Price discovery in dual-class shares across multiple markets

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Abstract: This paper proposes a new measure of price discovery that uses the spectral decomposition. The methodology is especially important in the context of large price systems, such as interest rate parities with spot and futures contracts or dual-class shares in multiple markets. We employ high frequency data to study price discovery in dual-class Brazilian stocks and their ADRs. We find that the foreign market is at least as informative as the home market and that shocks in the dual-class premium entail a permanent effect in normal times, but transitory in periods of financial distress.

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1 Introduction

Financial markets have the important function of discovering the price of a given asset. The increasing availability of high-frequency data allows us to study how efficiently and timely each market reacts to news in a much more precise manner. There is a large number of companies being listed in more than one exchange mainly because cross-listing allows raising equity capital, at a lower cost and increasing liquidity. Besides, it may also improve minority shareholder protection (Stulz, 1999), increase firm value (Doidge, Karolyi, and Stulz, 2004), and lower private benefits of control (Doidge, 2004). There are 498 foreign companies from 46 different countries listed on NYSE as of June 30, 2016. As a result, geographical price discovery is of major interest for domestic stock exchanges and companies (see, for instance, Eun and Sabherwal, 2003).

This paper extends the standard price discovery methodology to deal with high-dimension portfolios. Applications include interest rate parities using both spot and future contracts in interest rate and exchange rate markets as well as dual-class assets traded on multiple markets. We focus on the latter. The idea is to exploit every piece of information about the fundamental value of a firm by looking at the prices of both common and preferred shares across different trading platforms. As a by-product, by looking at the difference between prices of common and preferred shares, we may also shed some light on the behavior of the dual-class premium.

The main technical difficulty is to contrive a unique price discovery measure that does not assume *a priori* which share class and/or market lead the impounding of new information. For the standard information share (IS) measure of price discovery (Hasbrouck (1995)), which gauges the fraction of the variance of the fundamental price innovation due to the variance of a given asset/market price innovation, one normally imposes a triangular structure from the most informative to the least informative market price in order to handle contemporaneous correlations. Information shares will thus depend on the specific ordering in the price system.

One solution is to consider the average IS across different orderings of market prices. This is a simple and effective solution if there are only a few market prices. However, as the number of assets/markets increase, one would have to average over thousands of information shares as there is a factorial number of possible ordering.[1] Additionally, the ordering becomes more crucial

[1] For instance, a system consisting of 7 market prices as presented in Section 4 would lead to the unreasonable amount of $7! = 5,040$ distinct orderings.

to determine importance as the number of markets/assets increase. In the context of a high-dimensional portfolio, the average of different values may turn the interpretation unclear.

This problematic feature of imposing a triangular structure for measuring price discovery has been already targeted in the literature. Lien and Shrestha (2009) present an appealing method that yields unique IS measures.[2] Their framework imposes a factor structure on the correlation matrix of the price innovations.

This paper proposes a different unique measure that focuses on the covariance matrix of the price innovations, rather than on the correlation matrix. Imposing a factor structure on the covariance matrix can lead to efficiency gains in finite samples because it is unnecessary to standardize the price innovations by their volatility if they are already on similar scales. Additionally, the magnitudes of the price innovations convey information about the price discovery process given that the information share depends on the covariance matrix rather than on the correlation matrix. Monte Carlo simulations show that our spectral-based IS measure indeed outperforms Lien and Shrestha (2009) in the settings we examine. The advantage of employing our measure is that it is unique and order invariant as opposed to the Choleski decomposition. As a result, our measure of information share is completely agnostic about which market price reacts first to new information. This is especially important for the case of dual-class shares because there is no reason to believe that one asset is relatively more informative than the others. Furthermore, as highlighted by Lien and Shrestha (2009), taking averages of the different IS measures obtained with the Cholesky decomposition cannot be the result of any factor structure.

As for our empirical contribution, this paper investigates price discovery in dual-class shares trading both at the Sao Paulo Stock Exchange (BM&FBovespa) and at the New York Stock Exchange (NYSE) through the American Depositary Receipt (ADR) program. Our setting considers a much richer data set than previous studies. In particular, it takes advantage of the fact that both common and preferred share prices depend on the latent efficient/fundamental stock price. The focus on Brazilian stocks and their ADRs is convenient for a number of reasons. First, the BM&FBovespa is the leading exchange in Latin America and among the 20 largest stock exchanges in the world. Second, the trading hours at the BM&FBovespa track to a large extent the trading

[2] Grammig and Peter (2012) also propose an unique identification for the IS measure by imposing tail dependence restrictions, however their method is only applied to daily data and it requires the econometrician to take a stand on how the shocks disseminate across markets.

hours at the NYSE, amounting to an overlap of 6.5 hours in the majority of the year. This comes as a huge advantage relative to most studies in price discovery which end up with only 2 to 3 hours of intersection when using European stocks and their ADR counterparts. Third, preferred shares are historically very liquid in the BM&FBovespa because Brazilian firms could issue two preferred shares for each common share before 2001 (now it is a one-to-one ratio). The number of common shares over the number of preferred shares is indeed about 0.75 for Petrobras and 0.65 for Vale. Fourth, quality transactions data from the BM&FBovespa are available from December 2007 to November 2009, allowing us to examine how price discovery works over different market cycles.

Due to liquidity issues, we focus on the two most actively traded stocks in Brazil, namely, Petrobras and Vale, whose common and preferred shares also trade as ADRs at the NYSE. Note that for Petrobras there is also the availability to employ ADR trades from Arca (previously known as Archipelago Exchange or ArcaEx), NYSE's Chicago-based electronic platform. The latter is the second largest electronic communication network in the world, accounting for roughly 10%and 20% of the traded volume of NYSE- and Nasdaq-listed securities, respectively. This amounts to a system of 7 variables: common and preferred share prices in the BM&FBovespa, Arca and NYSE, plus the exchange rate. The latter is included in order to gauge how stock prices adjust to exchange-rate shocks. To validate our results, we also examine some additional cross-listed Brazilian stocks from different sectors.

Our price discovery analysis yields some interesting findings. First, the U.S. market is at least as informative as the home market for both Petrobras and Vale. This is not so surprising given that these Brazilian giant firms are commodity exporters and hence more sensitive to international (rather than local) market conditions. Second, Petrobras' common shares are more informative than the preferred ones in the U.S. and vice-versa in Brazil. This reflects well the liquidity and trading intensity pattern found on both common and preferred stocks in the two markets. In contrast, common and preferred shares have a similar role in Vale's price discovery process. This illustrates the fact that Vale's common shares may actually entail control power, as opposed to the case of the state-owned Petrobras. Third, shocks in the dual-class premium entail a permanent impact in normal times, whereas their effects are transitory during the financial crisis. We argue that the latter is consistent with a dual-class premium as a function of private benefits that shareholders may obtain for holding voting rights (see Zingales 1994, 1995). As there are fewer opportunities to extract private benefits, investors cease to price the dual-class premium as an asset in periods of financial distress.

The remainder of this paper is as follows. Section 2 develops the spectral-based information share measure which is more suitable to study price discovery in large price systems and present the Monte Carlo study. Section 3 discusses the institutional background of the BM&FBovespa and the handling of the high-frequency data. Section 4 documents the empirical price discovery analyses for Petrobras and Vale and some external validation tests using other dual-listed Brazilian stocks. Section 5 offers some concluding remarks.

2 Information share in a large price system

2.1 Computing information shares

To contemplate common and preferred shares in both domestic and foreign markets, this paper extends the three-variable model proposed by Grammig, Melvin, and Schlag (2005). The setup is such that every stock price in the system shares a common component given by the fundamental value of the firm (i.e. the present value of the firm's expected cash flow). This means that these prices are cointegrated. They should not diverge too much from each other as they both track the implicit efficient price. However, the latter is not the only common factor driving the system dynamics. To make stock prices in the foreign market comparable to stock prices in the domestic market, one must include the exchange rate in the system. This results in another common factor, which relates to the efficient exchange rate.

In our setup, the dual-class premium stands for another potential common factor. In that case, the gap between common and preferred share prices gauges the dual-class premium up to transient effects (e.g. liquidity issues). In principle, it stands for the price of voting rights (see Zingales 1994, 1995). It thus relates to the fundamental value of the firm through two channels. First, it depends on whether the controlling shareholder is able to extract private benefits from the firm. Such opportunities are more likely in boom periods given the higher ability to generate cash flows.[3]

[3] The argument relies on the optimal behavior of controllers. Saving the firm is more important than extracting private benefits in the short run because otherwise they will have no private benefits to extract in the future. Under good shareholder protection, the controlling shareholder would have limited access to private benefits and hence a crisis would not affect much dual-class premium.

Second, it also reflects the expected takeover premium paid to shareholders outside the control block. This implies a premium that increases with voting power, but decreases with ownership, size and trading liquidity (Smith and Amoako-Adu (1995)).

Regardless of the number of common factors governing the price dynamics, it remains a fact that common and preferred share prices must not drift apart, otherwise arbitrage opportunities would persist. There are several ways to represent such a cointegrated system. For instance, the vector error correction model (VECM) posits that

$$\Delta y_t = \xi_0\, y_{t-1} + \xi_1\, \Delta y_{t-1} + \xi_2\, \Delta y_{t-2} + \ldots + \xi_p\, \Delta y_{t-p} + \zeta + \epsilon_t, \tag{1}$$

where $\xi_0 = \alpha\beta'$, α is the error correction term with dimension $(K \times r)$, β is the $(K \times r)$ cointegrating vector, y_t is a $(K \times 1)$ vector of prices for both share classes and markets (including the exchange rate), and K and r account for the number of variables and cointegrating vectors in (1), respectively. We further assume that ϵ_t is a zero-mean white noise with a covariance matrix given by Ω and that ζ is such that cumulative price changes feature no deterministic time trends.

The error-correction representation easily leads to the vector moving average (VMA) representation:

$$\Delta y_t = \epsilon_t + \psi_1\, \epsilon_{t-1} + \psi_2\, \epsilon_{t-2} + \ldots = \Psi(L)\, \epsilon_t. \tag{2}$$

Considering Hasbrouck (1995) setup, applying Beveridge Nelson decomposition into (2) yields a permanent component which is the relevant term to compute the information share.

$$y_t = y_0 + \psi \left(\sum_{s=1}^{t} \epsilon_s \right) \iota + \Psi^*(L)\, \epsilon_t, \tag{3}$$

where ψ = common row vector of $\Psi(1) = I + \psi_1 + \psi_2 + ...$ and ι is a vector of ones. The term $\psi \left(\sum_{s=1}^{t} \epsilon_s \right)$ is common to all prices and it is seen as the efficient price. This results in $\psi\, \epsilon_t$ as the vector of common factor innovations. The covariance matrix of the innovation vector then is $\psi\Omega\psi'$.

Hasbrouck (1995) defines the information shares as the relative contributions of each share class/market to the total variance of the innovation in the permanent common factor. Considering a model which has more than one common factor and hence $\Psi(1)$ does not have a common row, the information share of each market is given by equation (4), ending up with a square matrix on

IS estimates, instead of a row:

$$IS_{ij} = \frac{\left(\left[\Psi\left(1\right)\Omega^{1/2}\right]_{ij}\right)^2}{\left(\Psi\left(1\right)\Omega\Psi\left(1\right)'\right)_{ii}}, \tag{4}$$

where i and j stand for row and column, respectively.[4]

If Ω is diagonal, $\Omega^{1/2}$ does not need any sort of decomposition. However, Ω is no longer diagonal in the presence of contemporaneous correlation between markets. To circumvent this, Hasbrouck (1995) applies a Cholesky decomposition to Ω. This amounts to assuming a lower-triangular structure in the system, which implies the markets ordered first and last as the most and least endogenous, respectively. As a result, the IS measure is not unique, varying with the ordering of the prices. If the researcher has some prior information regarding importance of prices in the system, this method can be very suitable. If this is not the case, averages across all possible permutations should be taken.

This solution has two drawbacks. First, the average IS measures do not sum up to one and they do not satisfy a factor structure as pointed out by Lien and Shrestha (2009). Second, averaging across permutations can also be inconvenient in the context of high-dimensional price systems. The number of permutations increases at a factorial rate with the system dimension. For instance, a liquid asset whose futures contracts trade with 4 different maturities would entail a system with five prices, implying over 1,000 different orderings. This is likely to entail a large gap between the minimum and maximum information shares, impairing any sort of meaningful price discovery analysis. Huang (2002), Hupperets and Menkveld (2002), Kim (2010a,b), and Grammig and Peter (2012) indeed report sizeable differences even for systems of only two/three market prices.

The literature has already targeted the issue of non unique information shares derived from the Cholesky decomposition. Lien and Shrestha (2009) come with a great contribution to this literature. They suggest an alternative IS measure which rests on the spectral decomposition of the correlation matrix, delivering the so desired unique information shares.

This paper proposes an alternative order-invariant IS measure, where a spectral decomposition of Ω is employed. The resulting IS measure is the ratio of $[\psi\, S]^2_{ij}$ to $[\psi\Omega\psi']_{ii}$, where $S = \Omega^{1/2} = V\Lambda^{1/2}V'$, with Λ and V respectively denoting the diagonal matrix with the eigenvalues along the

[4] The setup analysed in the empirical part of this paper consists on up to three common factors, namely the efficient price, the efficient exchange rate and the dual class premium.

principal diagonal and the matrix with the corresponding eigenvalues in the columns.[5] In contrast with the Cholesky factorization, the spectral decomposition does not impose a lower-triangular structure in the system, hence not implying an order of importance.

In response to its absence of economic content, the spectral decomposition is agnostic about lead-lag patterns, imposing no assumption about which share class or market is more informative. Because the only restriction imposed by the spectral decomposition is that S is symmetric, there is no need to take average out of different permutations, as S is unique no matter the order of variables. This makes our framework particularly suitable to identify which markets are dominant in setting the price (Garbade and Silber (1983)).

Comparing our methodology with the one of Lien and Shrestha (2009), the symmetric structure of the system makes economic interpretation as easy as in the triangular structure associated with the Cholesky decomposition. The structure imposed by decomposing the correlation matrix as Lien and Shrestha (2009) implement is not as clear. Monte Carlo simulations in the next section indeed show that it pays off to take a more direct approach and use the covariance matrix. This is not surprising given that information shares depend on the covariance matrix and hence, by focusing on the correlation matrix, Lien and Shrestha (2009) IS estimate ignores to some extent the information in the volatility of the price innovations.

Alternatively to the IS metric of Hasbrouck (1995), the price discovery literature has adopted an alternative metric based on the Gonzalo and Granger's (1995) Permanent-Transitory (PT) decomposition (see Lehmann, 2002). The so-called component share uses the orthogonal projections of the adjustments coefficients in the VECM model, $\alpha_\perp$, as a measure of price discovery, where the market that presents the highest element of $\alpha_\perp$ is the most important in the price discovery process (see, among others, Harris, McInish, Shoesmith, and Wood, 1995; deB. Harris, McInish, and Wood, 2002). More recently, Figuerola-Ferretti and Gonzalo (2010) go one step further and relate the component share measure to the number of market participants (proxied by trading volume). Using a theoretical model that encompasses spot and future commodities prices, they show that the number of market participants enters directly into the VECM model in the form of the speed of adjustment parameters. The price discovery measures are thus obtained through the PT decomposition, so that the component share is explicitly given as function of the number

[5] In the case of more common factors, this representation turns into $[\Psi(1)\, S]_{ij}^2$ to $[\Psi(1)\Omega\Psi(1)']_{ii}$.

of market participants. Their theoretical result is consistent with previous empirical work, which suggests that price discovery is determined by trading volume and liquidity (see, among others, Eun and Sabherwal, 2003; Frijns, Gilbert, and Tourani-Rad, 2015).

On the relation between the IS and the component share, Jong (2002) and Baillie, Booth, Tse, and Zabotina (2002) highlight the fact that both measures are closely related, but coincide only if the VECM residuals are uncorrelated across markets. However, innovations across markets are likely to correlate even at the ultra high frequency, implying that these price discovery measures should provide different results. In particular, Baillie, Booth, Tse, and Zabotina (2002) provide some analytical examples to study these differences in the event of substantial correlation. The use of the factorized covariance matrix of a cointegrated system requires the employment of upper and lower bounds on the IS measure, in contrast with the component share. Baillie, Booth, Tse, and Zabotina (2002) also briefly review a number of studies in the literature that find correlations as high as they find across markets in their empirical application at the 1-minute frequency.

2.2 Comparison between spectral-based measures

This section examines the benefits of implementing the spectral decomposition in the covariance matrix of the price innovations rather than in the correlation matrix as proposed by Lien and Shrestha (2009) (called by the authors as MIS measure). A simple simulation exercise based on the following model is performed:

$$\begin{aligned}
e_t &= e_{t-1} + u_t^e \\
p_{p,t}^h &= p_{p,t-1}^h + u_{p,t}^h \\
p_{c,t}^h &= p_{p,t-1}^h + d + u_{c,t}^h \\
p_{p,t}^f &= p_{p,t-1}^h + e_{t-1} + u_{p,t}^f \\
p_{c,t}^f &= p_{p,t-1}^h + d + e_{t-1} + u_{c,t}^f,
\end{aligned}$$

where all prices are in logs and $(u_t^e, u_{p,t}^h, u_{c,t}^h, u_{p,t}^f, u_{c,t}^f)$ is a vector of Gaussian white noises. The exchange rate e_t is entirely exogenous, the ADR price p_t^f follows the share price p_t^h at the home market. We also consider that there are both common and preferred shares (indexed by subscripts

c and p, respectively) at the home and foreign markets. Note that the model assumes a constant dual-class premium of d for the sake of simplicity.

We simulate 1,000 replications with a sample size of 10,000 observations. Note that the first 500 observations are discarded in order to alleviate any dependence on the initial values. We contemplate five scenarios for the covariance matrix of the errors. In particular, Scenarios 1 and 2 have higher correlation across markets, whereas Scenarios 3 and 4 consider lower correlation. Scenario 5 employs the sample correlation matrix of the VECM residuals in Section 4. To generate the data, we employ a factorization of the covariance matrix Ω.

It is important to stress that it is not very clear what sort of restrictions that Lien and Shrestha's (2009) MIS measure imply. In this matter, the Cholesky and spectral decompositions of the covariance matrix are more transparent in that we know exactly the restrictions they impose. More specifically, the Cholesky decomposition delivers a lower triangular matrix, whereas the spectral decomposition of the covariance matrix imposes a symmetric matrix.

To avoid biasing the results in our favor, we generate data using asymmetric decomposed matrices in Scenarios 1 to 4. As it is not clear what sort of restrictions MIS methodology imposes, we cannot know whether the factorizations we use for the covariance matrix favor or harm the MIS estimator. This is one of the main reasons why we also entertain a decomposed matrix consistent with the sample estimate of Ω. In this case, as we do not known the true information shares, we report as the theoretical information shares the values we get based on the spectral decomposition of the original sample covariance matrix.

Tables 1 to 3 report the mean and standard errors of the IS estimates for the five scenarios, whereas Table 4 exhibits the relative root mean squared error of each IS across the different scenarios. We find strong evidence that our spectral-based IS measure outperforms Lien and Shrestha's (2009) MIS measure in most situations (that is, parameter-scenario combination). Interestingly, our spectral-based IS measure performs much better whenever they do better, whereas their MIS measure perform only slightly better whenever they do outperform ours. As every parameter should carry the same importance, Table 4 also displays the average relative root mean squared errors (RRMSE) as a summary statistic. It is always lower than one, confirming the better performance of our spectral-based IS relative to the MIS.

3 Data description

BM&FBovespa is the only stock exchange in Brazil and the leading exchange in Latin America. It is a fully electronic exchange since 2005, operating under supervision of the CVM (Brazilian Securities Exchange Commission). It proportionates a central clearing for equity, commodity, derivatives, and foreign exchange markets as well as a trading platform for exchange-traded funds. With a market capitalization of USD 0.5 trillion, BM&FBovespa is among the 20 largest stock exchanges in the world.

We focus on the two most liquid stocks in the BM&FBovespa, namely, Petrobras and Vale, though Subsection 4.3 also looks at additional cross-listed Brazilian stocks to check for external validity. Petrobras and Vale are both constituents of the IBOVESPA, the main benchmark indicator of the Brazilian capital markets. Petrobras is a publicly-traded integrated oil and gas multinational, whose main stockholder is the Brazilian government with over 55% of the common shares. At the time, it was the fifth largest energy company in the world, with presence in 28 countries. It focuses on exploration and production of oil and gas in offshore fields, though it also operates in many other segments of the energy sector, e.g. include petrochemicals and biofuel. As for Vale, it is a former state mining giant, which has been private since 1997. It was at the time the second biggest metals-and-mining company in the world with the largest production of iron ore and pellets.

Petrobras and Vale issue both common and preferred shares at the BM&FBovespa. In addition, they are also present at the NYSE through the ADR program at the highest level a foreign company may sponsor (i.e. level 3, allowing for listing and public offering). Petrobras and Vale are the most active ADRs in the NYSE, both by trading value and volume. The ADRs respond for about 30% of the Petrobras outstanding shares (26% for common shares and 34% for preferred ones), whereas these figures for Vale are about 25% for common shares and 40% for preferred shares. Our data set includes the prices of both common and preferred shares of Petrobras and Vale in Brazil as well as their ADR prices in the U.S. from January 2008 to November 2009. This gives way to a system of 5 market prices for Vale: exchange rate, common and preferred share prices in Brazil and in U.S. For Petrobras, we are also able to distinguish trades on the NYSE from transaction at the NYSE Arca, leading to a system of 7 market prices.

The trading hours at the BM&FBovespa follow to a large extent the trading hours at the NYSE.

This results in 6.5 hours of overlapping from mid-February to mid-November and in 5.5 hours of overlap from mid-November to mid-February. This comes as a great advantage of the use of this data set compared to European stocks and their ADR counterparts, namely due to time difference, the intersection is of only 2 to 3 trading hours.

Given that the goal is to check how timely markets react to news incorporating them into prices, it is paramount to work with high frequency data. Sampling data at a lower frequency could well blur some lead-lag patterns between different assets and/or trading platforms. On the other hand, employing tick data raises a number of data handling issues. To control for reporting errors and delays as well as, to some extent, for microstructure effects (e.g. bid-ask bounce), we first purge the data from observations that seem implausible not only given the usual market conditions, but also given the market activity at the time. In particular, the filter proposed by Brownlees and Gallo (2006) is used.[6] Table 5 reports the initial number of observations and the number of outliers discarded for each price series as well as the resulting sample sizes after the filtering.

The next step is to deal with the nonsynchronicity of tick data. The data is aggregated into regular intervals of time, so as to employ the VMA machinery that permeates Hasbrouck's (1995) information share framework. As for the sampling frequency, the literature documents a trade-off between market microstructure noise and contemporaneous correlation between markets. As data frequency increases, microstructure effects become more apparent, whereas the contemporaneous correlation presumably declines. As the spectral-based IS measure is robust to contemporaneous correlation, more weight to alleviating market microstructure effects is given. In particular, prices are sampled at intervals of 30 and 60 seconds by considering the most recent trades on each market.

A first requirement to implement the price discovery framework discussed in Section 2.1 is that log-prices are unit root processes. Table 6 reports the p-values from the Augmented Dickey-Fuller (ADF) test. Under the null hypothesis, the ADF test states that log-prices have a unit root. As expected, we do not reject the null at 5% significance level for all assets in all sample frequencies and periods.[7] Prior to the estimation of the VECM model, it is necessary to define the number of cointegrating vectors and the lag length of the VECM models. The lag length is determined by minimizing the Bayesian information criterion (BIC) criterion, while the number of cointegrating

[6] Details on the cleaning parameters are on Table 5 and more information is available upon request.

[7] We also perform the KPSS test for the null hypothesis of no unit root. We reject the null at the 5% significance level, which reinforces the ADF test results.

vectors is chosen by means of Johansen's (1996) trace and maximum eigenvalue tests. Table 7 displays the p-values computed with the computer program of MacKinnon, Haug, and Michelis (1999) for both the trace and maximum eigenvalue tests for all firms and different sample periods. We overall find that prices are cointegrated, with the number of cointegrating vectors changing over the different sample periods. Section 4 discusses in details the economic interpretation of these results.

4 Which share class leads, and in which market?

We expect the dynamics of share and ADR prices to feature no more than three common factors. The first corresponds to the efficient exchange rate in view that the system must include the BRL/USD exchange rate to make ADR prices in U.S. dollars comparable to share prices at the BM&FBovespa. The second refers to the fundamental values of Petrobras and Vale given by the present value of their expected cash flow. Note that CVM normally requires preferred shares to pay 10% more of preferential dividends relative to common shares (as calculated from a *minimum* dividend payment of 25% of the adjusted net income). However, both Petrobras and Vale distribute systematically more dividends than the minimum payment that CVM requires during the sample period. As such, their common and preferred shares end up receiving the same amount of dividends and hence the same present value of expected cash flow.

The dual-class premium may stand as a third stochastic trend in the system. Note that the Brazilian government detains the vast majority of Petrobras voting shares and hence it makes no sense to speak about takeover premium. As for the private benefits story, it seems to fit the bill for both Petrobras and Vale. The Brazilian government has been imposing a gasoline price cap on Petrobras since 2006 to help control inflation (see e.g. The Economist, "The perils of Petrobras: How Graça Foster plans to get Brazil's oil giant back on track", November 17, 2012). Surprisingly, the same arguments also apply to Vale. Although it has been privatized in 1997, the Brazilian government indirectly detains the majority of the voting rights through a consortium of state pension funds. This not only makes takeovers very unlikely, but also raises the issue that the government may exert sway on Vale against the interest of the minority shareholders. For instance, the former CEO of Vale, Roger Agnelli, was ousted in 2011 by the state pension funds because he did not invest enough at home, particularly in low-margin industries such as steel and shipbuilding (see e.g. The Economist, "Vale dumps its boss: Roger and out", April 1, 2011).

Petrobras' and Vale's common share prices are higher than preferred prices both in Brazil and in the U.S. Relative liquidity can not explain this dual-class premium because preferred shares are much more actively traded at the BM&FBovespa than common shares for both stocks, whereas the opposite is true for their ADR counterparts. The fact that the difference between the common and preferred share prices is positive regardless of the trading platform perhaps indicates that the foreign market leads the process of impounding information for Petrobras and Vale.

In what follows, we first describe the results for Petrobras and then discuss the findings for Vale. Note that the main difference between the two analyses is that we only observe prices at the NYSE for Vale. The price system for Vale thus consists of the prices of common and preferred shares at the BM&FBovespa as well as their ADRs at the NYSE (i.e. 5 variables, including the exchange rate), whereas the Petrobras system also includes the ADR counterparts at the Arca trading platform. Note that we actually expect Arca to impound information more timely for Petrobras than the NYSE. Arca's smart order router does not restrict attention exclusively to NYSE's quotes, executing orders at the trading venue with the best available quote across all stock exchanges in the U.S. (including NASDAQ). Finally, to better understand how the price discovery mechanism changes across different market cycles, IS measures are estimated for the periods ranging from January to June 2008, July to December 2008, January to June 2009, and July to November 2009. Blanchard (1981) and Veronesi (1999) bring a theoretical analysis of asset value changes and states of the economy.

4.1 Petrobras

Figure 1 plots the prices of the Petrobras shares at the BM&FBovespa (in both BRL and USD) as well as their corresponding ADR prices in the U.S. market. It is striking how prices move in tandem, even if not surprising, given that they respond to the same fundamental value. The subperiods considered are separated by dashed lines so as to highlight their differences. Petrobras share prices are clearly trending up in the first subsample running from January to June 2008, but then stock prices plummet in the second half of 2008 as a reaction to the steady decline in the price of oil. Petrobras share prices show some recovery in the last two subsamples, reflecting to some extent the steady rise in oil prices as from January 2009.

For each subperiod, we carry out a price discovery analysis relying on the spectral-based IS measure of Section 2. We compute bootstrap-based standard errors for the IS measures based on 1,000 artificial samples from the VECM residuals as in Li and Maddala (1997). Tables 9 and 10 report the results. In particular, we report the $(K \times K)$-matrices of IS estimates, where K is the number of variables (7 for Petrobras and 5 for Vale). The cointegrating vectors are presented as a $(K \times r)$ matrix, where r is the number of cointegrating vectors (4 to 5 for Petrobras and 2 to 3 for Vale). Because there are multiple common factors, $\Psi(1)$ does not have a common row; see (4) in Section 2.1. Accordingly, we make use of the entire matrix $\Psi(1)$ as in Grammig, Melvin, and Schlag (2005) to extract the $(K \times K)$ matrix of information shares. Element (i, j) of the IS matrix corresponds to the relative contribution of market j to the total variance of the innovation in market i, where $i = 1, 2, ..., K$ and $j = 1, 2, ..., K$. As a result, every row must always sum up to one.

The top panel of Table 9 reports the results for the first half of 2008. There are 4 cointegrating vectors and hence 3 common factors. The first cointegrating vector takes essentially the difference between NYSE and Arca prices of Petrobras common shares since both of these ADRs have voting rights, their price difference essentially eliminates the common factor given by the fundamental value of Petrobras.[8] Voting rights aside, the same reasoning applies to the second cointegrating vector, which considers the difference between the prices of the preferred ADRs at the NYSE and Arca trading platforms. The third cointegrating vector corresponds to the difference between the BM&FBovespa and NYSE dual-class premia. This means that the dual-premium class indeed is a common factor driving the price dynamics, otherwise one would not have to take the difference between the observed dual-class premia in Brazil and in the U.S. to get stationarity. This may come as a surprise, especially at such a high frequency. However, it is consistent with the Brazilian government expropriating preferred shareholders as a class during this period. Finally, the fourth cointegrating vector dictates that prices in Brazil and in the U.S. must not drift apart once they are considered in the same currency. This indicates that the remaining common factor is attributable

[8] We say 'essentially' because there are some small nonzero coefficients in the cointegrating vector. These coefficient estimates are not only economically insignificant, but also jointly insignificant at the usual 95% confidence level. Indeed, we obtain a p-value of 0.0971 for a log-likelihood ratio test for the null that the cointegrating vector is $(0.00\ 0.00\ 0.00\ 1.00\ 0.00\ \text{-}1.00\ 0.00)'$.

to the efficient exchange rate.

The IS estimates show the importance of each market on price discovery. For instance, consider the preferred shares at BM&FBovespa (PETR4). The Brazilian market for preferred shares itself contributes 38% for total variance of innovations, whereas NYSE contributes 22% (12%+10%) and Arca, 36% (25%+11%). The higher the contribution of a market to the total variance of innovations, the higher its importance for price discovery. Accordingly, every row sums one. The IS estimates show that the preferred share is much more informative than the common share in Brazil, whereas the opposite is true in the U.S. This may sound puzzling, but it actually reflects well the difference in liquidity. Table 8 presents the number of trades per trading venue as well as its participation on the total trades of the asset for each period.[9] For the common share of Petrobras, Brazil accounts for only 9% of the trades in the first half of 2008, whereas NYSE and Arca have 45% and 47%, respectively. Eun and Sabherwal (2003) and Frijns, Gilbert, and Tourani-Rad (2015) indeed show that liquidity is one of the main drivers of the price discovery mechanism. It is also in line with Figuerola-Ferretti and Gonzalo (2010), who formally show how to formulate price discovery measures as a function of the number of market participants in each market. Table 9 also confirms our prediction that Arca's smart order route contributes more to the impounding of information into security prices than the NYSE.

We find that the exchange rate is not completely exogenous. This is actually similar to what Grammig, Melvin, and Schlag (2005) obtain if not ordering the exchange rate first in their Cholesky decomposition. The high correlation with the changes in the BRL/USD exchange rate (e.g. circa -0.40) is also not very surprising given that the share prices of Petrobras proxy well for the prospects of the Brazilian economy and hence for the strength of the local currency (see Krugman (1991) and Ma and Kanas (2000)).[10] Perhaps more interestingly, shocks in the efficient exchange rate are mostly absorbed by the share prices in the U.S. rather than in the local market.

The bottom panel of Table 9 reports the estimates of the spectral IS measures as well as of the cointegrating vectors for the second half of 2008. This is when the financial crisis finally hits Brazil: the Ibovespa drops about one third of its value and the Brazilian real devaluates over 50% against the U.S. dollar in this period. The financial distress seems to strongly affect the price discovery

[9] We use the number of trades in each trading venue as a proxy for liquidity and trading intensity.
[10] See also Tabak (2006) for further evidence that stock market returns in Brazil indeed lead the BRL/USD exchange rate.

process. To begin with, there are now 5 cointegrating vectors and hence only two common factors. In particular, the dual-class premium becomes stationary, characterizing the fifth cointegrating vector.[11] The fact that investors do not price the voting premium anymore remains consistent with our private benefit story. It is much easier to expropriate the shareholders with no control power in periods of boom. As crises shut down most opportunities for extracting private benefits, the difference between common and preferred shares starts to reflect much more transient liquidity issues than anything else. Furthermore, periods of financial distress are usually associated with higher correlation among stock returns. In other words, the idiosyncratic component of returns tends to be smaller in such periods. Scherrer and Fernandes (2016) find that the dual class premium relates mostly to idiosyncratic factors rather than market common factors. Additionally, the contribution of Petrobras' shares at the BM&FBovespa to the price discovery mechanism reduces in this period. This drop is particularly strong for the preferred shares. At the same time, the Arca platform gains in importance. This is in accordance with the trading activity in Table 8. Although trading intensity has increased in all trading venues, Arca's participation peaks up in this period.

Table 10 documents a similar pattern for the first half of 2009 in that the dual-class premium remains stationary and the BM&FBovespa keeps losing importance in the price discovery process. In turn, the second half of 2009 resembles more the pre-crisis period, with the efficient exchange rate, the fundamental value of the company and the dual-class premium driving the stochastic trends in the system. The only difference is that the BM&FBovespa does not recover its relative importance (especially for preferred shares), whereas Arca has kept its gain in importance, reflecting the increased relative liquidity. NYSE starts playing a more significant role, even if relative trading intensity does not increase in the same proportion. We conjecture that this reflects the increase in institutional trading as from September 2009, when Brazil obtained the investment grade rating from Moody's, allowing foreign pension funds to invest in Brazilian ADRs.

We perform a number of robustness checks. First, we estimate the IS measures at the 60-second interval. The results are very similar with no qualitative change. Second, we carry out the price discovery analysis using the complete sample period (i.e. January 2008 to November 2009) as well as by years (i.e. January to December 2008 and January to November 2009).[12] We find similar

[11] Note that the dual-class premium with the opposite sign is not present in the fifth cointegrating vector, differently from what is observed in the third cointegrating vector on the first half of 2008.

[12] All results are available upon request.

information shares, confirming that the foreign market contributes more to the price discovery mechanism than the home market. Third, we also look at the price discovery mechanism during the recent plunge in oil prices. Table 11 documents the IS estimates for the period running from June 2014 to February 2016. The first and second cointegrating vectors have the two common and preferred shares in the US, respectively, the third has the voting rights with opposite sign canceling each other out, and finally, the last cointegrating vector has the common shares in Brazil and US adjusted by the exchange rate. We find that, although the Brazilian market is more informative than before, the US market still dominates (mainly due to the higher importance of Arca). The increase in market importance for the home market is not surprising given that this periods coincides with Petrobras' corruption scandal, and hence Petrobras' news became more locally driven. Finally, we estimate the IS measures from a system that additionally contains the SPDR S&P Oil & Gas Exploration & Production ETF (XOP). The idea is to control for the effect of oil prices to our price discovery analysis. Again, the results are very similar with no qualitative change.[13]

4.2 Vale

In the absence of heavy trading at the Arca platform for Vale, we focus on a 5-dimensional system consisting of common and preferred shares at the BM&FBovespa (VALE3 and VALE5, respectively) and their corresponding ADRs at the NYSE (RIO.N and RIOp.N, respectively), plus the exchange rate. We expect Vale to feature a price discovery process similar to Petrobras. The extension code 5 in Vale's preferred shares implies they are 'class A', meaning that preferred shareholders have the right to vote in general assembly deliberations, just as common shareholders. The only difference is that preferred shareholders do not have a say in the composition of the board of directors. We thus expect Vale's preferred shares to contribute relatively more to the price discovery than Petrobras' preferred ones.

Figure 2 displays the prices of Vale's common and preferred shares and of their ADRs. The pattern it depicts is very similar to that of Petrobras in that the second half of 2008 witnesses a huge drop in prices with a slow recovery afterwards. Table 12 reveals the information shares for each half of 2008 and 2009, respectively. As in the case of Petrobras, the dual-class premium is

[13] We thank an anonymous referee for suggesting the last two robustness analyses. All results are available from the authors upon request.

a common factor in the first half of 2008, but then becomes stationary from July 2008 to June 2009. In this turbulent period, the preferred shares lose most of their importance (especially in the NYSE) and hence the price discovery takes place mainly through common shares. Shocks in the dual-premium class become permanent again only as from July 2009. As before, the NYSE is more informative than the BM&FBovespa regardless of the share class. The contribution of the NYSE to the price discovery actually increases over time. We also reject the exogeneity of the exchange rate, reflecting the fact that Vale's share prices also act as a good proxy for the local economy. As before, it is the ADR prices that absorb most of the shocks in the efficient exchange rate.

The main difference relative to Petrobras is that preferred shares play a much more significant role for Vale. The higher information shares uncovered for the preferred ADRs are likely due to the 'class A' nature of VALE5. In contrast, common and preferred shares at the BM&FBovespa entail similar contributions to the price discovery. The financial crisis seems to have a strong impact in this pattern. The information shares of the preferred shares are indeed much lower from July to December 2008, though they start to recover in the first half of 2009, regaining their full importance in the price discovery mechanism only by the second half of 2009. Also, we observe that, similarly to what happens with Petrobras, the relevance of the BM&FBovespa in the price discovery for Vale shares reduces after the financial crisis. Finally, the second half of 2009 marks the return of the dual-class premium as a common factor driving the price dynamics.

Table 8 shows that there is more liquidity for common shares at the NYSE than at the BM&FBovespa. This is in line with the information shares results. In turn, relative trading intensity is reasonably stable for preferred shares in the two halves of 2008 (56% and 57% at the BM&FBovespa versus 44% and 43% at the NYSE, respectively), whereas market informativeness decreases considerably in both markets in the second half of 2008. Brazil's participation for preferred shares increases in the second half of 2009, just as the information shares, though the latter never quite recover the levels from the beginning of 2008.

All in all, we conclude that (1) common shares have higher contributions to the impounding of information into securities prices during turbulent periods, (2) the exchange rate is not entirely exogenous to changes in share prices, (3) the dual-class premium is a common factor only in normal times, and (4) the foreign market impounds more information than the home market. The latter seems a new result given that the literature has found the home market more important to the price discovery process. For instance, Grammig, Melvin, and Schlag (2005) concludes that the German market contributes more than the NYSE for three cross-listed German firms; Hupperets and Menkveld (2002) finds that the Dutch market is more important for most cross-listed shares they analyse; and Pascual, Pascual-Fuster, and Climent (2006) find that the NYSE has little importance for five Spanish firms. All firms in these studies present considerably lower trading intensity in the U.S. when compared to the home markets. However, it seems that controlling for relative liquidity suffices to explain these differences. For instance, the firms for which the home market does not lead the price discovery process in Hupperets and Menkveld (2002) are exactly those with the smallest difference on the average number of trades per day between the Dutch and American markets. In the next section, we also show that the home market does not lose much importance during the financial crisis for a stock mainly traded at the BM&FBovespa.

Finally, apart from redoing the analysis at the 60-second frequency, we also compute information shares for each year and for the whole sample period.[14] Table 13 presents the results for the whole sample. We observe no qualitative change in the results, in that foreign market impounds more information than the home market, the exchange rate is not entirely exogenous to changes in share prices and common shares seem to be more important to the information process.

4.3 External validity

The main goal of this subsection is to check whether the information flow across markets also changes over time for other stocks. We carry out the information share analysis for some additional dual-listed Brazilian stocks, namely, Bradesco (banking), Gerdau (steel), BrTelecom (telecommunication), and Ambev (beverage). These stocks do not have enough liquidity for both classes of shares in both markets, thereof yielding price systems with a lower dimension.

Tables 14 to 17 report our findings. First, it is interesting to observe that the exchange rate looks much more exogenous for Ambev and BrTelecom. These are the stocks with the lowest weight in the Ibovespa index in our sample. Petrobras and Vale respond for about 20% of the stock market index during the sample period, whereas Ambev and BrTelecom answer for only 2.4% and 0.7%, respectively. This is consistent with our conjecture that the lack of exogeneity of the exchange rate in Petrobras' and Vale's price systems relates to their influence in the country's economy and their

[14] Additional results are available upon request.

significance on the stock market. Second, we find that Arca gains importance for every stock after the recent financial crisis. The only exception is the information share of BrTelecom, which seems pretty stable over time, probably because of liquidity reasons. BrTelecom does not trade as often on Arca as it trades on other exchanges (see Table 8). It is especially liquid at BM&FBovespa, yielding a relative higher importance of this exchange to the price discovery process. In summary, the results corroborate the evidence that the price discovery role of NYSE increased in 2009.

The trading intensity figures are in line with the price informativeness measures for these stocks. Arca is the most important trading venue for Bradesco in every period, with the highest trading intensity participation. It does not present the highest trading participation for Gerdau in every period, but it increases considerably during the crises just as its information share does. Ambev's liquidity is relatively higher at the NYSE, and hence it is not surprising that information share is relatively higher there as well, especially for the first halves of 2008 and 2009. Arca also experiences a significant decrease in relative liquidity for BrTelecom in 2009 and, accordingly, the same happens with its information share. To conclude, it is important to stress that liquidity matters, but it is certainly not the only driver in the price discovery process. For instance, although noise traders bring liquidity to markets, they do not make prices more informative.

5 Conclusion

We perform a price discovery analysis for dual-class shares that trade on different markets. In particular, we focus on the common and preferred shares of Petrobras and Vale at the BM&FBovespa and their ADR counterparts at the NYSE. This leads to a system with 5 variables for Vale and 7 variables for Petrobras given that transactions at Arca are also observed for the latter. We gauge the contribution of each share class and market by means of Hasbrouck's (1995) information share measure. Unfortunately, the standard framework does not work well for large systems because the Cholesky decomposition imposes *ex-ante* restrictions on which share class and market leads the price discovery process. We thus estimate a unique spectral-based IS measure that rests on the order-invariant eigen-decomposition of the covariance matrix of the reduced-form errors. This ensures that we remain agnostic about any lead-lag pattern in the price dynamics. Backing out unique price discovery measures is particularly important for high-dimensional price systems, such as spot and futures contracts with different maturities or dual-class shares in multiple markets. The usual fix of averaging the IS measure across all different price orderings may make the interpretation difficult and, sometimes, even ambiguous.

Examining both common and preferred shares allows us not only to gather more information about the fundamental value of the company, but also to analyze the dual-class premium and the role of the exchange rate. We find that the dual-class premium is a common factor governing the dynamics of the system only in normal times in that it becomes stationary in periods of financial distress. We also conclude that the foreign market is more important than the home market for the price discovery for both Petrobras and Vale. In fact, the IS estimates increase with the trade intensity of the corresponding price and hence the dominance of the NYSE. This pattern actually becomes more pronounced in the aftermath of the financial crisis with the BM&FBovespa losing much of its importance for Petrobras and Vale in this period.

References

Baillie, Richard T., G. Geoffrey Booth, Yiuman Tse, and Tatyana Zabotina, 2002, Price discovery and common factor models, *Journal of Financial Markets* 5, 309–321.

Blanchard, Olivier J., 1981, Output, the Stock Market, and Interest Rates, *The American Economic Review* 71, 132–143.

Brownlees, Christian T., and Giampiero M. Gallo, 2006, Financial econometric analysis at ultra-high frequency: Data handling concerns, *Computational Statistics and Data Analysis* 51, 2232–2245.

deB. Harris, Frederick H., Thomas H. McInish, and Robert A. Wood, 2002, Security price adjustment across exchanges: An investigation of common factor components for Dow stocks, *Journal of Financial Markets* 5, 277–308.

Doidge, Craig, 2004, U.S. cross-listings and the private benefits of control: evidence from dual-class firms, *Journal of Financial Economics* 72, 519–553.

Doidge, Craig, G. Andrew Karolyi, and Rene M. Stulz, 2004, Why are foreign firms listed in the U.S. worth more?, *Journal of Financial Economics* 71, 205–238.

Eun, Cheol S., and Sanjiv Sabherwal, 2003, Cross-Border Listings and Price Discovery: Evidence from U.S.-Listed Canadian Stocks, *The Journal of Finance* 58, 549–575.

Figuerola-Ferretti, Isabel, and Jess Gonzalo, 2010, Modelling and measuring price discovery in commodity markets, *Journal of Econometrics* 158, 95–107.

Frijns, Bart, Aaron Gilbert, and Alireza Tourani-Rad, 2015, The determinants of price discovery: Evidence from US-Canadian cross-listed shares, *Journal of Banking & Finance* pp. 457–468.

Garbade, Kenneth D., and William L. Silber, 1983, Price movements and price discovery in futures and cash markets, *Review of Economics and Statistics* 65, 289–297.

Gonzalo, Jesus, and Clive Granger, 1995, Estimation of common long-memory components in cointegrated systems, *Journal of Business and Statistics* 13, 27–35.

Grammig, Joachim, Michael Melvin, and Christian Schlag, 2005, Internationally cross-listed stock prices during overlapping tradinghours: Price discovery and exchange rate effects, *Journal of Empirical Finance* 12, 139–164.

Grammig, Joachim, and Franziska J. Peter, 2012, Tell-tale tails: A data-driven approach to estimate unique market information shares, *Journal of Financial and Quantitative Analysis* 48, 459–488.

Harris, Frederick H., Thomas H. McInish, Gary L. Shoesmith, and Robert A. Wood, 1995, Cointegration, error correction, and price discovery on informationally linked security markets, *Journal of Financial and Quantitative Analysis* 30, 563–578.

Hasbrouck, Joel, 1995, One security, many markets: Determining the contributions to price discovery, *Journal of Finance* 50, 1175–1198.

Huang, Roger D., 2002, The quality of ECN and Nasdaq market maker quotes, *Journal of Finance* 57, 1285–1319.

Hupperets, Erik C., and Albert J. Menkveld, 2002, Intraday analysis of market integration: Dutch blue chips traded in Amsterdam and New York, *Journal of Financial Markets* 53, 57–82.

Johansen, Søren, 1996, *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models.* (Oxford University Press, Oxford).

Jong, Frank De, 2002, Measures of contributions to price discovery: A comparison, *Journal of Financial Markets* 5, 323–327.

Kim, Cherl Hyun, 2010a, Cross-border listing and price discovery: Canadian blue chips traded in TSX and NYSE, Working paper, University of Washington.

Kim, Cherl Hyun, 2010b, Price discovery in crude oil prices, Working paper, University of Washington.

Krugman, Paul, 1991, Target zones and exchange rate dynamics, *Quartely Journal of Economics* 106, 669–682.

Lehmann, Bruce N., 2002, Some desiderata for the measurement of price discovery across markets, *Journal of Financial Markets* 5, 259–276.

Li, Hongyi, and G. S. Maddala, 1997, Bootstrapping cointegrating regressions, *Journal of Econometrics* 80, 297–318.

Lien, Donald, and Keshab Shrestha, 2009, A new information share measure, *Journal of Futures Markets* 4, 377–395.

Ma, Y., and A. Kanas, 2000, Testing for a nonlinear relationship among fundamentals and exchange rates in the ERM, *Journal of International Money and Finance* 19, 135–152.

MacKinnon, James G., Alfred A. Haug, and Leo Michelis, 1999, Numerical distribution functions of likelihood ratio tests for cointegration, *Journal of Applied Econometrics* 14, 563–577.

Pascual, Roberto, Bartolome Pascual-Fuster, and Francisco Climent, 2006, Cross-listing, price discovery and the informativeness of the trading process, *Journal of Financial Markets* 9, 144–161.

Scherrer, Cristina M., and Marcelo Fernandes, 2016, Disentangling the effect of private and public cash flows on firm value, Working paper, Queen Mary Working Paper No. 800 ISSN 1473-0278.

Smith, Brian F., and Ben Amoako-Adu, 1995, Relative prices of dual class shares, *Journal of Financial and Quantitative Analysis* 30, 563–578.

Stulz, R., 1999, Globalization, corporate finance, and the cost of capital, *Journal of Applied Corporate Finance* 12, 8–25.

Tabak, Benjamim, 2006, The dynamic relationship between stock prices and exchange rates: Evidence for Brazil, Working paper, Banco Central do Brasil (Brazilian Central Bank).

Veronesi, Pietro, 1999, Stock market overreaction to bad news in good times: A rational expectations equilibrium model, *The Review of Financial Studies* 12, 975–1007.

Zingales, Luigi, 1994, The value of the voting right: A study of the Milan Stock Exchange experience, *Review of Financial Studies* 7, 125–148.

Zingales, Luigi, 1995, What determines the value of corporate votes?, *Quarterly Journal of Economics* 104, 1047–1073.

Figure 1
The prices of Petrobras' shares and their ADR counterparts

The first plot depicts share prices in Brazilian reais and ADR prices in US dollars, whereas the second chart displays all prices in US dollars. PETR3 and PETR4 correspond to common and preferred shares at the BM&FBovespa, respectively. Similarly, PBR and PBRa are the symbols for Petrobras' common and preferred ADRs, with extensions indicating the trading platform: N for NYSE and P for Arca. The horizontal axes present the number of observations. To guide visualization dashed lines delimit the 6-month trading period.

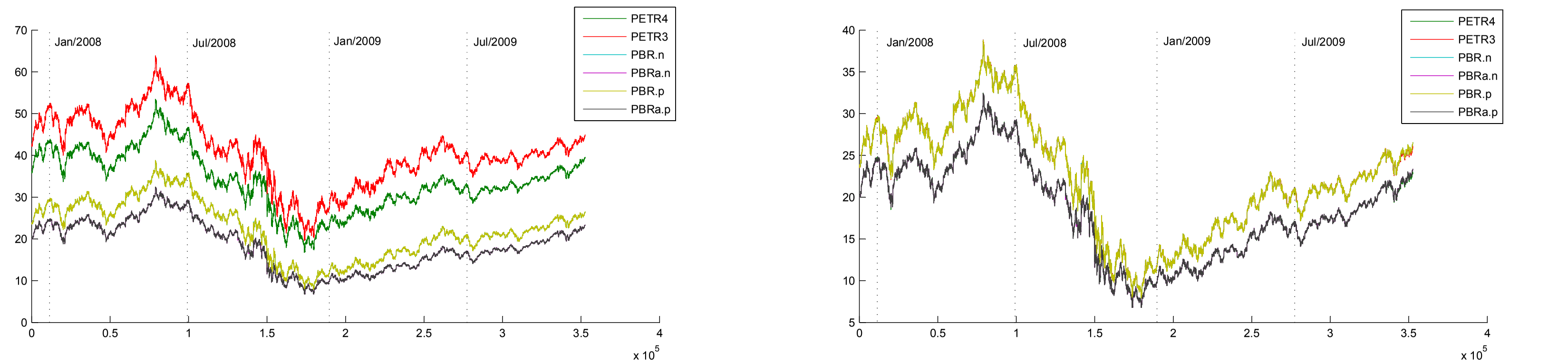

Figure 2
The prices of Vale's shares and their ADR counterparts

The first plot depicts share prices in Brazilian reais and ADR prices in US dollars, whereas the second chart displays all prices in US dollars. VALE3 and VALE5 correspond to common and preferred shares at the BM&FBovespa, respectively. Similarly, RIO and RIOp are the symbols for Vale's common and preferred ADRs at the NYSE. The horizontal axes present the number of observations. To guide visualization dashed lines delimit the 6-month trading period.

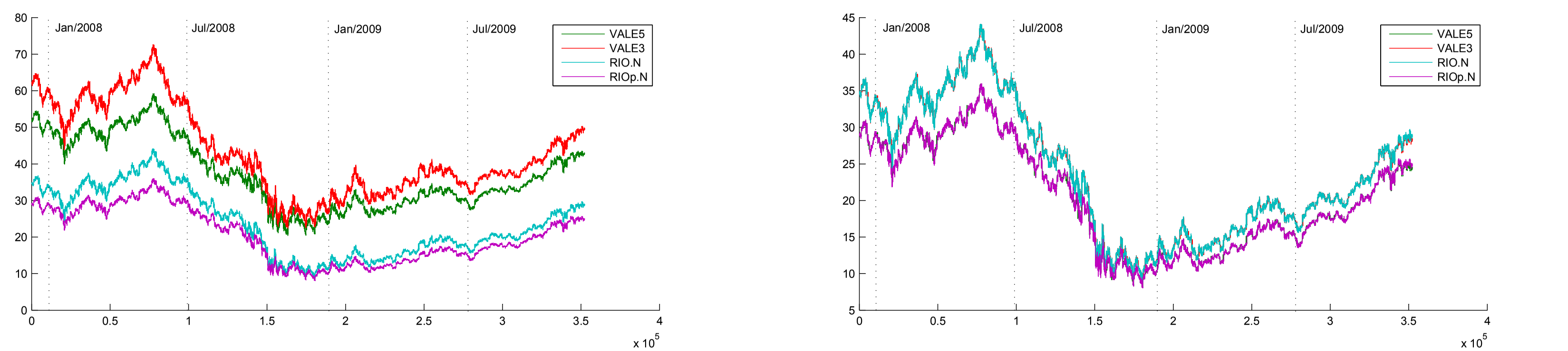

Table 1
Benefits of our methodology: high correlation

We report the mean estimates of the information shares using the spectral decomposition as well as using the MIS methodology proposed by Lien and Shrestha (2009), with their standard errors within parentheses. All results rest on 1,000 samples of 10,0000 observations of model M_3. The decomposed matrix used to generate the data is not symmetric, in order not to give advantage to our methodology. The first table has the results for scenario 1 where the decomposed matrix used is (1 0.8 0.9 0.8 0.7; 0.8 1 0.9 0.8 0.7; 0.9 0.7 1 0.8 0.7; 0.9 0.8 0.7 1 0.7; 0.9 0.8 0.7 0.8 1), followed by scenario 2 with (1 0.6 0.7 0.6 0.5; 0.6 1 0.7 0.6 0.5; 0.7 0.5 1 0.6 0.5; 0.7 0.6 0.5 1 0.5; 0.7 0.6 0.5 0.6 1).

	true					spectral					MIS				
	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h
e	0.28	0.18	0.23	0.18	0.14	0.29 (0.029)	0.16 (0.024)	0.20 (0.026)	0.20 (0.026)	0.16 (0.024)	0.25 (0.028)	0.17 (0.024)	0.19 (0.026)	0.20 (0.026)	0.19 (0.026)
p_c^h	0.23	0.18	0.14	0.18	0.28	0.22 (0.026)	0.17 (0.025)	0.15 (0.024)	0.17 (0.025)	0.29 (0.031)	0.19 (0.025)	0.18 (0.025)	0.15 (0.023)	0.17 (0.025)	0.32 (0.032)
p_p^f	0.25	0.18	0.18	0.18	0.20	0.26 (0.028)	0.17 (0.024)	0.17 (0.025)	0.19 (0.026)	0.21 (0.028)	0.22 (0.027)	0.17 (0.024)	0.17 (0.025)	0.19 (0.026)	0.25 (0.029)
p_c^f	0.25	0.18	0.18	0.18	0.20	0.26 (0.028)	0.17 (0.024)	0.17 (0.025)	0.19 (0.026)	0.21 (0.028)	0.22 (0.027)	0.17 (0.024)	0.17 (0.025)	0.19 (0.026)	0.25 (0.029)
p_p^h	0.23	0.18	0.14	0.18	0.28	0.22 (0.026)	0.17 (0.025)	0.15 (0.024)	0.17 (0.025)	0.29 (0.031)	0.19 (0.025)	0.18 (0.025)	0.15 (0.023)	0.17 (0.025)	0.32 (0.032)

	true					spectral					MIS				
	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h
e	0.41	0.15	0.20	0.15	0.10	0.40 (0.032)	0.13 (0.022)	0.18 (0.024)	0.17 (0.024)	0.13 (0.022)	0.36 (0.032)	0.14 (0.023)	0.18 (0.024)	0.17 (0.024)	0.15 (0.024)
p_c^h	0.20	0.15	0.10	0.15	0.41	0.18 (0.025)	0.13 (0.023)	0.11 (0.019)	0.14 (0.022)	0.44 (0.031)	0.15 (0.023)	0.14 (0.023)	0.11 (0.019)	0.13 (0.022)	0.48 (0.031)
p_p^f	0.31	0.15	0.15	0.15	0.24	0.30 (0.03)	0.14 (0.023)	0.15 (0.022)	0.16 (0.024)	0.26 (0.028)	0.26 (0.029)	0.14 (0.023)	0.15 (0.022)	0.16 (0.024)	0.29 (0.029)
p_c^f	0.31	0.15	0.15	0.15	0.24	0.30 (0.03)	0.14 (0.023)	0.15 (0.022)	0.16 (0.024)	0.26 (0.028)	0.26 (0.029)	0.14 (0.023)	0.15 (0.022)	0.16 (0.024)	0.29 (0.029)
p_p^h	0.20	0.15	0.10	0.15	0.41	0.18 (0.025)	0.13 (0.023)	0.11 (0.019)	0.14 (0.022)	0.44 (0.031)	0.15 (0.023)	0.14 (0.023)	0.11 (0.019)	0.13 (0.022)	0.48 (0.031)

Table 2
Benefits of our methodology: low correlation

We report the mean estimates of the information shares using the spectral decomposition as well as using the MIS methodology proposed by Lien and Shrestha (2009), with their standard errors within parentheses. All results rest on 1,000 samples of 10,0000 observations of model M_3. The decomposed matrix used to generate the data is not symmetric, in order not to give advantage to our methodology. The first table has the results for scenario 3 where the decomposed matrix used is (1 0.4 0.5 0.4 0.3; 0.4 1 0.5 0.4 0.3; 0.5 0.3 1 0.4 0.3; 0.5 0.4 0.3 1 0.3; 0.5 0.4 0.3 0.4 1), followed by scenario 4 with (1 0.2 0.3 0.2 0.1; 0.2 1 0.3 0.2 0.1; 0.3 0.1 1 0.2 0.1; 0.3 0.2 0.1 1 0.1; 0.3 0.2 0.1 0.2 1).

	true					spectral					MIS				
	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h
e	0.60	0.10	0.15	0.10	0.05	0.57 (0.031)	0.09 (0.018)	0.14 (0.023)	0.12 (0.021)	0.08 (0.017)	0.55 (0.031)	0.09 (0.019)	0.14 (0.023)	0.13 (0.021)	0.10 (0.019)
p_c^h	0.15	0.10	0.05	0.10	0.60	0.12 (0.021)	0.08 (0.018)	0.06 (0.015)	0.08 (0.018)	0.67 (0.031)	0.10 (0.019)	0.08 (0.018)	0.05 (0.015)	0.08 (0.018)	0.69 (0.031)
p_p^f	0.38	0.11	0.11	0.11	0.29	0.36 (0.03)	0.09 (0.019)	0.11 (0.02)	0.11 (0.021)	0.32 (0.03)	0.34 (0.03)	0.10 (0.02)	0.11 (0.02)	0.12 (0.021)	0.34 (0.031)
p_c^f	0.38	0.11	0.11	0.11	0.29	0.36 (0.03)	0.09 (0.019)	0.11 (0.02)	0.11 (0.021)	0.32 (0.03)	0.34 (0.03)	0.10 (0.02)	0.11 (0.02)	0.12 (0.021)	0.34 (0.031)
p_p^h	0.15	0.10	0.05	0.10	0.60	0.12 (0.021)	0.08 (0.018)	0.06 (0.015)	0.08 (0.018)	0.67 (0.031)	0.10 (0.019)	0.08 (0.018)	0.05 (0.015)	0.08 (0.018)	0.69 (0.031)

	true					spectral					MIS				
	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h
e	0.85	0.03	0.08	0.03	0.01	0.81 (0.025)	0.03 (0.011)	0.07 (0.017)	0.06 (0.015)	0.03 (0.011)	0.80 (0.025)	0.03 (0.011)	0.07 (0.017)	0.06 (0.016)	0.03 (0.011)
p_c^h	0.08	0.03	0.01	0.03	0.85	0.04 (0.012)	0.02 (0.009)	0.01 (0.005)	0.02 (0.009)	0.92 (0.019)	0.03 (0.012)	0.02 (0.009)	0.01 (0.005)	0.02 (0.009)	0.92 (0.018)
p_p^f	0.50	0.05	0.05	0.05	0.36	0.46 (0.031)	0.03 (0.012)	0.04 (0.013)	0.05 (0.014)	0.41 (0.032)	0.44 (0.031)	0.04 (0.012)	0.05 (0.014)	0.05 (0.014)	0.42 (0.032)
p_c^f	0.50	0.05	0.05	0.05	0.36	0.46 (0.031)	0.03 (0.012)	0.04 (0.013)	0.05 (0.014)	0.41 (0.032)	0.44 (0.031)	0.04 (0.012)	0.05 (0.014)	0.05 (0.014)	0.42 (0.032)
p_p^h	0.08	0.03	0.01	0.03	0.85	0.04 (0.012)	0.02 (0.009)	0.01 (0.005)	0.02 (0.009)	0.92 (0.019)	0.03 (0.012)	0.02 (0.009)	0.01 (0.005)	0.02 (0.009)	0.92 (0.018)

Table 3
Benefits of our methodology: empirical correlation

We report the mean estimates of the information shares using the spectral decomposition as well as using the MIS methodology proposed by Lien and Shrestha (2009), with their standard errors within parentheses. All results rest on 1,000 samples of 10,0000 observations of model M_3. In scenario 5, the empirical covariance matrix is retrieved, and decomposed using the spectral methodology. The small differences between the theoretical value and the spectral estimates come from rounding effects (for instance 0.19 on position 24 of the theoretical value is actually 0.1852, whereas on the spectral estimate is 0.1849).

	theoretical					spectral					MIS				
	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h	e	p_c^h	p_p^f	p_c^f	p_p^h
e	0.90	0.01	0.00	0.06	0.03	0.90 (0.019)	0.01 (0.006)	0.00 (0.003)	0.06 (0.015)	0.03 (0.012)	0.94 (0.015)	0.01 (0.006)	0.00 (0.003)	0.03 (0.01)	0.02 (0.009)
p_c^h	0.01	0.10	0.08	0.19	0.63	0.01 (0.005)	0.10 (0.019)	0.08 (0.018)	0.18 (0.025)	0.63 (0.031)	0.02 (0.009)	0.12 (0.021)	0.08 (0.018)	0.15 (0.023)	0.64 (0.031)
p_p^f	0.12	0.08	0.08	0.12	0.59	0.12 (0.022)	0.08 (0.018)	0.08 (0.018)	0.12 (0.021)	0.59 (0.034)	0.08 (0.018)	0.11 (0.02)	0.08 (0.018)	0.11 (0.02)	0.62 (0.033)
p_c^f	0.12	0.08	0.08	0.12	0.59	0.12 (0.021)	0.08 (0.018)	0.08 (0.018)	0.12 (0.021)	0.59 (0.034)	0.08 (0.018)	0.11 (0.02)	0.08 (0.018)	0.11 (0.02)	0.62 (0.033)
p_p^h	0.01	0.10	0.08	0.19	0.63	0.01 (0.005)	0.10 (0.019)	0.08 (0.018)	0.18 (0.025)	0.63 (0.031)	0.02 (0.009)	0.12 (0.021)	0.08 (0.018)	0.15 (0.023)	0.64 (0.031)

Table 4
Relative Root Mean Squared Error

We report the root mean squared error of our spectral-based IS measure relative to the MIS measure. Values lower than 1 indicate better performance of our estimtor, whereas values inferior to one indicate otherwise. For each scenario, we estimate 25 information shares given that we consider a 5-dimensional price system. The first columns labels them by IS_{ij}, whereas the following columns report the corresponding RRMSE for each scenario.

	scenario				
information share	1	2	3	4	5
IS_{11}	0.65	0.39	0.44	0.69	0.23
IS_{21}	0.35	0.36	0.53	0.84	0.09
IS_{31}	0.46	0.38	0.41	0.69	0.24
IS_{41}	0.46	0.38	0.41	0.69	0.24
IS_{51}	0.35	0.36	0.53	0.84	0.09
IS_{12}	1.19	1.19	1.11	0.99	1.06
IS_{22}	1.02	1.03	0.98	0.98	0.44
IS_{32}	1.11	1.13	1.12	1.08	0.40
IS_{42}	1.11	1.13	1.12	1.08	0.40
IS_{52}	1.02	1.03	0.98	0.98	0.44
IS_{13}	0.95	1.04	1.11	1.05	0.58
IS_{23}	1.11	1.12	1.04	1.01	0.99
IS_{33}	0.97	0.99	1.00	1.00	1.03
IS_{43}	0.97	0.99	1.00	1.00	1.03
IS_{53}	1.11	1.12	1.04	1.01	0.99
IS_{14}	1.00	0.92	0.82	0.85	0.24
IS_{24}	0.95	0.89	0.88	0.97	0.30
IS_{34}	1.03	1.02	0.98	0.95	0.82
IS_{44}	1.03	1.02	0.98	0.95	0.82
IS_{54}	0.95	0.89	0.88	0.97	0.30
IS_{15}	0.33	0.37	0.49	0.77	0.61
IS_{25}	0.36	0.39	0.61	0.88	1.02
IS_{35}	0.34	0.35	0.47	0.78	0.54
IS_{45}	0.34	0.35	0.47	0.78	0.54
IS_{55}	0.36	0.39	0.61	0.88	1.02
average	0.78	0.77	0.80	0.91	0.58

Table 5
Sample sizes before and after discarding outliers

We filter out any price entry p_i that does not conform to $|p_i - \bar{p}_i(k, 0.10)| < 3s_i(k) + 0.01$, where $\bar{p}_i(k, 0.10)$ and $s_i(k)$ are respectively the 10%-trimmed sample mean and the sample standard deviation of a neighborhood of k observations around i. We fix k according to the trade intensity, ranging from 20 to 60 observations. The column 'trading platform' informs the market at which the asset trades, 'company' reports the name of the stock, 'class' reveals whether the share class is common (ON) or preferred (PN), and 'symbol' documents the asset symbol in the trading platform. We report the sample sizes (in millions) for both raw and clean data, i.e., respectively before and after excluding outliers (in thousands).

trading platform	company	class	symbol	raw data	outliers	clean data
BM&FBovespa	Petrobras	ON	PETR3	2.11	4,812	2.10
		PN	PETR4	9.07	7,353	9.06
	Vale	ON	VALE3	2.07	8,139	2.06
		PN	VALE5	6.39	5,236	6.38
	Ambev	PN	AMBV4	0.72	4,109	0.72
	BR Telecom	PN	BRTO4	0.56	1,564	0.55
	Gerdau	PN	GGBR4	3.24	3,000	3.23
	Bradesco	PN	BBDC4	2.96	3,909	2.95
NYSE	Petrobras	ON	PBR.N	7.91	3,318	7.91
		PN	PBRa.N	5.02	4,485	5.02
	Vale	ON	Rio.N	6.93	1,159	6.93
		PN	Riop.N	3.58	1,823	3.58
	Ambev	PN	ABV.N	1.12	1,645	1.12
	BR Telecom	PN	BTM.N	0.20	521	0.20
	Gerdau	PN	GGB.N	2.83	723	2.83
	Bradesco	PN	BBD.N	3.60	959	3.60
Arca	Petrobras	ON	PBR.P	11.82	3,460	11.82
		PN	PBRa.P	4.87	2,501	4.87
	Ambev	PN	ABV.P	0.51	1,190	0.50
	BR Telecom	PN	BTM.P	0.11	391	0.11
	Gerdau	PN	GGB.P	4.16	871	4.16
	Bradesco	PN	BBD.P	6.09	1,038	6.09
exchange rate			BRLUSD	4.09	600	4.09

Table 6
P-values of the augmented Dickey-Fuller test for unit root

We report the p-values of the augmented Dickey-Fuller (ADF) test, with the lag length that minimizes the Bayesian information criterion. We do not allow for deterministic trends. The null hypothesis of the ADF test is that the variable contains a unit root. We consider 4 subsamples, namely, the first and second halves of 2008 and 2009.

frequency	trading platform	company	class	2008:01	2008:02	2009:01	2009:02
30”	BM&FBovespa	Petrobras	ON	0.76	0.19	0.92	0.84
			PN	0.74	0.17	0.93	0.94
		Vale	ON	0.66	0.32	0.73	0.93
			PN	0.64	0.33	0.76	0.93
		Gerdau	PN	0.94	0.12	0.86	0.93
		Bradesco	PN	0.48	0.42	0.86	0.93
60”		Ambev	PN	0.28	0.70	0.92	0.99
240”		BR Telecom	PN	0.60	0.51	0.56	0.93
30”	NYSE	Petrobras	ON	0.82	0.11	0.94	0.91
			PN	0.81	0.11	0.94	0.96
		Vale	ON	0.72	0.23	0.77	0.92
			PN	0.70	0.24	0.80	0.93
		Gerdau	PN	0.95	0.07	0.91	0.94
		Bradesco	PN	0.59	0.25	0.92	0.95
60”		Ambev	PN	0.49	0.42	0.96	0.99
240”		BR Telecom	PN	0.68	0.32	0.78	0.97
30”	Arca	Petrobras	ON	0.82	0.10	0.94	0.92
			PN	0.81	0.10	0.94	0.96
		Gerdau	PN	0.95	0.07	0.92	0.94
		Bradesco	PN	0.59	0.25	0.92	0.95
60”		Ambev	PN	0.50	0.42	0.96	0.99
240”		BR Telecom	PN	0.68	0.31	0.77	0.96
30”	exchange rate	BRLUSD		0.22	0.93	0.16	0.22
60”				0.22	0.93	0.16	0.22
240”				0.23	0.94	0.16	0.21

Table 7

P-values of the trace and maximum eigenvalue tests of cointegration rank

We report the p-values of the trace and maximum eigenvalue cointegration rank tests, with the lag structure that minimizes the . Bayesian information criterion. We compute p-values using the compiled version of the computer program in MacKinnon, Haug, and Michelis (1999). We do not allow for deterministic trends. The first column inform the sampling frequency at which we perform the cointegration analysis. The third column presents the number of cointegrating vectors under the null hypothesis, where r stands for the rank of $\xi_0 = \alpha\beta'$ matrix. We highlight in bold every p-value above 5%

			trace				maximum eigenvalue			
frequency	company	null	2009:01	2008:02	2009:01	2009:02	2009:01	2008:02	2009:01	2009:02
30”	Petrobras	r=1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		r=2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		r=3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		r=4	**0.63**	0.01	0.00	**0.30**	**0.43**	0.00	0.00	**0.62**
		r=5		**0.89**	**0.59**			**0.84**	**0.62**	
30”	Vale	r=1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		r=2	**0.53**	0.00	0.00	**0.14**	**0.73**	0.00	0.00	**0.08**
		r=3		**0.85**	**0.79**			**0.79**	**0.78**	
30”	Gerdau	r=1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		r=2	**0.88**	**0.74**	**0.93**	**0.67**	**0.83**	**0.66**	**0.91**	**0.73**
30”	Bradesco	r=1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		r=2	**0.64**	**0.89**	**0.93**	**0.77**	**0.83**	**0.84**	**0.93**	**0.79**
60’	Ambev	r=1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		r=2	**0.16**	**0.84**	**0.88**	**0.35**	**0.28**	**0.79**	**0.86**	**0.51**
240”	BR Telecom	r=1	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
		r=2	**0.56**	**0.90**	**0.80**	**0.57**	**0.68**	**0.86**	**0.89**	**0.76**

Table 8
Trading activity

We report the number of trades (in millions) per trading venue for each period, as well as the participation share of each trading venue on the total trades of each stock share. For example, the participation of BM&FBovespa on the common share of Petrobras is computed as trades of PETR3 over the sum of trades of PETR3, PBR.N and PBR.P.

trading platform	company	class	symbol	2008:01	2008:02	2009:01	2009:02
BM&FBovespa	Petrobras	ON	PETR3	0.34	0.61	0.64	0.51
				9%	8%	10%	16%
		PN	PETR4	1.95	2.96	2.41	1.75
				55%	44%	46%	51%
	Vale	ON	VALE3	0.36	0.60	0.63	0.47
				16%	23%	23%	33%
		PN	VALE5	1.23	1.71	1.89	1.55
				56%	57%	68%	77%
	Ambev	PN	AMBV4	0.15	0.22	0.18	0.17
				27%	29%	30%	39%
	BR Telecom	PN	BRTO4	0.13	0.14	0.14	0.15
				54%	56%	74%	79%
	Gerdau	PN	GGBR4	0.45	0.84	1.06	0.88
				29%	27%	33%	38%
	Bradesco	PN	BBDC4	0.54	0.78	0.83	0.80
				20%	18%	25%	34%
NYSE	Petrobras	ON	PBR.N	1.77	2.46	2.50	1.17
				45%	31%	37%	36%
		PN	PBRa.N	1.07	1.78	1.39	0.79
				30%	27%	26%	23%
	Vale	ON	Rio.N	1.93	1.98	2.06	0.96
				84%	77%	77%	67%
		PN	Riop.N	0.95	1.27	0.88	0.46
				44%	43%	32%	23%
	Ambev	PN	ABV.N	0.30	0.36	0.30	0.16
				53%	48%	50%	38%
	BR Telecom	PN	BTM.N	0.07	0.07	0.03	0.03
				30%	27%	17%	16%
	Gerdau	PN	GGB.N	0.69	0.80	0.73	0.61
				45%	25%	23%	26%
	Bradesco	PN	BBD.N	1.03	0.88	0.98	0.70
				38%	21%	30%	30%
Arca	Petrobras	ON	PBR.P	1.85	4.80	3.59	1.57
				47%	61%	53%	48%
		PN	PBRa.P	0.51	1.95	1.48	0.93
				15%	29%	28%	27%
	Ambev	PN	ABV.P	0.11	0.18	0.12	0.10
				19%	24%	20%	23%
	BR Telecom	PN	BTM.P	0.04	0.04	0.02	0.01
				16%	17%	9%	5%
	Gerdau	PN	GGB.P	0.40	1.50	1.43	0.82
				26%	48%	44%	36%
	Bradesco	PN	BBD.P	1.16	2.62	1.48	0.83
				43%	61%	45%	35%

Table 9
Information shares for Petrobras in 2008

We report the IS estimates based on the eigendecomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. The first subsample covers 87,781 observations from the first half of 2008, while the second subsample has 90,431 observations from the second half of 2008. BRLUSD refers to the exchange rate, PETR3 and PETR4 are the common and preferred shares of Petrobras at the BM&FBovespa, PBR and PBRa are the common and preferred ADRs of Petrobras. The extensions N and P are for NYSE and Arca, respectively. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors.

January to June	information shares							$\widehat{\beta}_{(K\times r)}$			
	BRLUSD	PETR4	PETR3	PBR.N	PBRa.N	PBR.P	PBRa.P				
BRLUSD	0.53 (0.027)	0.00 (0.002)	0.04 (0.007)	0.05 (0.008)	0.07 (0.009)	0.16 (0.017)	0.15 (0.014)	-0.02	0.00	0.01	1.00
PETR4	0.00 (0.001)	0.38 (0.025)	0.03 (0.007)	0.12 (0.015)	0.10 (0.012)	0.25 (0.023)	0.11 (0.015)	0.02	0.00	1.00	-0.01
PETR3	0.00 (0.002)	0.23 (0.019)	0.06 (0.010)	0.20 (0.023)	0.06 (0.008)	0.39 (0.026)	0.06 (0.010)	0.00	0.00	-1.00	-1.00
PBR.N	0.02 (0.004)	0.16 (0.017)	0.03 (0.006)	0.20 (0.023)	0.07 (0.010)	0.42 (0.027)	0.09 (0.014)	1.00	0.00	1.01	1.00
PBRa.N	0.04 (0.006)	0.26 (0.021)	0.01 (0.004)	0.13 (0.016)	0.12 (0.014)	0.29 (0.024)	0.15 (0.017)	0.00	1.00	-1.00	0.00
PBR.P	0.02 (0.004)	0.16 (0.017)	0.03 (0.006)	0.20 (0.023)	0.07 (0.010)	0.42 (0.027)	0.09 (0.014)	-1.00	0.00	0.00	-0.04
PBRa.P	0.04 (0.006)	0.26 (0.021)	0.01 (0.004)	0.13 (0.016)	0.12 (0.014)	0.29 (0.024)	0.15 (0.017)	-0.02	-1.00	0.00	0.04

July to December	information shares							$\widehat{\beta}_{(K\times r)}$				
	BRLUSD	PETR4	PETR3	PBR.N	PBRa.N	PBR.P	PBRa.P					
BRLUSD	0.38 (0.071)	0.04 (0.027)	0.00 (0.006)	0.09 (0.045)	0.04 (0.029)	0.29 (0.105)	0.14 (0.082)	0.00	0.00	1.00	1.00	-0.04
PETR4	0.04 (0.016)	0.14 (0.056)	0.05 (0.017)	0.17 (0.053)	0.07 (0.037)	0.36 (0.105)	0.17 (0.094)	0.00	0.00	-0.01	-1.00	0.00
PETR3	0.04 (0.015)	0.14 (0.057)	0.05 (0.017)	0.17 (0.053)	0.07 (0.037)	0.36 (0.105)	0.17 (0.093)	0.00	0.00	-1.00	-0.01	0.00
PBR.N	0.00 (0.007)	0.05 (0.032)	0.02 (0.013)	0.19 (0.057)	0.08 (0.041)	0.45 (0.114)	0.21 (0.107)	1.00	0.00	0.00	0.00	1.00
PBRa.N	0.01 (0.007)	0.05 (0.031)	0.02 (0.013)	0.19 (0.057)	0.08 (0.041)	0.45 (0.114)	0.21 (0.108)	0.00	1.00	0.00	0.00	-1.00
PBR.P	0.00 (0.007)	0.05 (0.032)	0.02 (0.013)	0.19 (0.057)	0.08 (0.041)	0.45 (0.114)	0.21 (0.107)	-1.00	0.00	1.00	0.00	0.00
PBRa.P	0.01 (0.007)	0.05 (0.031)	0.02 (0.013)	0.19 (0.057)	0.08 (0.041)	0.45 (0.114)	0.21 (0.108)	0.00	-1.00	0.00	1.00	-0.04

Table 10
Information shares for Petrobras in 2009

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. The first subsample covers 87,797 observations from the first half of 2009, while the second subsample has 75,010 observations from July to November 2009. BRLUSD refers to the exchange rate, PETR3 and PETR4 are the common and preferred shares of Petrobras at the BM&FBovespa, PBR and PBRa are the common and preferred ADRs of Petrobras. The extensions N and P are for NYSE and Arca, respectively. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors.

January to June	information shares							$\widehat{\beta}_{(K\times r)}$				
	BRLUSD	PETR4	PETR3	PBR.N	PBRa.N	PBR.P	PBRa.P					
BRLUSD	0.39 (0.042)	0.00 (0.017)	0.02 (0.004)	0.10 (0.022)	0.10 (0.031)	0.14 (0.044)	0.25 (0.056)	0.00	0.00	1.00	1.00	0.00
PETR4	0.00 (0.002)	0.08 (0.053)	0.05 (0.008)	0.26 (0.037)	0.06 (0.024)	0.41 (0.078)	0.14 (0.042)	0.00	0.00	-0.03	-1.00	0.00
PETR3	0.00 (0.004)	0.09 (0.056)	0.06 (0.010)	0.26 (0.037)	0.06 (0.022)	0.41 (0.078)	0.12 (0.038)	0.00	0.00	-1.00	-0.01	0.00
PBR.N	0.03 (0.010)	0.06 (0.048)	0.02 (0.005)	0.24 (0.037)	0.08 (0.027)	0.38 (0.075)	0.18 (0.047)	1.00	0.00	0.00	0.00	1.00
PBRa.N	0.06 (0.016)	0.05 (0.044)	0.01 (0.004)	0.23 (0.035)	0.09 (0.029)	0.35 (0.072)	0.21 (0.050)	0.00	1.00	0.00	0.00	0.00
PBR.P	0.03 (0.010)	0.06 (0.048)	0.02 (0.005)	0.24 (0.037)	0.08 (0.027)	0.38 (0.075)	0.18 (0.047)	-1.00	0.00	1.00	0.00	-1.00
PBRa.P	0.06 (0.016)	0.05 (0.044)	0.01 (0.004)	0.23 (0.035)	0.09 (0.029)	0.35 (0.072)	0.21 (0.050)	0.00	-1.00	0.02	1.00	0.00

July to November	information shares							$\widehat{\beta}_{(K\times r)}$			
	BRLUSD	PETR4	PETR3	PBR.N	PBRa.N	PBR.P	PBRa.P				
BRLUSD	0.47 (0.072)	0.00 (0.010)	0.01 (0.011)	0.13 (0.015)	0.09 (0.017)	0.15 (0.019)	0.15 (0.021)	0.01	0.00	0.00	1.00
PETR4	0.02 (0.014)	0.09 (0.046)	0.01 (0.005)	0.21 (0.014)	0.18 (0.017)	0.24 (0.017)	0.25 (0.018)	-0.01	0.00	1.00	-0.17
PETR3	0.03 (0.016)	0.03 (0.025)	0.02 (0.013)	0.33 (0.017)	0.09 (0.014)	0.37 (0.018)	0.13 (0.015)	0.00	0.00	-1.00	-1.00
PBR.N	0.01 (0.008)	0.02 (0.019)	0.01 (0.007)	0.32 (0.017)	0.12 (0.015)	0.36 (0.018)	0.18 (0.016)	1.00	0.00	1.01	1.00
PBRa.N	0.02 (0.012)	0.05 (0.033)	0.00 (0.005)	0.22 (0.015)	0.19 (0.018)	0.25 (0.017)	0.27 (0.019)	0.00	1.00	-1.00	0.00
PBR.P	0.01 (0.008)	0.02 (0.019)	0.01 (0.007)	0.32 (0.017)	0.12 (0.015)	0.36 (0.018)	0.18 (0.016)	-1.00	0.00	0.00	0.03
PBRa.P	0.02 (0.012)	0.05 (0.033)	0.00 (0.005)	0.22 (0.015)	0.19 (0.018)	0.25 (0.017)	0.27 (0.019)	0.01	-1.00	0.00	0.06

Table 11
Information shares for Petrobras June 2014 to February 2016

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. The subsample covers 291,751 observations from June 2014 to February 2016. BRLUSD refers to the exchange rate, PETR3 and PETR4 are the common and preferred shares of Petrobras at the BM&FBovespa, PBR and PBRa are the common and preferred ADRs of Petrobras. The extensions N and P are for NYSE and Arca, respectively. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors. In the next column, we report the p-values from the Augmented Dickey-Fuller (ADF) test, where no deterministic trend is allowed. The null hypothesis of the ADF test is that the variable contains a unit root. Finally, the last two columns report the p-values from the trace (Trace) and maximum eigenvalue (M.E.) cointegration rank tests. The p-values are computed using the compiled version of the computer program in MacKinnon, Haug, and Michelis (1999). The optimal lag length is obtained as the one which minimizes the BIC criterion. The deterministic elements in the VEC are restricted such that no deterministic trends are allowed. The Null column presents the number of cointegrating vectors under the null hypothesis, where r stands for the rank of $\xi_0 = \alpha\beta'$ matrix.

	information shares							$\widehat{\beta}_{(K\times r)}$				ADF	Null	Trace	M. E.
	BRLUSD	Petr4	Petr3	PBR.N	PBRa.N	PBR.P	PBRa.P								
BRLUSD	0.72 (0.022)	0.00 (0.001)	0.00 (0.003)	0.03 (0.006)	0.01 (0.004)	0.16 (0.014)	0.07 (0.009)	0.0	0.0	0.0	1.0	0.88	r=0	0.00	0.00
Petr4	0.00 (0.001)	0.28 (0.014)	0.15 (0.015)	0.12 (0.012)	0.09 (0.011)	0.23 (0.016)	0.13 (0.011)	0.0	0.0	1.0	0.0	0.93	r=1	0.00	0.00
Petr3	0.00 (0.002)	0.12 (0.01)	0.26 (0.021)	0.19 (0.015)	0.04 (0.007)	0.31 (0.017)	0.08 (0.009)	0.0	0.0	-1.0	-1.0	0.77	r=2	0.00	0.00
PBR.N	0.03 (0.004)	0.10 (0.009)	0.19 (0.017)	0.18 (0.016)	0.05 (0.008)	0.35 (0.018)	0.10 (0.01)	1.0	0.0	1.0	1.0	0.87	r=3	0.04	0.04
PBRa.N	0.04 (0.005)	0.22 (0.012)	0.11 (0.013)	0.12 (0.013)	0.09 (0.011)	0.27 (0.017)	0.15 (0.012)	0.0	1.0	-1.0	0.0	0.95	r=4	0.34	0.27
PBR.P	0.03 (0.004)	0.10 (0.009)	0.19 (0.017)	0.18 (0.016)	0.05 (0.008)	0.35 (0.018)	0.10 (0.01)	-1.0	0.0	0.0	0.0	0.87	-	-	-
PBRa.P	0.04 (0.005)	0.22 (0.012)	0.11 (0.013)	0.12 (0.013)	0.09 (0.011)	0.27 (0.017)	0.15 (0.012)	0.0	-1.0	0.0	0.0	0.94	-	-	-

Table 12
Information shares for Vale

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. There are 87,966 observations in the first half of 2008, 90,143 in the second half of 2008, 86,256 in the first half of 2009, and 76,311 observations from July to November 2009. BRLUSD refers to the exchange rate, VALE3 and VALE5 are the common and preferred shares of Vale at the BM&FBovespa, RIO.N and RIOp.N are the common and preferred ADRs of Vale at the NYSE. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors.

January to June 2008	information shares					$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	VALE5	VALE3	RIO.N	RIOp.N		
BRLUSD	0.42	0.01	0.00	0.21	0.36	1.00	0.00
	(0.039)	(0.006)	(0.002)	(0.021)	(0.029)		
VALE5	0.00	0.24	0.14	0.22	0.41	0.04	1.00
	(0.003)	(0.028)	(0.019)	(0.020)	(0.029)		
VALE3	0.00	0.10	0.25	0.42	0.23	-1.05	-0.99
	(0.003)	(0.018)	(0.026)	(0.026)	(0.023)		
RIO.N	0.02	0.06	0.18	0.45	0.30	1.02	0.99
	(0.007)	(0.014)	(0.022)	(0.026)	(0.026)		
RIOp.N	0.02	0.14	0.09	0.27	0.48	0.00	-0.99
	(0.008)	(0.022)	(0.017)	(0.021)	(0.031)		

July to December 2008	information shares					$\widehat{\beta}_{(K\times r)}$		
	BRLUSD	VALE5	VALE3	RIO.N	RIOp.N			
BRLUSD	0.54	0.04	0.01	0.32	0.10	1.00	-1.02	0.14
	(0.050)	(0.032)	(0.006)	(0.053)	(0.070)			
VALE5	0.00	0.03	0.23	0.71	0.03	0.00	1.00	1.00
	(0.002)	(0.032)	(0.026)	(0.064)	(0.034)			
VALE3	0.00	0.04	0.24	0.69	0.02	-0.99	0.00	-0.84
	(0.005)	(0.036)	(0.027)	(0.065)	(0.030)			
RIO.N	0.05	0.01	0.17	0.72	0.05	0.99	0.00	0.00
	(0.016)	(0.018)	(0.022)	(0.059)	(0.047)			
RIOp.N	0.09	0.00	0.14	0.71	0.06	0.00	-1.01	0.00
	(0.022)	(0.014)	(0.021)	(0.058)	(0.051)			

January to June 2009	information shares					$\widehat{\beta}_{(K\times r)}$		
	BRLUSD	VALE5	VALE3	RIO.N	RIOp.N			
BRLUSD	0.40	0.04	0.01	0.26	0.29	1.00	-1.00	0.16
	(0.043)	(0.044)	(0.012)	(0.051)	(0.083)			
VALE5	0.01	0.00	0.27	0.48	0.23	0.00	1.00	1.00
	(0.006)	(0.020)	(0.035)	(0.064)	(0.076)			
VALE3	0.02	0.01	0.29	0.47	0.22	-1.02	0.00	-0.78
	(0.009)	(0.022)	(0.036)	(0.064)	(0.072)			
RIO.N	0.01	0.00	0.22	0.49	0.28	1.01	0.00	0.00
	(0.004)	(0.016)	(0.032)	(0.063)	(0.085)			
RIOp.N	0.02	0.00	0.19	0.49	0.30	0.00	-1.00	0.00
	(0.009)	(0.017)	(0.031)	(0.062)	(0.089)			

July to November 2009	information shares					$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	VALE5	VALE3	RIO.N	RIOp.N		
BRLUSD	0.49	0.00	0.01	0.19	0.31	1.00	0.00
	(0.046)	(0.006)	(0.012)	(0.020)	(0.045)		
VALE5	0.00	0.19	0.08	0.28	0.44	0.15	1.00
	(0.003)	(0.041)	(0.029)	(0.022)	(0.052)		
VALE3	0.00	0.08	0.11	0.49	0.31	-1.36	-1.02
	(0.005)	(0.026)	(0.035)	(0.027)	(0.042)		
RIO.N	0.02	0.05	0.09	0.48	0.36	1.13	0.99
	(0.009)	(0.021)	(0.032)	(0.026)	(0.045)		
RIOp.N	0.04	0.11	0.07	0.33	0.46	0.00	-0.97
	(0.011)	(0.031)	(0.027)	(0.022)	(0.052)		

Table 13
Information shares for Vale January 2008 to November 2009

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. The subsample covers 340,676 observations from January 2008 to November 2009. BRLUSD refers to the exchange rate, VALE3 and VALE5 are the common and preferred shares of Vale at the BM&FBovespa, RIO.N and RIOp.N are the common and preferred ADRs of Vale at the NYSE. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors. In the next column, we report the p-values from the Augmented Dickey-Fuller (ADF) test, where no deterministic trend is allowed. The null hypothesis of the ADF test is that the variable contains a unit root. Finally, the last two columns report the p-values from the trace (Trace) and maximum eigenvalue (M.E.) cointegration rank tests. The p-values are computed using the compiled version of the computer program in MacKinnon, Haug, and Michelis (1999). The optimal lag length is obtained as the one which minimizes the BIC criterion. The deterministic elements in the VEC are restricted such that no deterministic trends are allowed. The Null column presents the number of cointegrating vectors under the null hypothesis, where r stands for the rank of $\xi_0 = \alpha\beta'$ matrix.

	information shares					$\widehat{\beta}_{(K\times r)}$			ADF	Null	Trace	M. E.
	BRLUSD	Vale5	Vale3	RIO.N	Riop.N							
BRLUSD	0.48 (0.032)	0.01 (0.017)	0.02 (0.015)	0.27 (0.062)	0.21 (0.079)	1.0	-1.0	0.1	0.58	r=0	0.00	0.00
Vale5	0.00 (0.003)	0.09 (0.051)	0.25 (0.044)	0.47 (0.08)	0.19 (0.07)	0.0	1.0	1.0	0.38	r=1	0.00	0.00
Vale3	0.01 (0.005)	0.10 (0.054)	0.26 (0.045)	0.46 (0.08)	0.17 (0.067)	-1.0	0.0	-0.9	0.35	r=2	0.00	0.00
RIO.N	0.02 (0.007)	0.05 (0.036)	0.19 (0.039)	0.50 (0.078)	0.23 (0.082)	1.0	0.0	0.0	0.36	r=3	0.73	0.66
Riop.N	0.04 (0.009)	0.03 (0.032)	0.18 (0.038)	0.50 (0.077)	0.24 (0.084)	0.0	-1.0	0.0	0.38	r=4	-	-

Table 14
Information shares for Gerdau

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. There are 87,928 observations in the first half of 2008, 88,555 in the second half of 2008, 87,734 in the first half of 2009, and 76,265 observations from July to November 2009. BRLUSD refers to the exchange rate, GGBR4 is the preferred share of Gerdau at the BM&FBovespa, GGB.N is the preferred ADR of Gerdau at the NYSE and GGB.P is the preferred ADR of Gerdau at the Arca. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors.

January to June 2008	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	GGBR4	GGB.N	GGB.P		
BRLUSD	0.78 (0.025)	0.00 (0.001)	0.09 (0.014)	0.13 (0.02)	0.00	-1.02
GGBR4	0.02 (0.004)	0.56 (0.032)	0.16 (0.017)	0.27 (0.025)	0.00	1.00
GGB.N	0.11 (0.011)	0.43 (0.03)	0.17 (0.018)	0.29 (0.026)	1.00	0.00
GGB.P	0.11 (0.011)	0.43 (0.03)	0.17 (0.018)	0.29 (0.026)	-1.00	-1.00

July to December 2008	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	GGBR4	GGB.N	GGB.P		
BRLUSD	0.74 (0.035)	0.00 (0.001)	0.03 (0.011)	0.22 (0.03)	0.00	-0.97
GGBR4	0.00 (0.002)	0.42 (0.038)	0.12 (0.017)	0.45 (0.033)	0.00	1.00
GGB.N	0.10 (0.012)	0.28 (0.034)	0.12 (0.018)	0.50 (0.034)	1.00	0.00
GGB.P	0.11 (0.012)	0.28 (0.034)	0.12 (0.018)	0.50 (0.034)	-1.00	-0.99

January to June 2009	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	GGBR4	GGB.N	GGB.P		
BRLUSD	0.73 (0.024)	0.01 (0.005)	0.07 (0.012)	0.20 (0.022)	0.00	-0.95
GGBR4	0.02 (0.004)	0.34 (0.032)	0.17 (0.019)	0.47 (0.026)	0.00	1.00
GGB.N	0.12 (0.011)	0.25 (0.028)	0.17 (0.019)	0.47 (0.026)	1.00	0.00
GGB.P	0.12 (0.011)	0.25 (0.028)	0.17 (0.019)	0.47 (0.026)	-1.00	-0.98

July to December 2009	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	GGBR4	GGB.N	GGB.P		
BRLUSD	0.66 (0.053)	0.00 (0.015)	0.15 (0.035)	0.19 (0.039)	0.00	-0.89
GGBR4	0.01 (0.005)	0.33 (0.085)	0.27 (0.045)	0.39 (0.05)	0.00	1.00
GGB.N	0.08 (0.018)	0.24 (0.075)	0.28 (0.047)	0.40 (0.051)	1.00	0.00
GGB.P	0.08 (0.018)	0.24 (0.075)	0.28 (0.047)	0.40 (0.051)	-1.00	-0.96

Table 15
Information shares for Bradesco

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. There are 87,956 observations in the first half of 2008, 88,568 in the second half of 2008, 87,718 in the first half of 2009, and 76,268 observations from July to November 2009. BRLUSD refers to the exchange rate, BBDC4 is the preferred share of Bradesco at the BM&FBovespa, BBD.N is the preferred ADR of Bradesco at the NYSE and BBD.P is the preferred ADR of Bradesco at the Arca. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors.

January to June 2008	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BBDC4	BBD.N	BBD.P		
BRLUSD	0.66 (0.034)	0.00 (0.003)	0.10 (0.016)	0.23 (0.026)	0.00	-1.00
BBDC4	0.00 (0.002)	0.38 (0.033)	0.16 (0.019)	0.46 (0.03)	0.00	1.00
BBD.N	0.07 (0.009)	0.26 (0.028)	0.18 (0.021)	0.50 (0.03)	1.00	0.00
BBD.P	0.07 (0.009)	0.26 (0.028)	0.18 (0.021)	0.50 (0.03)	-1.00	-1.00
July to December 2008	**information shares**				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BBDC4	BBD.N	BBD.P		
BRLUSD	0.64 (0.038)	0.01 (0.005)	0.13 (0.02)	0.22 (0.021)	0.00	-1.00
BBDC4	0.00 (0.001)	0.32 (0.032)	0.14 (0.02)	0.54 (0.028)	0.00	1.00
BBD.N	0.09 (0.011)	0.16 (0.023)	0.19 (0.023)	0.57 (0.026)	1.00	0.00
BBD.P	0.09 (0.011)	0.16 (0.023)	0.19 (0.023)	0.57 (0.026)	-1.00	-1.00
January to June 2009	**information shares**				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BBDC4	BBD.N	BBD.P		
BRLUSD	0.66 (0.025)	0.00 (0.001)	0.16 (0.018)	0.18 (0.018)	0.00	-1.00
BBDC4	0.00 (0.002)	0.35 (0.025)	0.31 (0.018)	0.33 (0.017)	0.00	1.00
BBD.N	0.12 (0.01)	0.21 (0.02)	0.32 (0.02)	0.35 (0.018)	1.00	0.00
BBD.P	0.12 (0.01)	0.21 (0.02)	0.32 (0.02)	0.35 (0.018)	-1.00	-1.00
July to December 2009	**information shares**				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BBDC4	BBD.N	BBD.P		
BRLUSD	0.69 (0.06)	0.00 (0.018)	0.03 (0.021)	0.27 (0.058)	0.00	-1.04
BBDC4	0.00 (0.006)	0.35 (0.099)	0.14 (0.045)	0.50 (0.066)	0.00	1.00
BBD.N	0.12 (0.034)	0.23 (0.078)	0.13 (0.044)	0.53 (0.074)	1.00	0.00
BBD.P	0.12 (0.034)	0.23 (0.078)	0.13 (0.044)	0.53 (0.074)	-1.00	-1.00

Table 16
Information shares for Ambev

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. There are 43,947 observations in the first half of 2008, 44,304 in the second half of 2008, 43,869 in the first half of 2009, and 38,131 observations from July to November 2009. BRLUSD refers to the exchange rate, AMBV4 is the preferred share of Ambev at the BM&FBovespa, ABV.N is the preferred ADR of Ambev at the NYSE and ABV.P is the preferred ADR of Ambev at the Arca. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors.

January to June 2008	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	AMBV4	ABV.N	ABV.P		
BRLUSD	0.80	0.00	0.15	0.04	0.00	-1.00
	(0.027)	(0.003)	(0.021)	(0.01)		
AMBV4	0.00	0.52	0.32	0.16	0.00	1.00
	(0.001)	(0.032)	(0.025)	(0.019)		
ABV.N	0.07	0.38	0.38	0.17	1.00	0.00
	(0.01)	(0.03)	(0.027)	(0.021)		
ABV.P	0.07	0.38	0.38	0.17	-1.00	-1.00
	(0.01)	(0.03)	(0.027)	(0.021)		

July to December 2008	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	AMBV4	ABV.N	ABV.P		
BRLUSD	0.80	0.00	0.04	0.17	0.00	-1.00
	(0.035)	(0.001)	(0.012)	(0.033)		
AMBV4	0.00	0.65	0.06	0.29	0.00	1.00
	(0.002)	(0.04)	(0.013)	(0.035)		
ABV.N	0.23	0.34	0.08	0.36	1.00	0.00
	(0.021)	(0.037)	(0.016)	(0.039)		
ABV.P	0.23	0.34	0.08	0.36	-1.00	-1.00
	(0.021)	(0.037)	(0.016)	(0.039)		

January to June 2009	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	AMBV4	ABV.N	ABV.P		
BRLUSD	0.78	0.00	0.11	0.10	0.00	-0.99
	(0.022)	(0.001)	(0.015)	(0.015)		
AMBV4	0.00	0.32	0.37	0.31	0.00	1.00
	(0.001)	(0.023)	(0.019)	(0.019)		
ABV.N	0.17	0.16	0.37	0.31	1.00	0.00
	(0.015)	(0.017)	(0.021)	(0.02)		
ABV.P	0.17	0.16	0.37	0.31	-1.00	-1.00
	(0.015)	(0.017)	(0.021)	(0.02)		

July to December 2009	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	AMBV4	ABV.N	ABV.P		
BRLUSD	0.86	0.00	0.06	0.08	0.00	-0.86
	(0.037)	(0.014)	(0.021)	(0.025)		
AMBV4	0.00	0.34	0.26	0.40	0.00	1.00
	(0.002)	(0.095)	(0.045)	(0.058)		
ABV.N	0.15	0.22	0.26	0.37	1.00	0.00
	(0.026)	(0.078)	(0.047)	(0.06)		
ABV.P	0.15	0.22	0.26	0.37	-1.00	-0.94
	(0.026)	(0.078)	(0.047)	(0.06)		

Table 17
Information shares for BrTelecom

We report the IS estimates based on the spectral decomposition of the covariance matrix of the reduced-form errors, and their bootstrap-based standard errors. There are 11,015 observations in the first half of 2008, 11,078 in the second half of 2008, 10,984 in the first half of 2009, and 9,564 observations from July to November 2009. BRLUSD refers to the exchange rate, BRTO4 is the preferred share of BrTelecom at the BM&FBovespa, BTM.N is the preferred ADR of BrTelecom at the NYSE and BTM.P is the preferred ADR of BrTelecom at the ARCA. Estimates of the cointegrating vector are denoted as $\widehat{\beta}_{(K\times r)}$. The subscript highlights that the cointegrating vector estimates are reported as a $(K \times r)$ matrix, where K is the number of variables in the system and r stands for the number of cointegrating vectors.

January to June 2008	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BRTO4	BTM.N	BTM.P		
BRLUSD	0.85 (0.027)	0.00 (0.006)	0.09 (0.021)	0.06 (0.018)	0.00	-0.98
BRTO4	0.00 (0.002)	0.36 (0.036)	0.23 (0.027)	0.41 (0.031)	0.00	1.00
BTM.N	0.06 (0.01)	0.29 (0.033)	0.25 (0.028)	0.40 (0.031)	1.00	0.00
BTM.P	0.06 (0.01)	0.29 (0.033)	0.25 (0.028)	0.40 (0.031)	-1.00	-1.00

July to December 2008	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BRTO4	BTM.N	BTM.P		
BRLUSD	0.86 (0.039)	0.00 (0.007)	0.06 (0.018)	0.08 (0.022)	0.00	-0.99
BRTO4	0.00 (0.002)	0.44 (0.052)	0.28 (0.031)	0.28 (0.031)	0.00	1.00
BTM.N	0.16 (0.023)	0.28 (0.044)	0.27 (0.031)	0.30 (0.033)	1.00	0.00
BTM.P	0.16 (0.023)	0.28 (0.044)	0.27 (0.031)	0.30 (0.033)	-1.00	-1.00

January to June 2009	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BRTO4	BTM.N	BTM.P		
BRLUSD	0.78 (0.033)	0.00 (0.002)	0.12 (0.025)	0.10 (0.023)	0.00	-0.98
BRTO4	0.02 (0.006)	0.46 (0.039)	0.34 (0.033)	0.18 (0.026)	0.00	1.00
BTM.N	0.18 (0.02)	0.28 (0.034)	0.33 (0.034)	0.20 (0.028)	1.00	0.00
BTM.P	0.18 (0.02)	0.28 (0.034)	0.33 (0.034)	0.20 (0.028)	-1.00	-0.98

July to December 2009	information shares				$\widehat{\beta}_{(K\times r)}$	
	BRLUSD	BRTO4	BTM.N	BTM.P		
BRLUSD	0.86 (0.039)	0.00 (0.008)	0.10 (0.031)	0.04 (0.018)	0.00	-0.93
BRTO4	0.01 (0.006)	0.38 (0.072)	0.43 (0.068)	0.18 (0.032)	0.00	1.00
BTM.N	0.16 (0.025)	0.24 (0.063)	0.42 (0.069)	0.17 (0.034)	1.00	0.00
BTM.P	0.16 (0.025)	0.24 (0.063)	0.42 (0.069)	0.17 (0.034)	-1.00	-0.96