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Christian Kellner, Mark T. Le Quement

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Endogenous Ambiguity in Cheap Talk[☆]

Christian Kellner[*]

Departement of Economics, University of Southampton, UK

Mark T. Le Quement

School of Economics, University of East Anglia, UK

Abstract

This paper proposes a model of ambiguous language. We consider a simple cheap talk game in which a sender who faces an ambiguity averse receiver is able to perform ambiguous randomization, i.e. to randomize according to unknown probabilities. We show that for any standard influential communication equilibrium there exists an equilibrium featuring an ambiguous communication strategy which Pareto-dominates it in terms of consistent planning ex ante utilities. Ambiguity, by triggering worst-case decision-making by the receiver, shifts the latter's response to information towards the sender's ideal action, thus encouraging more information transmission.

Keywords: Keywords: cheap talk, ambiguity JEL classification: D81, D83.

1. Introduction

Ambiguous language is a recurrent feature of economic and political communication. The term *Fedspeak* for example refers to the cryptic language used by chairmen of the Federal Reserve Board. On the face of it, the phenomenon of ambiguous language is puzzling because it appears to gratuitously decrease the precision of transmitted information. Within the standard cheap talk game à la Crawford and Sobel (1982) (CS in what follows), we find that ambiguous language on the contrary increases the payoffs achievable by both parties.

An informed sender (S) faces an uninformed receiver (R) and S is known to favour a higher action than R for any realization of the state. R is ambiguity averse and applies

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[*]Corresponding author

Email addresses: `c.kellner@soton.ac.uk` (Christian Kellner), `m.le-quement@uea.ac.uk` (Mark T. Le Quement)

Max-Min expected utility in the presence of ambiguity. For the canonical Uniform-Quadratic specification of the CS model, we find that S and R can both benefit from the use of an ambiguous communication strategy according to which S conditions her messages on a private draw from an Ellsberg urn. For any standard influential equilibrium, there exists an ambiguous communication equilibrium which strictly Pareto-dominates it. Ambiguity mitigates conflict by shifting upwards R's response to information, which encourages greater information transmission. S gains as she effectively faces a less misaligned receiver. R also benefits because her suboptimal response to information is more than compensated by more information transmission.

In CS, preference misalignment (i.e. bias) causes imprecise communication. Any equilibrium outcome can be implemented via a so-called partitional equilibrium. The state space is divided into adjacent intervals $1, ..., N$ and S reveals the interval in which the state is located by sending m_i when the state is in interval i. Reducing bias causes the largest equilibrium partition to become more informative (i.e. to have more and/or better distributed intervals), yielding a higher expected payoff for both parties.

We propose a new communication strategy which exploits the dynamic inconsistency of R's behavior in the presence of ambiguity. By generating local ambiguity, communication leads R to act as if her preferences were less misaligned than they are. Given a set of standard intervals $1, ..., N$, S subdivides every standard interval i into two adjacent subintervals i_- and i_+. If S draws a red ball from the Ellsberg urn, she sends m_i^A if $\omega \in i_-$ and m_i^B if $\omega \in i_+$. If instead S draws a blue ball, she uses the reciprocal rule, i.e. she sends m_i^B if $\omega \in i_-$ and m_i^A if $\omega \in i_+$. Upon observing m_i^A and m_i^B, R is now Knighteanly uncertain as to whether the state is situated in i_- or i_+. We model ambiguity aversion by assuming Max-Min preferences. This involves evaluating every action according to its lowest expected utility under all possible priors (i.e. all possible compositions of the urn) and picking the action that maximizes the thus-constructed objective function. The key mechanism is that if the left subinterval i_- is significantly larger than the right subinterval i_+, so that the state is ex ante much more likely to be situated in i_- than in i_+, R (driven by worst-case thinking) evaluates all low and middle actions as if certain that the state is in i_+, no matter how unlikely this event. R thus acts as if subjectively overweighting the event that the state is in i_+. As a result, she takes a higher action than the expected utility maximizing action conditional on the event that the state is located in the standard interval i.

Our main contribution is to study Ellsbergian strategies within the classical Crawford and Sobel (1982) cheap talk game. In so doing, we build on Bade (2010), Riedel and Sass (2011), Azrieli and Teper (2011) and Riedel (2017), who introduce ambiguous strategies and equilibrium under such strategies.[1] The ambiguous communication strategy that we introduce builds on Bose and Renou (2014). The latter state a revelation principle for setups where the principal can use ambiguous communication to expand each agent's prior about the opponent's type into a set of priors, thereby generating ambiguity and advantageously affecting the behavior of the (ambiguity averse) agents. Our paper similarly uses ambiguous communication to strategically generate ambiguity, but we examine communication without

[1]See also earlier work by Lo (1996) and Klibanoff (1996) on equilibrium in ambiguous beliefs.

commitment within the Crawford and Sobel (1982) game with a focus on welfare. Our results also relate closely to Chen and Gordon (2015), who show that any method that shifts upwards the best response of R to any given interval (a property called "nestedness") will improve communication. In this light, a contribution of our paper is to add a new method (ambiguous communication strategies) for producing nestedness.

Our paper also contributes to a literature studying how vagueness alters the set of outcomes in sender-receiver games. Blume, Board and Kawamura (2007), Blume and Board (2014) as well as Gordon and Nöldeke (2015) consider cheap talk games with a noisy communication channel that generates some randomness in the message received by R. They find that channel noise helps foster communication by forcing pooling of S-types, which helps R react less adversarially to messages, in turn encouraging S to reveal more. In the partitional equilibria of Blume, Board and Kawamura (2007), R's expectation given a message is a weighted average of the conditional expectation without transmission error and the ex ante mean. R's expectation given a low message is distorted upwards, implying a reduction in de facto preference misalignment. Blume and Board (2014) features a $\{0, 1\}$ state space and a continuum message space $[0, 1]$. While maximal informativeness entails sending extreme messages (0 or 1), in equilibrium S uses an interior message in state 0, thereby adding intentional vagueness to the exogenous vagueness (channel noise). Gordon and Nöldeke (2015) find that S uses truth-distorting figures of speech (exaggeration, understatement). Our paper identifies benefits from noise as the above papers, differences being that noise is exclusively voluntarily added by S, comes in a different form (ambiguity) and affects R's actions via a different motive (hedging). Finally, Lipman (2009) examines cheap talk with aligned preferences and concludes that vagueness can be efficient only if the informed party has "vague views of the world". In this spirit, Kellner and Le Quement (2017b) examines cheap talk with an ambiguous prior about the state. The communication strategy featured in S-optimal equilibria is not a partition but a randomization over partitions, as in the present paper. Randomization obeys a known distribution in Kellner and Le Quement (2017b) and instead an ambiguous one in this paper. It hedges S against exogenous ambiguity in the former paper, while its function here is to induce hedging by R against the endogenously generated ambiguity.

The paper proceeds as follows. Section 2 introduces the model. Section 3 contains our main results. Section 4 discusses generalizations.

2. Model

There are two players, a sender S and a receiver R and only S has private information. S privately observes the value of two random variables ω and θ. The random variable ω, which corresponds to the payoff-relevant state of the world, is drawn from the uniform distribution on $[0, 1]$. The variable θ is the payoff-irrelevant color of a ball drawn from an Ellsberg urn containing balls of colors 1 and 2, where $\theta = \theta_i$ if the drawn ball has color i. The proportion ρ of balls of color 1 is Knighteanly unknown to S and R and the urn is maximally ambiguous, any ρ in $[0, 1]$ being considered possible. The timing of the game is as follows. S observes ω and θ. She then picks a message $m \in M$, where M is a rich message space of cardinality $|M|$. After observing m, R picks an action $a \in \mathbb{R}$. S has a utility function denoted by $U^S(a, \omega, b) = -(\omega + b - a)^2$, where $b > 0$. R's utility function is given by $U^R(a, \omega) = -(\omega - a)^2$. R is ambiguity averse and applies the Max-Min decision rule (Gilboa (1987), Gilboa and Schmeidler (1989)). In contrast, the ambiguity attitude of S can be arbitrary. A standard communication strategy is given by a family $q(.\,|\omega)$, $\omega \in [0, 1]$, of distributions. Such a family defines a distribution over M for each value of ω and is thus a mapping $[0, 1] \mapsto \Delta^{|M|}$, where $\Delta^{|M|}$ is the set of distributions over M. An Ellsbergian communication strategy is given by a pair of standard communication strategies denoted $(q_1(.\,|\omega), q_2(.\,|\omega))$. S plays such a strategy by conditioning her choice of q_i on the value of θ, more precisely by using q_i if $\theta = \theta_i$.[2] A (mixed) strategy of R specifies a distribution $\delta(.\,|m)$ over pure actions for any $m \in M$. Letting $\Delta^{\mathbb{R}}$ denote the set of distributions over $\mathbb{R}$, a strategy of R is a mapping $M \mapsto \Delta^{\mathbb{R}}$.

Our equilibrium concept takes into account the possibility that S uses an Ellsbergian strategy. We require consistent beliefs in the sense that for messages used on the equilibrium path, R performs prior by prior Bayesian updating conditional on S's equilibrium strategy.[3] We furthermore require that at each information set at which a player is called upon to act, the action chosen is optimal given the player's beliefs and the other player's equilibrium strategy. S never faces ambiguity when called upon to act as she observes θ before choosing a message and she hence simply maximizes expected utility. As to R, who is assumed ambiguity averse, our optimality requirement is that she chooses a Max-Min action conditional on her beliefs given the message received. Note that in the absence of ambiguity, Max-Min decision making reduces to expected utility maximization.

Formally, a strategy profile $(q_1^*(m\,|\omega), q_2^*(m\,|\omega))$, $\delta^*(a\,|m)$ and a belief system constitute an equilibrium if the following conditions hold. First, $\forall\,(\omega, i) \in [0, 1] \times \{1, 2\}$, any m^* in the support of $q_i^*(m\,|\omega)$ solves

$$\max_{m \in M} \int_{a \in \mathbb{R}} U^S(a, \omega, b)\delta^*(a\,|m)da. \tag{1}$$

[2]While we refer to ω as the state and θ as the draw from an urn, an equivalent approach (see Hanany, Klibanoff and Mukerji (2016)) would be to define the state as (ω, θ), in which case the strategy of S would be given by a distribution over M for each state $(\omega, \theta) \in \{\theta_1, \theta_2\} \times [0, 1]$.

[3]Note that prior by prior updating may entail dynamically inconsistent behavior. The latter refers to behavior featuring ex post decision-making conditional on given information that is suboptimal from an ex ante point of view. Hanany and Klibanoff (2007,2011) propose an alternative updating rule which satisfies dynamic consistency. This rule however violates consequentialism, which requires that updated preferences conditional on a given event only depend on the subevents that remain possible. This for example entails that past choices do not affect the way in which an agent updates his preferences given an event. Without restrictions on preferences, ambiguity aversion entails either violations of consequentialism or dynamic consistency (or both). We refer to Siniscalchi (2011) for further theoretical discussion of this issue. A dynamic Ellsberg experiment by Dominiak, Dürsch and Lefort (2012) finds that more subjects satisfy consequentialism than dynamic consistency.

Second, for each m, δ^* solves

$$\max_{\delta \in \Delta^{\mathbb{R}}} \min_{\rho \in [0,1]} \int_0^1 \left(\int_{a \in \mathbb{R}} U^R(a, \omega) \delta(a \,|m) da \right) p(\omega \,|m, \rho) d\omega, \tag{2}$$

where, if m is an equilibrium message, we have

$$p(\omega \,|m, \rho) = \frac{\sum\limits_{i=1,2} p(\theta_i \,|\rho) q_i^*(m \,|\omega) f(\omega)}{\int_0^1 \sum\limits_{i=1,2} p(\theta_i \,|\rho) q_i^*(m \,|t) f(t) dt}.$$

The expression $p(\omega \,|m, \rho)$ denotes R's posterior belief given message m and urn composition ρ, where the pdf of ω is denoted by f.

Regarding out of equilibirum beliefs, we simply assume that there is some equilibrium message $\widetilde{m}$ s.t. any out of equilibrium message gives rise to the same beliefs as $\widetilde{m}$. Note that by a standard argument, for any equilibrium featuring out of equilibrium messages, one can trivially construct an outcome equivalent equilibrium featuring no out of equilibrium messages.

3. Analysis

We know from CS that absent Ellsbergian strategies, any equilibrium is equivalent to one featuring a partitional communication strategy. Given messages $\{m_i\}_{i=0}^{N-1}$, such a strategy is described by $t_0 = 0 < t_1 < ... < t_N = 1$ s.t. S sends m_i if $\omega \in (t_i, t_{i+1}]$, $\forall i \in \{0, ..., N-1\}$ (and m_0 if $\omega = 0$). Denote R's optimal action given $\omega \in (t_{i-1}, t_i]$ by $a_{ne}^*(t_{i-1}, t_i)$. The profile $\{t_i\}_{i=1}^{N-1}$ constitutes a standard partitional equilibrium (of size N) iff

$$U^S(a_{ne}^*(t_{i-1}, t_i), t_i, b) = U^S(a_{ne}^*(t_i, t_{i+1}), t_i, b), \ i = 1, ..., N-1. \tag{3}$$

If $b \geq \frac{1}{4}$, no informative communication is feasible whereas if $b < \frac{1}{4}$, there is a finite $\overline{N}(b) \geq 2$ s.t. the maximal equilibrium size is $\overline{N}(b)$. For each $N = 2, ..., \overline{N}(b)$, there exists a unique standard equilibrium of size N, denoted $E(b, N)$ and featuring threshold profile $\{t_i(b, N)\}_{i=1}^{N-1}$. The maximal size $\overline{N}(b)$ is weakly decreasing in b. Holding constant the equilibrium size N, S and R's expected payoff decreases in b. Holding constant the bias b, S and R favor equilibria of greater size. Given that S's bias is b, we denote S and R's expected payoff given $E(b, N)$ by respectively $\pi^S(E(b, N))$ and $\pi^R(E(b, N))$.

We now introduce the notion of Ellsbergian partitional communication strategies.

Definition 1. *An Ellsbergian partitional communication strategy is defined as follows. Let there be two profiles of thresholds $\{t_i\}_{i=0}^{N}$ and $\{c_i\}_{i=0}^{N-1}$ s.t. $t_0 = 0 < t_1 < ... < t_{N-1} < t_N = 1$ and $c_i \in (t_i, t_{i+1}]$, $i = 0, ..., N-1$. Given $\theta = \theta_1$, S sends m_i^A with probability one if $\omega \in (t_i, c_i)$ and instead m_i^B with probability one if $\omega \in [c_i, t_{i+1}]$, $i = 0, ..., N-1$. Given $\theta = \theta_2$, S sends m_i^B with probability one if $\omega \in (t_i, c_i)$ and instead m_i^A with probability one if $\omega \in [c_i, t_{i+1}]$, $i = 0, ..., N-1$.*

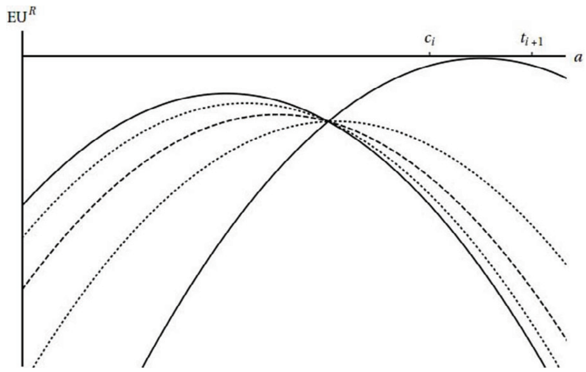

Figure 1: Expected utility over actions given various urn compositions.

An Ellsbergian communication strategy summarized by $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ adds Ellsbergian randomization to a standard partitional strategy and involves two steps. First, determine the interval i to which ω belongs. Second, randomize over two "local" and reciprocal partitional strategies defined on $(t_i, t_{i+1}]$ by conditioning on a draw from an Ellsberg urn. One local partitional strategy sends m_i^A if $\omega \in (t_i, c_i)$ and m_i^B if $\omega \in [c_i, t_{i+1}]$. The other local partitional strategy does the opposite. We refer to N as the size of a given Ellsbergian partitional strategy featuring $\{t_i\}_{i=1}^{N-1}$. We now characterize R's best response to messages given an Ellsbergian partitional strategy.

Lemma 1. *Given the Ellsbergian partitional communication strategy* $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$, *$R$'s best response to m_i^A and m_i^B is identical. Denote it by $a_e^*(t_i, t_{i+1}, c_i)$.*
a) a_e^* satisfies $E\left[U^R(a_e^*, \omega)\,|\omega \in (t_i, c_i)\right] = E\left[U^R(a_e^*, \omega)\,|\omega \in [c_i, t_{i+1}]\right]$,
b) $a_e^*(t_i, t_{i+1}, c_i) = \frac{t_i + t_{i+1} + c_i}{3}$.

Proof: see Appendix.

While the formal proof of the above result is in the Appendix, we now provide an intuition as to how the Max-Min action is constructed and how it can be higher than the standard best response. Given the concavity of U^R, it follows from Jensen's inequality that we can restrict ourselves w.l.o.g. to pure actions by R (see step 1 in the proof of Lemma 1). Figure 1 below shows the expected utility of (pure) actions given various urn compositions (ρs). We set $t_i = 0$, $t_{i+1} = .75$ and $c_i = .6$. Continuous curves correspond to $E\left[U^R(a, \omega)\,\middle|m_i^A, \rho = 1\right]$ and $E\left[U^R(a, \omega)\,\middle|m_i^A, \rho = 0\right]$ while dashed curves correspond to interior values of ρ $\left(\frac{1}{4}, \frac{1}{2} \text{ and } \frac{3}{4}\right)$. The Max-Min best response to m_i^A corresponds to the intersection of the two continuous curves, as explained below.

Upon observing (say) m_i^A, R performs prior by prior updating. R now knows that ω belongs to the interval $(t_i, t_{i+1}]$ and ρ affects the probability attributed to the subintervals (t_i, c_i) and $[c_i, t_{i+1}]$. If $\rho = 1$ (0), m_i^A implies that $\omega \in (t_i, c_i)$ $([c_i, t_{i+1}])$. If instead $\rho \in (0, 1)$, both subintervals are assigned positive probability. R's Max-Min action after m_i^A is obtained by maximizing a lower envelope. Consider the set of expected utility curves implied by different ρs given m_i^A, each of which is a concave and single-peaked function of a. For any a, the highest and lowest expected utility across ρs corresponds to either $\rho = 0$ or $\rho = 1$. The Max-Min action is at the unique intersection of these two latter curves and thus fully hedges R against ambiguity, i.e. yields the same expected utility for any ρ. Given m_i^B, the $\rho = 0$ and the $\rho = 1$ curves are interchanged w.r.t. the case of m_i^A. The Max-Min best response given m_i^B is thus the same as given m_i^A and also fully hedges R.

To see that one can have $a_e^*(t_i, t_{i+1}, c_i) > a_{ne}^*(t_i, t_{i+1})$, consider c_i very close to t_{i+1}. While $a_{ne}^*(t_i, t_{i+1})$ maximizes $E\left[U^R(a,\omega)\,|\omega \in (t_i, t_{i+1}]\right]$, $a_e^*(t_i, t_{i+1}, c_i)$ instead roughly corresponds to the action at which $E\left[U^R(a,\omega)\,|\omega \in (t_i, t_{i+1})\right]$ and $E\left[U^R(a,\omega)\,|\omega = t_{i+1}\right]$ intersect. If (t_i, c_i) is significantly larger than $[c_i, t_{i+1}]$, so that ω is much more likely ex ante to belong to the former than to the latter, R acts as if subjectively overweighting the event $\omega \in [c_i, t_{i+1}]$. Worst-case thinking leads her to evaluate all low and middle actions as if certain that $\omega \in [c_i, t_{i+1}]$.

Summarizing formally, for any $i \in \{0, ..., N-1\}$ it holds true that:

$$E\left[U^R(a_e^*(t_i, t_{i+1}, c_i), \omega)\,\middle|m_i^A, \rho\right] = E\left[U^R(a_e^*(t_i, t_{i+1}, c_i), \omega)\,\middle|m_i^A, \rho'\right],\ \forall \rho, \rho' \in [0,1]\,, \tag{4}$$

and

$$E\left[U^R(a_e^*(t_i, t_{i+1}, c_i), \omega)\,\middle|m_i^A, \rho\right] = E\left[U^R(a_e^*(t_i, t_{i+1}, c_i), \omega)\,\middle|m_i^B, \rho\right],\ \forall \rho \in [0,1]\,. \tag{5}$$

Note that the set of priors in (4) contains $\frac{1}{2}$. Setting $\rho = \frac{1}{2}$, m_i^A implies a uniform conditional distribution of ω on $(t_i, t_{i+1}]$ so that the LHS in (4) reduces to

$$-\frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} (\omega - a_e^*(t_i, t_{i+1}, c_i))^2\, d\omega.$$

We speak of the above expression as R's interim payoff given m_i^A. We now characterize incentive conditions for S. The key aspect here is that randomization by S involves messages which trigger identical actions by R.

Lemma 2. *An equilibrium featuring the Ellsbergian partitional strategy* $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ *exists if and only if*

$$U^S(a_e^*(t_{i-1}, t_i, c_{i-1}), t_i, b) = U^S(a_e^*(t_i, t_{i+1}, c_i), t_i, b),\ \ i = 1, ..., N-1. \tag{6}$$

Proof: For all $i \in \{0, ..., N-1\}$, m_i^A and m_i^B trigger an identical best response, so S is indifferent between m_i^A and m_i^B for any $\omega \in (t_i, t_{i+1}]$. We thus only need to consider deviations across messages carrying different subscripts. Condition (6) ensures that $\forall i \in \{1, ..., N-1\}$, S is indifferent between messages m_{i-1}^A and m_i^A given $\omega = t_i$. The condition also ensures that S prefers m_i^A to m_{i-1}^A if $\omega \in (t_i, t_{i+1}]$ and prefers m_{i-1}^A to m_i^A if $\omega \in (t_{i-1}, t_i]$. ■

Note that (6) is identical to the standard equilibrium condition in CS (see (3)), except that R's best response is now $a_e^*(t_i, t_{i+1}, c_i)$ instead of $a_{ne}^*(t_i, t_{i+1})$.

The examined Ellsbergian equilibria feature no ex ante ambiguity: Conditional on $\omega \in (t_i, t_{i+1}]$, for any θ the action $a_e^*(t_i, t_{i+1}, c_i)$ is chosen with probability one. In an Ellsbergian equilibrium featuring $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$, our measure of S's welfare is simply the following expected payoff function:

$$\begin{aligned}\sum_{i=0}^{N-1} P(\omega &\in (t_i, t_{i+1}])E\left[U^S(a_e^*(t_i, t_{i+1}, c_i), \omega, b)\,|\omega \in (t_i, t_{i+1})\right] \\ &= -\sum_{i=0}^{N-1}\int_{t_i}^{t_{i+1}} ((\omega + b) - a_e^*(t_i, t_{i+1}, c_i))^2\, d\omega.\end{aligned} \tag{7}$$

Our measure of R's welfare is her consistent planning ex ante utility, i.e. her ex ante utility anticipating that she responds to m_i^j, $j \in \{A, B\}$, with action $a_e^*(t_i, t_{i+1}, c_i)$. It is given as follows

$$\begin{aligned}\sum_{i=0}^{N-1} P(\omega &\in (t_i, t_{i+1}])E\left[U^R(a_e^*(t_i, t_{i+1}, c_i), \omega)\,|\omega \in (t_i, t_{i+1})\right] \\ &= -\sum_{i=0}^{N-1}\int_{t_i}^{t_{i+1}} (\omega - a_e^*(t_i, t_{i+1}, c_i))^2\, d\omega.\end{aligned} \tag{8}$$

Note that R's equilibrium action profile does not maximize her ex ante utility given S's equilibrium communication strategy, as choosing $a_{ne}^*(t_i, t_{i+1})$ for every i would yield a strictly higher ex ante utility. Hence, Ellsbergian communication generates dynamically inconsistent behavior by R. Expression (8) is also R's expected payoff if S uses a standard partitional strategy $\{t_i\}_{i=1}^{N-1}$ and R can commit to respond to m_i with $a_e^*(t_i, t_{i+1}, c_i)$, $i = 0, ..., N-1$. Finally, note that (8) also corresponds to the expected interim payoff of R, given the arguments appearing above Lemma 2. We simply speak of R's expected payoff in what follows. We now state our main result.

Proposition 1. *Given b and $N \geq 2$, if the standard equilibrium $E(b, N)$ exists, there is an $\overline{\varepsilon} > 0$ s.t. $\forall \varepsilon \leq \overline{\varepsilon}$, there exists an Ellsbergian equilibrium $\widetilde{E}$ summarized by $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ which satisfies:*

a) $t_i = t_i(b - \varepsilon, N)$, $i = 1, ..., N-1$.

b) $a_e^*(t_i, t_{i+1}, c_i) = a_{ne}^*(t_i, t_{i+1}) + \varepsilon$, $i = 0, ..., N-1$.

c) S's expected payoff in $\widetilde{E}$ equals $\pi^S(E(b-\varepsilon, N))$, i.e. is the same as the (strictly larger) expected payoff obtained in $E(b-\varepsilon, N)$ by a sender with bias $b-\varepsilon$.

d) R's expected payoff in $\widetilde{E}$ is strictly larger than her expected payoff in $E(b, N)$.

Proof: See Appendix.

The result states that if there exists an informative standard equilibrium, then there exists an Ellsbergian equilibrium that Pareto-dominates it. Ellsbergian randomization has two effects. The first (and direct) effect is to distort upwards R's response to given intervals, making her behave as if her preferences were closer to S's than they actually are. The second (and indirect) effect is to generate new equilibria in which more information is transmitted. S benefits from both of these effects, this being an instance of Theorem 2 (part ii)) in Chen and Gordon (2015) which states that S prefers to face a receiver with more aligned interests. The direct effect hurts R as her response to any given standard interval becomes ex ante suboptimal. The indirect effect is beneficial to R and echoes Theorem 3 (part ii)) in Chen and Gordon (2015), which states that R prefers to face the largest equilibrium partition corresponding to a different R with more aligned interests than herself. An Envelope Theorem argument ensures that the trade-off between the two effects is resolved positively for ε low enough (i.e. if ambiguity is sufficiently small), the direct negative effect being only second-order.

Conceptually, our result is reminiscent of insights on delegation appearing in Chen and Gordon (2015) and in Dessein (2002). In the former, R prefers to delegate decision making to another receiver who has the same preferences but a more optimistic belief than herself. The indirect informational gains outweigh the direct cost of delegating to an intermediary whose prior is different. In Dessein (2002), R may prefer to delegate the decision to a third party with preferences in between hers and S's. One way to interpret Proposition 1 (w.r.t. R's welfare) is that an ambiguity neutral R, on which the Ellsergian randomization introduced in this paper has no effect, prefers to delegate to an ambiguity-averse receiver.

We now add three more specific results concerning respectively 1) the existence of equilibria with equally-sized intervals, 2) the fact that Ellsbergian randomization can generate the possibility of informative communication and 3) the existence of so-called ambiguous babbling equilibria.

Consider the following Ellsbergian communication strategy. Set $\{t_i\}_{i=1}^{N-1}$ so as to create N equally sized standard intervals and set $c_i \in (t_i, t_{i+1}]$ such that $a_e^*(t_i, t_{i+1}, c_i) = \frac{t_i+t_{i+1}}{2} + b$ (assuming that this is feasible). For each $(t_i, t_{i+1}]$, R thus picks S's optimal action conditional on $\omega \in (t_i, t_{i+1}]$. If such a strategy profile constitutes an equilibrium, we call it an equal intervals equilibrium of size N. The appeal of such an equilibrium is that it implements S's optimal decision rule given a restriction to (at most) N different actions being taken with positive probability.

Following Sobel (2013), we say that a given Ellsbergian equilibrium features informative communication if messages affect beliefs, i.e. if it is not true that for each $\rho \in [0, 1]$, $p(. |m, \rho)$ is constant across equilibrium messages. Denote $\langle x \rangle$ as the largest integer smaller than x.

Proposition 2. *1. For all* $b \leq \frac{1}{12}$ *the largest equal intervals equilibrium has size* $\widehat{N}(b) \equiv \left\langle \frac{1}{6b} \right\rangle$ *and there exists an equal intervals equilibrium of every size* $N \in \left\{2, ..., \widehat{N}(b)\right\}$. *If* $b > \frac{1}{12}$, *there exists no informative equal intervals equilibrium. 2. For all* $b \leq \frac{1}{18}$, *larger equal intervals equilibria Pareto-dominate smaller ones. 3. For all* $b \leq \frac{1}{18}$, *the largest equal intervals equilibrium Pareto-dominates the largest standard equilibrium.*

Proof: See Appendix.

Unlike in Proposition 1, the positive welfare result for R stated in Point 2 does not rely on an Envelope Theorem argument. In the considered equilibrium construction, the shift in

R's response to information away from the ex ante optimal response is potentially large, so that the direct negative effect of biasing R's actions is not a priori guaranteed to be second order as compared to the indirect and positive (informational) effect achieved. The result shows that for this class of equilibria, as long as $b \leq \frac{1}{18}$, this latter effect dominates.

Next, we show that Ellsbergian equilibria can help overcome babbling in a way that is beneficial to one or both parties. Recall that for $b > \frac{1}{4}$, no informative standard equilibrium exists.

Remark 1. *1. For all $b \in \left(\frac{1}{4}, \frac{1}{3}\right)$, there exists an informative Ellsbergian partitional equilibrium in which S's expected payoff is strictly larger than in the standard babbling equilibrium.*
2. For all $b < \frac{(1+\sqrt{6})}{12} \simeq 0.28$, there exists an informative Ellsbergian partitional equilibrium in which both S and R's expected payoff is strictly larger than in the standard babbling equilibrium.

Proof: See Appendix.

We conclude our main analysis with a remark on the role of communication in our setup. Following Sobel (2013), we define the notions of influential and payoff-relevant communication. Communication is influential if it affects actions, i.e. if $\delta(.\,|m)$ is not constant across equilibrium messages. Communication is payoff-relevant if at least one agent's expected payoff differs from that implied by R's ex ante payoff maximizing action. Sobel (2013) writes for the standard case: "*In order for communication to be payoff-relevant for R it must be both informative and influential.*" and "*Relative to babbling, payoff-relevant communication must increase R's expected utility but may make S worse off.*". We now show that both of these properties break down once allowing for Ellsbergian strategies. Equilibrium communication can be payoff-relevant without being either informative or influential and it can be payoff-relevant while making S better-off and R worse-off.

Remark 2. *There exists an equilibrium featuring non-informative, non-influential and payoff-relevant communication, in which S (R) obtains an expected payoff that is strictly larger (smaller) than in the standard babbling equilibrium.*

Proof: By Lemma 1, there exists $\overline{\varepsilon} > 0$ s.t. for any $\varepsilon \leq \overline{\varepsilon}$, one can find a $c \in [0,1]$ yielding $a_e^*(0,1,c) = a_{ne}^*(0,1) + \varepsilon$. Also, given $b > 0$, for any $\varepsilon \leq b$ it holds true that

$$\int_0^1 U^S\left(a_{ne}^*(0,1) + \varepsilon, \omega, b\right) d\omega > \int_0^1 U^S\left(a_{ne}^*(0,1), \omega, b\right) d\omega.$$

As to R, note that she necessarily loses in ex ante terms whenever $a_e^*(0,1,c)$ shifts away from $a_{ne}^*(0,1)$. ■

We call the constructed equilibrium an Ellsbergian babbling equilibrium. The different equilibrium messages do not generate different sets of posteriors. Communication is thus non-informative, implying that it is non-influential. It however generates a set of posteriors leading R to pick $a_e^*(0,1,c) \neq a_{ne}^*(0,1)$. R loses ex ante because $a_e^*(0,1,c) > a_{ne}^*(0,1)$. Conversely, S gains ex ante as long as $a_e^*(0,1,c)$ is not too high. The equilibrium can be refined away if we make listening optional for R, i.e. add a participation constraint requiring that listening increases R's ex ante payoff.[4] Our main Proposition concerns equilibria that satisfy such a constraint.

4. Generalizations and robustness

As shown in a working paper version of this paper (Kellner and Le Quement, 2017a), our results extend beyond the uniform-quadratic specification to environments satisfying Condition M (introduced in CS) and an added condition.

A key issue is to which extent Ellsbergian randomization can shift R's actions upwards if the urn available to S is not fully ambiguous but instead characterized by $\rho \in [\underline{\rho}, \overline{\rho}]$ with $0 < \underline{\rho} < \overline{\rho} < 1$. We provide a partial positive answer. Assuming that S uses the Ellsbergian strategy $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$, we identify sufficient conditions on $\underline{\rho}, \overline{\rho}$ such that for every $i \in \{0, ..., N-1\}$, R's best response to m_i^A and m_i^B is still given by $a_e^*(t_i, t_{i+1}, c_i) = \frac{t_i+t_{i+1}+c_i}{3}$. In what follows, let $a^*_{[\underline{\rho},\overline{\rho}]}(t_i, t_{i+1}, c_i, m_i^j)$ denote the Max-Min action of R conditional on m_i^j, $j \in \{A, B\}$, given an urn characterized by $\rho \in [\underline{\rho}, \overline{\rho}]$ and assuming that S uses the Ellsbergian strategy $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$.

Remark 3. *Let $\underline{\rho} < 1/2 < \overline{\rho}$ and let $\gamma\left(\underline{\rho}, \overline{\rho}\right) \equiv \min_{\rho \in \{\underline{\rho}, 1-\overline{\rho}\}} \frac{\sqrt{(1-3(1-\rho)\rho)}-1/2}{1-2\rho}$, so that $\gamma\left(\underline{\rho}, \overline{\rho}\right) \in \left(0, \frac{1}{2}\right)$. Assume that S uses the Ellsbergian strategy $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$. For any $i \in \{0, ..., N-1\}$, if*

$$(t_i + t_{i+1})/2 \leq c_i \leq (t_i + t_{i+1})/2 + \gamma\left(\underline{\rho}, \overline{\rho}\right)(t_{i+1} - t_i),$$

then it holds true that

$$a^*_{[\underline{\rho},\overline{\rho}]}(t_i, t_{i+1}, c_i, m_i^A) = a^*_{[\underline{\rho},\overline{\rho}]}(t_i, t_{i+1}, c_i, m_i^B) = \frac{t_i + t_{i+1} + c_i}{3}.$$

Proof: See Appendix.

Figure 1 illustrates the result. Suppose that $\underline{\rho} = \frac{1}{4}$ and $\overline{\rho} = \frac{3}{4}$ as in the dotted curves, meaning that the continuous curves corresponding to $\rho \in \{0, 1\}$ should be ignored. The Max-Min action after m_i^A (and m_i^B) is the same as in the case of $\underline{\rho} = 0$ and $\overline{\rho} = 1$. It is given by the intersection of the $\underline{\rho} = \frac{1}{4}$ and $\overline{\rho} = \frac{3}{4}$ curves. Note that if the set of probabilities shrinks further, R's response no longer fully hedges against ambiguity.

Finally, we would expect our main result to survive under alternative decision rules such as the α-Max-Min or the smooth ambiguity model. The key is that these would not entirely neutralize Ellsbergian randomization's ability to induce a shift in R's actions away from the standard best responses.[5]

[4] The argument relies on the fact that the decision maker is sophisticated in the sense that she anticipates her future preference reversal. See Siniscalchi (2011).

[5] Since the smooth ambiguity model is a second-order model of ambiguity (see Lang (2017)), R's best response would however not entirely hedge her against ambiguity. This would raise new issues w.r.t. the evaluation of R's equilibrium welfare.

5. Appendix

5.1. Proof of Lemma 1

Step 1 We show in this step that it is without loss of generality to assume that the Max-Min action of R is a pure action. Consider any (non-degenerate) mixed action $\widetilde{a}$ of R given by a distribution $\widetilde{g}$ over $\mathbb{R}$. Denote by $\overline{a}\left(\widetilde{a}\right)$ the pure action satisfying $\overline{a}\left(\widetilde{a}\right) = \int_{\mathbb{R}} a\widetilde{g}(a)da$. Recall that the payoff function U^R is concave. It follows by Jensen's inequality that the expected payoff of $\widetilde{a}$ is weakly smaller than that of $\overline{a}\left(\widetilde{a}\right)$ given any state ω, i.e.$\int_{\mathbb{R}} U^R(a,\omega)\widetilde{g}(a)da \leq U^R(\overline{a}\left(\widetilde{a}\right), \omega)$ $\forall\omega$. Hence, for any distribution of the state with pdf $\widehat{f}$, it holds true that

$$\int_0^1 \left(\int_{\mathbb{R}} U^R(a,\omega)\widetilde{g}(a)da\right)\widehat{f}(\omega)d\omega \leq \int_0^1 U^R(\overline{a}\left(\widetilde{a}\right), \omega)\widehat{f}(\omega)d\omega.$$

It follows that

$$\min_{\widehat{f}} \int_0^1 \left(\int_{\mathbb{R}} U^R(a,\omega)\widetilde{g}(a)da\right)\widehat{f}(\omega)d\omega \leq \min_{\widehat{f}} \int_0^1 U^R(\overline{a}\left(\widetilde{a}\right), \omega)\widehat{f}(\omega)d\omega.$$

We may conclude that for any mixed action (identified by $\widetilde{g}(a)$) there is a pure action that yields a weakly higher conditional Max-Min expected utility. Focusing on pure actions is thus without loss of generality.

Step 2 This proves Point a). Let $E\left[U^R(a,\omega)|m_i^A,\rho\right]$ denote the expected utility implied by action a conditional on message m_i^A and ρ, assuming that S uses $\left(\{t_r\}_{r=1}^{N-1}, \{c_r\}_{r=0}^{N-1}\right)$. Observe that

$$E\left[U^R(a,\omega)|m_i^A,\rho\right] = \begin{array}{c} \Pr(\theta_1|m_i^A,\rho)E\left[U^R(a,\omega)|\omega\in(t_i,c_i)\right] + \\ (1-\Pr(\theta_1|m_i^A,\rho))E\left[U^R(a,\omega)|\omega\in[c_i,t_{i+1}]\right] \end{array},$$

where $\Pr(\theta_1|m_i^A,\rho) = \frac{\rho(c_i-t_i)}{\rho(c_i-t_i)+(1-\rho)(t_{i+1}-c_i)}$. Hence $E\left[U^R(a,\omega)|m_i^A,\rho\right]$ is strictly increasing in ρ if $E\left[U^R(a,\omega)|\omega\in(t_i,c_i)\right] > E\left[U^R(a,\omega)|\omega\in[c_i,t_{i+1}]\right]$. One easily computes that the latter inequality holds true if $a < \frac{t_i+t_{i+1}+c_i}{3}$. Instead, $E\left[U^R(a,\omega)|m_i^A,\rho\right]$ is strictly decreasing in ρ if $a > \frac{t_i+t_{i+1}+c_i}{3}$. Finally, it is constant in ρ if $E\left[U^R(a,\omega)|\omega\in(t_i,c_i)\right] = E\left[U^R(a,\omega)|\omega\in[c_i,t_{i+1}]\right]$, i.e. if $a = \frac{t_i+t_{i+1}+c_i}{3}$. Thus if $a < \frac{t_i+t_{i+1}+c_i}{3}$, then

$$\min_{\rho} E\left[U^R(a,\omega)|m_i^A,\rho\right] = E\left[U^R(a,\omega)|m_i^A,0\right] = E\left[U^R(a,\omega)|\omega\in[c_i,t_{i+1}]\right],$$

while if $a > \frac{t_i+t_{i+1}+c_i}{3}$, then

$$\min_{\rho} E\left[U^R(a,\omega)|m_i^A,\rho\right] = E\left[U^R(a,\omega)|m_i^A,1\right] = E\left[U^R(a,\omega)|\omega\in(t_i,c_i)\right].$$

Note furthermore that $E\left[U^R(a,\omega)|\omega\in(t_i,c_i)\right]$ is convex with a maximum at $a = \frac{t_i+c_i}{2} < \frac{t_i+t_{i+1}+c_i}{3}$. Hence $E\left[U^R(a,\omega)|\omega\in(t_i,c_i)\right]$ is decreasing if $a > \frac{t_i+t_{i+1}+c_i}{3}$. By a similar argument, $E\left[U^R(a,\omega)|\omega\in[c_i,t_{i+1}]\right]$ is increasing if $a < \frac{t_i+t_{i+1}+c_i}{3}$. Thus, $\min_\rho E\left[U^R(a,\omega)|m_i^A,\rho\right]$

is maximized at $a^* = \frac{t_i+t_{i+1}+c_i}{3}$, so that it holds true that $E\left[U^R(a^*,\omega)|\omega \in (t_i,c_i)\right] = E\left[U^R(a^*,\omega)|\omega \in [c_i,t_{i+1}]\right]$.

Step 3 This proves that the best response to respectively m_i^A and m_i^B is identical. Note that $E\left[U^R(a,\omega)|m_i^A,\rho\right] = E[U^R(a,\omega)|m_i^B,1-\rho]$ and thus for all a, $\min_\rho E\left[U^R(a,\omega)|m_i^B,\rho\right] = \min_\rho E\left[U^R(a,\omega)|m_i^A,\rho\right]$. Hence, the objective function maximized by R, and thus the implied optimal action, is the same after m_i^A and m_i^B. ■

5.2. Proof of Proposition 1

Outline Step 1 defines $\overline{\varepsilon}$. Step 2 introduces an Ellsbergian communication strategy satisfying a) and ensuring that R's best response satisfies b). Step 3 shows that S's strategy is indeed an equilibrium strategy. Steps 4 and 5 respectively prove c) and d).

Step 1 Assume that $E(b,N)$ exists. As stated in CS, $t_i(b,N) = \frac{1}{N} - 2bi(N-i)$ and $t_i(b,N) - t_{i-1}(b,N) = \frac{1}{N} + 2b(2i-N-1)$. Hence, for any b,N, it is true that $\min_{i\in\{0,...,N-1\}}\{t_{i+1}(b,N) - t_i(b,N)\} = t_1(b,N) - t_0(b,N) = \frac{1}{N} - 2b(N-1)$ and thus $\min_{i\in\{0,...,N-1\}}\{t_{i+1}(b,N)-t_i(b,N)\}$ is strictly decreasing in b. Let $\overline{\varepsilon} = \min\left\{\frac{1}{6N} - \frac{1}{3}b(N-1), b\frac{N^2-1}{N^2+2}\right\}$, which implies $\overline{\varepsilon} < b$.

Step 2 Consider the Ellsbergian communication strategy $\left(\{t_i(b-\varepsilon,N)\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$, where $c_i = \frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2} + 3\varepsilon$ for all $i \in \{0,...,N-1\}$. Consider any $\varepsilon \leq \overline{\varepsilon}$. Clearly, $c_i > t_i(b-\varepsilon,N)$. Since $\varepsilon \leq \min_{i\in\{0,...,N-1\}}\{t_{i+1}(b,N) - t_i(b,N)\}/6 < \min_{i\in\{0,...,N-1\}}\{t_{i+1}(b-\varepsilon,N) - t_i(b-\varepsilon,N)\}/6$, it follows that $3\varepsilon < (t_{i+1}(b-\varepsilon,N) - t_i(b-\varepsilon,N))/2$ for all $i \in \{0,...,N-1\}$. Hence $c_i < t_{i+1}(b-\varepsilon,N)$, so that all c_i satisfy $c_i \in (t_i(b-\varepsilon,N), t_{i+1}(b-\varepsilon,N)]$. Furthermore, we have $a_e^*(t_i(b-\varepsilon,N), t_{i+1}(b-\varepsilon,N), c_i) = \frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2} + \varepsilon = a_{ne}^*(t_i(b-\varepsilon,N), t_{i+1}(b-\varepsilon,N)) + \varepsilon$.

Step 3 Note that $U^S(a,\omega,b-\varepsilon) = -(\omega + (b-\varepsilon) - a)^2 = -(\omega + b - (a+\varepsilon))^2 = U^S(a+\varepsilon,\omega,b)$. Recall that given a sender with bias $b-\varepsilon$, a necessary condition for the existence of the non-Ellsbergian equilibrium $E(b-\varepsilon,N)$ is:

$$\begin{aligned} & U^S\left(a_{ne}^*(t_{i-1}(b-\varepsilon,N), t_i(b-\varepsilon,N)), t_i(b-\varepsilon,N), b-\varepsilon\right) \\ = \; & U^S\left(a_{ne}^*(t_i(b-\varepsilon,N), t_{i+1}(b-\varepsilon,N)), t_i(b-\varepsilon,N), b-\varepsilon\right), \forall i \in \{1,...,N-1\}. \end{aligned}$$

The above set of equalities, combined with Point b), implies that

$$\begin{aligned} & U^S\left(a_e^*(t_{i-1}(b-\varepsilon,N), t_i(b-\varepsilon,N), c_{i-1}), t_i(b-\varepsilon,N), b\right) \\ = \; & U^S\left(a_e^*(t_i(b-\varepsilon,N), t_{i+1}(b-\varepsilon,N), c_i), t_i(b-\varepsilon,N), b\right), \forall i \in \{1,...,N-1\}. \end{aligned}$$

By Lemma 2, it follows that for $\varepsilon \leq \overline{\varepsilon}$, an equilibrium featuring the Ellsbergian communication strategy $\left(\{t_i(b-\varepsilon)\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ exists.

Step 4 The expected payoff of S in $\widetilde{E}$ is given by

$$
\begin{aligned}
& -\sum_{i=0}^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left((\omega+b)-\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)+c_i}{3}\right)^2 d\omega \\
= & -\sum_{i=0}^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left(\omega+(b-\varepsilon)-\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2}\right)^2 d\omega \\
= & \ \pi^S(E(b-\varepsilon,N))=-\frac{1}{12N^2}-\frac{(b-\varepsilon)^2(N^2+2)}{3}>\pi^S(E(b,N)),
\end{aligned}
$$

where the inequality follows since $\varepsilon \leq \overline{\varepsilon} < b$.
Step 5 The expected payoff of R in $\widetilde{E}$ is given by

$$
\begin{aligned}
& -\sum_{i=0}^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left(\omega-\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)+c_i}{3}\right)^2 d\omega \\
= & -\sum_{i=0}^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left(\omega-\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2}-\varepsilon\right)^2 d\omega \\
= & -\frac{1}{12N^2}-\frac{(b-\varepsilon)^2(N^2-1)}{3}-\varepsilon^2.
\end{aligned}
$$

The above expected payoff is higher than $\pi^R(E(b,N)) = -\frac{1}{12N^2} - \frac{b^2(N^2-1)}{3}$ if (and only if) $\varepsilon < 2b\frac{N^2-1}{N^2+2}$, which is true since we assumed $\varepsilon \leq \overline{\varepsilon} \leq b\frac{N^2-1}{N^2+2}$. ■

5.3. Proof of Proposition 2

Step 1 This proves Point 1). For $i \in \{0, ..., N-1\}$, let $t_i = \frac{i}{N}$ and let c_i be such that

$$\frac{t_i + t_{i+1} + c_i}{3} = \frac{t_i + t_{i+1}}{2} + b. \tag{9}$$

For any $i \in \{0, ..., N-1\}$, condition (9) is feasible iff

$$\frac{t_i + t_{i+1}}{2} + b \leq \frac{t_i + 2t_{i+1}}{3},$$

which is equivalent to $b \leq \frac{t_{i+1}-t_i}{6}$, which in turn simplifies to $b \leq \frac{1}{6N}$. Assuming $b \leq \frac{1}{6N}$, the constructed strategy is an equilibrium iff for any $i \in \{1, ..., N-1\}$, we have

$$-\left(\frac{t_{i-1}+t_i+c_{i-1}}{3}-t_i-b\right)^2 = -\left(\frac{t_i+t_{i+1}+c_i}{3}-t_i-b\right)^2.$$

The above, using (9) and $t_i = \frac{i}{N}$, simplifies to $\left(\frac{1}{2N}\right)^2 = \left(-\frac{1}{2N}\right)^2$, which is always true. The condition $6Nb \leq 1$ means that for given b, there exists an equal intervals equilibrium of size N (denoted $\widehat{E}(b,N)$) if and only if $N \leq \widehat{N}(b) = \left\langle \frac{1}{6b} \right\rangle$.

Step 2 This proves Point 2). Denote by $\pi^S(\widehat{E}(b,N))$ the expected payoff of S in $\widehat{E}(b,N)$ and denote by $\pi^R(\widehat{E}(b,N))$ the expected payoff of R in $\widehat{E}(b,N)$.

$$\begin{aligned}\pi^R(\widehat{E}(b,N)) &= -\sum_{i=1}^{N}\int_{\frac{i-1}{N}}^{\frac{i}{N}}\left(\omega-\left(\frac{\frac{i-1}{N}+\frac{i}{N}}{2}+b\right)\right)^2 d\omega \\ &= -\sum_{i=1}^{N}\left(-\frac{1}{12}\left(\frac{i-1}{N}-\frac{i}{N}\right)^3-\left(\frac{i-1}{N}-\frac{i}{N}\right)b^2\right)=-b^2-\frac{1}{12N^2}.\end{aligned}$$

It can similarly be shown that $\pi^S(\widehat{E}(b,N))=-\frac{1}{12N^2}$. Both $\pi^S(\widehat{E}(b,N))$ and $\pi^R(\widehat{E}(b,N))$ are increasing in N. The condition $b\leq\frac{1}{18}$ guarantees that $\widehat{E}(b,3)$ exists.

Step 3 This proves Point 3). For $b\leq\frac{1}{18}$, $\widehat{N}(b)\geq\overline{N}(b)\geq 3$. It is immediate that $\pi^S(\widehat{E}(b,N'))>\pi^S(E(b,N))$ given $N'\geq N$. On the other hand, $\pi^R(\widehat{E}(b,N))-\pi^R(E(b,N))$ equals

$$-b^2-\frac{1}{12N^2}-\left(-\frac{1}{12N^2}-\frac{b^2(N^2-1)}{3}\right)=\frac{1}{3}b^2\left(N^2-4\right),$$

which is strictly positive for any $N>2$. Thus, $\pi^R(\widehat{E}(b,\overline{N}(b)))>\pi^R(E(b,\overline{N}(b)))$, for any $b\leq\frac{1}{18}$. In turn, Point 2) implies $\pi^R(\widehat{E}(b,\widehat{N}(b)))>\pi^R(\widehat{E}(b,\overline{N}(b)))$. ■

5.4. Proof of Remark 1

Let $E(b,1)$ be the standard babbling equilibrium. We consider an Ellsbergian partitional equilibrium $\widetilde{E}(b,2)$ featuring a two-intervals partition. We consider the Ellsbergian strategy featuring $t_1=\left(\frac{2}{3}-\frac{2}{3}\varepsilon-2b\right)\in(0,1)$, $c_0=t_1-\varepsilon$ and $c_1=1-\varepsilon$. It must hold true that $a_e^*(0,t_1,c_0)=\frac{t_1+t_1-\varepsilon}{3}$ and $a_e^*(t_1,1,c_1)=\frac{t_1+1+1-\varepsilon}{3}$. By Lemma 2, this strategy is an equilibrium strategy if t_1 satisfies

$$\frac{t_1+1+1-\varepsilon}{3}-t_1-b=t_1+b-\frac{t_1+t_1-\varepsilon}{3},$$

which is true for $t_1=\frac{2}{3}-\frac{2}{3}\varepsilon-2b$.

Denote by $\pi^S(\widetilde{E}(b,2))$ the expected payoff of S in $\widetilde{E}(b,2)$ and denote by $\pi^R(\widetilde{E}(b,2))$ the expected payoff of R in $\widetilde{E}(b,2)$. $\pi^S\left(\widetilde{E}(b,2)\right)$ is given by:

$$\begin{aligned}&-\int_0^{t_1}\left((\omega+b)-\left(\frac{t_1+t_1-\varepsilon}{3}\right)\right)^2 d\omega-\int_{t_1}^{1}\left((\omega+b)-\left(\frac{t_1+1+1-\varepsilon}{3}\right)\right)^2 d\omega \\ =\ &\frac{8}{3}b^3+\frac{8}{3}b^2\varepsilon-\frac{25}{9}b^2+\frac{8}{9}b\varepsilon^2-\frac{50}{27}b\varepsilon+\frac{11}{27}b+\frac{8}{81}\varepsilon^3-\frac{25}{81}\varepsilon^2+\frac{11}{81}\varepsilon-\frac{1}{27}.\end{aligned}$$

For any $b\in\left[\frac{1}{4},\frac{1}{3}\right)$, for ε small enough $\pi^S\left(\widetilde{E}(b,2)\right)$ is thus strictly larger than $\pi^S(E(b,1))=-b^2-\frac{1}{12}$.

On the other hand, $\pi^R\left(\widetilde{E}(b,2)\right)$ is given by

$$
\begin{aligned}
& -\int_0^{t_1}\left(\omega-\left(\frac{t_1+t_1-\varepsilon}{3}\right)\right)^2 d\omega-\int_{t_1}^1\left(\omega-\left(\frac{t_1+1+1-\varepsilon}{3}\right)\right)^2 d\omega \\
= \; & \frac{8}{9}b^2\varepsilon-\frac{4}{3}b^2+\frac{16}{27}b\varepsilon^2-\frac{28}{27}b\varepsilon+\frac{2}{9}b+\frac{8}{81}\varepsilon^3-\frac{25}{81}\varepsilon^2+\frac{11}{81}\varepsilon-\frac{1}{27}.
\end{aligned}
$$

For any $b<\frac{1}{12}(1+\sqrt{6})\simeq 0.28$, for ε small enough $\pi^R\left(\widetilde{E}(b,2)\right)$ is thus strictly larger than $\pi^R(E(b,1))=-\frac{1}{12}$. ■

5.5. Proof of Remark 3

Outline We state and prove a sequence of results which together prove Remark 3.

Step 1 *$\min_\rho E\left[U^R(a,\omega)|m_i^A,\rho\right]$ equals $E\left[U^R(a,\omega)|m_i^A,\underline{\rho}\right]$ if $a<\frac{t_i+t_{i+1}+c_i}{3}$ while it instead equals $E\left[U^R(a,\omega)|m_i^A,\overline{\rho}\right]$ if $a>\frac{t_i+t_{i+1}+c_i}{3}$.*

Proof: This was proved in Step 2 of the proof of Lemma 1.

Step 2 *Let $a_\rho^*(t_i,t_{i+1},c_i,m_i^A)$ be the maximizer of $E\left[U^R(a,\omega)|m_i^A,\rho\right]$. Then a_ρ^* is decreasing in ρ.*

Proof: The FOC $\partial E\left[U^R\left(a,\omega\right)|m_i^A,\rho\right]/\partial a=0$ yields $a_\rho^*(t_i,t_{i+1},c_i,m_i^A)=\frac{c_i^2(1-2\rho)+t_i^2\rho-t_{i+1}^2(1-\rho)}{2(c_i(1-2\rho)+t_i\rho-t_{i+1}(1-\rho))}$. Note that $\partial a_\rho^*(t_i,t_{i+1},c_i,m_i^A)/\partial\rho=-\frac{(t_{i+1}-t_i)(c_i-t_i)(t_{i+1}-c_i)}{2(c_i(1-2\rho)+t_i\rho-t_{i+1}(1-\rho))^2}<0$.

Step 3 *Given $c_i>(t_i+t_{i+1})/2$, it holds true that $a_{[\underline{\rho},\overline{\rho}]}^*(t_i,t_{i+1},c_i,m_i^A)=\frac{t_i+c_i+t_{i+1}}{3}$ if $a_{\underline{\rho}}^*(t_i,t_{i+1},c_i,m_i^A)\geq\frac{t_i+c_i+t_{i+1}}{3}$.*

Proof: Recall first that $a_{1/2}^*(t_i,t_{i+1},c_i,m_i^A)=\frac{t_i+t_{i+1}}{2}$. By Step 2, $a_{\overline{\rho}}^*(t_i,t_{i+1},c_i,m_i^A)<\frac{t_i+t_{i+1}}{2}$. Since $\frac{t_i+c_i+t_{i+1}}{3}>\frac{t_i+t_{i+1}}{2}$, $a_{\overline{\rho}}^*(t_i,t_{i+1},c_i,m_i^A)<\frac{t_i+c_i+t_{i+1}}{3}$. Concavity of $E\left[U^R(a,\omega)|m_i^A,\rho\right]$ in a implies that $E\left[U^R(a,\omega)|m_i^A,\overline{\rho}\right]$ is decreasing in a if $a>\frac{t_i+c_i+t_{i+1}}{3}$. Concavity of $E\left[U^R(a,\omega)|m_i^A,\rho\right]$ in a also implies that if $a_{\underline{\rho}}^*(t_i,t_{i+1},c_i,m_i^A)\geq\frac{t_i+c_i+t_{i+1}}{3}$, then $E\left[U^R(a,\omega)|m_i^A,\underline{\rho}\right]$ is increasing in a for $a<\frac{t_i+c_i+t_{i+1}}{3}$. Recalling Step 1, we may conclude that if $a_{\underline{\rho}}^*(t_i,t_{i+1},c_i,m_i^A)\geq\frac{t_i+c_i+t_{i+1}}{3}$, then $a_{[\underline{\rho},\overline{\rho}]}^*(t_i,t_{i+1},c_i,m_i^A)=\frac{t_i+c_i+t_{i+1}}{3}$.

Step 4 *Let $\widehat{\gamma}(\rho)=\frac{\sqrt{(1-3(1-\rho)\rho)}-1/2}{1-2\rho}$* and let $\rho\in(0,\frac{1}{2})$. *Then: a) $a_\rho^*(t_i,t_{i+1},c_i,m_i^A)=\frac{t_i+c_i+t_{i+1}}{3}$ iff $c_i=(t_i+t_{i+1})/2+(t_{i+1}-t_i)\widehat{\gamma}(\rho)$. b) $a_{\underline{\rho}}^*(t_i,t_{i+1},c_i,m_i^A)>\frac{t_i+c_i+t_{i+1}}{3}$ if $c_i\in\left[\frac{t_i+t_{i+1}}{2},(t_i+t_{i+1})/2+(t_{i+1}-t_i)\widehat{\gamma}(\underline{\rho})\right)$.*

Proof: Concerning Part a), straightforward algebra shows that $c_i=(t_i+t_{i+1})/2+(t_{i+1}-t_i)\widehat{\gamma}(\rho)$ is the only value of c_i in $\left(\frac{t_i+t_{+1}}{2},t_{i+1}\right]$ that satisfies the equality $a_\rho^*(t_i,t_{i+1},c_i,m_i^A)=\frac{t_i+c_i+t_{i+1}}{3}$. Part b) is proved as follows. First, note that $a_{\underline{\rho}}^*(t_i,t_{i+1},(t_i+t_{i+1})/2,m_i^A)=(t_i+t_{i+1})/2+(t_{i+1}-t_i)(1-2\underline{\rho})/4>(t_i+t_{i+1}+(t_i+t_{i+1})/2)/3$. Combining this with Part a) and the fact that $a_\rho^*(t_i,t_{i+1},c_i,m_i^A)-\frac{t_i+c_i+t_{i+1}}{3}$ is continuous in c_i on $\left[\frac{t_i+t_{+1}}{2},t_{i+1}\right]$, the statement of Part b) follows.

Step 5 Since m_i^B only reverses the role of the two ball colors θ_1 and θ_2, we may conclude that $a_{[\underline{\rho},\overline{\rho}]}^*(t_i,t_{i+1},c_i,m_i^B)=\frac{t_i+c_i+t_{i+1}}{3}$ if $(t_i+t_{i+1})/2\leq c_i\leq(t_i+t_{i+1})/2+(t_{i+1}-t_i)\widehat{\gamma}(1-\overline{\rho})$.

Step 6 Remark 3 follows from steps 3-5 if one sets $\gamma=\min_{\rho\in\{\underline{\rho},1-\overline{\rho}\}}\widehat{\gamma}(\rho)$. ■

References

[1] Azrieli, Y. & Teper, R., 2011, Uncertainty aversion and equilibrium existence in games with incomplete information, Games and Economic Behavior, 73(2)
[2] Bade, S., 2010, Ambiguous act equilibria, Games and Economic Behavior, 71(2)
[3] Blume, A., Board, O. & Kawamura, K., 2007, Noisy talk, Theoretical Economics, 2(4)
[4] Blume, A. & Board, O., 2014. Intentional vagueness. Erkenntnis, 79(4)
[5] Bose, S. & Renou, L., 2014, Mechanism design with ambiguous communication devices, Econometrica, 82(5)
[6] Chen, Y. & Gordon, S., 2015, Information transmission in nested sender–receiver games, Economic Theory, 58(3)
[7] Crawford, V. & Sobel, J., 1982, Strategic information transmission, Econometrica, 50(6)
[8] Dessein, W., 2002, Authority and communication in organizations, Review of Economic Studies, 69(4)
[9] Dominiak, A., Dürsch, P. & Lefort, J.P., 2012, A dynamic Ellsberg urn experiment, Games and Economic Behavior, 75(2)
[10] Gilboa, I., 1987, Expected utility with purely subjective non-additive probabilities, Journal of Mathematical Economics, 16(1)
[11] Gilboa, I. & Schmeidler, D., 1989, Maxmin expected utility with a non-unique prior, Journal of Mathematical Economics, 18(2)
[12] Gordon, S. & Nöldeke, G., 2015, Figures of speech in strategic communication, mimeo
[13] Hanany, E. & Klibanoff, P., 2007, Updating preferences with multiple priors, Theoretical Economics, 2(3)
[14] Hanany, E. & Klibanoff, P., 2009, Updating ambiguity averse preferences, The BE Journal of Theoretical Economics, 9(1)
[15] Hanany, E., Klibanoff, P., & Mukerji, S, 2016, Incomplete information games with ambiguity averse players, mimeo
[16] Kellner, C. & Le Quement, M. T., 2017a, Endogeneous ambiguity in cheap talk, Department of Economics Working Paper 1701, Southampton University
[17] Kellner, C. & Le Quement, M. T., 2017b, Modes of ambiguous communication, Games and Economic Behavior, 104
[18] Klibanoff, P., 1996, Uncertainty, decision and normal form games, mimeo
[19] Klibanoff, P., Marinacci, M. & Mukerji, S., 2005, A smooth model of decision making under ambiguity, Econometrica, 73(6)
[20] Lipman, B. L., 2009, Why is language vague?, mimeo
[21] Lang, M., 2017, First and second-order ambiguity aversion, Management Science, 63(4)
[22] Lo, K.C., 1996, Equilibrium in beliefs under uncertainty, Journal of Economic Theory, 71(2)
[23] Riedel, F. & Sass, L., 2014, Ellsberg games, Theory and Decision, 76 (4)
[24] Riedel, F., 2017, Uncertain acts in games, Working Paper, Center for Mathematical Economics, Universität Bielefeld
[25] Siniscalchi, M., 2011, Dynamic choice under ambiguity, Theoretical Economics, 6(3)
[26] Sobel, J., 2013, Giving and receiving advice, in Acemoglu, D., Arellano, M., and Dekel, E., Advances in Economics and Econometrics: Tenth World Congress, Cambridge: Cambridge University Press