From resource to document: Scaffolding content and organising student learning in teachers' documentation work on the teaching of series

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We examine teachers' use of resources as they prepare to teach the topic of numerical series of real numbers in order to identify how their *personal relationship* with mathematical content—and its teaching—interacts with their use of a commonly used textbook. We describe this interplay between textbook and personal relationship, a term coined in the Anthropological Theory of the Didactic (ATD, Chevallard, 2003), in the terms of *documentation work* (*resources*, *aims*, *rules of action*, *operational invariants*), a key construct from the documentational approach (DA, Gueudet & Trouche, 2009). We do so in the case of five post-secondary teachers who use the same textbook as a main resource to teach the topic. Documentational analysis of interviews with the teachers led to the identification of their aims and rules of action (the *what* and *how* of their resource use as they organise their teaching of the topic) as well as the operational invariants (the *why* for this organisation of their teaching). We describe the teachers' documentation work in two sets of aims/rules of action: scaffolding mathematical content (series as a stepping stone to learning about Taylor polynomials and Maclaurin series) and organising student learning about series through drill exercises, visualisation, examples, and applications. Our bridging (networking) of theoretical constructs originating in one theoretical framework (personal relationship, ATD) with the constructs of a different, yet compatible, framework (documentation work, DA) aims to enrich the latter (teachers' documentation work) with the individual agency (teachers' personal relationship with the topic) provided by the former.

Keywords: documentational approach; documentation work; ATD; personal relationship; series; textbook use.

Textbooks in the teaching and learning of post-secondary mathematics

Textbooks play a crucial role in the school environment (Gueudet, 2015), and they are an essential tool in long- and short-term planning (Pepin, Grevholm & Sträßer, 2006). However, in spite of their role in teaching planning, little research has focused on textbook use by teachers. This is particularly acute in the case of advanced mathematical topics (Gueudet, 2017; Mesa & Griffiths, 2012; Randahl, 2012a, 2012b). To address this gap, we analyse textbook use by post-secondary teachers in Quebec (Canada) and we focus our analyses on the teaching of a specific mathematical topic: infinite series of real numbers (series, hereafter).

Research on textbooks and their use is important because textbooks have an influence on *what* and *how* a topic should be taught (Love & Pimm, 1996). They convey specific views of mathematics and its organisation (Raman, 2004), and can shape what and how teachers teach (Weinberg, Wiesner, Benesh, & Boester, 2012). Among the few studies of textbook use by university teachers is that of Mesa and Griffiths (2012). In their data, the textbook appears as a crucial artefact in the instruction preparation process for all instructors participating in their study. The textbook has different pragmatic mediations including generating the syllabus, preparing classes, and designing homework. Mesa and Griffiths also identify three main ways these instructors use textbooks: "to use the same information that the textbook had (*offloading*), […] to supplement with alternative examples they themselves designed or from other textbooks (*adapting*), or […] to change the presentation altogether, including using different notation (a form of *improvising*)[1]" (p. 100). They also observe that: 1) the reliance on the textbook diminished with the number of times a course was given; 2) instructors saw the textbooks they used as written *for* students and not as a tool from which they themselves could learn; and 3) the textbook features that instructors perceived to be the most helpful were problems and examples. This behaviour is quite different from that of teachers at earlier levels, for whom the use of curricular materials has been reported to play a more significant role in their learning (Nicol & Crespo, 2006).

[1] The terms *offloading*, *adapting*, and *improvising* come originally from Brown (2009).

Mesa and Griffiths' results mirror those of Randahl (2012a) which report that university teachers state that they choose a textbook based on the clarity of the presentation and treatment, and on the number of tasks required of students. Maybe as a consequence, Randahl observes, undergraduate students perceive the textbook mainly as a source of tasks.

In this paper, we explore textbook use by post-secondary teachers in the institutional context of *collégial* studies in Quebec, having previously analysed how the topic of series is presented in textbooks especially used in this context (González-Martín, Nardi & Biza, 2011). We focus on teachers' use of a specific textbook as they prepare to teach series, in order to identify how their views of this topic—and its pedagogy—interact with their use of this commonly used textbook. We do so by drawing on the *documentational approach* (Gueudet, 2013, 2017; Gueudet, Buteau, Mesa & Misfeldt, 2014; Gueudet & Trouche, 2009) to identify teachers' *aims* in the teaching of series, their *rules of action* (i.e., ways of using the textbook towards these aims) and the main *operational invariants* (i.e., the warrants, justifications) that underlie their lesson planning and execution. Especially in relation to *operational invariants*, we draw on preliminary analyses (González-Martín, 2015), in which we identified teachers' *personal relationship*—a term coined in the Anthropological Theory of the Didactic (ATD, Chevallard, 2003)—with the topic of series and its teaching. In what follows, we outline the theoretical underpinnings of the study.

Documentation Work and what ATD's Personal Relationship may tell us about it

The interaction of teachers with resources, as well as how these interactions play a central role in a teacher's professional activity, are the focus of the recently developed documentational approach (DA – Gueudet, 2013, 2017; Gueudet *et al.*, 2014; Gueudet

& Trouche, 2009). This approach considers resources in a broad way: a resource is anything that can possibly be used during individual activity, or "anything likely to re-source the teacher's practice" (Adler, 2000, in Gueudet, 2017, p. 201). This includes material and non-material elements (social, cultural, and human resources such as a discussion with a colleague). An individual's own training and knowledge about the content to teach is also a resource. A *document* is "a mixed entity, associating resources and utilization schemes of these resources" (Gueudet, 2013, p. 2336). These utilisation schemes have four elements (Gueudet, 2017, p. 200):

- *Aims* can be very general ("I aim to teach this topic"), for which more specific sub-aims can be identified ("I aim to demonstrate how a particular approach can be used to solve a particular type of problem");
- *Rules of action* govern—usually in regular ways—the action towards the aim ("I will select parts from the textbook that prioritise what I see as the most pertinent elements of this topic");
- *Operational invariants* include propositions considered by the individual as true and relevant (e.g., "this topic has such and such importance in mathematics and in students' subsequent courses in their degree") and may act as implicit justifications for the rules of action; and,
- *Inferences*—which may or may not be made by the individual—allow adaptation to the features of a given situation corresponding to the same aim (e.g., "if the students exhibit difficulties with this aspect of the topic, I will intensify the use of this part of the textbook").

Teachers may use resources in class, modify them (on the spot or afterwards), or share them. This use constitutes a teacher's *documentation work* (Gueudet *et al.*, 2014, p. 142).

Gueudet (2017) highlights that the operational invariants are discourses which justify the rules of actions. These discourses can concern a given resource or the mathematical content to be taught. This approach therefore acknowledges that resources, the use of these resources and teachers' mathematical and pedagogical discourses interact in their documentation work. However, teachers' mathematical and pedagogical discourses related to a topic, and consequently their utilisation schemes of resources to teach this topic, are mainly influenced by their overall experiences as students and teachers of mathematics and its teaching, what Chevallard has termed within ATD (2003) as teachers' *personal relationship* with this topic.

We note that ATD provides tools that facilitate the interpretation of how an "institution" (ibid, p.82; in our case *collégial* studies in Quebec) shapes the teaching of mathematical topics in many and varied ways, as well as the possible effects that these ways may have on an individual teacher's practices. The construct of personal relationship is intended by Chevallard as a means to elaborate how individuals interact with and respond to the requirements placed by institutions. In ATD, the *personal relationship* of an individual with any entity (material or immaterial) is defined as the product of all the interactions (using it, speaking of it, etc.) that one can have, or have had, in different *institutions*.

For instance, a postsecondary teacher has a *personal relationship* with series which is created, mostly, through the presentation and use of this notion in his/her undergraduate courses as a student. However, when becoming a teacher and having to teach series, preparing activities and answering students' questions may modify this *personal relationship*: for example, the use of hitherto unknown resources to prepare his/her courses may also modify this *personal relationship*. As a result of all the interactions that an individual can have with a given object in different institutions, his/her personal relationship includes what Chevallard (2003) designates as a teacher's evolving positions throughout the accumulation of his/her experiences from different institutions, their 'knowledge', 'know-how', 'conceptions', 'competencies', 'mastery', 'mental images', 'representations' and 'attitudes' (1989, p. 227).

Analogous to the mathematical elements of this personal relationship, there are pedagogical elements too. Institutional restrictions influence what can be done and how it can be done. For instance, to prove a theorem as a student in an undergraduate mathematics course, a future post-secondary teacher may resort to the advanced mathematical techniques that are part of the course (e.g., through reference to a previously proven lemma). However, the same individual years later, when teaching post-secondary students, may choose to prove this theorem by using different approaches based on professional experience (e.g., "it is better to do it with an image"). In our research, we draw on the documentational approach to study teachers' use of a specific resource. In previous work of the first author (González-Martín, 2015), we conjectured that teachers' personal relationship is an important element in the utilisation schemes of the resources. Pursuing this conjecture further, we connect the operational invariants—which guide this resource use—to salient elements of the teachers' personal relationship with the topic of series. Specifically, in this paper we address the following research question: *How does a commonly used textbook become a document for post-secondary teaching of series in calculus?* We do so in the context of the institution these teachers operate in. We now introduce this context.

The topic of series in the institutional context of the *collégial*

In Canada, each province develops its own official curricula. In the province of Quebec, compulsory education ends at age 16 and students who intend to pursue tertiary studies must first complete two years of post-secondary *collégial* studies. These prepare them for university study. For students pursuing scientific or technical careers, the topic of series is introduced in calculus, during their *collégial* studies. As we elaborate in previous work (González-Martín *et al*., 2011) this topic has played an important role in the historical development of calculus, is present in calculus courses in many countries, and it has received little attention from researchers in mathematics education. Some of the main difficulties we report in that work as faced by students learning this topic include the notion of convergence and the use of potential infinity, the process-object duality of the notion of series, the confusion between sequences and series, and the use of notation. The main institutional characteristic of the *collégial* likely to influence teachers' work in ways that are relevant to the study we report in this paper is that, the *collégial* aims to prepare students for university entrance and therefore content is introduced in a more formal way than in secondary school; and, that a textbook is seen as necessary to achieve this aim: for each course, the teacher (usually in consultation with his or her department) chooses one textbook that becomes the course's guide. We now introduce briefly the five participants and the data collection and analyses procedures of our study.

The teacher participants, the interviews, and the analysis of the interview data

The project we report from in this paper was organised in two stages. In the first stage, we identified the main characteristics of the introduction of series in a sample of 17 *collégial* textbooks used in Québec between 1993 and 2008 (González-Martín *et al*., 2011). In the second stage, we interviewed five *collégial* teachers. The participants were volunteers invited from a range of *collegial* establishments that use the same textbook (Charron & Parent, 2004) in the teaching of integral calculus (and series). This textbook has been published in different editions, and many *collégial* establishments in Québec choose it as a guide for their integral calculus course. The five teachers are male, come from three different *collégials* with a variety of departmental policies, and have different degrees of teaching experience (Table 1). None of them is an active research mathematician. The interviews lasted about one hour each and their overall aim was to identify key characteristics of the teachers' personal relationship with the topic of series and its teaching—and to explore any associations between these characteristics and those teachers' use of the particular textbook.

TABLE I

Profile of the five participants. T1 and T4 teach in the same *collegial* establishment, as do T2 and T3.

Teacher	**T1**	**T2**	**T3**	**T4**	**T5**
Years of experience teaching at this level	5	20	32	6	7
Years of experience teaching series	5	4	Teaches series every 4-5 years	4	2

The interview protocol—see Supplementary Materials—was designed considering the main elements identified in the textbook analyses (González-Martín *et al*., 2011): introduction of the topic; use of examples and applications; use of visualisation; tasks that support the learning of series; key elements for the learning of series; awareness of student difficulties within and beyond the topic and how the participants' teaching takes these into account; and, how the participants would ideally like to teach the topic. The interviews were designed with key elements of ATD in mind, particularly what in that theory are labelled as "the six moments of didactic praxeologies" (Barbé, Bosch, Espinoza, & Gascón, 2005): how the teachers organise the first encounter with the new content on series (e.g., in relation to the textbook prescribed by their institution); what teaching tasks they deploy in their teaching (e.g., what exercises they ask students to engage with in order to investigate the convergence or divergence of particular series) and how they elaborate the techniques that these tasks aim to bring to the fore (e.g., introducing and applying the convergence tests); how they establish the theoretical underpinnings, called "technologies" in ATD, relative to these techniques (e.g., introducing the proofs for these convergence tests); the technical work (e.g., how they demonstrate to the students ways to detect the convergence or divergence of a series through algebraic or graphical means); the institutionalisation (e.g., how they plan their lessons on series in ways that are embedded in the overall plan for the course, the broader curriculum etc.); and, the evaluation (e.g., how they reflect on the effectiveness of their approach on all above).

The transcripts of the five interviews (translated from French to English) were read by the three authors, in order to identify and classify statements that related to each teacher's personal relationship with series (focusing on their techniques to teach series, and the reasons given to support these techniques, e.g., statements about the key features of the topic or its importance in the students' studies and mathematics at large). The identification of the teachers' main aims, actions/techniques to tackle them, and the underlying justifications ("technologies" in ATD) given, allowed us to identify and summarise the salient elements of each teacher's personal relationship with series.

As we were working towards identifying the salient elements of each teacher's personal relationship with series, we noted that the interviews also allowed us insight into important elements of the teachers' documentation work (Gueudet & Trouche, 2009), namely: what *resources* teachers *use* (e.g., the textbook); what their *aims* are in different moments of their teaching; what actions characterise their activity (*rules of action*) towards meeting these aims; and what justifications (*operational invariants*) underlie these actions. In this paper, we explore two conjectures that appeared first in González-Martín (2015):

- an important part of textbook use is mediated by teachers' views on the content (their personal relationship); and,
- different personal relationships result in different forms of documentation work.

The analysis we present in this paper elaborates the variations in the teachers' personal relationship and how these variations are reflected in their documentation work—and also contributes to answering the research question stated earlier..

In order to explore these conjectures we re-analysed the transcripts to identify statements that related to the teachers' documentation work as follows:

- We identified the main aims or sub-aims that guide the teachers' action and grouped them into categories: for example, "prepare the overall structure of the course," "prepare students to learn Taylor polynomials," "introduce the new content," "help students overcome their difficulties."
- For each sub-aim, we identified rules of action that include (or not) the use of the textbook. For instance, in T3's statement

  > "[we start and <u>present the new content] from examples</u> in particular related to geometric series. So we add, for example, the geometric series that has a rate of one-half […]. And then, we can compare with the harmonic series […]. <u>To show the students that … you must be prudent</u> with infinite sums, intuition is often a bad advisor."

  we identify a specific technique ('offer some examples of the new content') associated to the sub-aim 'introduce the new content'.

- For each of these rules of action, we associated relevant (explicit or implicit) operational invariants—identified during the characterisation of the personal relationship—that seem to guide these rules of action. For instance, in T3's statement "At university, the emphasis is put on proofs and theorems, here it's the intuitive approach…" we detect the operational invariant, the justification that underlies his use of examples (the proliferation and acceptability of intuitive approaches in the students' pre-university studies).

In what follows, we present our data analysis in three parts. First, we outline briefly how the topic of series is presented in the textbook used by the participants. We do so through reference to the analysis of the 17 *collégial* textbooks in González-Martín *et al*. (2011) and to make the link between our account of teachers' documentation work in relation to the contents of the textbook transparent to the reader. Second, we present brief profiles of the five teachers' personal relationship with the topic of series with a particular focus on aspects of their documentation work (especially, operational invariants) that are pertinent in the fuller account of the teachers' documentation work that follows. Third, we present an account of the teachers' documentation work, associating their main rules of action with the operational invariants identified in the second section. Our analyses are followed by our reflections on how the teachers' pedagogical awareness—in the sense elaborated by Nardi, Jaworski and Hegedus (2005): spectrum of pedagogical awareness in terms of varying degrees of sensitivity to students' needs—manifests itself across the teachers' documentation work. We note that interactions with students also become resources for teachers, and some of their actions appear to be guided by these interactions.

The topic of series in Charron & Parent (2004)

Charron & Parent (2004) is one of the 17 textbooks used in *collégial* studies in Québec between 1993 and 2008 for teaching series. This textbook is representative of the sample's main characteristics: our analyses showed that series are usually introduced without reference to their applications or their *raison d'être* in mathematics (González-Martín et al., 2011). They are introduced broadly as a tool that will be needed later to introduce functional series, and the presentation in the texts largely seems oblivious to students' difficulties with the topic as identified in previous research. Moreover, the vast majority of tasks concerning series are related to the application of convergence criteria, or to the application of algorithmic procedures (ibid). Our critique of the textbook resonates with that on post-secondary textbooks by Weinberg *et al.* (2012) and Randahl (2012b) who note the absence of links to practical situations and the heavy reliance on topics with which students have prior, and often unresolved, difficulties.

This textbook places series just before the chapter about Taylor series (and after covering integration techniques and applications of integrals), with a great emphasis on classical tasks. Among the 167 tasks presented, 78 concern studying the convergence or divergence of given series, 30 concern calculating the sum of given series, and 24 concern determining the convergence and calculating the sum (in the case of convergence). These three types of tasks altogether represent 79% of the tasks proposed to students. Only two tasks are given as applications, one of them fictional (calculating the total distance covered by a ball that bounces infinitely) and the other real-life (series used to model the distribution of a drug in a patient's blood, knowing s/he has to take it during his/her whole life). Series appear just after sequences, and the textbook makes a connection with integrals:

Some mathematical problems require calculating the sum of an infinite number of terms. For instance, in chapter 2, we have evaluated the area of closed regions by calculating an infinite sum of areas of rectangles, through a limit. In this section, we are going to study the addition of an infinite number of terms (Charron & Parent, 2004, p. 295, our translation).

This first encounter with the new topic makes a connection with a topic studied previously (integrals). However, the connection is not followed through as what follows immediately after the above quotation are three examples of series (divergent: $\sum_{i=1}^{\infty} i$; convergent: $\sum_{i=1}^{\infty} \frac{1}{2^i}$; and divergent with undefined sum: $\sum_{i=1}^{\infty}(-1)^i$), with no apparent connection to integrals. The introduction of the sequence of partial sums follows and is associated with techniques to explore convergence and divergence. What follows is structured in the subsections: arithmetic series, geometric series, series with positive terms (where several criteria are introduced—including the integral test—as well as Riemann series), and alternating series (absolute and conditional convergence). However, in this chapter, series are not used to calculate integrals (despite the aforementioned note in the beginning).

Regarding the text's visual elements, there are two graphs (Figure 1) in the section on the integral test, denoted with "see the graph opposite" and with no further explanation on how these graphs relate to the formal presentation of the test in the text.

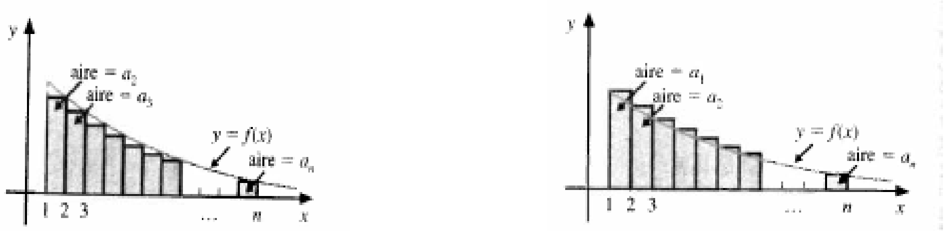

Figure 1: The two graphs for series in Charron & Parent (2004), pp. 313-314.

There are three more images in the analysed text (54.5 pages in total) (Figure 2).

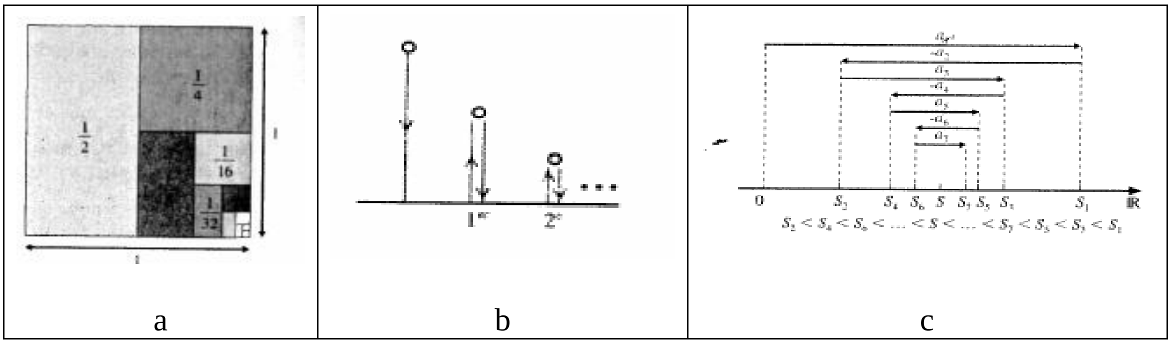

Figure 2: The three images related to series in Charron & Parent (2004), p. 296, 308, 330 respectively.

Figure 2a appears in the second example of the chapter. The text of this example says:

> We can consider the sum of the terms on the right [referring to $\sum_{i=1}^{\infty}\frac{1}{2^i}=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots+\frac{1}{2^n}+\cdots$] as the equivalent to the area of a square of side length 1, subdivided as in the representation opposite (p. 296).

Figure 2b appears in a fictional example of modelling (a ball that bounces infinitely), but there is no sentence in the text that connects the text with the image (p. 308). The same applies to Figure 2c: it appears next to the calculations in the proof of Leibniz's criterion for alternate series, but there is no text connecting it to the proof (p. 330).

The teachers' personal relationship with the topic of series and its teaching, with a particular focus on teachers' operational invariants

We now summarise the salient elements of each teacher's personal relationship with the topic of series and its teaching as evidenced in their interview statements. Our focus is particularly skewed towards evidence from the interviews of the teachers' operational invariants, namely how the teachers ground their aims and rules of action regarding the teaching of series. Table 2 sums up these operational invariants. To facilitate reading across the text and Table 2, we have inserted—indicatively, not exhaustively—in square brackets the respective operational invariant code in the text.

TABLE II

Operational invariants identified in our account of the five teachers' personal relationships with the topic of series and its teaching

Operational Invariants	**Code**	**Participants**
The order of the textbook is good.	OB	T1, T3, T4, T5
Series is a stepping stone for more important parts of the content (e.g., power, Taylor and Maclaurin series, approximation).	SS	T1, T2
Including series in the Integral Calculus curriculum is debatable.	SD	T2, T3, T5
Series is about convergence criteria.	CC	T1, T2, T3, T5
Series is about convergence (arithmetic of infinity, convergence of partial sums).	C	T1, T2, T3, T4
Series is a counterintuitive concept.	CI	T3
It is difficult to visualise series.	HV	T1, T2, T4, T5
It is difficult to find real-life applications of series.	HA	T1, T2, T3, T4, T5
It is important to practise with many drill exercises.	DE	T2, T3, T4, T5
Using criteria does not necessarily imply grasping the topic.	AntiCC	T4
The content of the course has an already established structure / this structure is logical and helps learning.	ES	T1
Time pressure and students' fatigue constrain the teaching approach.	TP	T1, T4
Visualisation is good for students' learning / helps develop intuition.	VG	T1, T3, T4, T5
Creating intuitions/a feeling about mathematical concepts is good for students' learning.	IG	T2, T3, T4

T1 praised the order of the course content and seemed to see series as a stepping stone [SS] to study polynomial series and approximation ("why do we teach that ... in the program we have to learn approximation"). He expressed appreciation for the textbook he uses [OB] and stated that the presented order facilitates learning [ES]: "It's difficult to deconstruct a logical structure that has been established for many years of teaching [...] It's well enchained, a pretty logical sequence." When asked for examples of applications that he uses when organising the first encounter with series for his students, he offered: "Certain functions cannot be integrated … I lead the students to … well, I give a classic example like […] e^{x^2}." Regarding real-life applications, T1 mentioned the fictional "two panels of glass and [...] one ray is refracted, but not completely; a part of the light passes through, but another part is reflected," adding later that "There are not many examples" [HA]. Finally, an important part of learning series for him is the convergence criteria [CC]: "It's also really, really centred on the convergence tests … understanding the convergence tests in order to eventually get to intervals of convergence and all that."

For T2, series also appear to be a stepping stone towards Taylor polynomials [SS]: "We do the different criteria for convergence of series to finally arrive at some more interesting things – the Taylor and Maclaurin series." He reinforced this view later, adding: "Numerical series in themselves are not sure to be of particular interest. It's mainly the development in Taylor and Maclaurin series." This comes across even more clearly when asked about how he would like to teach series had he no constraints: "One of the approaches that we'd take is to completely drop numerical series and go directly to Taylor and Maclaurin developments." [SD] When asked about applications of series, he did not mention any [HA]. Rather, he turned to the Taylor series and mentioned "finite polynomial approximations." Although he believes that "it's quite hard to make graphic representations with series in general" [HV], he showed sensitivity to the importance of the differences between the arithmetic of finite and infinite elements ("we can expand what we know about arithmetic with infinite terms […], we can only expand for convergent series, so […] it's important to understand the difference between a converging series and a non-converging one") [C]. Finally, he added that students need to develop the experience (or feeling) [IG] to know the right criterion to apply, and therefore insisted on the importance of doing many exercises to master the criteria [DE].

T3, the most experienced among the five teachers, spoke early in the interview about the counterintuitive aspect of series [CI]: "You must be very prudent with infinite sums; intuition is often a bad advisor." Unlike the other four teachers, he stressed the counterintuitive element inherent in series: "It may seem paradoxical […] it's because basically it depends on the speed with which the terms diminish. It’s a bit on that level, if we remain at the intuitive level.” Also, unlike the other four teachers, he mentioned two real-life applications: one fictional (a ball that bounces infinitely) and one real-life (investments and accumulated interests). Like T2, he also evoked the arithmetic of infinity [C], and the importance of selecting the appropriate test of convergence [CC]. When asked about his ideal teaching of series, had he no constraints, he replied: “I could even ask myself the question ‘would I teach series?’ Because the title of the course is Integral Calculus. It’s definitely debatable. What are series doing in this course?” [SD]

T4 stated early in his interview—and repeated several times—that he “believe[s] a lot in drill exercises.” [DE] He also highlighted the importance of the sequence of partial sums [C] as a key element of the notion of series, using expressions such as “you have to feel the sequence of partial sums,” and “comparisons that can be felt.” Using the criteria, he stressed, does not necessarily imply grasping the topic [AntiCC]. When asked about applications, he cited two: the case of eating successively half of a cheese, and, “how a calculator would calculate naturally the root of one radian.” He commented that “series […] complete the topic of integrals […] If not, I can’t think of any other direct applications.” [HA] When asked about the use of visualisation, he stated that “for series, I’m more numerical than graphical […] and I find that to be more adequate,” and that “the graphical doesn’t have its place [in teaching this topic].” [HV] Finally, when asked about how he would teach series had he no constraints, he stressed that “it’s a topic that I love” and that he would happily spend substantially more time on it.

Finally, T5’s—the less experienced teacher among the five—personal relationship with series seemed to be closely associated to what appears in the textbook. He referred to the arithmetic of infinity as “we add the terms … we know there’s an infinity of terms … But we add them anyway.” [DE] He places the weight of grasping series on mastering the criteria for convergence [CC]: “The practice is more on the criteria than on the concept of series. Because the criteria help to be more at ease with the series.” With regard to visualisation, he noted that “we don’t have a graphic for [the sum of a series],” and that “for geometric series, yes it works. Visually, we can make a square, but there is not a lot of material to visualise the concept of infinite series …” [HV] With regard to applications, he stated: “I would say that there are not really any applications.” [HA] However, he made many connections to integrals, where he highlighted the importance of visualisation [VG]. Overall, his personal relationship with series seems to be captured by this statement: “I am like the students, I don’t pay much attention to the part on series.” [SD]

The above account demonstrates that the personal relationships of the five teachers are quite diverse. There were, however, some commonalities, too: in spite of their evident awareness of students’ difficulties, the teachers overall view the content as easily followed by the students. This impression is strongly grounded on their expectation that this ease emanates from “the logical structure of the course” (T1) or from the fact that the students “have already met [series] when we presented infinite sums […] when we did the definite integral” (T5). Their confidence that what they see as “logical” sequencing of the course content (what is done) must result in pedagogical effectiveness (what students learn) seems to lead to an equilibrium point of confidence and contentment.

This reverence for the logical flow and structure of the textbook appears as a strong operational invariant in the teachers’ documentation work. We return to this point in our presentation of the teachers’ documentation work in what follows.

The teachers' documentation work

To present the teachers' documentation work, we quote their statements in the interviews that evidence how the operational invariants —identified during the characterisation of teachers' personal relationship with the topic of series and its teaching—are associated to their aims and rules of action regarding the teaching of this topic. We focus on the teachers' use of the textbook as a main resource, although our data suggest that other resources are also considered: previous experience with the teaching and learning of series, and overall pedagogical awareness emergent from their prior teaching experiences and their sense of curricular and institutional constraints, such as time pressure and content coverage obligation. The teachers' documentation work is summed up in Table 3.

We present the teachers' aims and rules of action in two themes: scaffolding content and support for students' learning (mainly through strategies to foster sense-making of series through visualisation and engagement with examples and applications). In brackets we use the abbreviations for the teachers' operational invariants listed in Table 3.

TABLE III. Summary of main rules of action and associated operational invariants identified in the five participants' statements.

Aims	Sub-aims	Rules of action (ways of using the textbook and other classroom activities)	Operational invariants	Participants	Example
Scaffolding content	Deliver the content of the textbook	Follow the structure of the textbook which covers the course content	The order of the textbook is good, namely key to the way the content is presented to the students (OB)	T1, T2, T3, T4, T5	"Certain sections are put aside and left out, but on the whole we almost follow the order …" (T2)
			The content of the course has an already established structure / this structure is logical and helps learning (ES)	T1	"The most simple reason [why I use this book] is that [it] contains the course content so it's a good work tool as much for the students as for the teacher." (T1) "It's difficult to deconstruct a logical structure that has been established for many years of teaching, at least in Québec." (T1)
		Go through the textbook's exercises (particularly drill exercises)	The order of the textbook is good (OB) Series is about convergence criteria (CC) It is important to practise with many drill exercises (DE)	T1, T2, T3, T4, T5	"The Charron & Parent textbook has an effective manner of grouping together the exercises. Meaning that, they are enumerated. At the beginning, they are numbered … inside the same number, there are many letters, so many … at the drill level. At the beginning, the drill exercises are grouped together so that one can associate a concept to many examples, so a kind of synthesis and then, they are mixed up. So, I find that it is a learning sequence that is favourable." (T4)
	Prepare students to learn Taylor polynomials	Give hints and organise the content in order to highlight Taylor polynomials	Series are a steppingstone for more important parts of the content (e.g., power, Taylor and Maclaurin series, approximation) (SS) Including series in Integral Calculus curriculum is debatable (SD)	T1, T2, T3, T5	"The objective is to get to power series and Taylor." (T1) "Our objective is the series of Taylor and Maclaurin; it is very important." (T2) "One of the approaches that we'd take is to completely drop numerical series and go directly to Taylor and Maclaurin developments." (T2)
	Develop competencies on the convergence criteria	Practise convergence criteria (taken from the textbook)	Series is about convergence criteria (CC) It is important to practise with many drill exercises (DE) Series is about convergence (arithmetic of infinity, convergence of partial sums) (C)	T1, T2, T3, T4, T5	"Because [using the right criterion] is the basis … it's the most crucial element so that the students will understand well." (T2) "We do not have a lot of time to make the student practise. They are the ones who have to practise. […] So, the time is short for that… and the students … all they have to do is practise with the exercises … to do more exercises and try to get familiar with series." (T5)
Support students' learning	Introduce the new content	Offer some examples (mostly from the textbook)	The order of the textbook is good (OB) Creating intuitions/a feeling about mathematical concepts is good for students' learning (IG)	T1, T2, T3, T4, T5	"So, in fact, I give these 3 classic examples of something that converges and something that does not and something that is ambiguous; for example, what happens when I do 1 – 1 + 1 – 1 … um rapidly I give… um… I give an intuitive idea and then I come back quickly and formalise everything." (T1)
			Series is a counterintuitive concept (CI)	T3	"Well [we start and present the new content], from examples in particular related to geometric series. So we add, for example, the geometric series that has a rate of ½. […] We can then wonder: […] does it converge? And then we realise that the answer is yes. And then, we can compare with the harmonic series: ½ + 1/3 + ¼ + 1/5… which diverges, the whole is not bounded. To show the students that… you must be prudent with infinite sums; intuition is often a bad advisor." (T3)
		Make students see that the new content is useful → not give (many) examples of real-life applications	It is difficult to find real-life applications of series (HA)	T1, T2, T3, T4, T5	"I introduce the whole thing by explaining that certain functions cannot be integrated […] I don't put much emphasis on [giving examples of applications], probably I get one or two examples from time to time, but [there are] not many examples." (T1) "Yes [I give examples of applications on the first day]. One is the one that I just mentioned [using Zeno's paradox to introduce ½ + ¼ + …]. If not, well, we have to wait a little bit because for instance … numerical series in themselves it's not sure they have a particular interest." (T2)
	Develop intuitions through visualisation	Use visual images (mostly from the textbook) → limited repertoire	Visualisation is good for student learning / helps develop intuition (VG) It is difficult to visualise series (HV) The order of the textbook is good (OB)	T1, T2, T3, T4, T5	"I really like the example ½ + ¼ + … that I can draw here […] When I usually present it, they don't have the… necessarily the intuition … they don't necessarily have the formal tools to demonstrate it, but they seem to feel relatively well that an example like that can converge towards 1." (T1) "The visual representations in the textbook, there are already some graphs." (T3) "Yes … but … I don't make very many… for example, to get to … at the moment where we get to alternate series, I demonstrate the theorem of an alternate series by using a graphic representation […] Well, of course the objective of this representation here is to show by drawing, I don't want to be too ambitious and use the term demonstrate, but to guide their intuition towards the theorem of alternate series, which works very well in that way. Furthermore, in the textbook that's how they do it." (T4)
	Develop competencies	Make students work through drill exercises → use the textbook as a guide	The order of the textbook is good (OB) Series is about convergence criteria (CC) Series is about convergence (arithmetic of infinity, convergence of partial sums) (C) It is important to practise with many drill exercises (DE)	T1, T2, T3, T4, T5	"There is a bank of problems. I give them almost all the problems. Well, I am a little … demanding, but I ask them to do all the problems, but it's also really, really centred on the convergence tests." (T1)
			Creating intuitions/a feeling about mathematical concepts is good for students' learning (IG)	T2, T3, T4	"The study of the convergence criteria …. So they will see many exercises in order to see if a series converges or not … and by which criterion we can arrive at … so to do a lot of, um … so that… when looking at a series … to say 'ah, what is the type of criterion that I'd need to apply, or which other'… it's necessary to have done many, so I'll give them many, many exercises on convergence." (T2)
	Help students overcome difficulties	Make reminders, try to summarise the content	Time pressure and students' fatigue constrain the teaching approach (TP)	T1	"I try to … well, to remind them that this is a series, it's an infinite sum and then I give them reminders, but it does not always work… […] maybe, there is also a question of … their attention is less there, because after an integral calculus course that has taken up an enormous amount of energy, that's what they bring up, there are many factors that we encounter." (T1)
		Make them practise a lot	Series is about convergence criteria (CC) It is important to practise with many drill exercises (DE)	T2, T3, T5	"Well, the best way [to help them overcome their difficulties] is to force them to work, to do the exercises." (T2)
		Use examples	Series is about convergence criteria (CC) It is important to practise with many drill exercises (DE)	T2, T3, T4, T5	"I would say that the best method is by example … um … and then … well, essentially, I do a lot in class. Each time that I do some, I develop the same reasoning that brought me to choose the convergence criterion that I have chosen, […] but it resembles or makes me think of another type of series that we have already seen and we know that it converges or we know that it diverges. So, um … it's really by example finally that we will get there, but there is still a part that the student must do, I mean he must do a lot too." (T2) "Well, exactly by giving examples where we ask the students, does this converge? We do an example that converges, we do another, which looks similar, well, ouf… it's sure that many students will want that 'it looks similar, it should converge'. Sometimes, just a small change suffices and then all of the sudden it does not converge." (T3)
		Develop their intuition	Using criteria does not necessarily imply grasping the topic (AntiCC) Creating intuitions/a feeling about mathematical concepts is good for students' learning (IG)	T4	"Concerning the theorems that look like artificial, I try to make them feel them before the statements, or after the statements, especially before. And after that, I will tell them, let's translate a bit and we'll see what it gives mathematically. […] Honestly, I plan to talk more about the sequence of partial sums and things that can be felt. There are students who don't retain many things from this course." (T4)

The teachers' aims and rules of action: Scaffolding mathematical content (towards learning about Taylor polynomials and Maclaurin series)

The five teachers see series in a way that mirrors the structure of the textbook—much like Weinberg *et al*.'s (2012) undergraduates—and as a stepping stone towards Taylor and Maclaurin series. This vision of series appears more prominently in T1 and T2, who give their students hints about what the 'objective' is. The mere scaffolding *raison d'être* for the topic of series is cast into doubt explicitly in the statements by three of the five participants, T2, T3 and T5, who stated that had they no restrictions, they would drop the content of series to directly study Taylor and Maclaurin polynomials.

The five teachers stated that to prepare the course, they mostly follow the textbook—where series appear almost as a prelude for polynomial series. Although some parts may be put aside, overall, they organise the content they present to the students in strict resonance with this resource (*offloading*). They stated that the textbook they use is a good resource for the teacher and some of the reasons they invoke for this are: the order and the way content is presented (T1, T5); that it covers the course content (T3); and, the number of exercises—particularly drill exercises—are grouped in such a way that students can identify their different types (T4). These criteria—clarity of the presentation and treatment (Randahl, 2012a), and type and range of exercises (Mesa & Griffiths, 2012; Randahl, 2012a)—have been identified as important for university teachers' selecting a textbook. Furthermore, it seems reasonable to suggest that the five teachers appreciate the presentation of the content in the textbook that reflects their personal relationships with the topic and this appreciation becomes an operational invariant (OB) that guides their documentation work. We also note that an important part of their overall organisation of the course consists of practising the use of the different convergence criteria. The textbook structure prioritises this drill practice of using the convergence criteria in agreement with the teachers' personal relationships—in particular their operational invariants about the importance of learning convergence criteria (CC) and other aspects related to convergence (C).

We see here what we interpret as an interplay between the teachers' personal relationships with series and their documentation work that relates to teaching this content. We see that some of the elements present in the teachers' personal relationships become elements they value in the textbook (such as the order of content and the number and type of drill exercises). For instance, we interpret T1's statement: "it's well linked up, a pretty logical sequence" as evidence of how the textbook resonates with his personal relationship with series. His documentation work then reflects this resonance.

The teachers' aims and rules of action: Organising student learning of the new topic of series (introducing the new content and addressing students' needs through drill exercises, visualisation, examples and applications)

The teachers' overall aim of supporting students' learning about the new topic of series (presented in Table 3 also in terms of several sub-aims) materialise through several rules of action: the teachers establish a number of sub-aims: offer them opportunities for drill practise of the convergence criteria; and, develop intuitions about convergence through visualisation, examples and applications.

The teachers stressed the importance visualisation has for mathematical learning overall, and four (T1, T3, T4, T5) explicitly commented on the advantages of visualising for students' learning (VG). However, four (T1, T2, T4, T5) stated that there are not many possible visual representations for series (HV) and this operational invariant in their personal relationships seems to result in reducing the number of images (such as graphs and diagrams) they consider in their documentation work. For instance, T1 explicitly stated that visualisation can create intuitions and he referred to one image present in the textbook (Figure 2a). However, he stated that he would not prioritise visualisation in his teaching due to institutional constraints (TP): he is compelled by the syllabus to reach Taylor polynomials (SS) quickly. He also feels that there are “quite limited” examples for visualisation (HV).

Therefore, although visualisation seems *generally* valued in the teachers’ personal relationships (VG), their reliance on the textbook which has limited repertoire of visualised examples—as well as other institutional constraints such as obligation to cover content at a certain speed and their own minimally visual personal relationships *specifically* with the topic of series—results in limited emphasis on visualisation in their documentation work.

For example, T4 offered the only occasion in these interviews of a direct and explicit pedagogical use of an image in the textbook (Figure 2c). He explained that he uses this image as a way to illustrate the theorem of convergence for alternate series “to guide [the students’] intuition” (IG and VG). He connected the sub-aim of developing students’ intuitions with how this sub-aim materialises in the textbook (“in the textbook, that’s how they do it”). However, he also described himself as being more numerically than graphically oriented and as not seeing graphs as dynamic (“it’s just a photo”). So, in spite of his generally stated appreciation for visualisation, we see that his personal relationship with images of series is static, exactly as the textbook does, restricting visualisation to a small number of isolated occasions.

We note that the textbook includes the square-based illustration of the sum of the geometric series (Figure 2a) that the teachers did invoke. It also includes the graph used for the integral test (Figure 1) to illustrate the connection between series and integrals. However, apart from T4, the teachers did not invoke two other images appearing in the textbook (Figure 2b, Figure 2c). Overall, they use the textbook mostly to select exercises and, in other words, their documentation work emerges as minimally inclusive of visualisation.

With regard to intra-mathematical applications of series, the five teachers referenced several: series are useful to integrate functions that do not have a primitive (T1 and T4); series are useful in calculating Riemann sums (T1); series underlie how a calculator calculates transcendental values (T4) and approximates functions by series (T5); and, series are key elements in power series and associated finite sums (T2 and T5). However, they did not explain how they would deploy these applications in their teaching. As mentioned earlier, the main intra-mathematical application they cited is the Taylor and Maclaurin series, for which series is seen as a stepping stone.

With regard to extra-mathematical applications, all teachers stated that they introduce the new content to their students by demonstrating its usefulness in real life within the constraints of the operational invariant “It is difficult to find real-life applications of series” (HA): two of the teachers (T1, T5) explicitly stated that “there are not many applications” or “there are not many at their level” (T4); other responses include “it’s not sure they have a particular interest” (T2) or not remembering (T1, T3). Only three teachers mentioned extra-mathematical applications and we see these as fictional: two glasses and a ray of light that is reflected infinitely (T1), ball that bounces infinitely (T3), and a cheese of which we eat half of what remains every time (T4). Only one teacher (T3) mentioned an extra-mathematical, real-life application of series (investments and accumulated interests). We note that none of the teachers mentioned the real-life application present in the textbook (distribution of a drug in a patient’s blood).

Overall, the five teachers did not seem to consider elements that are present in the resource they use, unless these are already present in their personal relationships. We interpret this as an indication that their documentation work is highly entangled with their personal relationships. This seems in agreement with Mesa and Griffiths' (2012) result that teachers—in their case university professors—use textbooks primarily as a resource for selecting exercises that are within their comfort zone.

Using examples is another rule of action that the five teachers resort to with the aim of introducing the new content and helping students overcome difficulties. As with applications, this action varies in the teachers' documentation work, although what the five teachers referred to in the interviews mirrors the textbook closely. For instance, T1 suggested introducing series through three types of "concrete" examples: convergence, divergence, and not easy to conclude (which mirrors what the textbook does in its second page about series). The use of Zeno's paradox as an example to introduce the topic was also mentioned (T2; T1 and T4 also mentioned it, but not for the introduction), being one of the few cases of *adapting* (Mesa & Griffiths, 2012) the main resource. All references to a specific series are to $\sum_{n=1}^{\infty} 1/_{2^n}$. All teachers suggested introduction to the topic through definitions and examples of convergence and divergence. However, they do not pay attention to the purpose of the new content of series and what type of mathematical, or other, problem series can address. We speculate that this absence of addressing the *raison d'être* for series is related to the teachers' personal relationships about series as mostly a stepping stone to study Taylor polynomials and to the operational invariant that series do not have (many) extra-mathematical applications (HA).

From resource to document: Factoring personal relationship into documentation work

Our data show that, although the five teachers overall have similar aims, there is some variability in their documentation work. First, the data show that not all share the same operational invariants that guide their actions, but also that, guided by their personal relationships with the topic and even when sharing the same invariants, they may resort to different rules of action—for instance, giving different examples of applications for series, or using different examples of visual images. Moreover, although the five teachers cited several features of the topic that they want their students to learn, they agreed that they appreciate the textbook they use because it caters for students' needs (e.g., "we have to learn approximation and it's well presented", T1). T4 and T5 noted that it is easy for students to practise with exercises on the use of the convergence criteria following the textbook.

The teachers also mentioned difficulties that students have with series and that they try to take these into account in their practice. They explained that they do so, for example, through encouraging students to practise, through activating students' intuition**,** or through examples or visualisation. This pedagogical awareness (Nardi *et al*., 2005) also may lead to scaffolding following the structure of the textbook: "[the paradox of adding infinite terms] is the reason why we start with sequences and series of partial sums, so seeing that a series converges is perhaps [an] easier [start]" (T2).

We discussed in the previous sections that, in spite of this evident awareness of students' difficulties, the teachers overall view the content as easily followed by the students. We conjecture that, in these cases, the personal relationship with the content yields operational invariants: the teachers' appreciation for the structure of the textbook from a mathematical point of view seems to transform into a view that this structure makes the content "easier" (T2) to understand. This way, the connection of the mathematical (this is the purpose series serves in the discipline of mathematics) and pedagogical (this is how and why the topic of series can be introduced to students) views seems to be the basis (the operational invariant) for the endorsement of the resource—and, hence, the basis for the aims and rules of action that we identified in the teachers' statements.

The research presented here aims to uncover an important part of the documentation work of teachers: how their personal relationships with the content interact with the way they view and plan their use of resources (what they prioritise and why—their aims and operational invariants) and how these priorities are mirrored into what they say they do (their rules of action). Although it is arguable that the use, during their mathematical training, of resources similar to the one they currently use may have played an important role in their personal relationships, our research focuses on how these already existing personal relationships interact with their documentation work.

Our results seem to reinforce previous recent results in post-secondary education: the main criteria to choose a textbook are perceived clarity of the presentation and treatment, and the number of tasks to engage students (Randahl, 2012a, pp. 248-249). Our teachers follow, overall, the structure and content of their textbook and, although some see the variety of exercises to be limited, they give almost all the exercises to the students. In this sense, their documentation work does not take a critical stance about their main resource and their textbook use seems more aligned to *offloading* and less to *adapting* (Mesa & Griffiths, 2012). This uncritical perspective on the textbook is compounded in our view also by the absence of any statement by the five interviewees that they see this resource as an instrument for their own learning (a point made also in Mesa & Griffiths, 2012), and we see that some elements it has to offer (such as a real-life application and visual images) are overall not considered by the teachers. The ways in which mathematics is presented in textbooks can present real difficulty for students (Kajander & Lovric, 2009, cited in Randahl, 2012a, p. 252) and teachers' documentation work needs to be sensitive to this difficulty. Research in this area, especially at post-secondary levels, is much needed (Weinberg et al., 2012, p. 157). We see our work as one such contribution.

From a theoretical point of view, we believe that seeing the personal relationship (an ATD construct) as a resource that the teachers utilise in their documentation work (a DA construct) is a helpful first step towards bridging (networking) elements of the two approaches as conjectured in González-Martín (2015). While prior research suggests that textbooks can shape what and how teachers teach (Weinberg *et al.*, 2012), our analyses help us see where the personal relationship may play an important role in the shaping of the teachers' rules of action and operational invariants.

In the analyses we present in this paper, we operationalise DA and ATD constructs in order to describe five teachers' documentation work in relation to their use of a particular textbook they deploy towards the teaching of the topic of series. We organised this presentation in terms of two themes: scaffolding content towards learning about Taylor polynomials and Maclaurin series and facilitating student engagement with the new topic of series through drill exercises, visualisation, examples**,** and applications. We aim that this presentation is a potent step in the bridging (networking) direction we mention above. While DA says much about the use of resources, but not about the user, we believe that individual agency needs to come into the accounts of teachers' documentation work. Taking into account the personal relationship with the topic to teach is one way towards fulfilling this need.

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SUPPLEMENTARY MATERIALS (to be available online):

Questions of the interview protocol:

1. In which establishment do you teach?
2. What is your experience teaching at *collégial* and teaching infinite sums or series?
3. What is the content that you teach in the course where you present series and in which order do you present the content?
4. What are the textbooks that you use currently in the course where you teach infinite sums?
5. Why do you use that textbook?
6. Do you follow the textbook's order when you present the content related to series during your course?
7. How do you organise your courses in order to teach series?
8. Do you find that the textbook that you use meets the needs of both teachers and students?
9. During the first day of teaching about series, how do you present the notion of infinite sum to your students?
10. The first day when you talk about series to your students, do you give examples of application of the concept?
11. When you teach series, do you give other examples of applications?
12. Do you try to make your students aware of the paradox of adding infinite terms?
13. What are the examples that you use?
14. Do you make any visual representations?
15. What kind of visual representation (graph or drawing) do you privilege and why?
16. Do you ask your students to produce visual representations?
17. Do you offer activities to students during which they are asked to interpret a graph or drawing?
18. What importance do you give to visualisation in your teaching in general and in the teaching of series in particular?
19. Do you use some aspects of the history of mathematics to teach series?
20. What are the tasks (exercises, problems …) that you ask your students to do during the lesson on series?

21. What are the four more pertinent tasks in the textbook you use?
22. During your teaching of series, what is the knowledge and ideas that you think your students should really retain? In other words, at the end of your teaching of series, what are the ideas about series that your students should really have retained?
23. According to you, what are the main difficulties in learning the notion of series?
24. How do you take into consideration these difficulties in your teaching?
25. During your teaching of series, what are the difficulties that you see most often among your students?
26. How do you help your students overcome those difficulties?
27. Are you aware of the difficulties and obstacles identified by research concerning the learning of series?
28. If you didn’t have any time or content constraints, how would you like to teach series?