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Top guns may not fire: Best-shot group contests with group-specific public good prizes☆,☆☆

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ABSTRACT

We analyze a group contest in which n groups compete to win a group-specific public good prize. Group sizes can be different and any player may value the prize differently within and across groups. Players exert costly efforts simultaneously and independently. Only the highest effort (the *best-shot*) within each group represents the group effort that determines the winning group. We fully characterize the set of equilibria and show that in any equilibrium at most one player in each group exerts strictly positive effort. There always exists an equilibrium in which only the highest value player in each active group exerts strictly positive effort. However, *perverse* equilibria may exist in which the highest value players completely free-ride on others by exerting no effort. We provide conditions under which the set of equilibria can be restricted and discuss contest design implications.

1. Introduction

Groups often confront each other in order to win a prize. Individual group members contribute costly efforts to the 'group effort' which can increase the probability of winning by their group. The prize can be of a public-good nature in the sense that every group member of the winning group earns the prize even if he does not contribute to the group effort at all. Examples

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of this setting include sports events between teams, rent-seeking contest between lobbying groups, electoral confrontation, or war between coalitions.

Most studies on contests between groups use a *perfect-substitutes group impact function* in which the efforts of the group members within a group are perfectly substitutable and hence a group's effort is determined by the sum of members' efforts in that group.[1] However, Lee (2012) analyzes a group contest with the *weakest-link group impact function* where the efforts of the members within a group are perfectly complementary and thus the minimum effort among the group members represents the group effort. Kolmar and Rommeswinkel (2013) further study group contests employing a *constant elasticity of substitution group impact function*, which allows the impact function of a group to become a perfect-substitute type or a weakest-link type with different degree of complementarity accordingly to the elasticity of substitution of efforts within the group. In the current paper we study a group contest with a *best-shot group impact function*, where individual group members exert costly efforts simultaneously and independently, but only the highest effort within each group represents the group effort.

Best-shot contests are readily observed in the field. Among sporting events, the *Fourball* golf format in which a team performance is noted as the lowest score between the two team members is a distinct example.[2] In industrial organization, the case of competing Research Joint Ventures (RJVs) can be a close example of the aforementioned setup. If an RJV member can make a high quality innovation then it benefits the whole RJV, while other lower quality innovations by other members of the same RJV get obsolete. Similar logic applies to the case of patent pools competing for industry standards (e.g. 4G mobiles), where the main patent (best-shot) provides most competing features of a particular patent pool. The best-shot public good structure is also well documented and discussed in the defense economics and system reliability literature (Conybeare et al., 1994; Varian, 2004).[3] The results show that in a defense or system reliability coalition, in the equilibrium the group member with the highest valuation exerts strictly positive effort and the others free-ride by exerting no effort. Our paper adds to this literature by introducing and characterizing the best-shot contest between groups. In contrast to the aforementioned literature, we show that, in addition to the standard equilibrium where only 'top guns' fire, multiple equilibria in which top guns do not fire may arise.

A behavioral background for the best-shot impact function is given in Baik and Shogren (1998). Clark and Konrad (2007) and Deck and Sheremeta (2012) use a best-shot impact function as the attack mechanism in the context of terrorism. Topolyan (2011) applies the best-shot technology to portray extreme form of free-riding behavior in an all-pay auction between two groups under symmetric valuations. Sheremeta (2011) uses the best-shot among other impact functions to experimentally investigate group contests, without general theoretical analysis. However, none of the existing studies provide a thorough analysis of the best-shot group contests.

We characterize the complete set of equilibria for the best-shot group contest and show that in each equilibrium at most one player in each group exerts strictly positive effort, whereas all the other players in the group free-ride by exerting no effort. Unlike Baik (1993, 2008), who finds that in the perfect-substitutes contest only the highest-valuation member in each group exerts positive effort, in our best-shot contest the unique and active member in each group is not always the highest-valuation member in that group. Specifically, there are *perverse* equilibria in which a member whose valuation is not the highest within a group exerts strictly positive effort, while other members in that group, including the highest-valuation member, free-ride by exerting no effort. These results are also different from the ones by Lee (2012), who shows that in equilibrium of the weakest-link contest there is no free-riding but there is a coordination problem due to the perfect complementarity of efforts among the group members. In contrast, in our best-shot contest there is severe free-riding in any equilibrium. Interestingly, the free-riders in each group may be the highest-valuation players as well as other low-valuation players.

We also rank the possible equilibria in our model and show the conditions under which *perverse* equilibria can be avoided from the perspective of a contest designer. Therefore, our paper is also related to the paper by Kolmar and Wagener (2012), who incorporate the contest mechanism into the public goods setting in order to solve two efficiency failures: (1) insufficient provision of public goods (quantity problem) and (2) unproductive players' contributions to the public goods (sorting problem). They examine the optimal structures of the CSF (contest success function) and prizes that solve these two problems. Focusing on the sorting problem as in Kolmar and Wagener (2012), we specify the structure of players' valuations that prevents *perverse* equilibria, i.e., equilibria in which the low-valuation players in each group exert positive efforts.

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[1] A function that translates the efforts of individual group members into the *group effort* is called a *group impact function*. The literature on group contest originated with the work of Katz et al. (1990). Katz et al. use symmetric players within each group, a perfect-substitutes group impact function (Bergstrom et al., 1986), and a lottery contest success function (Tullock, 1980). Most follow-up studies on group contests use the perfect-substitutes group impact function (Baik, 1993, 2008; Baik and Shogren, 1998; Baik et al., 2001; Münster, 2009). There is also a growing experimental literature examining contests between groups (Abbink et al., 2010; Sheremeta and Zhang, 2010; Ahn et al., 2011; Sheremeta, 2011; Cason et al., 2012), for a review see Dechenaux et al. (2012).

[2] Cycling team events like *Tour de France* have some aspects of best-shot contests. Although each member of a team participates separately in the contest, if any member of the team finishes first, then it records the win of the whole team. Team archery or shooting contests have similar features in which literally the *best-shot* among the team members determines the performance of the whole team.

[3] Competing defense coalitions, such as the *NATO* and the *Warsaw Pact*, may follow the best-shot technology in which the best performance of the coalition member determines the performance of the whole coalition (Conybeare et al., 1994). The same story holds for inland security coalition comprised of different independent bodies such as *CIA* and *FBI*, or in the context of system reliability (Varian, 2004).

The paper proceeds as follows. In Section 2 we present and in Section 3 we analyze our best-shot contest model. In Section 4, as an example, we consider a contest with three two-player groups. In Section 5, we discuss contest design implications and conclude by suggesting avenues for future research. Technical proofs are in the Appendix.

2. The model

Consider a contest in which $n \geq 2$ groups compete to win a group-specific public-good prize. Group $g \in N$, where $N = \{1, 2, \ldots, n\}$ is a set of groups, consists of $m_g \geq 2$ risk-neutral players who exert costly efforts to win the prize.

The individual group members' valuation for the prize may differ within group and across groups. This intra-group asymmetry in values can be a result of player asymmetry, but it can also be interpreted as an exogenous sharing rule of the group-specific prize, in which the prize-shares among the members of a group are different. Let $v_{gi} > 0$ represent the valuation for the prize of player i in group g. Without loss of generality, assume $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$, and $v_{(k-1)1} \geq v_{k1}$ for $n \geq k > 1$. Let $x_{gi} \geq 0$, measured in the same unit as the prize values, represent the effort level exerted by player i in group g.

Next, we specify the *group impact function* as $f_g : \mathbb{R}_+^{m_g} \to \mathbb{R}_+$, such that the group effort of group g is given by $X_g = f_g(x_{g1}, x_{g2}, \ldots, x_{gm_g})$. The following assumption defines the best-shot technology:

Assumption 1. The group effort of group g is represented by the maximum effort level exerted by the players in group g, i.e., $X_g = \max\left\{x_{g1}, x_{g2}, \ldots, x_{gm_g}\right\}$.

To specify the winning probability of group g, denote $p_g(X_1, X_2, \ldots, X_n) : \mathbb{R}_+^n \to [0, 1]$ as a *contest success function* (Münster, 2009). Assumption 2 specifies the regularity conditions for p_g.

Assumption 2. $p_g(0, 0, \ldots, 0) = 1/n$, $\sum_{g=1}^{n} p_g = 1$, $\partial p_g/\partial X_g \geq 0$, $\partial^2 p_g/\partial X_g^2 \leq 0$, $\partial p_g/\partial X_k \leq 0$, $\partial^2 p_g/\partial X_k^2 \geq 0$ where $k, g \in N$ and $k \neq g$. Furthermore, $\partial p_g/\partial X_g > 0$, $\partial^2 p_g/\partial X_g^2 < 0$ for some $X_k > 0$, and $\partial p_g/\partial X_k < 0$, $\partial^2 p_g/\partial X_k^2 > 0$ for $X_g > 0$.

We assume that all players forgo their efforts and they have a common cost function as described by Assumption 3.

Assumption 3. $c : \mathbb{R}_+ \to \mathbb{R}_+$ is the common cost function of effort with the following properties: $c(0) = 0$, $\partial c(x_{gi})/\partial x_{gi} > 0$, $\partial^2 c(x_{gi})/\partial x_{gi}^2 \geq 0$.

Only the members of the winning group receive the prize, while all other players receive nothing. Let π_{gi} represent the payoff for player i in group g. Under the above assumptions, the payoff for player i in group g is

$$\pi_{gi} = v_{gi} p_g(X_1, X_2, \ldots, X_n) - c(x_{gi}). \tag{1}$$

Eq. (1) along with the three assumptions represents the *best-shot group contest*. To close the structure we assume that all players in the contest choose their effort levels independently and simultaneously, and that all of the above (including the valuations, group compositions, impact functions, and the contest success function) is common knowledge.

We use the following definitions throughout the paper.

Definition 1. If player i in group g exerts strictly positive effort, i.e., $x_{gi} > 0$, then the player is called *active*. Otherwise (when $x_{gi} = 0$) the player is called *inactive*.

Definition 2. If at least one player in group g exerts strictly positive effort, i.e., $X_g > 0$, then group g is called *active*. Otherwise (when $X_g = 0$) the group is called *inactive*.

3. The equilibria of the game

We employ Nash equilibrium as our solution concept and begin by stating Lemma 1. This lemma points out that there are always at least two groups that actively participate in a best-shot contest.

Lemma 1. *In any equilibrium at least two groups are active.*

Assumption 1 gives rise to Lemma 2.

Lemma 2. *In an equilibrium only one player in each group, if any, is active.*

Next, let $x_{gi}^b \in \mathbb{R}_+$ denote the best-response of player i in group g in a situation where player i is a unique player in group g. Lemma 3 follows.

Lemma 3. *Given the effort levels of other groups, $x_{g(t-1)}^b \geq x_{gt}^b$ for $m_g \geq t > 1$.*

Lemmas 1 and 2 transform the game into a generalized version of an asymmetric value individual contest, whereas Lemma 3 provides restrictions on the participation of the players. A combination of the three lemmas gives Corollary 1 that we state without proof.

Corollary 1. *An equilibrium in which player 1s in group 1 and 2 are active always exists.*

It follows from Lemma 3 that $x^b_{g1} > 0$ for some $g \le k \le n$ and $x^b_{g1} = 0$ for $g > k$, i.e., given the distribution of values, there can be instances in which the best response for every player in a group is to exert no effort, as each of their valuation is low enough and exerting any positive effort will result in a negative payoff.

The general nature of the current setup restricts us from finding closed form solution for participation and equilibrium effort. To make the problem tractable and attain closed form solutions we make the following restrictive assumptions. First, following the axiomatic foundation of Münster (2009) we apply a logit (Tullock, 1980) form group contest success function. Second, we use linear cost function with unit marginal cost.

Assumption 2′. The probability of winning the prize for group g is

$$p_g(X_1, X_2, \ldots, X_n) = \begin{cases} \dfrac{X_g}{\sum_{k=1}^{n} X_k} & \text{if } \sum_{k=1}^{n} X_k > 0 \\ \dfrac{1}{n} & \text{if } \sum_{k=1}^{n} X_k = 0 \end{cases}.$$

Assumption 3′. The common cost function is $c(x_{gi}) = x_{gi}$.

Lemmas 1–3 under these two assumptions convert the best-shot group contest into a generalized asymmetric individual lottery contest in which each group behaves like an individual contestant, but the valuation of the individual contestant may change depending on which group member within a group is active. We use the results on asymmetric individual contest by Stein (2002), who shows that in an n-player asymmetric contest some players may not be active in an equilibrium. Using Proposition 1 of Stein (2002) we state the condition in Lemma 4 for active participation in the contest.

Lemma 4. *Suppose each player E_j in group $j = 1, 2, \ldots, k-1 (\le n-1)$ is already active. Then for at least one more player from any other group to be active, the following condition has to be satisfied:*

$$v_{k1} > \frac{(k-2)\Pi_{j<k} v_{jE_j}}{\sum_{j<k} \Pi_{t \ne j, t<k} v_{tE_t}}.$$

Lemma 4 gives us a set of sufficient conditions to exclude one or more groups from participating in the best-shot contest. This lemma also gives Corollary 2 that shows the needed parametric restriction which, in turn, ensures participation of all the groups in an equilibrium.

Corollary 2. *If $v_{n1} > ((n-2)\Pi_{j<n} v_{j1}) / \left(\sum_{j<n} \Pi_{t \ne j, t<n} v_{t1}\right)$ then, in an equilibrium, all n groups are active.*

Based on Corollary 2, assume that first n_1 ($<n$) groups are active in an equilibrium where each player E_j in group $j = 1, 2, \ldots, n_1$ is active, i.e.,

$$v_{n_1 1} > \frac{(n_1-2)\Pi_{j<n_1} v_{jE_j}}{\sum_{j<n_1} \Pi_{t \ne j, t<n_1} v_{tE_t}} \quad \text{and} \quad v_{(n_1+1)1} < \frac{((n_1+1)-2)\Pi_{j<(n_1+1)} v_{jm_j}}{\sum_{j<(n_1+1)} \Pi_{t \ne j, t<(n_1+1)} v_{tm_t}}.$$

From Lemmas 1–4, one can expect that, under certain restrictions, there exists an equilibrium in which player 1 (the highest-valuation player) in each active group exerts strictly positive effort and the other group members free-ride by exerting no effort, i.e., $E_j = 1$ for each active group j. In such a case, each highest-valuation active player exerts the equilibrium effort in an n_1-player individual contest with asymmetric values as in Stein (2002). It is easy to verify that this constitutes an equilibrium as no player, exerting strictly positive or zero effort, has an incentive to deviate from the effort level. This is summarized in Proposition 1.

Proposition 1. *Suppose*

$$v_{n_1 1} < \frac{(n_1-2)\Pi_{j<n_1} v_{j1}}{\sum_{j<n_1} \Pi_{t \ne j, t<n_1} v_{t1}} \quad \textit{and} \quad v_{(n_1+1)1} < \frac{((n_1+1)-2)\Pi_{j<(n_1+1)} v_{jm_j}}{\sum_{j<(n_1+1)} \Pi_{t \ne j, t<(n_1+1)} v_{tm_t}},$$

i.e., only first n_1 groups are active. Define

$$x^*_{g1} = \frac{n_1 - 1}{\sum_{k=1}^{n_1} v_{k1}^{-1}} \left(1 - (n_1 - 1)\frac{v_{g1}^{-1}}{\sum_{k=1}^{n_1} v_{k1}^{-1}}\right).$$

Then a profile $((x^*_{11}, 0, \ldots, 0), (x^*_{21}, 0, \ldots, 0), \ldots, (x^*_{n_1 1}, 0, \ldots, 0), (0, \ldots, 0), \ldots, (0, \ldots, 0))$ is a Nash equilibrium of the best-shot group contest.

The implication of Proposition 1 is that in the equilibrium only one of the players with the highest stake, or the most efficient player, exerts effort on behalf of the active group and all other group members free-ride by exerting no effort.[4] This particular equilibrium indicates that in a market coalition such as an RJV or a defense coalition such as *NATO*, the most efficient player does not benefit from being a member of the coalition, instead of contesting as an individual player. Therefore, the equilibrium described by Proposition 1 does not justify the inclusion of the most efficient player in a coalition formation due to the *exploitation of the great by the small* (Olson, 1965), i.e., exploitation of the most efficient players by other members.[5]

Coalition formation may be justified if there are *perverse* equilibria in which the most efficient players can free-ride on other players' efforts. Or, in other words, in those equilibria the 'top guns' do not fire and there is *exploitation of the small by the great*. Proposition 2 shows that indeed *perverse* equilibria exist, justifying the existence of market, political and international coalitions under best-shot technology.

Proposition 2. *Suppose*

$$v_{n_1 1} > \frac{(n_1-2)\Pi_{j<n_1} v_{j1}}{\sum_{j<n_1} \Pi_{t\neq j, t<n_1} v_{t1}} \quad \text{and} \quad v_{(n_1+1)1} < \frac{((n_1+1)-2)\Pi_{j<(n_1+1)} v_{jm_j}}{\sum_{j<(n_1+1)} \Pi_{t\neq j, t<(n_1+1)} v_{tm_t}}.$$

Define

$$x^*_{gE_g} = \frac{n_1-1}{\sum_{k=1}^{n_1} v^{-1}_{kE_k}} \left(1-(n_1-1)\frac{v^{-1}_{gE_g}}{\sum_{k=1}^{n_1} v^{-1}_{kE_k}}\right)$$

where $E_g > 1\ \forall g \in N_1 = \{1, 2, \ldots, n_1\}$. *Then a profile* $((0,\ldots,0,x^*_{1E_1},0,\ldots,0),\ldots,(0,\ldots,0,x^*_{n_1E_{n_1}},0,\ldots,0),\ldots,(0,\ldots,0))$ *is a Nash equilibrium of the best-shot group contest if*

$$\frac{v_{g1}}{v_{gE_g}} \le \left(1+\sqrt{1-(n_1-1)\frac{v^{-1}_{gE_g}}{\sum_{k=1}^{n_1} v^{-1}_{kE_k}}}\right) \forall g \in N_1.$$

Proposition 2 fully characterizes the set of equilibria for the best-shot group contest. This proposition shows that it is possible for a coalition to exist in which there is *exploitation of the small by the great*, and in which the most efficient player can earn a higher payoff as an inactive member of a group rather than contesting as an individual player. Nevertheless, the final payoff of each player, including the most efficient players, crucially depends on the equilibrium selection and the coordination between group members.

The rent dissipation results are very different in the current analysis compared to the group contests with other group impact functions or the best-shot public good games. We know that in the case of a perfect-substitutes impact function, the total rent dissipation is uniquely determined (Baik, 1993) and a coalition-proof equilibrium is unique in the case of a weakest-link impact function (Lee, 2012). It is not trivial to fully rank the equilibria in the best-shot case in terms of rent dissipation. The following corollary points out the highest and the lowest possible rent dissipation in the best-shot group contest.

Corollary 3. *The highest (lowest) possible equilibrium rent is dissipated for a given number of active groups, when only the highest (lowest) value players in each active group exert strictly positive effort. The rent dissipation is intermediate otherwise.*

4. An example with three two-player groups

To portray a simple diagrammatic explanation of our general results we consider an example with three two-player groups, where there are three groups and each group consists of two players. First, we show conditions for which one group becomes inactive and the contest is reduced to a contest between two two-player groups. Then we characterize the set of equilibria for this reduced form contest.

As shown in Lemmas 1 and 2, in any equilibrium at least two groups will be active and only one player in each group will be active. One can show that there can be 20 equilibria in which one player in each group is active and at least two groups are active. Now we impose condition $v_{31} < (v_{12}v_{22})/(v_{12}+v_{22})$. From Lemma 4, this means that both members of group 3 have relatively low valuations of the prize and active participation in the contest ensures loss to them. Hence, both players in group 3 exert no effort and group 3 is always inactive. As a result the set of equilibria reduces to only 4. Let us denote the equilibrium in which the player *i* from group 1 and the player *j* from group 2 are active as N_{ij}. Then the four possible equilibria

[4] This equilibrium is equivalent to the equilibrium in a group contest with perfect-substitutes impact function as in Baik (1993). The equilibrium strategies are also similar to the equilibrium strategies in best-shot public good games (Hirshleifer, 1983).

[5] One may argue, however, that the most efficient player may still be better off being a member of the coalition, instead of contesting as an individual player, because it prevents the other members from exerting effort as potential opponents.

Table 1
Equilibrium effort and corresponding condition (two active groups).

Equilibrium	x_{11}	x_{12}	x_{21}	x_{22}	Equilibrium conditions
N_{11}	$\frac{v_{11}^2 v_{21}}{(v_{11}+v_{21})^2}$	0	$\frac{v_{11} v_{21}^2}{(v_{11}+v_{21})^2}$	0	No condition required
N_{12}	$\frac{v_{11}^2 v_{22}}{(v_{11}+v_{22})^2}$	0	0	$\frac{v_{11} v_{22}^2}{(v_{11}+v_{22})^2}$	$\alpha_2 \le \left(1+\sqrt{\frac{v_{22}}{\alpha_1 v_{12}+v_{22}}}\right)^2$
N_{21}	0	$\frac{v_{12}^2 v_{21}}{(v_{12}+v_{21})^2}$	$\frac{v_{12} v_{21}^2}{(v_{12}+v_{21})^2}$	0	$\alpha_1 \le \left(1+\sqrt{\frac{v_{12}}{v_{12}+\alpha_2 v_{22}}}\right)^2$
N_{22}	0	$\frac{v_{12}^2 v_{22}}{(v_{12}+v_{22})^2}$	0	$\frac{v_{12} v_{22}^2}{(v_{12}+v_{22})^2}$	$\alpha_1 \le \left(1+\sqrt{\frac{v_{12}}{v_{12}+v_{22}}}\right)^2$
					$\alpha_2 \le \left(1+\sqrt{\frac{v_{22}}{v_{12}+v_{22}}}\right)^2$

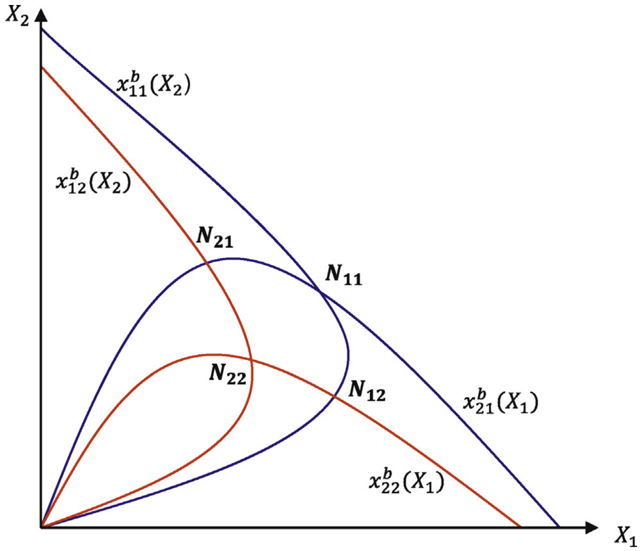

Fig. 1. The equilibria.

are N_{11}, N_{12}, N_{21}, and N_{22}. Also, define $\alpha_1 \equiv v_{11}/v_{12}(\ge 1)$ and $\alpha_2 \equiv v_{21}/v_{22}(\ge 1)$. The necessary restrictions on parameters and the resulting equilibria are summarized in Table 1 and are shown diagrammatically in Fig. 1.

As stated in Corollary 3, the rent dissipation is highest for N_{11}, lowest for N_{22} and intermediate for the other two equilibria. This gives the flexibility to a contest designer not only to select the equilibrium in terms of active players but also in terms of rent dissipation. Depending on the distribution of values different equilibria may exist, and thus the designer may want to impose certain restrictions to select a set of desirable equilibria. Fig. 2 summarizes the required equilibrium conditions for N_{11}, N_{12}, N_{21}, and N_{22} equilibria in $\alpha_1 - \alpha_2$ graph. Note that the equilibrium N_{11} always exists in which 'top guns' fire. However, there are six segments of the graph where, in addition to N_{11}, *perverse* equilibria exist in which 'top guns' do not fire.

5. Discussion

In this paper we construct and analyze the best-shot group contest. We find that depending on the distribution of values there can be multiple equilibria, but in each equilibrium only one player from each group exerts strictly positive effort.[6] This result is robust to the number of groups, the number of players in each group, and the valuations of players. However, the exact equilibrium strategies and rent dissipation are not uniquely determined and are not robust.

We identify the conditions on the distribution of values that can give rise to multiple equilibria. A contest designer may use such conditions to achieve a given objective. For example, if individual values portray the within-group prize sharing rules, then based on the fairness principle (Phillips, 1997) a designer can assign players to the groups such that most efficient players always exert positive efforts. On the other hand, the objective of a designer can be to either achieve the highest efficiency (Barr, 2004) or the lowest rent dissipation (Tullock, 1980). In any case, the appropriate distribution of values can be selected to meet the objective.

[6] Multiple equilibria have also been documented in simple two-player lottery contests with spillovers (Chowdhury and Sheremeta, 2011).

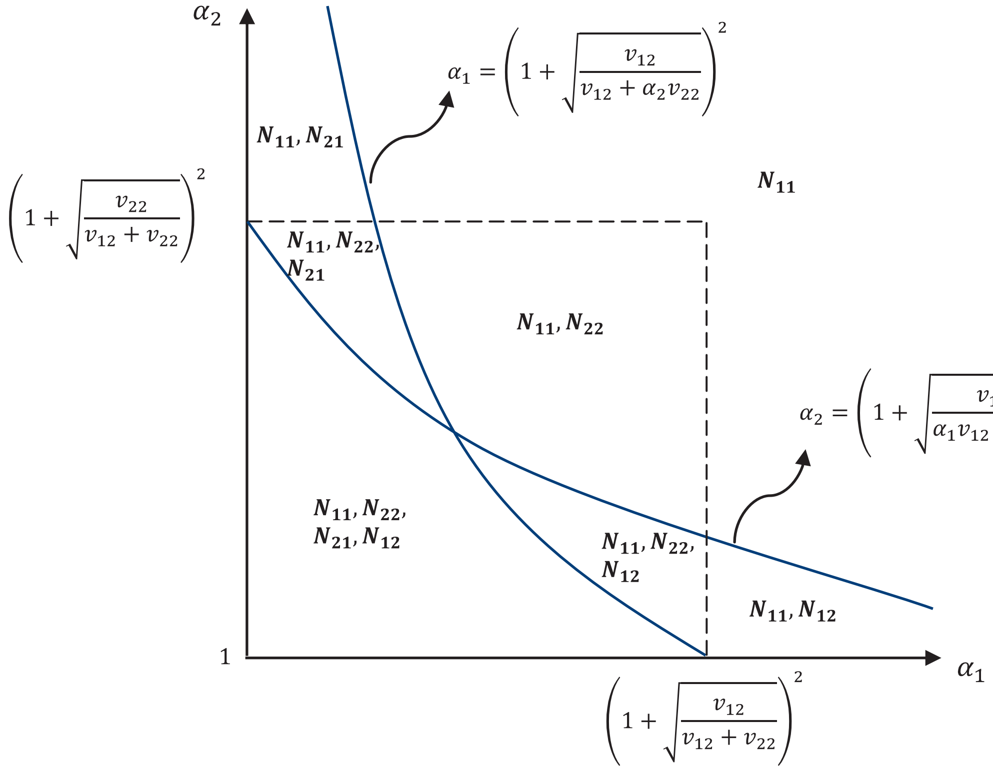

Fig. 2. The equilibria conditions.

In the context of public goods and defense economics literature (Hirshleifer, 1983, 1985; Bliss and Nalebuff, 1984; Harrison and Hirshleifer, 1989; Cornes, 1993) it has been well recognized that using best-shot technology leads to equilibria in which the most able players contribute to the public good. This result has been well known as the *exploitation of the great by the small* (Olson, 1965), and it has been used to argue that it is difficult to sustain market or defense coalitions because of the low incentives (or none) for efficient players to join the coalition. The current study shows that, there may be scenarios where there is *exploitation of the small by the great*, i.e., *perverse* equilibria in which 'top guns' do not fire and the most efficient players free-ride on other players' efforts. This result has important implications, as it justifies coalition formation under the best-shot structure.

The existence of multiple equilibria in the best-shot group contest also raises the issue of equilibrium selection (Galeotti et al., 2010; Dall'Asta et al., 2011). As in the existing literature, if one models the issues of system reliability or defense mechanisms in terms of best-shot public good game, then a unique equilibrium ensures no equilibrium selection concerns. However, if one models the aforementioned situations in a more realistic contest setting, then the existence of multiple equilibria will affect the comparative statics results. Hence, our findings reinforce the need for further research on equilibrium selection in contests with best-shot impact functions. Even in the simplest case with two two-player groups presented in Section 4, we show that depending on the distribution of the players' valuations for the prize, the number of possible equilibria ranges from one to four. Furthermore, if we consider the possibility of mixed strategies (i.e., when active players in the same group have the same valuations), or pre-contest coordination strategies, then the equilibrium selection becomes even more complicated. Refining the equilibria based on different equilibrium selection criteria is an interesting avenue for the future research.

Appendix A.

Proof of Lemma 1. Suppose there exists an equilibrium in which all the groups are inactive. In such a case, the payoff of player i in group g is v_{gi}/n. Now, suppose player i in group g exerts an infinitesimal effort $\varepsilon>0$ instead of being inactive. Then the payoff becomes $v_{gi} - c(\varepsilon) > v_{gi}/n$. Hence, all groups being inactive can never be an equilibrium. Now, suppose there exists an equilibrium in which only group k is active. In such a case, from equation (1), the payoff of player i in group g is 0. Now suppose player i in group g exerts an effort $x^*_{gi} > 0$ instead of being inactive, where x^*_{gi} is the best response of player i in group g against X_k in the two-player individual contest with CSF defined in *Assumption 2*. Consequently, the payoff becomes $p_g v_{gi} - x^*_{gi} > 0$. Thus, a single active group cannot constitute an equilibrium either. Hence in any equilibrium there are at least two active groups.

Proof of Lemma 2. Suppose in an equilibrium more than one player, say players i and j, in group g exert strictly positive efforts with $x_{gi} \geq x_{gj} > 0$. Hence, the payoff of player j, under *Assumption 1*, is $\pi'_{gj} = v_{gj} p_g(X_1, \ldots, X_{g-1}, x_{gi}, X_{g+1}, \ldots, X_n) - c(x_{gj})$. In such a case it is always beneficial for player j to reduce effort to zero and increase payoff to: $x''_{gj} = v_{gj} p_g(X_1, \ldots, X_{g-1}, x_{gi}, X_{g+1}, \ldots, X_n) > \pi'_{gj}$. Hence, more than one player in the same group exerting strictly positive effort cannot be an equilibrium.

Proof of Lemma 3. Suppose that player i in group g is a unique player in the group. Then the best-response of player i in group g is the non-negative effort level that maximizes its (non-negative) payoff, given the effort levels of the players in the other groups. Specifically, x^b_{gi} is the effort level that maximizes the payoff $\pi^b_{gi} = v_{gi} p_g(X_1, \ldots, X_{g-1}, x_{gi}, X_{g+1}, \ldots, X_n) - c(x_{gi})$ subject to the non-negativity constraint $x_{gi} \geq 0$, and the participation constraint $\pi^b_{gi}(x^b_{gi}, x_{-g}) \geq 0$, where $X_{-g} = (X_1, \ldots, X_{g-1}, X_{g+1}, \ldots, X_n)$ Since the payoff function π^b_{gi} is strictly concave, the solution x^b_{gi} is unique, and satisfies the first-order condition for maximizing π^b_{gi}, $c'(x_{gi})/(\partial p_g/\partial x_{gi}) \geq v_{gi}$. The first-order condition, jointly with *Assumptions 2* and *3*, ensure that x^b_{gi} is monotonically increasing in v_{gi}, given X_{-g}. Finally, this observation along with the assumption $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$ implies that $x^b_{g(t-1)} \geq x^b_{gt}$ for all $X_{-g} \geq (0, \ldots 0)$.

Proof of Lemma 4. By assumptions $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$ and $v_{(k-1)1} \geq v_{k1}$ for $n \geq k > 1$, when each player E_j in group $j = 1, 2, \ldots, k-1$ is already active. If player 1 of group k cannot earn positive payoff by exerting strictly positive effort, it is impossible for any other player E_l in group $l = k, k+1, \ldots, n$ to earn positive payoff by being active. Now, from *Proposition 1* of Stein (2002) we know that when each player E_j in group $j = 1, 2, \ldots, k-1$ is active, then player 1 of group k will exert $x_{k1} > 0$ and earn strictly positive payoff only if the condition $v_{k1} > \frac{(k-2)\prod_{j<k} v_{jE_j}}{\sum_{j<k}\prod_{t \neq j, t<k} v_{jE_t}}$ is satisfied.

Proof of Corollary 2. From *Lemma 4*, condition $v_{n1} > \frac{(n-2)\prod_{j<n} v_{j1}}{\sum_{j<n}\prod_{t \neq j, t<n} v_{t1}}$ ensures that at least player 1 in group n is always active in an equilibrium even when player 1s in all the other groups are active. This, along with the assumption $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$, means that at least player 1 in group n is always active in an equilibrium when players other than player 1s from all the other groups are active. Also, if it is participation compatible for player 1 in group n, then assumption $v_{(k-1)1} \geq v_{k1}$ for $n \geq k > 1$ and the properties of harmonic mean imply that it is participation compatible for player 1s in all the other groups.

Proof of Proposition 1. Note that $x^*_{g1} = \frac{n_1 - 1}{\sum_{k=1}^{n_1} v_{k1}^{-1}}\left(1 - (n_1 - 1)\frac{v_{g1}^{-1}}{\sum_{k=1}^{n_1} v_{k1}^{-1}}\right)$ is the equilibrium effort of player 1 in group g when the original contest is reduced to an n_1-player contest which consists of the highest-valuation players in groups 1, 2, …, n_1. From *Lemma* 4, the conditions $v_{n_1 1} > \frac{(n_1-2)\prod_{j<n_1} v_{j1}}{\sum_{j<n_1}\prod_{t \neq j, t<n_1} v_{t1}}$ and $v_{(n_1+1)1} < \frac{((n_1+1)-2)\prod_{j<(n_1+1)} v_{jm_j}}{\sum_{j<(n_1+1)}\prod_{t \neq j, t<(n_1+1)} v_{tm_t}}$ restrict only the first n_1 groups to be active in an equilibrium. Hence, the following profile $\left((x^*_{11}, 0, \ldots, 0), (x^*_{21}, 0, \ldots, 0), \ldots, (x^*_{n1}, 0, \ldots, 0), (0, \ldots, 0), \ldots, (0, \ldots, 0)\right)$ is an equilibrium.

Proof of Proposition 2. Note that $x^*_{gE_g} = \frac{n_1 - 1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\left(1 - (n_1 - 1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)$ is the equilibrium effort of player $E_g > 1$ in group g when the original contest is reduced to the n_1-player contest consisting of the E_gth highest-valuation players in each group $g \in N_1$. Hence, in a profile $\left((0, \ldots, 0, x^*_{1E_1}, 0, \cdots, 0), \ldots, (0, \cdots, 0, x^*_{n_1 E_{n_1}}, 0, \cdots, 0), \ldots, (0, \ldots, 0)\right)$ player E_g in group g exerts strictly positive reduced-form-individual-contest effort and the others in the group exert no effort. For this profile to be a Nash equilibrium, no player in the contest should have an incentive to deviate from this profile. It is straightforward to show that any player $i = E_g + 1, E_g + 2, \ldots, m_g$ in group g does not have an incentive to deviate according to *Lemma 3*.

Similarly, it is straightforward to show that player 1 in group g has the highest incentive, if any, to deviate from this profile because $x^b_{g1}(X_{-g}) \geq x^b_{g2}(X_{-g}) \geq \cdots \geq x^b_{gE_g}(X_{-g})$. This means that if player 1 in group g does not have an incentive to deviate from this profile, then players 2, 3, …, $(E_g - 1)$ in group g do not have such incentives, either. Therefore, in order to determine if this profile is an equilibrium, it is enough to check if player 1 in each group g has any incentive to deviate from it.

Now we show the conditions under which player 1 in group g does not deviate from the profile $\left((0, \cdots, 0, x^*_{1E_1}, 0, \cdots, 0), \ldots, (0, \cdots, 0, x^*_{n_1 E_{n_1}}, 0, \cdots, 0), \ldots, (0, \ldots, 0)\right)$. Under this profile, player 1 in group g exerts no effort, and the winning probability of group g is $p_g = \left(1-(n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)$. Hence, the payoff of player 1 in group g is $\pi_{g1} = v_{g1}\left(1-(n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)$.

If player 1 in group g deviates from the profile, his optimal effort level is $x_{g1}^b(X_{-g}) = \sqrt{v_{g1}\sum_{k\neq g}^{n_1} X_k} - \sum_{k\neq g}^{n_1} X_k = \sqrt{v_{g1}\sum_{k\neq g}^{n_1} x_{kE_k}^*} - \sum_{k\neq g}^{n_1} x_{kE_k}^* = \frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\left(\sqrt{\frac{v_{g1}}{v_{gE_g}}} - (n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)$. However, by deviating from the profile, this player earns the payoff $\pi_{g1}^d = v_{g1}\left(\sqrt{\frac{v_{g1}}{v_{gE_g}}} - (n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)\left(\sqrt{\frac{v_{gE_g}}{v_{g1}}} - (n_1-1)\frac{v_{g1}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)$. Then, for player 1 in group g not to deviate from the profile, we need $\pi_{g1} \geq \pi_{g1}^d$, i.e., $\pi_{g1} - \pi_{g1}^d = -\frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\left(\frac{v_{g1}}{v_{gE_g}} - 2\sqrt{\frac{v_{g1}}{v_{gE_g}}} + (n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right) \geq 0$. The inequality can be rewritten as $\pi_{g1} - \pi_{g1}^d = -\frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \times \left(\sqrt{\frac{v_{g1}}{v_{gE_g}}} - 1 - \sqrt{1-(n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}}\right) \times \left(\sqrt{\frac{v_{g1}}{v_{gE_g}}} - 1 + \sqrt{1-(n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}}\right) \leq 0$.

Therefore, the following profile $\left((0,\cdots,0,\ x_{1E_1}^*,0,\cdots,0),\ldots,(0,\cdots,0,\ x_{n_1E_{n_1}}^*,0,\cdots,0),\ldots,(0,\cdots,0)\right)$ is a Nash equilibrium if $\forall g \in N_1$, $\frac{v_{g1}}{v_{gE_g}} \leq \left(1+\sqrt{1-(n_1-1)\frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}}\right)$.

Proof of Corollary 3. Suppose n_1 groups are active. From *Proposition* 2, the total equilibrium rent dissipation is $\sum_{g=1}^{n_1} x_{gE_g}^* = \sum_{g=1}^{n_1}\left(\frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} - \left[\frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right]^2 (v_{gE_g}^{-1})\right) = (n_1-1)\frac{\prod_{k=1}^{n_1} v_{kE_k}}{\sum_{l=1}^{n_1}\prod_{k\neq l}^{n_1} v_{kE_k}}$. Differentiating with respect to the value of an active player t we get $\frac{\partial \sum_{g=1}^{n} x_{gE_g}^*}{\partial v_{tE_t}} = \frac{(n_1-1)}{\left(\sum_{l=1}^{n_1}\prod_{k\neq l}^{n_1} v_{kE_k}\right)^2}\left[\left(\sum_{l=1}^{n_1}\prod_{k\neq l}^{n_1} v_{kE_k}\right)\prod_{k\neq t}^{n_1} v_{kE_k} - \left(\prod_{k=1}^{n_1} v_{kE_k}\right)\sum_{l=1}^{n_1}\prod_{k\neq l,t}^{n_1} v_{kE_k}\right] = \frac{(n_1-1)\prod_{k\neq t}^{n_1} v_{kE_k}}{\left(\sum_{l=1}^{n_1}\prod_{k\neq l}^{n_1} v_{kE_k}\right)^2}\left[\left(\sum_{l=1}^{n_1}\prod_{k\neq l}^{n_1} v_{kE_k}\right) - v_{tE_t}\sum_{l=1}^{n_1}\prod_{k\neq l,t}^{n_1} v_{kE_k}\right] = \frac{(n_1-1)\left(\prod_{k\neq t}^{n_1} v_{kE_k}\right)^2}{\left(\sum_{l=1}^{n_1}\prod_{k\neq l}^{n_1} v_{kE_k}\right)^2} > 0$, i.e., the total rent dissipation is monotonically increasing in the values of the active players. Hence, the equilibrium in which only the highest valuation player in each group exerts positive effort results in the highest rent dissipation among the set of equilibria of the best-shot group contest. Following the same logic, the equilibrium in which only the lowest valuation player in each group exerts positive effort, dissipates the lowest rent and the intermediate cases will result in intermediate rent dissipation and the ranking will depend on the distribution of values within and between groups.

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