# Mean and variance equation dynamics: Time deformation, GARCH and a robust analysis of the London housing market

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**ABSTRACT**

The potential relationship between time deformation and generalised autoregressive conditional heteroscedasticity (GARCH) is examined. Despite time deformation and GARCH being mean and variance equation phenomena respectively, they are argued herein to share a common motivation relating to the examination of changes in the temporal evolution of time series processes. Via extensive simulation analysis, a close connection between the two concepts is established. It is found that the presence of GARCH can result in the spurious detection of time deformation, particularly when examining the heavy-tailed distributions and volatile data typically considered in empirical finance. It is shown that although the application of heteroskedasticity corrected covariance matrix estimators often increases, rather than corrects, the detected oversizing of the tests of time deformation, the application of GARCH filters does provide a solution to size distortion. The findings of the experimental analysis are drawn upon to provide a robust empirical examination of the London housing market where evidence of overwhelming and widespread nonlinearity is detected in the form of time deformation. The implications of these findings for the conduct of future, and the interpretation of previous, research are discussed.

# 1 Introduction

Changes in the nature of the evolution of financial and economic variables have long been of interest to empiricists in both finance and economics. Following Engle (1982), Bollerslev (1986) and Taylor (1986), the notions of autoregressive conditional heteroscedasticity (ARCH) and generalised ARCH (GARCH) have come to occupy prominent positions within finance as means of examining such changes in the form of movements in the volatility of a series. Within economics, consideration of changes in the evolution of variables has often occurred in connection with the business cycle where a long history exists depicting, typically, a contrast between the rapid movement of series during cyclical downturns and their slow steady growth during recovery periods (see Burns and Mitchell, 1946; Hicks, 1950; Keynes, 1936). A particular example of this form of analysis is provided by the studies of Stock (1987, 1988) where the notion of time deformation has been introduced to consider the possibility that variables evolve on an operational time scale rather than the calendar time scale from which observations are drawn, thereby permitting the capture of potentially differing speeds of evolution of a series through time. The resulting diagnostic tests associated with this concept consider the relationship between the change in a series and indicator functions capturing the operational-calendar time transformation noted above, thus allowing the increasingly popular issue of nonlinear adjustment to be explored. Alternatively expressed, the above depiction of the contrasting stances adopted in economics and finance presents a situation in which the changes in evolution of a series are explored via consideration of the *mean equation* by the former discipline, but via the *variance equation* within the latter.

The aim of the present study is to explore the link between mean equations and variance equations within the context of the temporal evolution of time series via examination of the relationship between time deformation and GARCH. Surprisingly, such a relationship has yet to be considered despite the prominence of the notions within their respective disciplines, the similarity of their underlying motivations and previous research employing a near shared language when considering their respective behaviour.[1] To consider this potential relationship, the present study examines

[1] As an example of this latter issue, Lamoureux and Lastrapes (1990) consider issues of temporal precedence and

the finite-sample properties of diagnostic tests of time deformation in the presence of GARCH. This analysis permits examination of the possibility that volatility in the variance equation can appear as spurious time deformation in the mean equation. It is shown that this mean-variance equation spillover effect can occur and that its effects can be very substantial in the form of severe size distortion of the diagnostic test of time deformation. The analysis proceeds to explore whether the application of appropriate GARCH filters can overcome this noted size distortion by removing conditional volatility to produce correctly sized tests. A comprehensive simulation analysis is conducted to examine these issues with a range of GARCH specifications (including heavy-tailed processes) considered in conjunction with the use of alternative variance-covariance estimators commonly employed in empirical research. The results obtained show the ability of prior GARCH filtering to permit the application of robust tests of time deformation. The analysis concludes by complementing the above simulation analysis with an empirical examination of time deformation in the London housing market. Aside from providing a vehicle for an empirical analysis of the issues considered theoretically via simulation analysis, the London housing market warrants attention for a variety of additional reasons including the extension of literatures concerning the volatility of house prices, the examination of a ripple effect in UK housing market and the role of housing in the recent financial crisis. With regard to the volatility of house prices, the examination of the London market herein adds to previous studies such as Dole and Tirtiroglu (1997), Crawford and Fratantoni (2003) and Lin and Fuerst (2014) which have considered the U.S. and Canada respectively.[2] Turning to the ripple effect (see, *inter alia*, MacDonald and Taylor 1993; Meen 1999, Cook and Watson 2016), the underlying hypothesis that movements in the UK housing market are driven by the London submarket emphasises the importance of understanding and examining its properties. Finally, recent research such as Miles (2011) and Barros *et al.* (2015) has illustrated the importance of housing markets in the recent financial crisis with the implications of their volatility for mortgage defaults and pre-payment, portfolio management, property taxation revenues, the

differential speeds of information flows in relation to ARCH, both of which are clearly related to time deformation.

[2] In addition to these empirical studies examining the presence of volatility clustering in house prices, the work of Case and Shiller (1988, 1989, 1990) has sought to explain underlying causes for this behaviour.

probability of losses at a level beyond that associated with modern portfolio theory and the pricing of mortgage-backed securities all noted.[3] The prominence of the London market and the extent of capital flows within it emphasise further in its obvious importance to the analysis of all of these issues. Drawing upon the insights offered by the simulation analysis and portmanteau nonlinearity testing, the results presented provide robust evidence of extensive time deformation within this submarket; a finding which has clear implications for subsequent analyses of this market and the interpretation of previous analyses.

## 2 Time deformation

Following Stock (1987), time deformation can be considered as a distinction between the calendar time ($t$) from which observations on a series are drawn and the operational time scale ($s$) on which the series evolves. Stock (1987) links operational and calendar time via the function $g(\cdot)$ with $s = g(t)$. As a result, an observed series of interest $x_t$ can be related to a latent process $\xi(s)$ evolving in operational time via $x_t = \xi(g(t))$.

To model the time scale transformation from operational to calendar time, the approach employed by Stock (1987) is based upon the use of the Heaviside indicator function to allow for the possibility that variables evolve at different speeds during different phases of the business cycle.[4] Two forms of indicator are considered. The first of these, $z_t$, referred to as cyclical expansion/contraction, is given as:

$$z_t = \begin{cases} 1 & \text{if } \Delta y_t \geq 0 \\ 0 & \text{if } \Delta y_t < 0 \end{cases} \tag{1}$$

where $y_t$ is the variable of interest under examination. The second form of indicator, $z_t^*$, referred to as cyclical expansion/contraction, is given as:

[3]Earlier discussion of the impact of housing markets upon these issues is provided by Crawford and Rosenblatt (1995), Foster and Van Order (1984) and LaCour *et al.* (2002).

[4]The use of the Heaviside indicator function is common in analysis of nonlinearity or asymmetry, being employed in studies such as, *inter alia*, Neftci (1984) and Granger and Lee (1989).

$$z_t^* = \begin{cases} 1 & \text{if } \Delta y_t \geq \overline{\Delta y} \\ 0 & \text{if } \Delta y_t < \overline{\Delta y} \end{cases} \tag{2}$$

Therefore the two indicator functions relate to positive/negative growth and growth above/below the average respectively. Although the analysis of Stock (1988) shows the estimation of time deformation models to be computationally demanding, diagnostic testing for the presence of the non-linear transformation between operational and calendar time is relatively straightforward. For unit root processes, the relevant diagnostic testing equations for cyclical and cyclical growth expansion/contraction are as given:

$$\Delta y_t = \mu + \lambda \Delta y_{t-1} + \sum_{i=1}^{k} \alpha_i \overline{z}_{t-i} + \sum_{i=1}^{k} \beta_i \, \overline{z}_{t-i} \, \Delta y_{t-i} + \epsilon_t \tag{3}$$

$$\Delta y_t = \mu^* + \lambda^* \Delta y_{t-1} + \sum_{i=1}^{k} \alpha_i^* \, \overline{z}_{t-i}^* + \sum_{i=1}^{k} \beta_i^* \, \overline{z}_{t-i}^* \, \Delta y_{t-i} + \epsilon_t \tag{4}$$

where:

$$\overline{z}_{t-i} = z_{t-i} - \overline{z} \tag{5}$$

$$\overline{z}_{t-i}^* = z_{t-i}^* - \overline{z}^* \tag{6}$$

$$\overline{z} = \sum_{i=1}^{T} z_{t-i} \tag{7}$$

$$\overline{z}^* = \sum_{i=1}^{T} z_{t-i}^* \tag{8}$$

Diagnostic testing of time deformation is then undertaken by consideration of the significance of the terms involving the time scale transformation indicator functions via the following null hypotheses of no time deformation:

$$\mathrm{H}_0 \; : \; \alpha_i = \beta_i = 0 \qquad \forall i \quad \text{in (3)} \tag{9}$$

$$\mathrm{H}_0 \; : \; \alpha_i^* = \beta_i^* = 0 \qquad \forall i \quad \text{in (4)} \tag{10}$$

The resulting test statistics associated with the above nulls of (9) and (10) are denoted here as $F$ and $F^G$ respectively and are referred to as examining time deformation and growth time deformation.

The notion of (growth) time deformation is clearly a mean equation phenomenon. However, given its reference to differential speeds of evolution of series, it has close parallels with the variance equation concepts of ARCH and GARCH which consider variation in the movement of series also.[5] This connection is alluded to in the work of Clark (1973) where time deformation is discussed in connection with the mixtures of distributions hypothesis which has been used to explain the emergence of (G)ARCH behaviour in time series. More recently, this potential link has appeared implicitly and indirectly in the research of Lamoureux and Lastrapes (1990) where reference is made to varying speeds of informational flows and their impact on the evolution of financial data. In this research, trading volume is treated as weakly exogenous mixing variable to represent differential speeds of information flow.[6] Given the apparent similarity in the motivations underlying time deformation and GARCH, and the previous research implicitly and indirectly drawing parallels between them, it is perhaps surprising that there has yet to be a formal examination of their potential linkages. In the following section, this issue is addressed by the examination of the impact of GARCH behaviour on the properties of diagnostic tests of time deformation. Alternatively expressed, it is examined whether apparent time deformation can emerge as a result of the presence of GARCH.

[5] The prominence of GARCH in financial analysis is clear from studies such as, *inter alia*, Anderson and Bollerslev (1998) and Engle and Patton (2001).

[6] Volume is treated as weakly exogenous in the sense of Engle *et al.* (1983). Despite recognising the uncertainty surrounding the use of volume following studies such as Ross (1987), Lamoureux and Lastrapes (1990) employ volume as a variance equation regressor showing its capture of varying informational flows to render ARCH coefficients insignificant.

## 3 Simulation analysis

### 3.1 Experimental Design

To explore the impact of the conditional volatility in the variance equation upon the detection of time deformation in the mean equation, the following data generation process (DGP) is employed:

$$y_t = y_{t-1} + w_t \qquad t = 1, ..., T \tag{11}$$

$$h_t^2 = \phi_0 + \phi_1 w_{t-1}^2 + \phi_2 h_{t-1}^2 \tag{12}$$

$$w_t = h_t v_t \tag{13}$$

$$v_t \sim \begin{cases} \text{N}(0,1) \text{ or,} \\ \text{GED}(r) \end{cases} \tag{14}$$

The DGP of (11)-(14) defines a variable denoted as $y_t$ which is a unit root process exhibiting conditional volatility in the form of a GARCH(1,1) specification. The above tests of time deformation are subsequently applied to this series of interest $y_t$. With regard to technicalities, the initial value of the unit root process $y_t$ is set to zero ($y_0 = 0$) without loss of generality while the initial value of the conditional variance is set equal to one ($h_0 = 1$) with the first 400 observations of the generated GARCH process discarded prior to its use in the generation of $y_t$. Considering the variance equation, empirical realistic values of the GARCH parameters $\{\phi_1, \phi_2\}$ are considered which correspond to near integration ($\phi_1 + \phi_2 = 0.99$), with $\phi_0 = 1 - \phi_1 - \phi_2$.[7] The precise values of $\{\phi_1, \phi_2\}$ employed are informed by those observed in empirical research for data of differing frequencies (see, *inter alia*, Drost and Nijman 1993; Engle and Patton 2001) and are $\{\phi_1, \phi_2\} = \{0.05, 0.94\}, \{0.10, 0.89\}, \{0.15, 0.84\}, \{0.20, 0.79\}, \{0.30, 0.69\}$.

To ensure the simulation analysis matches empirical research further, two additional considerations are incorporated in the experimental design. First, alternative specifications are employed for the error process $v_t$. Given the frequent observation of heavy tails in financial data and their attendant implications (see, *inter alia*, Bali and Demiritas 2006; Cook 2009; Deo 2000; Granger and Orr, 1972; Harvey 2013; Loretan and Phillips 1994; Resnick 2006; So *et al.* 2008), different distributions are employed for $v_t$ in (14). In addition to the generation of $v_t$ as a pseudo *i.i.d.*

[7] In this paper the less empirically realistic, or relevant, cases of degenerate GARCH ($\phi_0 = 0$) and integrated GARCH ($\phi_1 + \phi_2 = 1$) are not considered.

$\mathsf{N}(0,1)$ process, the generalised error distribution (GED) is utilised to generate heavy-tailed distributions. Following its introduction into empirical finance by Nelson (1991), the GED has proved popular in empirical research and is employed in the present analysis using alternative values for the tail parameter ($r$) determining the thickness of the tails. More specifically, the three values $r = \{0.5, 1.0, 1.5\}$ are employed. The chosen values of $r < 2$ provide distributions with heavier tails than the Normal distribution (for which $r = 2$) and reflect those noted in empirical research.[8]

The second feature incorporated to allow the simulation design to more closely reflect empirical research is the use of alternative variance-covariance matrix estimators in the application of the time deformation tests. In addition to the standard OLS variance-covariance estimator, the heteroscedasticity corrected variance-covariance matrix estimators (HCCMEs) of White (1980) and Newey-West (1987) are employed.[9] Such a development of the experimental design is warranted given the tendency of investigators to utilise such estimators. Previously the test statistics for time deformation and growth time deformation were denoted as $F$ and $F^G$ respectively. To denote utilisation of the White and Newey-West HCCMEs, the subscripts '$w$' and '$n$' respectively are included to produce six test statistics $\left(F, F_w, F_n, F^G, F_w^G, F_n^G\right)$ for consideration, where the absence of a subscript indicates use of the standard OLS covariance matrix estimator.[10]

The empirical rejection frequencies of the above six tests are considered at the 5% nominal level of significance for a range of sample sizes $T = \{100, 250, 500, 1000\}$ over 100,000 simulations for each of the experimental designs above. As a result, the size distortion of the tests and hence potential spurious time deformation as a result of the presence of GARCH are examined.

### 3.2 Standardised residuals and GARCH filtering

The above experimental framework permits consideration of time deformation effectively acting as a proxy for unconsidered GARCH. To consider this issue more closely, the properties of the

[8]For example, the selected values for $\lambda$ correspond closely to those noted for inflation rate data for a number of economies in Cook (2009).

[9]Obviously the Newey-West (1987) estimator corrects for serial correlation in addition to heteroskedasticity.

[10]It should be noted that a decision is required concerning the value of $k$ in equations in (3) and (4). It can be seen that $k$ determines the degree of augmentation of the testing equations. A value of $k = 4$ is chosen for the simulation analysis.

$F$-tests of (growth) time deformation are considered for not only for $y_t$ but for GARCH filtered versions of $y_t$ also. That is, the generated series $y_t$ is filtered using a GARCH(1,1) model to produce a standardised residual $\left(\widehat{v}_t = \widehat{h}_t^{-1}\widehat{w}_t\right)$. Applying diagnostic tests of time deformation to the standardised residual series permits examination of the extent to which time deformation disappears once the volatility in the series generated by the GARCH process is removed. As the residual series is stationary, diagnostic testing of time deformation requires the application of testing equations of an alternative form to those of (4) and (5) above which are appropriate for examination of unit root processes. Following Stock (1987), the appropriate specifications for time deformation testing equations when considering the non-trending stationary standardised residual series are as below:

$$\widehat{v}_t = \pi + \theta\widehat{v}_{t-1} + \sum_{i=1}^{\tau} \delta_i \,\overline{z}_{t-i} + \sum_{i=1}^{\tau} \gamma_i \,\overline{z}_{t-i}\,\widehat{v}_{t-i} + e_t \quad (15)$$

$$\widehat{v}_t = \pi^* + \theta^*\widehat{v}_{t-1} + \sum_{i=1}^{\tau} \delta_i^* \,\overline{z}_{t-i}^* + \sum_{i=1i}^{\tau} \gamma^* \,\overline{z}_{t-i}^*\,\widehat{v}_{t-i} + e_t \quad (16)$$

where:

$$z_t = \begin{cases} 1 & \text{if } \Delta\widehat{v}_t \geq 0 \\ 0 & \text{if } \Delta\widehat{v}_t < 0 \end{cases} \quad (17)$$

$$z_t^* = \begin{cases} 1 & \text{if } \Delta\widehat{v}_t \geq \overline{\Delta\widehat{v}} \\ 0 & \text{if } \Delta\widehat{v}_t < \overline{\Delta\widehat{v}} \end{cases} \quad (18)$$

$$\overline{z}_{t-i} = z_{t-i} - \overline{z} \quad (19)$$

$$\overline{z}_{t-i}^* = z_{t-i}^* - \overline{z}^* \quad (20)$$

$$\overline{z} = \sum_{i=1}^{T} z_{t-i} \quad (21)$$

$$\overline{z}^* = \sum_{i=1}^{T} z^*_{t-i} \tag{22}$$

and the null hypotheses of no time deformation are given as:

$$\text{H}_0 \ : \ \delta_i = \gamma_i = 0 \qquad \forall i \quad \text{in (15)} \tag{23}$$

$$\text{H}_0 \ : \ \delta^*_i = \gamma^*_i = 0 \qquad \forall i \quad \text{in (16)} \tag{24}$$

# 4 Simulation Results

The results obtained from the simulation analysis outlined above are reported in Tables One to Eight. To ease consideration of the wealth of results generated, findings for the series of interest ($y_t$) and the standardised residuals ($\widehat{v}_t$) are considered separately.

## 4.1 Size distortion when examining $y_t$

Considering the results obtained for a unit root process exhibiting GARCH behaviour as provided in Tables One to Four, it can be seen that spurious rejection of the null of no time deformation occurs frequently. From closer inspection, it can be seen that time deformation tests is dependent upon a number of factors included in the experimental design, namely the values of the GARCH coefficients, the thickness or heaviness of the underlying error distribution, the sample size and variance-covariance matrix estimator employed. With regard to the first factor, higher values of the volatility parameter ($\phi_1$) result in greater oversizing of the time deformation tests irrespective of the sample size, type of error distribution or sample size considered. To illustrate this, consider the empirical size of the $F_w$ test for a sample of $T = 250$ and an underlying error $v_t$ distributed as GED(1.5) as $\phi_1$ increases from 0.01 to 0.30. The resulting empirical sizes noted during this progression are 7.54%, 8.32%, 9.65%, 12.66% and 15.41%, this illustrating a steady increase in size distortion.

Considering the effects of the underlying error distribution, these are not as straightforward to interpret as those for the GARCH parameters. More precisely, while heavier tailed distributions, *ceteris paribus*, have a general tendency to result in greater size distortion with oversizing reducing when moving from the GED(0.5) distribution to the $N(0,1)$, this is not always the case. In particular, the results under application of White's HCCME show oversizing to be reduced when in mnay cases when heavier tailed distributions are considered. Turning to the other factors or design parameters in the simulation analysis, namely the sample size and the covariance matrix, their effects are more complicated in nature. For example, the ordering of the $F$ and $F^G$ by size across covariance matrix estimators changes when different error series, GARCH coefficients and sample sizes are considered. While for smaller values of the volatility parameter the OLS covariance matrix estimator results in a test with less size distortion leading to a test size ordering of $F < F_w < F_n$ (and similarly $F^G < F^G_w < F^G_n$), this does not hold for greater values of $\phi_1$ nor does it continue as the sample size increases where it can be seen that the performance of tests employing the OLS estimator worsens as that of the tests utilising the White and Newey-West estimators improves. That is, while increased size distortion is noted under application of the standard OLS estimator as the sample size increases, the benefits of the HCCMEs become apparent under such circumstances. To illustrate these issues, consider the tests of time deformation for the following four sets of design parameters: $\{\phi_1, T, v_t\} = \{0.05, 250, GED\,(0.5)\}\,;\ \{0.05, 250, GED\,(1.5)\}\,;\ \{0.30, 1000, GED\,(0.5)\}\,; \{0.05, 1000, GED\,(1.5)\}$. The empirical percentage sizes for $\{F, F_w, F_n\}$ under these combinations of design parameters are found to be $\{12.40, 7.63, 18.55\}$, $\{7.46, 8.32, 14.30\}$, $\{67.99, 12.76, 25.33\}$ and $\{11.52, 6.30, 8.67\}$ respectively. These results show the test using the OLS estimator is the best performing test only when considering a smaller sample, small value of the volatility parameter and a thinner tailed error distribution. Similarly, the worsening performance of the test employing the OLS estimator when the sample size increases is apparent (7.46% to 11.52% when the only change is from $T = 250$ to $T = 1000$). Conversely, the sizes of tests under the HCCMEs improve under these circumstances.

[ **TABLES ONE TO FOUR ABOUT HERE** ]

In summary, the results show the time deformation tests to exhibit size distortion in the presence of GARCH, resulting in spurious rejection. When employing the standard OLS covariance matrix estimator, this oversizing can be very substantial and is found to increase with the sample size, the value of the volatility parameter and the heaviness of the tails of the error distribution. These are problematic findings for empirical research where the time deformation tests will often be employed without incorporation of an HCCME using relatively small samples of observations for series exhibiting higher levels of volatility- in short, the combination of circumstances under which the tests can be seen to suffer substantial size distortion. While the use of HCCMEs does result in reduced size distortion in many instances, it does not eradicate oversizing and its benefits are more apparent for larger samples, with thinner tailed distributions and lower levels of volatility (as measured by the volatility parameter $\phi_1$). In other words, application of HCCMEs can be beneficial in some circumstances, especially White's HCCME rather than the Newey-West alternative, but they are by no means a panacea. Indeed, in many cases the application of an HCCME can increase size distortion, when perversely its application is motivated by a desire to increase robustness. The obvious issue arising from the above analyses is simply how an investigator be confident in the robustness of apparent time deformation in series which exhibit volatility in the form of GARCH. The following section reviews the possible use of GARCH filtering as a means of providing such confidence in combination with the results presented above.

### 4.2 Size distortion when examining $\widehat{v}_t$

The results obtained from application of tests of time deformation to the GARCH filtered version of the $y_t$ series are presented in Tables Five to Eight. The outstanding feature apparent in these tables is that GARCH filtering, or consideration of the standardised residual, returns the time deformation tests to near correct size (i.e. the empirical sizes are very close to their nominal level) under use of the standard OLS covariance matrix estimator. That is, prior estimation of a GARCH model partials out the volatility causing size inflation. Considering analogous results obtained under use of HCCMEs, application of White's estimator is seen to result in oversizing, but this is marginal and reduces with the use of larger sample sizes. In contrast, application of the Newey-West HCCME is found to result in poor test size, albeit that the results herein for the standardised residuals are not of the magnitude observed when considering the series of interest ($y_t$) itself. Considering the results of Tables Five to Eight further, it can be seen that neither the heaviness of the underlying error distribution ($v_t$) nor the value of the volatility parameter ($\phi_1$) influence test size unduly. Again, these results for the standardised residual are in contrast to the findings presented for the series itself where both factors were found to be highly influential in determining the size of the time deformation tests.

[ **TABLES FIVE TO EIGHT ABOUT HERE** ]

# 5 Empirical analysis of the London housing market

## 5.1 Data

The house price data considered here are for the 32 boroughs of the London available from the UK Land Registry.[11] The data are monthly observations on the boroughs Barking, Barnet, Bexley, Brent, Bromley, Camden, City of Westminster, Croydon, Ealing, Enfield, Greenwich, Hackney, Hammersmith and Fulham, Haringey, Harrow, Havering, Hillingdon, Hounslow, Islington, Kensington and Chelsea, Kingston upon Thames, Lambeth, Lewisham, Merton, Newham, Redbridge, Richmond upon Thames, Southwark, Sutton, Tower Hamlets, Waltham Forest and Wandsworth over the period January 1995 to March 2015. These data on the London housing market have become the subject of recent attention in the literature where work such as Abbott and DeVita (2012) has extended previous regional analysis at a more aggregated level to explore highly important submarket dynamics.

## 5.2 Basic temporal properties of the house series

Before considering the application of portmanteau tests of nonlinearity and diagnostic tests of time deformation to the house price series and their GARCH filtered versions, the orders of integration

[11] The data set is available from http://landregistry.data.gov.uk/app/hpi

of the series are examined.[12] Denoting the natural logarithmic values of the house price series as $p_{it}$, with $i = 1, ..., 32$ indexing the alternative boroughs under examination, the integrated natures of $p_{it}$ and $\Delta p_{it}$ are considered to ensure the correct specifications of the time deformation testing equations are employed. The method followed is a twofold approach. First, augmented Dickey-Fuller (ADF) tests are applied to $p_{it}$ and $\Delta p_{it}$. Given the trending nature of $p_{it}$, ADF tests for these series are performed with a deterministic trend term included in the testing equation. Conversely, a deterministic trend is not included in the testing equations employed for the non-trending $\Delta p_{it}$. The resulting ADF test statistics are denoted herein as $\tau$. Following best practice, the degrees of augmentation of the testing equations for the $\tau$ statistics are determined using the modified Akaike Information Criterion (MAIC). Second, panel unit root testing is applied to exploit the informational content offered by the cross-sectional dimension of the 32 regional series for both $p_{it}$ and $\Delta p_{it}$. This analysis is undertaken using the W-statistic of Im *et al.* (IPS) (2003). In contrast to panel unit root tests such as those of, *inter alia*, Breitung (2000) and Levin *et al.* (2002), the IPS W-statistic has the advantage of avoiding the restrictive assumption of a common autoregressive coefficient for all series under the alternative. Application of the IPS tests is undertaken using the same approaches adopted for the ADF tests with regard to the inclusion of deterministic trend terms and the degree of augmentation of the testing equations.

From inspection of the unit root test results in Table Nine, it can be seen that the univariate ADF test outcomes do not reject the null of a unit root in the house price series. In the interests of brevity, the range of p-values associated with the tests are reported rather than each of the 32 values. Given the p-values range from just over 10% to just over 65%, the null is not rejected for any series at conventional levels. This non-rejection of the unit root hypothesis is found to occur also when using the IPS panel unit root test with a p-value of 99% observed. In light of the

[12]It is recognised that the voluminous literature on the properties of unit root tests has created something of a minefield in which structural change, variance breaks, initial conditions, outliers, asymmetric and nonlinear alternatives, the incorporation of of conditional volatility, and the allowance for explosive bubbles are but a few potentially counteracting issues which can influence the properties and behaviour of tests relative to those under 'standard' conditions. As such, the seminal univariate test and a preferred panel test offering increased power are considered here despite a myriad of other tests being recognised as available. In essence, the testing of the unit root hypothesis is not the focus of the current study but rather a stepping stone to the application of the subsequent analysis.

overwhelming rejection of the unit root null for the $\Delta p_{it}$ series under both univariate and panel tests (all percentage p-values being zero to 2 decimal places), it is concluded that the $p_{it}$ series are I(1). As a consequence, time deformation testing equations of the form of (3) and (4) are appropriate for the examination of London borough house prices.

### 5.3 Time Deformation in the London Housing Market

The results obtained from application of the time deformation and growth time deformation tests of Stock (1987) to the London house price series are presented in Table Ten.[13] Following the approach adopted in the simulation analysis, each test is applied using the standard OLS covariance matrix estimator and the HCCMEs of White (1980) and Newey-West (1987). The results presented show an overwhelming presence of time deformation in the London housing market with evidence of both forms of time deformation observed beyond the conventional 5% level of significance for 29 of the 32 boroughs examined. The three exceptions to this are Ealing and Richmond where time deformation but not growth time deformation is detected, and Wandsworth where growth time deformation but not time deformation is detected. Alternatively expressed, 3 of the boroughs exhibit one form of time deformation while the remaining 29 exhibit both forms. However, following the simulation analysis above, it is clear that despite these findings indicating overwhelming evidence of time deformation, such findings might arise spuriously as a result of GARCH being present in the series. Given the series under examination are asset prices, it is to be expected that substantial GARCH effects might be present. To explore this issue, GARCH(1,1) models were fitted to all series with the results obtained showing strong GARCH effects to indeed be present.[14] Following estimation of the GARCH models, time deformation tests using testing equations of the form of (15) and (16) were applied to the resulting standardised residuals. The results from obtained from application of time deformation tests to the or GARCH filtered series are reported in Table Eleven. The findings

[13]The time deformation tests are applied using a four period lag length for the augmented component of the testing equations of (3) and (4). Alternative lag lengths were considered and quantitatively similar results were obtained. However, given the potential dimension of the testing equation and the sample size available for the empirical analysis ($T = 243$) results for models with four lags are reported.

[14]In the interests of brevity the specific values for the GARCH coefficients for the 32 estimated GARCH models are not reported. However, all models produced large significant volatility and GARCH (i.e. $\phi_1$ and $\phi_2$) parameters.

presented can be seen to be more conclusive than those for the house price series themselves as time deformation is observed at the 5% level in every case considered. Indeed, for the 192 test outcomes for the standardised residuals for the 32 regions, the maximum p-value observed is 4.14% with the vast majority of p-values being zero to two decimal places.[15] It can therefore be concluded the extensive evidence of time deformation noted for the house price series is not spurious and detected as a result of the GARCH behaviour present in the series, but rather reflective of genuine nonlinearity present in the data. These findings have a number of implications as they indicate consideration of linear methods often employed may fail to detect relationships or portray an misleading view of relationships detected. Instances where this may occur in the present context relate to, *inter alia*, the examination of dynamic interrelationships between the boroughs and policy analyses where empirical analysis of housing market interactions or base rate (and hence mortgage rate) changes may be explored.

As a further check of the robustness of the results obtained, portmanteau nonlinearity testing is undertaken for both the house prices series and their standardised residuals.

### 5.4 Portmanteau Nonlinearity Testing

As a means of further examining nonlinearity, the portmanteau BDS test of Brock *et al.* (1996) is applied. The BDS test examines the null hypothesis that data are identically and independently distributed using the correlation integral based upon the 'histories' of the series considered.[16] To implement the test, values are required for the embedding dimension ($m$) and the distance parameter ($\epsilon$). Following standard practice, the values $m = \{2, 5\}$ and $\epsilon/\sigma = 1$, where $\sigma$ denotes the standard deviation of the data under examination, are employed. Application of the test to the house price series ($p_{it}$) produced test statistics with percentage p-values of zero for all regions for tests using both $m = 2$ and $m = 5$, thus clearly indicating the presence of nonlinearity in the data. Subsequent application of the BDS test to the standardised residuals of, or GARCH filtered, house

[15] Note that the figure of 192 arises as a result of 2 tests of time deformation (non-growth and growth) being considered for 32 regions using 3 covariance matrix estimators. That is, $192 = 2 \times 32 \times 3$.

[16] In the interests of brevity, the structure of the BDS test is outlined here. Further details are available from a range of sources including the seminal work of Brock *et al.* (1996).

price series produced similar results indicating highly significant and widespread nonlinearity. More precisely, 58 of the 64 BDS test statistics possessed p-values of zero.[17] Four of the remaining test statistics (Greenwich and Hackney for both $m = 2$ and $m = 5$) had p-values between 0.02% to 1.97% indicating that while the p-values may be non-zero, the respective nulls are overwhelmingly rejected nonetheless at conventionally considered levels of significance. The final two statistics relate to Brent where the BDS test statistics have p-values of 22.27% and 0.34% for $m = 2$ and $m = 5$ respectively. Again, the null of is rejected overwhelmingly at conventional levels of significance in the latter case.

The results obtained from application of the BDS test to the house price series and their GARCH filtered forms has produced overwhelming evidence of nonlinearity for all of the 32 boroughs. However, to interpret fully the results obtained, consideration must be paid to the findings of the studies of Brooks and Heravi (1999) and Brooks and Henry (2000) where the finite-sample properties of the BDS test are examined. In summary, these studies have shown the BDS to possess low power when applied to series possessing nonlinear or asymmetric behaviour and, even more noticeably, exceptionally low power when applied to GARCH-filtered standardised residuals. These results therefore reinforce the current findings for the London housing market as overwhelming rejection has occurred in circumstances where there is an inherent bias against rejection. In combination with the previous results for time deformation, the BDS test results show nonlinearity in the data is detected and that it is not caused by GARCH behaviour as it remains after GARCH filtering. Beyond this, the current findings have shown the generic nonlinearity identified by the BDS tests takes the form of time deformation as evidenced by the results obtained using the diagnostic tests of Stock (1987).

[17] 64 test statistics arise as a result of employing the BDS twice (once for each of two values of $m$) to each of the 32 London boroughs. Given the number of results, full details are not reported here in the interests of brevity. Particularly as their clearcut nature makes discussion straightforward.

## 6 Concluding remarks

This paper has considered the relationship between time deformation and GARCH. Despite clear parallels between these two notions in terms of their focus on changes in the evolution of time series processes and their apparent connection via issues such as the mixtures of distributions hypothesis of Clark (1973), the potential relationship between them has not been considered formally in previous research. The results of a comprehensive simulation analysis showed a clear connection between the two notions with the presence of GARCH capable of generating spurious time deformation. It was found that the oversizing was particularly apparent for heavy-tailed and more volatile GARCH processes and that the use of the corrected covariance matrix estimators of either White (1980) or Newey-West (1987) did not restore the tests to their correct (nominal) size. Importantly, it was found that the degree of oversizing of time deformation tests employing the standard OLS covariance matrix estimator did not diminish as larger sample sizes were considered, but rather size distortion increased. In response to these findings, it was considered whether GARCH filtering might provide a solution to the problem of size distortion and the spurious detection of time deformation. The results obtained showed GARCH filtering to restore the correct size of time deformation testing using the standard OLS covariance estimator across a range of GARCH processes and heavy (and non-heavy) tailed error distributions. However, the application of corrected covariance matrix estimators was seen to produce a less robust test in these circumstances. The analysis concluded with an examination of the London housing market with pronounced time deformation detected in all of its 32 boroughs. Using the results of the simulation analysis, the finding of nonlinearity in the form of time deformation was robust as it drew upon the issues of GARCH filtering in particular.

The current analysis has a number of important implications. It has been shown that alternative concepts considering the temporal evolution of time series in finance and economics are in fact closely related. In addition, the fragility of testing for time deformation has been exposed and evaluated while, importantly, a solution to this has been proposed and confirmed. The message to practitioners in light of these findings is clear: when considering time deformation for volatile series, the application of HCCMEs is not a solution to potential size distortion, but the consideration, and addressing, of volatility is. Further to this, the empirical analysis has provided practitioners with clear evidence of nonlinearity within the London housing market indicating the importance of considering methods other than standard linear, symmetric approaches when modelling relationships, undertaking policy analyses or interpreting previous research.

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**Table One: The finite-sample sizes of tests of time deformation** ($T = 100, 250$)

| | $T=100$ | | | | | $T=250$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F$ | 4.82 | 7.03 | 10.38 | 16.32 | 20.49 | 5.97 | 12.40 | 20.55 | 32.00 | 38.47 |
| $F_w$ | 7.95 | 9.52 | 11.47 | 14.69 | 17.06 | 5.99 | 7.63 | 9.50 | 12.49 | 14.73 |
| $F_n$ | 21.58 | 23.87 | 26.70 | 30.95 | 33.60 | 15.52 | 18.55 | 21.83 | 26.60 | 29.57 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F$ | 4.77 | 6.20 | 8.97 | 14.89 | 20.00 | 5.16 | 8.88 | 15.81 | 28.47 | 37.15 |
| $F_w$ | 10.14 | 11.25 | 12.85 | 15.91 | 18.67 | 6.97 | 8.16 | 9.75 | 12.74 | 15.47 |
| $F_n$ | 22.42 | 23.61 | 25.63 | 29.49 | 32.52 | 13.83 | 15.32 | 17.79 | 22.53 | 26.27 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F$ | 4.90 | 5.84 | 7.93 | 13.29 | 18.15 | 5.03 | 7.46 | 12.73 | 24.39 | 33.77 |
| $F_w$ | 11.32 | 12.19 | 13.63 | 16.67 | 19.22 | 7.54 | 8.32 | 9.65 | 12.66 | 15.41 |
| $F_n$ | 22.67 | 23.51 | 25.02 | 28.45 | 31.31 | 13.25 | 14.30 | 16.25 | 20.53 | 24.19 |
| *(iv)* $v_t \sim N(0,1)$ | | | | | | | | | | |
| $F$ | 4.81 | 5.51 | 7.03 | 11.71 | 16.57 | 5.02 | 6.72 | 10.98 | 21.90 | 31.35 |
| $F_w$ | 11.86 | 12.48 | 13.76 | 16.67 | 19.42 | 7.61 | 8.24 | 9.45 | 12.42 | 15.41 |
| $F_n$ | 22.85 | 23.30 | 24.57 | 27.65 | 30.59 | 12.94 | 13.72 | 15.53 | 19.48 | 23.23 |

*Notes*: The figures presented above are percentage empirical sizes of the tests of time deformation associated with the null hypothesis of (9). Results relate to rejection at the 5% nominal level of significance. The tests are applied to series generated by the DGP of (11)-(14) under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of the relevant experimental designs.

**Table Two: The finite-sample sizes of tests of time deformation** ($T = 500, 1000$)

| | $T = 500$ | | | | | $T = 1000$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F$ | 7.22 | 19.32 | 31.77 | 46.62 | 53.92 | 8.51 | 27.64 | 44.62 | 60.88 | 67.99 |
| $F_w$ | 5.57 | 7.10 | 8.68 | 11.34 | 13.42 | 5.42 | 6.55 | 7.98 | 10.49 | 12.76 |
| $F_n$ | 12.62 | 15.76 | 19.20 | 24.08 | 27.17 | 10.17 | 13.09 | 16.52 | 21.87 | 25.33 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F$ | 5.56 | 12.14 | 23.88 | 41.76 | 52.00 | 5.95 | 16.08 | 33.54 | 55.44 | 66.01 |
| $F_w$ | 6.12 | 7.04 | 8.47 | 11.26 | 14.04 | 5.71 | 6.51 | 7.74 | 10.41 | 13.10 |
| $F_n$ | 10.45 | 11.94 | 14.42 | 19.26 | 23.25 | 8.15 | 9.63 | 12.04 | 16.98 | 21.10 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F$ | 5.15 | 9.10 | 18.53 | 36.61 | 48.12 | 5.47 | 11.52 | 25.88 | 49.42 | 62.14 |
| $F_w$ | 6.23 | 6.89 | 8.06 | 10.90 | 13.71 | 5.71 | 6.30 | 7.29 | 9.81 | 12.42 |
| $F_n$ | 9.69 | 10.71 | 12.74 | 17.12 | 21.10 | 7.78 | 8.67 | 10.42 | 14.84 | 18.79 |
| *(iv)* $v_t \sim N(0,1)$ | | | | | | | | | | |
| $F$ | 5.19 | 8.05 | 15.79 | 32.76 | 44.99 | 5.38 | 9.70 | 21.84 | 45.12 | 58.87 |
| $F_w$ | 6.40 | 6.88 | 7.95 | 10.58 | 13.50 | 5.79 | 6.14 | 7.00 | 9.39 | 11.93 |
| $F_n$ | 9.50 | 10.31 | 11.96 | 15.86 | 19.88 | 7.61 | 8.30 | 9.76 | 13.58 | 17.47 |

*Notes*: The figures presented above are percentage empirical sizes of the tests of time deformation associated with the null hypothesis of (9). Results relate to rejection at the 5% nominal level of significance. The tests are applied to series generated by the DGP of (11)-(14) under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of relevant experimental designs.

**Table Three: The finite-sample sizes of tests of growth time deformation** ($T = 100, 250$)

| | $T=100$ | | | | | $T=250$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F^G$ | 4.83 | 7.20 | 10.83 | 16.93 | 21.27 | 6.10 | 12.69 | 20.88 | 32.58 | 39.22 |
| $F_w^G$ | 9.18 | 10.74 | 12.99 | 16.59 | 19.31 | 6.44 | 8.16 | 10.15 | 13.48 | 15.97 |
| $F_n^G$ | 23.17 | 25.68 | 28.79 | 33.41 | 36.35 | 16.13 | 19.49 | 23.08 | 28.28 | 31.32 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F^G$ | 4.75 | 6.22 | 9.09 | 15.12 | 20.17 | 5.16 | 8.93 | 15.85 | 28.57 | 37.30 |
| $F_w^G$ | 10.14 | 11.37 | 13.06 | 16.31 | 19.07 | 7.10 | 8.24 | 9.73 | 12.77 | 15.70 |
| $F_n^G$ | 22.36 | 23.73 | 25.85 | 29.77 | 33.01 | 13.84 | 15.35 | 17.78 | 22.77 | 26.65 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F^G$ | 4.83 | 5.79 | 7.89 | 13.36 | 18.31 | 5.06 | 7.40 | 12.61 | 24.52 | 33.88 |
| $F_w^G$ | 11.13 | 12.08 | 13.60 | 16.56 | 19.30 | 7.53 | 8.28 | 9.61 | 12.56 | 15.40 |
| $F_n^G$ | 22.72 | 23.54 | 25.06 | 28.51 | 31.58 | 13.25 | 14.28 | 16.22 | 20.55 | 24.46 |
| *(i)* $v_t \sim N(0,1)$ | | | | | | | | | | |
| $F^G$ | 4.86 | 5.54 | 7.08 | 11.81 | 16.61 | 5.05 | 6.77 | 11.00 | 22.03 | 31.40 |
| $F_w^G$ | 11.77 | 12.37 | 13.66 | 16.61 | 19.29 | 7.65 | 8.24 | 9.49 | 12.30 | 15.43 |
| $F_n^G$ | 22.83 | 23.25 | 24.46 | 27.59 | 30.55 | 13.09 | 13.79 | 15.54 | 19.50 | 23.32 |

*Notes*: The figures presented above are percentage empirical sizes of the tests of growth time deformation associated with the null hypothesis of (10). Results relate to rejection at the 5% nominal level of significance. The tests are applied to series generated by the DGP of (11)-(14) under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of relevant experimental designs.

**Table Four: The finite-sample sizes of tests of growth time deformation** ($T = 500, 1000$)

| | $T = 500$ | | | | | $T = 1000$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F^G$ | 7.42 | 19.68 | 32.31 | 46.96 | 54.34 | 8.59 | 27.86 | 44.85 | 61.05 | 68.16 |
| $F_w^G$ | 5.80 | 7.37 | 9.02 | 11.85 | 14.26 | 5.43 | 6.64 | 8.05 | 10.89 | 13.36 |
| $F_n^G$ | 13.01 | 16.37 | 20.08 | 25.43 | 28.73 | 10.35 | 13.40 | 17.06 | 22.63 | 26.19 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F^G$ | 5.54 | 12.19 | 23.98 | 41.83 | 52.12 | 5.98 | 16.18 | 33.62 | 55.42 | 66.01 |
| $F_w^G$ | 6.14 | 6.98 | 8.47 | 11.36 | 14.22 | 5.72 | 6.51 | 7.80 | 10.50 | 13.13 |
| $F_n^G$ | 10.47 | 11.93 | 14.52 | 19.57 | 23.72 | 8.17 | 9.59 | 12.12 | 17.14 | 21.37 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F^G$ | 5.17 | 9.13 | 18.66 | 36.74 | 48.21 | 5.34 | 11.46 | 26.00 | 49.36 | 62.17 |
| $F_w^G$ | 6.22 | 6.91 | 8.02 | 10.81 | 13.71 | 5.62 | 6.23 | 7.24 | 9.78 | 12.42 |
| $F_n^G$ | 9.73 | 10.76 | 12.73 | 17.18 | 21.29 | 7.74 | 8.64 | 10.48 | 14.84 | 18.86 |
| *(iv)* $v_t \sim N(0,1)$ | | | | | | | | | | |
| $F^G$ | 5.20 | 7.98 | 15.78 | 32.77 | 44.97 | 5.38 | 9.65 | 21.84 | 45.08 | 58.81 |
| $F_w^G$ | 6.30 | 6.85 | 7.91 | 10.57 | 13.51 | 5.76 | 6.11 | 6.95 | 9.32 | 11.99 |
| $F_n^G$ | 9.43 | 10.22 | 11.93 | 15.87 | 19.86 | 7.59 | 8.22 | 9.69 | 13.53 | 17.55 |

The figures presented above are percentage empirical sizes of the tests of growth time deformation associated with the null hypothesis of (10). Results relate to rejection at the nominal level of significance of 5%. The tests are applied to series generated by the DGP of (11)-(14) under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of relevant experimental designs.

**Table Five: The finite-sample sizes of tests of time deformation when applied to GARCH filtered series** ($T = 100, 250$)

| | $T = 100$ | | | | | $T = 250$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F$ | 3.80 | 4.14 | 4.39 | 4.66 | 4.86 | 4.18 | 4.49 | 4.78 | 4.97 | 5.10 |
| $F_w$ | 7.02 | 7.33 | 7.67 | 7.83 | 8.00 | 5.30 | 5.38 | 5.47 | 5.39 | 5.46 |
| $F_n$ | 20.64 | 20.98 | 21.46 | 21.99 | 22.26 | 14.18 | 14.31 | 14.34 | 14.54 | 14.58 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F$ | 4.63 | 4.85 | 5.03 | 5.14 | 5.08 | 4.72 | 4.84 | 4.90 | 4.81 | 4.83 |
| $F_w$ | 10.27 | 10.48 | 10.51 | 10.40 | 10.27 | 6.85 | 6.84 | 6.86 | 6.82 | 6.82 |
| $F_n$ | 22.21 | 22.41 | 22.47 | 22.40 | 22.28 | 13.04 | 12.99 | 13.07 | 13.06 | 12.99 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F$ | 5.02 | 5.18 | 5.28 | 5.31 | 5.22 | 5.03 | 5.20 | 5.25 | 5.07 | 4.90 |
| $F_w$ | 12.05 | 12.00 | 12.06 | 11.87 | 11.81 | 7.44 | 7.51 | 7.51 | 7.40 | 7.34 |
| $F_n$ | 23.11 | 23.19 | 23.24 | 23.17 | 23.07 | 12.95 | 13.01 | 13.02 | 12.92 | 12.78 |
| *(iv)* $v_t \sim N(0, 1)$ | | | | | | | | | | |
| $F$ | 4.97 | 5.10 | 5.16 | 5.17 | 5.09 | 5.10 | 5.33 | 5.37 | 5.26 | 5.10 |
| $F_w$ | 12.64 | 12.72 | 12.67 | 12.64 | 12.44 | 7.91 | 7.90 | 7.88 | 7.80 | 7.72 |
| $F_n$ | 23.68 | 23.76 | 23.88 | 23.79 | 23.58 | 13.01 | 13.12 | 13.13 | 13.07 | 12.98 |

*Notes*: The figures presented above are percentage empirical sizes of the tests of time deformation associated with the null hypothesis of (23). Results relate to rejection at the 5% nominal level of significance. The tests are applied to series generated by the DGP of (11)-(14) subjected to subsequent GARCH filtering under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of relevant experimental designs.

**Table Six: The finite-sample sizes of tests of time deformation when applied to GARCH filtered series** ($T = 500, 1000$)

| | $T=500$ | | | | | $T=1000$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F$ | 4.60 | 4.91 | 4.94 | 5.14 | 5.25 | 4.91 | 5.02 | 5.08 | 5.15 | 5.19 |
| $F_w$ | 4.90 | 4.86 | 4.88 | 4.89 | 4.86 | 4.86 | 4.82 | 4.88 | 4.85 | 4.85 |
| $F_n$ | 11.44 | 11.40 | 11.43 | 11.48 | 11.50 | 9.18 | 9.13 | 9.14 | 9.14 | 9.18 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F$ | 5.02 | 5.04 | 4.97 | 4.92 | 4.98 | 5.07 | 4.97 | 4.96 | 4.96 | 5.01 |
| $F_w$ | 5.97 | 5.93 | 5.91 | 5.87 | 5.84 | 5.54 | 5.49 | 5.50 | 5.52 | 5.47 |
| $F_n$ | 9.83 | 9.80 | 9.80 | 9.74 | 9.69 | 7.81 | 7.74 | 7.78 | 7.77 | 7.73 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F$ | 5.08 | 5.28 | 5.13 | 5.01 | 4.94 | 5.19 | 5.25 | 5.10 | 5.02 | 4.99 |
| $F_w$ | 6.29 | 6.35 | 6.27 | 6.27 | 6.25 | 5.73 | 5.72 | 5.71 | 5.68 | 5.63 |
| $F_n$ | 9.56 | 9.61 | 9.63 | 9.57 | 9.57 | 7.62 | 7.62 | 7.55 | 7.48 | 7.48 |
| *(iv)* $v_t \sim N(0,1)$ | | | | | | | | | | |
| $F$ | 5.13 | 5.36 | 5.25 | 5.04 | 4.96 | 5.10 | 5.24 | 5.11 | 4.97 | 4.95 |
| $F_w$ | 6.37 | 6.38 | 6.39 | 6.38 | 6.28 | 5.68 | 5.76 | 5.66 | 5.64 | 5.64 |
| $F_n$ | 9.47 | 9.42 | 9.43 | 9.38 | 9.33 | 7.41 | 7.39 | 7.36 | 7.33 | 7.31 |

*Notes*: The figures presented above are percentage empirical sizes of the tests of time deformation of associated with the null hypothesis of (23). Results relate to rejection at the 5% nominal level of significance. The tests are applied to series generated by the DGP of (11)-(14) subjected to subsequent GARCH filtering under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of relevant experimental designs.

**Table Seven: The finite-sample sizes of tests of growth time deformation when applied to GARCH filtered series** ($T = 100, 250$)

| | $T=100$ | | | | | $T=250$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F^G$ | 3.77 | 4.11 | 4.40 | 4.68 | 4.87 | 4.18 | 4.47 | 4.76 | 4.96 | 5.10 |
| $F^G_w$ | 7.01 | 7.34 | 7.63 | 7.86 | 7.98 | 5.30 | 5.42 | 5.48 | 5.37 | 5.45 |
| $F^G_n$ | 20.59 | 21.07 | 21.37 | 21.96 | 22.25 | 14.21 | 14.28 | 14.37 | 14.51 | 14.61 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F^G$ | 4.65 | 4.84 | 5.05 | 5.13 | 5.08 | 4.69 | 4.87 | 4.90 | 4.81 | 4.90 |
| $F^G_w$ | 10.33 | 10.52 | 10.50 | 10.40 | 10.24 | 6.86 | 6.84 | 6.87 | 6.82 | 6.84 |
| $F^G_n$ | 22.18 | 22.47 | 22.51 | 22.50 | 22.27 | 13.05 | 12.97 | 13.08 | 13.05 | 12.95 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F^G$ | 5.02 | 5.17 | 5.34 | 5.29 | 5.18 | 5.01 | 5.21 | 5.26 | 5.08 | 4.90 |
| $F^G_w$ | 12.05 | 12.07 | 12.07 | 11.89 | 11.76 | 7.46 | 7.51 | 7.45 | 7.39 | 7.31 |
| $F^G_n$ | 23.11 | 23.24 | 23.22 | 23.19 | 23.01 | 12.97 | 13.05 | 13.03 | 12.89 | 12.79 |
| *(iv)* $v_t \sim N(0,1)$ | | | | | | | | | | |
| $F^G$ | 4.98 | 5.07 | 5.17 | 5.18 | 5.03 | 5.09 | 5.33 | 5.39 | 5.25 | 5.10 |
| $F^G_w$ | 12.63 | 12.76 | 12.66 | 12.61 | 12.46 | 7.89 | 7.92 | 7.86 | 7.79 | 7.72 |
| $F^G_n$ | 23.67 | 23.79 | 23.86 | 23.68 | 23.53 | 13.02 | 13.12 | 13.14 | 13.04 | 12.99 |

*Notes*: The figures presented above are percentage empirical sizes of the tests of growth time deformation associated with the null hypothesis of (24). Results relate to rejection at the nominal level of significance of 5%. The tests are applied to series generated by the DGP of (11)-(14) subjected to subsequent GARCH filtering under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of relevant experimental designs.

**Table Eight: The finite-sample sizes of tests of growth time deformation when applied to GARCH filtered series** ($T = 500, 100$)

| | $T = 500$ | | | | | $T = 1000$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ : | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 | 0.01 | 0.05 | 0.1 | 0.20 | 0.30 |
| *(i)* $v_t \sim ged(0.5)$ | | | | | | | | | | |
| $F^G$ | 4.58 | 4.92 | 4.98 | 5.17 | 5.27 | 4.93 | 5.00 | 5.09 | 5.16 | 5.18 |
| $F^G_w$ | 4.92 | 4.85 | 4.88 | 4.89 | 4.88 | 4.86 | 4.81 | 4.86 | 4.84 | 4.84 |
| $F^G_n$ | 11.45 | 11.42 | 11.42 | 11.46 | 11.53 | 9.18 | 9.13 | 9.13 | 9.13 | 9.15 |
| *(ii)* $v_t \sim ged(1.0)$ | | | | | | | | | | |
| $F^G$ | 5.03 | 5.04 | 4.97 | 4.94 | 4.96 | 5.08 | 5.00 | 4.96 | 4.96 | 4.99 |
| $F^G_w$ | 5.99 | 5.94 | 5.91 | 5.86 | 5.84 | 5.53 | 5.50 | 5.50 | 5.50 | 5.48 |
| $F^G_n$ | 9.84 | 9.76 | 9.75 | 9.74 | 9.71 | 7.82 | 7.76 | 7.78 | 7.78 | 7.73 |
| *(iii)* $v_t \sim ged(1.5)$ | | | | | | | | | | |
| $F^G$ | 5.06 | 5.27 | 5.14 | 5.02 | 4.97 | 5.17 | 5.23 | 5.09 | 5.00 | 4.99 |
| $F^G_w$ | 6.28 | 6.35 | 6.25 | 6.26 | 6.25 | 5.73 | 5.73 | 5.73 | 5.65 | 5.63 |
| $F^G_n$ | 9.51 | 9.58 | 9.64 | 9.58 | 9.56 | 7.62 | 7.60 | 7.57 | 7.49 | 7.48 |
| *(iv)* $v_t \sim N(0,1)$ | | | | | | | | | | |
| $F^G$ | 5.11 | 5.35 | 5.25 | 5.03 | 4.95 | 5.12 | 5.24 | 5.11 | 4.97 | 4.95 |
| $F^G_w$ | 6.36 | 6.36 | 6.40 | 6.36 | 6.29 | 5.69 | 5.74 | 5.64 | 5.63 | 5.64 |
| $F^G_n$ | 9.43 | 9.43 | 9.45 | 9.35 | 9.35 | 7.40 | 7.40 | 7.37 | 7.33 | 7.31 |

*Notes*: The figures presented above are percentage empirical sizes of the tests of growth time deformation of associated with the null hypothesis of (24). Results relate to rejection at the 5% nominal level of significance. The tests are applied to series generated by the DGP of (11)-(14) subjected to subsequent GARCH filtering under the use of alternative covariance matrix estimators. All results were derived over 100,000 simulations of relevant experimental designs.

**Table Nine: Univariate and Panel Unit Root Tests**

| | $p_{it}$ | $\Delta p_{it}$ |
|---|---|---|
| *ADF* $\tau$ | $10.58 - 65.03$ | 0.00 |
| *IPS W* | 99.43 | 0.00 |

*Notes*: The above tabulated values represent p-values associated with univariate (ADF) and panel (IPS W) unit root test statistics obtained from analysis of London house prices.

**Table Ten: Time Deformation in London House prices**

| Borough | $F^G$ | $F^G_w$ | $F^G_n$ | $F^G$ | $F^G_w$ | $F^G_n$ |
|---|---|---|---|---|---|---|
| Barking and Dagenham | 0.00 | 0.01 | 0.05 | 0.00 | 0.03 | 0.49 |
| Barnet | 0.84 | 2.04 | 1.60 | 1.04 | 1.10 | 4.23 |
| Bexley | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| Brent | 0.00 | 0.02 | 0.02 | 0.00 | 0.00 | 0.03 |
| Bromley | 0.11 | 13.70 | 12.36 | 0.01 | 0.38 | 0.94 |
| Camden | 2.47 | 1.60 | 0.08 | 0.00 | 0.00 | 0.02 |
| City of Westminster | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Croydon | 0.01 | 0.56 | 1.10 | 0.00 | 0.07 | 0.52 |
| Ealing | 2.62 | 20.81 | 52.21 | 29.94 | 43.72 | 34.77 |
| Enfield | 0.04 | 0.49 | 1.30 | 0.16 | 0.02 | 0.06 |
| Greenwich | 0.11 | 0.68 | 0.80 | 0.25 | 2.00 | 1.40 |
| Hackney | 0.75 | 1.48 | 3.56 | 0.01 | 0.02 | 0.05 |
| Hammersmith and Fulham | 0.33 | 0.28 | 0.00 | 0.26 | 0.17 | 2.19 |
| Haringey | 0.00 | 0.03 | 0.14 | 0.00 | 0.00 | 0.00 |
| Harrow | 0.33 | 0.09 | 0.02 | 0.07 | 0.03 | 0.10 |
| Havering | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Hillingdon | 0.09 | 0.86 | 3.33 | 0.10 | 0.90 | 6.60 |
| Hounslow | 0.00 | 0.06 | 0.00 | 0.00 | 0.24 | 0.45 |
| Islington | 0.29 | 4.90 | 0.27 | 0.22 | 2.62 | 0.70 |
| Kensington and Chelsea | 0.01 | 0.01 | 0.06 | 0.01 | 0.01 | 0.03 |
| Kingston upon Thames | 0.00 | 0.00 | 0.00 | 0.21 | 0.08 | 0.45 |
| Lambeth | 0.77 | 0.29 | 0.01 | 0.00 | 0.01 | 0.01 |
| Lewisham | 1.18 | 2.75 | 0.17 | 0.15 | 0.00 | 0.00 |
| Merton | 0.00 | 0.57 | 0.36 | 0.04 | 1.18 | 2.49 |
| Newham | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| Redbridge | 0.31 | 0.12 | 0.36 | 1.86 | 1.78 | 8.80 |
| Richmond upon Thames | 0.86 | 43.32 | 49.48 | 9.29 | 55.99 | 72.47 |
| Southwark | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Sutton | 0.00 | 0.00 | 0.00 | 0.19 | 0.77 | 4.23 |
| Tower Hamlets | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Waltham Forest | 0.53 | 0.05 | 0.10 | 0.21 | 0.00 | 0.01 |
| Wandsworth | 83.50 | 83.13 | 60.18 | 1.53 | 0.80 | 5.72 |

*Notes*: The above figures are percentage p-values obtained from application of time deformation and growth time deformation tests to London house prices using alternative covariance matrix estimators.

**Table Eleven: Time Deformation in GARCH filtered London house prices**

| Borough | $F^G$ | $F^G_w$ | $F^G_n$ | $F^G$ | $F^G_w$ | $F^G_n$ |
|---|---|---|---|---|---|---|
| Barking and Dagenham | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Barnet | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Bexley | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Brent | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Bromley | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Camden | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| City of Westminster | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Croydon | 0.01 | 0.04 | 0.02 | 0.01 | 0.04 | 0.02 |
| Ealing | 0.30 | 0.30 | 4.00 | 0.34 | 0.38 | 4.14 |
| Enfield | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Greenwich | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Hackney | 0.14 | 0.20 | 0.04 | 0.17 | 0.21 | 0.05 |
| Hammersmith and Fulham | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Haringey | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Harrow | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Havering | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Hillingdon | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Hounslow | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Islington | 1.38 | 0.85 | 0.90 | 1.38 | 0.85 | 0.90 |
| Kensington and Chelsea | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Kingston upon Thames | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Lambeth | 0.02 | 0.02 | 0.20 | 0.02 | 0.02 | 0.20 |
| Lewisham | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Merton | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Newham | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Redbridge | 0.02 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 |
| Richmond upon Thames | 0.13 | 0.01 | 0.00 | 0.13 | 0.01 | 0.00 |
| Southwark | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Sutton | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Tower Hamlets | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Waltham Forest | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Wandsworth | 0.19 | 0.10 | 0.37 | 0.19 | 0.10 | 0.37 |

*Notes*: The above figures are percentage p-values obtained from application of time deformation and growth time deformation tests to standardised residuals of, or GARCH-filtered, London house prices using alternative covariance matrix estimators.