BOOK REVIEW

Youngsters solving mathematical problems with technology: the results and implications of the Problem@Web project, by Susana Carreira, Keith Jones, Nélia AQ1 Amado, Hélia Jacinto, Sandra Nobre; Springer; Cham, Heidelberg, New York, Dordrecht, London; 2016; pp. 276; Hardcover: £74.50, ISBN 978-3-319-24908-7; eBook: £58.99, ISBN 978-3-319-24910-0

Digital technology is now an important part of our life that is affecting the ways we communicate, access information and organise our social activities. Youngsters grow up with this technology and they are considered competent users, especially of mobile communication devices which are a fundamental component of their everyday life. Prensky (2010) characterises the youngest generation as digital natives, especially in relation to previous generations to which their parents or teachers belong, who might be considered as digital immigrants. However not all young people are proficient digital users (e.g. Akcayir, Dundar & Akcayir, 2016) and, even more, when they use digital technology for educational purposes they do not share amongst themselves the same characteristics of learning (Thompson, 2013). It seems that the connection between digital technology and systematic learning is still unclear. Artigue (2010) in her closing lecture on The Future of Teaching and Learning Mathematics with Digital Technology at the ICMI Study Conference in Hanoi mentioned that in the mid-1980s there was the impression that informatics was likely to have an important impact on mathematics education but mathematicians could not see its use in their research activities. Thirty years later we can see exactly the opposite: although the influence of technology on the professional practices of mathematicians is undeniable, its influence on learning mathematics is less clear or at least debatable. Regardless of this debate though, digital technologies are the tools of modern life and we cannot ignore their potential and affordances for learning purposes. The learning purposes for the book I discuss in this review concern the development of problem-solving skills, an area that has gained more and more importance for employers (see the list of twenty-first century skills in Economic Intelligent Unit, 2015) and has important position in the objectives of school curricula. For all these reasons, the book by Carreira, Jones, Amado, Jacinto and Nobre addresses a topical area in mathematics education.

As it is described in Chapter 1, the Problem@Web project (Mathematical problem solving: perspectives on an interactive web-based competition) is a three-year study in the context of two mathematical problem-solving competitions (SUB12 and SUB14) for students (SUB12 for 10–12 years of age and SUB14 for 12–14 years of age) covering the southern region of Portugal. These competitions have two phases: (1) the qualifying phase in which 10 moderate challenge mathematical problems are posted online each fortnight and students (individually or in groups) are asked to submit their solutions online or by email, receive feedback and improve their solutions towards their qualification for the final phase; and (2) the final phase where a half-day competition is held on campus at the

University of Algarve. During the qualifying period students engage with problem-solving by using any means available including digital tools of their choice and potentially seeking help from peers, parents and teachers. The project followed three rounds of both competitions over three years by: collecting students' solutions; interviewing students, their teachers and parents/relatives; and attending classes in which teachers and students work on similar problems. The study focuses mainly on the solutions that employed digital means, both those involving tools of general use such as word processors or spreadsheets and those involving specialised software for mathematics such as Digital Geometry Environments (DGE), and aims at better understanding of youngsters' problem-solving approaches with technology.

Chapter 2 provides findings on participating students' views and experiences of digital technology and problem-solving from their responses to an online survey, interviews with a sample of them, their submitted solution and email exchanges between students and the organisers during the competition. Students suggested a range of interesting solutions but their engagement with technology varied. Many of them were comfortable with hand writing their response and then submitting a digitalised version while others were familiar with the general use of digital environments (e.g. word processors or spreadsheets) but they were less at ease with specialised environments (e.g. DGE).

Chapter 3 discusses teachers' perspectives by drawing on a series of interviews with teachers who supported their students in both competitions. Teachers acknowledged the benefits on students' development of problem-solving and technological skills as well as the opportunities of working beyond the official curriculum regardless of students' level of attainment. They could see the competition as a motivating factor for students' participation in mathematical activities. Teachers also reported their own difficulties with handling technology, lack of resources and support in schools.

Chapter 4 draws on a range of theoretical perspectives related to mathematical cognition and communication in the context of problem-solving with and without technology. Although not all the theoretical constructs reviewed in this chapter are fully used in the study discussed in this book, there are some main ideas that are more relevant and recur in this and the following chapters. For example, according to the authors, problem-solving is seen as a ground to develop mathematical understanding and especially "as a synchronous process of mathematisation and of expressing mathematical thinking" (p. 84). Mathematisation is an action of organising information related to a problem situation from the mathematical perspective drawn on Realistic Mathematics Education (Doorman & Gravemeijer, 2009). Students engaging with problem-solving build conceptual models in order to solve a specific problem, which does not necessarily have the sophistication of a mathematical model. The communication of the solution of a problem is important and the solving stage and the reporting the solution stage are inseparable. When digital tools are used, individuals and digital means co-act in a symbiotic relation. Thus, we cannot separate people from technology in a sense that technology transforms people reasoning and people are continuously transforming technology (humans-with-media, Borba & Villarreal, 2005).

Chapters 5, 6 and 7 discuss the affordances of digital tools in relation to youngsters' problem-solving production in specific problem cases: (1) geometrical invariance; (2) quantity variance; and (3) co-variation.

Specifically, in Chapter 5 we find four solutions offered by different groups of students who all used DGE (Geogebra, https://www.geogebra.org) to tackle the same geometrical problem, i.e. the area of a family of triangles constructed in a certain way remains the same when the high and the base do not change. Different responses to the same geometrical problem using the same software originated different solutions. The authors claim that these solutions reflect different "conceptual models of geometrical invariance" and explain that "the main difference among the solutions lies in the relationship between the aptitude of the solver and their perception of the affordances [of the digital tool]" (p. 138).

Then, Chapter 6 discusses responses to two algebraic word problems in which an unknown value should be found under a given set of conditions. The analysis draws on solutions that use spreadsheets for functional reasoning and pattern-finding strategies and were either submitted for the competition or collected during a classroom activity in which students' engagement with the problem was recorded through a screen recorder (Camtasia, https://www.techsmith.com/camtasia.html). Findings suggest a range of "conceptual models" of algebraic problem solutions by proposing that "[t]he spreadsheet and the user are involved in a co-action process in constructing relations between the several variables involved and in expressing conditions and restrictions appropriately" (p. 170).

Chapter 7 deals with a one-word problem involving a journey of two friends departing from different points, walk in a straight path with different speeds and meet halfway. The modelling of motion includes space, time and speed. This part of the study is different to the other two (presented in Chapters 5 and 6) as all students' solutions were analysed – not only those that used digital means. Also, the method of analysis was different. Five experts, all university mathematics teachers, were invited to solve the same problem at first and then to suggest other alternative solutions. Sixteen solutions were collected and classified in "two major categories of conceptual models that underpin the process of solving the problem" (p. 183): (1) the completed journey where the whole journey is considered by looking at the past (e.g. the two friends met after t hours); and (2) the developing journey where the journey is considered step by step by looking at the future (e.g. in the first hour the two friends will cover s_1 and s_2 distance, respectively). Students' responses were then classified into these two categories. Also, responses were characterised according to the used representations: algebraic/ symbolic representation (4%); textual/descriptive representations (44%); tables/tabular representations (23%); diagrammatic/schematic representations (20%); and pictorial/figurative (9%). Findings indicate that the developing journey conceptual model was more popular (92%) amongst students' solution and that the textual/descriptive form had a clear dominance among the choices of representations.

Finally, Chapter 8 summarises the findings from the previous chapters by proposing an approach that sees the "solving and expressing with technology as a core concept" in relation to four components: “conceptual models”, “expressing thinking”, “digital expressivity” and “unity of solver and tool” (p. 223). Thus, “[m]athematical problem-solving with technology [can be seen] as an output of the joint development of technological skills and mathematical skills” (p. 233). I can see the articulation of youngsters’ problem-solving engagement, the affordances of digital environments and the mediation of these affordances to the communication of this problem-solving as an important contribution of the book.

Throughout the book I had the opportunity to see a variety of solutions suggested by young students to moderate challenge problems that demand mathematical content which they had not yet met at school. I was impressed by the sophistication of students’ solutions which, although not aligned with formal mathematical discourses, proposed alternative and coherent ways of communication with innovative representational modes. Different modes of communication were used in these solutions such as verbal or figural descriptions, colours, diagrams, numerical manipulations, dynamic images in DGE or tables constructed in word processors or spreadsheets. Apart from the observations in Chapter 6 in which we can follow students solving problems, we do not have direct access to students’ thinking development but we can see the final product of the solution. This solution might be the result of a collaborative work with peers, parents or teachers or even be influenced by the feedback of competition organisers. Students, for example, worked in groups, teachers organised relevant sessions in school and parents supported the students. As one student said in an interview: “My father eventually taught me how Excel works” (p. 35). Also, students might have ended up with a solution by using paper and pencil and, then, reproduced it in a digital environment for communication purposes. For the above reasons, I cannot see how we can attribute the suggested responses to specific individuals. I would say that, instead of discussing students’ “conceptual models”, “conceptual development” or “cognitive constructs”, I would consider each one of these solutions as the outcome of a collective development and I would like to focus even further on how these solutions have been communicated in relation to and in interaction with the affordances of digital environments.

As I mentioned earlier, some of the suggested solutions address the problems and conclude with a reasonable result without necessarily using the common school mathematical symbolisation. I draw on an example from a student response that introduced the “‘heart’ operation” (pp. 216–217) where “♥” indicates multiplication and 14 may mean either the number 14 or the outcome of the addition 1+4=5. As a result, the notation “14♥31=20” is used to describe “(1+4)×(3+1)=20”. It seems that a new set of symbols and a use of these symbols have been created in order to address the problem effectively, i.e. it suggests an acceptable solution in a clear, coherent and consistent way. The authors “envisage solving a mathematical problem as a synchronous process of mathematisation and expression of mathematical thinking in which digital tools play a key role” (p. 224). In this example, the use of symbols is controversial and one may challenge its “mathematisation”. However, regardless of this controversy, I can see how the structure of the solution has the potential to shift to a mathematical accurate solution that eventually will become a "mathematisation" of the problem. Can we attribute this "creativity" to the digital environment? The role of the digital environment, in some cases, is just supportive – e.g. the word processor becomes the writing environment that contributes with new symbols (e.g. "♥"). In other examples, the digital environment has a more operational role in the solution – e.g. when the spreadsheet supports calculations and tabularisation of information and facilitates trial and improvement approaches.

The authors suggest the idea of a "digital-mathematical discourse" (p. 85, p. 217 and else- where in the book) focusing especially on how the digital affects the mathematical discourse and vice-versa. This reminded me a recent study by Ng (2016) on how touchscreen DGE environments, in comparison to textbook environments, changed students' communication about calculus. Students who used the DGE environment employed different modes of communication such as utterances, gestures and touchscreen dragging. In an earlier study, Sinclair and Yurita (2008) identified changes in the mathematics teacher's discourses when a DGE environment was employed in a geometry class. So, it is evident in the examples suggested in the book and in other studies that there is a discursive shift when digital environments and their affordances mediate the mathematical communication.

On the other hand, what are these affordances? Are they a set of specific characteristics associated with a certain technological tool by the designer of this tool? Can we see these affordances disconnected from the user? In the examples presented in the book (apart from some instances in Chapter 6) we do not have access to individuals while producing the mathematical solutions. We have access to their solutions, though, and we can see how the communication of a solution and the environment in which this is presented co-exist. In this sense, the digital environment shapes the solution (e.g. the images in DGE visualise examples of polygons) while the affordances of the environments go beyond the intentions of their designers (e.g. solutions employ figural representations by colouring cells in a spreadsheet). So, in my view, we cannot see the affordances of the digital environment disconnected from the suggested solutions.

From a different perspective, I would like to return to the range of solutions students offered to problems they had not met in school before. Although they were not aware of the corresponding mathematical methods (e.g. solving simultaneous equations) they created their own approaches to address the problems. As I mentioned earlier, although these approaches are very often far away from the mathematical symbols, words, terms and methods met in the school curriculum they indicate coherent problem-solving strategies and suggest correct solutions. This is indicative of young students' potential development and what they can do while engaging with mathematics both in and out of school and with the support of peers and knowledgeable others.

My final thought from the reading of this book regards competitions; the authors claim that these competitions are opportunities for many students and not for the small number of gifted ones. Because these events engage parents and relatives, they have the potential to raise the public appreciation of mathematics. However, I am wondering how a competition for students can be inclusive – at the end of the process a small number of students will get the trophy. Furthermore, how can be avoided that a competition of this type leads to a competition between parents and teachers with high aspirations for youngsters having potential effect on students' self-confidence and self-esteem? The book is very supportive of the positive influence of these competitions and does not address sensitive social factors that might be involved in this experience. This is a missing opportunity that I hope will be addressed in future studies.

In conclusion, I enjoyed reading the book and I enjoyed my time solving the problems also by putting myself in young students' shoes. I can see the benefit of using these problems as well as the suggested solutions in secondary mathematics problem-solving activities. I am planning to use some of these problems (and suggested solutions) with undergraduate, graduate and teacher students as a trigger for a discussion on problem-solving also with digital environments.

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