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Interest Rate Volatility and Risk Management: Evidence from CBOE Treasury Options

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Highlights

- We investigate US Treasury market volatility.
- We find substantial interest rate volatility risk for medium-term instruments
- We show that it has a time-varying relationship with equity volatility risk.
- It is affected by macroeconomic and monetary news.
- It is only partially spanned by information contained in the yield curve.

Abstract

This paper investigates US Treasury market volatility and develops new ways of dealing with the underlying interest rate volatility risk. We adopt an innovative approach which is based on a class of model-free interest rate volatility (VXI) indices we derive from options traded on the CBOE. The empirical analysis indicates substantial interest rate volatility risk for medium-term instruments which declines to the levels of the equity market only as the tenor increases to 30 years. We show that this risk appears to be priced in the market and has a significant time-varying relationship with equity volatility risk. US Treasury market volatility is appealing from an investment diversification perspective since the VXI indices are negatively correlated with the levels of interest rates and of equity market implied volatility indices, respectively. Although VXI indices are affected by macroeconomic and monetary news, they are only partially spanned by information contained in the yield curve. Motivated by our results on the magnitude and the nature of interest rate volatility risk and by the phenomenal recent growth of the equity volatility derivative market, we propose the use of our VXI indices as benchmarks for monitoring, securitizing, managing and trading interest rate volatility risk. As a first step in this direction, we describe a framework of one-factor equilibrium models for pricing VXI futures and options on the basis of empirically favored mean-reverting jump-diffusions.

Keywords: Interest rate volatility; Volatility indices; Volatility risk premium; Level effect; Unspanned stochastic volatility; Macroeconomic news.

Classification codes: C51, E44, G12, G13, G14.

1. Introduction

The volatility of interest rates is of prime importance to monetary authorities, financial institutions, policy makers and journalists since interest rates have such a central position in economic theories, models and systems. Bond and foreign exchange market participants are also particularly concerned about the future evolution and variability of interest rates since volatility is a protagonist in the pricing, hedging and risk management of financial instruments involving interest rates (Chapman & Pearson 2001; Ederington & Lee 2007). Researchers have examined various hypotheses and uncovered stylized facts, but there is no strong consensus yet in the empirical literature on how interest volatility should be measured and modeled. More importantly, although we now understand well that interest rate volatility exhibits large swings (Ait-Sahalia 1996; Andersen & Lund 1997; Amin & Ng, 1997), we know much less about how this particular risk should be monitored and dealt with (existing risk management practices are reviewed by Ho 2007).

Expanding on an idea originally mentioned in Brenner and Galai (1989), we take a fresh look at interest rate volatility by employing ideas and tools from the extensive recent research on volatility indices in equity markets. This allows us to make a number of significant extensions to the literature. Specifically, using a well-established model-free methodology, which was first used for the VIX equity market volatility index, we build a set of new metrics for interest rate volatility. These metrics are employed as proxies of expected volatility for Treasury market instruments on the basis of information contained in interest rate options traded on the Chicago Board Options Exchange (CBOE). We study the daily empirical behaviour of three VXI indices with maturities of 5 years, 10 years and 30 years, respectively, over a twelve year period. The results indicate that implied volatility is substantial in magnitude and variation. For example, in the case of the 5-year instrument, volatility is almost double that of the VIX equity volatility index (39.34% *vs*. 20.41%).

Over the recent credit crisis, levels of implied interest volatility of medium-term rates increase sharply, more than fourfold relative to the past. The negative premium which corresponds to interest rate volatility risk is found to be much higher in magnitude than estimates reported for the equity market. An important new result is that our estimates of interest rate volatility risk premia have a significant time varying-correlation with equity market volatility risk premia. Another interesting finding is that the VXI indices, as is the case with the VIX index, offer valuable diversification opportunities to bond and equity investors. Specifically, our measures of interest rate volatility have a strong negative correlation with interest rate levels (up to -85.8%) and equity market implied volatility index levels (up to -23.5%). In line with previous research, we show that macroeconomic and monetary announcements affect significantly implied interest rate volatility by decreasing (increasing) it the day before (after) an announcement. A new result is that this effect varies across the term structure and becomes more prominent at the longer maturities studied. We confirm previous findings in the literature that interest rate implied volatility is not fully spanned by the information which is contained in the underlying yield curve. Finally, motivated by our results and the rapid development of the equity volatility derivative market, we propose our VXI indices as vehicles for developing options and futures which can be used for managing and trading interest rate volatility risk. On the basis of a horserace amongst popular continuous time models for representing the VXI index empirical behavior, we describe a single-factor pricing framework using autonomous mean-reverting jump-diffusions.

Our study is closely related to the recent studies of Claes, De Ceuster, Lópezc and Navarroc (2010), and of Choi, Mueller & Vedolin (2017), where both studies propose the construction of Treasury bond implied volatility indices. The first study uses data from the U.S. cap (floor) market, which are portfolios of options on interest rates traded in the over-the-counter (OTC) interest rate derivatives market, and the second study uses data from options on Treasury futures. The empirical results in our paper are based data from interest rate options traded on the CBOE (for more details on the specifications of these products see Longstaff, 1990; Christiansen & Hansen, 2002). Although this market may not be as large as the interest rate option markets used by Choi et al. (2017), these options are of particular interest because of two reasons. First, since they are written directly on the rates (yields), they allow the construction of indices that measure the model free implied volatility of treasury rates, rather than the implied volatility of Treasury Bonds (Claes et al., 2010; Choi et al., 2017). Second, they provide direct insight into the information context of the interest rate, which is implicit in other contingent claims (e.g. treasury futures or Eurodollar futures).

The rest of the paper is organized as follows. Section 2 describes the dataset and the methodology used to construct the interest rate implied volatility indices. Section 3 presents the results of our empirical study, Section 4 develops valuation formulae for futures and options written on interest rate implied volatility and discusses some of their key properties, and Section 5 concludes.

2. Methodology and Data: Interest Rate Implied Volatility Indices (VXI).

Two main approaches are most popular amongst academics and practitioners for the estimation of interest rate volatility. The first resorts to historical time series of interest rates in order to derive estimates of "historical volatility" using unconditional moment estimators, exponential moving averages, ARCH models, stochastic volatility models, etc. (see Ederington & Lee, 2007). The second approach aims at calculating the "implied volatility" that equates actual prices of interest rate options with those given by some theoretical pricing model. Since this estimate reflects market data, it incorporates investor expectations, behaviors and risk attitudes about the future evolution of volatility. Although there is controversy in the empirical literature about which approach is superior, most researchers seem to agree that implied volatility is better than historical volatility in terms of forecasting power (Christensen & Prabhala, 1998; Poon & Granger, 2003; Szakmary, Ors, Kim & Davidson, 2003; Li & Yang, 2009). A third nonparametric approach employs intraday price data to derive so-called realized volatility measures (Andersen & Benzoni, 2010). Although this last approach is known now to be theoretically and empirically appealing, it is still not widely applied due to the significant data requirements it has.

Turning now to interest rates in particular, previous empirical research on implied volatility is based on estimates derived by inverting observed option prices using a specific option pricing model. Amin and Morton (1994), and Amin and Ng (1997) are using the Heath, Jarrow and Morton (1992) model, while the studies by Christiansen and Hansen (2002), and Claes et al. (2010) estimate implied volatility via the LIBOR market model. Unfortunately, the implied volatility estimation approach used by the above studies comes with two important disadvantages. First, the accuracy of the estimates depends critically on the validity of the option pricing model assumed. Second, at any particular moment, there are as many implied volatility estimates as strike prices of the options. In order to overcome such problems, Britten-Jones and Neuberger (2000) and Jiang and Tian (2005) propose a general model-free methodology that calculates implied volatility using the entire set of the option prices at a certain point of time. In both studies, the authors provide evidence that the model-free implied volatility is better than both historical volatility and model-driven implied volatility.

In recent years there has been a great deal of research also on the construction and the properties of equity implied volatility indices (Carr & Wu, 2006; Gio, 2005; Whaley 1993; 2009). The first volatility index (VIX) was introduced in 1993 by the CBOE. Soon after the introduction of the index, CBOE received strong criticism regarding the methodology used for the calculation of VIX. Originally the VIX was calculated as an average of the Black-Scholes at-the-money (ATM) option implied volatility, according to the methodology proposed by Whaley (1993). As a response, on September 22, 2003, the CBOE changed the Black-Scholes based methodology of VIX calculation. The new VIX methodology is model-free and allows VIX to be robustly replicated by a portfolio of options (see CBOE, White Paper, 2009 and Carr & Wu, 2006, for a detailed description of the "new" VIX methodology and for a comparison of the two methodologies).[1] Specifically, the new VIX implied volatility index is constructed as the weighted sum of out-of-the-money (OTM) call and put option closing prices at two nearby maturities across all available strikes. The implied volatility index captures the implied volatility of a synthetically created ATM option with a constant maturity of 30 days. Several other equity implied volatility indices have also been developed. These include the VXN, the VXD and the RVX in the CBOE, which are the equivalent to VIX implied volatility indices for the NASDAQ, Dow Jones Industrial Average and Russell 2000 Index, respectively. Similarly, we have the DAX-30 volatility index (VDAX-NEW) in Germany, the CAC-40 volatility index (VCAC) in France and the Dow Jones EURO STOXX 50 volatility index (VSTOXX) in the Eurex. Given the great success of equity implied volatility indices and the rapidly expanding market for volatility futures and options, CBOE recently decided to launch three more implied volatility indices in different asset classes than equity: the Crude Oil Volatility Index (OVX), the EuroCurrency Volatility Index (FVX) and the Gold Volatility Index (GVX).

Although the model-free methodology and index construction has been widely applied to equities, to the best of our knowledge our research is the first to be undertaken in the interest rate

[1] CBOE still quotes the "old" VIX, which is calculated with the old methodology, under the ticker "VXO". All volatility indices, apart from VXO, quoted in CBOE are calculated with the new model free methodology.

literature. The relevant studies of Claes et al. (2010) and of Choi et al. (2017) construct implied volatility indices written on bond prices. As such, they implicitly assume that the log-return of the bond equals the yield to maturity. In the present study, we closely follow the model-free methodology of VIX, in order to construct the interest rate implied volatility index (VXI). The VXI represents the risk-neutral expectation of the annualized volatility of the underlying interest rate over the next 30 calendar days. As in the case of equity implied volatility indices, where each implied volatility index corresponds to the implied volatility of a particular stock index, we can construct a different interest rate implied volatility index for each interest rate maturity available (e.g. 13 weeks, 5 years, 10 years, 30 years). Moreover, a similar encapsulation technique is applicable to all interest options, e.g., options on Eurodollar futures traded at CME, treasury option traded at CBOE or at CBOT, etc.

We employ daily market prices for interest rate call and put options traded on the CBOE over the period 1/4/96 to 8/29/08, a total of 3,159 trading days. The sample covers over 12 years of data which allows statistical significance and a variety of market environments. The starting point corresponds to the earliest point for which we have available data. The cut-off date corresponds roughly to the beginning of the financial crisis which was followed by the zero interest-rate policy in the US. We do not use more recent data in order to avoid the effects this very unusual interest rate environment. One can argue that interest rate volatility is not a major concern in this period in comparison to systemic or equity risk. The introduction of unconventional monetary policies such as quantitative easing after 2008, means that the sample may not be homogeneous with respect to the past. We recognise this as a limitation of our study which can be addressed through future research which can look at the management of interest rate risk at very low interest rate environments.

The options used are cash-settled European style and are written on the spot yield of U.S. Treasury securities. There are 4 different contracts available. The first is written on: the annualized discount rate of the most recently auctioned 13-week Treasury bill. The other three are written on the yield-to-maturity of the most recently auctioned 5-year Treasury note, the 10-year Treasury note and the 30-year Treasury bond, respectively. The ticker symbols of the underlying instruments are IRX, FVX, TNX and TYX, respectively. According to the construction of the IRX, FVX, TNX and TYX, each day the underlying asset of the options used is always the same. Say at issuance, the option is written on the yield-to-maturity of 30-year Treasury bond (TYX). Tomorrow, this 30-

year Treasury bond, which now has a maturity of 30 years minus one day, will be replaced by the yield-to-maturity of a new Treasury bond with exactly 30 years maturity. So, the underlying asset is always the same (see the website of CBOE for more details). Other studies of interest rate implied volatility use data from swaptions (see, among others, Trolle & Schwartz 2009), option contracts written on Eurodollar futures (Amin & Ng 1997), options on Treasury futures (Choi et al. 2017), and OTC interest rate options (Claes et al. 2010). To the best of our knowledge there are only two studies that use similar data. Firstly, Christiansen and Hansen (2002) use interest rate options data from CBOE to analyse the IRX rate. Secondly, Heuson and Su (2003) use option data written on FVX, TNX and TYX, and examine the intra-day reaction of implied interest rate volatility around macroeconomic new announcements. However, since in both studies the implied volatilities are not model free (Christiansen & Hansen, 2002, use the LIBOR market model, and Heuson & Su, 2003, use Hull-White model to extract the implied volatilities), their estimates are subject to model misspecification. Our dataset offers four main advantages. First, it gives us the opportunity to provide empirical evidence on a relatively unexplored market. Second, the interest rate options we analyse are much simpler than options on Eurodollar futures, since the former are written directly on interest rates. In this manner, we deal directly with the quantity of interest and avoid any irrelevant effects. Third, since we are dealing with new volatility metrics and derivatives, it makes sense to base our study around the CBOE which is leader in the volatility securitisation and monitoring industry. Finally, since all the data are provided by the CBOE, we preserve homogeneity and minimize the errors that may result from asynchronous trading and variations in data quality.

Using the model-free methodology of VIX we calculate the implied volatility from the option contracts on FVX, TNX and TYX and coin the corresponding indices as VXI-5Y, VXI-10Y and VXI-30Y, respectively. We choose not to construct an index written on the shortest rate (i.e. IRX) for two reasons. First, options written on IRX are the least liquid. Second, IRX is not directly comparable with the other three rates, since IRX is the discount yield of the most recently auctioned 13-week T-Bill, while the others are yield to maturities of T-Bonds. In the cases where the value of the implied volatility index cannot be computed due to low liquidity or missing values, that is around 5% of the sample, we use the value of the previous trading day as an approximation.[2]

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[2] However, a recent study of Andersen et al. (2011). proposes an alternative methodology for volatility index construction using a limited range of options, and develop a so-called corridor implied volatility index.

We estimate also simple logarithmic returns for these indices and denote them as: ΔVXI-5Y, ΔVXI-10Y and ΔVXI-30Y (hereafter these will be referred to as returns or changes).

3. Empirical Results

3.1 Descriptive Statistics

Time series plots of the indices and underlying yields are presented in Figure 1. Descriptive statistics of index levels and returns are given in Table 1. We also include some results for the VIX, S&P500 and the Merrill Option Volatility Expectations (MOVE) indices in order to facilitate comparative inferences. The MOVE is calculated by Merrill Lynch as the weighted average of the normalized implied volatility from constant one-month at-the-money OTC Treasury options written on benchmark Treasury securities with maturity periods of 2 years, 5 years, 10 years and 30 years, respectively. The yields of these maturity periods are equally weighted with 20% except for the 10 year which has a weight of 40%. Since the maturity of the options are constant up to one month, the MOVE measures implied volatility over a forecast horizon similar to VXI indices. However, in the case of the MOVE index, implied volatility is model depended, as it is calculated only from at-the-money options using the Black (1976) model.

[Insert Table 1 and Figure 1 here]

In line with previous studies (Ait-Sahalia, 1996; Amin & Ng, 1997; Andersen & Lund, 1997), the plots indicate that interest rate volatility is substantial and varies significantly across time. The averages and standard deviations are very different between the indices analysed. The index referring to the volatility of the shortest maturity interest rate (VXI-5Y) has the highest level of mean ($MEAN_{VXI\text{-}5Y}$ = 39.34) and variability ($CV_{VXI\text{-}5Y}$ = 83.69%), much higher than the corresponding values for the VIX ($MEAN_{VIX}$ = 20.41, CV_{VIX}= 32.83%). The remaining two interest rate volatility indices (VXI-10Y, and VXI-30Y) have a similar average level and variability compared to the VIX. A significant shift took place in volatility as the credit crisis unfolded. As shown in Figure 1, the VXI-5Y increased more than fourfold since June 2008 and went from an average level of 33.56 and 37.02 to 135.71 and 177.86, respectively. The extreme positive and negative returns confirm what can be seen visually in the plots as violent and abrupt changes. The shift in volatility in 2008 reflects the drop in interest rates to almost zero and a period of very low volatility which last up to present. All VXI indices are non-normally distributed with a positive skewness and excess kurtosis. Again, these characteristics are more prominent for the shortest term index studied. Finally, we see that the behavior of the VXI indices to the MOVE is quite different. The differences in magnitude are justified by the fact that MOVE is based on some unknown normalization scheme and this index may not have a direct correspondence to volatility. MOVE changes are far more smooth and closer to a Gaussian distribution.

Similar to previous studies such as Chapman and Pearson (2001), and Litterman, Scheinkman and Weiss (1991), our results across the 3 maturities studied suggest that the implied volatility term structure is hump-shaped with a peak at the 5-year period. In addition to the level of volatility, we find that the variability of the indices also has a similar hump-shaped pattern. As in Ball and Torous (1999), the preliminary analysis for all series demonstrates that interest rate volatilities are highly persistent but stationary. A first indication for this is given by the time series plots and the fact that autocorrelation coefficients of levels at lag 1 are just below unity. These results are confirmed by both the augmented Dickey and Fuller and Phillips and Perron unit root tests (see results in Appendix, Table 10.).

In contrast to Ball and Torous (1999), we find that interest rate volatility displays similar persistence to that of the equity market with the first lag autoregressive coefficients being just below unity. An exception is the VXI-30Y which has a considerably smaller coefficient.

3.2 Interest Rate Volatility Risk Premium

A crucial step in understanding interest rate volatility is to examine if it is priced by investors. In other words, if a volatility risk premium (*VRP*) is demanded as a compensation for assuming interest rate volatility risk. Following Almeida and Vicente (2009), Bollerslev, Tauchen, and Zhou (2009), Carr and Wu (2009), (2016), and Fornari (2010) the *VRP* over the next month can be defined as the difference between the realized volatility for the same period ($RV_{t,t+30}$) and the risk neutral interest rate expected volatility (VXI_t):

$$VRP_t = RV_{t,t+30} - VXI_t \tag{1}$$

$RV_{t,t+30}$ is calculated from the following formula:

$$RV_{t,t+30} = \sqrt{\frac{365}{30}\sum_{i=1}^{30}\left(\frac{r_{t+i} - r_{t+i-1}}{r_{t+i-1}}\right)^2} \tag{2}$$

where r_t denotes the time t treasury rate. In approximating the second term, we use volatility estimates based on the sum of squared interest rate daily returns over a period of one month. [3]

The method used for the calculation of the volatility risk premium, equation (1), directly uses the model-free realized and implied volatilities to extract the volatility risk premium. Both measures and in turn the premium are directly observable at time *t* in a completely model-free fashion. As such, it is easier to implement than other methods, which rely on the joint estimation of both the underlying asset return and the prices of one or more of its derivatives. The latter requires complicated modeling and estimation procedures (see, e.g., Pan, 2002; Jones, 2003; Eraker, 2004; Garcia, Lewis, Pastorello & Renault, 2011, among many others). There is clearly a trade-off between model-free and model-based approaches to recover implied volatilities. While a model-free approach is robust to misspecification it requires theoretically continuous strikes for option prices or practically a very liquid market like the S&P500 option market. On the contrast, model-based approaches are sensitive to misspecification but they require only a few option prices. Fortunately, however, even with relatively few different options, model-free approach tends to provide a fairly accurate approximation to the true (unobserved) risk-neutral expectation of the future return variation (Jiang & Tian, 2005)

In addition, our use of the volatility difference as a simple proxy for the volatility risk premium implicitly assumes that the volatility follows a random walk, or equivalently that the best predictor of the expected realized volatility $E(RV_{t,t+30})$ is the current realized volatility, $RV_{t,t+30}$. Toward this end, Bollerslev et al. (2009) and Bollerslev, Marone, Xu and Zu (2014) also use the Expected or Forward *VRP* (FVRP), which is obtained by replacing the model-free monthly realized variances with forward looking model-based expectations, i.e.:

$$FVRP_t = E\left(RV_{t,t+30}\right) - VXI_t \qquad (3)$$

Unfortunately, in contrast to the *VRP* defined in equation (1), *FVRP* necessitates the use of a model for generating the forward expectations $E(RV_{t,t+30})$, and as such *FRVP* abolishes both the simplicity

[3] Bollerslevet al. (2009). and Carr and Wu (2009). construct variance risk premia, which are defined as the difference between the implied variance and the risk neutral variance. However, some of their estimation procedures used in our study, can easily be extended to the case of volatility risk premia.

and the “model-free” nature of *VRP*[4]. Still, as shown in the Bollerslev et al. (2009) and Bollerslev, et al. (2014), perform fairly similar.

A number of theoretical and practical implications of the *VRP* are discussed in the literature. Chernov (2007) emphasizes the role of the *VRP* for portfolio managers and policymakers in allowing them to form better forecasts of future volatility. Accordingly, Bollerslev et al. (2009) show that the *VRP* explains a significant portion of the variation in the post-1990 aggregate stock market returns. Bekaert and Hoerova (2014) present empirical evidence that the *VRP* predicts stock returns. Almeida and Vicente (2009) argue that the VRP is crucial in reconciling option market implied volatilities with spot market historical volatilities. Joslin (2007) discuss the importance of the market price of volatility risk for matching the option price dynamics. Bakshi and Kappadia (2003) point out that a negative *VRP* implies that option prices are higher than those that would be realized if volatility risk was not priced. A variety of option pricing models incorporate explicitly a volatility risk premium and account for its stylized facts (Christoffersen, Heston & Jacobs, 2013; Papantonis, 2016).

Empirical studies in equity (Bakshi & Kappadia, 2003; Bollerslev et al. 2009; Carr & Wu, 2009; Todorov, 2010), and fixed income markets (Almeida & Vicente, 2009; Fornari 2010; Joslin 2007; Choi et al. 2017; Trolle & Schwartz, 2014) give evidence that the volatility risk premium is negative, time-varying and dependent on the level of volatility. Estimates of the *VRP* range between -2% to -3% and -4% to -5% for developed equity and fixed income markets, respectively. The negative sign of the premium suggests that investors are willing to pay large premiums to hedge volatility (Cieslak & Povala, 2016). More recently, Duyvesteyn and de Zwart (2015) has demonstrated that the volatility risk premium depends also on the maturity of the underlying interest rates. Specifically, the volatility risk premium is more negative for short-term maturities than for longer maturities.

The estimated risk premia are depicted in Figure 2 while Table 2 gives some summary statistics. Interest rate volatility risk premia VRP.FVX-5Y, VRP.TNX-10Y and VRP.TYX-30Y denote the 5, 10 and 30 years volatility risk premium, respectively. In order to facilitate comparisons, we also include descriptive statistics for the volatility risk premium of the S&P500 index *VRP.SP500*, which is calculated using the same methodology. In this calculation, the VIX

[4] Numerous parametric and nonparametric volatility forecasting procedures are proposed in the literature (see, for example, Andersen et al. 2006)

index is used as a proxy of the S&P500 risk neutral expected volatility, while realized volatility is estimated over the next month using the sum of squared S&P500 index daily returns. The results indicate that the interest rate premia are time varying and subject to violent upward and downward shifts. The median values range between -5.52% for the 10-year maturity down to -6.31% for the 5-year instrument. These estimates are higher than the -4% premium obtained for US interest rates by Fornari (2010) using a different methodology and dataset. The interest rate *VRP* are also clearly much higher than the -2.45% premium estimate obtained for the equity market using the VIX and S&P500 returns. Our premia estimates have high variability, especially for the two shortest tenors examined (5 and 10 years), while the distributions of premia and their changes are highly nonnormal with many violent upward and downward jumps. The plots also suggest that premia are increasing over the recent past. Specifically, since June 2008 median premia have increased in magnitude by a factor of 2.36, 1.3 and 1.15 when compared to the previous period for the case of the FVX, TNX and TYX, respectively. It is interesting to note that the VIX volatility risk premium decreased in magnitude by a factor of only 0.47 since June 2008. Comparable results are obtained if means are used rather than the outlier-robust median measures of central tendency.

[Insert Table 2 and Figure 2 here]

3.3 Correlation Analysis

A correlation analysis of the *VRP* with respect to the underlying interest rates is shown at Table 3. The inspection of the table allows several interesting insights. First, interest rate *VRP* are interrelated between them, especially at the longer maturities considered (e.g.. the *VRP.TNX-10Y* and *VRP.TYX-30Y* have a correlation of 78.4%). Second, there is a positive "level effect" in that VXI-derived *VRP* are correlated to the levels of the underlying interest rates. In other words, risk premia are higher at higher levels of interest rates. Third, there appears to be a weak negative correlation between equity and interest rate volatility risk premia. The relationship between these two premia should be examined in the context of the voluminous literature on the association between bond and equity markets (see Jubinski & Lipton, 2012; Baele et al. 2010; Guidolin & Timmermann, 2006, *inter alia*). Volatility risk premia, as proxies of investor risk aversion and attitudes, should be equal between these two markets according to most asset pricing frameworks. However, the "flight to quality" and "flight from quality" phenomena predict an inverse relationship between the risk premia in the two markets. Specifically, when the stock market crashes (rallies) then risk aversion towards equities (bonds) increases (decreases) and investors move to bonds (stocks). Empirical evidence with respect to the direction of the relationship between stocks and bonds has been conflicting. However, recent studies suggest that this relationship is time-varying and depends on a variety of macroeconomic and microeconomic variables (see Baele et al. 2010; Guidolin & Timmermann, 2006). In light of this evidence, a rolling correlation analysis is used with a window of 125 trading days which corresponds to a calendar period of 6 moths (for a similar approach see, for example, Connolly et al. 2005). The results for the VRP.TYX-30Y, depicted in Figure 3, suggest that the relationship is indeed time-varying with correlation being negative over most of the sample period under study[5].

[Insert Table 3 and Figure 3 here]

It must be noted also that the level effect, positive or negative, has not been justified yet in the interest rate literature. A possible explanation of the negative level-effect could be derived by inverting the leverage-effect arguments from equity markets. The leverage effect hypothesis proposed by Black (1976) and Christie (1982), postulates that negative returns will usually reduce the stock price and market value of the firm, which in turn means an increase in financial leverage, i.e., a higher debt to equity ratio. The latter will ultimately lead to an increase in risk and equity volatility (see Bae, Kim, & Nelson, 2007, inter alia, for recent advances on leverage effect). However, from a debt market perspective, higher levels of interest rates mean that the market value of debt decreases. As with the inverse link between bond prices and yields, this is because we are discounting with a higher rate. Financial leverage will decrease with the market value of debut which in turn suggests less interest rate risk and volatility. In practical terms, our results concerning the negative association of VXI indices with other variables have important practical implications. They suggest that interest rate volatility can act as significant hedge against variations in the underlying interest rate levels and equity market volatility. A similar picture to that painted above, although correlation coefficients are much smaller in magnitude, is drawn if changes rather than levels in volatility and interest rates are used in the analysis.

In order to examine the static relationship between the VXI indices and other series, we undertake a correlation analysis. The results, shown in Table 4, demonstrate clearly that a strong positive relationship (all correlations above 75%) exists between the VXI indices at the three maturities studied. The MOVE index is positively related to VXI levels with correlation

[5] Similar patterns observed for all maturities and for rolling windows size of 3 months or 1 year.

coefficients ranging between 29.7% and 49%. The negative correlation of -10.9% between the VIX and S&P500 returns confirms what is widely known in the financial industry with respect to the hedging benefits of implied equity volatility. Although we find that the VXI indices have no linear correlation with SP500 returns, they are negatively correlated with VIX levels, especially for the 5-year and 30-year maturity studied (-20.7% and -23.5, respectively). A strikingly significant result is the strong negative relationship between levels of VXI and interest rates with the correlation coefficients ranging between -52.3% (VXI-5Y with TYX-30Y) and -85.8% (VXI-10Y with FVX-5Y). A graphical inspection, as shown for instance in the scatter-plot of Figure 4 for the TYX-30Y, indicates that this relationship negative curvilinear.

[Insert Table 4 and Figure 4 here]

A negative relationship is consistent with findings throughout the interest rate literature on the so-called "level effect" according to which interest rate volatility is sensitive to the level of interest rates. However, there is controversy with respect to the size and sign of this level effect. Earlier studies characterize this relationship as strongly positive, whereby high volatility is associated with high interest rate levels (see, for example Chan et al. 1992). Later studies, which account for properties of the series such as autocorrelation and heteroskedasticty, find a much weaker positive relationship (see, for example, Andersen & Lund, 1997; Ball & Torous, 1999). More recently, Trolle and Schwartz (2009) use interest rate implied volatility estimates from swaptions and caps, and report both positive and negative relationships between interest rate implied volatility and interest rate levels, depending on the model used to back-out implied volatilities. Our model-free estimates of volatility provide new empirical evidences in favor of a nonlinear negative relationship between volatility and interest rates.

3.4 Granger Causality Analysis

In order to assess the possible spillovers between the variables and markets under consideration we employ Granger-causality analysis. Both levels and changes of the volatility indices and interest rates are considered in order to capture possible dynamic level effects. The null hypothesis in the test is that "Variable A" does not cause "Variable B" and it is evaluated using an F-statistic. The statistically significant results using a single lag in the test specification are summarized in Table 5. The single lag was selected on the basis that most of the dynamics will be captured with this as we are dealing with daily data. We also used a longer five lag or weekly structure in the test specification which produced comparable results (available upon request from the authors). As an example in interpreting the results, we reject at the 99% level (F-statistic is 7.92**) the null hypothesis that ΔVXI-5Y causes ΔFVX-5Y (in the Granger causality sense). Two main conclusions can be drawn from the results. First, various intermarket spillovers exist in the Treasury market with interest rates Granger-causing volatility and vice versa. As Amin and Ng (1997) suggest, it appears that implied volatility is useful in predicting future interest rate implied volatilities. Second, the intermarket spillover effects that can be observed between the S&P500 and the Treasury rates involve some of the volatility variables studied. Moreover, the VIX and *Δ*VIX appear to lead Treasury market rate variations, levels and volatilities. The only exception concerns the relationship of *Δ*VXI-30Y with *Δ*VIX. For the other significant intermarket dynamic relationships, we can see that *Δ*VIX (VIX). Granger-causes *Δ*FVX-5Y, *Δ*TNX-10Y, FVX-5Y and TNX-10Y (*Δ*VIX and VXI-30Y).

[Insert Table 5 here]

Our results are consistent with evidence of volatility spillover between equity and bond markets (see, for example, Fleming et al. 1998 for historical volatility spillover, and Wang 2009 for implied volatility spillover). These spillovers can be justified on the basis of commonalities in the information set that simultaneously affects expectations in both markets, i.e., changes in the macroeconomic variables (see, for example Harvey & Huang 1991). Another explanation is based on cross-market hedging which dictates securing a position in one asset class by taking an offsetting position in another asset class with similar price movements. Portfolio managers often shift funds from stocks into bonds and vice versa due to new information arrival that alters their expectations about stock or bond returns. In this manner, a shock in one market is transferred to the other market due to trading activity and this is consistent with volatility spillover (see, for example, Fleming et al. 1998).

3.5 Principal Component Analysis

Another interesting issue that receives much attention in the empirical literature is if the volatility implied from interest rate derivatives contains important unspanned components. This is of great practical concern since it ultimately determines if bonds should be used to hedge interest rate volatility as is predicted by most 'afine' term structure models. Most of the previous studies use data on LIBOR, swap rates and Eurodollars with mixed results (Andersen & Benzoni, 2010). For example, Collin-Dufresne and Goldstein (2002), and Li and Zhao (2006) report unspanned stochastic volatility factors which drive interest rate derivatives without affecting the term structure. Heidari and Wu (2003) demonstrate that three factors - the level, slope, and curvature term-structure - manage to explain only around 60% of the cross-sectional variability in option-implied volatilities. This finding is puzzling since these same three factors explain over 95% of the variation in the underlying interest rates (see, for example, Litterman and Scheinkman 1991). Andersen and Benzoni (2010) also find unspanned factors in realized interest rate volatility. In an attempt to shed more light on this issue, we undertake a principal component analysis of the three interest rate series under consideration. The results, presented in Table 6a, indicate that the first two factors are able to explain almost all (>99%) of the variation in the three interest rate series. The next step is to test if these two factors are able to explain the variation in the implied volatility indices. As shown in Table 6b, although the first two yield curve principal components are always significant regressands of the VXI indices, they are able to explain only a portion of the variability in the volatility ranging from 37.2% in the case of the VXI-5Y up to 73.5% in the case of the VXI-10Y. These results support to the hypothesis that interest rate volatility are not fully explained by information contained in the yield curve. The analysis is preliminary and could be extended using other approaches from the literature including GARCH and Kalman filter modelling, which is left for future research.

[Insert Table 6a and Table 6b here]

3.6 The effect of news announcements

Finally, we examine the relationship between VXI and four types of news announcements over the period 1/4/96 to 8/29/08. Specifically, we studied CPI and PPI announcements (152 and 150 events, respectively), Federal Open Market Committee (FOMC) meetings (137 events) and employment announcements (148 events). The meeting dates are downloaded from the website of the Federal Reserve. Since Goodhart and Smith (1985), several papers over the years present mixed evidence regarding the impact of such announcements on equity market returns and volatility (for a recent overview of this literature see Chen and Clements 2007). Three recent papers focus on implied volatility market and give more conclusive results. Kearney and Lombra (2004) show that the VIX increases along with the surprise element in employment announcements. Nikkinen and Sahlström (2004) find that the VIX rises prior to and falls after announcements related to the CPI, PPI and FOMC meetings. Chen and Clements (2007) find that the VIX makes a significant drop only on the day of FOMC meetings[6]. Motivated by this research, we attempt to investigate if macroeconomic and monetary news constitute a significant factor in the fixed income market. Following Nikkinen and Sahlström (2004), we adopt the following regression framework in order to examine the impact of news on interest rate volatility:

$$\Delta VXI_t = \alpha + \phi \Delta VXI_{t-1} + \sum_{i=-1}^{1} \beta_i D_{i,t}^{CPI} + \sum_{i=-1}^{1} \gamma_i D_{i,t}^{PPI} + \sum_{i=-1}^{1} \delta_i D_{i,t}^{FOMC} + \sum_{i=-1}^{1} \zeta_i D_{i,t}^{Empl} + \varepsilon_t$$

$$\log(\sigma_t^2) = \omega + \lambda_1 \left| \varepsilon_{t-1} / \sigma_{t-1} \right| + \lambda_2 \, \varepsilon_{t-1} / \sigma_{t-1} + \lambda_3 \log(\sigma_{t-1}^2)$$

Where $D_{-1,t}^{CPI}$ ($D_{+1,t}^{CPI}$) is a dummy variable which takes the value of 1 one day prior (after) to the employment report release day and zero otherwise. On the release day $D_{0,t}^{CPI}$ assumes a value of 1 and zero otherwise. The other dummies are defined accordingly. As Nikkinen and Sahlström (2004), a lagged ΔVXI_t term is used in order to capture persistence in the dependent variable. However, rather than using the GARCH(1,1) specification with normally distributed errors, which Nikkinen and Sahlström (2004) and Chen and Clements (2007) use, we adopt the more flexible EGARCH(1,1) with errors following a Generalized Error Distribution (GED).

The estimation results are presented in Table 7. In general, the announcements studied have a significant effect on the volatility of all the series. In most cases for the CPI and FOCM, this effect is negative on both the day of the announcement and the day before. In most instances for the PPI and EMPL, the effect is positive for the day before the announcement and positive on the day. Implied volatility tends to increase following the announcement day for the CPI, PPI and FOCM. These results are broadly in line with those reported by previous researchers for implied equity volatility and suggest that derivative market investors consider the meetings studied as significant for fixed income pricing. For example, Chen and Clements (2007) report a 2% drop in the VIX on the day of FOCM meetings. Here we find a somewhat milder effect with the VXI-10Y and VXI-30Y falling by 1.18%and 0.92% on the FOCM meeting day. The estimation results in Table 7 offer some further insights on the dynamics of the VXI series. The GED parameter is statistically

[6] There are also the studies of Heuson and Su (2003), and Arnold and Vrugt (2010) that examines the effect of such announcements to the intra-day interest rate volatility and Treasury bond volatility, respectively. Although our results are in line with their results, they are not directly comparable. The first study used a model depended implied volatility and examines the intra-day effect. The second study uses Treasury bond volatility (which is backward looking, in contrast to the model free and forward looking VXI) in order to show that monetary policy is the most significant predictor of the uncertainty in Treasury markets.

significant and equal or less than 2 in all cases suggesting that the errors have a fat-tailed distribution. Implied volatility is highly persistent since all the λ_3 GARCH coefficients in the conditional variance equation are well above zero. The effect of news is asymmetric for all series ($\lambda_2 \neq 0$) and in all cases there is a leverage effect ($\lambda_2 < 0$).

[Insert Table 7 here]

4. Interest Rate Volatility Risk Management

Following the large success of equity implied volatility indices, CBOE introduced volatility futures and options written on the VIX (March 2004 and February 2006, respectively). Futures on the VXD were introduced in April 2005 and European options followed soon. According to a recent CBOE Futures Exchange press release (December 3, 2009), year-to-date through November 2009, almost 29 million VIX options have changed hands, making VIX options the second most-actively traded index option at the exchange. Motivated by the success of the VIX market and the relative magnitude of interest rate volatility risk demonstrated in the present study, we believe that it is useful to discuss relevant solutions for trading and managing interest rate volatility risk.

A first step in the direction of building pricing and risk management models is to understand and approximate empirically the continuous time dynamics of the volatility processes considered. The models under consideration are nested in the following stochastic differential equation, under the real probability measure *P*:

$$dV_t = \mu(V_t, t)\,dt + \sigma(V_t, t)\,dW_t + y(V_t, t)\,dq_t \tag{4}$$

where, V_t is the value of *VXI* at time *t*, W_t is a standard Wiener process, and $\mu(V_t, t)$, $\sigma(V_t, t)$ and $y(V_t, t)$ are the drift, the diffusion and the jump amplitude coefficients, respectively. The jump component is driven by a Poisson process q_t with constant arrival parameter λ, i.e. $\Pr\{dq_t = 1\} = \lambda dt$ and $\Pr\{dq_t = 0\} = 1 - \lambda dt$. dW_t, dq_t and y are assumed to be mutually independent. We allow $\mu(V_t, t)$, $\sigma(V_t, t)$ and $y(V_t, t)$ to be general functions of time and the interest rate volatility. Hence, by changing the specification of the above coefficients we come up with the following five models:

Mean Reverting Square-Root process (MRSRP) $$dV_t = \kappa(\theta - V_t)\,dt + \sigma\sqrt{V_t}\,dW_t \tag{5}$$

Mean Reverting Logarithmic process (MRLP) $$d\ln(V_t) = \kappa(\theta - \ln(V_t))\,dt + \sigma dW_t \tag{6}$$

Constant Elasticity of Variance (CEV) $dV_t = \kappa\left(\theta - V_t\right)dt + \sigma V_t^{\gamma} dW_t$ (7)

MRSRP with Jumps (MRSRPJ) $dS_t = \kappa\left(\theta - V_t\right)dt + \sigma\sqrt{V_t}dW_t + \left(y-1\right)V_t dq_t$ (8)

MRLP with Jumps (MRLPJ) $d\ln\left(V_t\right) = \kappa\left(\theta - \ln\left(V_t\right)\right)dt + \sigma dW_t + \left(y-1\right)V_t dq_t$ (9)

The choice of models is based on four criteria: economic intuition, stationarity, mathematical tractability, and popularity among the researchers. Random walk processes make no economic sense, as they imply that volatility can drift off to arbitrarily high levels. The inclusion of jump diffusions is motivated by our empirical findings concerning abrupt upward and downward changes in the VXI indices. All jump-diffusion processes are the natural extensions to their diffusion analogues, so as to facilitate a direct comparison. The jump size distribution is assumed exponential, which allows for the derivation of the characteristic function of the examined processes (see Duffie et al., 2000; Psychoyios et al., 2010 for more details on the specifications of the jump-diffusions processes under consideration).[7] Without bounded lower support on the jump size distribution, it is possible that in some of the models the volatility becomes negative. We could restrict the jump sizes to be positive to avoid such problems (for similar assumptions see Broadie et al., 2007; Eraker, 2004). However, we deliberately use "unrestricted" jump-diffusion models in order to account for the empirically observed negative jumps in implied volatility. The models under consideration are widely used to model the dynamics of the instantaneous and implied volatility and variance for equities in continuous time setting (see, among others, Brenner et al, 2006; Chan et al., 1992; Eraker, 2004; Jones, 2003).

Estimation is done in MATLAB using a Maximum Likelihood (ML) approach (see Psychoyios et al. 2010 for details on the estimation methodology). The ML results for the three indices under study are given in Table 8. The table also provides two performance measures: the likelihood ratio test and the Bayes Information Criterion (BIC). The likelihood ratio test can be used only for comparisons between nested models, i.e., between MRLP/MRLPJ, CEV/MRSRP/MRSRPJ, respectively.[8] Comparison of the non-nested models can be made using the BIC

[7] The derivation of the characteristic functions can be provided by the authors upon request.

[8] The likelihood ratio test statistic for comparing the nested models is given by: $LR = -2\times\left(\Im_R - \Im_U\right) - \chi^2(df)$, where df is the number of parameter restrictions and $\Im_R$, $\Im_U$ are the log-likelihoods of the restricted and unrestricted model, respectively. The 5% level critical values are: $\chi^2(df) = \left[3.84\,(df=1), 7.82\,(df=3), 9.49\,(df=4)\right]$. In order to facilitate the direct comparison of the

criterion. The results provide several interesting insights. First, the MRSRPJ is the best performing model. Second, the jump-diffusion processes significantly outperform their diffusion counterparts. Third, although MRSRPJ is the best performing process, its diffusion counterpart (MRSRP) is one the worst performing models among the diffusion processes. In this case, CEV process dominates all the other models, closely followed by the MRLP process. The only exception occurs in the case of the VXI-5Y, where MRSRP performs better than CEV. A further investigation of the results, regarding the diffusion processes, reveals that the higher the dependence of the volatility of volatility parameter (σ) on the current level of interest rate implied volatility (i.e., MRLP and CEV), the higher the fitting performance.[9] In general, the findings indicate that interest rate implied volatility has a proportional, mean reverting structure with jumps, i.e., they are subject to large movements that cannot be explained by standard diffusion processes. These three main conclusions are supported by all the performance criteria used and hold for all interest rate implied volatility indices. Moreover, they are consistent with the descriptive analysis findings from the previous section, namely: the existence of jumps, the nonormality of returns and the stationarity of the interest rate implied volatility processes.

[Insert Table 8 here]

Apart from improving the fitting performance, the introduction of the jump component also has two more effects. First, it significantly reduces the diffusion volatility parameter (σ), suggesting that jumps account for a substantial component of volatility and help to capture additional skewness. For example, in Table 8, in all three volatility indices the diffusion volatility drops on average to one-third its prior level (see also Das, 2002 for similar results regarding interest rate levels). Second, it significantly reduces the speed of the mean reversion parameter. This is caused by the fact that many jumps, as it can be seen also in Figure 1, have a persistent effect and the process does not pull back to its long run mean. The latter may imply that models with non-linear long run mean, or regime-switching jump diffusion models, may be more appropriate to

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logarithmic processes (i.e., MRLPJ and MRLP). with the rest of the processes, we apply the following change of variable to the log-likelihoods of the MRLP and MRLPJ: $\Im'_R = \sum_{t=1}^{T} \ln(V_{t+\tau}) + \Im_R$.

[9] Two additional specifications are also examined: $dV_t = \kappa(\theta - V_t)dt + \sigma dW_t$, and its counterpart augmented by jumps $dS_t = \kappa(\theta - V_t)dt + \sigma dW_t + (y-1)V_t dq_t$. However, the subsequent analysis indicates that the processes are misspecified and their performance is inferior in relation to the other models. Due to space limitations neither the processes nor the results are presented.

capture the characteristics of the interest rate implied volatility (see, for example, Bakshi et al., 2006). However, these models are beyond the scope of this research since they require too many parameters to be estimated and are accompanied by substantial mathematical complexity, both of which make derivative pricing challenging. In order to check for the stability of the above general results we estimate all the all the processes again over the period from 1/4/96 to 31/12/07. We eliminate all 2008 data, which correspond to the latest credit crash and could bias the results in favor of finding jumps. The ranking of the processes as well as the main conclusions remain the same (due to space limitations we do not include the table with the estimated parameters; however, the results are available from the authors upon request).

We have to note that the estimated parameters of the processes that are used to model the dynamics of the interest rate implied volatility index cannot be used as a proxy for the parameters of the instantaneous volatility process. This is because implied and instantaneous volatility processes do not share the same structure. However, for the processes and assumptions underhand it can be proved that under the risk-adjusted probability measure the parameters of the implied variance process (i.e., VXI^2) are related to those of the instantaneous variance (see also Wu, 2010).[10]

To test the out-of-sample performance of the estimated models, we use the time period from Oct/2006 to Aug/2008 (about 470 observations or 15% of the total sample). Table 9 shows the unconditional mean square error of each process (MSE). We can see that the results are similar to those obtained from the MLE (Table 8). In general, the processes augmented by jumps perform better than their continuous counterparts. In particular, MRSRPJ performs best in all of the cases, while MRSRP and CEV perform equally well along the diffusion processes. Surprisingly enough, the of MRLP and MRLPJ has by far the worst performance. A closest inspection of the Table 9 reveals that the differences in the out-of-sample performance between jump-diffusion and diffusion declines as the “maturity” of the index increases. The latter is expected, since the indices with the highest maturity (VXI-30Y and VXI-10Y) are less volatile than the VXI 5Y (Figure 1 and Table 8), as such the comparative advantage of the jump-diffusion processes is less important.

[Insert Table 9 here]

Before proceeding to futures valuation, we must rewrite equation (8) under the risk neutral probability measure *Q*. Following Pan (2002) we assume that the volatility risk is proportional to

[10] The proofs of these statements are available from the authors upon request

the current level of interest rate implied volatility, i.e., ζV_t. We also assume that there is no "volatility of volatility", "jump" risk, and model risk.[11] So, the volatility process under the risk neutral measure is given by:

$$dV_t = \left(k(\theta - V_t) - \zeta V_t\right)dt + \sigma\sqrt{V_t}dz + \left(y-1\right)dq \qquad (10)$$

or, equivalently,

$$dV_t = k^*(\theta^* - V_t) + \sigma\sqrt{V_t}dz + \left(y-1\right)dq \qquad (11)$$

where $k^* = k + \zeta$ and $\theta^* = \dfrac{k\theta}{k+\zeta}$.

Denote $F_t(V,T)$ the price of a futures contract on V_t at time t with maturity T. Under the risk-adjusted equivalent martingale measure Q, $F_t(V,T)$ equals the conditional on the information up to time t expectation of V_T at time T, or:

$$F_t = E_t^Q(V_T) \qquad (12)$$

Since the MRSRPJ process does not have a known density, $E_t^Q(V_T)$ is derived by differentiating the characteristic function once with respect to s and then evaluating the derivative at $s= -i$ (see Psychoyios *et al.*, 2010 for the derivation of the characteristic function):

$$E_t^Q\left(V_T\right) = V_t e^{-k^*(T-t)} + \theta^*(1-e^{-k^*(T-t)}) + \frac{\lambda}{k^*}(1-e^{-k^*(T-t)})\frac{1}{\eta} \qquad (13)$$

In order to obtain the valuation formula for a European volatility call, we follow the approach of Bakshi and Madan (2000). The price $C(V_t,\tau;K)$ of the call option with strike price K and τ time to maturity is given by:

$$C(V_t,\tau;K) = W(V_t,\tau)\Pi_1(t,\tau) - e^{-r\tau}K\Pi_2(t,\tau) \qquad (14)$$

where $W(V_t,\tau) = e^{-\left(r+k^*\right)\tau}V_t(t,\tau) + e^{-r\tau}(1-e^{-k^*\tau})\left(\theta^* + \dfrac{\lambda}{k^*\eta}\right)$

The Π1 and Π2 probabilities are given by the equation 13 at Bakshi and Madan (2000), and the characteristic function of V_t is given by the equation 26 at Psychoyios et. al 2010.

[11] We cannot use no-arbitrage arguments to price interest rate volatility derivatives since the market is not complete. Only for the case of VXI-futures we can derive arbitrage free bounds, following the same methodology as in the case of VIX-futures of Carr and Wu (2006). However, in order to do so, Carr and Wu assume a very liquid market of plain vanilla options and exotic OTC derivatives, such as forward-start at-the-money forward call options, written on the underlying of each VXI index.

5. Summary

This paper concentrates on how to measure and manage interest rate volatility risk. We use model-free estimators in order to analyze the implied volatility of the US treasury rates using data from CBOE on interest rate options. Over a 12-year period, we constructed three daily interest rate implied volatility indices with 5 years, 10 years and 30 years maturities. The results suggest that interest rate volatility is substantial in magnitude and variation, and comparable to equity implied volatility as measured by VIX. We derive negative interest rate volatility risk premium, which is priced in the market and significantly related to equity volatility risk. An important new result is that our estimates of interest rate volatility risk premia have a significant time varying-correlation with equity market volatility risk premia. Another interesting finding is that the interest rate implied volatility indices, as is the case with the VIX index, offer valuable diversification opportunities to bond and equity investors. This is because the proposed indices have a strong negative correlation with both interest rate levels and equity market implied volatility index levels. We also show that interest rate volatility is linked to macroeconomic and monetary news announcements although it is only partially spanned by information contained in the yield curve. In particular, macroeconomic and monetary announcements decrease (increases) the implied interest rate volatility the day before (after) an announcement. A new result is that this effect varies across the term structure and becomes more prominent at the longer maturities studied. Finally, our VXI model-free estimators of interest rate volatility allow investment diversification potential and used as market indicators and benchmarks in order to develop equilibrium models for pricing futures and options derivative assets.

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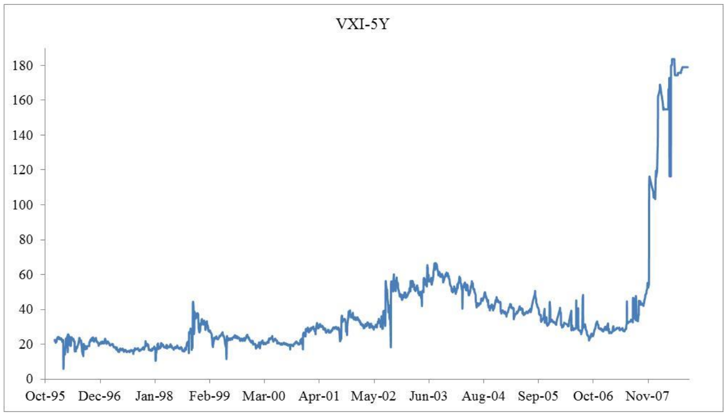
VXI-5Y
180
160
140
120
100
80
60
40
20
0
Oct-95 Dec-96 Jan-98 Feb-99 Mar-00 Apr-01 May-02 Jun-03 Aug-04 Sep-05 Oct-06 Nov-07


FVX-5Y
0.08
0.07
0.06
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Oct-95 Dec-96 Jan-98 Feb-99 Mar-00 Apr-01 May-02 Jun-03 Aug-04 Sep-05 Oct-06 Nov-07


VXI-10Y
180
160
140
120
100
80
60
40
20
0
Oct-95 Dec-96 Jan-98 Feb-99 Mar-00 Apr-01 May-02 Jun-03 Aug-04 Sep-05 Oct-06 Nov-07


TNX-10Y
0.08
0.07
0.06
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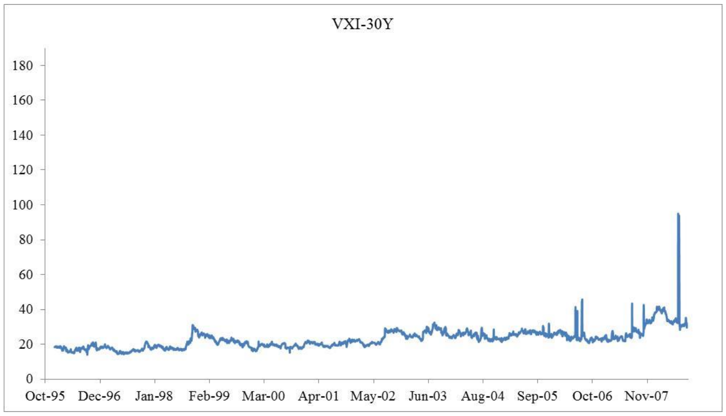

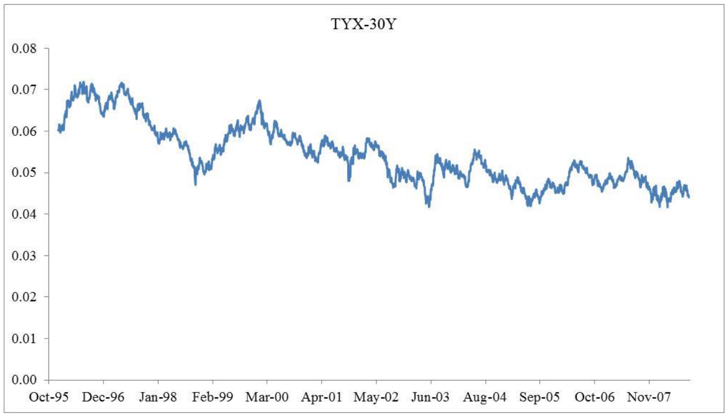

Figure 1. VXI Indices and their corresponding treasury rates

This figure plots the time-series of the VXI-5Y, VXI-10Y and VXI-30Y interest rate implied volatility indices, respectively, (1st column), and their corresponding treasury rates, FVX, TNX, TYX, respectively (2nd column). Data is daily and the sample spans the period 1/4/96 to 8/29/08.

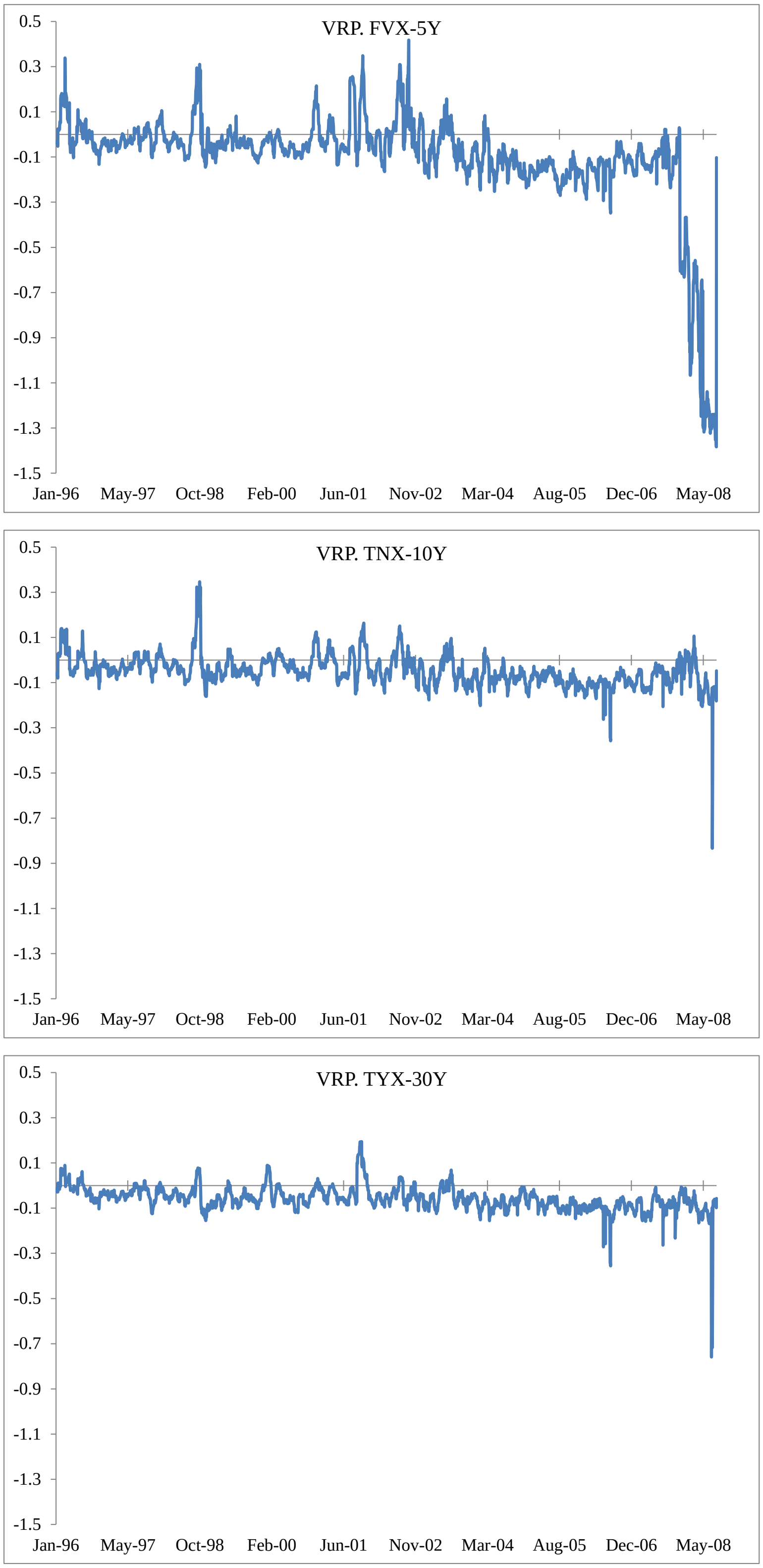

Figure 2. Interest rate volatility risk premia.

This figure plots the time-series of the interest rate volatility risk premia. VRP.FVX-5Y, VRP.TNX-10Y and VRP.TYX-30Y denotes the 5, 10 and 30 years volatility risk premium, respectively.Each volatility risk premium is calculated as the difference between the realized volatility (*RV*) and the risk neutral interest rate expected volatility (*VXI*). Data is daily and the sample spans the period from 1/4/96 to 8/29/08

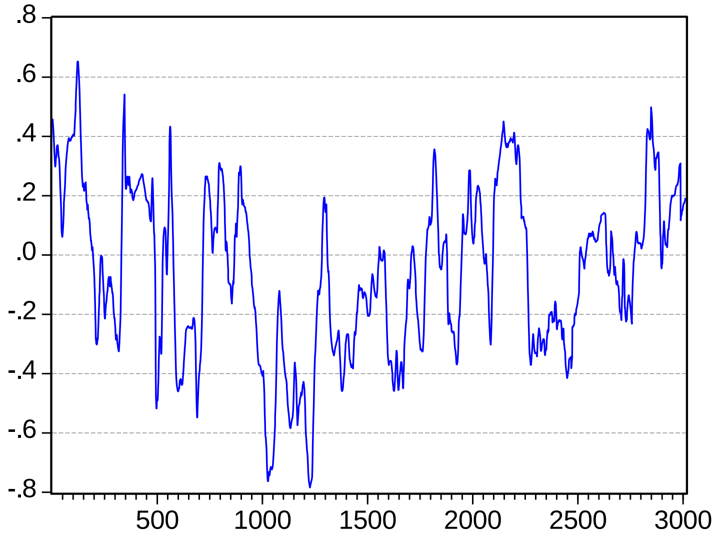

Figure 3. Correlation window between VRP.TYX-30Y and VRP.SP500

The figure plots the correlation coefficient between VRP.TYX-30Y and VRP.SP500 using a rolling window of 125 trading days (6 months). Data is daily and the sample spans the period from 1/4/96 to 8/29/08. The two-tailed 5% and 1% critical values for the absolute value of the correlation coefficient are 17.5% and 22.87%, respectively.

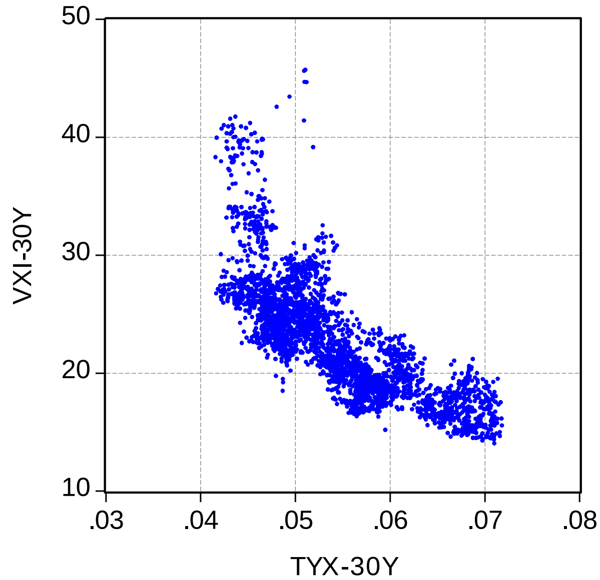

Figure 4. VXI-30Y against underlying TYX-30Y Treasury rates.

The figure plots VXI-30Y with respect to TYX-30Y. VXI-30Y is expressed in percent, while TYX-30Y is expressed in absolute numbers. Data is daily and the sample spans the period from 1/4/96 to 8/29/08.

Table 1: Descriptive statistics of volatility indices.

Columns 2, 3 and 4 report descriptive statistics of the proposed interest rate volatility indices. VXI-5Y denotes the model free implied volatility of the yield to maturity of the 5 year Treasury note (FVX), VXI-10Y and VXI-30Y denote the model free implied volatility of the yield to maturity of the 10 year and 30Y Treasury bond, respectively (TNX and TYX, respectively). Columns 5 and 6 report the descriptive statistics of MOVE and VIX, respectively. The MOVE is obtained from Bloomberg, while the VIX is obtained from CBOE. MOVE index is quoted in basis points whereas all the other indices are expressed in percent. All data is daily and the sample spans the period from 1/4/96 to 8/29/08. Jarque-Bera is a test of normality. $\rho(1)$ is the coefficient of an AR(1) model with a constant.

	VXI-5Y	**VXI-10Y**	**VXI-30Y**	**MOVE**	**VIX**
Observations	3,159	3,159	3,159	3,159	3,159
Mean	39.3429	27.0728	22.9346	99.3071	20.4110
Median	29.7287	25.3819	22.4687	99.6800	19.7800
Max	183.6313	114.4094	94.9674	195.0000	45.7400
Min	5.9831	6.4852	13.9961	51.2000	9.8900
St. Deviation	32.9273	8.3109	5.3406	22.8868	6.7018
CV	0.8369	0.3070	0.2329	0.2305	0.3283
Skewness	3.0927	1.0918	2.2918	0.3165	0.7741
Kurtosis	12.5641	6.7058	23.7647	2.9542	3.4807
Jarque-Bera	17076.00	2435.30	59518.00	53.01	345.92
$\rho(1)$	0.9891	0.9624	0.8897	0.9834	0.9832
	ΔVXI-5Y	**ΔVXI-10Y**	**ΔVXI-30Y**	**ΔMOVE**	**ΔVIX**
Mean	0.0028	0.0019	0.0018	0.0009	0.0016
Median	0.0000	0.0000	0.0011	-0.0009	0.0000
Max	1.8604	1.5804	1.9344	0.2875	0.6422
Min	-0.7241	-0.6800	-0.6755	-0.1653	-0.2591
St. Deviation	0.0729	0.0623	0.0665	0.0413	0.0550
Skewness	9.8764	7.8762	16.3295	0.9183	1.1050
Kurtosis	212.8932	193.5014	467.8288	8.2960	11.1054
Jarque-Bera	5848300.00	4807900.00	28571000.00	53.01	9287.30
$\rho(1)$	-0.1698	-0.2293	-0.2745	0.0312	-0.0482

Table 2: Descriptive statistics of volatility risk premia

The first three columns report descriptive statistics of the interest rate volatility risk premia. VRP.FVX-5Y, VRP.TNX-10Y and VRP.TYX-30Y denotes the 5, 10 and 30 years volatility risk premium, respectively. The last column reports the descriptive statistics of the S&P500 volatility risk premium (VRP.SP500). VRP.SP500 is calculated as the difference between S&P500 realized volatility and the VIX index, which is used as a proxy of the S&P500 risk neutral expected volatility. S&P500 realized volatility is estimated over the next month using the sum of squared S&P500 index daily returns. All data is daily and the sample spans the period from 1/4/96 to 8/29/08. Jarque-Bera is a test of normality. $\rho(1)$ is the coefficient of an AR(1) model with a constant.

	VRP.FVX-5Y	**VRP.TNX-10Y**	**VRP.TYX-30Y**	**VRP.SP500**
Mean	-0.1029	-0.0477	-0.0571	-0.0194
Median	-0.0631	-0.0552	-0.0601	-0.0245
Max	0.4114	0.3468	0.1944	0.2386
Min	-1.3833	-0.8336	-0.7588	-0.2045
St. Deviation	0.2300	0.0661	0.0522	0.0543
Skewness	-3.3782	0.2354	-0.8980	0.9781
Kurtosis	16.7485	11.9236	24.3514	6.0989
Jarque-Bera	31292.00	10503.00	59352.00	1784.60
$\rho(1)$	0.9866	0.9232	0.8668	0.9564
	ΔVRP.FVX-5Y	**ΔVRP.TNX-10Y**	**ΔVRP.TYX-30Y**	**ΔVRP.SP500**
Mean	-0.0714	0.0445	-0.0681	0.3869
Median	-0.0013	-0.0074	0.0002	-0.0254
Max	22.7400	197.4842	77.6308	1973.2390
Min	-117.1709	-114.1927	-83.1226	-1951.7070
St. Deviation	3.1409	5.4068	2.7911	52.5937
Skewness	-27.9670	21.8505	-5.4509	1.6669
Kurtosis	962.1677	930.5794	510.7815	1252.4380
Jarque-Bera	121000000.00	113000000.00	33707000.00	204000000.00

Table 3. Correlation analysis of the *VRP* with respect to the underlying interest rates.

The table reports the correlation coefficients (%) of volatility risk premia with respect to underlying interest rates. Data is daily and the sample spans the period from 1/4/96 to 8/29/08. The two-tailed 5% and 1% critical values for the absolute value of the correlation coefficient are 3.49% and 4.59%, respectively.

	VRP.FVX-5Y	**VRP.TNX-10Y**	**VRP.TYX-30Y**	**VRP.SP500**	**FVX**	**TNX**
VRP.FVX-5Y	100.0	51.2	46.4	-5.9	32.1	35.1
VRP.TNX-10Y	51.2	100.0	78.4	-2.8	18.0	22.3
VRP.TYX-30Y	46.4	78.4	100.0	-7.7	17.6	25.4
VRP.SP500	-5.9	-2.8	-7.7	100.0	-7.9	-8.7
FVX	32.1	18.0	17.6	-7.9	100.0	96.9
TNX	35.1	22.3	25.4	-8.7	96.9	100.0
TYX	37.6	28.0	34.5	-10.1	85.1	95.0

Table 4. Correlation analysis of VXI levels and changes with other variables.

The table reports the correlation coefficients (%) of the VXI indices with respect to underlying interest rates, the MOVE and the S&P500 index. Panel A reports the correlation coefficients of the levels, while Panel B reports the correlation coefficients of the changes. Data is daily and the sample spans the period from 1/4/96 to 8/29/08. The two-tailed 5% and 1% critical values for the absolute value of the correlation coefficient are 3.49% and 4.59%, respectively.

PANEL a									
	VXI-5Y	**VXI-10Y**	**VXI-30Y**	**MOVE**	**FVX-5Y**	**TNX-10Y**	**TYX-30Y**	**SP500**	**VIX**
VXI-5Y	100.0	80.3	75.0	44.4	-61.3	-58.7	-52.3	-1.0	-20.7
VXI-10Y	80.3	100.0	86.2	49.0	-85.8	-80.3	-68.9	-2.2	-3.7
VXI-30Y	75.0	86.2	100.0	29.7	-71.8	-74.8	-74.4	-1.5	-23.5
MOVE	44.4	49.0	29.7	100.0	-27.2	-11.2	7.7	-1.8	21.6
FVX-5Y	-61.3	-85.8	-71.8	-27.2	100.0	96.9	85.1	2.5	-2.3
TNX-10Y	-58.7	-80.3	-74.8	-11.2	96.9	100.0	95.0	2.2	6.3
TYX-30Y	-52.3	-68.9	-74.4	7.7	85.1	95.0	100.0	2.0	19.1
SP500	-0.9	-2.0	-1.2	-1.9	2.2	1.8	1.6	100.0	-10.9
VIX	-20.7	-3.7	-23.5	21.6	-2.3	6.3	19.1	-10.2	100.0

***P*ANEL B**									
	ΔVXI-5Y	**ΔVXI-10Y**	**ΔVXI-30Y**	**ΔMOVE**	**ΔFVX-5Y**	**ΔTNX-10Y**	**ΔTYX-30Y**	**ΔSP500**	**ΔVIX**
ΔVXI-5Y	100.0	38.8	19.1	4.7	0.3	0.6	-1.0	0.1	-1.8
ΔVXI-10Y	38.8	100.0	57.1	11.9	3.5	4.0	2.7	0.7	-2.3
ΔVXI-30Y	19.1	57.1	100.0	8.3	1.5	0.5	-0.4	1.6	-3.2
ΔMOVE	4.7	11.9	8.3	100.0	10.2	13.5	15.4	0.2	-0.5
ΔFVX-5Y	0.3	3.5	1.5	10.2	100.0	94.6	82.5	0.3	-2.0
ΔTNX-10Y	0.6	4.0	0.5	13.5	94.6	100.0	93.1	0.4	-2.0
ΔTYX-30Y	-1.0	2.7	-0.4	15.4	82.5	93.1	100.0	0.3	-1.3
ΔSP500	0.1	0.7	1.6	0.2	0.3	0.4	0.3	100.0	-71.6
ΔVIX	-1.8	-2.3	-3.2	-0.5	-2.0	-2.0	-1.3	-71.6	100.0

Table 5. Granger-causality analysis.

The table reports the results of Granger-causality tests for 1 lag of the null hypothesis that “Variable A” does not cause “Variable B”. Both levels and changes of the volatility indices and interest rates are considered in order to capture possible dynamic level effects. One (two) stars denote statistical significance at the 95% (99%) level.

Variable A	**Variable B**	**F-Statistic**	**Variable A**	**Variable B**	**F-Statistic**
ΔVXI-5Y	ΔFVX-5Y	7.92**	ΔVIX	ΔTNX-10Y	6.76**
ΔVXI-5Y	ΔTNX-10Y	10.72**	ΔVIX	FVX-5Y	5.06*
ΔVXI-5Y	ΔTYX-30Y	7.74**	ΔVIX	TNX-10Y	5.36*
ΔVXI-5Y	FVX-5Y	6.99**	VIX	ΔVIX	19.53**
ΔVXI-5Y	VXI-10Y	5.55*	VIX	VXI-30Y	11.61**
ΔVXI-5Y	VXI-5Y	14.69**	ΔFVX-5Y	ΔVXI-10Y	20.45**
ΔVXI-5Y	TNX-10Y	9.67**	ΔFVX-5Y	ΔVXI-30Y	9.60**
ΔVXI-5Y	TYX-30Y	8.04**	ΔFVX-5Y	ΔTNX-10Y	4.17*
VXI-5Y	ΔVXI-30Y	9.86**	ΔFVX-5Y	ΔTYX-30Y	9.81**
VXI-5Y	VXI-10Y	90.48**	ΔFVX-5Y	VXI-10Y	14.19**
VXI-5Y	VXI-30Y	229.26**	ΔFVX-5Y	VXI-30Y	6.28*
VXI-10Y	VXI-30Y	154.98**	FVX-5Y	VXI-10Y	188.57**
VXI-10Y	MOVE	8.17**	FVX-5Y	VXI-30Y	210.97**
ΔVXI-30Y	ΔVIX	4.98*	FVX-5Y	MOVE	5.24*
ΔVXI-30Y	VXI-10Y	387.4**	ΔTNX-10Y	ΔVXI-10Y	14.30**
ΔVXI-30Y	VXI-30Y	1130.28**	ΔTNX-10Y	ΔVXI-30Y	6.21*
VXI-30Y	VXI-10Y	14.30**	ΔTNX-10Y	VXI-10Y	7.55**
VXI-30Y	MOVE	5.13*	ΔTNX-10Y	TNX-10Y	4.16*
ΔMOVE	ΔVXI-10Y	31.21**	TNX-10Y	VXI-10Y	125.39**
ΔMOVE	ΔVXI-30Y	5.50*	TNX-10Y	VXI-30Y	255.74**
ΔMOVE	ΔVXI-5Y	50.38**	TNX-10Y	MOVE	5.96*
ΔMOVE	VXI-5Y	12.4**	ΔTYX-30Y	ΔVXI-10Y	12.69**
ΔMOVE	MOVE	7.58**	ΔTYX-30Y	ΔVXI-30Y	6.46*
MOVE	ΔVXI-10Y	6.89**	ΔTYX-30Y	VXI-10Y	4.82*
MOVE	ΔVXI-5Y	10.76**	ΔTYX-30Y	TNX-10Y	6.04*
MOVE	VXI-30Y	5.92*	TYX-30Y	VXI-10Y	62.90**
MOVE	ΔMOVE	34.66**	TYX-30Y	VXI-30Y	250.64**
ΔVIX	ΔFVX-5Y	7.97**	TYX-30Y	MOVE	6.15*

Table 6a. Principal Components Analysis of interest rate levels.

The first three rows report the eigenvector of each variable with respect to the three components, PC1, PC2 and PC3 The last three rows report the eigenvalue, the percentage of the explained variation and the cumulative explained variation, respectively. Data is daily and the sample spans the period from 1/4/96 to 8/29/08.

	PC1	**PC2**	**PC3**
FVX-5Y	-0.572	0.672	0.470
TNX-10Y	-0.592	0.058	-0.804
TYX-30Y	-0.568	-0.738	0.365
Eigenvalue	2.848	0.150	0.002
Variance Explained (%)	94.917	5.004	0.079
Cumulative Variance Explained (%)	94.917	99.921	100.000

Table 6b. Regression of the VXI indices against the yield curve principal components

We run the following regression for the VXI indices: $VXI=b_1PC1+ b_2PC2+\varepsilon$. Data is daily and the sample spans the period from 1/4/96 to 8/29/08. One (two) stars denote statistical significance at the 95% (99%) level. Heteroskedasticity and autocorrelation consistent covariances and standard errors are estimated using the Newey and West (1987) approach.

	VXI-5Y	**VXI-10Y**	**VXI-30Y**
PC1	11.510**	3.959**	2.392**
PC2	-13.228**	-6.369**	0.814**
Constant	39.343**	27.073**	22.935**
R-squared	0.372	0.735	0.575

Table 7. Relationship between VXI indices and macroeconomic news announcements.

The table reports the coefficients of regression between the changes of the VXI indices and the dummies for four types of macroeconomic news announcements over the period from 1/4/96 to 8/29/08. One (two) stars denote statistical significance at the 95% (99%) level.

	ΔVXI-5Y	**ΔVXI-10Y**	**ΔVXI-30Y**
α	9.75E-09	0.0010**	0.0016**
ϕ	0.0090	-0.0324**	-0.0867**
β_{-1}	-0.0022**	-0.0021**	-0.0010
β_0	-0.0017**	-0.0059**	-0.0083**
β_{+1}	4.07E-04	0.0023*	0.0032**
γ_{-1}	0.0027**	0.0051**	0.0009
γ_0	0.0006	-0.0081**	-0.0053**
γ_{+1}	0.0015	0.0055**	0.0013
δ_{-1}	0.0004	-0.0074**	-0.0028**
δ_0	-0.0032**	-0.0044**	-0.0064**
δ_{+1}	0.0080**	0.0022*	0.0108**
ζ_{-1}	0.0063**	0.0074**	0.0034**
ζ_0	-0.0121**	-0.0117**	-0.0065**
ζ_{+1}	0.0001	-0.0010	-0.0019
ω	-0.6282**	-2.5272**	-1.7718**
λ_1	0.2338**	0.3372**	0.1400**
λ_2	-0.0763**	-0.1294**	-0.0486*
λ_3	0.9295**	0.6616**	0.7637**
GED paremeter	0.6251**	0.7012**	0.7449**

Table 8. Estimation results of diffusion and jump diffusion processes over the period from 1/4/96 to 8/29/08 for the three interest rate implied volatility indices.

	VXI-5Y					VXI-10Y					VXI-30Y				
Paramet	MRS	MRL	CEV	MRSR	MRL	MRS	MRL	CEV	MRSR	MRL	MRS	MRL	CEV	MRSR	MRL
k	3.7186	5.3604	4.0004	1.3520	2.0751	6.0155	4.9855	4.5465	1.0961	2.0642	5.6594	4.5085	2.6467	1.3444	2.0643
	(1.981	(2.575	(3.851	(17.988	(2.869	(5.973	(5.462	(1.882	(19.967	(2.1909	(5.853	(5.246	1.4347	(0.8349)	(8.281
θ	56.705	3.6057	33.262	21.7440	3.8925	27.330	3.2644	24.663	19.5048	3.7099	23.047	3.1165	25.279	22.0672	3.9128
	(2.571	(16.75	(6.614	(10.881	(23.46	(20.99	(61.17	(28.12	(16.418	(15.016	(28.39	(78.29	0.6939	(3.4991)	(85.57
σ	5.5099	1.0109	1.0831	1.3841	0.2680	5.2769	0.9411	0.4000	1.6389	0.3258	3.3725	0.6345	0.8208	1.2561	0.2812
	(79.19	(79.17	(75.50	(156.05	(51.75	(78.46	(78.64	(76.16	(120.39	(44.481	(78.51	(78.69	2.4081	(90.560	(55.20
γ	0.5	1.0	0.9857	0.5	1.0	0.5	1.0	1.1104	0.5	1.0	0.5	1.0	0.9105	0.5	1.0
			(1.275					(1.741					(1.848		
λ	-	-	-	33.1044	39.176	-	-	-	46.8874	44.484	-	-	-	90.3816	39.445
				(4.0786	(10.79				(3.4737)	(5.4977				(2.8870)	(6.165
$1/\eta$	-	-	-	2.0315	11.454	-	-	-	2.2845	10.484	-	-	-	3.3237	11.571
				(3.6637	(4.806				(2.0673)	(3.4654				(2.2993)	(9.823
BIC	13,295	13,584	13,560	6,700	8,302	12,231	11,635	11,628	6,266	7,033	8,940	8,220	8,188	4,672	5,270
$\Im$	-6,636	-6,780	-6,764	-3,330	-4,131	-6,103	-5,805	-5,798	-3,105	-3,3513	-4,458	-4,098	-4,078	-2,315	-2,550

Numbers in brackets denote t-statistics. The table also gives the Log-Likelihood value ($\Im$) and the Bayes Information Criterion (BIC)

Table 9: Out of sample performance.

To test the out-of-sample performance of the estimated models, the following approach is adopted. For a given implied volatility index, every estimated model is used to generate 5,000 simulation paths of implied volatility over the period Oct/2006 to Aug/2008. To eliminate the dependence on the simulated path, an average path is calculated over the 5,000 ones. The average path is compared with the actual implied volatility path. The squared percentage error $(\frac{VXI_t^S - VXI_t^A}{VXI_t^A})^2$ is recorded for any point in time t where VXI_t^S, VXI_t^A, are the average simulated implied volatility and the actual implied volatility, respectively. Then, the average squared percentage error is calculated.

	MRSRP	MRLP	CEV		MRSRPJ	MRLPJ
VXI-5Y	0.368	0.452	0.369		0.249	0.398
VXI-10Y	0.174	0.287	0.196		0.148	0.269
VXI-30Y	0.158	0.261	0.140		0.069	0.268

Appendix: Unit Root tests

Table 10: Unit Root tests of Interest rate volatility indices

The null hypothesis assumes a unit root with individual unit root processes. Tests were performed assuming a constant and a deterministic trend. The probabilities reflect Fisher tests computed using an asymptotic Chi-square distribution. Lag length selection for the tests was based on the SIC while the bandwidth for the spectral estimation was selected using a Newey-West approach and a Bartlett kernel. For a description of the Augmented Dickey Fuller test (ADF) and the Phillips-Perron (PP), see Mills and Markellos (2008).

Test	Statistic	Probability
ADF	18.7924	0.0045
PP	326.335	0.0000