Density is more peaked than the Real-World density, which indicates the more leptokurtic in Risk-Neutral Density. In economic theory, we can associate the kurtosis with the fat-tailed property. This means we can also document that people have put more probabilities on the tails when we estimate the risk-neutral density. In reality, higher probabilities have been given in the middle portion of the market expectation.

Table 4.6: Correlation matrix of Risk-Neutral and Real-World Density Moments

		Risk-Neutral Density (RND)				Real-World Density (RWD)			
		Mean	Variance	Skewness	Kurtosis	Mean	Variance	Skewness	Kurtosis
RND	Mean	1.0000							
	Variance	0.8098	1.0000						
	Skewness	0.1517	0.3155	1.0000					
	Kurtosis	-0.3783	-0.2528	0.5899	1.0000				
RWD	Mean	0.9916	0.8662	0.2113	-0.339	1.0000			
	Variance	0.8547	0.9823	0.3362	-0.2564	0.9112	1.0000		
	Skewness	-0.1118	-0.0308	0.757	0.4554	-0.0905	-0.0102	1.0000	
	Kurtosis	-0.9143	-0.898	-0.1981	0.4408	-0.9456	-0.9459	0.0226	1.0000

We see from the Table 4.6, the correlation between the means from two measurements is 0.9916, as well as highly correlated between the variances.

However, for both skewness and kurtosis, the correlations are not as high as those for the first two moments, especially for that between the kurtosis. When looking at the correlation matrix, the rolling window is still a good method to adopt, even though the main objective of current study is to examine how successful the Ross recovery theorem can apply in the stock option. As in Table 4.6, which shows the correlation matrix of risk neutral density and real-world density for Adidas AG option prices trading on 15 March, 2013, we extend our results by using the rolling window. I try with the window of 5, the result shows in the following figure.

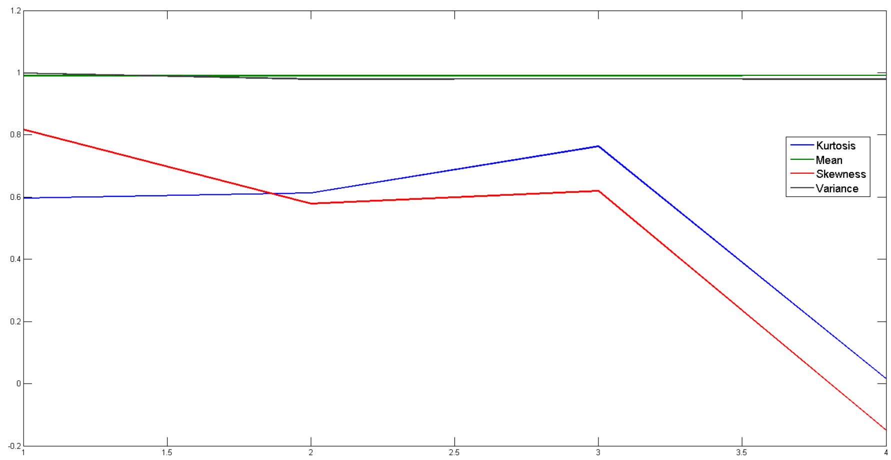


Figure 4.9: Correlation between the risk-neutral and real-world moments

From the figure, we find that recovered skewness and kurtosis are significantly different with the higher moments in the risk-neutral measure. This is reasonable, although we have successfully derive the real-world density, due to the insufficient data for stock options, comparing to the S&P 500 index options, it is difficult to get an accurate estimator (see Kiriu and Hibiki, 2019).

4.7 Conclusions

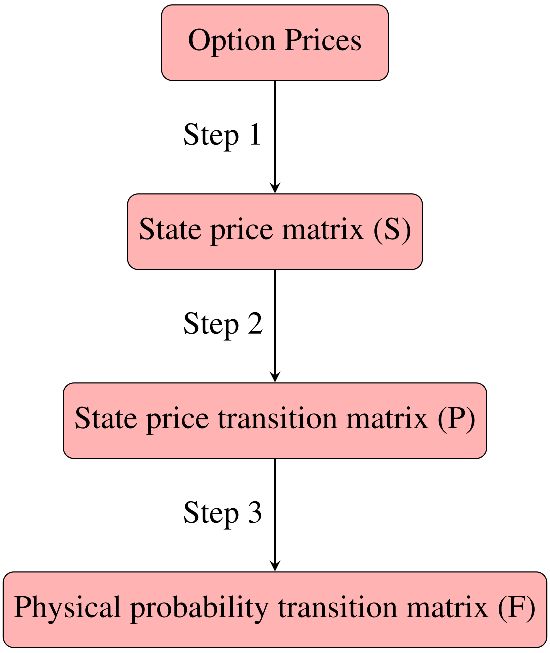
The nature of the forward-looking feature in options market is of interest to the academia and investors, who is in the field of asset pricing, risk management and portfolio allocation. The conventional argument indicates that we might be able to determine the physical measure from the risk-neutral measure. Summing up, current study has discussed the Recovery Theorem and adopted the Ross' recovery theorem to decode the physical probability and risk preferences from the Adidas AG option prices.

In this study, we analyse the implied volatility surface and find that the implied volatility decreases with time to expiry and decreases with strikes, which are consistent with the traditional financial theory and most of the previous empirical studies, such as Mixon (2002) and Frijns, Tallau and Tourani-Rad (2010). Furthermore, when defining the moneyness as a ratio of underlying price to strike price, there is a positive relationship between implied volatility and moneyness. As stated in the Ross (2015), based on the Perron-Frobenius Theorem, F can be uniquely determined by the P. Also in this study, we have successfully found the real-world density from the Adidas AG data. We also discussed the evolution of the risk-neutral density and the real-world density.

Appendices

Appendix I: Steps of Ross Recovery

The following flow chart shows the procedures how the Ross recovery theorem implements. The details can be found in the Estimation Framework section.

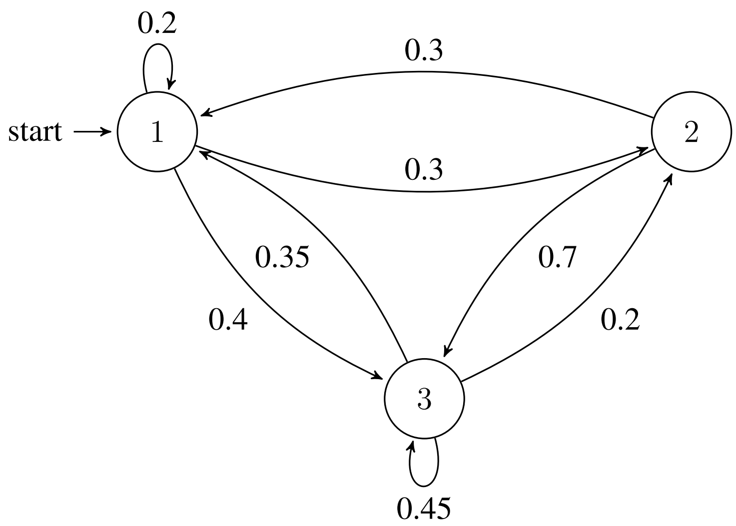


Appendix II: Transition Matrix Diagram

Consider a system with three states, i.e., 1, 2 and 3, and transition matrix is presented below,

$$P = \begin{bmatrix} 0.20 & 0.30 & 0.50 \\ 0.30 & 0.00 & 0.70 \\ 0.35 & 0.20 & 0.45 \end{bmatrix}$$

The following transition matrix diagram shows the three possible states 1, 2 and 3, and the arrows indicate the transition probabilities from state i to other state j.



Appendix III: Perron Theorem and Frobenius Theorem

Theorem 3. *(Perron Theorem) Let A be an $n \times n$ square matrix, where each entry is strictly positive. Then the spectral radius $\rho(A)$ of A is greater than the other eigenvalue λ, i.e., $\rho(A) > |\lambda|$. The $\rho(A)$ has a positive eigenvector v.*

Theorem 4. *(Frobenius Theorem or Frobenius's extended version) Let A be an $n \times n$ non-negative and irreducible matrix with spectral radius $\rho(A) = \lambda$. Then, λ is a positive real number and an eigenvalue of A; all components of the associated right eigenstate z are strictly positive. Moreover, the eigenspace associated to λ is one-dimensional.*

Appendix IV: Table of Smoothed Implied Volatility Surface

Table 4.7: Table of Smoothed Implied Volatility Surface

Moneyness	1.5047	1.4663	1.4280	1.3896	1.3513	1.3129	1.2746	1.2362	1.1979	1.1595	1.1212	1.0828	1.0444	1.0061	0.9677	0.9294	0.8910	0.8527
τ																		
0.0959	0.2600	0.2464	0.2205	0.1967	0.1876	0.1848	0.1831	0.1818	0.1802	0.1779	0.1750	0.1724	0.1699	0.1684	0.1677	0.1666	0.1654	0.1648
0.3700	0.2561	0.2450	0.2245	0.2007	0.1885	0.1846	0.1827	0.1810	0.1790	0.1764	0.1741	0.1721	0.1698	0.1684	0.1676	0.1665	0.1653	0.1648
0.6442	0.2511	0.2414	0.2248	0.2035	0.1895	0.1843	0.1820	0.1800	0.1776	0.1752	0.1735	0.1719	0.1697	0.1683	0.1675	0.1664	0.1652	0.1647
0.9183	0.2455	0.2366	0.2226	0.2043	0.1901	0.1838	0.1810	0.1788	0.1764	0.1744	0.1731	0.1717	0.1696	0.1682	0.1674	0.1663	0.1651	0.1645
1.1924	0.2394	0.2311	0.2189	0.2033	0.1901	0.1832	0.1801	0.1778	0.1756	0.1739	0.1729	0.1715	0.1694	0.1680	0.1672	0.1661	0.1649	0.1644
1.4666	0.2330	0.2253	0.2146	0.2014	0.1895	0.1826	0.1794	0.1770	0.1749	0.1735	0.1726	0.1713	0.1692	0.1678	0.1670	0.1659	0.1647	0.1641
1.7407	0.2264	0.2194	0.2101	0.1990	0.1886	0.1820	0.1787	0.1764	0.1744	0.1731	0.1722	0.1709	0.1689	0.1674	0.1667	0.1656	0.1643	0.1638
2.0148	0.2196	0.2136	0.2057	0.1963	0.1874	0.1813	0.1780	0.1758	0.1738	0.1725	0.1717	0.1705	0.1684	0.1670	0.1662	0.1651	0.1639	0.1633
2.2890	0.2129	0.2077	0.2012	0.1935	0.1859	0.1803	0.1772	0.1750	0.1730	0.1718	0.1710	0.1698	0.1677	0.1663	0.1655	0.1644	0.1632	0.1626
2.5631	0.2062	0.2020	0.1967	0.1905	0.1841	0.1790	0.1760	0.1739	0.1719	0.1707	0.1700	0.1688	0.1668	0.1654	0.1646	0.1635	0.1623	0.1617
2.8372	0.1999	0.1964	0.1923	0.1873	0.1819	0.1772	0.1744	0.1723	0.1705	0.1693	0.1687	0.1675	0.1655	0.1641	0.1633	0.1623	0.1611	0.1605
3.1114	0.1940	0.1912	0.1879	0.1839	0.1792	0.1749	0.1722	0.1702	0.1684	0.1674	0.1669	0.1658	0.1639	0.1625	0.1617	0.1606	0.1594	0.1589
3.3855	0.1887	0.1864	0.1837	0.1803	0.1759	0.1718	0.1694	0.1676	0.1659	0.1649	0.1646	0.1636	0.1617	0.1604	0.1596	0.1586	0.1575	0.1569
3.6596	0.1842	0.1822	0.1798	0.1765	0.1721	0.1680	0.1659	0.1643	0.1628	0.1620	0.1618	0.1611	0.1593	0.1579	0.1572	0.1562	0.1551	0.1546
3.9338	0.1806	0.1785	0.1761	0.1726	0.1678	0.1637	0.1618	0.1606	0.1593	0.1588	0.1588	0.1582	0.1565	0.1552	0.1545	0.1536	0.1526	0.1522
4.2079	0.1776	0.1754	0.1726	0.1686	0.1633	0.1590	0.1575	0.1567	0.1557	0.1554	0.1556	0.1553	0.1537	0.1525	0.1518	0.1510	0.1501	0.1496
4.4820	0.1753	0.1727	0.1694	0.1646	0.1586	0.1543	0.1533	0.1529	0.1522	0.1521	0.1526	0.1524	0.1510	0.1499	0.1492	0.1485	0.1476	0.1472
4.7562	0.1736	0.1704	0.1663	0.1607	0.1540	0.1497	0.1493	0.1494	0.1489	0.1491	0.1498	0.1498	0.1486	0.1475	0.1469	0.1462	0.1454	0.1451

Appendix V: Risk-Neutral Density and Real-World Density for Each Expiration Date

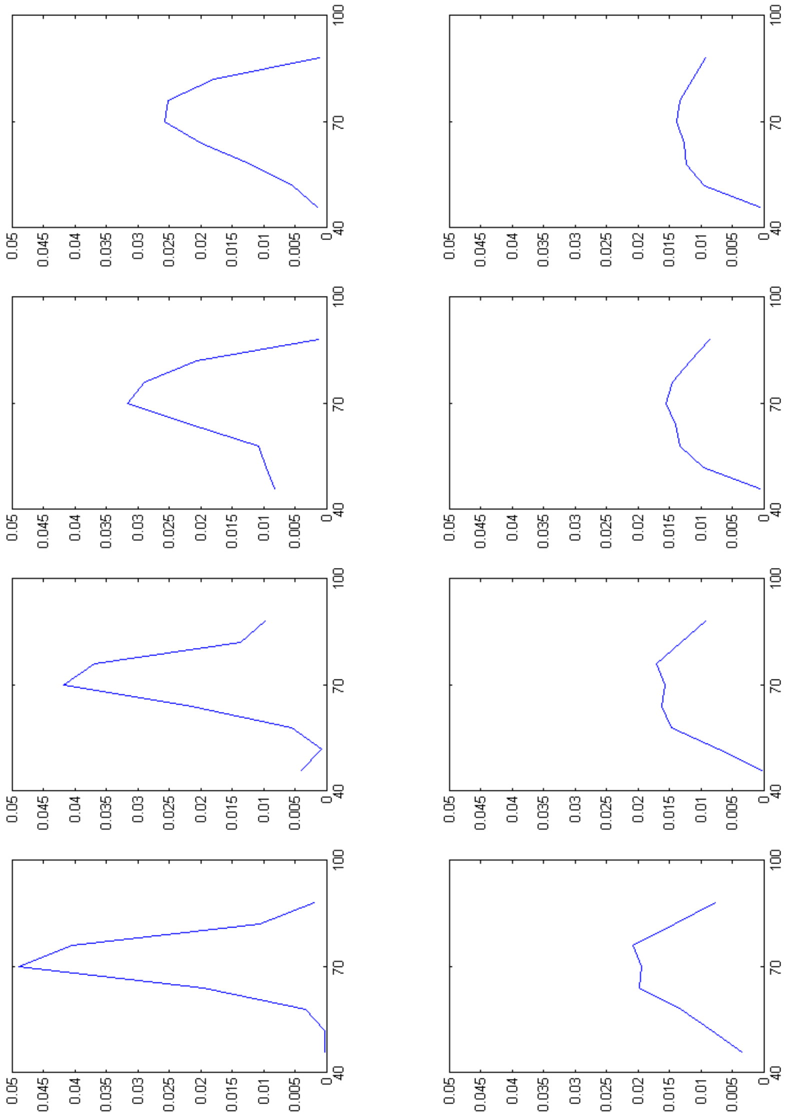


Figure 4.10: Risk-Neutral Densities

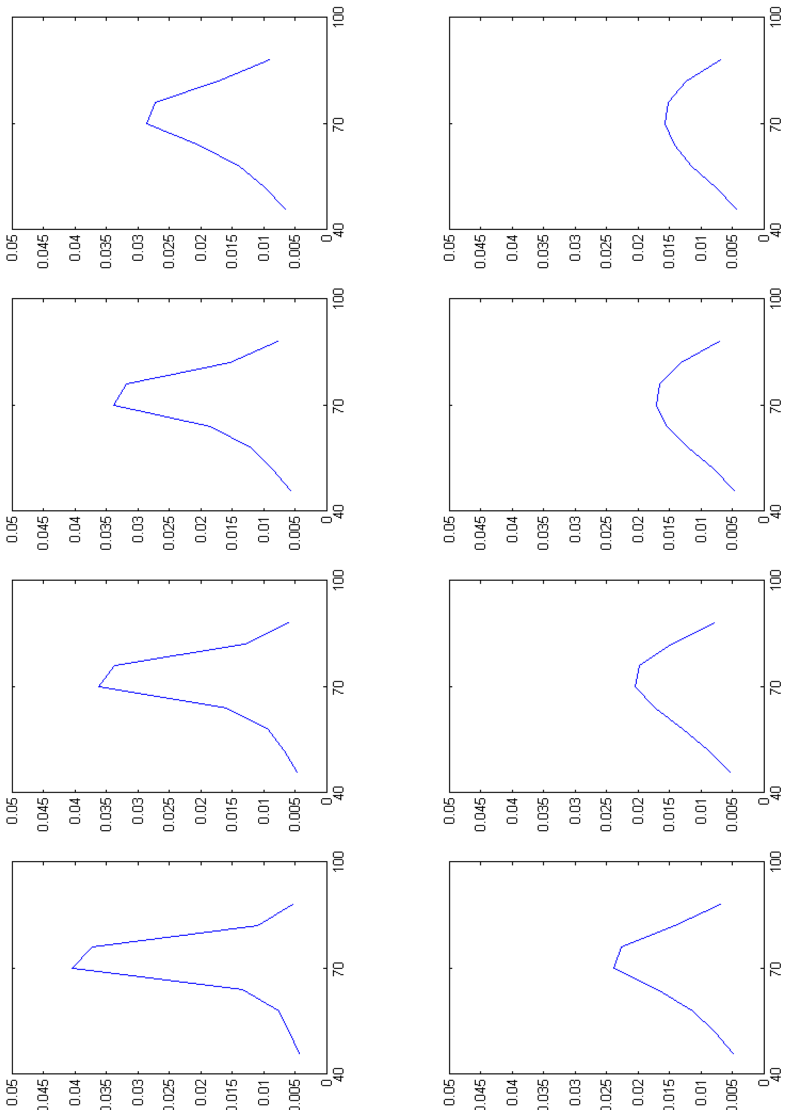


Figure 4.11: Real-World Densities

Chapter 5

Conclusions

5.1 Summary of the Thesis

Option markets by their nature contain abundant information to investors and regulators. They also contain useful information for academic research. This thesis has mainly focused on the use of option data in the study of the risk-neutral density and the real-world density and contained four individual essays on options market research.

The preceding four chapters have concerned four different objectives using financial option data. Chapter 2 adopted three alternative approaches to estimate the risk-neutral density: 1) Single lognomal; 2) Mixture of two lognormals; and 3) Smoothing implied volatility smile (IVS) method. The smoothing IVS method was found to be the best of these three in the context of the pricing errors. When looking into the risk-neutral moments, we found that the variances declined with the maturity date approaches, which was in line with Figlewski (2012). Due to investors' risk aversion, consistent with previous studies, the densities were left-skewed in normal period. The fat-tailed property has also been confirmed. We also found that the higher moments of the densities are dramatically linked to the underlying index and the index returns

In Chapter 3, we investigated the relationship between the option market and the macroeconomy in Chinese market. Intuitively, the options market should have reflected the macroeconomic news. This study particularly aimed to find out how the market response to the macroeconomic announcements. We, empirically, examined this issue in the context of both good and bad news and the surprised and unsurprised news. More interestingly, we found out different results for the whole sample period and the sub-sample periods.

Chapter 4 recovered the real-world density and risk aversion from the Adidas AG option prices through the recovery theorem of Ross (2015). Two main nonparametric assumptions have been made in this study. Here we examined the state price density (surface). Moreover, we have empirically and successfully estimated the real-world density for the Adidas AG stock options. We also discussed the evolution of the real-world density.

5.2 Directions for Further Research

Our studies do have some shortcomings, which have been indicated in each chapter, mainly due to the limited access to the use of the data. Consequently, there are a number of different areas in which further research could be beneficial In particular, we are keen on the following direction for future research.

5.2.1 Risk-neutral density from American options

In the thesis, we have successfully extracted the risk-neutral density from S&P 500 index options. To the best of our knowledge and according to the description from CBOE website, the S&P 500 index options are European style, which normally can be exercised only on a particular date, the maturity day. With the right to exercise their options before the maturity, the American options, such as OEX options and Exchange Traded Products Options might contain more valuable information than European options. How about the densities from the American options? Can we distil the information from them? Could they provide more flexible and more valuable information to the investors and policy-maker? These questions are largely unanswered. To date, studies to find the risk-neutral densities for American options have been conducted (e.g. Abken, Madan and Ramamurtie, 1996; Melick and Thomas, 1997; Bates, 2000; Broadie, Chernov and Johannes, 2007; Gemmill and Saflekos, 2000; Tian, 2011; Arismendi and Prokopczuk, 2016). Regarding this idea, we aim to find out the densities from the American options and also to find what are the expectations of the investors to the market.

5.2.2 Does investor sentiment matter for option markets?

Going back to 1990s, Barberis, Shleifer and Vishny (1998) develop a novel investor sentiment estimator and examine the reactions of the stock prices to the news. A number of studies have also been done to investigate the relationship between investor sentiment and stock markets. To name a few, the reaction of market price of risk to the market sentiment (see Verma and Soydemir, 2009), the relationship between the sentiment and the stock returns (e.g. Chung, Hung and Yeh, 2012; Kim and Kim, 2014), the investor sentiment during the bear market (see Garcia, 2013). Our focus will be on the investor sentiment in options market. Han (2007) and Lemmon and Ni (2009) document that the investor sentiment plays an important determinant of the time variation in the slope of the volatility smile. Han (2005) and Coakley, et al. (2014) also examine the effect of investor sentiment on the option prices. Extending our studies in Chapter 2 and 3, I intend to find out whether investor sentiment can partly explain the time-varying of the risk neutral moments. More interestingly, allowing us to compute the real-world densities (see Ross, 2015), our focus on the physical moments would be also of interest.

5.2.3 The determinants of the time variant in pricing kernel

A number of studies have examined the empirical pricing kernel from option markets (see Dittmar, 2002; Rosenberg and Engle, 2002; Brandt and Wang, 2003; Bollerslev, Gibson and Zhou, 2011; Drechsler, 2013). In line with previous studies, we would like to investigate the time-varying pricing kernel. Only a few has attempted to examine the drivers of the movements. I aim to find out some potential factors that might have affected the pricing kernel, such as the macroeconomic indicators, investor sentiments, realized variances, economic policy uncertainty, business cycle, the stochastic volatility, the jumps of financial returns, etc. Recent work in this area includes Scheicher (2003), Kurz, Jin and Motolese (2005), Damodaran (2009), Guiso, Sapienza and Zingales (2013), to name a few.

5.2.4 Systematic risk and option prices

We also motived by the study Backus, Chernov and Martin (2011), Acharya, Pedersen, Philippon and Richardson (2017), Kelly, Lustig and Van Nieuwerburgh (2016), Diavatopoulos, Doran and Peterson (2008), who investigate the systematic risk in options market. I intend to document whether the systematic risk has been priced in option prices, which is close to the impressive study Duan and Wei (2008). Their study examine the states that the influence of the systematic risk on equity options, i.e. the S&P 100 index and 30 component stocks. Evidence shows that the systematic risk does affect the implied volatility level and its slope.

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