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The Savings Multiplier

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Abstract

A theory of macroeconomic development based on the novel concept of *savings multiplier* is developed. Capital accumulation changes relative prices, amplifying incentives to save as the economy grows. The savings multiplier hinges on two mechanisms. First, accumulation raises wages and leads to redistribution from the consuming old to the saving young. Second, higher wages raise the price of old-age care and, in anticipation of this, the young save more. Our theory captures important aspects of China's development and suggests new channels through which the one child policy and the dismantling of social benefits have fueled China's savings rates.

Keywords: Intertemporal choices; China's savings puzzle; Overlapping generations.

JEL classification: O11, D91, E21

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1. Introduction

A theory of macroeconomic development based on the novel concept of *savings multiplier* is developed: capital accumulation sparks output growth but also induces changes in relative prices and in intergenerational income shares that create further incentives to accumulate, implying rising saving rates as the economy develops. The savings multiplier creates a feedback effect of growth on savings that magnifies the impact of exogenous shocks – such as demographic change, policy reforms, productivity shocks – on capital per capita in the long run. The scope of our results is twofold. First, the savings multiplier introduces circular causality in the savings-growth relationship and thus provides a new explanation for rising saving rates in developing countries. Second, our theory captures important aspects of China's economic performance and suggests new channels through which the one child policy and the dismantling of cradle-to-grave social benefits have fueled China's savings and accumulation rates. Each point is discussed below.

Rising saving rates characterized the growth process of most developed economies. Lewis (1954) provides an early recognition of this stylized fact, stressing that

> "The central problem in the theory of economic development is to understand the process by which a community which was previously saving and investing 4 or 5 per cent of its national income or less, converts itself into an economy where voluntary saving is running at about 12 to 15 per cent of national income or more. [...] We cannot explain any industrial revolution [...] until we can explain why saving increased relatively to national income." (Lewis, 1954: p.155).

The issue of causality in the relationship between growth and saving rates is still an open question (see Deaton, 2010). Standard growth theories tell us that saving rates drive development but empirical evidence suggests that causality may run in the opposite direction (Attanasio et al. 2000; Rodrik, 2000). The topic received attention in the growth literature of the late 1990s – mostly dedicated to the stunning performance of East Asian economies – but only a few contributions attempted at developing new theories to explain the effects of growth on saving rates. One of these contributions is the theory of Relative Consumption, where households' utility depends on current consumption relative to a benchmark level which may reflect habit formation (Carroll et al. 2000), interpersonal comparisons (Alvarez-Cuadrado et al. 2004), or international status seeking (Valente, 2009). In Relative Consumption models, economic growth raises the benchmark consumption level over time and the agents' willingness to catch-up with the benchmark prompts households to adjust savings accordingly. Our theory of the savings multiplier is different because the feedback effects of growth on saving rates hinge on the economy's demographic structure, which comprises overlapping generations, and on the allocation of labor between different production sectors.

In our model, the first channel through which growth affects saving rates is the *intergenerational distribution effect*. Higher savings imply both higher capital stock and increased demand for care by the old, both fueling wage increases. The income distribution shifts in favor of the wage earners – that is, accumulation raises the income share of savers relative to the old agents – which stimulates further savings and capital accumulation.

The second channel is the *old-age requirement effect*. Increased savings and capital accumulation push the anticipated future wage up, making old-age care more expensive. To compensate for the increased future costs of care, young agents increase their savings relative to current income. This gives an additional channel whereby savings and capital accumulation stimulate further savings and capital accumulation. During the transition to the long-run equilibrium, savings rates increase over time, the share of employment in the manufacturing sector declines, the income distribution shifts in favor of the young, and an increasing share of private expenditures is allocated to the purchase of services.[1]

Although our contribution is theoretical, the key motivation of our analysis lies in the empirical literature on Asian economies, and on the experience of China in particular. Since 1978, real per capita GDP in China has increased tenfold, and fast output growth was accompanied by massive capital accumulation. After drastic policy changes in the late 1970s, savings and investment as a share of GDP increased sharply. Importantly, savings and investment rates continued to grow thereafter: graph (a) in Figure 1 shows that more than 40% of GDP has been invested, while more than 50% of GDP has been saved, over the last years.

China's saving behavior inspired a huge body of empirical literature but there is a lack of new theories that could explain the most puzzling fact, namely, that households have increased their savings rate, despite being quite poor, having fast income growth, and receiving low returns on their savings.[2] In this respect, our model provides a theory of savings that is consistent with four relevant facts that characterized China's development – most of which are direct consequences of the reforms enacted in the last forty years.

First, saving rates increased while fertility sharply declined (*Fact 1*). China's fertility rate decreased from 4.9 in 1975 to 1.7 in 2007, while life expectancy increased by ten years in the same period (Litao and Sixin, 2009). A major trigger of this acceleration in population aging was the one-child policy implemented since 1978, which changed family composition and reduced the number of births.

Second, Chinese workers face an increased need to provide for old age with their own resources (*Fact 2*). A prominent cause is the reform of the industry sector implemented since the late 1980s, which gradually dismantled state owned enterprises and deleted cradle-to-grave social benefits for a huge fraction of workers (Ma and Yi, 2010).[3] Meanwhile, the private provision of old-age security is neither efficient nor pervasive: less than 30% of all employees are covered by pension schemes (Oksanen, 2010).

Third, a growing share of health care services is, and will increasingly need to be, purchased in the market (*Fact 3*). The share of health spending that households pay themselves increased from 16% in 1980 to 61% in

[1]This mechanism clearly distinguishes our notion of savings multiplier, which operates on the supply side under full employment conditions, from the traditional concept of demand multiplier according to which income is pushed up from the side of demand when factors of production are not fully utilized. To our knowledge, neither the term 'savings multiplier' nor its underlying concept have been previously introduced in the literature.

[2]The high savings rate reported in graph (a) of Figure 1 reflects the sum of high corporate savings and high household savings. Song et al. (2011) provide a theoretical explanation for high *corporate* savings based on the existence of capital market imperfections that generate high shares of firms' retained profits. Our claim on the lack of theories refers, instead, to the analysis of *household* savings, which is the focus of our model. At present, household savings is the single largest component of total savings and according to Yang (2012), the increase in the rate of household savings from 2000 to 2008 is the most important contribution to the overall increase in the Chinese savings rate in the same period.

[3]The reform implied massive layoffs, and the enterprise-based social safety net shrank rapidly as a result (Ma and Yi, 2010). In the pre-reform system, instead, each state enterprise provided housing, medical care and old-age security to its workers and pensioners (James 2002).

2001 (Blanchard and Giavazzi, 2006), and the growth in China's health spending is "one of the most rapid in world history" (Eggleston, 2012: p.4). The rising importance of private provision may itself be a side-effect of the one-child policy through changes in family composition.[4] But beyond its causes, the relevant consequence for our analysis is that the increased share of care services in private expenditures is driving structural change in production sectors. Graph (b) in Figure 1 shows that the share of employment in health and social work relative to that in manufacturing has doubled over 15 years.[5] Such sectoral change has been neglected as a possible determinant of China's saving rates whereas it plays an important role in our model.

Fourth, the income distribution is shifting in favor of young wage earners and in disfavor of the old (*Fact 4*). The share of labor income in GDP has increased (Bai and Qian, 2010) and, since 1998, real wage growth has exceeded GDP growth (Li et al., 2012). This induced a shift in the income distribution towards young workers (Song and Yang, 2010).

Our model produces equilibrium dynamics that are fully consistent with Facts 1-4: capital accumulation in the manufacturing sector raises wages and shifts labor into the care sector, boosting saving rates via both higher income for young cohorts and higher expected future cost of care services. In particular, exogenous shocks that plausibly capture the effects of China's past reforms – namely, a reduction in the population growth rate, an increase in the minimum level of care to be purchased – induce higher capital per capita and raise saving rates during the transition because capital accumulation is accelerated by the savings multiplier. A calibration exercise shows that, despite the small relative size of China's care sector, the accumulation effects of the savings multiplier account for 34% of the capital stock per worker in the steady state. These results suggest that the one-child policy and the dismantling of cradle-to-grave social benefits have fueled China's saving rates and capital accumulation in the last decades. By the same token, the counteracting reforms that China's government recently announced – namely, the abandonment of the one-child policy as well as the intention to expand the welfare system – are predicted to reduce savings and capital accumulation. In this respect, the calibrated model quantifies the impact of exogenous fertility shocks and shows that a reform introducing coverage of basic care would reduce capital per worker by more than 20%.

With respect to the existing literature, a specific value added of our analysis is the use of the general equilibrium framework. In our model, the economy's equilibrium path brings together Facts 1-4 and combines them with a precise causal order. The existing empirical literature – e.g., Kraay (2000), Modigliani and Cao (2004), Chamon and Prasad (2010) – provides very valuable information on each of these facts but typically focusing on one single mechanism in isolation from the others, thus failing to deliver a complete picture.[6] Our paper is different, but complementary, to this line of research: none of the above mentioned contributions

[4] The one-child policy drastically reduced the scope for family provided care during a period in which the need for such care was rapidly increasing. More and more families now consist of four grandparents, two parents and one child, making the market provision of care a necessity.

[5] From 1993 to 2008, the employment share of manufacturing decreased from 37% to 29% while the employment share of health and social work increased from 2.8% to 4.7% (ILO, 2015).

[6] Kraay (2000) documents the link between the increased need to provide for old age and the dismantling of state-owned enterprises; Modigliani and Cao (2004) find a strong effect of the one-child policy on the needs to save for retirement; Blanchard and Giavazzi (2006) and Chamon and Prasad (2010) explain increased saving rates with the rising burden of expenditures such as health care and education; Song and Yang (2010) argue that the main reason for the rising saving rate is the shift in the income distribution in favor of young workers.

develops a general equilibrium model where capital accumulation affects subsequent saving rates, or note any of the two mechanisms behind the savings multiplier.

2. The Model

The key features of the model are the overlapping-generations (OLG) structure, the hypothesis of age-dependent needs, and the existence of two production sectors. The first set of firms produces the *generic good* which is partly saved as physical capital, and partly consumed by both young and old agents. The second set of firms provides services that are exclusively purchased by the old and may be interpreted as *old-age care.* The one-good OLG framework pioneered by Diamond (1965) – henceforth termed the *canonical model* – may be viewed as a special case of our model.[7]

2.1. Consumers

Each agent lives two periods $(t, t+1)$. Total population, denoted N_t, consists of N_t^y young and N_t^o old agents, and grows at the exogenous net rate $n > -1$;

$$N_t = N_t^y + N_t^o, \quad N_t^y = N_t^o \cdot (1+n), \quad N_{t+1} = N_t \cdot (1+n). \tag{1}$$

Agents purchase two types of goods over their life-cycle: the generic consumption good is enjoyed in both periods of life whereas old-age care services are only purchased in the second period of life. The lifetime utility of an agent born at the beginning of period t is

$$U_t \equiv u(c_t) + \beta \cdot v\left(d_{t+1}, h_{t+1} - \bar{h}\right), \tag{2}$$

where c_t and d_{t+1} represent consumption levels of the generic good in the first and second period of life, respectively, h_{t+1} is the amount of old-age care consumed when old, $\bar{h} \geqslant 0$ is the *minimum requirement* – i.e., the minimum amount of old-age care required by old agents – and $\beta \in (0,1)$ is the private discount factor between young and old age. The consumer problem is subject to the constraint that the minimum requirement, $h_{t+1} - \bar{h} \geqslant 0$, is at least weakly satisfied.[8] The case with zero minimum requirement, $\bar{h} = 0$, is of special interest since it will allow us to separate the two central mechanisms of the model, the 'intergenerational distribution' and the 'old-age requirement' effects (cf. Section 4.).

Young agents supply inelastically one unit of homogeneous labor and save part of their labor income. Old agents do not work and spend all their interest income in purchasing consumption goods and old-age care. The individual budget constraints read

$$c_t = w_t - s_t, \tag{3}$$

$$s_t R_{t+1} = d_{t+1} + p_{t+1} h_{t+1}, \tag{4}$$

[7] Detailed derivations and long proofs are collected in the separate online appendix.

[8] As is standard, the focus will be on interior equilibria where $h_{t+1} > \bar{h}$ and verification of ex-post the conditions under which this strict inequality holds. The analysis shows that there always exists a unique equilibrium in which the allocation of labor between generic-good and health-care production is consistent with the interior solution $h_{t+1} > \bar{h}$.

where the generic good is taken as the numeraire, w_t is the wage rate, s_t is savings, R_{t+1} is the gross rate of return to saving, and p_{t+1} is the price of old-age care. Savings consist of physical capital, which is homogeneous with the generic good. Assuming full depreciation within one period, market clearing requires that aggregate capital at the beginning of period $t+1$ equals aggregate savings of the young agents in the previous period, $K_{t+1} = N_t^y s_t$.

In order to make the analysis transparent, consider a specific form of preferences:

$$u(c_t) \equiv \log c_t, \tag{5}$$

$$v\left(d_{t+1}, h_{t+1} - \bar{h}\right) \equiv \log\left[\gamma\left(d_{t+1}\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(h_{t+1} - \bar{h}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where $\gamma \in [0,1]$ is a weighting parameter and $\sigma > 0$ is the elasticity of substitution between consumption goods and care services in the second period of life: d_{t+1} and h_{t+1} are strict complements if $\sigma < 1$, strict substitutes if $\sigma > 1$. In the limiting case $\sigma \to 1$, the term in square brackets reduces to the Cobb-Douglas form $(d_{t+1})^{\gamma}(h_{t+1})^{1-\gamma}$. The empirical literature shows that, when h is interpreted as health care, the most plausible case is that of strict complementarity with a positive requirement, $\sigma < 1$ and $\bar{h} > 0$.[9] The case of substitutability is nonetheless also studied. Preferences (5)-(6) exhibit two essential properties. First, they allow us to treat the canonical OLG model as a special case: setting $\gamma = 1$ and $\bar{h} = 0$, old-age care services do not yield utility and, hence, are not produced in equilibrium. Second, the utility functions (5)-(6) exhibit a unit elasticity of intertemporal substitution. Therefore, setting $\gamma = 1$ yields the log-linear version of the canonical model in which the saving rate is constant over time. This implies that, in the general case $0 < \gamma < 1$, any departure from the canonical result of 'constant saving rate' must be induced by our distinctive hypothesis, namely, the fact that old agents need dedicated care services.

2.2. Production Sectors

From a technological viewpoint, the different nature of generic goods – which may be interpreted as manufactured products – and old-age care services – which include health care as well as personal assistance – is captured by Baumol's (1967) hypothesis: the production of care services is strongly labor intensive because, differently from what happens in manufacturing industries, capital cannot be used as a substitute for labor. Hartwig (2008) tests this hypothesis on recent data, obtaining strong empirical support to Baumol's view and showing that health care expenditure is mainly driven by wage increases. Our model captures these aspects by assuming that care services are produced with labor as the only factor of production. The consumption good, instead, is produced by means of capital and labor as in Diamond's (1965) canonical model.[10] The fraction of workers employed in the generic sector is denoted ℓ_t while $1 - \ell_t$ denotes the fraction employed in the care sector. Perfect labor mobility and perfectly competitive conditions in the labor market ensure wage

[9] When $\bar{h} > 0$, function (6) implies that the income elasticity of old-age care falls short of unity, in line with Acemoglu et al. (2013) that estimate the income elasticity of health spending to 0.7. Finkelstein et al. (2012) estimate an elasticity of substitution between health and non-health consumption equal to $\sigma = 0.2$.

[10] For a two-sector OLG model with capital in both sectors, as well as the existence and stability properties of such models, see Galor (1992).

equalization in equilibrium. The old-age care sector exhibits a simple constant returns to scale technology,

$$H_t \equiv \eta \cdot (1-\ell_t) \cdot N_t^y, \tag{7}$$

where H_t is the aggregate output of care services, and $\eta > 0$ is a constant labor productivity parameter. In the generic good sector, aggregate sectoral output X_t is given by

$$X_t = B \cdot (K_t)^\alpha \left(\ell_t N_t^y\right)^{1-\alpha} \tag{8}$$

where $B > 0$ is an exogenous productivity parameter, K_t is aggregate capital, and $\alpha \in (0,1)$ is an elasticity parameter.

3. Static Equilibrium

This section discusses the static equilibrium conditions holding in each period for a given stock of capital per worker. The profit-maximizing conditions for firms and the utility-maximizing conditions for households determine the joint static equilibrium of the labor and goods markets, and bear precise implications for the saving rate and for the co-movements of employment and capital.

3.1. Firms

In the service sector, technology (7) implies that the wage is proportional to the market price of care services,

$$w_t = \eta p_t. \tag{9}$$

Market clearing requires that total output of old-age care services matches aggregate demand by old agents, $H_t = N_t^o h_t$. The existence of a minimum requirement, $h_t \geqslant \bar{h}$, implies that total production H_t must exceed $N_t^o \bar{h}$. This imposes an upper bound on the employment share of the generic sector: given technology (7), an interior equilibrium requires

$$\ell_t \leqslant \frac{\eta(1+n)-\bar{h}}{\eta(1+n)} \equiv \ell^{\max}, \tag{10}$$

where $\ell^{\max}$ is the maximum level of employment in the generic sector that is compatible with a level of old-age care output equal to the minimum requirement.[11] In the remainder of the analysis, the restriction $\bar{h} \leqslant \eta(1+n)$ is assumed to hold, which implies $\ell^{\max} \geqslant 0$. When the minimum requirement is $\bar{h} = 0$, it follows that $\ell^{\max} = 1$.

In the generic good sector, factor prices equal marginal productivities,

$$w_t = B(1-\alpha)(\kappa_t/\ell_t)^\alpha = (1-\alpha)(x_t/\ell_t), \tag{11}$$

$$R_t = B\alpha(\ell_t/\kappa_t)^{1-\alpha} = \alpha(x_t/\kappa_t), \tag{12}$$

where $x_t \equiv X_t/N_t^y$ is sectoral output *per young*. Aggregating incomes between sectors yields

$$\frac{Y_t}{N_t^y} = w_t + R_t\kappa_t = x_t\left(\frac{1-\alpha}{\ell_t}+\alpha\right), \tag{13}$$

[11] The level of care supply equal to the minimum requirement is $H_t^{\min} \equiv \eta(1-\ell^{\max}) N_t^y = N_t^o \bar{h}$.

where Y_t is aggregate income, which coincides with the total value of goods and services produced in the economy, $Y_t \equiv X_t + p_t H_t$.

3.2. Consumers

Each agent maximizes (2) subject to the budget constraints (3)-(4). Using the standard notation for derivatives – i.e., $u_{c_t} \equiv \partial u / \partial c_t$ – the solution to the consumer problem yields two familiar first order conditions: the Keynes-Ramsey rule, $u_{c_t} = \beta R_{t+1} v_{d_{t+1}}$, and an efficiency condition establishing the equality between the price of care services and the marginal rate of substitution with second-period generic goods consumption, $v_{h_{t+1}} / v_{d_{t+1}} = p_{t+1}$. Under preferences (5)-(6), these conditions determine the following relationships (see online appendix). Consumption and savings of young agents are given by

$$c_t = \frac{1}{1+\beta}\left(w_t - \frac{p_{t+1}}{R_{t+1}}\bar{h}\right) \text{ and } s_t = \frac{1}{1+\beta}\left(\beta w_t + \frac{p_{t+1}}{R_{t+1}}\bar{h}\right). \tag{14}$$

When $\bar{h} = 0$, these expressions are similar to those holding in the canonical model, where young agents save a constant fraction of their wage income. This similarity does not imply, however, the same accumulation dynamics: as shown in section 3.7. below, our model predicts that, even with $\bar{h} = 0$, the aggregate saving rate is not constant because the intergenerational distribution of income changes over time. In the more general case with $\bar{h} > 0$, consumption and savings are not fixed proportions of labor income: in the first period of life, consumption is lower and savings are higher the larger is $\bar{h}$. The reason is that young agents take into account the future cost of the minimum care to be purchased in the second period of life. The magnitude of this effect on savings depends on the *future* price of care in present-value terms, p_{t+1}/R_{t+1}, which is in turn determined by the future wage since $p_{t+1}/R_{t+1} = \eta w_{t+1}/R_{t+1}$. This mechanism, henceforth labeled the *old-age requirement effect*, establishes a precise channel through which relative factor prices affect present savings: high future wages w_{t+1} and low returns on savings R_{t+1} induce higher savings today in order to purchase the minimum amount of care tomorrow.

Considering generic consumption in the second period of life, each old agent purchases

$$d_t = (1+n)\left[\ell_t - (1-\alpha)\right] B \left(\kappa_t/\ell_t\right)^{\alpha}, \tag{15}$$

which is the residual (per-old) output of the generic sector after consumption and savings of young agents have been subtracted. Result (15) implies that second-period consumption is positive only if $\ell_t > 1-\alpha$, which is always the case in equilibrium, as shown below.

The last condition for utility maximization links the old agents' expenditure shares over the two goods to their relative price:

$$\frac{p_t \cdot \left(h_t - \bar{h}\right)}{d_t} = \left(\frac{1-\gamma}{\gamma}\right)^{\sigma} p_t^{1-\sigma}. \tag{16}$$

Expression (16) shows that the expenditure share of net care services increases (decreases) with the price when the two goods are complements (substitutes). The reason is that the effect of a ceteris paribus increase in p_t on the expenditure ratio $p_t\left(h_t - \bar{h}\right)/d_t$ depends on the elasticity of the relative demand for care services.

Under complementarity, demand is relatively rigid and the increase in p_t raises the expenditure share of net care. Under substitutability, instead, demand is elastic and the opposite happens. These substitution effects bear crucial consequences for the allocation of labor, as shown below.[12]

3.3. Labor Market

The labor demand schedules of the two production sectors determine a unique equilibrium in the labor market. From (9) and (11), wage equalization between sectors implies

$$p_t = (B/\eta)(1-\alpha)(\kappa_t/\ell_t)^{\alpha} \equiv \Phi(\ell_t, \kappa_t). \tag{17}$$

Condition (17) defines p_t as the level of the price of care ensuring wage equalization for given levels of sectoral employment, capital per worker, and productivity. In particular, function $p_t = \Phi(\ell_t, \kappa_t)$ is strictly decreasing in ℓ_t. The intuition is that for a given capital per young κ_t, higher employment in the generic sector decreases the marginal productivity of labor, implying a lower wage, and thus a lower price of care.

3.4. Goods Markets

The equilibrium in the goods market is characterized by solving the demand relationship (16) for the price of care, and substituting $p_t h_t/d_t$ with the market-clearing and zero-profit conditions holding for the producing firms, obtaining (see online appendix)

$$p_t = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\sigma-1}} \left[\frac{(1-\alpha)(\ell^{\max}-\ell_t)}{\ell_t-(1-\alpha)}\right]^{\frac{1}{1-\sigma}} \equiv \Psi(\ell_t). \tag{18}$$

Expression (18) defines p_t as the price of care that ensures equilibrium in the goods market.[13] The most important insight is that the function $p_t = \Psi(\ell_t)$ is strictly decreasing when $\sigma < 1$, and strictly increasing when $\sigma > 1$. When $\sigma < 1$ the price of care is positively related to the employment share in the care sector $1-\ell_t$. The reason is that a ceteris paribus increase in p_t increases the expenditure share that old consumers devote to care services, attracting labor in the care sector. When $\sigma > 1$, in contrast, a higher price of care induces a lower expenditure share of care, and thus more labor in the generic sector.[14]

3.5. Employment and Capital Co-Movements

Consider now the joint equilibrium of the markets for labor and for goods. The two relevant conditions, (17) and (18), imply that the price of care and sectoral employment levels in each period t depend on current capital per worker, κ_t. Formally, the employment share of the generic sector for a given level of κ_t, denoted by $\ell_t = \ell(\kappa_t)$, is the fixed point

$$\ell(\kappa_t) \equiv \arg\operatorname{solve}_{\{\ell_t \in (1-\alpha, \ell^{\max})\}} \left[\Phi(\ell_t, \kappa_t) = \Psi(\ell_t)\right]. \tag{19}$$

[12] Substitution effects only disappear with Cobb-Douglas preferences: when $\sigma = 1$, relative expenditure shares are exclusively determined by the taste parameter γ and do not depend on the relative price p_t.

[13] Function $\Psi(\ell_t)$ does not depend on capital per worker because, with Cobb-Douglas technologies, the sector allocation of labor alone determines the sectoral output ratio $X_t/p_t H_t$.

[14] It should be noted that, in the special case of unit elasticity of substitution, $\sigma = 1$, expression (18) does not hold because price and quantity effects on the demand side balance each other. As a result, the equilibrium between demand and supply in the goods market is characterized by constant employment shares, with $\ell_t = \frac{(1-\alpha)(\gamma\ell^{\max}+1-\gamma)}{\gamma(1-\alpha)+1-\gamma}$ at each t.

The existence and uniqueness of this fixed point can be verified in graphical terms in Figure 2 (see the online appendix for a formal proof). On the one hand, the function $\Phi(\ell_t, \kappa_t)$ is strictly decreasing in ℓ_t and exhibits positive vertical intercepts at the boundaries of the relevant interval $\ell_t \in (1-\alpha, \ell^{\max})$. On the other hand, the function $\Psi(\ell_t)$ is decreasing (increasing) under complementarity (substitutability), and display asymptotic properties that ensure the existence and uniqueness of the fixed point $\Psi(\ell_t) = \Phi(\ell_t, \kappa_t)$ within the relevant interval $\ell \in (1-\alpha, \ell^{\max})$.[15] The fixed point (19) simultaneously determines employment shares and the price of care. Substituting $\ell(\kappa_t)$ in $\Psi(\ell_t)$ or in $\Phi(\ell_t, \kappa_t)$ the equilibrium price of care for given capital per worker follows as

$$p(\kappa_t) \equiv \Psi(\ell(\kappa_t)) = \Phi(\ell(\kappa_t), \kappa_t). \tag{20}$$

Even though it is not yet specified whether and how capital grows, result (20) clarifies how capital accumulation affects the price of care and employment shares:

Proposition 1 *An equilibrium trajectory with positive accumulation implies a rising price of care. Under complementarity, the employment share in the generic sector is decreasing. Under substitutability, the employment share in the generic sector is increasing;*

$$\kappa_{t+1} > \kappa_t \qquad \Longrightarrow \qquad p_{t+1} > p_t$$

and

$$\kappa_{t+1} > \kappa_t \Rightarrow \left\{ \begin{array}{l} \ell_{t+1} < \ell_t \text{ if } \sigma < 1 \\ \ell_{t+1} > \ell_t \text{ if } \sigma > 1 \end{array} \right\}$$

Proof. The proposition can be proved in graphical terms.[16] Since $\partial\Phi(\ell, \kappa)/\partial\kappa > 0$, an increase in κ shifts the $\Phi(\ell, \kappa)$ curve up-rightward in Figure 2. The resulting equilibrium price $p(\kappa)$ is necessarily higher but $\ell(\kappa)$ reacts differently depending on the value of σ. The employment share $\ell(\kappa)$ increases under complementarity, decreases under substitutability:

$$\ell'_\kappa \equiv \frac{\mathrm{d}\ell(\kappa_t)}{\mathrm{d}\kappa_t} < 0 \quad \text{if } \sigma < 1; \quad > 0 \quad \text{if } \sigma > 1 \ .$$

■

The intuition is that an increase in capital per young increases the equilibrium wage and thereby the price of care. Under complementarity, old agents react to the price increase by raising the share of expenditure on net care, which decreases the employment share in the generic sector $\ell(\kappa)$. Under substitutability, instead, old agents reduce the expenditure share on net care and employment in the generic sector rises.

[15] See the online appendix for further details.

[16] Proposition 1 is equivalently proved by differentiating the equilibrium condition $\Psi(\ell(\kappa_t)) = \Phi(\ell(\kappa_t), \kappa_t)$. The exact relationship between κ and ℓ is reported in expression (30) below, and indeed implies that $\ell'_\kappa \equiv \mathrm{d}\ell(\kappa_t)/\mathrm{d}\kappa_t$ is strictly negative (positive) under complementarity (substitutability).

3.6. Static Equilibrium Comparative Statics

For a given capital stock, the static equilibrium labor allocation depends on the parameters of the model. The following Proposition establishes how the employment share of the generic sector, denoted by $\ell(\kappa_t) = \ell(\kappa_t; B, n, \bar{h})$, depends on productivity B, on population growth n, and on the level of the minimum requirement $\bar{h}$.

Proposition 2 *In the static equilibrium with given κ_t,*

$$\frac{d\ell(\kappa_t; B, n, \bar{h})}{dB} \equiv \ell'_B < 0 \quad \text{if } \sigma < 1; \quad > 0 \quad \text{if } \sigma > 1 \;, \tag{21}$$

$$\frac{d\ell(\kappa_t; B, n, \bar{h})}{d\bar{h}} \equiv \ell'_{\bar{h}} < 0, \tag{22}$$

$$\frac{d\ell(\kappa_t; B, n, \bar{h})}{dn} \equiv \ell'_n > 0 \text{ if } \bar{h} > 0 \quad (= 0 \text{ if } \bar{h} = 0). \tag{23}$$

Proof. The proposition may be proved in graphical terms. An increase in B shifts $\Phi(\ell, \kappa)$ upward in Figure 2. The employment share ℓ increases when $\sigma < 1$, and decreases when $\sigma > 1$. Changes in n and in $\bar{h}$ operate through $\ell^{\max}$ in the expression for $\Psi(\ell)$ in equation (18). An increase in $\ell^{\max}$ shifts $\Psi(\ell)$ to the right, increasing ℓ. Provided $\bar{h} > 0$, A higher n and a lower $\bar{h}$ both imply a higher $\ell^{\max} \equiv 1 - \frac{\bar{h}}{\eta(1+n)}$. ■

A higher B expands production possibilities in the generic sector and affects the labor allocation depending on the value of σ. Under complementarity, consumers wish to exploit the productivity gain to purchase more care, and such higher demand pushes labor into the care sector. Under substitutability, instead, labor is drawn into the generic sector as old agents increase their relative demand for consumption goods. The effects of changes in $\ell^{\max}$ are more clear-cut: when a larger fraction of workers is needed to satisfy the minimum care requirement, the care sector will employ more workers.

3.7. Saving Rates and Accumulation

The general relationships linking savings rates, capital accumulation, and sectoral employment shares can be summarized as follows. Considering the economy's aggregate income (13) and the wage rate (11), the labor share accruing to young agents is

$$\frac{w_t N_t^y}{Y_t} = \frac{(1-\alpha)\frac{x_t}{\ell_t}}{x_t\left(\frac{1-\alpha}{\ell_t} + \alpha\right)} = \frac{1-\alpha}{1-\alpha(1-\ell_t)}, \tag{24}$$

Equation (24) shows that, in static equilibrium, an increase in the generic sector employment share ℓ_t reduces the total income share of young agents. The intuition is that if labor moves from the care sector to generic production, the return to capital increases relative to the wage rate, and this implies a shift in the income distribution away from the young towards the old. This result is referred to as the *intergenerational distribution effect.*

Since only young agents save, the intergenerational distribution directly influences the economy's saving rate and, hence, capital accumulation. The savings rate is denoted by θ_t and is defined as aggregate savings relative to the total value of production. Combining the saving function in (14) with expression (24), and substituting $\ell^{\max}$ by (10), yields

$$\theta_t \equiv \frac{N_t^y s_t}{Y_t} = \underbrace{\frac{\beta(1-\alpha)}{1+\beta}}_{\text{Canonical model}} \cdot \underbrace{\frac{1}{1-\alpha\cdot(1-\ell_t)}}_{\text{Intergenerational Distribution}} \cdot \underbrace{\Gamma\left(\frac{\bar{h}}{\ell_{t+1}}\right)}_{\text{Old-age Requirement}}, \tag{25}$$

where

$$\Gamma\left(\frac{\bar{h}}{\ell_{t+1}}\right) \equiv \left[1 - \frac{(1-\alpha)}{\alpha(1+\beta)\eta(1+n)}\frac{\bar{h}}{\ell_{t+1}}\right]^{-1}, \quad \Gamma'(\cdot) > 0, \quad \Gamma(0) = 1. \tag{26}$$

Expression (25) shows that the savings rate is negatively related to both ℓ_t and ℓ_{t+1}. The *current* employment share of the generic sector, ℓ_t, affects the saving rate through the intergenerational distribution channel described above. The anticipated *future* employment share, ℓ_{t+1}, affects the saving rate through the function $\Gamma(\cdot)$, which captures the old-age requirement effect – i.e., extra savings induced by the existence of a minimum care requirement: being increasing in $\bar{h}$, the term $\Gamma(\cdot)$ equals unity when $\bar{h} = 0$ and strictly exceeds unity when $\bar{h} > 0$.[17] The comparison with the canonical model is straightforward. Without the care sector, the last two terms in (25) reduce to unity, and the saving rate equals the fraction of income saved by the young, $\beta/(1+\beta)$, times the income share of the young, $1-\alpha$.

Our preliminary conclusion is twofold. First, both the intergenerational distribution and the old-age requirement effects push the saving rate *above* the level predicted by the canonical model. Second, the saving rate is, in general, *not constant* over time and in particular, it will be *increasing over time* if the economy follows an equilibrium path along which the employment share of the generic sector ℓ_t grows over time.

4. Dynamic General Equilibrium

Since the generic consumption good is produced by means of a neoclassical technology, the dynamic equilibrium path of the economy admits a long-run steady state in which capital per worker is constant, and generic production grows at the exogenous rate of population growth. This section derives the stability properties of the long-run steady state and shows that the transitional dynamics arising under complementarity match qualitatively the stylized facts that inspire our analysis (cf. Introduction). In the long run, the intergenerational distribution and the old-age requirement effects affect, through distinct channels, the steady-state level of capital per worker which is thus higher than in the canonical model.

4.1. Accumulation Law

The equality between investment and savings implies that capital per worker is determined by previous savings according to

$$\kappa_{t+1} = \frac{\theta_t Y_t}{1+n}. \tag{27}$$

[17] The online appendix shows that the static equilibrium conditions imply $(1-\alpha)\bar{h} < \alpha(1+\beta)\eta(1+n)\ell_{t+1}$, from which it follows that $\Gamma(\bar{h}/\ell_{t+1}) > 1$ for any $\bar{h} > 0$.

This market clearing condition, combined with the saving decisions of young agents, yields the dynamic law that governs capital accumulation in the economy: by substituting (25) and (13) in the right hand side of (27), the accumulation law reads

$$\kappa_{t+1} = \underbrace{\frac{B\beta(1-\alpha)}{(1+\beta)(1+n)}\kappa_t^{\alpha}}_{\text{Canonical model}} \cdot \underbrace{\ell_t^{-\alpha}}_{\text{Intergen. Distr.}} \cdot \underbrace{\Gamma\left(\frac{\bar{h}}{\ell_{t+1}}\right)}_{\text{Old-age requirement effect}}. \tag{28}$$

Expression (28) decomposes the accumulation law of capital per worker in three parts. The first term on the right hand side is the dynamic law in the canonical one-good model. The second and third terms on the right hand side of (28) directly follow from the intergenerational distribution effect and the old-age requirement effect. An increase in ℓ_t reduces κ_{t+1} because a lower current wage reduces young agents' income, and thereby, current savings. An increase in ℓ_{t+1} reduces κ_{t+1} because a lower future wage reduces the expected future cost of health care, and thereby, current savings.

The presence of anticipated future variables in the right hand side of (28) implies that further work is needed to characterize the equilibrium path. Recalling result (19), equilibrium employment shares are a function of the capital stock per worker in each period. By substituting $\ell_t = \ell(\kappa_t)$ and $\ell_{t+1} = \ell(\kappa_{t+1})$ into (28), the accumulation law follows as

$$\kappa_{t+1} = \frac{B\beta(1-\alpha)}{(1+\beta)(1+n)}\kappa_t^{\alpha}\left[\ell(\kappa_t)\right]^{-\alpha}\Gamma\left(\frac{\bar{h}}{\ell(\kappa_{t+1})}\right). \tag{29}$$

Expression (29) implies that capital dynamics crucially depend on how sectoral employment shares react to variations in capital per worker. In this respect, the relevant elasticity is[18]

$$\frac{\ell'_{\kappa}(\kappa_t)\,\kappa_t}{\ell(\kappa_t)} = \frac{1}{1-\frac{1}{1-\sigma}\frac{1}{\alpha}Q_1}\left\{\begin{array}{l} <0 \text{ if } \sigma<1 \\ >0 \text{ if } \sigma>1 \end{array}\right\}, \tag{30}$$

where $Q_1 \equiv \frac{\ell_t}{\ell_t-(1-\alpha)}\cdot\frac{\ell^{\max}-(1-\alpha)}{\ell^{\max}-\ell_t} > 1$. The slope of the accumulation law can be found by taking the elasticity of (29) with respect to κ_t and κ_{t+1}, which yields[19]

$$\frac{\mathrm{d}\kappa_{t+1}}{\mathrm{d}\kappa_t}\frac{\kappa_t}{\kappa_{t+1}} = \frac{\alpha-\alpha\frac{\ell'_{\kappa}(\kappa_t)\kappa_t}{\ell(\kappa_t)}}{1+\frac{\Gamma'}{\Gamma}\frac{\bar{h}}{\ell(\kappa_{t+1})}\frac{\ell'_{\kappa}(\kappa_{t+1})\kappa_{t+1}}{\ell(\kappa_{t+1})}}. \tag{31}$$

In the numerator of (31), the direct effect on κ_{t+1} of an increase in κ_t is larger under complementarity, i.e., when $\ell'_{\kappa}(\kappa_t) < 0$. When $\bar{h} > 0$, there is also an indirect effect via the increase in $\ell(\kappa_{t+1})$, captured in the denominator. The possibility of (local) instability and multiple steady states, however, turns out to be remote: non-uniqueness and instability might only occur under unreasonable parameter values (see online appendix). Armed with these results, the equilibrium path of the economy can be fully characterized. The following subsections show that the intergenerational distribution and the old-age requirement effects raise the long-run capital stock above the canonical level through distinct channels. In order to obtain transparent results,

[18] Expression (30) is obtained by differentiating the equilibrium condition $\Psi(\ell(\kappa_t)) = \Phi(\ell(\kappa_t), \kappa_t)$ and is fully derived in the online appendix. The fact that $Q_1 > 1$ directly follows from the requirement $1-\alpha < \ell_t < \ell^{\max}$ and it implies the signs reported in (30). Note that (30) yields an alternative proof of Proposition 1.

[19] Totally differentiating (29) yields $\frac{\mathrm{d}\kappa_{t+1}}{\kappa_{t+1}} = \alpha\frac{\mathrm{d}\kappa_t}{\kappa_t} - \alpha\frac{\partial\ell(\kappa_t)}{\partial\kappa_t}\frac{1}{\ell(\kappa_t)}\mathrm{d}\kappa_t - \frac{\Gamma'}{\Gamma}\frac{\bar{h}}{\ell_{t+1}}\frac{\partial\ell(\kappa_{t+1})}{\partial\kappa_{t+1}}\frac{1}{\ell(\kappa_{t+1})}\mathrm{d}\kappa_{t+1}$, which can be rearranged to obtain (31).

subsection 4.2. investigates the case without minimum care requirement, $\bar{h} = 0$. Subsection 4.3. extends the analysis to the more general case with $\bar{h} > 0$.

4.2. Dynamics without Minimum Requirement

When there is no minimum care requirement for old agents, capital accumulation obeys a fairly simple dynamic law. This subsection assumes for simplicity that the elasticity of capital in generic production is not too high:

Assumption 1: $\alpha < \frac{3}{4}$.

This assumption is sufficient but not necessary for the steady state to be unique.[20] The next Proposition establishes that the steady state is globally stable under both complementarity and substitutability: the economy converges towards a long-run equilibrium in which capital per worker, the price of health care and employment shares are constant.

Proposition 3 *In the neoclassical case with $\bar{h} = 0$, capital per worker obeys*

$$\kappa_{t+1} = \frac{\beta\eta}{(1+n)(1+\beta)} p(\kappa_t), \tag{32}$$

where $p(\kappa_t)$ is the price of health care determined by (20). Under Assumption 1 the steady state $\kappa_{ss} = \frac{\beta\eta}{(1+n)(1+\beta)} p(\kappa_{ss})$ is unique and globally stable:

$$\lim_{t\to\infty} \kappa_t = \kappa_{ss}, \quad \lim_{t\to\infty} \ell_t = \ell(\kappa_{ss}), \quad \lim_{t\to\infty} p_t = p(\kappa_{ss}).$$

During the transition, given a positive initial stock $\kappa_0 < \kappa_{ss}$, both capital per worker and the price of health care increase; under complementarity (substitutability), employment in the generic sector declines (increases) and the saving rate increases (declines):

$$\kappa_{t+1} > \kappa_t, \quad p_{t+1} > p_t, \quad \left\{ \begin{array}{l} \ell_{t+1} < \ell_t \text{ and } \theta_{t+1} > \theta_t \text{ if } \sigma < 1 \\ \ell_{t+1} > \ell_t \text{ and } \theta_{t+1} < \theta_t \text{ if } \sigma > 1 \end{array} \right\}. \tag{33}$$

Proof. Expression (32) follows from setting $\bar{h} = 0$ in (29) and substituting (17) and (20). Result (33) follows from Proposition 1 combined with (25) that establishes θ_t be decreasing in ℓ_t. For κ_{ss} to be stable and unique, the elasticity (31) evaluated in κ_{ss} must be less than unity. Inserting $\kappa_t = \kappa_{t+1} = \kappa_{ss}$ and $\Gamma = 1$ and $\Gamma' = 0$ in (31), the elasticity reduces to

$$\frac{\mathrm{d}\kappa_{t+1}}{\mathrm{d}\kappa_t} = \alpha - \alpha \frac{\ell'_\kappa(\kappa_{ss})\kappa_{ss}}{\ell(\kappa_{ss})},$$

where the right hand side is less than unity if and only if $m_1 < 1$, where

$$m_1(\kappa_{ss}) \equiv -\frac{\ell'_\kappa(\kappa_{ss})\kappa_{ss}}{\ell(\kappa_{ss})} \frac{\alpha}{1-\alpha} \tag{34}$$

In the online appendix it is shown that Assumption 1 is a sufficient condition for $m_1 < 1$. ■

[20] In the online appendix, part B, the model is solved for the case where Assumption 1 is not satisfied. Moreover, under substitutability, the steady state is unique and stable independently of the parameter values.

Proposition 3 suggests three remarks. First, the dynamic law (32) shows that, with no minimum care requirement, investment per young is proportional to the price of care. The reason is that, when $\bar{h} = 0$, savings only depend on current wages. Second, given that capital per worker grows monotonically, both the wage and the price of care increase over time. Employment shares, however, move in opposite directions depending on the value of σ, which determines whether the expenditure share of care services increases or decreases in response to increasing prices. The third remark is that, under complementarity, the savings rate θ_t increases during the transition because rising care prices attract labor in the care sector and the income share of young agents then grows – i.e., the intergenerational distribution effect.

The long-run consequences of the intergenerational distribution effect become evident by comparing the steady-state level of the capital stock, κ_{ss}, with that arising in the canonical model, denoted by $\kappa_{ss}^{\text{canonical}}$. From (28), imposing $\bar{h} = 0$ and $\kappa_{t+1} = \kappa_t = \kappa_{ss}$ yields

$$\kappa_{ss} = \frac{1}{\ell\left(\kappa_{ss}\right)^{\frac{\alpha}{1-\alpha}}} \left[\frac{B\beta\left(1-\alpha\right)}{\left(1+\beta\right)\left(1+n\right)}\right]^{\frac{1}{1-\alpha}} = \kappa_{ss}^{\text{canonical}} \cdot \frac{1}{\ell\left(\kappa_{ss}\right)^{\frac{\alpha}{1-\alpha}}}, \tag{35}$$

where $\kappa_{ss}^{\text{canonical}}$ is obtained by setting $\ell_t = 1$ in each period, and equals

$$\kappa_{ss}^{\text{canonical}} = \left[\frac{B\beta\left(1-\alpha\right)}{\left(1+\beta\right)\left(1+n\right)}\right]^{\frac{1}{1-\alpha}}. \tag{36}$$

It follows from (35) that $\kappa_{ss} > \kappa_{ss}^{\text{canonical}}$ always holds as long as $\ell\left(\kappa_{ss}\right) < 1$. Therefore, capital per worker in the long run is higher than in the canonical model independently of whether generic goods and care services are complements or substitutes: for any value of σ, the need for care services increases the demand for labor, pushing up the income share of young cohorts and thereby the saving rate.

4.3. Dynamics with Minimum Care Requirement

When the minimum old-age care requirement is strictly positive, $\bar{h} > 0$, the accumulation law (28) includes the dependency of current savings on future employment shares, i.e. the old-age requirement effect. This dynamic law determines the steady state of the system and the associated stability properties. Under substitutability, $\sigma > 1$, there always exists a unique steady state. The case of complementarity, $\sigma < 1$, can be studied more easily by assuming, again, that the production elasticity of capital is not too high:

Assumption 2: $\alpha < \frac{1-\alpha}{1-\sigma}$.

This assumption is sufficient but not necessary for the steady state to be unique.[21]

Proposition 4 *Under Assumption 2, equation (29) exhibits a unique steady state $\bar{\kappa}_{ss}$ that is globally stable. The transitional dynamics of $p\left(\kappa_t\right)$ and $\ell\left(\kappa_t\right)$ comply with Proposition 1.*

Proof. For $\bar{\kappa}_{ss}$ to be stable and unique, the elasticity (31) evaluated in $\bar{\kappa}_{ss}$ must be less than unity. Inserting $\kappa_t = \kappa_{t+1} = \bar{\kappa}_{ss}$ in (31), the elasticity reduces to

$$\frac{\mathrm{d}\kappa_{t+1}}{\mathrm{d}\kappa_t} = \frac{\alpha - \alpha\frac{\ell'_\kappa(\bar{\kappa}_{ss})\bar{\kappa}_{ss}}{\ell(\bar{\kappa}_{ss})}}{1 + \frac{\Gamma'}{\Gamma}\frac{\bar{h}}{\ell(\bar{\kappa}_{ss})}\frac{\ell'_\kappa(\bar{\kappa}_{ss})\bar{\kappa}_{ss}}{\ell(\bar{\kappa}_{ss})}},$$

[21] In the online appendix, part B, the model is soved for the case where Assumption 2 is not satisfied.

where the right hand side is less than unity if and only if

$$m_1(\bar{\kappa}_{ss}) + m_2(\bar{\kappa}_{ss}) < 1, \tag{37}$$

with

$$m_2(\bar{\kappa}_{ss}) \equiv -\frac{\ell'_\kappa(\bar{\kappa}_{ss})\bar{\kappa}_{ss}}{\ell(\bar{\kappa}_{ss})}\frac{\Gamma'}{\Gamma}\frac{\bar{h}}{\ell(\bar{\kappa}_{ss})}\frac{1}{1-\alpha}\begin{cases} <1 \text{ if } \sigma<1 \\ <0 \text{ if } \sigma>1 \end{cases}. \tag{38}$$

In the online appendix Assumption 2 is shown to be a sufficient condition for (37) to be satisfied. ■

Proposition 4 establishes that, even in the general case with positive minimum care requirement, $\bar{h} > 0$, complementarity is associated with increasing savings rates during the transition. This is the combined result of the old-age requirement and intergenerational distribution effects. By imposing $\kappa_{t+1} = \kappa_t = \bar{\kappa}_{ss}$ in (28), the steady-state level of capital per worker equals

$$\bar{\kappa}_{ss} = \kappa_{ss}^{\text{canonical}} \cdot \frac{1}{\ell(\bar{\kappa}_{ss})^{\frac{\alpha}{1-\alpha}}} \cdot \Gamma\left(\frac{\bar{h}}{\ell(\bar{\kappa}_{ss})}\right)^{\frac{1}{1-\alpha}}. \tag{39}$$

Since $\Gamma(\cdot)$ strictly exceeds one when $\bar{h} > 0$, result (39) establishes that $\bar{\kappa}_{ss} > \kappa_{ss} > \kappa_{ss}^{\text{canonical}}$. That is, the long-run level of capital per worker is higher when there is a positive minimum requirement of old-age care, which prompts young agents to save more during the transition in response to the continuous increase of the price of care services. Expression (39) will be exploited in the quantitative analysis of section 6. to calculate the impact of exogenous shocks on capital per worker in a calibrated version of our model.

Our main remark is that, under complementarity, $\sigma < 1$, the transitional dynamics of our model capture very well the stylized facts that inspired the analysis. During the transition to the steady state, the saving rate grows, the price of care services and the wage rate increase over time, the income distribution shifts in favor of young workers, and the employment share of the generic sector declines. Several developing countries, and in particular, China in the last two decades, experienced the same qualitative dynamics as documented in the Introduction. Since the hypothesis $\sigma < 1$ is also empirically plausible (Finkelstein et al. 2012), the remainder of the analysis will focus on the case of complementarity.

5. Savings Multipliers

This section introduces the concept of *savings multiplier* (subsection 5.1.) and describes its use in the analysis of three types of exogenous shocks: increased productivity (subsect. 5.2.), reduced fertility (subsect. 5.3.) and increased minimum care requirement (subsect. 5.4.). The nature of these shocks may be conceptually linked to the effects of past reforms in China, in particular, the one-child policy and the dismantling of social benefits.

5.1. Conceptual Definition

The intergenerational distribution and the old-age requirement effects create *feedback mechanisms* whereby capital accumulation stimulates further savings and, hence, further accumulation. These feedback effects bear major consequences for the economy's response to exogenous shocks: following a change in the value of a parameter, the resulting change in the long-run level of capital per worker must include the cumulative impact of all the feedback effects that operate during the transition to the new steady state. Therefore, in our model with complementarity, the long-run effects of exogenous shocks are always amplified by a 'savings multiplier', which measures the impact of the feedback effects that raise savings during the transition.

5.2. Productivity Shocks

It is henceforth assumed $\sigma < 1$ for the reasons explained in the previous section.[22] Consider a productivity shock taking the form of an exogenous increase in B. In the canonical model, this shock would increase the long-run level of (log) capital per worker in (36) by

$$\frac{\mathrm{d}\log \kappa_{ss}^{\text{canonical}}}{\mathrm{d}B} = \frac{1}{B\,(1-\alpha)}. \tag{40}$$

In our model, the impact of the shock is magnified by both the intergenerational distribution and the old-age requirement effects. To preserve expositional clarity, first consider the case with zero minimum requirement.

Zero minimum requirement. With $\bar{h} = 0$, the steady-state capital per worker is κ_{ss} defined in (35), and the impact of the productivity shock is determined by

$$\frac{\mathrm{d}\log \kappa_{ss}}{\mathrm{d}B} = \underbrace{\frac{1}{1-m_1\left(\kappa_{ss}\right)}}_{\text{Savings Multiplier}} \left(\frac{\mathrm{d}\log \kappa_{ss}^{\text{canonical}}}{\mathrm{d}B} + m_1\left(\kappa_{ss}\right) \frac{\ell_B'\left(\kappa_{ss}\right)}{\ell_\kappa'\left(\kappa_{ss}\right)\cdot \kappa_{ss}} \right), \tag{41}$$

The crucial element in (41) is the savings multiplier, where m_1 is already defined in (34). Under complementarity, m_1 is strictly positive, and is less than unity in view of the stability of the steady state.[23] Since $0 < m_1 < 1$, the savings multiplier in (41) is strictly higher than unity. Combining this result with $\ell_\kappa' < 0$ and $\ell_B' < 0$,[24] it follows that the impact of a productivity shock on steady-state capital per worker is stronger than that predicted by the canonical model. There are two reasons for this, both related to the intergenerational distribution effect. First, the productivity increase modifies the static equilibrium of the labor market: workers move out of generic production and into the care sector, increasing the wage further relative to the canonical model. This 'static reallocation effect', represented by the term $m_1 \ell_B' / \left(\ell_\kappa' \kappa\right) > 0$, increases firms' demand for capital and current savings. Second, as the capital stock starts to grow, further labor is pushed out of generic production and into care, increasing the wage even further and thus magnifying the initial increase in savings: the cumulative impact of such 'dynamic feedback effects' is represented by the savings multiplier, $1/\left(1-m_1\right)$. The combination of these static and dynamic reallocation effects thus yields a larger overall impact of productivity shocks than in the canonical model.

[22] All the equations that follow are identical under substitutability, the only difference being in the strength of the effects: the saving multipliers exceed unity when $\sigma < 1$ and fall short of unity when $\sigma > 1$. Hence, shocks that are magnified with complementarity are instead dampened with substitutability.

[23] Under complementarity, m_1 is positive because $\ell_\kappa' < 0$ – see expression (30) – and is strictly less than unity in view of the stability condition proven in Proposition 3. Under substitutability, instead, expression (30) implies $\ell_\kappa' > 0$ and therefore $m_1 < 0$.

[24] Under complementarity, $\ell_\kappa' < 0$ follows from (30) whereas $\ell_B' < 0$ is established in Proposition 2.

Positive minimum requirement. With $\bar{h} > 0$, the savings multiplier is modified by the old-age requirement effect. From (39), the effect of increased productivity on long-run capital is now given by

$$\frac{\mathrm{d}\log\bar{\kappa}_{ss}}{\mathrm{d}B} = \underbrace{\frac{1}{1-m_1\left(\bar{\kappa}_{ss}\right)-m_2\left(\bar{\kappa}_{ss}\right)}}_{\text{Savings multiplier}}\left[\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{\mathrm{d}B}+\frac{\left(m_1\left(\bar{\kappa}_{ss}\right)+m_2\left(\bar{\kappa}_{ss}\right)\right)\ell_B'\left(\bar{\kappa}_{ss}\right)}{\ell_\kappa'\left(\bar{\kappa}_{ss}\right)\bar{\kappa}_{ss}}\right], \tag{42}$$

where m_2 is defined in (38). Under complementarity, the term m_1+m_2 is strictly positive, and is less than unity in view of the stability of the steady state.[25] Since $0<m_1+m_2<1$, the savings multiplier in (42) exceeds unity. Compared to the case with zero requirement – cf. expression (41) – the impact of increased productivity on steady-state capital is now strengthened in two respects. First, the 'static reallocation effect' that raises the equilibrium wage now induces larger savings because higher wages also mean a higher anticipated cost of minimum care in the second period of life: the additional increase in savings is determined by the presence of m_2 inside the last term of (42). Second, the 'dynamic feedback effects' are stronger because rising wages during the transition prompt young agents to raise their savings further due, again, to the old-age requirement mechanism: this is why the savings multiplier, $1/\left(1-m_1-m_2\right)$, is larger than in the previous case with $\bar{h}=0$.

5.3. Reduced Fertility

In the canonical model, a lower growth rate of population increases the steady-state level of capital per worker: from (36) it follows that

$$\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{-\mathrm{d}n} = \frac{1}{\left(1+n\right)\left(1-\alpha\right)} > 0. \tag{43}$$

In contrast, from (39), the effect of reduced fertility in our model is given by

$$\frac{\mathrm{d}\log\bar{\kappa}_{ss}}{-\mathrm{d}n} = \frac{1}{1-m_1-m_2}\left[\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{-\mathrm{d}n}+\frac{\ell_n'}{(-\ell_\kappa')\bar{\kappa}_{ss}}(m_1+m_2)+\frac{\ell}{\left(1+n\right)(-\ell_\kappa')\bar{\kappa}_{ss}}m_2\right], \tag{44}$$

where the argument $\bar{\kappa}_{ss}$ is suppressed to simplify the notation.[26] Expression (44) incorporates five effects that do not arise in the canonical model. The first two are included in the multiplier: as explained before, the term $1/\left(1-m_1-m_2\right)>1$ represents the positive feedbacks that capital growth exerts on itself due to the intergenerational distribution and the old-age requirement effects. The second and third effects are contained in the term $\frac{\ell_n'}{(-\ell_\kappa')\bar{\kappa}_{ss}}(m_1+m_2)$, which represents the change in the static equilibrium of the labor market: the reduction in fertility increases the fraction of old agents in total population, pushing workers out of generic production and into care services; the resulting wage increase raises the savings rate through both the intergenerational distribution and the old-age requirement effects. The fifth effect is the last term appearing (44), which represents a dilution effect: lower population growth increases labor scarcity even for a fixed labor allocation. The implied rise in wages triggers further savings through the old-age requirement effect.

[25] Given $\sigma<1$, both m_1 and m_2 are positive because $\ell_\kappa'<0$ – see expression (30) – and m_1+m_2 is strictly less than unity in view of the stability condition (37) proven in Proposition 4. Under substitutability, instead, expression (30) would imply $\ell_\kappa'>0$, $m_1+m_2<0$ and, hence, a multiplier below unity.

[26] In (44), the terms $m_1, m_2, \ell, \ell_n', \ell_\kappa'$ are all evaluated in the steady state $\bar{\kappa}_{ss}$. Also, the derivation of (44) exploits the fact that $\frac{d\Gamma}{dn}=-\Gamma'\frac{\bar{h}}{(1+n)\ell}$ from expression (26). See the online appendix for full derivations.

5.4. Increased need for care

In the model, a higher $\bar{h}$ represent an increased need to purchase care services through the market. Obviously, this draws resources out of generic production and into the care sector. By (39), the effect on steady-state capital is

$$\frac{\mathrm{d}\log\bar{\kappa}_{ss}}{\mathrm{d}\bar{h}} = \frac{1}{1-m_1-m_2}\left[\frac{\ell'_{\bar{h}}}{\ell'_{\kappa}\cdot\bar{\kappa}_{ss}}(m_1+m_2) - \frac{\ell}{\bar{h}\cdot\ell'_{\kappa}\bar{\kappa}_{ss}}m_2\right]. \tag{45}$$

Besides the now familiar savings multiplier, a higher minimum requirement induces two types of static effects. First, there is a direct positive effect on the cost of care, represented by the term $-\frac{\ell}{\bar{h}\ell'_{\kappa}\bar{\kappa}_{ss}}m_2$, which increases savings. Second, the static equilibrium of the labor market changes since higher demand for care pulls workers out of generic production and drives up the wage. This effect, represented by the term $\frac{\ell'_{\bar{h}}}{\ell'_{\kappa}\bar{\kappa}_{ss}}(m_1+m_2)$, generates higher savings through both the intergenerational distribution and the old-age requirement effects. With the additional stimulus of the savings multiplier, $1/(1-m_1-m_2)$, the increased need for market-provided care may thus have a strong positive impact on capital accumulation. This possibility is confirmed by our quantitative analysis in section 6.

6. Quantitative Analysis

This section presents a quantitative assessment of the theoretical results taking China's economy as empirical reference. The model parameters are calibrated to obtain steady-state values that match the most recent data, and the effects of exogenous shocks are evaluated.

6.1. Calibration

The model is calibrated so as to match, in the steady state, four target values of the endogenous variables reported in the first line of Table 1. The value $\theta = 0.28$ reflects China's saving rate (Prasad, 2015). Variable TES is the share of total expenditures devoted to care services, with target value 0.083 given by conservative projections based on Chamon and Prasad (2010).[27] Variable CIS is the capital income share, with target value 0.46 given by one minus the long-run labor share in GDP net of production tax calculated by Bai and Qian (2010).[28] The target value of the employment share of manufacturing relative to care services, $\frac{1-\ell}{\ell} = 0.19$, corresponds to paid employment in "Health and Social Work" plus "Social and Personal Service Activities" divided by paid employment in "Manufacturing" in China (ILO, 2015: Table 2.E), and implies an employment share in the generic sector $\ell = 0.84$.[29] Given these target values, the exogenous parameters

[27] Chamon and Prasad (2010: Table A2, p.129) report the 1992-2004 time series of health versus non-health expenditures: the implied TES goes from 2.5% in 1992 to 7.4% in 2004. More recent data on sectoral GDP shares show that, during the 2005-2014 decade, total spending in services went from from 42.9% to 48.2% of GDP (World Development Indicators, 2015). Under the conservative hypothesis that, during the 2005-2014 decade, health expenditures grew at the same rate as total expenditures in services, the implied figure for 2004 is $TES = 8.3\%$.

[28] For the mathematical definitions of TES and CIS in our model see the online appendix.

[29] The 'generic good' sector of the model is here interpreted as a real-world sector that includes both 'manufacturing' and 'services' excluding 'care services'. The reason is that our aim is to assess to what extent the intergenerational distribution and the old-age requirement effects influence long-run capital even though the care services sector is quantitatively small in terms of both employment and expenditure shares.

are chosen as follows (cf. Table 1, second and third panels). The values of α and σ are given by empirical evidence: $\alpha = 0.5$ is the baseline used in most calibrated models of China (e.g., Song et al. 2011), and $\sigma = 0.2$ is the elasticity of substitution between consumption and health care services estimated by Finkelstein et al. (2013). Next, parameters $(\eta, \bar{h}, n)$ are restricted by numerical combinations that yield a threshold employment share $\ell^{\max} = 0.9$ that is consistent with the target value $\ell = 0.84$. The remaining parameters (β, γ, B) are then chosen so as to obtain, in the steady state equilibrium, the four target values of the endogenous variables discussed above. The actual steady state values are reported in the fourth panel of Table 1.

The last panel in Table 1 evaluates the determinants of capital per worker according to the decomposition reported in equation (39): the steady-state stock $\bar{\kappa}_{ss}$ is determined by the canonical component ($\kappa_{ss}^{\text{canonical}}$), the intergenerational distribution effect (IDE), and the old-age requirement (OAR).[30] The numbers show that $\bar{\kappa}_{ss}$ exceeds the canonical level by 34%, which is a noticeable quantitative result: despite the small size of the care sector, the existence of needs for old-age care ultimately raises the capital stock by one third relative to the canonical model's baseline. The IDE and the OAR factors reported in Table 1 show that the intergenerational distribution effect alone raises capital by 18% above the canonical level, and the old-age requirement effect adds a further 13% gain. The product of IDE and OAR factors, 1.34, determines the overall distance between $\bar{\kappa}_{ss}$ and the canonical level.

6.2. Exogenous shocks and transitional dynamics

The calibrated model can be used to evaluate the impact of exogenous events of different nature. This subsection considers a negative *requirement shock*, represented by a decline in the value of $\bar{h}$, and positive *fertility shocks* represented by exogenous increases in the value of n. Conceptually, the exercise is related to the important reforms recently announced by China's government, namely, the introduction of welfare benefits that will cover the minimum care services required by the old, and the abandonment of the one-child policy that will arguably raise China's fertility rate in the future. In quantitative terms, the requirement shock is a decline of $\bar{h}$ from the baseline value 1 to zero. Fertility shocks consist of two scenarios in which the population growth rate increases from the baseline value of zero to $n = 0.133$ and to $n = 0.282$: these numbers may be interpreted as permanent shocks raising the fertility rate by, respectively, 0.5% and 1% per annum (given an OLG model with 25 years per period).

Table 2 reports the results obtained using the baseline simulation (i.e., the parameter values reported in Table 1) in the upper panel, as well as two robustness checks in which parameters are changed to allow for weaker complementarity (middle panel, with higher values of σ) and a higher output elasticity to capital (lower panel, with higher values of α). The impact of all the shocks on steady-state capital per worker is quite robust to such alternative parametrization. The requirement shock would reduce the long-run stock of capital per worker by 21%-24%. This effect is quantitatively close to that of the permanent fertility shock $n = 0.133$, which would reduce $\bar{\kappa}_{ss}$ by 25%-31% (the stronger fertility shock $n = 0.282$ would instead reduce $\bar{\kappa}_{ss}$ by 43%-

[30] From (39), the IDE factor equals $1/\ell^{\frac{\alpha}{1-\alpha}}$ and the OAR factor is $\Gamma\left(\bar{h}/\ell_{ss}\right)^{\frac{1}{1-\alpha}}$.

51%). The crucial difference between requirement and fertility shocks lies in the transmission channels. The elimination of the care requirement $\bar{h}$ involves pure *non-canonical mechanisms*, with strong labor reallocation and sensible reductions in saving rates: in the baseline simulation, ℓ_{ss} goes from 0.847 to 0.941 and θ_{ss} drops from 28% to 25%. The fertility shock, instead, activates the canonical dilution effect of population on capital per worker and, hence, generates modest variations in employment shares and in saving rates. The scope of the reallocation effects arising after the shocks is further emphasized by the transitional dynamics depicted in Figure 3, which refer to the baseline simulation.

7. Conclusion

This paper introduced the concept of savings multiplier, a general equilibrium mechanism that induces rising saving rates over time and that magnifies the impact of exogenous shocks on capital per capita in the long run. In our theory, capital accumulation yields positive feedbacks on saving rates via two channels. First, real wages increase as the capital stock grows at the same time as workers move from the manufacturing sector to the labor-intensive service sector, implying a shift of the income distribution in favor of young savers (*intergenerational distribution effect*). Second, growth in real wages raises the anticipated cost of providing for the old age, prompting the currently young to save a higher fraction current income (*old-age requirement effect*). Both these mechanisms provide a novel explanation for rising saving rates in developing countries and, more specifically, are consistent with the stylized facts that characterize China's economic performance.

Our analysis of exogenous shocks suggests that China's past reforms – in particular, the one-child policy and the dismantling of cradle-to-grave social benefits – have fueled China's saving rates in the past decades. Our quantitative analysis shows that capital in the long run may be quite sensitive to changes in the minimum care services required by old agents even though the care sector is small relative to manufacturing and other services. This suggests that the recently announced policy reforms, i.e., the abandonment of the one-child policy and the introduction of welfare benefits, may reduce savings and long-run capital to a much larger extent than what the traditional neoclassical model would predict.

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1 **Tables**

Target values for endogenous variables (from data)			
$\theta = 0.28$	$TES = 0.083$	$CIS = 0.46$	$\frac{1-\ell}{\ell} = 0.19$
Parameter values (from empirical evidence)			
$\alpha = 0.5$	$\sigma = 0.2$		
Parameter values (free)			
$n = 0$	$\bar{h} = 1$	$\eta = 10$	
$B = 1.628$	$\beta = 0.95$	$\gamma = 0.5$	
Simulations results: targeted steady state values			
$\theta_{ss} = 0.281$	$TES_{ss} = 0.083$	$CIS_{ss} = 0.468$	$\frac{1-\ell_{ss}}{\ell_{ss}} = 0.182$
Simulations results: sectoral labor			
$\ell_{ss} = 0.846$	$\ell^{\max} = 0.90$		
Simulations results: capital per worker			
$\bar{\kappa}_{ss} = 0.210$			
$\kappa_{ss}^{\text{canonical}} = 0.157$	$IDE = 1.181$	$OAR = 1.133$	

2 Caption Table 1:

3 **Table 1**. Baseline calibration, parameter values and steady state results (see the procedure described in
4 subsection 6.1. for details).

Baseline simulation	n	$\bar{h}$	σ	α	ℓ_{ss}	θ_{ss}	$\bar{\kappa}_{ss}$ (change)
Before shock	0.000	1	0.2	0.5	0.847	0.281	0.210
Requirement shock	0.000	0	0.2	0.5	0.941	0.251	0.167 (-21%)
Fertility shock (0.5% p.a.)	0.133	1	0.2	0.5	0.862	0.276	0.158 (-25%)
Fertility shock (1% p.a.)	0.282	1	0.2	0.5	0.876	0.272	0.120 (-43%)

Robustness (higher σ)	n	$\bar{h}$	σ	α	ℓ_{ss}	θ_{ss}	$\bar{\kappa}_{ss}$ (change)
Before shock	0.000	1	0.5	0.5	0.782	0.293	0.230
Requirement shock	0.000	0	0.5	0.5	0.860	0.262	0.183 (-21%)
Fertility shock (0.5% p.a.)	0.133	1	0.5	0.5	0.796	0.288	0.173 (-25%)
Fertility shock (1% p.a.)	0.282	1	0.5	0.5	0.810	0.283	0.131 (-43%)

Robustness (higher α)	n	$\bar{h}$	σ	α	ℓ_{ss}	θ_{ss}	$\bar{\kappa}_{ss}$ (change)
Before shock	0.000	1	0.5	0.6	0.786	0.234	0.071
Requirement shock	0.000	0	0.5	0.6	0.875	0.211	0.054 (-24%)
Fertility shock (0.5% p.a.)	0.133	1	0.5	0.6	0.805	0.229	0.050 (-31%)
Fertility shock (1% p.a.)	0.282	1	0.5	0.6	0.823	0.225	0.035 (-51%)

Caption Table 2:

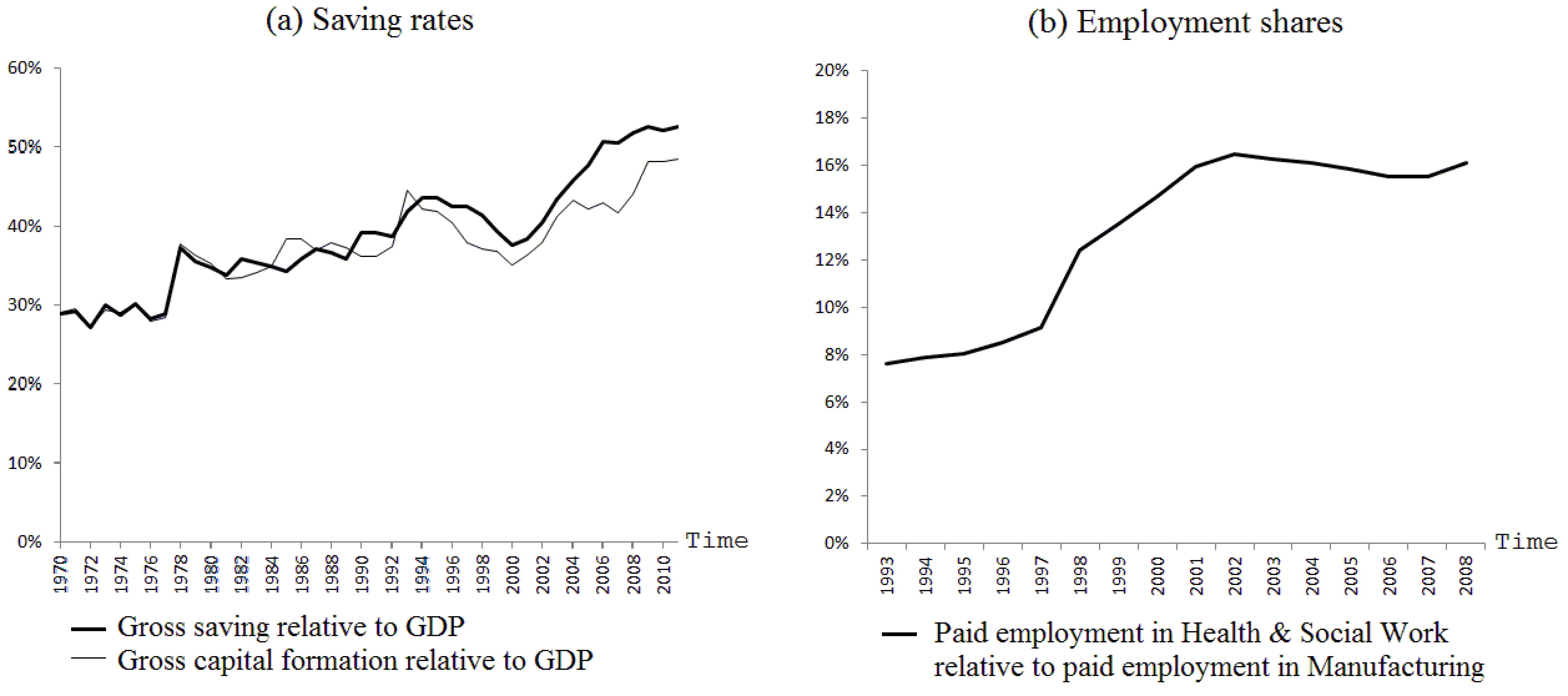
Table 2. Impact of exogenous variations in $\bar{h}$ or n on capital per worker (see subsection 6.2. for details). In the upper panel, the row 'Before shock' reports results obtained in the baseline calibration described in Table 1. The middle and lower panels perform the same exercise under stronger complementarity in preferences (higher σ) and higher elasticity to capital (higher α), respectively.

Figures Captions

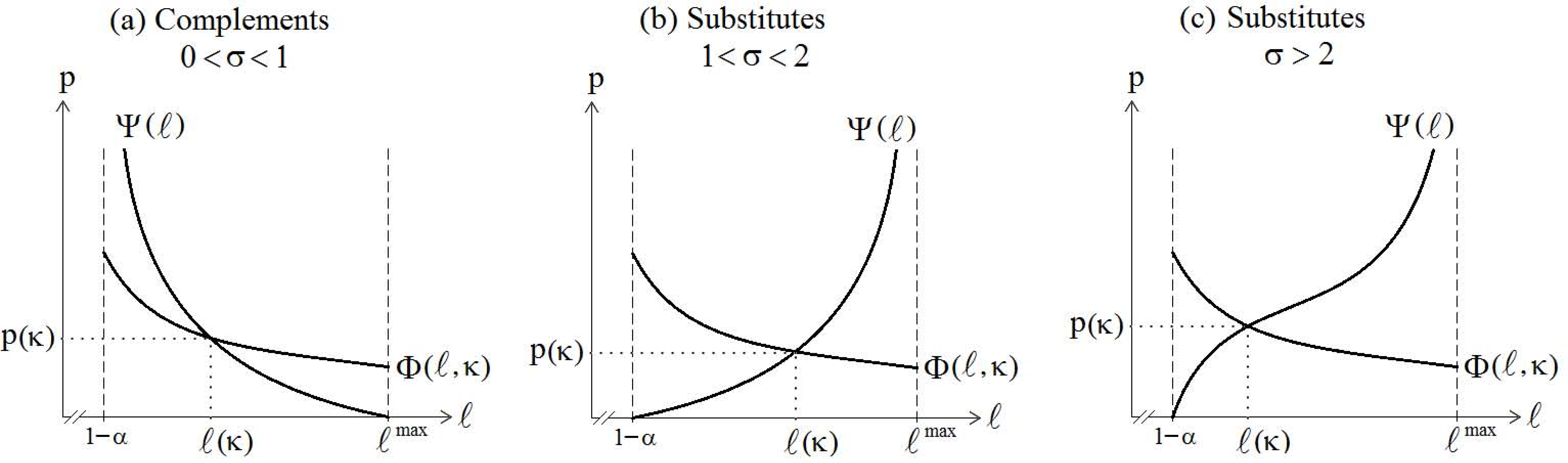
Caption Figure 1: Graph (a): saving and investment shares of GDP in China 1970-2010 (source: World Bank). Graph (b): paid employment in Health and Social Work relative to paid employment in Manufacturing in China 1993-2008 (source: authors calculations on LABORSTA Table 2E, International Labor Organization).

Caption Figure 2: Static equilibrium: determination of ℓ_t and p_t for given κ_t. The case of strong substitution ($\sigma > 2$) implies local concavity of $\Psi(\ell)$ for low ℓ without altering existence, uniqueness, and comparative-statics properties.

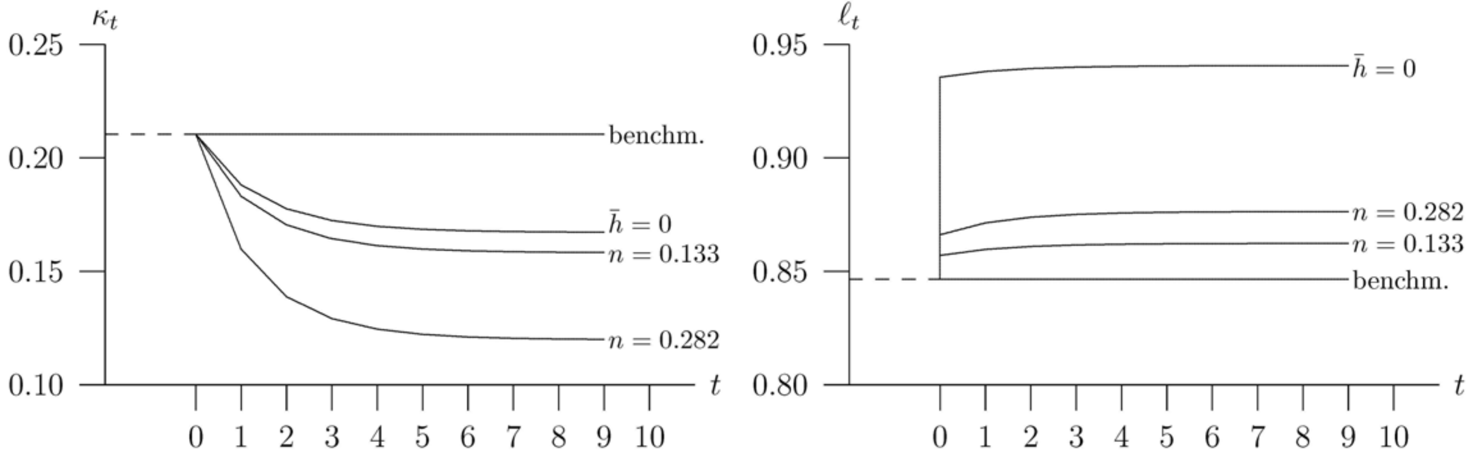
Caption Figure 3: Transitional dynamics induced by fertility and requirement shocks in the baseline simulation (see Section 6. for details). The 'benchmark' paths represent steady state values before the shocks.



(a) Saving rates
60%
50%
40%
30%
20%
10%
0%
Time
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
Gross saving relative to GDP
Gross capital formation relative to GDP
(b) Employment shares
20%
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
Time
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
Paid employment in Health & Social Work
relative to paid employment in Manufacturing



(a) Complements
0<σ<1
(b) Substitutes
1<σ<2
(c) Substitutes
σ>2
p
Ψ(ℓ)
Φ(ℓ,κ)
p(κ)
1−α
ℓ(κ)
ℓ max
ℓ



κ_t
0.25
0.20
0.15
0.10
benchm.
$\bar{h} = 0$
$n = 0.133$
$n = 0.282$
t
0 1 2 3 4 5 6 7 8 9 10
ℓ_t
0.95
0.90
0.85
0.80
$\bar{h} = 0$
$n = 0.282$
$n = 0.133$
benchm.
t
0 1 2 3 4 5 6 7 8 9 10