Guilt and Participation*

Amrish Patel[†] and Alec Smith[‡]

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Abstract

How does guilt affect participation in providing public goods? We characterise and analyse completely mixed symmetric equilibria (CMSE) in participation games where players are guilt averse. We find that relative to material preferences, guilt aversion can: facilitate the existence of CMSE; increase or decrease participation; and imply that group size has a non-monotonic effect on participation. Using our equilibrium characterisation we also re-analyse experimental data on participation games and find a low, but positive, guilt sensitivity parameter.

Keywords: Participation, threshold public good, volunteer's dilemma, psychological games, guilt aversion. **JEL codes:** C72, D91, H41

1 Introduction

Social dilemmas involve conflicts between private and group incentives. One important class of social dilemma involves the decision to participate in the provision of a discrete public good (Palfrey and Rosenthal 1984, P&R). In this setting, which we refer to as a participation game, each individual simultaneously decides whether to participate, and if a threshold number of participants is reached, the public good is provided. Social dilemmas such as public goods games and participation games typically have Nash equilibria involving no cooperation. Yet evidence from both the lab and field suggests that people do contribute to public goods, volunteer, and participate.

Participation games can capture strategic and economic situations ranging from referendum voting to international agreements such as the Kyoto Protocol. These games can

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[†]University of East Anglia; amrish.patel@uea.ac.uk.

[‡]Virginia Tech University; alecsmith@vt.edu.

be used to model situations where a certain number of volunteers are needed from a group. For a concrete example, consider the problem of an economics department that needs, say, four faculty members to serve on a committee (e.g. for hiring, or for a university-wide initiative). Participation on the committee benefits the whole department, but at a cost to the volunteers. If a faculty member expects that the committee is full, or if there will only be two or fewer volunteers, then it is better not to join, but if he expects that there are only three other participants, then the faculty member would prefer to join the committee.

One motivation for participation may be the desire to avoid disappointing or "letting down" one's co-players, also known as guilt aversion (Dufwenberg 2002; Charness and Dufwenberg 2006; Battigalli and Dufwenberg 2007, 2009). Emotions such as guilt are thought by psychologists to be central to the facilitation of social behaviour (Chang and Smith 2015). In addition, guilt aversion may provide a microfoundation for the decision to participate, as the shared expectation of contributing to the public good may constitute a moral expectation or social norm (Charness and Dufwenberg 2006). Because guilt aversion is a belief-dependent motivation, in this paper we employ the toolbox of psychological game theory (Geanakoplos, Pearce and Stacchetti 1989, Battigalli and Dufwenberg 2009) to study how guilt affects the decision to participate.

In general, participation games have many asymmetric Nash equilibria, both in pure and mixed strategies (see P&R 1984). Much of the literature studying participation games has focussed primarily or exclusively on completely mixed symmetric equilibria (CMSE); see e.g., Dixit and Olson (2000) and Hong and Karp (2012). This focus on CMSE has been justified on at least three grounds. First, it is unlikely that otherwise identical agents playing non-cooperatively can coordinate such that only a precise subset of agents participate, as required by asymmetric pure strategy equilibria; or, that a precise subset play a different mixed strategy, as required by asymmetric mixed strategy equilibria (see Dixit and Olson 2000, p. 318). Second, the only mixed strategy equilibrium that identical players can learn via repeated play is a CMSE (Crawford and Haller 1990). Finally, if there are sufficiently many players, the CMSE of the complete information game is an approximation for the equilibrium of a participation game where there is a little uncertainty over players' preferences (Makris 2009).

Given the previous interest in CMSE, we examine how such equilibria are affected by guilt. Specifically, we explore three issues: equilibrium existence; comparative statics; and empirical estimates of guilt aversion.

For the general class of participation games with an arbitrary provision threshold and number of players, we show that, as with material preferences, with guilt aversion, there exist at most two CMSE. With a few exceptions that we discuss below, comparative statics results carry through from the participation game with material preferences to the participation game with guilt aversion. Our analysis demonstrates that despite the additional complexities resulting from belief-dependent motivations, guilt aversion generates largely intuitive results.

In the volunteer's dilemma (the participation game with a provision threshold of one), guilt aversion implies a unique CMSE in which the probability of participation is increasing in the guilt sensitivity parameter. The equilibrium participation rate is decreasing in the participation cost and increasing in the public good benefit. In contrast to the game with material preferences, for high cost-benefit ratios, the equilibrium probability of participating can be increasing in the number of players in the game. However, for lower cost-benefit ratios, equilibrium participation rates are decreasing in the number of players, as in the game with material preferences.

In participation games with a provision threshold of two or more, for high cost-benefit ratios where no CMSE exist in the game with material preferences, guilt aversion can imply existence of generically two CMSE. As in the game with material preferences, when there are two equilibria an increase in the cost-benefit ratio decreases (increases) the equilibrium participation rate in the high (low) participation equilibrium; an increase in the number of players reduces equilibrium participation rates; and an increase in the provision threshold increases equilibrium participation rates. An increase in guilt sensitivity decreases participation in the low participation equilibrium and increases it in the high participation equilibrium.

Using our equilibrium characterisation, we re-analyse existing experiments on participation games to estimate the average guilt sensitivity parameter. We contrast our estimates with existing measures in the literature to comment on the portability of guilt aversion across different strategic environments. Although our estimates vary widely, they are, largely, positive and in a range between 0 and 1. The empirical analysis is especially straightforward for the volunteer's dilemma, for which both the game with material preferences and the game with guilt aversion have a single CMSE. Participation rates in these games are somewhat greater than the material CMSE prediction, consistent with the idea that guilt aversion and the concern for other's disappointment may motivate participation. We do not suggest that guilt aversion is the sole factor driving participation, but this analysis provides empirical support for the idea that guilt aversion motivates the decision to participate.

Our work adds to a literature studying behavioural preferences in participation games. Existing contributions include Palfrey and Rosenthal (1988) on altruism, Pérez-Martí and Tomás (2004) on warm-glow and regret, and Dufwenberg and Patel (2017) on reciprocity. In addition, a small literature looks at the implications of guilt in linear public good games including Dufwenberg *et al.* (2011) and Dhami *et al.* (forthcoming).

Rothenhäusler *et al.* (2018) also study guilt in participation games. Their notion of shared guilt is, however, substantially different from ours. They model agents who experience (belief-independent) guilt from supporting an immoral activity, where preferences are private information. By contrast, we apply a psychologically grounded, complete information model of belief-dependent guilt to a standard participation game.

Experimental studies of guilt aversion, including Charness and Dufwenberg (2006), Vanberg (2008), Ellingsen *et al.* (2010), Khalmetski *et al.* (2015), and Bellemare *et al.* (2017) largely focus on reduced-form analyses of the relationship between second-order beliefs and behaviour in trust or dictator games. A few papers estimate guilt sensitivities directly using games other than the participation game (Bellemare *et al.* (2011); Attanasi *et al.* (2013); Peeters and Vorsatz (2018); Bellemare *et al.* (2018); Attanasi *et al.* (2018)). We will discuss how our estimates of guilt sensitivity compare with such studies later.

We proceed as follows. Section 2 presents P&R's participation game and Section 3 our model of guilt based on Battigalli and Dufwenberg (2007). Section 4 contains our theoretical results on the effect of guilt on participation: implicitly characterising equilibria (4.1), understanding existence (4.2) and comparative statics (4.3). Section 5 applies our analysis to existing experimental data to estimate guilt sensitivities and Section 6 concludes.

2 The participation game

Consider the following participation game as in P&R. Let $N = \{1, \ldots, n\}$ denote the set of players. The set of strategies available to each player *i* is $S_i = \{0, 1\}$. Each player *i* chooses a binary strategy $s_i \in S_i$, with strategy profiles given by $s = (s_1, \ldots, s_n)$. We refer to $s_i = 1$ as *participate* and $s_i = 0$ as *abstain*. The threshold for provision of the public good is $w \in [1, n)$.¹ Let $\sum_i s_i = m$ refer to the number of participants.

A mixed strategy for player *i* is a probability distribution $\sigma_i \in \Delta(S_i)$, where $\Delta(X)$ denotes the collection of probability measures over the set *X*. The profile of strategies of all players but player *i* is given by $\sigma_{-i} = (\sigma_j)_{j \neq i}$, and the complete profile of mixed strategies for all players is given by $\sigma = (\sigma_i)_{i \in N}$. A mixed strategy Nash equilibrium is a strategy profile σ^* such that for every player *i*, every action in the support of σ_i^* is a best response to σ_{-i}^* . We assume that players do not actually randomise, but that randomised choices may be interpreted as an expression of players' beliefs. We defer a formal presentation of beliefs to the next section, where we make this interpretation explicit.

Payoffs are given by $\pi_i(s)$ as follows:

$$\pi_{i}(s_{i}, s_{-i}) = \begin{cases} v & \text{if } s_{i} = 0 \text{ and } m \ge w \\ 0 & \text{if } s_{i} = 0 \text{ and } m < w \\ v - c & \text{if } s_{i} = 1 \text{ and } m \ge w \\ -c & \text{if } s_{i} = 1 \text{ and } m < w \end{cases}$$
(1)

where 0 < c < v.

¹Assuming w < n ensures that there are multiple efficient pure strategy Nash equilibria.

The participation game has many pure strategy Nash equilibria where exactly m = wplayers choose to participate and the rest abstain; if w > 1, there is also one where all players abstain. It also has many mixed strategy Nash equilibria where there are three types of players: those who participate with probability 1, those who abstain with probability 1, and those who participate with probability $\sigma_i \in (0, 1)$.

Given the many asymmetric equilibria, we focus on completely mixed symmetric equilibria throughout (CMSE): that is, we consider equilibria where players view the strategy choices of their co-players as independent and identically distributed random variables. To define the CMSE of P&R's game, we assume that every player *i* believes that each of his n - 1 co-players independently chooses to participate with probability p, so that $\sigma_i = p$ for all *i*, and denote the probability of abstaining by q = 1 - p. Under these assumptions, the number of participants (m) has a binomial distribution. Let $\rho(w; n, p)$ be the probability of a player being pivotal for provision of the public good (i.e., the probability that exactly m = w - 1 of *i*'s n - 1 co-players choose to participate). Using the binomial probability mass function,

$$\rho(w;n,p) = \binom{n-1}{w-1} p^{w-1} (1-p)^{n-w}.$$
(2)

Let F(k; r, p) be the probability that out of r players, k or fewer participate when each participates with probability p. Using the CDF of a random variable that follows a binomial distribution,

$$F(k;r,p) = \sum_{i=0}^{k} {\binom{r}{i}} p^{i} (1-p)^{r-i}.$$
(3)

Observation 1 (Participation games, cf. P&R equation 1.3). For all $n \ge 2$, $n > w \ge 1$ and v > c > 0, a CMSE is a probability p^* satisfying

$$\rho(w; n, p^*) = \frac{c}{v}.$$
(4)

Proof: All proofs are found in the appendix.

Equation (4) shows that in equilibrium, the probability of being pivotal for provision of the public good is equal to the cost-benefit ratio. This makes intuitive and economic sense: in equilibrium, the expected benefit from participating is equal to the (certain) cost and players are indifferent between participating and abstaining.

It is useful to distinguish between a special case of the model where w = 1, referred to as the *volunteer's dilemma*; and cases where $w \ge 2$, which we refer to as *threshold participation games*. The term *threshold* is added to refer specifically to games with thresholds greater than 1. Throughout the text we refer generically to both of these settings as participation games. Our next observation concerns the volunteer's dilemma. **Observation 2** (Volunteer's dilemma). In the volunteer's dilemma, for all $n \ge 2$ and v > c > 0, there exists a unique CMSE. It is described by

$$p^* = 1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}}.$$
 (5)

In the volunteer's dilemma with material preferences, a unique CMSE exists. To understand equation (5) more intuitively, rewrite it as $q^{n-1} = \frac{c}{v}$. Thus in a CMSE, the cost-benefit ratio of participating is equal to the probability of being pivotal for provision (i.e., the probability that n-1 players do not participate).

Comparative statics are very intuitive. The equilibrium probability of participating is increasing in the benefit of the public good and decreasing in the cost of participation and the number of players (a higher n implies a lower probability of being pivotal for provision, thus less incentive to participate).

For threshold participation games $(w \ge 2)$, there may be zero, one or two CMSE.

Observation 3 (Threshold participation games, cf. P&R proposition 2). For all $n \ge 2$, $n > w \ge 2$ and v > c > 0,

- **a.** if $\frac{c}{v} > \rho(w; n, \frac{w-1}{n-1})$, then there exists no CMSE;
- **b.** if $\frac{c}{v} = \rho(w; n, \frac{w-1}{n-1})$, then there exists one CMSE;
- c. if $\frac{c}{v} < \rho(w; n, \frac{w-1}{n-1})$, then there exist two CMSE.

For some intuition, reason as follows. If co-players participate with probability one, i should abstain. If they abstain with probability one, i should also abstain. Whether there will exist some participation probability in the middle such that i is indifferent between his two actions depends on c relative to v. If c is too large then although the probability of being pivotal does increase as the probability that co-players participate increases, it does not increase enough, thus no equilibrium exists.

The figure below illustrates the three possible cases.

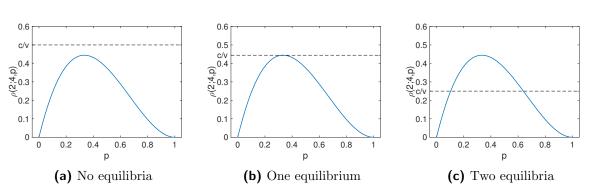


Figure 1: Equilibria of the threshold participation game with n = 4 and threshold w = 2.

Note: For n = 4 and w = 2, the solid line in each panel plots the probability of being pivotal, $\rho(w; n, p)$, over p. A CMSE is where $\rho(w; n, p) = \frac{c}{v}$. The dashed line in each panel illustrates a different value of $\frac{c}{v}$.

In panel (a) the cost-benefit ratio is so high that no CMSE exist, in panel (b) it is lower so one CMSE exists, and in panel (c) it is lower still so two CMSE exist. Overall, non-existence of CMSE is caused by players not having enough incentive to participate.

Finally, note the comparative statics. Where two CMSE exist, an increase in $\frac{c}{v}$ increases participation in the CMSE with lower participation and decreases it in the CMSE with higher participation. An increase in n implies a reduction in participation in both CMSE. To see why, note that for a given p, more players implies an increased probability of provision, thus the expected benefit of participation increases. To ensure that the expected benefit of participation decreases. To ensure that the expected benefit of participation decreases in w increases participation in both CMSE. To see why, note that for a given p, an increase in w increases participation in both s, it must thus be that participation decreases. An increase in w decreases the probability of provision and thus the expected benefit of participation decreases. To ensure that the expected benefit of participation equals its cost, it must thus be that participation in both CMSE. To see why, note that for a given p, an increase in w decreases the probability of provision and thus the expected benefit of participation decreases. To ensure that the expected benefit of participation equals its cost, it must thus be that participation in both CMSE. To see why, note that for a given p, an increase in w decreases. To ensure that the expected benefit of participation decreases.

3 Guilt aversion and psychological Nash equilibrium

Guilt aversion captures the psychological disutility from disappointing one's co-players. In this section we review the definitions of guilt aversion and psychological Nash equilibrium (Geanakoplos *et al.* 1989) as they apply in our setting. We focus on the model of *simple guilt* from Battigalli and Dufwenberg (2007).

3.1 Guilt aversion

To model guilt aversion, we need to specify players' beliefs. Each player *i* has a first-order belief $\alpha_i \in \Delta(S_{-i})$ about the strategies of the other players. Let $\alpha_{-i} = (\alpha_j)_{j \neq i}$ denote the first-order beliefs of all players save player *i*, then $\alpha = (\alpha_i, \alpha_{-i})$ is a profile of first-order beliefs.

Player *i* also has second-order beliefs β_i about the first-order beliefs of each co-player. Most generally, player *i*'s second-order beliefs might allow for correlation between the actions of his co-players and their beliefs. Here, we assume that higher-order beliefs are degenerate point beliefs, so that β_i is identified with an array of first-order beliefs: $\beta_i = (\beta_{ij})_{j \neq i}$, where $\beta_{ij} \in \Delta(S_{-j})$. Then $\beta = (\beta_i)_{i \in N}$ is a profile of second-order beliefs.²

Before the game is played, player j can calculate his expected payoff

$$\bar{\pi}_{j}^{0}(s_{j},\alpha_{j}) = \sum_{s_{-j}\in S_{-j}} \alpha_{j}(s_{-j})\pi_{j}(s_{j},s_{-j}), \tag{6}$$

given his strategy s_j and his first-order beliefs α_j about the strategies s_{-j} of the other players. The expression

$$D_j((s_j, s_{-j}), \alpha_j) = \max\{0, \bar{\pi}_j^0(s_j, \alpha_j) - \pi_j(s)\}$$
(7)

measures how much player j is disappointed or "let down" at the end of the game. After the game is played, if i knew j's beliefs α_j , he could calculate how much of D_j is due to his own behaviour:

$$G_{ij}(s,\alpha_j) = D_j(s,\alpha_j) - \min_{s'_i} D_j((s'_i, s_{-i}), \alpha_j).$$
(8)

A guilt averse player i then chooses s_i to maximize the expected value of

$$u_i((s_i, s_{-i}), \alpha_{-i}) = \pi_i(s) - \sum_{j \neq i} \theta_{ij} G_{ij}(s, \alpha_j),$$

$$\tag{9}$$

with respect to player *i*'s second-order beliefs, where $\theta_{ij} \ge 0$ is a parameter capturing player *i*'s guilt from disappointing player j.³

 $^{^2 {\}rm Rabin}$ (1993), Dufwenberg and Kirchsteiger (2004) and Battigalli and Dufwenberg (2007) impose a similar condition.

³As noted by Battigalli and Dufwenberg (2007, 2009), a simpler formulation where players dislike other's disappointment: $u_i((s_i, s_{-i}), \alpha_{-i}) = \pi_i(s) - \sum_{j \neq i} \theta_{ij} D_j(s, \alpha_j)$ results in the same best response correspondence. We focus on the definition of guilt in Equations 8 and 9, though we view both guilt and concern for other's disappointment as plausible motivations in participation games.

3.2 Psychological Nash equilibria with guilt aversion

A psychological Nash equilibrium (Geanakoplos, Pearce and Stacchetti 1989) with guilt aversion is a profile of behaviour and beliefs such that strategies are best responses to beliefs, and beliefs are correct.

Definition 1. A psychological Nash equilibrium with guilt aversion is a tuple (σ, α, β) such that

1. For each player i and beliefs α_i , for all s_i in the support of σ_i ,

$$s_i \in \arg\max \operatorname{E}_{\alpha_i,\beta_i}[u_i((s_i, s_{-i}), \alpha_{-i})].$$
(10)

2. For all $i \in N$, and for all $j \neq i$, $\alpha_i = \sigma_{-i} = \beta_{ji}$.

As noted above, our focus is on completely mixed symmetric equilibria with guilt aversion.⁴ Therefore, we consider a game where each player i's preferences are captured by the following utility function with homogeneous guilt sensitivities:

$$u_i((s_i, s_{-i}), \alpha_{-i}) = \pi_i(s) - \theta G(s, \alpha_{-i}), \tag{11}$$

where for all $i, j, \theta = \theta_{ij}$ and $G(s, \alpha_{-i}) = \sum_{j \neq i} G_{ij}(s, \alpha_j)$.

Definition 2. A completely mixed symmetric psychological Nash equilibrium with guilt aversion is a psychological Nash equilibrium with guilt aversion (σ, α, β) such that each player believes that all co-players independently choose their strategies, and that for all *i*, *j*, and $k \in N$ such that $i \neq j$ and $j \neq k$, $\sigma_i = \alpha_{ji} = \beta_{kji} = p^* \in (0, 1)$, where α_{ji} denotes the entry corresponding to player *i* in α_j and β_{kji} is the entry corresponding to player *i* in β_{kj} .

Definition 2 says that a CMSE of the participation game with guilt aversion is a psychological Nash equilibrium where each player believes that each other player's participation decision is an independent Bernoulli random variable with probability p. Notice that this definition also includes CMSE of the participation game with material preferences, which we obtain by setting $\theta = 0$.

4 Guilt and participation

In this section we characterise and provide comparative statics results for completely mixed symmetric psychological Nash equilibria of the participation game with guilt aversion.

⁴Observation 2 of Battigalli and Dufwenberg (2007) establishes that all of the pure strategy equilibria of the material payoff game are also equilibria of the associated psychological game with guilt aversion.

4.1 CMSE with guilt

Definition 2 says that as in CMSE of the participation game with material payoffs, in any CMSE involving guilt aversion, each player *i* believes that each co-player $j \neq i$ participates with the independent probability $\alpha_{ij} = p^*$, and higher-order beliefs are correct. Given his beliefs, in any CMSE player *i* must be indifferent between participating and abstaining.

If player i participates then by equation (8) the guilt he feels towards all other players is 0, because participating minimises the disappointment of all co-players. Therefore the expected payoff from participating is

$$(1 - F(w - 2; n - 1, p^*))v - c.$$
(12)

If player *i* abstains, then in the CMSE his expected material payoff is $(1 - F(w - 2; n - 1, p^*))v$. Abstaining players feels guilty only if they are pivotal, meaning that exactly w-1 other players participate. In that case, the n-w abstainers are disappointed in the amount $(1 - F(w - 1; n - 1, p^*))v$, while the w - 1 participants are disappointed by the greater amount $(1 - F(w - 2; n - 1, p^*))v$. When *i* abstains and is pivotal, his psychological payoff is the sum of the guilt he feels towards both abstainers and participants. Let \overline{s} denote a strategy profile where *i* is pivotal and abstains (so m = w - 1); then by slight abuse of notation we can write:

$$G(\underline{\overline{s}}, p^*) = v \left[(w-1)(1 - F(w-2; n-1, p^*)) + (n-w)(1 - F(w-1; n-1, p^*)) \right]$$
(13)

and his expected total utility is

$$(1 - F(w - 1; n - 1, p))v - \rho\theta G(\underline{\overline{s}}, p^*)$$

$$(14)$$

Our first result allowing for guilt aversion characterises the CMSE of the participation game. The result holds for all participation games, including both volunteer's dilemmas (w = 1) and threshold participation games $(w \ge 2)$.

Proposition 1 (Participation games with guilt). For all $n \ge 2$, $n > w \ge 1$, v > c > 0and $\theta \ge 0$, a CMSE is a p^* satisfying

$$\rho(w; n, p^*)(1 + \theta \widetilde{G}) = \frac{c}{v}, \tag{15}$$

where $\widetilde{G} = G(\overline{s}, p^*)/v = (w-1)(1 - F(w-2; n-1, p^*)) + (n-w)(1 - F(w-1; n-1, p^*)).$

The left hand side of equation (15) is the expected marginal benefit of participating normalised by v. If player i is not pivotal, his marginal benefit of participating is zero. If

i is pivotal (which occurs with probability $\rho(w; n, p^*)$) and he participates, his normalised material marginal benefit is 1 and his normalised guilt-alleviation marginal benefit is $\theta \tilde{G}$. The right hand side of equation (15) is the normalised marginal cost of participating.

Notice how the equilibrium condition compares to that with material preferences, condition (4). If $\theta = 0$ the two conditions are identical. Each player must be indifferent between participating and not, so the equilibrium probability equates the probability of being pivotal with the material cost-benefit ratio. If $\theta > 0$, conditions (4) and (15) are identical other than $\theta \tilde{G} > 0$ being included in the left-hand side of condition (15).

To understand \tilde{G} , consider when *i* would feel guilty. The only way that *j* can feel disappointed is if *j* expected the good to be provided and it is not. Player *i* will not feel guilty for *j*'s disappointment if *i* participated as there is nothing more *i* could do to ensure provision. If *i* abstained, then he only feels guilty for *j*'s disappointment if *i* is pivotal for provision (i.e., w - 1 other players participate), otherwise *i*'s choice could not affect provision. Guilt aversion thus endogenously increases the value of participation as by participating *i* can avoid the possibility of feeling guilty for *j*'s disappointment.

Notice how G reflects the asymmetry in the guilt that i would feel towards participating and abstaining co-players. If j abstained then he expected that the good would be provided with probability $(1 - F(w - 1; n - 1, p^*))$; if j participated then he expected provision with probability $(1 - F(w - 1; n - 1, p^*)) + \rho(w; n, p^*)$. Thus i would feel more guilty towards each of the w - 1 participants than each of the n - w abstainers.

It is difficult to see the implications of guilt for the nature of equilibria in terms of existence, multiplicity and comparative statics from condition (15). We now examine these issues more closely.

4.2 Existence of CMSE with guilt

We will first examine existence and multiplicity for volunteer's dilemmas (w = 1) and then for threshold participation games ($w \ge 2$).

With material preferences we noted that there exists a unique CMSE in the volunteer's dilemma (Observation 2). The same is true when players are guilt averse.

Proposition 2 (Volunteer's dilemma with guilt). In the volunteer's dilemma, for all $n \ge 2, v > c > 0$ and $\theta \ge 0$ there exists a unique CMSE. This CMSE is characterised by

$$p^{*}(c, n, v, \theta) = 1 - \left(\frac{2c}{v\left(1 + \theta\left(n - 1\right)\right) + \sqrt{v(v + \theta(n - 1)\left[v(n - 1)\theta - 4(c - \frac{v}{2})\right])}}\right)^{\frac{1}{n - 1}}.$$
 (16)

Thus within the class of CMSE, guilt does not create an equilibrium multiplicity problem in the volunteer's dilemma. The effect of guilt in participation games with a higher threshold is more interesting. Observation 3 stated that with material preferences and $w \ge 2$ there may be zero, one or two CMSE. Allowing for guilt, we have the following result.

Proposition 3 (Threshold participation games with guilt). For all $n \ge 2$, $n > w \ge 2$ and v > c > 0, there exists $\theta^* > 0$ such that

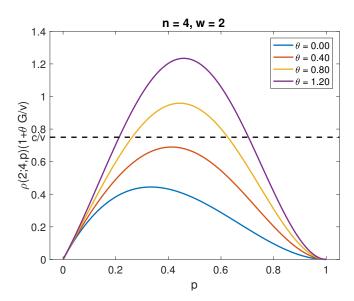
- **a.** if $\frac{c}{v} \ge \rho(w; n, \frac{w-1}{n-1})$, then for all $\theta > 0$ there exist two CMSE;
- **b.** if $\frac{c}{v} < \rho(w; n, \frac{w-1}{n-1})$, then
 - *i.* for $\theta > \theta^*$ there exist two CMSE;
 - *ii.* for $\theta = \theta^*$ there exists one CMSE;
 - *iii.* for $\theta < \theta^*$ there exist no CMSE.

Contrast how this result differs from Observation 3. There are three cases. If there are two CMSE in the game with material preferences, then there remain two CMSE in the game with guilt. If there is one CMSE in the game with material preferences then there are two CMSE in the game with guilt. If there are no CMSE in the game with material preferences, then if players are only slightly guilt averse then there are no CMSE. However if they are sufficiently guilt averse then there are generically two CMSE (a unique CMSE exists only for the knife-edge case of $\theta = \theta^*$). Thus guilt aversion can help the existence of CMSE.

The intuition as to why guilt facilitates existence of CMSE is straightforward. As discussed after Observation 3, non-existence of CMSE with material preferences was caused by too little incentive to participate. Guilt aversion endogenously increases incentives to participate as not doing so can disappoint co-players if the good is not provided and the player is pivotal. If players are sufficiently sensitive to guilt ($\theta \ge \theta^*$), then these incentives to participate are sufficiently high and the existence of a CMSE follows.

The figure below illustrates the point graphically.

Figure 2: The effect of guilt on participation



Note: For n = 4 and w = 2, this figure plots the normalised marginal benefit of participating over p for various values of θ . A CMSE is a p such that the function equals $\frac{c}{v}$ (an illustrative value of $\frac{c}{v} = \frac{3}{4}$ is depicted).

For the game with n = 4 and w = 2, the figure plots the normalised expected marginal benefit of participation (LHS of equilibrium condition (15)) over p for various values of θ . A CMSE is a p where this intersects with the normalised marginal cost of participation ($\frac{c}{v}$, RHS of equilibrium condition (15)). For the marginal cost depicted, $\frac{c}{v} = \frac{3}{4}$, and material preferences, i.e., $\theta = 0$, or very low guilt sensitivity, e.g., $\theta = 0.4$, there exist no CMSE. However, when players are more sensitive to guilt, e.g., $\theta = 0.8$, there are two CMSE.

While it is often the case that belief-dependent preferences worsen coordination problems, in our case, guilt aversion may actually foster coordination. To understand why, recall that in the introduction we noted that with material preferences, coordination is difficult in participation games due to the large number of asymmetric pure and mixed strategy Nash equilibria. Previous literature has thus argued that CMSE are easier to coordinate on (e.g., Dixit and Olson 2000; Hong and Karp 2012; see the reasons given in our introduction). We have found that guilt aversion can imply existence of CMSE for parameters where there exist none with material preferences (Proposition 3/Figure 2). It follows that guilt averse players may be better able to coordinate and thus more efficiently provide discrete public goods.

4.3 Comparative statics

Next we consider comparative statics. Once again, we will examine the volunteer's dilemma first (w = 1), then threshold participation games $(w \ge 2)$.

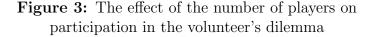
Proposition 4 (Volunteer's dilemma comparative statics). In the volunteer's dilemma, for all $n \ge 2$, v > c > 0 and $\theta \ge 0$,

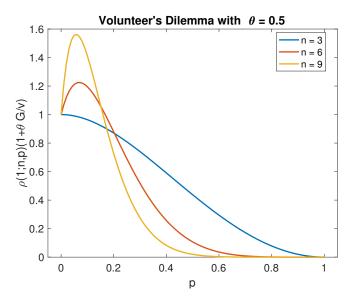
- **a.** p^* is strictly decreasing in $\frac{c}{v}$,
- **b.** p^* is strictly decreasing in n, for all $c \leq \overline{c}$,
- c. p^* is strictly increasing in θ .

The cost-benefit ratio of the public good affects equilibrium participation similarly to how it does with material preferences: As the cost increases relative to the benefit, equilibrium participation decreases (Proposition 4a).

Recall that with material preferences, CMSE participation in the volunteer's dilemma was strictly decreasing in the number of players (see Observation 2). With guilt, the same is true when the cost is low (Proposition 4b). Note that Proposition 4b does not state how equilibrium participation varies with n when the cost is high. For high costs, the effect of n on participation can be qualitatively different from that when the cost is low. We illustrate this via example.

For the example depicted in Figure 3, notice that when the cost is high, the relationship between CMSE participation and n is non-monontonic. That is, participation is strictly increasing in n for some interval of n.





Note: For w = 1 and $\theta = 0.5$, this figure plots the normalised marginal benefit of participating over p for various values of n. A CMSE is a p such that the function equals $\frac{c}{n}$.

For $\theta = 0.5$, the figure plots the normalised expected marginal benefit of participation (LHS of equilibrium condition (15)) in the volunteer's dilemma over p for different values of n. A CMSE is a value of p such that the plotted function equals the normalised marginal cost of participation, $\frac{c}{v}$. For the example in the figure, if $\frac{c}{v} < 0.8$ then indeed, CMSE participation is decreasing in the number of players. However, for higher values of $\frac{c}{v}$, CMSE participation can be increasing in the number of players.

To understand the intuition behind why guilt aversion gives rise to the non-monotonic relationship, consider the following two opposing effects. On the one hand, when there are more players, there is a higher probability that someone else will participate, therefore a lower incentive to participate. On the other hand, when there are more players, there are more people that will be disappointed if the good is not provided, thus increasing the guilt that players experience and providing more incentive to participate. When the cost is low, then the first effect always dominates as the material incentive to participate is very high. When the cost is high then which of the two effects is larger depends on the number of players. When there are few players, the first effect dominates. When there are many players, the second effect dominates. This explains the non-monotonic effect of additional players on equilibrium participation in a volunteer's dilemma with a high cost.

For the intuition behind the last part of the result (Proposition 4c), recall that we discussed in Section 4.2 how guilt aversion increases incentives to participate so as to avoid disappointing co-players. This is reflected Proposition 4 in that equilibrium participation increasing in the guilt sensitivity.

Having understood the nature of comparative statics in a volunteer's dilemma (w = 1) with guilt, we now consider comparative statics in threshold participation game ($w \ge 2$) with guilt. In order to make a precise statement, we restrict attention to parameters where there exist two CMSE (see Proposition 3 for when this is the case).

Proposition 5 (Threshold participation games comparative statics). For all $n \ge 2$, $n > w \ge 2$, v > c > 0 and $\theta \ge 0$ where there exist two CMSE, let $p_H^*(c, n, v, w, \theta)$ and $p_L^*(c, n, v, w, \theta)$ denote the CMSE participation probabilities where $p_L^* < p_H^*$, then

- **a.** p_L^* is strictly increasing in $\frac{c}{v}$ and p_H^* is strictly decreasing in $\frac{c}{v}$,
- **b.** p_L^* and p_H^* are strictly decreasing in n,
- **c.** p_L^* and p_H^* are strictly increasing in w,
- **d.** p_L^* is strictly decreasing in θ and p_H^* is strictly increasing in θ .

When there are two CMSE, factors that unambiguously increase the value of participation (i.e., a decrease in the material cost-benefit ratio or an increase in guilt sensitivity) decrease the participation probability for the low participation equilibrium and increase that of the high participation equilibrium. For an illustration of how the equilibria change, see Figure 2. As in the volunteer's dilemma, an increase in the number of players has opposing effects. On the one hand, i has more co-players whose disappointment he could feel guilty for, thus increasing his incentive to participate. On the other hand, more co-players means a higher probability of provision, thus a material incentive to decrease participation. The higher probability of provision also implies a lower chance of i feeling guilty, further decreasing his incentive to participate. The latter two effects dominate.

If the provision threshold increases, the probability of provision decreases. Players thus have a material incentive to increase participation in order to compensate for this. Guilt provides additional incentives to increase participation. As discussed after Proposition 1, i only feels guilty if w-1 players participate and feels more guilty towards participants than abstainers. An increase in w increases the number of participants towards whom i feels guilty, thus providing larger guilt-alleviation incentives for i to increase his participation.

5 Estimating guilt sensitivity from experiments

In this section we revisit data from existing experimental studies on participation games and consider whether our theory can explain observed behaviour. More specifically, we assume that all subjects play CMSE, and estimate the value of the (homogenous) guilt sensitivity term, θ , that is consistent with the participation rates seen in experiments. In doing so, we make two separate contributions to the literature.

First, experimental studies on participation games have tested whether behaviour conforms to the prediction of CMSE of the game with material preferences (Diekmann 1993) and the quantal-response equilibrium of the game with material preferences (Goeree *et al.* 2017).⁵ Since the prediction of CMSE with guilt aversion had not been derived until our study, we add to the literature trying to explain behaviour in experimental participation games by examining whether CMSE with guilt aversion can explain experimental data.

Second, experimental work estimating guilt sensitivities has demonstrated that it can be role dependent (e.g., Attanasi *et al.* 2016). Subjects playing different games will typically face different strategic incentives and thus one would imagine different guilt sensitivities would be triggered. Existing research has only estimated guilt sensitivity in relatively simple games with few players (e.g., binary dictator games, as in Bellemare *et al.* 2017). Our estimates thus adds to a literature understanding how different games trigger different guilt sensitivities.

We found nine studies that ran experimental games identical to that described in Section 2. Table 1 lists these studies in the first column; the relevant treatment is stated in the second column; the parameter values used (n, w, v and c) follow in columns three to six and column seven states the number of subjects.

⁵Non-game theoretic explanations of behaviour have also been explored. For example, Offerman *et al.* (1996) examine whether subjects' value orientations affect behaviour.

Column 8, Observed p, states the share of times subjects chose to participate. It is this figure that we shall try to explain with our model. The penultimate column, Nash, substitutes the experimental parameters into equilibrium condition (4) to calculate the probability with which a subject should participate if they play according to the CMSE of the game with material preferences. Notice how the CMSE reflect Observations 2 and 3. There may be no CMSE (e.g., Dawes *et al.* 1986 in Table 1), as noted in Observation 3a. Volunteer's dilemmas (w = 1) always have exactly one CMSE (e.g., Diekmann 1993 in Table 1), as noted in Observation 2. Threshold participation games ($w \ge 2$) can have two CMSE (e.g., Offerman *et al.* 1996, treatment High 7, in Table 1), as noted in Observation 3c.

The final column of Table 1, θ , is the imputed guilt sensitivity, i.e., the value of θ that can generate the observed participation rate. This value is calculated by first substituting the parameter values used in the treatment into the equilibrium condition for a CMSE with guilt aversion, condition (15), and then solving the condition for θ .

While participation games (with either w = 1 or $w \ge 2$) may have 0, 1, or 2 CMSE in general (Propositions 2 and 3), for a given set of parameters the assumption that the empirically observed participation rate is a CMSE implies a unique value of θ . To see this, assume that the observed participation rate is equal to p^* in equation (15). Then, because the terms n, w, c, v and p^* are known, the terms $\rho(w; n, p^*)$, \tilde{G} , and c/v are constant. The equation is linear in θ , the only unknown term, so the solution is unique.

If the empirically observed participation rate in a volunteer's dilemma is higher than the CMSE participation rate of a material game (e.g., Feldhaus and Stauf 2016 in Table 1), only a negative value of θ can explain the data. An imputed negative guilt sensitivity could imply that a subject's utility is increasing in how disappointed his co-players are if he is responsible for that disappointment. While we have not modelled such antisocial motives (note that we assumed $\theta \ge 0$ in Sections 3-4), they may be relevant empirically.⁶ Thus where the imputed θ is negative in the volunteer's dilemma it is reported as such (e.g., Feldhaus and Stauf 2016 in Table 1).

Given how θ affects the normalised material benefit of participation in threshold participation games (as illustrated in Figure 2), the sign of the θ that implies the observed participation rate can be deduced by comparing the observed participation rate to that predicted by the CMSE of the game with material preferences. If there are no CMSE in the game with material preferences (e.g., Dawes *et al.* 1986 in Table 1), then clearly a positive θ is needed to increase incentives to participate and thus existence of CMSE. If there exist two CMSE in the game with material preferences, and the observed participation rate is either less than the lower equilibrium probability or greater than the higher equilibrium probability (e.g., Cadsby and Maynes 1999, YB4 & GB4), then once again, a positive θ can explain the data. If however the empirical participation rate is between the lower and higher CMSE probabilities of the game with material preferences

⁶See e.g., Herrmann *et al.* (2008), for evidence of antisocial motivations for behaviour.

(e.g., Offerman *et al.* 1996, High 5), then only a negative θ can explain the data, as there is a need to reduce incentives to participate relative to those under material preferences.

$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Study	Treatment	n	w	v	c	Subj.	Observed p	Nash	θ
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Dawes et al. 1986	Standard	7	3	10	5	70	0.51	none	0.267
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		dilemma	7	5	10	5	70	0.64	none	0.131
1990 None 0.429 none 0.205 Diekmann 1993 A 2 1 100 50 33 0.61 0.5 0.429 Offerman et al. Low 7 7 3 180 60 63 0.198 none 0.254 1996 High 7 7 3 245 60 63 0.41 0.199 -0.054 1996 High 57 5 3 245 60 63 0.41 0.199 -0.054 Cadsby YB1 & GB1 10 5 11 10 20 0.136 none 34.5 K Maynes YB2 & GB2 10 5 20 10 20 0.184 none 3.148 999 YB3 & GB3 10 5 30 10 20 0.366 0.398 0.0261 WB4 & GB4 10 5 40 10 20 0.361 0.0261 0.036 0.491 0.2651 -0.033 0.026 0.491 0.2651 -0.033 0.026 0.491 0.2551 </td <td>1 1</td> <td>F&G</td> <td>5</td> <td>3</td> <td>5</td> <td>2</td> <td>60</td> <td>0.365</td> <td>none</td> <td>0.152</td>	1 1	F&G	5	3	5	2	60	0.365	none	0.152
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	* *	Simultaneous	5	3	6	3	35	0.429	none	0.205
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Diekmann 1993	А	2	1	100	50	33	0.61	0.5	0.462
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		Η	5	1	100	50	25	0.28	0.159	0.294
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Offerman <i>et al.</i>	Low 7	7	3	180	60	63	0.198	none	0.254
High 5'532456060 0.504 $\& 0.719$ -0.153 CadsbyYB1 & GB1105111020 0.136 none 34.5 & MaynesYB2 & GB2105201020 0.243 none 1.53 1999YB3 & GB3105301020 0.184 none 3.148 YB4 & GB4105401020 0.366 $\begin{array}{c} 0.398 \\ \& 0.491 \end{array}$ $0.026 \\ \& 0.491 \end{array}$ YB5105851010 0.611 $\begin{array}{c} 0.251 \\ \& 0.651 \end{array}$ $-0.033 \\ \& 0.651 \end{array}$ Healy & PateA, all cost 2021 0.8 0.2 18 0.711 0.75 $-0.194 \\ \& 0.651 \end{array}$ 2009A, all cost 6021 0.8 0.6 18 0.259 0.255 $0.047 \\ B, all cost 60$ 61 0.8 0.6 18 0.137 0.056 $0.217 \\ 0.217 \end{bmatrix}$ Feldhaus & Stauf 2016Baseline31 12 660 0.288 0.293 $-0.014 \\ 0.5 $ Goeree et al. $n = 2$ 21 0.8 0.2 34 0.52 0.75 $-0.921 \\ 0.75 $ 2017 $n = 3$ 31 0.8 0.2 48 0.28 0.242 0.722 $n = 6$ 61 0.8 0.2 48 0.28 0.242 0.722	1996	${\rm High}\ 7$	7	3	245	60	63	0.41		-0.054
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		High 5^7	5	3	245	60	60	0.504		-0.153
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Cadsby	YB1 & GB1	10	5	11	10	20	0.136	none	34.5
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	& Maynes	YB2 & GB2	10	5	20	10	20	0.243	none	1.53
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1999	YB3 & GB3	10	5	30	10	20	0.184	none	3.148
YB5105851010 0.61 $\& 0.651$ -0.033 Healy & PateA, all cost 20210.80.2180.710.75 -0.194 2009A, all cost 60210.80.6180.2590.25 0.047 B, all cost 20610.80.2180.396 0.242 0.459 B, all cost 60610.80.618 0.137 0.056 0.217 Feldhaus & StaufBaseline3112660 0.288 0.293 -0.014 Goeree et al. $n = 2$ 21 0.8 0.2 34 0.52 0.75 -0.921 2017 $n = 3$ 31 0.8 0.2 48 0.28 0.242 0.072 $n = 6$ 61 0.8 0.2 48 0.28 0.242 0.072		YB4 & GB4	10	5	40	10	20	0.36		0.026
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		YB5	10	5	85	10	10	0.61		-0.033
B, all cost 20610.80.2180.3960.2420.459B, all cost 60610.80.6180.1370.0560.217Feldhaus & Stauf 2016Baseline31126600.2880.293-0.014Goeree et al. $n = 2$ 210.80.2340.520.75-0.9212017 $n = 3$ 310.80.2360.40.5-0.239 $n = 6$ 610.80.2480.280.2420.072	Healy & Pate	A, all cost 20	2	1	0.8	0.2	18	0.71	0.75	-0.194
B, all cost 60610.80.6180.1370.0560.217Feldhaus & Stauf 2016Baseline31126600.2880.293-0.014Goeree et al. $n = 2$ 210.80.2340.520.75-0.9212017 $n = 3$ 310.80.2360.40.5-0.238 $n = 6$ 610.80.2480.280.2420.072	2009	A, all cost 60	2	1	0.8	0.6	18	0.259	0.25	0.047
Feldhaus & Stauf 2016Baseline31126600.2880.293-0.014Goeree et al. $n = 2$ 210.80.2340.520.75-0.9212017 $n = 3$ 310.80.2360.40.5-0.239 $n = 6$ 610.80.2480.280.2420.072		B, all cost 20	6	1	0.8	0.2	18	0.396	0.242	0.459
2016 Baseline 3 1 12 6 60 0.288 0.293 -0.014 Goeree et al. $n = 2$ 2 1 0.8 0.2 34 0.52 0.75 -0.921 2017 $n = 3$ 3 1 0.8 0.2 36 0.4 0.5 -0.239 $n = 6$ 6 1 0.8 0.2 48 0.28 0.242 0.072		B, all cost 60	6	1	0.8	0.6	18	0.137	0.056	0.217
2017 $n = 3$ 310.80.2360.40.5-0.239 $n = 6$ 610.80.2480.280.2420.072		Baseline	3	1	12	6	60	0.288	0.293	-0.014
n = 6 6 1 0.8 0.2 48 0.28 0.242 0.072	Goeree <i>et al.</i>	n=2	2	1	0.8	0.2	34	0.52	0.75	-0.921
	2017	n = 3	3	1	0.8	0.2	36	0.4	0.5	-0.239
		n = 6	6	1	0.8	0.2	48	0.28	0.242	0.072
n = 9 9 1 0.8 0.2 50 0.19 0.159 0.054		n = 9	9	1	0.8	0.2	36	0.19	0.159	0.054
n = 12 12 1 0.8 0.2 48 0.61 0.118 0.155		n = 12	12	1	0.8	0.2	48	0.61	0.118	0.155

Table 1: Imputed guilt sensitivities

Notes: This table summarises the parameters and participation rates in experiments that run the P&R game. Subj. refers to the number of subjects in the study. Observed p denotes the share of choices where subjects chose to participate. *Nash* refers to the equilibrium probability of participation in the CMSE (where it exists) with material preferences given the game parameters. A value of "none" in this column implies no CMSE exists. θ calculates the value of θ needed to generate the observed participation rate assuming subjects play CMSE and given the game parameters.

7. This treatment also appears in Sonnemans *et al.* (1998) as treatment "Public good" and in Offerman *et al.* (2001) as treatment "PGP".

Using the imputed estimates from Table 1, we find that our mean estimate of θ is 1.68 and the median is 0.142. The subject-weighted mean estimate of θ (a weighted mean estimate of theta where the weight a particular study receives is proportional to the number of subjects it has) is 0.925.

Table 1 contains one imputed value of θ that is several orders of magnitude larger than any other study: Cadsby and Maynes (1999), treatment YB1 & GB1, where $\theta = 34.5$. Given that all other estimates range from -0.921 to 3.1, this is an obvious outlier. Since the other treatments in Cadsby and Maynes (1999) imply plausible estimates of θ , the reason for this outlier must be the parameters used in treatment YB1 & GB1. Recall that $\frac{c}{v} \in (0, 1)$ and note that for YB1 & GB1, $\frac{c}{v} = 0.9$. The value of $\frac{c}{v}$ used for almost all other studies and treatments analysed in Table 1 is no more than 0.5.⁸ Thus it seems that when the public good cost-benefit ratio is very high, the theory performs poorly. Subjects should theoretically have little incentive to participate, but empirically observed participation rates are not extremely low, thus an implausibly large guilt sensitivity is needed to explain participation.

If our objective is to understand behaviour in the mean participation game, it makes sense to exclude extreme parameter calibrations such that found in treatment YB1 & GB1 in Cadsby and Maynes (1999). Excluding this treatment, our mean estimate of θ is 0.255, the median is 0.131 and the subject-weighted mean is 0.165.

While one cannot over-infer from these back-of-the-envelope calculations for many reasons, a low but positive estimate of guilt sensitivity is broadly consistent with previous work estimating the parameter. Our precise estimates of guilt sensitivity are very similar to some studies, but somewhat lower than others. As discussed earlier, different strategic environments may trigger different guilt sensitivities and it is useful to identify contexts that trigger similar sensitivities.

Several studies have found somewhat higher estimates of θ than ours. Using a fourplayer game, Bellemare *et al.* (2011) report θ estimates in the range of 0.4 to 0.8.⁹ Peeters and Vorsatz (2018) examine a two player prisoner's dilemma and find estimates of θ between 1.8 and 3.5. Bellemare *et al.* (2018) use mini-dictator games and find θ in the range of 0.4 to 1.0 when estimates are "stake dependent".

At least two studies have found θ estimates remarkably close to ours, despite studying very different games. For their "stake independent" estimate, Bellemare *et al.* (2018) find a mean θ of 0.1. Attanasi *et al.* (2018) also find an estimate of 0.1 in their embezzlement game. Recall that our own median estimates of θ are also close to 0.1.

The factors that determine whether the guilt sensitivity is around 0.1 or somewhat higher are not obvious. We hope our comparisons of sensitivity estimates are a useful

⁸The exception being Healy and Pate 2009, B, all cost 60, where $\frac{c}{v} = 0.75$.

⁹Ederer and Stremitzer (2017) find θ estimates in the range of 0 to 20. However their study is less comparable as they allow utilities to be concave in guilt and their model allows promises to be made. Attanasi *et al.* (2013) also estimate guilt sensitivities, however they are once again less comparable to ours due to the modelling assumptions used.

starting point for future work that more systematically studies guilt aversion.

6 Discussion and conclusion

Participation is necessary for the provision of public goods the world over. Threshold public goods come with an additional challenge, the coordination problem that results from equilibrium multiplicity. We studied the effect of an important motivation that may explain participation in such situations, guilt aversion. Despite the additional complexities such preferences introduce, many of the results from the material preference game carry over to that with guilt. Some new and intuitive results also emerge.

Guilt may help coordination as it facilitates the existence of completely mixed symmetric equilibria (CMSE), which are easier to coordinate on than many other types of equilibria. Guilt increases (decreases) participation when players are coordinated on high (low) participation equilibria. In a volunteer's dilemma with guilt, group size has a nonmonotonic effect on participation in games with high participation costs. Re-analysing existing experimental data, our equilibrium characterisation suggests that a low but positive guilt sensitivity parameter can explain observed participation rates.

To appreciate the relevance of our results it is important to note the critical assumptions that they are predicated on. One such assumption is the solution concept. Given our focus on understanding coordination, our theoretical analysis was restricted to CMSE. While we have fully characterised the effects of guilt for this class of equilibria, the intuitions driving our results do not necessarily extend to other classes of equilibria. To see this, consider pure strategy equilibria in the participation game.

With material preferences in threshold participation games,¹⁰ the only Nash equilibria are where no-one participates or exactly the threshold number of players needed for provision, w, participate. With guilt aversion, regardless of the guilt sensitivity, the set of psychological Nash equilibria is identical to the set of Nash equilibria with material preferences. To see the simple intuition behind this, reason as follows.

First, note that if no-one participates, *i*'s participation choice cannot influence *j*'s material payoff, thus *j* provides *i* no guilt incentives to participate. Hence no-one participating is an equilibrium. Second, suppose that exactly *w* players participate. Participant *i* deviating to non-participation would lower *j*'s material payoffs as the good is not provided, thus guilt discourages participant deviation. Non-participant *i* cannot influence *j*'s material payoff, thus a non-participant has no guilt incentives. Hence exactly *w* participants is an equilibrium in pure strategies.¹¹ Third, suppose that the equilibrium number of participants is less than or equal to w - 1. Participant *i* cannot lower *j*'s material payoff through his action choice, thus *i* faces no guilt incentives. Since material incentives

¹⁰Arguments analogous to those that follow can be made for the volunteer's dilemma.

¹¹As noted previously, that all NE of the game with material preferences are also psychological NE of the game with guilt follows directly from Battigalli and Dufwenberg (2007, p. 173, Observation 2).

motivate i to deviate, he does so and the profile is not an equilibrium. Finally, suppose that at least w + 1 players participate in equilibrium. Deviation by participant i does not affect any j's material payoff, thus i feels no guilt from deviating. Since deviation increases i's material payoff, he does so and thus the profile is not an equilibrium.

Our theoretical results identifying the effect of guilt on CMSE demonstrated that the effect of guilt was potentially rather significant. As the above reasoning demonstrates, guilt may have no effect at all on other classes of equilibria.

Throughout this study we have focussed on the effects of guilt on participation, however other motivations for participation may also be relevant. By understanding the incentives behind our results on guilt one can deduce the effect of other preferences in participation games. For instance, the reason for non-existence of CMSE in the game with material preferences was insufficient incentives to participate. Guilt aversion overcame this, but presumably so could a model with altruistic agents, for example. It would be interesting to more fully compare and contrast the differences between different preference models and test between them experimentally.

To the best of our knowledge, no other paper has empirically tried to identify the effect of guilt in participation games. Our empirical estimates provide a useful check of the portability of guilt aversion estimates and its empirical relevance for participation games; however, they are far from conclusive. For example, it may be that rather than playing CMSE, subjects are playing asymmetric pure strategy equilibria and alternating between them such that they generate the same overall participation rate as a CMSE. Future empirical work should study individual subject choices rather than aggregate participation rates as we do.

On the topic of how experimental participation game data could be viewed as arising from equilibria other than CMSE, it would be interesting to understand the implications of asymmetric mixed strategy equilibria with guilt. We do not provide an exhaustive analysis here but simply illustrate that if subjects play asymmetric mixed strategy equilibria, the guilt sensitivity, θ , required to generate a particular participation rate may be lower than that required in CMSE. To see the logic behind this, consider the following example.

Take a three-player volunteer's dilemma. With material preferences, there exists an equilibrium where player 1 does not participate, while players 2 and 3 play the same non-degenerate mixed strategy.¹² Clearly the probability with which 2 and 3 participate in such an equilibrium is higher than that of the CMSE of the game (recall Observation 2).

Now suppose that players are guilt averse. In an equilibrium where player 1 does not participate and the other two players play a non-degenerate mixed strategy, player 2 and 3's probability of participation must be strictly higher than in the game without guilt

¹²Given that player 1 does not participate, the game reduces to a two-player volunteer's dilemma. Using (5), players 2 and 3 each participating with probability $1 - \frac{c}{v}$ implies that they are best responding. Given this, it is straightforward to show that non-participation is a best response for player 1.

for two reasons. First, player 1 now feels guilt for not participating and will deviate to participation unless 2 and 3 increase their probability of participation in order to increase the chance that the good is provided. Second, players 2 and 3 feel guilty if the good is not provided, thus they have an additional incentive to participate. Asymmetric equilibria can thus involve additional material and guilt motivations to participate, and for such equilibria a lower guilt sensitivity, θ , can generate a given participation rate relative to the CMSE.¹³ Thus, if experimental subjects play asymmetric equilibria, like the one just described, the implied θ estimates would be lower than those we calculated assuming that they play CMSE (see Table 1). Note, however, that since there are typically many mixed strategy asymmetric equilibria it is not obvious that the θ estimate implied by all such equilibria is lower than that implied by a CMSE.

While we are able to draw some inferences by relating our theory to existing experimental data, there are some predictions that just cannot be tested based on existing data. For instance, our model suggests that in a volunteer's dilemma with guilt, group size can have a non-monotonic effect on participation. However, since we are not aware of any existing experimental work that has implemented a high cost volunteer's dilemma where group size is the varied by treatment, our novel conjecture remains untested.

Finally, our model assumes that players have identical preferences, involving the same guilt sensitivities towards each co-player. It seems likely that guilt sensitivities vary, both within and across people. We do not model such variation in preferences and it remains to be seen how heterogeneous guilt sensitivities might affect play in many settings including participation games. We leave this analysis for future work.

There is much to be understood on how guilt affects participation in the provision of public goods. We hope that the equilibrium characterisation presented here provides a useful starting point for further study, both theoretical and empirical.

Appendix: Proofs

Proof of Observation 1

If player *i* chooses to abstain, then at least *w* of the remaining n-1 players must choose participate for the good to be provided. Thus *i*'s expected payoff from abstaining is (1 - F(w - 1; n - 1, p))v. If *i* chooses to participate, then at least w - 1 of the remaining n - 1 players must choose to participate for the good to be provided. Thus *i*'s expected payoff of participating is (1 - F(w - 2; n - 1, p))v - c. In a CMSE *i* must be indifferent between his two options. Equating the two expected payoffs gives condition (4).

¹³Example available on request.

Proof of Observation 2

Substitute w = 1 into (4) and solve for p.

Proof of Observation 3

Recall equilibrium condition (4), $\rho(w; n, p^*) = \frac{c}{v}$. Since $\rho(w; n, p)$ is the probability mass function of a discrete variable following a binomial distribution, defined by (2), $\rho(w; n, 0) = \rho(w; n, 1) = 0$, $\rho(w; n, p)$ is strictly increasing for all $p \in [0, \frac{w-1}{n-1})$ and strictly decreasing for all $p \in (\frac{w-1}{n-1}, 1]$. As $\frac{c}{v} \in (0, 1)$, simply compare $\rho(w; n, \frac{w-1}{n-1})$ with $\frac{c}{v}$ to determine for how many values of p condition (4) holds.

Proof of Proposition 1

First, in a CMSE *i*'s expected utilities from participating and abstaining must be equal.

To calculate the expected utility from participating, note that if player i participates, then he cannot feel guilty. Thus in any CMSE, the expected utility from participating is

$$(1 - F(w - 2; n - 1, p))v - c.$$
(17)

To calculate the expected utility from abstaining, note that if player j chooses to participate and the threshold is not met (so m < w), then j's material payoff is -c. Let \underline{S} denote the set of strategy profiles such that the public good is not provided: $\underline{S} \equiv \{s \in S | m = \sum s_i < w\}$. Let $\underline{s} \in \underline{S}$. Then player j's disappointment, if the public good provision threshold is not reached, is

$$D_j(\underline{s}, 1, \alpha_j) = \max\{0, (1 - F(w - 2; n - 1, p))v - c - (-c)\} \\ = (1 - F(w - 2; n - 1, p))v.$$

If player j chooses to abstain, his expected material payoff is (1 - F(w - 1; n - 1, p))v. When m < w, j's material payoff is 0 thus his disappointment is

$$D_j(\underline{s}, 0, \alpha_j) = \max\{0, (1 - F(w - 1; n - 1, p))v - 0\}$$

= $(1 - F(w - 1; n - 1, p))v.$

To calculate *i*'s expected utility from abstaining, note first that if *i* abstains, his expected material payoff is (1 - F(w - 1; n - 1, p))v. To calculate *i*'s total expected psychological payoff from abstaining, note again that *i* only feels guilty when he is pivotal (else the two components of equation (8) are equal). If *i* abstains and is pivotal, then m = w - 1 and there are w - 1 players participating and n - w others abstaining. The probability that *i* is pivotal when players choose In with probability *p* is $\rho(w; n; p)$. In the CMSE, this implies *i*'s expected total guilt from abstaining is

$$E_{\alpha_{i},\beta_{i}} [G(s, \alpha_{-i})] = \rho \left[\sum_{j \neq i} G_{ij}(s, \alpha_{j}) \right]$$
$$= \rho v \left[(w-1)(1 - F(w-2; n-1, p)) + (n-w)(1 - F(w-1; n-1, p)) \right]$$
(18)

Then the expected utility from abstaining is

$$(1 - F(w - 1; n - 1, p))v - \rho(w; n, p)\theta v\tilde{G}$$
(19)

where $\tilde{G} = G/v = \left[(w-1)(1 - F(w-2; n-1, p)) + (n-w)(1 - F(w-1; n-1, p)) \right].$ Equating equations (17) and (19) and simplifying gives

$$\rho(w; n, p)(1 + \theta \widetilde{G}) = \frac{c}{v}, \qquad (20)$$

as shown in Equation (15) in the main text. \blacksquare

Proof of Proposition 2

Substituting w = 1 into (15) and using q = 1 - p gives,

$$\left(1 + \theta(n-1)(1-q^{n-1})\right)q^{n-1} = \frac{c}{v}.$$
(21)

We first find an explicit expression for $q^*(c, n, v, \theta)$ then demonstrate its image lies in the unit interval. Implicitly differentiating (21) with respect to c gives

$$\frac{\partial q^*(c,n,v,\theta)}{\partial c} = \frac{1}{v(n-1)q^*(c,n,v,\theta)^{n-2} \left[1 + \theta(n-1)(1 - 2q^*(c,n,v,\theta)^{n-1})\right]}.$$
 (22)

Solving partial differential equation (22) gives the general solution

$$q^{*}(c, n, v, \theta) = \left(\frac{2(K(n, v, \theta) + c)}{v\left(1 + \theta\left(n - 1\right)\right) + \sqrt{v(v + \theta(n - 1)\left[v(n - 1)\theta - 4(K(n, v, \theta) + c - \frac{v}{2})\right])}}\right)^{\frac{1}{n - 1}}$$
(23)

where K(n, v, c) is a function. Substituting (23) into (21) and solving for $K(n, v, \theta)$, establishes that $K(n, v, \theta) = 0$. The particular solution of (22) is thus

$$q^{*}(c,n,v,\theta) = \left(\frac{2c}{v\left(1+\theta\left(n-1\right)\right) + \sqrt{v(v+\theta(n-1)\left[v(n-1)\theta - 4(c-\frac{v}{2})\right])}}\right)^{\frac{1}{n-1}}.$$
 (24)

We now demonstrate that $q^* \in (0, 1)$. Suppose $q^* \leq 0$. This would imply that

$$v(n-1)^{2}\theta^{2} + (1-4c+2v)(n-1)\theta + v + 1 \le 0.$$
(25)

The LHS of (25) is a quadratic in θ . If the discriminant were negative then (25) would be false. So suppose that it is non-negative; the larger root of the quadratic is then

$$\frac{4c - 2v - 1 + \sqrt{16c^2 - (16v + 8)c + 1}}{2v(n - 1)}.$$

For this root to be non-negative it must be that $4v(1+v) \leq 0$, which is false. Thus $q^* > 0$. Now suppose $q^* > 1$. This would imply that 4c(c-v) > 0, which is false since c < v by assumption, thus $q^* < 1$.

Proof of Proposition 3

We first establish some properties of the LHS of equilibrium condition (15), then state how these can be used to identify when equilibria exist.

Condition (15) can be written as

$$\binom{n-1}{w-1}p^{w-1}(1-p)^{n-w}\left(1+\theta\left[(n-1)\left(1-\sum_{i=0}^{w-1}\binom{n-1}{i}p^{i}(1-p)^{n-i}\right)\right.\\\left.+(w-1)\binom{n-1}{w-1}p^{w-1}(1-p)^{n-w}\right]\right)=\frac{c}{v}.$$
 (26)

Let $g(p,\theta)$ denote the LHS of (26). Clearly $g(0,\theta) = g(1,\theta) = 0$. We can show that

$$\begin{aligned} \frac{\partial g(p,\theta)}{\partial p} &= \binom{n-1}{w-1} p^{w-2} (1-p)^{n-w-1} \left[\alpha + \theta \left[(n-w)\alpha + (n-1) \sum_{i=0}^{w-1} \binom{n-1}{i} p^i (1-p)^{n-i} (np-i-\alpha) \right. \right. \\ &+ 2(w-1) \binom{n-1}{w-1} p^{w-1} (1-p)^{n-w} \alpha \right] \right], \end{aligned}$$

where $\alpha = w - 1 - (n - 1)p$. Note that when p is arbitrarily close to zero, $\alpha > 0$ and $(np - i - \alpha) \leq 0$. However since the $(np - i - \alpha)$ is multiplied by an arbitrarily small number, it follows that for p arbitrarily close to zero, $\partial g(p, \theta)/\partial p > 0$. Note also that when p is arbitrarily close to one, $\alpha < 0$ and $(np - i - \alpha) > 0$. However since the $(np - i - \alpha)$ is multiplied by an arbitrarily small number, it follows that for p arbitrarily small number, it follows that for p = 0.

Next we argue that $g(p, \theta)$ is strictly quasiconcave in p. Rewrite equilibrium condition (15) as

$$\rho(1+\theta\left[(n-w)(1-F(w-1;n-1,p^*))+(w-1)(1-F(w-2;n-1,p^*)\right])=\frac{c}{v}.$$

Note that the LHS equals $g(p,\theta)$. In order to show that $g(p,\theta)$ is strictly quasiconcave in p, reason as follows. Clearly ρ is a strictly quasiconcave function of p on [0,1]. Note that $H(p) = (1+\theta [(n-w)(1-F(w-1;n-1,p^*)) + (w-1)(1-F(w-2;n-1,p^*)]$ is a monotone increasing function of p, since 1-F is increasing in p. Note also that H(p) is bounded above by $1+\theta(n-1)$ and below by 1. This is sufficient for $g(p,\theta)$ to be strictly quasiconcave in p for $p < \arg \max \rho = \frac{w-1}{n-1}$. In order to guarantee $g(p,\theta)$ is strictly quasiconcave in p for $p > \arg \max \rho = \frac{w-1}{n-1}$, we demonstrate that the inflection point of H(p) equals the argmax of ρ . The second derivative of H(p) at $\frac{w-1}{n-1}$ is

$$H''\left(\frac{w-1}{n-1}\right) = \left(-\frac{1}{w(w^2-1)}\right)(n-1)^{3-n}\left((w-1)(n-1)\sum_{k=0}^{\infty}\frac{(2)_k(w+1-n)_k(1-w)^k}{(w+2)_k(n-w)^kk!}\right) + \theta(n-w)^{n-2-w}(w-1)^w\sum_{k=0}^{\infty}\frac{(1)_k(w-n)_k(1-w)^k}{(w+1)_k(n-w)^kk!}\right) \left[(n-w)\binom{n-1}{w-1} - w\binom{n-1}{w}\right]$$

where $(c)_0 = 1$ and $(c)_k = \prod_{i=0}^{k-1} (c+i)$. Note that we can rewrite $[\cdot]$ as follows

$$(n-w)\binom{n-1}{w-1} - w\binom{n-1}{w} = \frac{(n-w)(n-1)!}{(w-1)!(n-w)!} - \frac{w(n-1)!}{w!(n-w-1)!}$$
$$= (n-1)! \left(\frac{1}{(w-1)!(n-w-1)!} - \frac{1}{(w-1)!(n-w-1)!}\right) = 0.$$

Thus $H''(\frac{w-1}{n-1}) = 0$ and the inflection point of H(p) equals the argmax of ρ . Hence $g(p, \theta)$ is strictly quasiconcave in p and thus intersects $\frac{c}{v}$, twice, once or zero times.

Finally, note that since $[\cdot]$ in (26) is strictly positive, $\partial g(p,\theta)/\partial \theta > 0$. Thus for θ sufficiently high, $g(p,\theta)$ will intersect $\frac{c}{v}$ twice.

(a) If $\frac{c}{v} \ge \rho(w; n, \frac{w-1}{n-1})$, at least one CMSE exists when $\theta = 0$ (Observation 3). Given the properties of g established in this proof, there will exist two CMSE for all $\theta > 0$.

(b) If $\frac{c}{v} < \rho(w; n, \frac{w-1}{n-1})$, then no CMSE exists when $\theta = 0$ (Observation 3). By the properties of g established in this proof, there exist two CMSE if and only if θ is sufficiently high.

Proof of Proposition 4

Consider how $q^*(c, n, v, \theta)$ varies with each of its arguments in turn.

(a) First consider $\partial q^*/\partial c$. Clearly $\partial q^*/\partial c \neq 0$ given (22). Suppose then that $\partial q^*/\partial c < 0$. This requires that $[\cdot]$ in the denominator of (22) is strictly negative. Substituting (24)

into $[\cdot], [\cdot] < 0$ implies

$$\frac{1}{2} + \frac{1}{2\theta(n-1)} < \frac{2c}{v(1+\theta(n-1)) + v(v+\theta(n-1)[v(n-1)\theta - 4(c-v/2)])}.$$
 (27)

The LHS of (27) is positive, thus if the denominator of the RHS of (27) were negative, (27) would not be true. The denominator of the RHS of (27) must thus be positive. Simplifying (27) given this implies

$$v < c \left[\frac{4(n-1)\theta}{(\theta(n-1)+1)^2} \right].$$

$$\tag{28}$$

Note that $[4(n-1)\theta/(\theta(n-1)+1)^2] < 1$ if $(n-1)^2\theta^2 - 2(n-1)\theta + 1 > 0$. Since the discriminant of this quadratic in θ equals 0 and $(n-1)^2 > 0$, $(n-1)^2\theta^2 - 2(n-1)\theta + 1 > 0$ holds for all c, n, v and θ . However, this implies that (28) cannot hold as $[\cdot] < 1$ and v > c by assumption. Thus our supposition is false and it must be that $\partial q^*/\partial c > 0$.

Also consider $\partial q^*(c, n, v, \theta) / \partial v$. Implicitly differentiating (21) with respect to v,

$$\frac{\partial q^*(c,n,v,\theta)}{\partial v} = \frac{q^*(c,n,v,\theta)^{n-1}[1+\theta(n-1)(1-q^*(c,n,v,\theta))^{n-1}]}{-v(n-1)q^*(c,n,v,\theta)^{n-2}[1+\theta(n-1)(1-2q^*(c,n,v,\theta)^{n-1}-1)]}.$$
 (29)

The numerator of (29) must be positive given (24). Since $\partial q^*/\partial v$ clearly cannot equal zero, suppose $\partial q^*/\partial v > 0$. This requires that [·] in the denominator of (29) be negative. Substituting (24) into [·] implies (27). However we have already established that (27) is false. Thus it must be that $\partial q^*/\partial v < 0$.

(b) Second consider how $q^*(c, n, v, \theta)$ varies with n. Assume that n is continuous to simplify the analysis. Implicitly differentiating (21) with respect to n gives

$$\frac{\partial q^*(c,v,n,\theta)}{\partial n} = -\frac{q^*(c,v,n,\theta)}{n-1} \left[\frac{\theta(1+q^*(c,n,v,\theta)^{n-1})}{1+\theta(n-1)(1-2q^*(c,n,v,\theta)^{n-1})} + \ln q^*(c,v,n,\theta) \right].$$
(30)

The sign of (30) clearly depends on the sign of $[\cdot]$. Sign each term in $[\cdot]$ as follows. The numerator of the first term is strictly positive. Note that the denominator of the first term is identical to $[\cdot]$ in (22), and we have demonstrated that this is strictly positive (see discussion around (27)). Thus the first term in $[\cdot]$ of (30) is strictly positive. The second term is strictly negative. Thus the sign of (30) depends on which term is larger. Note that since $\lim_{c\to 0} q^* = 0$, it must be that $\lim_{c\to 0} [\cdot] = -\infty$. Thus there exists \bar{c} , such that if $c \leq \bar{c}$, then $\partial q^*/\partial n > 0$.

(c) Finally, consider $\partial q^*(c, n, v, \theta) / \partial \theta$. Implicitly differentiating (21) with respect to θ gives

$$\frac{\partial q^*(c,n,v,\theta)}{\partial \theta} = \frac{q^*(c,n,v,\theta)(q^*(c,n,v,\theta)^{n-1}-1)}{1+\theta(n-1)(1-2q^*(c,n,v,\theta)^{n-1})}.$$
(31)

Clearly this cannot equal zero. Suppose that $\partial q^*/\partial \theta > 0$. Since the numerator of (31) is negative, it must then be that the denominator is also negative. Substituting (24) into the denominator of (31), this requires that (27) is true. However, we have already established that (27) is false. Therefore $\partial q^*/\partial \theta < 0$.

Proof of Proposition 5

Let $g(n, w, \theta, p)$ denote the LHS of equilibrium condition (15). Recall that in the proof of proposition 3 we demonstrated that $g(n, w, \theta, 0) = g(n, w, \theta, 1) = 0$, that g is strictly quasiconcave and that $\arg \max_p g \in (0, 1)$.

(a) Equilibrium requires $g(n, w, \theta, p) = \frac{c}{v}$. Note that the LHS is independent of $\frac{c}{v}$. Given the properties of g stated at the start of this proof, an increase in $\frac{c}{v}$ increases p_L^* and decreases p_H^* .

(b) Let $p^*(n)$ denote an equilibrium probability as a function of n (with $p_L^*(n)$ and $p_H^*(n)$ defined analogously). We will demonstrate that $p_L^*(n+1) < p_L^*(n)$.

Given that we are interested in cases where there are two equilibria and given the properties of g stated at the start of this proof, it must be that $p_L^*(n) < \arg \max_p g(n + 1, w, \theta, p)$. For all $p < \arg \max_p g(n + 1, w, \theta, p)$, consider $g(n + 1, w, \theta, p) - g(n, w, \theta, p)$.

$$g(n+1, w, \theta, p) - g(n, w, \theta, p)$$

= $\rho(w; n+1, p) - \rho(w; n, p)$
+ $\theta n[\rho(w; n+1, p)(1 - F(w-1, n, p)) - \rho(w; n, p)(1 - F(w-1, n-1, p))]$
+ $\theta \rho(w; n, p)(1 - F(w-1, n-1, p)) + \theta(w-1)[\rho(w; n+1, p)^2 - \rho(w; n, p)^2] > 0.$

To understand why the expression is strictly positive consider each line in turn. The first line on the RHS is the first-difference of the pmf of a binomial distribution, which is strictly increasing in n for the interval of p of interest; thus the first line is strictly positive. Using the same property of the pmf, note that $\rho(w; n + 1, p) > \rho(w; n, p)$ in the second line on the RHS. Also, since the cdf of the binomial distribution is decreasing in n, 1 - F(w - 1, n, p) > 1 - F(w - 1, n - 1, p). Taken together, this implies that the second line of the RHS is strictly positive. The first term on the third line of the RHS is the product of three strictly positive numbers, thus is strictly positive itself. Finally, the aforementioned property of the pmf implies that the final term is strictly positive.

Equilibrium condition (15) requires that $g(n + 1, w, \theta, p_L^*(n + 1)) = g(n, w, \theta, p_L^*(n))$. Given that $g(n+1, w, \theta, p) - g(n, w, \theta, p) > 0$, it must be that $p_L^*(n+1) < p_L^*(n)$ since g is strictly increasing in p for the range of interest. Applying the above reasoning recursively establishes the result for arbitrary increases in n.

Analogous reasoning shows that $p_H^*(n+1) - p_H^*(n) < 0$.

(c) Let $p^*(w)$ denote an equilibrium probability as a function of w (with $p_L^*(w)$ and $p_H^*(w)$ defined analogously). We will demonstrate that $p_L^*(w+1) > p_L^*(w)$.

Given that we are interested in cases where there are two equilibria and given the properties of g stated at the start of this proof, it must be that $p_L^*(w) < \arg \max_p g(n, w, \theta, p)$. For all $p < \arg \max_p g(n, w, \theta, p)$, consider $g(n, w + 1, \theta, p) - g(n, w, \theta, p)$.

$$\begin{split} g(n, w+1, \theta, p) &= g(n, w, \theta, p) \\ &= \rho(w+1; n, p) - \rho(w; n, p) \\ &+ \theta(n-1)[\rho(w+1; n, p)(1-F(w, n-1, p)) - \rho(w; n, p)(1-F(w-1, n-1, p))] \\ &+ \theta(n-1)[\rho(w+1; n, p)^2 - \rho(w; n, p)^2] + \theta\rho(w; n, p)^2, \\ &= f(w; n-1, p) - f(w-1; n-1, p) \\ &+ \theta(n-1)\left[f(w; n-1, p)\left(\sum_{k=w+1}^{n-1} f(k; n-1, p)\right) - f(w-1; n-1, p)\left(\sum_{k=w}^{n-1} f(k; n-1, p)\right) \\ &+ \theta w[f(w; n-1, p)^2 - f(w-1; n-1, p)^2] \\ &+ \theta f(w-1; n-1, p) - f(w-1; n-1, p) \\ &+ \theta(n-1)\left[f(w; n-1, p)\left(\sum_{k=w+1}^{n-1} f(k; n-1, p)\right) \\ &- f(w-1; n-1, p)\left(f(w; n-1, p) + \sum_{k=w+1}^{n-1} f(k; n-1, p)\right)\right] \\ &+ \theta w[f(w; n-1, p)^2 - f(w-1; n-1, p)^2] \\ &+ \theta f(w-1; n-1, p) - f(w-1; n-1, p) \\ &+ \theta(n-1)\left(f(w; n-1, p) - f(w-1; n-1, p)\right)\left(\sum_{k=w+1}^{n-1} f(k; n-1, p)\right) \\ &+ \theta(n-1)\left(f(w; n-1, p) - f(w-1; n-1, p)\right) \\ &+ \theta(n-1)\left(f(w; n-1, p) - f(w-1; n-1, p)\right) \\ &+ \theta wf(w; n-1, p)^2 - \theta(n-1)f(w-1; n-1, p)f(w; n-1, p) \\ &+ \theta f(w-1; n-1, p)^2 - \theta wf(w-1; n-1, p)^2 < 0. \end{split}$$

To understand why the expression is strictly negative consider each line in turn. The fourth and third from last lines are strictly negative as the pmf of the binomial is strictly decreasing in w for relevant p. Using the same property and that $n - 1 \ge w$ implies the penultimate line is strictly negative. The final line is strictly negative as w > 1.

Equilibrium condition (15) requires that $g(n, w + 1, \theta, p_L^*(w + 1)) = g(n, w, \theta, p_L^*(w))$. Given that $g(n, w + 1, \theta, p) - g(n, w, \theta, p) < 0$, it must be that $p_L^*(w + 1) > p_L^*(w)$ since g is strictly increasing in p for the range of interest. Applying the above reasoning recursively establishes the result for arbitrary increases in w. Analogous reasoning shows that $p_H^*(w+1) - p_H^*(w) > 0$.

(d) Given that $\partial g/\partial \theta > 0$ and the properties of g stated at the start of this proof it must be that p_L^* is decreasing in θ and p_H^* is increasing in θ .

References

- [1] Attanasi, G., Battigalli, P. and E. Manzoni (2016) "Incomplete information models of guilt aversion in the trust game," *Management Science*, 62(3), pp. 648-667.
- [2] Attanasi, G., Battigalli, P., and R. Nagel (2013) "Disclosure of belief-dependent preferences in a trust game," *IGIER Working Paper*, no. 506.
- [3] Attanasi, G., Rimbaud, C. and M. C. Villeval (2018) "Embezzlement and guilt aversion," GATE Working Paper, WP 1807.
- [4] Battigalli, P. and M. Dufwenberg (2007) "Guilt in games," American Economic Review, 97(2), pp. 170-176.
- [5] Battigalli, P. and M. Dufwenberg (2009) "Dynamic psychological games," Journal of Economic Theory, 144, pp. 1–35.
- [6] Bellemare, C., Sebald, A. and M. Strobel (2011) "Measuring the willingness to pay to avoid guilt: Estimation using equilibrium and stated belief models," *Journal of Applied Econometrics*, 26(3), pp. 437–453.
- [7] Bellemare, C., Sebald, A. and S. Suetens (2017) "A note on testing guilt-aversion," Games and Economic Behavior, 102, pp. 233–239.
- [8] Bellemare, C., Sebald, A. and S. Suetens (2018) "Heterogenous guilt sensitivities and incentive effects," *Experimental Economics*, forthcoming.
- Cadsby, C. B. and E. Maynes (1999) "Voluntary provision of threshold public goods with continuous contributions: Experimental evidence," *Journal of Public Economics*, 71, pp. 53-73.
- [10] Chang, L. J. and A. Smith (2015) "Social emotions and psychological games," Current Opinion in Behavioral Sciences, 5, pp. 133–140.
- [11] Charness, G. and M. Dufwenberg (2006) "Partnerships and promises," *Econometrica*, 74, pp. 1579–1601.
- [12] Crawford, V. P. and H. Haller (1990) "Learning how to cooperate: Optimal play in repeated coordination games," *Econometrica*, 58(3), pp. 571–595.
- [13] Dawes, R. M., Orbell, J. M., Simmons, R. T. and Alphons, J. C. Van de Kragt (1986)
 "Organizing groups for collective action," *American Political Science Review*, 80(4), pp. 1171–1185.

- [14] Dhami, S., Mengxing, W. and A. al-Nowaihi (2018) "Public goods games and psychological utility: Theory and evidence," *Journal of Economic Behavior and Organization*, forthcoming.
- [15] Dixit, A. and M. Olson (2000) "Does voluntary participation undermine the Coase Theorem?" Journal of Public Economics, 76, pp. 309-335.
- [16] Diekmann, A. (1993) "Cooperation in an asymmetric volunteer's dilemma game: Theory and experimental evidence," *International Journal of Game Theory*, 22, pp. 75–85.
- [17] Dufwenberg, M. (2002) "Marital investments, time consistency and emotions," Journal Economic Behavior and Organization, 48, pp. 57–69.
- [18] Dufwenberg, M., Gaechter, S. and H. Hennig-Schmidt (2011) "The framing of games and the psychology of play," *Games and Economic Behavior*, 73, pp. 459–478.
- [19] Dufwenberg, M. and G. Kirchsteiger (2004) "A theory of reciprocity," Games and Economic Behavior, 47, pp. 268–298.
- [20] Dufwenberg, M. and A. Patel (2017) "Reciprocity networks and the participation problem," Games and Economic Behavior, 101, pp. 260–272.
- [21] Ederer, F. and A. Stremitzer (2017) "Promises and expectations," Games and Economic Behavior, 106, pp. 161–178.
- [22] Ellingsen, T., Johannesson, M., Tjøtta, S. and G. Torskvik (2010) "Testing guilt aversion," Games and Economic Behavior, 68(1), pp. 95–107.
- [23] Erev, I. and A. Rapoport (1990) "Provision of step-level public goods: The sequential contribution mechanism," *Journal of Conflict Resolution*, 34(3), pp. 401–425.
- [24] Feldhaus, C. and J. Stauf (2016) "More than words: The effects of cheap-talk in a volunteers dilemma," *Experimental Economics*, 19, pp. 342–359.
- [25] Geanakoplos, J., D. Pearce, and E. Stacchetti (1989): "Psychological games and sequential rationality," *Games and Economic Behavior*, 1, 60–79.
- [26] Goeree, J. K., Holt, C. A. and A. M. Smith (2017) "An experimental examination of the volunteer's dilemma," *Games and Economic Behavior*, 102, pp. 303–315.
- [27] Healy, A. and J. Pate (2009) "Asymmetry and incomplete information in an experimental volunteeer's dilemma," in Anderssen, R. S., Braddock, R. D. and L. T. H. Newtham (Eds.), 18th World IMACS Congress and MODSIM09 International Congress on Modelling and Simulation, pp. 1459–1462.
- [28] Herrmann, B., Thöni, C. and S. Gächter (2008) "Antisocial punishment across societies," *Science*, 319, pp. 1362-1367.
- [29] Hong, F. and L. Karp (2012) "International Environmental Agreements with mixed strategies and investment," *Journal of Public Economics*, 96, pp. 685–697.

- [30] Khalmetski, K., Ockenfels, A. and P. Werner (2015) "Surprising gifts Theory and laboratory evidence," *Journal of Economic Theory*, 159(A), pp. 163–208.
- [31] Makris, M. (2009) "Private provision of discrete public goods," Games and Economic Behavior, 67, pp. 292–299.
- [32] Offerman, T., Sonnemans, J. and A. Schram (1996) "Value orientations, expectations and voluntary contributions in public goods," *Economic Journal*, 106(437), pp. 817–845.
- [33] Offerman, T., Sonnemans, J. and A. Schram (2001) "Expectation formation in step-level public good games," *Economic Inquiry*, 39(2), pp. 250–269.
- [34] Palfrey, T. and H. Rosenthal (1984) "Participation and the provision of discrete public goods," *Journal of Public Economics*, 24(2), pp. 171–193.
- [35] Palfrey, T. and H. Rosenthal (1988) "Private incentives in social dilemmas: The effects of incomplete information and altruism," *Journal of Public Economics*, 35(3), pp. 309– 332.
- [36] Peeters, R. and M. Vorsatz (2018) "Simple guilt and cooperation," University of Otago Economics Discussion Papers, no. 1801, January 2018.
- [37] Pérez-Martí, F. and J. Tomás (2004) "Regret, warm-glow and bounded rationality in experiments on binary public goods," *Journal of Economic Behavior and Organization*, 55, pp. 343–353.
- [38] Rabin, M. (1993) "Incorporating fairness into game theory and economics," American Economic Review, 83, pp. 1281–1302.
- [39] Rapoport, A. and D. Eshed-Levy (1989) "Provision of step-level public goods: Effects of greed and fear of being gypped," Organizational Behavior and Human Decision Processes, 44, pp. 325–344.
- [40] Rothenhäusler, D., Schweizer, N. and N. Szech (2018) "Guilt in voting and public good games," *European Economic Review*, 101, pp. 664-681.
- [41] Sonnemans, J., Schram, A. and T. Offerman (1998) "Public good provision and public bad prevention: The effect of framing," *Journal of Economic Behavior and Organization*, 34, pp. 143–161.
- [42] Vanberg, C. (2008) "Why do people keep their promises? An experimental test of two explanations," *Econometrica*, 76(6), pp. 1467-1480.