

REVISITING NICHE FUNDAMENTALS WITH TUKEY DEPTH

J. Orestes Cerdeira, Tiago Monteiro-Henriques, M. João Martins, Pedro C. Silva, Diogo Alagador, Aldina M. A. Franco, Manuel L. Campagnolo, Pedro Arsénio, Francisca C. Aguiar, Mar Cabeza

Jorge Orestes Cerdeira

Departamento de Matemática and Centro de Matemática e Aplicações, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Quinta da Torre, 2829-516 Caparica, Portugal.

jo.cerdeira@fct.unl.pt

Tiago Monteiro Henriques

Centro de Estudos Florestais, CEF, Instituto Superior de Agronomia, Universidade de Lisboa, Tapada da Ajuda, 1349-017 Lisboa, Portugal.

Centro de Investigação e de Tecnologias Agroambientais e Biológicas, CITAB, Universidade de Trás-os-Montes e Alto Douro, Apartado 1013, 5001-801 Vila Real, Portugal.

tmh@isa.ulisboa.pt

M. João Martins

Centro de Estudos Florestais, CEF, Instituto Superior de Agronomia, Universidade de Lisboa, Tapada da Ajuda, 1349-017 Lisboa, Portugal.

mjmartins@isa.ulisboa.pt

Pedro C. Silva

Centro de Estudos Florestais, CEF, Instituto Superior de Agronomia, Universidade de Lisboa, Tapada da Ajuda, 1349-017 Lisboa, Portugal.

pcsilva@isa.ulisboa.pt

Diogo Alagador

CIBIO/InBio-UE: Centro de Investigação em Biodiversidade e Recursos Genéticos, Universidade de Évora, 7000-890 Évora, Portugal.

alagador@uevora.pt

Aldina M. A. Franco

School of Environmental Sciences, University of East Anglia, Norwich NR4 7TJ, UK.

a.franco@uea.ac.uk

Manuel L. Campagnolo

Centro de Estudos Florestais, CEF, Instituto Superior de Agronomia, Universidade de Lisboa, Tapada da Ajuda, 1349-017 Lisboa, Portugal.

mlc@isa.ulisboa.pt

Pedro Arsénio

Linking Landscape, Environment, Agriculture and Food, LEAF, Instituto Superior de Agronomia, Universidade de Lisboa, Tapada da Ajuda, 1349-017 Lisboa, Portugal.
arseniop@isa.ulisboa.pt

Francisca C. Aguiar

Centro de Estudos Florestais, CEF, Instituto Superior de Agronomia, Universidade de Lisboa, Tapada da Ajuda, 1349-017 Lisboa, Portugal.
fraguiar@isa.ulisboa.pt

Mar Cabeza

Centre of Excellence in Metapopulation Research, Department of Biosciences, University of Helsinki, Finland.
mar.cabeza@helsinki.fi

Corresponding author: Jorge Orestes Cerdeira

Type of article: Research article

Running headline: Revisiting niche fundamentals

1 **ABSTRACT**

2 1. The first attempts to describe species ecological niches were simple geometric
3 procedures that depict the niche boundaries directly from environmental data. The
4 convex hull was one of such procedures, popular for its simplicity, clear
5 ecological rational and precise definition of the niche. However, it lacked the
6 ability to differentiate areas of the niche with different probabilities of occurrence
7 according to environmental suitability.

8 2. We incorporate the Tukey depth, a mathematical tool to measure the centrality of
9 a point within a cloud of points on a multidimensional space, in the convex hull
10 approach to (i) propose a new procedure (CH-Tukey) to estimate species'
11 environmental suitability, and (ii) estimate niche overlap coherently. In addition
12 to a clear ecological rational and simplicity the CH-Tukey procedure has a
13 number of attractive features: use of presence-only data; independence from
14 background data; invariance to scale; robustness to outliers; and the
15 decomposition of the niche into a finite number of iso-suitability levels,
16 permitting the computation of consistent overlap indices. We illustrate the use of
17 CH-Tukey, using occurrence data of the main *Quercus* species and subspecies
18 from Western Mediterranean Europe, comparing its outputs with BIOCLIM and
19 MaxEnt.

20 3. Results showed distinct niche geometries among the different approaches.
21 BIOCLIM produced rectilinear niches reflecting the assumption that ecological
22 variables are independent in their action on the species. CH-Tukey, relaxing this
23 assumption, adjusts niche outer boundary and the inner suitability levels to the

1 known occurrences. MaxEnt produced unbounded niche geometries, showing
2 abrupt shifts in the species response to the environmental variables.

3 4. The niche predictions obtained with geometric approaches, BIOCLIM and
4 CH-Tukey, are simpler but better aligned with Hutchinson's niche concept than
5 those obtained with MaxEnt, this latter showing ecologically implausible
6 relationships with the environmental variables. CH-Tukey and the related overlap
7 measures provide an adequate tool to explore niche properties and species-
8 environment relationships.

9

10 **Keywords:** Depth functions, environmental envelope, niche modelling, geometric
11 procedures, Hutchinsonian niche, niche overlap, Quercus species.

12

13 **INTRODUCTION**

14 Hutchinson's niche concept (Hutchinson, 1957) inspired ecological analyses immediately
15 after its proposal (Whittaker & Levin, 1975), with numerous applications and methods
16 emerging after Hutchinson's seminal paper. Applications are broad, including: studies of
17 resource partitioning in ecological communities (Schoener, 1974), methods on predictive
18 vegetation mapping (Franklin, 1995), explorations of the relation between genome
19 changes and niche differentiation (Rocap *et al.*, 2003), reflections on plant coexistence
20 theories (Silvertown, 2004), or even in taxonomy (Raxworthy *et al.*, 2007). Several other
21 associated ecological concepts followed from Hutchinson's niche concept, e.g.: the guild
22 (Root, 1967), niche breadth and niche overlap (see Colwell & Futuyma, 1971; Hurlbert,

1 1978), niche width (see Roughgarden, 1972), the regeneration niche (Grubb, 1977), the
2 ontogenetic niche (Werner & Gilliam, 1984), persistence niche (Bond & Midgley, 2001),
3 niche conservatism/dynamics (Peterson *et al.*, 1999; Pearman *et al.*, 2008). The uses of
4 the Hutchinson's n-dimensional hypervolume concept across ecology and evolution have
5 been recently surveyed and discussed by Blonder (2017).

6 In recent years, the most conspicuous application of the niche concept has been the
7 plethora of methods developed to estimate the suitability of ecological conditions to
8 species, or eventually, species distributions (Guisan & Zimmermann, 2000; Franklin,
9 2010; Ahmed *et al.*, 2015). Generalized Linear Models (GLM), Generalized Additive
10 Models (GAM), Maximum Entropy Modelling (MaxEnt), Mahalanobis distance, point-
11 to-point using Gower metric (DOMAIN), Ecological-Niche Factor Analysis (ENFA),
12 among others, produce continuous predictions, raising further challenges. For instance,
13 they depend on thresholds to delineate a precise niche boundary (see Elith *et al.*, 2006;
14 Tsoar *et al.*, 2007; Blonder *et al.*, 2014). Some methods, such as GLM, GAM, MaxEnt,
15 Genetic Algorithm for Rule-set Production (GARP) and artificial neural networks, even
16 after applying user-defined suitability/probability thresholds, may produce unlimited
17 hypervolumes (see Blonder *et al.*, 2014). Although some of those methods fit data well,
18 they may exhibit species-environment relationships with lack of ecological plausibility
19 (Brewer *et al.*, 2016). Addressing this critical issue and aiming at a better relationship
20 with ecological theory, Brewer *et al.* (2016) proposed a new parametric approach, the
21 Plateau climate envelope. A non-parametric procedure, also ecologically meaningful, is
22 BIOCLIM, the very first geometric approach to the niche. Revisiting and improving such
23 geometric approaches can result interesting, as will be shown hereafter.

1 BIOCLIM (Nix, 1986; Busby, 1991) was one of the first attempts to formalize the
2 Hutchinsonian niche of a species and to produce maps of species potential distributions in
3 a systematic way (Booth *et al.*, 2014). It is a simple geometric procedure, which identifies
4 the range of suitable values for each environmental variable, independently of the other
5 variables, thus, defining the niche as the hyperrectangle bounded by the minimum and
6 maximum values of the environmental variables for the species known occurrences. This
7 "range" notion within each environmental variable is implicit in Hutchinson's article and
8 is central to the niche concept (i.e. the species is expected to survive and reproduce within
9 some known pair of limiting values for each variable, and not outside those conditions).
10 To distinguish suitability levels within the niche, Nix (1986) and Busby (1991) used the
11 nested hyperrectangles obtained from percentiles of each environmental variable
12 individually. In this way they incorporated Hutchinson's perception of an optimal part of
13 the niche versus a suboptimal one near the boundaries (Hutchinson, 1957). The most
14 suitable regions are those corresponding to the environmental conditions lying in the
15 innermost hyperrectangles.

16 The simple approach of BIOCLIM exhibits some attractive properties: i) performs
17 geometric operations with straightforward ecological interpretation; ii) delineates precise
18 (limited) boundaries of the niche; iii) only uses presences and does not depend on the
19 background; iv) is invariant to scale (suitability levels do not change under linear
20 transformations of data); v) is robust to outliers (while the outer regions are sensitive to
21 outliers their influence vanishes for the interior); and vi) produces ordinal predictions
22 decomposing the niche into a finite number of iso-suitability regions.

1 BIOCLIM presents however a major drawback: it relies on the assumption that ecological
2 variables are independent in their action on the species. With the aim of relaxing this
3 independence assumption, the HABITAT procedure (Walker & Cocks, 1991) replaced
4 the bounding box envelop used in BIOCLIM by the convex hull of the occurrence points.
5 The convex hull of a set of points defines the minimum convex hypervolume containing
6 all points. Hereafter the construction of the convex hull of occurrences in the
7 environmental space will be referred as the CH-approach. The CH-approach is a
8 comprehensible multidimensional geometric way to circumscribe the niche, but no
9 geometric procedure was used to discriminate suitability levels within it. However, this is
10 achievable in a similar way to the percentile procedure implemented in BIOCLIM by using
11 depth functions, which are procedures to measure the centrality of a point within a data
12 cloud on a multi-dimensional space (Zuo & Serfling, 2000). Cerdeira *et al.* (2014)
13 suggested the use of depth functions to estimate suitability levels for species in the CH-
14 approach. Here we describe and discuss a new method that adds normalized Tukey depth
15 to the CH-approach, and give an application of this proposal to *Quercus* taxa occurring in
16 the Iberian Peninsula, illustrating it further by assessing the overlap of the produced
17 climatic niches. Finally we compare the geometry of the environmental niches produced
18 by CH-approach using Tukey depth with those of BIOCLIM and MaxEnt.

19

20 **METHODOLOGIES**

21 **Tukey depth**

22 The Tukey depth of a point (also called location depth or halfspace depth) is a measure,
23 introduced by Tukey (1975), to generalize the order of quantiles for the multivariate case.

1 Consider a set P of k points of an n -dimensional space (\mathbb{R}^n), with $n > 1$, and an arbitrary
 2 point q . The Tukey depth of point q with respect to P is defined as $d_q(P) = m/k$, where m is
 3 the minimum number of points to remove from P so that q is not in the convex hull of the
 4 remaining points of P . We call m the (Tukey) interiority of point q in P .

5 **(Figure 1)**

6 Figure 1 depicts a set P of 20 points of \mathbb{R}^2 represented by “•”, and three points x , y and
 7 z . The lines passing through each of the points x , y or z , show that removing 1, 3 and 9
 8 points from P (those that are on one of the sides of the line through the point) will leave x ,
 9 y and z outside the convex hull of remaining points of P (those that are on the other side
 10 of the line). It can be easily verified that removing less points would leave x , y or z ,
 11 respectively, inside the convex hull of the remaining points of P . We can therefore
 12 conclude that the interiorities of points x , y and z are 1, 3 and 9, respectively, and $d_x(P) =$
 13 $1/20$, $d_y(P) = 3/20$, and $d_z(P) = 9/20$.

14 From the definition, it follows that the maximum interiority cannot exceed

$$= \frac{n + 1}{2} \tag{1}$$

15 A point q with maximum interiority is a Tukey median point. Point z has interiority 9
 16 and, since $M = 9$, it is a Tukey median of P .

17 Depth functions define a partition of \mathbb{R}^n into iso-depth regions, i.e., regions of points
 18 having equal depth values. Figure 2A shows the iso-depth regions corresponding to the
 19 set of 10 points of \mathbb{R}^2 represented by “•”. Note that the regions of points with Tukey
 20 depth greater than or equal to m/k , $\mathbb{R}^n = \mathcal{D}_1 \supset \mathcal{D}_2 \supset \dots \supset \mathcal{D}_m$,

1 with increasing depth toward the innermost region. The region ($m \geq 1$) could
2 alternatively be defined as the intersection of all convex hulls of the subsets of P with k -
3 ($m-1$) points.

4 Depth values of the points of a set P can give indication on how P is arranged,
5 distinguishing between configurations of points that are concentrated in the “interior”,
6 from those which occur mainly on the “margins”. Silva et al. (2014) have recently
7 established that, if the points in P are uniformly distributed, which is a borderline
8 between the two above configurations, the average $d_q(P)$, for q in P , approximates $1/2^{n+1}$.
9 This value, which only depends on the dimension (n) of the representation space,
10 provides a sound threshold to distinguish configurations where the concentration is in the
11 “interior” (average $d_q(P) > 1/2^{n+1}$), from configurations where points occur mainly on the
12 “margins” (average $d_q(P) < 1/2^{n+1}$).

13

14 **Assessing environmental suitability**

15 Let P be the set of k points in \mathbb{R}^n , each point indicating the n environmental conditions of
16 a location where a species occurs. To every point q of P we calculate the depth $d_q(P)$ and
17 divide it by M/k . With this rescaling, points with interiority M (if any) score 1. We
18 propose to interpret these normalized depth values as a measure of environmental
19 suitability for the species. The value 1 corresponds to environmental conditions of sites
20 that belong to the most inner region of the estimated niche. Note that such
21 environmental conditions might not exist.

1 We call this the CH-Tukey procedure. Under the equilibrium assumption, areas of greater
 2 $d_q(P)$ values correspond to the optimal parts of the niche (sensu Hutchinson 1957), i.e.
 3 areas where species performance is expected to be higher (e.g. growth rate or species
 4 fitness). The innermost region, where $d_q(P)$ reach its maximum value, is the set of Tukey
 5 median points.

6 It should be clear that CH-Tukey procedure assumes unimodal species responses, in
 7 relation to the considered environmental variables. More specifically, CH-Tukey
 8 procedure assumes that the suitability of every point of the line connecting two points q_1
 9 and q_2 of the environmental space, will be at least as large as the minimum between the
 10 suitability of q_1 and q_2 .

11 Our proposal can be viewed as a refinement of the procedure of BIOCLIM to define
 12 different levels of suitability, relaxing the independence assumption. Indeed, the way
 13 BIOCLIM differentiates suitability levels in the bounding box of P , using percentiles of
 14 each environmental variable individually, is actually the output of a depth function that
 15 can be read as follows. The environmental suitability of a location corresponding to point
 16 q on the environmental space, is twice the value of $d'_q(P) = m'/k$, where m' is the
 17 minimum number of points to remove from P so that q is not in the minimum
 18 hyperrectangle containing the remaining points of P . The multiplication by two is a
 19 rescaling so that the environmental suitability of points in the most inner hyperrectangle
 20 equals one. Figure 2B depicts the regions of points with equal d' values corresponding to
 21 the set of 10 points of \mathbb{R} represented by “ ” (the same points as in Figure 2A). Note
 22 that the regions of points for which d' is greater than or equal to $m/k, ' ,$ are also
 23 nested, i.e., $\mathbb{R} = ' \supset ' \supset \dots \supset ' ,$ where $' =$ \dots . The region $' (m \geq 1)$ is

1 the intersection of all bounding boxes of the subsets of P with $k-(m-1)$ points. Therefore,
2 we have: i) $M'=M$ if $n=1$ and $M'>M$ for $n \geq 2$; ii) $M' \supset M$, for $m=1, \dots, M$, since the
3 bounding box of a set of points always contains the convex hull of the same set; or,
4 equivalently, for any q , $d'_q(P) = m'/k \geq m/k = d_q(P)$.

5 (Figure 2)

6 Both procedures differentiate the outer polygon in nested regions of increasing suitability
7 for the interior, CH-Tukey producing tighter regions than BIOCLIM. Points outside the
8 outer polygon are considered unsuitable; points in the margins have small suitability,
9 while the largest suitability is assumed to occur in the inner regions.

10

11 **Assessing the overlap of niches**

12 Several indices have been proposed to assess the niche overlap between two species, by
13 comparing the respective suitability (or presence/absence) on a finite number of points,
14 which correspond to cells of the geographical area (see Warren *et al.*, 2008, for a survey).
15 However, to assess the overlap of the environmental niches of two species all points of
16 the environmental space should be considered and not only the projection into the cells of
17 a map. This is an issue that is particularly important in climate change scenarios. Villéger
18 *et al.* (2011) proposes to evaluate the functional dissimilarity between two (or more)
19 species assemblages, considering the volumes of the union and of the intersection of the
20 convex hulls of the points representing each species assemblage in the space of their
21 functional traits. This approach can be easily transposed to the evaluation of the overlap
22 between the environmental niches of two species, giving an overlap index defined as the

1 ratio between the volume of the intersection and the volume of the union of the convex
 2 hulls of the occurrences of both species. This measure can be interpreted as the Jaccard
 3 index for the volume and is the natural way to estimate niche overlap when using the CH-
 4 approach to describe the niche. It has the disadvantage of not distinguishing between
 5 intersections occurring in regions of high suitability and those occurring in low suitability
 6 areas of the niches.

7 As Tukey depth divides the niche into a finite number of iso-suitability regions, it allows
 8 to generalize Pianka and Jaccard indices incorporating volumes and suitability.

9 Consider two species A and B, and the partition of the environmental space into n_A and
 10 n_B regions of (non zero) iso-depth for each species. Denote by $0 = s_1 < s_2 < \dots < s_{n_A}$
 11 $\dots < s_{n_B}$ and $0 = t_1 < t_2 < \dots < t_{n_B}$ all possible values of suitability for
 12 species A and B, respectively, and let v_{ij} , with $(i,j) \neq (0,0)$, be the volume of the (possibly
 13 disconnected) region composed by all the environments having suitability s_i for
 14 species A and t_j for species B, and v^* arbitrary. The volumes can be arranged in a
 15 $(n_A + 1)$ by $(n_B + 1)$ matrix $M_V = v_{ij}/v^*$. This matrix contains all the information on
 16 intersections of the environmental niches, taking into account distinct levels of suitability
 17 for both species. Note that, if s_i and t_j only take 0-1 values, i.e. resulting from
 18 binary prediction models, such as the CH-approach, $M_V = v_{ij}/v^*$ is a 2 by 2 matrix, where
 19 v_{11} is the volume of the intersection of the niches, and $v_{10} + v_{01}$ is the volume of the
 20 niche of species A (B) not shared by species B (A).

21 From M_V we define the matrix M_A (M_B) of the asymmetric *overlap of species B (A) on A*
 22 *(B)*, multiplying the row i of M_V by s_i (the column j of M_V by t_j). Each entry (i,j)

1 of matrix M_A (M_B) is the volume V_{ij} , of the region with iso-suitability V_{ij} , for species A
 2 and V_{ij} for species B, weighted by V_{ij} (V_{ij}).

3 We propose to use the matrices above to define two overlap indices. The first is

$$wJ = \frac{\sum_{i,j} M_A(i,j) M_B(i,j)}{\sum_{i,j} M_A(i,j) + \sum_{i,j} M_B(i,j) - \sum_{i,j} M_V(i,j)}$$

4

5 Index wJ incorporates the different levels of suitability on the ratio of the volumes of the
 6 intersection and of the union of the niches. If V_{ij} and V_{ij} only assumes values 0 or 1,

7

$$wJ = \frac{M_V(1,1)}{M_V(1,0) + M_V(0,1) + M_V(1,1)}$$

8

9

10 which is the Jaccard index for the volume. For the particular case when suitability is
 11 given by CH-approach, the niches are estimated by the convex hulls of the occurrences of
 12 both species, and wJ becomes the index of Villéger *et al.* (2011).

13 When suitability is obtained with Tukey depth, wJ distinguishes between intersections
 14 occurring in regions of high suitability and those occurring in low suitability areas of the
 15 niches.

1 Figure 3 illustrates iso-suitability regions for each of three hypothetical species A, B and
 2 B' (top) in the same environmental space, and how they are arranged together (bottom).
 3 The volumes of the intersections/unions of the convex hulls for A and B, and for A and
 4 B' are exactly the same. Thus, Jaccard index returns the same value (0.2179) for both
 5 pairs. Index wJ gives 0.3579 for pair (A, B), where intersection occurs mainly in the
 6 regions of high suitability, and 0.1604 for pair (A, B') where the regions of high
 7 suitability almost do not overlap.

8 **(Figure 3)**

9 The second overlap index we propose is a *cosine* index, which is an adaptation of Pianka
 10 index used to measure resource overlap between two species (Pianka, 1973). Consider
 11 matrices M_A and M_B as vectors of length $(+1) \times (+1)$. The *cos* index is defined as the
 12 cosine of the angle between these vectors.

$$\text{cos} = \frac{\sum_{i=0}^{k_A} \sum_{j=0}^{k_B} M_A(i, j) M_B(i, j)}{\sqrt{\sum_{i=0}^{k_A} \sum_{j=0}^{k_B} M_A(i, j)^2} \sqrt{\sum_{i=0}^{k_A} \sum_{j=0}^{k_B} M_B(i, j)^2}}$$

$$\frac{\sum_{i=0}^{k_A} \sum_{j=0}^{k_B} s_i(A) s_j(B) (M_V(i, j))^2}{\sqrt{\sum_{i=0}^{k_A} \sum_{j=0}^{k_B} s_i(A) M_V(i, j)^2} \sqrt{\sum_{i=0}^{k_A} \sum_{j=0}^{k_B} s_j(B) M_V(i, j)^2}}$$

14
 15
 16 When the niches do not overlap, vectors are orthogonal and $\text{cos}=0$. When high and low
 17 suitability occur in the same regions for both species, $\text{cos}\approx 1$.

1

2 **ILLUSTRATIVE EXAMPLE**

3 **The climatic suitability of *Quercus* taxa in the Iberian Peninsula**

4 As an example of application of the above-described CH-Tukey procedure, we computed
5 climatic suitability for *Quercus* taxa occurring in the Iberian Peninsula.

6 *Quercus* spp. are widespread in the Northern Hemisphere (Manos *et al.*, 1999). In the
7 Iberian Peninsula *Quercus* taxa occur in most of the territory being an important
8 economic and conservation target (see Bugalho *et al.*, 2011; Correia *et al.*, 2017). For
9 example, several of the local economies are dependent on cork extraction, cork represents
10 2.5% of Portugal exports, and acorns from *Quercus rotundifolia* and *Q. suber* are
11 particularly important in many animals' diet (Rodríguez-Estévez *et al.*, 2009; Bugalho *et*
12 *al.*, 2011). *Q. robur* is an important forestry species. *Q. faginea* and *Q. pyrenaica* are
13 valuable to local communities for the production of firewood (Gonçalves *et al.*, 2012;
14 Cerqueira *et al.*, 2013).

15 In this paper we focus our analyses on six species/subspecies: *Q. rotundifolia* (= *Q. ilex*
16 subsp. *ballota*), *Q. suber*, *Q. pyrenaica*, *Q. faginea* subsp. *faginea*, *Q. faginea* subsp.
17 *broteroi* and *Q. robur*. Occurrences were obtained from the Proyecto Anthos (Anthos,
18 2011) and from the Herbarium “João de Carvalho e Vasconcellos” (Lisbon, LISI) and
19 include a mix of point and grid-based data of several natures and types. Positional
20 accuracy of the species observations data was analysed and improved when appropriate.

21 We used four climatic variables known to affect and limit species distributions,
22 downloaded from the Worldclim website (Hijmans *et al.*, 2005;

1 <http://www.worldclim.org>) at the $1\text{ km} \times 1\text{ km}$ grid cell resolution. Two variables involve
2 temperature (isothermality and the minimum temperature of coldest month) and two
3 variables involve precipitation (annual precipitation and the precipitation of driest
4 quarter).

5 For each selected taxon, we applied the CH-Tukey and the BIOCLIM procedures in order
6 to produce geographical maps distinguishing suitability levels. The Tukey depth of each
7 site within the Iberian Peninsula (ca. $1\text{ km} \times 1\text{ km}$ resolution), with respect to the
8 occurrences of each taxon, was determined using the function *depth* from the R package
9 *depth* version 2.0-0 (Genest *et al.*, 2012). The depth values were then rescaled dividing
10 by $0 - 3k/23$, where k is the number of occurrences of the taxon, thus obtaining a
11 normalized Tukey depth. The BIOCLIM suitability associated to each site was obtained
12 using the functions *bc* and *predict* from the R package *dismo* version 1.1-1 (Hijmans *et*
13 *al.*, 2016).

14 The maps clearly show that the inland non-white areas for CH-Tukey, (Figure 4) are
15 contained in the inland non-white areas for BIOCLIM, ' (Figure 5), denoting the fact
16 that the convex hull is a tighter envelope than the hyperrectangle.

17 A robust validation of these models imply controlled experiments using a small set of
18 factors considered limitative of species fitness. Given that these experiments are typically
19 time-consuming, expensive and therefore are seldom performed, expert knowledge is a
20 valuable source of information to discuss niche predictions.

21 The obtained spatial extents are quite consistent with experts' knowledge (see e.g. Rivas-
22 Martínez & Sáenz Laín, 1991), specially for the CH-Tukey approach (e.g. *Q. robur* in the
23 NW of the Peninsula, *Q. faginea* subsp. *broteroi* in the SW, *Q. suber* mainly in the W,

1 while the other three species occur mostly in the interior parts of the Peninsula). For the
2 BIOCLIM procedure such spatial coincidence is not so evident and can only be
3 approximated if the lower levels of suitability are discarded.

4 For some taxa (*Q. suber*, *Q. rotundifolia* and both *Q. faginea* subspecies) the maps show
5 a low density of occurrences in regions of predicted high suitability. This may result,
6 among other possible reasons, from: i) missing relevant environmental variables (e.g. soil
7 characteristics or other climatic variables) in the model; ii) incomplete sampling of
8 occurrences in the corresponding geographic areas; iii) some ecological factors that lead
9 to the exclusion of the species from that area (e.g. competition, parasitism, disease); or
10 even iv) local extinction due to human overuse of *Quercus* forests, conversion to
11 agricultural fields, managed forests, pastures or urban fabric. In the case of *Q. faginea*
12 subsp. *faginea* and *Q. suber* it is quite plausible that a soil-related environmental variable
13 is missing in the model, as the former is known to prefer base-rich or clay-rich soils,
14 while the latter is calcifugous (Franco, 1990). In the case of *Q. faginea* subsp. *broteroi*,
15 the biogeographic disjunction that the occurrence points show (i.e. one cloud of
16 occurrences concentrated in the west of Portugal and a second cloud in the central part of
17 Spain) may lead to an unoccupied niche core. The findings of Silva et al. (2014),
18 mentioned above, can be used to detect those less frequent distribution patterns within the
19 niche, previously to modelling.

20 In general, with the CH-Tukey approach, the maximum suitability obtained for each of
21 the studied *Quercus* taxa is relatively low. This is not problematic *a priori* and might give
22 further structural information on the species niche and on the study area, as: i) the
23 maximum suitability level of 1, might not be achieved for some configurations of

1 occurrence points in the environmental space; ii) when achieved, the volume of the
2 region with suitability 1 can be very small; iii) even when the volume of the Tukey
3 median region is conspicuous, it may correspond to combinations of the n variables that
4 do not come into existence within the study area.

5 **(Figure 4)**

6 **(Figure 5)**

7

8 **Quantifying the environmental niche overlaps of Iberian *Quercus* taxa**

9 In the space defined by the four environmental variables (isothermality, minimum
10 temperature of coldest month, annual precipitation and precipitation of driest quarter), we
11 uniformly generated $N=45 \times 10^6$ points within the bounding box of the occurrences of all
12 taxa, and computed the Tukey suitability of each of these points with respect to the
13 occurrences of each taxon. To assess niche overlap for each pair of taxa, A and B, we
14 estimated the volumes V_{+} , for $V_{+} \neq 0,0$, as the number of points with Tukey
15 suitability V_{+} , for taxon A, and Tukey suitability V_{+} , for taxon B, divided by N .
16 Results for the two proposed overlap indices (wJ and cos) are presented in Table 1.

17 **(Table 1)**

18 Both wJ and cos showed that *Q. robur* and *Q. pyrenaica* presented the highest overlap,
19 while *Q. robur* and *Q. faginea* subsp. *broteroi* presented the lower overlap.

20 In order to visualize graphically the relative positions of occurrences and iso-depth
21 regions of the pairs of taxa that, in the 4-dimensional space, were more separated (*Q.*
22 *robur* vs. *Q. faginea* subsp. *broteroi*), and less separated (*Q. robur* vs. *Q. pyrenaica*)

1 (Table 1), we projected the occurrences of each of these taxa in the 2-dimensional
2 environmental space of variables minimum temperature of the coldest month and
3 precipitation of the driest quarter (Figure 6). Computing the overlap indices in this 2-
4 dimensional space, we obtained $wJ= 0.9177$ and $cos= 0.3450$ for the first pair (*Q. robur*,
5 *Q. pyrenaica*) and $wJ= 0.3274$ and $cos= 0.0283$ for the second pair (*Q. robur*, *Q. faginea*
6 subsp. *broteroi*). These values agree with the graphs of Figure 6, where it is shown a clear
7 overlap of the niches of *Q. robur* and *Q. pyrenaica* while the niche of *Q. faginea* subsp.
8 *broteroi* is mainly concentrated in the peripheral areas of the *Q. robur* niche.

9 In general, Table 1 shows values that are coherent with the known regional co-occurrence
10 patterns of the studied *Quercus* (see e.g. Rivas-Martínez & Sáenz Laín, 1991; Capelo *et*
11 *al.*, 2007). For example, *Q. robur* co-occurs frequently with *Q. pyrenaica* in the highlands
12 of the northwest of the Iberian Peninsula, and with *Q. suber* in the lowlands. *Q. pyrenaica*
13 co-occurs with *Q. rotundifolia* in the continental plateaux of the Iberian Meseta and with
14 *Q. suber* in the warmer areas of the centre-south. *Q. rotundifolia* and *Q. suber* co-occur in
15 the SW of the Peninsula. The referred pairs correspond both to the highest values of the
16 indices shown in Table 1 and to intermingled vegetation series mapped as a mosaic in the
17 work of Capelo *et al.* (2007).

18 (Figure 6)

19 Comparing niche geometries

20 In order to illustrate and compare niche geometries produced by different approaches, we
21 compared outputs from CH-Tukey to BIOCLIM, and also MaxEnt (Phillips *et al.*, 2006).
22 MaxEnt is one of the most used modelling algorithms, to obtain species environmental
23 suitability maps (Ahmed *et al.*, 2015). We ran these three models using the two variables

1 referred above (minimum temperature of the coldest month and precipitation of the driest
2 quarter) and depicted their outputs both in the environmental and in the geographic space
3 (Figure 7) for *Q. robur*, *Q. pyrenaica* and *Q. faginea* subsp. *broteroi*. Default MaxEnt
4 parameterization was used.

5 Inspecting Figure 7 (B, C, D) we can visualize how BIOCLIM relies on the assumption
6 that ecological variables are independent in their action on the species, producing
7 rectilinear niche geometries, while CH-Tukey relaxes it, adjusting its outer boundary and
8 the inner suitability levels to the known occurrences. Both BIOCLIM and CH-Tukey are
9 built upon the "range" notion, implicit in Hutchinson's seminal work, i.e., if you select
10 two points from the species niche representing environmental conditions known to be
11 suitable for that species, you should not expect a decrease in suitability between these two
12 points, at least while only those variables are considered. MaxEnt neglects this, producing
13 not only unbounded geometries, but also allowing (visible for the three tested taxa) a
14 decrease in suitability between two points of the niche. Although relying on unimodal
15 geometries, BIOCLIM and CH-Tukey approximate better Hutchinson's notion of optimal
16 versus suboptimal niche regions than the geometry produced by MaxEnt, which proposes
17 high suitability values for unbounded or extrapolative areas (see "clamping" in Merow *et*
18 *al.*, 2013; check *Q. faginea* subsp. *broteroi* output in Figure 7D, values of min
19 Temperature greater than 10.0 °C or smaller than -10.0 °C and Precipitation around 50
20 mm). This is in agreement with the known limitations of correlative models concerning
21 extrapolation (Merow *et al.*, 2013). According to the same authors, MaxEnt can produce
22 intricate model outputs of hard ecological interpretation, especially when regularization is
23 used. This is apparent in the sudden shifts in suitability observed in the MaxEnt output

1 (within the species niche). Even if some shifts could relate to interesting or interpretable
2 facts (e.g. coming from the background data), we believe that they deviate this modelling
3 technique from the seminal ecological theory.

4

5 **(Figure 7)**

6 A thorough and careful setting of the input parameters in MaxEnt can improve the
7 ecological interpretation of the output (Merow *et al.*, 2013), but such parameterization is
8 not always achievable using presence-only data coming from usual databases. Even so,
9 MaxEnt produces geographical maps that are closely linked to the known occurrences of
10 points (see Figure 7A for the occurrence points of each taxa), which is expected from a
11 correlative approach. This tends to hinder fundamental discussions, such as: i) niche
12 boundaries and niche optimality; ii) interpolation vs. extrapolation of the observations;
13 and iii) possible discrepancies between niche optimality and the geographic distribution
14 (e.g. niche is highly suitable according to the studied environmental variables, but there
15 are no occurrences in the corresponding geographic regions, possibly indicating that other
16 important variables lack in the model).

17 Incorporating putative niche voids directly in the modelling approaches has been
18 considered useful, especially while addressing niches using real-world examples (Walker
19 & Cocks, 1991; Blonder *et al.*, 2014). Nevertheless, even when dealing with real-world
20 examples, general niche premises and properties should hold (e.g. the "range" notion,
21 niche boundaries and optimality) and these can be consistently addressed and explored
22 using appropriate methodological approaches such as BIOCLIM and CH-Tukey.

1

2 **CONCLUDING REMARKS**

3 The CH-Tukey procedure allows the construction of environmental suitability maps, as
4 shown for the selected *Quercus* taxa, differentiating iso-suitability levels within the CH-
5 approach. Such levels are drawn from occurrence data only, in the selected environmental
6 hyperspace, using normalized Tukey depth, and can, subsequently, be used to derive
7 overlap measures coherently.

8 Carpenter *et al.* (1993) listed four disadvantages linked with the CH-approach, two
9 related to computational difficulties which are no longer a problem for current computers.

10 The other two were related to the tightly constrained envelope produced by the CH-
11 approach and the exclusion of many similar sites (close to, but outside, the convex hull
12 boundary). But this may be arguable since the convex hull is a natural boundary between
13 interpolation and extrapolation (see Hirzel & Le Lay, 2008). We stress that the CH-Tukey
14 procedure is appropriate to interpolate from the input data, as the model simply considers
15 unsuitable all environmental combinations outside the convex hull (i.e. it does not
16 extrapolate). Nevertheless, similar sites in the immediate outside neighbourhood of the
17 convex hull, which are considered unsuitable in the CH-Tukey procedure, could be
18 modelled separately with other techniques, in a coupled approach, assuming that
19 extrapolation is acceptable.

20 As shown in the maps obtained with CH-Tukey methodology, it is possible that higher
21 suitability regions have low density of occurrences (see e.g. *Q. faginea* subspecies). This
22 results from the combinatorial-geometrical nature of the approach that does not rely so
23 strongly on point density to predict suitability. As the procedure is geometrically

1 interpretable, hypotheses can be sketched from the obtained patterns, given the used
2 environmental variables, and tested subsequently. This interpretability ensures a seamless
3 relationship with Hutchinsonian niche concept, a feature frequently asked by ecologists
4 and epistemologists (Austin, 2002; Elith & Leathwick, 2009; Brewer *et al.*, 2016).

5 In conclusion, the CH-Tukey procedure presents several relevant features that can be
6 considered advantages in some ecological niche modelling contexts, such as:

7 i) The modelling process is easily interpretable geometrically, with precise definition of
8 the estimated niche boundaries.

9 ii) It uses only presence data and does not depend on the background data.

10 iii) The suitability levels are invariable with respect to the ecological variables units.

11 iv) The regions of the multivariate space corresponding to high Tukey depth values are
12 robust to outliers, in accordance to what occurs with the univariate median of a set of
13 values.

14 v) It produces ordinal predictions, decomposing the niche into a finite number of iso-
15 suitability levels, permitting the computation of consistent overlap indices.

16 vi) A tight envelope is adjusted to the points cloud and, consequently, the independence
17 of the ecological variables action on the species is not assumed (i.e., allowing for
18 interactions between variables); moreover, the output of the procedure results uniquely
19 from interpolations of the input data.

20 The main disadvantage of the CH-Tukey procedure is that depth computation is time
21 demanding, and heuristic approximations may be required depending on the number of

1 occurrence points used as input data and on the spatial resolution of the predictions.
2 Heuristics are already implemented in package *depth* (Genest *et al.*, 2012).

3

4 **ACKNOWLEDGMENTS**

5 This work was supported by the Portuguese Foundation for Science and Technology
6 (FCT) through the projects UID/MAT/00297/2013, CMA (Centro de Matemática
7 Aplicada); UID/AGR/00239/2013, CEF (Centro de Estudos Florestais) under
8 FEDER/POCI; and PTDC/AAC-AMB/ 113394/2009. T. Monteiro-Henriques was
9 partially funded by a post-doctoral grant within Project SUSTAINSYS: Environmental
10 Sustainable Agro-Forestry Systems - NORTE-07-0124-FEDER-0000044 (ON.2-QREN-
11 FEDER and PIDDAC-FCT-MEC). D. Alagador was funded through the project
12 PTDC/AAG-GLO/3979/2014 via FEDER through the COMPETE program and national
13 funding. D. Alagador and F. C. Aguiar are also supported with FCT post-doctoral
14 fellowships (SFRH/BPD/104077/2014 and SFRH/BPD/112417/2015, respectively).

15

16 **AUTHORS' CONTRIBUTIONS**

17 JOC, TMH, MJM and PCS conceived the ideas and designed methodology; MJM, TMH
18 and MLC dealt with computational issues; TMH, PA, MJM and FCA collected, analysed
19 and interpreted the data; JOC, TMH, MJM and PCS led the writing of the manuscript.
20 DA, AF and MC revised it and contributed to the development of ideas. All authors
21 contributed critically to the manuscript.

22

1 **REFERENCES**

- 2 Ahmed, S.E., McInerny, G., O'Hara, K., Harper, R., Salido, L., Emmott, S. & Joppa, L.N.
3 (2015) Scientists and software – surveying the species distribution modelling
4 community. *Diversity and Distributions*, **21**, 258–267.
- 5 Anthos (2011) Sistema de información de las plantas de España. Real Jardín Botánico,
6 CSIC- Fundación Biodiversidad. Downloaded in November 16, 2011.
- 7 Austin, M.P. (2002) Spatial prediction of species distribution: an interface between
8 ecological theory and statistical modelling. *Ecological Modelling*, **157**, 101–118.
- 9 Blonder, B. (2017) Hypervolume concepts in niche- and trait-based ecology. *Ecography*,
10 **40**, 001–013.
- 11 Blonder, B., Lamanna, C., Violle, C. & Enquist, B.J. (2014) The n-dimensional
12 hypervolume. *Global Ecology and Biogeography*, **23**, 595–609.
- 13 Bond, W.J. & Midgley, J.J. (2001) Ecology of sprouting in woody plants: the persistence
14 niche. *Trends in Ecology & Evolution*, **16**, 45–51.
- 15 Booth, T.H., Nix, H.A, Busby, J.R. & Hutchinson, M.F. (2014) BIOCLIM: the first
16 species distribution modelling package, its early applications and relevance to
17 most current MAXENT studies. *Diversity and Distributions*, **20**, 1–9.
- 18 Brewer, M.J., O'Hara, R.B., Anderson, B.J., & Ohlemüller, R. Plateau: A New Method
19 for Ecologically Plausible Climate Envelopes for Species Distribution Modelling.
20 *Methods in Ecology and Evolution*, **7**, 1489–1502.

- 1 Bugalho, M.N., Caldeira, M.C., Pereira, J.S., Aronson, J. & Pausas, J.G. (2011)
2 Mediterranean cork oak savannas require human use to sustain biodiversity and
3 ecosystem services. *Frontiers in Ecology and the Environment*, **9**, 278–286.
- 4 Busby, J.R. (1991) *BIOCLIM - A Bioclimate Analysis and Prediction System*. *Nature*
5 *Conservation: Cost Effective Biological Surveys and Data Analysis* (ed. by C.R.
6 Margules and M.P. Austin), pp. 64–68. CSIRO.
- 7 Capelo, J., Mesquita, S., Costa, J.C., Ribeiro, S., Arsénio, P., Neto, C., Monteiro-
8 Henriques, T., Aguiar, C., Honrado, J., Espírito-Santo, D. & Lousã, M.F. (2007)
9 A methodological approach to potential vegetation modeling using GIS
10 techniques and phytosociological expert-knowledge: application to mainland
11 Portugal. *Phytocoenologia*, **37**, 399–415.
- 12 Carpenter, G., Gillison, A.N. & Winter, J. (1993) DOMAIN: a flexible modelling
13 procedure for mapping potential distributions of plants and animals. *Biodiversity*
14 *and Conservation*, **2**, 667–680.
- 15 Cerdeira, J.O., Monteiro-Henriques, T., Martins, M.J., Silva, P.C., Alagador, D. &
16 Franco, A.M.A. (2014) Mathematical contributions to link biota with
17 environment. *Journal of Vegetation Science*, **25**, 1148–1153.
- 18 Cerqueira, M., Gomes, L., Tarelho, L. & Pio, C. (2013) Formaldehyde and acetaldehyde
19 emissions from residential wood combustion in Portugal. *Atmospheric*
20 *Environment*, **72**, 171–176.
- 21 Colwell, R. & Futuyma, D. (1971) Measurement of Niche Breadth and Overlap. *Ecology*,
22 **52**, 567–576.

- 1 Correia, R.A., Bugalho M.N., Franco A.M.A. & Palmeirim J.M. Contribution of Spatially
2 Explicit Models to Climate Change Adaptation and Mitigation Plans for a Priority
3 Forest Habitat. *Mitigation and Adaptation Strategies for Global Change, in press*
4 1–16.
- 5 Elith, J., Graham, C.H., Anderson, R.P., Dudík, M., Ferrier, S., Guisan, A., Hijmans, R.J.,
6 Huettmann, F., Leathwick, J.R., Lehmann, A., Jin Li, Lohmann, L.G., Loiselle,
7 B.A., Manion, G., Moritz, C., Nakamura, M., Nakazawa, Y., McC. Overton, J.,
8 Peterson, A.T. & Phillips, S.J. (2006) Novel methods improve prediction of
9 species' distributions from occurrence data. *Ecography*, **29**, 129–151.
- 10 Elith, J. & Leathwick, J.R. (2009) Species Distribution Models: Ecological Explanation
11 and Prediction Across Space and Time. *Annual Review of Ecology, Evolution, and*
12 *Systematics*, **40**, 677–697.
- 13 Franco, J. do A. (1990) 3. *Quercus L.*. Flora iberica: Plantas vasculares de la Península*
14 *Ibérica e Islas Baleares* (ed. by S. Castroviejo, M. Laínz, G. López González, P.
15 Montserrat, F. Muñoz Garmendia, J. Paiva, and L. Villar), pp. 15–36. Real Jardín
16 Botánico - Consejo Superior de Investigaciones Científicas, Madrid.
- 17 Franklin, J. (1995) Predictive vegetation mapping: geographic modelling of biospatial
18 patterns in relation to environmental gradients. *Progress in Physical Geography*,
19 **19**, 474–499.
- 20 Franklin, J. (2010) *Mapping Species Distributions: Spatial Inference and Prediction*,
21 Cambridge University Press, Cambridge.

- 1 Genest, M., Masse, J.-C. & Plante, J.-F. (2012) *depth: Depth functions tools for*
2 *multivariate analysis (version 2.0-0)*, <http://CRAN.R-project.org/package=depth>.
- 3 Gonçalves, C., Alves, C. & Pio, C. (2012) Inventory of fine particulate organic
4 compound emissions from residential wood combustion in Portugal. *Atmospheric*
5 *Environment*, **50**, 297–306.
- 6 Grubb, P.J. (1977) The maintenance of species-richness in plant communities: the
7 importance of the regeneration niche. *Biological Reviews*, **52**, 107–145.
- 8 Guisan, A. & Zimmermann, N.E. (2000) Predictive habitat distribution models in
9 ecology. *Ecological Modelling*, **135**, 147–186.
- 10 Hijmans, R.J., Cameron, S.E., Parra, J.L., Jones, P.G. & Jarvis, A. (2005) Very High
11 Resolution Interpolated Climate Surfaces for Global Land Areas. *International*
12 *Journal of Climatology*, **25**, 1965–1978.
- 13 Hijmans, R.J., Phillips, S., Leathwick, J. & Elith, J. (2016) *dismo: Species distribution*
14 *modeling*, <http://CRAN.R-project.org/package=dismo>.
- 15 Hirzel, A.H. & Le Lay, G. (2008) Habitat suitability modelling and niche theory. *Journal*
16 *of Applied Ecology*, **45**, 1372–1381.
- 17 Hurlbert, S. (1978) Measurement of Niche Overlap and Some Relatives. *Ecology*, **59**, 67–
18 77.
- 19 Hutchinson, G.E. (1957) Concluding remarks. *Cold Spring Harbor Symposia on*
20 *Quantitative Biology*, **22**, 415–427.

- 1 Manos, P.S., Doyle, J.J. & Nixon, K.C. (1999) Phylogeny, biogeography, and processes
2 of molecular differentiation in *Quercus* subgenus *Quercus* (Fagaceae). *Molecular*
3 *Phylogenetics and Evolution*, **12**, 333–349.
- 4 Merow, C., Smith, M.J., Silander Jr, J.A. 2013 A practical guide to MaxEnt for modeling
5 species' distributions: what it does, and why inputs and settings matter.
6 *Ecography*, **36**, 1058-1069.
- 7 Nix, H.A. (1986) *A biogeographic analysis of Australian elapid snakes. Atlas of elapid*
8 *snakes of Australia* (ed. by R. Longmore), pp. 4–15. Australian Government
9 Publishing Service, Canberra.
- 10 Pearman, P.B., Guisan, A., Broennimann, O. & Randin, C.F. (2008) Niche dynamics in
11 space and time. *Trends in Ecology & Evolution*, **23**, 149–158.
- 12 Peterson, A.T., Soberon, J. & Sanchez-Cordero, V. (1999) Conservatism of ecological
13 niches in evolutionary time. *Science*, **285**, 1265–1267.
- 14 Phillips, S.J., Anderson, R.P., Schapire, R.E. (2006) Maximum entropy modeling of
15 species geographic distributions. *Ecological Modelling*, **190**, 231-259.
- 16 Pianka, E.R. (1973) The Structure of Lizard Communities. *Annual Review of Ecology*
17 *and Systematics*, **4**, 53–74.
- 18 Raxworthy, C.J., Ingram, C.M., Rabibisoa, N. & Pearson, R.G. (2007) Applications of
19 Ecological Niche Modeling for Species Delimitation: A Review and Empirical
20 Evaluation Using Day Geckos (*Phelsuma*) from Madagascar. *Systematic Biology*,
21 **56**, 907.

- 1 Rivas-Martínez, S. & Sáenz Laín, C. (1991) Enumeración de los Quercus de la Península
2 Ibérica. *Rivasgodaya*, **6**, 101–110.
- 3 Rocap, G., Larimer, F.W., Lamerdin, J., Malfatti, S., Chain, P., Ahlgren, N.A., Arellano,
4 A., Coleman, M., Hauser, L., Hess, W.R., Johnson, Z.I., Land, M., Lindell, D.,
5 Post, A.F., Regala, W., Shah, M., Shaw, S.L., Steglich, C., Sullivan, M.B., Ting,
6 C.S., Tolonen, A., Webb, E.A., Zinser, E.R. & Chisholm, S.W. (2003) Genome
7 divergence in two Prochlorococcus ecotypes reflects oceanic niche differentiation.
8 *Nature*, **424**, 1042–1047.
- 9 Rodríguez-Estévez, V., García, A. & Gómez, A.G. (2009) Characteristics of the acorns
10 selected by free range Iberian pigs during the montanera season. *Livestock
11 Science*, **122**, 169–176.
- 12 Root, R.B. (1967) The Niche Exploitation Pattern of Blue-Gray Gnatcatcher. *Ecological
13 Monographs*, **37**, 317–350.
- 14 Roughgarden, J. (1972) Evolution of Niche Width. *The American Naturalist*, **106**, 683–
15 718.
- 16 Schoener, T. (1974) Resource Partitioning in Ecological Communities. *Science*, **185**, 27–
17 39.
- 18 Silva, P.C., Cerdeira, J.O., Martins, M.J. & Monteiro-Henriques, T. (2014) Data depth for
19 the uniform distribution. *Environmental and Ecological Statistics*, **21**, 27–39.
- 20 Silvertown, J. (2004) Plant coexistence and the niche. *Trends in Ecology & Evolution*, **19**,
21 605–611.

- 1 Tsoar, A., Allouche, O., Steinitz, O., Rotem, D. & Kadmon, R. (2007) A comparative
2 evaluation of presence-only methods for modelling species distribution. *Diversity*
3 & *Distributions*, **13**, 397–405.
- 4 Tukey, J.W. (1975) Mathematics and the picturing of data. *Proceedings of the*
5 *International Congress of Mathematicians*, **2**, 523–531.
- 6 Villéger, S., Novack-Gottshall, P.M. & Mouillot, D. (2011) The multidimensionality of
7 the niche reveals functional diversity changes in benthic marine biotas across
8 geological time. *Ecology Letters*, **14**, 561–568.
- 9 Walker, P.A. & Cocks, K.D. (1991) HABITAT: A Procedure for Modelling a Disjoint
10 Environmental Envelope for a Plant or Animal Species. *Global Ecology and*
11 *Biogeography Letters*, **1**, 108–118.
- 12 Warren, D.L., Glor, R.E. & Turelli, M. (2008) Environmental Niche Equivalency Versus
13 Conservatism: Quantitative Approaches to Niche Evolution. *Evolution*, **62**, 2868–
14 2883.
- 15 Werner, E. & Gilliam, J. (1984) The Ontogenetic Niche and Species Interactions in Size
16 Structured Populations. *Annual Review of Ecology and Systematics*, **15**, 393–425.
- 17 Whittaker, R.H. & Levin, S.A. eds. (1975) *Niche: Theory and Application*, Doven,
18 Hutchinson & Ross, Pennsylvania.
- 19 Zuo, Y. & Serfling, R. (2000) General notions of statistical depth function. *The Annals of*
20 *Statistics*, **28**, 461–482.
- 21
- 22

1 **BIOSKETCH**

2

3 The authors include mathematicians, ecologists and biologists from several Universities
4 and Research Centers, involved in a three-year multidisciplinary project (S-PerforMA:
5 Species Performance Modelling Algorithm - <http://www.isa.utl.pt/proj/S-PerforMa/>)
6 aiming at exploring issues related with the concept of niche using combinatorial and
7 geometric tools.

8

9

10

1 **TABLES**

2

3 **Table 1.** Overlap (*wJ* and *cos* indices) of all pairs of *Quercus* taxa in the four dimensional
 4 space defined by the used environmental variables: isothermality, minimum temperature
 5 of coldest month, annual precipitation and the precipitation of driest quarter.

6

		<i>wJ</i>				
<i>cos</i>	<i>faginea broteroi</i>	0.3388	0.3238	0.1128	0.0460	0.2190
	0.0349	<i>faginea faginea</i>	0.5419	0.2802	0.1214	0.2876
	0.0700	0.2044	<i>rotundifolia</i>	0.8391	0.5262	0.7920
	0.0130	0.0463	0.3041	<i>pyrenaica</i>	0.8874	0.8810
	0.0043	0.0084	0.0516	0.3542	<i>robur</i>	0.7605
	0.0772	0.0426	0.1509	0.3141	0.2079	<i>suber</i>

7

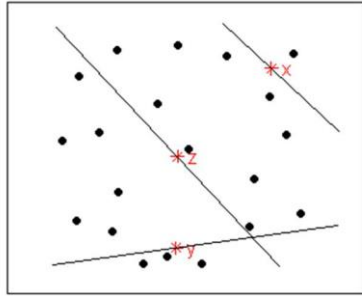


Figure 1. Illustration of the Tukey interiority of points x , y and z (represented by $*$), with respect to the set P of 20 points (represented by \bullet), as the number of points of P in the side of the line passing through $*$ which has the minimum number of points. Interiorities of x , y and z are 1, 3 and 9, respectively; $d_x(P) = 1/20$, $d_y(P) = 3/20$, and $d_z(P) = 9/20$.

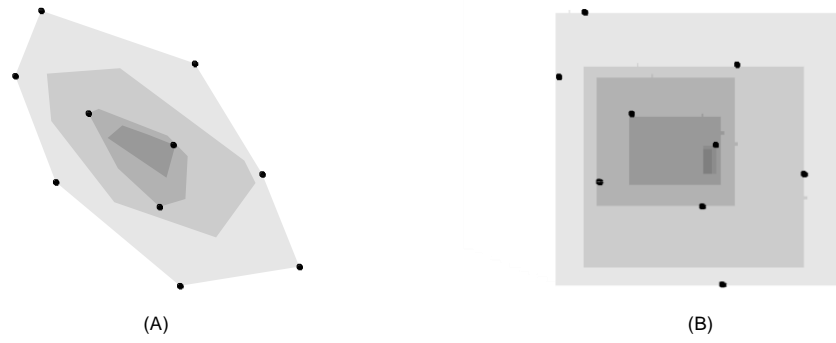


Figure 2. The 10 points represent occurrences of a species in a 2-dimensional environmental space. Shades of grey define areas of “iso-suitability”. The suitability levels increase from the white (=0) to the dark grey areas. The left panel (A) refers to CH-Tukey and the right panel (B) refers to BIOCLIM.

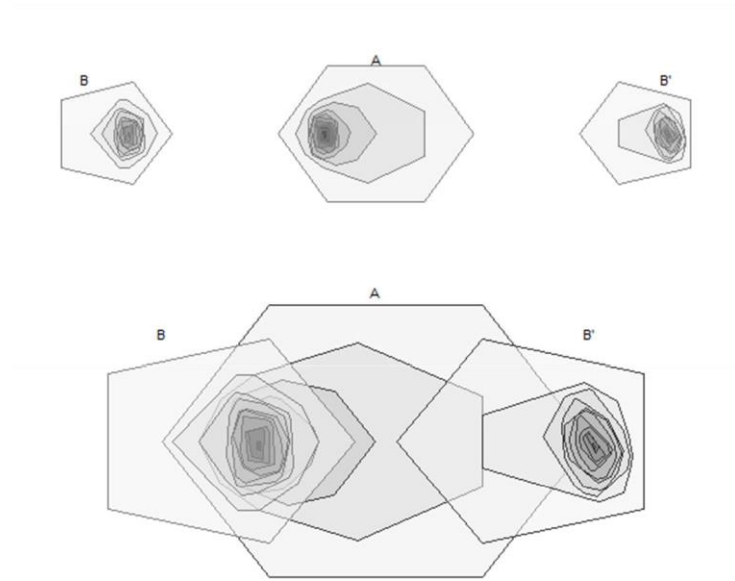


Figure 3. Niches and iso-depth regions of three species (A, B and B') in the same environmental space. Top row: niches depicted isolated. Bottom row: overlapped niches.

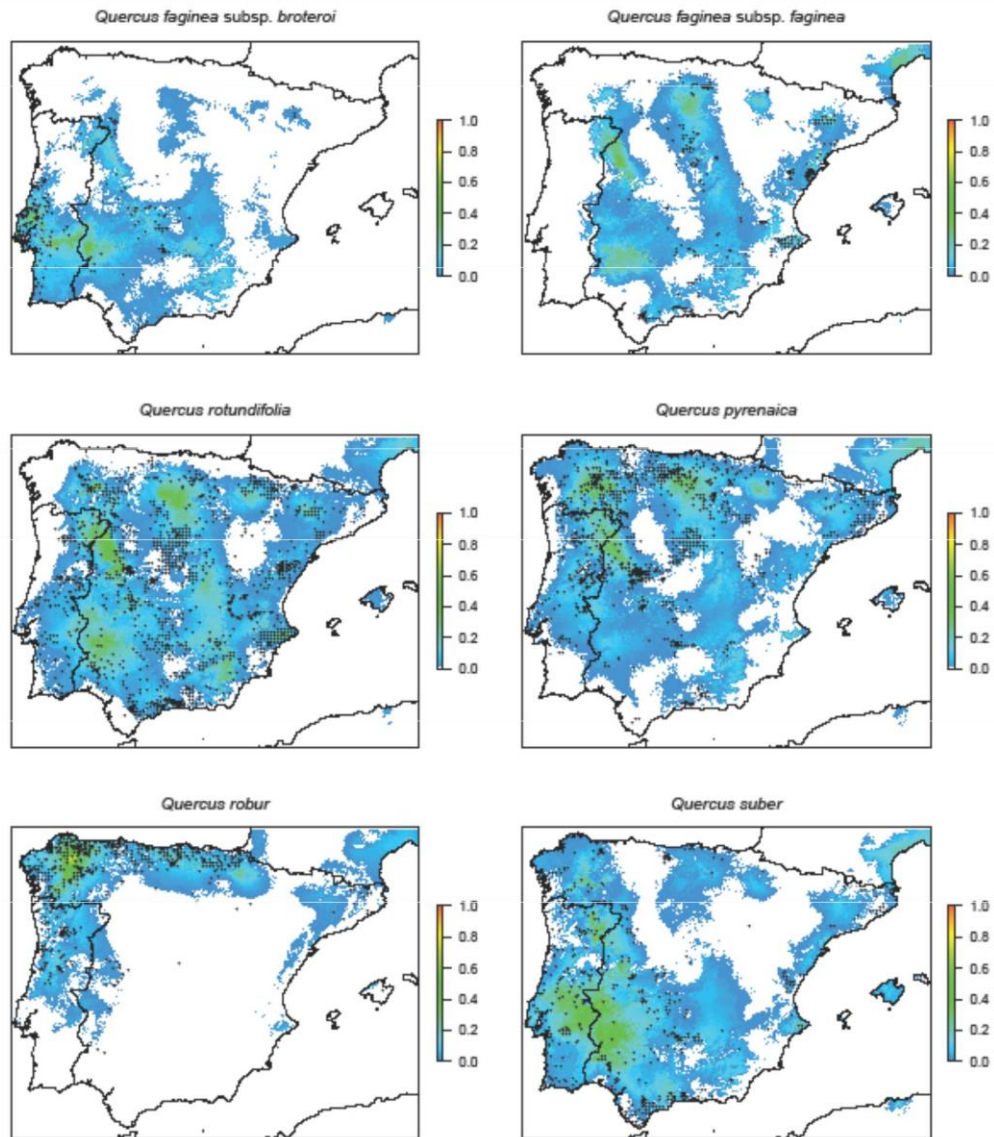


Figure 4. Environmental suitability maps for six *Quercus* taxa in the Iberian Peninsula obtained using the CH- Tukey procedure, with four climatic variables. Dots represent occurrences. The white inland areas refer to points outside the convex hull of the occurrences of the corresponding taxon, in the considered 4-dimensional environmental space.

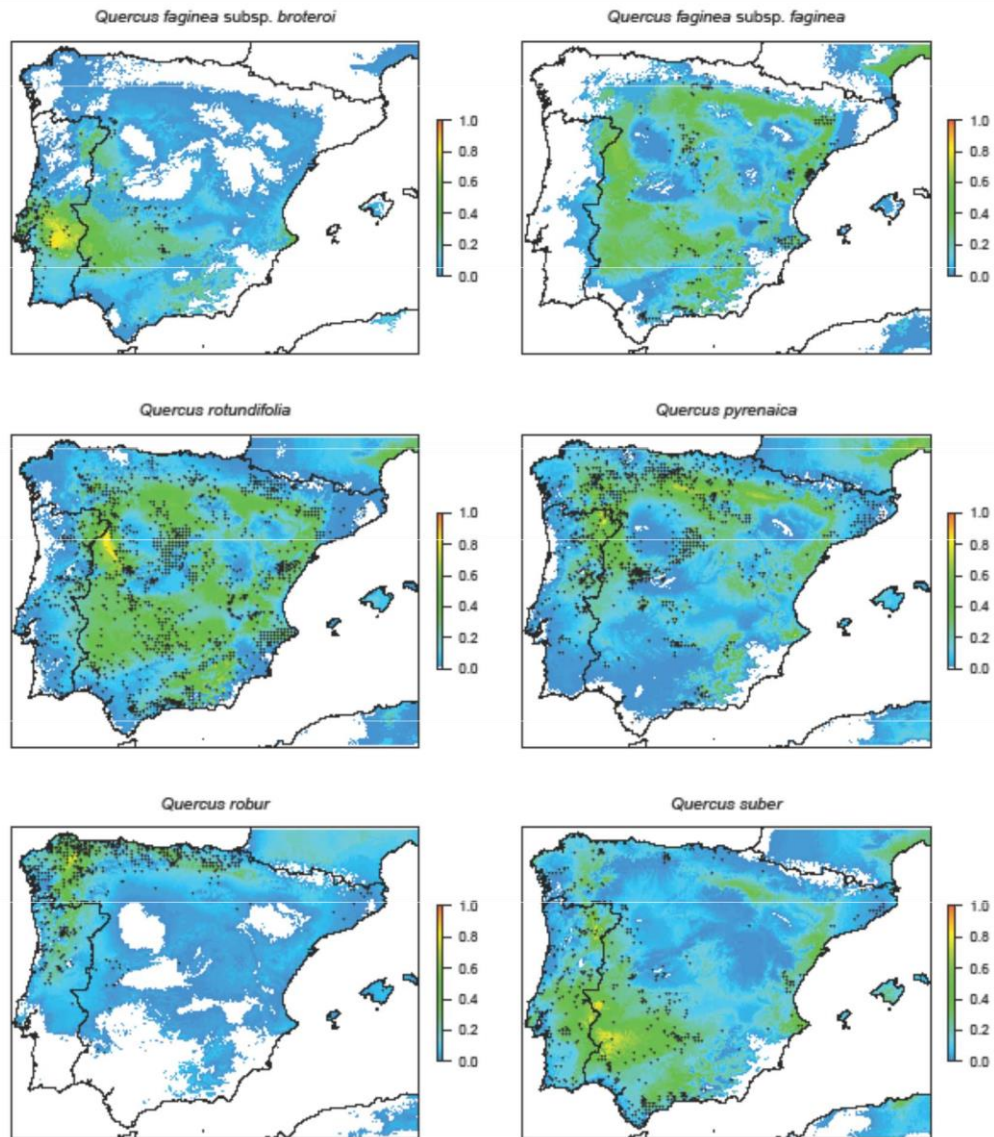


Figure 5. Environmental suitability maps for six *Quercus* taxa in the Iberian Peninsula obtained using the BIOCLIM model, with four climatic variables. Dots represent occurrences. The white inland areas refer to points outside the bounding box of the occurrences of the corresponding taxon, in the considered 4-dimensional environmental space.

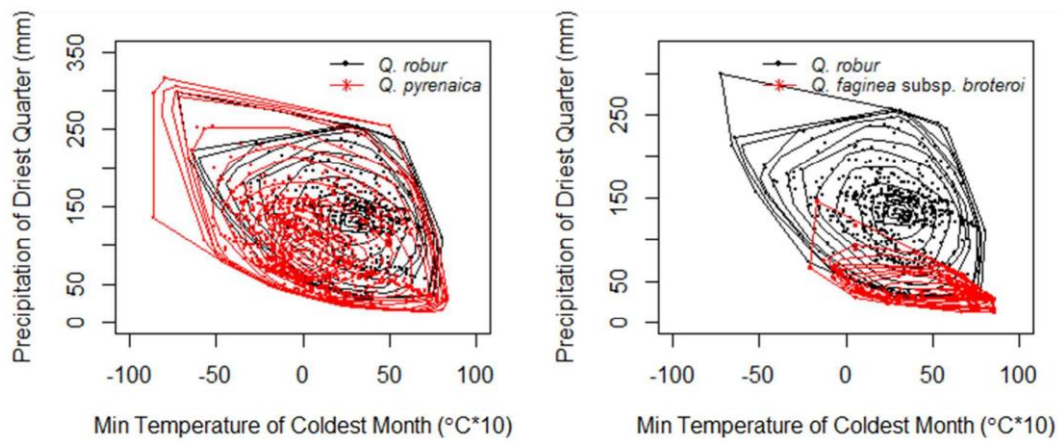


Figure 6. Occurrences and iso-depth regions of *Q. robur* and *Q. pyrenaica* in the left panel and of *Q. robur* and *Q. faginea* subsp. *broteroi* in the right panel, in the environmental space defined by the minimum temperature of the coldest month and precipitation of the driest quarter. Dots and stars correspond to occurrences, and the lines delimit iso-depth regions (regions with the same Tukey depth).

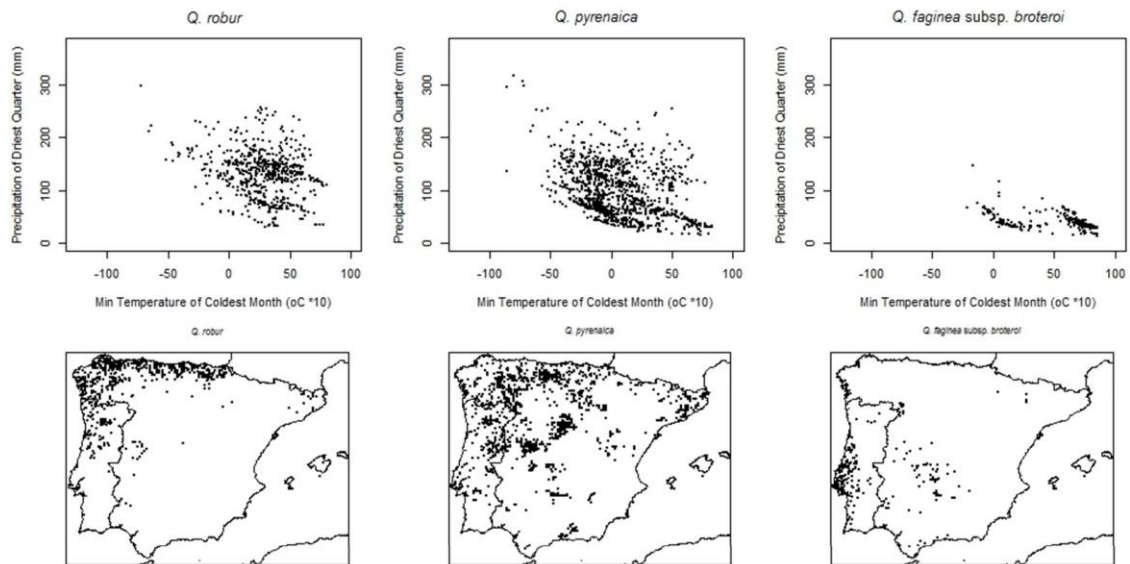


Figure 7A. Occurrences of *Q. robur*, *Q. pyrenaica* and *Q. faginea* subsp. *broteroi* in the environmental and the geographical spaces.

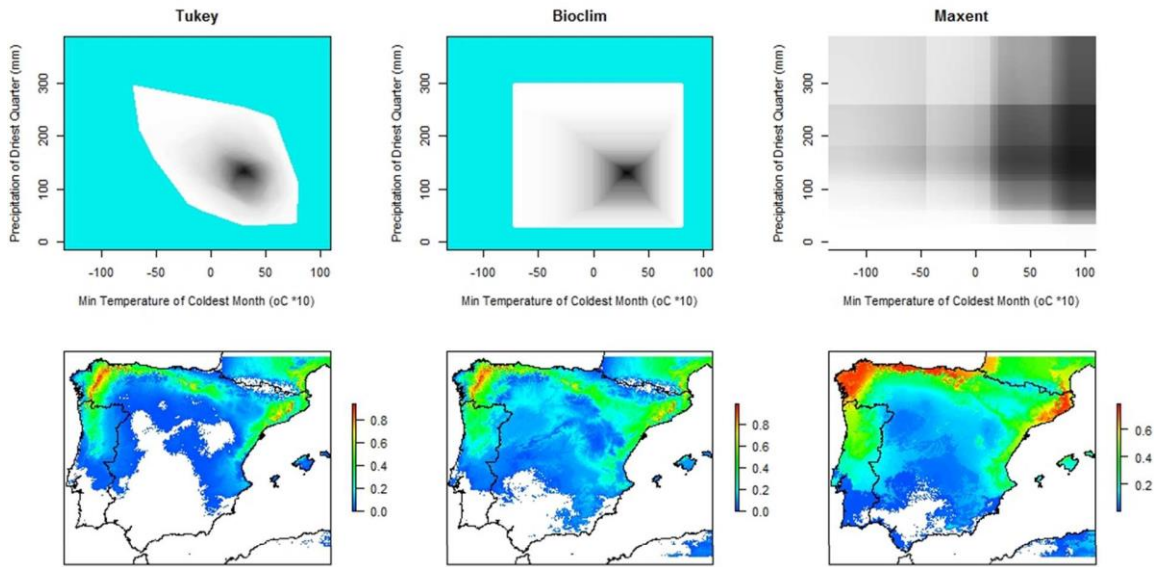


Figure 7B. Niches and suitability maps produced by CH-Tukey, BIOCLIM and MaxEnt for *Q. robur*.

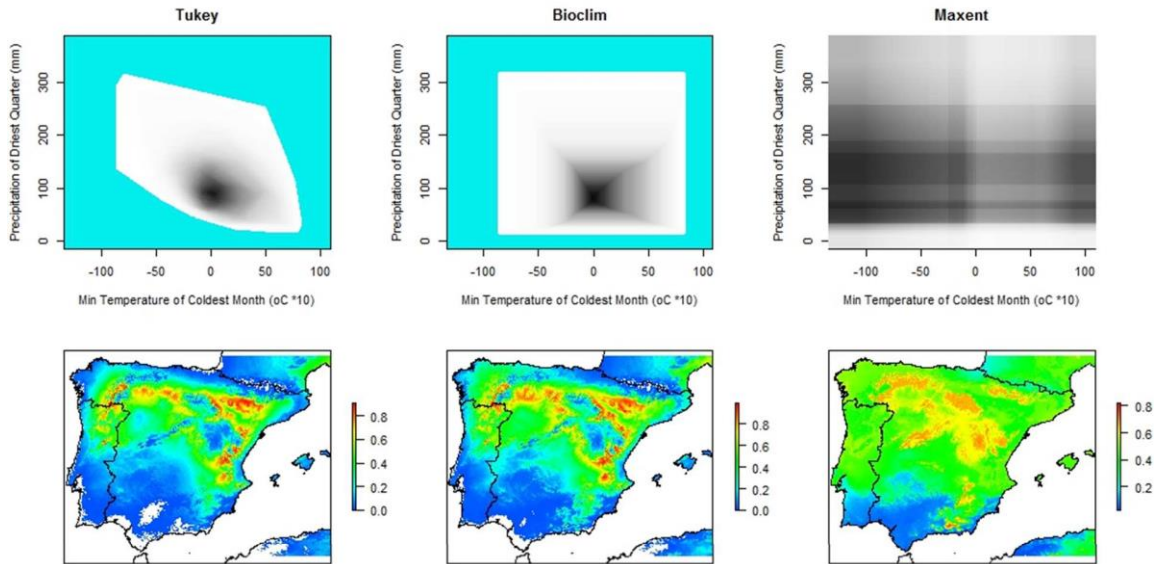


Figure 7C. Niches and suitability maps produced by CH-Tukey, BIOCLIM and MaxEnt for *Q. pyrenaica*.

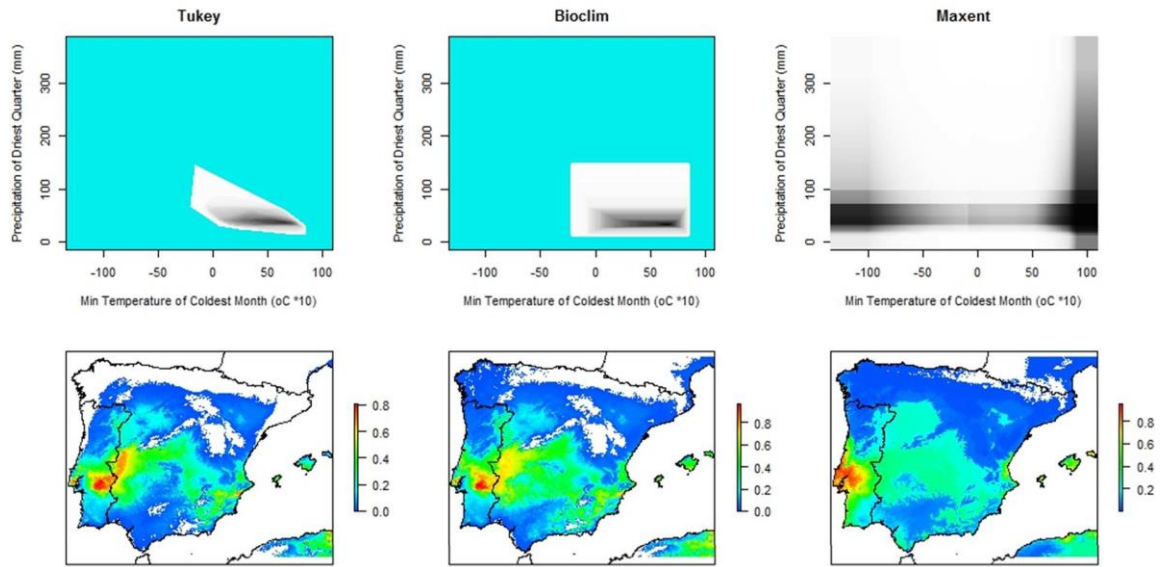


Figure 7D. Niches and suitability maps produced by CH-Tukey, BIOCLIM and MaxEnt for *Q. faginea* subsp. *broteroi*.