Scripting the experience of mathematics teaching: The value of student teacher participation in identifying and reflecting on critical classroom incidents

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In this paper, we propose, and evaluate, a proactive reflective activity for mathematics student teachers in which they consider mathematical content and its teaching in highly specific classroom situations. Student teachers identified, scripted and reflected upon critical classroom incidents soon after their first block of school placement. Twelve such scripts were produced and then discussed by seventeen student teachers in group and plenary sessions. Discussions were audio-recorded. Scripts and discussions were analysed according to a typology of four characteristics that emerged in our prior research: *consistency* between stated pedagogical priorities and intended practices*; specificity* of the reflection to the classroom situation reported in the scripts; *reification of pedagogical discourse* that the student teachers became familiar with during the programme; and, *reification of mathematical discourse* that the student teachers were familiar with through prior mathematical studies. Here, we exemplify student teachers' insights that emerged from the analysis of the scripts through the typology of the four characteristics and we observe that their insights mirror the complexity and richness of the mathematics classrooms they face. Our examples, and their evaluation through the aforementioned typology of the four characteristics, illustrate the potency of student teacher participation in producing, and reflecting upon, individually and collectively, on critical incidents of their inaugural experiences in the classroom.

Keywords: Mathematics teacher education, critical incidents, mathematical and pedagogical discourse, participatory research, reflection, MathTASK

INTRODUCTION

Teacher education and professional development programmes often invite teachers to reflect on their classroom experiences, either as observers of lessons or as class teachers. In these reflections, teachers are invited to draw on their professional experience and practice-based discourses, locally contextualised, as well as on educational theory that may be part of the teacher education or professional development programmes they attend. The interaction of these different practical and theoretical discourses becomes even more apparent when teachers engage with developmental research in which they enquire their own practice (e.g. Labaree, 2003). We see this interaction between practice-based and theory-based discourses as a critical element of teachers' engagement with reflection and research on their practice. Tensions rooted in this interaction can explain teachers' challenges with

activities such as the identification of what is worth investigating, or reflecting upon, in their classroom (e.g. Goodell, 2006); structuring their reflection (e.g. Pultorak, 1993); or looking reflectively at school-based experiences and benefiting from them (Labaree, 2003). As these activities take place in the context of teacher education, professional development or developmental research environments, an additional challenge is to generate robust and informative evaluation of teachers' engagement with reflection and research on their practice (e.g. Rodgers, 2002). Our study takes on this challenge in the context of a mathematics teacher education programme in the UK: we propose, and evaluate, a proactive reflective activity for mathematics student teachers in which they consider mathematical content and its teaching in highly specific classroom situations.

The study we report from in this paper draws on MathTASK[1], a research and development programme that engages teachers in reflection on mathematics teaching, theirs and that of others, through written responses to, and discussions of said responses, to tasks built around critical incidents from the mathematics classroom. MathTASK sets out from research evidence that reports overt discrepancy between theoretically and out of context expressed teacher beliefs about mathematics and pedagogy and actual practice (e.g. Speer 2005; Thompson 1992). Instead of discussing teaching practices and experiences with teachers in the abstract, we do so by starting from specific classroom situations that can provide a trigger for exchanges, and for building shared insights, between researchers and teachers.

Specifically, we invite pre- and in- service teachers to reflect on fictional but realistic and research grounded classroom situations (what we call *mathtasks*). A situation in a mathtask is triggered by a mathematical problem given to students in class. Responses from one or more students to the problem and a dialogue between students and teacher or/and a reaction from a teacher to these responses follow. A list of questions at the end of the text invites teachers to reflect on issues emerging from aforementioned classroom situation and to consider how they would react in a similar case (e.g. Biza, Nardi & Zachariades 2007, 2018; Nardi, Biza & Zachariades 2012). Teachers' written responses to mathtasks, followed by group discussion or interviews with individual teachers, reveal a complex set of considerations (e.g. personal, professional, institutional) that teachers take into account when they determine their actions (Nardi et al. 2012). Mathtasks aim to address the needs of mathematics student teachers through steering them towards considering the needs of mathematics learners, key elements of mathematics and the curriculum as

[1]MathTASK: Using situation-specific tasks to transform mathematics teachers' pedagogical and mathematical aspirations, https://www.uea.ac.uk/education/mathtask

well as classroom management issues. We intend that the exposure of student teachers to said issues prepares them for addressing these issues when they face them in actual teaching practice. Our approach sits within the now established tradition of teacher development research based on *critical incident* approaches (e.g. Skott, 2001).

In our writing from the study so far, we have reported analyses of data collected in the course of teachers' engagement with *mathtasks* that we, with other members of our team, created in advance. In this paper, we turn to a hitherto unreported part of our work within the MathTASK programme in which student teachers participate actively in the *production* of mathtasks themselves. In a nutshell: first, we engaged student teachers with *mathtasks* created by us; then, we asked the participants to identify and create their own, inspired by their first school placement experience. Twelve scripts were produced and then discussed in a three-hour plenary session. How potent this activity was, and how this potency was determined through an evaluative instrument we recently devised (a typology of four characteristics, Biza et al, 2018) is the focus of this paper. In what follows, we outline briefly the prior works that have influenced the MathTASK programme and the methodology of the part of the MathTASK programme that is the focus of this paper and the aforementioned typology of four characteristics. We then present a selection of scripts and group discussion episodes analysed through the typology in order to illustrate the insightful student teacher productions that emerged in the process. We conclude with implications for research and teacher education practice.

MATHEMATICS TEACHERS IDENTIFY AND REFLECT ON CRITICAL CLASSROOM INCIDENTS: IMPORTANCE AND CHALLENGES

We are interested in what teachers identify as important in their practice to reflect on or to enquire upon, which is not always what the researchers or teacher educators identify as essential. As Labaree (2003) notes, there are differences in the world-views of teachers and researchers: engagement with education research invites teachers to "transform their cultural orientation" (p. 16) from normative to analytical, from personal to intellectual, from particular to universal, and from experiential to theoretical. This transformation is in no way trivial.

There is a substantial body of research on teachers' – and, for the purposes of the study we report here, mathematics teachers'– *noticing* (e.g. Schack, Fisher, & Wilhelm, 2017)*, professional vision* (e.g. Goodwin, 1994) or *diagnostic* skills (e.g. Leuders, Leuders, & Philipp, 2018). What teachers identify as important in their in-class actions or observations as well as in their out-of-class reflection is not expertise that develops automatically and does not necessarily develop proportionately through the accumulation of teaching experience. The part of the MathTASK programme from which we report in this paper is embedded in the broad

area of research which proposes that this expertise can develop through teacher engagement with artefacts of practice such as videos (e.g. Sherin & van Es, 2009), excerpts of students' written work (e.g. Nardi, 2008) or fictional classroom situations (e.g. Biza et al. 2007) as well as through identification by them, and discussion of, critical incidents (e.g. Goodell, 2006; Potari & Psycharis, 2018).

Critical incidents have been used extensively in teacher education programmes in the form of brief reflective accounts, written by teachers, on classroom situations they have observed or experienced as part of their training (e.g., Goodell, 2006). A critical incident is a moment or an episode, when the teacher makes a decision taking into account contextual and institutional information (Skott, 2001). An incident becomes critical when someone chooses to see it as such, therefore the choice and characterisation of an incident as critical is a matter of interpretation (Tripp, 2012). Goodell (2006) sees these incidents as everyday events that make the teacher question the event and the decisions that were made in its course.

Reflection on critical incidents has been shown to play an important role for teachers' learning and professional development (Goodell, 2006; Hole & McEntee, 1999; Potari & Psycharis, 2018; Skott, 2001; Tripp, 2012). It can "provide a window on the role of teachers' school mathematical priorities" and "may prove significant for the long-term development of a teacher's school mathematical priorities" (Skott, 2001, p. 19). Reflection on critical incidents inspires teachers to think of what happened, why it happened, what it could mean and what its implications are (Hole & McEntee, 1999). Furthermore, asking teachers to identify critical incidents and produce reflective accounts followed up by group discussions provides a concrete basis for teacher reflection (Goodell, 2006; Pultorak, 1993).

In the work we report in this paper we argue further that by familiarising student teachers with examples of critical incidents first – in our case, through engaging them with mathtasks – we introduce them to a practice of identification and communication of what might be critical for them. We then ask them to identify, reflect upon and discuss their own selection of incidents – and we finally evaluate their productions as well as their reflections and group discussion on these productions through a typology of characteristics that we see as pertinent. In what follows, we outline briefly this typology and then present evidence that illustrates the potency that lies within teacher engagement with this production and reflection.

EVALUATING MATHEMATICS TEACHER REFLECTION ON SELF-IDENTIFIED CRITICAL INCIDENTS

Mathematics teachers' identification of what is critical and worth addressing in their classroom has been discussed extensively in the literature with different foci, such as: teacher knowledge, e.g., *pedagogical content knowledge* (Shulman, 1986) or

mathematical knowledge for teaching (Ball, Thames & Phelps, 2008); the enactment of this knowledge in teachers' actions, e.g., the *foundation, transformation, connection* and *contingency* dimensions of the *knowledge quartet* (Turner & Rowland, 2011); the discourses teachers engage with, e.g., *mathematical discourse for teaching* (Cooper 2014); and rationale for teacher choices, e.g., *practical rationality of teaching* (Herbst and Chazan, 2003), *professional vision* (van Es, 2011), or classification of *teacher warrants* (Nardi, et al. 2012).

Our interest is mostly in the characterisation of teacher engagement with out-of-class activities in which they identify and reflect upon on what they consider as critical. For example, van Es (2011) characterised mathematics teachers' *professional vision* development in their interaction with videos from their own lessons with attention to *what* teachers notice – including *whom* a teacher may focus on (the whole class, individual students or groups, the teacher) and *which topics and issues* they identify (such as pedagogical strategies, behaviour, mathematical thinking, and classroom climate) – as well as *how* teachers analyse what they notice (descriptive, interpretive or evaluative) and the depth of their analysis (e.g., whether they provide details or ground their comments on evidence) when they decide about their potential actions. The *what* and *how* aforementioned dimensions were also used by Lee and Choy (2017) in the tracing of teachers' noticing during the planning, teaching, and reviewing phases of a Lesson Study process. Recently, van Es, Cashen, Barnhart and Auger (2017) proposed a coding framework for teachers' noticing in videos that includes attention to: mathematical content and learning goal; student thinking; teacher actions to make student thinking visible; classroom discourse norms; specificity; and, making connections (p. 172). van Es et al's (ibid) coding framework resonates particularly with our work which aims to contribute a robust set of lenses through which teacher responses can be examined in relation to both mathematical and pedagogical priorities.

Specifically, in Biza, et al. (2018) we analysed teachers' responses to mathtasks in order to identify evidence of their competences in diagnosing issues in students' responses and in responding to these issues. The analysis suggested a typology of four interrelated characteristics of teachers' responses (ibid, p. 64):

- *Consistency:* how consistent a response is in the way it conveys the link between the respondent's stated pedagogical priorities and their intended practice? For example, does a teacher who states their prioritising student participation in class propose a response to a classroom situation that involves such participation of students? Or, does their proposed response involve only telling students the expected answer to a mathematical problem?
- *Specificity:* how contextualised and specific a response is to the teaching

situation under consideration? For example, does a teacher who writes generally about their valuing the use of vivid, visual imagery in mathematics teaching, propose a response to a classroom situation that involves specific examples of such imagery? Or, does the teacher response include only a general statement of their preference?

- *Reification of pedagogical discourse:* how reified is the pedagogical discourse that student teachers have become familiar with through the course in their responses? For example, how productively are terms such as "relational understanding" (Skemp, 1976) or "sociomathematical norms" (Cobb and Yackel, 1996) used in the responses?
- *Reification of mathematical discourse:* how reified is the mathematical discourse that student teachers are familiar with through prior mathematical studies in their responses? For example, how productively does prior familiarity with natural, integer, rational and real numbers inform a student teacher's discussion about fractions in a primary classroom situation?

We note that we use the term *reification* in the sense encountered with discursive theoretical perspectives such as Sfard's (2008). There, reification is defined as the gradual turning of processes into objects. Discourses, Sfard writes, change in a "chain of intermittent expansion and compression" (p. 118). Reification is the key element of compression which can be endogenous – resulting from saming within one particular discourse – and exogenous which 'conflates several discourses into one' (p. 122). We concur with Sfard that reification is evidence of productivity in a teacher's discursive activity. In our case, we look for evidence of reification in the way that participants deploy terms from mathematics education research and from mathematics.

In this paper, we evaluate the proposed activity for mathematics student teachers in which they produce and reflect upon their own critical incidents and we deploy the aforementioned typology of four characteristics in the analysis of these incidents and the follow up discussion. Our analysis aims to answer the following research questions:

1. What do teachers prioritise, mathematically and pedagogically, as they engage with a proactive reflective activity in which they identify, script and discuss critical incidents from their school experience?
2. Can, and if so in what ways, the four characteristics of consistency, specificity, reification of pedagogical discourse, reification of mathematical discourse be used towards the identification and analysis of said priorities in the written and oral evidence of the teachers' participation in the activity?

We now introduce the context, participants and methodology of the part of the MathTASK programme that is the focus of this paper.

CONTEXT, PARTICIPANTS AND METHODS OF DATA COLLECTION AND ANALYSIS

We conducted this part of the MathTASK programme – thereafter "the study" – in the context of the mathematics Initial Teacher Education (ITE) programme in our institution. This is a thirty-six-week programme that comprises a minimum twenty-four weeks work placement in two schools, in two blocks, and twelve weeks study at university. The university-based component aims to provide students with the academic, professional and personal skills and competencies needed for teaching. The study took place between the two blocks of school placement. Participants were invited from the whole cohort of twenty-two student teachers – of whom twelve contributed their own mathtask scripts and then together with five more student teachers from their cohort (a total of seventeen) participated in the plenary session from which we report in this paper.

We invited the student teachers to compose a script (approximately one page) drawing on incidents they saw as critical in their classroom experience soon after their first block of school placement. We specified that the script could be an account of a fictional incident but had to be based on their experiences. We suggested that the scripts include: (a) a title; (b) a brief description of the context; (c) a story, possibly in a dialogic format, reflecting the situation; and, (d) their reflection on this situation (e.g. the reasons they chose it, their concerns, their questions, etc.). The approach of narrating teaching situations is close to the identification of critical incidents in other studies (e.g., Goodell, 2006; Hole & McEntee, 1999; Potari & Psycharis, 2018; Skott, 2001; Tripp, 2012), as we mentioned earlier, with the difference that the student teachers were at liberty to compose a fictional incident inspired by their experience. This approach reduces the load of reporting the details of the real situation and the exposure of potentially sensitive issues. We see this as a liberating approach that allows teachers to say what they feel is worth discussing. We have seen similar approaches in other works such as the *extended imagined classroom dialogues* in Zazkis, Sinclair and Liljedahl's (2013) *lesson plays* and recently in Marmur and Zazkis's (2018) *scripting tasks*. The participants were familiar with mathtasks from earlier sessions in the academic year (Biza et al., 2015). This was the first time they were asked to produce their own scripts.

Twelve student teachers sent their scripts via email. We then invited discussion of these scripts in a whole class session that lasted three hours in total and in which seventeen student teachers participated. They discussed the scripts in four groups, each dedicated to common issues raised in the scripts grouped in the themes: "student (dis)engagement with mathematical activities (including boredom)"; "relational/instrumental understanding in the secondary mathematics classroom";

"students' (mis)conceptions in particular mathematical topics"; and, "secondary mathematics classroom management and students' (mis)behaviour". Each group prepared a poster of the main points of their group discussion (two hours) and presented these points in a plenary discussion with the whole class (one hour). All discussions, small group and plenary, were audio-recorded and transcribed. The study is approved by our institution's Research Ethics Committee. Participants were informed about the aims of the study and consented to the use of their data for research purposes.

The twelve scripts, the transcriptions of the discussions and the group posters, were analysed according to the aforementioned typology of four characteristics: *consistency, specificity*, *reification of pedagogical discourse* and *reification of mathematical discourse*. The datasets from each source (teacher scripts, discussion transcripts, posters) were first scrutinised for evidence of each characteristic. We then identified patterns in this evidence and collated examples of these patterns through drawing on evidence from across the different sources. In what follows, we present four such examples, each telling a story that we see as a pertinent reflection of the participants' priorities and key concerns (research question one). In doing so, we address also the second research question by looking how the operationalisation of the four characteristics in the analysis contributes to the identification and analysis of said priorities.

STUDENT TEACHERS' INSIGHTS AS THEY SELECT AND REFLECT UPON CRITICAL INCIDENTS OF MATHEMATICS TEACHING

In the four examples that follow, we draw on the twelve scripts produced by the student teachers and the subsequent discussions in small groups first and then with the whole class. All names are pseudonyms.

Overall, the proposed scripts were quite *consistent*. Student teachers wrote about their experiences from the classroom and reflected on them without contradictions between what they stated as an appropriate way to deal with the described issues and the way they have acted and would act in a similar case. We note however that most written reflections were quite brief (see Figures 1-3), restricting our capacity as researchers to trace potential patterns of consistency or lack thereof. Where reflection seemed to flourish was mostly in the group and plenary discussions where it was possible to identify patterns of (in)consistency, or shifts, in the teachers' arguments. "Mark's …" episode we sample in what follows, for example, was an episode from the plenary discussion.

Specificity was also high across the twelve scripts. Student teachers described incidents that challenged them or triggered their reflection on their teaching experiences. However, there was fluctuation between lower and higher specificity.

Examples of this fluctuation are two scripts that we characterised as less specific in terms of the mathematical content and the aims of the lesson, but quite specific in relation to the pedagogical issue they aimed to address. Both scripts, were about students' engagement with classroom activities in general. The first, is the script produced by Emma and describes the boredom of students who complete mathematical tasks more quickly than their peers. Emma did so with no reference to specific classroom activities or circumstances and, in her comments, she seemed to address the perceived tedium of mathematics lessons overall, and broadly. The other episode is "Zina's…" episode we sample in what follows.

With regard to *reified mathematical discourse*, we discerned substantial challenges that student teachers face when they deal with certain mathematical content in the secondary mathematics classroom. "Cathy's…" episode in what follows is an example of said challenges.

With regard to *reified pedagogical discourse*, terms such as "relational / instrumental understanding" in mathematics that had proliferated in the university sessions of the teacher education programme the student teachers were enrolled in, feature, albeit modestly, in the student teacher responses. "Mary's…" episode in what follows is an example of this modest but decisive presence.

Mark's consistent view on a student engagement conundrum: class over individuals!

We draw on an example from the plenary discussion that followed Sue's summary of the main points raised in the "student (dis)engagement with mathematical activities" group, when Sue concludes with a list of questions such as:

> […] do the individualised tasks help engage challenging students? And should you - if you got one challenging student that isn't getting engaged with the work, should you spend time trying to get them engaged? Or, is your time best spent monitoring the rest of the class's progress? […]

Following the tutor's invitation to discuss and share ideas on the questions posed by Sue, Mark says that "is the responsibility of the teacher to change one un-engaged student in the expense of the rest of the class" and he added that if a student is "in a bad mood or they are not feeling very well" the teacher cannot do much either than "have a word" with the student to do what they can and catch up later. He added that there is no point insisting with an un-engaged student "at the expense of the rest of the class".

The discussion then shifts to lack of participation of good students out of boredom, and Mark highlights that it is the teacher's responsibility to deal with those students: "If they [teachers] know that they are high ability or gifted and talented student, it is […] almost their fault not planning extension activities so, [students] they've always have got something to do".

A further shift of the discussion follows towards mathematics classroom practices, even since primary school, where students are encouraged to compete with each other for quick, yet not necessarily well-justified, responses. Mike is very concerned when he describes an incident concerning one of his students who refused to engage with a task that "was too easy for him […] how can you motivate him to try to […] take part?", Mike wonders.

Mark and other student teachers agree that some students are "so confident in their own ability to answer the question [that] they don't bother" and some added that students are "lazy" to "show their work". Later, Mark returned to his initial point that there is no point in trying to engage a student who does not want to; trying to do so may be at the expense of the rest of the class.

In this episode from the plenary discussion around student engagement and teacher responsibility, we see the attention fluctuating between student / teacher responsibilities and institutional characteristics often germane to deeply rooted ways to teach and learn mathematics. For some of the student teachers, in our example Mike, this is a real concern and he is wondering how he would motivate his students. Mark, and others, state their priority of class against individual students. Mark also then claims that it is the teacher's responsibility to prepare individualised activities for students who need them. Later, he blames students for lack of interest and returns to his initial point that his priority is the class. Looking at Mark's arguments across the episode, we can see some internal consistency in his prioritisation of the class and the transferring of responsibility to individual students that was shifted temporarily when the class discussion turned to the boredom of good students. We note that this rich exchange between the student teachers – layered, toing and froing between various social and other norms (Cobb and Yackel, 1996) that govern the work in mathematics classrooms – was triggered by the questions put forward by Sue. While, the interlocutors in these exchanges seem to fluctuate in their attribution of classroom events to teacher agency, student perceptions of their role in the classroom and much else, the overall theme of who is in charge of, and responsible for, maintaining high levels of engagement in the mathematics classroom remains sharply in focus throughout.

Zina's triggering a varied and insightful conversation around adolescents' classroom (dis)engagement patterns: Questioning technique

In her script (Figure 1), Zina describes an incident where a "capable" student who has answered all the "starter" activities refuses to contribute to the class when the teacher invites her.

Title: Questioning Technique

Context:

Whilst marking a starter activity the teacher picked students randomly to answer. A capable student, who had answered all the starter questions (seen whilst circulating), denied they had in order to avoid participating in the lesson. The teacher subtly informed the student they were not allowed to avoid answering, without causing a disruption, or allowing room for argument.

The story in dialogic format:

Teacher: What did you get for number 5?

Student: I don't know, I didn't get that far.

Teacher: Okay, you have 1 minute and then we'll come back to you.

My reflection:

Found it very difficult at the beginning of placement A to know how to respond to students who don't want to participate in the lesson. When a student who you know is capable of answering a question doesn't want to give an answer it's difficult to know whether to push them. We've been told a lot to consider extra factors to do with the student's mood and the last thing you want to do is further upset or anger any student. When observing I recognised this was a technique which didn't overly pressure or embarrass the student. It also allows the student some time to think, allows the pace and lesson continue and doesn't make a big issue out of the situation.

This technique is also applicable for students who genuinely have yet to answer the question and shows that participation in the lesson is very important, and not optional.

Figure 1: Zina's script ("(dis)engagement with mathematical activities" theme)

In her reflection, Zina is wondering about ways "to respond to students who don't want to participate in the lesson", especially when you know that this student is "capable". She mentions "extra factors to do with student's mood" and "a technique which didn't overly pressure or embarrass the student". However, although she expresses her difficulty to deal with such situation, she does not describe the specific situation of her incident and she does not elaborate on said factors or technique in her reflection. This script expresses a genuine concern of Zina, but it is expressed and reflected upon with what we saw as little specificity.

In the small group discussion on Zina's script, however, Mike, Emma, and Zina search for more elaboration. Zina says that it is students' "fear of getting it wrong and being laughed at" and Emma adds that students do not feel the need to repeat what they have already done. Then Mike adds that it might be because of "shyness" and that there are different ways of engaging students "which doesn't involve talking in front of the class". Zina mentions gender: "[…] cause it is a girl", but this point is not elaborated further and the group discussion focus shifts to student's "confidence". Later, Emma points student "attitude" that is followed by Mike's comment that non-responding might be a way of the student to "become more popular with friends" and, as Zina follows up, when the teacher invites the class to

contribute, the student may attract more attention by non-responding instead by "literally just answered and just moved on".

As we can see in these exchanges, even a less specific reflection on a classroom situation can generate a quite focused and detailed analysis of the underpinning causes of students' behaviour. Despite its being rough and generic, Zina's script can be credited for eliciting a broad and insightful conversation around adolescents' classroom (dis)engagement patterns – in this case in mathematics, even though mathematics comes across as a little incidental in the script and its discussion. Mathematics is far more at the forefront in the example that follows.

Cathy's mystifying characterisation of zero as a "special" number

In her script (Figure 2), Cathy describes an incident where students "a year 10 middle ability set" work on the rearranging of a quadratic equation.

Title: Combining like terms
Context: This scenario involves a year 10 middle ability set. The class had been working on solving quadratics by factorising and were moving on to using the quadratic formula. We were discussing the need for the equation to be equal to zero before using the quadratic formula and going through some examples of rearranging equations and changing the subject of the equations.
The story in dialogic format: Teacher: to make x^2 the subject of $x^2+5x=0$ we just need to subtract $5x$ for both sides giving us? Pupil A: $x^2 = -5x$ Pupil B: you can't add the $-5x$ to the zero as they are different terms Pupil A: but we don't need to write $x^2 = 0 - 5x$ it's just $x^2 = -5x$ Teacher: pupil B you are right, we never combine unlike terms when adding or subtraction, but pupil A's answer is correct because zero is special. We wouldn't write $0-5$ we would just right -5; it's the same with algebra.
My reflection: Pupil A has an understanding of the rules used to combine terms but there was confusion when zero was involved. This is a small misconception that can be easily overlooked and something I would not have considered without the question from Pupil B. I think the explanation could be improved so there is not further confusion.

Figure 1: Cathy's script ("(mis) conception in particular mathematical topics" theme)

Cathy's observation is apt: algebraic operations that involve zero often challenge students (Kieran, 1992). However, the way the teacher in the script responds to this

student conflict may raise concern. Let's see Cathy's response from a mathematical perspective. Although Cathy describes that the equation should "be equal to zero", she suggests rearranging the equation $x^2+5x=0$ in terms of x^2. This action contradicts her stated aim. Then, instead of using the zero property of multiplication and explaining that "$0\text{-}5x$" can be written as "$0x\text{-}5x$", which cannot be done with other numbers such as $2\text{-}5x$, Cathy says that, although we cannot "combine unlike terms", we can do it with zero, "because zero is special". She then adds that "We wouldn't write $0\text{-}5$ we would just write -5". In her reflection, she characterises student challenges with zero as a "small misconception", most likely around zero. By characterising zero as a "special" number without explaining why, she attempts addressing a student difficulty albeit by maintaining a mystified perspective on zero.

Later, in the group discussion, she describes her insecurity whether students will be able to deal with similar problems in the future by saying: "I don't know whether they'd remember it, I don't know whether they actually understood why … or they just accepted it". In practice, we cannot see her intention to resolve what she called "small misconception". We conjecture that Cathy herself has not reified the mathematical properties of zero in the set of real number (zero is the neutral element for addition and its multiplication with any real number gives zero). As a result, in this instance, Cathy's characterisation of zero as a "special" number without explanation obfuscates the number and its properties and misses the opportunity to go beyond unreflective mathematical action.

Overall, and beyond this brief analysis of Cathy's mathematical discourse, we can see clear evidence of the pedagogical discourse dominating the teaching profession and the teacher education course she attends endorsed consistently in her script and in the follow up discussion. For example, we see the use of the "ability" descriptor that is commonly used in UK schools and policy documents – the mainstream practice in English schools is the grouping of the students according to their ability, usually to low, middle and high ability groups. Also, Cathy praises pupils A and B, highlights her common practice of attending to student needs and reflects on how to improve her practice ("explanation") as consistently instructed during the teacher education course. Finally, she deploys the term "misconception", mirroring the literature in this area commonly used in the course. We discuss further the reification of such pedagogical discourse in the next, and final example.

Mary's "relational understanding" aspirations clash with school norms

In her script (Figure 3), Mary describes an incident on operations with fractions in a "year 9 top set" class.

Title: Conceptual vs instrumental understanding
Context:

This scenario was with a year 9 group top set in which we started our fractions, decimals and percentages topic. The specific incident happens in our lesson on the four operations of fractions in which the class had told me they had covered some of in year 8 but could not really remember. The incident occurred after I put up a fraction addition question in which I asked a student to answer explain to me the answer and how they did it. The names of students have been changed for ethical reasons

The story in dialogic format:

The question on the board was 2/3 + 1/4

"That's easy miss, we did this last year."

"Okay Joe, come to the board and show us what the answer and how to do it"

"Easy, you times the two bottom numbers together, then you cross multiply the top number with the bottom number and do it on the other side, add them together coz' it's an add question."

"Hands up who think that Joe is right"

Most of the class at this point put their hands in the air, with only about 2-3 students not putting their hands in the air.

"Now keep your hands up if you have an alternative way as to how you might go about adding these two fractions together".

In response to this, no one was able to show me another method. I then asked someone to show me how and why this was a way in which we could add fractions together.

"Because this is the way we got taught it last year miss."

After this, I led the students through different examples to discover as to *why* [her emphasis] this specific algorithm they were following worked (lowest common denominator) and the class was able to make the connections. I then asked the class honestly as to whether that helped them, half of the class agreed that it did and the other students I asked why not and they said because they were still getting the questions right doing it the way they were and that was what mattered.

My reflection:

I reflected about this for a while and thought had it been taught the other way round, e.g. me teaching them the lowest common denominator method and then asking them to try and discover a short cut method to remember this, would that have received different responses? It got me worried that the students didn't really care why the cross multiply method worked they just cared that they got the answer right. They had been taught in a very instrumental approach and I wonder that if it had been taught the other way round, would further the students even further.

Figure 2: Mary's script ("relational/instrumental understanding" theme)

In Mary's script, students do not see the value in the teacher's attempt to introduce a more "relational" approach. This triggers Mary's frustration in the small group discussion and an explanation from Jack: "with top sets […] there is a level of arrogance […] they take joy from the subject in doing well […] not joy from actually their subject […] they measure their success by grades and nothing else". His

suggestion is to try “to teach them [students] every single method within ten minutes” and leave them to choose one method.

Later, Jack reports on Mary’s script by saying: “[Mary] was like caught up between two areas […] just encourage them [the students] to go for higher grades but actually understand the stuff that they are learning”.

In Mary’s script, we see what a significant influence the construct of instrumental/relational understanding (Skemp, 1976) is on her as a way to distinguish the instrumental approach to mathematics (following rules) from the relational approach to mathematics (knowing the reasons behind these rules). Also, in the exchanges that followed in the group discussion, we see how, according to the student teachers, Mary’s priorities contradict usual school practice especially if students decidedly focus on exam success. In principle, the theoretical constructs she was introduced to during the university part of the course have been reified in this teacher’s pedagogical discourse and form a seminal influence on her classroom priorities. However, the constructs are used as a rather over-simplified dichotomy, an either-or-approach, which does not reflect the complexity of mathematical learning. Additionally, Jack’s comment on teaching “every single method” may suggest a menu of options that does not necessarily indicate a more relational approach, rather a more expansive instrumental approach.

TOWARDS A ROBUST AND INFORMATIVE EVALUATION OF MATHEMATICS TEACHER PRIORITIES AND REFLECTIONS

In this paper, we propose, and evaluate, a proactive reflective activity for mathematics student teachers in which they consider mathematical content and its teaching in highly specific classroom situations. Specifically, we offer the student teachers the opportunity to script their own mathematical teaching experience from placement in schools; to select incidents; to write them up; to reflect on them individually and then to discuss them in small group and plenary sessions. Regarding our first research question, we investigated whether and how this activity works in relation to what teachers prioritise, mathematically and pedagogically. Our analysis, illustrated here through four examples, indicates rich and varied insights into mathematics teaching. The first example addresses the tension between the prioritisation of the collective mathematical good of the class, over that of individuals. This is a tension that has been reported in the literature, for example in Cobb & Yackel’s (1996) renowned work on the social and sociomathematical norms that govern classrooms; yet articulating and reflecting on this tension might yet be a challenge for teachers. The second example addresses the affective issue of how successful mathematical learners may disengage from classroom activity, bearing in mind classroom environment (social) factors as well as and (psychological) perspectives on adolescence – and what classroom management techniques, in this

example questioning techniques, may be deployed to tackle this issue. The third example indicates how teachers' own ambivalence about mathematical content, here 'zero' in the algebraic expression, may result in mystifying, rather than clarifying, explanations. Finally, in the fourth example, we see how teacher aspiration for relational understanding (Skemp, 1976) also inspired by the teacher education programme they attend, may clash with established school norms.

Our work resonates with works on critical incidents (e.g. Skott, 2001) with the difference that student teachers are at liberty to create a fictional incident inspired by their experience. Our work also resonates with other works (Goodell, 2006; Pultorak, 1993) in that group discussion following the identification of incidents offers a concrete basis for teacher reflection. The insights we exemplified in this paper demonstrate also that, beyond written responses, teachers' reflection develops during this discussion (see "Mark's" and "Zina's" episodes); and that there is added value in the familiarisation of teachers with incidents created by others. In our case working on mathtasks first provided an opportunity to experience, and practise with, incidents created by others, and then identify, write up and reflect upon their own incidents.

Furthermore, our study also evaluates student teacher insights as they emerge across all phases of the activity and we do so through a typology of four characteristics (*consistency*, *specificity*, *reification of pedagogical discourse*, *reification of mathematical discourse*; our second research question). Each one of the four examples in this paper was commented through this lens. Our analysis has allowed a targeted and concrete evaluation of evidence of student teacher participation in the reflective activity that is at the heart of our study. We see this opportunity to script their inaugural experiences of mathematics teaching as a valuable exercise in preparing for the classroom – and an essential stepping stone towards embracing a participatory research gaze onto professional practice, their own as well as that of their peers.

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