TRANSFORMING TRAINEES' ASPIRATIONAL THINKING INTO SOLID PRACTICE

Irene Biza, Gareth Joel and Elena Nardi explore with their trainees how to establish aspirational social and sociomathematical norms in the secondary mathematics classroom.

Introduction:

Gareth, the member of our team who, since 2006, is deeply involved in initial teacher training (ITT) claims that working on an ITT course is an enviable position in which to find oneself; the passion and optimism that beginner teachers bring to their studies is inspiring. The vibrancy that trainees hope to bring to a classroom is one that educationalists have always endeavoured to sustain. However, in his experience, trainees do sometimes *lose themselves* in the emotional roller coaster of the training year. This may be due to classroom challenges such as behaviour or wider challenges such as the ever-changing political and administrative landscape. Throughout the six years of his experience so far the key thing that Gareth has learnt in all this time is that for beginner teachers to be successful is for them to recognise classroom dynamics but, more importantly, to *stop* and *reflect*. From the first years Gareth found himself in this post, he remembers how trainees were speaking about general university courses and how the teacher training year had been the first time where they had to fully consider and evaluate their own progress. In all previous years they had been told *what to think*, *when to complete tasks* and *what to value* in their studies. With this in mind Gareth has aimed to make *support for meta-cognitive practices* and *consideration of values* the cornerstones of the ITT programme he is leading.

Although Gareth would suggest that the most successful trainees show depth and astuteness in reflection on practice, some trainees' reflections remain simplistic and organisational in nature rather than showing an understanding of why certain classroom events occur. For example, some trainees would state that student behaviour is an issue within a lesson but would not seek to unpack the cause of this behaviour. There have also been trainees who find self-criticism a difficult task with which to engage, particularly when related to a consideration of values; or when a passionate mathematician does not understand why students cannot engage; or, a teacher who themselves have a preferred learning style that seems not to fit with the students they are teaching. This is the point which made Gareth think that the time was apt for introducing deeper reflection in the programme's sessions, *away from the busy-ness* of classrooms; a reflection that can ensure good teacher training. And this is the point when he started looking for *tools* that can trigger this reflection – and brought the two other members of our team into play.

Irene and Elena are researchers in mathematics education, with Irene having a lengthy secondary school teaching experience. Since 2005, both are involved in a long term project in which they design tasks for teacher reflection. These tasks are based on classroom scenarios which are fictional but realistic and based on research outcomes as well as actual classroom practice. Until recently the scenarios had focused on teaching situations that capture key *mathematical issues* (such as formation of mathematical concepts, use of definitions, visualisation, mathematical argumentation, etc.) and *forms of mathematical thinking* (such as instrumental and relational understanding, Skemp, 1976) – and less on *other* key issues such as classroom management. When our team came together we decided to refine our study in a way that addresses the complexity of classroom situations within which the teacher needs to deal *concurrently* with issues that pertain to mathematical learning as well as to classroom management. To this aim we drew on theoretical perspectives that deal with the classroom environment, most pronouncedly with the constructs of *social* and *sociomathematical norms* (Cobb & Yackel, 1996). *Social norms* govern the overall interaction in the classroom, including rules regarding students' participation in discussion, group work, and critique of other students or teacher. *Sociomathematical norms* govern the classroom interactions that are specific to mathematics, and they include rules such as "what count as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution and an acceptable mathematical solution" (ibid., p. 178).

With this in mind we designed tasks based on scenarios that combine subject knowledge and behaviour management and we invited 21 trainee teachers attending the SY PGCE Mathematics

Programme in our institution to respond to these tasks first and then to participate in a whole-class discussion of their responses.

In the following sections we firstly present two examples of these scenarios and the corresponding tasks. Then we exemplify how trainee teachers responded to these tasks in terms of the *social* and *sociomathematical norms* they aspire to establish in their classroom. We are very interested in finding out not only *the aspirations* expressed in the trainees' responses but also *how* they would *transform this aspiration into practice*.

Engaging trainees with classroom scenarios

In the first scenario on which the *polygon task* (Figure 1) is based, the teacher is entering a class that is used to an instrumental and competitive working style. The teacher aims to introduce a more investigative and relational approach. The class is invited to investigate in a Dynamic Geometry environment the sum of the angles of the polygon and conclude with the formula. When the teacher invites the class to justify the formula they derived through the investigation, the students cannot see the necessity of doing so: they consider the teacher's request as a waste of time and react rather rudely to the teacher.

In the second scenario on which the *simplification task* (Figure 2) is based, a class is asked to solve the problem: "When p=2.8 and c=1.2, calculate the expression: $3c^2+5p-3c(c-2)-4p$". Two students reach the result (10) in different ways: student A substitutes the values for p and c and carries out the calculation; student B simplifies the expression first and then substitutes the values for p and c. When student A acknowledges her difficulty with simplifying expressions, student B retorts offensively ("you are thick") and dismissively ("what can I expect from you anyway?"). Both solutions are correct and student B's approach particularly demonstrates proficiency in important algebraic skills. But student B's behaviour is questionable.

Class X is a high attaining group which you have taken over at the start of Year 10. So far Class X has been taught mathematics as a list of rules and they have been practising the application of these rules in a range of examples. These students have learnt to perform well in a competitive classroom environment in which they work on tasks and they are rewarded for the correctness and rapidness of their work. In your teaching you aim to instigate a different approach that includes justifications for the used rules and the relations amongst them.

In a session on the sum of the angles of a polygon, you have asked the students to

- work with a Dynamic Geometry software in order to sketch polygons with 3, 4, 5, 6, 7, … sides and
- report the number of sides and the sum of the angles in a table, in order to conclude with a general rule about the **sum of the angles of a polygon**.

After a couple of trials the students conclude that the sum equals 180° multiplied by the number of sides minus two and verify this rule with trials of polygons with several numbers of sides.

At that point you ask the students to explain why this rule is correct and the dialogue below follows:

YOU: Why is this formula correct? Can you give any explanation?
STUDENT A: It works for all the polygons we tried.
YOU: How do you know that this will work for *all* polygons?
STUDENT B: It isn't necessary. What we need is a formula that works.
STUDENT C: Yes, we spent so much time playing with the software. If you had *given* us the formula and a list of problems to work on, by now we would have got *more done*.
STUDENT A: Practice makes perfect.

Questions for Scenario 1:

a. What do you think are the issues in this situation?
b. What are you going to say to each one of these students?
c. Are you going to change your approach? Justify your response.

Figure 1: Polygon Task

In a Year 10 middle attaining class you have invited the students to solve the following problem:

When p=2.8 and c=1.2, calculate the expression: $3c^2+5p-3c(c-2)-4p$.

After working on the problem for some time you invite the students to share their solution with the class. The dialogue below follows:

YOU: Ok, let's see what we can do with this question. Who wants to share their answer with me?
[Student A and Student B raise their hands at the same time.]
YOU: Student A?
STUDENT A: I found 10.
YOU: How did you find 10?
STUDENT A: I substituted the values 2.8 and 1.2 in the expression. It took me ages.
YOU: Thank you Student A! [To the class] Does everyone agree?
STUDENT B: I have the same answer but I did it so much quicker.
YOU: Go on…

STUDENT B:	I worked out the expression before substituting the numbers and I ended up with a much simpler expression: *p*+6*c*. Then I substituted the values 2.8 and 1.2 and I found 10, *easy*!
STUDENT A:	I like the way I did it; I don't like simplifying.
STUDENT B:	My solution is brilliant, yours takes ages. You cannot work with letters because you are *thick* [Some students are giggling] … what can I expect from you anyway? [Some students are laughing].

You heard what Student B said …

Questions:
a. How are you going to respond to Student A, to Student B and to the whole class?
b. What do you think are the issues in this situation?
c. How are you going to deal with these issues in the future?

Figure 1: Simplification Task

We analysed the responses to these two tasks in our effort to find out *the issues* trainees can see in these situations and *how* they would react in a similar instance. In the examples we present in the next sections we are particularly interested in the *norms* (*social* and *sociomathematical*) trainees want to establish in their classroom and *how* they would establish these norms. Although we note that it is not always possible to distinguish *social* from *sociomathematical* norms with watertight precision, in the next two sections we exemplify the former from the responses to the *simplification task* and the latter from the responses to the *polygon task*.

My classroom is a *no put down zone*: Social norms in the mathematics classroom

All respondents addressed student B's ill-behaved reaction to student A in the *simplification task* (Figure 2). In fact, about 10 respondents focus almost exclusively on behavioural aspects of the incident, as we report in more detail in (Biza, Joel & Nardi, 2014). Almost all respondents reprimand student B for disrespecting student A, some of them in public and some other in private. Some trainees reflected on the situation as an indication of individual student misbehaviour and suggested *disciplinary measures* for student B's *punishment,* such as: "consequences", "sanctions", "warnings", "sent out", "exclude from class", "peer courts" or "detention". Trainee [9] for example, considers this as a "fairly common misbehavior in a lesson, due to the level of disruption and attention that pupil can steal away from a member of staff" and she feels the need to protect the class from this disruption through penalizing student B. Other trainees reflect on the situation not only as an indication of individual student misbehaviour but also as a sign of a concerning classroom culture and address this issue as such. In general, across the scripts there is a clear priority to establish certain *social norms* in the classroom. The norm that these teachers aspire to establish is of a respectful classroom, a "no put down zone" (trainee [18]) with a teacher who "will not tolerate classroom bullying" (trainee [1]). But how *exactly* would trainees react in a similar situation and, especially, *how would* they establish this particular *social norm*? We scrutinised the trainees' responses further to this aim.

Characteristically, trainee [3] in her response to question (a), she would praise student B for the efficiency of their approach but she would reprimand them sternly for their treatment of student A: "Both solutions to the problem are good solutions they both gave correct answers, student A's solution took a lot longer as they were working with really complicated arithmetic rather than simplifying this doesn't make student A thick so I don't want to hear you use that again". Also, she would enrol student B towards helping student A: "I like how you have simplified the expression to get a quicker easier method so maybe you could try and help student A with simplifying as it's something student A doesn't like and it will help with your understanding too". In her response to question (b) she identifies issues related to the class environment with impressive precision such as lack of "respect" and "self-esteem". Later, in question (c), her tackling of these issues is similarly concretely targeted. She would "constantly instil a positive respectful classroom environment encouraging all students to offer answers" and she would "try to encourage students to help another and discuss methods". We were impressed by the pedagogical specificity and consistency of this response, especially regarding the establishment of respect through collaborative work. This collaboration appears as panacea to the disrespect mind-set. However, her approach, namely to invite student B to help student A after their offensive behaviour, is perhaps a little questionable: how would we expect student B to collaborate with student A after what happened between them?

In the same spirit, trainee [4] wants to "create a culture of discussing, sharing and involving each other and make sure no student goes against this culture". Similarly, trainee [12] "would always reiterate that students should be supportive of each other's views and give constructive criticism". Neither [4] nor [12] suggest *how* they would establish this culture in their class though. Trainee [11], on the other hand, is more specific when she suggests "prompting cards/discussion templates so that the students are aware of how to argue their point without being disrespectful".

Generally speaking the commendable pedagogical goal of establishing a collaborative, participatory and engaging culture in the classroom is transparent in most of the responses we received. What is not always transparent, however, is *how* this culture would be established (see for example [4] and [12]). Or, as we saw in trainee [3]'s response, the intended approach is questionable. As we know, in practice, implementation of pedagogical goals in the class is always a challenge that trainee teachers need to be prepared to face, react to and reflect on. We return to this in the concluding section.

Asking *why* makes you *into a mathematician, not just a technician*: Sociomathematical norms in the mathematics classroom.

In their responses to the *polygon task* (Figure 1), *all* trainees highlighted that students are not used to or expected to – and therefore reluctant to – ask *why* in mathematics. Instead, they are conditioned to an instrumental approach to mathematics and unwilling to engage with mathematics in an exploratory manner. In the *polygon task* the class is asked to identify a formula that works for all polygons and establish why the formula works (Figure 1). In most responses, which used Skemp's (1976) language extensively, a juxtaposition is offered between the *instrumental understanding* in which students are used to, and trained in, and the *relational understanding* that will give them more insight into how things work. Also, a reflection is offered on respondents' intentions to introduce a teaching approach towards the relational understanding and to launch a shared understanding (between the students and themselves) of what mathematics is and how the truth is secured. Trainee [20], for example, emphasises the different perspectives between teacher (understanding) and students (application of rules) and notes the abrupt shift from one to the other: "[I would] perhaps have a clear discussion with the students of what maths is (for me) and what it is for them and why. Also, discuss then what the expectations are and that I'm the teacher now and it can't be all as before". Clearly, trainees aim to establish *sociomathematical norms* in their class and acknowledge their responsibility in doing so. As in the previous section, here we would also like to see *how* trainees would establish a *sociomathematical* norm of an investigative classroom which aspires at identifying mathematical justification and relational understanding.

Trainee [02], for example, diagnoses students' unwillingness to investigate mathematical ideas and their perception of mathematics as a repetitive use of ready-made rules: "They don't see the point and value in investigating mathematical concepts when someone has already done this for you. They see maths as a repetition of using ready-made rules to calculate given questions". She suggests "testing with a shape of 100 sides" with the help of a computer software. Generally, she praises students for self-discovery and states her resilience that the beauty of mathematical discovery will prevail over instrumentalism.

Trainee [16] mentions that students are unsettled by the "change of tack" in the lesson. The "teacher should have explained the investigative nature of the task BEFORE the event". She would give "tiny" or "huge" polygons to trial. She would encourage deriving, not just using, the formula. This "make you into a mathematician, not just a technician", she stresses. She insists that a pre-introduced "framework of discovery" is necessary. This script stands out for its insistence on the need for an explicit articulation of the new norm. She makes a case for it too: when the norm changes, the students will justifiably resist!

Similarly, trainee [01] notes that students are used to the pursuit of quick answers and they "are not interested in why and thus won't develop a deeper understanding". This will "hinder them further on in their relationship with mathematics". She will not relent under the pressure for the students and will strive for a smoother move away from instrumental to "deeper" understanding. A similar consideration on the gradual transition towards such understanding comes also from trainee [21]: "If they are used to one way of thinking/one type of maths class maybe changes should be introduced slowly". However neither trainee [01] nor [21] offer more insight on *how* this gradual transition can be achieved.

From a different perspective trainee [15] is uncertain on the prospect of changing an approach that is efficient for high exam results, especially if these results – even at the expense of understanding and enjoyment – is the aim of schooling: "Previous teaching was prescriptive: learn the rule, apply it. While this may be appropriate if the sole goal is to get students through their exams, it means their understanding (and probably enjoyment) of mathematics is very limited. Is it possible to change the way that the students learn? Will they accept learning by investigation?".

Trainee [18] on the other hand observes that students' understanding is instrumental and that they have no interest in conceptual or relational understanding. He would respond to them with stressing the importance of knowing why, of having a formula that works for all (including large) polygons: "[in his response to student C] You may have had more practice, but I think you will learn more this way if you understand the theory behind the rule"[his emphasis]. It seems that he appreciates the contribution to students' learning that comes from understanding the theory behind the rules. However, interestingly, later in question (c) he doubts this contribution in students' progress when he claims that although he would "persist with the relational teaching for a while" he "would definitely be prepared to return to a more instrumental style of teaching if [he] felt the progress of the students was being negatively affected in any way". This is one of many scripts where regressing to an instrumental approach – if this is what students want – is a priority. This is also one of the many scripts where the two approaches are perceived in a rather simplistically dichotomous manner.

A natural question that comes to our mind is: would it be actually possible for trainees to attempt this change when they will join the school as NQTs the year after their training? How are they going to see themselves in a sensitive situation in which they have to align with the rules without having the opportunity to be critical? How are they going to deal with potential conflicts with the students? Trainee [11], for example, raised the issue of teacher undermining by the students: "The students are too used to being in a very instrumental learning environment. So, when asked to investigate and think more in depth about their explanations they struggle. I think that because they're struggling, and they're not necessarily used to not being able to answer questions, they start to undermine the teacher with their comments". This is a very subtle comment on a situation in which successful students are facing a novel for them experience of not being able to respond to a mathematical problem and deal with this uncomfortable position by undermining the teacher. This undermining might be distracting for an NQT who has not developed robust confidence yet and is not always trained to anticipate similar situations.

Establishing social and sociomathematical norms in the mathematics classroom: The role of the NQT

From the trainee responses we sampled from in the preceding section we can see their intentions to establish pedagogical and mathematical commendable practices in a respectful environment that supports relational understanding. However, establishing these norms is a key challenge for NQTs. This observation brings us back to the question that motivated this project: how could a training course assist teachers to transform commendable aspirations to solid practice?

The recent rhetoric of government policy is that training should be solely school based. However, we would suggest that a research-informed *and* practice-related teacher training programmes that allow time for reflection ensures the generation of most self-aware and subsequently successful teachers. Could we consider a HEI as a suitable place for this to occur? The classroom scenarios developed for this study engaged a cohort of twenty-one Mathematics trainees all placed at a range of diverse schools which brought to these trainees a wealth of experiences – and brought them in contact with a range of views and approaches that each one of our partnership schools take. It is through peer discussion and reflection on this wealth of experiences that the teaching 'values' of the trainees become defined. We see this approach as avoiding the pitfall of single-minded enculturation into the practices that may occur within one, and only, particular school. With the development of more classroom scenarios, informed by both research and everyday classroom events that all teachers are likely to face, we hope to take this teacher training programme further towards the fruits of such reflection. We now recognise that the refined social constructivism that underlies engagement with such classroom scenarios, is the best way to get the most out of trainees and get them *thinking!*

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References

Biza, I., Nardi, E., & Joel, G. (2014, in press). What are prospective teachers' considerations regarding their intended practice when management interferes with mathematical learning? In X, Y. (Ed). *Proceedings of the British Society for Research into Learning Mathematics x*(y), June 2014.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*(3/4), 175-190.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.