Essays on Lottery contests: Theory and Experiments

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Abstract

This thesis presents four studies on a variety of lottery contests. A contest is a game in which players incur irreversible expenditure of costly resources to win some valuable reward. The contest success function, which maps the resource investments into corresponding success probabilities, is the mathematical representation of the selection mechanism in a contest. This thesis focuses on lottery contests where chance plays a crucial role in determining the outcome. The first chapter, called "That's the ticket: Explicit lottery randomisation and learning in Tullock contests" looks at how mere difference in representation can affect behaviour in an experimental lottery contest. A simple lottery contest with a single winner prize is implemented using two different frameworks one more abstract, following convention in majority of existing literature in this field, and one more operational, closely resembling the familiar institution of a lottery. Behaviour in the operational framework is closer to the theoretical prediction and reflects quicker adjustment towards best-responses. The findings have been replicated in two university locations in two continents. The second chapter, "A combinatorial multi-winner contest with package-designer preferences" provides a general framework for modelling a special type of contest that has been discretely studied in the existing literature. This is a highly stylized model of a multi-winner contest characterized by a group lottery contest among overlapping groups with perfect substitution of effort within a group. Best-responses are characterized for any connected network, and equilibrium properties have been discussed for certain types of networks. The third chapter, titled "Equivalent multi-winner contests: An experiment", shows the strategic redundancy of three multi-winner contest mechanisms and proves outcome equivalence through laboratory implementation. This chapter also argues that such lottery-based multi-winner mechanisms generate similar contest investment as in a single-winner contest for suitable parameter values. The fourth chapter, "In-group and out-group motives in group conflicts: An experimental study", employs a novel design to distinguish between different group-related attitudes in an inter-group conflict. Depending on the treatment, financial as well psychological consequences for in-group or out-group members are removed by matching current experimental subjects with the pre-recorded decisions made by previously participating subjects. This has been crossed with minimal identity conditions. No significant evidence of group bias is found. Observed behaviour shows higher support for conformity with others in decision-making. Overall, the findings from the three experiments indicate that decision-making patterns in lottery contests are largely unaffected by strategy-preserving procedural variations, but are responsive to representational differences.

My most sincere love and earnest apologies to those who missed me terribly, suffered alone, and still chose to love me only deeper through these long four years...

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Introduction

This thesis presents a collection of essays on the importance of different structural elements in different contest settings. A contest is any game where the players make irreversible investments of valuable resources to win some reward which is obtainable in a limited amount. Sports tournaments, electoral competition, lobbying for government licenses, advertising to capture a product market, or R&D investments for patents... in other words, any situation depicting a race for success can be modelled as a contest game. This reward may have intrinsic or extrinsic value to the players, but the investments and returns must be translated into comparable units of measurement for the modelling purpose. The basic contest mechanism underlying all different settings considered in this thesis is a simple lottery contest. A lottery contest is unique in featuring the role of luck and choice side by side. Given a set of contest entries, in whichever way a contest entry may be defined, the success probability of a given entry is simply the ratio of the resource content of the given entry to the aggregated resource content of all the entries. This mapping of the resource investments into corresponding success probabilities is referred to as the contest success function (CSF henceforth).

There exist certain other contest mechanisms too. The different types of contests are classified into two broad categories of perfectly discriminating and imperfectly discriminating contests. Any contest mechanism that certainly rewards the highest entry is called a perfectly discriminating contest. These type of contests are mostly equivalent to the all-pay auction or its affine transformations. An imperfectly discriminating contest on the other hand, assigns the reward to the highest entry with the highest probability, but a lower entry can win with positive probability. Gordon Tullock (1980) theorized a general class of such contests based on the principle of a simple lottery. The Tullock CSF is equivalent to a lottery CSF applied on each entry raised to a certain fixed exponent. Therefore, the lottery contest is commonly described as a special case of the Tullock contest where the exponent is one. The third type of contests are known as tournaments (Lazear and Rosen, 1981), where a player's final entry is a function of the resource investment with a given error distribution and the player with the highest final entry wins the reward with certainty. However, the mapping of the investments in the output space is not necessarily monotonic due to the error distributions being different across players. We adopt the simple lottery contest for all our investigations because of two reasons; firstly, most real world contests involve some random elements which is often simplified as luck, and secondly, the lottery contest is very easy to understand. It is important to note that a lottery contest doesn't capture unobserved elements that might affect a contest, that's precisely the point of a tournament CSF. Instead, a lottery contest models the randomness inherent to the selection mechanism. Advertising for market share is a befitting example of a lottery contest. Suppose multiple firms in an industry are simultaneously spending on advertising. The more money a firm spends the more it gets exhibited. Now supposing there is a unit mass of population who can observe (or pay attention to) only one advertisement, the firm spending the highest amount on advertising has the highest chance of being observed but it's not guaranteed the population's attention.

Given the underlying lottery mechanism, we look at three different contest settings in different chapters - a single-winner contest, a multi-winner contest, and an inter-group contest. In a single winner contest there is an indivisible fixed reward which can be assigned to exactly one player among the entire pool of contesting players. In a multi-winner contest, there are k number of fixed rewards which can be assigned to exactly k number of players out of the entire pool of players. In an inter-group contest, there are multiple groups of players and the reward can be assigned to exactly one such group. We consider a bilateral inter-group contest where all players in the winning group equally benefit from the reward.

Chapter 1 exclusively concentrates on the experimental implementation of the lottery contest. There is a very popular finding in the experimental contest literature that players in a lottery contest typically spend above the theoretical prediction, and sometimes to the extent that the aggregated investment from all players surpasses the reward value. There are multiple behavioural explanations of such over-expenditure, which includes joy of winning, risk preferences and imitation mechanism [0.1]. Our experiment implements the simple lottery contest under two different protocols and shows that using a more transparent protocol significantly reduces expenditure and expedites learning. Both protocols have similar information content, but presents the information in two different manners. We argue that making the process more transparent results in behaviour which is closer to theoretical prediction.

Both **Chapter 2** and **Chapter 3**, look at multi-winner contests. **Chapter 2** develops a general theoretical framework for a special type of multi-winner contests characterized by network-based spillover of contest investments. The typical problem considered is choosing k number of continuously connected players form a space of n players forming a connected graph. This is a highly stylized model of contests observed in thematic academic publications,

[0.1] Sheremeta, R.M. (2018). Impulsive behavior in competition: Testing theories of overbidding in ret-seeking contests.

markets with closely connected products, and selection of small close-knit project teams from a wide choice of candidates. We provide best responses for linear effort aggregation, and discuss equilibrium properties for some regular and irregular networks. **Chapter 3** implements three best-response equivalent multi-winner contests using the explicit randomization protocol from chapter 1. All three mechanisms have been discussed by different existing studies in the literature. The contribution of this chapter is to prove their strategic equivalence and further, the outcome equivalence in terms of observed behaviour. The three multi-winner mechanisms are also compared to a best-response equivalent single-winner contest and average behaviour is not significantly different. The findings from this chapter suggests higher flexibility in contest designing.

Chapter 4 addresses the much-discussed issue of group bias in an inter-group conflict. We use an inter-group lottery contest over a public good prize (Katz, 1990) to model the conflict situation. This chapter presents experimental evidences against theoretical conjectures based on common behavioural economics principles and existing findings from social psychology. This chapter also claims methodological contribution in studying group biases by removing all payoff consequences, both material and psychological, for the relevant in-group and/or out-group members. Historical player decisions constitute the crucial instrument for achieving this design goal. A 'historical player' is a subject who has participated and made their decisions in a previous session and has already been paid. If the current players' decisions are matched with such historical players, then all consequences are born by the current players only. In the control treatment, all players in the two contesting groups are current players - with the possible existence of both in-group and out-group motives, since one's decision affects the payoffs to both in-group and out-group members. Then a second treatment removes out-group motives by matching a group of current players with a group of historical players, so that there are no payoff consequences for any out-group member. The third treatment also removes the in-group motives by matching a current player with historical in-group members and against historical out-group members. Any consequences of one's decisions therefore, are born by oneself only. These three treatments are repeated under two separate conditions - the contest groups are formed on the basis of some minimal identity assignment and two contesting groups always have different identities - in one condition the subjects know this and in the other they are unaware. No treatment differences are found in the average contest investments. The findings suggest minimal group bias. On the other hand, subjects seem to conform to other current participants' behaviour whether in-group or out-group members.

Chapter 1

That's the ticket: Explicit lottery randomisation and learning in Tullock contests

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Abstract

Most laboratory experiments studying Tullock contest games find bids significantly exceed risk-neutral equilibrium predictions. We test the generalisability of these results by comparing a typical experimental implementation of a contest against the familiar institution of a lottery. We find that in the lottery (1) initial bid levels are significantly lower and (2) bids adjust more rapidly towards expected-earnings best responses. We demonstrate the robustness of our results by replicating them across two continents at two universities with contrasting student profiles. Care is therefore required in mapping the features of the experimental protocol to the features of the target field application(s) of Tullock contests.

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1.1 Introduction

> *"The hero ... is condemned because he doesn't play the game. [...] But to get a more accurate picture of his character, [...] you must ask yourself in what way (the hero) doesn't play the game".*
>
> — Albert Camus, in the afterword of *The Outsider* (Camus, 1982, p. 118)

There is an active literature studying contest games using laboratory experiments. (Dechenaux et al., 2015) The workhorse game form in these experiments has been the model of Tullock (1980). Under the Tullock contest success function (CSF), the ratio of the chances of victory of any two contestants is given by the ratio of their bids, raised to some exponent. Most experimental studies using the Tullock form report that bids in the contests exceed the levels predicted by the Nash equilibrium for risk-neutral contestants, both initially and after repeated experience with the game.

The maturity of this literature on experimental contests is reflected by the standardisation of papers in the area. Most papers follow a standard formula of motivating the interest of the experimental design and the Tullock CSF to applications in one or more areas such as rent-seeking, electoral competition, advertising, research and development, or sports. This versatility of potential applications is indeed one of the reasons for the interest in behaviour in contests. However, in the real world, the contest success function is rarely observable, and most experimental papers do not explore in detail the link between their motivating example(s) and the choice of the Tullock CSF in the experiment. The external validity of these experiments can be supported by an implicit argument that the Tullock CSF, for sufficiently small exponent, represents a generic environment in which success in the contest depends significantly *both* on the choices (hereinbelow *bids*) of the contestants, and on luck. The resulting game is analytically tractable; for example in symmetric setting with risk-neutral contestants, no spillovers, and the exponent set to one, there is a unique Nash equilibrium in which the contestants play pure strategies. (Szidarovszky and Okuguchi, 1997; Chowdhury and Sheremeta, 2011)

Laboratory studies using the Tullock CSF also vary in their use of instructions, examples, and choice architecture in the experimental protocol. In Section 1.2 we summarise a review of recent protocols. Experimenters tend to design protocols with features parallel to other recent papers. This increases the comparability of results with the existing literature, at the potential risk of converging on one set of conventions for the protocol design. However, the Tullock CSF is interesting precisely because of its wide range of applicability. Although the game form generated by the Tullock CSF abstracts away from implementation detail, it is well-established that implementation details can matter because human decision-makers do attend to details of language and process. Careful empirical support is required for confident claims about robust behavioural findings across "contests" in all their domains of application.

There is a familiar and naturally-occurring institution which implements the Tullock CSF with exponent one in a transparent way: the lottery or raffle. We conduct a series of experiments contrasting this institution with a protocol based on typical features of the experiments reported in our survey. We demonstrate that both the levels and the dynamics of bids differ significantly as a function of the institutions. In the lottery, initial bids are significantly lower, and adjustment towards (risk-neutral) best-responses are faster, with median group bids approaching the risk-neutral prediction. Although our two treatments implement the same Tullock CSF and have the same financial incentives as a function of the bids of the players, they give rise to visibly different patterns of behaviour. We replicate our results in two participant pools in two countries, at two universities with undergraduate student bodies with contrasting profiles.

We begin by surveying the details of recent experiments using the Tullock CSF in Section 1.2. We present the formal description of the Tullock contest game, our experimental design, and hypotheses in Section 1.3. The summary of the data and the results are included in Section 1.4. We conclude in Section 1.5 with further discussion.

1.2 Survey of protocols

Table 1.1 updates and extends the list of Tullock contest studies compiled in Sheremeta (2013). The rightmost column in this table shows the ratio of observed bids in the experiment to the risk-neutral Nash equilibrium bid. Most studies report bids to be above the Nash prediction. Sheremeta (2013) reports that the median experiment generates bids 1.72 times that of the Nash prediction, on the basis of his meta-analysis of 30 different contest experiments involving 39 experimental treatments. Several papers (Herrmann and Orzen, 2008; Abbink et al., 2010; Cason et al., 2012a; Cohen and Shavit, 2012; Mago et al., 2013) report average bids more than double the Nash level. However, bids relative to the Nash prediction do vary substantially, with some studies finding bids below or close to the Nash benchmark. This variability, which has not been carefully studied, could be attributable to differences in parameters, in terminology and choice architecture, or participant pools.

We augment the survey of Sheremeta by collecting information on the experimental protocols used for presenting the game.[4] We focus on two qualitative features of these

[4]We were unable to obtain instructions for a few of the studies listed. Cells with entries marked — indicate studies we were not able to classify.

protocols: (1) whether the instructions refer to a ratio of bids mapping into probabilities of winning the prize, and (2) what concrete randomisation mechanism, if any, is mentioned in the instructions.

The column "ratio rule" in Table 1.1 indicates whether the experiment's instructions made an explicit mention that the probability of winning is given by the *ratio* of the contestant's own bid to the total bids of all contestants. A majority of studies do discuss this ratio, with many, including Fallucchi et al. (2013), Ke et al. (2013), and Lim et al. (2014) explicitly using a displayed mathematical formula similar to (1.1).[5]

Many designs supplement mention of a ratio or probability with a concrete description of a mechanism capable of generating the probability. For example, instructions may state that it is "as if" bids translate into lottery tickets, or other objects such as balls or tokens (Potters et al., 1998; Fonseca, 2009; Masiliunas et al., 2014; Godoy et al., 2015), which are then placed together in a container, with one drawn at random to determine the winner. We record this practice as "Lottery" in the "Example" column of Table 1.1. However, in carrying out the experiment itself, experimenters rarely resolve the outcome of each period using an explicit lottery draw presentation.

An alternative approach used in a minority of studies (Shupp et al., 2013; Herrmann and Orzen, 2008; Morgan et al., 2012; Ke et al., 2013) is a spinning lottery wheel. On this wheel, bids are mapped proportionally onto wedges of a circle. Morgan et al. (2012) and Ke et al. (2013) report bids around 1.5 times that predicted by the equilibrium, while Herrmann and Orzen (2008) report bids just above equilibrium and Schmidt et al. (2006) find bids below the equilibrium prediction.

Based on this survey, the emerging standard of recent years, which we refer to as the *conventional* approach, is for a Tullock contest experiment to

1. introduce the game in terms of a probability or ratio;

2. give a lottery as an example mechanism, but

3. not to play out the lottery explicitly when determining the winner of the prize in each game.

The introduction of the conceit of the lottery mechanism suggests that the total *counts* or frequencies of tickets are helpful for participants to understand and process the strategic situation. Differences in processing probabilistic (ratio) versus frequency (count) information have been studied extensively in psychology. For example, Gigerenzer and Hoffrage (1995)

[5] Another alternative, giving a full payoff table as used by Shogren and Baik (1991), is a rather rare device.

Study	Treatment	Ratio rule	Example	Actual-to-Nash
Millner and Pratt (1989)	Lottery	Yes	Lottery	1.12
Millner and Pratt (1991)	Less risk averse	Yes	Lottery	1.23
Shogren and Baik (1991)	Lottery	No (payoff table)	—	1.01
Davis and Reilly (1998)	Lottery	—	—	1.46
Potters et al. (1998)	Lottery	Yes	Lottery	1.68
Anderson and Stafford (2003)	One-shot (n=2)	No	Lottery	1.79
	One-shot (n=3)	No	Lottery	1.50
	One-shot (n=4)	No	Lottery	1.87
	One-shot (n=5)	No	Lottery	3.00
	One-shot (n=10)	No	Lottery	3.46
Schmitt et al. (2004)	Static	Yes	Lottery	1.76
Herrmann and Orzen (2008)	Direct repeated	No	Wheel	2.05
Kong (2008)	Loss aversion	Yes	Lottery	1.81
Fonseca (2009)	Simultaneous	No	Lottery	2.00
Abbink et al. (2010)	1:1	No	Lottery	2.05
Sheremeta (2010)	Single	Yes	Lottery	1.52
Sheremeta and Zhang (2010)	Individual	Yes	Lottery	1.95
Ahn et al. (2011)	1V1	—	—	1.37
Price and Sheremeta (2011)	P treatment	Yes	—	1.90
Sheremeta (2011)	GC	Yes	Lottery	1.33
	SC	Yes	Lottery	1.31
Cason et al. (2012b)	Individual	No	—	1.26
Faravelli and Stanca (2012)	Standard Lottery	Yes	Lottery	1.10
Morgan et al. (2012)	Small Prize	No	Wheel	1.45
	Large Prize	No	Wheel	1.19
Cohen and Shavit (2012)	One-shot (w/o refund)	Yes	—	2.52
Mago et al. (2013)	High r	Yes	Lottery	1.90
	High r + IP	Yes	Lottery	1.51
	Low r + IP	Yes	Lottery	2.73
Ke et al. (2013)	Share/fight	Yes	Wheel	1.50
Kimbrough and Sheremeta (2013)	Baseline	Yes	Lottery	1.95
Cason et al. (2012a)	Blue & Green	Yes	Lottery	2.23
Shupp et al. (2013)	Single-prize (real lottery)	Yes	Wheel	0.73
Brookins and Ryvkin (2014)	Complete symmetric	Yes	—	1.42
Chowdhury et al. (2014)	PL	Yes	—	1.75
Lim et al. (2014)	n=2	Yes	Lottery	1.30
	n=4	Yes	Lottery	1.61
	n=9	Yes	Lottery	3.30
Fallucchi et al. (2013)	Lottery-Full	Yes	Lottery	1.25
Kimbrough et al. (2014)	Baseline unbalanced	Yes	Lottery	1.41
	Baseline balanced	Yes	Lottery	1.15
	Random unbalanced	Yes	Lottery	1.17
	Random balanced	Yes	Lottery	1.25
Masiliunas et al. (2014)	N1S1	Yes	Lottery	1.25
Deck and Jahedi (2015)	Experiment 1	Yes	—	1.64
Sheremeta (2015)	One-shot	Yes	Lottery	1.81
Price and Sheremeta (2015)	Gift / Yardstick	Yes	Lottery	1.92
Godoy et al. (2015)	Partner-Random	No	Lottery	0.66
	Partner-No allocation	No	Lottery	1.37
Baik et al. (2015)	Medium	Yes	Lottery	1.73
Baik et al. (2016)	Partner (n=3)	Yes	Lottery	1.53
	Partner (n=2)	Yes	Lottery	1.20
Mago et al. (2016)	NP-NI	Yes	Lottery	1.94
	NP-I	Yes	Lottery	1.89

Table 1.1: Summary of Tullock contest experiments. “Ratio rule” indicates whether the study gives chances of winning in terms of ratios of bids. “Example” describes whether and how the game is additionally explained in terms of a (pseudo-)physical mechanism. “Actual-to-Nash” is the reported ratio of average bids to the Nash baseline prediction.

proposed humans are well-adapted to manipulating frequency-based information as this is the format in which information arises in nature.

However, the lottery is precisely the institution which occurs in the field, with which participants will typically be familiar, and which implements exactly the Tullock CSF. Our *ticket* treatment implements this institution throughout the instructions and play of the game. The instructions introduce the game only as a lottery, in which bids translate into individually-numbered tickets. When playing the game, ticket numbers are assigned, and in realising the outcome of the game the number of the winning ticket is revealed. The play of the game therefore matches the description in the instructions. It likewise corresponds closely to the implementation of real-world lotteries, in which tickets are often collected into a container such as a large drum, with one selected at random to determine the winner.

1.3 Experimental design

The Tullock lottery contest game we study is formally an n-player simultaneous-move game. There is one indivisible prize, which each player values at $v > 0$. Each player i has an endowment $\omega \geq v$, and chooses a bid $b_i \in [0, \omega]$. Given a vector of bids $b = (b_1, \ldots, b_n)$, the probability player i receives the prize is given by

$$p_i(b) = \begin{cases} \frac{b_i}{\sum_{j=1}^n b_j} & \text{if } \sum_{j=1}^n b_j > 0 \\ \frac{1}{n} & \text{otherwise} \end{cases} \tag{1.1}$$

If players are risk-neutral, they maximise their expected payoff $u_i(b) = vp_i(b) + (\omega - b_i)$. The unique Nash equilibrium is in pure strategies, with $b_i^{NE} = \min\left\{\frac{n-1}{n^2}v, \omega\right\}$ for all players i.

In our experiment, we choose $n = 4$ and $\omega = v = 160$. We restrict the bids to be drawn from the discrete set of integers, $\{0, 1, \ldots, 159, 160\}$. With these parameters, the unique Nash equilibrium has $b_i^{NE} = 30$.

Participants played 30 contest periods, with the number of periods announced in the instructions. The groups of $n = 4$ participants were fixed throughout the session. Within a group, members were referred to anonymously by ID numbers 1, 2, 3, and 4; these ID numbers were randomised after each period. All interaction was mediated through computer terminals, using zTree (Fischbacher, 2007). A participant's complete history of their own bids and their earnings in each period was provided throughout the experiment. Formally, therefore, the 4 participants in a group play a repeated game of 30 periods, with a common public history.[6]

[6]That is, all players in the group share the same history of which bids were submitted in each period. Because

By standard arguments, the unique subgame-perfect equilibrium of this supergame interaction is to play the $b_i^{NE} = 30$ in all periods irrespective of the history of play.[7]

We contrast two treatments, the *conventional* treatment and the *ticket* treatment, in a between-sessions design. The instructions for both treatments introduce the game as "bidding for a reward."[8] In the conventional treatment, the instructions explain the relationship between bids and chances of receiving the reward using the mathematical formula first, with a subsequent sentence mentioning that bids could be thought of as lottery tickets. Our explanation follows the most common pattern found across the studies surveyed in Table 1.1. In the ticket treatment, each penny bid purchases an individually-numbered lottery ticket. To determine the winner of the prize, the number of one of those lottery tickets is selected and displayed on the screen.

The randomisation in each period was presented to participants in line with the explanations in the instructions. In conventional treatment sessions, after bids were made but before realising the outcome of the lottery, participants saw a summary screen (Figure 1.1a), detailing the bids of each of the participants in the group. In sessions using the ticket treatment, the explicit ticket metaphor was played out by providing the identifying numbers for each ticket purchased (Figure 1.1b).

There are two channels through which the implementation of the ticket treatment could lead to behaviour different from that in the conventional treatment.

1. While both mention the lottery ticket metaphor, the conventional treatment instructions discuss the chances of receiving the prize, whereas the ticket treatment uses counts of tickets purchased. An effect due to this change would be identifiable in the first-period bids, which are taken when participants have not had any experience with the mechanism or information about the behaviour of others.

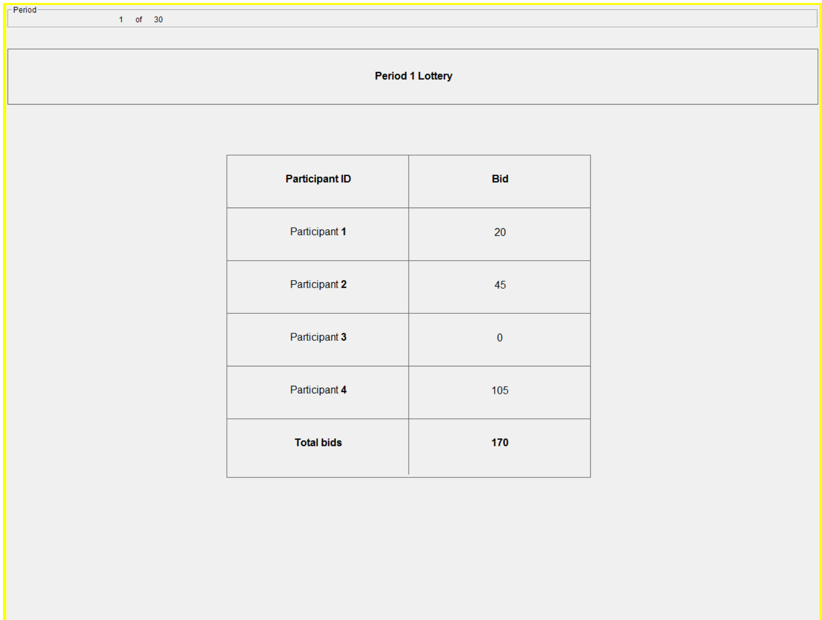
2. Feedback is structured in terms of the individually-identifiable tickets. The number of the winning ticket conveys no additional payoff-relevant information beyond the identity of its owner. An effect due to this feedback structure will be identifiable by looking at the evolution of play within each fixed group over the course of the 30 periods of the session.

We structure our analysis to look for treatment effects via both of these possible channels.

of the re-assignment of IDs in each period, the private histories differ, because each participant knows which of the four bids in each period was the one they submitted, and whether or not they were the winner in each period.

[7] Table 1 in Fallucchi et al. (2013) shows that both fixed-groups and random-groups designs are common in the literature. We choose a fixed-groups design to facilitate the focus in our analysis on the dynamics of behaviour as the supergame is played out.

[8] We provide full text of the instructions in Appendix 1.A.



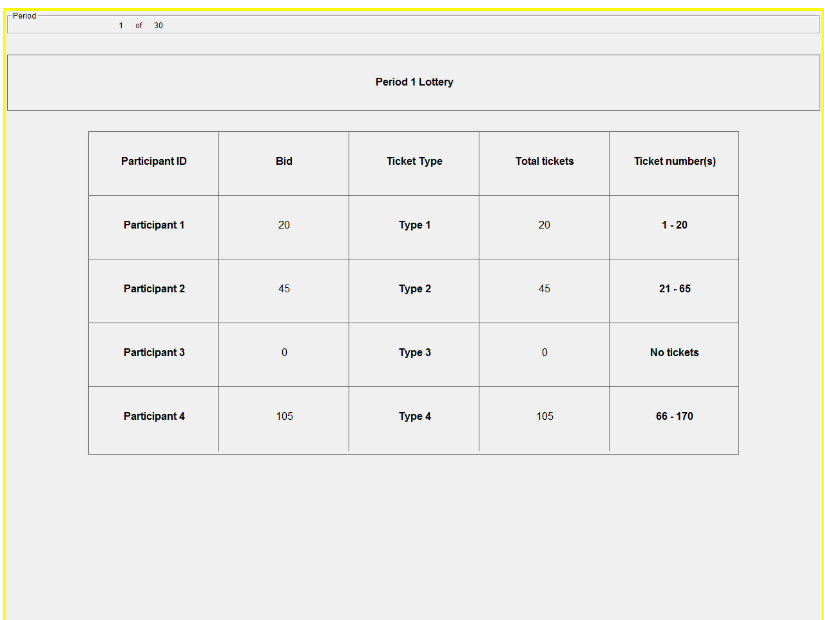
Period

1 of 30

Period 1 Lottery

Participant ID	Bid
Participant 1	20
Participant 2	45
Participant 3	0
Participant 4	105
Total bids	170

(a) Conventional treatment



Period

1 of 30

Period 1 Lottery

Participant ID	Bid	Ticket Type	Total tickets	Ticket number(s)
Participant 1	20	Type 1	20	1 - 20
Participant 2	45	Type 2	45	21 - 65
Participant 3	0	Type 3	0	No tickets
Participant 4	105	Type 4	105	66 - 170

(b) Ticket treatment

Figure 1.1: Comparison of bid summary screens

We conducted a total of 14 experimental sessions. Eight of the sessions took place at the Centre for Behavioural and Experimental Social Science at University of East Anglia in the United Kingdom, using the hRoot recruitment system (Bock et al., 2012), and six at the Vernon Smith Experimental Economics Laboratory at Purdue University in the United States, using ORSEE (Greiner, 2015). We refer to the samples as UK and US, respectively. In the UK, there were four sessions of each treatment with 12 participants (3 fixed groups) per session; in the US, there were three sessions of each treatment with 16 participants (4 fixed groups) per session. We therefore have data on a total of 48 participants (12 fixed groups) in each treatment at each site.

The units of currency in the experiment were pence. In the UK sessions, these are UK pence. In the US sessions, we had an exchange rate, announced prior to the session, of 1.5 US cents per pence. We selected this as being close to the average exchange rate between the currencies in the year prior to the experiment, rounded to 1.5 for simplicity.

Participants received payment for 5 of the 30 periods. The five periods which were paid were selected publicly at random at the end of the experiment, and were the same for all participants in a session.[9] Sessions lasted about an hour, and average payments were approximately £10 in the UK and $15 in the US.

1.4 Results

We begin with an overview of all 5,760 bids in our sample. Figure 1.2 displays dotplots for the bids made in each period, broken out by subject pool and treatment. Table 1.2 provides summary statistics on the individual bids for each treatment and subject pool. Both the figure and table suggest a treatment difference. Aggressive bids at or near the maximum of 160 are infrequent in the ticket treatment after the first few periods, but persist in the conventional treatment. Figure 1.3 summarises the distribution of mean bids by group over time. The aggregate patterns of behaviour are similar in the UK and US.

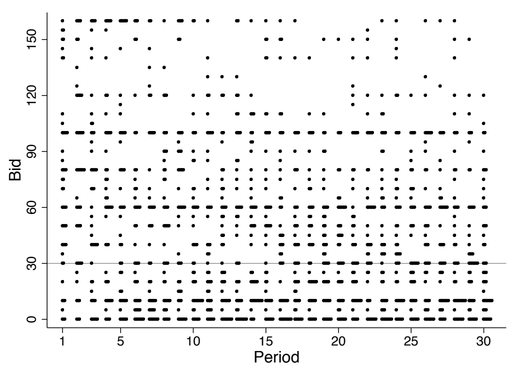
Result 1.1. *In each treatment, there are no significant differences between the distributions of bids in the UK versus in the US.*

Support. We use the group as the unit of independent observation, and compute, for each group, the average bid over the course of the experiment. The Mann-Whitney-Wilcoxon (MWW) rank-sum test does not reject the null hypothesis of equal distributions of these group means between the UK and US pools ($p = 0.86$, $r = 0.479$ for the conventional and

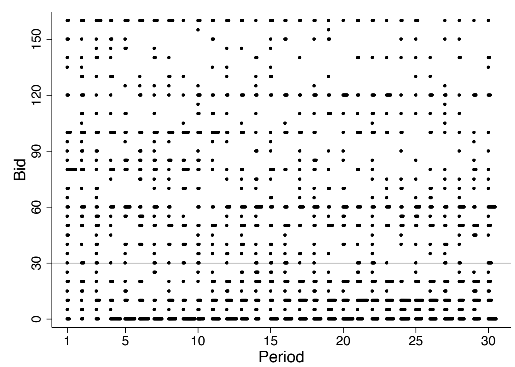
[9]The US participants also received a $5 participation payment on top of their contingent payment, to be consistent with conventions at Purdue.

	UK	US	All
Conventional	49.85	53.56	51.70
	(43.77)	(48.41)	(46.18)
	$N = 1440$	$N = 1440$	$N = 2880$
Ticket	40.35	41.08	40.72
	(35.96)	(35.36)	(35.65)
	$N = 1440$	$N = 1440$	$N = 2880$

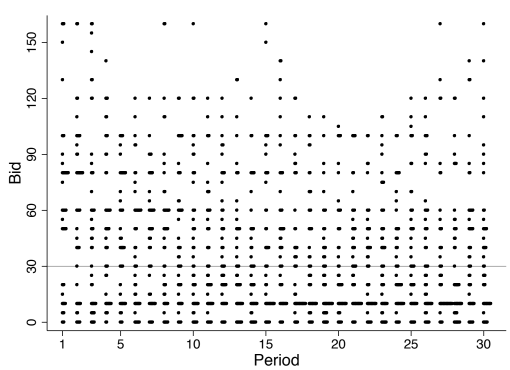
Table 1.2: Descriptive statistics on individual bids. Each cell contains the mean, standard deviation (in parentheses), and total number of bids. The column All pools the bids from the two sites.



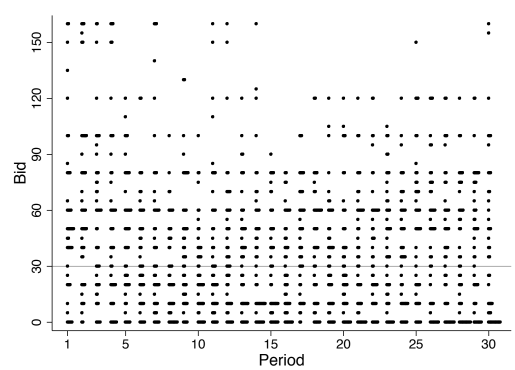
(a) Conventional, UK



(b) Conventional, US

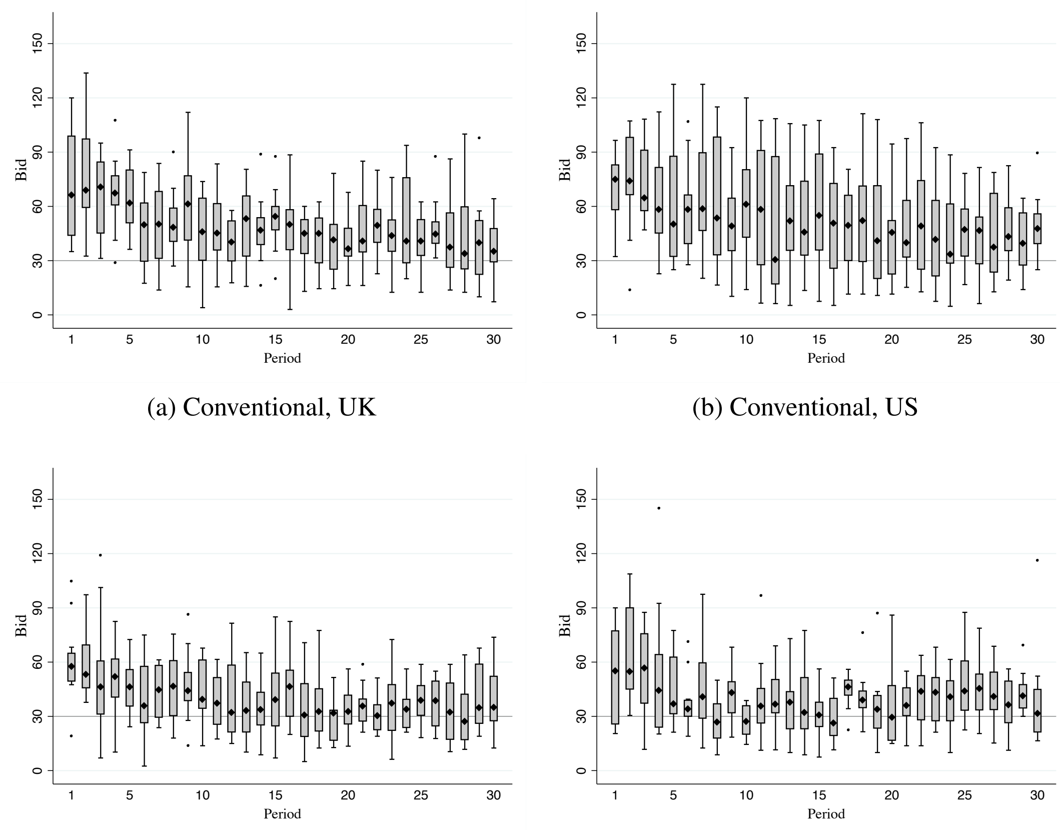


(c) Ticket, UK



(d) Ticket, US

Figure 1.2: All bids by period, grouped by subject pool and treatment. Each dot represents the bid of one participant in one period. The Nash equilibrium bid of 30 is indicated by a horizontal line.



(a) Conventional, UK

(b) Conventional, US

(c) Ticket, UK

(d) Ticket, US

Figure 1.3: Distribution of group mean bids, by subject pool and treatment. For each period, the vertical boxes plot the interquartile range of average bids across groups. The black diamond indicates the median of the group averages.

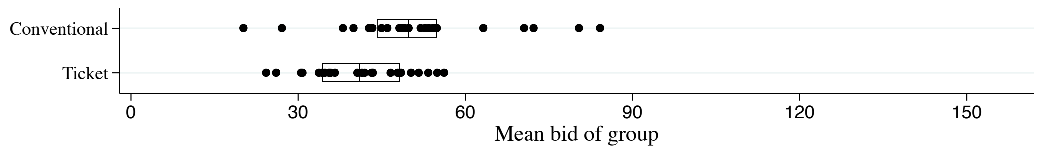


Figure 1.4: Distribution of mean bids for each group over the experiment. Each dot represents the mean bid of one group.

$p = 0.91$, $r = 0.486$ for the ticket treatment).[10] Similarly, the MWW test does not reject the null hypothesis if the group means are computed based only on periods 1-10, 11-20, or 21-30. □

This result speaks against a hypothesis that a significant amount of the variability observed in Table 1.1 is attributable to differences across subject pools. In view of the similarities between the data from our two subject pools, we continue by using combined sample for our subsequent analysis. Our next result treats the full 30-period supergame as a single unit for each group, and compares behaviour to the benchmark of the unique subgame-perfect Nash equilibrium in which the stage game equilibrium is played in each period.

Result 1.2. *Bids are significantly lower over the course of the experiment in the ticket treatment than in the conventional treatment. Both treatments significantly exceed the Nash equilibrium prediction.*

***Support*.** For each group we compute the mean bid over the course of the experiment. The mean over groups is 51.7 in the conventional treatment (standard deviation 14.8) and 40.7 in the ticket (standard deviation 9.1). Figure 1.4 plots the full distribution of these group means; the boxes indicate the locations of the median and upper and lower quartiles of the distributions. Using the MWW rank-sum test, we reject the hypothesis that the distribution under the ticket treatment is the same as that under the conventional treatment ($p = 0.0036$, $r = 0.255$). □

The difference between the treatments could be attributable to some difference in how experiential learning takes place because of the feedback mechanism in playing out the lottery explicitly, or simply because participants process the explanation of the game differently. We can look for evidence of the latter by considering only the first-period bids.

[10]The MWW test statistic between samples A and B is equivalent to the probability that a randomly-selected value from sample A exceeds a randomly-selected value from sample B. We report this effect size probability as r, with the convention that the sample called A is the one mentioned first in the description of the hypothesis in the text. Under the MWW null hypothesis of the same distribution, $r = 0.5$.

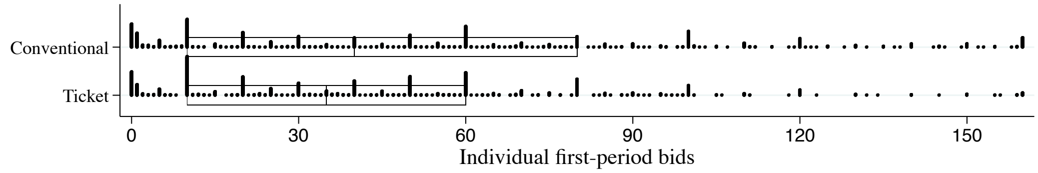


Figure 1.5: Distribution of first-period bids for all participants. Each dot represents the bid of one participant. For each distribution, the superimposed box indicates the median and the lower and upper quartiles.

Result 1.3. *First-period bids are significantly lower in the ticket treatment than the conventional treatment, and therefore are closer to the Nash equilibrium prediction.*

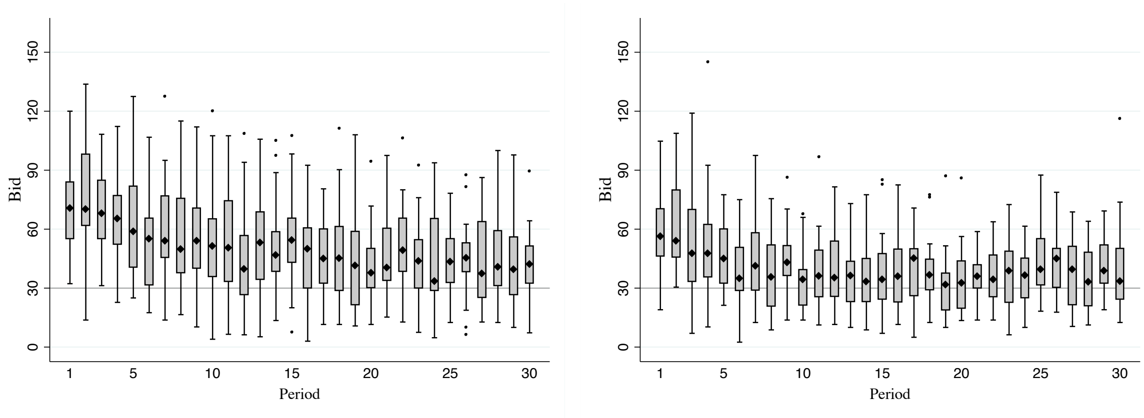
Support. Figure 1.5 displays the distribution of first-period bids for all 192 bidders (96 in each treatment). Because at the time of the first-period bids participants have had no interaction, we can treat these as independent observations. The mean first-period bid in the conventional treatment is 71.1, versus 56.8 in the ticket treatment. Put another way, as a point estimate approximately 35% of the observed overbidding relative to the Nash prediction is explained in the first period by the treatment difference. Using the MWW rank-sum test, we reject the hypothesis that the distribution under the ticket treatment is the same as that under the conventional treatment ($p = 0.020$, $r = 0.403$). □

First-period bids on average are above the equilibrium prediction in both treatments. We therefore turn to the dynamics of bidding over the course of the session. Returning to the group as the unit of independent observation, Figure 1.6 displays boxplots of the distribution of group average bids period-by-period for each treatment. Bid levels are higher in the conventional treatment in the first period, and both treatments exhibit a trend of average bids decreasing towards the Nash equilibrium prediction.

We are interested in determining whether the ticket-based implementation also has an effect on the dynamics of behaviour over the experiment. Under the maintained assumption that participants are interested in the earnings consequences of their actions, we organise our analysis of dynamics in terms of payoffs, rather than bids themselves. Consider a group g in session s of treatment $c \in \{\text{conventional}, \text{ticket}\}$. We construct for this group, for each period t, a measure of disequilibrium based on ε-equilibrium. (Radner, 1980) In each period t, each bidder i in the group submitted a bid b_{it}. Given these bids, bid b_{it} had an expected payoff to i of

$$\pi_{it} = \frac{b_{it}}{\sum_{j \in g} b_{jt}} \times 160 + (160 - b_{it}).$$

For comparison, we can consider bidder i's best response to the other bids of his group.



(a) Conventional (b) Ticket

Figure 1.6: Evolution of group average bids over time. For each period, the vertical boxes plot the interquartile range of average bids across groups. The black diamond indicates the median of the group averages.

Letting $B_{it} = \sum_{j \in g : j \neq i} b_{jt}$, the best response, if bids were permitted to be continuous, would be given by

$$\tilde{b}^{\star}_{it} = \max\{0, \sqrt{160B_{it}} - B_{it}\}.$$

Bids are required to be discrete in our experiment; the quasiconcavity of the expected payoff function ensures that the discretised best response $b^{\star}_{it} \in \{\lceil \tilde{b}^{\star}_{it} \rceil, \lfloor \tilde{b}^{\star}_{it} \rfloor\}$. This discretised best response then generates an expected payoff to i of

$$\pi^{\star}_{it} = \frac{b^{\star}_{it}}{b^{\star}_{it} + B_{it}} \times 160 + (160 - b^{\star}_{it}).$$

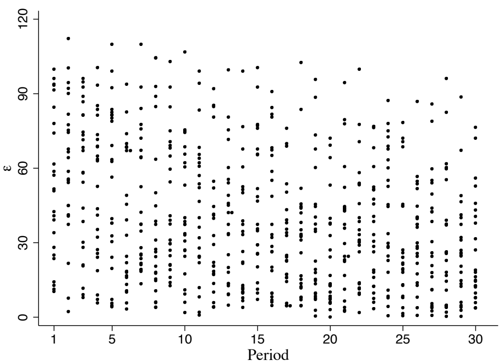
We then write[11]

$$\varepsilon_{csgt} = \max_{i \in g}\{\pi^{\star}_{it} - \pi_{it}\}.$$

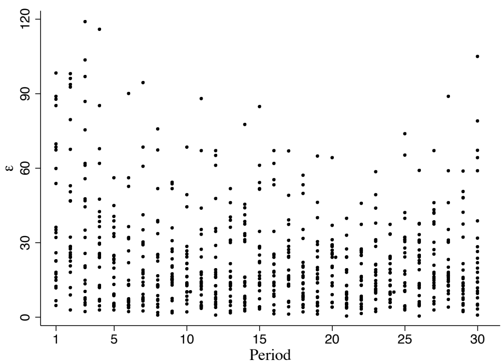
By construction, $\varepsilon_{csgt} \geq 0$, with $\varepsilon_{csgt} = 0$ only at the Nash equilibrium.

Conducting the analysis in the payoff space measures behaviour in terms of potential earnings. The marginal earnings consequences of an incremental change in bid depends on both b_{it} and B_{it}, so a solely bid-based analysis would not adequately capture incentives. In addition, although in general bids are high enough that the best response in most groups in most periods is to bid low, there are many instances in which the best response for a bidder would have been to bid higher than they actually did. A focus on payoffs allows us to track the dynamics without having to account for directional learning in the bid space.

[11]Taking the maximum to define the metric ε_{csgt} gives the standard definition of ε-equilibrium. Our results about the treatment effect on dynamics also hold if ε_{csgt} is defined as the average or the median in each group.



(a) Conventional



(b) Ticket

Figure 1.7: Ex-post measure of disequilibrium ε within groups, by period. Each dot corresponds to the value of the measure for one group in one period.

Figure 1.7 shows the evolution of the disequilibrium measure ε over the experiment. The clustering of this measure at lower values, especially below about 30, is evident in the ticket treatment throughout the experiment, while any convergence in the conventional treatment is slower. While suggestive, these dot plots alone are not enough to establish whether the evolution of play differs between the treatments, because it does not take into account the dynamics of each individual group. Result 1.3 implies that values of ε in Period 1 are lower in the ticket treatment. Therefore, the difference seen in Figure 1.7 could be attributable to the different initial conditions rather than different dynamics, as there is simply less room for ε to decrease among the groups in the ticket treatment given their first-period decisions.

We control for this by investigating the evolution of ε within-group over the experiment. As a first graphical investigation, we plot the average value of $\varepsilon_{csg(t+1)}$ as a function of ε_{csgt} for both treatments in Figure 1.8.[12] Consider two groups, one in the conventional treatment and one in the ticket treatment, who happen to have the same ε in some period. Figure 1.8 says that in the subsequent period, on average, the ε measure of the group in the ticket treatment will be lower, that is, they will move further towards an approximate mutual (expected-earnings) best response.[13]

Result 1.4. *Convergence towards equilibrium, as measured by ε-equilibrium, is significantly faster in the ticket treatment than in the conventional treatment.*

Support. To formalise the intuition provided by Figure 1.8, we estimate a random-effects

[12] For the purposes of Figure 1.8 we aggregate observations by rounding ε_{csgt} to the nearest multiple of five, and taking the average over all observations with the same rounded value.

[13] There are very few groups in either treatment with values of ε above about 75, accounting for the instability in the graph for large ε.

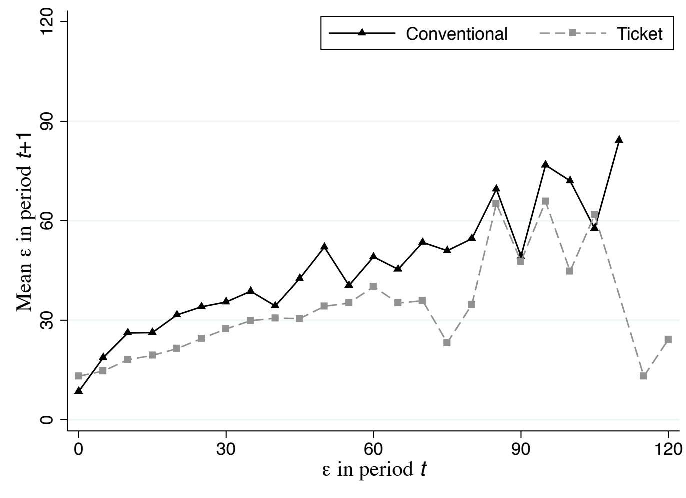


Figure 1.8: Expected value of disequilibrium measure ε in next period, as a function of a group's current ε.

	Coefficient	Standard error	p-value
Intercept	18.77	1.39	<0.001***
ε_{csgt}	0.50	0.03	<0.001***
$\mathbf{1}_{c=\text{ticket}}$	-4.90	1.85	0.008**
$\mathbf{1}_{c=\text{ticket}} \times \varepsilon_{csgt}$	-0.11	0.05	0.024*

Table 1.3: Random-effects panel regression of evolution of disequilibrium measure ε over time. Dependent variable is $\varepsilon_{csg(t+1)}$; standard errors clustered at the session level. Overall $R^2 = 0.2955$. *** denotes significantly different from zero at 0.1%; ** at 1%, * at 5%.

panel regression

$$\varepsilon_{csg(t+1)} = \alpha + \beta_0 \varepsilon_{csgt} + \beta_1 \mathbf{1}_{c=\text{ticket}} + \beta_2 \mathbf{1}_{c=\text{ticket}} \times \varepsilon_{csgt},$$

where $\mathbf{1}_{c=\text{ticket}}$ is a dummy variable which takes on the value 1 for groups in sessions using the ticket treatment and 0 otherwise. The resulting parameter estimates, with standard errors clustered at the session level, are reported in Table 1.3. Both the intercept and slope parameters for the ticket treatment are significantly lower than for the conventional treatment, indicating a faster rate of convergence for groups in the ticket treatment from any given initial value of ε. □

Figure 1.8 and Table 1.3 show that in both treatments, groups that have very small values of ε in one period tend to increase ε in the subsequent period; that is, they move away from equilibrium. The fixed point for ε using the point estimates is about 37.5 for the conventional treatment, and 22.5 for the ticket treatment. This observation is consistent with previous studies (Chowdhury et al., 2014) which have demonstrated a difference between the Tullock contest with a random outcome, as studied here, and a version in which the prize is shared deterministically among the contestants in proportion to their bids. When behaviour is close to equilibrium, small deviations in bids have small consequences in terms of expected payoffs, leaving open the door for other behavioural factors to come into play. For example, although the outcome of the randomisation contains no new information for participants, they may nevertheless base their bids in subsequent periods in part on the outcomes of previous random draws. The presence of this or similar heuristics would introduce an underlying level of noise in play consistent with the positive intercepts obtained in Table 1.3.

1.5 Discussion

Our results show that generic claims about behaviour in "contests" are not yet well-founded by experimental evidence. The literature commonly takes the risk-neutral Nash equilibrium as a benchmark and defines "overbidding" to be bids in excess of this amount. The lottery implementation lowers overbidding by approximately one-third in initial bids, and one-half over the course of 30 periods, measured relative to the conventional implementation. Although we may identify a variety of laboratory environments as implementing a Tullock CSF, the prediction that these environments are behaviourally equivalent is not supported by our data.

Our literature survey summarised in Table 1.2 illustrates that while greater-than-Nash average bids are the norm, there is substantial variation reported across studies. These studies have been conducted over years, with different implementation details as well as in different participant pools. Our design focuses on two institutional implementations and explicitly compares behaviour across two universities in two countries. Usefully these universities have distinct academic profiles and reputations. Purdue University is noted for its strengths in particular in engineering and other quantitative disciplines, whereas University of East Anglia has no engineering programme but is strong in areas such as creative writing.

Our results therefore suggest that much of the variation observed in Table 1.2 may be attributable to features of experimental implementation. To address this research question our experimental design compared two existing implementations, a typical one drawn from the literature and a lottery-based implementation which we designed to be similar to the explanation and implentation of a real-world raffle. Our study does not go on to construct additional new auxiliary implementations as additional ex-post treatments not previously observed in an attempt to identify detailed mechanisms for the differences. Nevertheless, our results do allow for some partial conclusions, and some questions for further research.[14]

[14]The ex-post addition of treatments to a study risks introducing a number of confounds. In particular, we

We observe differences both in initial bids and in the dynamics of adjustment over the course of the experiment. The concrete feedback mechanism of a lottery with individually-numbered tickets is indeed different from a more vague mechanism in which only a winner is determined. Both mechanisms appear in applications of contest theory. The winner of a real-world lottery holds a real ticket with a specific number, and this is "why" that person is the winner, while it is not possible to identify "why" one participant defeated another in a sporting contest modeled with a Tullock CSF. The concreteness of the feedback mechanism may influence the faster dynamics of adjustment we observe in the lottery.

However, we likewise observe significantly different and lower bids in the initial period, when participants have not had any concrete feedback experience. Unless the mere anticipation of getting the feedback influences participants' introspection in their initial bids, other candidate explanations for this difference are required. The ticket institution uses a frequency-based representation, which Gigerenzer (1996) and others suggest leads to systematically different judgments relative to the probability-based representation in the conventional treatment. Experiments using an implementation like the conventional treatment often present the probability of winning using a displayed formula patterned after (1.1). Kapeller and Steinerberger (2013) argue that the mere introduction of a mathematical representation of an argument may hinder understanding among a significant proportion of people. Xue et al. (2017) show that people who self-report that they are not "good at math" make earnings-maximising choices substantially less often in a riskless decision task. As quantitative researchers, experimental economists may suppose that the precision of a mathematical display encourages comprehension and improves control, whereas the opposite may occur.

The literature on contest experiments is inspired by the many and varied potential applications of contest models, and the Tullock model specifically, to the real world. The claim, and the hope, is that the substantial body of experimental results accumulated in recent years is not merely an exercise in testing the predictions of (classical or behavioural) game theory, but helps to reveal something about the way people approach given real competitive situations. Our results show that, in a laboratory setting, different ways of implementing a Tullock CSF lead to significantly different behaviour.

Although groups in our ticket treatment converge more rapidly and closely towards the

alternated sessions of each treatment to try to make unobservable participant characteristics as similar as possible between the two treatments. To add a treatment ex-post means tapping into a potentially different segment of the participant pool. For example, Xue et al. (2017) find that participants who responded to a treatment conducted later than the others were significantly less likely to report they considered themselves "good at math." A valid exploration of new hypotheses arising from our results must await a new design with a fresh selection of participants.

risk-neutral Nash equilibrium benchmark, we do not interpret our results as favouring the ticket implementation as superior. Care must be taken when speaking of “overbidding” in contests, as the received results in the contests literature could be characterised equally as the risk-neutral Nash equilibrium “underpredicting” bidding behaviour. Our comparison is not a horse race; to understand more about how people behave in real-world contest situations, experimental designs should be selected based on the mapping of the features of the experiment to the real-world target environment, and not on the ability of the design to return behaviour close to a given theoretical benchmark. To the extent that real contests may have features, such as the opaqueness of the resolution of the randomness in the contest, designs with features of the conventional treatment will be useful; our ticket-based design would be a dubious choice for an experiment which hoped to speak to behaviour in a sporting contest. The external validity of laboratory experiments on contests depends on considered replication of the key features of the real-world contest in the laboratory implementation.

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Appendix

1.A Instructions

The session consists of 30 decision-making periods. At the conclusion, any 5 of the 30 periods will be chosen at random, and your earnings from this part of the experiment will be calculated as the sum of your earnings from those 5 selected periods.

You will be randomly and anonymously placed into a group of 4 participants. Within each group, one participant will have ID number 1, one ID number 2, one ID number 3, and one ID number 4. The composition of your group remains the same for all 30 periods but the individual ID numbers within a group are randomly reassigned in every period.

In each period, you may bid for a reward worth 160 pence. In your group, one of the four participants will receive a reward. You begin each period with an endowment of 160 pence. You may bid any whole number of pence from 0 to 160; fractions or decimals may not be used.

If you receive a reward in a period, your earnings will be calculated as:

$$\text{Your payoff in pence} = \text{your endowment} - \text{your bid} + \text{the reward}.$$

That is, your payoff in pence = 160 – your bid + 160.

If you do not receive a reward in a period, your earnings will be calculated as:

$$\text{Your payoff in pence} = \text{your endowment} - \text{your bid}.$$

That is, your payoff in pence = 160 – your bid.

Portion for conventional treatment only

The more you bid, the more likely you are to receive the reward. The more the other participants in your group bid, the less likely you are to receive the reward. Specifically, your chance of receiving the reward is given by your bid divided by the sum of all 4 bids in your group:

$$\text{Chance of receiving the reward} = \frac{\text{Your bid}}{\text{Sum of all 4 bids in your group}}.$$

You can consider the amounts of the bids to be equivalent to numbers of lottery tickets. The computer will draw one ticket from those entered by you and the other participants, and assign the reward to one of the participants through a random draw.

An example. Suppose participant 1 bids 80 pence, participant 2 bids 6 pence, participant 3 bids 124 pence, and participant 4 bids 45 pence. Therefore, the computer assigns 80 lottery tickets to participant 1, 6 lottery tickets to participant 2, 124 lottery tickets to participant 3, and 45 lottery tickets for participant 4. Then the computer randomly draws one lottery ticket out of 255 (80 + 6 + 124 + 45). As you can see, participant 3 has the highest chance of receiving the reward: 0.49 = 124/255. Participant 1 has a 0.31 = 80/255 chance, participant 4 has a 0.18 = 45/255 chance and participant 2 has the lowest, 0.05 = 6/255 chance of receiving the reward.

After all participants have made their decisions, all four bids in your group as well as the total of those bids will be shown on your screen.

Participant ID	**Bid**
Participant **1**	80
Participant **2**	6
Participant **3**	124
Participant **4**	45
Total bids	**255**

Interpretation of the table: The horizontal rows in the left column of the above table contain the ID numbers of the four participants in every period. The right column lists their corresponding bids. The last row shows the total of the four bids. The summary of the bids, the outcome of the draw and your earnings will be reported at the end of each period.

At the end of 30 periods, the experimenter will approach a random participant and will ask him/her to pick up five balls from a sack containing 30 balls numbered from 1 to 30. The numbers on those five balls will indicate the 5 periods, for which you will be paid in Part 2. Your earnings from all the preceding periods will be throughout present on your screen.

Portion for ticket treatment only

The chance that you receive a reward in a period depends on how much you bid, and also how much the other participants in your group bid. At the start of each period, all four participants of each group will decide how much to bid. Once the bids are determined, a computerised lottery will be conducted to determine which participant in the group will receive the reward. In this lottery draw, there are four types of tickets: Type 1, Type 2, Type 3 and Type 4. Each type of ticket corresponds to the participant who will receive the reward if a ticket of that type is drawn. So, if a Type 1 ticket is drawn, then participant 1 will receive the reward; if a Type 2 ticket is drawn, then participant 2 will receive the reward; and so on.

The number of tickets of each type depends on the bids of the corresponding participant:

- Number of Type 1 tickets = Bid of participant 1
- Number of Type 2 tickets = Bid of participant 2
- Number of Type 3 tickets = Bid of participant 3
- Number of Type 4 tickets = Bid of participant 4

Each ticket is equally likely to be drawn by the computer. If the ticket type that is drawn has your ID number, then you will receive a reward for that period.

An example. Suppose participant 1 bids 80 pence, participant 2 bids 6 pence, participant 3 bids 124 pence, and participant 4 bids 45 pence. Then:

- Number of Type 1 tickets = Bid of participant 1 = 80
- Number of Type 2 tickets = Bid of participant 2 = 6
- Number of Type 3 tickets = Bid of participant 3 = 124
- Number of Type 4 tickets = Bid of participant 4 = 45

There will therefore be a total of 80 + 6 + 124 + 45 = 255 tickets in the lottery. Each ticket is equally likely to be selected.

In each period, the calculations above will be summarised for you on your screen, using a table like the one in this screenshot:

Participant ID	Bid	Ticket Type	Total tickets	Ticket number(s)
Participant 1	80	Type 1	80	1 - 80
Participant 2	6	Type 2	6	81 - 86
Participant 3	124	Type 3	124	87 - 210
Participant 4	45	Type 4	45	211 - 255

Interpretation of the table: The horizontal rows in the above table contain the ID numbers of the four participants in every period. The vertical columns list the participants' bids, the corresponding ticket types, total number of each type of ticket (second column from right) and the range of ticket numbers for each type of ticket (last column). Note that the total number of each ticket type is exactly same as the corresponding participant's bid. For example, the total number of Type 1 tickets is equal to Participant 1's bid.

The last column gives the range of ticket numbers for each ticket type. Any ticket number that lies within that range is a ticket of the corresponding type. That is, all the ticket numbers from 81 to 86 are tickets of Type 2, which implies a total of 6 tickets of Type 2, as appears from the 'Total Tickets' column. In case a participant bids zero, there will be no ticket that contains his or her ID number. In such a case, the last column will show 'No tickets' for that particular ticket type.

The computer then selects one ticket at random. The number and the type of the drawn ticket will appear below the table. The ID number on the ticket type indicate the participant receiving the reward.

At the end of 30 periods, the experimenter will approach a random participant and will ask him/her to pick up five balls from a sack containing 30 balls numbered from 1 to 30. The numbers on those five balls will indicate the 5 periods, for which you will be paid in Part 2. Your earnings from all the preceding periods will be throughout present on your screen.

Chapter 2

A Combinatorial multiple winner contest with package designer preferences

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Abstract

We introduce a multiple winner contest in which players are located on a pre-specified network. Players expend costly effort, and a fixed number of adjacent players are selected as winners by applying a lottery contest success function to the sum of the individual bids of all possible fixed-size groups of adjacent players. We formalize the generic best response for such a contest for any arbitrary network and discuss equilibrium properties for certain regular and irregular networks.

JEL Classifications: C72, D72, D74, D85.

Keywords: Multi-winner contest, combinatorial contest, networks.

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2.1 Introduction

Various situations such as conflict, war, rent-seeking, grant applications, sports, job interviews etc., where players invest costly resources (effort, henceforth) to win prizes, are called contests. A contest success function (CSF) is the mapping of these resource investments into corresponding winning probabilities. Contests on various networks often exhibit different sort of spillovers (Dziubiński et al., 2016). In this study we model multiple winner contests in which the contest designer exhibits a package preference over the combined efforts of all winning players such that only certain subsets of the players, connected to each other through the designer's preferred network structure, can be winners. A package preference means that the contest designer can observe how the different players are connected to each other based on certain characteristics, and prefers the successful players to constitute a closely connected subset. Given such a mapping, at least some players can be grouped in multiple ways rather than being a member of one explicit group, and coalitions are formed endogenously. The purpose of the contest is to choose one combination out of many possible combinations of players. Therefore, we call this a combinatorial contest.

The cardinality of the set of successful players, alternatively the number of prizes, is fixed in our setting, and is common knowledge. Suppose the pre-specified cardinality of the set of successful players is k. Any size-k subset of players that satisfies the package preference and fulfills any additional threshold criteria (e.g. any required effort cut-off) is called an eligible subset. One of these eligible subsets then emerge as the winning set as a result of the contest. We limit our attention to a case where an individual player does not distinguish between the k winning positions. An alternative interpretation is that the k players in the winning set receive a public-good prize. All players in the winning set enjoy the same benefits. The probability of success of a given eligible subset of players depends on the total outlay from all eligible subsets. Thus, individual players in any given eligible subset enjoy positive externalities from the efforts of all other players in the same subset. Given the distribution of players on a connected network, and the resulting overlap between different size-k subsets of connected players, any given individual may belong to many such size-k subsets.

From a given individual's perspective, any other player belonging to an eligible subset common to both of them is a potential collaborator, and any player who does not belong to a common eligible subset is a pure competitor. An individual's effort in the contest can be perfectly substituted by a potential collaborator's effort. We use the Tullock lottery contest success function (Tullock, 1980) to define the success probability of any potential subset of players. We then characterize the individual best response for such a contest for any arbitrary network and demonstrate how equilibrium outlays in such a contest crucially depend on the associated network on which the players are positioned. We discuss equilibrium properties on certain regular (the complete and the ring network) and irregular (star, fan-shaped, and a line) networks.[4] The final section concludes.

This highly stylized model roughly reflects various situations of real contests. One example is the selection of papers submitted to an academic conference. Assuming the organizer's objective is to maximize productive discussion among the participating academicians, the organizer is most likely to select a subset of papers with similar research questions. Researchers are aware of this objective and of the areas that other prominent researchers are working on. Given such preferences, it is possible that a few papers, which are qualitatively mediocre but investigate areas that are studied by other good papers, are selected for presentation, while some methodologically sound paper on an unpopular topic may not be selected.

Another example is the Hobbesian anarchy (Hobbes, 2006) in which the agents are related to each other through a network based on geographical position or religious belief or political ideology. Coalitions emerge endogenously in this case, but only among a concentrated cluster of nodes on the network (similar ideology or belief, neighbours etc.).

Note that the package preference in our model can also be interpreted as the existing norms of choice in a certain market context which are such that the fruits of victory is shared by closely connected players. Post-election coalition formation in multi-party Westminster systems, for example, can be modeled as a combinatorial contest, and it may fit both a linear (Downs, 1957) and a circular (Lester, 1996) model of political spectrum. In a multi-party electoral contest where all parties may contest each other making irreversible investments during the pre-election campaign, a post-election coalition is more likely among two or more parties that are relatively close on an ideological space, than among parties with very different political agendas. In the United Kingdom for instance, a post-election coalition between Labour (left/ Centre-left) and SNP (Centre-left) is more likely than a coalition between Labour and the Conservatives (right/ Centre-right). But the Lib Dems (centrist) can run into a coalition with either of the Labour or Conservatives.

A combinatorial contest may also be observed in a market with a spectrum of horizontally differentiated products. Each firm makes costly investment on their products, a set of related varieties with some common feature(s) become popular among the customers, and only a subset of the firms that are producing these related products are able to break even in the market. The examples include network-markets (Besen and Farrell, 1994) such as video games, digital entertainment sector, or street foods. Consider the online streaming of YouTube videos for instance. The app suggests videos using an algorithm that depends on both individual search history and popular viewing trends. If one video (e.g., the Gangnam Style)

[4] A regular network is one in which all players have equal number of connections. An irregular network is one in which at least two players have different number of connections.

gets very popular, then other videos of similar type (K-pop, instead of American Country) are also likely to be viewed more frequently by users around the world.

It must be noted that in this chapter presents a highly stylised model of a combinatorial contest. While the above examples are scenarios where a combinatorial contest may be observed, the scope of modelling these scenarios with our model is limited by the pre-specified cardinality of the eligible set. The conference organizer's example may seem most suitable as there are fixed number of slots in a session. However, there being multiple sessions in a conference, this example is also not an ideal depiction of our combinatorial contest.

It is also important to point out that in contrast to ex-post spillovers in network-markets (Thum, 1996) or rank-order tournaments (Baye et al., 2012), we look at ex-ante spillover of contest efforts. Also, in contrast to many studies about ex-ante alliance formation (Konrad and Kovenock, 2009; Esteban and Sákovics, 2003), our model implies an ex-post alliance formation. This study contributes to the literature on contests and networks by identifying the potential existence of combinatorial preferences which may lead to endogenous ex-post coalitions in political and commercial market contests. Our setting is a compromise between multi-winner contests with no substitutability of effort, such as Clark and Riis (1996) or Fu and Lu (2009), and the group contests over group specific public good prizes with perfect substitutability of effort within pre-specified group of players, as in Katz et al. (1990). We elaborate this in the next section by discussing related studies, and specifying our contributions in relation to the existing literature. We introduce the model and derive the results in Section 2.3. Section 2.4 concludes.

2.2 Related Literature

We model network-based spillovers resulting from the contest designer's preferences about player positions (or, characteristics) in a multiple winner contest. We borrow the coinage of "combinatorial contests" from combinatorial auctions which was first proposed by Rassenti et al. (1982), and are used in the auctions of estates, bus routes, transportation, radio spectrums etc.[5] Contrary to our model, the combinatorial aspect in such auctions arise from the preferences of the bidders themselves.

The foundation of the present model is borrowed from Berry (1993), who was the first to study multiple winner contests. He proposes a CSF for simultaneous selection of k winners out of $N(> k)$ players, where any individual players winning probability equals the aggregated winning probabilities of all possible k-player combinations containing this individual. We modify Berry's success function in consideration of the network-based restrictions.

[5] see Cramton et al. (2007) for a survey of the combinatorial auction literature.

Clark and Riis (1996) propose a model of sequential assignment of winning positions based on the simultaneously decided initial outlays. They criticize Berry's model on the ground of being tantamount to a situation where rent-seeking outlays affect the distribution of only the first prize and the remaining $k-1$ prizes are allocated randomly. But de Palma and Munshi (2013) argue that the simultaneous assignment mechanism underlying the Berry CSF has a probabilistic foundation which is undermined in Clark and Riis's interpretation. Chowdhury and Kim (2014) propose a sequential assignment mechanism based on Berry's mechanism where $N-k$ selection rounds are used to obtain the k final winners. However, in contrast to Clark and Riis (1996)'s mechanism of selecting out one winner every round, Chowdhury and Kim (2014) devise a sequential elimination by selecting in multiple players every round. Thus exactly $N-t$ players survive after the t-th round for all $t \in [1, N-k]$. They arrive at the same equilibrium outcome as Berry's simultaneous assignment CSF under homogeneity of players. Finally, Fu and Lu (2009) suggest another sequential mechanism which drops the player exerting the least effort with the highest probability in every round, and selecting the k winners takes a total of $N-k$ such rounds. These are the only mechanisms for lottery CSF that devise ways of assigning k winning positions to distinct players among a total of n players. We generalize Berry's mechanism to a simultaneous multiple-winner contest with network-based spillovers, and show that Berry's CSF can be derived as a special case of our model.

As pointed out before, our study also contributes to the literature of contests under network externalities through its consideration of a setting where ex-post coalitions emerge endogenously. A number of studies have analyzed network effects in contests, but majority of them consider single winner contests, and players in those models interact with neighbours who are either friend or foe. Grandjean et al. (2017) study endogenous formation of cooperative networks and welfare consequences under alternative network structures in the standard Tullock contest. Huremovic (2016) analyses dynamic stability in simultaneous bilateral Tullock contests among neighbours in a network. Network formation has been studied widely in the domain of R&D contests. Konovalov (2014) finds that firms invest more in bilateral joint projects than full cooperation, and bilateral cooperation increases (decreases) the profits of more (less) connected firms. Marinucci (2014) finds that the presence of heterogeneous players increases the number of possible pairwise stable networks. Bozbay and Vesperoni (2018) axiomatize a contest where players are connected in complex networks of both friends and enemies and shows that for any symmetric effort profile, the peace network (where all players are friends) is the unique pairwise stable network. All these studies either pre-specify a conflicting or coercive relation between neighbours on a network (Franke and Öztürk, 2015; Huremovic, 2016; Hiller, 2017), or assume that neighbouring competitors form a cooperative alliance (Grandjean et al., 2017; Konovalov, 2014; Marinucci, 2014). We refrain from a-priori specification of alliance or antagonism among neighbouring players, but the nature of the relation depends on the network structure. The marginal effect of a neighbouring player's effort on an individual's success probability depends on the number of common eligible subsets relative to the total number of eligible subsets this neighbour belongs to. Given k and the network structure, the effort decisions of only some potential neighbours' may be strategic substitutes.

Another type of contest with multiple winners is a contest among pre-specified group of players. One group emerges as the winning group and all the members of this winning group receive a group-specific public good prize. The literature on group contests with lottery CSF starts with Katz et al. (1990). They assume symmetric players within each group and a perfect substitute impact function. Consequently, equilibrium total effort at the group level is uniquely determined but multiple equilibria exist as far as individual effort is concerned. Baik (2008) generalizes this structure by considering asymmetric valuation among players in a pre-specified group. In the equilibrium, only the group member with the highest valuation in each group spends positive effort and the rest expend zero effort. If multiple players within a group have the highest valuation, then multiple equilibria exist. Konrad (2004) compares single-stage and hierarchical contests under heterogeneous valuation among members of the different contest groups. He finds that rent dissipation depends on the degree of heterogeneity within and across groups, but the hierarchical contest is inefficient as it increases the probability of a low-value candidate emerging as a winner.

To our knowledge, there are only two studies that consider situations in between, where there is partial ex-ante restriction regarding the formation of the winning group. Fu and Lu (2009) show that if the players are homogeneous and are divided into several sub-contests, then under the Clark and Riis (1996) winner selection mechanism a grand contest always generates more equilibrium rent dissipation than any set of sub-contests. Chowdhury and Kim (2017) show that this result depends crucially on the contest mechanism implemented and a loser elimination mechanism (instead of a winner selection one) may revert the result. The current study fits in this area, albeit from a very different perspective. First, since all the players in the winning set equally share the winning benefits, this contest can be interpreted as a contest over group specific public good prize with a perfectly substitute impact function and overlapping groups. Second, it investigates the case in which there is only partial ex-ante restriction on the formation of the winning group. A unique feature of this model is that any player's membership is not limited to a single group. Instead, the players know the possible formations from their relative positions on the network. The network structure and all possible coalitions are common knowledge.

2.3 Model

A set of n players is denoted as $N = \{1, 2, ..., n-1, n\}$. Suppose, the n players form a connected graph $G = (N, E)$. The connectivity among the players at the different nodes is given by E which denotes the set of all edges. The number of available winning positions are $k \leq N$. A package preference means that these k positions can be secured by any k players as long as there exists a path on G between any two winners. An induced subgraph is a subset of vertices of a graph together with any edges whose endpoints are both in this subset. Formally, if $S \subseteq N$ is any subset of vertices of G, then the induced subgraph $G[S]$ is the graph whose vertex set is S and whose edge set consists of all the edges in E that have both endpoints in S (Diestel, 2006). A connected subgraph G', on the other hand, is a subgraph of G whose edge set is such that there exists a path on G' between any two elements in its vertex set. Let $G'[S]$ denote a connected induced subgraph with vertex set $S \subseteq G$. Formally, $G'[S] = (S \subseteq G, E' \subseteq E)$ is such that there exists a path in $G'[S]$ between any two elements in S and E' includes all the edges in E that have both endpoints in S.[6] A combinatorial contest $\Phi(k, (N, E), M)$ allocates k equally valued indivisible rewards to n players on graph $G = (N, E)$ using contest mechanism M such that the winning players form a connected induced subgraph of G.[7] The following definitions are useful for further exposition.

Definition 1. ***(k-distance neighbours):*** *If there are exactly k number of edges along the shortest path on G between any pair of nodes (players) $\{i, j\} \subset N$, then i and j are called k-distance neighbours. That is, i and j are k-distance neighbours if $d(i, j) = k$.*

Definition 2. ***(k-neighbouring group):*** *A k-neighbouring group is a set of k players the members of which constitute a connected induced subgraph of G with a maximum diameter of $k-1$. That is, $S \subset N$ is a k-neighbouring group if $|S| = k$ and $G'[S]$ is a connected induced subgraph of G with $d(i, j) \leq k-1$ for any $\{i, j\} \subset S$.*

Therefore, two k-distance neighbours in G can never belong to a common k-neighbouring group. In other words, the maximum number of edges along the shortest path between two players in a k-neighbouring group is $k-1$. Any eligible subset in our model must be a k-neighbouring group. All players in the network simultaneously exert effort and one of the

[6] A connected subgraph doesn't necessarily include all the edges among the nodes. For example, looking at Figure 2.1, $\{\{1, 2, 4\}, \{1-2, 2-4\}\}$ will qualify for a subgraph, when $\{\{1, 2, 4\}, \{1-4, 2-4\}\}$ is a different subgraph. $\{3, 5, 6\}$ on the other hand will form an induced subgraph without any connecting edges. Therefore, what we need to define a k-neighbouring group is an induced subgraph which is connected.

[7] If G is disconnected, then due to fixed number of winning positions and the package preference restrictions, any disconnected node(s) will be ineligible to be a winner. If there are $m < k$ mutually connected players, and the package preference still holds, then different possibilities exist depending on the nature of the graph and whether the contest organizer decides to withhold or reduce the number of rewards.

eligible subsets is selected as the winning set. We consider a mechanism which determines the winning k-neighbouring group using a lottery contest success function over the outlays from all possible k-neighbouring groups, and denote this as $\Phi(k,(N,E),L)$ where L stands for lottery. The effort outlay from a k-neighbouring group is the aggregated effort from all k members (we limit this discussion to linear aggregation technology, as discussed later) in that group. The success probability of a given k-neighbouring group is the ratio of the aggregate outlay of this group to the sum of the aggregate outlays from all possible k-neighbouring groups.

The reward from the contest is either a public good prize assigned to the winning k-neighbouring group, or k equivalent prizes distributed among the members of the winning group. Suppose player i values the reward at $v_i > 0$ and $x_i \in R_+$ is her effort outlay. The positions and valuations of all players are common knowledge. The probability that player i ends up in the winning set is denoted by $P_i(x_i, \mathbf{x_{-i}})$, where $\mathbf{x_{-i}} \in R_+^{n-1}$ is the vector of contest outlays from all $j \in N, j \neq i$.

Suppose there are N_g distinct k-neighbouring groups that can be defined on G. All k-neighbouring groups are arbitrarily and uniquely indexed, the t-th k-neighbouring group being denoted as S_t. Total outlay from group t is $X_t \in R_+$ for all $t \in \{1,\cdots,N_g\}$. The aggregated outlay from all k-neighbouring groups is denoted by $X = \sum_{t=1}^{N_g} X_t = \sum_{t=1}^{N_g}\sum_{i\in S_t} f(x_i)$, where $f(\cdot)$ denotes the effort impact function. Defining the standard Tullock lottery CSF over the group outlays, the probability that the t-th k-neighbouring group emerges as the winning set in $\Phi(k,(N,E),L)$ is given by

$$\text{Prob}(S_t) = \begin{cases} \frac{X_t}{X} & \text{if } X > 0 \\ \frac{1}{N_g} & \text{if } X = 0 \end{cases} \tag{2.1}$$

Hence, the probability that player i is a winner is given by $P(x_i, \mathbf{x_{-i}}) = \sum_{t:i\in S_t} \text{Prob}(S_t)$, the sum of the success probabilities of all groups containing i. Expected payoff to player i is given by

$$E(\pi_i) = \sum_{t:i\in S_t} \text{Prob}(S_t)v_i - x_i = \begin{cases} \frac{\sum_{t:i\in S_t} X_t}{X} v_i - x_i & \text{if } X > 0 \\ \frac{|\{t: i\in S_t\}|}{N_g} v_i & \text{if } X = 0 \end{cases} \tag{2.2}$$

The CSF is continuous everywhere except when all players exert zero amount of effort.[8]

[8]This model can be easily extended to accommodate a minimum effort cut-off $c \geq 0$. In such a model, no

But this can never happen in equilibrium because any player can strictly increase her winning chances by spending an infinitesimally small amount of resources.

The precise form of the CSF will depend on the group effort impact function as generated from the underlying aggregation technology. In what follows, we consider two types of linear aggregation, simple and degree-weighted. The following definitions are useful for brevity.

Definition 3. ***(Membership):*** *The membership of player i, denoted as w_i, is the number of k-neighbouring groups that contain player i. That is, $w_i = |\{t : i \in S_t\}|$, $\forall i \in N$.*

$w_i(m)$ is the number of k-neighbouring groups such that player i has exactly m direct neighbours within that group, for any $m \leq k-1$. That is, $w_i(m) = |\{t : deg(i)_{G'[S_t]} = m\}|$ for any given $i \in N$ and any given $m \leq k-1$, where $deg(i)_{G'[S_t]}$ denotes the degree of i within the connected induced subgraph $G'[S_t]$.

Note that player i may be connected to more than m players on the n-player space. $w_i(m)$ only gives the number of k-neighbouring groups such that player i has exactly m number of direct neighbours within each group. Therefore, $\sum_{m=1}^{k-1} w_i(m) = w_i$. Let us demonstrate this with the following graph

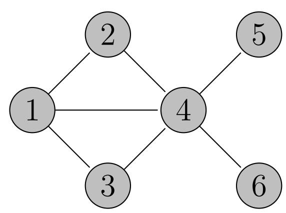

Figure 2.1: An example network

Suppose we are considering a 3-winner contest on the Figure 2.1 network of 6 players. Note that player 1 is a direct neighbour to 3 players, 2, 3, and 4. The 3-neighbouring groups containing player 1 are $\{2, 1, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{1, 4, 5\}$ and $\{1, 4, 6\}$. Therefore $w_1 = 5$. The degree of player 1 is 2 in the subgraphs induced by groups $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$ and 1 in the subgraphs induced by the remaining two groups. Hence, on this network, $w_1(1) = 2$ and $w_1(2) = 3$.

Definition 4. ***(Joint Membership):*** *For any pair of players $\{i, j\} \subset N$, the joint membership w_{ij} is the number of k-neighbouring groups that contain both i and j. That is, $w_{ij} = |\{t : i \in S_t \text{ and } j \in S_t\}|$, for any $\{i, j\} \subset N$.*

Additionally, for any pair of players $\{i, j\} \subset N$ and any given $m \leq k-1$, $w_j(m, i)$ gives the total number of k-neighbouring groups in which player j has exactly m number

player exerts $x_i \in (0, c)$ in equilibrium. For the present analysis however, we assume the effort cut-off to be zero.

of direct neighbours, and i is one member, whether a direct neighbour to j or not. That is, $w_j(m,i) = |\{t : deg(j)_{G'[S_t]} = m$ and $\{i,j\} \subset S_t\}|$ for any $\{i,j\} \subset N$ and any given $m \leq k-1$. Therefore, $w_{ij} = \sum_{m=1}^{k-1} w_j(m,i)$.

Consider Figure 2.1 again. The 3-neighbouring groups commonly shared by both players 1 and 2 are $\{2,1,3\}$ and $\{1,2,4\}$. Player 2 has a single connection in the subgraph induced by $\{2,1,3\}$, and two connections in the subgraph induced by $\{1,2,4\}$. Therefore, $w_2(1,1) = 1$, and $w_2(2,1) = 1$. Similarly, $w_4(1,1) = 0$ and $w_4(2,1) = 4$.

Given the above definitions, we can define the expected payoff functions for two types of linear aggregation in a combinatorial contest.

Definition 5. ***Simple Aggregation:*** *The effective aggregate outlay of any given k-neighbouring group is the sum of the individual outlays of its members. That is, $X_t = \sum_{i \in S_t} x_i$. Contest efforts from all members in a k-neighbouring group are perfect substitutes. The expected payoff function under simple aggregation can be written as*

$$E(\pi_i|SA) = \begin{cases} \frac{w_i x_i + \sum_{j \neq i} w_{ij} x_j}{\sum_{i \in N} w_i x_i} v_i - x_i & \text{if } \sum_{i \in N} w_i x_i > 0 \\ \frac{w_i}{N_g} v_i & \text{if } \sum_{i \in N} w_i x_i = 0 \end{cases} \tag{2.3}$$

Definition 6. ***Weighted Aggregation:*** *The effective aggregate outlay of any given k-neighbouring group is the sum of individual outlays weighted by the degrees of the respective individuals in the corresponding connected induced subgraph. That is, $X_t = \sum_{i \in S_t} deg(i)_{G'[S_t]} x_i$, where $deg(i)_{G'[S_t]}$ is the degree of player i in $G'[S_t]$, or the number of direct neighbours player i has within S_t.*

Let us define $w'_i = \sum_{m \in [1,k-1]} m w_i(m)$, $\forall i \in N$. Using definitions 3 and 4, the expected payoff function under weighted aggregation can thus be written as

$$E(\pi_i|WA) = \begin{cases} \frac{w'_i x_i + \sum_{j \neq i} x_j \sum_{m=1}^{k-1} m.w_j(m,i)}{\sum_{i \in N} x_i w'_i} v_i - x_i & \text{if } \sum_{i \in N} x_i w'_i > 0 \\ \frac{w_i}{N_g} v_i & \text{if } \sum_{i \in N} x_i w'_i = 0 \end{cases} \tag{2.4}$$

Effort of all members in an eligible subset are not perfect substitutes under weighted aggregation. The effective aggregate outlay of any k-neighbouring group heavily depends on the central players. A densely-connected cluster of players have a higher aggregate outlay than a sparsely connected cluster of players when individual outlays remain the same across the two groups.

2.3.1 Best Responses

From expression 2.2, player i's optimization problem in $\Phi(k,(N,E),L)$, when at least some player in the network spends positive effort, is given by

$$\max_{x_i \geq 0} \frac{\sum_{t:i\in G_t} X_t}{X} v_i - x_i$$

Proposition 2.1. *In the combinatorial contest $\Phi(k,(N,E),L)$, first order conditions for optimization under linear aggregation gives the following best-response outlays for player i.*

$$x_i^{BR}(\mathit{SA}) = max\left\{0, \frac{1}{w_i}\left[\left\{v_i w_i \sum_{j\neq i}(w_j - w_{ij})x_j\right\}^{1/2} - \sum_{j\neq i} w_j x_j\right]\right\} \tag{2.5}$$

$$x_i^{BR}(\mathit{WA}) = max\left\{0, \frac{1}{w_i'}\left[\left\{v_i w_i' \sum_{j\neq i} x_j \sum_{m=1}^{k-1} m(w_j(m) - w_j(m,i))\right\}^{1/2} - \sum_{j\neq i} x_j w_j'\right]\right\} \tag{2.6}$$

Corollary 2.1. *Suppose player i belongs to all possible k-neighbouring groups in a given n-player network. Then for any vector of effort outlays $\mathbf{x_{-i}}$ from other players, player i's best-response is $x_i = 0$.*

The expected payoff function under simple aggregation is strictly concave except when $w_j = w_{ij}\ \forall j \in N$. In other words, the strict concavity of the payoff function requires that w_{ij}, the number of eligible subsets shared by players i and j, must be less than the total number of eligible subsets including player j, at least for some $j \in N$. This condition is generally satisfied in any network structure, the only exception being the central player in the star network.

Similarly, concavity under weighted aggregation requires $\sum_{j\neq i} x_j \sum_{m=1}^{k-1} m(w_j(m) - w_j(m,i)) > 0$. In other words, the number of k-neighbouring groups commonly shared by i and j, in which player j has m connections, must be less than the number of k-neighbouring groups in which player j has m connections, for some $m \in [1, k-1]$ and some $j \in N$. Again, this is true for any arbitrary player in any network structure except the centre of a star. Thus, unless player i belongs to all possible k-neighbouring groups on the given network of players, the strict concavity of the expected payoff function renders the first order conditions necessary and sufficient for optimization.

Under either aggregation rule, if all players spend zero amount of resources, then one k-neighbouring group is picked up at random as the winning group. Hence, connectivity doesn't matter in that case. However, as indicated earlier, this cannot be an equilibrium.

2.3.2 Equilibrium Analysis

Lemma 2.1. *Suppose, $\mathbf{x}^*$ is a vector of equilibrium efforts in $\Phi(k,(N,E),L)$. Under linear aggregation, any feasible reallocation of efforts, which keeps the sum of aggregate outlays constant across all k-neighbouring groups, also constitutes an equilibrium.*

Proof. Suppose, $X > 0$. From expression 2.2, the first order condition for optimization requires

$$\begin{aligned}
\frac{\partial E(\pi_i)}{\partial x_i} &= \frac{\sum_{t:i\in G_t}\frac{\partial X_t}{\partial x_i}\left(X-\sum_{t:i\in G_t}X_t\right)}{X^2}v_i - 1 = 0\\
\implies X &= \left[v_i\sum_{t:i\notin G_t}X_t\sum_{t:i\in G_t}\frac{\partial X_t}{\partial x_i}\right]^{1/2}\\
\implies \sum_{t:i\in G_t}X_t &= \left[v_i\sum_{t:i\notin G_t}X_t\sum_{t:i\in G_t}\frac{\partial X_t}{\partial x_i}\right]^{1/2} - \sum_{t:i\notin G_t}X_t
\end{aligned} \tag{2.7}$$

In a given network for a linear aggregation rule, $\sum_{t:i\in G_t}\frac{\partial X_t}{\partial x_i}$ is constant for every $i \in N$. For example, under SA, $\sum_{t:i\in G_t}\frac{\partial X_t}{\partial x_i} = w_i$, and under WA, $\sum_{t:i\in G_t}\frac{\partial X_t}{\partial x_i} = w_i'$. If $\mathbf{x}^*$ is an equilibrium effort profile, then it satisfies the above condition for all $i \in N$. So, any $\mathbf{x}'$, that keeps $\sum_{t:i\in G_t}X_t$ and $\sum_{t:i\notin G_t}X_t$ intact for all $i \in N$, also satisfies the above condition for all $i \in N$. □

The feasibility of such reallocation depends on the network properties and the number of prizes. This is demonstrated with the following examples of simple aggregation in a set of small networks.

Let us first consider a 4-player ring network (Figure 2.2.a) with two winning positions and simple aggregation of efforts. All winning positions are valued symmetrically within and across players.[9] Two winning positions imply $w_i = 2$ and $w_{ij} \in \{0,1\}$ for all i. Setting $v_i = v$, the best-responses can be obtained from 2.5. An equilibrium effort profile must

[9]We consider asymmetric valuation across players in the Appendix

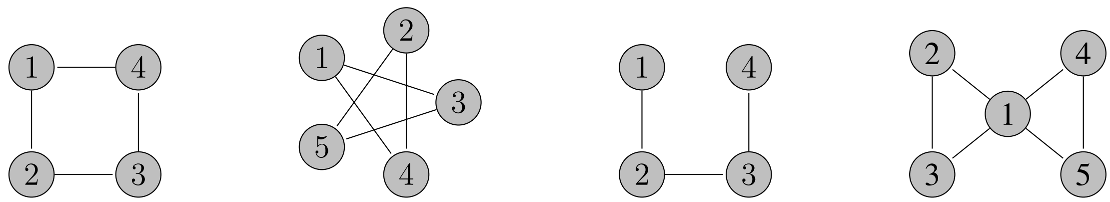

Figure 2.2: Examples of simple networks. with $k = 2$ and homogeneous valuation, examples (a) and (c) are characterized by multiple equilibria

satisfy $2(x_1^* + x_2^* + x_3^* + x_4^*)^2 = v(x_2^* + 2x_3^* + x_4^*) = v(x_1^* + x_3^* + 2x_4^*) = v(2x_1^* + x_2^* + x_4^*) = v(x_1^* + 2x_2^* + x_3^*)$ and therefore $\sum x^* = v/2$. In the ex-post symmetric equilibrium $x_i^* = v/8 \quad \forall i \in \{1, 2, 3, 4\}$. Total effort from each of the four coalitions is $v/4$. However, note that any alternative effort profile $\mathbf{x}' = (\frac{v}{2} + \varepsilon, \frac{v}{2} - \varepsilon, \frac{v}{2} + \varepsilon, \frac{v}{2} - \varepsilon) \quad \forall \varepsilon \in [-v/2, v/2]$ also satisfies the equilibrium conditions. Next, consider $k = 3$ on the same network. The first order conditions for equilibrium require $3(x_1 + x_2 + x_3 + x_4)^2 = (x_2 + x_3 + x_4) = (x_1 + x_3 + x_4) = (x_1 + x_2 + x_4) = (x_1 + x_2 + x_3)$. In this case, the unique equilibrium effort profile in pure strategies is the ex-post symmetric equilibrium with $x_i^* = v/16$. There exist no other effort profile in pure strategies that will keep the coalition efforts same and also satisfy the first order conditions. Similarly, for the 5-player network in Figure 2.2.b, which is equivalent to a 5-player ring network as each player has only two direct neighbours, the only effort profile in pure strategies that satisfy the first order conditions for interior equilibrium for any $k \in (1, 5)$ is the symmetric equilibrium effort profile. Proposition 2.4 extends this analysis to ring networks of larger dimensions and with varying k. However, as demonstrated in the above example, although the ex-post symmetric equilibrium is unique, this may not be the unique equilibrium effort profile even under ex-ante symmetry. This result is similar to what is pointed out in Konrad et al. (2009). A more comprehensive conclusion about existence of multiple equilibria is provided in proposition 2.4.

To look at irregular networks next, first let us consider a 4-player line network (Figure 2.2.c) under simple aggregation and two winning positions. By solving best-responses from Proposition 2.1, we see that an equilibrium effort profile must satisfy $x_1^* = x_3^*$, $x_2^* = x_4^*$ and $x_1^* + x_2^* = x_3^* + x_4^* = 2v/9$. Consequently, $(v/9, v/9, v/9, v/9)$, $(2v/9, 0, 2v/9, 0)$, and $(0, 2v/9, 0, 2v/9)$ all qualify as equilibria, and so does any $(v/9 - \varepsilon, v/9 + \varepsilon, v/9 - \varepsilon, v/9 + \varepsilon) \quad \forall \varepsilon \in [-v/9, v/9]$. Changing the number of winning positions from $k = 2$ to $k = 3$ does not return a unique symmetric equilibrium as in the ring network though. Instead, it reduces the contest to a single-winner contest between the two marginal players. player 1 and player 4, while player 2 and 3 spend zero effort as any 3-player neighbourhood on the line necessarily contains both of them. The corresponding equilibrium effort profile is uniquely determined at

$\mathbf{x}^* = (v/4, 0, 0, v/4)$.

Lastly, the fan-shaped network (star with alternated peripheral nodes connected) in Figure 2.2.d exhibits a counter-intuitive scenario. For $k = 2$, an ex-post symmetry assumption among players 2, 3, 4 and 5 yields the unique equilibrium effort profile $\mathbf{x}^* = (6v/49, 4v/49, 4v/49, 4v/49, 4v/49)$. This is interesting in comparison with the star network with player 1 in the centre. In absence of the links between players 2 and 3 and that between players 4 and 5, every 2-player network necessarily includes player 1, thus eliminating her incentive to put forth positive effort. Each of the other four players spend $3v/16$ at the unique equilibrium. Each of the peripheral players put less effort and earn higher payoffs in the fan-shape, while the central player puts more effort and earns less compared to the star network. The fan-shape generates higher total payoff. However, the loss in the central players payoff (and the increase in her effort) is of a higher magnitude than the gain in any of the peripheral player's payoff (decrease in her effort). The fan-shape is also suboptimal from an effort-maximizing organizer's point of view, as contest revenue (or, total effort) is lower under the connected star. This example proposes a situation where free-riding by one player may be desirable from the organizer's perspective, and the organizer's interest coincides with the central player's.

Regular networks

A graph where all the nodes have the same number of edges is called a regular graph in contrast to an irregular graph where at least two nodes have different number of edges. A q-regular graph is a graph where each node lies on exactly q number of edges. Let us denote the edge set of a q-regular graph as E_q. Using this notation, a combinatorial contest on a ring network is denoted by $\Phi(k, (N, E_2), L)$, and that on a complete network is denoted by $\Phi(k, (N, E_{n-1}), L)$, and so on. Same number of edges also means all nodes in a regular graph belong to the same number of k-neighbouring groups, and the total number of joint memberships enjoyed by each node is the same.

The non-zero best-response of player i under simple aggregation implies $\left(w_i x_i^{BR} + \sum_j w_j x_j\right)^2 = v_i w_i \sum_j (w_j - w_{ij}) x_j$, where $w_i = w_j = w \quad \forall \{i, j\} \subset N$ if G is a regular graph and $w_i \neq w_j$ for at least some $\{i, j\} \subset N$ if G is an irregular graph. Consequently, the best reply functions of player i and player j have different parameters for at least some $\{i, j\} \subset N$ in an irregular graph. But on a regular graph $\sum_j w_{ij} = \bar{w}$ for all $i \in N$, $j \neq i$.

Lemma 2.2. *Suppose $G = (N, E_q)$ is a q-regular graph. Then (i) $w_i = w$, (ii) $\sum_j w_{ij} = \bar{w}$, and (iii) $\sum_j \sum_{m=1}^{k-1} m w_i(m, j) = \bar{w}'$ for all $i \in N$, where w, $\bar{w}$, and $\bar{w}'$ are constants for given k.*

Proposition 2.2. *The equilibrium total effort from* $\Phi(k,(N,E),L)$ *is*
(i) $\sum_i x_i^* = \frac{(n-1)w-\bar{w}}{nw}v$ *under simple aggregation, and*
(ii) $\sum_i x_i^* = \frac{(n-1)w'-\bar{w}'}{nw'}v$ *under weighted aggregation.*

Proof. (i) Using lemma 2.2, one can easily show that $\sum_i \sum_j w_{ij}x_j = \sum_i \left(\sum_j w_{ij}\right) x_i = \sum_i \bar{w}x_i$. From equation 2.7, $X^2 = vw\sum_{t:i\notin G_t} X_t$. Summing up over all $i \in N$

$$
\begin{aligned}
nX^2 &= vw\sum_i \sum_{t:i\notin G_t} X_t \\
&= vw\sum_i (X - \sum_{t:i\in G_t} X_t) \\
&= vw\sum_i (X - wx_i - \sum_j w_{ij}x_j) \\
&= vw(nX - X - \sum_i \sum_j w_{ij}x_j) \\
&= vw((n-1)X - \bar{w}\sum_i x_i) \\
&= v((n-1)w - \bar{w})X \\
\implies nw\sum_i x_i^* &= v((n-1)w - \bar{w}) \\
\implies \sum_i x_i^* &= \frac{(n-1)w-\bar{w}}{nw}v
\end{aligned}
$$

(ii) Using lemma 2.2, one can also show that
$\sum_i \sum_j \sum_{m=1}^{k-1} mw_j(m,i)x_j = \sum_i \left(\sum_j \sum_{m=1}^{k-1} mw_i(m,j)\right) x_i$.
Similar expansion as in (i) obtains $\sum_i x_i^* = \frac{(n-1)w'-\bar{w}'}{nw'}v$. □

It is crucial to note that the above argument does not claim existence of multiple equilibria for all regular networks, or impossibility of multiple equilibria for all irregular network.

The next two propositions relate to equilibrium analysis on two specific regular networks - the complete and ring network. The first proposition interprets Berry's model as a combinatorial contest on a complete network and obtains the same equilibrium outcome. The second proposition discusses general properties for a ring network.

Proposition 2.3. *Berry's (1993) model of simultaneous multi-winner contest is equivalent to* $\Phi(k,(N,E_{n-1}),L_{SA})$*, the combinatorial contest for allocating* k *prizes among a complete network of* n *players with symmetric effort aggregation.*

Proof. In the multi-winner contest described by Berry (1993), any player can be a joint winner with any other player. This contest, with n players and k winning positions, can be interpreted as a combinatorial contest under symmetric aggregation on a complete network. Note that any player $i \in N$ belongs to a total of $w_i = \binom{n-1}{k-1}$ coalitions. the number of coalitions that player i shares with another player j is $w_{ij} = \binom{n-1}{k-1}$ coalitions. The number of coalitions that player i shares with another player j is $w_{ij} = \binom{n-2}{k-2}$. Therefore, using expression 2.5 from Proposition 2.1,

$$w_i x_i^{BR} = [v_i w_i \sum_{j \neq i} (w_j - w_{ij}) x_j]^{\frac{1}{2}} - \sum_{j \neq i} w_j x_j$$

$$\implies \frac{n-1}{k-1}\binom{n-2}{k-2}(x_i + \sum_{j \neq i} x_j) = \left[v \left(\frac{n-1}{k-1}\right) \binom{n-2}{k-2} \sum_{j \neq i} \left(\frac{n-k}{k-1}\right) \binom{n-2}{k-2} \sum_{j \neq i} x_j \right]^{\frac{1}{2}}$$

$$\implies \left(\frac{n-1}{k-1}\right)^2 \binom{n-2}{k-2}^2 (x_i + \sum_{j \neq i} x_j)^2 = v \left(\frac{n-1}{k-1}\right) \left(\frac{n-k}{k-1}\right) \binom{n-2}{k-2}^2 \sum_{j \neq i} x_j$$

$$\implies \left(\frac{n-1}{n-k}\right) (x_i + \sum_{j \neq i} x_j)^2 = v \sum_{j \neq i} x_j$$

$v_i = v \quad \forall i \in N$ for homogeneous players. Under ex-post symmetry, $x_i = x_j = x$ for all $\{i, j\} \subset N$. Imputing this in the above expression obtains $x^* = (n-k)v/n^2$, which is the same as the symmetric effort in Berry(1993). Moreover, this is the unique pure strategy equilibrium as no reallocation of efforts would satisfy all the first order conditions. This follows directly from the first order condition expanded above. The first order condition for player l for instance requires replacing x_l with x_i on the right hand side. The left-hand side being constant, it implies $x_i = x_l$ for all $l \in N, \ l \neq i$. □

Next, we consider $\Phi(k, (N, E_2), L_{SA})$ to be a combinatorial contest on a ring network under simple aggregation, where all players are ex-ante symmetric. Suppose n players are arranged around the circumference of a ring. For the sake of simplicity, let us index the players in a clock-wise manner into different k-neighbouring groups, as in Figure 2.3.

For graphical exposition, let G_t denote the k-neighbouring group with t being the first (left-most) member, for any $t \in N$. Therefore, beginning from any player i, there are exactly n distinct k-neighbouring groups that can be defined along the circumference of the ring, namely $G_i, G_{i+1}, \cdots, G_{i+n-1}$. Player i belongs to exactly k such groups named $G_{i+n-k+1}$, $G_{i+n-k+2}$,$\cdots$, G_{i+n-1} and G_i, the indices being interpreted *modulo* n. Consequently, w_i is equal to k for all $i \in N$. This provides us with the following equilibrium results.

Proposition 2.4. *In a combinatorial contest of n homogeneous players and k winners with simple aggregation along the circumference of a ring network,*
(i) There exists a symmetric pure strategy Nash equilibrium in which every player spends

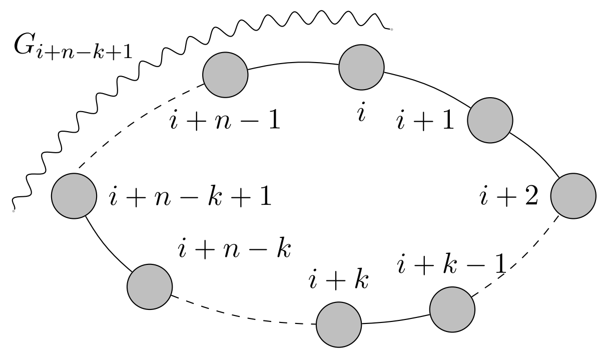

Figure 2.3: Ring network for an (n,k) combinatorial contest

$(n-k)v/n^2$ *for an expected payoff equal to* $(k+n(k-1))v/n^2$.
(ii) A unique coalition effort $X^*(n,k) = (n-k)kv/n^2$ *is obtained in every feasible equilibrium. (iii) There exist one symmetric and infinitely many asymmetric equilibria in pure strategies if* n *is a multiple of* k.

Proof. (i) Distance between any two nodes on a ring network can be calculated in clockwise and anti-clockwise manner, and the maximum distance between two players in a winning coalition is $k-1$. Therefore, an individual player i shares only one possible coalition with player $i+k-1$, and exactly two possible coalitions, G_i and G_{i+n-1}, with player $i+k-2$. There is exactly one coalition $G_{i+n-k+1}$, that includes both player i and player $i+n-k+1$, and exactly two possible coalitions, $G_{i+n-k+1}$ and $G_{i+n-k+2}$, that include both player i and player $i+n-k+2$. Accordingly, any player at d units from player i is part of exactly $k-d$ distinct coalitions that also include player i. The number of coalitions the $(i+t)$-th player shares with player i equals $k-t$ for any $t \in \{1, 2, \cdots, k-1\}$ and $k-n+t$ for any $t \in \{n-k+1, n-k+2, \cdots, n-1\}$, all indices defined *modulo* n. Therefore,

$$\sum_{j \neq i} w_{ij} x_j = \sum_{t=1}^{k-1} (k-t) x_{i+t} + \sum_{t=n-k+1}^{n-1} (k-n+t) x_{i+t}$$

The above expression and $w_j = k \quad \forall j \in N$ means

$$\sum_{j \neq i} (w_i - w_{ij}) x_j = \sum_{t=1}^{k-1} t x_{i+t} + \sum_{t=k}^{n-k} k x_{i+t} + \sum_{t=n-k+1}^{n-1} (n-t) x_{i+t}$$

Given this, the non-zero best reply of any $i \in N$ is easily obtained from equation 2.5 as

$$x_i^{BR} = \left(\frac{v_i}{k}\right)^{\frac{1}{2}} \left(\sum_{t=1}^{k-1} t x_{i+t} + \sum_{t=k}^{n-k} k x_{i+t} + \sum_{t=n-k+1}^{n-1} (n-t) x_{i+t} \right)^{\frac{1}{2}} - \frac{1}{k} \left(\sum_{t=1}^{n-1} k x_{i+t} \right) \quad (2.8)$$

Note that the ring network is symmetric in terms of connectivity. If all the players have a valuation $v_i = v \quad \forall i \in N$, then under ex-post symmetry, i.e. $x_i = x_j = x \quad \forall\{i,j\} \subset N$ in the above equation, the equilibrium effort is obtained as $x = (n-k)v/n^2$. Imputing this in the payoff function as given in expression (2.2) gives symmetric equilibrium expected payoff equal to $(k + n(k-1)v)/n^2$.

(ii) Any given player i on a n-player ring network belongs to exactly k number of distinct coalitions, and the total number of coalitions on the network is n. Further, simple linear aggregation implies $\frac{\partial X_t}{\partial x_i} = 1 \quad \forall t : i \in G_t \implies \sum_{t:i\in G_t} \frac{\partial X_t}{\partial x_i} = k$.
Adding up the left-hand and right-hand sides of the first order condition from expression (2.8) over all $i \in N$, one can write $n\left(k\sum_i x_i^*\right)^2 = vk\left[(n-2k+1) + 2\sum_{d=1}^{k-1} d\right]\sum_i x_i^*$, simplifying which obtains $\sum_i x_i^* = (n-k)v/n$. This is the unique total effort in any equilibrium.

From expression (2.7), we have $\left(\sum_{t:i\in G_t} X_t + \sum_{t:i\notin G_t} X_t\right)^2 = kv\sum_{t:i\notin G_t} X_t \implies \left(k\sum_i x_i\right)^2 = kv\sum_{t:i\notin G_t} X_t$. Imputing the unique equilibrium total effort, and simplifying, we must have

$$\sum_{t:i\notin G_t} X_t^* = (n-k)\left(\frac{n-k}{n^2}kv\right)$$

for all $i \in N$ in equilibrium. Note that $\sum_{t:i\notin G_t} X_t = X_{i+1} + X_{i+2} + ... + X_{i+n-k}$ and $|t : i \notin G_t| = (n-k)$. Starting from any $i \in N$, the first order condition for player $(i+1)$ requires replacing X_{i+1} with $X_{i+n-k+1}$ in the left-hand side. The constant right hand side implies $X^*_{i+n-k+1} = X^*_{i+1}$. Similarly, $X_i^* = X^*_{i+k}$. This is true for every i, implying every two k-neighbouring groups, the left-most (or, right-most) members of which are k edges apart, must have the same total efforts.

If N is a multiple of k then for any $i \in N$ one can always define (n/k) number of disjoint k-neighbouring groups with equal total efforts in equilibrium, such that the sum of effort is equal to $\sum_i x_i^*$. Suppose, $\rho = n/k$. Then, $X_i^* = X^*_{i+k} = \cdots = X^*_{i+(\rho-1)k}$ for any $i \in N$, but due to these groups being disjoint $X_i^* + X^*_{i+k} + \cdots + X^*_{i+(\rho-1)k} = \sum_i x_i^* = (n-k)v/n$ is true for any $i \in N$. Hence, $\rho X_i = (n-k)v/n$, or $X_i = (n-k)kv/n^2$ for all $i \in N$.

If N is not a perfect multiple of k, then let us define $\rho = \lfloor n/k \rfloor$. Then $X_i^* = X^*_{i+k} = \cdots = X^*_{i+\rho k+1}$. But also $X^*_{i+\rho k+1} = X^*_{i+k-(n\, mod\, k)} = ... = X^*_{i+\rho k+2}$ and so on. Remembering that all indices are defined *modulo* n, one can establish that X_i^* is same across all $i \in N$. On the other hand, $G_{i+\rho k+1}$ includes $k - (n\ mod\ k)$ players already included in G_i. Then we can write $(\rho+1)X_i^* - (x_i^* + x^*_{i+1} + \cdots + x^*_{i+k-(n\, mod\, k)-1}) = \sum_i x_i^*$. Note that replacing X_i^* on the left hand side by X^*_{i+1} also require replacing x_i^* by $x^*_{i+k-(n\, mod\, k)}$. Since $X_i^* = X^*_{i+1}$ and the right-hand side is constant, we must have $x_i^* = x_{i+k-(n\,mod\,k)}^* = x_{i+k}^*$ which implies $x_i^* = x_j^* \quad \forall\{i,j\} \subset N$. Consequently, the $(\rho+1)X_i^* - (x_i^* + x_{i+1}^* + \cdots + x_{i+k-(n\,mod\,k)-1}^*) = \sum_i x_i^*$ can be simplified to $(\rho+1)kx_i^* - [k-(n\ mod\ k)]x_i^* = (n-k)v/n$. But, $(n\ mod\ k) = n - \rho k$. Substituting in the previous expression one obtains $x_i^* = (n-k)v/n^2$ for all $i \in N$, and $X_i^* = (n-k)kv/n^2$ is the unique equilibrium total effort in every possible k-neighbouring group.

(iii) $X_i^* = X_{i+1}^*$ implies $x_i^* = x_{i+k}^*$. Any redistribution of equilibrium efforts within each G_t satisfying ex-post symmetry between every pair of k-distance neighbours and preserving $X_t^* = (n-k)kv/n^2$ for all $t \in N$ also constitute an equilibrium. It follows from the argument in the proof of 2.4(ii) that such a redistribution is feasible only when n is a multiple of k, otherwise any deviation from the symmetric equilibrium effort profile results in $X_t^* \neq (n-k)kv/n^2$ for at least some $t \in N$. Let us look at Figure 2.3 for an intuition. Suppose n is a multiple of 3 and not of 4. If $k = 3$, then starting from the ex-post symmetric equilibrium profile, if player $i+1$ spends ε more and $i+2$ spends ε less and i does not change her effort, the total effort in G_i remains the same. If efforts are changed the same way for the respective k-neighbours of these three players and their respective k-distance neighbours, and so on, then total effort in all possible k-neighbouring groups remain the same and so is the total effort. On the other hand, if $k = 4$, player i keeps her effort intact at the ex-post symmetric equilibrium level, and players $i+1$, $i+2$, and $i+3$ change their efforts in such a manner that total effort is same in G_i, then keeping total efforts same across all k-neighbouring groups will require similar effort readjustment in all possible streams of k-distance neighbours. But due to $\lfloor n \text{ mod } k \rfloor > 0$ and players being counted *modulo* n, player i will belong to the sequence of k-distance neighbours of one of $i+1$, $i+2$, or $i+3$, which contradicts with the beginning assumption. $\square$

Despite the conditional existence of multiplicity of equilibria, total effort from a combinatorial contest to allocate k prizes among $n$players on a ring network with symmetric effort aggregation is always $(n-k)v/n$ which follows from the first order conditions.

Corollary 2.2. $\Phi(k,(N,E_2),L_{SA})$ *generates the same total equilibrium effort as* $\Phi(k,(N,E_{n-1}),L_{SA})$ *for any given values of* n *and* k.

Irregular Networks

Irregular networks are difficult to analyse because of the numerous possibilities. As demonstrated in the examples, a line network for $n = 4$ and $k = 2$ is characterized by multiple equilibria. For $k = 2$, this result is true for any line network with even value of $n(> 3)$. The first order conditions require

$$\begin{aligned}
(i) \quad & x_1^* + x_2^* = x_3^* + x_4^* = x_5^* + x_6^* = \cdots \\
(ii) \quad & x_2^* + x_3^* = x_4^* + x_5^* = x_6^* + x_7^* = \cdots \\
(iii) \quad & n(x_1^* + 2x_2^* + \cdots + 2x_{n-1}^* + x_n^*)^2 = v[(2n-5)(x_1^* + x_n^*) \\
& + (4n-11)(x_2^* + x_{n-1}^*) + (4n-10)(x_3^* + \cdots + x_{n-2}^*)]
\end{aligned}$$

Given $\mathbf{x}*$ is an equilibrium effort profile, consider an alternative profile $\mathbf{x}'$ such that every odd player's effort is increased by ε and every even player's effort is decreased by ε. The first two conditions are satisfied regardless the value of n, but the last condition is satisfied only for even values of n, such that both sides of the last condition remain unchanged. This result can be extended to show multiplicity of equilibria for any n and k such that $\lfloor n \bmod k \rfloor = 0$. However, unlike a ring network, existence of equilibrium on a line network depends on the relative values of n and k. It is important to note that despite all non-marginal players on a line network having same number of connections, the degree of irregularity depends on the contest parameters. A detailed analysis is out of the scope of the present study.

Lastly, a star network is an extreme example of an irregular network where the central player has $n - 1$ connections and all but the central player has exactly one connection. However, due to all peripheral players on a star being ex-ante symmetric, the equilibrium analysis is straightforward.

Proposition 2.5. *A combinatorial contest on a star network with n homogeneous players generates the same total effort in equilibrium as*
(i) a single-winner contest with $n - 1$ homogeneous players, for $k = 2$.
(ii) a combinatorial contest with $k - 1$ rewards on a complete network of $n - 1$ players, for $k \geq 3$.

It is straightforward to argue that a k-winner combinatorial contest on an n player fan-shaped network (a star with every alternate pair of peripheral players connected, possible only for an odd value of n, such that there are even number of peripheral players) is equivalent to a $(k-1)$-player combinatorial contest on an n-player complete network for any $k \geq 3$. The central player is part of all possible k-neighbouring groups, and therefore spends zero effort. For $n = 2$, the central player spends more effort than the peripheral players in equilibrium and but still obtains a larger payoff compared to the peripheral players.

2.4 Discussion

We present a highly stylized model of multiple winner contest with effort substitution among neighbouring players. Players in this model are connected through a network and exert costly effort, but only a subset of the connected players can emerge as winners. To our knowledge, this is the very first attempt to investigate such combinatorial contests. The study contributes to the literature on multiple winner contests as well as on contests under network-based spillover. It also provides a connection between the multiple winner contest and the group contest literature.

We derive the best response correspondence for a combinatorial contest on any arbitrary network, and discuss equilibrium properties for common networks like a ring, a star, and a complete network. The general model can be extended to virtually any network structure with given number of total players. We show that the combinatorial contest on a complete network is equivalent to Berry's (1993) model. An extreme example of an asymmetric network is the star network. It is easy to observe that the central player in a star network will never exert a positive effort in the combinatorial contest, since he will be a winner regardless of which final combination of players becomes the winning coalition. Effort of the other players will depend on individual valuation of rewards. Another irregular network, a line, is briefly discussed but equilibrium properties on a line network has multiple cases depending on the number of winning positions relative to the total number of players, and therefore avoided in the current study.

There are many possibgle applications of the results. The generic best responses can be used to analyze contests with any network structure. The observations about different network structures have implication for contest designing. Considering a four-player combinatorial contest with two prizes on a ring network, for example, it follows that if half of the players are low-value type and half are high-value type, then players expend more equilibrium effort if the two high value players are placed diagonally instead of adjacently.

However, this is a very simple framework for analysing a combinatorial contest, and has strong assumptions of homogeneity in terms of value and ability, and fixed size of the winning coalition. A particularly interesting and realistic extension will be to impose a limit on the maximum permissible distance between two players on a winning coalition (which is k in our model, and equal to the total number of positions in the winning coalition) and model the size of the winning coalition as dependent on the network topology.

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Appendix

2.A Asymmetric Valuation in combinatorial contests on small networks

Observation 1: With players numbered clockwise on a 4-player ring and reward valuation of player i given by $i \in \{1, 2, 3, 4\}$,
(i) Only a single player, say player 1, spend zero effort in the equilibrium if $\frac{1}{v_1} \geq \left(\frac{1}{v_2} + \frac{1}{v_4}\right)$, given that $2v_2v_4$, v_2v_3 and v_3v_4 satisfy triangle inequality.
(ii) Two adjacent players, say players 1 and 2, spend zero effort in equilibrium if $\frac{1}{v_1} \geq \left(\frac{2}{v_4} + \frac{1}{v_3}\right)$ and $\frac{1}{v_2} \geq \left(\frac{2}{v_3} + \frac{1}{v_4}\right)$.
(iii) Any two diagonally positioned players (say player 1 and player 3) spend zero effort in equilibrium if $\frac{2}{v_1} \geq \left(\frac{1}{v_2} + \frac{1}{v_4}\right)$ and $\frac{2}{v_3} \geq \left(\frac{1}{v_2} + \frac{1}{v_4}\right)$.

Proof. Player $i \in \{1, 2, 3, 4\}$ maximizes his or her expected payoff

$$\pi_i = \frac{(x_i + x_{i-1}) + (x_i + x_{i+1})}{2\sum_i x_i} v_i - x_i \qquad \text{s.t.} \quad x_i \geq 0$$

The first order conditions require
$v_i(2\sum_j x_j - x_{i+1} - x_{i-1}) = 2(\sum_i x_i)$, or, $x_i = 0$ for each $i \in N$, where $j \in N, j \neq i$.
The second order condition is satisfied for at least two players spending strictly positive efforts. Assuming only player 1 spends zero effort, $x_1^* = 0$ and $x_i \geq 0 \forall i \in \{2, 3, 4\}$. From the first order conditions, we must have $\frac{\partial \pi_1}{\partial x_1} \leq 0$ and $\frac{\partial \pi_i}{\partial x_i} = 0 \forall i \in \{2, 3, 4\}$.
Setting $x_1 = 0$ in the first order conditions give (i) $(2v_2 - v_3)x_4 = v_3x_2 - v_2x_3$, (ii) $(2v_4 - v_3)x_2 = v_3x_4 - v_4x_3$, and (iii)$(v_2 - v_4)x_3 = 2(v_4x_2 - v_2x_4)$ Evaluating x_3 from (iii) and imputing in (ii) we have

$$x_2 = \frac{2v_2v_4 - v_3v_4 + v_2v_3}{2v_2v_4 - v_2v_3 + v_3v_4} x_4$$

Substituting this back into (iii) gives

$$x_3 = \frac{v_2v_3 + v_3v_4 - 2v_2v_4}{2v_2v_4 - v_2v_3 + v_3v_4}x_4$$

Therefore

$$x_1 + x_2 + x_3 + x_4 = \frac{2v_2v_4 + v_2v_3 + v_3v_4}{2v_2v_4 - v_2v_3 + v_2v_4}x_4$$

Using the above to solve for x_4, we have

$$x_4 = \frac{2v_2v_4 - v_2v_3 + v_3v_4}{(2v_2v_4 + v_2v_3 + v_3v_4)^2}v_2v_3v_4$$

Consequently, $\frac{\partial \pi_1}{\partial x_1} \leq 0$ requires $\frac{1}{v_1} \geq \left(\frac{1}{v_2} + \frac{1}{v_4}\right)$.

Similarly, assuming $x_1 = 0$ and $x_2 = 0$, the first order conditions give $x_3 = \frac{v_3^2 v_4}{2(v_3+v_4)^2}$ and $x_4 = \frac{v_3 v_4^2}{2(v_3+v_4)^2}$.

$\frac{\partial \pi_1}{\partial x_1} \leq 0$ and $\frac{\partial \pi_2}{\partial x_2} \leq 0$ are both satisfied only when $\frac{1}{v_1} \geq \frac{2}{v_4} + \frac{1}{v_3}$ and $\frac{1}{v_2} \geq \frac{2}{v_3} + \frac{1}{v_4}$.

And analogously for $x_1 = x_3 = 0$. □

Observation 2: With players numbered sequentially on a 4-player line network and reward valuation for player i given by v_i for all $i \in \{1, 2, 3, 4\}$,
(i) Only a single player at a marginal position (say, player 1) spend zero effort in equilibrium if (a)$v_3 \geq v_4$, (b)$\frac{1}{v_2} \geq \frac{1}{2}\left(\frac{1}{v_4} - \frac{1}{v_3}\right)$, and(c) $\left(\frac{1}{v_1} - \frac{1}{v_4}\right) \geq 2\left(\frac{1}{v_2} - \frac{1}{v_3}\right)$.
(ii) Only a single player at a central position (say, player 2) spends zero effort in the equilibrium if (a) $v_3 \geq v_4/2$, (b) $\left(\frac{1}{v_1} + \frac{1}{v_3} \geq \frac{2}{v_4}\right)$, and (c) $\left(\frac{1}{v_2} - \frac{1}{v_3}\right) \geq 2\left(\frac{1}{v_1} - \frac{1}{v_4}\right)$.
(iii) One marginal and the corresponding non-adjacent central player (say, players 1 and 3) together spend zero effort if (a) $v_4 \geq v_3$, and (b) $\frac{1}{v_3} \geq \left(\frac{1}{v_2} + \frac{1}{v_4}\right)$.
(iv) Both the central players (players 2 and 3) spend zero effort in equilibrium if (a) $2v_2 \leq v_1$, and (b) $2v_4 \leq v_3$.
(v) Both players at the marginal positions (players 1 and 4) spend zero effort if (a) $2\left(\frac{1}{v_1} - \frac{1}{v_2}\right) \geq \frac{1}{v_3}$, and (b) $2\left(\frac{1}{v_4} - \frac{1}{v_3}\right) \geq \frac{1}{v_2}$.

The above observation can be verified following the same technique as observation 1. It is apparent from both observations that low value players are more likely to free ride if there's a high value player as their immediate neighbour. This is useful when designing such a contest.

Chapter 3

Equivalent multi-winner contests: An experiment

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Abstract

In contests with more than one winning positions, different mechanisms can be used to pick up the final set of winning players. We implement three such mechanisms in a laboratory experiment. The three mechanisms follow explicitly different selection procedures but generates the same success function when all winning positions are equally valued and players are homogeneous. Our experiment confirms the strategic equivalence in terms of observed behaviour. Further, we compare these multi-winner mechanisms to a standard Tullock lottery which is best-response equivalent for our chosen parameter values and predicts the same equilibrium effort as the multi-winner contests. Average expenditure from the single winner contest turns out to be statistically similar to the multi-winner treatments. The results indicate practical robustness of strategic equivalence, and suggest higher flexibility in contest designing.

JEL classifications: C72, C91, D72, D82.

Keywords: lottery contest, multi-winner contests, learning, experiment, strategic equivalence.

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3.1 Introduction

Economic contests refer to games in which players incur irreversible expenditure of costly resources (summarised as 'effort') in order to increase their success probabilities. Majority of contest literature focuses on contests that offer final success to only one player. Considering different real-world examples of contest games however, multiple winning positions are not very uncommon.[4] Some contests rank players into different positions and allocate rank-specific rewards to the players holding the first few positions. The sprint race is an example of such a contest. There are other contests that offer similar rewards to a certain number of players, such as university admissions, allocation of trading licenses, or cadre selection for military or civil services. Multi-winner contests can often be viewed as intra-group contests, where a subset of players chosen from a larger set of players constitutes the set of winners. Different mechanisms can be followed to choose the members of this winning set. An individual's success probability in the contest will depend on the particular mechanism used for choosing the winners. We consider three mechanisms that follow explicitly different selection procedures, but can be characterized by the same contest success function, leading to the same equilibrium total effort and expected payoffs under complete information and player homogeneity. Our experimental investigation of these three mechanisms render the procedural differences irrelevant for the final outcome.

The key component in the formal analysis of contests is the mapping of players' efforts into individual success probabilities, which is called the contest success function (CSF). The classic Tullock CSF (Tullock, 1980) defines individual success probability in a lottery contest as the ratio of own effort to total effort. This CSF is conventionally used for many contest scenarios where success is not perfectly correlated with effort. Maximum effort from player i does not guarantee her success in such games, though higher effort implies higher success probability. The Tullock CSF is defined over the vector of individual efforts. Berry (1993) devised a parallel CSF for multi-winner contests with n players and k ($k \leq n$) rewards, which is a Tullock CSF defined over the vector of joint efforts from each potential k-player subset. Individual success probability is the sum of the success probabilities of all such potential subsets that include the given individual.

Clark and Riis (1996) criticise the Berry CSF on the ground of being effort-dependent for the allocation of the first winning position only, and a random allocation of the other positions thereafter. However, de Palma and Munshi (2013) refutes this criticism by arguing that Clark and Riis undermined the simultaneity of selection inherent to Berry's mechanism. They also derive the Berry CSF as a limiting case from a generalised probability space defined over

[4] See Sisak (2009) for a complete survey on multi-winner contests.

all possible outcomes under a simultaneous distribution of rewards. The acceptability of the Berry CSF for simultaneous selection of multiple winners is thus restored in the literature.

One of our treatments directly implements the Berry CSF. Two other treatments implement modified versions of this mechanism. One of these is developed by Chowdhury and Kim (2014), who implement the Berry CSF for one by one elimination of the losers by selecting gradually shrinking sets of survivors. The other mechanism is a combinatorial contest on a ring network as discussed in the previous chapter.[5] Both the mechanisms preserve the Berry CSF for individual players, while changing the selection procedure at the outset. The observed contest behaviour in our experiment confirms the theoretical equivalence. In addition to these, we also run a single-winner contest which is characterized by a different CSF leading to a different best-response function. For our chosen parameter values however, this best-response function is similar to the best-response function from the multi-winner mechanisms, and predicts the same equilibrium outcome. We find that average bid in this contest is statistically no different from average bids in the multi-winner mechanisms.

The practical implication of this result is for designing selection mechanism in contests. The observed equivalence among the three multi-winner mechanisms and the best-response equivalent single-winner mechanism provides high level of flexibility as far as contest designing is concerned. Different procedures of selection may be preferable given the particular context or the logistic costs of a contest, and the contest designer can choose among a number of procedures without violating the expected contest behaviour. Each of the three multi-winner mechanisms discussed here selects multiple winners at a time, as opposed to a one by one selection of the winners (Clark and Riis, 1996). A simultaneous selection mechanism may be particularly preferable in wasteful contests as the expected equilibrium effort under these contests is lower than the expected equilibrium effort under one by one selection or a single winner contest with the same reward budget. The observed convergence to equilibrium also suggests more predictable behaviour among experienced players.

Section 3.2 summarises related theoretical as well experimental literature. Section 3.3 establishes the strategic equivalence of the three multi-winner mechanisms. Section 3.4 describes the implementation of the three mechanisms in the laboratory. Section 3.5 reports the results from the experiment, followed by a brief discussion in section 3.6.

[5]Chowdhury and Kovenock (2012) exclusively discussed the ring network in an unpublished manuscript. However, a generally defined combinatorial contest restricts the number of potential k-player subsets via an imposed adjacency requirement, and should accommodate any network structure. The previous chapter in this thesis extends Chowdhury and Kovenock (2012)'s idea for any arbitrary network. The previous chapter also shows that a combinatorial contest with risk-neutral homogeneous palyers on the ring network can be characterized by multiple equilibria. However, under the assumption of ex-post symmetry, it yields the same equilibrium effort as the unique symmetric equilibrium in Berry (1993). The present chapter shows that they also give rise to the same CSF under uncertainty about player positions on the ring.

3.2 Related Literature

3.2.1 Theoretical studies

Berry's model, to our knowledge, is the first model to talk about contests with multiple prizes. Katz (1990) considers a group contest with a group-specific public good prize, where all individuals in a given group receives the same value from winning. Pre-specified groups fight each other for a reward but individuals within the same group do not fight among themselves. Katz (1990) considers variable group sizes with symmetric values for all members within a group. Baik (2008) extends the model by also allowing asymmetric values among members within a given group. A multi-winner contest may not have ex-ante specified groups.[6] Clark and Riis (1996) suggest an alternative CSF that interprets a multi-winner contest as a series of single-winner Tullock contests on a gradually receding pool of contestants (a sequence of draws without replacements). The pool of contestants shrink by dropping out one winner after each draw, and the next draw is conducted with the remaining contestants. Note that all players expend effort only once, at the beginning of the contest, and all subsequent draws are conducted over these one-time efforts. The strategy set of players is therefore entirely different from a sequential selection contest with multiple rounds of effort inputs, where one winner is selected in each round and remaining players have to incur effort costs for the next round of selection (Clark and Riis, 1998).

Fu et al. (2014) argue that in contrast to the sequential allocation of winning positions in Clark and Riis (1996), 'many real world competitions, however, implement an opposite procedure in their decision processes,... one wins a prize only when he is not excluded'. They propose a reverse nested lottery in which losers are selected out one by one. Again, players decide effort only at the beginning, and successive lotteries are conducted over the inverted outlays of the players who survive the previous elimination. The mechanism by Chowdhury and Kim (2014) that we implement in one of our treatments, suggest a parallel mechanism for eliminating the losers. Like Fu et al. (2014), their mechanism consists of $(n-k)$ consecutive draws to select k winners out of n contestants. However, unlike Fu et al., each draw involves a multi-winner lottery to select in multiple survivors instead of selecting out a single loser.[7] In the first round, $(n-1)$ survivors are chosen out of n contestants using Berry's CSF, the

[6] By ex-ante specified groups, we mean groups that are mutually exclusive. As is apparent from the set-up of the combinatorial contest in chapter 2, multiple size-k groups can be ex-ante defined in relation to the connectivity structure. However, multiple groups are overlapping in the sense that any given player can belong to more than one such group.

[7] Both the mechanisms have the same practical implication of dropping one loser after each draw. Fu et al. (2014) directly picks out the loser by employing a logit CSF on reverse outlays, and Chowdhury and Kim (2014) determines the loser by selecting in the rest of the players into the next draw using Berry's multi-winner CSF.

remaining player being removed from the pool of contestants as the first loser. In the second round, $(n-2)$ survivors are chosen out of those $(n-1)$ previous-round survivors using Berry's CSF, and the remaining player is eliminated from the pool. The process continues until there are exactly k survivors left. The corresponding contest success function is the sum of the unconditional probabilities of not being eliminated in one of the $(n-k)$ rounds. Chowdhury and Kim shows the CSF to be similar to Berry's CSF when all players spend similar efforts. The symmetric equilibrium effort is also similar to that from Berry's simultaneous selection mechanism. The similarity of the CSF holds for asymmetric efforts too, as is shown in section 3.3. Both Clark and Riis (1996) and Fu et al. (2014) generates higher equilibrium efforts.

A few other studies analyse multiple prizes from a desirability perspective. Yates and Heckelman (2001) propose a model in which the number of rewards is endogenously determined from the organiser's preferences about the long-term competitiveness among winners. Szymanski and Valletti (2005) examine the desirability of second prizes when players have different abilities. Fu and Lu (2009) invoke the desirability of a grand contest over a set of sub-contests in terms of higher effort generation. The combinatorial contest that we implement in one of our treatments can be classified along with these studies in their consideration of organiser's preference about connectivity among the winners.

3.2.2 Experimental studies

Coming to the experimental literature involving multi-winner contests, nearly all of the existing experiments look at multiple winners or multiple prizes in comparison to a standard single-winner contest. Most experiments find multiple winning positions to have a positive effect on effort provision.

Herbring and Irlenbusch (2003) investigate the effect of variation in number of contestants as well as in the number of winning prizes in a perfectly discriminating contest where players choose between spending all or nothing, and find average effort increasing in the ratio of winning prizes to number of contestants. Based on two laboratory and field experiments on sales persons, Lim et al. (2009) invokes multiple prizes over a single prize from an incentive point of view. The theoretical propositions from Moldovanu and Sela (2001) inspire a laboratory experiment (Muller and Schotter, 2010) about the relation between the shape of effort-cost function and the number of prizes in a perfectly discriminating contest. The experiment reports that two prizes generate higher effort than a single prize in the presence of a quadratic cost component, which is in line with the theoretical findings. Chen et al. (2011) show effort to be increasing in number of prizes in an imperfectly discriminating heterogeneous contest with two favourites and one underdog. Stracke et al. (2014) considers a dynamic elimination contest with two unequal prizes and concludes that multiple prizes dominate a single prize if participants are risk-averse. A few studies also find the opposite effect though. Orrison et al. (2004) examine the effect of variation in tournament size and prize structure in imperfectly discriminating rank-order tournaments, and find that increase in the number of winning prizes relative to the number of loser prizes significantly decrease effort. Anderson and Freeborn (2010) also shows that expenditure falls with reduced competition.

Two other experiments have motivations that are more similar to ours. Sheremeta (2011) draws an experimental comparison of one grand-prize contest, two multiple prize contests using the Clark and Riis (1996) mechanism (one with equal and another with unequal prize values), and one contest with multiple single-prize sub-contests (the Fu and Lu (2009) mechanism). The results show that in terms of effort generation, the grand prize dominates multiple unequal prizes, which dominates multiple equal prizes, which in turn generates significantly higher expenditure than the sub-contest based mechanism. Over-dissipation is observed in all four treatments. Shupp et al. (2014) tests effort equivalence between single-prize and multi-prize contests (Sequential draws with replacement, a la Clark and Riis (1998)) under risk neutrality, though their experimental findings indicate that aggregate expenditure tends to be significantly lower in a single-prize contest than in a multi-prize contest. In contrat, our experimental findings are in line with Sheremeta (2011), and invokes desirability of multiple prizes over a single prize in wasteful contests. However, besides the different mechanisms followed for the draws, Shupp et al. (2014) also differed in allowing the possibility that all prizes can go to the same individual. Moreover, they were testing equivalence in equilibrium outcome, whereas the multi-winner mechanisms in our experiment are also best-response equivalent to the single-winner contest.

The multiple prizes in majority of these experiments are allocated either by ranking players' effort decisions (Herbring and Irlenbusch, 2003; Orrison et al., 2004; Muller and Schotter, 2010; Chen et al., 2011), or are based on ordered random draws on the effort entries (Anderson and Freeborn, 2010; Sheremeta, 2011; Shupp et al., 2014). So, none of these experiments are comparable to ours where the alternative selection mechanisms are all based on simultaneous draw of multiple effort entries. Besides comparing single-winner and multi-winner contests in terms of effort generation, we have parallel interest in examining strategic equivalence among the multi-winner mechanisms, which is under-explored in contest experiments.

3.3 Theoretical equivalence of alternative mechanisms

Consider a set of n players denoted by $\mathbf{N} = \{1, 2, \cdots, n\}$. The traditional single-winner Tullock contest gives the success probability of an individual as the ratio of own effort to total effort from all players. That is,

$$p_i(e_i, \sum_{j \neq i} e_j) = \frac{e_i}{e_i + \sum_{j \neq i} e_j} \quad \text{when } e_i + \sum_{j \neq i} e_j > 0$$

where e_i denotes the effort from player i, for all $i \in \mathbf{N}$, and $p_i(\cdot)$ is her probability of winning. The winning probability strictly increases in the effort of player i, and strictly decreases in other players' efforts. An individual spending no effort can never win the contest as long as any other player is spending a positive effort. This is also true for the sequential selection contests discussed by Clark and Riis (1996), Clark and Riis (1998), and Fu et al. (2014). In both sequential winner selection (Clark and Riis, 1996) and sequential loser elimination (Fu et al., 2014), one player is selected at a time. The probability of this one time selection (or elimination) necessarily depends on how the player's own effort compares to the total effort of the remaining players in the contest at the time of a draw. This ensures that any player spending zero effort cannot be a winner as long as there are at least k players spending positive efforts. In a simultaneous selection mechanism however, it is the total expenditure of an arbitrarily given set of players that determines the probability of that particular set qualifying as the winning set. More specifically, the ratio of the total effort of any feasible k-partition (size-k subset of players) to the sum of the total efforts from all feasible k-partitions gives the probability of that particular k-partition qualifying as the winning set. The probability that a given player i belongs to the winning k-partition is the sum of the probabilities of every favourable k-partition being chosen, where a favourable k-partition is any k-player combination containing player i.

Chowdhury and Kim (2014) shows that their mechanism, in spite of resembling a sequential contest in structure, returns the same symmetric equilibrium outcome as Berry's one-shot mechanism. The previous chapter in this thesis shows that the additional restriction of close connectivity among the winners on a ring network can have multiple equilibria depending on the number of prizes relative to number of players, but returns the same equilibrium effort profile (as Berry's symmetric equilibrium) under the assumption of ex-post symmetry.

In what follows we argue that the three mechanisms considered in this experiment are not only equivalent in terms of equilibrium outcome, but also generate the same normal form game. That is, the three mechanisms give rise to the exact same contest success function.

Suppose players in $\mathbf{N}$ are contesting for $k(< n)$ number of rewards each worth v.

Let us first look at Berry's mechanism. A given player i can be part of a total number of $\binom{n-1}{k-1}$ distinct k-partitions, and player i can be in the same k-partition with any other player j for $\binom{n-2}{k-2}$ k-partitions. This is true for all $j \in \mathbf{N}$, such that $j \neq i$. Hence, player i's own effort will be counted $\binom{n-1}{k-1}$ times and each player j's ($\forall\ j \in \mathbf{N}\ s.t.\ j \neq i$) effort will be counted $\binom{n-2}{k-2}$ times in the numerator of the CSF for player i. The denominator of the CSF is the sum of efforts of all possible k-partitions, and hence will count each player's effort exactly $\binom{n-1}{k-1}$ times. Berry's CSF, or the CSF for our Joint Selection treatment can therefore be written as

$$
\begin{aligned}
p_i^{JS}(e_i, \sum_{j\neq i} e_j) &= \frac{\binom{n-1}{k-1}e_i + \binom{n-2}{k-2}\sum_{j\neq i} e_j}{\binom{n-1}{k-1}\left(e_i + \sum_{j\neq i} e_j\right)} \\
&= \frac{\binom{n-1}{k-1}e_i + \frac{k-1}{n-1}\binom{n-1}{k-1}\sum_{j\neq i} e_j}{\binom{n-1}{k-1}\left(e_i + \sum_{j\neq i} e_j\right)} \\
&= \frac{(n-1)\,e_i + (k-1)\sum_{j\neq i} e_j}{(n-1)\left(e_i + \sum_{j\neq i} e_j\right)}
\end{aligned}
$$

Next, consider the combinatorial contest on a ring network as discussed in the previous chapter. Consider that there are n total number of players and exactly n positions on the ring network. The previous chapter considers known arrangement of players on the network. Consequently, only those players who are at a distance of less than k units from player i, can share at least one common k-partition with player i. The resultant contest success function looks like

$$
p_i(e_i, e_{-i}) = \frac{ke_i + \sum_{d=1}^{k-1}(k-d)\,(e_{i-d} + e_{i+d})}{k\left(e_i + \sum_{j\neq i} e_j\right)}
$$

Note that, in order to avoid reputation building during the laboratory implementation of this mechanism, we randomly assign players to different positions on the network at the beginning of each period. Though the above formulation is valid for an ex-ante given circular arrangement of players, the ID-reassignment feature in our experiment requires considering the possibility of the occurrence of a particular arrangement of players on the ring. This makes all possible arrangements equally likely. That is to say, if A, B, C and D are the four players forming a circular network, then the list of possible distinct arrangements comprises ABCD, ABDC and ACBD[8] and all these three circular arrangements are equally likely to occur under random assignment of positions. This renders the CSF for Restricted Coalition treatment exactly the same as the Joint Selection CSF.

[8] Note that arrangements ABCD and ADCB are equivalent. So are ABDC and ACDB, or ACBD and ADBC.

Number of free[9] circular permutations of order n is $(n-1)!/2$. A given player i can belong to k distinct k-partitions for each of this arrangements. Therefore, the total number of possible k-partitions containing player i is $k(n-1)!/2$. Every other player j (s.t. $j \in \mathbf{N}$ and $j \neq i$) could be at a distance of d units from player i for $(n-2)!$ strictly distinct permutations. For $d < k$, player j will share $(k-d)$ common k-partitions with player i. The CSF for restricted coalition treatment can therefore be written as

$$\begin{aligned}
p_i^{RC}(e_i, \sum_{j\neq i} e_j) &= \frac{\frac{k(n-1)!}{2} e_i + \sum_{j\neq i}(n-2)! \sum_{d=1}^{k-1}(k-d)e_j}{\frac{k(n-1)!}{2}\left(e_i + \sum_{j\neq i} e_j\right)} \\
&= \frac{\frac{k(n-1)!}{2} e_i + (n-2)! \sum_{j\neq i} \frac{k(k-1)}{2} e_j}{\frac{k(n-1)!}{2}\left(e_i + \sum_{j\neq i} e_j\right)} \\
&= \frac{k(n-1)! e_i + k(k-1)(n-2)! \sum_{j\neq i} e_j}{k(n-1)!\left(e_i + \sum_{j\neq i} e_j\right)} \\
&= \frac{(n-1)e_i + (k-1)\sum_{j\neq i} e_j}{(n-1)\left(e_i + \sum_{j\neq i} e_j\right)}
\end{aligned}$$

Chowdhury and Kim (2014) shows that their mechanism yields the same contest success function as Berry's under the assumption that every player other than player i spends the same amount of effort. However, that assumption is not necessary for the equivalence of the contest success functions. Given that there are $\binom{n-1}{k-1}$ distinct k-partitions containing player i, and $\binom{n-2}{k-2}$ among these contain any j ($j \in \mathbf{N}$ and $j \neq i$), we may look at the number of possible sequences in which any given k-partition can emerge as the final set of winners. The first set of survivors with $(n-1)$ players that also contains our given k-partition can be chosen in $(n-k)$ ways. The second set of survivors with $(n-2)$ remaining players that also contains this given k partition can be chosen in $(n-k-1)$ ways. So on and so forth, and finally the set of $(k+1)$ survivors that also contains our given k-partition can be chosen in only 2 possible ways. Therefore, the total number of possible survivor sequences through which any given k-partition can get selected is $(n-k)!$. So, the total number of instances in which player i can be a part of the final set of survivors is $(n-k)!\binom{n-1}{k-1}$ and the total number of instances in which player i will share a k-partition with player j is $(n-k)!\binom{n-2}{k-2}$. This is true for every $i, j \in \mathbf{N}$. The contest success function for our survivor selection treatment can therefore be

[9] strictly distinct circular permutations, or the distinct circular permutations obtained after allowing for two flipped circles to be equivalent.

written as

$$\begin{aligned}
p_i^{SS}(e_i, \sum_{j\neq i} e_j) &= \frac{(n-k)!\left(\binom{n-1}{k-1}e_i + \sum_{j\neq i}\binom{n-2}{k-2}e_j\right)}{(n-k)!\binom{n-1}{k-1}\left(e_i + \sum_{j\neq i} e_j\right)} \\
&= \frac{(n-k)!\binom{n-1}{k-1}\left(e_i + \frac{k-1}{n-1}\sum_{j\neq i} e_j\right)}{(n-k)!\binom{n-1}{k-1}\left(e_i + \sum_{j\neq i} e_j\right)} \\
&= \frac{(n-1)e_i + (k-1)\sum_{j\neq i} e_j}{(n-1)\left(e_i + \sum_{j\neq i} e_j\right)}
\end{aligned}$$

The similarity of the contest success functions imply similar payoff functions for all three multi-winner mechanisms, which is given by

$$\pi_i = \begin{cases} \frac{(n-1)e_i + (k-1)\sum_{j\neq i} e_j}{(n-1)\left(e_i + \sum_{j\neq i} e_j\right)} v - e_i & \text{if } e_i > 0 \text{ for some } i \in \mathbf{N} \\ \\ \frac{k}{n} v & \text{otherwise} \end{cases}$$

The corresponding best-response function is given by

$$e_i^{\text{BR}}(\mathbf{e_j})|_{\text{MW}} = \max\left\{0, \sqrt{\left(\frac{n-k}{n-1}\right) v \sum_{j\neq i} e_j} - \sum_{j\neq i} e_j\right\}$$

The symmetric equilibrium effort follows to be $(n-k)v/n^2$.

In our experiment, we implement the above-discussed multi-winner contest mechanisms with four players and two winning positions in every contest group. Each winner is rewarded with 240 pence. Symmetric equilibrium effort is therefore

$$\frac{n-k}{n^2} v = \frac{4-2}{4^2} \text{ x 240 pence } = 30 \text{ pence}$$

As mentioned before, we also implement a single-winner Tullock lottery with comparable equilibrium effort. The payoff function in a standard single-winner Tullock lottery is given by

$$\pi_i = \begin{cases} \frac{e_i}{e_i + \sum_{j\neq i} e_j} v - e_i & \text{if } e_i > 0 \text{ for some } i \in \mathbf{N} \\ \\ \frac{1}{n} v & \text{otherwise} \end{cases}$$

the corresponding best-response being

$$e_i^{\text{BR}}(\mathbf{e_j})|_{\text{SW}} = \max\left\{0, \sqrt{v\sum_{j\neq i} e_j} - \sum_{j\neq i} e_j\right\}$$

Consequently, the symmetric equilibrium effort is $(n-1)v/n^2$. Note that this equals 30 pence when $n = 4$ and $v = 160$ pence. Note that the best response functions $e_i^{\text{BR}}(\mathbf{e_j})$ are different in the single-winner and the multi-winner contests, but for $n = 4$, $v_{\text{MW}} = 240$ and $v_{\text{SW}} = 160$, we have $e_i^{\text{BR}}|_{\text{SW}} = e_i^{\text{BR}}|_{\text{MW}}$, as well as $e_i^*|_{\text{SW}} = e_i^*|_{\text{MW}}$.

3.4 Experiment

3.4.1 Experimental Design

Each of the three simultaneous selection multi-winner mechanisms described in Section 3.3 have been implemented in the laboratory using a direct lottery metaphor.[10] Additionally, we implemented a standard single-winner lottery which has a comparable best-response function for individual players and predicts the same equilibrium as the multi-winner contests for the chosen parameter values.[11]

There are four participants in each contest group and they are randomly assigned ID numbers 1 to 4 in any given contest period. The participants are asked to bid integer amounts of money for lottery tickets. There are different types of lottery tickets. A participant receives a reward when the selected lottery ticket bears his or her ID number in its type name. The treatments vary in the method of selecting the winner(s) of the contest, and this is achieved by varying the procedure followed in generating the lottery tickets. The ticket generation process in each treatment reflects the contest success function underlying the corresponding mechanism.

Single Winner (SW): This treatment implements the conventional Tullock CSF using explicit lottery randomization. The number of lottery tickets assigned to each participant equals the amount of their bid. All tickets assigned to a participant is marked with his or her ID number. Then the computer picks up one ticket at random which gives the winning ID number.

Joint Selection (JS): This treatment implements the mechanism underlying Berry's contest success function. There are six types of lottery tickets - Type $1\&2$, Type $1\&3$, Type

[10]Please refer to Chapter 1 in this thesis.

[11]The single-winner contest sessions are same as the 'Ticket' treatment sessions in Chapter 1. The findings are reproduced to compare them with the multi-winner treatments.

1&4, Type 2&3, Type 2&4 and Type 3&4. The two digits in each type correspond to the participants with the same ID numbers. Number of each ticket type equals the total amount (in pence) bid by the respective pair of participants. For instance, the bids made by participant 1 and participant 2 are added together to determine the number of Type 1&2 tickets and so on. Consequently, the total number of tickets in any contest period is thrice the total amount bid by the four participants. One ticket is selected through a simple computerized lottery as in the baseline and the ticket type indicates the winning participants.

Restricted Coalition (RC): This treatment implements the combinatorial contest on a ring network (Chapter 2). There are four type of tickets - Type 1&2, Type 1&4, Type 2&3, and Type 3&4. The total number of each type of ticket is decided in the same way as in Joint Selection. The two winners are decided from the type of the drawn ticket.

Survivor Selection (SS): This treatment implements the sequential selection proposed by Chowdhury and Kim (2014). Accordingly, two phases of lottery draws are conducted within each contest period to select in the two final survivors. However, bids are made only once at the beginning of each period. For the first phase draw, there are four types of tickets - Type 1&2&3, Type 1&2&4, Type 1&3&4, and Type 2&3&4. Number of each ticket type is determined by summing the bid amounts from the three corresponding participants. For example, the total number of Type 2&3&4 is equal to the sum of the individual bids from participants 2, 3 and 4. One ticket is drawn by the computer to decide the surviving participants. Only the surviving participants' bids are considered for the following phase. Accordingly, there are three types of tickets in the second phase draw, each type corresponding to one possible pair from the three survivors. The two winners are decided from the ticket type drawn in the second phase.

Figure 3.1 gives a precise description of the treatments. The second column gives a graphical demonstration of the selection mechanisms. A dashed lines between any two players indicate a possible winning pair. In single winner, there is only one winning position and all the participants' efforts are considered individually. In Joint Selection, anybody can be a joint winner with anybody else and therefore each player is connected to all others with a dashed line. But in Restricted Coalition, only certain pairs can win the two rewards at the same time, and the dashed lines show the permitted pairing possibilities only. The first phase in the survivor selection treatment selects the three survivors by replicating the procedure in joint selection to select three players, and the second phase again selects the two final survivors from the three players chosen in the first draw.

It has been shown that the three multi-winner mechanisms, though different in the selection procedure, can be described with the same contest success function and the same expected payoff function therefore. However, this similarity may not be as straightforward to observe.

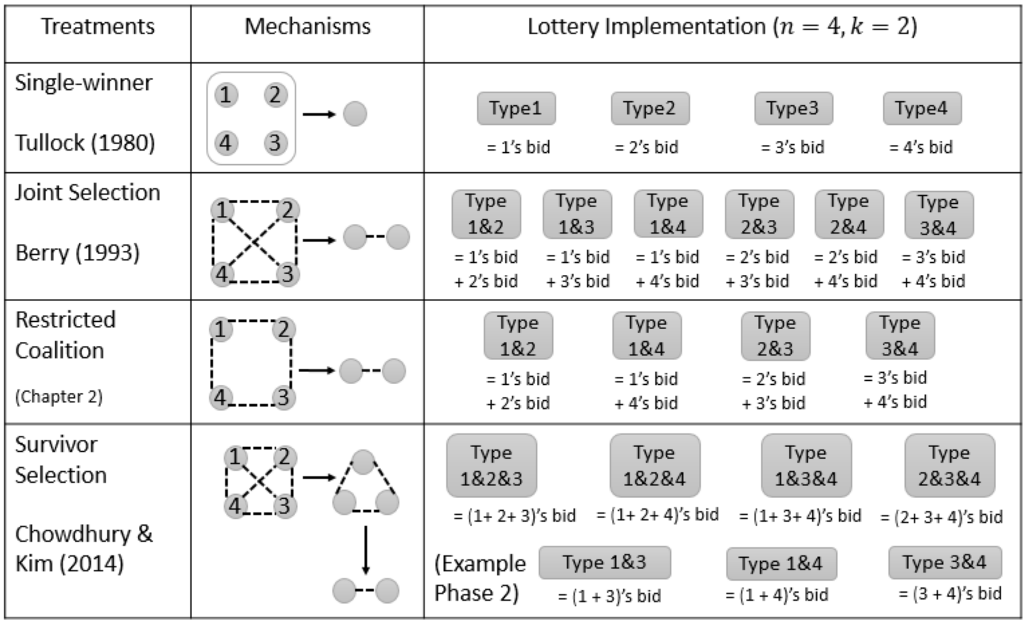


Figure 3.1: Experimental treatments and ticket generation procedures

Human subjects participating in decision-making experiments are hardly expected to go through the painstaking algebraic calculations as in Section 3.3, nor do the experimental instructions provide them with any mathematical formulation of the winning probabilities. There can be yet other differences in the behavioural implication of the three mechanisms, as is briefly discussed in Appendix A, which may result in possible deviations from the theoretical predictions.

Our main hypothesis concerns the comparability of bidding behaviour between the multi-winner mechanisms and a best-response equivalent single-winner contest. The reward value in our single-winner contest being 160, and that in all three multi-winner mechanisms being 240, the resulting best response function is unique, which is

$$e_i^* = \sqrt{160 \sum_{j \neq i} e_j} - \sum_{j \neq i} e_j$$

Accordingly, overall bidding behaviour in the single-winner contest is expected to be statistically similar to any or all of the multi-winner mechanisms.

3.4.2 Experimental Procedure

We conducted four sessions for each treatment. Sessions typically consisted of 12 participants. Any such session implemented a single treatment, which was repeated 30 times. All sessions were conducted at the Center for Behavioural and Experimental Social Science (CBESS) using student subjects of University of East Anglia. All the sessions were fully computerized and were coded and run using z-Tree (Fischbacher, 2007).

Each experimental session was divided into two parts. In Part I, the subjects were asked to perform a simple risk elicitation task (eckel and Grossman, 2008), 2008), the result of which was disclosed only after the completion of Part II. After the instructions for Part II were read out, all subjects were asked to answer a quiz (available from the author on demand) to ensure subjects' understanding of the lottery procedures. The mechanism underlying the ticket generation process was explicitly shown to the subjects in each period under each treatment (Please refer to Appendix C for an example screen-shot of the feedback table). The control quiz also ensured subjects' understanding of a sample feedback table. Individual performance history remained on the screen during decision-making, and were regularly updated.

All 12 participants in a typical session were anonymously matched into groups of four and the matching remained the same for all 30 contest periods. However, the four participants within any particular group were assigned identification numbers 1 to 4, and these identification numbers were randomly reassigned within a group at the beginning of each period in order to reduce reputation building.

In the multi-winner sessions, any participant could bid an integer amount of pence between 0 and 240. In each contest period, there were two rewards to be assigned to any two participants in each contest group under the multi-winner treatments. Each reward was worth 240 pence. However, the strategy space and the reward value were different in the single-winner sessions to keep the equilibrium value comparable with the multi-winner treatments. Participants in any of the single-winner sessions could bid any integer amount between 0 and 160, and every period one reward worth 160 pence was assigned to one member in each 4-person group.

Payments were made for randomly selected 5 periods. Single-winner sessions lasted about an hour and average payment was 9, while multi-winner sessions lasted about an hour and a half with an average payment of 17.50.

3.5 Results

Let us begin by comparing the average bids across the four treatments. Table 3.1 provides the summary statistics for individual bids in all treatments. Median bids in SW and JS treatments are similar and less than the median bids in RC and SS. Average individual bid in all three multi-winner treatments are marginally higher than the average individual bid in SW treatment. However, using the four-person groups as the unit of independent observation, a comparison based on the Mann-Whitney-Wilcoxon (MWW) rank-sum test of the average bids (over all 30 periods) do not indicate this differences to be significant.

Treatment	Mean	Median	sd	Observations
SW	40.35	30	35.96	1440
JS	43.08	30	41.90	1440
RC	42.63	36	42.66	1440
SS	43.07	40	41.98	1440

Table 3.1: Descriptive statistics on individual bids.

Result 3.1. *Average bids in the three multi-winner mechanisms are not statistically different.*

Support. The independent observations are obtained by taking the average bid from the four group members of each contest group over the course of the experiment. A k-sample median test cannot reject the null-hypothesis that the distribution of the average group bids have the same population median in all three treatments. A Kruskal-Wallis test for equality of average group bids among all three multi-winner treatments is insignificant. Also, comparison of average group bids in each pair of treatments based on Mann-Whitney-Wilcoxon rank-sum test fails to reject the null-hypothesis of equal distribution of the group means in any pair of treatments ($p = 0.81$, $r = 0.52$ for JS vis-á-vis RC, $p = 0.95$, $r = 0.50$ for JS vis-á-vis SS, and $p = 0.81$, $r = 0.47$ for RC vis-á-vis SS). The pairwise treatment comparison performed separately on average group bids for periods 1-10, periods 11-20, and periods 21-30, all support the same conclusion. □

Result 3.2. *Average bids in any of the multi-winner mechanisms is not statistically different from average bids in the best-response equivalent single-winner contest.*

Support. A Kruskal-Wallis test of the average bids at the group-level do not indicate any significant difference among the four treatments. Pairwise comparisons between each multi-winner treatment and the best-response equivalent single-winner treatment is carried out following the same statistical procedures as in Result 3.1 and no significant differences are observed. The effect size, interpreted as the probability that the average bid in a randomly selected group from one treatment will exceed the average bid in a randomly selected group from another treatment, is close to 50% for every pair of treatments. Figure 3.2 plots the interquartile range of average group bids under the four treatments. □

The above two results [12] confirm the theoretical equivalence of the three multi-winner mechanisms and their consequential equivalence to a best-response equivalent single-winner

[12] These results remain unchanged if the median bids in each group are considered as the independent observations.

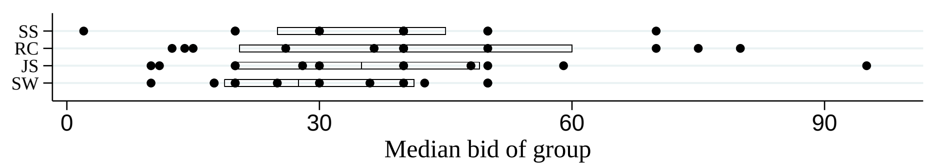


Figure 3.2: Each dot represents the median bid of one group. The quartile range of these group medians are indicated by the horizontal boxes bisected at the median.

contest. In what follows, we look at how the observed bids in different treatments compare to the theoretically predicted Nash equilibrium.

Result 3.3. *There is significant overbidding in the initial periods in all treatments, but average bids converge to the Nash equilibrium in later periods. The median group bids over the entire duration of the experiment are not significantly different from the Nash equilibrium.*

Support. The percentage of individual bids exceeding the Nash benchmark is actually higher in the SS treatment compared to all other treatments and lowest in the SW treatment. However, a large percentage of individual bids in the SS treatment are only slightly higher than the Nash benchmark, while the bids exceeding Nash in the SW treatment are more dispersed across a wider range in the strategy space. Individual bids in the first few periods are notably higher than bids in the later periods. We account for this by finding the median bid in each group in each period, and then taking the median of these medians for each group over all periods as our independent observation. As indicated in 3.2, a one-sample signed rank test cannot reject the null-hypothesis that median group bids are equal to the Nash benchmark of 30 pence when the entire duration of the experiment is considered.

Table 3.2: The reported values indicate the probability that the median bid in a randomly selected group from a treatment exceeds zero, the figures in the parentheses showing the corresponding p-values. * indicates significant difference at 5%.

	Periods 1-10	Periods 11-20	Periods 21-30	All periods
SW	0.72^* (0.01)	0.00 (1.0)	0.22 (0.43)	0.09 (0.75)
JS	0.68^* (0.02)	0.36 (0.2)	0.26 (0.36)	0.36 (0.21)
RC	0.64^* (0.02)	0.01 (0.9)	0.01 (0.96)	0.22 (0.43)
SS	0.65^* (0.02)	0.00 (1.0)	0.03 (0.91)	0.19 (0.50)

Median group bids in the first 10 periods are significantly higher than the Nash benchmark in all four treatments, but bids in later periods are not statistically different from Nash. Figure 3.3 shows this in a simple line diagram of the median of group medians in each period. Note however, that using the mean bids for independent observations will result in significant deviation from Nash for the first 10 periods as well as the whole duration, but not for the later two-third of the course of the experiment. □

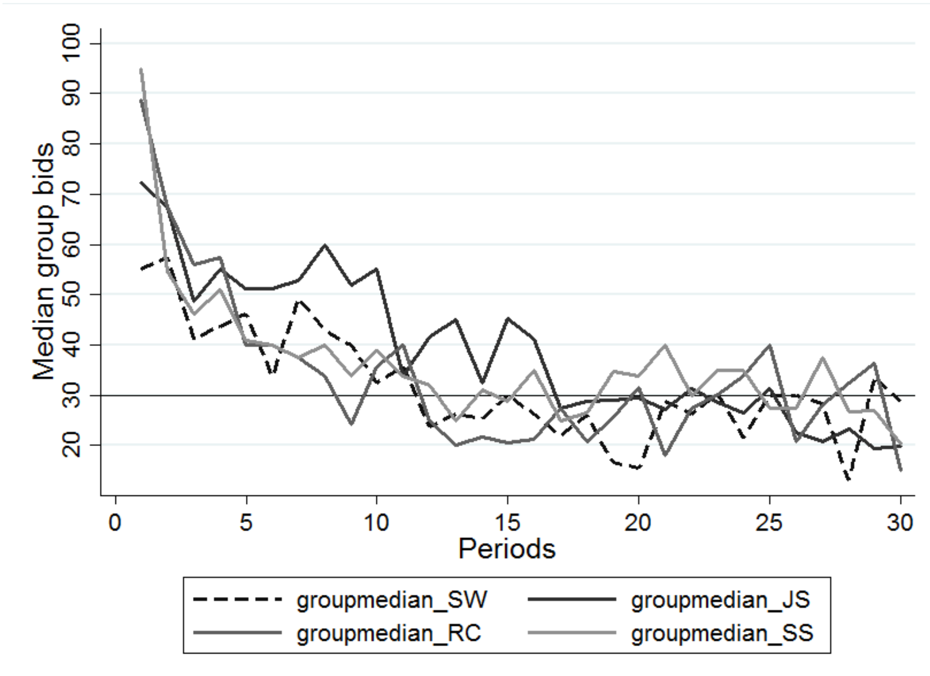
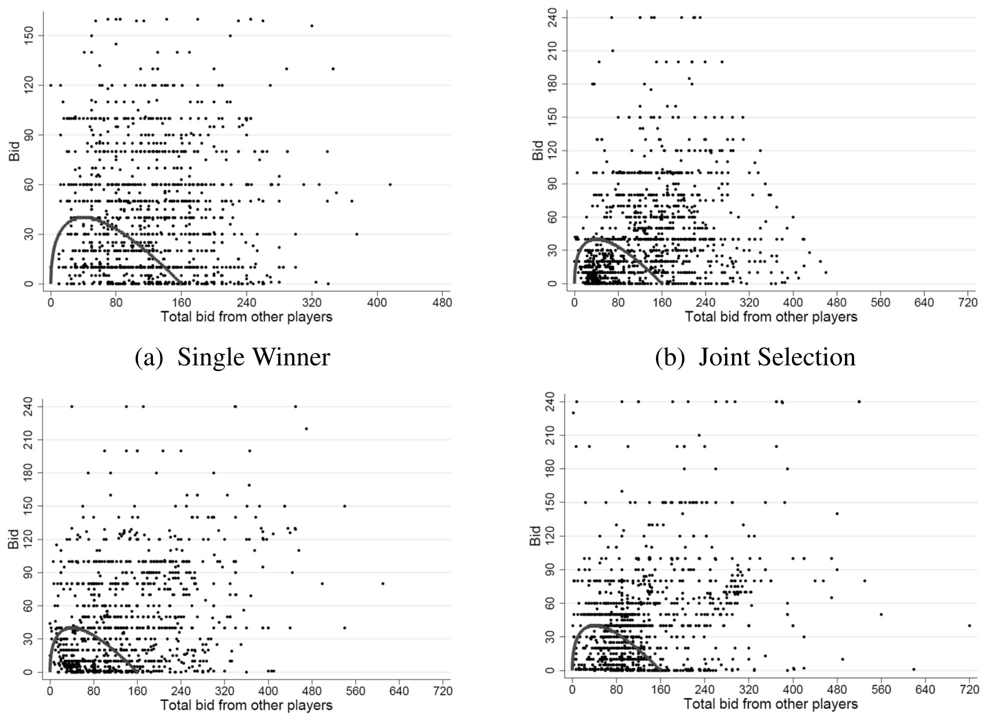


Figure 3.3: The vertical axis plots the median of the median bids in each period across all groups in a treatment. Bids start high but converges to the ex-post symmetric Nash equilibrium benchmark after the first ten periods.

However, reducing the clustered individual bid decisions into independent observations by summarizing them into mean or median may result in loss of important properties about the evolution of bidding behaviour. Also, summarizing them at the group level do not show how observed individual bids compare to the best responses. Figure 3.4 plots individual bids against total bids from the remaining players in the same group in the same period. The computed median absolute difference between bid and best-response is lowest in the SS treatment, as may also be evident from a higher concentration of bids in close proximity of the best-response function in the SS treatment. However, we fail to find any significant treatment difference in the median absolute deviations from best-responses.

The problem with comparing bids and best responses is that bids can be lower or higher than the best response. Thanks to the best response being closer to the lower bound of the strategy space at any level of total bid from other players, there are usually more bids exceeding the best response than bids lower than the best response. Further, in our experiment, the strategy space in SW being smaller than the strategy space in the multi-winner treatments, downward adjustments can be expected to be more frequent in the multi-winner treatments.



(a) Single Winner

(b) Joint Selection

(c) Restricted Coalition

(d) Survivor Selection

Figure 3.4: The y-coordinate of each dot indicates an individual bid, the x-coordinate indicating the total bid from the other players in the same group in the same period. The positive best responses are indicated with the continuous curve.

In other words, an absolute distance between bid and best response would fail to take care of the higher scope of variation in bids in the multi-winner treatments resulting from a wider strategy space. Thus, by looking at the absolute distance between bid and best-response, one would run the risk of overestimating learning under the multi-winner mechanisms. In order to avoid accounting for such directional adjustments and to balance for the differences in the strategy space, we consider deviations on the expected payoff space.

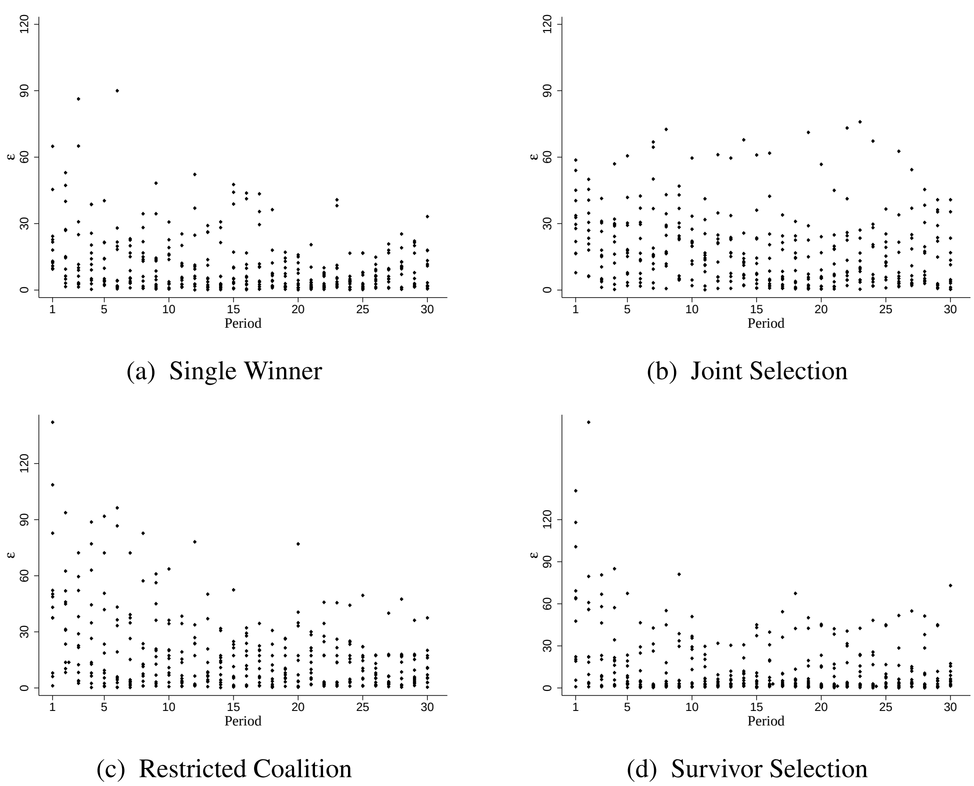
The expected payoff corresponding to the best-response is higher than the expected payoff corresponding to any other bid. If π_{it}^{e*} indicate the expected payoff from best-response and π_{it}^{e} stands for the expected payoff from the actual bid, then $\varepsilon = \pi_{it}^{e*} - \pi_{it}^{e}$ is non-negative by definition. We use this measure to analyse the period by period adjustment in bidding. Note that,

$$\pi_{it}^{e*} - \pi_{it}^{e} = \frac{(b_{it}^{*} - b_{it})^2}{b_{it} + \sum_{j \neq i} b_{jt}}$$

where b_{it} is the observed bid of subject i in period t, and following the notation and definition used in Chapter 1, b_{it}^{*} is the discretised best-response of player i in period t given other players' bids in period t, i.e. $b_{it}^{*} = \{\lceil \tilde{b}_{it}^{*} \rceil, \lfloor \tilde{b}_{it}^{*} \rfloor\}$ where $\tilde{b}_{it}^{*}$ is the true best-response if continuous bidding were allowed.

This measure has the advantage of looking at the distance between bid and best-response as a proportion of the total bids in the group and could therefore be considered as a more standardized measure of disequilibrium. This is conventionally denoted as ε-equilibrium (Radner, 2005). We adopt this measure from Chapter 1, where the maximum deviation between expected best-response payoff and expected payoff resulting from the actual observed bid in each group has been used to construct a group-level measure of disequilibrium. However, as the maximum deviation is more susceptible to one player in the group making a large mistake in one period, especially given the asymmetry in strategy spaces across treatments, we consider the median deviation in each group as a group-level disequilibrium measure for the present analysis. That is, for a group g in session s of treatment c, the disequilibrium measure in period t is given by $\varepsilon_{csgt} = \text{median}_{i \in g} \{\pi_{it}^{e*} - \pi_{it}^{e}\}$.

Figure 3.5 shows the distribution of this disequilibrium measure for all groups in the four treatments. As is evident from the four plots in figure 3.5, distribution of ε_{csgt} is more or



(a) Single Winner

(b) Joint Selection

(c) Restricted Coalition

(d) Survivor Selection

Figure 3.5: Ex-post measure of disequilibrium within groups (ε_{csgt}), by period. Each dot indicates the value of the measure for one group in the corresponding period.

less similar among SW, JS and RC treatments, while in SS, ε_{csgt} approaches zero value for a higher number of groups in all periods. As argued in Chapter 1, a lower value of ε_{csg} in period t could be attributable to a lower value of ε_{csg} in period $t-1$, as there is less room for adjustment. However, that doesn't look like the scenario here. A closer look at the plots in figure 3.5 reveals that the values of ε_{csgt} is rather higher in SS and RC compared to SW and JS in the beginning periods. To formalize this result, we also run a random effect panel regression with $\varepsilon_{csg(t+1)}$ as the dependent variable and ε_{csgt} as an independent variable. As high bids are more frequent in the first few periods and the varying distribution of the beginning bids across the four treatments can possibly overstate the differences in disequilibrium measure, we drop the first 10 periods from the regression.

Result 3.4. *Convergence towards best response, as measured by ε_{csgt}, is significantly more frequent in the survivor selection treatment than in any other treatment.*

Dependent variable $\varepsilon_{csg(t+1)}$	Coefficient	Standard error	95% CI interval
Intercept	6.19^{***}	0.93	$[4.36, 8.01]$
ε_{csgt}	0.29^{***}	0.09	$[0.12, 0.47]$
JS	-1.65	1.82	$[-5.23, 1.92]$
RC	0.09	1.16	$[-2.18, 2.38]$
SS	-4.07^{***}	1.48	$[-6.97, -1.16]$
ε_{csgt}# JS	0.39^{***}	0.14	$[0.11, 0.67]$
ε_{csgt}# RC	0.22^{***}	0.09	$[0.04, 0.40]$
ε_{csgt}# SS	0.52^{***}	0.11	$[0.29, 0.74]$

Table 3.3: Random effect panel regression of disequilibrium measure over time for the last 20 periods. Random errors adjusted at the session level. *** denotes significance at 1%, ** at 5% and * at 10%.

Proof. The regression in Table 3.3 shows that the disequilibrium measure in period $t+1$ for a given level of disequilibrium measure in period t is significantly lower in the survivor selection treatment than in the single-winner. Note that the coefficient for ε_{csgt} is positive in the SW treatment. While the very small value of the coefficient may be interpreted as groups in the SW treatment moving somewhat away from equilibrium, the coefficient of ε_{csgt} remains largely negative in the SS treatment even after accounting for the net effect. One may conclude that the SS treatment results in faster convergence towards equilibrium compared to the SW treatment. Note however, that dropping the SW observations and repeating the regression with one of the multi-winner treatments as the baseline does not exhibit any significant evidence of faster convergence. □

We consider the previous period contest outcome to understand the possible channels through which the SS treatment may induce a faster convergence towards equilibrium. First, for each period t, we recorded $\varepsilon_{i,t}$ and $\varepsilon_{i,t+1}$ for each subject i. Then we identified the winners and losers in every period t, and constructed a separate dataset for all winning outcomes and another one consisting of all defeat outcomes.[13] We also record each subjects risk preference, further filter the observations according to the participants' risk-attitudes, and finally take the median ε_t and ε_{t+1} from each group in the reduced dataset and perform the regression as in result 3.4. Unlike in the result 3.4 regression, the ε_{csgt} here is the median disequilibrium measure from the similar type of players, and $\varepsilon_{csg(t+1)}$ is the median disequilibrium measure from among those exact players in the following period. This means in each period in each group we are considering people having similar risk-preference and having faced similar contest outcome.

A cross-treatment comparison of ε_{csgt} conditional upon losing in the previous period shows that both previous period winners and losers further diverge from their best-response payoffs in the SW treatment. We repeat the analysis separately for subset of participants winning and losing in period t and find that faster convergence in SS treatment holds for both losers and winners, but more pronounced for the former. We repeat this analysis after classifying the previous period winners and losers according to their risk preferences, the findings for which are summarized in Table 3.4. The number of risk-averse (risk-prone) subjects is 30 (18) in SW, 27 (21) in JS, 25 (23) in RC, and 21 (27) in SS. A first look at this table may instantly suggest that faster convergence in the SS treatment is largely brought about by the relatively risk averse losers in the previous period. In interpreting this finding one should also note that the total number of losers in any period is higher than the total number of winners as there are always three losers in the SW treatment. However, the SS treatment is the only treatment where the number of relatively risk averse people fell short of the number of relatively risk-prone people which supports this interpretation.

	Previous period losers	Previous period winners	Any outcome
Risk averse	Faster convergence SS (no. observations = 1,170)	No treatment effect (no. observations = 1,049)	Faster convergence SS (no.observations = 1,380)
Risk prone	No treatment effect (no. observations = 1,079)	No treatment effect (no. observations = 927)	No treatment effect (no.observations = 1,302)
Any type	Faster convergence SS (no. observations = 1,440)	Faster convergence SS (no. observations = 1.438)	Faster convergence SS (no. observations = 1,440)

Table 3.4: Validity of Result 3.4 for different classification of people according to their risk attitude and previous period outcome.

[13] Thus, if participant i is a winner in period t, a loser in period $t+1$, and again wins in period $t+2$, then the winner dataset contains observations from participant i in periods t and $t+2$, but not for period $t+1$ which gets recorded in the loser dataset.

3.6 Discussion

We compared contest behaviour in three multi-winner mechanisms that are different from each other in terms of the execution structure but could be described by the same mathematical success probability. These three mechanisms are then compared with a standard single winner contest that give rise to a different success probability but the same best responses due to some manipulation in the reward value. The four treatments generate the same average contest expenditure which steadily converges to the equilibrium over time. The only significant treatment effect observed is that of a faster adjustment towards best responses in the survivor selection treatment. Probing further we find that the previous period losers are significantly closer to their respective best responses in the SS and the SW treatment than in the other two treatments. No such treatment difference was found among the winners. The winners however, converge faster to their respective best-responses in the SS treatment only.

The two important findings from this experiment are the fairly quick convergence of average bids to the equilibrium prediction and that of the theoretical equivalence between procedurally different mechanisms being imported into the lab. The first finding is in contrary with many other contest experiments that report significant overbidding in the lab. We claim that this is attributable to our typical framing of the game as a virtual lottery (as evident from Chapter 1). However, the fact remains that subjects do not differ significantly from the equilibrium even in the relatively complicated multi-winner mechanisms. The second finding is the main contribution of this study that has potential implications for contest designing, as mentioned in Section 3.1.

This is the first study to our knowledge that explicitly looks at alternative mechanisms in multi-winner contests. The experimental support of the theoretical conclusion of strategic equivalence indicates behavioural consistency of a purely mathematical result, which may not be so obvious for boundedly rational subjects. Wu et al. (2009) finds discrepancy in rent-seeking behaviour in a lottery presented in motor-form that is mathematically equivalent to a classical economic lottery. Masatlioglu et al. (2012) also reject outcome equivalence in strategically equivalent auction mechanisms. Experimental evidences found by Lucking-Reiley (2004) and a field experiment by Hossain and Morgan (2003) also fails to support the theoretical conclusion of revenue equivalence among auction mechanisms. Our study, on the other hand, indicate that subjects exhibit similar responses to explicitly different selection procedures as long as the underlying success probabilities are similar. This establishes the behavioural robustness of Berry's contest success function.

Even though the three multi-winner contests could be described by the same normal form game, the single winner Tullock contest is a different game, but generates the same best-response function due to manipulation in the parameter value. The statistical similarity in subjects' responses to all these mechanisms points towards the strength of the best-response function in predicting behaviour. Our results thus also support Grimm and Mengel (2012)s' argument that strategically equivalent games are played in the same way and that "behaviour can be explained using the tools and languages of game theory", indicating greater reliability of strategic equivalence or best-response equivalence in contest mechanisms.

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Appendix

3.A Possible deviation from the theoretical predictions.

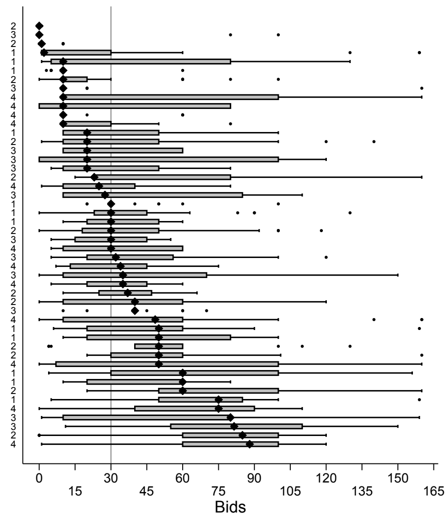
The sequential elimination in the survivor selection treatment may affect bidding behaviour through difference in perceived importance of the order of elimination for example. If players derive different utilities from losing in the first selection and second selection, then the best response function will look like

$$e_i^*|_{ss} = \sqrt{\frac{1}{3}\left(2v - L_1 - L_2\right)\sum_{j\neq i} e_j} - \sum_{j\neq i} e_j$$

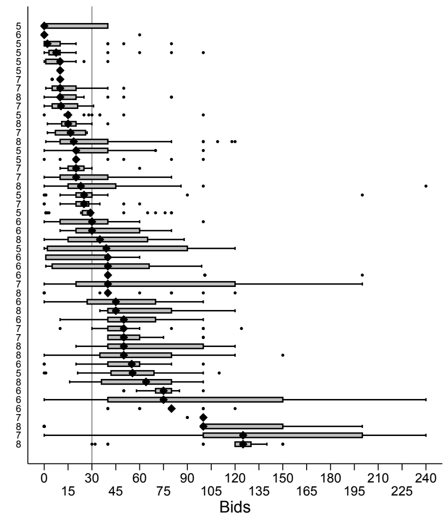
where L_1 and L_2 are the utilities for losing in the first selection and second selection respectively. One can assume L_1 to be zero, that is being the first loser is comparable to simply losing the contest under a purely simultaneous mechanism. Being the second loser however, may make the subject feel more disappointed in which case L_2 will be negative and the best-response bid higher. On the other hand, L_2 could be positive if being the second loser raises hope about the winning potential and the best-response in that case would be lower. Further, L_1 and L_2 being strictly private information, one can expect to see higher variability in bids across subjects.

We look at the variability of bids in the four treatments in order to find evidence of heterogeneous private values in the SS treatment. We use an across-subject measure of variability as defined in Chowdhury et al. (2014). First we take the median bids of each subject participating in a session. The standard deviation of this individual medians constitute the measure of across-subject variability in that session. MWW test on each pair of treatment indicates that across-subject variability is similar in all four treatments. The similarity in across-subject variability is also apparent from Figure 3.A1.

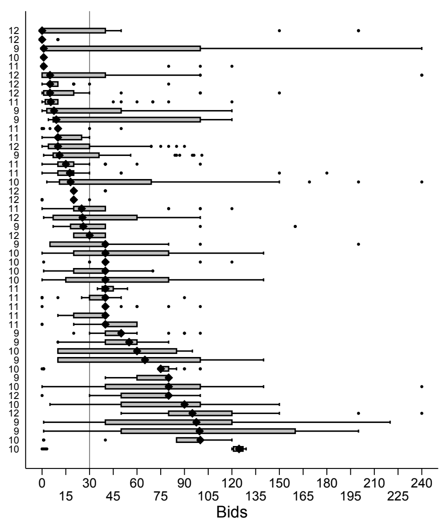
A random effect panel regression of individual bids in period t on total bid from other members of the group in period $t-1$, as presented in Table 3.A1, indicates others contestants' bids to be highly significant in both RC and SS treatments. However, the coefficients for both



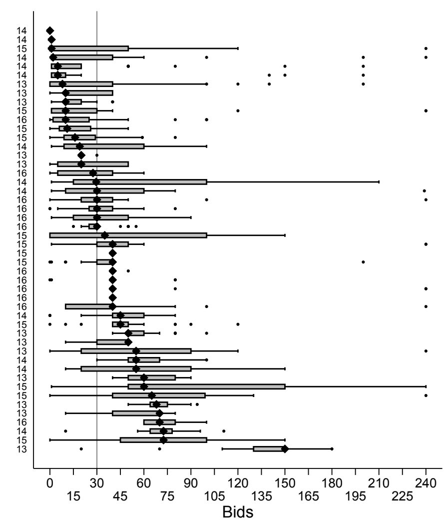
(a) Single Winner



(b) Joint Selection



(c) Restricted Coalition



(d) Survivor Selection

Figure 3.A1: The horizontal boxplots show the interquartile range of bids for an individual subject. The small black diamonds indicate the median bids from individuals. The distribution of these medians across all subjects in a treatment gives a graphical measure of across-subject variability.

RC and SS being similar, we may not infer a higher emphasis on other contestant' bid in one of the two treatments. In an alternative regression model we included risk attitudes as an independent variable but didn't find any significant effect.

Bid in Period t	Coefficient	Standard error	[95% CI]
Intercept	33.99^{***}	3.75	$[26.6, 41.3]$
$\sum_j b_{j,t-1}$	0.05	0.03	$[-0.00, 0.1]$
JS	-8.66	7.57	$[-23.49, 6.17]$
RC	-13.26^{**}	4.76	$[-22.59, -3.9]$
SS	-13.64^{***}	4.75	$[-22.95, -4.3]$
$\sum_j b_{j,t-1}$*JS	0.08^{**}	0.04	$[0.00, 0.16]$
$\sum_j b_{j,t-1}$*RC	0.11^{***}	0.03	$[0.05, 0.17]$
$\sum_j b_{j,t-1}$*SS	0.11^{***}	0.03	$[0.05, 0.18]$

Table 3.A1: Random effect panel regression on other players' bids in previous period. Standard errors clustered at the session level. *** indicates significance at 1% and ** at 5%.

In the RC treatment, a given player i always faces one player who can never be a joint winner. Increase in the latter's bid decreases i's winning probability more than how much it would fall due to increase in any of the other two players' efforts. That is, if player i can not win jointly with another player s, then $\left|\frac{\partial p_i}{\partial e_s}\right| \geq \left|\frac{\partial p_i}{\partial e_j}\right|$ where $j, s \in \mathbf{N}$ and $j \neq i,\ s \neq i$ Let us call player s as player i's strict opponent. With fixed circular arrangement, increase in strict opponent's effort will have a different effect on best response than an increase in another player's effort. In particular, $\left|\frac{\partial e_i^{\mathrm{BR}}}{\partial e_j}\right| < \left|\frac{\partial e_i^{\mathrm{BR}}}{\partial e_s}\right|$ whaere s is the strict opponent of i and $j \neq t$ In our experiment however, the identity of the strict opponent changes every period and renders the above two inequalities irrelevant for next period decision-making. So, increase in another player's bid should always have the same marginal effect on own bid regardless of the player's identity.

Nevertheless, we consider our conjecture about reaction to strict opponents' bids in order to examine the significantly negative coefficient of previous period bid from other players in RC. For each participant i in period t, we take the average bid from the two other participants $i+1$ and $i-1$, who could be joint winners with participant i. Then we regress player i's bid in period $t+1$ on this average bid from player $i-1$ and $i+1$ in period t and the strict opponent's (denoted by s) bid in period t (Table 3.A2). Both the coefficients are significant and positive, with very low values. However, coefficients are not significantly different from each other (the coefficient for the average bid from previous period's potential joint winners is higher at 10% level of significance), which could be seen as subjects' correct understanding of the mechanism under random reassignment of id numbers.

$b_{i,t+1}$	Coefficient	Standard error	[95% CI]
Intercept	20.7***	4.97	$[10.9, 30.4]$
$(b_{i-1,t} + b_{i+1,t})$	0.3***	0.06	$[0.17, 0.42]$
$b_{s,t}$	0.17***	0.03	$[0.10, 0.24]$

Table 3.A2: Random effect panel regression of bids in the RC treatment on strict opponent's bid and average bid from potential joint winners in the previous period. Standard errors clustered at group level. *** indicates significance at 1%.

3.B Instructions

General Instructions

Welcome! You are about to participate in an experiment in the economics of decision-making.

If you follow the instructions and make appropriate decisions, you can earn an appreciable amount of money. At the end of todays session you will be paid in private and in cash.

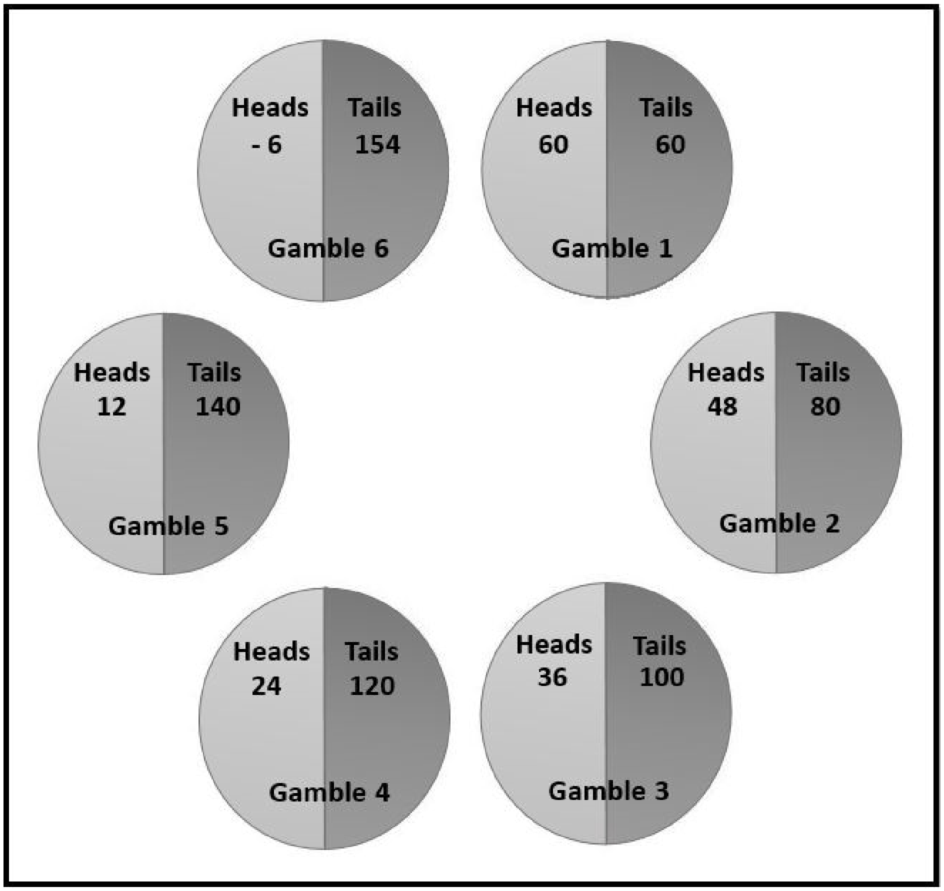
It is important that you remain silent and do not look at other peoples work. If you have any questions, or need assistance of any kind, please raise your hand and an experimenter will come to you. If you talk, laugh, exclaim out loud, etc., you will be asked to leave and you will not be paid. We expect and appreciate your cooperation.

Todays session consists of two parts. The decisions you make in the two parts are completely unrelated to each other. Your earnings for the session will be the total of your earnings from the two parts.

Baseline Instructions

Part I Instructions

In this task, you will be asked to choose from six different gambles (as shown below). Each circle represents a different gamble from which you must choose the one that you prefer. Each circle is divided into two halves. The two halves are marked with different shades, each showing one possible earning (in pence) from the respective gamble. The two different shades represent the two possible outcomes of a coin toss, the light grey standing for a Head and the dark grey standing for a Tail. The experimenter will toss a coin at the end of todays experiment. If the result is a Head, each of you will receive the amount of pence written in the light grey part of your chosen circle. Alternatively, if the result is a Tail, each of you will receive the amount written in the dark grey part of your chosen circle. Note that no matter which gamble you pick, you have a 50-50 chance of earning either of the two amounts written on that circle.



Please select the gamble of your choice by clicking one of the Select this buttons that will appear on each circle in the picture. Once you have made your choice, please click on the Confirm button at the bottom of the screen. We will move to part 2 after all the participants have made their choices. Your earning from this session will be decided only after part 2 is finished.

Part II Instructions

Part 2 of the session consists of 30 decision-making periods. At the conclusion of Part 2, any 5 of the 30 periods will be chosen at random, and your earnings from this part of the experiment will be calculated as the sum of your earnings from those 5 selected periods.

At the beginning of Part 2, you will be randomly and anonymously placed into a group of 4 participants. Within each group, one participant will have ID number 1, one ID number 2, one ID number 3, and one ID number 4. The composition of your group remains the same for all 30 periods but the individual ID numbers within a group are randomly reassigned in every period.

In each period, you may bid for a reward worth 160 pence. In your group, one of the four participants will receive a reward. You begin each period with an endowment of 160 pence. You may bid any whole number of pence from 0 to 160; fractions or decimals may not be used.

If you receive a reward in a period, your earnings will be calculated as:

Your payoff in pence = your endowment your bid + the reward.

That is,

Your payoff in pence = 160 your bid + 160.

If you do not receive a reward in a period, your earnings will be calculated as:

Your payoff in pence = your endowment your bid

That is,

Your payoff in pence = your endowment your bid

The chance that you receive a reward in a period depends on how much you bid, and also how much the other participants in your group bid. At the start of each period, all four participants of each group will decide how much to bid. Once the bids are determined, a computerised lottery will be conducted to determine which participant in the group will receive the reward. In this lottery draw, there are four types of tickets: Type 1, Type 2, Type 3 and Type 4. Each type of ticket corresponds to the participant who will receive the reward if a ticket of that type is drawn. So, if a Type 1 ticket is drawn, then participant 1 will receive the reward; if a Type 2 ticket is drawn, then participant 2 will receive the reward; and so on.
Number of each type of ticket depends on the bids of the corresponding participant:
Number of Type 1 tickets = Bid of participant 1
Number of Type 2 tickets = Bid of participant 2
Number of Type 3 tickets = Bid of participant 3
Number of Type 4 tickets = Bid of participant 4
Each ticket is equally likely to be drawn by the computer. If the ticket type that is drawn has your ID number, then you will receive a reward for that period.
We will now work through an example of how the numbers of lottery tickets are computed, and what you will see during a typical period of the session.

An example

Suppose participant 1 bids 80 pence, participant 2 bids 6 pence, participant 3 bids 124 pence, and participant 4 bids 45 pence. Then:
Number of Type 1 tickets = Bid of participant 1 = 80
Number of Type 2 tickets = Bid of participant 2 = 6
Number of Type 3 tickets = Bid of participant 3 = 124
Number of Type 4 tickets = Bid of participant 4 = 45

There will therefore be a total of 80 + 6 + 124 + 45 = 255 tickets in the lottery. Each ticket is equally likely to be selected. In each period, the calculations above will be summarised for you on your screen, using a table like the one in this screenshot:

Participant ID	Bid	Ticket Type	Total tickets	Ticket number(s)
Participant 1	80	Type 1	80	1 - 80
Participant 2	6	Type 2	6	81 - 86
Participant 3	124	Type 3	124	87 - 210
Participant 4	45	Type 4	45	211 - 255

Interpretation of the table: The horizontal rows in the above table contain the ID numbers of the four participants in every period. The vertical columns list the participants bids, the corresponding ticket types, total number of each type of ticket (second column from right) and the range of ticket numbers for each type of ticket (last column). Note that the total number of each ticket type is exactly same as the corresponding participants bid. For example, the total number of Type 1 tickets is equal to Participant 1s bid.

The last column gives the range of ticket numbers for each ticket type. Any ticket number that lies within that range is a ticket of the corresponding type. That is, all the ticket numbers from 81 to 86 are tickets of Type 2, which implies a total of 6 tickets of Type 2, as appears from the Total Tickets column. In case a participant bids zero, there will be no ticket that contains his or her ID number. In such a case, the last column will show No tickets for that particular ticket type.

The computer then selects one ticket at random. The number and the type of the drawn ticket will appear below the table. The ID number on the ticket type indicate the participant receiving the reward.

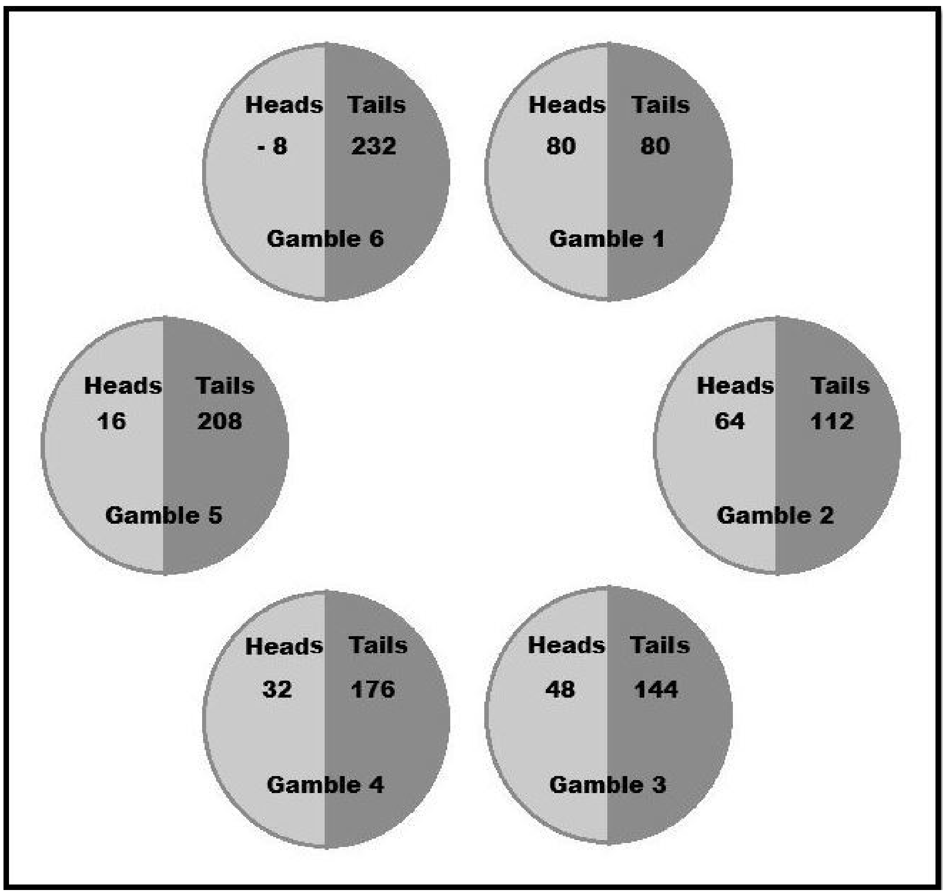
At the end of 30 periods, the experimenter will approach a random participant and will ask him/her to pick up five balls from a sack containing 30 balls numbered from 1 to 30. The numbers on those five balls will indicate the 5 periods, for which you will be paid in Part 2. Your earnings from all the preceding periods will be throughout present on your screen.

Earnings from Part 1 of the experiment will be decided after that. Your earnings from both parts of this experiment will be summarized on your screen.

Multi-winner Instructions

Part I Instructions

In this task, you will be asked to choose from six different gambles (as shown below). Each circle represents a different gamble from which you must choose the one that you prefer. Each circle is divided into two halves. The two halves are marked with different shades, each showing one possible earning (in pence) from the respective gamble. The two different shades represent the two possible outcomes of a coin toss, the light grey standing for a Head and the dark grey standing for a Tail. The experimenter will toss a coin at the end of todays



experiment. If the result is a Head, each of you will receive the amount of pence written in the light grey part of your chosen circle. Alternatively, if the result is a Tail, each of you will receive the amount written in the dark grey part of your chosen circle. Note that no matter which gamble you pick, you have a 50-50 chance of earning either of the two amounts written on that circle.

Please select the gamble of your choice by clicking one of the Select this buttons that will appear on each circle in the picture. Once you have made your choice, please click on the Confirm button at the bottom of the screen. We will move to part 2 after all the participants have made their choices. Your earning from this session will be decided only after part 2 is finished.

Part II Instructions

Part 2 of the session consists of 30 decision-making periods. At the conclusion of Part 2, any 5 of the 30 periods will be chosen at random, and your earnings from this part of the experiment will be calculated as the sum of your earnings from those 5 selected periods.

At the beginning of Part 2, you will be randomly and anonymously placed into a group of 4 participants. Within each group, one participant will have ID number 1, one ID number 2, one ID number 3, and one ID number 4. The composition of your group remains the same for all 30 periods but the individual ID numbers within a group are randomly reassigned in every period.

In each period, you may bid for a reward worth 240 pence. In your group, two of the four participants will receive a reward. You begin each period with an endowment of 240 pence. You may bid any whole number of pence from 0 to 240; fractions or decimals may not be used.

If you receive a reward in a period, your earnings will be calculated as:

Your payoff in pence = your endowment your bid + the reward.

That is,

Your payoff in pence = 240 your bid + 240.

If you do not receive a reward in a period, your earnings will be calculated as:

Your payoff in pence = your endowment your bid

That is,

Your payoff in pence = your endowment your bid

The chance that you receive a reward in a period depends on how much you bid, and also how much the other participants in your group bid. At the start of each period, all four participants of each group will decide how much to bid. Once the bids are determined, a computerised lottery will be conducted to determine which two participants in the group will receive the rewards.

{*The Part II instructions for all the treatments are the same until this part. Since, the treatments vary only in terms of the draw procedure, the treatment-specific instructions are as below*}

(Baseline: Joint Selection)

In this lottery draw, there are six types of tickets: Type 1&2, Type 1&3, Type 1&4, Type 2&3, Type 2&4, and Type 3&4. Each type of ticket corresponds to the two participants who will receive the rewards if a ticket of that type is drawn. So, if a Type 1&2 ticket is drawn, then participants 1 and 2 will receive the rewards; if a Type 1&3 ticket is drawn, then participants 1 and 3 will receive the rewards; and so on.

The number of tickets of each type depends on the bids of the corresponding two participants:

Number of Type 1&2 tickets = Bid of participant 1 + Bid of participant 2

Number of Type 1&3 tickets = Bid of participant 1 + Bid of participant 3

Number of Type 1&4 tickets = Bid of participant 1 + Bid of participant 4

Number of Type 2&3 tickets = Bid of participant 2 + Bid of participant 3

Number of Type 2&4 tickets = Bid of participant 2 + Bid of participant 4

Number of Type 3&4 tickets = Bid of participant 3 + Bid of participant 4

Each ticket is equally likely to be drawn by the computer. If the ticket type that is drawn includes your ID number, then you will receive a reward for that period.

We will now work through an example of how the numbers of lottery tickets are computed, and what you will see during a typical period of the session.

An example

Suppose participant 1 bids 80 pence, participant 2 bids 6 pence, participant 3 bids 124 pence, and participant 4 bids 45 pence. Then:

Number of Type 1&2 tickets = Bid of participant 1 + Bid of participant 2 = 80 + 6 = 86

Number of Type 1&3 tickets = Bid of participant 1 + Bid of participant 3 = 80 + 124 = 204

Number of Type 1&4 tickets = Bid of participant 1 + Bid of participant 4 = 80 + 45 = 125

Number of Type 2&3 tickets = Bid of participant 2 + Bid of participant 3 = 6 + 124 = 130

Number of Type 2&4 tickets = Bid of participant 2 + Bid of participant 4 = 6 + 45 = 51

Number of Type 3&4 tickets = Bid of participant 3 + Bid of participant 4 = 124 + 45 = 169

There will therefore be a total of 86 + 204 + 125 + 130 + 51 + 169 = 765 tickets in the lottery. Each ticket is equally likely to be selected. In each period, the calculations above will be summarised for you on your screen, using a table like the one in the following screenshot.

Interpretation of the table: The horizontal rows in the above table show the different types of lottery tickets that are generated by the computer in every period. The vertical columns list the participants bids, total number of each type of ticket (second column from right) and the range of ticket numbers for each type of ticket (last column). Note that the total number of each ticket type is the sum of the two corresponding participants bids. For example, total number of Type 1&2 tickets is the sum total of Participant 1s bid and participant 2s bid. Therefore, the table cell corresponding to Type 1&2 and Participant 4s bid is kept blank,

Ticket Types	Participant 1's bid	Participant 2's bid	Participant 3's bid	Participant 4's bid	Total tickets	Ticket number(s)
Type 1&2	80	6	_	_	86	1 - 86
Type 1&3	80	_	124	_	204	87 - 290
Type 1&4	80	_	_	45	125	291 - 415
Type 2&3	_	6	124	_	130	416 - 545
Type 2&4	_	6	_	45	51	546 - 596
Type 3&4	_	_	124	45	169	597 - 765

and so is the table cell corresponding to Type 1&2 and Participant 3s bid. Similarly, the table cell corresponding to Type 2&3 and Participant 1s bid is kept blank, and so is the one corresponding to Type 2&3 and Participant 4s bid.
The last column gives the range of ticket numbers for each ticket type. Any ticket number that lies within that range is a ticket of the corresponding type. That is, all the ticket numbers from 87 to 290 are tickets of Type 1&3, which implies a total of 204 tickets of Type 1&3, as appears from the Total Tickets column. In case any three participants all bid zero, there will be no ticket that contains those three ID numbers together. In such a case, the last column will show No tickets for that particular ticket type.

The computer then selects one ticket at random. The number and the type of the drawn ticket will appear below the table. The two ID numbers on the ticket type indicate the two participants receiving the rewards.

(Treatment I: Restricted Coalition)

In this lottery draw, there are four types of tickets: Type 1&2, Type 2&3, Type 3&4, and Type 1&4. Each type of ticket corresponds to the two participants who will receive the rewards if a ticket of that type is drawn. So, if a Type 1&2 ticket is drawn, then participants 1 and 2 will receive the rewards; if a Type 2&3 ticket is drawn, then participants 2 and 3 will receive the rewards; and so on.

The number of tickets of each type depends on the bids of the corresponding two participants:
Number of Type 1&2 tickets = Bid of participant 1 + Bid of participant 2
Number of Type 2&3 tickets = Bid of participant 2 + Bid of participant 3

Number of Type 3&4 tickets = Bid of participant 3 + Bid of participant 4
Number of Type 1&4 tickets = Bid of participant 1 + Bid of participant 4

Each ticket is equally likely to be drawn by the computer. If the ticket type that is drawn includes your ID number, then you will receive a reward for that round.

We will now work through an example of how the numbers of lottery tickets are computed, and what you will see during a typical period of the session.

An example

Suppose participant 1 bids 80 pence, participant 2 bids 6 pence, participant 3 bids 124 pence, and participant 4 bids 45 pence. Then:

Number of Type 1&2 tickets = Bid of participant 1 + Bid of participant 2 = 80 + 6 = 86
Number of Type 2&3 tickets = Bid of participant 2 + Bid of participant 3 = 6 + 124 = 130
Number of Type 3&4 tickets = Bid of participant 3 + Bid of participant 4 = 124 + 45 = 169
Number of Type 1&4 tickets = Bid of participant 1 + Bid of participant 4 = 80 + 45 = 125

There will therefore be a total of 86 + 130 + 169 + 125 = 510 tickets in the lottery. Each ticket is equally likely to be selected.

In each period, the calculations above will be summarised for you on your screen, using a table like the one in the following screenshot.

Ticket Types	Participant 1's bid	Participant 2's bid	Participant 3's bid	Participant 4's bid	Total tickets	Ticket numbers
Type 1&2	80	6	—	—	86	1 - 86
Type 2&3	—	6	124	—	130	87 - 216
Type 3&4	—	—	124	45	169	217 - 385
Type 1&4	80	—	—	45	125	386 - 510

Interpretation of the table: The horizontal rows in the above table show the different types of lottery tickets that are generated by the computer in every period. The vertical columns list the participants bids, total number of each type of ticket (second column from right) and the range of ticket numbers for each type of ticket (last column). Note that the total number of each ticket type is the sum of the two corresponding participants bids. For example, total number of Type 1&2 tickets is the sum total of Participant 1s bid and participant 2s bid. Therefore, the table cell corresponding to Type 1&2 and Participant 4s bid is kept blank, and so is the table cell corresponding to Type 1&2 and Participant 3s bid. Similarly, the table cell corresponding to Type 2&3 and Participant 1s bid is kept blank, and so is the one corresponding to Type 2&3 and Participant 4s bid.

The last column gives the range of ticket numbers for each ticket type. Any ticket number that lies within that range is a ticket of the corresponding type. That is, all the ticket numbers from 87 to 216 are tickets of Type 2&3, which implies a total of 130 tickets of Type 2&3, as appears from the Total Tickets column. In case any three participants all bid zero, there will be no ticket that contains those two ID numbers together. In such a case, the last column will show No tickets for that particular ticket type.

The computer then selects one ticket at random. The number and the type of the drawn ticket will appear below the table. The two ID numbers on the ticket type indicate the two participants receiving the rewards.

(Treatment II: Survivor Selection)

This lottery will be conducted in two phases. In the first phase, there are four types of tickets: Type 1&2&3, Type 1&2&4, Type 1&3&4, and Type 2&3&4. Each type of ticket corresponds to the three participants who will continue on to the second phase if a ticket of that type is drawn. So, if a Type 1&2&3 ticket is drawn, then participants 1, 2, and 3 will continue to the second phase; if a Type 1&3&4 ticket is drawn, then participants 1, 3, and 4 will continue to the second phase; and so on.

The number of tickets of each type depends on the bids of the corresponding three participants:

Number of Type 1&2&3 tickets = Bid of participant 1 + Bid of participant 2 + Bid of participant 3

Number of Type 1&2&4 tickets = Bid of participant 1 + Bid of participant 2 + Bid of participant 4

Number of Type 1&3&4 tickets = Bid of participant 1 + Bid of participant 3 + Bid of participant 4

Number of Type 2&3&4 tickets = Bid of participant 2 + Bid of participant 3 + Bid of participant 4

Each ticket is equally likely to be drawn by the computer. If the ticket type that is drawn includes your ID number, then you will continue to the second phase.

In the second phase, there are three types of tickets. The types of tickets depend on which three participants have continued on to the second phase:

If Participants 1, 2, and 3 have continued, then the types will be Type 1&2, Type 1&3, and

Type 2&3;
If Participants 1, 2, and 4 have continued, then the types will be Type 1&2, Type 1&4, and Type 2&4;
If Participants 1, 3, and 4 have continued, then the types will be Type 1&3, Type 1&4, and Type 3&4;
If Participants 2, 3, and 4 have continued, then the types will be Type 2&3, Type 2&4, and Type 3&4.

Each type of ticket corresponds to the two participants who will receive the two rewards if a ticket of that type is drawn. So, if a Type 1&2 ticket is drawn, then participants 1 and 2 will receive the rewards; if a Type 1&3 ticket is drawn, then participants 1 and 3 will receive the rewards; and so on. The number of each type of tickets will be computed using a formula similar to the one used in the first phase. Suppose, for example, that in the first phase a Type 1&2&3 ticket was chosen, and Participants 1, 2, and 3 have continued to the second phase. Then, the number of tickets of each type depends on the bids of the corresponding participants as follows:
Number of Type 1&2 tickets = Bid of participant 1 + Bid of participant 2
Number of Type 1&3 tickets = Bid of participant 1 + Bid of participant 3
Number of Type 2&3 tickets = Bid of participant 2 + Bid of participant 3.

The formulas for the cases when a Type 1&2&4, Type 1&3&4, or Type 2&3&4 ticket is chosen in the first phase are similar. We will now work through an example of how the numbers of lottery tickets are computed, and what you will see during a typical period of the session.

An example

Suppose participant 1 bids 80 pence, participant 2 bids 6 pence, participant 3 bids 124 pence, and participant 4 bids 45 pence. Then, in the first phase:
Number of Type 1&2&3 tickets = Bid of participant 1 + Bid of participant 2 + Bid of participant 3 = 80 + 6 + 124 = 210
Number of Type 1&2&4 tickets = Bid of participant 1 + Bid of participant 2 + Bid of participant 4 = 80 + 6 + 45 = 131
Number of Type 1&3&4 tickets = Bid of participant 1 + Bid of participant 3 + Bid of participant 4 = 80 + 124 + 45 = 249
Number of Type 2&3&4 tickets = Bid of participant 2 + Bid of participant 3 + Bid of participant 4 = 6 + 124 + 45 = 175

There will therefore be a total of 210 + 131 + 249 + 175 = 765 tickets in the first phase lottery. Each ticket is equally likely to be selected. In each period, the calculations above will be summarised for you on your screen, using a table like the one in this screenshot:

Ticket Types	Participant 1's bid	Participant 2's bid	Participant 3's bid	Participant 4's bid	Total tickets	Ticket numbers
Type 1&2&3	80	6	124	_	210	**1 - 210**
Type 1&2&4	80	6	_	45	131	**211 - 341**
Type 1&3&4	80	_	124	45	249	**342 - 590**
Type 2&3&4	_	6	124	45	175	**591 - 765**

Interpretation of the table: The horizontal rows in the above table shows the different types of lottery tickets that are generated by the computer in every period. The vertical columns lists the participants bids, total number of each type of ticket (second column from right) and the range of ticket numbers for each type of ticket (last column). Note that the total number of each ticket type is the sum of the three corresponding participants bids. For example, total number of Type 1&2&3 tickets is the sum total of Participant 1s bid, Participant 2s bid and participant 3s bid. Therefore, the table cell corresponding to Type 1&2&3 and Participant 4s bid is kept blank. Similarly, the table cell corresponding to Type 2&3&4 and Participant 1s bid is blank. The last column gives the range of ticket numbers for each ticket type. Any ticket number that lies within that range is a ticket of the corresponding type. That is, all the ticket numbers from 211 to 341 are tickets of Type 1&2&4, which implies a total of 131 tickets of Type 1&2&4, as appears from the Total Tickets column. In case any three participants all bid zero, there will be no ticket that contains those three ID numbers together. In such a case, the last column will show No numbers for that particular ticket type.

The computer then selects one ticket at random. The number and the type of the drawn ticket will appear below the table. The three ID numbers on the ticket type indicate the three participants continuing to Phase II.

Suppose a ticket of Type 1&2&3 is selected in the first phase. Then, in the second phase, there will be Type 1&2, Type 1&3, and Type 2&3 tickets. The number of tickets of each type will be:

Number of Type 1&2 tickets = Bid of participant 1 + Bid of participant 2 = 80 + 6 = 86

Number of Type 1&3 tickets = Bid of participant 1 + Bid of participant 3 = 80 + 124 = 204
Number of Type 2&3 tickets = Bid of participant 2 + Bid of participant 3 = 6 + 124 = 130.

There will therefore be a total of 86 + 204 + 130 = 420 tickets in the second phase lottery. Each ticket is equally likely to be selected.

In each period, the calculations above will be summarised for you on your screen, using a table like the one in the following screenshot.

Ticket Types	Participant 1's bid	Participant 2's bid	Participant 3's bid	Total tickets	Ticket numbers
Type 1&2	80	6	---	86	**1 - 86**
Type 1&3	80	---	124	204	**87 - 290**
Type 2&3	---	6	124	130	**291 - 420**

The interpretation of this table is same as the table shown in phase 1. Since only three participants survive for phase 2, this table contains three rows for the ticket types. The columns and the interpretation of the cells are the same. Again the computer selects one ticket at random. The number and the type of the drawn ticket will appear below the table. The two ID numbers on the ticket type indicate the two participants receiving the rewards.

{*The Part II instructions for all the treatments are again the same for the final paragraph that states the payment procedure.*}

At the end of 30 periods, the experimenter will approach a random participant and will ask him/her to pick up five balls from a sack containing 30 balls numbered from 1 to 30. The numbers on those five balls will indicate the 5 periods, for which you will be paid in Part 2. Your earnings from all the preceding periods will be throughout present on your screen.

Earnings from Part 1 of the experiment will be decided after that. Your earnings from both parts of this experiment will be summarized on your screen.

3.C Feedback screen

Below is the screenshot of a sample feedback table for the baseline treatment. Such feedbacks were given in each contest period for all three treatments. In the survivor selection treatment, two such tables were used within the same period to explain the two consecutive draws.

Ticket Types	Participant 1's bid	Participant 2's bid	Participant 3's bid	Participant 4's bid	Total tickets	Ticket numbers
Type 1&2	90	35	_	_	125	**1 - 125**
Type 1&3	90	_	1	_	91	**126 - 216**
Type 1&4	90	_	_	184	274	**217 - 490**
Type 2&3	_	35	1	_	36	**491 - 526**
Type 2&4	_	35	_	184	219	**527 - 745**
Type 3&4	_	_	1	184	185	**746 - 930**

The range of ticket numbers on which the draw was performed in this period was **1 - 930**

The computer has drawn the ticket number **526** , which is a ticket of **Type 2&3** .

Chapter 4

In-group and out-group motives in group conflicts: An experimental study

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Abstract

Identity in a group conflict is a complex element. Being grouped with other people and engaging in a conflict against another group of people can generate group-related motivations, which may further depend on the social context of a given conflict. Our experimental design disentangles group-related motivations in an inter-group lottery contest over a public good prize. We develop our hypothesis based on a relative-payoff maximisation framework. Group-related motives are manipulated by matching participants with historical subjects - that is, stored decisions from previous participants who are not going to be informed of the outcomes of the present session or receive any payoffs. These manipulations are crossed with two identity conditions - concealed and revealed - to emphasize the role of social context. No significant difference in average contributions are observed. We find weak evidence of individuals adjusting their contributions to avoid within-group exploitation and to match contributions at both intra-group and inter-group levels.

JEL Classifications: C91, C92, D74, D91.

Keywords: group conflict, contest, identity, group attitude

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4.1 Introduction

> "in speaking of Social or Group Psychology it has become usual ...to isolate as a subject of inquiry the influencing of an individual by a large number of people simultaneously, people with whom he is connected by something, though otherwise they may be strangers to him. Group Psychology is therefore concerned with the individual as a component part of a crowd of people who have been organized into a group at some particular time for some definite purpose."

- S.Freud, 1922, Group Psychology and the Analysis of the Ego

The scientific analysis of social conflicts has long ago started recognizing the implications of treating a person or group of people as intrinsically different from oneself, the 'othering behaviour (Canales, 2000; Dervin, 2012), which is recently becoming a popular jargon in the social and commercial media. There are ample historical examples of 'othering behaviour associated with violent group conflicts, whether religious (the Hindus and Muslims in the Indian subcontinent), ethnic (the Slovenes and the Croats in former Yugoslavia), or political (the civil wars in El Salvador or Guatemala between 1970-1990). The core of group psychology comprises the two reciprocally related emotions of ingroup love and outgroup hate (Sumner, 2013; Allport, 1954; Sherif, 1966; Brewer, 1999). Economists model the impact of such emotions on individual choice as other-regarding preferences (Cooper and Kagel, 2009).

Social psychologists suggest some closely entwined motives behind pro-group behaviour. One is the common fate (goal/task) identity (Campbell, 1958; Gaertner and Dovidio, 2014) that generates from the consequential commonality shared by the individuals on the same side in a conflict - if their group wins, they all win; if their group lose, so do all of them. Another is role identity which refers to the fact that being in a group, one is expected to contribute to the group in a certain way (Charng et al., 1988). Both role-identity and common fate identity are consequences of being grouped together with other individuals for a certain purpose, but the concepts themselves are independent of any specific social context, and present in any group-based activity. We refer to them as context-independent identity concepts.

Additionally, in a social conflict, often the two sides also bear different social or categorical identities - whether ethnic, political, racial, or gender - which are specific to the context of interaction and create an additional sense of belongingness among the members on the same side.

Traditional economic models of decision-making abstract away from such context dependent identity labels arguing them to be payoff-irrelevant, but the rapidly expanding field of identity economics demonstrate otherwise (Akerlof and Kranton, 2000; Chen and Li, 2009;

Chen and Chen, 2011; Weisel and Zultan, 2016). While context-independent identities are sufficient for the development of group-related emotions like in-group love and/or out-group hate, the simultaneous presence of context-dependent identities may affect such emotions. We are interested in studying how group-related motives interact with context dependent identities in inter-group conflicts. It is important to point out however that due to the presence of conflicting interests at both inter-group and intra-group level, as will be demonstrated later, we refrain from using the terms ‘in-group love’ and ‘out-group hate’. Instead, any motives related to other people in the same group is referred to as ‘in-group motives’, and any motives related to people in the opponent group is referred to as ‘outgroup motives’.

The experiment in the present study investigates (i) how the in-group and out-group motives affect behaviour in inter-group conflicts and (ii) how such motives are impacted when the conflicting groups have different categorical identities.

For the rest of this chapter, group motives generated from context-independent identities are called group attitudes. We approach the first question with a set of experimental treatments which remove the in-group or out-group motives in a group interaction by completely removing payoff consequences for the in-group or outgroup members through manipulating the group composition. To completely remove payoff consequences, subjects’ decisions in some treatments are matched with stored decisions of participants from previous sessions. For exposition purposes, let us refer to a participant in a previous session as a historical subject. Since participants from previous sessions have already been paid, and will face no further consequence from the present events, interaction with such historical ‘others’ would remove all possible financial and psychological payoffs that apply to real ‘others’. In the IN-OUT treatment, all in-group and out-group members are real-time participants (i.e. participants who are making decisions at the same time in the same session and are getting paid at the end of the present session). In the IN treatment, only the in-group members are real-time participants, and all out-group members are historical subjects who do not get paid or informed of the outcomes in the present session. Finally, in the NONE treatment, the individual subject alone is a real-time participant, while all in-group and out-group members are historical subjects. Thus, compared to the IN-OUT treatment, the IN treatment removes all out-group motives, which include any possible joy or excitement from defeating others or disappointment from losing to others.[4] Similarly, compared to the IN treatment, the NONE treatment removes all in-group motives, which include any positive synergy and reciprocation incentives in working with others.

Group motives in the second question may include biases originated from both context-independent and context dependent identities. To address this question, we group the subjects

[4]The pure joy of winning or disappointment from losing might still play a role, but is not related to group-bias.

based on minimal identities (Tajfel, 1970) assigned to them in the lab, and control the effect of social identity through either revealing or not revealing this matching criterion. The subjects in any given group always share the same identity and interact with a group having the other identity. The subjects are aware of this fact only in the Revealed (R) condition, and not in the Concealed (C) condition. The IN-OUT, IN, and NONE treatments are elicited separately under the two conditions. Comparison between the R and C treatments can tell us if group-related motives get strengthened in presence of a context-dependent identity. Procedural details of the treatments are explained in Section 4.4.

Figure 4.1 summarizes the design principle. The prominent figures represent real-time participants and the watermarked ones represent historical subjects. Defining in-group and out-group members from the perspective of the relatively bigger figure in the left-side groups in each of the six treatments, out-group members are real-time participants in the IN-OUT treatments only, and in-group members are real-time participants in both IN-OUT and IN treatments. Groups in the concealed identity condition are not distinguishable from one another in terms of any exogenous parameter. The rival groups in the revealed identity condition however, have different exogenous identities, as represented by the white and grey figurines.

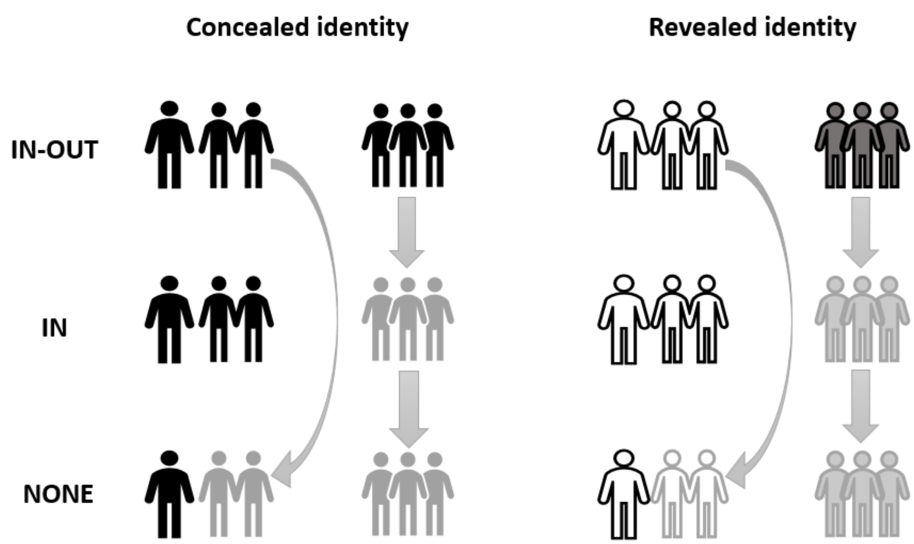


Figure 4.1: In the Concealed identity treatments, participants in both groups are homogeneous as indicated by the similar black figurines in both groups. In the Revealed identity treatment, the two groups have different identities as represented by white and grey figurines.

As we are interested in group related motives in a conflict, we engage our subjects in an inter-group contest for a public good prize (Katz et al., 1990). The contest game, in which players incur irreversible expenditure of costly resources to win some valuable reward, is one of the major tools used by researchers to model costly conflicts (Garfinkel and Skaperdas, 2007). A simple lottery contest models a conflict with a probabilistic component. In an inter-group lottery contest, two groups participate in a lottery for a prize that can be won by only one of the two groups. Each member within a group can choose to contribute some of his/her endowment towards the intergroup lottery. A higher total contribution means a higher chance of winning for the group. All the members of the winning group receive the same value irrespective of their individual contributions.

An interesting feature of this game is that equilibrium spending at the group level can be uniquely defined by a pure strategy Nash equilibrium, but there exists a continuum of pure strategy equilibria at the individual level if all players attach the same value to the reward, or at least two players attach the highest value (Baik, 1993). Consequently, some player(s) may free ride with positive probability in equilibrium. Experimental evidences fail to support this conjecture however, with many studies reporting observed group-level spending at more than double the equilibrium prediction (Abbink et al., 2010; Ahn et al., 2011; Sheremeta, 2011; Eisenkopf, 2014; Chowdhury et al., 2016).

Sheremeta (2017) offers three possible explanations of this observed overspending in group contests - joy-of-winning, social preferences, and parochial altruism. Joy-of-winning can drive excessive investment in one-to-one conflicts as well (Sheremeta, 2018). The significance of social preferences can be understood with reference to the evidence from public good experiments demonstrating individual willingness to contribute to the public good and the underlying aversion to free-riding. Parochial altruism takes shape in subjects' desire to increase their own group's payoff over the opponent group's. Note that pure joy-of-winning comes from individual competitiveness, and can exist even in absence of others, whereas social preferences and parochial altruism are necessarily group related motivations, the latter being relevant only in the presence of some out-group individual(s). The observed aggressive spending in inter-group contests is perhaps a combined consequence of all these motives and it is difficult to distinguish the relative impact of one from the other.

Inter-group conflicts also involve an intra-group conflict of interest which stems from the fact that all members in the winning group equally share the benefits irrespective of their individual contributions (Bornstein and Ben-Yossef, 1994). This two-level dilemma offers individuals a scope to free ride on their own-group members.[5] The extent of such free-riding is likely to diminish the stronger the group motives are.

The games of inter-group public good (Bornstein et al., 1989; Bornstein, 1992; Bornstein et al., 1994; Bornstein, 2003) and inter-group prisoners dilemma (Bornstein, 1992; Bornstein

[5]In the inter-group lottery contest, this is characterized by the lack of unique equilibrium strategy for the individual decision-maker, which allows individuals with lower valuations to free ride on high-valuation players.

and Ben-Yossef, 1994; Bornstein et al., 1994; Probst et al., 1999; Goren and Bornstein, 2000; Juvina et al., 2012; Coen, 2013; Weisel and Zultan, 2016) are among the conventional tools for studying group dynamics by observing the extent of intra-group free-riding in inter-group conflicts. In both games, the group-optimal strategy is for all members to contribute. However, these games offer individuals with binary choices of contributing all or nothing. Besides, they have been widely used for studying the composite group motives but couldn't distinguish between in-group and out-group motives. Halevy et al. (2008) allowed continuity in contribution and proposed a unique method (the inter-group prisoner's dilemma maximizing difference, or IPD-MD game) of isolating the sole impact of in-group motives from the impact of out-group motives. But their design abstracts away from the composite manifestation aspect by providing individuals with clear choices of only benefiting in-groups versus also harming out-groups. Weisel and Zultan (2016) use the inter-group prisoner's dilemma to propose a threat-based explanation behind the emergence of pro-group behaviour.

The inter-group lottery is appropriate for modelling the resource-investment and uncertainty aspects of a violent conflict. There being only one group account where subjects can make their contributions, any group motives should be reflected in their contributions to this group account. We expose the subjects to the inter-group lottery in two phases. The first phase consists of only one such lottery, and the second phase constitutes 20 repetitions of the same game without any carry-over of earnings from oner period to the next. This allows us to investigate how subjects might adjust their decisions as they become more experienced. The subjects do not know about the second phase when they participate in the first phase one-shot game.

The effect of the group motives may not be perfectly additive. If social identity strengthens the group motives, then one would expect the contributions in the R treatments to exceed the contributions in the C treatments. However, the behavioural implication of the group-related motives may be more complex. Abbink et al. (2012) interprets parochial altruism as the preference for maximizing the payoff difference between one's own group and the rival group. The motive of relative group-payoff maximization (Jervis, 1978; Snidal, 1991; Powell, 1991) can be interpreted as a composite manifestation of both in-group love and out-group hate - in-group love being realized in the added utility from higher payoff to in-group individuals, and out-group hate being realized in the loss of utility due to higher payoff to out-group individuals. One would expect that the simple motive of in-group love, if present, will inspire contributions in the IN treatment, and out-group hate, if present, may further raise the contributions in the IN-OUT treatment (as is argued in Abbink et al. (2012)). However, there may be other motives - such as, dislike for being a sucker - which can moderate the induced contribution following from in-group love. We model such motives with relative payoff maximizing agents in section 4.2. Section 4.3 outlines our hypotheses regarding behaviour in the experimental treatments as derived from Section 4.2 and from existing findings in the social identity theory literature. Section 4.5 present the results, followed by a general discussion in Section 4.6.

The observed contest contributions do not exhibit much across-treatment variation. We find similar average contributions across the six treatments. Observed variations in the contribution amounts are also not significantly different across treatments. In all treatments, contributions adjust downwards over time, and subjects in a winning group is more likely to stick to their present contribution levels. Cross-treatment comparison of dynamic adjustments in contribution indicates that comparison with in-groups lowers contribution. However, comparison with out-groups doesn't increase contribution at a statistically significant level. Groups with higher contribution compared to their out-groups reduce contribution, while groups with lower contributions compared to their out-groups increase contribution only under revealed identity. The cross-treatment similarity of adjustment patterns seems to provide a stronger support for the conformity bias than group bias.

4.2 Theoretical Framework

The inter-group lottery contest over a public good prize (Katz et al., 1990) is applicable to any finite number of groups. Suppose N denotes the set of n contesting groups. There are m_G members in group G. Suppose player i belongs to group $G \in N$, and has an endowment of w from which (s)he contributes $e_i \in [0, w]$ to the group account. Any remaining endowment is part of her final earning.

The total contribution in a group G is the sum of the individual contributions for all $i \in G$. Therefore, the total contribution in group G is $e_G = \sum_{i \in G} e_i$. Group G wins the contest with probability

$$P_G = \begin{cases} \frac{e_G}{\sum_{G \in N} e_G} & \text{if } e_G > 0 \text{ for some } G \in N \\ \\ \frac{1}{n} & \text{if } e_G = 0 \quad \forall G \in N \end{cases} \tag{4.1}$$

Each player in the winning group receives a prize $v_i > 0$. Therefore, the expected earnings of any player $i \in G$ is given by

$$\pi_i = P_G v_i + (w - e_i) \tag{4.2}$$

Substituting P_G from (4.1) into (4.2), and solving for pure strategy Nash equilibrium for homogeneous players leads to the unique equilibrium group contribution $e_G^* = (n-1)v/n^2$. The inter-group contest is finitely repeated without any carry-over of earnings. Therefore, the unique pure-strategy group-level equilibrium of the stage game is also the sub-game perfect equilibrium of the finitely repeated game.

Individual contribution in the equilibrium, however, is not necessarily unique. If a single player in a group has the highest valuation for the reward, this player spends as much as e_G^* in equilibrium, and others spend nothing. If two or more players have the highest valuation in a group, then individual contribution is characterized by multiplicity of equilibria. (Baik, 2008)

In our experiment, there are 3 members in each of the two contesting groups, and the value of the reward is 60 Experimental currency units (ECU) which is common knowledge. The group level equilibrium contribution is $e_G^* = 15$ ECU. Since all group members are homogeneous in terms of reward valuation and endowment, and this is common knowledge, we may use the symmetric contribution profile ($e_i^* = e_G^*/m_G$) as an analytical benchmark (Abbink et al., 2010). This is true for risk-neutral players without other-regarding preferences.

In a group conflict however, players may have concerns beyond their own absolute payoffs. In what follows, we consider a relative-payoff maximizing player as modelled by Riechmann (2007). This lets us easily distinguish between our different treatments from a strategic point of view. Riechmann (2007) showed that in a one-to-one rent-seeking contest with homogeneous relative-payoff maximizing players, the pure strategy equilibrium expenditure is higher than the standard contest. In the inter-group contest over a public good prize, there are two levels of payoff comparison. The relative payoff of player i within group G will be given by

$$\pi_i^r = \pi_i - \bar{\pi}_{i\in G} = \pi_i - \frac{\sum_{i\in G}\pi_i}{m_G} = \frac{m_G-1}{m_G}\pi_i - \frac{1}{m_G}\sum_{j\in G}\pi_j = \frac{1}{m_G}\sum_j e_j - \frac{m_G-1}{m_G}e_i \tag{4.3}$$

where $j \in G$ and $j \neq i$. This is irrespective of the outcome of the inter-group contest. In conformity with the public good characteristic of the reward, π_i^r is maximized at $e_i = 0$. The relative payoff of group G is

$$\pi_G^r = \pi_G - \bar{\pi}_G = \frac{e_G}{e_G + e_{G^-}}v - \frac{v}{2} - \frac{1}{2}(e_G - e_{G^-}) \tag{4.4}$$

The design principle followed in our experiment is that in-group (out-group) motivations do not exist in the absence of payoff consequences to in-group (out-group) members. Accordingly, the inter-group relative payoff (4.4) is irrelevant in the IN and NONE conditions. Similarly, the intra-group relative payoff (4.3) is irrelevant in the NONE condition.

Proposition 4.1. *Under the motive of relative-payoff maximization, the group-level best response to a given level of out-group contribution*
(i) is higher under IN-OUT condition than under IN condition.
(ii) can be higher under IN-OUT than under NONE, only if out-group contribution is below certain threshold given by the reward value and group-size.
(iii) can be higher under IN than under NONE if players are maximizing inter-group relative payoff subject to intra-group constraint and out-group contribution is below certain threshold given by the reward value and group-size, and is otherwise lower under IN than under NONE.

Proof. Relegated to the appendix. □

4.3 Hypotheses

As mentioned before, the goal of this study is to disentangle the group-related attitudes from purely self-related motivations in a conflict situation, and to see how these elements get strengthened or weakened in presence of exogenous context-dependent identities.

The treatment in the 3X2 factorial design varies in two dimensions; the group composition, and identity information. The group composition manipulates the group-related motives, while identity information manipulates the exogenous identity aspect. Our two main hypotheses, drawn form Proposition 4.1 in Section 4.2, and existing research in social psychology, are as follows.

Hypothesis 4.1. *For any given identity condition (C or R), average group-level contribution in the IN-OUT treatment is higher than the average group-level contribution in the IN treatment. Average contribution in the NONE treatment is expected to be weakly lower than the other two treatments for a low level of average out-group contribution.*

This is in conformity with the theoretical prediction for relative payoff maximizing players as presented in Section 4.2. There are other studies suggesting that subjects behave at least as aggressively in the presence of an additional motive (Abbink et al., 2010; Cason et al., 2017). In the IN-OUT treatment, both in-group and out-group motives are present. Only in-group motives are present in the IN treatment, and no group-related motives are present in the NONE treatment. There is experimental evidence of individuals contributing higher amounts of resources in group decision-making contexts relative to strategically comparable individual decision-making contexts (Kocher and Sutter, 2005; Cooper and Kagel, 2005; Morone and

Temerario, 2018). However, considering the theoretical arguments about relative-payoff maximizing players Section 4.2, we refrain from proposing adirectional hypothesis about contributions in the NONE treatment relative to the other treatments.

Hypothesis 4.2. *For any given group composition (IN-OUT, IN, or NONE), average group-level contribution in the R treatment is at least as much as the average contribution in the C treatment.*

Our second hypothesis comes from the standard proposition of social identity theory (Tajfel, 1970), and also findings in economics experiments (Chen and Li, 2009; Chowdhury et al., 2016), suggesting that the mere presence of different categorical identities enhance group identification, and thus drives group conflict.

Both these hypotheses are tested by comparing contributions from groups and individuals who experienced the same out-group conflict across the different treatments along the relevant treatment dimension (group-motive or identity). Two additional hypotheses are as follows,

Hypothesis 4.3. *Contribution to the group account falls over time.*

This is in conformity with the downward trend in spending observed in experimental public good games and inter-group contests over public goods (Abbink et al., 2010; Chowdhury et al., 2016). This is attributable to the public good characteristic of the reward, and learning from repeated interaction with in-group members. Bornstein et al. (1994) report that subjects in the inter-group public good game contribute at the same rate over time, but subjects in that game had only binary choice of contributing or not. Subjects in our experiment face the same in-group members (whether real-time or historical) across periods, and imperfect stranger[6] out-group members in different periods. This minimizes the possibility of escalation of group contributions or collusive behaviour, while preserving the salience of group-related motives. Given a continuous choice of contribution amounts on a discretized strategy space, we expect contributions to damp down over time, albeit in possible jumps between multiples of five or ten as is often observed in experimental choices.

Hypothesis 4.4. *Positive group attitude is associated with higher contribution.*

We construct a Likert-scale index for group attitude from the group-attitude questionnaire, a high measure on this scale indicating higher group cohesion. The post-treatment questionnaire does not capture the gradually evolving expectations. However, such questionnaires

[6] each group was matched with one of five different opponent groups in a random order, so they face the same opponents in more than one period but do not know which group they face in which period.

are a standard practice in social psychology literature (Campbell, 1958; Hinkle et al., 1989; Insko et al., 2013) and have been reported to be significant predictors of group behaviour. The Cronbach α for the group attitude questionnaire was 0.83, indicating high correlation among the items in the questionnaire, and the consequent reliability of the Likert-scale measure.

4.4 Design and Procedure

As described in Section 4.1, we use a 32 between-subject factorial design with a total of 6 treatments and 180 subjects, as summarized in Table 1. [7] There are 3 group-motive conditions elicited through different group matching variations - IN-OUT, IN, and NONE, each allowing for a different set of group related motivations. All three variations are conducted separately under each of the 2 identity conditions, namely, the C (concealed) and the R (revelation) conditions. Part I of the experiment consists of a painting choice task (Tajfel, 1970) which is used for assigning the minimal identities in the R treatments. This task is implemented in the C treatments also, but neither the minimal identities nor the use of these identities for group assignment in part II are revealed to the subjects therein. The inter-group contest in Part II has two phases; phase I constitutes a one-shot inter-group lottery and phase II consists of 20 repetitions of the same game. The subjects are not aware of phase II when making their decisions in Phase I.

At the beginning of the experiment (Part I), each participant is shown five pairs of paintings, each pair consisting of one painting by Paul Klee and another by Wassily Kandinsky. The participants are told the name of the two painters but the paintings are never identified with a painter's name. Participants are asked to select their most preferred painting from each pair. Depending on the median preference, participants are assigned to one of the two identity clusters, namely the KLEE cluster and the KANDINSKY cluster. Participants completed this task in both C and R treatments, but in the C treatment they were explicitly told that decision-making and outcomes in the following parts of the experiment are independent of this task.

These lab-generated KLEE and KANDINSKY identities are used for assigning participants to their respective contest groups. Each participant is allocated to a three-player group that engages in a lottery contest against another three-player group. All participants in the same contest group always belong to the same identity cluster. Each participant is endowed with 60 experimental currency units (ECU) which could be allocated between a private and a

[7] The experiment was coded and conducted using the z-Tree software (Fischbacher, 2007) at the Centre for Experimental and Behavioural Social Science (CBESS) in University of East Anglia between May 2017 and February 2018.

Table 4.1: Experimental treatments (number of subjects in parentheses)

Identity	Group-related Motivation IN-OUT	IN	NONE
Concealed (C)	IN-OUT-C (30)	IN-C (30)	NONE-C (30)
Revealed (R)	IN-OUT-R (30)	IN-R (30)	NONE-R (30)

group account. The participants independently decide how much to allocate to their respective group accounts. Once all participants have made their decisions, a computerized lottery is conducted to select the winning group. A simple Tullock lottery contest success function is used so that the winning probabilities depend only on the total amounts in the two group accounts. Each participant in the winning group is rewarded with 60 ECU. This lottery contest is then repeated for another 20 periods without any carry-over of earnings. The outcome from each period is shown before the next period begins. Outcome or payoffs from any of the previous periods are not observable at any point. We also restricted subjects' access to any paper during the experiment to avoid heterogeneity in observability of contest history.

The composition of any given group remains the same throughout, but the opponent group randomly changes in each period (see the detailed description of the treatment variations below). Any two groups contesting each other always have different identities. This information is concealed in the C treatments and is revealed in the R treatments. Both exogenous and endogenous identities are activated in the R treatments, while C treatments allow only the endogenous dimension through the different group-matching variations. In what follows, we describe the treatment variations used to distinguish between the different group related motives.

IN-OUT: In phase I, participants from the same identity cluster are randomly matched into different three-player groups and two groups with different identities are matched against each other. That is, a KLEE group of 3 participants is matched with a KANDINSKY group of 3 participants. However, this information is revealed in the R condition only, while participants in the C condition only know that they have been randomly assigned into 3-player contest groups. Each of the six participants simultaneously and independently decides how much to allocate to their respective group accounts. All three participants in the winning group receives 60 ECU each. There were 30 subjects in each of the two IN-OUT treatments (IN-OUT-C and IN-OUT-R). All 30 subjects participated in the same session, and depending on their identity clusters, as derived from the painting choice task in Part I, each subject was assigned to one of the 5 KLEE and 5 KANDINSKY groups. Composition of each group remained unchanged for all periods in phase II. However, a given KLEE group was randomly matched against one of the 5 different KANDINSKY groups in each subsequent period. Participants had no access to the decision or outcome history from earlier periods.

IN: Participants from the same identity cluster are randomly matched into three-player groups. Unlike IN-OUT however, the KLEE groups are not matched with any of the KANDINSKY groups in the current session. Instead, each group replaces a previously active group that participated in the IN-OUT treatment (this is common knowledge under both C and R) and had the same identity (This is common knowledge only in R). That is, a currently active KLEE group replaces a previously active KLEE group from the IN-OUT treatment and competes against the decisions made by the previously active KANDINSKY groups from the same previous session and in the same order as encountered by the original KLEE group they replace. If the active group wins, each player in that group receives the 60 ECU reward. If the inactive group (from the IN-OUT treatment) wins, nobody gets the reward. Therefore, any participant's decision doesn't have any consequence for their respective out-groups. Only in-group members' payoffs are affected.

NONE: Each participant is randomly assigned to replace one member from a previously active 3-player group with the same identity who participated in the IN-OUT treatment. In any given period, the remaining two members' contribution amounts from the respective period of the IN-OUT session are added to the individual participant's contribution to determine the total contribution in his or her group. The rival group's decisions are constructed the same way as in the IN treatments. In every period, an individual subject in the NONE treatment faces the out-group as well as in-group decisions exactly as experienced by the previously active same-identity group-member whom she replaces.

Note that the across-session matching of contribution decisions is always confined within the respective identity condition (C or R). Thus, for any $j \in \{1, 2, 3, 4, 5\}$ and any $i \in \{1, 2, 3\}$, the KLEE-j (KANDINSKY-j) groups in IN-OUT-C, IN-C, and NONE-C face the same out-group decisions. Note that each member i of group j in the NONE treatment participates separately and are neither aware nor affected by another member in the same treatment. [8] Any i in KLEE-j group also face the same in-group decision in IN-OUT-C and NONE-C, but not in IN-C. Therefore, denoting the contribution decision of member i in group j of treatment k as e_{ijk}, we can find the relative impact of the in-group motive by comparing $\sum_{i \in j} e_{ij(\text{NONE-C})}$ with $\sum_{i \in j} e_{ij(\text{IN-C})}$, and the effect of the out-group motive by comparing $\sum_{i \in j} e_{ij(\text{IN-C})}$ with $\sum_{i \in j} e_{ij(\text{IN-OUT-C})}$, across all i and all j. Similarly, for the R condition.

[8] To illustrate, one subject in NONE-C replaces member 1 of the KLEE-j group in IN-OUT-C and faces member 2's and 3's decisions, as well as the rival group's decisions as experienced by the original member 1 in group KLEE-j in IN-OUT-C. Another subject replaces member 2 of the KLEE-j group in IN-OUT-C and faces the same in-group and out-group decisions as experienced by member 2 in KLEE-j of IN-OUT-C. So on and so forth.

At the end of the experiment, any 5 of the 20 periods are randomly chosen for payment. Subjects also answer a cognitive reflection test (CRT), a questionnaire about perceived group attitudes, a questionnaire about the reasons behind their decisions, and a demographic questionnaire. The use of CRT was motivated by Sheremeta (2018) and Cox (2017), both of whom reported significant negative correlation between CRT scores and spending in one-shot contests. The group attitude questionnaire was adapted from Hinkle et al. (1989) and Insko et al. (2013). The choice reason questionnaire was used to extract perceived cooperation from own-group members. Although, both Sheremeta (2011) and Shupp et al. (2013) report that attitudes towards risk and ambiguity do not explain contest expenditure, we controlled for both these attitudes.

4.5 Results

Average contribution from participants in all treatments were very similar across both group and identity dimensions. Table 4.2 summarizes the mean and standard deviation of contributions in each treatment, separately for the one-shot contest and the continued contest of 20 periods. The second and the fourth column aggregates the observations from all participants in each treatment. Each treatment had 30 participants, with 3 participants in each of the 10 groups (5 KLEE groups and 5 KANDINSKY groups).

Table 4.2: Summary statistics for one-shot and repeated contest (std. dv. in parentheses)

Treatments	One-shot (Period 1)		Repeated contest (Period 2-21)	
	No. Obs.	Average Contribution	No. Obs.	Average contribution
IN-OUT-C	30	21.23 (10.62)	600	14.90 (11.34)
IN-OUT-R	30	20.60 (12.77)	600	15.64 (10.81)
IN-C	30	19.23 (09.86)	600	14.36 (12.19)
IN-R	30	21.40 (11.26)	600	15.05 (12.23)
NONE-C	30	19.93 (10.86)	600	14.83 (12.78)
NONE-R	30	21.66 (13.12)	600	15.50 (15.29)

In the IN-OUT treatment, all 3 participants in a group made their decisions simultaneously. Every period, each of the five KLEE (KANDINSKY) groups were randomly matched with one of the five KANDINSKY (KLEE) groups. Hence neither a group nor a pair of contesting groups can be treated as an independent unit of observation. The IN treatment on the other hand, matches the 3-player groups with previously active groups, and therefore each 3-player group constitutes an independent unit of observation. Similarly, each participant in the NONE treatment faces decisions from previously active participants, and never interacts with another subject participating in the same session as her. Each subject therefore comprises an independent unit of observation. However, for the across-treatment comparison of group level contributions, we will sum up the contributions from the 3 individual members in the NONE treatment who are matched against the same rival group.[9]

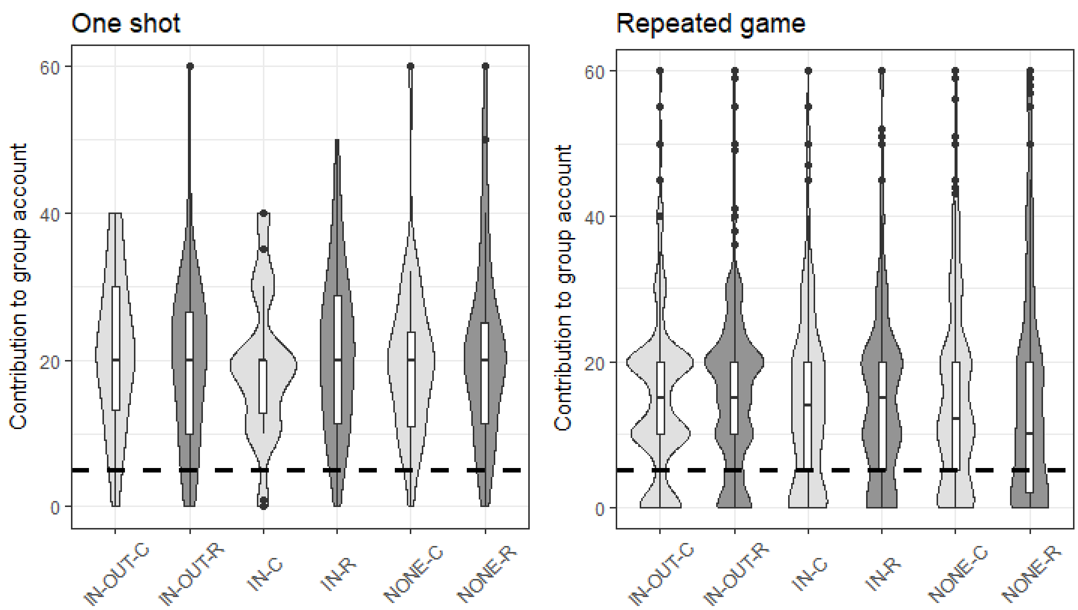


Figure 4.2: The box-plots are embedded on the violin plots to indicate summary statistics for individual contributions. The grey areas inside the violins show the distribution of contribution amounts, the white box-plots give the interquartile range, the thick horizontal section marks denote the medians. The dashed line indicates the individual-level benchmark in the ex-post symmetric equilibrium ($e_i^* = e_G^*/3$) for risk-neutral players without any social preferences.

4.5.1 Overall contribution in One-shot vis-á-vis repeated contests

Figure 4.2 plots the average contribution in all treatments in the one-shot and the repeated contests, along with the distribution of contribution amounts. For each treatment, the distribution of all contributions is shown with the shaded violin-shaped areas. The dark grey violins indicate the R treatments and the light grey ones indicate the C treatments. The white box-plots embedded inside the grey violins give the inter-quartile range of individual contributions, median contributions being highlighted with the thick black horizontal sections of the box-plots. The medians for all treatments are similar in the one-shot game. Medians in

[9]As illustrated in Footnote 8, we sum up the contribution decisions from the three individual subjects in the NONE treatment who replace three members in the same IN-OUT group and consider this sum of contributions as the group contribution of the corresponding group under NONE condition for comparison with IN-OUT and IN.

the NONE treatments are slightly lower compared to other treatments in the repeated game, although this difference is statistically insignificant. Also, medians in the repeated game are lower than in the one-shot game for all treatments. The dashed line indicates the individual-level benchmark for ex-post symmetric risk-neutral equilibrium effort in the absence of any social preferences. The modal contribution in the one-shot treatment is 20 ECUs for all treatments. Contributions in the repeated game are multi-modal around zero contribution, 10 ECUs, and 20 ECUs, except in the NONE-R treatment. Lower contributions have higher density in the repeated game in all treatments. Despite the absence of any strategic interaction in the NONE treatments, there were enough within-subject variation in both NONE-C and NONE-R treatments.

4.5.2 Comparable groups in the repeated game

Note that for a given identity condition, C or R, member i in group j in the NONE treatment replaced member i in group j of the IN-OUT treatment, for all $i \in \{1, 2, 3\}$ and all $j \in \{\text{KLEE1}, \cdots, \text{KLEE5}, \text{KAND1}, \cdots, \text{KAND5}\}$. Thus, every member in the NONE treatments experienced the exact same history of in-group and out-group contributions as the corresponding member in the corresponding group of the IN-OUT treatment.[10] Similarly, for any given identity condition, all players in group j of the IN treatment experienced the same out-group contributions but different in-group contributions as the members in group j of the IN-OUT and the NONE treatments, for all $j \in \{\text{KLEE1}, \cdots, \text{KLEE5}, \text{KAND1}, \cdots, \text{KAND5}\}$. We consider data from repeated contest over Periods 2-21 for the exposition in this section. Figure 4.3 presents the box-plots of average contribution across periods 2-21 for each group $j \in \{\text{KLEE1}, \cdots, \text{KLEE5}, \text{KAND1}, \cdots, \text{KAND5}\}$. There is no observed difference between the KLEE and KANDINSKY clusters in terms of average contribution pattern.

Standard non-parametric tests like the Kruskal-Wallis test, and the Wilcoxon-Mann-Whitney rank-sum test for comparison between a pair of treatments fail to exhibit any difference in average contributions in the one-shot contest.

To compare contributions from the repeated game, we take the difference in total contributions from two comparable groups in two treatments for each period, and take the average of these differences over the 20 periods. Note that these differences can be treated as independent observations. Next, we test the null-hypothesis of zero average difference using a Wilcoxon signed-rank test. All comparisons turn out statistically insignificant as presented in Table 4.3.

Similarly, regression-based comparison of group contributions averaged over periods 2-21 (allowing for group-level random effects) fail to indicate any difference across treatments.[11]

[10] The history of the game is not exactly comparable though, because of the different outcome histories.

[11] A Brown-Forsythe test for equality of variances reveal that the across-group variance in NONE-R is

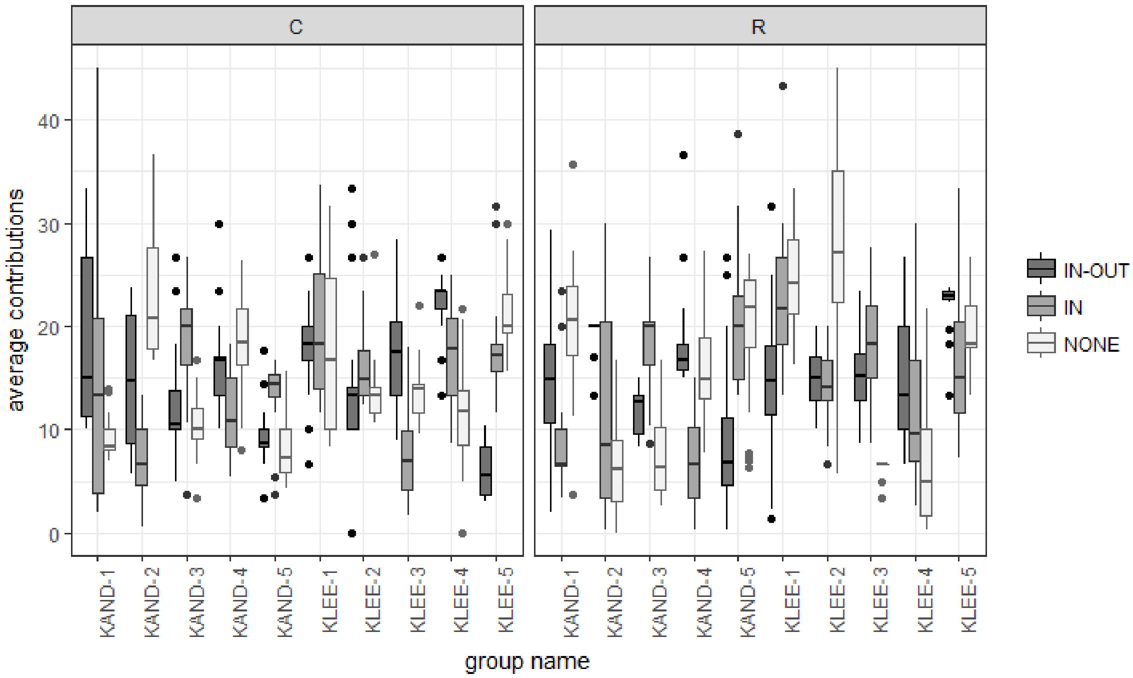


Figure 4.3: Average contribution in the comparable groups across treatment conditions.

Lastly, there was no significant difference in the group-level occurrence of zero contributions in different treatments. Therefore, when looking at the group contributions averaged over all periods in the repeated contest, neither of hypotheses 1 or 2 holds true in our experiment. In contrast to hypothesis 1, average contribution is similar in all treatments. The next section looks at the dynamic adjustment pattern in contributions.

4.5.3 Dynamic adjustment in individual contribution

The analysis in this section considers only periods 2 to 21. The adjustments in individual contributions can be observed from the 2d density plot in Figure 4.4 which maps individual contributions in period $t+1$ against contributions in period t, for any $t \in [2, 21]$.

Each square represent a block of 5 ECU, and the colour of the square indicates the count of individual adjustments in contribution between subsequent periods (any two subsequent periods over the whole duration of the session) that fall in these block. The square at the extreme top-right corner for example, indicates the number of within-subject adjustments in contribution which are such that the contributions in both period $t+1$ and period t were in the range of 55-60 ECUs. Similarly, the square at the extreme top-left corner indicates the number of individual adjustments between subsequent periods such that contribution in period

significantly higher in comparison to all three C treatments and the IN-OUT-R, but difference in average contribution is again rejected with a heteroskedastic F-test.

Table 4.3: Observations from two treatments are directly compared for the one-shot game using the MWW rank-sum test. For the repeated game, we take the differences between the group totals for the comparable groups in two treatments for each period, take the average of these differences over the 20 periods and test the null-hypothesis of a zero average difference using the Wilcoxon signed rank test. The values indicate the effect sizes with p-values in parentheses.

Pair of treatments	One-shot		Reeated	
	C	R	C	R
IN-OUT vs. IN	0.37 (0.34)	0.53 (0.82)	0.32 (0.68)	0.29 (0.91)
IN vs. NONE	0.49 (0.97)	0.51 (0.97)	0.27 (1.00)	0.26 (0.92)
IN-OUT vs. NONE	0.58 (0.54)	0.45 (0.73)	0.31 (0.76)	0.27 (1.00)

t was in the range of 0-5 ECUs, but contribution in period $t + 1$ ranged between 55-60 ECUs. The darker the colour of a square, the higher is the count of adjustments in the corresponding region. The darkest regions are distributed along the 45-degree line in all treatments, and density is lower in the off-diagonal areas, which implies relatively fewer period-to-period adjustments in contribution. The multiple modes around 0, 10 and 20, and a strong serial correlation in individual contribution are apparent from these plots.

Result 4.1. *Contributions to the group account falls over time in all treatments.*

Support. We regress individual contributions on contest periods for each treatment. Table 4.4 reports the regression coefficients with panel corrected standard errors (PCSE). The

Table 4.4: Random effect panel regression of individual contribution on contest periods (panel corrected standard errors in parentheses)

Individual Contribution	Coefficient (std. error)					
	IN-OUT-C	IN-OUT-R	IN-C	IN-R	NONE-C	NONE-R
Period	-0.49^{***}	-0.16	-0.55^{***}	-0.57^{***}	-0.27^{***}	-0.44^{***}
	(0.11)	(0.08)	(0.13)	(0.15)	(0.12)	(0.12)
Constant	20.11^{***}	17.32^{***}	20.11^{***}	21.04^{***}	17.65^{***}	20.15^{***}
	(1.81)	(1.44)	(1.94)	(2.38)	(2.16)	(2.76)

negative period trend is significant in all treatments, with a weaker coefficient for the NONE-C treatment. We also conduct a mixed effect regression with error clustering at contest-group level, and observe a significant negative period trend. However, the trends for different treatments were not significantly different. □

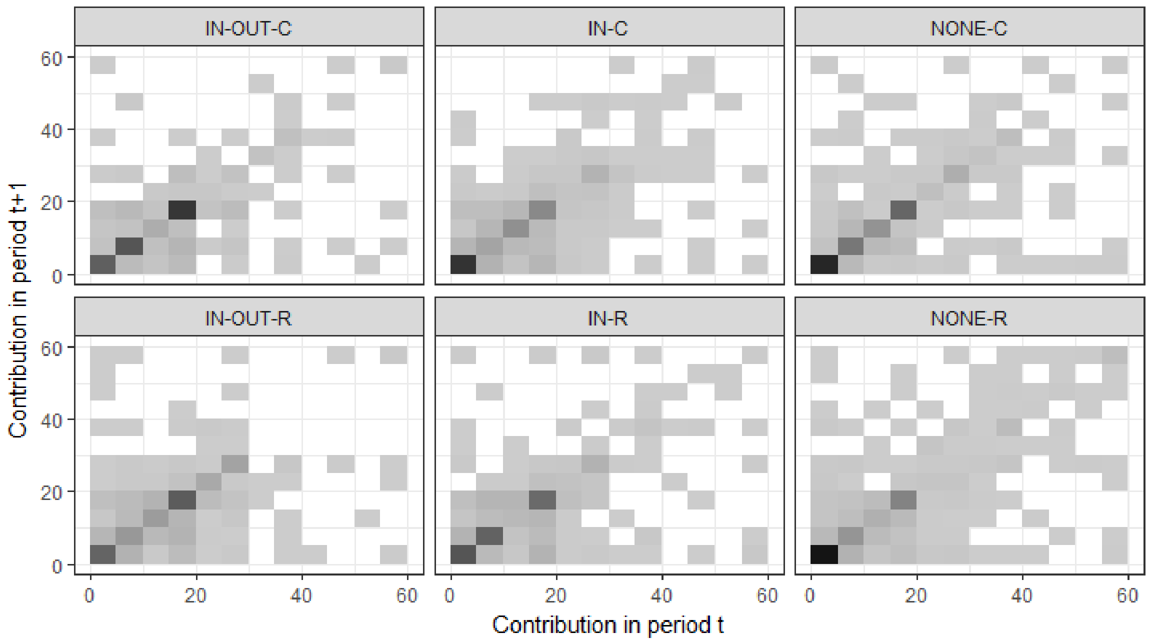


Figure 4.4: Two-dimensional density map of period-to-period adjustments in individual contributions. The colour of each block indicates the number of occurrences of successive-period adjustments in individual contribution that fall in the corresponding region of contributions in periods t and $t+1$. Darker regions indicate higher counts of occurrences.

Participants in our experiment were shown only the total contributions in their group and the other group, and the lottery outcome at the end of each period. They did not have any access to previous period outcomes. However, immediate memory can play an important role in adjustment of contributions over time.

Result 4.2. *Winning in the previous period reduces the possibility of upward adjustment in contributions, and contributions are very likely to be reduced (increased) following a high (low) relative contribution to the group account.*

Support. We perform a two-step generalized logit regression to observe the impact of possible comparison with in-group vis--vis out-group members on individual decisions regarding contribution to the group account. In step I, the dependent variable takes the value 1 (0) if individual contribution in Period $t+1$ is different from (same as) the contribution in Period t. If y_i denotes the log odds of a non-zero adjustment in contribution, we can specify the following model

$$y_i = \beta_0 + \beta_1 \text{win}_{G,t} + \beta_2 e_{i,t} + \beta_3 \text{lead}_{G,t} + \beta_4 |(e_{G^-} - e_G)_t| + \beta_5 * \text{lead}_{G,t} * |(e_{G^-} - e_G)_t| \\ + \beta_6 \text{groupshare}_{i,t} + q_i + q_G \tag{4.5}$$

The independent variables in the above specification include an indicator for the period t outcome for the group ($\text{win}_{G,t}$), own contribution in period t, a binary dummy ($\text{groupshare}_{i,t}$) that takes the value 1 if own contribution in period t is higher than one-third, and the absolute difference in contribution between the two contesting groups as denoted by $|(e_G - e_{G^-})_t|$. The last variable is interacted with a leading group dummy ($\text{lead}_{G,t}$) which takes value 1 if total allocation in own group account exceeds total allocation in the rival group's account, and zero otherwise.[12] The odds ratio (OR) and 95% confidence intervals (CI) are reported in Table 4.5. An OR less than 1 indicates that the outcome associated with value 1 of the dependent variable is more likely (i.e. contribution in period $t+1$ is different from contribution in period t).

The results indicate that winning the inter-group contest in period t is less likely to be associated with an adjustment in contribution in period $t+1$. In general, higher contribution in period t is likely to be followed by an adjustment in the subsequent period.[13] The difference in group contributions also turn out to be insignificant for the decision to adjust contribution. The highest number of period-to-period adjustment in contribution is observed in the IN-C treatment. Both IN-OUT and NONE treatments exhibited higher number of period-to-period adjustments under the R condition, while the IN treatments exhibited higher number of adjustments under the C condition. The step I model doesn't say anything about the direction of adjustments.

In contrast, the Step II model considers only those observations for which there is an adjustment in contribution from the previous period. The dependent variable in the mixed effect logit regression presented in Table 5 takes a value of 1 (0) if contributions are adjusted upwards (downwards). The set of explanatory variables remain the same as the step I model except for lagged own contribution, which has a consistent negative effect on subsequent contribution but has been dropped for the sake of computational convenience.[14]

[12] We also ran a more detailed model with three-way interaction between period t contest outcome, the leading group dummy, and the absolute difference in group contributions, which returned a qualitatively similar result.

[13] An alternative model where lagged own contribution was dropped indicated predictive power of the in-group share dummy in most treatments. However, this significance was lost once controlled for lagged own contribution.

[14] The nested mixed effect logistic regression including lagged own contribution fails to achieve convergence when error covariances are clustered at the group level. A random effects logit including lagged own contribution and error clustering at the individual participant level indicates that a higher contribution in the previous period

Table 4.5: The binary outcome variable indicates dynamic adjustment in individual contribution. Each cell gives the corresponding Odds Ratio from mixed logit regression, with confidence intervals inside parentheses.

1 if $e_{i,t+1} \neq e_{i,t}$	C			R		
	IN-OUT	IN	NONE	IN-OUT	IN	NONE
Intercept	0.69 $(0.2-1.7)$	0.9 $(0.3-2.4)$	0.54 $(0.2-1.6)$	0.73 $(0.3-1.9)$	0.95 $(0.4-2.5)$	1.38 $(0.5-4.1)$
$win_{G,t}$ (win=1, loss=0)	0.36*** $(0.2-0.6)$	0.66 $(0.4-1.0)$	0.40*** $(0.2-0.6)$	0.52*** $(0.3-0.8)$	0.47*** $(0.3-0.7)$	0.33*** $(0.2-0.5)$
$e_{i,t}$	1.01 $(0.9-1.0)$	1.07*** $(1.0-1.1)$	1.05*** $(1.0-1.1)$	1.05*** $(1.0-1.1)$	1.05*** $(1.0-1.1)$	1.01 $(0.9-1.0)$
$lead_{G,t}$	0.65 $(0.3-1.3)$	0.97 $(0.4-2.3)$	0.63 $(0.3-1.3)$	1.39 $(0.6-3.0)$	1.02 $(0.5-2.1)$	0.87 $(0.4-2.0)$
$\|(e_G - e_{G^-})_t\|$	1.00 $(0.9-1.0)$	1.01 $(0.9-1.0)$	0.99 $(0.9-1.0)$	1.01 $(0.9-1.0)$	0.99 $(0.9-1.0)$	1.00 $(0.9-1.0)$
$\|(e_G - e_{G^-})_t\|$ * $lead_{G,t}$	1.02 $(0.9-1.0)$	0.99 $(0.9-1.0)$	1.01 $(0.9-1.0)$	0.97 $(0.9-1.0)$	1.00 $(0.9-1.0)$	1.00 $(0.9-1.0)$
$groupshare_{i,t}$	1.99*** $(1.0-3.8)$	0.67 $(0.3-01.4)$	1.17 $(0.6-2.3)$	0.74 $(0.4-1.3)$	1.03 $(1.5-2.0)$	1.79 $(0.9-3.6)$

The main result from step II is that individuals with higher within-group share of contribution have a significantly lower intercept in all treatments. An alternative model replacing the within-group share dummy with the exact magnitudes of the lagged within-group shares also indicates a negative slope for all treatments, with a steeper slope in the IN treatments.[15] Favourable outcome in the previous period was likely to reduce contribution in the IN-C treatment only, and insignificant for all other treatments. A higher intergroup difference increased contribution among members of the trailing groups in IN-R and reduced contribution among members of the leading groups in the IN-OUT-C treatment. Members in the leading group

significantly reduces contributions in the following period in all treatments except IN-OUT-C. The within-group share dummy for higher-than-average contributors is still significant in all treatments except NONE-C and indicates stronger marginal effect under the R condition.

[15]The ideal model would include both the within-group share dummy, and the exact magnitude of share. However, the nested logistic regression fails to converge under this specification. Running two separate regressions for the two levels of the within-group share dummy indicates a steeper slope for the group with lower-than-average contribution.

Table 4.6: The binary outcome variable indicates the direction of dynamic adjustment in individual contribution. Each cell gives the corresponding Odds Ratio from mixed logit regression, with confidence intervals inside parentheses.

1 if $e_{i,t+1} > e_{i,t}$	C			R		
	IN-OUT	IN	NONE	IN-OUT	IN	NONE
Intercept	3.43^{***} (1.4 – 8.4)	4.07^{***} (1.9 – 8.3)	2.28^{**} (1.05 – 4.9)	3.23^{***} (1.6 – 6.5)	4.92^{***} (2.2 – 10.8)	2.30^{**} (1.1 – 4.7)
$win_{G,t}$ (win=1, loss=0)	1.09 (0.6 – 2.1)	0.52^{**} (0.3 – 0.9)	0.61 (0.4 – 1.0)	0.65 (0.4 – 1.1)	0.58 (0.3 – 1.0)	1.6 (0.9 – 2.6)
$lead_{G,t}$	1.21 (0.4 – 3.4)	0.72 (0.3 – 1.7)	0.78 (0.3 – 1.9)	0.58 (0.2 – 1.5)	0.60 (0.2 – 1.5)	0.32^{**} (0.1 – 0.8)
$\lvert(e_G - e_{G^-})_t\rvert$	1.02 (0.9 – 1.0)	1.02 (0.9 – 1.0)	1.0 (0.9 – 1.0)	1.0 (0.9 – 1.0)	1.02^{**} (1.0 – 1.1)	1.0 (0.9 – 1.0)
$\lvert(e_G - e_{G^-})_t\rvert$ * $lead_{G,t}$	0.93^{***} (0.8 – 0.9)	0.98 (0.9 – 1.0)	0.98 (0.9 – 1.0)	0.98 (0.9 – 1.0)	0.96 (0.9 – 1.0)	1.00 (0.9 – 1.0)
$groupshare_{i,t}$	0.14^{***} (0.1 – 0.3)	0.11^{***} (0.0 – 0.2)	0.41^{***} (0.2 – 0.8)	0.15^{***} (0.1 – 0.2)	0.11^{***} (0.0 – 0.2)	0.28^{***} (0.1 – 0.5)

were likely to reduce contributions in the NONE-R treatment. The models are not comparable across treatments and the findings do not provide much evidence in support of our hypotheses, nevertheless they indicate towards a conforming-to-the-average behaviour at both intra-group and inter-group level, which is consistent with the intra-group constraint. □

Figures 4.5 plots non-zero adjustments in individual contributions against own share in group contribution in the previous period. The dashed vertical line divides the plot at one-third share. The darker dots on the right of the divider indicate individuals with a higher-than-average contribution, and the lighter ones on the left indicate individuals with lower-than-average contribution. The fitted regressions and the distribution of the scatter points across the four quadrants clearly show that individuals with a higher(lower)-than-average contribution in the last period were more likely to reduce (increase) contribution in the current period. The slopes are similar in all treatments. Whether these adjustments in response to the lagged intra-group share indicate aversion to exploitation by other in-group members or a preference for conforming to the average is unclear however.

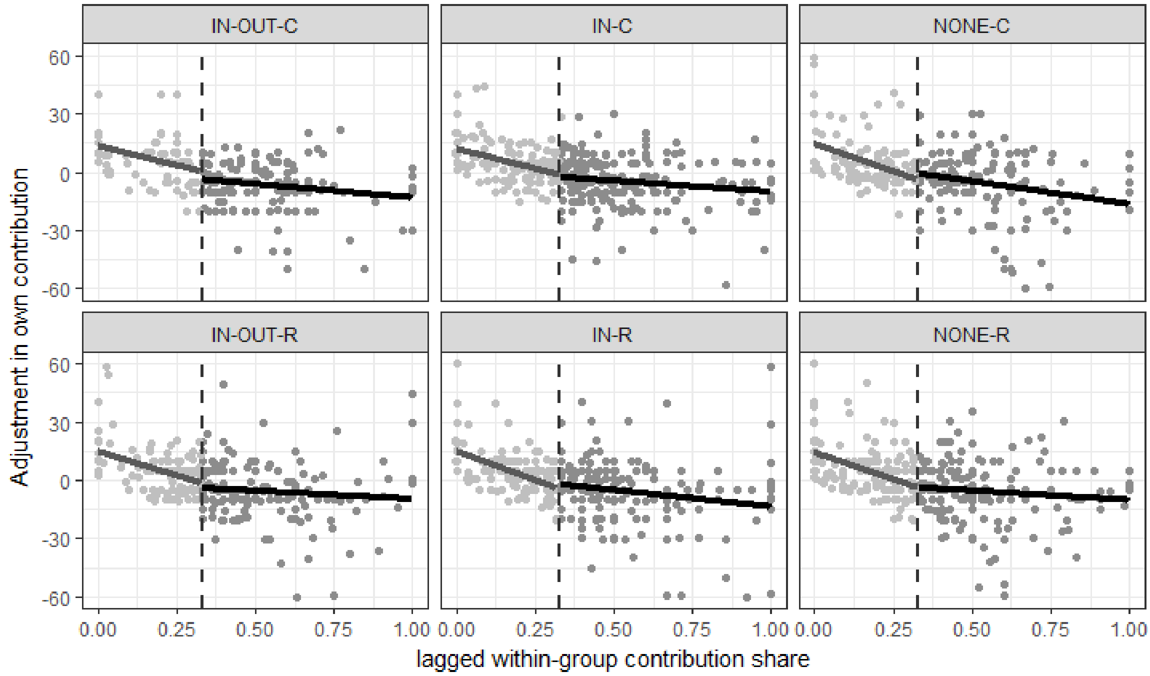


Figure 4.5: Non-zero adjustment in contributions plotted against own share in group contribution in the previous period.

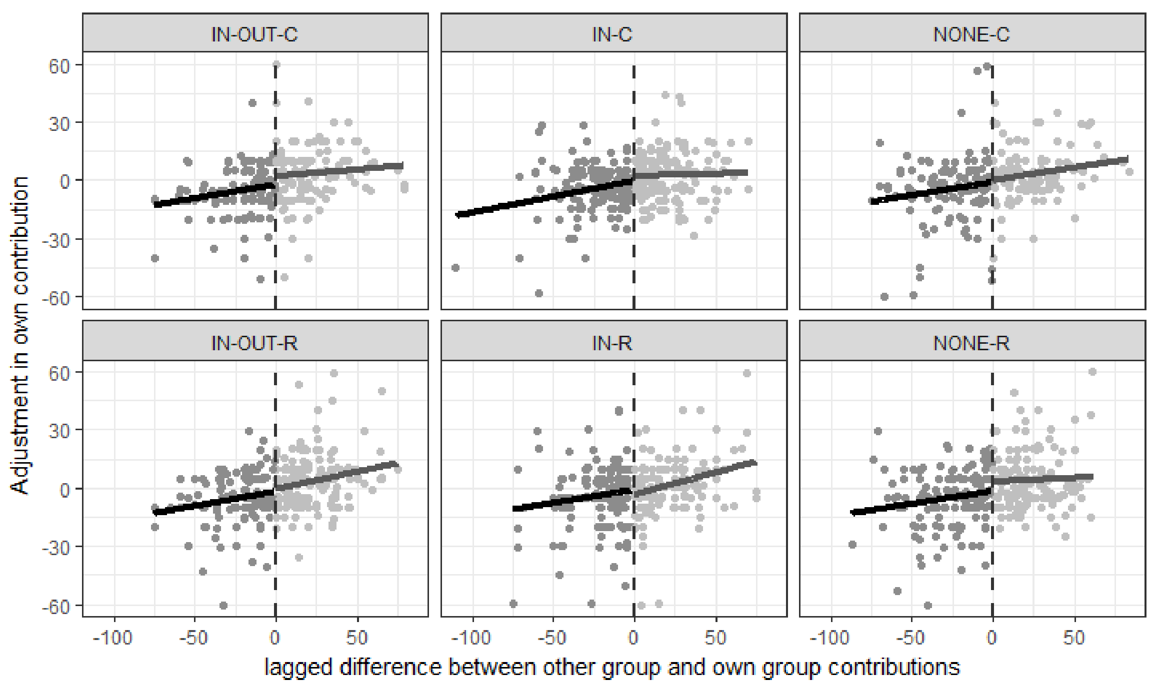


Figure 4.6: Non-zero adjustments in contributions plotted against the lagged difference between opponent group and own group contributions.

Figure 4.6 plots non-zero adjustments in individual contributions against the lagged difference between opponent group and own group contributions on the horizontal axis. A dashed vertical line divides the plot at zero lagged intergroup difference. The darker dots on the left of the divider indicate individuals in leading groups and the lighter ones on the right indicate individuals in the trailing groups. The regression line fitted to the scatter points to the left of the vertical divider shows that contributions in all treatments adjust downwards as own group contribution exceeds opponent group contribution by a higher margin. A weak upward adjustment is observed among members of the trailing groups. The upward slope among the trailing groups is stronger in the R condition for IN-OUT and IN variations, but weaker for the NONE variation. The negative slope among the leading groups is similar across all treatments.

4.5.4 Impact of group motives on dynamic contribution (for a given identity condition)

Next, we investigate the effect of the different sets of group motives as elicited by the three matching variations for any given identity condition. Figure 4.7 plots average contributions over time for all three group-motive conditions under each identity condition. Contrary to hypothesis 1, contributions are not highest in the IN-OUT treatment. Under concealed identity, average contributions in all treatments are very similar across all periods.

Under revealed identity, average contributions in the IN treatment start high but falls steeply and stay below average contributions in IN-OUT from Period 7 onwards. But there is no significant difference in average contributions across treatments. Unlike under concealed identity, contributions under revealed identity exhibit a weaker and insignificant negative time trend in the IN-OUT treatment, as is also evident form Table 4.4. One possible explanation is that comparison with in-group members reduce contributions over time when identity information is unavailable, but under revealed identity, the presence of a salient out-group has a moderating impact on this downward adjustment. This explanation is also supported by the findings from a modified specification of the step II regression for each identity condition. This specification incorporates interactions between the within-group share dummy and the matching variation dummy, and also a three-way interaction between intergroup difference, the leading group dummy and the matching variation dummy. A higher-than-average group share significantly reduces the probability of an upward adjustment under both C and R conditions, but the lagged intergroup difference is likely to result in a lower contribution only under C.

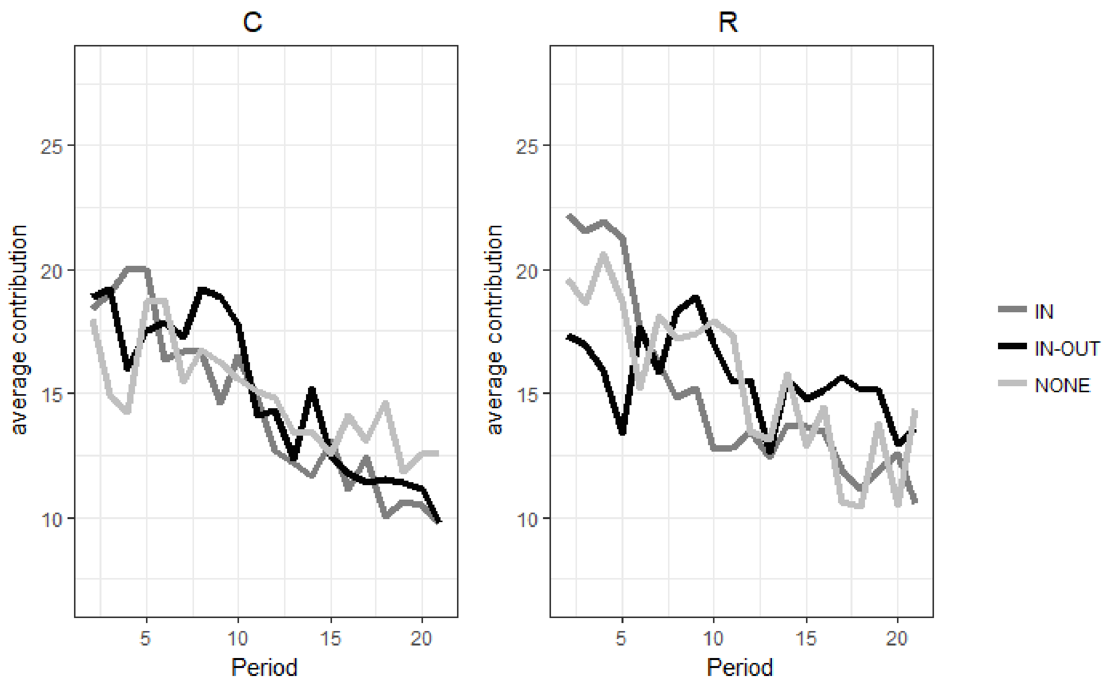


Figure 4.7: Average contribution from comparable players in the different group motive treatments, for a given identity condition.

4.5.5 Impact of identity on dynamic contribution (for a given group-motive condition)

Figure 4.8 plots average contributions over time for both identity conditions under each group motive condition. Under concealed identity there is a steeper fall in contributions over time in the IN-OUT treatment. Contributions across C and R conditions are very similar in both IN and NONE treatments.

The lack of variations across identity conditions may be attributable to the minimal identity paradigm. From the findings in Result 2, we find stronger downward adjustment in all treatments under the R condition, but the similarity of coefficients across C and R in all matching variations also indicate that social identity has no effect on the out-group bias. It is possible that the findings will differ in the presence of a stronger social identity.

4.5.6 Relative payoff maximisation vis-á-vis conformity with average behaviour

To further elaborate upon the findings from the regressions in subsection 4.5.3, we look at the average deviation of observed individual contributions from three separate benchmarks, namely the RPM individual best-response (assuming ex-post symmetry under IN-OUT and

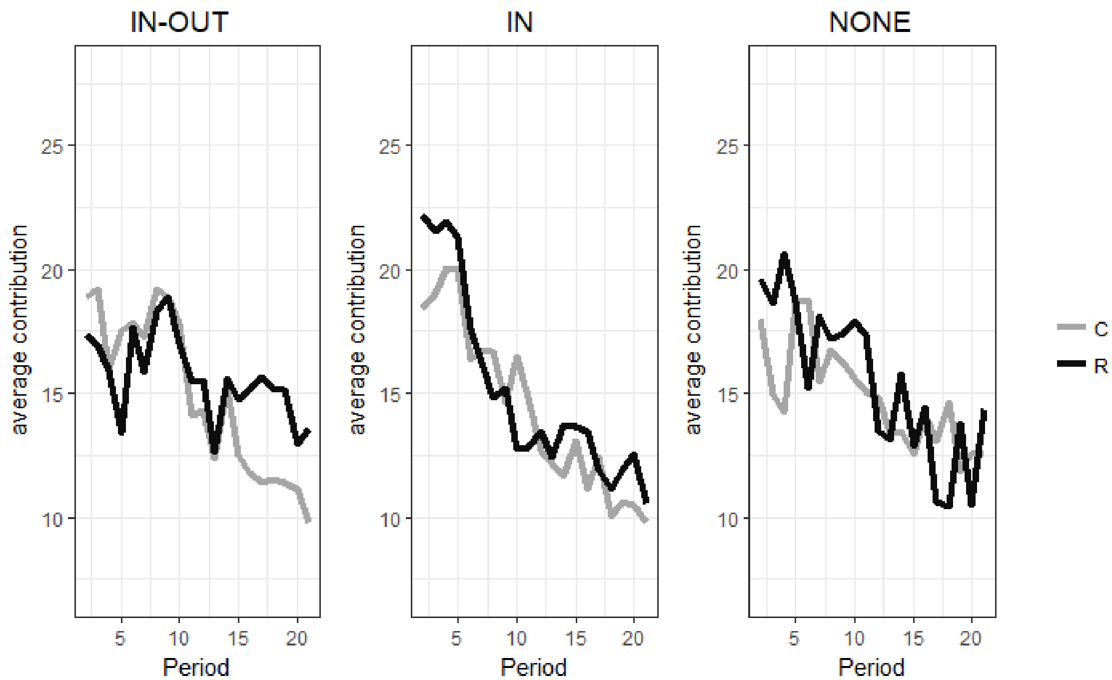


Figure 4.8: Average contribution from comparable players for varying identity conditions, for given group motive conditions.

IN), average contribution across all other in-group and out-group members (global average), and average contributions from the two in-group members alone (in-group average). Figure 4.9 compares each of these measures across the three group-motive conditions for each identity condition.

Average deviation from RPM best-responses is fairly large in all group-motive conditions, though decreasing over time. Given the comparable rival group contributions, the claim in proposition 4.1 (i) that the group best-response in IN-OUT condition exceeds the group best-response in the IN condition, is consistent with the lower deviations under IN-OUT in the top panel. Average deviation from the best-responses is mostly lower in IN than NONE, which is also in line with Proposition 4.1(iii), although the measure of individual best responses under NONE are calculated using the actual contributions from the historical in-group members.

The middle and the lower panel consider average absolute deviations from the global and in-group averages respectively. The deviations are always positive therefore. However, we are only interested in comparing the deviations across the group-motive conditions. Such deviations are highest in the NONE condition under both C and R, while average deviations are similar in the IN-OUT and IN conditions.

Wilcoxon rank sum tests of the average deviation from the respective benchmarks indicate that average deviations from the best-response benchmark are significantly different from zero in all conditions, while average deviations from the average contribution benchmarks are not significantly different in any treatment. However, the patterns in average deviations from the global and in-group averages across the matching variations, as observed in figure 4.9 could not be verified with the standard statistical procedure. Based on this weak evidence and the lack of evidence in support of group-bias, our conjecture is that subjects use the decisions from others, whether present in real-time or not, as a cue for making their own contribution decisions.

4.5.7 group attitudes and other factors

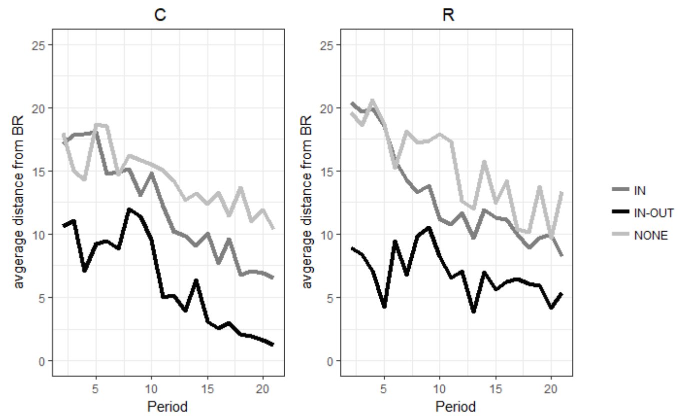
Result 4.3. *Under concealed condition, individual contribution in the IN treatment exhibits negative association with the group attitude index. Under revealed condition, individual contribution is positively associated with the group attitude index in the IN treatment and negatively associated with the group attitude index in the IN-OUT treatments.*

Support: The group attitude index (GA) is built as a Likert-scale measure from the group attitude questionnaire (refer to Appendix C). The Cronbach's α is sufficiently high (0.81), indicating internal consistency. We regress average individual contributions in each group-motive condition on the group attitude scale, identity condition, and the interaction between the two (Table 4.7). The group attitude index is found to be negatively associated with individual contribution in the IN treatment, but not significant in either of IN-OUT or NONE variations. Individual contributions averaged over all periods exhibit positive association with revealed identity in the IN-OUT variation and negative association with revealed identity in the IN condition, but these are not statistically significant. However, it is important to note that the respective coefficients for the interaction between revealed identity and the GA scale are significant for both IN-OUT and IN, and indicate association in opposite directions. That is, any positive effect of revealed identity in the IN-OUT condition is moderated by a stronger group attitude whereas the negative impact of group identity in the IN condition is also moderated by a stronger group attitude. It is important to note in this relation that the median group attitude was relatively lower in the IN treatments.[16] Neither identity condition nor group attitude were significant in the NONE variations.

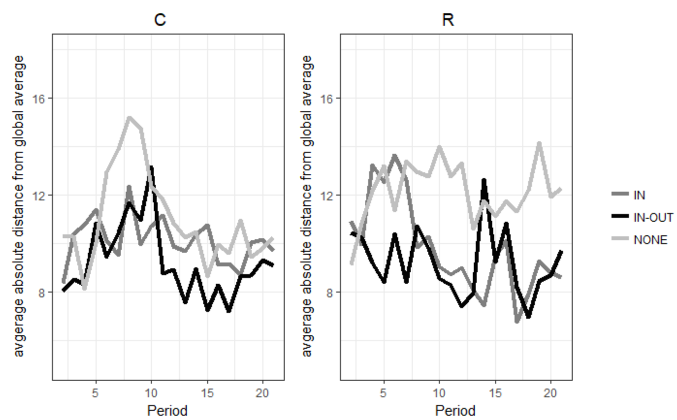
□

A random effect regression of individual contributions averaged over all periods on demographic factors such including risk attitude, ambiguity attitude, gender and CRT indicate

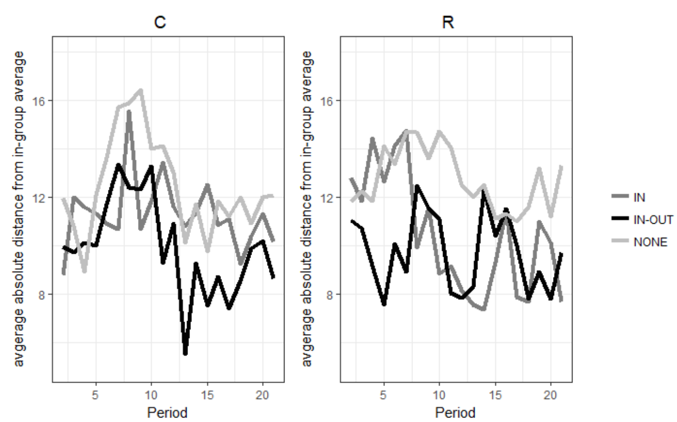
[16]The median score on the GA scale were 3.16 and 3.25 for the IN-OUT-C and IN-OUT-R treatments respectively, and 3.00 and 2.91 in the IN-C and IN-R treatments respectively.



(a) Average deviation from RPM best responses



(b) Average absolute deviation from global averages



(c) Average absolute deviation from in-group averages

Figure 4.9: The above panels show the average difference between actual individual contribution and several benchmarks for individual contribution. In the first panel, individual best response (BR) is calculated assuming ex-post in-group symmetry within the RPM framework under IN-OUT and IN. Global average means the average contribution from the in-group and out-group members, while in-group average means the average contribution from in-group members only.

Table 4.7: Mixed-effect regression of individual contributions on identity condition and group attitude index

Group Condition	IN-OUT	IN	NONE
Revealed (R)	1.71 (1.51)	0.86 (2.71)	-0.66 (3.05)
GA	0.19 (0.26)	-1.01^{***} (0.24)	0.19 (0.46)
R X GA	-0.83^{**} (0.35)	0.86^{**} (0.37)	0.31 (0.62)
Intercept	13.91^{***} (1.51)	12.59^{***} (1.91)	13.01^{***} (2.16)

that individual contributions fall with CRT, which is consistent with evidence in existing literature (Cox, 2017; Sheremeta, 2018). A weak negative association of contributions is also found with higher ambiguity aversion. But no impact of risk attitude or gender were observed in our data.

4.6 Discussion

Conflicts are very different from other social dilemmas like a public good game or a prisoner's dilemma. Although the latter games can address the conflict between individual and group interests, only contests highlight the additional prospect of a 'win' or a 'loss', which has been proven to significantly impact behaviour. The desire to win a contest often makes people fight relentlessly despite the possible uncertainty of outcome and sunk effort costs. Worldwide evidences of human conflicts demonstrate how aggression in conflict often surpasses any rational bounds. The existing literature in economics and social psychology overlooks this contextual difference between a conflict and a social dilemma, and lacks suitable methods for studying co-evolving motivations in a conflict environment. We believe that the present study can make important observations about group attitudes in a group conflict environment rather than general cooperative and non-cooperative games as is more common in the literature. Our experiment also claims methodological contribution to the experimental economics literature. There are other experiments that study social preferences by pairing up active human participants with inactive (Ahmed, 2007) or previously active participants (Herrmann and Orzen, 2008; Cox, 2017), but this design element has not been used to study group attitudes. Our design explores the possibility of employing this technique to control for the presence of in-group or out-group participants. The strategic situation remains the same but without any monetary or non-monetary consequences for in-groups or out-groups, depending on what the respective treatment is.

Our design of complete removal of others is partially inspired by the economic interpretation of other-regarding preferences as a derived utility from the payoff consequences for the respective others. Another possible design choice could be simply removing any payoff consequences of one's decision on others by assigning constant payoffs. However, Sheremeta (2018) argues that value of winning is a significant factor behind aggression in conflict, which may not be removed even if we avoid payoff consequences by assigning constant payoff to the relevant others, as the motive of defeating the outgroup could still be present. Our design has limited external validity as it is never possible to remove in-group or out-group members in a real-world conflict. However, this allows us to completely remove the other-regarding preferences and incentives.

In contrary to our hypotheses based on a plausible theoretical argument and the standard findings in social psychology literature or other group experiments in economics, the results indicate little support for any group-related bias. The presence of real-time active in-group members brings down contest expenditure due to the conformity bias, but we do not find any statistically significant impact of the opponent group's contribution except for weak evidence of conformity bias. Contributions in the one-shot contest is perfectly similar across treatments. Result 2 and 3 discuss the patterns of dynamic adjustments in contributions. Revealing identity in the IN-OUT treatment weakly (though not statistically significant) increases contributions. The adjustment pattern in all treatments provide strong evidence for the conformity bias rather than any group-related bias. However, we do not claim the findings of our experiment to be fully conclusive. The lack of impact of social identity may be due to our minimal identity paradigm. Implementing the present design under a stronger identity elicitation may exhibit stronger group-related bias. The absence of group-related bias is more remarkable. It shows that common fate and common interest among different individuals is not sufficient to induce in-group bias. Individuals grouped under a common goal are more likely conform to the average behaviour in the group rather than seeking to achieve more for the group.

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Appendix

4.A Proof of Proposition for relative-payoff maximizing players

Let us first assume that the a typical player wants to maximize the relative payoff of her own group but is also concerned about not being a sucker within her own group. In other words, we model individuals who have two types of relative payoff concern, they are concerned about their intra-group relative payoff as well as their group's payoff relative to the other group. [17] Player $i \in G$ maximizes her own group's relative payoff subject to her intra-group relative payoff being no less than γ_i. We allow γ_i to be positive or negative. A positive value of γ_i indicates that the player prefers to earn strictly more than the average payoff to her in-group members, while a negative value indicates that the player can tolerate her payoff to be somewhat less than her in-group players' payoffs. A zero value indicates that the player is happy with at least as much as the average payoff to her in-group members. Note that, $\gamma_i = 0$ also imply equal contribution under within-group homogeneity.

Setting $\pi_i^r \geq \gamma_i$, and restricting our attention to a bilateral group conflict, the optimization problem of a relative-payoff maximizing player $i \in G$ can be written as

$$\max_{e_i} \pi_{i \in G}^r = \frac{e_G}{e_G + e_{G^-}} v - \frac{v}{2} - \frac{1}{2}(e_G - e_{G^-}), \quad \text{where } e_G = e_i + \sum_j e_j$$

[17] We assume that individuals prioritise one concern over the other. An alternative modelling approach could be to assign decision-weights to the two concerns,i.e. total utility = own expected payoff + inter-group weight (own groups expected payoff other groups expected payoff) + intra-group weight (own expected payoff other group members expected payoff). However, the problem with assigning decision weights in this fashion is that the weights will be highly correlated, because the intra-group relative payoffs are embedded in the inter-group relative payoff. The qualitative outcome of this model will depend on the inter-group and intra-group weights. This model will be more appropriate for testing group-bias at the individual level, which is not the primary interest here. We are not interested in estimating decision weights. As inter-group weight is zero in the IN treatment, and both inter-group and intra-group weights are zero in NONE treatment by design (that is if the group-matching variations do remove the motives), this model will not be very helpful for constructing our hypotheses. Also, our intuition is that people act based on prioritised motives rather than assigning weights to individual motives.

$$s.t. \sum_j e_j - (m_G - 1)e_i - m_G\gamma_i \geq 0 \text{ and } e_i \geq 0$$

where $j \in G, j \neq i$, and G^- denotes the out-group. All in-group and out-group members being real-time participants in the IN-OUT treatment, we can assume the above optimization problem to be relevant here. Under a weakly binding intra-group constraint ($\sum_j e_j - (m_G - 1)e_i \geq m_G\gamma_i$), the unique group-level best response is given by

$$e^{\text{BR}}_{\text{G,IN-OUT}} = \sqrt{2ve_{G^-}} - e_{G^-}$$

Individual $i \in G$ spends zero effort in equilibrium if $\sum_j e_j \geq \sqrt{2ve_{G^-}} - e_{G^-}$, and spends $e_i \in \left(0, \frac{\sum_j e_j - m_G\gamma_i}{(m_G-1)}\right)$ otherwise.[18]

Note that under the assumption of ex-post symmetry at the inter-group level, which can be justified in view of complete across-group homogeneity, $e_G = e_G^-$ in equilibrium. Under the further assumption of ex-post symmetry, the intra-group constraint $\sum_j e_j - (m_G - 1)e_i \geq m_G\gamma_i$ means $\gamma_i \leq 0$. It is straightforward to show that the pure-strategy equilibrium under the assumption of both within-group and across-group ex-post symmetry calls for a total effort of $e^*_{\text{G,IN-OUT}} = v/2$, with a symmetric individual benchmark of $e^*_{i,\text{IN-OUT}} = v/6$ for $m_G = 3$.

In absence of the inter-group payoff comparison, as in the IN condition, the optimization problem reduces to

$$\max_{e_i} \pi_{i \in G} = \frac{e_G}{e_G + e_{G^-}} v - e_i \quad \text{where } e_G = e_i + \sum_j e_j$$
$$s.t. \sum_j e_j - (m_G - 1)e_i - m_G\gamma_i \geq 0 \text{ and } e_i \geq 0$$

The unique group level best response under a weakly binding intra-group constraint is given by

$$e^{\text{BR}}_{\text{G,IN}} = \sqrt{ve_{G^-}} - e_{G^-}$$

Individual $i \in G$ spends zero effort in equilibrium if $\sum_j e_j \geq \sqrt{ve_{G^-}} - e_{G^-}$, and spends $e_i \in \left(0, \frac{\sum_j e_j - m_G\gamma_i}{(m_G-1)}\right)$ otherwise.

Groups in the IN treatment are matched against groups from the IN-OUT treatment. Had they been playing simultaneously, that is, if a group of individuals with only intra-

[18]If the constraint is strictly binding, then the equilibrium group effort is lower, and complete free-riding by an individual is unlikely unless $m_G\gamma_i = e^*_G|_{(\pi_i = \bar{\pi}_i)}$

group motives were playing with a group of individuals with both intra-group and inter-group motives, then the equilibrium group contributions could be solved for by imputing $e_{G^-} = e^{\text{BR}}_{\text{G,IN-OUT}}$ in the expression for $e^{\text{BR}}_{\text{G,IN}}$. This would give us the equilibrium expenditure in the group of individuals with only intra-group motives as $e^*_{\text{G,IN}} = 2v/9$, and a symmetric individual benchmark of $e^*_{i,\text{IN}} = 2v/27$ for $m_G = 3$.

In the absence of both intra-group and inter-group payoff comparison, an otherwise payoff-maximizing player maximizes the standard group contest payoff function given in (2.2). In other words, (s)he solves the same optimization problem as in the IN condition but without being subject to the intra-group constraint. The individual best response for given levels of e_{G^-} and $\sum_j e_j$ is given by $e^{\text{BR}}_{i,\text{NONE}} = max(0, \sqrt{ve_{G^-}} - e_{G^-} - \sum_j e_j)$.

The individual in the NONE condition is matched with in-group and out-group memebers from the IN-OUT condition. Had they been playing simultaneously, that is, if a group of individuals with both intra-group and inter-group motives were playing against a group of individuals where all but one members have both motives, then we could set $e_{G^-} = e^{\text{BR}}_{\text{G,IN-OUT}}$ and $\sum_j e_j = \left(\frac{m_G-1}{m_G}\right) e^{\text{BR}}_{\text{G,IN-OUT}}$ to solve for the equilibrium. The equilibrium contribution of the single player with no group motives would be given by $e^*_{i,\text{NONE}} = 4v/27$ for $m_G = 3$.

However, only the best responses are of practical interest for our experiment. This is because different individuals or group of individuals in the IN and NONE treatments only make decisions in the face of previously made (unobserved) decisions of individuals with dual motives. Comparison of group-level best responses indicate $e^{\text{BR}}_{\text{G,IN-OUT}} > e^{\text{BR}}_{\text{G,IN}}$ for a given level of e_{G^-} and a weakly binding intra-group constraint under IN-OUT and IN. This is due to the added utility from group level payoff comparison. Comparison with $e^{\text{BR}}_{G,\text{NONE}}$ is not obvious due to $e^{\text{BR}}_{i,\text{NONE}}$ being a function of $\sum_{j\neq i} e_j$. However, if it is assumed that the other two group members had best-responded to the out-group contribution, that is, $\sum_{i\neq j} e_j = \left(\frac{m_G-1}{m_G}\right) e^{\text{BR}}_{\text{G,IN-OUT}}$, then

$$\begin{aligned} e^{\text{BR}}_{i,\text{NONE}} &= \sqrt{ve_{G^-}} - e_{G^-} - \frac{m_G-1}{m_G}(\sqrt{2ve_{G^-}} - e_{G^-}) \\ &= \frac{1}{m_G}\left(\sqrt{ve_{G^-}}\left(m_G - \sqrt{2}(m_G-1)\right) + e_{G^-}\right) \end{aligned}$$

Note that given $e_{G^-} > 0$, a positive best-response is guaranteed for any $m_G \leq 3$, and requires $\sqrt{\frac{e_{G^-}}{v}} > \sqrt{2}(m_G-1) - m_G$ otherwise. The additional assumption of $e_G \leq vm_G$ doesn't rule out the existence of positive best-response for any group size. Further, defining group-level best-response as $e^{\text{BR}}_{G,\text{NONE}} = \sum_{i\in G} e^{\text{BR}}_{i,\text{NONE}}$, which is equivalent to $e^{\text{BR}}_{G,\text{NONE}} = m_G e^{\text{BR}}_{i,\text{NONE}}$ under the assumption of the other two members having best-responded, comparison with the best-response under IN shows that $e^{\text{BR}}_{\text{G,IN}} \geq e^{\text{BR}}_{\text{G,NONE}}$ requires

$$\begin{aligned} \sqrt{ve_{G^-}} - e_{G^-} &\geq \sqrt{ve_{G^-}}\Big(m_G - \sqrt(2))(m_G - 1)\Big) + e_{G^-} \\ \implies \frac{(\sqrt{2}-1)}{2}(m_G - 1) &\geq \sqrt{\frac{e_{G^-}}{v}} \end{aligned}$$

Similarly, comparison with the best-response under IN-OUT shows that $e^{\text{BR}}_{\text{G,IN-OUT}} \geq e^{\text{BR}}_{\text{G,NONE}}$ is true only if

$$\frac{(\sqrt{2}-1)}{2} m_G \geq \sqrt{\frac{e_{G^-}}{v}}$$

Hence, for given group size and value of reward, the group-level best-response under IN-OUT (or, IN) is higher than the group best-response under NONE only if the out-group contribution is not too large.

If, on the other hand, the objective motive under IN-OUT is to maximize intra-group relative payoff π^r_i, subject to an inter-group constraint $\pi^r_G \geq 0$, then the unique pure strategy group-level best-response is greater than the group-level best-response in the former case for $m_G < 5$ and smaller otherwise. In the IN condition, the individual maximizes the intra-group payoff without the inter-group contest, and spends zero effort in equilibrium. So, (i) immediately follows. The intra-group relative payoff comparison being irrelevant in NONE, best response under NONE should be similar to the former case, and so is its comparison with IN-OUT. However, group-level best-response under NONE is higher than that under IN.

4.B Sample instructions

Part I Instruction:

In part I, everyone will be shown 5 pairs of paintings by two artists Paul Klee and Wassily Kandinsky. Each pair will contain one painting by Klee and one painting by Kandinsky. However, there will be no information on which painting belongs to which artist. Each pair of paintings will also be shown in the same order to all the participants. Please state your preferences over each pair of paintings by clicking on the 'Select' button corresponding to your preferred painting. A red check mark appears against your selected painting. Please note that you may not select both the paintings in a pair at the same time and you may not go back to a previous pair at any point. So, please make sure that the red check mark is visible against your preferred painting while confirming your choices by clicking on the 'Confirm' button that appears below each screen. The next pair of paintings will be shown only after you have confirmed your preference over the current pair.

After everybody submits their preferences over all 5 pairs, each of you will be classified as either a KLEE person or a KANDINSKY person. All participants will be privately informed about their individual classification.

Part II Instruction sample (NONE-C)

The second part of today's experiment consists of a decision-making task that is closely described by the following situation. Your role in the actual task will be instructed later. Suppose, there are 2 groups, Group A and Group B, and there is a reward that can be won by only one of the two groups. There are 3 members in each group and each member has 60 ECU in their individual accounts. Each member of a group independently decides how many ECU to allocate to the group account, the allocation can be any number of ECU from 1 to 60 inclusive. The total allocations in the two group's accounts determine which group receives the reward. Each member in the reward-receiving group earns another 60 ECU in their individual account as reward. The final earnings of an individual is the final amount of ECU in their individual accounts.

How is the receiver of the reward determined?

Each penny in Group A's account is exchanged for an 'A' token. Each penny in Group B's account is exchanged for a 'B' token. All 'A' and all 'B' tokens are put into the same box and shuffled well. One token is drawn blindly. If it is an 'A' token, Group A receives the reward. If it is a 'B' token, Group B receives the reward. A computer program performs this blind draw. Number of a group's tokens = Member 1's allocation + Member 2's allocation +

Member 3's allocation.

$$\text{A group's chance of receiving the reward} = \frac{\text{own group tokens}}{owngruptokens + othergrouptokens}$$

Individual Earnings:

$60 -$ Allocation $+ 60$ (reward) ECU, if the group receives the reward.

$60 -$ Allocation ECU, if the group does not receive te reward.

Click 'Continue' to see a hypothetical example illustrating the entire process.

Example

Group A	Initial ECU	Allocation	Group B	Initial ECU	Allocation
Member 1	60	30	Member 1	60	35
Member 2	60	18	Member 2	60	5
Member 3	60	42	Member 3	60	0
Total Allocation		90	Total Allocation		40

Number of A tokens = Total allocation in Group A's account = 90

Number of B tokens = Total allocation in Group B's account = 40

Total number of tokens in the box = $90 + 40$ = 130

Suppose an 'A' token has been drawn. Group A receives the reward. The earnings of the two group's members will be as follows.

Group A	**Final earnings** ECU	Group B	Final Earnings
Member 1	$60 - 30 + 60 = 90$ ECU	**Member 1**	$60 - 35 + 0 = 25$ ECU
Member 2	$60 - 18 + 60 = 102$ ECU	**Member 2**	$60 - 5 + 0 = 55$ ECU
Member 3	$60 - 42 + 60 = 78$ ECU	**Member 3**	$60 - 0 + 0 = 60$ ECU

Part II General Quiz:

1) How many members are there in each group? (Answer: 3)

2) If, in some period, member 1 of Group A allocates 50 ECU and each of the other two members allocates 10 ECU each, who will have the highest earnings in Group A in that period?

a) Member 1

b) Member 2 or Member 3

c) Member 2 and Member 3 (Answer)

3) What is the maximum amount one can allocate to their group account? (Answer: 60 ECU)

4) If, in some period, total allocation in Group As account is 40 ECU and total allocation in

Group B's account is 60 ECU, what chance does Group B have of receiving the reward?

a) 40 out of 100

b) 20 out of 100

c) 60 out of 100 (Answer)

Your Task: Phase I

Other people have previously participated in the decision-making task described above. These previous participants were matched into 3-member groups and each group was matched with another 3-member group as explained before. They had decided their allocations to their respective group accounts, the draw was performed and rewards were allocated.

All participants were paid and the computer had recorded their decisions.

In this session, you will take part in a similar decision-making task, but without any other participants either in your group or in the other group (the group that your group is matched with).

Then what does my group and the other group mean?

Note that any matched pair of groups in the previous session had 6 participants - 3 members in each group. However, for some matched pair of groups, the computer can recall only 2 member' allocations in one group's account and all 3 members' allocations in the other group's account. The computer will place you in the missing person's position for one such pair of groups.

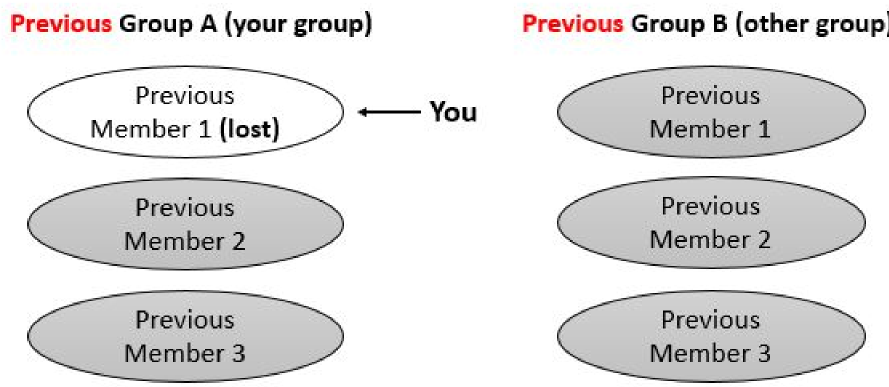
How does it work exactly

Every group is named with a letter of the English alphabet. Think of two groups that were matched in a previous session, say Group A and Group B. The computer can recall the allocation decisions of all members in the two groups except Member 1 in Group A. You will replace that member for the present draw and decide your allocation to the group account.

The following picture demonstrates this.

Your allocation will be added to the pre-existing allocations of the two members of Group A to determine total allocation in your group account. The allocation in Group B's account remains as it was.

Then the two groups' allocations are used for the blind draw. There will be as many 'A'



tokens as the total allocation in your group account and as many ‘B’ tokens as was the total allocation in the previously active Group B’s account. If an ‘A’ token is drawn, you will receive a 60 ECU reward in your individual account. If a ‘B’ token is drawn, you will not receive any extra money. Your earnings in a period will be

$60 -$ Allocation $+ 60$ (reward) ECU, if one of your group’s tokens is drawn.

$60 -$ Allocation ECU, if one of the other group’s tokens is drawn.

Since none of the other members in your group or the other group are participating at present and have already been paid for their previous participation, they will not gain or lose anything due to the outcomes in this session.

Phase I quiz:

Suppose, members 2 and 3 in the previously active Group A had allocated 20 and 30 ECU to the Group account. You become member 1 in this group. Total allocation in the previously active Group B’s account was 40 ECU. If you allocate 10 ECU,

1) How many ECU will there be in your group account? (Answer: 60)

2) What chance will you have of receiving the reward?

   10 out of 50

   b) 10 out of 60

   c) 60 out of 100 (Answer)

   d)10 out of 40

3) How much will member 3 in your group earn if your group receives the reward?

   a) 60 30 + 60 = 90 ECU

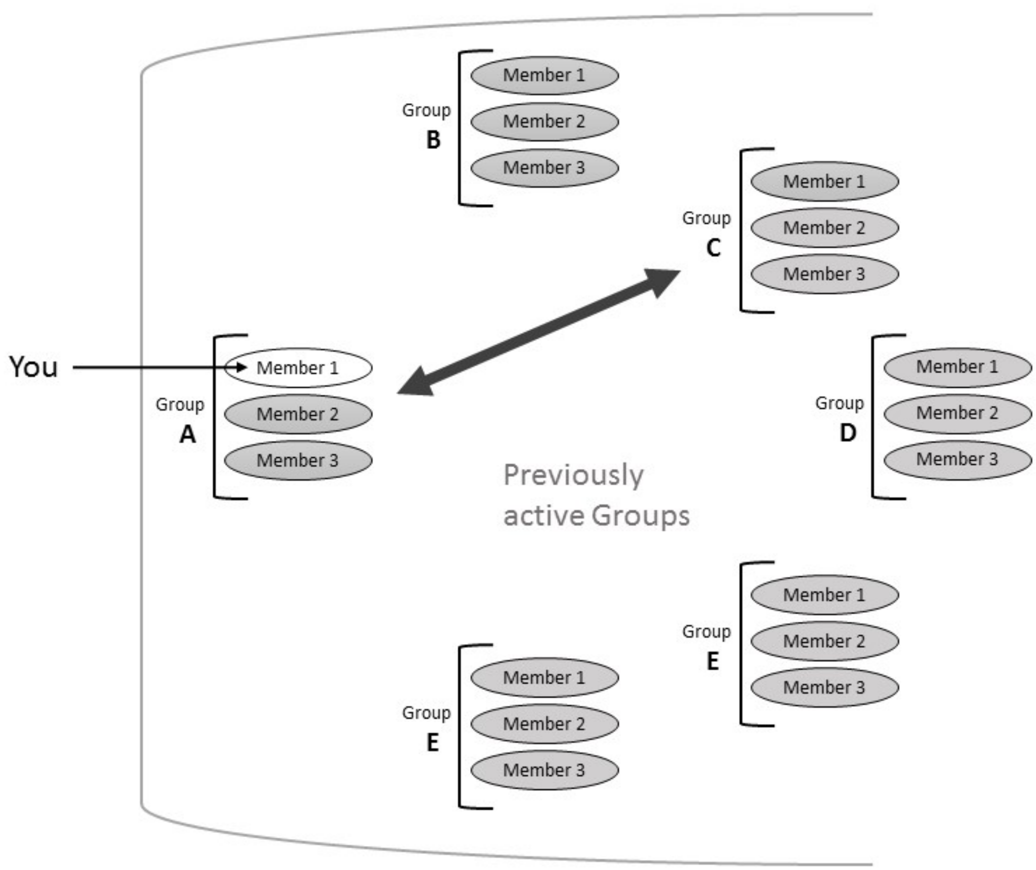
   b) 60 20 + 60 = 100 ECU

   c) Will not earn anything (Answer)

Your task: Phase II

You will participate in the same decision-making task for another 20 periods, though you will be paid for only 4 periods. Those 4 periods will be randomly decided at the end of the experiment. Therefore, each of the 20 decision making periods are equally important.

The previous participants also made their decisions for 20 consecutive periods. Composition of each group remained the same for all 20 periods. Each group was randomly matched with one of five other groups in different periods. The following image shows for example, how Group A was randomly matched with different other groups in different periods.



Suppose, the computer has lost the decisions of Member 1 in the previously active Group A. Your allocation decision will replace that lost decision in every period.

In each period, the pre-existing allocation decisions from members 2 and 3 of Group A will be added to your allocation to determine total allocation in your group account. Allocation in the other group's account will stay as it was in the corresponding period of that session.

You will make your allocation decisions without knowing the previous participants' decisions and you may not be told which other group your group was matched with in any period.

Your earnings from phase II:

At the end of the experiment, a random participant will be asked to pick up 4 balls from a sack containing 20 balls numbered from 1 to 20. The numbers of those 4 balls will determine the 4 periods that will be considered for actual payment. Your period earnings in ECU for those 4 periods will be added to your earnings from Phase I of your task and converted to cash at the end of the session.

Phase II Quiz:

Suppose, following are the total allocations from 5 groups (B, C, D, E and F) in 3 periods in the previous session. The last column gives the total allocation from the two members of Group A, whose decisions the computer can recall. You are placed in this group.

Groups	**B**	**C**	**D**	**E**	**F**	**Group A (member 2 and 3 total)**
Period 1	20	60	5	100	70	10
Period 2	40	60	30	50	50	20
Period 3	50	40	45	30	0	0

Suppose in the previous session, Group A was matched with Group D, F and B in periods 1, 2 and 3 respectively. Please answer the following questions.

1) If you allocate 10 ECU to the group account in period 1, what will be the total allocation in your group account? (Answer: 20)

2) How many tokens would belong to the other group in period 3? (Answer: 50)

3) Suppose your allocation to the group account in period 2 was such that your chance of receiving the reward was 50:50. How many ECU did you allocate? (Answer: 30)

This is a hypothetical scenario. In the actual task, you cannot observe the pre-existing allocations before making your own decisions.

4.C Group Attitude Index

The group attitude questionnaire asked the following questions
•GA1: I am glad to belong to this group.
•GA2: I feel held back by this group.
•GA3: I think this group worked well together.
•GA4: I saw myself as an important part of this group.
•GA5: I didnt consider the group to be important.
•GA6: I think the other members of this group acted as if we were (a) one group (b) separate individuals.

Question 1-5 were adapted from Hinkle et al. (1989) and 6 adapted from Insko et al. (2013). GA4 exhibited a lower correlation with the other items, the overall reliability improved considerably when excluded (Overall Cronbach was 0.74 with and 0.81 without GA4). Also, it was negatively correlated with the scale, while Hinkle et al. (1989) considered it to be a positive item. Thats because in our experiment, the more one contributed the more important one thought of oneself, but that corresponded to relatively lower contribution from the other members. This explains why this item was negatively correlated with the group attitude scale.

The Cronbach reliability of the individual items was as follows

	Std. α	Std.r	r.cor	r.drop	Mean	Non-missing response frequency				
						1	2	3	4	5
GA1	0.60	0.84	0.84	0.75	3.3	0.07	0.13	0.38	0.29	0.13
GA2 (-)	0.70	0.68	0.58	0.51	3.3	0.12	0.37	0.36	0.18	0.08
GA3	0.66	0.80	0.78	0.67	3.1	0.10	0.21	0.29	0.31	0.09
GA4 (-)	0.82	0.17	0.00	- 0.60	2.7	0.09	0.15	0.25	0.36	0.16
GA5 (-)	0.80	0.27	0.11	0.04	3.4	0.16	0.35	0.25	0.16	0.08
GA6a	0.65	0.82	0.84	0.71	2.8	0.14	0.29	0.22	0.30	0.05
GA6b (-)	0.65	0.81	0.82	0.72	2.6	0.03	0.22	0.23	0.36	0.16

Std. is the standardised measure calculated from the inter-item correlations and indicate the overall reliability of the scale if the corresponding item is dropped. The signal/noise ratio for all he items are above 1 but highest for GA4 followed by GA5. *Raw* (omitted) for each item is exactly one percentage point less than the standardised alpha. *Std.r* is the item correlation with the entire scale if each item were standardised, *r.cor* corrects for any item overlap by subtracting the item variance but then replacing this with the best estimate of common variance, and *r.drop* is the correlation of the item with the scale composed by the remaining items. We decided to drop G4 as this negatively correlated with the overall scale, even after reversal.

Conclusion

The four chapters in this thesis look at different contest settings. The second chapter elaborates on a theoretical concept which is used for the experiment contained in the third chapter. The three experimental chapters re-examine established prior findings in experimental behavioural economics in the specific context of simple lottery contests. The first chapter questions the nearly unanimous finding of overbidding in experimental rent-seeking contests. The third chapter addresses the question of behavioural relevance of strategic equivalence results. The fourth question examines the group-bias using a relatively novel experimental design. All three experiments produced results that were contrary to our prior expectations based on existing experimental findings in the related literature.

Chapter 1 showed that an experimental protocol that closely represent a familiar real-world environment increases efficiency in decision-making. This result can be related to existing findings from decision-making experiments which show that a common-language representation of problems often improves decision-making (citations in the main chapter). On the other hand, this questions the reliability of overbidding in contests as a universal phenomenon. The two protocols do not differ in terms of any relevant information content. Instead of relying on any technical exposition of the concept of 'chance', the explicit randomisation protocol exhibits the chosen ticket in each lottery period. This helps quick convergence of bids to the theoretical benchmark, thus increasing expected payoffs to the bidders.

Chapter 3 uses this explicit randomisation to compare bidding behaviour in three strategically equivalent multi-winner contest mechanisms. The three mechanisms, although different in terms of the explicit procedure followed to select the winners, can be described by the same expected payoff function. We ask if these mechanisms can be relied on to produce similar bidding behaviour on the same strategy space. The experimental answer to this question is positive. We also compare these mechanisms with a single-winner contest which have a different strategy space, and generates a different expected payoff function leading to a different best-response function for individuals. However, we set the parameter values in our experiment in a such way that this contest becomes best-response equivalent to the multi-winner contests. Average behaviour is similar across all four contests. This finding indicates behavioural robustness of strategic equivalence in the lottery contest framework.

The finding in chapter 3 should be viewed in light of the finding from Chapter 1 however. The combined findings from these two chapters tell us that the explicit randomisation protocol generates more predictable behaviour, increases efficiency in decision-making, and successfully translates strategic equivalence into outcome equivalence. But is it possible that the three mechanisms will produce different behaviour when implemented using a conventional experimental protocol? - we will need another experiment to find that. The experiments in Chapter 3 of this thesis were conducted first, and we were surprised to observe the outcome equivalence as well as equilibrium behaviour in these treatments. It is then that we decided to specifically examine the explicit randomisation protocol. The decision to replicate this in two different locations among different population was a further attempt to verify its robustness which was also successful.

In Chapter 4, we try to disentangle the group-related motives in an inter-group lottery contest over a public good prize. The experimental removes practical consequences for in-group and/or out-group members to manipulate the group-related motives. This design feature has been used by only two other studies (as cited in the main chapter) to investigate different research questions. This feature itself presented some critical designing challenges. One challenge was to keep the strategic elements unchanged across the treatments. We used a random matching among contest groups which partially protect us against reputation and reciprocation concerns at an inter-group level. In addition, it is important to note that it is in-group motives that we are interested in, not specifically in-group love which has been widely examined in the social psychology literature. In-group motives include all concerns related to in-groups. We present a theoretical model of how group-related concerns may render the three treatments strategically different, However, observed behaviour, which is very similar across all group-motive conditions, do not support our theoretical conjectures. The manipulation in social identity through conditional revelation of the group-matching criterion does not make any difference in observed behaviour either. But this is a very minimal identity manipulation which has worked in some existing experiments and failed in others. Replicating this experiment under stronger social identity manipulations may be a worthwhile idea.