Modeling and Analysis of Channel Holding Time and Handoff Rate for Packet Sessions in All-IP Cellular Networks

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Abstract—It is essential to model channel holding time (CHT), cell residence time (CRT), and handoff rate for performance analysis and algorithm evaluation in mobile cellular networks. The problem has been extensively studied in the past for circuit-switched (CS) cellular networks. However, little research has been done on packet-switched (PS) cellular networks. Unlike that a call occupies a dedicated channel during its whole lifetime in CS networks, an active session in PS networks occupies and releases channels iteratively due to discontinuous reception (DRX) mechanism. In this paper, we investigate the key quantities in PS cellular networks. We present a set of comprehensive new models to characterize the quantities and their relationship in PS networks. The models shed light on the relationship between CHT and CRT and handoff rate. The analytical results enable wide applicability in various scenarios and therefore have important theoretical significance. Moreover, the analytical results provide a quick way to evaluate traffic performance and system design in PS cellular networks without wide deployment, which can save cost and time.

Index Terms—Cell residence time (CRT), channel holding time (CHT), handoff rate, mobility, modeling and analysis, packet session.

I. INTRODUCTION

MOBILE cellular networks have become one of the essential requirements in our daily life. A recent study from Ericsson reports that in Q3 of 2015, the total mobile subscriptions and global mobile penetration rate have reached 7.3 billion and 95%, respectively [1]. Mobile services are moving from the traditional circuit-switched (CS) toward packet-switched (PS), such as VoLTE or voice over Long-Term Evolution (LTE).

The modeling of user equipment (UE) mobility, which can characterize mobile cellular network dynamics with appropriate accuracy, has wide usage in network planning, resource optimization, network performance analysis, and new service evaluations. With UE mobility models, mobile operators can quickly obtain the performance of their new system designs (e.g., tracking area and location update area) and evaluate new algorithms to reduce handoff cost without wide implementation and deployment. It can save cost and time.

Channel holding time (CHT), cell residence time (CRT), and handoff rate are key quantities for UE mobility modeling. These quantities are highly related. Specifically, CHT allows one to derive key network parameters such as new call/session blocking probability, handoff call/session blocking probability [2], etc. It has been widely used for the evaluation of the mobile routing strategy [3] and resource allocation [4], and models for a data charging system [5], spectrum sensing [6], and minimizing unnecessary handoffs [7], [8]. The CHT depends on UE mobility, which can be characterized by CRT, which is determined by cell size, UE speed, and movement direction. The handoff rate is also related to CHT. It is defined as the mean number of handoffs during an active call and one of the key quantities to model CHT distribution. Therefore, it is important to find an appropriate model to characterize these quantities and their relationship, which is the focus of this paper.

In CS cellular networks, extensive studies have been carried out for handoff performance in terms of CHT, CRT, and handoff rate [6]–[26]. For the sake of convenience and analytical tractability, a common assumption in these studies is that the CHT and/or CRT is exponentially distributed. Hong and Rappaport in [9] proposed the first traffic model for cellular mobile telephone networks, indicating that CHT could be approximated by an exponential distribution when CRT is exponentially distributed. Thereafter, plenty of research studies (e.g., [21]–[23]) have been conducted under the exponential distribution assumption. Recent studies [17], [18] have addressed that exponential distribution cannot capture the overall effects of UE mobility. Instead, the use of general distributions to model such quantities has been proposed, such as Erlang [11], [15], [17], Gamma [20], Uniform [9], deterministic [26], hyper-Erlang [15], [19], lognormal [25], Pareto [24], Weibull distribution [18], and sum of hyperexponentials [10].

However, the work [6]–[26] developed for CS cellular networks cannot be applied to PS cellular networks directly, due to the following reasons.

- Unlike CS networks in that an active call holds its channel during its whole lifetime, a packet session in the PS
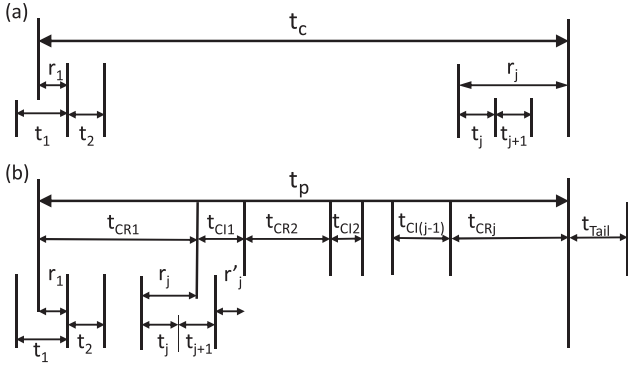
Fig. 1. (a) Timing diagram for CS cellular networks. (b) Timing diagram for PS cellular networks.

network occupies and releases its channel iteratively for the duration of the packet session to save UE energy.

- An active call in the CS network releases its channel when it is terminated. However, in the PS network, the UE does not release Radio Resource Control (RRC) connection until there is no data transmission for an *extended period of time*.

Fig. 1(a) shows the timing diagram for the CHT (denoted as t_c) in CS cellular networks. Specifically, in traditional CS networks, voice traffic is transmitted in a dedicated channel in which the resource is not released until the call connection terminates. However, during a packet session, the UE may *sleep* in some periods and partially release its resources. In comparison, Fig. 1(b) shows the timing diagram for PS cellular networks. t_p denotes the time interval between the time data transmission starts until the time it terminates. During t_p, the UE occupies the channel in $t_{CR1}, t_{CR2}, \ldots, t_{CRj}$, and sleeps in t_{CI1}, $t_{CI2}, \ldots, t_{CIj-1}$. Moreover, even if there is no packet transmission after t_p, the resources are still held for the UE during an extended time interval t_{Tail}. Considering the discontinuous reception (DRX) in LTE as an example, the UE turns on its receiver at certain predefined time points and sleeps at other periods (will be discussed in Section II-B) to save energy. During the sleep periods, the resource (i.e., LTE-U_u interface [27]) between the UE and the evolved Node B (eNB) is released. The differences between CS and PS networks motivate us to study UE mobility modeling for packet sessions. More details about Fig. 1 will be discussed later.

In this paper, we investigate the modeling of CHT, CRT, and handoff rate in PS cellular networks. As in [11], [13], [15], [19], [28], and [29], CHT depends on the user's mobility, which is characterized by CRT. We further define an effect CHT (eCHT) to model the extended resource holding time for UE devices in t_{CIj} and t_{Tail}, where the eCHT experiences active and idle periods in a session. We are the first to introduce eCHT and address the relationship between eCHT and CRT in packet sessions. Based on our analytical results, their relationship is as follows.

- If CRT is exponentially distributed, eCHT is exponentially distributed as well, and *vice versa*.
- Again, if CRT is exponentially distributed, the new session's eCHT and the handoff session's eCHT will have the same distribution, and *vice versa*.

These results provide critical guidelines for UE mobility modeling and can prevent researchers from making inappropriate assumptions on eCHT and CRT, e.g., assuming eCHT and CRT as an Erlang distribution and an exponential distribution, respectively. Moreover, we demonstrate that the quantity of handoff rate is not only used in characterizing the eCHT distribution but also to be tied to the service quality and cost parameters. We derive a general formula for the computation of the handoff rate under general conditions. The results enable wide applicability in various scenarios and therefore have important theoretical significance. The models presented in this paper can be used as a theoretical guideline for network designers and performance analysts.

The rest of this paper is organized as follows. In Section II, we introduce the hyper-Erlang distribution and review the background of DRX mode in LTE. Section III presents a new mathematical model to analyze CRT, (conditional) CHT with DRX mode, and handoff rate in a packet session. Numerical results are shown in Section IV. Section V offers conclusions.

II. PRELIMINARIES AND BACKGROUND

Here, we first briefly introduce the Erlang distribution and the hyper-Erlang distribution and then present the background of the LTE DRX mechanism.

A. Erlang Distribution and Hyper-Erlang Distribution

Erlang distribution has the following probability density function (pdf) and Laplace transform (LT):

$$f(t) = \frac{\beta^m t^{m-1}}{(m-1)!} e^{-\beta t}, f^*(s) = \left(\frac{\beta}{s+\beta}\right)^m. \quad (1)$$

Note that when $m = 1$, $f(t)$ is an exponential distribution.

According to the work in [11], [15], and [19], the pdf and LT of the *hyper-Erlang* distribution with M parameters are as follows:

$$f_{he}(t) = \sum_{i=1}^{M} \alpha_i \frac{(m_i \eta_i)^{m_i} t^{m_i-1}}{(m_i - 1)!} e^{-m_i \eta_i t}, \; t \geq 0$$

$$f_{he}^*(s) = \sum_{i=1}^{M} \alpha_i \left(\frac{m_i \eta_i}{s + m_i \eta_i}\right)^{m_i}. \quad (2)$$

Note that when $M = 1$ (i.e., hyper-Erlang distribution with one parameter), $f_{he}(t)$ is an Erlang distribution.

A hyper-Erlang distribution model with two parameters is used in our numerical study, where hyper-Erlang distribution with two parameters is as follows:

$$f^*(s) = \alpha_1 \left(\frac{m_1 \eta}{s + m_1 \eta}\right)^{m_1} + \alpha_2 \left(\frac{m_2 \eta}{s + m_2 \eta}\right)^{m_2}.$$

B. DRX Mode in LTE

The 3rd Generation Partnership Project has defined DRX [27] to increase the UE's battery lifetime and resource utilization and to minimize packet delay. With DRX, the UE
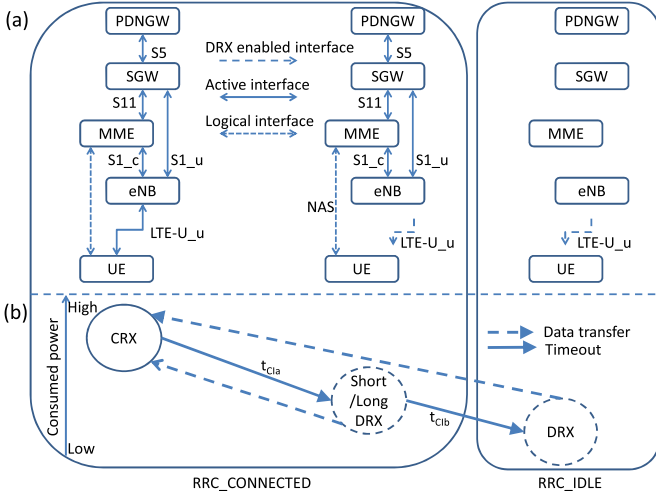
Fig. 2. DRX in LTE.

turns on the receiver at some predefined time points and sleeps at other time periods. The UE periodically wakes up to listen to the downlink (DL) transmissions, following the DRX cycle.

Fig. 2(a) shows the network architecture in which the Evolved Packet System (EPS) network interfaces are depicted. When a UE device is in DRX enabled/RRC_IDLE state, the eNB may initiate the release of the UE's RRC connection and request the Mobility Management Entity (MME) to release the UE's S1 connection, i.e., the S1, NAS, and RRC connections are removed.

The DRX state machine in LTE is shown in Fig. 2(b), in which a UE device has two different modes: RRC_CONNECTED and RRC_IDLE. In the RRC_CONNECTED state, the UE moves from continuous reception (CRX) to short DRX state and then shifts to long DRX state when the UE/eNB is waiting for data packets for an extended period after the last transmitted/ received packet (i.e., DRX inactivity timer expires [30]). If the channel is idle longer than the predefined DRX inactivity timer, which is denoted as a, the RRC will then move from RRC_ CONNECTED to RRC_IDLE [31].

From the viewpoint of holding resource in this paper, we consider the eCHT when a UE device is in the RRC_ CONNECTED state. Even in RRC_CONNECTED, a UE device may still *sleep* in long/short DRX state.

III. MODELING CHT, CRT, AND HANDOFF RATE IN PS CELLULAR NETWORKS

Here, we propose analytical models for UE mobility modeling in PS cellular networks. Moreover, we derive the general formula for handoff rates. The model of handoff rate can be used to characterize handoff session arrival rate, eCHT distribution, service quality, and cost parameters.

Section III-A extends the work in [11] to PS networks and proves that its results are also valid in PS networks, i.e., the eCHTs are exponentially distributed if and only if the CRTs are exponentially distributed. This result reveals the fundamental relationship between eCHT and CRT. Section III-B relaxes the

distribution assumption of CRT (i.e., the CRTs are not exponentially distributed) to derive the eCHT distribution. It provides a very general model for UE mobility modeling in PS networks. Later, Section III-C derives the model of conditional eCHT and shows its opposite results compared with the model in a CS network. Finally, another important quantity (handoff rate) for eCHT modeling is presented in Section III-D.

A. Modeling eCHT for Exponentially Distributed CRT

Here, we present our analytical model for eCHT considering that CRT is exponentially distributed. Recall that in the RRC_CONNECTED state, a UE device changes between active and sleep modes from time to time. We denote them as CRX time t_{CRj} $(j = 1, 2, \ldots)$ and Connected-Idle time t_{CIj} $(j = 1, 2, \ldots)$, respectively, as shown in Fig. 1(b). During CRX time, i.e., t_{CRj}, the channel is used for packet transmission. If there is no packet transmission, the UE enters sleep mode, i.e., t_{CIj} $(j = 1, 2, \ldots)$. Once there is a packet arrival, the UE changes to t_{CRj}. The t_{CRj} corresponds to the CRX state in Fig. 2(b). The t_{CIj} includes short DRX and long DRX, in which the channel is idle. The maximum idle time is denoted as t_{Tail}. If the idle time of the channel is longer than t_{Tail}, the UE enters RRC_IDLE mode, and we consider that the packet session is finished.

During t_{CIj}, a UE device has no activity in both DL (forward link) and uplink (reverse link). The maximum value of t_{CIj} is the length of the idle timer t_{Tail}. Let r_1 be the period between the time a new packet session starts and the time the session leaves the serving cell but is not finished yet. Let t_j be the CRT and r_j $(j \geq 2)$ be the residual lifetime of the eCHT after the packet session completes the $j - 1$th handoff successfully.

Let t_e represent the *eCHT*. We then can obtain

$$t_e = t_p + t_{\text{Tail}}, t_{CIj} > t_{\text{Tail}}. \tag{3}$$

In Fig. 1(b), we have the following four eCHTs.

1) t_{pnh}: The new session *eCHT* [15], [16] is shown in Fig. 3(a), i.e.,

$$t_{pnh} = \min\{t_p + t_{\text{Tail}}, r_1\}. \tag{4}$$

Specifically, there are two cases for t_{pnh}: 1) A new session starts at point A and hands off to another cell at point B. For this case, $t_{pnh} = r_1$; 2) a new session initiates at C and terminates at D, i.e., $t_{pnh} = t_p + t_{\text{Tail}}$. Here, we assume that λ is the new session arrival rate to a cell.

2) t_{phh_busy}: The handoff session *eCHT* in *busy mode* [15], [16] is shown in Fig. 3(b), i.e.,

$$t_{phh\_busy} = \min\{r_j + t_{\text{Tail}}, t_j\}. \tag{5}$$

Here, the busy mode means that the UE is in CRX state when the handoff is conducted. Similarly, the session either hands off to another cell at point B or terminates at D. We assume that λ_{hb} is the arrival rate of handoff sessions in busy mode.
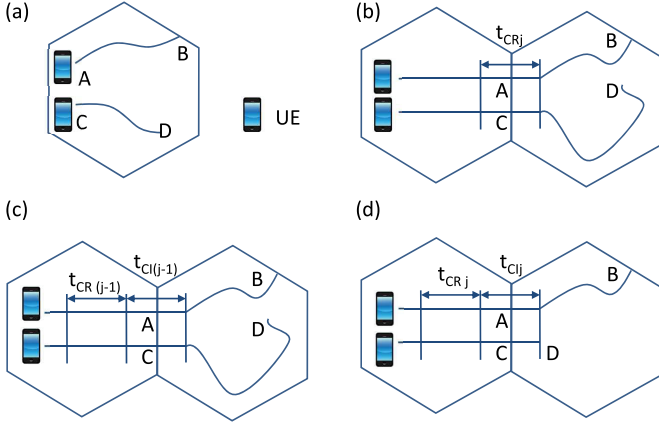
Fig. 3. (a) CRT ($\overline{AB} \equiv r_1$) and residual lifetime ($\overline{CD} \equiv t_p + t_{\text{Tail}}$) of a new session, (b) that of a handoff session in busy mode with a new session, (c) that of a handoff session in idle mode with a new session, and (d) that of a handoff session in idle mode without new sessions.

3) $t_{phh\_idle\_new}$: Fig. 3(c) shows that the handoff session *eCHT* is in idle mode and then enters busy mode, i.e.,

$$t_{phh\_idle\_new} = \min\{r_j + t_{\text{Tail}}, t_j\}. \tag{6}$$

Here, the idle mode means that the UE is in short/long DRX state when the handoff is conducted. We assume that λ_{hda} is the arrival rate of handoff sessions in idle mode. *Discussion*: An interesting observation is that t_{phh_busy} and $t_{phh\_idle\_new}$ have the same formulas, i.e., (5) is the same as (6). The reasons are as follows. For the handoff session in busy mode, as shown in Fig. 3(b), when the UE completes the data transmission in the right cell, the UE then changes to idle mode. In Fig. 3(c), the UE shifts from idle mode to busy mode again in the right cell. Because r_j is the residual lifetime after the jth successful handoff and the user will finally be in long DRX mode (i.e., the maximum value of $t_{CIj} = t_{\text{Tail}}$), we have (5) $\equiv$ (6).

4) $t_{phh\_idle\_no}$: The handoff session *eCHT* with *no arrival packet in idle mode* is shown in Fig. 3(d), i.e.,

$$t_{phh\_idle\_no} = \min\{r'_j + t_{\text{Tail}}, t_j\}. \tag{7}$$

We assume that λ_{hdb} is the arrival rate of handoff sessions in idle mode.

Here, we define pdfs $f_p(t)$, $f_{CIj}(t)$, $f_d(t)$, $f_r(t)$, $f_{pnh}(t)$, $f_{phh\_busy}(t)$, $f_{phh\_idle\_new}(t)$, and $f_{phh\_idle\_no}(t)$; cumulative distribution functions (cdfs) $F_p(t)$, $F_{CIj}(t)$, $F_d(t)$, $F_r(t)$, $F_{pnh}(t)$, $F_{phh\_busy}(t)$, $F_{phh\_idle\_new}(t)$, and $F_{phh\_idle\_no}(t)$; and LTs $f_p^*(s)$, $f_{CIj}^*(s)$, $f_d^*(s)$, $f_r^*(s)$, $f_{pnh}^*(s)$, $f_{phh\_busy}^*(s)$, $f_{phh\_idle\_new}^*(s)$, and $f_{phh\_idle\_no}^*(s)$ for t_p, t_{CIj}, t_j, r_1, t_{pnh}, t_{phh_busy}, $t_{phh\_idle\_new}$, and $t_{phh\_idle\_no}$, respectively (see Table I for details).

In the following, we will answer the question of how to model the eCHT for t_{pnh}, t_{phh_busy}, $t_{phh\_idle\_new}$, and $t_{phh\_idle\_no}$, respectively, if the CRT (t_j) is assumed to be exponentially distributed.

1) Handoff Session in Busy Mode: First, we can obtain the conditional probability of (5) as

$$P(t_{phh\_busy} \leq t \mid t_{CIj} > t_{\text{Tail}})$$
$$= P(r_j + t_{\text{Tail}} \leq t \text{ or } t_j \leq t \mid t_{CIj} > t_{\text{Tail}})$$
$$= P((r_j + t_{\text{Tail}}) \leq t \mid t_{CIj} > t_{\text{Tail}})$$
$$\quad + P(t_j \leq t \mid t_{CIj} > t_{\text{Tail}})$$
$$\quad - P(((r_j + t_{\text{Tail}}) \leq t, \, t_j \leq t) \mid t_{CIj} > t_{\text{Tail}})$$
$$= P((r_j + t_{\text{Tail}}) \leq t) + P(t_j \leq t)$$
$$\quad - P((r_j + t_{\text{Tail}}) \leq t) P(t_j \leq t)$$
$$= P(t_p \leq t - t_{\text{Tail}}) + P(t_j \leq t)$$
$$\quad - P(t_p \leq t - t_{\text{Tail}})P(t_j \leq t) \tag{8}$$

where we have $P(r_j + t_{\text{Tail}} \leq t, \, t_j \leq t) = P(r_j + t_{\text{Tail}} \leq t) P(t_j \leq t)$ from the independence of r_j, t_j, and t_{CIj} [11], [15], [16]. From the Residual Life Theorem [32] or the argument in [33], we also have $P(r_j + t_{\text{Tail}} \leq t) = P(t_p \leq t - t_{\text{Tail}})$ based on the memoryless property of the exponential distribution, where r_j has the same distribution as t_p.

By differentiating (8), we have the following conditional pdf:

$$f_{phh\_busy}(t) = f_p(t - t_{\text{Tail}}) + f_d(t) - f_p(t - t_{\text{Tail}})P(t_j \leq t)$$
$$\quad - P(t_p \leq t - t_{\text{Tail}})f_d(t)$$
$$= f_p(t - t_{\text{Tail}}) - f_p(t - t_{\text{Tail}}) \cdot \int_0^t f_d(\tau)d\tau$$
$$\quad + f_d(t) - f_d(t) \cdot \int_0^t f_p(\tau - t_{\text{Tail}})d\tau. \tag{9}$$

Suppose that the CRTs t_j are exponentially distributed with parameter η, then, from (9), we obtain

$$f_{phh\_busy}(t) = \mu e^{-\mu(t-t_{\text{Tail}})} \times e^{-\eta t} + \eta e^{\eta t} \times e^{-\mu(t-t_{\text{Tail}})}$$
$$= (\mu + \eta)e^{-[\mu(t-t_{\text{Tail}})+\eta t]}$$
$$= (\mu + \eta)e^{-(\mu+\eta)\left(t - \frac{\mu t_{\text{Tail}}}{\mu+\eta}\right)} \tag{10}$$

which is a *two-parameter exponential distribution* [34]. Here, we have shown that if the CRT is exponentially distributed, t_{phh_busy}, the handoff session eCHT in busy mode is exponentially distributed.

In contrast, suppose that t_{phh_busy} is exponentially distributed with parameter γ. Let $Z(t) = \int_t^\infty f_d(\tau)d\tau$, then $Z(t)' = -f_d(t)$. From (9), we get

$$\mu e^{-\mu(t-t_{\text{Tail}})}Z(t) + e^{-\mu(t-t_{\text{Tail}})}f_d(t) = \gamma e^{-\gamma \tau}. \tag{11}$$

From (11), we can derive

$$Z(t) = e^{\mu(t-t_{\text{Tail}})}Z(0) + \int_0^t e^{\mu[(t-t_{\text{Tail}})-\tau]}\left[-\gamma e^{-(\gamma-\mu)\tau}\right]d\tau$$
$$= e^{\mu(t-t_{\text{Tail}})}\left\{Z(0) - \gamma \int_0^t e^{-\gamma \tau}d\tau\right\}$$
$$= e^{\mu t_{\text{Tail}}} \cdot e^{-(\gamma-\mu)t}. \tag{12}$$

Variable	pdf	Laplace transform	CDF	Definition
t_p	$f_p(t)$	$f_p^*(s)$	$F_p(t)$	Packet session CHT
t_{CIj}	$f_{CIj}(t)$	$f_{CIj}^*(s)$	$F_{CIj}(t)$	Connected-Idle time, the maximum t_{CIj} is t_{Tail}
t_{CRj}	-	-	-	Continuous reception time (CRX)
r_1	$f_r(t)$	$f_r^*(s)$	$F_r(t)$	Time period from a new session starts to leave the serving cell
r_j	-	-		Residual life time after the jth successful handoff
t_j	$f_d(t)$	$f_d^*(s)$	$F_d(t)$	Cell Residence Time (CRT)
t_{pnh}	$f_{pnh}(t)$	$f_{pnh}^*(s)$	$F_{pnh}(t)$	new session effective CHT (eCHT)
t_{phh_busy}	$f_{phh\_busy}(t)$	$f_{phh\_busy}^*(s)$	$F_{phh\_busy}(t)$	handoff session eCHT in busy mode
$t_{phh\_idle\_new}$	$f_{phh\_idle\_new}(t)$	$f_{phh\_idle\_new}^*(s)$	$F_{phh\_idle\_new}(t)$	handoff session eCHT with new arrival packets in idle mode
$t_{phh\_idle\_no}$	$f_{phh\_idle\_no}(t)$	$f_{phh\_idle\_no}^*(s)$	$F_{phh\_idle\_no}(t)$	handoff session eCHT without arrival packet in idle mode
t_c				Circuit call CHT

Thus, $Z(t)' = -(\gamma - \mu)e^{[-(\gamma-\mu)t+\mu t_{Tail}]}$. Finally, we obtain

$$f_{phh\_busy}(t) = (\gamma - \mu)e^{[-(\gamma-\mu)t+\mu t_{Tail}]}$$
$$= (\gamma - \mu)e^{-(\gamma-\mu)\left(t-\frac{\mu t_{Tail}}{\gamma-\mu}\right)}. \quad (13)$$

As a conclusion, (13) and (10) show a strong result that the handoff session eCHT in busy mode t_{phh_busy} is exponentially distributed if the CRT is assumed to be exponentially distributed, and *vice versa*.

2) New Session: Similarly, the conditional probability of (4) is

$$P(t_{pnh} \leq t \mid t_{CIj} > t_{Tail})$$
$$= P(t_p + t_{Tail} \leq t \text{ or } r_1 \leq t \mid t_{CIj} > t_{Tail})$$
$$= P(t_p \leq t - t_{Tail}) + P(r_1 \leq t)$$
$$\quad - P(t_p \leq t - t_{Tail})P(r_1 \leq t). \quad (14)$$

By differentiating the given equation, we obtain

$$f_{pnh}(t) = f_p(t - t_{Tail}) + f_r(t) - f_p(t - t_{Tail})P(r_1 \leq t)$$
$$\quad - P(t_p \leq t - t_{Tail})f_r(t)$$
$$= f_p(t - t_{Tail}) - f_p(t - t_{Tail}) \times \int_0^t f_r(\tau)d\tau + f_r(t)$$
$$\quad - f_r(t) \cdot \int_0^t f_p(\tau - t_{Tail})d\tau. \quad (15)$$

Suppose that the new session eCHT is exponentially distributed with parameter μ_1. From a similar derivation of t_{phh_busy}, we can deduce that

$$f_r(t) = (\mu_1 - \mu)e^{-(\mu_1-\mu)\left(t-\frac{\mu t_{Tail}}{\mu_1-\mu}\right)} \quad (16)$$

which is also an exponential distribution.

3) Handoff Session in Idle With New Arrival Packets: Recall that the handoff session eCHT with new arrival packets in idle mode, i.e., $t_{phh\_idle\_new}$, is the same as the handoff session eCHT in busy mode, i.e., t_{phh_busy}. That is, (5) $\equiv$ (6). Based on Section III-A.1, we can derive that $t_{phh\_idle\_new}$ is also exponentially distributed if and only if the CRT is an exponential distribution.

4) Handoff Session in Idle Without Arrival Packet: From (7), we get the following conditional probability:

$$P(t_{phh\_idle\_no} \leq t \mid t_{CIj} > t_{Tail})$$
$$= P\left(r_j' + t_{Tail} \leq t \text{ or } t_j \leq t \mid t_{CIj} > t_{Tail}\right)$$
$$= P\left(r_j' + t_{Tail} \leq t \mid t_{CIj} > t_{Tail}\right) + P(t_j \leq t \mid t_{CIj} > t_{Tail})$$
$$\quad - P\left(\left(r_j' + t_{Tail} \leq t, \, t_j \leq t\right) \mid t_{CIj} > t_{Tail}\right)$$
$$= P\left(r_j' + t_{Tail} \leq t\right) + P(t_j \leq t) - P\left(r_j' + t_{Tail} \leq t, \, t_j \leq t\right)$$
$$= P(t_p \leq t - t_{Tail}) + P(t_j \leq t) - P(t_p \leq t - t_{Tail})P(t_j \leq t) \quad (17)$$

where we have the independence of t_{CIj} and t_j and the independence of t_j and r_j' [15], [16]. We also apply $P((r_j' + t_{Tail}) \leq t) = P(t_p \leq t - t_{Tail})$ from the memoryless property of the exponential distribution. Thus, r_j' has the same distribution as t_p. Therefore, we have the same conditional pdf and the LT with (8) and (20), respectively. As a conclusion, we have the following theorem.

Theorem 1: For a PS network with exponential CHT and Poisson new session arrivals, the following three statements are held if and only if the CRT is exponentially distributed.

1) The *new session's* eCHT is exponentially distributed.
2) The *handoff session's* eCHT in busy mode is exponentially distributed.
3) The *handoff session's* eCHT in idle mode with new or no arrival packets is exponentially distributed.

In summary, Theorem 1 reveals the relationship between eCHT and CRT if either eCHT or CRT is exponentially distributed. The results are the first step in the investigation of eCHT and CRT. For convenience and tractability, many traffic analytical works (e.g., [21]–[23]) in the CS network used the assumption that either CHT or CRT is exponentially distributed. Theorem 1 shows that the assumption also works if both eCHT and CRT are exponentially distributed in the PS network. In the following section, we will relax the restriction of the assumption on exponential CRT to find the associated distribution of CRT.

B. Modeling eCHT by Using LT

In wired telephone traffic, it is reasonable to assume that the CHTs are exponential distributions. However, the assumption

of exponential eCHTs in the PS cellular network is only held for the case when the CRTs are exponential distributions (based on our results in Section III-A). In the following, we relax the assumption on exponentially distributed CRTs ($f_d(t)$) and then use the inverse LT to see how close the exponential distribution is to the eCHT distribution.

Recall that we defined λ, λ_{hb}, λ_{hda}, and λ_{hdb} as the arrival rate for new sessions, handoff sessions in busy mode, handoff sessions in idle mode with new arrival packet, and handoff sessions in idle mode without arrival packet, respectively. Thus, according to the weighted average of probability, we can show that $t_p = t_{pnh}$ with probability $\lambda/(\lambda + \lambda_{hb} + \lambda_{hda} + \lambda_{hdb})$, $t_p = t_{phh\_busy}$ with probability $\lambda_{hb}/(\lambda + \lambda_{hb} + \lambda_{hda} + \lambda_{hdb})$, and so on. We obtain that

$$
\begin{aligned}
f_p^*(s) =\ & \frac{\lambda}{\lambda + \lambda_{hb} + \lambda_{hda} + \lambda_{hdb}} f_{pnh}^*(s) \\
& + \frac{\lambda_{hb}}{\lambda + \lambda_{hb} + \lambda_{hda} + \lambda_{hdb}} f_{phh-busy}^*(s) \\
& + \frac{\lambda_{hda}}{\lambda + \lambda_{hb} + \lambda_{hda} + \lambda_{hdb}} f_{phh-idle-new}^*(s) \\
& + \frac{\lambda_{hdb}}{\lambda + \lambda_{hb} + \lambda_{hda} + \lambda_{hdb}} f_{phh-idle-no}^*(s). \quad (18)
\end{aligned}
$$

We assume that $\lambda_{hda} = \lambda_{hb}$ and $\lambda_{hdb} = 0$. From (18), we have

$$
\begin{aligned}
f_p^*(s) =\ & \frac{\lambda}{2\lambda + \lambda_{hb}} \left[f_{pnh}^*(s) + f_{phh\_idle\_new}^*(s) \right] \\
& + \frac{\lambda_{hb}}{2\lambda + \lambda_{hb}} f_{phh\_busy}^*(s). \quad (19)
\end{aligned}
$$

1) Handoff Session in Busy Mode: Applying LT to (9), we obtain

$$
\begin{aligned}
& f_{phh\_busy}^*(s) \\
&= f_d^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - \int_0^\infty e^{-st} \left[f_d(t) \int_0^t f_p(\tau - t_{\text{Tail}}) d\tau \right] dt \\
&\quad - \int_0^\infty e^{-st} \left[f_p(t - t_{\text{Tail}}) \int_0^t f_d(\tau) d\tau \right] dt \\
&= f_d^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - \int_0^\infty e^{-st} \left(1 - e^{-\mu(t - t_{\text{Tail}})} \right) f_d(t) dt \\
&\quad - \mu e^{\mu t_{\text{Tail}}} \int_0^\infty e^{-(s+\mu)t} \int_0^t f_d(\tau) d\tau\, dt \\
&= f_d^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - \mu e^{\mu t_{\text{Tail}}} \int_0^\infty e^{-(s+\mu)t} F_d(t) dt \\
&\quad - \left[\int_0^\infty e^{-st} f_d(t) dt - e^{\mu t_{\text{Tail}}} \int_0^\infty e^{-(s+\mu)t} f_d(t) dt \right] \\
&= f_d^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - f_d^*(s) + e^{\mu t_{\text{Tail}}} f_d^*(s + \mu) \\
&\quad - \mu e^{\mu t_{\text{Tail}}} \frac{1}{s + \mu} f_d^*(s + \mu) \\
&= e^{-t_{\text{Tail}}s} \frac{\mu}{s + \mu} + e^{\mu t_{\text{Tail}}} \left(\frac{s}{s + \mu} \right) f_d^*(s + \mu) \quad (20)
\end{aligned}
$$

where we apply the property of LT that $\mathcal{L}(g'(t)) = s\mathcal{L}(g(t)) - g(0)$, if g is continuous at $t = 0$. Here, $g(t) = F_d(t)$, $g'(t) = f_d(t)$, and $f_d(0) = 0$.

2) New Session: Thus, the LT of (15) is

$$
\begin{aligned}
& f_{pnh}^*(s) \\
&= f_r^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - \int_0^\infty e^{-st} \left[f_r(t) \int_0^t f_p(\tau - t_{\text{Tail}}) d\tau \right] dt \\
&\quad - \int_0^\infty e^{-st} \left[f_p(t - t_{\text{Tail}}) \int_0^t f_r(\tau) d\tau \right] dt \\
&= f_r^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - \int_0^\infty e^{-st} \left(1 - e^{-\mu(t - t_{\text{Tail}})} \right) f_r(t) dt \\
&\quad - \mu e^{\mu t_{\text{Tail}}} \int_0^\infty e^{-(s+\mu)t} \int_0^t f_r(\tau) d\tau\, dt \\
&= f_r^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - \mu e^{\mu t_{\text{Tail}}} \int_0^\infty e^{-(s+\mu)t} F_r(t) dt \\
&\quad - \left[\int_0^\infty e^{-st} f_r(t) dt - e^{\mu t_{\text{Tail}}} \int_0^\infty e^{-(s+\mu)t} f_r(t) dt \right] \\
&= f_r^*(s) + e^{-t_{\text{Tail}}s} f_p^*(s) - f_r^*(s) + e^{\mu t_{\text{Tail}}} f_r^*(s + \mu) \\
&\quad - \mu e^{\mu t_{\text{Tail}}} \frac{1}{s + \mu} f_r^*(s + \mu) \\
&= e^{-t_{\text{Tail}}s} \frac{\mu}{s + \mu} + e^{\mu t_{\text{Tail}}} \left(\frac{s}{s + \mu} \right) f_r^*(s + \mu). \quad (21)
\end{aligned}
$$

Furthermore, we need to investigate the relationship between $f_r^*(s)$ and $f_d^*(d)$. Because the residual lifetime r_1 can be viewed as the residual life of the CRT, from the Residual Life Theorem [32], [35], we obtain

$$
f_r^*(s) = \frac{\eta \left[1 - f_d^*(d)\right]}{s}
$$

where $\eta = [\sum_{i=1}^M \alpha_i/\eta_i]^{-1}$.

If the residual lifetime r_1 of t_1 is exponentially distributed with parameter μ_r, the LT of r_1 ($f_r^*(s)$) is $\mu_r/(s + \mu_r)$. Taking this into (21), we obtain

$$
\begin{aligned}
f_{pnh}^*(s) &= e^{-t_{\text{Tail}}s} \frac{\mu}{s + \mu} + e^{\mu t_{\text{Tail}}} \left(\frac{s}{s + \mu} \right) f_r^*(s + \mu) \\
&= e^{-t_{\text{Tail}}s} \frac{\mu}{s + \mu} + e^{\mu t_{\text{Tail}}} \frac{\mu_r s}{(s + \mu)(s + \mu + \mu_r)}. \\
& \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (22)
\end{aligned}
$$

Applying *simple poles* (see Appendix) or looking up (21) in the table of inverse LTs, we can obtain

$$
\begin{aligned}
& f_{pnh}(t) \\
&= u_a(t) \mu e^{-\mu(t - t_{\text{Tail}})} + e^{\mu t_{\text{Tail}}} \frac{\mu_r \left(-\mu e^{-\mu t} - (-\mu - \mu_r) e^{-(\mu + \mu_r)t} \right)}{-\mu - (-\mu - \mu_r)} \\
&= u_a(t) \mu e^{-\mu(t - t_{\text{Tail}})} - e^{\mu t_{\text{Tail}}} \mu e^{-\mu t} + e^{\mu t_{\text{Tail}}} (\mu + \mu_r) e^{-(\mu + \mu_r)t}. \\
& \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (23)
\end{aligned}
$$

Similarly, if t_j is exponentially distributed with parameter μ_d, we have

$$f_{phh}(t)$$

$$= u_a(t)\mu e^{-\mu(t-t_{\text{Tail}})} + e^{\mu t_{\text{Tail}}}\frac{\mu_d\left(-\mu e^{-\mu t}-(-\mu-\mu_d)e^{-(\mu+\mu_d)t}\right)}{-\mu-(-\mu-\mu_d)}$$

$$= u_a(t)\mu e^{-\mu(t-t_{\text{Tail}})} - e^{\mu t_{\text{Tail}}}\mu e^{-\mu t} + e^{\mu t_{\text{Tail}}}(\mu+\mu_d)e^{-(\mu+\mu_d)t}. \tag{24}$$

3) Handoff Session in Idle With New Arrival Packets: Similarly, $t_{phh\_idle\_new}$ in (6) is the same as t_{phh_busy} in (5). We have the same conditional pdf and the LT with (8) and (20), respectively.

4) Handoff Session in Idle Without Arrival Packet: Because the conditional pdf of $t_{phh\_idle\_no}$ in (17) is the same as the conditional pdf of t_{phh_busy} in (8), the LT of (17) is also the same as that of (20). We conclude our results in Theorem 2 as follows.

Theorem 2: For a PS network with exponential eCHT and Poisson new session arrival rate λ, we have the following statements.

1) For a new session, the LT of the eCHT's pdf is given by

$$f_{pnh}^*(s) = e^{-t_{\text{Tail}}s}\frac{\mu}{s+\mu} + e^{\mu t_{\text{Tail}}}\left(\frac{s}{s+\mu}\right)f_r^*(s+\mu). \tag{25}$$

2) The LT of the eCHT's pdf for a handoff session in busy mode or a handoff session with new arrival packets or a handoff session without arrival packet in idle mode is given by

$$f_{phh\_busy}^*(s) = f_{phh\_idle\_no}^*(s) = f_{phh\_idle\_new}^*(s)$$

$$= e^{-t_{\text{Tail}}s}\frac{\mu}{s+\mu} + e^{\mu t_{\text{Tail}}}\left(\frac{s}{s+\mu}\right)f_d^*(s+\mu). \tag{26}$$

3) The LT of the eCHT's pdf is given by

$$f_p^*(s) = \frac{\lambda}{2\lambda+\lambda_{hb}}\left[f_{pnh}^*(s) + f_{phh\_idle\_new}^*(s)\right]$$

$$+ \frac{\lambda_{hb}}{2\lambda+\lambda_{hb}}f_{phh\_busy}^*(s). \tag{27}$$

4) If CRT is exponentially distributed, the eCHT of the new session and the eCHT of the handoff session have the same distribution, and *vice versa*.

Using Theorem 2, we can obtain the eCHT's pdf by the inverse LT, from which the distribution functions of CRT can be obtained.

C. Analytical Model of Conditional Distribution With Packet Session's eCHT

In the CS network, conditional CHT, which is easier to obtain, is proved to have the same distribution as CHT and has been widely used to further reduce computational complexity for the approximation of CHT distribution [10], [15], [36].

Orlik and Rappaport in [10] and [36] derived the results for conditional distributions for the CHT when the CRT is the sum of the hyper-Exponential (SOHYP) distribution. The work in [15] showed that the conditional CHT has the same distribution as the unconditional CHT when CRT is exponentially distributed. The work significantly reduces the CHT's computational complexity and has been widely used (e.g., [37] and [38]). Here, however, we will point out that *the results are no longer valid in PS networks*.

Let $f_{cnh}(t)$ and $f_{chh}(t)$ denote the conditional pdf of the new session's eCHT and the conditional pdf of the handoff session's eCHT in busy mode, respectively. Note that the conditional pdf of the handoff session's eCHT in idle mode with (without) new arrival packets ($f_{phh\_idle\_new}(t)$ and $f_{phh\_idle\_no}(t)$) is the same as the handoff session's eCHT in busy mode ($f_{phh\_busy}(t)$) because (5), (6), and (7) are the same. The corresponding LTs of these pdfs are $f_{cnh}^*(t)$ and $f_{chh}^*(t)$, and with cdfs $F_{cnh}(t)$ and $F_{chh}(t)$, respectively. We first start from $f_{chh}(t)$ conditioning on the timer, which is a timeout, and $t_j \geq r_j + t_{\text{Tail}}$. We then obtain the following expressions.

1) Handoff Session (in Busy Mode):

$$F_{chh}(h) = \Pr(t_{phh\_busy} \leq h|r_j + t_{\text{Tail}} \leq t_j)$$

$$= \frac{\Pr(r_j + t_{\text{Tail}} \leq h, r_j + t_{\text{Tail}} \leq t_j)}{\Pr(r_j + t_{\text{Tail}} \leq t_j)}$$

$$= \frac{\displaystyle\int_0^{h-t_{\text{Tail}}} f_p(t)\int_{t+t_{\text{Tail}}}^\infty f(\tau)d\tau dt}{\Pr(r_j + t_{\text{Tail}} \leq t_j)}$$

$$= \frac{\displaystyle\int_0^{h-t_{\text{Tail}}} f_p(t)\left[1 - F_d(t + t_{\text{Tail}})\right]dt}{\Pr(r_j + t_{\text{Tail}} \leq t_j)}. \tag{28}$$

Differentiating both sides, we obtain the following conditional pdf:

$$f_{chh}(h) = \frac{f_p(h - t_{\text{Tail}})\left[1 - F_d(h)\right]}{\Pr(r_j + t_{\text{Tail}} \leq t_j)}. \tag{29}$$

We observe that

$$\Pr(r_j + t_{\text{Tail}} \leq t_j) = \int_{t=t_{\text{Tail}}}^\infty \int_0^{t-t_{\text{Tail}}} f_d(t)f_p(\tau)d\tau dt$$

$$= \int_{t=t_{\text{Tail}}}^\infty f_d(t)\left[1 - e^{-\mu(t-t_{\text{Tail}})}\right]dt$$

$$= 1 - \int_{t=t_{\text{Tail}}}^\infty f_d(t)e^{-\mu(t-t_{\text{Tail}})}dt$$

$$= 1 - F_d(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}}f_d^*(\mu).$$

Taking this into (29), we obtain

$$f_{chh}(h) = \frac{\left[1 - F_d(h)\right]\mu e^{-\mu(h-t_{\text{Tail}})}}{1 - F_d(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}}f_d^*(\mu)}. \tag{30}$$

Hence

$$f_{chh}^*(s) = \frac{\mu \int_0^\infty e^{-(s+\mu)(h-t_{\text{Tail}})}\left[1 - F_d(h)\right]dh}{1 - F_d(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_d^*(\mu)}$$
$$= \frac{\mu e^{(s+\mu)t_{\text{Tail}}}}{s+\mu} \frac{1 - f_d^*(s+\mu)}{1 - F_d(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_d^*(\mu)}. \quad (31)$$

2) New Session: In a similar fashion, we obtain the following result for the new session's eCHT:

$$f_{cnh}(h) = \frac{[1 - F_r(h)]\,\mu e^{-\mu(h-t_{\text{Tail}})}}{1 - F_r(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_r^*(\mu)}$$

$$f_{cnh}^*(s) = \frac{\mu \int_0^\infty e^{-(s+\mu)(h-t_{\text{Tail}})}\left[1 - F_r(h)\right]dh}{1 - F_r(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_r^*(\mu)}$$
$$= \frac{\mu e^{(s+\mu)t_{\text{Tail}}}}{s+\mu} \times \frac{1 - f_r^*(s+\mu)}{1 - F_r(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_r^*(\mu)}.$$

The conditional eCHT distribution $f_{ch}(t)$ is the (weighted) average of the conditional new session and handoff session eCHT distribution. Thus

$$f_{ch}^*(s) = \frac{\mu e^{(s+\mu)t_{\text{Tail}}}}{s+\mu}$$
$$\times \left[\frac{\lambda}{2\lambda + \lambda_h} \cdot \frac{1 - f_r^*(s+\mu)}{1 - F_r(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_r^*(\mu)} \right.$$
$$\left. + \frac{\lambda_h}{2\lambda + \lambda_h} \cdot \frac{1 - f_d^*(s+\mu)}{1 - F_d(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_d^*(\mu)} \right].$$

Let T_{cnh}, T_{chh}, and T_{ch} denote the expected conditional new session eCHT, the expected conditional handoff session eCHT, and the expected conditional eCHT, respectively. We then have

$$T_{chh} = \frac{-e^{\mu t_{\text{Tail}}}}{\mu} \left[\frac{1 - \mu t_{\text{Tail}} - f^*(\mu)}{1 - e^{\mu t_{\text{Tail}}} f^*(\mu)} + f^{*(1)}(\mu) \right]$$

$$T_{cnh} = \frac{-e^{\mu t_{\text{Tail}}}}{\mu} \left[\frac{1 - \mu t_{\text{Tail}} - f_r^*(\mu)}{1 - e^{\mu t_{\text{Tail}}} f_r^*(\mu)} + f_r^{*(1)}(\mu) \right]$$

$$T_{ch} = \frac{-e^{\mu t_{\text{Tail}}}}{\mu} \left\{ \frac{1 - \mu t_{\text{Tail}} - f_r^*(\mu)}{1 - e^{\mu t_{\text{Tail}}} f_r^*(\mu)} + f_r^{*(1)}(\mu) \right.$$
$$\left. + \frac{\lambda_h}{2\lambda + \lambda_h} \left[\frac{1 - \mu t_{\text{Tail}} - f^*(\mu)}{1 - e^{\mu t_{\text{Tail}}} f^*(\mu)} + f^{*(1)}(\mu) \right] \right\}.$$

When the residual lifetime r_1 of t_1 is exponentially distributed with parameter μ_r, then its LT is $f_r^*(s) = \mu_r/(s+\mu_r)$. From (31), we obtain

$$f_{cnh}^*(s) = \frac{\mu e^{(s+\mu)t_{\text{Tail}}}}{s+\mu} \cdot \frac{1 - f_r^*(s+\mu)}{1 - F_r(t_{\text{Tail}}) - e^{\mu t_{\text{Tail}}} f_r^*(\mu)}$$
$$= \frac{\mu e^{(s+\mu)t_{\text{Tail}}}}{s+\mu} \times \frac{1 - \frac{\mu_r}{s+\mu+\mu_r}}{1 - (1 - \mu e^{-\mu t_{\text{Tail}}})\,e^{\mu t_{\text{Tail}}} \frac{\mu_r}{\mu+\mu_r}}$$
$$= \frac{\mu e^{(s+\mu)t_{\text{Tail}}}}{s+\mu} \cdot \frac{\frac{s+\mu}{s+\mu+\mu_r}}{\frac{\mu\mu_r}{\mu+\mu_r}}$$
$$= \frac{e^{(s+\mu)t_{\text{Tail}}}}{s+\mu+\mu_r} \cdot \frac{\mu+\mu_r}{\mu_r}.$$

D. Modeling of Handoff Rate for General eCHTs and CRTs

As pointed out in the introduction, it is necessary to study the handoff rate and its impact on eCHT. Here, we study the handoff rate for the general case where the eCHTs and CRTs are generally distributed. The handoff rate is defined as the average number of handoffs during an ongoing session. This value is used to characterize the eCHT distribution and is related to the service quality or cost parameters.

Let $t_1, t_2, \ldots$ denote the CRT (they are independent and identically distributed (i.i.d.) nonlattice distributions) and r_1 denote the residual life of a new session. Let t_p denote the CHT for the packet session. Let H be the number of handoffs of a packet session (without blocking) during the session connection.

$H = 0$ means the session is not blocked, and the eCHT $t_p + t_{\text{Tail}}$ is shorter than the residual life r_1 (the session completes before the portable moves out of the cell). If $H = 1$ means the session is not initially blocked, the UE either makes a successful handoff and then completes the session successfully in the new cell or is forced to terminate because of the first handoff failure, and so on. If the blocking probability for a new session is p_0 and the probability for a handoff session to be forced to terminate is p_f, we have

$$\Pr(H = 0) = (1 - p_0)\Pr(r_1 \ge t_p + t_{\text{Tail}})$$
$$\Pr(H = 1) = (1 - p_0)\Pr(r_1 < t_p + t_{\text{Tail}} \le r_1 + t_2)(1 - p_f)$$
$$\quad + (1 - p_0)\Pr(t_p + t_{\text{Tail}} > r_1 + t_2)p_f$$
$$\vdots$$
$$\Pr(H = k) = B_1$$
$$\vdots$$

where $B_1 = (1 - p_0)\Pr(r_1 + t_2 + \cdots + t_k < t_p + t_{\text{Tail}} \le r_1 + t_2 + \cdots + t_{k+1})(1 - p_f)^k + (1 - p_0)\Pr(t_p + t_{\text{Tail}} > r_1 + t_2 + \cdots + t_k)(1 - p_f)^{k-1}p_f$.

We first calculate $\Pr(H = 0)$ as follows:

$$\Pr(H = 0)$$
$$= (1 - p_0) \int_0^\infty \Pr(r_1 \ge t + t_{\text{Tail}})f_p(t)dt$$
$$= (1 - p_0) \int_0^\infty \int_{t+t_{\text{Tail}}}^\infty f_r(\tau)d\tau f_p(t)dt$$
$$= (1 - p_0) \int_0^\infty \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{1 - f_r^*(s - t_{\text{Tail}})}{s - t_{\text{Tail}}} e^{st}ds f_p(t)dt$$
$$= \frac{(1 - p_0)}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{1 - f_r^*(s - t_{\text{Tail}})}{s - t_{\text{Tail}}} \int_0^\infty f_p(t)e^{st}dt ds$$
$$= \frac{(1 - p_0)}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{1 - f_r^*(s - t_{\text{Tail}})}{s - t_{\text{Tail}}} f_p^*(-s)ds. \quad (32)$$

For $k > 0$, let $\xi = r_1 + t_2 + \cdots + t_k$, we have

$$f_\xi^*(s) = E[e^{-s\xi}] = E\left[e^{-sr_1}\right] \Pi_{i=2}^k E\left[e^{-st_i}\right]$$
$$= f_r^*(s) \left(f^*(s)\right)^{k-1}. \quad (33)$$

The pdf is given by

$$f_\xi(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} f_r^*(s) \left(f^*(s)\right)^{k-1} e^{st}ds \quad (34)$$

so $L(\Pr(\xi \le t + t_{\text{Tail}})) = (f_\xi^*(s - t_{\text{Tail}})/s - t_{\text{Tail}})$. Thus, we have

$$\Pr(r_1 + t_2 + \cdots + t_k \le t_p + t_{\text{Tail}})$$

$$= \int_0^\infty \Pr(\xi \le t + t_{\text{Tail}}) f_p(t) dt$$

$$= \int_0^\infty \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{f_r^*(s - t_{\text{Tail}}) \left[f^*(s - t_{\text{Tail}})\right]^{k-1}}{s - t_{\text{Tail}}} e^{st} ds f_p(t) dt$$

$$= \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \times \frac{f_r^*(s - t_{\text{Tail}}) \left[f^*(s - t_{\text{Tail}})\right]^{k-1}}{s - t_{\text{Tail}}} f_p^*(-s) ds.$$

We can obtain

$$\Pr(H = k)$$

$$= (1 - p_0) \left[\Pr(t_p + t_{\text{Tail}} \ge r_1 + t_2 + \cdots + t_k)\right.$$

$$\left. - \Pr(t_p + t_{\text{Tail}} \ge r_1 + t_2 + \cdots + t_{k+1}) (1 - p_f)^k\right.$$

$$\left. + (1 - p_0)\Pr(t_p + t_{\text{Tail}} \ge r_1 + t_2 + \cdots + t_k)(1 - p_f)^{k-1} p_f\right.$$

$$= \frac{(1 - p_0)(1 - p_f)^{k-1}}{2\pi j} \times \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{f_r^*(s - t_{\text{Tail}}) \left[f^*(s - t_{\text{Tail}})\right]^{k-1}}{s - t_{\text{Tail}}}$$

$$\times \left[1 - (1 - p_f) f^*(s - t_{\text{Tail}})\right] f_p^*(-s) ds. \quad (35)$$

Now, we find the Z-transform for the number of handoffs. Let $H(z)$ be the moment-generating function, then we have

$$H[z] = E[z^H] = \sum_{k=0}^\infty z^k \Pr(H = k)$$

$$= \Pr(H = 0) + \sum_{k=1}^\infty z^k \Pr(H = k)$$

$$= \Pr(H = 0) + \frac{(1 - p_0)z}{2\pi j}$$

$$\times \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{f_r^*(s - t_{\text{Tail}}) \left[1 - (1 - p_f) f^*(s - t_{\text{Tail}})\right]}{s - t_{\text{Tail}}}$$

$$\times \left\{\sum_{k=1}^\infty \left[(1 - p_f) z f^*(s - t_{\text{Tail}})\right]^{k-1}\right\} f_p^*(-s) ds$$

$$= \frac{1 - p_0}{2\pi j} \cdot \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{A_1 \times f_p^*(-s)}{(s - t_{\text{Tail}})[1 - (1 - p_f) z f^*(s - t_{\text{Tail}})]} ds$$

$$= \frac{1 - p_0}{2\pi j} \cdot \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{A_2 \times f_p^*(-s)}{(s - t_{\text{Tail}})^2 [1 - (1 - p_f) z f^*(s - t_{\text{Tail}})]} ds$$

$$(36)$$

where $A_1 = 1 + (z - 1 + z(1 - (1 - p_f) f^*(s - t_{\text{Tail}}))) f_r^*(s - t_{\text{Tail}}) - z(1 - p_f) f^*(s - t_{\text{Tail}})$, and $A_2 = (s - t_{\text{Tail}}) + (2z - 1 - $

$z(1 - p_f) f^*(s - t_{\text{Tail}})) \eta(1 - f^*(s - t_{\text{Tail}})) - z(s - t_{\text{Tail}})(1 - p_f) f^*(s - t_{\text{Tail}})$.

If we use σ_p to denote the singular points of $f_p^*(-s)$ in the right half complex plane, from Cauchy's residue theorem, we obtain

$$H(z) = -(1 - p_0)$$

$$\times \sum_{p \in \sigma_p} \text{Res}_{s=p} \frac{A_3 \times f_p^*(-s)}{(s - t_{\text{Tail}})^2 \left[1 - (1 - p_f) z f^*(s - t_{\text{Tail}})\right]}$$

where $A_3 = (s - t_{\text{Tail}}) + (2z - 1 - z(1 - p_f) f^*(s - t_{\text{Tail}})) \eta(1 - f^*(s - t_{\text{Tail}})) - z(s - t_{\text{Tail}})(1 - p_f) f^*(s - t_{\text{Tail}})$, and Res_p denotes the residue at singular point $s = p$. Differentiating $H(z)$ in (36) at $z = 1$, we obtain

$$E[H] = \frac{1 - p_0}{2\pi j}$$

$$\times \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{A_4}{(s - t_{\text{Tail}})^2 \left[1 - (1 - p_f) f^*(s - t_{\text{Tail}})\right]} f_p^*(-s) ds$$

$$(37)$$

where $A_4 = (s - t_{\text{Tail}}) + (1 - (1 - p_f) f^*(s - t_{\text{Tail}})) \eta(1 - f^*(s - t_{\text{Tail}})) - (s - t_{\text{Tail}})(1 - p_f) f^*(s - t_{\text{Tail}})$.

Applying Cauchy's residue theorem, we obtain

$$E[H] = -(1 - p_0)$$

$$\times \sum_{p \in \sigma_p} \text{Res}_{s=p} \frac{A_4 \times f_p^*(-s)}{(s - t_{\text{Tail}})^2 \left[1 - (1 - p_f) f^*(s - t_{\text{Tail}})\right]}. \quad (38)$$

If the eCHTs are exponentially distributed with parameter μ, $f_p^*(-s) = \mu/(-s + \mu)$, which has a singular point, and $\sigma_p = \{\mu\}$, we obtain

$$E[H] = \mu(1 - p_0) \frac{A_5}{(\mu - t_{\text{Tail}})^2 \left[1 - (1 - p_f) f^*(\mu - t_{\text{Tail}})\right]} \quad (39)$$

where $A_5 = (\mu - t_{\text{Tail}}) - (\mu - t_{\text{Tail}})(1 - p_f) f^*(\mu - t_{\text{Tail}}) + (1 - (1 - p_f) f^*(\mu - t_{\text{Tail}})) \eta(1 - f^*(\mu - t_{\text{Tail}}))$.

If there is no blocking and no forced termination, $p_0 = p_f = 0$, and from (39), $E[H] = 1$. If there are handoff failures, the expected number of handoffs is intuitively smaller.

Let us assume that the eCHTs are i.i.d. with an Erlang distribution, i.e.,

$$f_p(t) = \frac{\alpha^m t^{m-1}}{(m-1)!} e^{-\alpha t}, \quad f_p^*(s) = \left(\frac{\alpha}{s + \alpha}\right)^m \quad (40)$$

where $\alpha = m\eta$ is the scale parameter, and m is a positive integer. When $m = 1$, it gives the exponential distribution. We can also find a simple formula for the handoff rate. Let

$$g(s) = \frac{A_4}{(s - t_{\text{Tail}})^2 \left[1 - (1 - p_f) f^*(s - t_{\text{Tail}})\right]}. \quad (41)$$

Then, $f_p^*(-s) = [\alpha/(-s+\alpha)]^m$ has a unique singular point $\sigma_p = \{\alpha\}$. From (38), we obtain

$$E[H] = (-1)^{m+1}(1-p_0)\alpha^m \frac{g^{(m-1)}(\alpha)}{(m-1)!}, \alpha = m\mu. \quad (42)$$

For this case, if there is no handoff failure, $(p_0 = p_f = 0)$, from (41), we have $g(s) = (s - t_{\text{Tail}} - \eta)/(s - t_{\text{Tail}})^2$, and $g^{(m-1)}(\alpha) = (-1)^{m-1}((m-1)!(\alpha - t_{\text{Tail}}) + \eta(m!))/(\alpha - t_{\text{Tail}})^{(m+1)}$. From (42), we have

$$E[H] = \frac{(-1)^{2m}\alpha^m(\alpha - t_{\text{Tail}} + m\eta)}{(\alpha - t_{\text{Tail}})^{m+1}}. \quad (43)$$

In fact, it can be shown that for the ideal case $(p_0 = p_f = 0)$, the expected number of handoffs for any CRT distribution and any eCHT distribution is $E[H] = (\alpha^m(\alpha - t_{\text{Tail}} + m\eta)/(\alpha - t_{\text{Tail}})^{m+1})$. Indeed, from (37), if $p_f = 0$, we obtain

$$E[H] = \frac{1-p_0}{2\pi j} \int\limits_{\sigma-j\infty}^{\sigma+j\infty} \frac{1}{s} f_p^*(-s)ds$$

$$= (1-p_0)\int_0^\infty f_p(t) \left[\frac{1}{2\pi j} \int\limits_{\sigma-j\infty}^{\sigma+j\infty} \frac{1}{s} f_p^*(-s)ds \right] dt$$

$$= (1-p_0)\int_0^\infty t f_p(t)dt = (1-p_0)\frac{1}{\mu}.$$

Next, we show how to find the handoff session arrival rate from the handoff rate. It is easy to observe that for each nonblocking new session, there will be on average $E[H]$ number of handoff sessions induced. Therefore, the handoff session traffic will have arrival rate $\lambda_h = \lambda E[H]$. From (38), we obtain

$$\lambda_h = -\lambda(1-p_0)$$

$$\times \sum_{p\in\sigma_p} \text{Res}_{s=p} \frac{A_4}{(s - t_{\text{Tail}})^2 [1 - (1 - p_f)f^*(s - t_{\text{Tail}})]} f_p^*(-s). \quad (44)$$

If the eCHTs are Erlang distributed according to (40), we have

$$\lambda_h = -\lambda(-1)^{m+1}(1-p_0)\alpha^m \frac{g^{(m-1)}(\alpha)}{(m-1)!}, \alpha = m\mu. \quad (45)$$

The cell traffic intensity can be computed from this formula and the expected CHT.

When the eCHTs are Erlang distributed, we have to compute the derivatives of $g(s)$ for the computation of handoff rate and handoff session arrival rate. However, the explicit expressions for the derivatives of $g(s)$ may be difficult. We therefore develop the following recursive algorithm for their computations. Let

$$h(s) = (s - t_{\text{Tail}})^2 [1 - (1 - p_f)f^*(s - t_{\text{Tail}})]. \quad (46)$$

We obtain

$$h^{(0)}(\alpha) = (\alpha - t_{\text{Tail}})^2 [1 - (1 - p_f)f^*(\alpha - t_{\text{Tail}})]$$

$$h^{(1)}(\alpha) = 2(\alpha - t_{\text{Tail}})(1 - (1 - p_f)f^*(\alpha - t_{\text{Tail}}))$$
$$- (\alpha - t_{\text{Tail}})^2 \left((1 - p_f)f^{*(1)}(\alpha - t_{\text{Tail}}) \right)$$

$$h^{(2)}(\alpha) = 2(1 - (1 - p_f)f^*(\alpha - t_{\text{Tail}}))$$
$$- 4(\alpha - t_{\text{Tail}})(1 - p_f)f^{*(1)}(\alpha - t_{\text{Tail}})$$
$$- (\alpha - t_{\text{Tail}})^2(1 - p_f)f^{*(2)}(\alpha - t_{\text{Tail}})$$

$$h^{(3)}(\alpha) = -6(1 - p_f)f^{*(1)}(\alpha - t_{\text{Tail}})$$
$$- 6(\alpha - t_{\text{Tail}})(1 - p_f)f^{*(2)}(\alpha - t_{\text{Tail}})$$
$$- (\alpha - t_{\text{Tail}})^2(1 - p_f)f^{*(3)}(\alpha - t_{\text{Tail}})$$

$$h^{(4)}(\alpha) = -12(1 - p_f)f^{*(2)}(\alpha - t_{\text{Tail}})$$
$$- 8(\alpha - t_{\text{Tail}})(1 - p_f)f^{*(3)}(\alpha - t_{\text{Tail}})$$
$$- (\alpha - t_{\text{Tail}})^2(1 - p_f)f^{*(4)}(\alpha - t_{\text{Tail}}).$$

Since $g(s)h(s) = (s - t_{\text{Tail}}) + (1 - (1 - p_f)f^*(s - t_{\text{Tail}}))\eta(1 - f^*(s - t_{\text{Tail}})) - (s - t_{\text{Tail}})(1 - p_f)f^*(s - t_{\text{Tail}})$, differentiating both sides, we obtain $(p > 0)$

$$\sum_0^p C_i^p g^{(i)}(s)h^{(p-i)}(s)$$

$$= \{[1 - (1 - p_f)f^*(s - t_{\text{Tail}})]$$
$$\times [s - t_{\text{Tail}} + \eta(1 - f^*(s - t_{\text{Tail}}))]\}^{(p)}.$$

From this, we obtain the following recursive algorithm to compute $g^{(m-1)}(\alpha)$:

$$g^{(0)}(\alpha) = \frac{(1 - (1 - p_f)f^*(\alpha - t_{\text{Tail}}))}{h(\alpha)}$$
$$\times (\alpha - t_{\text{Tail}} + \eta(1 - f^*(\alpha - t_{\text{Tail}})))$$

$$g^{(p)}(\alpha) = -\frac{A_5}{h(\alpha)}, p > 0$$

where $A_5 = \sum_0^{p-1} C_i^p g^{(i)}(\alpha)h^{(p-i)}(\alpha) - [(1 - (1 - p_f)f^*(\alpha - t_{\text{Tail}}))(\alpha - t_{\text{Tail}} + \eta(1 - f^*(\alpha - t_{\text{Tail}})))]^{(p)}$.

IV. NUMERICAL RESULTS

Here, we present our numerical results on how the CRT distribution affects the eCHT distribution and handoff rate. For the sake of easy calculation, general applicability, and universal approximation property [32], we use the *hyper-Erlang* distribution model [11], [15], [19] to analyze the eCHT. Another reason is that some pdf functions may not exist as closed-form LT, e.g., lognormal distribution.
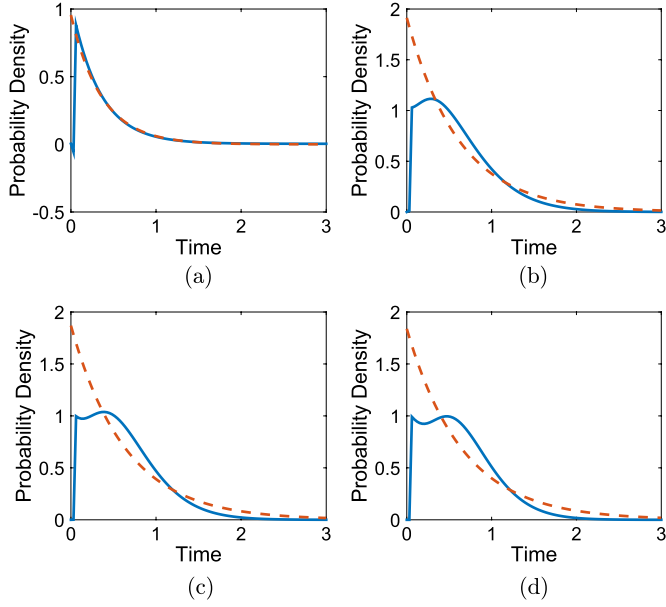
Fig. 4. PDF of handoff session eCHT (solid line) and its exponential fitting (dashed line) when CRT is Erlang distributed with parameters (μ, η). (a) $m = 2$. (b) $m = 3$. (c) $m = 4$. (d) $m = 5$.
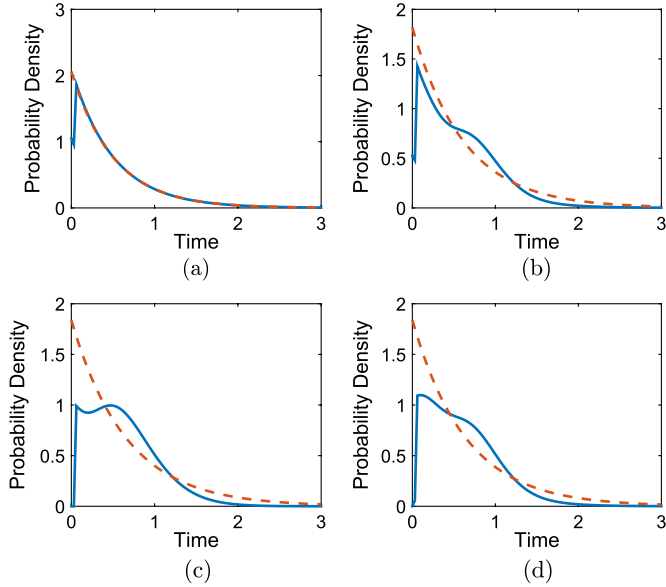

Fig. 5. PDF of handoff session eCHT (solid line) and its exponential fitting (dashed line) when CRT is hyper-Erlang distributed with two parameters (m_1, m_2, η). (a) $m_1 = 1$, $m_2 = 1$. (b) $m_1 = 1$, $m_2 = 10$. (c) $m_1 = 5$, $m_2 = 5$. (d) $m_1 = 2$, $m_2 = 10$.

A. Numerical Results for Section III-B

We take the Erlang, hyper-Erlang, and exponential (shown in Fig. 4 with $m = 1$) CRT into Theorem 2. We then can obtain the pdf of the handoff session eCHT from our analytical results.

In Figs. 4 and 5, we show the handoff session's eCHT when CRT is Erlang and hyper-Erlang distributed, respectively. We vary the shape parameter (m or m_1, m_2). It can be seen that the exponential model does not have a good fit for the real handoff session's eCHT distribution when the shape parameter increases.
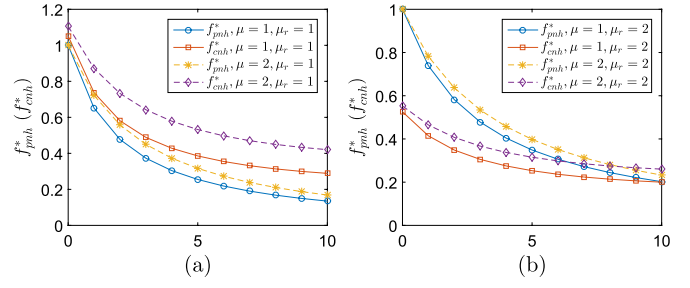

Fig. 6. Comparison of the LT functions of unconditional (f_{pnh}^*) and conditional (f_{cnh}^*) pdfs of eCHT on (0,10). (a) s. (b) s.
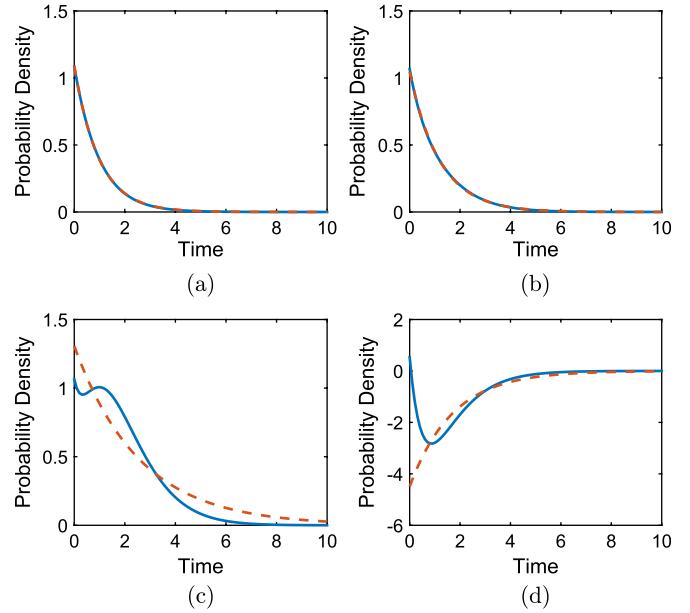

Fig. 7. Conditional pdf of handoff session eCHT (solid line) and its exponential fitting (dashed line) when CRT is Erlang distributed with parameters (m, η). (a) $m = 2$. (b) $m = 3$. (c) $m = 4$. (d) $m = 5$.

B. Numerical Results for Section III-C

Fig. 6 shows the results of the LT functions of unconditional (f_{pnh}^*) and conditional (f_{cnh}^*) pdfs of eCHT, respectively. It shows that the conditional pdf of eCHT cannot be used for the approximation of eCHT as CHT does in the CS network. The finding may prevent researchers from making an inappropriate approximation for eCHT by using its conditional pdf as they usually do in the CS network.

Figs. 7 and 8 show the conditional pdf for the handoff session's eCHT when the CRT is Erlang and hyper-Erlang distributed. From this example, we observe that the conditional distribution for the handoff session's eCHT is a better match to the exponential distribution when the variance is large. However, when the variance becomes small, i.e., the CRT is less spread, this match disappears.

C. Numerical Results for Section III-D

Finally, we turn our attention to the handoff rate. Assume that the eCHTs are hyper-Erlang distributed with one parameter. We shall use (42) and the recursive algorithms given at the end of
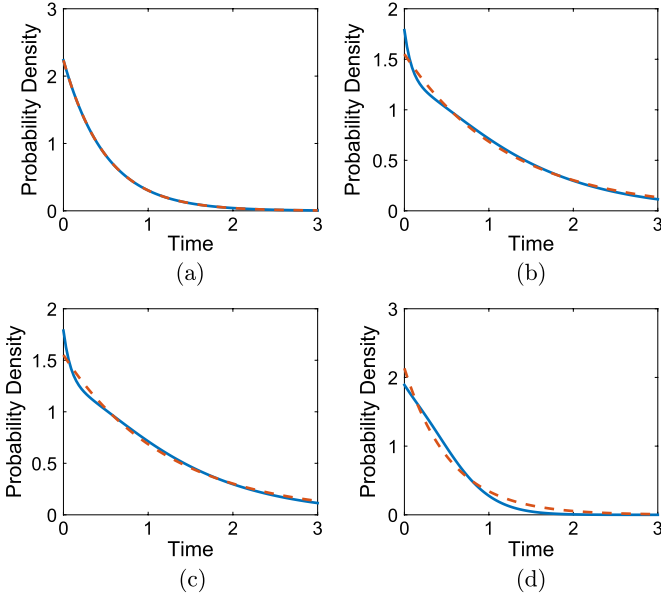
Fig. 8. Conditional pdf of handoff session eCHT (solid line) and its exponential fitting (dashed line) when CRT is hyper-Erlang distributed with two parameters (m_1, m_2, η). (a) $m_1 = 1$, $m_2 = 1$. (b) $m_1 = 1$, $m_2 = 10$. (c) $m_1 = 5$, $m_2 = 5$. (d) $m_1 = 2$, $m_2 = 10$.

the previous section. The CRTs are Gamma distributed with the pdf, i.e.,

$$f(t) = \frac{\beta^\gamma t^{\gamma-1}}{\Gamma(\gamma)} e^{-\beta t}, f^*(s) = \left(\frac{\beta}{s+\beta}\right)^\gamma, \beta = \gamma\eta \quad (47)$$

where γ is the shape parameter, β is the scale parameter, and $\Gamma(\gamma)$ is the Gamma function. The mean and variance of this distribution are $1/\eta$ and $1/(\gamma\eta^2)$, respectively.

Figs. 9 and 10 show the handoff rate for different blocking probabilities and forced termination probabilities. We have the following observations.

- The handoff rate for fixed variance of the CRTs (i.e., fixed m) is increasing when mobility η/μ increases.
- The handoff rate is decreasing for fixed mobility when the variance of the CRTs decreases (i.e., m is increasing).
- The handoff rate is smaller than that in the ideal case (when there is no blocking and no forced termination).
- The handoff rate is insensitive to the variance of the CRTs when mobility η/μ is small, whereas it is very sensitive to the variance of the CRT when mobility η/μ is high.

Since the handoff session arrival rate is equal to the new session arrival rate times the handoff rate in a session, we observe from Figs. 9 and 10 that for highly moving users the handoff rate is much smaller for the hyper-Erlang distributed with one parameter eCHT ($m > 1$) than that for exponential eCHT ($m = 1$). This implies that the exponential assumption for eCHT overestimates the handoff rate. This suggests that PS network designers have to carefully consider the distribution model for eCHT and CRT to meet the blocking probability requirement: Exponential distribution approximation for CRTs and eCHTs may not be enough for the real PS network design.
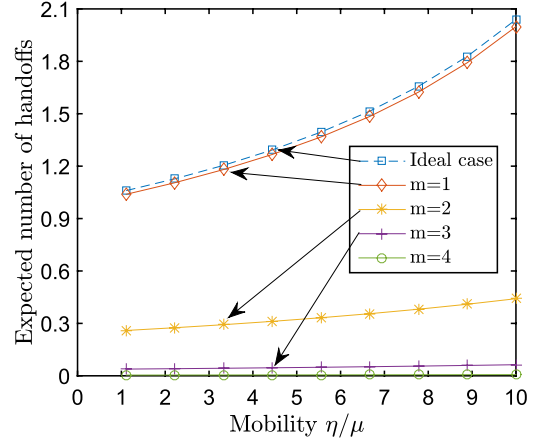

Fig. 9. Handoff rate: dashed line for the ideal case ($p_0 = p_f = 0$) and solid lines for $m = 1 \sim 4$ with $p_f = p_0 = 0.02$.
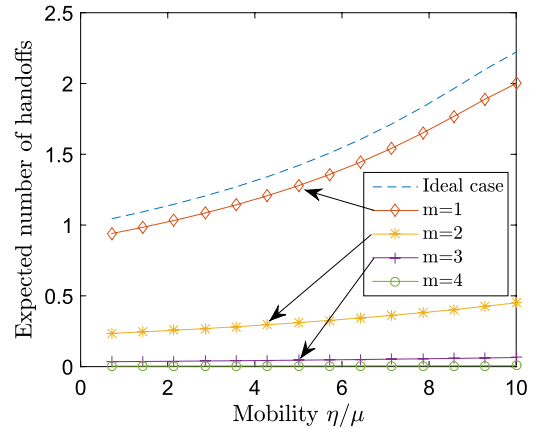

Fig. 10. Handoff rate: dashed line for the ideal case ($p_0 = p_f = 0$) and solid lines for $m = 1 \sim 4$ with $p_f = 0.05$ and $p_0 = 0.01$.

V. Conclusion

In this paper, we have proposed new models with DRX mode to analyze the impacts of eCHT for packet sessions in PS cellular networks. The new models reveal the essential relationship between eCHT, CRT, and handoff rate for traffic modeling and performance analysis in PS networks. Our analytical results show that some characteristics of eCHT are similar with those of CHT in CS networks so that researchers can use the same assumptions and results derived in the CS network for their performance modeling. However, we also point out the inapplicable assumptions and results of CHT in PS networks. The results can prevent researchers from making inappropriate assumptions on performance modeling and are essential for the traffic study, management, and performance evaluation of PS cellular networks. The results also enable wide applicability in various scenarios and therefore have important theoretical significance. The models presented in this paper can be used as a theoretical guideline for network designers and performance analysts. Our future work would be to study the models for configuring the appropriate parameters for mobile cellular networks.

APPENDIX

Simple poles [39]: Suppose that we have LT $F(S) = \mathcal{L}(f(t))$ for

$$F(s) = \frac{P(s)}{Q(s)}$$
$$= \frac{P(s)}{(s - \alpha_1)(s - \alpha_2) \cdots (s - \alpha_n)}, \ \alpha_i \neq \alpha_j$$

where $P(s)$ is a polynomial of degree less than n. The α_i's are known as *simple poles* of $F(s)$ in the terminology of complex variables. Partial-fraction decomposition is

$$F(s) = \frac{A_1}{(s - \alpha_1)} + \frac{A_2}{(s - \alpha_2)} + \cdots + \frac{A_n}{(s - \alpha_n)}. \quad (48)$$

By multiplying both sides of (48) by $s - \alpha_i$ and letting $s \to \alpha_i$, we obtain

$$A_i = \lim_{s \to \alpha_i} (s - \alpha_i) F(s). \quad (49)$$

Thus

$$f(t) = \mathcal{L}^{-1}\left(F(s)\right)$$
$$= \sum_{i=1}^{n} \mathcal{L}^{-1}\left(\frac{A_i}{s - \alpha_i}\right) = \sum_{i=1}^{n} A_i e^{\alpha_i t}.$$

According to (49), A_i gives a quick method, i.e.,

$$f(t) = \mathcal{L}^{-1}\left(F(s)\right) = \sum_{i=1}^{n} \lim_{s \to \alpha_i} (s - \alpha_i) F(s) e^{\alpha_i t}. \quad (50)$$

After applying it to (22), we can find

$$\mathcal{L}^{-1}\left[\frac{\mu_r s}{(s + \mu)(s + \mu + \mu_r)}\right]$$
$$= \mathcal{L}^{-1}\left[\frac{\mu_r s}{(s - (-\mu))(s - (\mu - \mu_r))}\right].$$

From (50), therefore, we can get

$$f(t) = \mathcal{L}^{-1}\left(F(s)\right)$$
$$= \lim_{s \to -\mu} (s + \mu) F(s) e^{-\mu t}$$
$$\quad + \lim_{s \to -\mu - \mu_r} (s + \mu + \mu_r) F(s) e^{-(\mu + \mu_r)t}$$
$$= \frac{-\mu \cdot \mu_r}{-\mu + \mu + \mu_r} \cdot e^{-\mu t} + \frac{\mu_r \cdot (-\mu - \mu_r)}{(-\mu - \mu_r) + \mu} \cdot e^{-(\mu + \mu_r)t}.$$
$$\quad (51)$$

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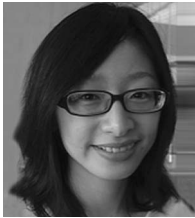
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