



## On a minimal model for estimating climate sensitivity

Gavin C. Cawley<sup>a,\*</sup>, Kevin Cowtan<sup>b</sup>, Robert G. Way<sup>c</sup>, Peter Jacobs<sup>d</sup>, Ari Jokimäki<sup>e</sup>

<sup>a</sup>*School of Computing Sciences, University of East Anglia, Norwich, U.K.*

<sup>b</sup>*Department of Chemistry, University of York, York, U.K.*

<sup>c</sup>*Department of Geography, University of Ottawa, Ottawa, Canada*

<sup>d</sup>*Department of Environmental Science and Policy, George Mason University, Fairfax, Virginia, U.S.A.*

<sup>e</sup>*Skeptical Science, Brisbane, Australia*

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### Abstract

In a recent issue of this journal, Loehle [1] presents a “minimal model” for estimating climate sensitivity, identical to that previously published by Loehle and Scafetta [2]. The novelty in the more recent paper lies in the straightforward calculation of an estimate of transient climate response based on the model and an estimate of equilibrium climate sensitivity derived therefrom, via a flawed methodology. We demonstrate that the Loehle and Scafetta model systematically underestimates the transient climate response, due to a number of unsupportable assumptions regarding the climate system. Once the flaws in Loehle and Scafetta’s model are addressed, the estimates of transient climate response and equilibrium climate sensitivity derived from the model are entirely consistent with those obtained from general circulation models, and indeed exclude the possibility of low climate sensitivity, directly contradicting the principal conclusion drawn by Loehle. Further, we present an even more parsimonious model for estimating climate sensitivity. Our model is based on observed changes in radiative forcings, and is therefore constrained by physics, unlike the Loehle model, which is little more than a curve-fitting exercise.

*Keywords:* Greenhouse effect, Forcing, CO<sub>2</sub>, Climate change

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### 1. The model of Loehle and Scafetta (2011)

Loehle and Scafetta [2] (hereafter LS11) model variations in the HadCRUT3-g1 annual global mean surface temperature anomaly dataset using a model comprised of a linear trend, and two cyclic components with periodicities of 20 and 60 years,

$$f(t) = \theta_0 + \theta_1 t + \theta_2 \cos \left\{ \frac{2\pi(t - \theta_3)}{20} \right\} + \theta_4 \cos \left\{ \frac{2\pi(t - \theta_5)}{60} \right\}, \quad (1)$$

where  $t$  is time, measured in years, and  $\theta = (\theta_0, \dots, \theta_5)$  is a vector of model parameters. The model is then fitted to the HadCRUT3-g1 annual GMST anomalies over a calibration period spanning the years 1850 to 1950. The linear component of the model, described by  $\theta_1$ , is intended to capture a supposed “long term warming since the Little Ice Age”. The cyclic components, with periods of 20 and 60 years, model observed cyclical variations in climate data, tentatively associated with variations in ocean circulation, namely the Pacific Decadal Oscillation (PDO) and with variation in solar activity. The magnitude and phase of these cyclical components, *but not their periodicities*, are tunable parameters of the model. The model is shown (in blue) in Figure 1(a), along with the HadCRUT3v-g1 annual

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*Email address:* [g.cawley@uea.ac.uk](mailto:g.cawley@uea.ac.uk) (Gavin C. Cawley)

temperature anomalies (depicted in green), the corresponding model residuals are shown in Figure 1(b). The model provides a subjectively reasonable fit to the observations during the calibration period, however the model does not explain the more rapid rise in temperature after 1950, and so this is modelled with an additional linear component, starting in 1942 and rising at a rate of  $0.66 \pm 0.08^\circ\text{C}$  per century, that is assumed to represent the anthropogenic influence on climate, as shown in Figure 1(b). Loehle [1] (hereafter LO14), uses this model to obtain estimates of the transient climate response and equilibrium climate sensitivity. Unfortunately, the LS11 model and hence the resulting estimates are fundamentally flawed, for reasons explored in the subsequent sections of this brief note.

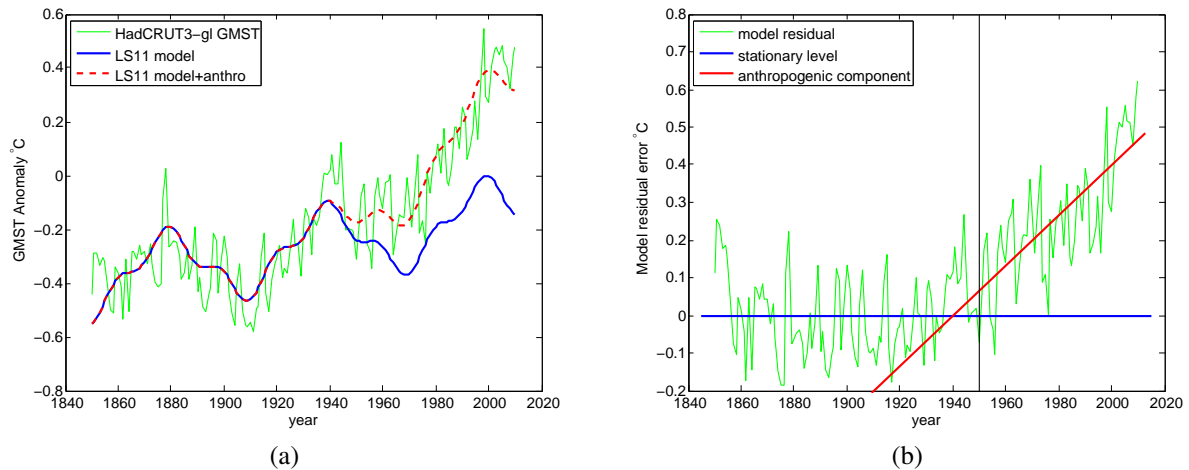


Figure 1. LS11 model fitted to HadCRUT3v-gI GMST anomalies from 1850-1950 and projection to 2013 (a) and model residual errors (b).

### 1.1. Understatement of the uncertainty in the estimates of climate sensitivity

LO14 gives a 95% confidence interval for the value of transient climate response of  $0.96 - 1.23^\circ\text{C}$  per doubling of  $\text{CO}_2$ , however this was derived from the slope of the linear “anthropogenic” component of the model, of  $0.66 \pm 0.08^\circ\text{C}$  per century, taken from LS11. The 95% confidence interval for this parameter in our MATLAB implementation of the Loehle-Scafetta model, using the `regress` routine of the statistics toolbox, is  $0.664 \pm 0.165^\circ\text{C}$  per century. The width of this interval is, to within the accuracy of rounding, twice that reported by LS11, so it seems likely that the interval reported in LS11 is a one standard deviation interval, rather than a 95% confidence interval. This (according to our MATLAB reimplementation) implies that the 95% confidence interval for the LO14 estimate of transient climate response should be  $1.100 \pm 0.274^\circ\text{C}$  for each doubling of atmospheric  $\text{CO}_2$  (implying an equilibrium climate sensitivity of  $2.000 \pm 0.498^\circ\text{C}$  per doubling, using the method employed by LO14, see Section 2). More importantly, this interval represents only the uncertainty in inferring the slope of the linear “anthropogenic” component from the residuals of the cyclic model. In reality, there is also considerable uncertainty in inferring the other parameters of the model,  $(\theta_1, \dots, \theta_5)$  from the calibration period, which also substantially broaden the confidence interval of the estimate of transient climate response. A Bayesian analysis of the model of LS11, described in the supplementary material in Appendix A, gives a highest posterior density (HPD) credible interval for the transient climate response of  $0.753$  to  $1.434^\circ\text{C}$  per doubling of  $\text{CO}_2$  with a corresponding estimate for equilibrium climate sensitivity of  $1.369$  to  $2.607^\circ\text{C}$  per doubling (using the method employed by LO14). These intervals are considerably broader than the corresponding intervals given in LO14.

### 1.2. The existence of 60 and 20-Year cyclic components is not well supported by the calibration period

Inspection of the residuals of the standard LS11 model, shown in Figure 1(b), suggest that the model is clearly deficient as large-scale structure is evident in the residuals for the calibration period (1850-1950), with the residuals exhibiting a downward trend from 1850 to around 1890 and an increasing trend from then onward. An extended model, where the periodicities of the cyclic components were also tunable parameters, fitted to the calibration data,

was also evaluated, where

$$f(t) = \theta_0 + \theta_1 t + \theta_2 \cos \left\{ \frac{2\pi(t - \theta_3)}{\theta_4} \right\} + \theta_5 \cos \left\{ \frac{2\pi(t - \theta_6)}{\theta_7} \right\}. \quad (2)$$

The results for this model are shown in Figure 2 (a) and (b), instead of the periodicities of 20 and 60 years used in the standard LS11 model, the extended model gives optimised periodicities of 21.76 and 69.65 years. In addition to improving the subjective fit of the model to the calibration period, the extended model clearly addresses the deficiency identified in the standard LS11 model as the residuals, shown in Figure 2 (b), no longer exhibit any clear structure during the calibration period. The model, however, now gives substantially higher estimates of transient climate response and equilibrium climate sensitivity (respectively  $1.191 \pm 0.262$  and  $2.164 \pm 0.476$  °C per doubling of atmospheric CO<sub>2</sub>).

A Bayesian analysis of the extended model, described in the supplementary material in Appendix A, was then conducted to determine the plausible periodicities of cycles within the calibration period and to obtain a credible interval on the estimates of climate sensitivity that reflect the uncertainties due to the estimation of all of the tunable parameters of the model from a finite calibration period. The 95% HPD credible interval on the periodicity of the shorter cycle,  $\theta_4$ , extends from 20.67 to 22.89 years, providing very little support for a periodicity as short as 20 years in the calibration period. The credible interval for the periodicity of the longer cycle,  $\theta_7$ , extends from 63.44 to 79.32 years, thus we conclude that the existence of a 60 year cycle in the calibration period is implausible. This result demonstrates that the standard LS11 model is inappropriate for use in attribution as a key modelling assumption is clearly invalid. The 95% HPD credible intervals on transient climate response and equilibrium climate sensitivity are 0.800 – 2.000 and 1.454 – 3.635 respectively.

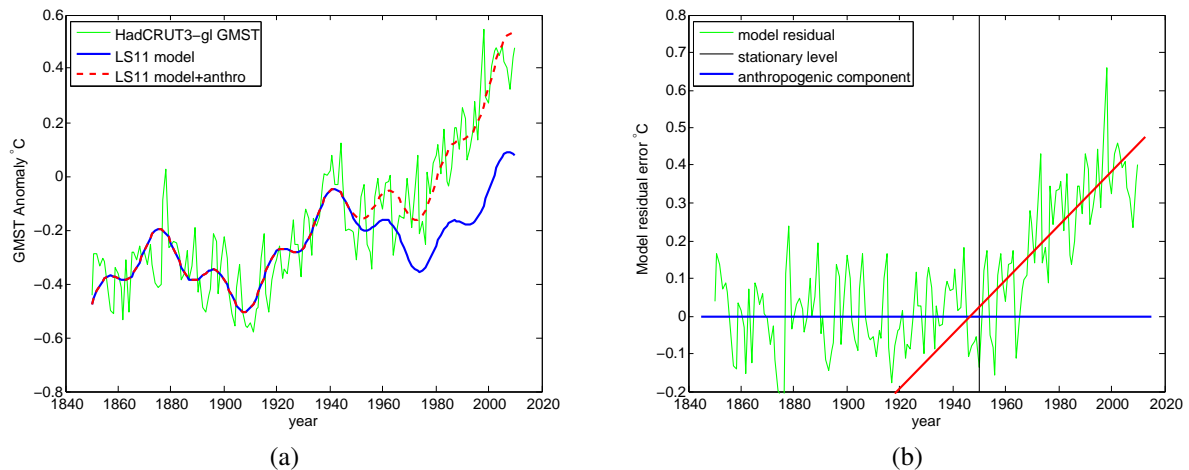


Figure 2. LS11 model fitted to HadCRUT3v-gI GMST anomalies from 1850–1950 and projection to 2013 (a) and model residual errors (b). In this case, the periodic components have been determined by fitting to the calibration period, rather than chosen “*a-priori*”.

LO14 states that “... the recent 17 year pause in warming was predicted based on data ending in 1950 (i.e. it performs a successful 60 year forecast)”. This claim is clearly incorrect as the modelling of the hiatus is entirely predicated on the existence of 20 and 60 year cycles, which were not based on data ending in 1950, but were chosen “*a-priori*”, and indeed, as we have shown, are effectively ruled out by the observations comprising the calibration period. If the periodicities are based on the observations up to 1950, the model no longer correctly predicts the 17 year pause in warming (see Figure 2).

### 1.3. The extension of the linear component representing natural warming is not justified

LO14 states that “The long term warming since the Little Ice Age is captured by the slow warming (linear) component, which we suggested could summarise longer cycles of solar activity”. It is true that solar forcing showed a modest increasing trend for the first half of the 20th century, however solar activity (e.g., as represented by sunspot

numbers or TSI observations) has declined since solar cycle 19 (April 1954 – October 1964). Therefore, there seems little justification for the continuation of the linear component past 1964, yet this is exactly what the model of LS11 does. This reduces the estimate of climate sensitivity given in LO14 as warming due to increasing GHG concentrations is potentially attributed to a long term trend in solar activity, which is not actually evident after solar cycle 19. It is also worth noting that there was a reduction in volcanic forcing, clearly evident in the total natural forcing shown in figure 3 (a) between 1920 and 1960, that should also be expected to result in a natural warming trend in the first half of the 20th century and a cooling after the resumption in large-scale volcanic activity in the 1960s (c.f. Figure 3 (c)). As a result, a cooling trend, starting in the second half of the 20th century, would be expected in the absence of anthropogenic forcing, as indicated by general circulation models (c.f. IPCC AR4 WG1 report [3], figure 9.5 and Frequently Asked Question 9.2), rather than continued natural warming. Note also that it is questionable to assume that all of the warming prior to 1950 is purely natural as anthropogenic GHG emissions from fossil fuel use and land use change have been increasing approximately exponentially from the start of the industrial revolution, and as a result, anthropogenic forcing shows no definite change in intensity in 1950. Current research situation on the issue suggests that anthropogenic forcing might indeed have been one factor in early 20th century warming (e.g. [4, 5, 6, 7, 8, 9, 10, 11, 12]). If the “slow warming (linear) component” is discontinued after 1950, when the uncertainty in estimating all of the parameters of the model are taken into account, the credible intervals for the transient climate response and equilibrium climate sensitivity become, 1.120–2.566 and 2.036 – 4.664°C, respectively. Note that this potentially underestimates climate sensitivity as the estimates of natural forcings suggest cooling from the 1960s onwards, rather than a levelling off of global mean surface temperatures in the absence of anthropogenic forcings.

#### 1.4. Total anthropogenic forcing was greater than CO<sub>2</sub> forcing since 1950

LO14 states that the “only simplifying assumption is that aerosols and non-CO<sub>2</sub> greenhouse gasses and other forcings (e.g., land use change) approximately cancel each other”, based on “the IPCC AR4 chart of forcings” [13] (presumably Figure SPM2 [3]). However, the IPCC AR4 chart of forcings gives estimates of forcings relative to 1750 (representing a pre-industrial baseline). This does not however imply that these forcings have approximately cancelled over the period from 1951 to 2010, used to estimate the anthropogenic warming after 1950. The RCP8.5 forcings [14], shown in Figure 3 (a) suggest that total anthropogenic forcing since 1950 has risen appreciably faster than the forcing from CO<sub>2</sub> alone by a ratio of approximately 1.145:1. As a result of this assumption, the method of LS14 underestimates climate sensitivity by about 13%, correcting the estimates from the previous section, the credible intervals for transient climate response and equilibrium climate sensitivity are 1.292 – 2.937 and 2.330 – 5.338 °C respectively. At this point we have a lower constraint on ECS that is higher than the upper constraint on ECS given by LO14, directly and diametrically contradicting the conclusions of LO14. The fact that the estimate of climate sensitivity obtained from the model is so sensitive to the assumptions made is an indication that the model is not a reliable or useful model of reality.

## 2. On the novel contribution of Loehle (2014)

The novel contribution of LO14 is rather limited, lying solely in the detail of the straightforward calculation of transient climate response (a figure of 1 – 1.5°C derived from the same model was previously given by LS11) and its scaling to give an estimate of equilibrium climate sensitivity (where the scaling factor is given by the average ratio of those quantities for a range of models used in the IPCC AR4 WG1 report). According to the IPCC AR4 WG1 report [3, section 9.6.2.3],

*“The TCR does not scale linearly with ECS because the transient response is strongly influenced by the speed with which the ocean transports heat into its interior, while the equilibrium sensitivity is governed by feedback strengths (discussion in Frame et al., 2005).”*

thus a naïve scaling of the TCR is not an acceptable method of estimating the equilibrium climate sensitivity. Even if a simple scaling were appropriate, LO14 argues that climate models over-estimate climate sensitivity, in which case the scaling factor derived from the GCMs could not be considered reliable, unless it were demonstrated that the GCMs overestimated both transient climate response and equilibrium climate sensitivity to an approximately equal degree. This seems unlikely (as the TCR and equilibrium sensitivity are more strongly influenced by different factors with

various sensitivities to the modelling of internal climate variability), and no such justification is provided. Thus the limited novel content of LO14 cannot be considered reliable.

### 3. An alternative minimal model for estimating climate sensitivity

We now present an even more parsimonious model, having only four parameters, that can also be used to estimate climate sensitivity. Unlike the model of LS11, the model is based on estimates of natural and anthropogenic forcings. The forced response of the climate system is modelled as the convolution of the forcings,  $F(t)$ , and an exponentially decaying impulse response,  $\psi(t)$ , an offset and an additive component representing internal climate variability,  $v(t)$ , i.e.

$$f(t) = \theta_0 + \theta_1(F * \psi)(t) + \theta_2v(t) \quad \text{where} \quad \psi(t) = \exp(-t/\theta_3),$$

and  $\theta = (\theta_0, \dots, \theta_3)$  is a vector of model parameters. The parameter  $\theta_3$  represents the characteristic timescale on which the climate system responds to a change in the forcings. In our model internal climate variability is represented by the NINO3.4 ENSO index, however in principle other sources of internal variability could also be included. Figure 3 (a) shows the forcings relative to those of 1750 [14], used in the model and (b) shows the output of the model where the parameters have been fitted to the calibration period 1856–1950 via the method of least-squares. Unlike the model of LS11, this model correctly predicts the rise in GMST after 1950 without having to add an additional component of the model to the test data.

As the model is linear, we can extract the contributions due to individual forcings; Figure 3 (c) shows the contribution due to natural forcings, due to total anthropogenic forcing and due to CO<sub>2</sub> radiative forcing. Like the GCMs discussed in the IPCC AR4 WG1 report [3] (figure 9.5 and Frequently Asked Question 9.2), this model suggests that GMSTs would have declined after 1960 in the absence of anthropogenic forcing. The model also suggests that the total anthropogenic warming since 1950 exceeds that due to CO<sub>2</sub> radiative forcing alone.

The transient climate response can be estimated as the change in temperature following a doubling in atmospheric CO<sub>2</sub> resulting from a steady annual increase of 1%. A Bayesian analysis of the model gives a 95% HPD credible interval on the transient climate response of 1.309 to 2.016 °C, with a maximum a-posteriori estimate of 1.662 °C. The posterior distribution for TCR is depicted in Figure 3 (d). Note the posterior distribution is comparable to estimates obtained using GCMs.

There are two important observations to make in contrasting this model with that of LS11. Firstly the model of LS11 is essentially a curve fitting exercise, as the cyclic and linear components are only anecdotally associated with potential physical processes, such as solar activity, the Pacific decadal oscillation and a “recovery from the Little Ice Age”. In the model described in this section, the physical processes are represented explicitly and *numerically* in the model via the estimates of the forcings. This means that we can directly test the effects of different assumptions regarding forcings or their uncertainties. The model of LS11 merely shows that the observations can be approximately represented as the sum of two cyclic and two linear components. However this does not imply that these components relate to distinct physical processes any more than fitting a polynomial model would imply that climate is governed by a physical system obeying a polynomial law. Secondly, both models are potentially susceptible to omitted variable bias. The model of LS11 estimates the anthropogenic influence on climate from the residual of the model that is left after what can be explained by the cyclic components and long term linear trend has been subtracted. However, if the anthropogenic contribution is correlated with any component of the LS11 model then some of that contribution will be falsely attributed to the LS11 model, and the model will potentially underestimate climate sensitivity. This bias means that such a model should not be used to argue that climate sensitivity is low, at least not without mention of this important caveat. The model presented here has the opposite bias, in that it attributes to internal climate variability what cannot be explained by the forcings (or ENSO in this case). As a result, a model of this type should not be used to uncritically argue that climate sensitivity is high (although if the incorrect assumptions of the LS11 model are addressed, it actually suggests that the plausible range of transient climate response extends well beyond that of our model).

### 4. Summary

LO14 estimates transient climate response and equilibrium climate sensitivity using a cyclic model, that systematically underestimates these quantities, due to flaws and unsupported modelling assumptions, summarised in Table 1.

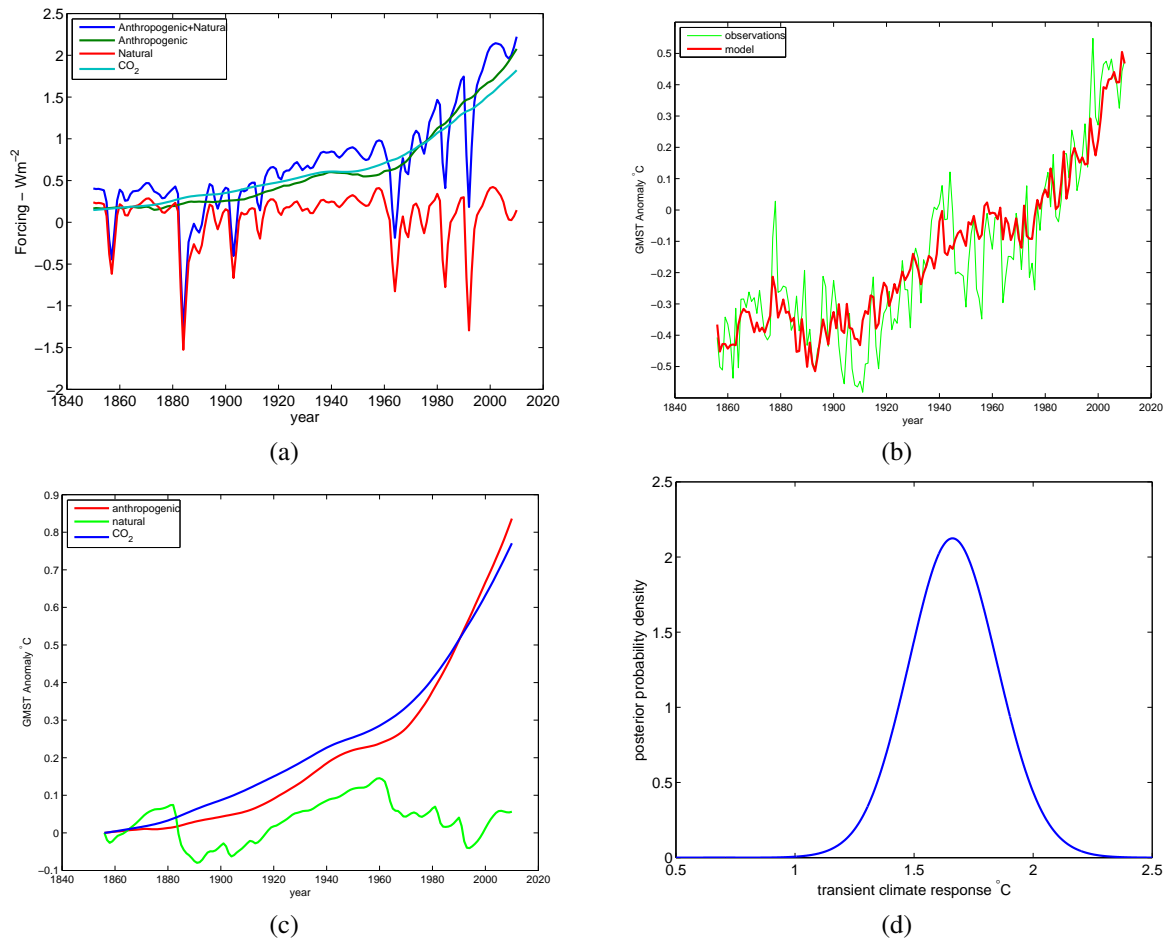


Figure 3. (a) Forcings relative to those of 1750 for RCP8.5 [14]; (b), model of the HadCRUT3v-gl annual GMST anomalies, based on total (anthropogenic + natural) forcings, using a simple one-box model; (c) model components representing the effects of total anthropogenic, CO<sub>2</sub> and natural forcings on GMST; (d) posterior distribution for transient climate response under the one-box model.

Addressing these issues results in a model that rules out the possibility of low climate sensitivity, directly contradicting the findings of LO14, and giving credible intervals on transient climate response and equilibrium climate sensitivity that are broadly consistent with those obtained from General Circulation Models criticised by LO14. Attributing climate change to natural and anthropogenic causes cannot be performed reliably using such a naïve correlative model, as the conclusions are so heavily dependent on the modelling assumptions. Instead a model more closely based on physics should be used, as the behaviour of the model is then more strongly constrained by physical plausibility.

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Table 1. Summary of the effects of the deficiencies of the LS11 model on estimates of transient climate response (TCR) and equilibrium climate sensitivity (ECS).

Description	TCR	ECS
Results presented in LO14	0.96 - 1.23	1.745 - 2.227
Faithful representation of uncertainty	0.753 - 1.434	1.369 - 2.607
Optimisation of periodicities	0.800 - 2.000	1.454 - 3.635
Continuation of natural warming trend	1.120 - 2.566	2.036 - 4.664
Assumption regarding anthropogenic forcings	1.292 - 2.937	2.330 - 5.338

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## Appendix A. Bayesian analysis of Cyclic Models

We begin by re-parameterising the model in order to allow the dependence between the amplitude parameters,  $\theta_2$  and  $\theta_4$ , and phase parameters,  $\theta_3$  and  $\theta_5$ , of the cyclic components to be more easily accounted for in post-processing, such that

$$f(t) = \theta_0 + \theta_1 t + \theta_2 \cos \left\{ \frac{2\pi t}{20} - \theta_3 \right\} + \theta_4 \cos \left\{ \frac{2\pi t}{60} - \theta_5 \right\}, \quad (\text{A.1})$$

A noise variance parameter,  $\theta_6$ , is added, to give a fully probabilistic model of the observations, i.e.

$$\text{gmst}(t) \sim \mathcal{N}(f(t), \theta_6). \quad (\text{A.2})$$

Adopting an uniform prior over  $\theta_1 \dots \theta_5$  and over  $\log \theta_6$  (as the noise variance is a strictly positive scale parameter), we can sample from the posterior distribution of the model parameters,  $\theta = (\theta_1, \dots, \theta_6)$  via the Hastings-Metropolis algorithm [15, 16], using the `metrop` routine of the NETLAB library [17]. This implementation uses a simple spherical Gaussian proposal distribution, so we transform the parameters by multiplying by the standard deviation of the sample from an exploratory run of the Markov Chain Monte Carlo simulation so that the transformed parameters have similar scale lengths, which improves the acceptance rate. The mixing of the chain was still quite slow, however as the number of parameters was low, the simplest solution was to run the simulation for a very long time and then downsample, as the computational expense involved remained acceptable. For the model where the periodicities are tunable parameters, the original parameterisation (2), with the addition of a noise variance parameter,  $\theta_8$ , was found to produce more rapid mixing. A uniform prior over  $\log \theta_4$ ,  $\log \theta_7$  and  $\log \theta_8$ , was used for these strictly positive quantities.