Bond yield modelling and its application in the European Union

by

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Abstract

Forecasting crises has always been an interesting and important topic for econometricians or statisticians. Literature suggests that government bond yields can be a valid leading indicator for this purpose. This thesis uses government bond yields and applies various models to forecast the crisis which happened recently.

Chapter 2 investigates a model utilising the term structure of interest rates to predict output growth and recession in the UK. In contrast to previous literature, information retrieved from the whole yield curve is used rather than just the yield spread. Using different methods, our models are found to outperform the yield spread models both in in-sample and out-of-sample forecasting. Notably, the B-spline fitting model is able to forecast the 2009-2010 recession. Moreover, the model with lag of growth shows great forecasting ability in out-of-sample output growth forecasting. In most cases, models based on B-spline perform better than the ones based on the Diebold-Li framework.

Chapter 3 examines the existence of time series non-linearities in the real output growth / recession-term spread relationship. Vector Autoregression (VAR), Threshold VAR (TVAR), Structural break VAR (SBVAR), Structural break threshold VAR (SBT-VAR) are applied in the analysis. The in-sample results indicate there are non-linear components in this relationship. And this non-linearity tends to be caused by structural breaks. The best in-sample model also shows its robustness on arrival of new information in the out-of-sample tests. Evidence shows the model with only structural break non-linearity outperforms linear models in 1-quarter, 3-quarter and 4-quarter ahead forecasting.

The European sovereign debt crisis has become a very popular topic since late 2009. In Chapter 4, the sovereign debt crisis is investigated by calculating the probabilities of the potential future crisis of 11 countries in the European Union. We use sovereign spreads of the European countries against Germany as targets and apply the GARCH based vine copula simulation technique. The methodology solves the difficulties of calculating the probabilities of rarely happening events and takes sovereign debt movement dependence, especially tail dependence, into consideration. Results indicate that Italy and Spain are the most likely next victims of the sovereign debt crisis, followed by Ireland, France and Belgium. The UK, Sweden and Denmark, which are outside EMU, are the most financially stable countries in the sample.

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Abbreviations

- 2SBVAR: Structural break Vector Autoregression with 2 structural breaks
- 2TVAR: Threshold Vector Autoregression with 2 thresholds
- AIC: Akaike Information Criterion
- BEL: Belgium
- C-vine: Canonical vine
- CDO: Collateralized Debt Obligation
- CDS: Credit Default Swap
- D-vine: Drawable vine
- VAR: Vector Autoregression
- DEN: Denmark
- ECB: European Central Bank
- EMU: European Monetary Union
- ES: Expected Shortfall
- fGARCH: Family GARCH
- FRA: France
- GARCH: Generalized Autoregressive conditional heteroskedasticity

- GARCH-M: GARCH-in-mean
- GER: Germany
- GDP: Gross Domestic Products
- GJR-GARCH: Glosten-Jagannathan-Runkle GARCH
- GRE: Greece
- i.i.d: independent and identically distributed
- TVAR: Threshold Vector Autoregression
- IRE: Ireland
- ITA: Italy
- NBER: National Bureau of Economic Research
- NET: Netherlands
- OTC: Over the Counter
- POR: Portugal
- RMSE: Root Mean Square Error
- SA: Seasonal Adjusted
- SBTVAR: Structural break Threshold Vector Autoregression with one structural break and one separate threshold in each broken regime
- SBTVARc: Structural break Threshold Vector Autoregression with one threshold and one structural break
- SBVAR: Structural break Vector Autoregression
- SPA: Spain
- SWE: Sweden

- TGARCH: Threshold GARCH
- TSP: Traveling Salesman Problem
- UK: United Kingdom
- VaR: Value at Risk

Chapter 1

Introduction

The bond market is a financial market where participants can issue new debt, and it provides a mechanism for long term funding of public and private expenditures. Government bonds, because of their size, liquidity, and relative lack of credit risk, are usually treated as the representative of the bond markets, although there are also bond markets for corporate bonds and financial instruments like mortgage bonds. The Bond market seems very useless on the surface, at least not as interesting as stock markets from most people's point of view. The mechanism of the bond market and the impact the bond market can make are rarely understood by the public. In reality, however, the bond market can be surprisingly powerful. It can change governments' policies or even can change governments. Here is a story regarding the power of the bond market and how it can change government policy. In the 1990s, President Clinton attempted to increase the US budget deficit, which led to a sell off of Government bonds. And the Clinton administration had to alter policy in order to secure the debt. Instead of an increasing budget deficit, Clinton managed a rare budget surplus. Most recently, the sovereign debt crisis shows us an almost extreme case of the impact bond market can cause – government bankruptcy. Because of the importance of the bond market, this thesis is exploring a way to use the information in the bond market to assess the economic situation of a country.

1.1 The Motivation of the Research

The global financial crisis, starting from the summer of 2007, is considered by many economists to be the worst financial crisis since the Great Depression of the 1930s. It resulted in the threat of total collapse of large financial institutions, the bailout of banks by national governments and downturns in stock markets around the world. The crisis played a significant role in the failure of key businesses, the decline in consumer wealth estimated to be in the trillions of US dollars, and a downturn in economic activity leading to the 2008-2012 global recession, as well as contributing to the European sovereign-debt crisis. Things got worse in late 2009. The fears of a sovereign debt crisis developed among investors as a result of the rising private and government debt levels around the world, together with a wave of downgrading of government debts in some European Monetary Union (EMU) members. The crisis started from Greece when investors began to worry about the potential default of Greek government bonds. And this fear spread to other EMU countries, especially peripheral members. Crises always make people ask questions like : 'What if I could see this coming?'. Economists are widely blamed in this crisis because of their failure of forecasting. This leads to the question of whether this crisis really could not be forecast and what leading indicator can be used. Regarding the sovereign debt crisis, it is even harder to forecast. This is because sovereign debt default is a rarely happening event. How to assess the risk level of sovereign debt is not an easy question to answer, and it is even harder to forecast the crisis.

In the literature, the yield curve has been proven a valid leading indicator in forecasting aggregate activities. The yield curve, also known as the term structure of interest rates, is a curve showing several yields or interest rates across different contract lengths (3 month, 5 year, 10 year, etc...) for a debt contract. The curve shows the relation between the interest rates (or borrowing cost) and the time to maturity (term of the debt contract). And government bond yield curve is generally treated as a benchmark curve among all. Since Kessel (1965) first brought the idea that different stages of the business cycle show different term structures of interest rates, researchers started to examine the predictive power of the term structure in predicting economic activities. And as the simplest form of term structure of interest rates, term spread is used as the leading indicator and it has been found useful for forecasting macroeconomic activities such as output growth, inflation, industrial production and recessions. Term spread here is a difference between a nominal long-term bond yield (usually 10-year government bond yield) and a nominal short-term bond yield (usually 3-month government bond yield). This suggests the these analysis are conducted by real GDP and nominal yields, which implies the assumption that inflation is constant across different terms. Despite the fact term spread makes a good indicator, the literature seems have trouble to reach an agreement on the theory to explain why this macroeconomic / term spread relationship exists. For later studies, they will just refer to this as a "stylized fact in search of theory" (Benati and Goodhart, 2008). Three main theories support this relationship so far. The first one is from Harvey (1989). He believes that when people expect a recession or a decreasing economy, they will change their investment behaviour. They will withdraw money from the short-term investment and put them into the long-term ones. In the bond market, this behaviour leads to a higher long-term bond price and a lower short-term bond price, so the yield of short-term bonds will rise and the yield on long-term bonds will drop. And of course, the spread is shrinking. And in the same manner, when people expect a booming of the economy, the term spread is increasing. The second theory is based on expectations hypothesis of the term structure of interest rates. The hypothesis explains long-term interest rates as sum of current and expected future short-term interest rates plus a term premium. This hypothesis well explained the reason why the long-term rate is larger than the short-term rate on most occasions, which makes the yield curve slope upwards. However, if the expectation about future short-term yield drops, the yield curves become flatten or even downward. This theory is closely related to the fact that monetary policy will be adjusted according to the market situation. And the third theory emphasizes the ability of term spread forecasting will be influenced by the reaction of a monetary authority on stabilizing the output growth. Feroli (2004), Estrella (2005), Estrella and Trubin (2006) noted in their research that whether the term spread is a good predictor depends on the monetary authority's policy objectives and reaction functions. This means if the monetary authorities focus on controlling inflation exclusively, the accuracy of term spread to forecast will be bad. While if the authorities are more responsive to the potential output growth, the term spread will become a better leading indicator. Term spreads did show a great forecast ability in the literature. And now the question is are they still reliable during the recent crisis and what can be done to improve this leading indicator?

The sovereign debt crisis became one of the most popular topics since late 2009. This is because the crisis happens in a monetary union so that it spreads quickly. Although when we talk about the sovereign debt, it can be issued with different terms to maturity, 10-year government bond is always treated as the benchmark in terms of the stability of the sovereign debt. The yield movement of 10-year government bonds represent the confidence of investors about government solvency in the bond markets. On 8th Dec, 2009, rating agency Fitch cut Greece's long-term debt from A- to BBB+. When the Greek government bond yield rose sharply as a result of the lack of confidence in investing in Greek government bonds, the bond yield of peripheral European countries Spain and Portugal also increased along with Greece. In Ireland and Italy, however, the yields decreased. The different reactions of different countries leads to another question: ‘ Is there any relationship between these countries' sovereign debt yield movements especially in a crisis?' And ‘Can this relationship be used as a tool for forecasting risk levels of sovereign debt in the European Union?' When a multi-dimensional method is needed to model this problem, vine copula becomes a valid and promising choice.

As a developed form of copula method, vine copula is a flexible graphical model for describing multivariate copulas built up using a cascade of bivariate copulas. Such pair-copula constructions decompose a multivariate probability density into bivariate copulas, where each pair-copula can be chosen independently from the others. It comes with all the properties a conventional copula has, which provides a method of isolating the description of the dependence structure and understanding the dependence at a deeper level. It expresses dependence on a quantile scale, which is useful for describing the dependence of extreme outcomes and is natural in a risk-management context. The method has been used in the stock market to analyse the risk of an investment portfolio (Joe, 1996, Nikoloulopoulos et al., 2012). And this leads us to ask the research question:‘Can one analyse government bond risks of the countries in the European union at one time using the vine copula method and how?'

1.2 Contributions of the Study

Chapter 2 and 3 make several contributions to the literature on output growth/recession - yield curve relationship from two perspectives.

Chapter 2 expands the research from the aspect of allowing more information to the leading indicator, and the contribution is threefold: First, conventionally, researchers use yield spreads to represent the yield curve. However, use of the yield spread is essentially based on the assumption that the yield curve is a straight line, while it may be that the non-linearity in the yield curve embodies predictive power. Therefore, the innovative feature of this chapter is that this assumption is relaxed by using the whole yield curve. Second, I use two different approaches, parametric and non-parametric, to model the yield curve in order to meet a great variety of forecasting purposes. Third, I demonstrate that the term structure forecasting model has an excellent forecast ability in recession forecasting. This is particularly relevant in the light of the major recession recently experienced.

The third chapter explores the relationship from the aspects of nonlinearity, and it contributes to the literature from the following five aspects. Firstly, I choose the UK as the target country to investigate the relationship between yield spread and real economic activities involving the time series non-linearity. Secondly, the presented research is conducted with data containing the recent Financial Crisis. The influence of this big recession to the relationship is tested. Thirdly, I apply Vector Autoregression (VARs) with non-linearity to forecast future real GDP growth as well as recessions. Fourthly, this chapter applied two more non-linear models and various autoregressive orders in the model search, which is more comprehensive compared to Galvão (2006)'s research. Last but not least, this study successfully identifies the non-linearity of the real growth-term spread relationship in the UK.

Chapter 4 analyses the sovereign debt crisis from a new perspective, and the contribution of this research is fourfold. Firstly, this is the first analysis of extreme value and tail dependence of sovereign debt spreads movements in the European Union. Secondly, this study conducts the comparison between 11 countries in the European Union at the same time. Thirdly, this chapter uses vine copulas to deal with large numbers of dimensions. The model satisfies the wide range of dependence, flexible range of upper and lower tail dependence, computationally feasible density for estimation, and closure property under marginalization simultaneously. Fourthly, which is also the key feature of the chapter, the research identifies the risk levels of sovereign debt in different countries in the European Union.

1.3 The Structure of the Thesis

The thesis is organized in five chapters, and the rest of the thesis is structured as follows:

Chapter 2: The yield curve as a leading indicator in economic forecasting in the UK

This chapter investigates a model utilising the term structure of interest rates to predict output growth and recession in the UK. In contrast to previous literature, information retrieved from the whole yield curve is used rather than just the yield spread. Using both parametric and non-parametric methods and analysing quarterly UK data from 1979q4 to 2009q4, our models are found to outperform the yield spread models both in in-sample and out-of-sample forecasting. Notably, the B-spline fitting model is able to forecast the 2009-2010 recession. Moreover, the model with lagged GDP growth (Model B in the chapter) shows great forecasting ability in out-of-sample output growth forecasting. In most cases, models based on B-spline perform better than the ones based on the Diebold-Li framework.

Chapter 3: Time series non-linearity in the real growth / recession-term spread relationship, some evidence from the UK

This chapter examines the existence of time series non-linearity in the real output growth / recession-term spread relationship. Vector Autoregression (VAR), Threshold VAR (TVAR), Structural break VAR (SBVAR), Structural break threshold VAR (SBTVAR) are applied to UK data from 1979q1 to 2013q1 in the analysis. The in-sample results indicate there are non-linear components in this relationship. And this non-linearity tends to be caused by structural breaks. The best in-sample model also shows its robustness on arrival of new information in the out-of-sample tests. I find evidence that the model with only structural break non-linearity outperforms linear models in 1-quarter, 3-quarter and 4-quarter ahead forecasting.

Chapter 4: Vine copulas and applications to the European Union sovereign debt analysis

The European sovereign debt crisis has become a very popular topic since late 2009. This chapter investigates sovereign debt crises by calculating the probabilities of the potential future crisis of 11 countries in the European Union. Sovereign spreads against Germany are used as targets and apply the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) based vine copula simulation technique. The methodology solves the difficulties of calculating the probabilities of rarely happening events and takes sovereign debt movement dependence, especially tail dependence, into consideration. Results confirm the situation that Greece and Portugal are in crises. They also indicate that Italy and Spain are the most likely next victims of the sovereign debt crisis, followed by Ireland, France and Belgium. The UK, Sweden and Denmark, which are outside the EMU, are the most financially stable countries in the sample.

Chapter 5: Conclusion

This Chapter concludes the thesis and discusses the limitations of the research as well as future research.

Chapter 2

The yield curve as a leading indicator in economic forecasting in the UK

2.1 Introduction

The practical objective for economic forecasting is to provide policymakers with new economic tools to estimate the impact on aggregate activities of their potential decisions. The importance of accurate forecasting was brought into sharp focus long ago by the painful experience of the Great Depression. However, this is not saying that a good forecaster can prevent a recession like the recent financial crisis happening, but a good forecaster may help to reduce the loss caused by recession to an acceptable level. The regulator is not the only party who can benefit from forecasting but also private agents such as practitioners, portfolio managers and risk managers whose future earnings and business strategies will be influenced by the quality of such forecasts.

The term structure of interest rates has been mentioned frequently in the context of monetary policy, particularly as an indicator of market expectations or of the position of policy. Although it is rarely viewed as a policy target, it is generally conceded to contain some information that may be of use to both market participants and to the monetary authority. There has already been a relatively extensive literature examination of the informational and predictive content of the term structure with regard to the conventional final targets of monetary policy, which are inflation and real activity. For instance, when people expect a recession, they will change their investment behaviour: withdraw money from short-term investing and put them into long-term investing. In the bond market this behaviour leads to a higher long-term bond price and a lower short-term bond price, so the yield of short-term bonds will rise and the yield on long-term bonds will drop, changing the shape of the yield curve.

This chapter looks at economic forecasting through the term structure of interest rates from a new perspective by examining the whole yield curve and using the information to forecast recessions and output growth.

In the literature, there has been little research using the whole yield curve. The majority use only the slope or both slope and level to investigate the relationship between yield curve and output growth. The contribution is threefold: First, conventionally, researchers use yield spread to represent the yield curve. However, use of the yield spread is essentially based on the assumption that the yield curve is a straight line, while it may be that the non-linearity in the yield curve embodies predictive power. Therefore, one innovative feature of this chapter is that this assumption is relaxed by using the whole yield curve. Second, two different approaches have been used, parametric and non-parametric, to model the yield curve in order to meet a great variety of forecasting purposes. Third, the chapter demonstrates that the term structure forecasting model has excellent forecast ability in recession forecasting. This is particularly relevant in the light of the major recession recently experienced.

The nominal zero-coupon rate and real GDP growth in the UK for the period from 1979q4 to 2009q4 are used in this chapter. We adopt the definition of recession used by the National Bureau of Economic Research (NBER) which is a period in which GDP falls (negative real economic growth) for at least two consecutive quarters. There are two striking features in the results of this study: 1. The B-spline and Diebold-Li frameworks forecast fit better than the yield spread model and achieve higher forecast performance in short-horizon forecasting especially 1-quarter ahead forecasts. According to both in-sample and out-of-sample tests, models based on the whole yield curve outperform those based on yield spread in output forecasting. 2. In terms of the forecasting ability of different forecasting approaches under two frameworks (Diebold-Li framework and B-spline framework), the out-of-sample forecasting results demonstrated that probit model base on B-spline approach exhibits extremely high forecast ability for forecasting recessions.

The rest of this chapter is structured as follows: Section 2.2 is a brief literature review about using the yield curve to forecast economic growth. In Section 2.3, methods of constructing a yield curve and the forecasting models are illustrated. Section 2.4 is the description of the data in the research. The results and findings of this research is presented in Section 2.5. Section 2.6 concludes.

2.2 Brief Literature Review

The use of interest rates and their term structure as a predictor of recession and GDP growth has been widely studied, and these literature show strong evidence that it is reliable (Estrella and Hardouvelis, 1991, Zagaglia, 2006, Bordo and Haubrich, 2008). However, evidence shows that not all the countries in the world can effectively use the term structure of interest rates as a leading indicator, while most of the literature find that the UK is one of those that can use it (Jorion and Mishkin, 1991, Harvey, 1991, Estrella and Mishkin, 1997, Plosser and Rouwenhorst, 1994, Bernard and Gerlach, 1998). Estrella and Mishkin (1997) conclude that term spread is not significant in most OECD countries, except the UK, France and Germany in recession forecasting. Schich (2000) finds in G-7 countries, US, German, UK and Canadian yield spreads are significant for output forecasting. Bonser and Morley (1997) show evidence that the UK Canada and Germany can effectively use the yield spread as a leading indicator, but weak evidence is shown in Japan and Switzerland.

The information included in the term structure successfully predicts recessions with discrete choice models, in which the recession is coded as 1 and other times coded 0 (Estrella and Mishkin, 1998, Wright, 2006). Although Dotsey (1998), and Stock and Watson (2003) report that the predictive power of the spread has decreased after 1985, Estrella and Mishkin (1998)'s work demonstrates that the spread is still better than other leading indicators in predicting recessions. In Stock and Watson (1999)'s work they include term spread as a very important element in their leading business cycle indicator index. Stock and Watson (2003), then imply a short-term interest rate (short-term commercial paper rate) to improve the predictive power of the whole model. By introducing monetary regime into the explanatory variables, Bordo and Haubrich (2004) successfully increase the predictive ability of the yield curve and show that this influence is changing over time. Ang et al. (2006) test the spreads between different long-term bonds to a 3-month bond together using a VAR approach. This approach avoids the limitation of using a 10-year and a 3-month spread and predicts that greater explanatory power should come from longer term spreads.

Term spread here is a difference between a nominal long-term bond yield (usually 10-year government bond yield) and a nominal short-term bond yield (usually 3-month government bond yield). This suggests the these analysis are conducted by real GDP and nominal yields, which implies the assumption that inflation is constant across different terms and time.

In order to add more elements into forecasting models, we apply a Nelson and Siegel (1987) exponential components framework modified by Diebold and Li (2006) and the B-spline model (de Boor, 1978) to include more information in the yield curve in the UK economy. We also assume that inflation is constant across different terms and time, in order to establish the relationship between the nominal term structure of interest rate and the real economics activities. The Diebold-Li framework and B-spline model can properly model yield from 3-month 6-month 9-month... to 15 year all in one daily yield curve. Moreover, from the Diebold-Li framework, this curve has three estimators which represent short-, medium- and long-term yield. The B-spline model is a non-parametric model which fits a curve very well. In this research, we use this whole yield curve to forecast both the real growth and the recession to identify the forecast ability of the whole yield curve.

2.3 Methodology

The econometric modelling approach adapted here consists of two stages: The first stage is to model the yield curve in each time period and the second stage is using the estimators obtained from the first stage to forecast recession and real output growth.

2.3.1 Stage 1: Model the yield curve

Several approaches have been developed for modeling the yield curve. The Bank of England for example have adapted a model developed by Mastronikola (1991) to estimate term structure in the early 1990s, later replacing it with a parametric model developed by Nelson and Siegel (1987), and then further improved by Svensson (1994). However, Fisher et al. (1995) and Waggoner (1997) construct term structure using non-parametric models based on cubic splines (B-splines) which later became part of the official model used by the Bank of England. Anderson and Sleath (2008) compared these models and conclude that the Nelson and Siegel (1987) model appears to be much more stable than the Svensson technique, while in all cases the Waggoner (1997) curve appears to perform well. Practically, the Nelson and Siegel model modified by Diebold and Li (2006) appears to be more stable than the Svensson model and more flexible than the Nelson-Siegel approach. Therefore, one parametric model which is Diebold-Li framework for explanatory purpose and one non-parametric model which uses the B-spline technique to construct yield curve for forecasting purposes have been chosen.

Diebold-Li framework

In general, when talking about the term structure of interest rates, economists usually refer to the zero-coupon yield which is because of the great data availability. There are three key theoretical constructs for yields: the discount curve, the forward curve, and the yield curve. Assume that a set of observable zero-coupon bond prices across a continuum of maturities, let $P_t(\tau)$ be the present value at time t of £1 receivable at maturity τ, let $y_t(\tau)$ be the yield to maturity of continuously compounded zero-coupon bond. From the yield to maturity, economists obtain the discount curve:

$$P_t(\tau) = e^{-\tau y_t(\tau)}. \tag{2.1}$$

The most basic type of information for estimating yield curve is the implied forward rates of interest at various horizons. Implied forward rates of interest are defined as the marginal rates of return that investors require in order to hold bonds of different maturities (Anderson and Sleath, 2001). The set of 'instantaneous' forward rates for period u, $f(u)$, are related to the price, $P(\tau)$, of a τ-maturity zero-coupon bond by:

$$P_t(\tau) = exp[\int_0^\tau f(u)\, du]. \tag{2.2}$$

Equation (2.1) shows a direct relationship between the bond prices which can be observed and zero coupon nominal yields which cannot be observed, while equation (2.2) shows a direct relationship between the bond prices and the instantaneous forward rates which cannot be observed.Hence a direct relationship between nominal yields and forward rates is obtained by both equation (2.1) and (2.2):

$$y_t(\tau) = \frac{1}{\tau}\int_0^{\tau} f(u)\,du, \tag{2.3}$$

where y_t denotes the yield at time t, and τ is the maturity of the bond. This equation implies that the zero-coupon yield is an average of all the implied one period forward rates. Since the discount curve, the forward curve and the yield curve are all algebraically related, any of these three can be computed by knowing one of the others. In practice, however none of the three can be computed by knowing one of the others. In practice, however, none of the three curves is directly observable, they must be estimated from observed coupon-bearing bond prices. The McCulloch (1975) and McCulloch and Kwon (1993), Vasicek and Fong (1982) constructing yields by estimating a smooth discount curve and then converting to yields at the relevant maturities via the above formulas. The problem of McCulloch (1975) and McCulloch and Kwon (1993) is fitted discount curve diverging from zero instead of converging to zero at long maturities. The problem of Vasicek and Fong (1982) is hard to restrict the implied forward rates to be positive. Instead of using an estimated discount curve, Fama and Bliss (1987) however, construct yields via estimated forward rates at different maturities. This method is called ‘unsmoothed Fama-Bliss yields’ is successfully fit the yield curves at longer-maturities bonds, and restricting the implied forward rates to be positive, is adapted by Diebold and Li (2006) for the United States government bond market.

The Diebold-Li method is a modified model from the Nelson-Siegel model (1987). Nelson-Siegel model is a parametric method and offers a conceptually convenient and parsimonious description of the term structure of interest rates. The yields are modeled as follows:

$$y_t(\tau) = b_{1t} + b_{2t}\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} + b_{3t}e^{-\lambda_t \tau}, \tag{2.4}$$

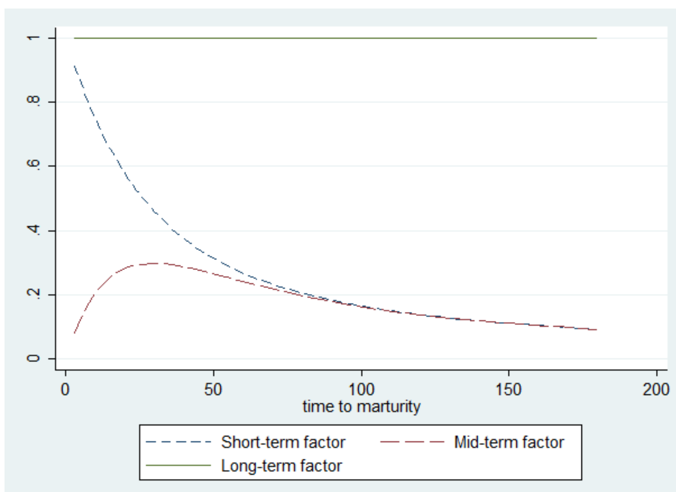
where $y_t(\tau)$ is yield at time t of a bond with time to maturity τ and the parameter λ determines the rate of exponential decay.

This model using $\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}$ and $e^{-\lambda_t \tau}$ have two problems: first, it is hard to explain the effect or significance of the two factors in the original Nelson-Siegel framework; second, it is difficult to estimate the factors precisely, because the high coherence in the factors produces multicollinearity. However, the model modified by Diebold and Li:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right), \tag{2.5}$$

is more preferable because the correlation between $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}$ and $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}$ is greatly reduced, compare to the two factors in Nelson-Siegel framework. The multicollinearity of the factors has been solved in the mean time. The four parameters β_{1t}, β_{2t}, β_{3t} and λ_t can be interpreted as long-, short-, medium-term factors and exponential decay rate respectively which represent the level, slope, curvature and proportion between slope and curvature of the curve respectively. Figure 2.1 shows how the model decomposes the yield curve into 3 factors.

Figure 2.1: Factor Loadings for Diebold and Li framework.



For simplicity, Diebold and Li (2006) fix λ_t at 0.0609. At this value the medium-term, or curvature, factor achieves its maximum at a maturity of 30 months. This is considered as standard.

B-spline model

In general a spline is a piecewise polynomial of degree k that is continuously differentiable $k - 1$ times, i.e. a curve constructed from individual polynomial segments joined at 'knot points', with coefficients chosen such that the curve and its first derivative are continuous at all points. The most commonly used polynomials are cubic functions – giving a cubic spline. The continuity constraints imply that any cubic spline can be written in the form:

$$S(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta + \sum_{i=1}^{N-1} \eta_i \left| x - k_i \right|^3 , \tag{2.6}$$

for some constants α, β, γ, δ, η_i,where k_i is the ith knot and N is the number of knots are chosen. It is the simplest expression for a cubic spline, but numerically unstable, and therefore a linear combination of cubic B-spline is preferred instead (de Boor, 1978). A B-spline is a piecewise cubic polynomial that is twice continuously differentiable as a cubic spline. The B-spline framework can be used to model a curve with following advantages. It provides a useful tool for the general construction of cubic splines and takes positive values over only four adjacent sub-intervals in the overall partition. On all other sub-intervals, cubic B-spline vanishes. Moreover, any cubic spline on $[a, b]$ can be constructed as a linear combination of this sequence of cubic B-splines. Finally, because these cubic B-splines are defined piecewise, this linear combination is easy to compute and numerically stable (Bolder and Gusba, 2002). This is a completely general transformation (any spline can be written as such a combination of B-splines of the appropriate order), which solves the numerically unstable problem. B-splines of order n are most simply represented by the following recurrence relation:

$$B_{i,n}(x) = \frac{x - k_i}{k_{i+n-1} - k_i} B_{i,n-1}(x) + \frac{k_{i+n} - x}{k_{i+n} - k_{i+1}} B_{i+1,n-1}(x), \tag{2.7}$$

with $B_{i,1}(x) = 1$ if $k_i \leq x < k_{i+1}$, and $B_{i,1}(x) = 0$ otherwise.[1]

In the present case one internal knot[2], $k_1 = 90$ is used and the yield curve can be

[1]For further details see Lancaster and Salkauskas (1986)

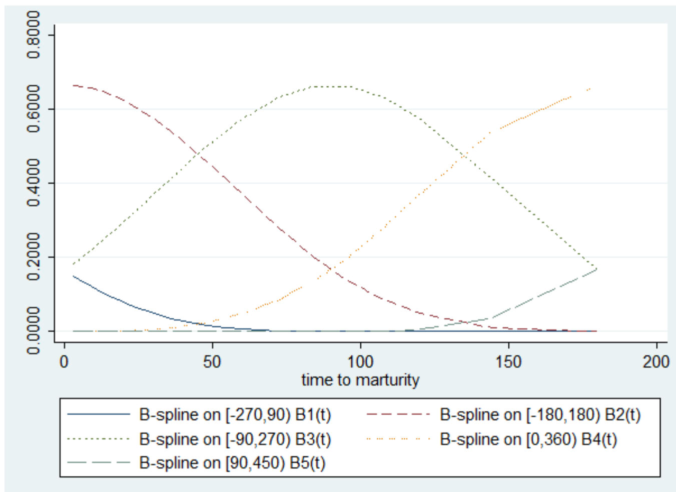
[2]One internal knot is chosen in B-spline model for the reason of balancing the accuracy of the curve modelling and the degree of freedom in the forecasting models.

written as:

$$y_t(\tau) = bs_{1t}B_1(\tau) + bs_{2t}B_2(\tau) + bs_{3t}B_3(\tau) + bs_{4t}B_4(\tau) + bs_{5t}B_5(\tau), \tag{2.8}$$

where $B_1(\tau)$, $B_2(\tau)$, $B_3(\tau)$, $B_4(\tau)$ and $B_5(\tau)$ are basis functions according to the internal knot. Figure 2.2 shows the decomposition of yield curve into basis functions.

Figure 2.2: Loading figure for B-spline framework.



For comparison reason, this paper also conduct forecast using the term spread in the same time. The term spread is calculated as follows,

$$S_t = lr_t - sr_t, \tag{2.9}$$

where S_t is the term spread, lr_t is 10-year government bond and sr_t is 3-month government bill.

2.3.2 Stage 2: Forecasting Model

By estimating β_{1t}, β_{2t} and β_{3t} from equation (2.5) and bs_{1t}, bs_{2t}, bs_{3t}, bs_{4t} and bs_{5t} from equation (2.8), information is extracted from the yield curve on the last day of the each quarter. We adapted the following forecasting models to draw a connection with the recession variable or real GDP growth and the yield curve.

Recession forecasting

As in the literature (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997, Dotsey, 1998, Wright, 2006), this research also suggests a probit model involves the prediction of whether or not the economy will be experiencing a recession k quarters ahead. This model abstracts from the actual magnitude of economic activity by focusing on the simple binary indicator variable. Although this forecast is in some sense less precise, the requirements on predictive power are in another sense less demanding and may increase the potential accuracy of the more limited forecast. Here the NBER definition of recession is used. The value of probability of recession is equal to 1 when the economy is in recession, and 0 when it is not in recession. And the models are as follows: For Diebold-Li framework:

$$P(\text{recession}_t) = \Phi(\alpha_1\beta_{1,t-h} + \alpha_2\beta_{2,t-h} + \alpha_3\beta_{3,t-h}), \tag{2.10}$$

where h is the forecasting horizon.

For B-spline model:

$$P(\text{recession}_t) = \Phi(\alpha_1 bs_{1,t-h} + \alpha_2 bs_{2,t-h} + \alpha_3 bs_{3,t-h} + \alpha_4 bs_{4,t-h} + \alpha_5 bs_{5,t-h}), \tag{2.11}$$

where h is the forecasting horizon.

Real GDP growth forecasting

Since the data is quarterly, annual GDP growth is calculated as follows:

$$\Delta y_t = \log(y_t) - \log(y_{t-4}) \tag{2.12}$$

Two different approaches to forecast real output growth are applied. They are identified as Model A and B. Model A is a model with the real GDP growth as the dependent variable and lagged yield curve representative variables being independent variables. And the equation is as follows:

$$\text{Model A:} \quad \Delta y_t = AX_{t-h} + \epsilon_t, \tag{2.13}$$

where matrix $X = (\beta_1, \beta_2, \beta_3)$ for Diebold-Li framework and matrix $X = (bs_1, bs_2, bs_3, bs_4, bs_5)$ for B-spline model.

Model B is a model modified from Model A by adding a lag term of the GDP growth, and it is set up as follows:

$$\text{Model B:} \quad \Delta y_t = \phi \Delta y_{t-h} + AX_{t-h} + \epsilon_t, \tag{2.14}$$

where matrix $X = (\beta_1, \beta_2, \beta_3)$ for Diebold-Li framework and matrix $X = (bs_1, bs_2, bs_3, bs_4, bs_5)$ for B-spline model

Models will be compared by both adjusted R^2 and Akaike information criterion (AIC). AIC is calculated as follows,

$$\text{AIC} = 2k - 2ln(L), \tag{2.15}$$

where k is the number of parameters, L is the maximized value of the likelihood function for the estimated model.

Because of the two-step setting of this analysis, the standard error of coefficients from the second step will be biased. In order to solve this, a bootstrap re-sampling technique is used. We re-sample the variables from the first step of the analysis for 1000 times, which improves the accuracy of the standard errors.

2.4 Data

In order to forecast quarterly macroeconomic activity, quarterly zero-coupon bond yield nominal spot rate in UK bond market data from fourth quarter of 1979 to the fourth quarter of 2009 is used in this research. For simplicity, maturities are fixed to 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 42, 48, 54, 60, 72, 84, 96, 108, 120, 144, 180 months. That is 21 yields in each time-period. There are 121 time-periods in total. The descriptive statistics for the yields is given in Table 2.1. The yield curves are wave like. The long rates are less volatile and more persistent than the short ones.

The UK Expenditure Approach Total GDP at Constant Prices, Seasonally Adjusted is used as the real GDP. The recession variable is using the NBER definition which is negative real GDP growth for at least two quarters and records 1 when the period is

Table 2.1: Descriptive statistics, nominal yield curves.

Maturity(Months)	Obs	Mean	Std. Dev.	Min	Max
3	121	7.936	3.723	0.435	17.029
6	121	7.777	3.521	0.410	15.129
9	121	7.719	3.414	0.496	14.795
12	121	7.710	3.353	0.665	14.627
15	121	7.718	3.303	0.878	14.563
18	121	7.732	3.263	1.077	14.530
21	121	7.749	3.231	1.221	14.513
24	121	7.768	3.206	1.368	14.605
30	121	7.805	3.166	1.644	14.804
36	121	7.8386	3.140	1.891	14.950
42	121	7.870	3.122	2.107	15.047
48	121	7.898	3.112	2.298	15.107
54	121	7.925	3.106	2.467	15.138
60	121	7.949	3.104	2.619	15.149
72	121	7.989	3.105	2.883	15.135
84	121	8.018	3.108	3.108	15.100
96	121	8.033	3.109	3.303	15.053
108	121	8.036	3.103	3.473	15.005
120	121	8.026	3.090	3.624	14.955
144	121	7.975	3.042	3.880	14.847
180	121	7.840	2.933	4.053	14.646

Note: The table summarizes the general information of the data is used, which is zero-coupon rate measured as a percentage in sample period 1979q4-2009q4.

experiencing recession, otherwise records 0. The term spread is calculated using “UK Yield 10-Year Central Government Securities" and “UK Yield Three-Month Treasury Bill". All data are collected from Thomson Reuters ECOWIN[3].

[3]Please see Appendix A for more information.

2.5 Results

2.5.1 Yield curve fitting

The first stage of the forecasting is to construct the yield curve. 121 regressions are run using both the Diebold-Li framework and the B-spline model one per time-period. Each regression has 21 observations on yields. The average adjusted R^2 results are 0.86 for Dielbold-Li framework and 0.99 for B-splines model (see Table 2.2). These results demonstrate that the modeled yield curves fit the original data well. However, the B-spline framework shows a better and more stable performance according to the smaller standard deviation (0.001) of the adjusted R^2 than the Diebold-Li frame work (0.196). From Table 2.2, the results show that the lowest adjusted R^2 for the Diebold-Li framework is 0.01. This means the Diebold-Li framework cannot model certain types of yield curve properly. For example, on 30 Sep, 1985, the performance of Deibold-Li framework fitting is not very good (see Figure 2.3a). On the other hand, B-spline is able to model the same yield curve accurately (see Figure 2.3b). On the other hand, Figure 2.3c and 2.3d show the examples on 30 June, 1997 that both models fit the yield curve accurately.

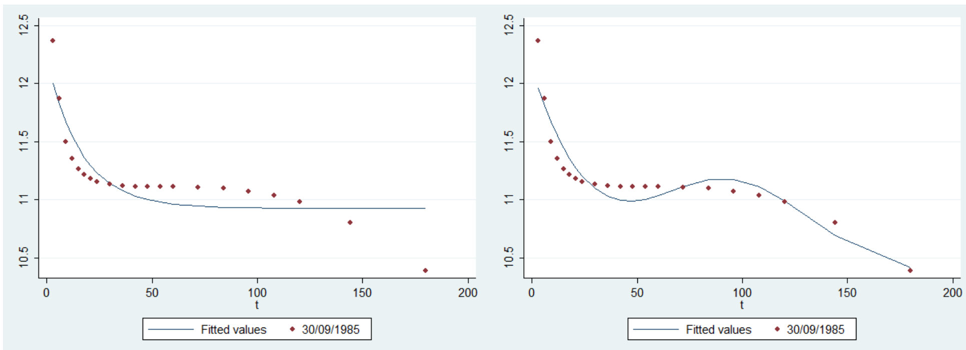
Table 2.2: Comparison of estimation of the yield curve based on Diebold-Li and B-spline frameworks.

Variable	Obs	Mean	Std. Dev.	Min	Max
Diebold-Li framework					
Adj R^2	121	0.86	0.196	0.01	0.99
B-spline framework					
Adj R^2	121	0.99	0.001	0.98	0.99

Figure 2.3: A comparison of Diebold-Li and B-spline framework fittings for selected dates.

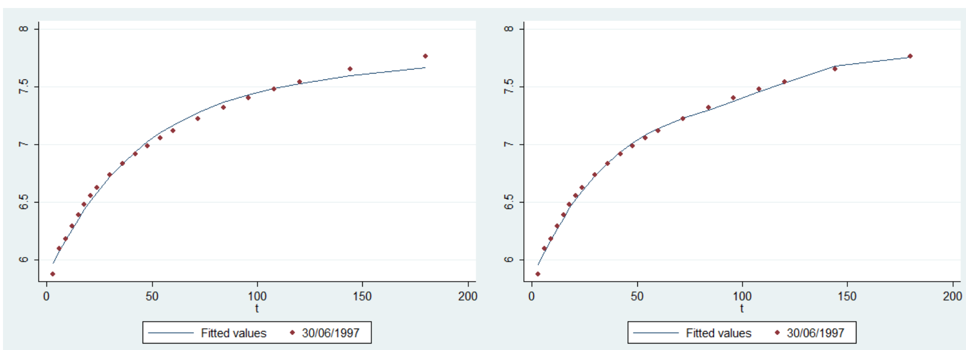
(a) The yield curve and fitting by Dieblod-Li framework on 30, Sep, 1985

(b) The yield curve and fitting by B-spline framework on 30, Sep, 1985



(c) The yield curve and fitting by Dieblod-Li framework on 30, Jun, 1997

(d) The yield curve and fitting by B-spline framework on 30, June, 1997



2.5.2 Recession forecasting

The results of Probit model in-sample tests are presented in Table 2.3 and Table 2.4. Pseudo R^2 is a value that is similar to the R^2 in OLS representing the fitting ability of the model. The results from both tables demonstrate that 6-quarters ahead forecasts have the biggest Pseudo R^2 and lowest AIC. All the coefficients of independent variables in Diebold-Li framework 6- and 7-quarters ahead are significant. Therefore, it means that the term structure explains the recession best by 6- and 7-quarters ahead forecasting, and all the independent variables can contribute to the explanation of the recessions. Long-term factor has a negative relationship with recession while short-term and mid-term factors have a positive relationship with the recession. This is consistent with the argument that when there is a recession looming people tend to sell short-term and mid-term bonds and change it into long-term bonds.

Table 2.3: Probit model forecasting recession for Diebold-Li framework.

qrt-ahead	α_1	α_2	α_3	constant	Pseudo R^2	AIC
1	0.0467	0.1286	-0.2751*	-1.9838*	0.25	75.40
2	0.0505	0.2287*	-0.2301*	-1.9499*	0.25	75.32
3	0.0487	0.3251*	-0.1774*	-1.9303*	0.28	72.51
4	0.0320	0.4704*	-0.1022*	-1.9041*	0.33	65.38
5	0.0165	0.5032*	-0.0329	-1.7868*	0.36	63.53
6	-0.1289*	0.5994*	0.0082*	-1.5781*	**0.52**	**40.47**
7	-0.1404*	0.8115*	0.0151*	-1.1326*	0.45	44.81
8	-0.2513*	0.9547*	0.1433*	-0.7472*	0.34	51.68

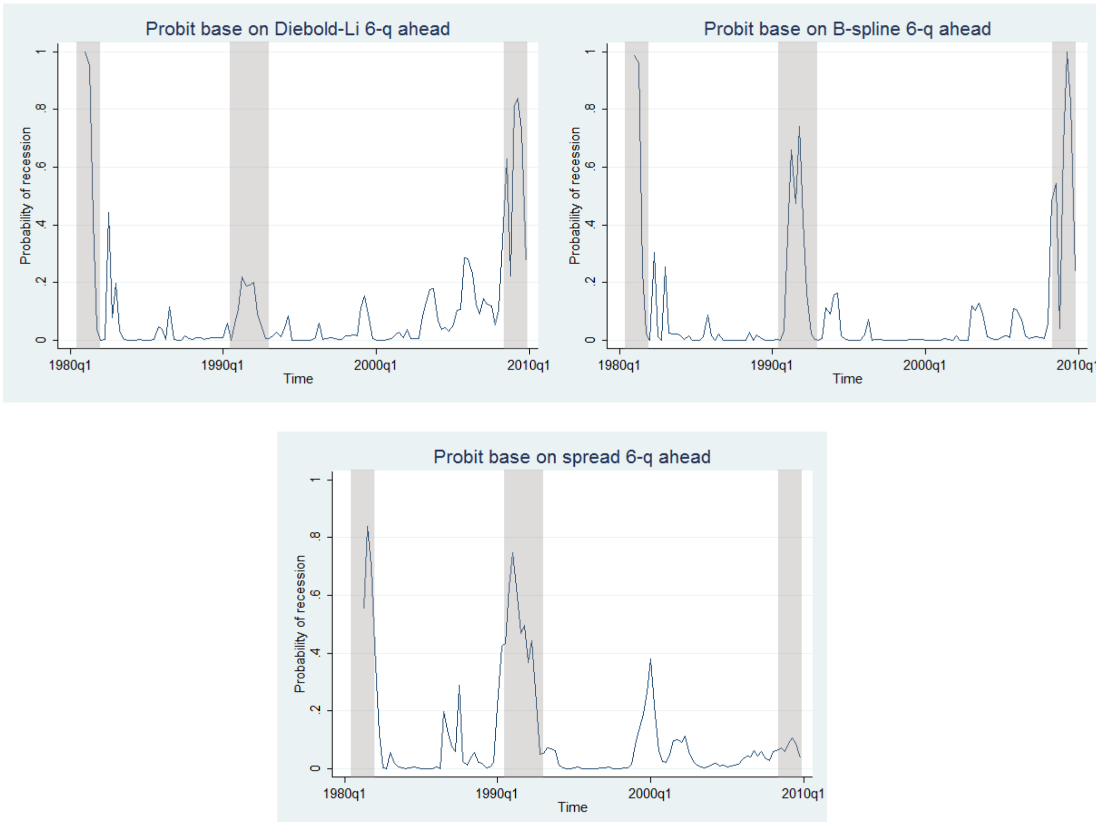
Note: * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times. Here α_1 is coefficient of b_1, α_2 is coefficient of b_2 and α_3 is coefficient of b_3.

Table 2.4: Probit model forecasting recession for B-spline.

qrt-ahead	α_{bs1}	α_{bs2}	α_{bs3}	α_{bs4}	α_{bs5}	constant	Pseudo R^2	AIC
1	0.0105	0.0110	-0.4576	0.0725	0.4584	-2.3411	0.38	67.94
2	0.0352	0.1424	-0.4295	-0.0589	0.3826	-2.0847	0.35	70.20
3	0.0699	0.1522	-0.4529	0.0748	0.2316	-2.0342	0.32	73.00
4	0.1128	0.3532	-0.7004	0.1042	0.1985	-2.0160	0.36	67.27
5	0.1124	0.2992	-0.3042	-0.0731	-0.0158	-1.7950	0.42	45.08
6	0.1408	0.3585	-0.4138	-0.0293	-0.0801	-1.5598	**0.60**	**39.71**
7	0.1793	0.3370	-0.1539	-0.2794	-0.2889	-0.7560	0.47	50.67
8	0.3190	-0.3056	2.1036	-1.5884	-1.7833	3.5564	0.34	60.14

Note: α_x represents the coefficient of variable x. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Figure 2.4: Comparison of model fitting using probit models 6-quarter ahead.

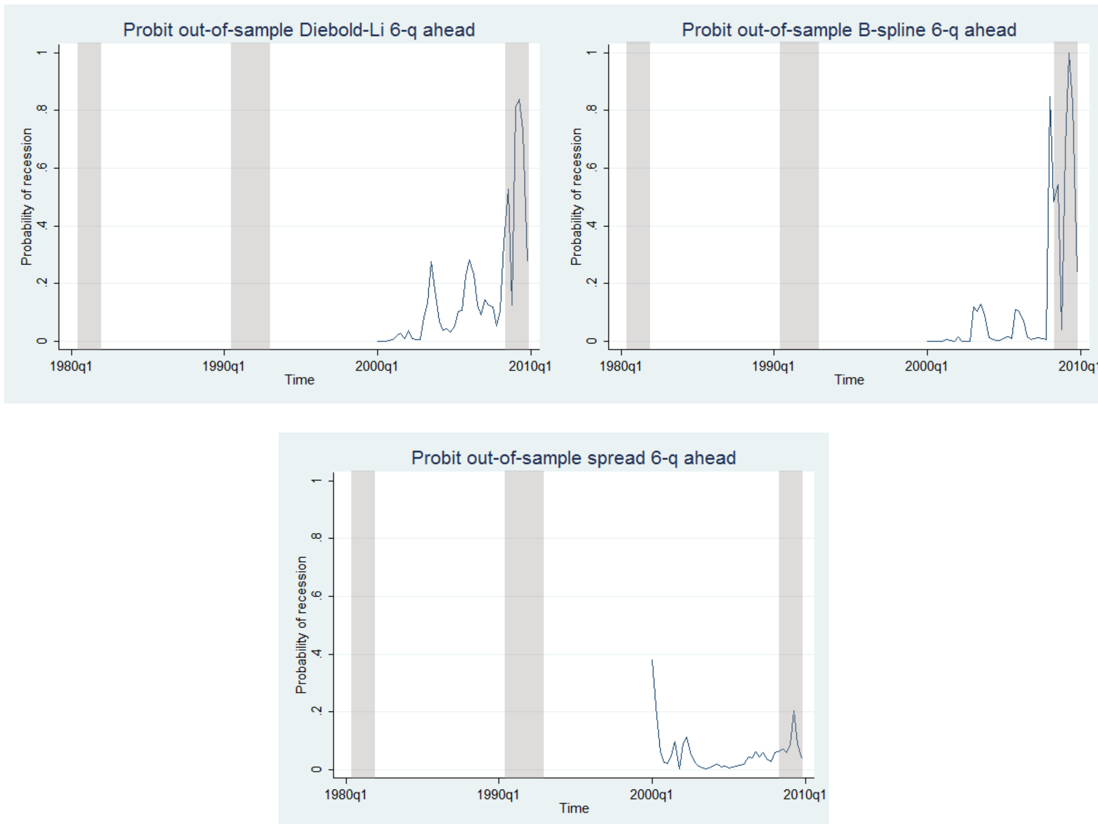


Note: Shaded areas are recessions under the NBER definition.

Figure 2.4 contains the graphs showing predicted probability among the Diebold-Li framework, B-spline model and spread model 6-quarter ahead. B-spline captures all the three recessions while the Diebold-Li framework captures the first and the recent ones. There are increasing probabilities during the second recession (the early 1990s), but they are smaller than 0.25 which is not high enough to alert the market. The spread model only captures the first two recessions and fail to predict the recent financial crisis. Moreover, the spread model also has a false alarm in the beginning of 2000.

Figure 2.5 presents the comparison of the out-of-sample forecasting results by using these models. Both Diebold-Li framework and B-spline forecast the recent financial crisis successfully 6-quarters ahead. However, the spread model fails to forecast the recent recession by giving out a very low probability (smaller than 0.2) of recession when we are experiencing the financial crisis. The spread model also show a fairly high probability of recession in the beginning of early 2000s. It is worth mentioning that both the Diebold-Li framework and B-spline model show a reducing probability of recession in 2009q3 while in reality the UK was still experiencing the recession.

Figure 2.5: Comparison of model out-of-sample using probit models 6-quarter ahead.



Note: Shaded areas are recessions under the NBER definition. In-sample period:1979q4-1999q4, out-of-sample period 2000q1-2009q4

2.5.3 GDP growth forecasting

In-sample test

By using real GDP growth as a dependent variable, firstly Model A with both the Diebold-Li method and B-spline method are constructed. The in-sample test results are given in Table 2.5 and Table 2.6. The model with the highest adjusted R^2 or lowest AIC is selected as best fit of the model. Therefore, Model A based on the Diebold-Li framework shows its best performance in 5-quarters ahead forecasting. Model A based on the B-spline framework performs the best in 4-quarters ahead. Actually from 1-quarter ahead to 4-quarters ahead models based on the B-spline framework show great forecasting ability. In the Diebold-Li framework coefficients of long-term and short-term factors are significant in the 3- to 5-quarters and 7- to 8 quarters ahead models. This shows that only short-term and long-term bonds of the yield curve contribute to the explanation of the real GDP growth in these forecasting horizons. The adjusted R^2 here can be interpreted as the percentage with which we can explain the GDP growth by using the model. Thus 26% of real GDP growth can be explained by the yield curve based on the Diebold-Li framework 5-quarters ahead. While in the B-spline model 4-quarters ahead explains 28% of real GDP growth, which is a better result compared to the yield spread, which can only explain a 22% real GDP growth base on the result of 5-quarter ahead modelling (see Table 2.7). This confirms that including more useful information into the model by using the whole yield curve improves the forecasting ability. Coefficients of the short-term factor (α_2) in 3- to 8-quarter ahead Diebold-Li framework based Model A are significant (see Table 2.5), which suggests that monetary policy is an important factor in explaining real growth. In the Diebold-Li framework based on Model B, coefficients of short-term factor in 3- to 8- ahead forecast are also significant, suggesting the same (see Table 2.8). The results from Model B (see Table 2.8 and Table 2.9) are better than spread model based on both their R^2 and AIC. One quarter ahead forecasting based on both the Diebold-Li framework (R^2 is 0.82 and AIC is -766.35) and B-spline model (R^2 is 0.82 and AIC is -762.85) are the best model compare to the other forecasting horizons. Table 2.10 examine the models performance based on AIC. According to Table 2.10, Model B based on Diebold-Li framework 1-quarter ahead forecast, is the best fitting model of all. For Diebold-Li framework, from 1- to 4- quarter ahead forecasts Model B are better than Model A, while from 5- to 8-quarters ahead forecasts Model A are better. This suggests including the lag term of the real growth is not always improves the model when it is based on the Diebold-Li framework. In most cases, models based on the B-spline framework achieve better performance than the ones based on the Diebold-Li framework. According to Table 2.8, the coefficients of lagged real GDP growth in 1-quarter to 6-quarters ahead forecast of Model B based on the Diebold-Li framework are significant. For Model B based on the B-spline framework, coefficients of lagged GDP of all the forecast horizons except 8-quarters ahead are significant. This means the lagged real GDP growth contribute to the explanation of GDP growth in short-term forecasting. This is also consistent with the increasing adjusted R^2 after applying the lagged real GDP.

Table 2.5: Model A based on Diebold-Li framework.

qrt-ahead	α_1	α_2	α_3	constant	R^2	AIC
1	-0.0006*	-0.0010	0.0031*	0.0278*	0.17	-584.58
2	-0.0007*	-0.0024	0.0022	0.0278*	0.14	-578.35
3	-0.0007*	-0.0038*	0.0015	0.0275*	0.17	-576.70
4	-0.0006*	-0.0047*	0.0004	0.0265*	0.20	-581.65
5	-0.0003*	-0.0047*	-0.0001	0.0242*	**0.26**	**-588.95**
6	0.0002*	-0.0046*	-0.0001	0.0208*	0.25	-586.24
7	0.0006*	-0.0043*	-0.0003	0.0178*	0.20	-585.08
8	0.0009*	-0.0039*	-0.0003	0.0153*	0.19	-583.39

Note: * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times. Here α_1 is coefficient of b_1, α_2 is coefficient of b_2 and α_3 is coefficient of b_3.

Table 2.6: Model A based on B-spline framework.

qrt-ahead	α_{bs1}	α_{bs2}	α_{bs3}	α_{bs4}	α_{bs5}	constant	R^2	AIC
1	-0.0002	-0.0003	0.0086*	-0.0044*	-0.0058*	0.0351*	0.27	-594.89
2	-0.0003	-0.0027*	0.0078*	-0.0015	-0.0054*	0.0347*	0.25	-590.92
3	-0.0005*	-0.0042*	0.0063*	0.0010	-0.0045	0.0333*	0.27	-596.90
4	-0.0007*	-0.0053*	0.0045*	0.0029	-0.0028	0.0302*	**0.28**	**-597.24**
5	-0.0007*	-0.0049*	0.0015	0.0047*	-0.0011	0.0257	0.21	-578.43
6	-0.0007*	-0.0036*	-0.0016	0.0058*	0.0006	0.0199	0.20	-580.06
7	-0.0008*	-0.0026*	-0.0027	0.0054*	0.0020	0.0151	0.21	-584.05
8	-0.0009*	-0.0017	-0.0022	0.0037	0.0027	0.0116	0.22	-583.48

Note: α_x represents the coefficient of variable x. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Table 2.7: Results based on spread model

qrt-ahead	α_1	constant	R^2
1	-0.002	0.0201*	0.01
2	-0.004*	0.021*	0.07
3	-0.005*	0.021*	0.12
4	-0.006*	0.021*	0.20
5	-0.007*	0.022*	**0.22**
6	-0.006*	0.023*	0.19
7	-0.006*	0.023*	0.18
8	-0.005*	0.023*	0.14

Note: α_x represents the coefficient of variable x. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Table 2.8: Model B based on Diebold-Li framework.

qrt-ahead	ϕ	α_1	α_2	α_3	constant	R^2	AIC
1	0.9486*	0.0002	-0.0008	0.0000	-0.0013	**0.82**	**-766.35**
2	0.8870*	0.0003*	-0.0017	-0.0003	-0.0019	0.64	-678.53
3	0.7070*	0.0003*	-0.0028*	-0.0001	0.0018	0.44	-621.40
4	0.4790*	0.0002*	-0.0038*	-0.0005*	0.0082	0.32	-599.22
5	0.3448*	0.0003*	-0.0041*	-0.0008*	0.0107	0.25	-586.70
6	0.2421*	0.0006*	-0.0041*	-0.0006*	0.0112	0.22	-585.28
7	0.1480	0.0008*	-0.0040*	-0.0006*	0.0120	0.21	-586.09
8	0.0383	0.0010*	-0.0039*	-0.0004	0.0138	0.19	-581.53

Note: * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times. Here ϕ is coefficient of lag of GDP growth, α_1 is coefficient of b_1, α_2 is coefficient of b_2 and α_3 is coefficient of b_3.

Table 2.9: Model B based on B-spline framework.

qrt-ahead	ϕ	α_{bs1}	α_{bs2}	α_{bs3}	α_{bs4}	α_{bs5}	constant	R^2	AIC
1	0.9518*	-0.0001	-0.0006	-0.0008*	0.0015*	0.0003	-0.0019*	**0.82**	**-762.85**
2	0.8594*	-0.0002	-0.0022	0.0000	0.0029	-0.0003	-0.0004	0.68	-695.88
3	0.6558*	-0.0003*	-0.0031*	0.0002	0.0041*	-0.0006	0.0045*	0.48	-629.88
4	0.4359*	-0.0005*	-0.0043*	0.0002	0.0050*	-0.0002	0.0102*	0.33	-606.71
5	0.3332*	-0.0006*	-0.0041*	-0.0017*	0.0062*	0.0009	0.0101*	0.26	-593.81
6	0.2841*	-0.0006*	-0.0029*	-0.0043*	0.0070*	0.0023	0.0065*	0.24	-587.67
7	0.2439*	-0.0007*	-0.0020	-0.0050*	0.0065*	0.003*4	0.0035*	0.24	-586.50
8	0.1634	-0.0008 *	-0.0013	-0.0037*	0.0045*	0.0037	0.0038*	0.23	-583.54

Note: α_x represents the coefficient of variable x. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Table 2.10: Model comparison based on AIC

	Model A		Model B	
qrt-ahead	DL	BS	DL	BS
1	-584.58	-594.89	**-766.35**	-762.85
2	-578.35	-590.92	-678.53	-695.88
3	-576.70	-596.90	-621.40	-629.88
4	-581.65	-597.24	-599.22	-606.71
5	-588.95	-578.43	-586.70	-593.81
6	-586.24	-580.06	-585.28	-587.67
7	-585.08	-584.05	-586.09	-586.50
8	-583.39	-583.48	-581.53	-583.54

Note: Selection rules: The lower AIC the model has, the better it fits the data. DL = Deibold-Li framework, BS = B-spline framework.

Out-of-sample test

A good forecasting model should not only fit well in-sample but also predict well out-of-sample. Out-of-sample tests are conducted using 1979q4 to 1999q4 as in-sample period. And in the mean time, the result is compared with the result from classic yield spread forecasting model. The forecast equation is:

$$\Delta y_t = \alpha + \beta S_{t-h} + \epsilon_t, \tag{2.16}$$

where S is the term spread and h is the forecasting horizon.

The results are shown in Table 2.11. RMSE (Root Mean Square Error) is a conventional tool to measure the efficiency of a forecast model in an out-of-sample test. It is calculated as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{T}(A_i - F_i)^2}{T}}, \tag{2.17}$$

where A is the actual value, and F is the forecast value.

According to the results, it is important to note that Model B 1-quarter ahead based on both the Diebold-Li framework and B-splines get the smallest RMSE which makes it the best forecast model of all. Generally speaking, all the models show an impressive out-of-sample forecasting ability across all of the forecasting horizons. In reference to both forecast ability and fitting ability, models based on the whole curve beat those based on the yield spread. Model B shows better fitting ability and forecasting performance as well as average stability in all the horizon based on $\bar{R}^2$ and RMSE, especially for 1- and 2-quarter ahead forecasting. This suggests that adding the term Δy_{t-h} to the model improves the model's forecasting performance. Models based on the B-spline framework get slightly better RMSE and better fitting ability.

Table 2.11: Out-of-sample test results.

qrt-ahead			Model A				Model B			
	Spread		D-L		B-spline		D-L		B-spline	
	RMSE	$\bar{R}^2$	RMSE	$\bar{R}^2$	RMSE	$\bar{R}^2$	RMSE	$\bar{R}^2$	RMSE	$\bar{R}^2$
1	0.029	0.13	**0.022**	0.45	0.022	0.50	**0.011**	**0.75**	**0.011**	**0.75**
2	0.028	0.21	**0.022**	**0.49**	0.023	**0.57**	0.015	0.70	0.018	0.70
3	0.027	0.27	0.023	0.48	**0.021**	0.56	0.020	0.60	0.020	0.63
4	0.026	0.32	0.024	0.43	0.024	0.53	0.023	0.47	0.023	0.56
5	**0.026**	**0.33**	0.025	0.36	0.024	0.43	0.024	0.42	0.024	0.49
6	0.026	0.29	0.026	0.32	0.027	0.43	0.026	0.39	0.025	0.44
7	0.027	0.27	0.027	0.32	0.028	0.38	0.027	0.31	0.026	0.39
8	0.028	0.22	0.027	0.30	0.028	0.32	0.026	0.25	0.027	0.30

Note: D-L represents Diebold-Li framework. Selection rules: Choose the model with smallest RMSE and biggest $\bar{R}^2$. In-sample period:1979q4-1999q4, out-of-sample period 2000q1-2009q4; RMSE: Root Mean Square Error is computed as in equation (2.17).

A comparison of out-of-sample results in this research and HM Treasury forecasts has been conducted. In the UK, the official forecast is the one published by HM Treasury, and HM Treasury does 1, 11 and 12 month-ahead forecasts annually. Because the presented forecasts are quarterly ahead, the only comparison can be conducted is 12-month ahead from the HM Treasury and 4-quarter ahead in the presented paper. The results are reviewed in Table 2.12. This League Table shows that all the models based on the whole yield curve come out with similar results with HM Treasury, and these models outperform the spread model.

Table 2.12: League Table of models out-of-sample 4-quarter ahead forecasting.

Forecastor	RMSE	Performance
HM Treasury forecast	0.0222	Best
Model B B-spline	0.0231	↓
Model B Diebold-Li	0.0233	↓
Model A B-spline	0.0240	↓
Model A Diebold-Li	0.0242	↓
Spread model	0.0262	Worst

Note: Selection rule: choose the model with the smallest RMSE. In-sample period: 1979q4-1999q4, out-of-sample period: 2000q1-2009q4.

The comparison shows that the presented models are able to get comparable results with HM Treasury forecasts, however, they are much more parsimonious.

2.6 Conclusion

This chapter proposes a model of the term structure forecasting output growth and recessions in the UK by using the whole yield curve rather than just the yield spread. By using the Diebold-Li framework and B-spline technique to fit yield curves other than traditional yield spread, this paper presents new models to forecast recession and economic growth. The results are satisfactory. According to the research there is strong evidence that in terms of recession forecasting the Diebold-Li framework does better than the yield spread and B-spline model does even better (see Fig 2.4). Secondly, from in-sample test results, short-term yields which represent monetary policy are playing a very important role in real GDP growth forecasting in Models A and B. Another finding of this chapter is that models based on the Diebold-Li and B-spline framework fit the yield curve better than the yield spread model and show a good forecasting ability in 5-quarters ahead for Model A based on the Diebold-Li framework, 4-quarters ahead for Model A based on the B-spline framework, and 1-quarter ahead for Model B based on both framworks. From out-of-sample tests Model B based on both the Diebold-Li framework and the B-spline framework achieve very satisfactory results and show a stable forecasting ability for all forecasting horizons. All models based on the whole yield curve beat the results from models based on yield spread. From the comparison, it is important to note that Model B based on B-spline generates closest forecast results to the one from HM Treasury. This means this chapter offers more parsimonious models which results are comparable to the HM Treasury forecasts.

Evidence found in this chapter can be used by the government, practitioners and investors to improve both their recession forecasting and output growth forecasting model, and adjust their policies efficiently and in a timely manner.

Chapter 3

Time series non-linearity in the real growth / recession-term spread relationship, some evidence from the UK

3.1 Introduction

The ability of the yield curve to predict future real activities has been an interest to academics since early 1990s. The intuition of these studies are, agents in the market will invest on assets based on the expectation of the economy, so that the price changing contains useful information about future economic growth. In the bond market, this behaviour will lead to a shape changing of the term structure of interest rates. Therefore, the simplest form of term structure of interest rates, term spread becomes a valid agent to investigate the theory.

Numerous studies try to understand how well the term spread explains and forecasts output growth and recessions. Evidence shows that term spread is a reliable predictor under linear analysis (Dotsey, 1998, Galbraith and Tkacz, 2000, Ang et al., 2006, Bordo and Haubrich, 2008). Nonetheless, the predictive power varies in different economies. It is a valid predictor in the UK and Germany as well as the US from most of the literature and it is not quite reliable in France, Canada and Japan. (Harvey, 1991, Estrella and

Mishkin, 1997, Plosser and Rouwenhorst, 1994, Bernard and Gerlach, 1998). From 2000, researchers started to look at this relationship from a non-linear perspective in order to enhance the explanatory power of yield spread. Galbraith and Tkacz (2000) find evidence of non-linearity in this relationship in the US and Canada. Venetis et al. (2003) discover threshold effects in the US, UK and Canada in the output growth-yield spread relationship. Duarte et al. (2005) find significant nonlinearity exists in the output growth-yield spread relation in the euro area as well as the US. Galvão (2006) uses structural break threshold VAR showing models with structural break and thresholds outperform linear models in recession forecasting in the US sample.

In the existing literature, the UK has not been tested comprehensively. This allows me to make several contributions to the literature on the output growth / recession-term spread relationship. Our contribution is fivefold. Firstly, we investigate the relationship between yield spread and real economic activities involving the time series non-linearity in the UK. Secondly, the presented research is conducted with data containing the recent financial crisis allowing tests of the influence of this big recession to the relationship. Thirdly, we apply VARs with non-linearity to forecast future real GDP growth as well as recessions. Fourthly, the chapter applies two more non-linear model and various autoregressive orders in the model search, which is more comprehensive compared to Galvão (2006)'s research. Last but not least, this study successfully identifies the non-linearity of the real growth-term spread relationship in the UK.

UK GDP at constant prices, 3-month UK government bond and 10-year UK government benchmark are chosen quarterly as data from the period between the first quarter of 1979 and the first quarter of 2013.

VARs, threshold VARs with one (TVAR) or two thresholds (2TVAR), structural break VARs with one (SBVAR) or two breaks (2SBVAR), structural break threshold VARs with one break and one threshold (SBTVARc) as well as structural break threshold VARs with one break and one threshold in each broken regime (SBTVAR) are applied to the real growth-term spread equation. The in-sample results confirm the existence of the nonlinearity and it tends to be structural break. The out-of-sample results show the robustness of the structural break model on arrival of new information. And they provide superior performance against the linear models in 1-quarter, 3-quarter and 4-quarter ahead forecasting.

The rest of this chapter is structured as follows: Section 3.2 is a brief literature review of yield curve forecasting. In Section 3.3, the structural break threshold VARs and introduction of methods applied for recession forecasting are presented. Section 3.4 gives the data description. Section 3.5 discusses the results and Section 3.6 concludes.

3.2 Literature Review

The yield spread containing information on future output growth and likelihood of future recessions has been studied widely in the literature. Wheelock and Wohar (2009) undertook a comprehensive survey regarding the ability of the term spread to forecast output growth and recessions. The survey covers 18 papers focusing on growth forecasting and 13 focusing on recession forecasting. All of the papers confirm the forecast ability of the term spreads. In terms of output growth forecasting, Harvey (1988, 1989, 1991) introduced this idea and examined the G-7 countries, and confirmed the forecast ability of term spread as a leading indicator. Later, Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Estrella and Mishkin (1997) and Estrella et al. (2003) start to use uni-variate or multi-variate linear models to examine the real growth-term spread relationship.

Galbraith and Tkacz (2000) use nonlinear models and the results show the significance of forecast ability of the yield spread as well as the non-linear behaviour in the relationship. This brings the research into a new chapter. Tkacz (2001) applied neural networks model on Canadian data and shows a greater forecast ability in 4-quarter ahead forecasting than in 1-quarter ahead forecasting. Venetis et al. (2003) conduct nonlinear procedures including smooth nonlinear transition models, regime-switching models and time-varying models using US, UK and Canadian data and show that threshold effects exist in the yield spread-output growth relationship. Duarte et al. (2005) use a change point model and a nonlinear threshold model and find that the nonlinear model outperforms the linear model and spreads achieve a better performance to predict output growth when output growth has slowed. Giacomini and Rossi (2006) present evidence of structural breaks in the US yield spread-output growth relationship. In Benati and Goodhart (2008)'s research, they find the forecasting ability of the yield spread varies in different periods of time by conducting time-varying parameters VARs. Regarding the recession forecasting, Probit models are widely used. Estrella and Hardouvelis (1991), Dotsey (1998), and Estrella and Mishkin (1998) prove the usefulness of the yield spread in recession forecasting using similar US data set. Bernard and Gerlach (1998) and Ahrens (2002) test the significance of yield spread as a leading indicator in eight industrialized countries. And structural break threshold VARs are delivered to predict recession by Galvão (2006) and it suggests 2-quarter ahead forecasts has the best performance in the US. According to the literature above, it is fair to say there are non-linearities in the yield spread-output growth and yield spread-recession relationships in the US. However, the literature that examines the UK is very limited, and the research have been done mostly using data from before 2007. It is important to know whether the most recent 2008 to 2010 recession has altered these relationships.

In this chapter, more comprehensive structural break threshold VARs are applied to examine the existence and influence of non-linearities in the real growth / recession relationship.

3.3 Methodology

3.3.1 Structural Break Threshold VARs

Structural break threshold VARs are combinations of Threshold VARs and Structural break VARs. Threshold VARs are piecewise linear models with different autoregressive matrices in each regime, determined by a transition variable (one of the endogenous variables), a delay and a threshold (Tsay, 1998). Structural break models also divide the sample into two or more regimes, but they are determined by one or more break-points and are not recurrent, allowing different dynamics before and after the break. Although non-linear models can capture some characteristics of structural break models ((Koop and Potter, 2000, 2001, Carrasco, 2002), it may be the case that the break also implies changes in the parameters that determine the non-linearity. Univariate time-varying smooth transition models have been proposed by Lundbergh et al. (2003) and have been applied to capture changes in seasonality of industrial production by van Dijk et al. (2003). Unlike time-varying parameters models, structural break threshold VARs are able to identify the break point from one regime to another so that one can analyse the cause of the changes.

Define x_t as a $m \times 1$ vector of m endogenous variables $X_t = (x_{1t}, x_{2t}, \ldots, x_{mt})'$ and define the $m \times (mp+1)$ matrix, $\boldsymbol{X}_{t-1} = (1, X_{t-1}, \ldots, X_{t-p}$ where p is the autoregressive order. A threshold VAR with one threshold (r) with a delay (d) can be written as:

$$X_t = (\boldsymbol{X}_{t-1}\beta_1)I_{z,t-d}(r) + (\boldsymbol{X}_{t-1}\beta_2)(1 - I_{z,t-d}(r)) + u_t, \tag{3.1}$$

where r should be allocated in one one m variables before the estimation. And $I_{z,t-d_i}(r_i)$ is an indicator function which depends on a transition variable z. For a threshold r_i and a delay d_i,

$$I_{z,t-d_i}(r_i) = \begin{cases} 1 & \text{if } (z_{t-d_i} \leq r_i) \\ 0 & \text{if } (z_{t-d_i} > r_i) \end{cases}$$

In the same manner, a structural break VAR with one break point (τ) can be written as:

$$X_t = (\boldsymbol{X}_{t-1}\beta_1)J_t(\tau) + (\boldsymbol{X}_{t-1}\beta_2)(1 - J_t(\tau)) + u_t, \tag{3.2}$$

where $J_t(\tau)$ is an indicator function which depends on a break-point τ,

$$J_t(\tau) = \begin{cases} 1 & \text{if } (t \leq \tau) \\ 0 & \text{if } (t > \tau) \end{cases}$$

A structural break threshold VAR with one break point and one threshold in each structural break regime can be written as:

$$\begin{aligned} X_t = [(\boldsymbol{X}_{t-1}\beta_1)I_{z,t-d_1}(r_1) + (\boldsymbol{X}_{t-1}\beta_2)(1 - I_{z,t-d_1}(r_1))]J_t(\tau) + \\ [(\boldsymbol{X}_{t-1}\beta_3)I_{z,t-d_2}(r_2) + (\boldsymbol{X}_{t-1}\beta_4)(1 - I_{z,t-d_2}(r_2))](1 - J_t(\tau)) + u_t, \end{aligned} \tag{3.3}$$

where β_i is a $(mp+1) \times m$ matrix of parameters. u_t is a $m \times 1$ vector of error term.

To estimate the threshold VAR models, structural break VAR models and structural break threshold VAR models, there are three methods. They are conditional least squares, which is suggested by Tsay (1998), maximum likelihood which is proposed by Hansen and Seo (2002), and maximum likelihood estimator that allows difference variance in each regime, introduced by Galvão (2006).

The conditional least squares estimation applies a grid search in part of the sample of threshold and delay and the estimator should be the one that minimizes the sum of squared residuals (Tsay, 1998). The sum of squared residuals can be calculated by the number of observations times the estimated covariance matrix of residuals for any given threshold. There is a limit on the sample in each regime for searching, and a proportion of π at either end of the data is excluded. And $0 < \pi < 1$. From the literature, 0.10 (Clements and Galvão, 2004) and 0.15 (Andrews, 1993) are usually chosen. Therefore, the conditional least square estimators$(\hat{r_1}, \hat{r_2}, \hat{\tau})$ can be obtained by:

$$\min(T * \text{trace}(\hat{\Sigma}(r_1, r_2, \tau))) \quad \forall \quad r_l \leq r_1 \leq r_u, r_l \leq r_2 \leq r_u, \tau_l \leq \tau \leq \tau_u.$$

The maximum likelihood estimator is calculated assuming the error term is normal distributed. Similar to the approach of conditional least squares, the estimator is obtained by a grid search in part of the sample in order to minimize $\log(\det(\Sigma(\hat{r})))$ (Hansen and Seo, 2002). Therefore, the maximum likelihood estimators $(\hat{r_1}, \hat{r_2}, \hat{\tau})$ can be obtained by:

$$\min(\log(\det(\hat{\Sigma}(r_1, r_2, \tau)))) \quad \forall \quad r_l \leq r_1 \leq r_u, r_l \leq r_2 \leq r_u, \tau_l \leq \tau \leq \tau_u.$$

Both of the estimators are based on the assumption that the covariance matrices are the same for each regime. However, in practice, especially when applying macroeconomic data, the variances are different for each regimes. Galvão (2006) suggests a maximum likelihood estimator with regime-switching variances. For instance, in a typical SBT-VAR (contains one break-point and one threshold in each break period) which has four separated regimes, the maximum likelihood estimator with regime-switching variances $(\hat{r_1}, \hat{r_2}, \hat{\tau})$ can be obtained by:

$$\min \begin{pmatrix} \frac{T_1}{2}\log(\det(\hat{\Sigma_1}(r_1, r_2, \tau))) + \frac{T_2}{2}\log(\det(\hat{\Sigma_2}(r_1, r_2, \tau))) \\ \frac{T_3}{2}\log(\det(\hat{\Sigma_3}(r_1, r_2, \tau))) + \frac{T_4}{2}\log(\det(\hat{\Sigma_4}(r_1, r_2, \tau))) \end{pmatrix}$$

$$\forall \quad r_l \leq r_1 \leq r_u, r_l \leq r_2 \leq r_u, \tau_l \leq \tau \leq \tau_u.$$

In this chapter the maximum likelihood estimator with regime-switching variances is applied in estimating the sample.

3.3.2 Forecasting Recessions

The definition of recession in this chapter is the NBER definition as defined in Section 2.1. It is a period in which GDP falls (negative real economic growth) for at least two consecutive quarters.

The probability of predicting recession is calculated by using estimated VARs to simulate future growth. And it is the proportion of number of events which have two consecutive periods of negative real growth over total simulated events. This procedure is first suggested by Anderson and Vahid (2001).

Define $X^{t-1} = \{X_{t-1}, X_{t-2}, \ldots, X_1\}$ as the history of X_t and $X_t = f(X^{t-1}; \Gamma, \beta) + u_t$ as the forecasting model where Γ is the matrix of parameters. In this chapter, they are thresholds and breaks. β is a k-vector of parameters. u_t is i.i.d. with $E(u_t) = 0$, $\mathrm{Var}(u_t) = \Sigma$. For the given value of $\hat{\beta}$ and $\hat{\Sigma}$, a forecast of pseudo sequence values for $\{x_t, x_{t+1}, x_{t+2}, x_{t+3}, x_{t+4}\}$ is conducted. The bootstrap re-sampling technique is used. We re-sample X^{t-1} for 2000 times using parametric bootstrap method. Given $\hat{\beta}$ we are able to generate 2000 $\hat{u}_t^*$. By applying these $\hat{u}_t^*$ back into $X_t = f(X^{t-1}; \Gamma, \beta) + u_t$, a sequence of $\hat{X}^t$ can be formed. Followed by a new draw of u_{t+1} using the same method, from the residuals and employed to calculated $\hat{X}_{t+1}^*$, given $\hat{X}^t$ and $\hat{\beta}$, so that $\hat{X}^{t+1}$ is formed. The procedure continues until the sequence $S_1 = \{\hat{X}_t^*, \hat{X}_{t+1}^*, \hat{X}_{t+2}^*, \hat{X}_{t+3}^*, \hat{X}_{t+4}^*\}$ has been generated. 2000 sequences of S_1 will be generated after the procedure. The probability of a recession h-quarter ahead is the proportion of these 2000 sequences, in which the $\hat{X}_{t+h-1}, \hat{X}_{t+h}$ are both negative.

In terms of threshold VARs and structural break VARs, the model can be transformed as $x_t^j = f^j(X^{t-1}; \Gamma^j, \beta^j) + u_t^j$, where $j = 1, 2$ for the two regimes. While in the case of structural break threshold VARs, $j = 1, 2, 3, 4$ for the four regimes. It is important to note that if there are thresholds in the model, all the regimes related need to be bootraped. However, the bootstrap procedure will be only applied to the latter regime related to the structural break if there are structural breaks in the model.

3.4 Data

The real GDP data used in this chapter is "gross domestic product (GDP) expenditure approach, at constant price, seasonal adjusted" (y_t). The GDP growth is calculated as

Equation 2.12 in Section 2.3.2.

Term spread is calculated by Equation 2.9 in Section 2.3 using “UK Yield 10-Year Central Government Securities" and “UK Yield Three-Month Treasury Bill".

All the data are quarterly data from 1979q1 to 2013q1 and they are collected from Thomson Reuters ECOWIN[1]. The software used in this chapter are GUASS and MATLAB.

3.5 Results

3.5.1 In-sample Estimation

In order to find the best estimation, VAR(1), VAR(2) and VAR(3) process with time series non-linearity is conducted. 7 models including VARs, Threshold VARs with one (TVAR) or two thresholds (2TVAR), Structural break VARs with one (SBVAR) or two breaks (2SBVAR), Structural Break threshold VARs with one break and one threshold (SBTVARc) and threshold VARs with one break and one threshold in each broken regime (SBTVAR) are estimated. For those threshold models the thresholds and delays will be chosen from the yield spreads.

In-sample Real Growth Estimation

The estimated parameters are shown in Table 3.1 to Table 3.4. In which, Table 3.1 to Table 3.3 show the delays, thresholds and break parameters as well as the information criteria. Models will be selected by comparing the AIC which is given in Equation 2.15.

According to the VAR(1) results (Table 3.1), it shows a increasing goodness-of-fit by introducing more regimes. Generally speaking models with only structural break(s)(AIC of SBVAR is -46.25 and AIC of 2SBVAR is -52.38) are better than the ones with threshold(s) (AIC of TVAR is -34.05 and AIC of 2TVAR is -39.89). Cross comparing all the AIC results from Table 3.1 to Table 3.3, 2SBVAR(2) gets the best result with a score of -122.16. This means there are 2 structural breaks in the real growth-term spread relationship. They are the first quarter of 1986 and the third quarter of 1991. In association with the results in Table 3.4 column 11, the dependency of real growth on its first lag

[1]Please see Appendix B for more information.

Table 3.1: VAR(1) Estimated Parameters

	VAR	TVAR	2TVAR	SBVAR	2SBVAR	SBTVARc	SBTVAR
d		4	4			4	4 4
$\hat{r}$		2.13	0.88 2.13			2.14	2.20 1.70
$\hat{\tau}$				1988q4	1986q1 1991q3	1991q1	1991q1
						3.10	3.10
${\sigma_Y}^2$	1.12	1.68	0.69	2.27	2.86	1.23	1.23
		1.22	0.91	1.53	1.84	0.55	0.55
			2.20		0.68	0.49	0.49
						0.92	0.92
${\sigma_S}^2$	0.64	0.64	1.58	0.76	0.34	0.24	0.24
		0.24	0.64	0.22	0.78	0.58	0.58
			0.24		0.20	0.14	0.14
T	133	50 83	27 22 84	31 102	20 22 91	17 23 33 60	18 22 28 65
AIC	-32.79	-34.05	-39.89	-46.25	-52.38	-80.39	**-81.89**

Note: Sample period is from 1980Q1 to 2012Q1. ${\sigma_Y}^2$ and ${\sigma_S}^2$ are the estimated variance of output-spread equations for each regime with T observations. Model with the lowest AIC (in bold format) is preferred.

Table 3.2: VAR(2) Estimated Parameters

	VAR	TVAR	2TVAR	SBVAR	2SBVAR	SBTVARc	SBTVAR
d		4	4			4	1 2
$\hat{r}$		2.47	0.92 2.13			2.14	1.04 0.87
$\hat{\tau}$				1989q2	1986q1 1991q3	1991q1	1991q1
						3.71	3.71
${\sigma_Y}^2$	1.02	1.54	0.57	2.06	2.52	1.26	1.26
		1.03	0.79	1.13	1.21	0.90	0.90
			0.94		0.66	0.69	0.69
						0.35	0.35
${\sigma_S}^2$	0.61	0.42	1.29	0.40	0.31	0.12	0.12
		0.24	0.56	0.17	0.37	0.30	0.30
			0.23		0.16	0.16	0.16
T	133	57 76	29 20 84	33 100	20 22 91	17 23 33 60	9 31 20 73
AIC	-42.37	-43.61	-90.56	-105.99	**-122.16**	-110.50	-103.74

Note: Sample period is from 1980Q1 to 2012Q1. ${\sigma_Y}^2$ and ${\sigma_S}^2$ are the estimated variance of output-spread equations for each regime with T observations. Model with the lowest AIC (in bold format) is preferred.

Table 3.3: VAR(3) Estimated Parameters

	VAR	TVAR	2TVAR	SBVAR	2SBVAR	SBTVARc	SBTVAR
d		2	2			4	1 4
$\hat{r}$		1.99	1.99 4.01			1.05	3.39 0.87
$\hat{\tau}$				1989q2	1986q1 1991q2	1991q1	1991q1
						1.61	1.61
${\sigma_Y}^2$	0.92	1.15	1.15	1.98	2.43	0.98	0.98
		0.69	0.69	1.03	1.15	0.24	0.24
			0.49		0.57	0.30	0.30
						0.34	0.34
${\sigma_S}^2$	0.55	0.52	0.19	0.39	0.19	0.16	0.16
		0.30	0.37	0.17	0.36	0.29	0.29
			0.17		0.16	0.14	0.14
T	133	48 85	48 63 22	33 100	20 21 92	8 32 23 70	32 8 20 73
AIC	-65.19	-88.92	-94.48	-103.03	**-119.62**	-107.89	-87.82

Note: Sample period is from 1980Q1 to 2012Q1. ${\sigma_Y}^2$ and ${\sigma_S}^2$ are the estimated variance of output-spread equations for each regime with T observations. Model with the lowest AIC (in bold format) is preferred.

dropped after 1986q1 (from 0.757 to 0.486) while increase significantly after 1991q3 (from 0.486 to 1.562). The dependence of first lag of spread is increasing through the first break (from 0.128 to 0.269) and dropping after the second break (from 0.269 to -0.292). The second lag of spread is increasing through both the breaks (from -0.404 to 0.122 to 0.235). The first break could be related to people's expectation changing after the whole UK economy has been fully recovered from the early 80's recession. While the second break could be associated with the government's inflation targeting policy which altered people's expectation using the information in the term spread and let people foresee a longer period.

From Table 3.1 to 3.3, it is important to note that a increasing the lag order of the model will increase models goodness-of-fit at first and then decrease. For VAR models' AICs are decreasing by increasing the lag orders. (which will peak at VAR(5)). So do the TVAR and the 2TVAR. While for SBVAR, 2SBVAR, SBTVARc, SBTVAR, AICs start to drop after lag orders being increased to 3. This could be led by the parsimonious problem. There are limited observations in some regimes.

Table 3.4: VAR(2) Estimated coefficients

		VAR		TVAR		2TVAR		SBVAR		2SBVAR		SBTVARc		SBTVAR	
		Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S
Regime 1	constant	0.349	0.395	0.216	0.350	0.774	0.695	0.473	0.949	0.602	1.092	0.184	1.502	0.646	1.364
	$\beta_{\Delta y,t-1}$	1.151	-0.055	1.024	-0.087	1.301	0.031	0.777	0.015	0.757	-0.094	0.692	0.467	0.421	0.277
	$\beta_{\Delta y,t-2}$	-0.287	0.043	-0.102	0.148	-0.566	-0.122	0.127	0.318	0.073	0.365	0.132	-0.060	-0.264	0.114
	$\beta_{S,t-1}$	0.127	1.025	0.155	0.930	-0.255	0.933	0.215	0.549	0.128	0.354	0.374	0.046	1.649	0.521
	$\beta_{S,t-2}$	-0.147	-0.174	-0.378	-0.191	-0.130	-0.234	-0.267	-0.306	-0.404	-0.379	-0.466	-0.251	-0.535	-0.364
Regime 2	constant			0.611	0.198	-0.238	-0.160	0.500	0.301	0.568	1.936	0.844	0.278	-0.289	-0.689
	$\beta_{\Delta y,t-1}$			1.276	-0.068	1.218	-0.423	1.509	-0.053	0.486	-0.189	0.665	0.012	0.771	-0.016
	$\beta_{\Delta y,t-2}$			-0.435	0.022	-0.067	0.657	-0.677	0.014	0.086	-0.125	-0.050	0.022	0.234	0.384
	$\beta_{S,t-1}$			-0.143	1.192	2.175	1.132	-0.297	1.399	0.269	0.942	-0.026	1.017	0.331	1.032
	$\beta_{S,t-2}$			0.096	-0.249	-2.651	-0.330	0.237	-0.485	0.122	-0.126	0.295	-0.215	-0.258	-0.372
Regime 3	constant					0.725	0.138			0.473	0.256	1.190	0.586	1.890	0.495
	$\beta_{\Delta y,t-1}$					1.115	0.012			1.562	-0.035	1.490	-0.095	0.877	0.032
	$\beta_{\Delta y,t-2}$					-0.261	-0.048			-0.725	0.001	-0.852	-0.027	-0.464	-0.129
	$\beta_{S,t-1}$					0.124	1.107			-0.292	1.373	-0.190	1.557	0.049	1.230
	$\beta_{S,t-2}$					-0.197	-0.162			0.235	-0.452	-0.098	-0.705	0.067	-0.775
Regime 4	constant											0.443	-0.032	0.334	0.261
	$\beta_{\Delta y,t-1}$											1.547	0.011	1.629	-0.030
	$\beta_{\Delta y,t-2}$											-0.690	-0.043	-0.779	0.001
	$\beta_{S,t-1}$											-0.175	1.190	-0.386	1.413
	$\beta_{S,t-2}$											0.135	-0.200	0.365	-0.490

Note: Δy is the real GDP growth, and S is the term spread.

In-sample Recession Estimation

Figures 3.1 to 3.3 show the models' in-sample estimation of recessions. The shaded areas show the actual recessions. From the graphs we can see that all the models can capture the three major recessions in the sample period (1980q1 to 2013q1). The graphs show that the models with threshold estimate the recession period smoothly but is bumpy during the non-recession period. In contrast, models with only structural break show some volatility during the recession period while smoother in the non-recession period. Linear models (VARs) show less stable in both periods. All the models display an increasing probability of recession after 2010. From Figure 3.1 and 3.2, there are small increasing in recessionary probabilities in VAR(1) and VAR(2) estimations with non-linearity in 2003. In Figure 3.3, it is much smoother in that period of time of VAR(3) estimation. By increasing the lag order, we can see more detailed movements of probabilities, however, models are much smoother in non-recession period. This is a sign of increasing accuracy in the recession modelling.

Summarizing, models with non-linearity are able to model real growth and recession very well. 2SBVAR(2) with break point 1986q1 and 1991q3 is the best in-sample estimation among the models. This is an evidence of non-linear behaviour in the real growth-term spread relationship.

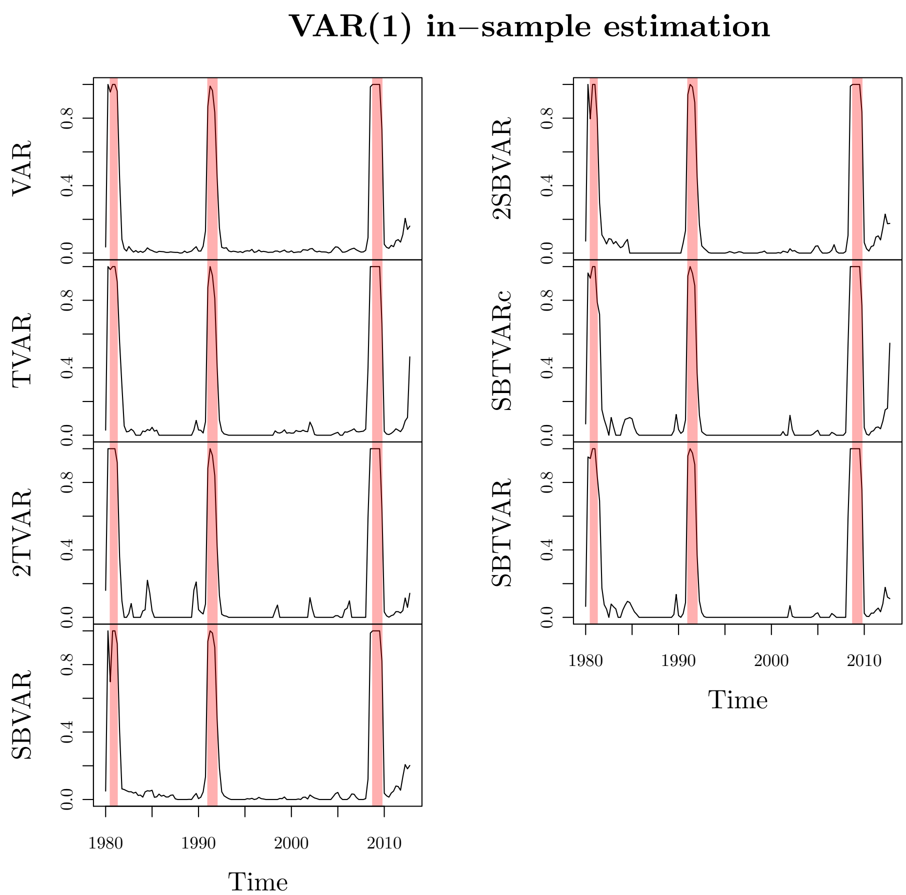


Figure 3.1: VAR(1) recession in-sample forecasting

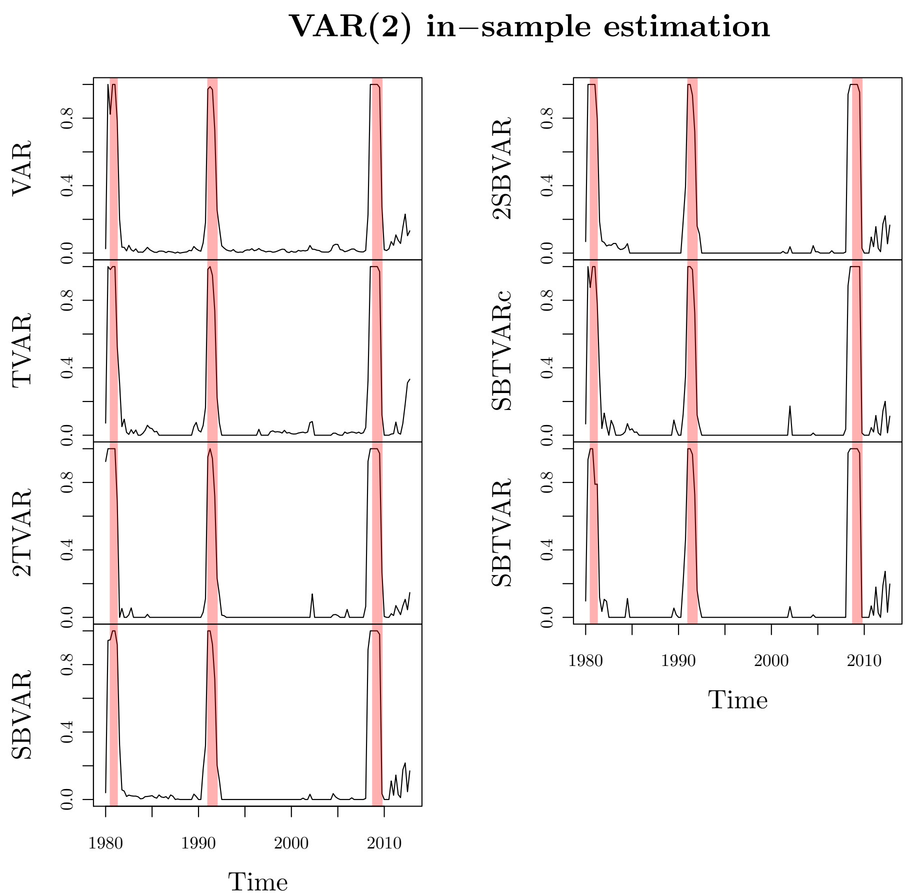


Figure 3.2: VAR(2) recession in-sample forecasting

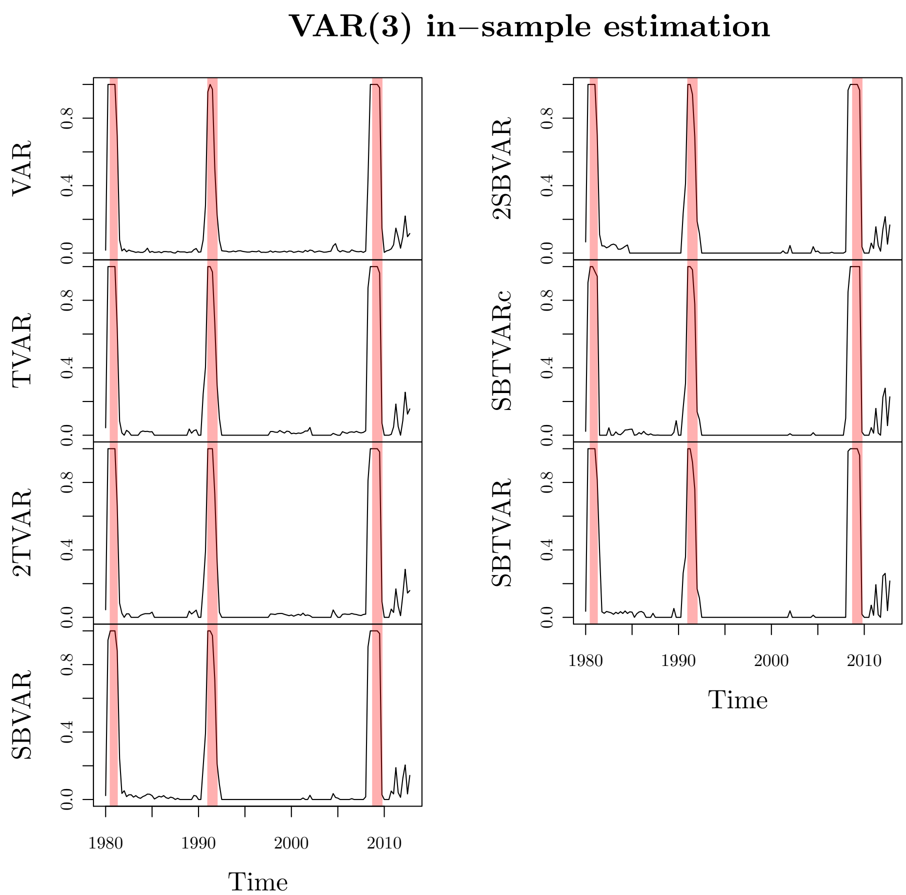


Figure 3.3: VAR(3) recession in-sample forecasting

3.5.2 Out-of-sample forecast

In order to examine the strength of the predicted link in the real growth-term spread relationship, out-of-sample tests have been conducted. Three aspects of the out-of-sample tests are considered: sensitivity to new information, performance of real growth forecasting and performance of recession forecasting.

Sensitivity to New Information

In the out-of-sample tests, thanks to the recursive estimation, one can record the dynamic relationship between the spread and output growth. Ideally, if the model is used for forecasting, then the parameters should be robust to the introduction of new information. Models are estimated recursively to examine the robustness.

The in-sample period is chosen from 1980q1 to 2001q2, and the out-of-sample period is 2001q3 to 2013q1. The model will be re-estimated each time a new time point joins the sample. Therefore the in-sample period is actually the first estimation sample. 1980q1 to 2001q3 will be the second estimation sample, and so on. The estimated parameters of TVAR, 2TVAR, SBVAR, 2SBVAR, SBTVARc, and SBTVAR are sorted into three sub-figures: delays, break points and thresholds.

Figures 3.4 to 3.6 show the sorted recursively estimated parameters of models with one, two and three autoregressive order(s) respectively. For VAR(1) (see Figure 3.4) delays are quite stable with models with thresholds only. Structural breaks are quite stable, one break is around late 1985 and gently increases on arrival of new information. And the second break and the break of SBTVAR(1) is 1991. There is only very short period of instability about the second break of 2SBVAR(1) during the financial crisis. Regarding the thresholds, the model with thresholds only show a very stable pattern, one is near 1 the other a little bit over 2. Figure 3.5 shows the parameters of model with 2 autoregressive orders. It is quite similar to the results from Figure 3.4. Delays and thresholds are not quite stable from all the models, especially after 2009. This may indicate that threshold model cannot digest information well in the recent financial crisis. It is worth mentioning that the structural breaks are more stable than VAR(1) models especially for the 2SBVAR. The instability that happens in 2SBVAR(1) does not show up here. This confirms 2SBVAR(2), which is the best model from in-sample estimation, enjoys the forecast ability's robustness as well. The two breaks estimated includes one in late

1985 and the other break around 1992. These breaks can be explained as it is stated in Section 3.5.1. This also indicates that the big recession that happened recently has had very limited influence on the relationship modeled by 2SBVAR(2). The break points of SBVAR are increasing from 1986 to 1990 along with the new information coming. This behaviour of recursively estimated structural breaks and the stability of the two breaks indicates there are two and only two breaks in the sample. The delay parameters of VAR(3) (see Figure 3.6) show a fairly stable pattern in the recursive estimation except the second threshold in SBTVAR(3). This can be the result of SBTVAR(3) being the model with the most estimated parameters. The degree of freedom in the model will drop significantly, which will influence the results. The threshold parameters are quite unstable in all models with threshold(s). This shows with the increase autoregressive order from 2 to 3, the performance of the model with threshold is decreasing. This is also because the limited sample size restricted the performance of model with high autoregressive orders. The structural break parameters for the SBTVAR(3) keep increasing after new information entering. This shows then model cannot identify a certain break point and this is evidence that the structural break parameter is lack of robustness in SBTVAR(3). In summary, the 2SBVAR(2) shows its stable performance on arrival of new information, which confirms the robustness of the model.

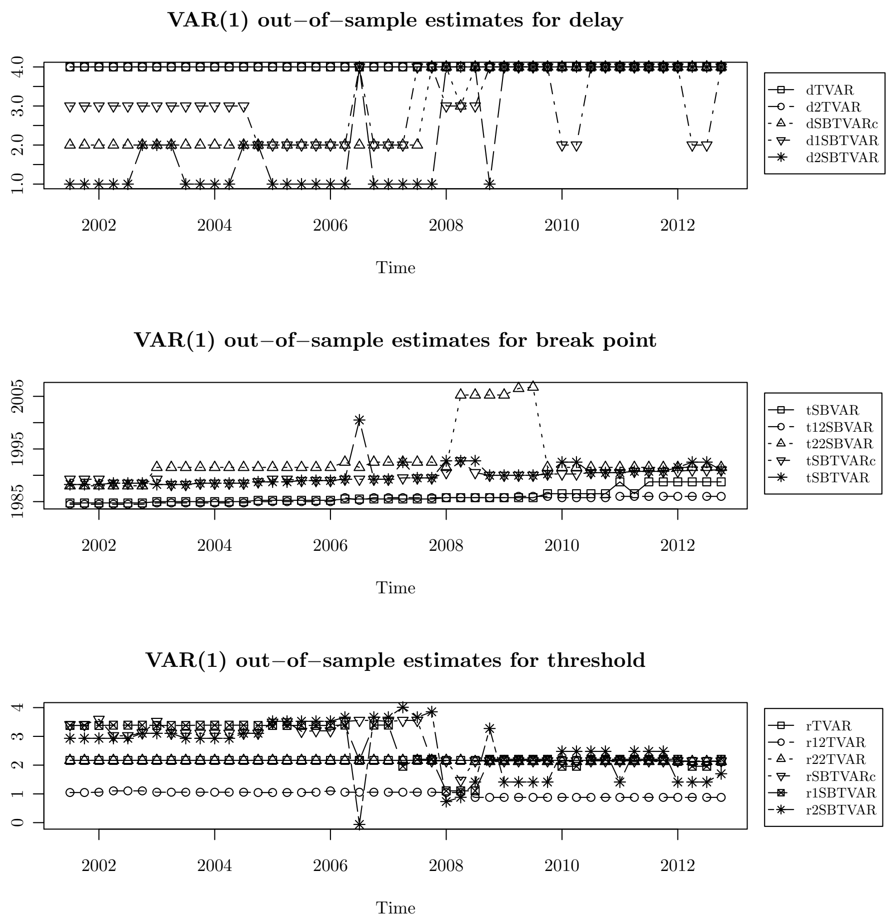


Figure 3.4: VAR(1) out-of-sample parameters movements with new information

Note: in-sample period is from 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

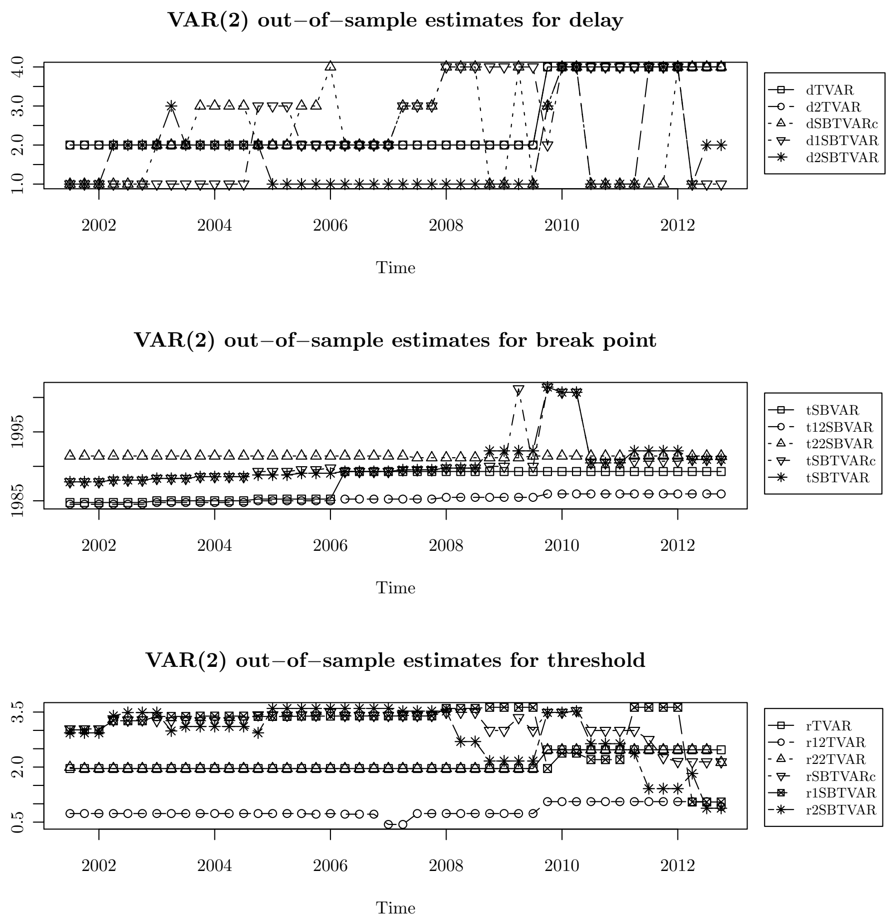


Figure 3.5: VAR(2) out-of-sample parameters movements with new information
Note: in-sample period is from 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

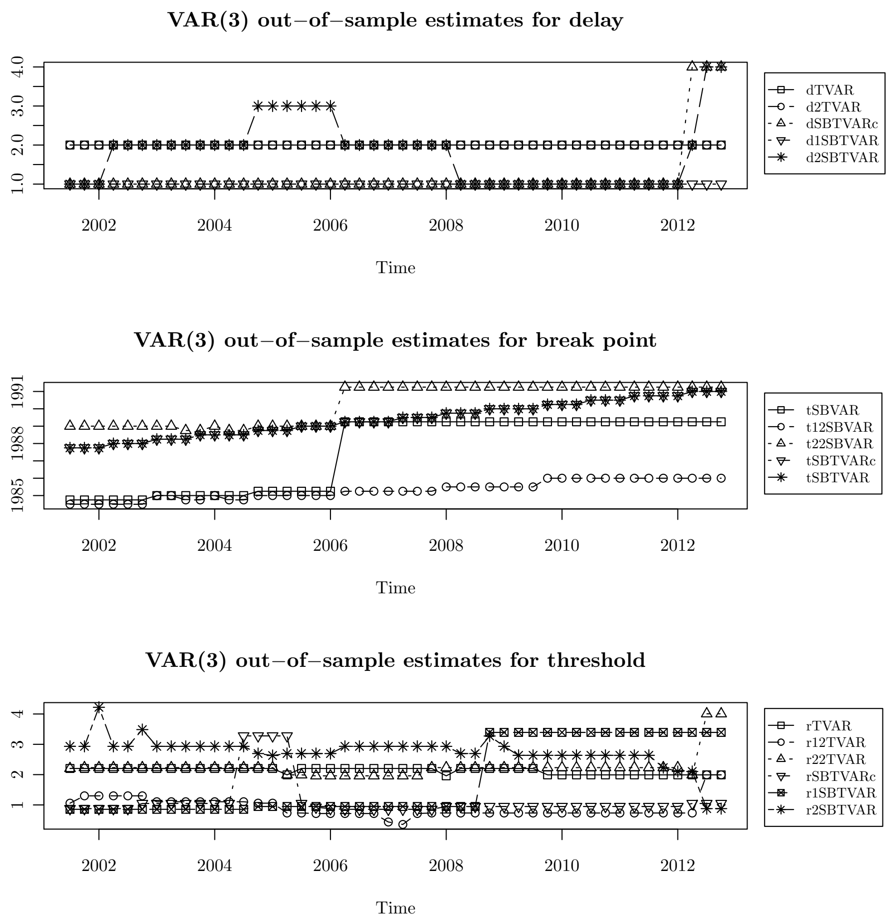


Figure 3.6: VAR(3) out-of-sample parameters movements with new information
Note: in-sample period is from 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

Real Growth Forecasting

Figure 3.7 to Figure 3.18 present the out-of-sample forecasting of all the model with forecasting horizon of 1 to 4 respectively.

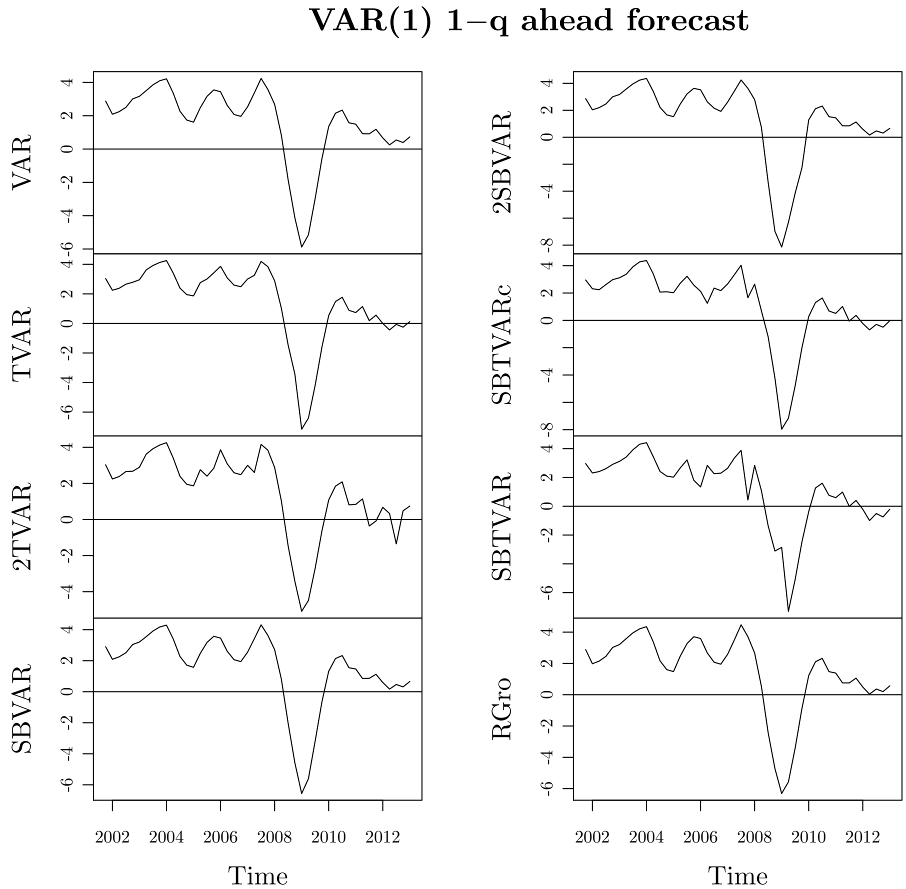


Figure 3.7: VAR(1) 1-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

The last sub-figure in each figure is the real growth (RGro) for comparison. Across the figures, we can see that the growth forecasted by threshold VARs are quite volatile especially with longer forecasting horizons. This is because, as mentioned in the last section (Section 3.5.2), the threshold parameters are quite unstable when the model absorbs new information. It is also interesting to note that, for SBTVARcs with 2 autoregressive order the predictions of real growth is quite abnormal in 2009 which pick around 25% (see Figure 3.11, 3.14 and 3.17). This could be the result of a parsimonious problem when there a small amount of observations in one of the regimes. This issue might also exist in SBTVAR estimation, nevertheless the impact is not as big. The linear models' (VARs) results are quite smooth in out-of-sample forecasts as well as structural

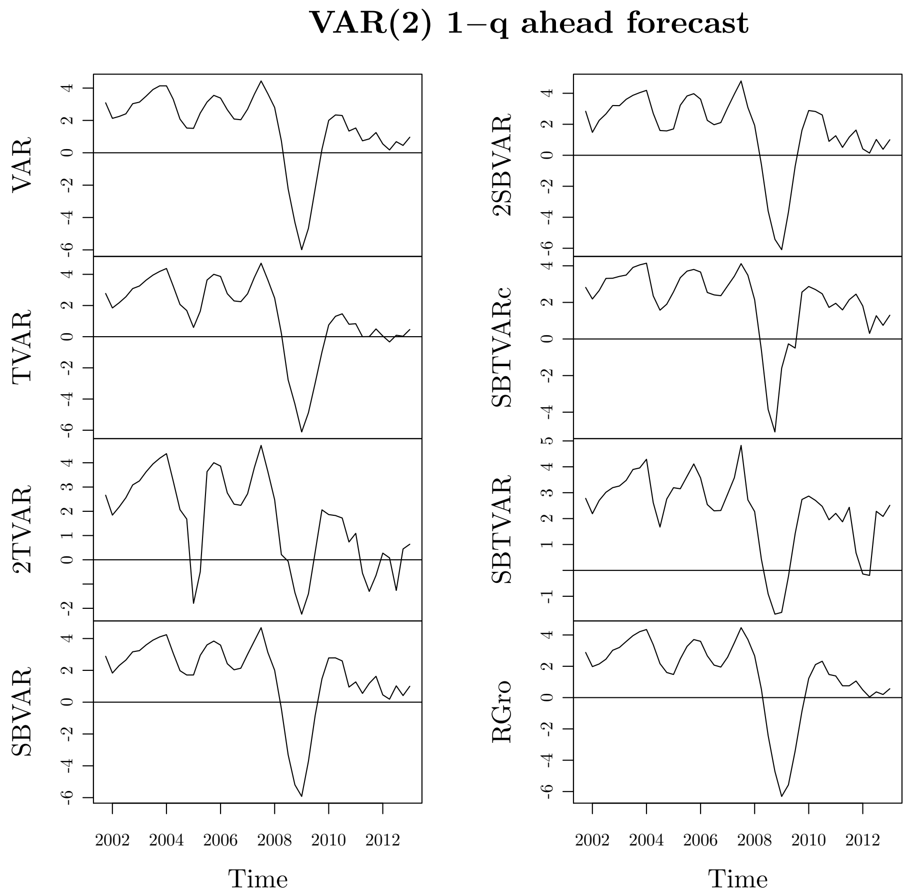


Figure 3.8: VAR(2) 1-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

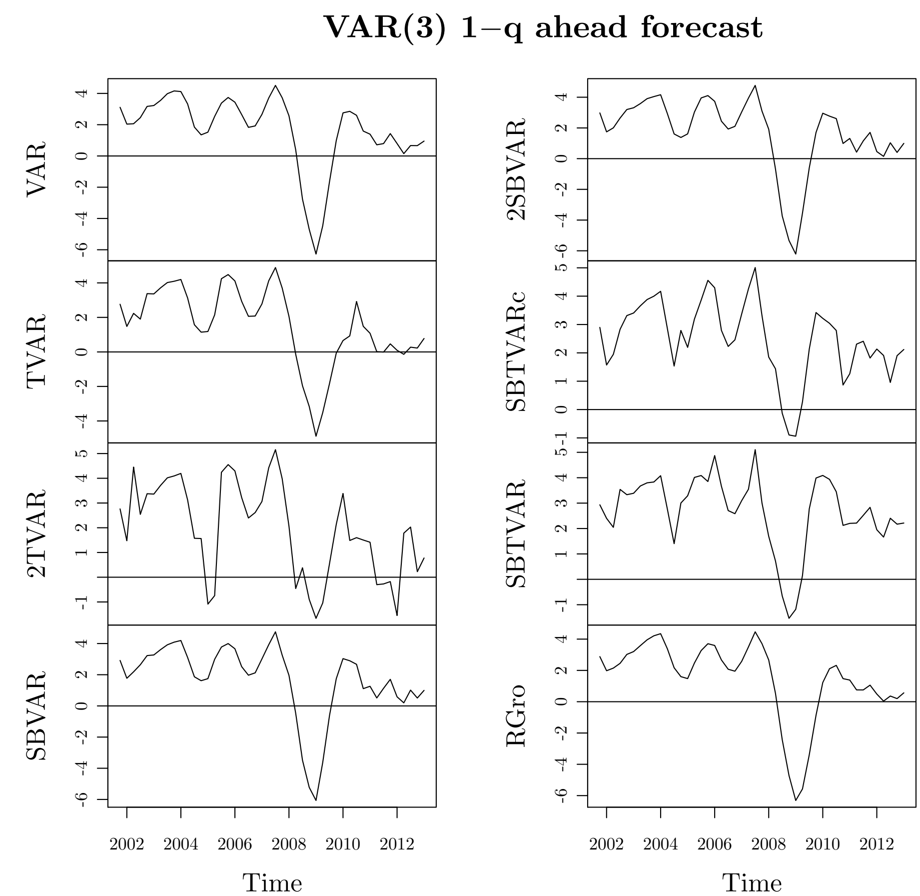


Figure 3.9: VAR(3) 1-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

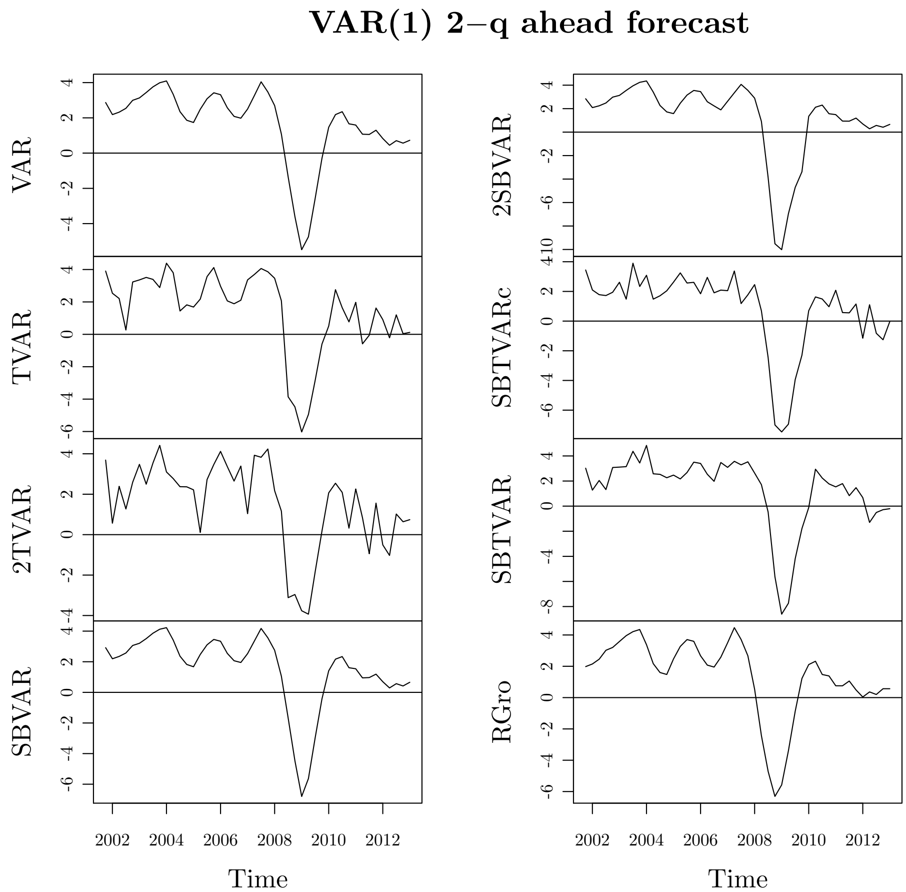


Figure 3.10: VAR(1) 2-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

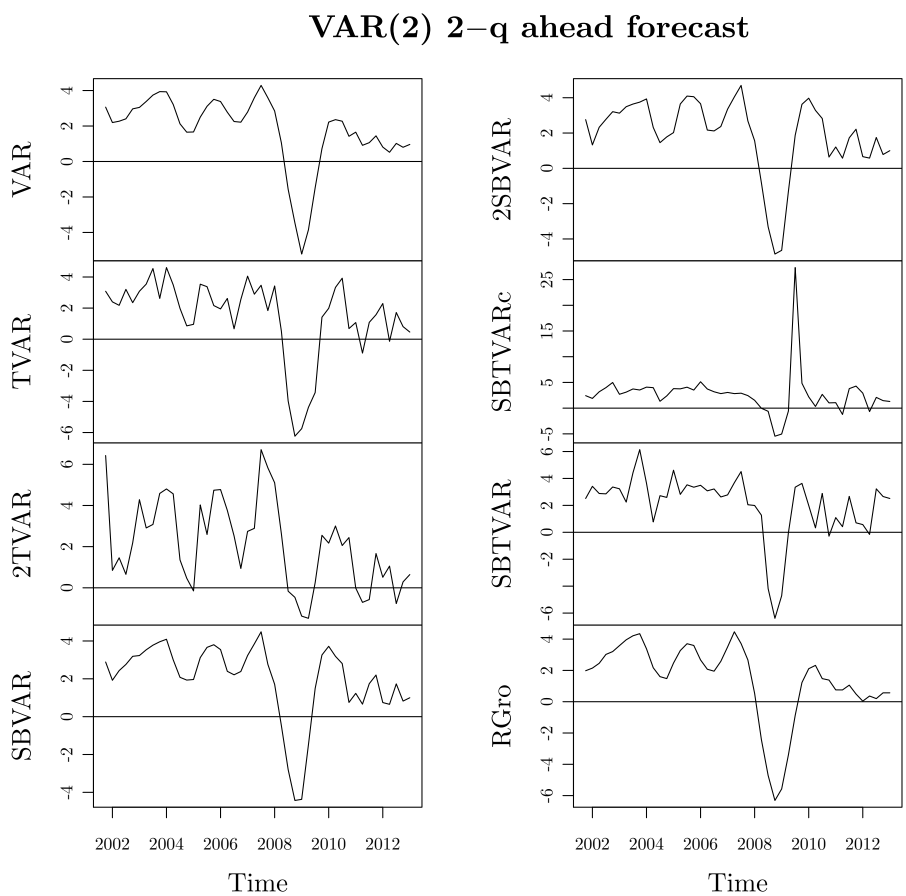


Figure 3.11: VAR(2) 2-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

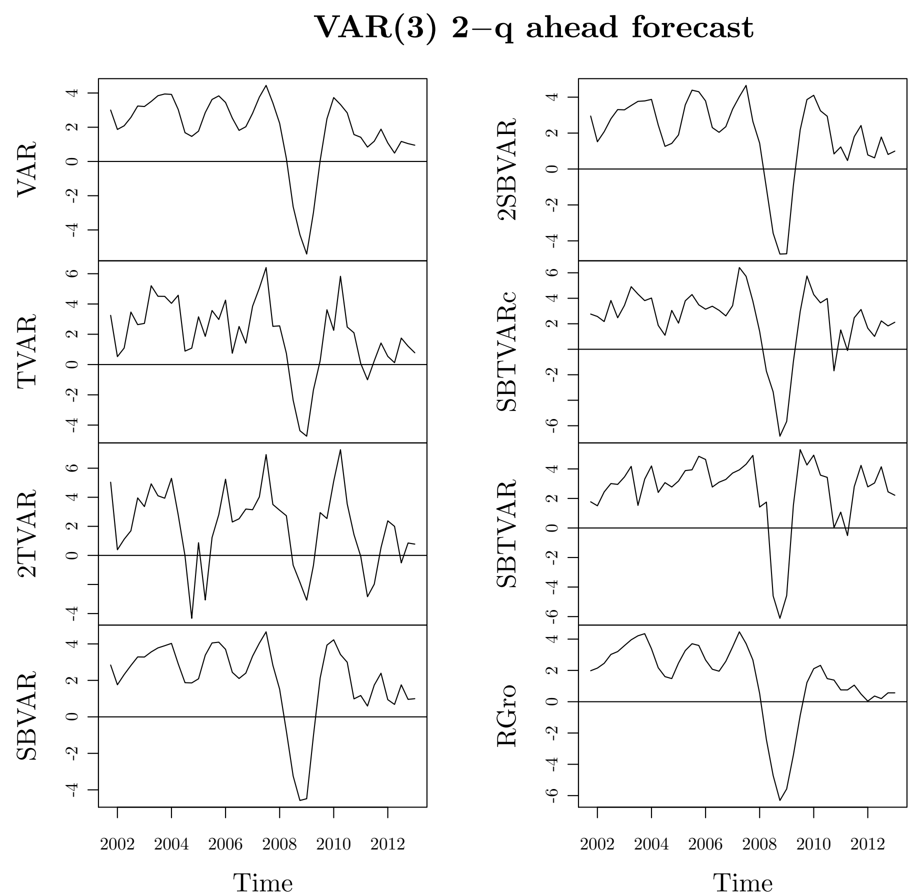


Figure 3.12: VAR(3) 2-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

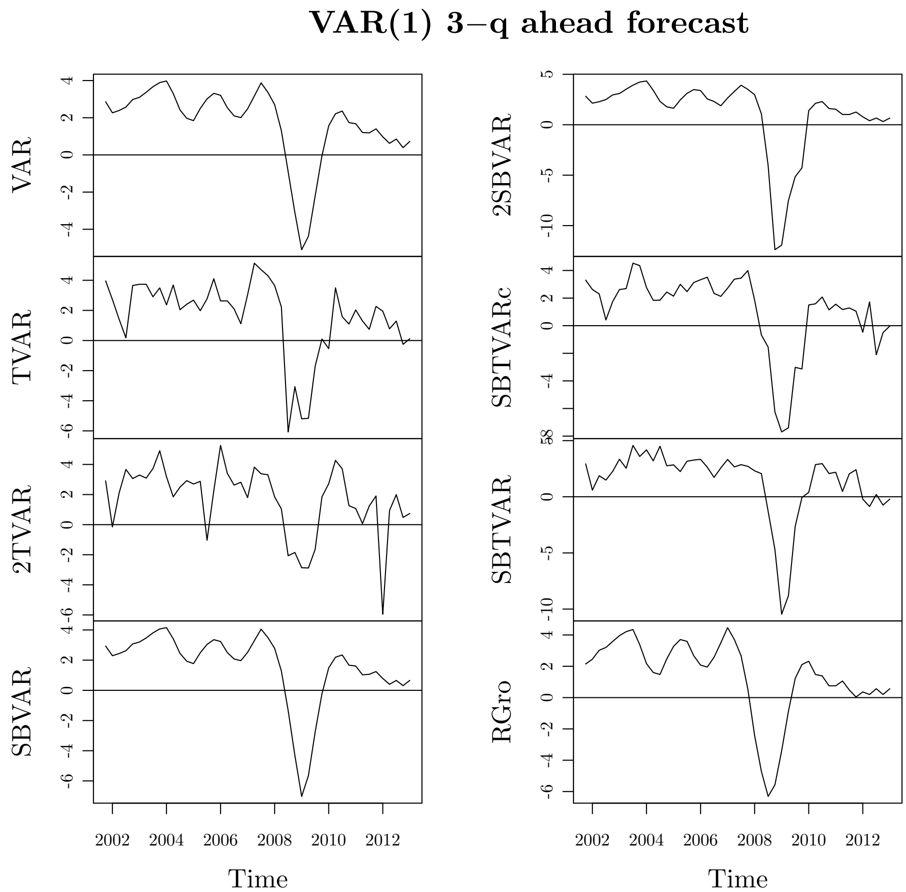


Figure 3.13: VAR(1) 3-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

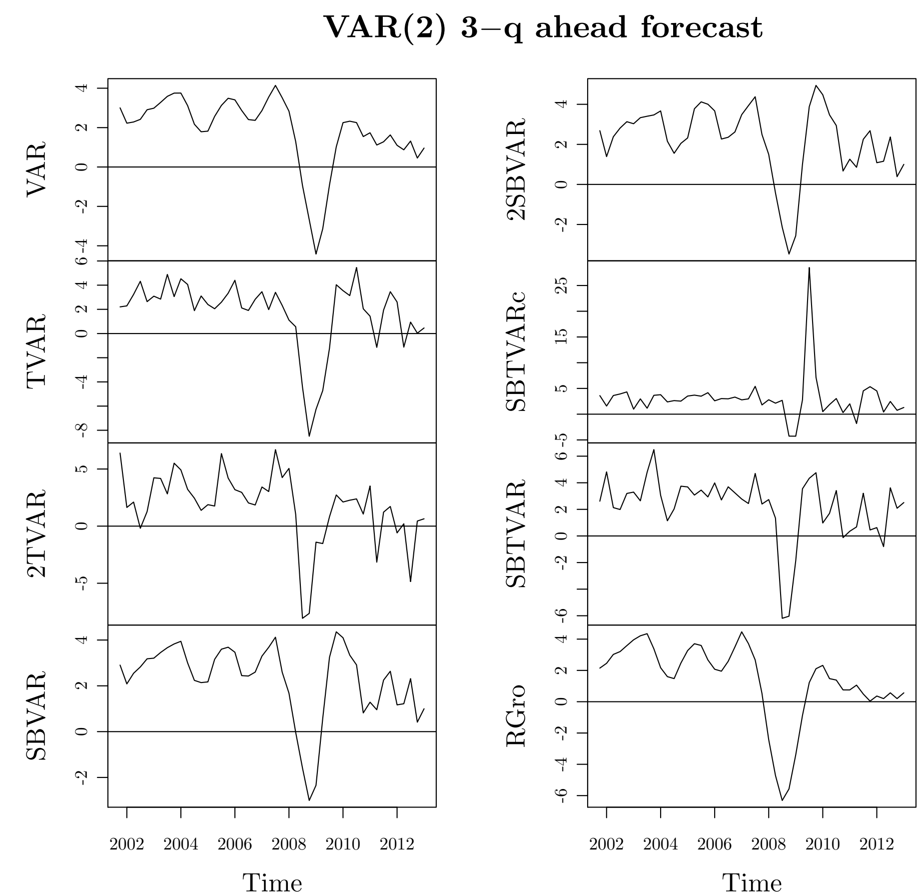


Figure 3.14: VAR(2) 3-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

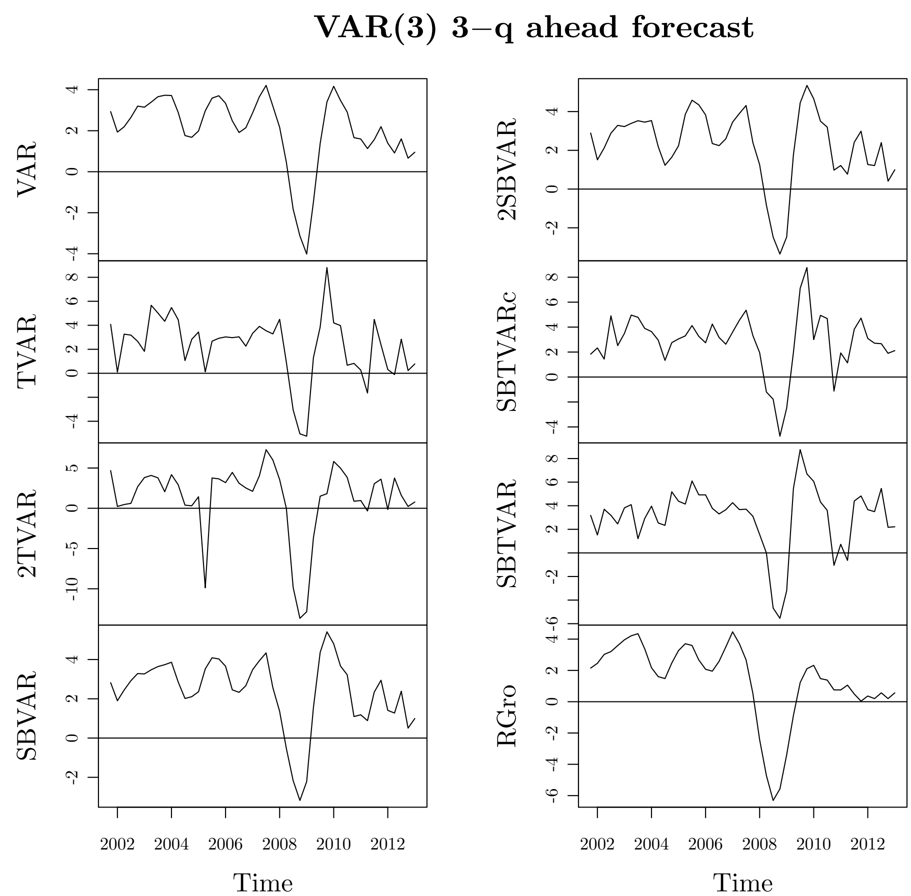


Figure 3.15: VAR(3) 3-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

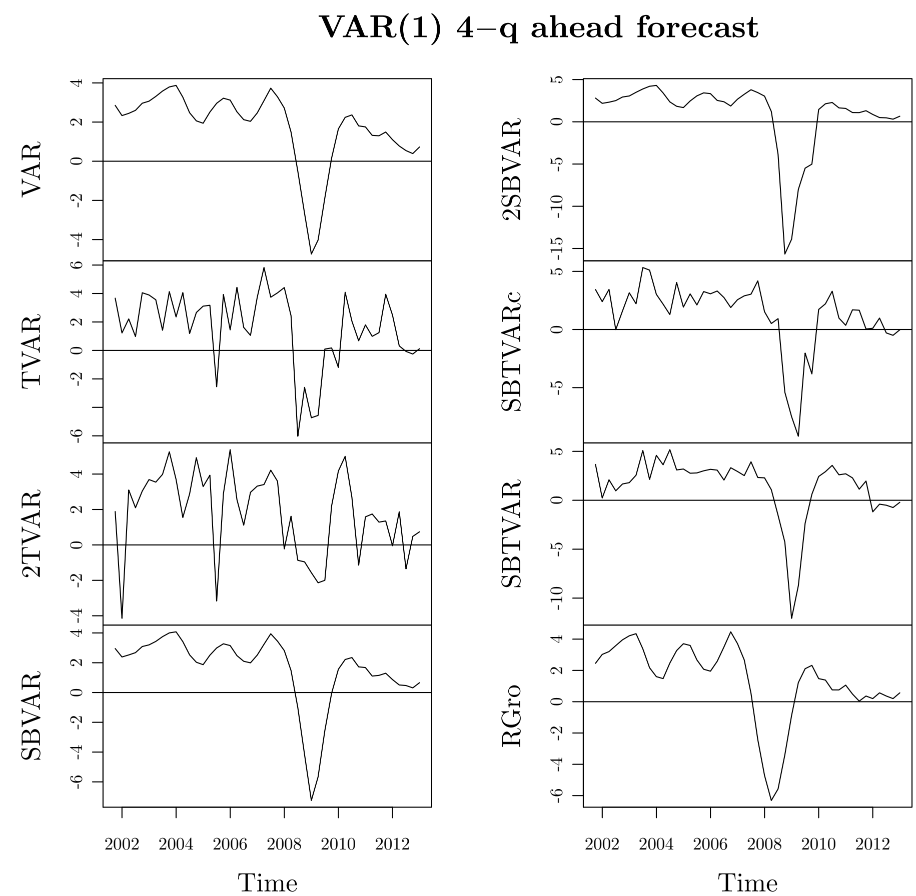


Figure 3.16: VAR(1) 4-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

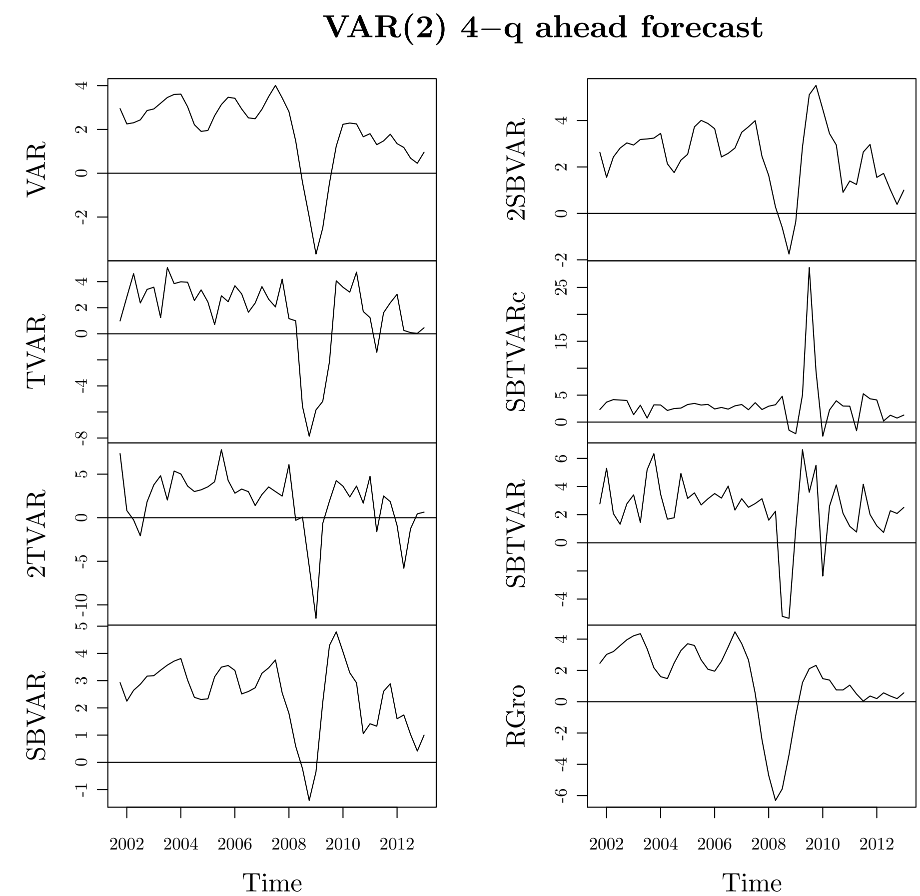


Figure 3.17: VAR(2) 4-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

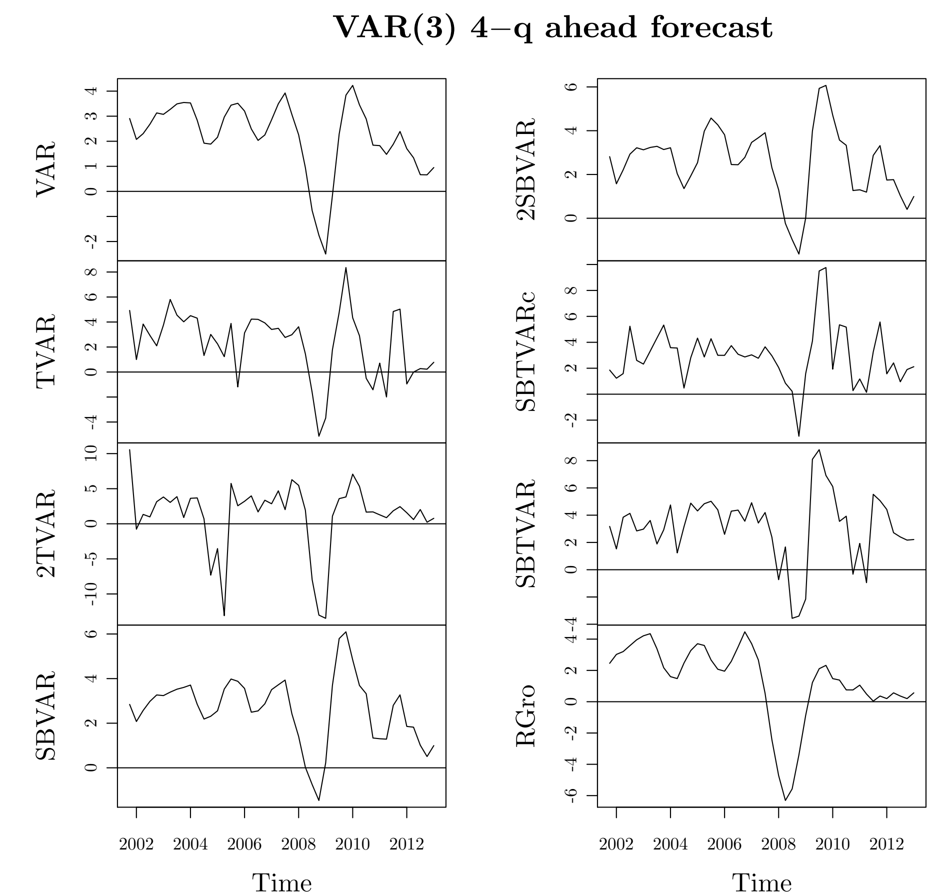


Figure 3.18: VAR(3) 4-quarter ahead real growth forecasts out-of-sample
Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3 to 2013q1.

break VARs. However, structural break models do better in longer horizon forecasting. The performance of forecasting are examined and compared by Root Mean Square Error (RMSE) in this chapter. RMSE is a conventional tool to measure the efficiency of a forecast model in out-of-sample testing. It is calculated as Equation 2.17 in Section 2.5.3

The results of forecasting performance comparison among the models are shown in Table 3.5. In terms of 1-quarter ahead forecasting, SBVAR(1) outperforms the others with a RMSE score of 0.1193. VAR(3) enjoys the best performance in 2-quarter ahead forecasting. For 3-quarter and 4-quarter ahead forecasting, 2SBVAR(2) and SBVAR(2) get the lowest RMSE respectively. Generally speaking, Structural break VARs outperform the linear models (VARs) out-of-sample. It is worth mentioning that SBVAR makes a better forecasting model than 2SBVAR on average in the out-of-sample test. By summarizing the out-of-sample results, the best in-sample model is not necessarily the best out-of-sample. However, 2SBVAR(2) did beat the others in 3-quarter ahead forecasting and is ranked second or third in 1-quarter 2-quarter and 4-quarter ahead forecasting. This means 2SBVAR(2) is a stable forecasting model for UK real growth forecast across different forecasting horizon.

Table 3.5: Comparison of out-of-sample forecast RMSE

Autoregressive order	1	2	3
		1-q ahead	
VAR	0.203719	0.330491	0.507225
TVAR	0.486583	0.401619	0.670783
2TVAR	0.602432	1.50678	1.713174
SBVAR	**0.119349**	0.713874	0.780364
2SBVAR	0.516948	0.786495	0.807101
SBTVARc	0.771787	1.394754	1.867266
SBTVAR	1.082038	1.638453	2.002732
		2-q ahead	
VAR	1.10935	1.040364	**0.912622**
TVAR	1.294086	1.182042	1.332358
2TVAR	1.358937	2.195417	2.154308
SBVAR	1.111953	0.98902	1.08046
2SBVAR	1.544483	0.994417	1.05831
SBTVARc	1.480936	4.460788	1.355875
SBTVAR	1.604809	1.482604	2.055407
		3-q ahead	
VAR	1.943813	1.849534	1.663777
TVAR	2.181866	1.944179	2.126007
2TVAR	2.237575	2.46949	3.087311
SBVAR	2.045546	1.678441	1.747401
2SBVAR	2.811096	**1.658131**	1.711788
SBTVARc	2.226587	4.874093	2.050893
SBTVAR	2.555783	1.94023	3.067268
		4-q ahead	
VAR	2.577424	2.458578	2.31837
TVAR	3.018232	2.7817	2.774163
2TVAR	2.82722	3.687261	3.749341
SBVAR	2.799693	**2.281595**	2.368796
2SBVAR	4.075094	2.300382	2.353885
SBTVARc	3.235476	5.209555	2.655144
SBTVAR	3.351514	2.642391	3.143473

Note: Preferred model indicated by bold type.

Recession Forecasting

For this analysis, the in-sample and out-of-sample periods are the same as the studies conducted in previous 2 sections (Section 3.5.2 and 3.5.2). Figure 3.19 to 3.21 show the 3-quarter ahead recession forecasting abilities of estimation models using 1 autoregressive order to three autoregressive orders. Generally speaking, all the models can identify the most recent financial crisis. And they are all showing an increasing probability of recession after 2010. 2TVAR(3) falsely predicts a recession in 2005, and both 2TVAR(2) and 2TVAR(3) predict another recession in late 2012. All the abnormal forecasts are predicted by models with only threshold(s). This result can be explained in association with the issue threshold VAR encountered in the previous section (Section 3.5.2). That is models with threshold(s) cannot digest new information very well in the UK output growth-term spread relationship.

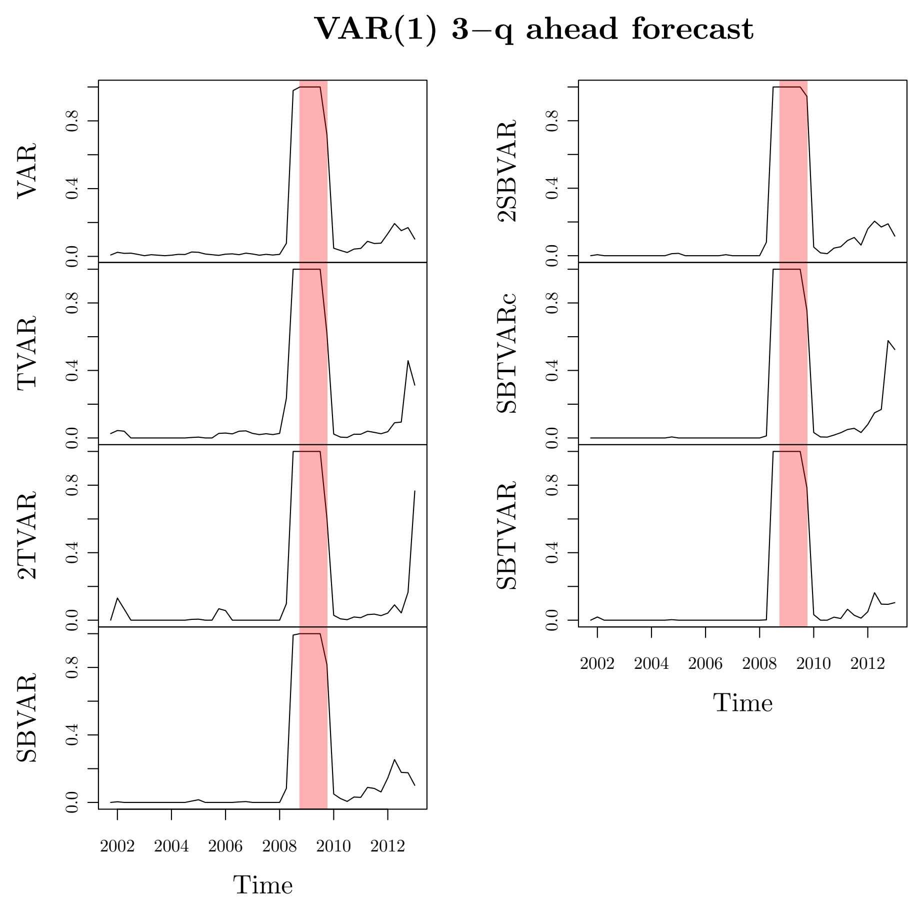


Figure 3.19: VAR(1) recession out-of-sample forecast 3-q ahead

To sum up, in terms of recession forecasting, both linear and non-linear models work well except threshold models.

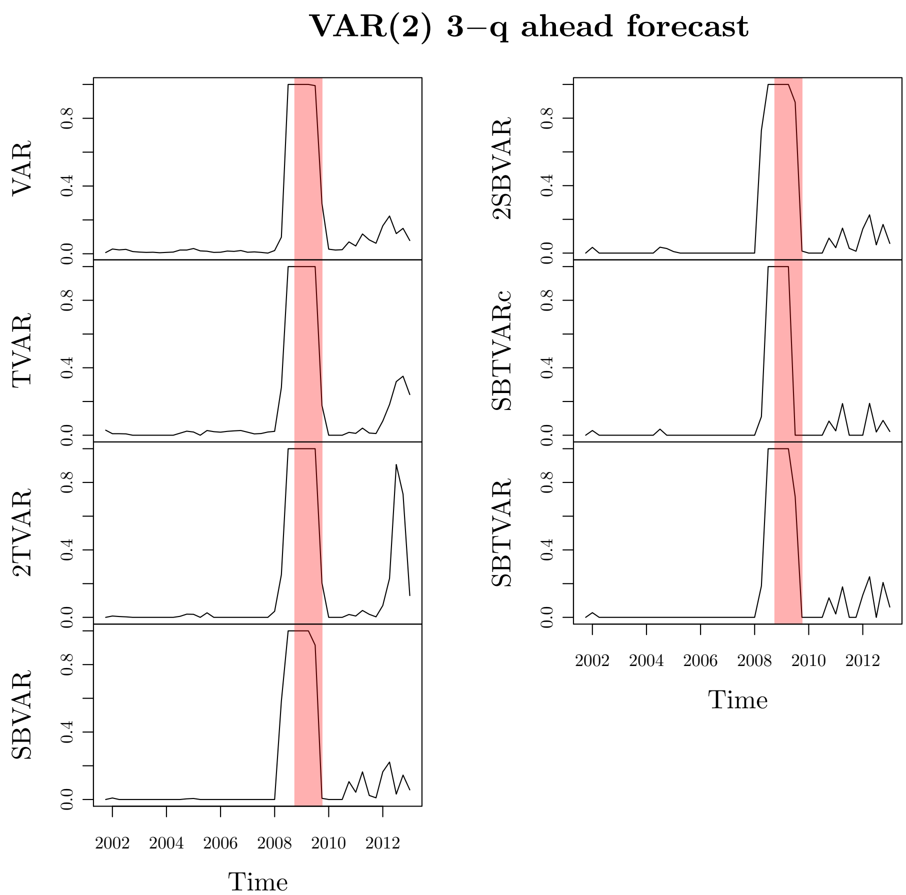


Figure 3.20: VAR(2) recession out-of-sample forecast 3-q ahead

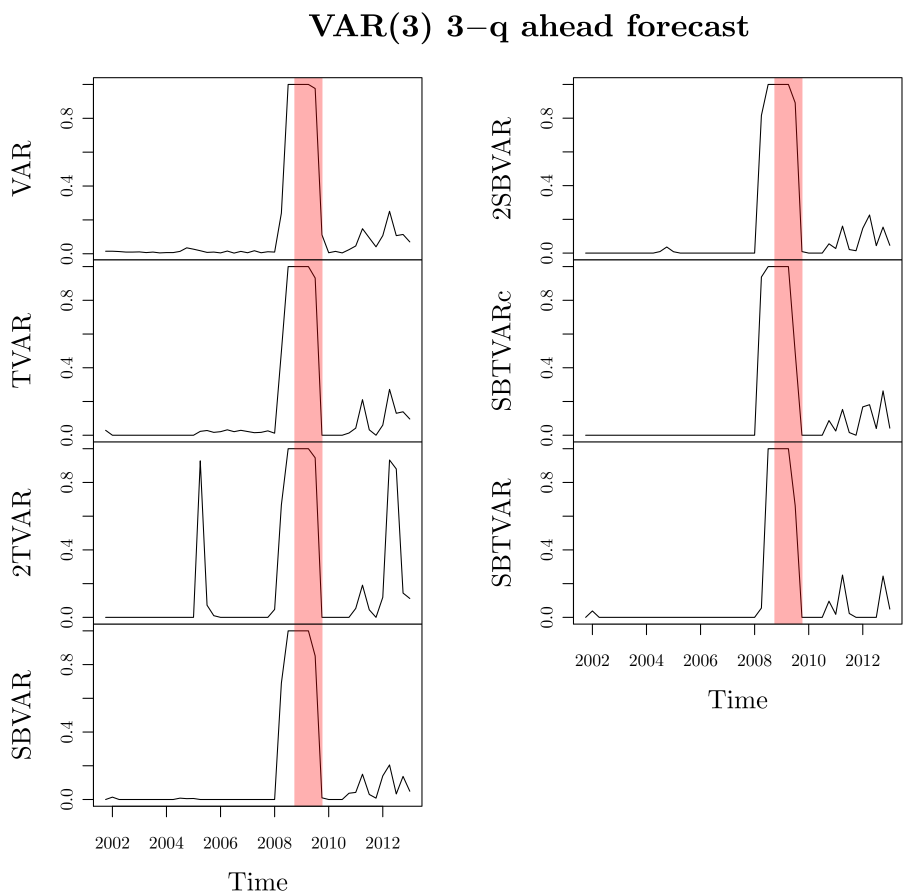


Figure 3.21: VAR(3) recession out-of-sample forecast 3-q ahead

3.6 Concluding Remarks

This chapter examines the non-linear behaviour in the output growth / recession-term spread relationship using UK data covering the last 34 years. The research conducts comparisons of VAR, TVAR, 2TVAR, SBVAR, 2SBVAR, SBTVARc, and SBTVAR with 1 to 3 autoregressive orders. The results suggest there are non-linearities in the relationship. And evidence shows that the type of this non-linearity is a structural break. Introducing structural break(s) into the model does improve the explanatory power of the output growth - yield spread relationship as well as the predictive power of the model. 2SBVAR(2) (model with 2 structural breaks and 2 autoregressive orders) is the tested best in-sample estimating model. The breaks are located at first quarter of 1986 and the third quarter of 1991. The first break can be explained by the expectation adjustment of people after recovery from the early 1980s' recession. And the second break is caused by the decision of the government to apply inflation targeting policy.

In out-of-sample tests, the models with structure break(s) enjoy the robustness on the arrival of new information. This also indicates the most recent financial crisis does not change the fundamental being of the relationship. The results from out-of-sample tests are slightly different from in-sample tests. SBVAR(1), VAR(3), 2SBVAR(2) and SBVAR(2) enjoy the best forecast ability in 1-quarter, 2-quarter, 3-quarter and 4-quarter ahead forecast respectively. Generally speaking SBVAR basically dominated the out-of-sample forecasts and 2SBVAR is almost as good. In terms of recession predicting, all models presented in the chapter except Threshold models give a fairly good performance.

Unavoidably in this study, there are limitations about the comparison. For models with more regimes (SBTVARc and SBTVAR) in this limited sample size, the disadvantage in the comparison is there might be regimes with small amounts of data in the model. This will trigger the parsimony problem of the VAR estimations, especially for models with higher autoregressive orders. We have seen that this problem can lead to abnormal forecasts.

Chapter 4

Vine copulas and applications to the European Union sovereign debt analysis

4.1 Introduction

The ongoing European sovereign debt crisis originated in Greece, but the impact has spread all over the European Union especially in the euro area. On 8th Dec, 2009, rating agency Fitch cut Greece's long-term debt from A- to BBB+. Because of the lack of confidence in investing in Greek government bonds, as one of the sovereign debt default indicators, the yield of 10-year government bonds jumped up significantly. In the mean time, the 10-year government bond yield of peripheral European countries Spain and Portugal also increased along with Greece. In Ireland and Italy, however, the yields decreased. This phenomenon shows that yield differentials across European bond markets have not been wiped out completely, although accelerated financial integration among euro bond markets has been widely expected, since the macroeconomic and fiscal indicators have shown significant improvement for the higher risk euro markets, creating a potential for those members to converge with lower risk members in terms of bond returns. **?** states that because the fragility for the governance of the eurozone, being a member of the monetary union could be easily be forced into default by financial markets. On the other hand, being non-member of the monetary union is not easily be forced into default because they are able to control over the currency in which they issue debt. Finding the relationship between the yields of these countries' sovereign bonds might be a useful way to understand how they will influence each other, especially in extreme events. This information could then be used to assess the risk level of a sovereign bond. In order to achieve this, a GARCH based vine copula simulation method to analyse the sovereign debts in the European Union is proposed in this chapter.

As a popular multivariate modeling tool, copula is widely used in many fields where the multivariate dependence matters, such as actuarial science (Frees et al., 1996), biomedical studies (Wang and Wells, 2000), engineering (Genest and Favre, 2007) and finance (Embrechts et al., 2003). In finance, the misuse of the copula method in the pricing of collateralized debt obligations (CDO) is considered by journalists to be one of the reasons that led to the global financial crisis of 2008 - 2009. The copula approach provides a method of isolating the description of the dependence structure and understanding the dependence at a deeper level. It expresses dependence on a quantile scale, which is useful for describing the dependence of extreme outcomes and is natural in a risk-management context. Due to the advantages of the copula method, it is an ideal tool for analysing the relationship of sovereign debts between countries in the European Union.

The main difficulty about sovereign debt crisis analysis is that the crisis rarely happens. It is extremely hard for statisticians to analyse an event which has never happened before. In order to solve this issue, this chapter uses simulation methods to create unknown situations. This chapter replicates 10000 iterations of a 365 future day simulation of sovereign spreads against Germany of 11 countries in the European Union. In the mean time, the relationships between the countries are considered. Then, the percentage chance of the crisis events is calculated, which is the probability of future crisis. In terms of defining crisis events, Sy (2004)'s definition of sovereign debt crisis is adopted, which is that sovereign spread against the US is more than 1000 basis points. In the same manner, an EU country experiencing a sovereign debt crisis is defined as being when its sovereign spread against Germany is greater than 1000 basis points in this research.

The contribution of this research is fourfold. Firstly, this is the first analysis of extreme value and tail dependence of sovereign debt spread movement in the European Union. Secondly, this study conducts the comparison between 11 countries in the European Union at the same time. Thirdly, this chapter uses vine copulas to deal with large numbers of dimensions and satisfies the wide range of dependence, flexible range of upper and lower tail dependence, computationally feasible density for estimation, and closure property under marginalization simultaneously. Fourthly, which is also the key feature of this chapter, the research identifies the risk level of sovereign debt in different countries in the European Union.

Daily 10-year government bond yields from 18/06/1997 to 12/03/2012 in Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and the UK are used in this research.

The results show that the estimated crisis probabilities of Greece and Portugal in the next 365 days are 100% and 99.77%, which is consistent with the situation that they are already in crisis. Spain and Italy show great potential to be the next victims in one year's time. France and Belgium show some instability in the results and the probability of crisis is fairly high: 63.13% and 60.14% respectively. Netherlands is next with an almost 1 in 4 chance of crisis and it is the most stable country in the euro area. In the mean time, countries outside the euro area in the sample which are the UK, Sweden and Denmark show the greatest stability in their sovereign bonds.

The remainder of the chapter is as follows. Section 4.2 is a literature review in sovereign debt analysis and copula methods. Section 4.3 is the data description. Section 4.4 discusses the bivariate relationships of these pairs of countries. Section 4.5 explains the vine copula approach. Section 4.6 shows the results of simulation and calculation of the risk levels of the countries. And Section 4.7 concludes.

4.2 Literature review

The literature on sovereign debt analysis generally uses sovereign bond spread between the target country and a benchmark country to assess the default risk level of the target country. Structural approaches developed from the Merton model (1974) and reduced form models such as the Jarrow and Turnbull (1995) approach are the two main streams.

The structural approaches explain the sovereign spread endogenously using enterprise value volatility and firm default definition (Diaz Weigel and Gemmill, 2006, Oshiro and Saruwatari, 2005). The pitfalls of these approaches are not only their difficulty and lack of accuracy to define appropriate country-specific proxy variables for the level of indebtedness, but also they disregard the fact that default incentives of a country are more complicated than those of enterprises. The reduced form approaches use different macro variables as the determinants of the sovereign default risk. Literature such as Reinhart et al. (2003), Eichengreen et al. (2003) and Goldstein and Turner (2004), analyse the sovereign debt risk of emerging market economies. Their focus is on the sustainability of the sovereign debt and the currency mismatches. They measure default risk by using country credit ratings. The disadvantage of these approaches is that these credit ratings are inefficient and cannot be adjusted in a timely manner to adapt to the market data when a big crisis is ongoing. Most recently, Dötz and Fischer (2010) use a GARCH-in-mean based on a reduced form model to analyse the factors triggering the sovereign spreads movement in the European Union and the result shows that the expectation of loss is the main reason sovereign spread widened during the recent global financial crisis. Both structural and reduced form approaches face a problem: they ignore the yields movement dependence with other countries, which is especially important inside the European Union.

Vine copula methods can solve these problems by modeling several sovereign yield spreads together in a single framework. They analyse countries sovereign debt risk, focusing on the interactions of countries and assess the crisis probabilities of countries simultaneously. There is a large body of literature using copulas in a financial context (Bouyé et al., 2000, Embrechts et al., 2003, Cherubini et al., 2004). Most of them are used to compute Value at Risk (VaR) and expected shortfall (ES) of the stock or bond portfolio by applying single copula families such as elliptical copulas and Archimedean copulas. There are lots of limitations on those copula families applied in the above literature. Elliptical copulas are widely used, but they cannot model the financial tail dependences very well (Patton, 2008). Archimedean copulas are not satisfactory for modeling with dimensions higher than two (Joe, 1997). Multivariate Archimedean copulas only allow exchangeable structure with a narrower range of negative dependence in a higher dimension (McNeil and Neslehova, 2009). Partially symmetric copulas extend Archimedean to a class with a non-exchangeable structure, but the dependence they provide are not particularly flexible (Joe, 1993). Mix-id copulas in Joe and Hu (1996) provide flexible positive dependence by construction, but only upper tail dependence is flexible not lower tail. Demarta and McNeil (2005) provides multivariate skewed-t copulas, which model well, but are computationally more involved. Vine copulas were first was proposed by Joe (1996) and explained in detail by Bedford and Cooke (2002). At that time, vine copulas model were a graphical model using bivariate copulas to construct multivariate copulas. Aas et al. (2009) conduct statistical inference on two types of vines: canonical vine (C-vine) and drawable vine (D-vine). This model has been improved by Nikoloulopoulos et al. (2012) which can satisfy most of the features that should be included in a copula model: firstly, a wide range of dependence including both positive and negative dependence; secondly, a flexible range of upper and lower tail dependence; thirdly, computationally feasible density for estimation and fourthly, closure property under marginalization.

In this chapter, a GARCH based Vine copula method is used to analyse the tail dependence and calculate probabilities of sovereign debt crisis of these countries in certain periods of time in the European Union.

4.3 Data

Daily 10-year government bond yields from 18/06/1997 to 12/03/2012 in Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and the UK are used in this research. All data are collected from Thomson Reuters ECOWIN[1].

4.4 Bivariate copula analysis

4.4.1 GARCH filter

Vine copula modeling proceeds in three stages. In the first stage, the model for the individual variables (i.i.d) is selected, which is the marginal distribution. For financial time series data, a GARCH filter with innovation being student-t distribution is applied for the purpose of making the data independent and identically distributed (Aas and Berg, 2009). Using Box-Jenkins analysis method (Box and Jenkins, 1970), all $\Delta(i_j - i^*)$

[1]Please see Appendix D for more information.

are determined to be MA(1) process. In order to find the best model to fit the series, MA(1)-GARCH(1,1), MA(1)-EGARCH(1,0)[2] and MA(1)-TGARCH(1,1) are proposed in this stage. Q-statistic (Ljung and Box, 1978) and ARCH LM test (Engle, 1982) are conducted at the same time for testing autocorrelation of residuals and squared residuals respectively.

The MA(1)-GARCH(1,1) model can be expressed as follows:

$$\Delta(i - i^*)_{t,j} = \mu_j + \epsilon_{t,j} + \theta\epsilon_{t-1,j}, \tag{4.1}$$

$$\epsilon_{t,j} = z_{t,j}\sigma_{t,j}, \tag{4.2}$$

$$\sigma^2_{t,j} = \alpha_{0,j} + \alpha_{1,j}\epsilon^2_{t-1,j} + \beta_{1,j}\sigma^2_{t-1,j}, \tag{4.3}$$

where $j = 1,\ldots,d$, $t = 1,\ldots,T$, $\Delta(i - i^*)$ is sovereign spread against Germany (i^*) of a target country (i), $z_t \sim T(0,1,\nu)$, The conditions of coefficients that ensure positive volatility and existence of second moment are $\alpha_1 > 0$, $\beta_1 > 0$ and $\alpha_1 + \beta_1 < 1$.

The MA(1)-EGARCH(1,0) model may generally be specified as follows:

$$\Delta(i - i^*)_{t,j} = \mu + \epsilon_{t,j} + \theta\epsilon_{t-1,j}, \tag{4.4}$$

$$\epsilon_{t,j} = z_{t,j}\sigma_{t,j}, \tag{4.5}$$

$$\ln\sigma^2_{t,j} = \alpha_{0,j} + \gamma_{1,j}\left(\left|\frac{\epsilon_{t-1,j}}{\sigma_{t-1,j}}\right| - E\left|\frac{\epsilon_{t-1,j}}{\sigma_{t-1,j}}\right|\right) + \beta_{1,j}\ln\sigma^2_{t-1,j}, \tag{4.6}$$

where $j = 1,\ldots,d$, $t = 1,\ldots,T$, $\Delta(i - i^*)$ is sovereign spread against Germany (i^*) of a target country (i), $z_t \sim T(0,1,\nu)$.

The MA(1)-TGARCH(1,1) model is represented by the expression:

$$\Delta(i - i^*)_{t,j} = \mu + \epsilon_{t,j} + \theta\epsilon_{t-1,j}, \tag{4.7}$$

$$\epsilon_{t,j} = z_{t,j}\sigma_{t,j}, \tag{4.8}$$

$$\sigma_{t,j} = \alpha_{0,j} + \alpha_{1,j}|z_{t-1,j}| + \beta_{1,j}\sigma_{t-1,j} + \delta_{1,j}z_{t-1,j}, \tag{4.9}$$

where $j = 1,\ldots,d$, $t = 1,\ldots,T$, $\Delta(i - i^*)$ is sovereign spread against Germany (i^*) of a target country (i), $z_t \sim T(0,1,\nu)$. The conditions of coefficients which guarantee positive conditional volatility are $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$, $|\delta_1| < \alpha_1$ and ${\alpha_1}^2 + {\beta_1}^2 +$

[2] MA(1)-EGARCH(1,1) was also considered, and all the coefficients α_1 are insignificant.

${\delta_1}^2 + 2\alpha_1\beta_1\nu_1 < 1$, where $\nu_1 = \sqrt{\frac{\nu-2}{\pi}}\frac{\Gamma(\frac{\nu-1}{2})}{\Gamma(\frac{\nu}{2})}$ for z_t is student-t distributed (Rodriguez and Ruiz, 2012).

Table 4.1,4.2 and 4.3 present the results of MA(1)-GARCH(1,1), MA(1)-EGARCH(1,0), MA(1)-TGARCH(1,1), respectively. In Table 4.1, all the coefficients satisfy the condition $\alpha_1 > 0$, $\beta_1 > 0$ and $\alpha_1 + \beta_1 < 1$, which ensure the positive conditional volatility and confirm the existence of second moment of a standard GARCH model. In Table 4.3, all the coefficients meet the requirements $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$,$|\delta_1| < \alpha 1$ and ${\alpha_1}^2 + {\beta_1}^2 + {\delta_1}^2 + 2\alpha_1\beta_1\nu_1 < 1$, which guarantees positive conditional volatility as well as the existence of the second moment of a TGARCH model. According to Akaike information criterion (Akaike, 1974), MA(1)-TGARCH(1,1) model fits the data the best, and then MA(1)-EGARCH(1,0), and last place is MA(1)-GARCH(1,1). However, in MA(1)-TGARCH(1,1) model, coefficients δ of DEN, FRA, and POR are insignificant in 95% confidence interval, which means there is no threshold effect in these models. In the mean time, ARCH LM tests of MA(1)-TGARCH(1,1) in FRA, POR and UK indicate autocorrelation of squared standardized residuals. The above results suggest that MA(1)-TGARCH(1,1) fit for BEL, GRE, IRE, ITA, NET, SPA, SWE the best. The next best model MA(1)-EGARCH(1,0) is considered for DEN, FRA, POR and UK. ARCH LM tests of MA(1)-EGARCH(1,0) imply that there are autocorrelations in squared standardized residuals for FRA and UK. With the insignificant coefficients of threshold parameter in MA(1)-TGARCH(1,1), this suggests the series of FRA and UK could be symmetric. Q-Statistics are mostly insignificant in 95% significance level, which represents no autocorrelation in the residuals.

In summary, the best model fit for BEL, GRE, IRE, ITA, NET, SPA and SWE is MA(1)-TGARCH(1,1); the best model fit for DEN and POR is MA(1)-EGARCH(1,0); and the best model fit for FRA and UK is MA(1)-GARCH(1,1).

Table 4.1: Results of MA(1)-GARCH(1,1)

	BEL	DEN	FRA	GRE	IRE	ITA	NET	POR	SPA	SWE	UK
μ	-0.0002	-0.00031	2.07E-05	-0.00031	-0.00024	-0.00027	-0.00014	-0.00016	-0.0003	-0.00046	2.32E-05
θ	-0.35547*	-0.46821*	-0.47978*	-0.27817*	-0.33931*	-0.28703*	-0.49628*	-0.35214*	-0.31153*	-0.23441*	-0.19524*
α_0	3.49E-06*	3.12E-05*	2.82E-06*	3.31E-05*	8.15E-06*	1.91E-06*	2.46E-06	2.15E-05*	2.83E-06*	5.02E-05*	2.13E-05*
α_1	0.142989*	0.169135*	0.113557*	0.192742*	0.154521*	0.098451*	0.135867*	0.206958*	0.117413*	0.125711*	0.055155*
β_1	0.856011*	0.806349*	0.885443*	0.806258*	0.844479*	0.900549*	0.863133*	0.792042*	0.881587*	0.835442*	0.930337*
ν	4.74224*	5.109619*	4.925748*	4.378324*	4.900171*	5.2668*	4.430659*	4.015165*	4.774563*	5.781193*	5.646772*
$\alpha_1 + \beta_1$	0.999	0.975484	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.961153	0.985493
AIC	-4.87767	-4.57883	-4.99516	-2.75439	-4.16988	-4.34567	-5.3233	-3.96323	-4.56947	-4.1455	-3.9174
Q-stat for standardized residuals											
lag1	2.653	0.406	0.307	2.998	6.952	0.008	0.005	1.299	1.367	1.368	0.645
lag3	2.999	0.828	0.772	4.339	7.494	0.586	0.793	2.442	3.087	3.603	2.808
lag7	10.323	7.547	6.528	6.456	10.531	4.343	3.125	4.442	5.409	9.304	6.06
ARCH LM test											
lag2	0.5451	0.059	4.324	0.003	1.692	0.826	0.047	1.497	1.244	5.369	3.268
lag5	1.3756	0.223	9.373	0.241	3.104	2.16	0.348	4.204	4.48	6.654	4.043
lag10	3.862	0.562	11.387	0.582	4.894	3.05	1.111	9.164	6.899	7.933	7.527

Note:* is significant in the 95% confidence interval.

Table 4.2: Results of MA(1)-EGARCH(1,0)

	BEL	DEN	FRA	GRE	IRE	ITA	NET	POR	SPA	SWE	UK
μ	-0.00019	-0.00029	1.92E-05	0.000187	-0.00014	-0.00017	-0.00018	-0.00014	-0.00026	-0.00039	-0.00011
θ	-0.34243*	-0.45268*	-0.48048*	-0.24924*	-0.33173*	-0.27014*	-0.48491*	-0.33622*	-0.30217*	-0.23033*	-0.18705*
$\alpha 0$	-0.05887*	-0.15162*	-0.06527*	-0.06603*	-0.03301*	-0.01592*	-0.03989	-0.05047*	-0.02427*	-0.24019*	-0.05975*
$\beta 1$	0.992167*	0.979542*	0.991508*	0.987961*	0.995293*	0.997969*	0.995033*	0.992279*	0.99662*	0.965449*	0.991166*
$\gamma 1$	0.276111*	0.173309*	0.257267*	0.265489*	0.198386*	0.162121*	0.189471*	0.273383*	0.208379*	0.200036*	0.092029*
ν	3.996966*	5.207032*	4.26362*	3.466955*	4.458685*	4.523871*	3.726167*	3.519717*	4.033725*	5.839347*	5.807454*
AIC	-4.88445	-4.58675	-5.00522	-2.78533	-4.16871	-4.36446	-5.33835	-3.97388	-4.58738	-4.14811	-3.92529
Q-stat for standardized residuals											
lag1	2.29	1.158	0.12	2.065	2.995	0.222	0.029	0.001	2.058	2.011	0.001
lag3	2.676	1.911	0.409	3.76	3.824	1.289	0.307	0.098	6.505	4.747*	1.731
lag7	10.442	8.312	6.89	7.761	8.618	5.067	2.124	2.291	7.725	10.026	5.45
ARCH LM test											
lag2	0.581	0.592	7.594*	0.173	0.611	2.05	0.0097	5.552	1.698	5.609	42.111*
lag5	0.914	0.72	12.381*	0.327	0.801	2.788	0.202	6.975	3.048	6.075	42.579*
lag10	2.151	1.147	18.847*	0.584	1.195	3.239	0.49	12.208	4.343	6.561	45.534*

Note:* is significant in the 95% confidence interval.

Table 4.3: Results of MA(1)-TGARCH(1,1)

	BEL	DEN	FRA	GRE	IRE	ITA	NET	POR	SPA	SWE	UK
μ	-0.00013	-0.00034	1.68E-05	4.19E-05	-0.0001	-0.00015	-0.00014	-0.00016	-0.00024	-0.00044	-0.00012
θ	-0.34019*	-0.44969*	-0.48155*	-0.24573*	-0.33402*	-0.26643*	-0.48293*	-0.33827*	-0.30044*	-0.22685*	-0.18653*
$\alpha 0$	0.000142*	0.000588*	0.000174*	0.000535*	0.000158*	6.12E-05*	0.000101	0.000345*	0.00011*	0.000913*	0.000357
$\alpha 1$	0.156665*	0.113818*	0.14433*	0.19591*	0.125661*	0.108066*	0.144631*	0.174769*	0.139214*	0.105767*	0.053928*
$\beta 1$	0.876154*	0.890765*	0.89129*	0.851958*	0.894857*	0.910654*	0.891647*	0.870463*	0.8955*	0.892559*	0.949712*
δ_1	-0.02407*	-0.0016	-0.01639	-0.03508*	-0.02487*	-0.02553*	-0.01252*	-0.01672	-0.02581*	0.00323*	0.00139*
ν	3.937861*	5.176186*	4.265713*	3.5826*	4.623747*	4.55136*	3.770753*	3.598538*	4.161302*	5.829874*	5.724568*
condition	0.986182	0.956137	0.999887	0.994651	0.980641	0.984313	0.995846	0.997733	0.999783	0.949083	0.981343
AIC	-4.89272	-4.5909	-5.00669	-2.7918	-4.17984	-4.37049	-5.35006	-3.98503	-4.59574	-4.15006	-3.92382
Q-stat for standardized residuals											
lag1	1.133	1.59	0.025	2.935	2.621	0.272	0.085	0.142	3.204	1.923	0.022
lag3	1.702	2.183	0.353	3.066	3.093	0.923	0.591	0.231	5.182	4.702	1.722
lag7	9.507	9.011	6.984	10.861	7.329	4.623	3.03	2.062	7.576	9.93	5.643
ARCH LM test											
lag2	1.818	1.086	10.45*	0.697	1.277	2.241	0.432	9.936*	1.269	5.387	42.88*
lag5	2.603	1.189	15.02*	0.907	1.601	3.219	0.555	11.554*	2.601	5.752	43.48*
lag10	4.145	1.479	21.17*	1.201	2.162	3.669	0.891	16.062*	3.963	6.123	46.99*

Note:* is significant in the 95% confidence interval. The “condition” is the calculated condition ${\alpha_1}^2 + {\beta_1}^2 + {\delta_1}^2 + 2\alpha_1\beta_1\nu_1$, where $\nu_1 = \sqrt{\frac{\nu-2}{\pi}}\frac{\Gamma(\frac{\nu-1}{2})}{\Gamma(\frac{\nu}{2})}$ and if it is smaller than 1, there will be guaranteed positive conditional volatility and second moment for TGARCH model.

4.4.2 Bivariate copula analysis

In the second stage, pairs of data using various families are modeled in order to select the proper copula family by goodness-of-fit tests. Different copula families have different characteristics of tail dependence that allow us to identify the tail-dependence between different pairs.

A copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform. Copulas are used to describe the dependence between random variables.

A d-variate copula $C(u_1, \ldots, u_d)$ is a cumulative distribution function (cdf) with uniform marginals on the unit interval. According to Sklar (1959), if $F_j(x_j)$ is the cdf of a univariate continuous random variable X_j, then $C(F_1(x_1), \ldots, F_d(x_d))$ is a d-variate distribution for $X = (X_1, \ldots, X_d)$ with marginal distributions $F_j, j = 1, \ldots, d$. Conversely, if $F_j, j = 1, \ldots, d$ is continuous, then there exists a unique copula C as

$$F(x) = C(F_1(x_1), \ldots, F_d(x_d)), \forall x = (x_1, \ldots, x_d), \tag{4.10}$$

which is called the theorem of Sklar (1959).

Sklar's Theorem states that any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a copula which describes the dependence structure between the variables.

In the first stage, the different GARCH filters are applied in this research. In the second stage, the Vuong (1989) test and the Clarke (2007) test are used to select the best copulas that fit the pairs as goodness-of-fit tests. These two tests compare two models against each other. Based on their null hypothesis, the tests will identify the better model by a statistically significant decision. Belgorodski (2010) proposes a method using these two tests for copula selection.

Using this method, a bivariate copula model A is compared with all other possible bivariate copula models. If copula model A outperforms another copula model, a score of "+1" is assigned to model A, and at the same time a score of "−1" will be added to the other copula model. No score will be added when the test cannot identify which model is better. There is a total score which sums up the scores we get from all these pairwise comparisons. Both the Vuong test and the Clarke test are likelihood ratio based and use the common Kullback-Leibler information criterion, which measures the distance between two statistical models. For instance, c_1 and c_2 are two bivariate copula with estimated parameters $\hat{\theta_1}$ and $\hat{\theta_2}$ respectively. The Vuong test requires a sum, ν, of the log differences of their point-wise likelihoods m_i. For observations $u_{i,j}$, $i = 1, \ldots, N$, $j = 1, 2$,

$$m_i = log\left[\frac{c_1(u_{i,1}, u_{i,2}|\hat{\theta_1})}{c_2(u_{i,1}, u_{i,2}|\hat{\theta_2})}\right], \tag{4.11}$$

and then

$$\nu = \frac{\frac{1}{n}\sum_{i=1}^{N} m_i}{\sqrt{\sum_{i=1}^{N}(m_i - \bar{m})^2}}. \tag{4.12}$$

The null hypothesis of the Vuong test is

$$H_0 : E(m_i) = 0, \forall i = 1, \ldots, N.$$

Vuong (1989) shows that ν is asymptotically standard normal distributed. Therefore, model A is preferred against model B at level α if

$$\nu > \Phi^{-1}\left(1 - \frac{\alpha}{2}\right). \tag{4.13}$$

In the same manner, if $\nu < -\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$,then model B is chosen. Nonetheless, if $|\nu| \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$, then the test cannot identify if there is a better one which will not reject the null hypothesis of the test as well.

On the other hand, the null hypothesis of the Clarke test is

$$H_0 : P(m_i > 0) = 0.5, \forall i = 1, \ldots, N,$$

and the test statistic is specified as

$$B = \sum_{i=1}^{N} \mathbf{1}_{(0,\infty)}(m_i), \tag{4.14}$$

where **1** is proposed by Clarke (2007) as the indicator of the function. It is binomial distributed with parameters N and $p = 0.5$. Based on this, the critical values can be obtained. Model A is considered statistically equivalent with model B if B is not significantly different from the expected value $Np = \frac{N}{2}$. Both test statistics from equations (4.13) and (4.14) can be corrected for the number of parameters used in the models by using AIC.

Table 4.4 and Table 4.5 show the goodness-of-fit test results of bivariate copula modelling. 11 copula families are chosen which include Gaussian, Student-t, Clayton, Gumbel, Frank, BB1, BB7, and the survival copulas of the Clayton (s.Clayton), Gumbel (s.Gumbel), BB1 (s.BB1) and BB7 (s.BB7)[3] in both tests. In these candidates, families represent various strengths of tail behaviour. For instance, Frank copulas show tail independence which is also considered as a benchmark for tail dependence, Gumbel copulas show only upper tail dependence while Clayton copulas show only lower tail dependence. Student-t copulas show reflection symmetric upper and lower tail dependence and BB families show different upper and lower tail dependence. From the results of the Vuong test, student-t copula family fits 53 out of 55 pairs best in all 11 copula families, although t copula of three pairs (NET.SWE, SPA.SWE, NET.DEN) share the highest score with both survival form of BB1 and survival form of BB7 families. Additionally, both survival form of BB1 and survival form of BB7 families indicate asymmetric upper and lower tail dependence. GRE.SWE and POR.SWE are modeled best by Frank copula, which shows no tail dependence of the pairs, according to Vuong test. On the other hand, the Clarke test shows that student-t copula family fits all 55 pairs better than the others, which means these pairs tend to have symmetric upper and lower tail dependence.

[3]In terms of bivariate copula families and their functions and properties, please see Appendix F.

Table 4.4: Bivariate goodness-of-fit Vuong test

Pairs	Gaussian	t	Clayton	Gumbel	Frank	BB1	BB7	s.Clayton	s.Gumbel	s.BB1	s.BB7
BEL.DEN	-6	**10**	-3	-5	-7	4	4	-9	4	4	4
BEL.FRA	-7	**10**	-8	-2	-3	6	2	-9	2	7	2
BEL.GRE	-7	**10**	-7	0	-7	4	6	-6	-2	4	5
BEL.IRE	-8	**10**	-7	-1	-5	4	4	-8	1	5	5
BEL.ITA	-6	**10**	-9	-1	-6	5	5	-7	-1	5	5
BEL.NET	-7	**10**	-8	0	2	5	0	-9	1	6	0
BEL.POR	-8	**10**	-8	-3	-3	4	2	-8	3	7	4
BEL.SPA	-8	**10**	-8	0	-4	5	4	-8	-1	5	5
BEL.SWE	-2	**7**	-2	-1	0	-1	-1	-1	1	1	-1
BEL.UK	-5	**8**	-5	-4	0	4	2	-10	3	5	2
DEN.FRA	-6	**10**	-6	-3	-7	4	4	-7	3	4	4
DEN.GRE	-6	**10**	-7	-5	3	3	-1	-9	4	6	2
DEN.IRE	-6	**10**	-6	-1	-4	4	2	-8	3	3	3
DEN.ITA	-7	**10**	-6	-3	-6	4	4	-7	3	4	4
DEN.NET	-4	**6**	-6	-3	-8	5	5	-8	1	**6**	**6**
DEN.POR	-7	**10**	-7	-3	3	4	0	-8	2	5	1
DEN.SPA	-6	**10**	-6	-4	-6	4	4	-7	3	4	4
DEN.SWE	-4	**10**	-9	-3	-4	6	2	-9	1	7	3
DEN.UK	-5	**10**	-8	0	-2	4	2	-6	-1	4	2
FRA.GRE	-8	**10**	-8	3	-4	3	3	-6	-1	4	4

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Table 4.4 –continued from previous page

Pairs	Gaussian	t	Clayton	Gumbel	Frank	BB1	BB7	s.Clayton	s.Gumbel	s.BB1	s.BB7
FRA.IRE	-8	**10**	-8	0	2	4	1	-8	1	5	1
FRA.ITA	-6	**10**	-9	-1	-5	5	5	-8	-1	5	5
FRA.NET	-7	**10**	-7	-2	-1	6	1	-10	3	6	1
FRA.POR	-7	**10**	-7	-2	2	5	1	-10	2	5	1
FRA.SPA	-8	**10**	-8	0	0	7	0	-8	0	7	0
FRA.SWE	-6	**9**	-3	-1	-1	0	-1	-3	2	3	1
FRA.UK	-5	**8**	-6	-4	-5	4	4	-9	4	5	4
GRE.IRE	-7	**10**	-7	0	-7	4	6	-7	-1	3	6
GRE.ITA	-8	**10**	-8	0	-5	5	3	-7	0	5	5
GRE.NET	-6	**10**	-6	-1	-4	3	2	-6	0	5	3
GRE.POR	-7	**10**	-10	4	-5	4	4	-5	-3	4	4
GRE.SPA	-7	**10**	-7	0	-7	4	6	-6	-2	4	5
GRE.SWE	-1	7	-6	-6	**8**	1	-4	-9	3	7	0
GRE.UK	-3	**9**	-5	0	0	3	-1	-6	0	3	0
IRE.ITA	-7	**10**	-7	-1	-7	4	4	-7	1	5	5
IRE.NET	-8	**10**	-8	1	2	5	1	-8	1	4	0
IRE.POR	-7	**10**	-7	-1	-7	5	5	-7	-1	5	5
IRE.SPA	-8	**10**	-7	-1	-5	4	4	-8	1	5	5
IRE.SWE	0	**7**	-1	-3	2	-1	-1	-3	-1	3	-2
IRE.UK	-6	**10**	-5	0	2	3	0	-8	2	3	-1

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Table 4.4 –continued from previous page

Pairs	Gaussian	t	Clayton	Gumbel	Frank	BB1	BB7	s.Clayton	s.Gumbel	s.BB1	s.BB7
ITA.NET	-8	**10**	-8	1	0	6	1	-8	1	5	0
ITA.POR	-8	**10**	-8	-2	-3	5	2	-8	2	7	3
ITA.SPA	-6	**10**	-9	-1	-6	5	6	-7	-1	5	4
ITA.SWE	-3	**10**	-7	0	0	2	-1	-6	0	3	2
ITA.UK	-6	**10**	-6	0	-6	3	3	-7	3	3	3
NET.POR	-7	**10**	-8	-1	3	5	0	-9	1	6	0
NET.SPA	-7	**10**	-8	-1	3	5	0	-9	2	5	0
NET.SWE	-6	**7**	-4	-5	-6	1	1	-6	4	**7**	**7**
NET.UK	-5	**10**	-6	-4	-6	4	4	-8	3	4	4
POR.SPA	-9	**10**	-7	-2	-3	5	2	-8	2	7	3
POR.SWE	2	6	-4	-4	**10**	0	-6	-5	1	3	-3
POR.UK	-4	**8**	-4	-4	7	3	-4	-10	6	5	-3
SPA.SWE	1	**2**	0	-3	1	0	0	-5	0	**2**	**2**
SPA.UK	-5	**8**	-5	-5	-1	4	2	-10	4	6	2
SWE.UK	-4	**7**	-6	0	-3	4	3	-6	0	4	1

Note: There are no order in the pair names.

Table 4.5: Bivariate goodness-of-fit Clarke test

Pairs	Gaussian	t	Clayton	Gumbel	Frank	BB1	BB7	s.Clayton	s.Gumbel	s.BB1	s.BB7
BEL.DEN	-9	**10**	-3	-3	-6	3	6	-9	2	1	8
BEL.FRA	-6	**10**	-8	-2	8	6	-3	-10	2	4	-1
BEL.GRE	-9	**10**	-8	0	-4	3	7	-7	-2	5	5
BEL.IRE	-9	**10**	-6	-4	6	5	2	-9	0	3	2
BEL.ITA	-6	**10**	-10	-1	8	5	-1	-8	-1	5	-1
BEL.NET	-6	**10**	-8	0	8	6	-3	-10	2	4	-3
BEL.POR	-9	**10**	-6	-4	8	6	0	-9	0	4	0
BEL.SPA	-6	**10**	-9	0	4	7	-1	-9	-1	7	-2
BEL.SWE	-10	**10**	-5	-1	-6	3	1	-6	6	2	6
BEL.UK	-8	**10**	-5	-4	-3	5	4	-10	3	3	5
DEN.FRA	-9	**10**	-5	-2	-5	3	6	-9	0	4	7
DEN.GRE	-8	**10**	-6	-3	4	3	-2	-10	5	4	3
DEN.IRE	-9	**10**	-6	-2	-5	4	5	-8	2	3	6
DEN.ITA	-9	**10**	-6	-2	-5	5	5	-8	0	3	7
DEN.NET	-8	**10**	-5	-2	-5	3	7	-10	0	4	6
DEN.POR	-9	**10**	-6	-3	4	2	-2	-9	5	5	3
DEN.SPA	-9	**10**	-4	-2	-7	3	6	-8	2	1	8
DEN.SWE	-6	**10**	-8	-3	8	6	1	-10	0	3	-1
DEN.UK	-8	**10**	-8	0	-3	4	4	-8	-1	6	4
FRA.GRE	-10	**10**	-7	-1	-4	4	6	-7	-1	6	4

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Table 4.5 –continued from previous page

Pairs	Gaussian	t	Clayton	Gumbel	Frank	BB1	BB7	s.Clayton	s.Gumbel	s.BB1	s.BB7
FRA.IRE	-9	**10**	-6	-3	8	4	0	-9	1	2	2
FRA.ITA	-6	**10**	-9	-1	8	3	1	-9	-1	4	0
FRA.NET	-6	**10**	-8	-1	8	5	-2	-10	2	5	-3
FRA.POR	-8	**10**	-6	-3	8	4	-3	-10	2	4	2
FRA.SPA	-6	**10**	-8	-1	8	5	-3	-10	2	5	-2
FRA.SWE	-10	**10**	-6	-1	-6	2	3	-6	5	2	7
FRA.UK	-8	**10**	-6	-3	-3	6	4	-10	1	4	5
GRE.IRE	-10	**10**	-8	0	-4	2	7	-6	-2	5	6
GRE.ITA	-10	**10**	-8	-2	7	3	2	-6	-1	4	1
GRE.NET	-10	**10**	-7	-1	-4	6	5	-7	-1	4	5
GRE.POR	-9	**10**	-9	0	7	4	2	-6	-4	3	2
GRE.SPA	-9	**10**	-9	0	-4	4	6	-6	-2	6	4
GRE.SWE	-7	**10**	-7	-3	7	2	-3	-10	4	5	2
GRE.UK	-9	**10**	-7	-1	-4	4	2	-8	6	3	4
IRE.ITA	-9	**10**	-6	-4	1	6	3	-9	0	4	4
IRE.NET	-9	**10**	-7	-1	8	6	0	-8	0	2	-1
IRE.POR	-10	**10**	-7	-2	1	5	3	-7	-1	5	3
IRE.SPA	-10	**10**	-6	-4	6	5	2	-8	0	3	2
IRE.SWE	-9	**10**	-6	-1	-4	2	0	-7	4	6	5
IRE.UK	-9	**10**	-6	-3	-1	3	1	-9	7	3	4

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Table 4.5 –continued from previous page

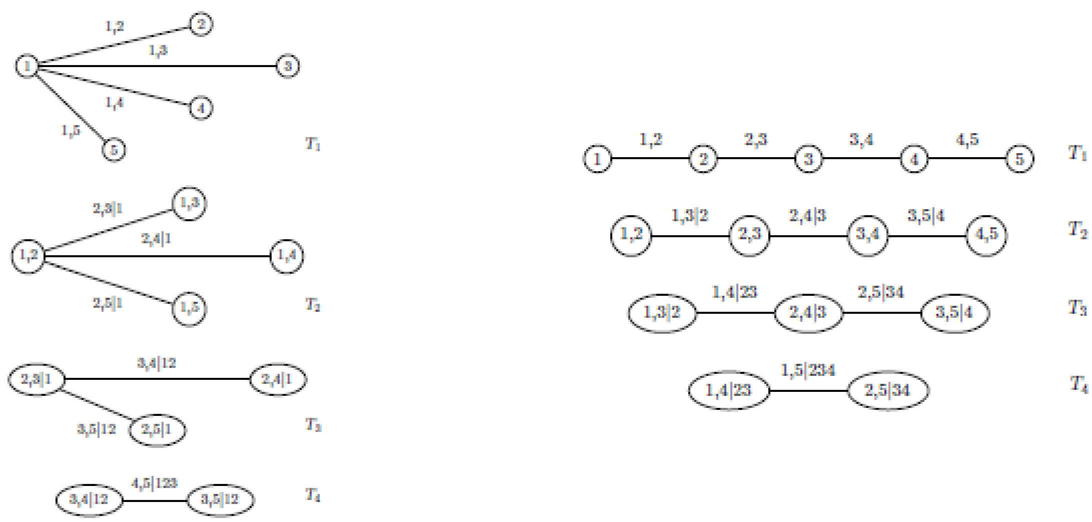
Pairs	Gaussian	t	Clayton	Gumbel	Frank	BB1	BB7	s.Clayton	s.Gumbel	s.BB1	s.BB7
ITA.NET	-7	**10**	-7	-2	8	5	-1	-10	0	4	0
ITA.POR	-9	**10**	-6	-2	8	6	-2	-9	0	4	0
ITA.SPA	-6	**10**	-10	-1	5	7	-1	-8	-2	6	0
ITA.SWE	-8	**10**	-8	-2	-4	6	4	-8	2	4	4
ITA.UK	-9	**10**	-5	-2	-5	3	6	-9	3	2	6
NET.POR	-8	**10**	-6	-2	8	4	-4	-10	2	5	1
NET.SPA	-8	**10**	-6	-1	8	5	-4	-10	2	5	-1
NET.SWE	-10	**10**	-4	-3	-7	1	5	-6	2	4	8
NET.UK	-8	**10**	-6	-2	-4	5	5	-10	0	3	7
POR.SPA	-10	**10**	-6	-3	8	6	-1	-8	0	4	0
POR.SWE	-7	**9**	-8	-2	7	2	-3	-9	6	4	1
POR.UK	-8	**10**	-6	-3	6	1	-3	-10	6	5	2
SPA.SWE	-9	**10**	-5	-2	-6	4	0	-7	4	5	6
SPA.UK	-8	**10**	-6	-3	-3	5	4	-10	2	4	5
SWE.UK	-8	**10**	-8	-1	-4	4	6	-8	0	5	4

Note: There are no order in the pair names.

4.5 Vine Copula approach

4.5.1 Introduction of Vine Copulas

In order to improve the copula method with regard to a wider range of dependence, a more flexible range of upper and lower tail dependence, a larger dimension and a computationally feasible density for estimation, vine copulas became a handy copula technique.



Source: Brechmann and Schepsmeier (2012)

Figure 4.1: Examples of 5-dimensional C- (left) and D-vine (right)

A d-dimensional vine copula are built by $d(d-1)$ bivariate copulas in a $d-1$-level tree form. There are different ways to construct a copula tree. C-vines and D-vines are the selected tree types in this chapter. In a C-vine tree, the dependence with respect to one particular variable, called first root node, is modeled using bivariate copulas for each pair. Conditioned on this variable, pair wise dependencies with respect to a second variable are modeled, which is called the second root node. In general, a root node is chosen in each tree and all pairwise dependencies with respect to this node are modeled conditioned on all previous root nodes (see Figure 4.1 left panel). According to Aas et al. (2009) this gives the following decomposition of a multivariate density,

$$f(x) = \prod_{k=1}^{d} f_k(x_k) \times \prod_{i=1}^{d-1} \prod_{j=1}^{d-i} c_{i,i+j|1:(i-1)}(F(x_i|x_1,\ldots,x_{i-1}),(F(x_{i+j}|x_1,\ldots,x_{i-1})|\boldsymbol{\theta}_{i,i+j|1:(i-1)}), \tag{4.15}$$

where $f_k, k = 1,\ldots,d$, denote the marginal densities and $c_{i,i+j|1:(i-1)}$ bivariate copula densities with parameter(s) $\boldsymbol{\theta}_{i,i+j|1:(i-1)}$ (here $i_k : i_m$ means $i_k,\ldots,i_m$. And the outer product runs over the $d-1$ trees and root nodes i, while the inner product refers to the $d-i$ pair-copulas in each tree $i = 1,\ldots,d-1$.

A D-vine chooses the order of these pairs in a different way (see Figure 4.1 right panel). In the first level of the tree, the dependence of the first and second variable, the second and the third, the third and the fourth, and so on, are used. That means in a 5-dimensional vine copula, in the first level of the tree, pairs $(1,2),(2,3),(3,4),(4,5)$ have been modeled. While in the second level of the tree, conditional dependence of the first and third given the second variable (pair $(1,3|2)$), the second and fourth given the third (pair $(2,4|3)$), and so on. In this way it continues to construct the third level up to the $d-1$ level. According to Aas et al. (2009) the density of a D-vine is,

$$f(x) = \prod_{k=1}^{d} f_k(x_k) \times$$
$$\prod_{i=1}^{d-1} \prod_{j=1}^{d-i} c_{j,j+i|(j+1):(j+i-1)}(F(x_j|x_{j+1},\ldots,x_{j+i-1}),(F(x_{j+i}|x_{j+1},\ldots,x_{j+i-1})|\theta_{j,j+i|x_{j+1},\ldots,x_{j+i-1}}), \tag{4.16}$$

where the outer product runs over the $d-1$ trees, while the pairs in each tree are designated by the inner product. In order to get the conditional distribution functions $F(x|\mathbf{v})$ for an m-dimensional vector $\mathbf{v}$, one can sequentially apply the following relationship,

$$h(x|\mathbf{v},\boldsymbol{\theta}) := F(x|\mathbf{v}) = \frac{\partial C_{xv_j|\mathbf{v}_{-j}}(F(x|\mathbf{v}_{-j}),F(v_j|\mathbf{v}_{-j})|\boldsymbol{\theta})}{\partial F(v_j|\mathbf{v}_{-j}} \tag{4.17}$$

where v_j is an arbitrary component of $\mathbf{v}$ and $\mathbf{v}_{-j}$ denotes the $(m-1)$-dimensional vector $\mathbf{v}$ excluding v_j. Further $C_{xv_j|\mathbf{v}_{-j}}$ is a bivariate copula distribution function with parameter(s) θ specified in tree m.

4.5.2 Vine copula estimation

Vine copulas can be constructed by the bivariate copulas estimated in section 4.4. 2 types of vine are chosen to be estimated, C-vine and D-vine, and then one will choose the better one base on their value of log-likelihood. First, a C-vine has been conducted. In order to achieve the best performance of the C-vine, $d-1$ pairs of countries should be carefully chosen. According to Aas and Berg (2009) empirical rules can be applied to select to vine order.

1. Select the first root node that has strong dependence with all other variables;
2. List the most dependent variables with the first root node as decreasing in dependence order;
3. List the least dependent variables with the first root node as increasing in dependence order;
4. Sequentially list the least dependent variable with the previous selected.

Table 4.6 shows the dependence of pairs according to Kendall's τ. Kendall's τ is a rank correlation coefficient which introduced by Kendall (1938). It is calculated as follows. Let (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) be a set of observations of the joint random variables X and Y respectively, such that all the values of (x_i) and (y_i) are unique. Any pair of observations (x_i, y_i) and (x_j, y_j) are said to be concordant if the ranks for both elements agree: that is, if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$. They are said to be discordant, if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant.

$$\tau = \frac{(\text{number of concordant pair}) - (\text{number of discordant pair})}{\frac{1}{2}n(n-1)}$$

The first root node should have strong dependence with all other variables. In this case Spain shows the strongest dependence with others. Applying the rest of rules the order

Table 4.6: Country-pair dependence base on Kendall's τ

Pair	τ	Pair	τ	Pair	τ
BEL.DEN	0.113469	FRA.GRE	0.165981	IRE.SWE	0.08461
BEL.FRA	0.526231	FRA.IRE	0.250222	IRE.UK	0.122281
BEL.GRE	0.193722	FRA.ITA	0.337805	ITA.NET	0.313732
BEL.IRE	0.29413	FRA.NET	0.56691	ITA.POR	0.358453
BEL.ITA	0.412142	FRA.POR	0.310606	ITA.SPA	0.507106
BEL.NET	0.523141	FRA.SPA	0.461769	ITA.SWE	0.126307
BEL.POR	0.356663	FRA.SWE	0.066616	ITA.UK	0.107109
BEL.SPA	0.537487	FRA.UK	0.154579	NET.POR	0.299719
BEL.SWE	0.058031	GRE.IRE	0.202541	NET.SPA	0.443678
BEL.UK	0.151617	GRE.ITA	0.299755	NET.SWE	0.071075
DEN.FRA	0.13127	GRE.NET	0.13684	NET.UK	0.160818
DEN.GRE	0.201002	GRE.POR	0.297087	POR.SPA	0.384536
DEN.IRE	0.112917	GRE.SPA	0.228495	POR.SWE	0.12385
DEN.ITA	0.123432	GRE.SWE	0.198992	POR.UK	0.193094
DEN.NET	0.16681	GRE.UK	0.114702	SPA.SWE	0.082487
DEN.POR	0.179232	IRE.ITA	0.274777	SPA.UK	0.139775
DEN.SPA	0.103206	IRE.NET	0.255291	SWE.UK	0.127193
DEN.SWE	0.348671	IRE.POR	0.337959		
DEN.UK	0.180609	IRE.SPA	0.314839		

Note: There are no order in the pair names.

of the C-vine is chosen as SPA, BEL, DEN, FRA, GRE, IRE, ITA, NET, POR, SWE, UK.[4] The log-likelihood function with parameter $\boldsymbol{\theta}_{CV}$ is as follows:

$$\ell_{CV}(\boldsymbol{\theta}_{CV}|\boldsymbol{u}) = \sum_{k=1}^{N}\sum_{i=1}^{d-1}\sum_{j=1}^{d-i} \log[c_{i,i+j|1:(i-1)}(F_{i|1:(i-1)}, F_{i+j|1:(i-1)}|\boldsymbol{\theta}_{i,i+j|1:(i-1)})], \quad (4.18)$$

where $F_{j|i_1:i_m} := F(u_{k,j}|u_{k,i_1}, \cdots, u_{k,i_m})$ and the marginal distribution are uniform.

In the case of D-vine, the empirical rule for first tree selection is choosing an order of the variables that intends to capture as much dependence as possible. According to It is equivalent to solve the Traveling Salesman Problem (TSP). The TSP is solved using the Cheapest Insertion Algorithm[5] . Using information from Table 4.6 with the algorithm,

[4]The estimated dependence parameters are shown in Table G.1. Figure H.1 to H.3 show the C-vine tree structure of each level.

[5]see Appendix E for details

the order of D-vine is chosen as IRE, POR, GRE, ITA, SPA, BEL, FRA, NET, UK, DEN, SWE.[6]

The log-likelihood with parameter $\boldsymbol{\theta}_{DV}$ is as follows:

$$\ell_{DV}(\boldsymbol{\theta}_{DV}|\boldsymbol{u}) = \sum_{k=1}^{N}\sum_{i=1}^{d-1}\sum_{j=1}^{d-i}\log[c_{j,j+i|(j+1):(j+i-1)}(F_{j|(j+1):(j+i-1)},F_{j+i|(j+1):(j+i-1)}|\boldsymbol{\theta}_{j,j+i|(j+1):(j+i-1)})]. \tag{4.19}$$

The estimatied log-likelihood of C-vine is 12934.83, while the log-likelihood of D-vine is 12805.13. Therefore, C-vine is superior to D-vine.

4.6 Simulation

In this chapter, we intend to forecast the probabilities of sovereign crisis in these 11 countries in the future year. Therefore, the sovereign spreads of each country for the next 365 days need to be generated. 365 groups of error terms based on the C-vine copula parameters are simulated. 365 is the forecast horizon of this research and it can be changed depending on the purpose of forecast. We apply these group of error terms back into the GARCH filters estimated in Section 4.4.1 to get the next 365 days' sovereign spreads movements of each country. Future sovereign spreads can be calculated by adding spreads movement to the spreads of previous day from 12/03/2012 which is the last day in the sample. We apply the definition of sovereign crisis is stated in Section 4.1, which is that sovereign spread against Germany is more than 1000 basis points. Therefore, if one or more of these simulated spreads are greater than 10%, the sovereign crisis in the following year will be counted. This process is repeated 10000 times, and the times with sovereign crisis divided by 10000 will be the probabilities of sovereign debt crisis. The relationship can be represented by the expression as follows:

$$k_i = \begin{cases} 1 & \text{if there is at least one crisis event in future } h\text{-day simulation} \\ 0 & \text{if there is no event in future } h\text{-day simulation} \end{cases}$$

[6]Table **??** shows the estimated dependence parameters. Figure **??** to **??** show the D-vine tree structure of each level.

The probability of the soveriegn debt crisis is expressed as

$$Pr = \frac{\sum_{i=1}^{N} k_i}{N},$$

where k is a dummy in order to identify whether there will be one or more crisis in the forecasting horizon, h is the forecast horizon (365 days) , i is the ith simulation, Pr is the probability of sovereign crisis in target country and N is the total number of simulations (10000).

Table 4.7: Probability of sovereign debt crisis in next 365 days

Countries	BEL	DEN	FRA	GRE	IRE	ITA	NET	POR	SPA	SWE	UK
Probability	60.14%	8.74%	62.13%	100%	71.60%	81.08%	25.86%	99.77%	87.17%	5.45%	12.87%

Table 4.7 presents the results of the estimated probabilities of sovereign crisis in the next 365 days. According to the results, Greece has the highest probability which is 100% and is followed by Portugal (99.77%), which are consistent with the fact that they are already in crisis. Spain (87.17%) and Italy (81.08%) have extremely high probabilities of entering crisis. Ireland has a 71.06% chance of entering crisis. The probability of crisis in France and Belgium are 62.13% and 60.14% which are fairly high, and for France it is higher than expected. Netherlands (25.86%) shows a fairly low probability of crisis, the most stable in the euro area. The probability of countries outside the euro area such as the UK (12.87%), Denmark (8.74%) and Sweden (5.45%) are very low which reveals the stability of sovereign debt in these countries. The results confirm the observation of **?** which is being non-members of a monetary union is not easily forced to default by the financial market. The reason is that these countries are able to control the currency by which they issue debt.

4.7 Conclusion

This chapter provides a method to calculate the probability of sovereign debt crisis which is an infrequent event. The sovereign spreads against Germany are simulated and the dependence of those time series is considered by applying vine copula models in the mean time. It is extremely useful in assessing the risk level of sovereign debt crisis in the European Union. We examined 11 countries in the European Union. Results show that Greece and Portugal have an extremely high probability of sovereign debt crisis. Spain and Italy are potentially the next victims of sovereign debt crisis. Unexpectedly, France and Belgium show a fairly high risk level. Netherlands enjoys the lowest probability of crisis in the euro area in the sample. The UK, Denmark and Sweden show strong stability of their sovereign debt and being outside the euro area might be the reason for this. According to the results, the probability calculated in this paper appears to be a very good indicator of sovereign debt default risk level. In addition, it is a better indicator than sovereign credit default swap (CDS), because sovereign CDS is an over the counter (OTC) traded financial instrument, which makes tracking all the trades difficult to achieve. This indicator can make a contribution to alerting the European Central Bank (ECB) or governments of those countries in the European Union, as well as ranking the risk level of each government bond in the European Union for investors.

Chapter 5

Conclusion

5.1 Main findings and contributions

This thesis makes a number of contributions to the literature in government bond yields analysis by exploring and making use of the information contained in them.

Chapter 2 proposes a model of the term structure of interest rates forecasting output growth and recessions in the UK by using the whole yield curve rather than just the yield spread. The idea is to use the Diebold-Li framework, which can extract short-, mid- and long-term factors as yield curve variables, and the B-spline technique to form a yield curve other than traditional yield spread to forecast recession and economic growth, and we get satisfactory results. The research shows strong evidence that in terms of recession forecasting the Diebold-Li framework does better than the yield spread and the B-spline model shows even better performance. Furthermore, from the economic explanation perspective, the in-sample test results suggest short-term yields, which represent monetary policy play a very important role in real GDP growth forecasting in both Model A and B. It is interesting to note that the Diebold-Li and B-spline frameworks enjoy better fitting of the yield curve than the yield spread model. This also leads to a better forecasting ability in mid horizon forecast especially, 5-quarter ahead for Model A based on the Diebold-Li framework and 4-quarter ahead fro Model A based on the B-spline framework. The forecast ability peaks 1-quarter ahead forecasting for Model B. From out-of-sample tests Model B based on both the Diebold-Li framework and the B-spline framework achieves very satisfactory results and shows a stable forecasting ability in all forecasting horizons (1- to 8-quarters ahead). The whole yield curve models outperform those results from the models based on yield spread. From the comparison, it is important to note that Model B based on the B-spline framework generates the closest forecast results to those from HM Treasury.

Following Chapter 2, Chapter 3 examines the non-linear behaviour in output growth / recession-term spread relationship using UK data covering the last 34 years. The research conducts comparisons of VAR, TVAR, 2TVAR, SBVAR, 2SBVAR, SBTVARc, SBTVAR with 1 to 3 autoregressive orders. The results suggest there are non-linearities in the relationship. And evidence shows that the type of this non-linearity is structural break. Introducing structural break(s) into the model does improve the explanatory power of the output growth - yield spread relationship, as well as the predictive power of the model. 2SBVAR(2) (model with 2 structural breaks and 2 autoregressive orders) is the best tested in-sample estimating model. The breaks are located at first quarter of 1986 and the third quarter of 1991. The first break can be explained by the expectation adjustment of people after recovery from the early 1980s' recession. And the second break is caused by the decision of the government to apply inflation targeting policy. In out-of-sample tests, the models with structure break(s) enjoy the robustness on arrival of new information. This also indicates the most recent financial crisis does not change the fundamental being of the relationship. The results from out-of-sample tests are slightly different from in-sample tests. SBVAR(1), VAR(3), 2SBVAR(2) and SBVAR(2) enjoy the best forecast ability in 1-quarter, 2-quarter, 3-quarter and 4-quarter ahead forecast respectively. Generally speaking SBVAR basically dominated the out-of-sample forecasts and 2SBVAR is almost as good. In terms of recession predicting, all models presented in the paper except Threshold models give a fairly good performance.

By analysing government bond yields, Chapter 4 proposes a method for calculating the probability of an infrequent event such as a sovereign debt crisis. A simulation of the sovereign spreads for European countries against Germany has been done in the chapter and with the consideration of the dependence of those time series by applying vine copula models in the mean time. This is extremely useful in assessing the risk level of sovereign debt crisis in the European Union which contains a one and only actual monetary union – the European Monetary Union (EMU). Eleven countries in the European Union are examined, and the results are quite consistent with the situation experienced at that moment as of March 2012. Results show that Greece and Portugal have an extremely high probability of sovereign debt crisis. Spain and Italy seem to be the next mostly likely victims of sovereign debt crisis from the end of the sample period. Unexpectedly, France and Belgium show a fairly high risk level. Netherlands enjoys the lowest probability of crisis in the Eurozone in the sample. The UK, Denmark and Sweden show strong stability of their sovereign debt and being outside the Eurozone might be the reason for this. According to the results, the probabilities calculated in this paper make very good indicators of sovereign debt default risk level. And it is a better indicator than sovereign CDS. After all, sovereign CDS is an over the counter (OTC) traded financial instrument, and it is hard to track all the trades. The proposed indicator can make a contribution to alert the ECB or government of those countries in the European Union, as well as ranking the risk level of each government bond in the European Union for investors.

5.2 Limitations and Future Research

It is unavoidable that in these studies there are limitations and these limitations form the basis of very good directions of further research about the subject.

In Chapter 2, regarding the comparison of forecasting with HM Treasury, there will be some of the difficulties in interpreting evidence in forecast performance comparisons. This is one of the limitations of this chapter. Because when we make this comparison, it is actually after-the-fact, in which the rules and objective of the competition were not specified ahead of time to the players. In this situation, there is an obvious potential risk that by selective reporting of results, one could give a misleading picture of the results for various reasons. This is especially true here, since different models, designed for different purposes, are specified at different levels of aggregation, and are used to forecast over various horizons. Secondly, the timing of the release of economic forecasts is another important consideration in any forecasting comparison. Forecasts are not generally published on the same date, so they will to some extent be based on slightly different information sets. It would be nice to have a model that outperforms the HM Treasury forecasts. It is hard to address this limitation, however, using real-time GDP growth could make the comparison more reasonable. After all, it is the real-time growth we are trying to forecast.

A second limitation of Chapter 2 is that in the sample period there could be non-linearity behavior about the relationship between the yield curve and recession or output growth. And this limitation has been researched in Chapter 3.

In Chapter 3, there are limitations about the comparison as well but from a different perspective. For models with more regimes (SBTVARc and SBTVAR) in this limited sample size research, the disadvantage in the comparison is there might be regimes with a small amount of data in the model. This will trigger the parsimony problem of the VAR estimations, especially for models with higher autoregressive orders. We have seen that this problem can lead to abnormal forecasts as was discussed in the results section. And also because of the sample size, the author is not able to use the model from Chapter 2 which uses a more advanced form of yield curve to do the forecasting. However, when the data availability is better, it could be a promising further research direction. From the future research perspective, the yield curve with non-linearities can be used as a leading indicator not only in GDP or recession forecasting but also other macro variable such as inflation, exchange rates and so on.

In Chapter 4, despite the result of GARCH based vine-copula method being satisfactory, there are two limitations worth mentioning. Firstly, GARCH may not be the best model to describe the volatility of yield movements of these countries. There are more sophisticated models such as GARCH-in-mean, fGARCH, TGARCH, GJR-GARCH which may enjoy a better goodness-of-fit. This is worth digging up in further research. Secondly, events like a sovereign debt crisis tend to have a contagious effect, which cannot be captured by the model brought in this chapter although the research does capture dependence. This dependence is without time difference and tends to be a one-time one-way dependence. It is more likely that contagion effects happen with lag of time and it can pass back in a manner similar to the transmission of a medical disease. This is an interesting topic to pursue in the future.

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Appendices

Appendix A

Data source of Chapter 2

Here are the data source code from ECOWIN.

ew:gbr01021 United Kingdom, Expenditure Approach, Gross Domestic Product, Total, Constant Prices, GBP, 2005 CHND PRC

ew:gbr40312 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 3 month (0.25 year), Yield, GBP

ew:gbr40315 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 6 month (0.50 year), Yield, GBP

ew:gbr40318 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 9 month (0.75 year), Yield, GBP

ew:gbr40321 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 12 month (1.00 year), Yield, GBP

ew:gbr40324 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 15 month (1.25 year), Yield, GBP

ew:gbr40327 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 18 month (1.50 year), Yield, GBP

ew:gbr40330 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 21 month (1.75 year), Yield, GBP

ew:gbr40333 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 24 month (2.00 year), Yield, GBP

ew:gbr40339 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 30 month (2.50 year), Yield, GBP ew:gbr40345 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 36 month (3.00 year), Yield, GBP

ew:gbr40351 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 42 month (3.50 year), Yield, GBP

ew:gbr40357 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 48 month (4.00 year), Yield, GBP

ew:gbr40363 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 54 month (4.50 year), Yield, GBP

ew:gbr40369 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 60 month (5.00 year), Yield, GBP

ew:gbr40381 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 6.0 year, Yield, GBP

ew:gbr40383 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 7.0 year, Yield, GBP

ew:gbr40385 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 8.0 year, Yield, GBP

ew:gbr40387 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 9.0 year, Yield, GBP

ew:gbr40389 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 10.0 year, Yield, GBP

ew:gbr40399 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 15.0 year, Yield, GBP

Appendix B

Data source of Chapter 3

Here are the data source code from ECOWIN.

ew:gbr01021 United Kingdom, Expenditure Approach, Gross Domestic Product, Total, Constant Prices, GBP, 2005 CHND PRC

ew:gbr14200 United Kingdom, Treasury Bills, Bid, 3 Month, Yield, Close, GBP

ew:gbr14130 United Kingdom, Government Benchmarks, Bid, 10 Year, Yield, Close, GBP

Appendix C

Other analysising results of Chapter 3

Table C.1: VAR(1) Estimated coefficients

		VAR		TVAR		2TVAR		SBVAR		2SBVAR		SBTVARc		SBTVAR	
		Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S
Regime 1	constant	0.203	0.355	-0.206	0.331	0.503	0.421	0.279	0.695	0.251	0.705	-0.403	1.135	-0.355	1.116
	$\beta_{\Delta y,t-1}$	0.896	-0.017	1.019	0.059	0.828	0.003	0.828	0.213	0.788	0.157	0.915	0.454	0.931	0.447
	$\beta_{S,t-1}$	0.008	0.871	-0.120	0.806	-0.110	0.797	0.101	0.479	-0.003	0.333	0.109	-0.046	0.115	-0.049
Regime 2	constant			0.610	0.041	-2.405	0.038	0.123	0.266	0.687	1.757	0.931	0.215	0.873	0.237
	$\beta_{\Delta y,t-1}$			0.867	-0.040	1.456	0.187	0.913	-0.047	0.600	-0.340	0.720	-0.037	0.720	-0.036
	$\beta_{S,t-1}$			-0.049	0.972	0.336	0.843	0.009	0.933	0.310	0.914	0.094	0.921	0.102	0.918
Regime 3	constant					0.603	0.061			0.130	0.223	-0.241	0.427	-0.446	0.301
	$\beta_{\Delta y,t-1}$					0.868	-0.041			0.921	-0.040	1.056	-0.078	1.114	-0.029
	$\beta_{S,t-1}$					-0.048	0.969			0.004	0.936	-0.182	1.022	-0.186	1.097
Regime 4	constant											0.248	-0.165	0.280	-0.154
	$\beta_{\Delta y,t-1}$											0.903	-0.033	0.897	-0.037
	$\beta_{S,t-1}$											0.017	1.024	0.004	1.024

Table C.2: VAR(3) Estimated coefficients

		VAR		TVAR		2TVAR		SBVAR		2SBVAR		SBTVARc		SBTVAR	
		Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S
Regime 1	constant	0.367	0.407	0.495	1.105	0.495	1.105	0.297	0.705	0.607	1.074	1.045	2.071	0.294	0.728
	$\beta_{\Delta y,t-1}$	1.102	-0.096	0.952	-0.182	0.952	-0.182	0.812	0.064	0.752	-0.092	0.820	1.669	0.773	0.181
	$\beta_{\Delta y,t-2}$	-0.029	0.260	-0.266	-0.014	-0.266	-0.014	0.189	0.368	0.195	0.466	-0.450	-4.030	-0.296	0.118
	$\beta_{\Delta y,t-3}$	-0.240	-0.203	0.115	-0.029	0.115	-0.029	-0.181	-0.198	-0.164	-0.144	0.301	1.478	0.078	-0.141
	$\beta_{S,t-1}$	0.177	1.068	0.424	0.635	0.424	0.635	0.318	0.678	0.177	0.406	0.009	-0.050	0.589	0.616
	$\beta_{S,t-2}$	-0.377	-0.378	-0.683	0.666	-0.683	0.666	-0.334	-0.417	-0.320	-0.315	0.212	-0.214	-0.083	-0.193
	$\beta_{S,t-3}$	0.205	0.183	0.015	-0.328	0.015	-0.328	0.130	0.196	-0.082	-0.054	-0.485	0.639	-0.016	0.056
Regime 2	constant			0.546	0.402	0.533	-0.118	0.516	0.319	0.974	2.524	-0.218	0.566	3.505	-0.563
	$\beta_{\Delta y,t-1}$			1.343	-0.250	1.438	-0.094	1.459	-0.048	0.370	-0.259	0.875	0.008	0.791	0.061
	$\beta_{\Delta y,t-2}$			-0.246	0.487	-0.468	0.172	-0.569	0.008	0.185	0.121	0.135	0.418	0.544	0.232
	$\beta_{\Delta y,t-3}$			-0.234	-0.296	-0.099	-0.109	-0.067	0.000	-0.264	-0.441	-0.216	-0.335	-0.466	-0.311
	$\beta_{S,t-1}$			-0.217	1.329	-0.240	1.297	-0.238	1.352	0.449	1.039	0.515	0.840	-0.453	1.009
	$\beta_{S,t-2}$			-0.148	-0.730	0.101	-0.262	0.065	-0.351	-0.424	-0.756	-0.690	-0.418	-0.276	-0.793
	$\beta_{S,t-3}$			0.301	0.297	0.066	0.000	0.115	-0.094	0.637	0.715	0.479	0.225	-0.027	0.891
Regime 3	constant					-0.665	3.120			0.438	0.249	2.713	-0.485	3.111	-0.315
	$\beta_{\Delta y,t-1}$					1.269	-0.328			1.566	0.008	1.111	0.153	0.754	0.023
	$\beta_{\Delta y,t-2}$					-0.192	0.684			-0.744	-0.079	-0.572	-0.320	-0.186	-0.183
	$\beta_{\Delta y,t-3}$					-0.230	-0.640			0.019	0.045	-0.343	0.369	-0.486	0.316
	$\beta_{S,t-1}$					-0.349	0.614			-0.232	1.343	-0.449	1.548	-0.604	1.511
	$\beta_{S,t-2}$					-0.037	-0.749			0.053	-0.316	-0.001	-0.466	0.368	-0.402
	$\beta_{S,t-3}$					0.598	0.529			0.132	-0.107	-0.025	-0.224	-0.295	-0.261
Regime 4	constant											0.223	0.108	0.219	0.108
	$\beta_{\Delta y,t-1}$											1.645	-0.042	1.723	-0.002
	$\beta_{\Delta y,t-2}$											-0.871	0.022	-0.998	-0.052
	$\beta_{\Delta y,t-3}$											0.098	-0.019	0.154	0.021
	$\beta_{S,t-1}$											-0.184	1.257	-0.194	1.270
	$\beta_{S,t-2}$											0.117	-0.259	0.085	-0.249
	$\beta_{S,t-3}$											0.073	-0.035	0.112	-0.060

Appendix D

Data source of Chapter 4

Here are the data source code from ECOWIN.

ew:bel14130 Belgium, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:dnk14130 Denmark, Government Benchmarks, Bid, 10 Year, Yield, Close, DKK

ew:fra14130 France, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:deu14130 Germany, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:grd14130 Greece, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:irl14130 Ireland, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:ita14130 Italy, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:nld14130 Netherlands, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:prt14130 Portugal, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:esp14130 Spain, Government Benchmarks, Bid, 10 Year, Yield, Close, EUR

ew:swe14130 Sweden, Government Benchmarks, Bid, 10 Year, Yield, Close, SEK

ew:gbr14130 United Kingdom, Government Benchmarks, Bid, 10 Year, Yield, Close, GBP

Appendix E

Cheapest Insertion Algorithm

Cheapest insertion algorithms for a symmetric and asymmetric TSP (Rosenkrantz et al., 1977).

The distances between cities are stored in a cost matrix E with elements $e(i, j)$. All insertion algorithms start with a tour consisting of an arbitrary city and choose in each step a city k not yet on the tour. This city is inserted into the existing tour between two consecutive cities i and j, such that

$$e(i,k) + e(k,j) - e(i,j)$$

is minimized. The algorithms stops when all cities are on the tour. Cheapest insertion chooses the city k such that the cost of inserting the new city (i.e., the increase in the tour's length) is minimal and tries to build the tour using cities which fit well into the partial tour constructed so far as well.

Appendix F

Properties of the Bivariate Copula Families

F.1 Elliptical copulas

Gaussian copula function is as follows:

$$C(u_1, u_2) = \Phi_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$$

Bivariate Student-t copula is as follows:

$$C(u_1, u_2) = t_{\rho,\nu}(t^{-1}(u_1), t^{-1}(u_2))$$

Table F.1: Properties of the elliptical copula families

Name	Parameter range	Kendall's τ	Tail dep.(l, u)
Gaussian	$\rho \in (-1, 1)$	$\frac{2}{\pi}\text{arcsin}(\rho)$	(0,0)
Student-t	$\rho \in (-1, 1), \nu > 2$	$\frac{2}{\pi}\text{arcsin}(\rho)$	$\left(2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right), 2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)\right)$

F.2 Archimedean copulas

The bivariate acrchimedean copulas function is:

Table F.2: Properties of bivariate Archimedean copula families

Name	Function	Para. range	Kendall's τ	Tail dep.(l,u)
Clayton	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta > 0$	$\frac{\theta}{\theta+2}$	$(2^{-\frac{1}{\theta}})$
Gumbel	$(-\log t)^{\theta}$	$\theta \geq 1$	$1-\frac{1}{\theta}$	$(0, 2-2^{\frac{1}{\theta}})$
Frank	$-\log\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right)$	$\theta \in \Re$	$1-\frac{4}{\theta}+\frac{4D_1(\theta)}{\theta}^{*}$	$(0,0)$
Joe	$-\log(1-(1-t)^t heta)$	$\theta > 1$	$1+\frac{4}{\theta^2}\int_0^1 t\log(t)(1-t)^{\frac{2(1-\theta)}{\theta}}dt$	$(0, 2-2^{\frac{1}{\theta}})$
BB1	$(t^{-\theta}-1)^{-\delta}$	$\theta > 0, \delta \geq 1$	$1-\frac{2}{\delta(\theta+2)}$	$(2^{-\frac{1}{\theta\delta}}, 2-2^{\frac{1}{\theta}})$
BB6	$(-\log(1-(1-t)^{\theta}))^{\delta}$	$\theta \geq 1, \delta \geq 1$	$1+\frac{4}{\theta\delta}\int_0^1(-\log(1-(1-t)^{\theta}) \times$ $(1-t)(1-(1-t^{-\theta})))dt$	$(0, 2-2^{\frac{1}{\theta\delta}})$
BB7	$(1-(1-t)^{\theta})^{-\delta}$	$\theta \geq 1, \delta > 0$	$1+\frac{4}{\theta\delta}\int_0^1(-(1-(1-t)^{\theta})^{\delta+1} \times$ $\frac{(1-(1-t)^{\theta})^{-\delta}-1}{(1-t)^{\theta}-1})dt$	$(2^{-\frac{1}{\theta}}, 2-2^{\frac{1}{\theta}})$
BB8	$-\log\left(\frac{1-(1-\delta t)^{\theta}}{1-(1-\delta)^{\theta}}\right)$	$\theta \geq 1, \delta \in (0,1]$	$1+\frac{4}{\theta\delta}\int_0^1(-\log\left(\frac{(1-t\delta)^{\theta}-1}{(1-\delta^{\theta}-1)}\right) \times$ $(1-t\delta)(1-(1-t\delta^{-\theta})))dt$	$(0,0)^{**}$

Note: * $D_1(\theta) = \int_0^{\theta} \frac{c/\theta}{\exp(x)-1}dx$ is the Debye function.

** For $\delta = 1$ the upper tail dependence coefficient is $2-2^{\frac{1}{\theta}}$.

$$C(u_1, u_2) = \varphi^{[-1]}(\varphi(u_1) + \varphi(u_2))$$

where $\varphi : [0,1] \rightarrow [0,\infty]$ is a continuous strictly decreasing convex such that $\varphi(1) = 0$ and $\varphi^{[-1]}$ is the pseudo-inverse as follows:

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{-1}(t), & 0 \leq t \leq \varphi(0), \\ 0, & \varphi(0) \leq t \leq \infty \end{cases}$$

F.3 Rotations of the copulas

In addition to the families presented in the last 2 sections, there are rotated versions of Clayton, Gumbel, Joe, BB1, BB6, BB7 and BB8 families in order to deal with more dependence structure. When the families are rotated by 180 degrees, they are also called the survival forms of the families. The copula function of these copulas will be calculated as follows:

$$
\begin{aligned}
C_{90}(u_1,u_2) &= u_2 - C(1-u_1,u_2), \\
C_{180}(u_1,u_2) &= u_1 + u_2 - 1 + C(1-u_1,1-u_2), \\
C_{270}(u_1,u_2) &= u_1 - C(u_1,1-u_2),
\end{aligned}
$$

Where C_{90},C_{180} and C_{270} are the copula C rotated by 90,180 and 270 degree respectively.

Appendix G

Estimated Parameters of C-vine and D-vine copulas

Table G.1: Estimated C-vine copula parameters (Log-likelihood = 12934.83)

	Level-1									
margin*:	91	92	93	94	95	96	97	98	9a	9b
family:	t	t	t	t	t	t	t	t	t	t
$\hat{\theta}_1$	0.785636	0.162843	0.692354	0.343322	0.508532	0.757037	0.65892	0.616534	0.124844	0.215362
$\hat{\theta}_2$	2.0001	4.599867	2.0001	2.126921	2.0001	2.0001	2.03829	2.0001	15.77393	7.204589
	Level-2									
margin:	12|9	13|9	14|9	15|9	16|9	17|9	18|9	1a|9	1b|9	
family:	t	t	t	t	t	t	t	t	t	
$\hat{\theta}_1$	0.078301	0.494936	0.045666	0.168873	0.181871	0.525246	0.206182	0.00662	0.109588	
$\hat{\theta}_2$	8.578611	3.110824	4.048904	3.658444	3.96442	3.452198	3.831343	16.10222	9.224822	
	Level-3									
margin:	23|19	24|19	25|19	26|19	27|19	28|19	2a|19	2b|19		
family:	t	t	t	t	t	t	t	t		
$\hat{\theta}_1$	0.079817	0.28738	0.112875	0.088872	0.129475	0.208854	0.525698	0.242231		
$\hat{\theta}_2$	11.87698	8.23072	11.3741	15.98879	11.707	9.281302	3.867485	7.188617		
	Level-4									
margin:	34|129	35|129	36|129	37|129	38|129	3a|129	3b|129			
family:	t	t	t	t	t	90.Clayton	Frank			
$\hat{\theta}_1$	-0.01066	0.021689	-0.00146	0.507135	0.025434	-0.01267	0.49029			
$\hat{\theta}_2$	8.751509	11.44785	8.198117	4.60616	8.407895	0	0			
	Level-5									
margin:	45|1239	46|1239	47|1239	48|1239	4a|1239	4b|1239				
family:	t	t	Frank	t	Frank	t				
$\hat{\theta}_1$	0.173071	0.234101	-0.41015	0.27752	1.047847	0.047483				
$\hat{\theta}_2$	6.216486	8.23026	0	4.326902	0	16.16467				
	Level-6					Level-7				
margin:	56|12349	57|12349	58|12349	5a|12349	5b|12349	67|123459	68|123459	6a|123459	6b|123459	
family:	Frank	t	t	270.Joe	t	t	t	t	t	
$\hat{\theta}_1$	0.553639	0.023637	0.283618	-1.01931	0.04726	-0.02059	0.117399	0.094077	-0.02442	
$\hat{\theta}_2$	0	23.86815	7.225896	0	20.07778	16.18606	12.6435	16.46889	16.44031	
	Level-8 (|9123456)			Level-9 (|91234567)		Level-10 (|912345678)				
margin:	78	7a	7b	8a	8b	ab				
family:	Joe	t	t	270.Clayton	Frank	Gaussian				
$\hat{\theta}_1$	1.003137	-0.00739	0.046393	-0.0234	0.807638	0.072738				
$\hat{\theta}_2$	0	25.25985	13.24346	0	0	0				

Note(*): It shows the bivariate margin under condition, and 1=Belgium, 2=Denmark, 3=France, 4=Greece, 5=Ireland, 6=Italy, 7=Netherlands, 8=Portugal, 9=Spain, a=Sweden and b=UK.

Appendix H

Estimated Vine Graphs

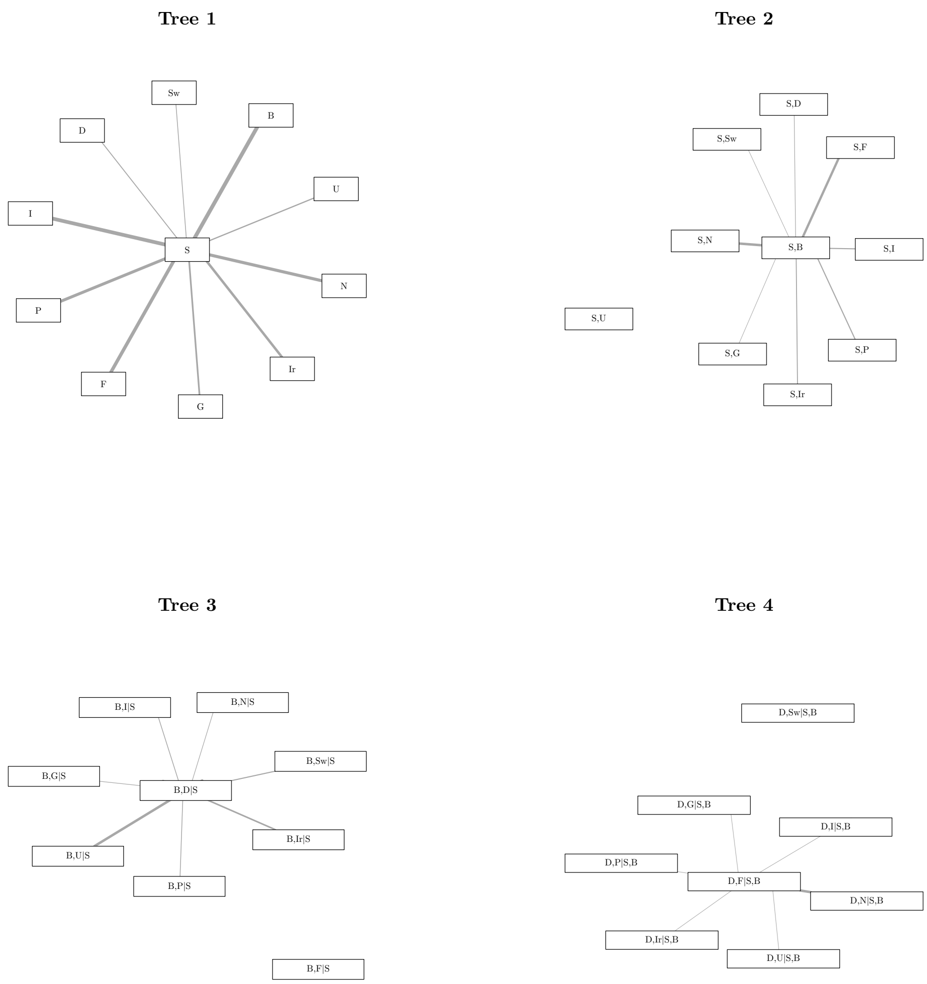

Figure H.1: C-vine tree structure Level 1 - 4

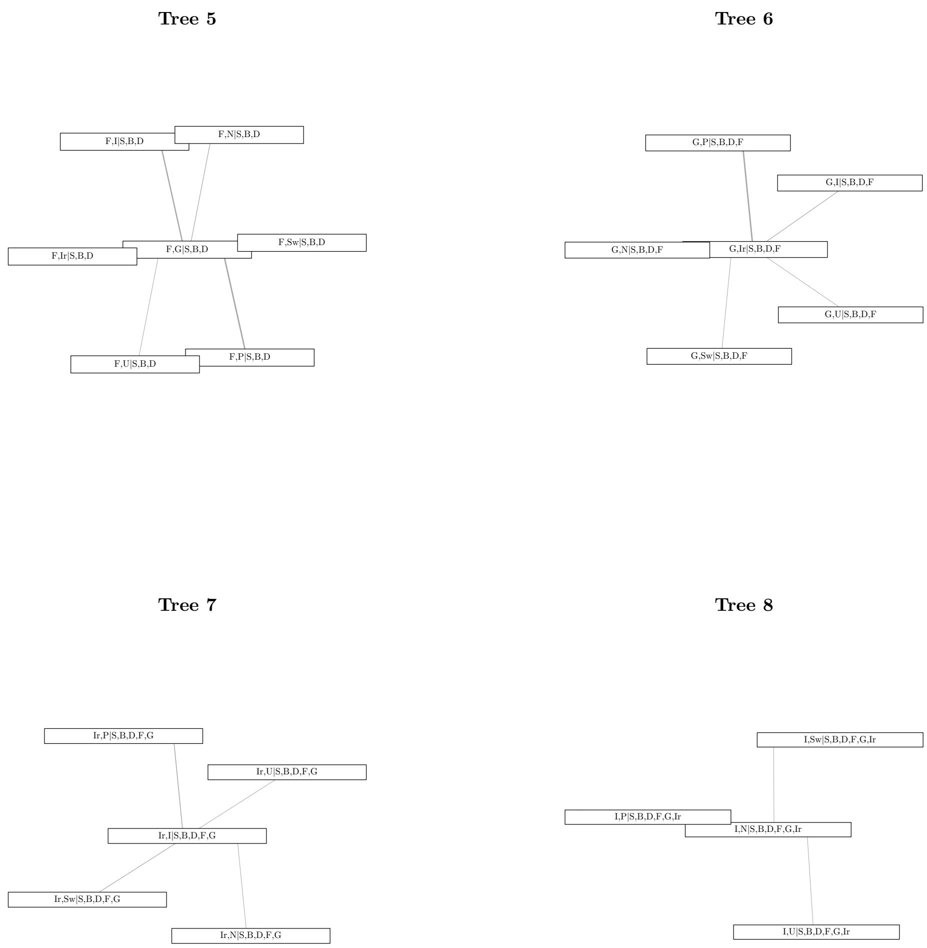

Figure H.2: C-vine tree structure Level 5 - 8

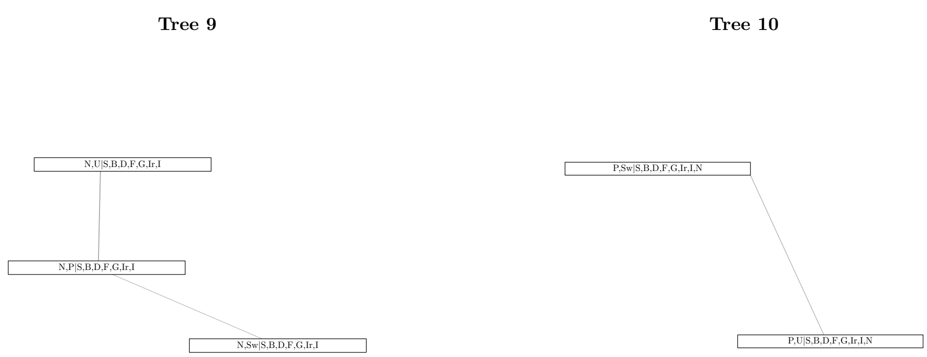

Figure H.3: C-vine tree structure Level 9 - 10

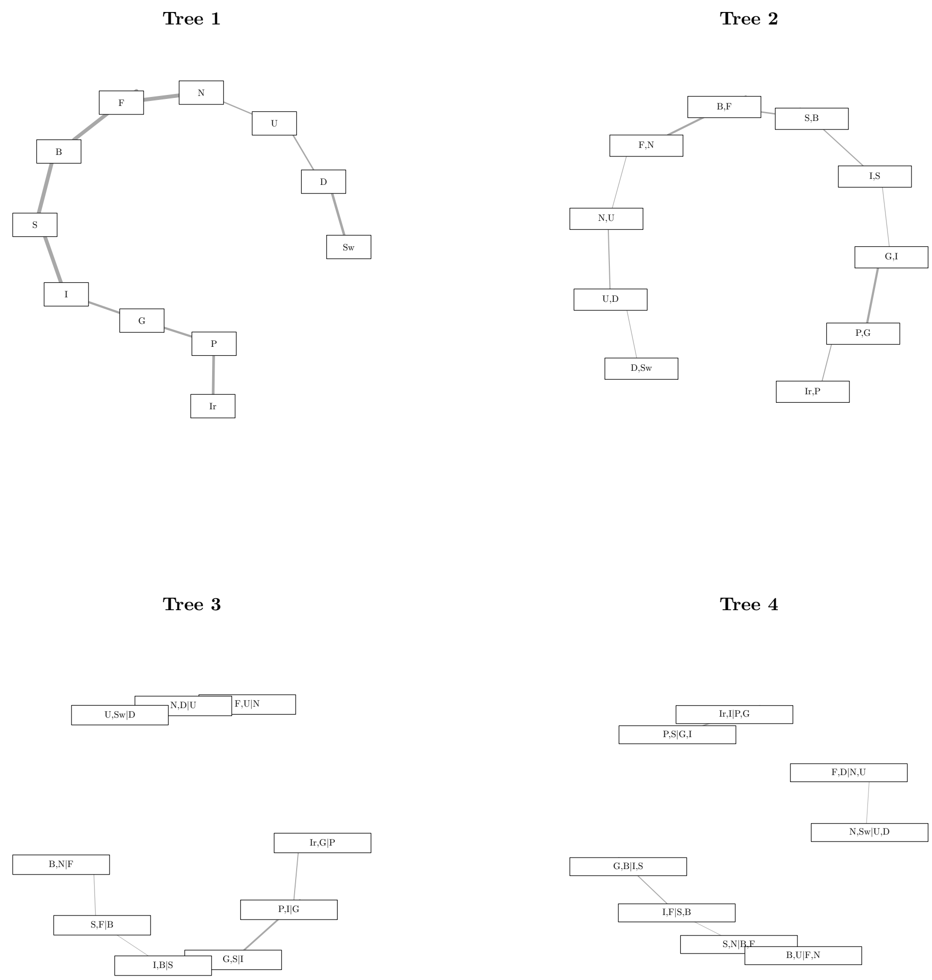

Figure H.4: D-vine tree structure Level 1 - 4

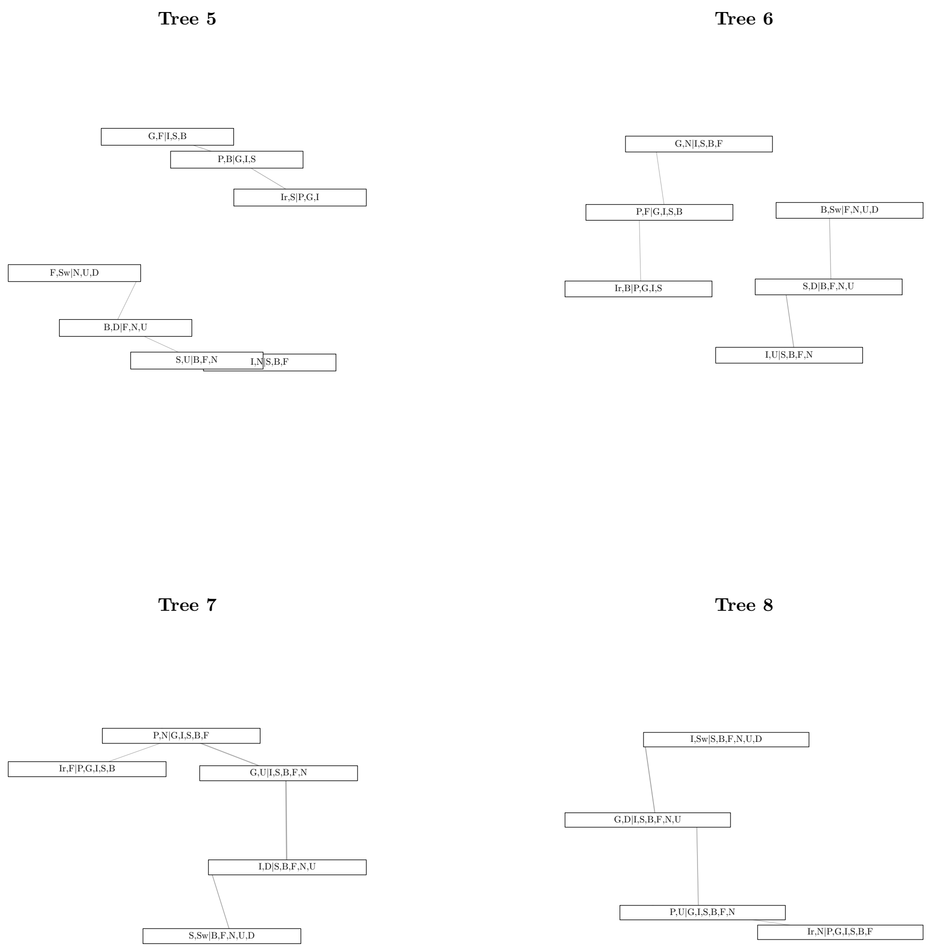

Figure H.5: D-vine tree structure Level 5 - 8

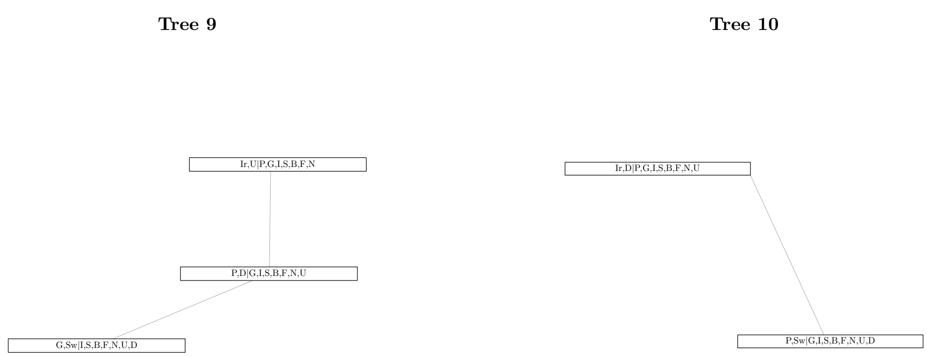

Figure H.6: D-vine tree structure Level 9 - 10