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Deflection of ice cover caused by an underwater body moving in channel

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Abstract. Deflections and strains in an ice cover of a frozen channel caused by an underwater body moving under the ice with a constant speed along the channel are studied. The channel is of rectangular cross section, the fluid in the channel is inviscid and incompressible. The ice cover is clamped to the channel walls. The ice cover is modeled by a thin viscoelastic plate. The underwater body is modeled by a three-dimensional dipole. The intensity of the dipole is related to the speed and size of the underwater body. The problem is considered within the linear theory of hydroelasticity. For small deflections of the ice cover the velocity potential of the dipole in the channel is obtained by the method of images in leading order without account for the deflection of the ice cover. The problem of moving dipole in the channel with rigid walls provides the hydrodynamic pressure on the upper boundary of the channel, which corresponds to the ice cover. This pressure distribution does not depend on the deflection of the ice cover in the leading approximation. The deflections of the ice and strains in the ice plate are independent of time in the coordinate system moving together with the dipole. The problem is solved numerically using the Fourier transform, method of the normal modes and the truncation method for infinite systems of algebraic equations.

1. Introduction

The problem of the ice cover response to applied external loads has been well studied for an unbounded ice cover [1] and for a semi-infinite ice cover clamped to a vertical wall [2]. The external load is usually modelled as a point pressure or smooth localized pressure distribution moving with constant speed along the ice cover. The problems were studied within the linear theory of hydroelasticity. The ice deflections and strain distributions in the ice cover were determined. A main practical goal of those studies was to answer the question whether ice can be broken or not by the applied moving pressure and where this would happen, close to the load or at a distance from it.

The ice deflection and strains in the ice plate covering a channel for a localized external load moving along the channel were investigated in [3, 4]. The effect of the channel parameters and characteristics of the external load on the ice deflections and strain distribution in the ice cover were studied. The hydroelastic waves propagating along a frozen channel were studied in [5, 6] within the model of elastic ice plate.

Small oscillations of a two-dimensional body in a liquid were studied by Sturova and Tkacheva [7, 8, 9] within the linear wave theory for finite floating ice plates. In their studies, the upper



boundary of the liquid consisted of either two semi-infinite intervals of free surfaces and a floating elastic plate or a finite interval of the free surface between two semi-infinite elastic plates of different thickness. The oscillating body was placed either under the ice or under the free surface. The problem was reduced to an integral equation for an unknown distribution of mass sources on the equilibrium position of the body surface.

In the present paper, the problem of ice cover deflection caused by an underwater body moving along a frozen channel is considered. This problem is approximately identical to the problem of a stationary body placed in a uniform current under the ice cover in a channel. The moving body is modelled by a three-dimensional dipole. A dipole moving at a constant speed in an unbounded fluid generates the flow and the pressure corresponding to a rigid sphere moving at the same speed. The radius of the sphere is related to the speed of the dipole and its intensity. A dipole moving in a channel with rigid walls also represents approximately a rigid sphere if the dipole intensity is small and the sphere radius is much smaller than the distance from the dipole to the walls of the channel. For a dipole of large intensity, a corresponding shape of a rigid body moving along the channel can be identified. This shape depends on the distances of the dipole from the walls. The velocity potential of a dipole placed in a rectangular channel with rigid walls is determined below by the method of images. The potential of several dipoles can be determined by superposition of the potentials of individual dipoles. A system of dipoles can be designed in such a way that these dipoles accurately represent a body of a given shape. By using the known velocity potential and the unsteady Bernoulli equation, the hydrodynamic loads acting on the channel walls are determined. For a dipole of small intensity, the linearized Bernoulli equation is used. This case corresponds to the motion of a small sphere along the channel covered with ice. The hydrodynamic loads on the ice cover caused by a moving underwater body are determined in the leading order without account for the ice deflection which is assumed small. The ice deflection is determined thereafter as the ice response to the applied loads caused by the moving body. The deflections of the ice cover are assumed small. However the strains in the cover can be large enough to cause ice breaking. The Kelvin–Voigt model of viscoelastic ice plate is used in the present study to estimate the strains in the ice cover. In this model, hydroelastic waves decay with a distance from a load due to the damping effect. The viscoelastic ice model is closer to reality than the elastic ice plate model.

2. Formulation of the problem

The deflection of an ice sheet caused by the motion of a dipole under the ice in the positive x -direction along a channel is considered. The channel is of rectangular cross section with depth H and width $2L$. The channel is of infinite extent in the x -direction. The channel is occupied with the inviscid and incompressible liquid of density ρ_l . The flow caused by both the dipole and the deflection of the ice is potential. The ice sheet is of constant thickness h_i and rigidity $D = Eh_i^3/[12(1-\nu^2)]$, where E is Young's modulus of ice and ν is Poisson's ratio. The ice sheet is clamped to the walls of the channel at $y = \pm L$.

The deflection of the ice sheet, $w(x, y, t)$, is governed by the equation of thin viscoelastic plate [10]

$$Mw_{tt} + D \left(1 + \tau \frac{\partial}{\partial t} \right) \nabla^4 w = p(x, y, 0, t) \quad (1)$$

$$(-\infty < x < \infty, -L < y < L, z = 0),$$

where $\tau = \eta/E$ is the retardation time, η is the viscosity of the ice, $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$, $M = \rho_i h_i$ is the mass of the ice plate per unit area, ρ_i is the ice density, $p(x, y, 0, t)$ is the hydrodynamic pressure acting on the lower surface of the ice plate, and t is the time. The ice sheet is frozen to the walls, which is modelled by the clamped conditions,

$$w = 0, \quad w_y = 0 \quad (-\infty < x < \infty, y = \pm L). \quad (2)$$

The hydrodynamic pressure on the ice-liquid interface is given by the linearized Bernoulli equation,

$$p(x, y, 0, t) = -\rho_l \varphi_t - \rho_l g w \quad (-\infty < x < \infty, -L < y < L), \quad (3)$$

where g is the gravitational acceleration and $\varphi(x, y, z, t) = \varphi^D(x, y, z, t) + \varphi^E(x, y, z, t)$ is the total potential of the flow, φ^D is the velocity potential of a 3D dipole moving in the channel with rigid walls and φ^E is a correction potential accounting for the deflection of the ice cover. The velocity potential $\varphi^E(x, y, z, t)$ satisfies Laplace's equation in the flow region, the linearized kinematic condition on the ice-liquid interface and the boundary conditions of impermeability on other boundaries of the channel:

$$\varphi_z^E = w_t(x, y, t) \quad (z = 0), \quad \varphi_y^E = 0 \quad (y = \pm L), \quad \varphi_z^E = 0 \quad (z = -H). \quad (4)$$

The potential $\varphi^D(x, y, z, t)$ represents the dipole of intensity I placed at the point with coordinates x_0, y_0, z_0 , which may depend on time t , between rigid walls at $z = -H$, $z = 0$, $y = L$ and $y = -L$. This potential satisfies the boundary conditions

$$\varphi_z^D = 0 \quad (z = 0, z = -H), \quad \varphi_y^D = 0 \quad (y = \pm L). \quad (5)$$

The potential of the dipole oriented in the negative x -direction in unbounded three-dimensional fluid is [11]

$$\phi(x, y, z, t) = \frac{I(x - x_0)}{4\pi r^3}, \quad r(x, x_0, y, y_0, z, z_0) = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}. \quad (6)$$

Parameters x_0, y_0, z_0 and I can be functions of time. The potential of the dipole in the channel with the boundary conditions (5) is derived using the method of images [12]. The potential which behaves as (6) when $r \rightarrow 0$ and satisfies the boundary conditions at $y \pm L$ is given by

$$\begin{aligned} \phi^y(x, x_0, y, y_0, z, z_0, t) = & -\frac{Ix}{4\pi} \left(\frac{1}{r^3(x, x_0, y, y_0, z, z_0)} + \sum_{n=1}^{\infty} \left(\frac{1}{r^3(x, x_0, y, y_0 + 4nL, z, z_0)} + \right. \right. \\ & + \frac{1}{r^3(x, x_0, y, 2L - y_0 + 4(n-1)L, z, z_0)} + \frac{1}{r^3(x, x_0, y, y_0 - 4nL, z, z_0)} + \\ & \left. \left. + \frac{1}{r^3(x, x_0, y, -2L - y_0 - 4(n-1)L, z, z_0)} \right) \right), \end{aligned}$$

where the method of images for two vertical walls was used. Then the potential $\varphi^D(x, y, z, t)$ is obtained as an infinite series of the potentials ϕ^y reflected from the walls $z = -H$ and $z = 0$:

$$\begin{aligned} \varphi^D(x, y, z, t) = & \phi^y(x, x_0, y, y_0, \hat{z}, \hat{z}_0, t) + \sum_{m=1}^{\infty} (\phi^y(x, x_0, y, y_0, \hat{z}, \hat{z}_0 + 2mH, t) + \\ & \phi^y(x, x_0, y, y_0, \hat{z}, H - \hat{z}_0 + 2(m-1)H, t) + \phi^y(x, x_0, y, y_0, \hat{z}, \hat{z}_0 - 2mH, t) + \\ & + \phi^y(x, x_0, y, y_0, \hat{z}, -H - \hat{z}_0 + 2(m-1)H, t)), \end{aligned} \quad (7)$$

where $\hat{z} = z + H/2$, $\hat{z}_0 = z_0 + H/2$. Below, y_0 and z_0 are constant and $x_0 = Ut$.

The plate equation (1) and the Bernoulli equation (3) give

$$Mw_{tt} + D \left(1 + \tau \frac{\partial}{\partial t} \right) \nabla^4 w = -\rho_l \varphi_t^D - \rho_l \varphi_t^E - \rho_l g w, \quad (8)$$

where $\varphi_t^D(x, y, 0, t)$ is obtained by differentiating (7) in time and φ_t^E is to be determined as a solution of the boundary problem for Laplace's equation with the boundary conditions (4). Therefore, equation (8) describes the ice deflection caused by the "external" load $-\rho\varphi_t^D(x, y, 0, t)$ moving along the channel. Equations (6) and (7) show that φ^D depends on $x - x_0$ but not on x and x_0 separately. Then

$$\varphi^D = \varphi_1^D(x - x_0, y, y_0, z, z_0)$$

and

$$\varphi_t^D = -U\varphi_{1x}^D(x - x_0, y, y_0, z, z_0)$$

for $x_0 = Ut$.

The term with $\tau\partial/\partial t$ in the equation of viscoelastic plate (1), (8) describes the damping of ice plate oscillations, so hydroelastic waves decay far away from the moving load, where $|(x - x_0)/L| \rightarrow \infty$.

The formulated problem is considered in the case of the dipole moving with constant speed along the channel in the x -direction, all other parameters of the dipole are constant. Let us introduce non-dimensional variables denoted by tilde. The half-width of the channel L is taken as the length scale, the ratio L/U as the time scale. The non-dimensional depth of the channel H/L is denoted by h . The moving system of nondimensional coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ with the origin on the upper channel wall at the centre of the dipole is introduced by

$$\tilde{y} = \frac{y}{L}, \quad \tilde{x} = \frac{x - Ut}{L}, \quad \tilde{z} = \frac{z}{L}, \quad \tilde{t} = \frac{U}{L}t.$$

We are concerned with a steady-state solution in the moving coordinate system,

$$w(x, y, t) = w(\tilde{x}L + Ut, L\tilde{y}, t) = w_{sc} \tilde{w}(\tilde{x}, \tilde{y}),$$

$$\varphi^E(x, y, z, t) = \varphi^E(\tilde{x}L + Ut, L\tilde{y}, L\tilde{z}, t) = \varphi_{sc}^E \tilde{\varphi}^E(\tilde{x}, \tilde{y}, \tilde{z}),$$

$$\varphi^D(x, y, z, t) = \varphi^D(\tilde{x}L + Ut, L\tilde{y}, L\tilde{z}, t) = \frac{I}{L^2} \tilde{\varphi}^D(\tilde{x}, \tilde{y}, \tilde{z}),$$

where w_{sc} , φ_{sc} are the scales of the ice deflection and the velocity potential φ^E correspondingly. The scales are chosen as $w_{sc} = (UI)/(L^3g)$ and $\varphi_{sc}^E = (U^2I)/(gL^3)$.

In the non-dimensional variables the problem reads (tildes are omitted below)

$$\alpha h \text{Fr}^2 w_{xx} + \beta \left(1 - \varepsilon \frac{\partial}{\partial x}\right) \nabla^4 w + w = h \text{Fr}^2 \varphi_x^E + \varphi_x^D \quad (-\infty < x < \infty, \quad -1 < y < 1, \quad z = 0), \quad (9)$$

$$w = 0, \quad w_y = 0 \quad (y = \pm 1), \quad (10)$$

$$\nabla^2 \varphi^E = 0 \quad (-\infty < x < \infty, \quad -1 < y < 1, \quad -h < z < 0), \quad (11)$$

$$\varphi_z^E = -w_x \quad (z = 0), \quad \varphi_y^E = 0 \quad (y = \pm 1), \quad \varphi_z^E = 0 \quad (z = -h), \quad (12)$$

$$\varphi_z^D = 0 \quad (z = 0, z = -h), \quad \varphi_y^D = 0 \quad (y = \pm 1), \quad (13)$$

$$w, \varphi \rightarrow 0 \quad (|x| \rightarrow \infty). \quad (14)$$

Here $\beta = D/(\rho_l g L^4)$, $\varepsilon = (\tau U)/L$, $\alpha = M/(\rho_l L) = (\rho_i h_i)/(\rho_l L)$ and $\text{Fr} = U/\sqrt{gH}$ is the Froude number. The solution of the problem (9) – (14) depends on five non-dimensional parameters h , α , β , ε , Fr , as well as on the position of the dipole (x_0, y_0, z_0) , which describe the aspect ratio of channel, characteristics of ice and the dipole. We shall determine the deflection w and strain distribution in the ice sheet for some given values of these parameters. Note that the scales of the deflection and the velocity potential directly proportional to the intensity of the dipole I .

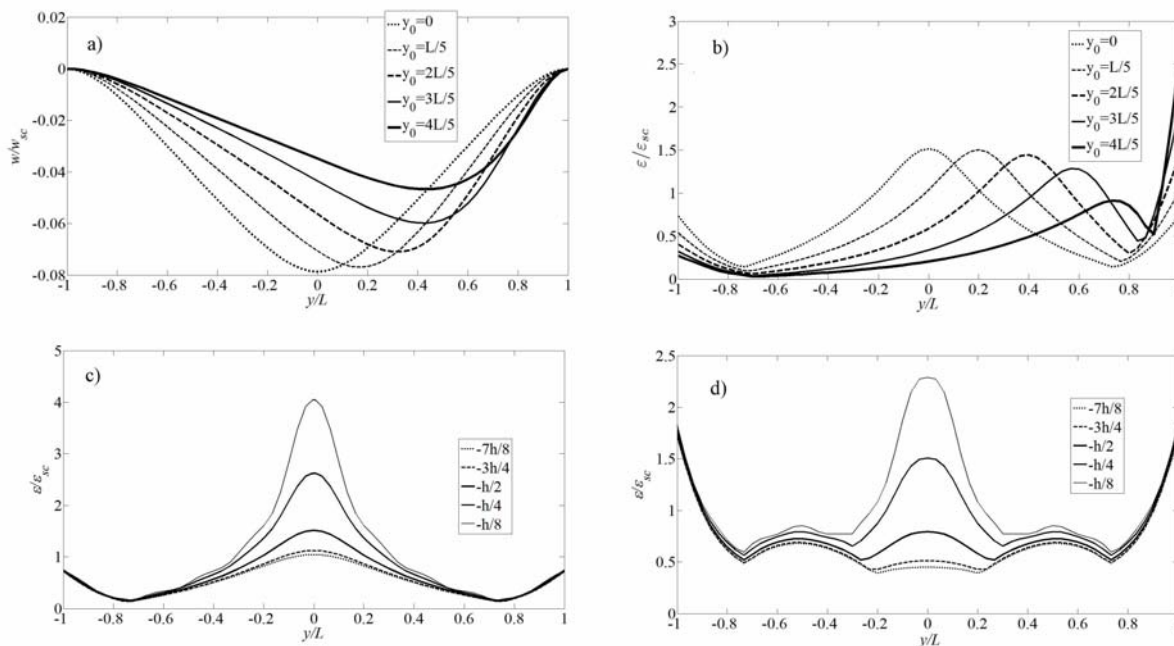


Figure 1. The nondimensional deflection *a)* and scaled strains *b)* in the ice cover across the channel at $x = 0$ for $z_0 = -h/2$ and different horizontal coordinate y_0 of the dipole, the speed of the dipole is $U = 3\text{m/s}$. Scaled strains in the ice cover across the channel at $x = 0$ for $y_0 = 0$ and different vertical positions of the dipole, the speed of the dipole is $U = 3\text{m/s}$ *c)* and $U = 6\text{m/s}$ *d)*.

3. Numerical results

The hydroelastic behaviour of the ice cover in a channel, which is caused by a dipole moving under the ice, is investigated numerically for a freshwater ice with density $\rho_i = 917\text{ kg/m}^3$, Young's modulus $E = 4.2 \cdot 10^9\text{ N/m}^2$, Poisson's ratio $\nu = 0.3$ and the retardation time $\tau = 0.1\text{ s}$. The thickness of the ice plate is 10 cm, the half width of the channel is 10 m and the water depth is 2 m. The coordinates of the dipole are (Ut, y_0, z_0) . The speed U and the position of the dipole with respect to the walls of the channel vary in the calculations. In contrast to the problem of a dipole moving under ice sheets of infinite extent, there are infinitely many critical speeds of hydroelastic waves in a frozen channel. Correspondingly, there are many values of the speeds of a moving dipole at which the stresses in the ice cover are amplified. The ranges of the speeds U and the widths L are chosen according to the critical speeds of hydroelastic waves propagating in the frozen channel, which were computed in [5], and the characteristic length $(D/\rho_l g)^{1/4}$ of the ice cover, which is equal to 2.48 m in the case under consideration. The lowest critical speed is 5.38 m/s for $\tau = 0\text{ s}$ and is equal approximately to 5.5 m/s for $\tau = 0.1\text{ s}$ for the channel considered in this paper and in [3]. We shall investigate elastic behavior of the ice cover for speeds of the dipole smaller or greater than the first critical speed.

The nondimensional deflection and strains scaled with $\epsilon_{sc} = (h_i UI)/(2gL^5)$ across the channel at $x = 0$ for the speed of the dipole $U = 3\text{ m/s}$ are shown in Figures 1*a* and 1*b*. The dipole is located at $y_0 = 0, L/5, 2L/5, 3L/5, 4L/5, z_0 = -h/2$ in these figures. For the subcritical speed, $U = 3\text{ m/s}$, the ice response is localised near the dipole, because the hydroelastic waves propagating with this speed along a channel do not exist [5]. When the dipole approaches the wall, the maximum deflection decreases, see Figure 1*a*, the strains decrease near the dipole and at the farther wall, and increase at the nearest wall, see Figure 1*b*, for this

subcritical speed. Scaled strains across the channel for different vertical coordinate of the dipole, $z_0 = -7h/8, -3h/4, -h/2, -h/4, -h/8$, and $y_0 = 0$ are shown in Figures 1c and 1d. The speed of the dipole is 3 m/s in Figure 1c and is 6 m/s in Figure 1d. This speed is greater than the first critical speed and is smaller than the second critical speed for the channel. Note that the strains are always maximum near the dipole for subcritical speed of the dipole. In contrast, for supercritical speeds of the dipole, the maximum strain can be achieved either near the dipole or on the wall depending on the position of the dipole in the channel.

4. Conclusion

The mathematical model of steady hydroelastic waves in the ice cover of a channel caused by a dipole moving in the liquid has been considered. The formulation of the problem was presented and discussed. The hydrodynamic pressure on the ice-liquid interface caused by the dipole moving in a rectangular channel with rigid walls has been derived. The ice deflection was neglected at leading order. This hydrodynamic pressure was treated as an external pressure on the upper surface of the ice cover. The problem of a moving underwater dipole was reduced to the problem of an external load moving on ice along the channel. The later problem was solved by the method described in [3]. With the help of the Fourier transform along the channel the original problem was reduced to the problem of the wave profile across the channel, which was solved by the method of normal modes.

The problem was solved numerically. It has been shown that displacement of the dipole across the channel towards a wall increases the strain on the wall nearest to the dipole and decreases the strain on the other wall. For subcritical speeds of the dipole strains are maximum near the dipole. For supercritical speeds the strains near the dipole decrease and can be smaller than strains on the walls when the dipole moves deeper from the ice cover.

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