

Introduction to the papers of TWG14:

University mathematics education

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Introduction

TWG14 (*University Mathematics Education*, hereafter UME) was launched in CERME7 (Nardi, González-Martín, Gueudet, Iannone, & Winsløw, 2011) in recognition of the growing area of research in university mathematics education research. This area, although sharing in many cases approaches, methods and research topics with other areas in mathematics education research, has its own distinctiveness: the institutional characteristics at university (or postsecondary education in general) are usually quite different from those in compulsory education and do not always follow national curricular guidelines; the training of teachers (when existing) is also different from the training followed by primary and secondary teachers; there are many cases of classes with a large number of students, and teaching approaches are usually different – the amount of personal work expected from the students is also much higher; students' personal experience and aims are different than in compulsory education; very often mathematical notions and reasoning are dealt with at a higher level of complexity and abstraction; etc.. The fast growth of this research area, both outside and inside CERME, is evident in the breadth of research publications in this field and was recognized by the ERME community, in inviting the three-year leader of this TWG to present a summary of UME research as a CERME10 plenary lecture (Nardi, 2017).

Within CERME, the number of papers submitted to TWG14 has been increasing steadily since its inception. This year, we received a record number of 64 papers, with 47 getting accepted for presentation (of which, 41 are published in the proceedings, together with 17 short contributions). This indicates a substantial increase in comparison to the 31 full and 14 short contributions in TWG14 in CERME9 proceedings (Nardi, Biza, González-Martín, Gueudet, Iannone, Viirman, & Winsløw, 2015). Additionally, the substantial number of papers led to the split of TWG14 into two *isomorphic* groups (TWG14A: 23 accepted papers and TWG14B: accepted 24 papers) which ran in parallel and common sessions during the conference. This introductory paper summarises the works presented in both groups, as well as our common discussions.

Outside of CERME, the number of handbook chapters focusing on UME or dedicating some sections to it (from the pioneering Artigue, Batanero, & Kent, 2007, to the most recent Coupland, Dunn, Galligan, Oates, & Trenholm, 2016; Larsen, Marrongelle, Bressoud, & Graham, in press; Rasmussen & Wawro, in press) acknowledges the recognition of this research area for its specificities, as does the launching of the [*International Journal of Research in Undergraduate Mathematics Education*](#). Moreover, the activities of TWG14 have also led to two major contributions: the creation of the *International Network for Didactic Research in University Mathematics* (INDRUM) with a bi-year ERME topic conference (see [INDRUM2016](#) and [INDRUM2018](#)); and, the publication of a [*Research in Mathematics Education special issue*](#) summarising some of the works presented during CERME7 and CERME8, and discussing the use of *institutional, sociocultural and discursive approaches to research in university mathematics education* (Nardi, Biza, González-Martín, Gueudet, & Winsløw, 2014).

In CERME10 we intended to cement and expand further this work, as well as welcoming contributions from across the board of research approaches and topics: the teaching and learning of advanced university mathematics topics (including proof); transition issues “at the entrance” to university mathematics, or beyond; the training of university mathematics teachers; challenges for, and novel approaches to, teaching mathematics at university level (including the teaching of students in non-mathematics degrees); the role of ICT tools and other resources (e.g. textbooks, books and other materials) in the teaching and learning of university mathematics; assessing the learning and teaching of mathematics at university level; collaborative research between university mathematics teachers and researchers in mathematics education; and, theoretical and methodological approaches to research into the teaching and learning of university mathematics.

In the large number of papers received, we identified some continuities, but also some ruptures, with previous iterations of TWG14. For instance, there is still a large number of papers following sociocultural, discursive, and institutional approaches, although a considerable number of papers using cognitive approaches was also present. Moreover, among the papers focusing on a specific mathematical notion, calculus and analysis are still predominant, although we also received papers discussing other topics (such as group theory, linear algebra, or logic). There is also a small but growing number of papers addressing the teaching and learning of more advanced topics, such as algebraic topology, ring theory, and quantum mechanics. The number of papers addressing the use of mathematics as a service course (i.e., mathematical course offered to non mathematics specialists) is still growing, and papers focusing on engineering were again predominant, although we also received papers addressing the use of mathematics by biologists, economists, and physicists. The number of papers addressing teachers’ practice, training, and knowledge has also grown considerably, showing the increasing interest that this area of research is gaining. Conversely, papers proposing experimental interventions (in particular, using technology) are still rare in the group. Finally, we note that a larger number of quantitative studies were presented at CERME10 in comparison to previous CERMEs. Among the accepted papers, we could identify six main themes (although we are aware that some papers fit in more than one theme): students’ learning of specific topics; students’ experience and affective issues; interventions; didactical transposition and use of resources; mathematics for non-mathematicians; and, teachers’ practices and knowledge. In what follows, we follow the structure of these themes to summarise the main results presented during the conference. Due to the large number of papers and to space limitation, we have not included the content of the short contributions in the summary.

Themes and paper contributions

Students’ learning of specific topics

Nine papers can be seen as contributing to this theme. Aaten, Deprez, Roorda and Goedhart show

the difficulties with applying Lithner's framework in order to analyze 'hybrid' types of students' reasoning when solving integration tasks in undergraduate calculus; the authors argue that this framework may need the addition of reasoning types that make use of some kinds of recall. Biza employs Sfard's commognitive framework to investigate first-year students' meaning-making of the tangent line, finding that they engage with analytical, geometrical and algebraic discourses in their substantiations about tangents, sometimes engaging with more than one discourse in the same response, and sometimes separating them across different responses. Chellougui uses Copi's system of natural deduction as a frame to investigate students' difficulties in producing a valid proof in mathematical contexts that involve multiply-quantified statements (e.g. the definition of an order relation in elementary set theory). Hanke and Schäfer use eight categories of mental images of continuity to show that students' mental images of real-valued continuous functions can be expressed by different forms of communication, which, in turn, depend on whether mental images are used or are explicitly verbalized. Juter investigates how students understand continuity and differentiability (and their links) during and after a calculus course, with a focus on students' choices of representations, both espoused and enacted; her study identifies that only students who preferred formal theoretical representations were able to produce formal proofs, as well as a strong coherence between students' espoused and enacted preferences of representations. Mai, Feudel and Biehler study first-year university students' personal concept definition of a vector; they identify several misconceptions and note that a vast majority of students state geometrical concept definitions that are not fully adequate and may cause conflicts in their learning of linear algebra. Ovodenko and Tsamir describe students' grasp of the notion of inflection point, and offer a detailed classification of reasons students offer to identify a point as an inflection point, and a point as a non-inflection point. In the field of abstract algebra, Ioannou studies discursive shifts in year-2 mathematics students' learning of group theory, drawing attention to some commognitive conflicts between the new discourse and other mathematical discourses, including advanced mathematics (e.g. set theory) and high school mathematics. Thoma and Nardi, also taking a commognitive approach, study first-year students' learning of the notion of variable, drawing attention to commognitive conflicts between the notion of variable, and even the notion of number, between school and university mathematics. Both these papers show that university instructors are aware of common student errors, yet they may not be aware of the conflicts that underlie these errors.

Students' experience and affective issues

Eight papers can be seen as contributing to this theme. Using data from questionnaires and exploratory factor analysis (EFA), Anastasakis produces a typology of seven different types of resources that engineering undergraduates use; he refers to the role of tools within Activity Theory and uses also Wartofsky's categorization of artefacts to propose an interpretation of these resources as seven different modes of action in students when studying mathematics, concluding that the way we usually classify resources does not necessarily reflect the way these resources are used by students. In a study of first-year engineering students' note-taking, Andrà uses a narrative approach where students' notes are seen as re-tellings of a story told by the teacher; she focuses on how the students condense the mathematical content in their notes, and what conditions might prompt students to act as 'scribblers' or 'thinkers'. Bampili, Zachariades and Sakonidis conduct an in-depth analysis of one student's process of transition from secondary to tertiary mathematics studies; they consider this transition from a rite of passage perspective, finding connections between the social, academic and mathematical dimensions of the transition, for instance, an interaction between emotions and the student's reconstruction of her mathematical thinking. Griese and Kallweit report on a quantitative analysis of the relationship between patterns of learning behaviour and examination outcome in first-year engineering courses. Kaspersen, Pepin and Sikko describe a quantitative analytical tool for evaluating students' mathematical identities and investigate the

relationships between mathematical identities and grades in university mathematical courses. Kürten reports on a preliminary course for engineering students, and shows how this course can be designed to influence students' self-efficacy. Liebendörfer, Hochmuth, Biehler, Schaper, Kuklinski, Khellaf, Colberg, Schürmann and Rothe propose a taxonomy of the goals of 44 mathematical learning support services offered by universities in Germany; the taxonomy suggests a range of educational goals (e.g. learning the language of mathematics or strategies for studying) and system related goals (e.g. reduce dropout rates or increase passing rates). Marmur and Koichu investigate the relationships between key affective events and the mathematical discourse in two university mathematics lessons where two similar problems were considered.

Interventions

Five papers can be seen as contributing to this theme. Fredriksen, Hadjerrouit, Monaghan and Rensaa study the introduction of a flipped classroom approach in an engineering course at a Norwegian university, focusing on the emerging tensions when students are introduced to a novel approach with videos and quizzes; using the Cultural Historical Activity Theory (CHAT) approach, they identify some tensions attributed to the changes of rules and expectations, as well as the lack of shared understanding in the community of students about the mathematical topic, their preparation for and participation in the sessions. Hogstad and Isabwe describe the use of a digital tool that combines mathematics and kinematics aiming to help students to better grasp integrals; using the theory of instrumental genesis, they investigate the activity of two groups of students with the tool, and identify the pragmatic and epistemic values of students' techniques for solving some given tasks. Kondratieva and Winsløw develop activities dedicated to helping students relate familiar practical tasks from calculus with theoretical ideas of more advanced courses in analysis; their approach is based on a theoretical model of the calculus-analysis transition, using the notion of praxeology from the anthropological theory of the didactic (ATD), and the associated strategies from Klein, to deal with students' challenges in this transition. Lecorre uses the scientific debate methodology developed by Legrand to design and implement, at the transition between high-school and university, specific tasks on double-quantified statements (the Q^2 -game) that may raise the need for conventions of interpretations before they are introduced through mathematical formalism. Schmitz and Schäfer investigate the potential of designing a course in linear algebra and in analysis using the Abstraction in Context framework to increase students' motivation and ability to engage in concept construction; their results indicate that the new courses seem to help students in the transition from school to university mathematics.

Didactical transposition and use of resources

Two papers can be seen as contributing to this theme. Ghedamsi investigates the mathematical organization of complex numbers in the official textbook in Tunisia at upper secondary level; mainly building on Sfard's three stages of cognitive development and on Duval's theory of semiotic representations, she identifies three didactical variables that can be used to efficiently influence students' activities and learning process of complex numbers when they enter tertiary levels. Jovignot, Hausberger and Durand-Guerrier analyze the implicit complexity of a proof presented in a textbook, which involves the concept of ideal in ring theory; using ATD's construct of praxeology, the mathematical organization related to abstract algebra is modeled into structuralist praxeologies, highlighting the intertwined relationships between algebraic, set-theoretic and logical praxeologies and, as a consequence, the inadequacy of such proof for students' self-learning.

Mathematics for non-mathematicians

Seven papers can be seen as contributing to this theme. Feudel analyses the use of derivatives in economics to introduce cost functions and marginal cost; his data indicate that many of the students

participating in his study, after their calculus course, just identified the derivative as an amount of change, without showing a clear understanding of the differences and connections between the derivative in mathematics and in economics contexts. González-Martín and Hernandes Gomes analyse the use of integrals in the Strength of Materials for Civil Engineering course to introduce the notion of bending moment in the study of beams; using tools from ATD, their analyses show that, even if bending moments are introduced as an integral, the proposed tasks do not mobilise elements related to integrals from a calculus course. Kortemeyer and Biehler investigate the mathematical skills and knowledge required in undergraduate engineering using quantitative and qualitative analytical tools developed particularly for this study. Quéré uses tools from ATD to study French engineers' mathematical needs in the workplace; using data from 237 French engineers, he identifies mathematical notions they use, but also the need of 'mathematical abilities' that allow them to use mathematics not only as a tool. Rensaa uses grounded theory techniques to investigate engineering students' own descriptions of what they mean by 'learning linear algebra'; she identifies an apparent contradiction: to describe what they have learned, students emphasize conceptual more than procedural approaches, but in order to know that they have learned something they refer to solving specific tasks in the discipline. Viirman and Nardi describe a series of activities designed for Norwegian students of biology on biology-related mathematical modeling, and follow the learners' path from ritualized participation in mathematical routines towards more explorative participation; they suggest that highly scaffolded tasks, that explicitly state which routines students should invoke, may inadvertently contribute to students' ritualized participation in mathematical discourse. Wawro, Watson and Christensen analyze one student's meta-representational competence as he engages in solving a quantum mechanics problem involving concepts from linear algebra; they correlate this type of competence with abilities to solve tasks that require thinking in, using, and relating different notation systems from physics to mathematics.

Teachers' practices and knowledge

Ten papers can be seen as contributing to this theme. Branchetti analyses the resources, orientation and goals in the intended practices of a high school mathematics teacher with a PhD in mathematics, in relation to the topic of real numbers; the analysis indicates that orientations concerning the epistemology of real numbers, the goals of mathematics education in the high school and students' conceptions and difficulties lead the teacher to choose a very intuitive approach, missing the opportunity to benefit from his knowledge and expertise as a research mathematician. Cooper and Zaslavsky analyse a case of a mathematician/mathematics educator co-teaching partnership in an undergraduate course on Mathematical Proof and Proving; they find that the mathematician's main concern was with the written proof and its "correctness", whereas the mathematics educator showed a sensitivity to the person behind the proof, and to pedagogical aspects of proof and proving, suggesting that this type of co-teaching might be a way of achieving relevance for teaching in mathematics courses. Farah combines an ATD perspective with a sociocultural approach to identify institutional features that influence and transform the working habits of students in the context of French preparatory classes for business schools; she finds a great stability among the teachers' practices she investigates, these practices being strongly linked to the specific institution in which they occur. Fernández-León, Toscano-Barragán and Gavilán-Izquierdo use the horizontal and vertical mathematisation to study the conjecturing and proving approaches of a research mathematician working in a Spanish university; the analysis suggests that these practices (both in a horizontal and a vertical way) interact with each other when mathematicians create new knowledge. Florensa, Bosch, Gascón and Ruiz-Munzon report on a professional development course for mathematics lecturers in engineering; using tools from ATD and the construct of study and research path, they carried out modelling activities with a group of lecturers, allowing the introduction of some ATD notions to empower lecturers to question and put under vigilance the

dominant epistemology at university. Jaworski, Potari and Petropoulou draw on their previous research to theorise and characterise university mathematics teaching within an Activity Theory perspective, developing an example concerning a lecture course in calculus with first year undergraduates; the Teaching Triad at the micro level of goals and actions, together with Activity Theory at the macro level, are used together to capture the complexity of the teaching situation, addressing for instance the ways the lecturer engages his students and provides for their needs. Meehan, O'Shea and Breen examine 'brief but vivid' accounts of their lectures they wrote during a first-year undergraduate calculus course, and investigate the kinds of decision points they faced and how these decisions were triggered. Pinto observes and analyses the decision making and the shift of choices of an experienced mathematics teacher (Alan Schoenfeld) while he teaches a mathematical problem solving course; the Teaching for Robust Understanding of Mathematics framework (TRUmath) is used to unpack the conflicts that may underlie teachers' dilemmas and to explain their decisions. Püschl investigates the discussion patterns of teaching assistants in Germany with specific focus on how they work on tasks in small group tutorials, suggesting a typology of five discussion patterns of tasks: heuristic, pragmatic, student-oriented, problem-oriented, and minimalistic. Stewart, Thompson and Brady examine one mathematician's thought processes as he taught a course on algebraic topology; adopting the perspective of Tall's three worlds, they investigate how the teacher moves between the formal, symbolic and embodied worlds, and how he uses written handouts to ease students' movement between the worlds, particularly from the embodied to the formal, which he sees as the most challenging for students.

Current developments in TWG14

As we mentioned earlier, one of the themes that has grown considerably in this CERME10 concerns teachers' practices and knowledge. The presented papers offer a variety of theoretical and methodological approaches to study teachers' practices and decision making in the preparation of their teaching as well as during their actual teaching. As evident in the papers discussed in the conference, the investigation of teaching in its complexity seems to demand the use of more than one theoretical perspective. Moreover, some of the papers have proposed ways of collaboration between researchers and teachers towards better research insight as well as further development of teaching through a research-based reflection of teachers on their practice. These works have facilitated the discussion on teacher education and professional development at university level; an area of significant teaching interest that seeks further research.

Regarding students' learning of specific topics, CERME10 contributions continue to deepen our understanding of aspects of students' learning. This year there was more interest in how learning of specific topics can be seen also in relation to students' studying practices that go beyond these topics and specific courses. For example, students' learning can be seen in relation to how they use resources, take notes or experience transition issues.

Furthermore, this year we discussed five papers proposing interventions and reporting on the evaluation of the implementation of these interventions. Also, there were studies of tensions between innovative approaches and students' experiences, especially when these approaches contradict students' expectations. As in previous CERMEs, the number of papers proposing interventions is not high (Winsløw, Gueudet, Hochmuth, & Nardi, in press), and the account of experimental uses of digital technologies is still low. Although a range of studies proposes innovative approaches, more research is needed in this area, particularly studies that go beyond specific contexts and groups of students.

Finally, another growing area in TWG14 concerns mathematics for non-mathematicians. The papers presented this year show different degrees of collaboration with experts from other disciplines, as well as the importance of understanding the needs of these disciplines and their use

of mathematical notions. From research about the learning of specific topics that happens to be conducted, for instance, with engineering students, the field has moved to study specific uses of mathematics by professionals (this year we have had examples from biology, economics, engineering, and physics). We believe that this is an important shift of focus, and we expect to see in the future more papers studying the use of mathematics (and the professional needs) of several categories of professionals, as well as how mathematics can be taught by targeting these professional needs.

Reflection and ways forward

In this concluding section we reflect on the research in UME so far and suggest ways forward in terms of two directions: general questions about teaching and learning at university level; and, the role of mathematics as a service subject.

Regarding the first direction, general questions about teaching and learning at university level, we have noticed that research in UME usually reports on studies conducted in a specific educational context. As a research community, we would like to see more research conducted in joint efforts by colleagues from different countries. Strengthening communication between mathematics educators and mathematicians is also necessary towards collaborative research projects that engage mathematicians and suggest innovative approaches for future practice. There are more occasions recently where researchers in mathematics education are invited by mathematics teachers to share experiences and views and to contribute to curricular development decisions; collaborations of this type are very welcomed by our community.

Furthermore, although there is a considerable number of studies connecting students with mathematics, as well as teachers with mathematics, we would like to see more studies connecting teachers and students (teaching with learning). Aiming towards this connection may lead to new theoretical and methodological developments. Finally, we also discussed that there is a growing body of research about mathematicians teaching non-mathematics students, but there is still little research on how non-mathematicians teach mathematics as a service subject.

Regarding the second direction, the role of mathematics as a service subject, we would like to see more research on the different challenges and priorities that may occur in service courses. To name some examples, in service courses teachers may encounter large and heterogeneous groups; the content is not necessarily in the teacher's research area; there can be consequences for teachers' promotions (between giving a course of his/her specialty or a general course), which may have an impact on their motivation and practices; etc.. There is also little research on the epistemological analyses of what it means to teach mathematics to other disciplines; what makes the use of mathematics necessary in other disciplines; and, why mathematics is used as it is used in other fields. These investigations may lead to the identification of possible ruptures – and conflicts for the students – with how the content is presented in the mathematics courses. Another way forward can come from the use of discursive approaches, which would allow studying the discursive difference between communities. In most of these cases, we see the value of the collaboration of UME researchers with experts of other disciplines towards a research agenda that can address these questions.

Finally, in a general way, we would like to see more research that goes beyond single case studies, as well as research projects that expand small-scale studies to a bigger scale. We also notice that most of the papers in our group address mainly one of the themes we listed earlier, but we see the benefit of research that connects these themes by addressing the complexity of the teaching and learning of mathematics at university level. In all these scenarios, it is possible that mixed-methods studies will become more necessary. Regarding contributions to practice, the accumulated body of

research results in UME should contribute to the development of research-based teacher training programmes for university teachers. Furthermore, there is a growth in the amount of mathematics learning support, and institutions are developing mechanisms to better guide and support students' learning of mathematics; we need to develop research about these new mechanisms offered to students, as well as about their impact and connections to what students learn in lectures.

This brief account of the presentations and discussions held in TWG14 during CERME10 aims to summarise our activities during the conference, as well as to invite the reader to explore the papers (long and short contributions) included in these proceedings. Our exchanges will continue in different fora and we hope to meet again the participants to pursue our discussions and reflections, and to foster collaboration. Until we meet again in CERME11, the next meeting will take place in April 2018; we invite all participants (as well as newcomers) to join us in [INDRUM2018](#).

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