

# Can Taylor rule fundamentals predict exchange rates?

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## Abstract

Recent research suggests that there are many favourable features of the asset-pricing model of exchange rates incorporating Taylor rules. Against this background, this thesis focuses on the relationship between the exchange rate and Taylor rule fundamentals. The introductory chapter provides a short summary of the most relevant literature, and explains the connections between the main chapters. In chapter 2, we mainly follow Engel and West's (2006) framework of the asset-pricing model of exchange rate incorporating Taylor rules to forecast the yen/dollar exchange rate. The central research question is whether this type of model has any predictive power with respect to the exchange rate.

In chapter 3, a more detailed analysis of the properties of Taylor rules is undertaken. The main idea derives from one of the assumptions made in chapter 2, concerning the structural stability of the Taylor rules. If there are unknown structural breaks, the estimation of the Taylor rule is likely to be biased. Furthermore, both theoretical and empirical studies suggest that the Taylor rule in advanced economies is asymmetric. If a central bank is minimizing an asymmetric loss function in which negative and positive inflation- and output-gap deviation are, respectively, assigned different weights, then a nonlinear Taylor rule is optimal. Hence we set out to identify any structural breaks in the Taylor rule, and to uncover the extent to which nonlinearity plays a role in Taylor rule modelling. In our empirical study, a threshold model introduced by Caner and Hansen (2004) is used to measure whether the Taylor rules are nonlinear or not, in order to explain the existence of asymmetry of Taylor rules.

Chapter 4 compares the performance of the traditional monetary model and the Taylor rule model in terms of out-of-sample forecasting performance. A key study is by Molodtsova and Papell (2009) who derive a simple version of the Taylor rule model and demonstrate that it can outperform a variety of monetary models as well as the naive random walk, on the basis of the state-of-the-art goodness-of-fit statistic developed by Clark and West (2006) (the CW statistic). It is of considerable interest to discover whether Molodtsova and Papell's (2009) results are driven by the superior predictability of the Taylor rule fundamentals, or by features of the CW statistic. To address this question, the sterling/dollar exchange rate for the period 1975-2010 is investigated. A detailed analysis of the CW statistic, including Monte-Carlo simulations, is conducted. In addition, a variety of estimators are used, including the Vector Error Correction Method (VECM) which is used to generate the out-of-sample forecast. Also, a number of goodness-of-fit measures (in addition to CW) are used for comparing the predictability of the Taylor rule model with traditional monetary models. The overall finding is that the out-of-sample forecasting predictability of the sterling/dollar

exchange rate obtained by the Taylor rule model is not as significant as we expect by using a variety of goodness-of-fit measures, but the traditional monetary models have certain predictive power if VECM is applied.

# Chapter 1

## Introduction

In the modern world, currencies are exchanged for many different reasons. If people intend to travel to another country, they may buy foreign currency in a bank in the home country. A commercial company may want to seek foreign currencies to pay for importing goods or services. Central banks may buy or sell foreign currencies in order to stabilize the economy. Hedge fund or investment firms may speculate one currency to make a huge profit.

An exchange rate between two currencies is the rate at which one currency will be exchanged for another. Exchange rates are determined in the foreign exchange rate market, which is the most liquid financial market in the world. It is open to a wide range of different types of buyers and sellers, and trading is continuous: 24 hours a day except weekends. According to the Bank of International Settlement, the average daily turnover in global foreign exchange rate markets is estimated over 4 trillion US dollar in 2008, which means this market represents the largest asset class in terms of the trading volume.

Due to the importance of exchange rates, both business and academia world are keen to understand this particular financial asset. One of the topics in this field is to forecast the future exchange rate. In the last four decades, numerous literature has been trying to seek appropriate determinants of the exchange rate for prediction purpose. However, at least in academia, this practice has not been very successful. A cornerstone work is made by Meese and Rogoff (1983a,b), they demonstrate that no structural models of exchange rates can outperform the random walk model in terms of out-of-sample prediction; the fundamental variables such as output, interest rate, and inflation rate have little correlations with exchange rates<sup>1</sup>. Cheung, Chinn and Pascual (2005) apply a wider set of empirical

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<sup>1</sup>A random walk means the series at time  $t$  is only related to its own lag at time  $t - 1$ , thus, no other variables can influence the series.

exchange rate models and come to similar conclusions. Sarno and Valente (2009) demonstrate that the the relationships between exchange rates and fundamentals vary across currencies. Therefore, it is difficult to find one empirical model suitable for all exchange rates.

Although the results provided by the above literature are disappointing, many researchers did continue studying the link between fundamentals and exchange rate determination. A new direction in macroeconomics is to combine Taylor rule fundamentals into exchange rate modelling. The Taylor rule is a monetary policy rule which is introduced by John Taylor in 1993, the rule is used to describe a central bank's behaviour in monetary policy making. The idea is that a central banks should consider to stabilize inflation and output as its main targets, in order to do so, the bank needs to set both inflation and output target, and adjusts its short-term nominal interest rate when inflation or output is deviated from its target. It is believed that the central bank behaviour in advanced economy can be best described by the Taylor rule.

Engel and West (2005) (hereafter EW05) first provide a theoretical framework to understand the link between exchange rate and Taylor rule fundamentals and derive a discounted present-value model by applying Taylor rules. Engel and West (2006) (hereafter EW06) apply this present-value model of EW05 for the Deutschmark/dollar real exchange rates. They find a positive correlation between actual and model-based exchange rates, their results can be considered to be a promising start for this type of research.(also see Mark, 2009; Molodtsova and Papell, 2009)

Based on this literature we started our journey in the exploration of the relationships between exchange rate modelling and Taylor rules. In this introductory

chapter we briefly introduce the motivation of and connection between each chapter.

As EW05 suggest, the asset approach of exchange rate modelling incorporating Taylor rules should be considered as a new direction in exchange rate forecasting, and the exchange rate especially should be viewed as a linear combination of a series of discounted future fundamentals. Chapter 2 first provides a short summary on the development of these type of models, and then follow this idea and try to demonstrate the validity of it in empirical study of the yen/dollar real exchange rate.

Our research, outlined in chapter 2, originated from EW06. They build up a present value model for the Deutschmark/dollar real exchange rate based on the assumptions that both Bundesbank and the Fed follow Taylor rules, and the discounted factors and parameter coefficients are imposed based on the results of Clarida et al. (1998) (CGG hereafter). The future fundamental variables are forecasted by using unrestricted VAR. EW06's forecasts show that there is a positive relationship between their fitted forecasts and the actual series.

A few changes are made in order to improve the out-of-sample predictability of the exchange rates. First of all, in EW06's paper, the parameter coefficients are imposed for the forecast, suggesting that a better method would be to search for the monetary policy rule parameters that lead to the best possible fit to the exchange rate. In chapter 2 our empirical study focuses on Japan and the US during the period between February 1971 and December 2006. Instead of using imposed parameters from CGG, we estimate the Taylor rules for the Bank of Japan (BOJ) and the Fed in different periods of time. It is of interest to find out whether the Taylor rules are used for both of these central banks, and to examine whether

or not both home and foreign central banks are using the same Taylor rules. Furthermore, we not only consider the baseline case of Taylor rules, but a more general form of the Taylor rule is discussed. For instance, we examine whether or not the lagged interest rate play a role in the interest rate reaction function. Since a series of lagged variables are used as the information set, assuming the number of instruments (orthogonality conditions) exceeds the parameter set, and the regressors are not orthogonal to the error term, the Generalized Method of Moments (GMM) is used as the estimator of Taylor rules. Naturally, we also need to test overidentifying restrictions, and Hansen's (1982) method is conducted for this purpose. Since the estimation period is over 3 decades, Hodrick and Srivastava's (1984) test is used to detect the validity of structural breaks. The results of the estimations is also used for the exchange rate forecast, the detailed discussion can be found in section 2.4.

The second contribution of chapter 2 is to forecast the yen/dollar real exchange rate based on our findings of the Taylor rule estimations. A variety of specifications are examined in order to investigate whether or not these specifications can improve out-of-sample predictability using this class of exchange rate model. The baseline case is to assume that both countries follow an identical Taylor rule without a smoothing parameter (the lagged interest rate). The alternative cases allow for a specification with smoothing parameters. we also discuss the specifications that the BOJ and the Fed follow different Taylor rules.

Third, we pay attention to the estimation of output gap. The output gap can be considered as the difference between actual output and the potential output the economy can achieve. Normally the output gap is obtained by the Quadratic Time Trend (QTT) or the Hodrick-Prescott filter in empirical study, but both these techniques have drawbacks. A new technique called B-spline method is

introduced in section 2.3. The idea of the B-spline method is to use several quadratic or cubic curves to smooth out the actual output and the left over is the output gap. All three output gap measures are used in chapter 2 to see if the choice of different output gaps makes a difference in the estimation of Taylor rules and the exchange rate forecasting<sup>2</sup>.

After finishing chapter 2 we began to realize the importance of the Taylor rule in terms of studying the relationship between exchange rate forecast and macro fundamental variables. Therefore, our interest in chapter 3 changed direction to studying whether or not a central bank's behaviour in an advanced economy can be described as a Taylor rule and how to improve Taylor rule estimation in empirical study. We focus on the Fed's behaviour in chapter 3.

A number of studies indicate that the central bank's behaviour is likely to be asymmetric. For instance, the former vice president Blinder said that the central bank tended to be more cautious about high inflation than high unemployment. (also see Persson and Tabellini, 1999). Moreover, there might be different priorities for a central bank in different time periods. For instance, in the 1980s the Fed put most of its resources into curbing the high rate of inflation whereas in the 2000s all the attention was turned towards responding to poor economic growth. If this is the case, then the study of the Taylor rule should focus on whether it is symmetric or asymmetric<sup>3</sup>.

Plenty of literature argues that a central bank's reaction function is asymmetric in both theoretical and empirical study. In empirical study there are two

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<sup>2</sup>The B-spline method is also used in chapter 3 and 4

<sup>3</sup>Asymmetric Taylor rule means that a central bank, for instance, is more worried about the output contraction than expansion. Therefore, the bank might set the interest reaction function to put a larger weight to output contractions than to output expansions to the same magnitude.

types of directions: one is to assume that the function follows a nonlinear form, so that the nonlinear techniques can be used to estimate the reaction function. The other is to assume that the instability is caused by economic structural changes, and that one needs to find out what these potential structural changes are, to estimate the reaction function in each subsample period, and then to test for whether these functions are linear or nonlinear. Regarding the second method, it is possible that the reaction function is still nonlinear even in the subsample period, and thus nonlinearity should be tested even in the sub regimes.

The first strand has been widely developed (see Carlin and Sosikice, 2006; Surico 2007; Castro 2010), whereas the second class has not been fully discussed. One of the issues is to estimate the potential multiple structural breaks. There are plenty of tests for detecting one or two structural breaks in the model (Perron, 1989; Clemente et al., 1998; Zivot and Andrews, 1992), but it is very difficult to find an appropriate test to identify more than two structural breaks. The one that has been used up to now is the Bai and Perron's (2003) test, which can only be used with OLS as the estimator. Section 3.3 modifies Andrews' (1993) method, which can be used under a GMM framework, in order to investigate the potential economic structural changes. The sample is then divided into several subsamples based on these change points and the Taylor rule is re-estimated under each subsample, thus showing us whether or not the monetary authorities behave symmetrically.

Another contribution in chapter 3 is to apply the GMM framework for the nonlinear Taylor rule estimation. The majority of literature on the subject of nonlinearity, in essence, uses Least Square (Qin and Enders, 2008; Surico, 2007; Kenneth, 2007). However, as we have explained in the previous section, GMM can be considered as a superior estimator for studying a Taylor rule. In sec-

tion 3.5, Caner and Hansen's (2004) (hereafter CH04) method is applied for the task. CH04 introduce a threshold model under a GMM framework, and also develop a method to identify the validity of the threshold estimation. We use their method to estimate Taylor rules of the Fed in different sub regimes and demonstrate whether there is nonlinearity in the estimation. We also estimate linear Taylor rules in different periods of time as a comparison set for the analysis.

Chapter 4 concentrates on the empirical study of the traditional monetary approach of exchange rates and the Taylor rule model in terms of out-of-sample forecasting, especially discussing their performance against a naive random walk. Three decades ago Meese and Rogoff (1983a,b) demonstrated that none of the traditional models can out-perform a random walk, and countless literature has confirmed their conclusions since then. However, there are still some interesting areas we would like to explore.

Our first inspiration comes from MacDonald and Taylor (1994), who re-examine the monetary model for the sterling/dollar exchange rate. Using the multivariate cointegration technique (or Vector Error Correction Method), they find that the unrestricted monetary model can outperform the random walk in an out-of-sample forecasting context. In Meese and Rogoff's (1983a) paper, the rolling regression is used for the forecast, which is basically OLS, but their results are not significant. In the following literature, Mark (1995) and Cheung et al. (2005), using the Error Correction Model, obtain similar conclusions. It appears that the choice of estimator can play a significant role in the forecasting. Thus, in chapter 4, a variety of estimators are used for out-of-sample sterling/dollar exchange rate forecasting. Three methods are included: VECM, VAR taking the first differences, and rolling regression in first differences. We expect that VECM can bring a better forecast since it loosens certain assumptions such as that only

the exchange rate should be endogenous in the estimation. Not only the long term relationship between the exchange rate and fundamentals are considered, but also the short term relationship between them. We are also interested in comparing the model with both homogeneous and heterogeneous coefficients. It is expected that allowing the coefficients between the home and the foreign country to be different can improve the forecasting performance.

In order to verify the validity of VECM, two types of tests are conducted (section 4.8 and 4.9). The first type of test is a cointegration test. Most of the macro fundamental variables are  $I(1)$  (see section 4.7), and if there is a cointegration equation in the specifications one should use VECM or ECM for the estimation. However, if there is no cointegration, one should consider VAR or regression taking the first difference of variables. We apply Johansen's Trace statistic and Maximum eigenvalue statistic to find the number of cointegrating equations in the VECM models. The second type of test is to identify the weak exogeneity of each variable. If the dependent variable is endogenous and the explanatory variable is weakly exogenous, then a partial system, such as ECM, is as efficient as a full system. However, if both the dependent variable and at least one of the explanatory variables are endogenous, then one should use VECM. The issue here is that if the dependent variable is weakly exogenous, then neither VECM nor ECM should be considered: one should consider ARIMA or VAR taking the first difference of variables, depending on whether the variable is also strongly exogenous. The detailed discussion is in section 4.3.

There is also a debate on which type of goodness-of-fit measures should be used for comparing the out-of-sample forecasting performance. Meese and Rogoff(1983a) choose Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) for out-of-sample forecast comparisons. Cheung et al (2005) use Diebold

and Mariano (1995) and West (1996) statistic (DMW statistic hereafter), the change of direction test, and the consistency test. Molodtsova and Papell (2009); Molodtsova et al. (2008) use a modified MSPE ratio test which is introduced by Clark and West (2006), the so called CW test. This comparison test can lead to different results. For instance, Clark and West (2006) demonstrate the DMW statistic fails to reject the null hypothesis that fundamental models cannot outperform random walks, whereas Rogoff and Stavrakeva (2008) imply that the CW statistic is not a minimum mean squared forecast error statistic.

Since there is a debate of the performance of the CW statistic and the DMW statistic, section 4.5 compare both techniques and find out which one is more appropriate to be considered as a better goodness-of-fit measure for the out-of-sample exchange rate forecasting. We use a variety of goodness-of-fit measures for the out-of-sample forecasting in the empirical study (see section 4.10) and these measures are introduced in section 4.4.

There is no independent literature review chapter in this thesis. This is because the main chapters are to a large extent independent of each other, and are therefore built on different literatures. For this reason, it seems more natural to provide literature reviews within each chapter. This is done in chapters 2 and 3 (see sections 2.2 and 3.1). In chapter 4, there is no section explicitly reviewing the literature, because the subject matter is such that it feels more natural to cite relevant literature where it is needed.

## Chapter 2

# Taylor rule and exchange rate modelling

## 2.1 Introduction

A long-standing puzzle in macroeconomics is the difficulty of demonstrating the linkage between the exchange rate and the fundamental variables such as money supply, interest rate and output. Meese and Rogoff (1983a,b) demonstrate that no time series or structural model of exchange rates can out-forecast the random walk model in terms of out-of-sample prediction; the fundamental variables have little correlation with exchange rates. Cheung et al. (2005) reassess a wider set of empirical exchange rate models and come to a similar conclusion. Both papers argue that, although there are some plausible models of exchange rate determination in these theories, exchange rates in advanced economies are more likely to follow a random walk process empirically.

Although the above papers are authoritative, many researchers continue studying the linkage between fundamentals and exchange rate determinations. Some researchers (Frenkel, 1981; Dornbusch, 1976; Mussa, 1982) argue that the exchange rate should be viewed as an asset price, for example, stock price, which is determined not only by the current fundamentals but also by the expectations of future fundamental variables. However, Meese and Rogoff (1983a,b) and Cheung et al. (2005), who examine the exchange rate movements at time  $t$ , find that they are determined by various combinations of fundamentals at time  $t$  or  $t + 1$ . One drawback of these models is that all the fundamentals on the right-hand side of the equations are very persistent, while the exchange rate is much more volatile than these variables. Thus it is not difficult to understand why these models are unable to do a good job in forecasting.

There are many ways of defining the exchange rate determination in international economics. One of them is to combine the exchange rate determination

with monetary policy. The traditional class of exchange rate models in combination with monetary policies derived from the money income model which assumes that central banks in advanced economies adjust the stock of money in the market to stabilize the economy. However, a recent belief is that the monetary policy can be better modeled by the Taylor rule. In brief, the Taylor rule sets a nominal interest rate in response to the development of inflation and output. Taylor (1993a) and Clarida et al. (1998, 2000) all demonstrate that it is more efficient to adjust the short term interest rates rather than the money supply in order to stabilize the economy.

Since the Taylor rule can be considered a monetary policy rule and the exchange rate should be viewed as an asset price, a number of models try to combine them with the exchange rate model. The Taylor rules are used by Engel and West (2005) as an example of present-value models where exchange rates will approach a random walk when the discount factor approaches 1. Engel and West (2006) (hereafter EW06) build a asset-pricing model for the Deutschmark/dollar real exchange rates. In this study the identical Taylor rules for Germany and the US are applied. They find a positive correlation between actual and model-based exchange rates. Mark (2009) challenges some of the assumptions in the EW06's specifications and obtains similar results to those of EW06.

There are many favourable properties in the asset-pricing model of exchange rates incorporating the Taylor rules in our discussions above. Nevertheless, few studies discuss whether or not the class of model can outperform other structural models and a random walk process in terms of the out-of-sample predictability. Therefore, the economic question we would like to pursue is whether or not the asset-pricing model can outperform other models. we intend to follow this line of enquiry and try to improve the asset-pricing model.

Our research approach is based mainly on the work of EW06. However, a few changes are made in order to improve the out-of-sample predictability of exchange rates. Firstly, one fundamental variable in the model is output gap. Normally the output gap is obtained by the Quadratic Time Trend or the Hodrick-Prescott filter, but both these techniques have their own drawbacks. We describe below, therefore, how new techniques might be needed. Secondly, it is important to find out if a central bank has applied the Taylor rules to its monetary policy decision process at specific periods of time. If the Taylor rule is used, it is also critical to examine whether or not the home and the foreign central banks are using this same rule. We also need to consider whether or not economic structural breaks have an impact on the changes in the Taylor rule at different periods of time, and whether other fundamentals such as lagged interest rate are important in the interest rate reaction function. Our third point regarding the exchange rate modelling concerns a variety of specifications which are examined in order to find out whether they can improve the out-of-sample predictability of this class of exchange rate models. In this empirical study we focus on the yen/dollar exchange rates, the data for Japan and the US are collected from the IMF International Financial Statistics database (IFS) and the Fed Reserve Bank of St. Louis Economic Dataset (FRED).

## **2.2 Literature review**

Since the breakdown of the Bretton Woods system in 1973 the major central banks in developed countries have adjusted their exchange rate regimes from ‘pegged to the US dollar’ to ‘controlled floating’. This significant innovation has brought in new characteristics, and these characteristics cannot be explained by

the traditional macroeconomic theories. Above all, in comparison to fixed exchange rate regimes, with the floating exchange rate regime the adjustments in exchange rates are much more volatile than other macroeconomic variables such as output and inflation. Also, changes in spot exchange rates are almost unpredictable. Exchange rates are more likely to follow a random walk process. The first problem indicates that it might not be appropriate to model exchange rates by using fundamental variables, as we explain later. Secondly, there has been a dramatic evolution in the relationship between the interest rate and the exchange rate. For instance, the empirical evidence shows that the rise in the federal fund rates caused the devaluation of the dollar in the 1970s, but this negative relationship between these two variables reversed in the 1980s. In other words, a rise in interest rate leads to the appreciation of the home currency. Finally, the purchasing power parity does not hold, i.e. there is a systematic deviation between exchange rates and aggregate price levels. Thus, economists have been trying to find new ways of explaining these empirical questions.

A new theory, called ‘the asset market approach to exchange rate’ has been developing in a great body of the literature in order to explain these new characteristics of the exchange rate under the controlled floating exchange rate regime. The theory suggests that

*“the exchange rate should be viewed as price of asset (like stock and commodity exchange) in which current prices reflect the markets expectation concerning present and future economic conditions relevant for determining the appropriate values of these durable assets, and in which price changes are largely unpredictable and reflect primarily new information that alters expectations concerning these present and future economic conditions.”* (Frenkel and Mussa, 1985)

Mussa (1976) explains why we should consider the exchange rate to be an asset price. The nominal exchange rate was excessively volatile after the 1970s compared to other macroeconomic fundamentals; therefore, it is hard to believe that we should apply the measures which are used to forecast fundamental variables to the exchange rate forecasting. On the other hand, asset prices are very volatile too; see, for example, movements in the stock prices. In addition, there is a consensus that both the asset price and the exchange rate are sensitive to future events. These coincidences imply that there are likely to be similarities between the two. Specifically, since the changes in an asset price are considerably influenced by the expectations of the market, it is possible that expectations also cause fluctuations of the exchange rate. For instance, if the market expects the home currency to devalue, its demand falls and the exchange rate will also fall because of such expectations.

Frenkel and Mussa (1980) have managed to build a theoretical model for the ‘asset market approach to exchange rates’. The simplest asset market model of the exchange rate can be written as:

$$s_t = z_t + bE[s_{t+1} - s_t] \quad (2.1)$$

where  $E[s_{t+1} - s_t]$  denotes the expected change in the nominal exchange rate between time  $t$  and  $t + 1$ , based on information available at  $t$ ,  $b$  measures the sensitivity of the current exchange rate to its expected rate of change, and  $z$  represents the factors of supply and demand<sup>1</sup> that affect the exchange rate at time  $t$ . If we consider the rational expectations and by forward iteration:

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<sup>1</sup>For instance, domestic and foreign money supply, income and output level, etc. Mussa (1976) develops a monetary approach to analyze exchange rates. He points out that the expectations of both the supply and demand of national currencies are significant in determining the exchange rates. Under the rational expectations, the supply and demand of currencies are influenced mainly by future monetary policy.

$$Es_{t+j} = \frac{1}{1+b} \sum_{k=0}^{\infty} \left( \frac{b}{1+b} \right)^k \times Ez_{t+j+k}, j = 0, 1, 2, \dots \quad (2.2)$$

According to Eq. (2.2), there is a connection between the current exchange rate ( $j = 0$ ) and the current expectations of future exchange rate ( $j > 0$ ), since both of them depend on the expectations of future fundamentals. If there is an expected change in the future fundamental variables  $z_{t+j+k}$ , the expectation of the exchange rate will change accordingly.

Eq. (2.2) can be viewed as a “present-value model”. The empirical facts for exchange rates above can be clarified by using this model. Firstly, the fact that the exchange rate responds not only to current, but also to future events, explains why we cannot find a link between exchange rates and past economic variables. Furthermore, the exchange rate should be much more volatile than other macroeconomic fundamentals such as output and inflation because the exchange rate responds to the ‘news’ quickly. Alongside this, it is difficult to predict exchange rates due to the unpredictability of certain future events.

A number of models try to explain the movements in exchange rates by finding the correlations between monetary policy and exchange rates. One critical issue is to decide the instrument of the monetary policy. Initially, money aggregates were considered to be the instrument of monetary policy (Dornbusch, 1976; Mussa, 1982; Frankel and Rose, 1995). In other words, the authorities controlled the aggregate money supply in order to stabilize the economy.<sup>2</sup> Nevertheless, empirical studies have shown that this class of exchange rate models that use the money supply as an instrument cannot beat the random walk model. Frankel and Rose (1995) build a monetary exchange rate model considering money sup-

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<sup>2</sup>Dornbusch (1976) provides a model linking exchange rates and money stock. He argues that the increase in money supply leads to the depreciation of exchange rates.

ply as an instrument. In this model they prove that it cannot do a better job than the random walk model irrespective of whether or not the price is sticky. Bergin (2003) attempts to estimate a structural general equilibrium model of a semi-small open economy. By adapting a ‘maximum-likelihood’ procedure, the behaviour of exchange rates is tested and estimated with monetary shocks, nominal price and wage rigidities. Bergin’s results show that the model cannot beat a random walk model in predicting movements in exchange rates.

Another class of models suggests that the short-term interest rate should be viewed as the instrument for monetary policy rather than the money supply because since the 1980s most central banks in advanced economies have become more and more sensitive to the change in expected inflation and so tend to control inflation by changing the short-run nominal interest rates. Many models demonstrate that the monetary policy models based on interest rate rules outperform those that focus on other instruments. For instance, Bryant et al. (1993) compare the performance of nine different monetary policy models. In all of them they consider the interest rate as being the monetary instruments. They find that the policy rules that focus on the price level and real output can deliver a more significant performance than policies that focus on the exchange rate or the money supply. Taylor (1993b) obtains similar results by simulating the economic performance of the G7 countries under a variety of interest reaction functions. Being inspired by these ideas and this evidence, Taylor (1993a) develops a simple monetary policy rule which calls for changes in the federal fund rates in response to a change in the level of real output and inflation. This reaction function rule is known as the ‘Taylor rule’<sup>3</sup>:

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<sup>3</sup>Although the Taylor rule is expressed as a mechanical function and its estimation of interest rate can imitate the actual federal fund rates from 1987-1992 quite accurately, even Taylor himself admits that this monetary policy rule is unable to work out or explain all circumstance in practice. He emphasizes that there are two reasons for this: Firstly, the monetary authorities might be aware of “the general instrument response that underlies the policy rule”, but they

$$i_t^* = r^* + \pi_t + \gamma_\pi(\pi_t - \pi^*) + \gamma_y(y_t - y^*) \quad (2.3)$$

where  $\gamma_\pi > 0, \gamma_y > 0$ . In the Taylor rule,  $i_t^*$  is the target of short-term nominal interest rate.  $r^*$  is the long-run real interest rate,  $\pi_t$  is the inflation rate,  $\pi^*$  is the inflation target,  $y_t$  is the real output,  $y^*$  is the output target, and  $\gamma_\pi$  and  $\gamma_y$  are the weights that the central bank uses in the interest rate reaction function.

There are different versions of Taylor rules. One is the forward-looking version developed by Clarida et al. (1998) (hereafter CGG). They assume that when the central bank chooses its target interest rate, it is possible that it does not have the information about GDP or price level at the time, so CGG specify the interest rate reaction function as follows:

$$i_t^* = i^* + \gamma_\pi(E[\pi_{t+n} | \Omega_t] - \pi^*) + \gamma_y(E[y_t | \Omega_t] - y^*) \quad (2.4)$$

where  $i^*$  is the long-run equilibrium nominal rate,  $E[\pi_{t+n} | \Omega_t]$  is the expected rate of inflation rate in period  $t+n$ , based on the information set  $\Omega$  available to the central bank in period  $t$ . Then we need to consider the implied real interest rate target <sup>4</sup>.

$$r_t^* = r^* + (\gamma_\pi - 1)(E[\pi_{t+n} | \Omega_t] - \pi^*) + \gamma_y(E[y_t | \Omega_t] - y^*) \quad (2.5)$$

where  $r_t^* = i_t^* - E[\pi_{t+n} | \Omega_t]$ ,  $r^* = i^* - \pi^*$ . According to Eq. (2.5), the target interest rate adjusts in response to the change in the deviations from the inflation

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tend to operate the rule using their own judgment rather than simply following the function. Secondly, there might be other real factors affecting the interest rate, but these factors are not included in the Taylor rule.

<sup>4</sup> $r_t^*$  is an ‘approximate’ real rate since the forecast horizon for inflation will generally differ from the maturity of the short-term nominal rate used as a monetary policy instrument. In practice this is of little relevance, given the high correlation between short-term rates at maturity associated with plausible target horizons.

target and the output target. The coefficient  $\gamma_\pi$  is critical in this specification. With  $\gamma_\pi > 1$ , the real interest rate can stabilize inflation. If  $\gamma_\pi < 1$ , it instead accommodates any changes in inflation. In other words, if the central bank does not raise the nominal interest rate enough when inflation is expected to rise, the real interest rate could still rise, which would in turn provoke a burst in the rise of inflation <sup>5</sup>.

The policy reaction function represented by Eq.(2.5) is too restrictive to describe actual changes in the interest rate. The specification assumes an immediate adjustment of the actual interest rate to its target level, and thus ignores the central bank's tendency to smooth changes in interest rates. In order to challenge this assumption, CGG specify the relationship for the actual nominal interest rate:

$$i_t = \rho i_{t-1} + (1 - \rho)i_t^* + v_t \quad (2.6)$$

where  $\rho \in [0, 1]$  captures the degree of interest rate smoothing.  $v_t$  is an exogenous random shock to the interest rate. CGG assume that it is i.i.d.

CGG define  $\alpha = i^* - \gamma_\pi \pi^*$ ,  $x_t = y_t - y^*$ ,  $\gamma_x = \gamma_y$ , and then rewrite Eq. 2.4 as:

$$i_t^* = \alpha + \gamma_\pi E[\pi_{t+n} | \Omega_t] + \gamma_x E[x_t | \Omega_t] \quad (2.7)$$

Combining Eq. (2.6) with the target model Eq. (2.7) yields the policy reaction function,

$$i_t = (1 - \rho)(\alpha + \gamma_\pi E[\pi_{t+n} | \Omega_t] + \gamma_x E[x_t | \Omega_t]) + \rho i_{t-1} + v_t \quad (2.8)$$

Finally, adding and subtracting  $(1 - \rho)\gamma_\pi \pi_{t+n}$  and  $(1 - \rho)\gamma_x x_t$  on the right

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<sup>5</sup>The idea that the real interest rate should be raised when inflation increases (requiring the nominal interest rate to increase more than inflation does) has sometimes been called the Taylor principle.

hand side:

$$i_t = (1 - \rho)(\alpha + \gamma_\pi \pi_{t+n} + \gamma_x x_t) + \rho i_{t-1} + \varepsilon_t \quad (2.9)$$

where the error term  $\varepsilon_t = -(1 - \rho)[\gamma_\pi(\pi_{t+n} - E[\pi_{t+n} | \Omega_t]) + \gamma_x(x_t - E[x_t | \Omega_t])] + v_t$  is a linear combination of the forecast errors of inflation, output and exogenous disturbance.

The Taylor rule can also be tested for backward-looking specifications, in which the central bank adjusts its interest rate in response to the lagged inflation rate rather than the current or future value of inflation. Clarida et al. (2000) also test the backward-looking specification and demonstrate that, in fact, in most industrialized countries, central banks tend to use the forward-looking one. We do not, therefore, consider the backward-looking version in detail because of space limitations.

Although it has not been verified in the theory that the nominal or real exchange rate should be considered as an explanatory variable, Clarida et al. (1998, 2000) find evidence that the Deutschmark/dollar and yen/dollar exchange rates play an important role in the interest rate for Germany and Japan. Based on their findings a considerable number of models have tried to develop the link between exchange rates modelling and Taylor rules.

At least five strands of literature specifically analyze nominal or real exchange rate modelling incorporating Taylor rules. The first strand of literature is from Engel and West (2004, 2005, 2006) and Engel et al. (2007), who emphasize that exchange rates should be considered as an asset price, being determined by the sum of present and discounted future macroeconomic fundamental variables. Moreover, they believe that the fundamentals are determined by the Taylor rules.

The second strand is proposed by Mark (2009), who builds a present-value model of the Deutschmark/dollar (euro/dollar) real exchange rate with learning, thereby challenging some of the assumptions of EW06. He argues that the economic agents know the structure of the economy and they also know that central banks use Taylor rules. He believes, however, that the true parameters used by policy makers are unknown to the public, and that these parameters also change over time. Thus agents have to behave like econometricians in order to estimate the parameters of the Taylor rules and to form their own present-value models of exchange rates. In the long run they find the true parameters in these learning environments.

The third strand of literature is mentioned in Engel et al. (2007). EMW07 demonstrate that panel data can increase the forecasting efficiency for the exchange rate model. They compare the out-of-sample predictive power of the Taylor rule, monetary and PPP models with two versions of the random walk model: with and without ‘drift’. The outcomes confirm that both the monetary and PPP models have some probability of beating the random walk model in the long run, when using panel data. However, the evidence for Taylor rule models is mixed. It is difficult to conclude that panel techniques could definitely improve the forecasting performance of Taylor rule models. One of the reasons could be that these models all assume that every country on the panel have the identical Taylor rules.

The fourth class of model follows Molodtsova et al. (2008) and Molodtsova and Papell (2009), who examine out-of-sample exchange rate predictability with Taylor rule fundamentals. They do not treat the exchange rate as an asset price. The Taylor rule for a foreign country is subtracted from the rule for a home

country (the US), the equation then having the interest rate differential on the left-hand side and a sequence of fundamentals on the right-hand side. Molodtsova et al. (2008) and Molodtsova and Papell (2009) assume that there is a negative relationship between a change in the exchange rate and the interest rate differential, and thus that the rise in domestic interest rate causes an appreciation in the home currency, and a forecasted appreciation in the following period.<sup>6</sup> The interest rate differential can then be replaced by the exchange rate differential. Therefore, they succeed in finding the missing link between exchange rates and Taylor rule fundamentals. The main difference between Molodtsova et al. (2008) and Molodtsova and Papell (2009) and others is that they consider a variety of specifications of the exchange rate model. They examine and compare the model with or without a constant, the real exchange rate and an interest rate smoothing variable (the lagged interest rate). They also consider the possibility that the central banks may have different Taylor rules (a heterogeneous model) or that they have the same coefficients in their Taylor rules (a homogeneous model). Their findings provide strong evidence for the short run predictability of the exchange rate model using Taylor rule fundamentals. Overall, the specification that produces the most evidence of exchange rate predictability is “a symmetric model with heterogeneous coefficients, smoothing and a constant”.

Finally, the last strand follows Wang and Wu (2012). In their model they use interval forecasting for exchange rates. Their method concentrates on interval forecasting rather than point forecasting. They apply semi-parametric forecast intervals to a group of Taylor rule exchange rate models for 12 OECD countries. The results show that the forecasting intervals generated by Taylor rule models are tighter than those obtained by the random walk model, especially in the long

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<sup>6</sup>Molodtsova et al. (2008) and Molodtsova and Papell (2009) assume that UIP does not hold in their model.

run. There is also evidence that the Taylor rule models outperform both the Monetary and PPP models in terms of out-of-sample interval forecasts.

Due to space limitations, in the following section, we only describe the first class of model in detail.

### **Engel Mark and West’s present-value model of exchange rates incorporating the Taylor rule**

Engel and West (2005) provide a theorem to explain why the exchange rate so nearly follows a random walk in low-inflation advanced economies. They argue that the exchange rate should be viewed as an asset and can be obtained by a discounted value of a linear combination of observable fundamentals and unobservable shocks. They also apply the Taylor rules to generate a present-value model of exchange rates.

The asset price follows a random walk if the following two requirements are satisfied: first, at least one explanatory variable is an  $I(1)$  process; and second, the discount factor is near unity. If the asset price is considered to be the discounted sum of current and expected future ‘fundamentals’, then a general asset price model could be expressed as follows:

$$s_t = (1 - b) \sum_{j=0}^{\infty} b^j E_t(a'_1 x_{t+j}) + b \sum_{j=0}^{\infty} b^j E_t(a'_2 x_{t+j}) \quad (2.10)$$

where  $s_t$  is the log of an asset price,  $x_t$  is an  $n \times 1$  vector of fundamental at time  $t$ , and  $b$  is a discount factor. With reference to Eq. (2.10), consider that either  $a'_1 x_{t+j} \sim I(1)$  and  $a_2 = 0$  or  $a'_2 x_{t+j} \sim I(1)$  and  $a'_1 x_{t+j}$  is unrestricted. Then, in either case, when the discount factor approaches unity, the first difference in

the nominal exchange rate approaches zero, which indicates a near-random walk behaviour in the exchange rate. Further tests conducted by EM05 suggest that the correlation between the change in exchange rate  $\Delta s_t$  and the change in fundamentals and the autocorrelation of  $\Delta s_t$  is near to zero when  $b$  approaches one. These findings correspond with the properties of a random walk.

If the exchange rate models can be viewed as an asset price, the general model of exchange rates is written as follows:

$$s_t = (1 - b)(f_{1t} + z_{1t}) + b(f_{2t} + z_{2t}) + bE_t s_{t+1} \quad (2.11)$$

Under the ‘no bubbles’ conditions,<sup>7</sup>  $b^j E_t s_{t+j} \rightarrow 0$  when  $j \rightarrow \infty$

$$s_t = (1 - b) \sum_{j=0}^{\infty} b^j E_t (f_{1t+j} + z_{1t+j}) + b \sum_{j=0}^{\infty} b^j E_t (f_{2t+j} + z_{2t+j}), \quad j = 0, 1, 2, 3, \dots \quad (2.12)$$

where  $a'_1 x_{t+j} = f_{1t+j} + z_{1t+j}$ ,  $a'_2 x_{t+j} = f_{2t+j} + z_{2t+j}$  and  $s_t$  is the log of the exchange rate which is the log of the price of foreign currency in terms of domestic currency;  $f_{it}$  are observable variables and  $z_{it}$  are unobservable shocks.

EW05 employs two sorts of specific models which are the ‘money income model’ and the ‘Taylor rule model’. Both these models can be transformed into the general form above. In EW05’s empirical study, bilateral U.S. exchange rates versus the currencies of six other countries in G7 are studied, using the quarterly

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<sup>7</sup>“(Rational) bubbles represent a divergence from the equilibrium associated with the market fundamentals. Bubbles could be considered as one possible explanation of the observed volatility of exchange rates.” (Copeland, 2005, p.372). If there is bubble in the function, it can be written as:

$$q_t = b \sum_{j=0}^{\infty} b^j E_t Z_{t+j} + B_t, \quad 0 < b < 1, \quad j = 0, 1, 2, 3, \dots$$

Where  $B_t$  is the bubble at time  $t$ . As long as it persists, the exchange rate deviates from its fundamental equilibrium. In other words, under the ‘no-bubbles’ condition, the exchange rate is the level dictated by the fundamentals.

data from January 1974 to March 2001. They find that all the observable variables<sup>8</sup>, which are very persistent  $I(1)$  or nearly so. In addition, there is evidence that the unobservable variables are also very persistent according to other papers<sup>9</sup>. Although whether  $b$  approaches one is open to debate, plenty of evidence proves that it is very near unity.<sup>10</sup> Therefore, both theories about asset price being a random walk process are vindicated in the exchange rate models. Thus it is reasonable to infer that exchange rates follow a near-random walk process.

An alternative explanation for the near-random walk behaviour of exchange rates is that the unobserved variables determine the changes in the exchange rates, and if these variables follow a random walk process, exchange rates are not far from a random walk. In 2004, Engel and West (hereafter EW04) provide a simple empirical study for their EW05 theorem, in which they conclude that both explanations are possible.

Engel and West (2006) (hereafter EW06) summarize that there are “at least four strands of literature” that analyze real or nominal exchange rate models and consider interest rate rules as the instrument of monetary policy. The first one is the literature on identified VARs<sup>11</sup>. Kim (2002) develops a structural VAR model to estimate the monetary policy reactions for European countries, espe-

<sup>8</sup>The fundamental variables in EW05’s paper are:  $m_t^h - m_t^f, p_t^h - p_t^f, i_t^h - i_t^f, y_t^h - y_t^f$  and  $m_t^h - y_t^h - (m_t^f - y_t^f)$ .

<sup>9</sup>For money demand model, see Sriram (2001); for PPP, see Rogoff (1996); for Interest Rate Parity, see Engel (1996).

<sup>10</sup>See Bilson (1978), Frankel (1979), Stock and Watson (1993), Clarida et al (1998).

<sup>11</sup>The VAR (vector autoregression) approach can be very helpful in examining the relationship between a set of economic variables. Moreover, it can be used for forecasting purpose. The following systems of equations are first-order vector autoregressions:

$$(1) y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}$$

$$(2) z_t = b_{20} - b_{22}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt}$$

In this ‘two variable’ case, the series of  $y$  can be affected by the current and past value of  $z$ ;  $z$  can be affected by  $y$  too. This is a structural VAR. However, if we want to estimate the coefficients of this model, we have to make restrictions (the restriction is based on the economic theory or event) in identifying the structural model. The model which is identified is called an ‘identified VAR’. (Enders, 2010, p.326)

cially the stabilization of exchange rates. A second strand of literature tests the interest parity, using an unstructured method, decomposing real exchange rate movements into components that can be linked to interest rates and those that cannot. Examples include Campbell and Clarida (1988), Edison and Pauls (1993), and Baxter (1994). A third strand of literature develops general equilibrium sticky-price models, and uses calibration. Examples include Benigno (2004) and Benigno and Benigno (2001). These papers do find a strong connection between interest rates and exchange rates.

In the last strand of models the exchange rate is considered as an asset price, which depends on the weighted average of the present value of future fundamentals. This kind of model was discussed in detail in the 1970s, as we mentioned above. But it is obvious that these models did not use the Taylor rule as a policy feedback rule. Thus, it is wise to examine the performance of the exchange rate present-value model with Taylor rule fundamentals.

EW06 define equations Eq.(2.13) and Eq.(2.14) as the Taylor rules for the home and the foreign country. This differs from EW05's specification in that in EW06 the expectation of inflation is applied to the specification rather than the actual inflation rate. Also, the dynamics of real Deutschmark/dollar exchange rates are discussed rather than nominal rates. Therefore, this exchange rate model deals with the relationship between the real exchange rate and the fundamental variables. The Taylor rule models are as follows<sup>12</sup>:

$$Home : i_t^h = \gamma_q q_t + \gamma_\pi E_t \pi_{t+1}^h + \gamma_x x_t^h + u_{mt}^h \quad (2.13)$$

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<sup>12</sup>In the EW06's model the smoothing parameter is not included, but in the following section, our model adds the lagged interest rate in the general case.

$$Foreign : i_t^f = \gamma_\pi E_t \pi_{t+1}^f + \gamma_x x_t^f + u_{mt}^f \quad (2.14)$$

where  $h$  and  $f$  represent home and foreign country, respectively. Subtracting Eq. (2.13) from Eq. (2.14):

$$i_t^h - i_t^f = \gamma_q q_t + \gamma_\pi (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) + \gamma_x (x_t^h - x_t^f) + (u_{mt}^h - u_{mt}^f) \quad (2.15)$$

Assuming UIP and PPP hold:

$$q_t = b E_t q_{t+1} + b(1 - \gamma_\pi)(E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) - b\gamma_x (x_t^h - x_t^f) - b(u_{mt}^h - u_{mt}^f) \quad (2.16)$$

where  $b = \frac{1}{1+\gamma_q}$ .

Under ‘no bubbles’ conditions:

$$q_t = b \sum_{j=0}^{\infty} b^j E_t [(1 - \gamma_\pi)(\pi_{t+1}^h - \pi_{t+1}^f) - \gamma_x (x_t^h - x_t^f) - (u_{mt}^h - u_{mt}^f)] \quad j = 0, 1, 2, 3, \dots \quad (2.17)$$

it should be noted that, in empirical study, EW06 quotes CGG’s paper and advises that the annual expected inflation should appear in the Taylor rules<sup>13</sup>. Therefore,

$$E_t \pi_{t+1} = E_t p_{t+12} - p_t \quad (2.18)$$

Then the Taylor rule becomes:

$$Home : i_t^h = \gamma_q q_t + \gamma_\pi (E_t p_{t+12}^h - p_t^h) + \gamma_x x_t^h + u_{mt}^h \quad (2.19)$$

$$Foreign : i_t^f = \gamma_\pi (E_t p_{t+12}^f - p_t^f) + \gamma_x x_t^f + u_{mt}^f \quad (2.20)$$

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<sup>13</sup>CGG argue that central bank is not sensitive to the monthly change in the inflation rate. Thus, an expectation of annual inflation is involved in their specification.

Subtracting Eq. (2.20) from Eq. (2.19),

$$i_t^h - i_t^f = \gamma_q q_t + \gamma_\pi [(E_t p_{t+12}^h - p_t^h) - (E_t p_{t+12}^f - p_t^f)] + \gamma_x (x_t^h - x_t^f) + (u_{mt}^h - u_{mt}^f) \quad (2.21)$$

In EW06 it is also emphasized that the Taylor rule places the annualized interest rate on the left-hand side. Since monthly data are used throughout, UIP should be expressed as:

$$(i_t^h - i_t^f)/12 = s_{t+1} - s_t \quad (2.22)$$

The expectation of inflation differential is subtracted on both sides, implying that in real terms

$$(i_t^h - i_t^f)/12 - (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) = q_{t+1} - q_t \quad (2.23)$$

Thus, the exchange rate model becomes <sup>14</sup>

$$q_t = (12 + \gamma_q)^{-1} \sum_{j=0}^{\infty} b^j E_t \{ 12(E_t \pi_{t+j+1}^h - E_t \pi_{t+j+1}^f) - \gamma_\pi [(p_{t+j+12}^h - p_{t+j}^h) - (p_{t+j+12}^f - p_{t+j}^f)] - \gamma_x (x_t^h - x_t^f) - (u_{mt}^h - u_{mt}^f) \} \quad j = 0, 1, 2, 3, \dots \quad (2.24)$$

where  $b = \frac{1}{1+\gamma_q}$ .

In the empirical work EW06 forecast the inflation and output gap with a fourth-order vector autoregression (VAR). The baseline autoregression relies on  $Z_t = (\pi_t, x_t, i_t)'$ .<sup>15</sup> EW06 do not estimate Taylor rules for both countries, but instead <sup>16</sup> define the value of coefficients  $(\gamma_q, \gamma_x, \gamma_\pi)$  according to CGG's finding. After calculating the forecasts of the fundamental variables and the relevant parameters, the forecasts of model-based exchange rates can be found.

<sup>14</sup>a more general case is introduced in Appendix A.

<sup>15</sup>In the more general case, they add the commodity prices in the autoregression.

<sup>16</sup>According to CGG, the following parameters are used in EW06's work:  $\gamma_q = 0.1$ ,  $\gamma_\pi = 1.75$ ,  $\gamma_x = 0.25$ .

EW06 use monthly data from January 1979 to December 1998. The data are obtained from International Financial Statistics (IFS). Output is the log of seasonally adjusted industrial production rather than the gross domestic product. The results show that the correlation between the model-based exchange rate and the actual one is 0.32, which can be considered a promising start.

## 2.3 Output gap method

‘Output gap’ is an important concept in macroeconomics. Technically, it is the difference between the output that can be achieved when the system is at its most efficient or working under full capacity, and the actual output. The belief is that without the nominal price rigidities and technological shocks, actual output and potential output should converge in the long run. When the output gap is positive, which means the actual output is above its potential level, it can be considered as a boom in the economy. When the output gap is negative, the actual output drops below its potential level; this can be seen as a recession. In theory the output gap can play a central role in monetary policy strategy. Firstly, one of the goals of the central banks is to maintain full employment, which corresponds to an output gap of zero. Secondly, the output gap is a key determinant of inflation. A positive output gap implies an economy which is overheating and putting upward pressure on inflation. A negative output gap implies a slack economy and downward pressure on inflation.

Since the ‘potential output’ is the output the economy would be producing if there were no nominal rigidities but all other real shocks remained unchanged, the potential output should not be too volatile or, as we say, it should follow a

certain trend. This (trend) potential output can be used to forecast the actual GDP. In empirical studies there are many ways to estimate output gap. For instance, Edge et al. (2008) use a dynamic stochastic general equilibrium (DSGE) model to estimate the output gap. However, these kinds of economic models are quite complicated and hard to operate in practice; instead, many studies tend to seek simple statistical methods to extract the trend line from its actual output data. Two statistic approaches are very popular. One is called the ‘Quadratic Time Trend’ method (QTT), and the other one is the Hodrick-Prescott filter. The advantage of these techniques is that they are easy to operate in practice. In the following parts, the log of the monthly data of Industrial Production Index (IPI) is taken as a proxy for GDP and the data from Japan and the US between January 1971 and December 2009 is applied in the comparison of the two techniques. Alongside that, a new output measure called B-spline is introduced, and we believe that it could prove to be a better technique than the other two.

The idea that trend output can be obtained by the QTT can easily be observed in the graph in which the log of output is plotted against time. If we plot the output with time, the output line roughly matches a quadratic function (see Fig 2.1(a) and Fig 2.1(c)). Therefore, in the regression model, let the log of output be the linear combination of time and time squared, then the error term (or residual) left in the model can be considered as the output gap (see Fig 2.1(b) and Fig 2.1(d)). The difference between the actual data and the residual is the trend curve (the dotted line in Fig 2.1(a) and 2.1(c)). The advantage of the QTT is that the method is easy to observe and measure in practice. Its disadvantage is its lack of flexibility. In other words, if there is a systematic shock in the output trend, the QTT may not be able to capture it. If we take the data of Japan and the US as examples, Fig 2.1(a) shows the output of Japan and its trend output by using the Quadratic Time Trend. Fig 2.1(b) shows the corresponding output

gap. As we can see from the actual GDP in Fig 2.1(a), there were several booms between 1979 and 1982, and again between 2000 and 2001, but the trend line obtained by a Quadratic Time Trend cannot capture them and consider all of them as recessions. In addition, from 2005 to 2007 GDP had an upward trend, but the QTT trend line was downward because it was affected by the recession from 2007 to 2009. Fig 2.1(c) and Fig 2.1(d) show the case of the US. From Fig 2.1(c) we can see the drawback of QTT clearly. Since the fluctuations of US GDP are slightly less volatile than those of Japan, the trend curve obtained by QTT becomes a straight line, failing to show the booms in 1985 and 1990. In addition, according to the Dickey-Fuller test and the Phillips Perron test, the outputs for both countries are  $I(1)$  without trend (not reported). After the detrending process we would expect the output gaps to become a stationary process. Statistics show, however, that the residuals in Fig 2.1(b) and 2.1(d) are still  $I(1)$  processes.

The second output gap measure is the Hodrick-Prescott filter (HP-filter), which is a mathematical tool used in macroeconomics, especially in business cycle theory. This filter can separate the trend component of the time series from the raw data. The adjustment of the sensitivity of the trend to short-term fluctuations is achieved by modifying a multiplier,  $\lambda$ . Normally the default multiplier is 1600. The benefit of this technique is that it does increase the sensitivity of the trend to short-term fluctuation, but its drawback is that it imitates the fluctuation of the time series too well. Thus, it is difficult to find the boom and bust by using this technique.

Fig 2.2(a) and Fig 2.2(c) show the GDP of Japan and the US and their trends obtained by HP-filter, respectively. Fig 2.2(b) and 2.2(d) show the corresponding output gaps. Although both output gaps are made stationary by using HP-filter,

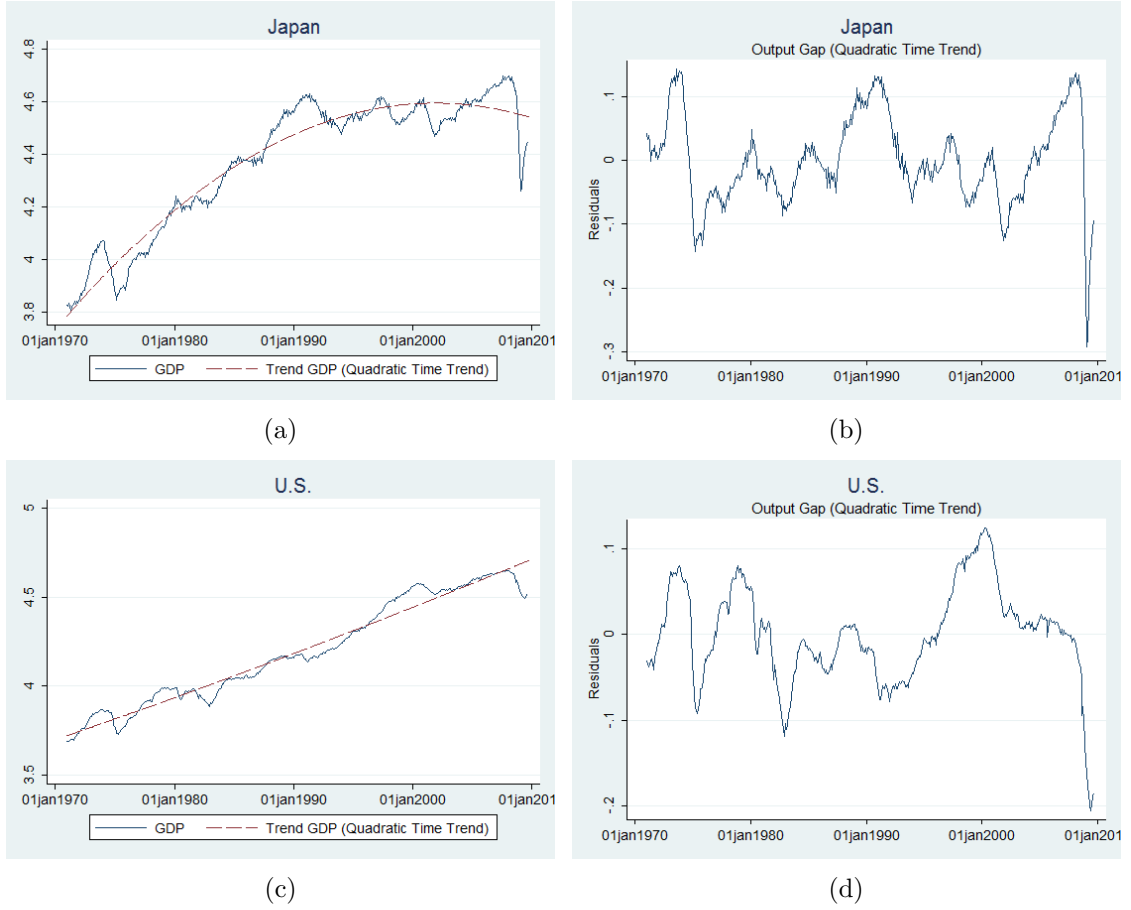


Figure 2.1: Output gap measurement (Quadratic Time Trend)

it is hard to distinguish the trend line from the actual data since the HP-filter trend line is too close to the actual data.

Having discussed both QTT and HP-filter, it seems that neither of them are perfect for our requirements and another statistical model might be needed. One that we would like to introduce is called B-spline. The definition is as follows:

*“B-spline is a spline function that has minimal support with respect to a given degree, smoothness, and domain partition. A fundamental theorem states that every spline function of a given degree, smoothness, and domain partition can be represented as a linear combination of B-splines of that same degree and smoothness, and over that same partition.” (de Boor, 1978).*

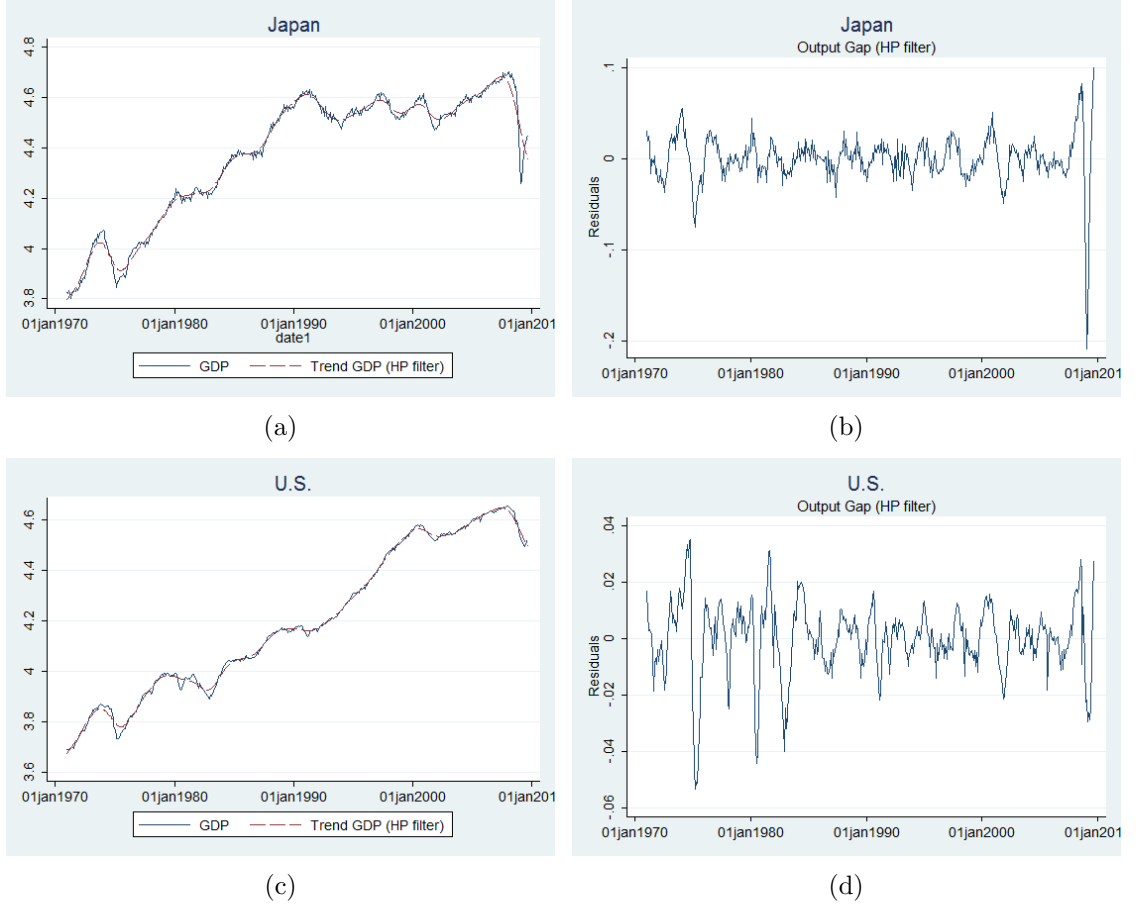


Figure 2.2: Output gap measurement (HP filter)

The idea of applying B-spline to the de-trending process is rather simple. Generally speaking, the QTT can be used to estimate trend output because the actual output curve roughly follows a quadratic curve, but the drawback of the technique is obvious. The Quadratic Time Trend line lacks flexibility. By using the B-spline approach, several quadratic or cubic curves can be used to characterize the shape of the output, which can produce a more flexible trend line. On the other hand, it is expected that the consequent trend line should not be too close to the actual time series; otherwise it fails to show the trend. Thus, how to define the appropriate B-spline is essential.

Specifically, as the number of the quadratic or cubic curves increase in the

de-trending process, the trend line will approach closer and closer to the actual series. Eventually, the trend line and the actual time series should converge. Our idea was to use the traditional way: trial and error. First of all, since both quadratic and cubic curves tend to provide fairly similar results in practice, and the cubic curve is commonly used, we suggested using a cubic curve in the estimation, while keeping the number of curves as small as possible. Meanwhile, we made as many observations as possible. For instance, there are 468 observations in our sample and, in order to use the minimum number of B-splines, the distance between knots had been set to 234. By doing this, no observation was missed in the process of generating the trend output. In other words, in order to reduce the missing variable in the sample, it would be better to keep the distance between knots close to a divisor of the number of observations. In this case, the minimum number of knots was 3, as can be observed in Fig 2.3. A regression was then run of log output on a range of B-splines (in our case bs1-bs5 were used); the trend line and residual were kept, and the actual time series and estimates observed. If the estimated line fairly matched the actual data, we could keep the results and use them in the following estimation. If not, the distance could be reduced to another divisor, in our case, 156. In general, we are quite satisfied with the outcome by using the divisors 234 and 156 in our sample data for Japan and the US respectively.

Fig 2.4(a) shows a comparison between Japan's output and its trend, obtained by B-spline. Fig 2.4(b) illustrates the output gap made by B-spline. We defined power 3 and 3 knots in the B-spline, which means that 4 or 5 cubic curves were used in the de-trending process and the width of the B-spline was 234 the distance between knots. Comparing Fig 2.4(b) with Fig 2.1(a) and Fig 2.2(a), it can be seen that the B-spline trend line tends to be more flexible and displays a more reasonable trend. In addition, the output gap obtained by B-spline is

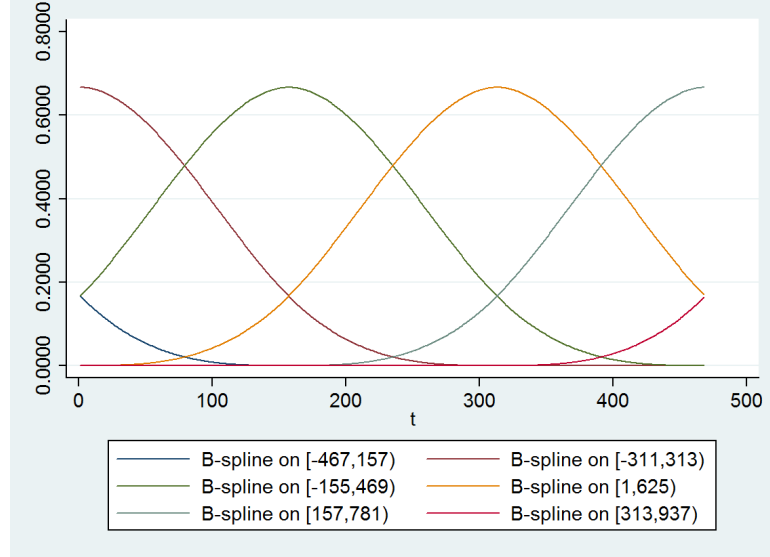


Figure 2.3: B-spline

slightly more stationary than that obtained by QTT, but less than that obtained by HP-filter.

The analysis of US output shows similar results. Fig 2.4(c) shows the comparison between US output and its trend obtained by B-spline, and Fig 2.4(d) illustrates the output gap. We define power 3 and 4 knots, which means the cubic curve is used in the filtering process and the distance between knots is 156. It is obvious that the B-spline trend curve is much more flexible and captures the performance of GDP much better than QTT. Meanwhile, the trend line is clearly distinguishable from the actual data, so that it performs more acceptably than the HP-filter. In terms of how stationary the output gap is, B-spline also falls between the other techniques.

To sum up, in employing the monthly IPI data of Japan and the US, the properties of various de-trending techniques are displayed clearly. The results of the B-spline approach fall between those of QTT and HP-filter. In terms of both flexibility and ‘stationarity’, it seems that B-spline can be considered an output

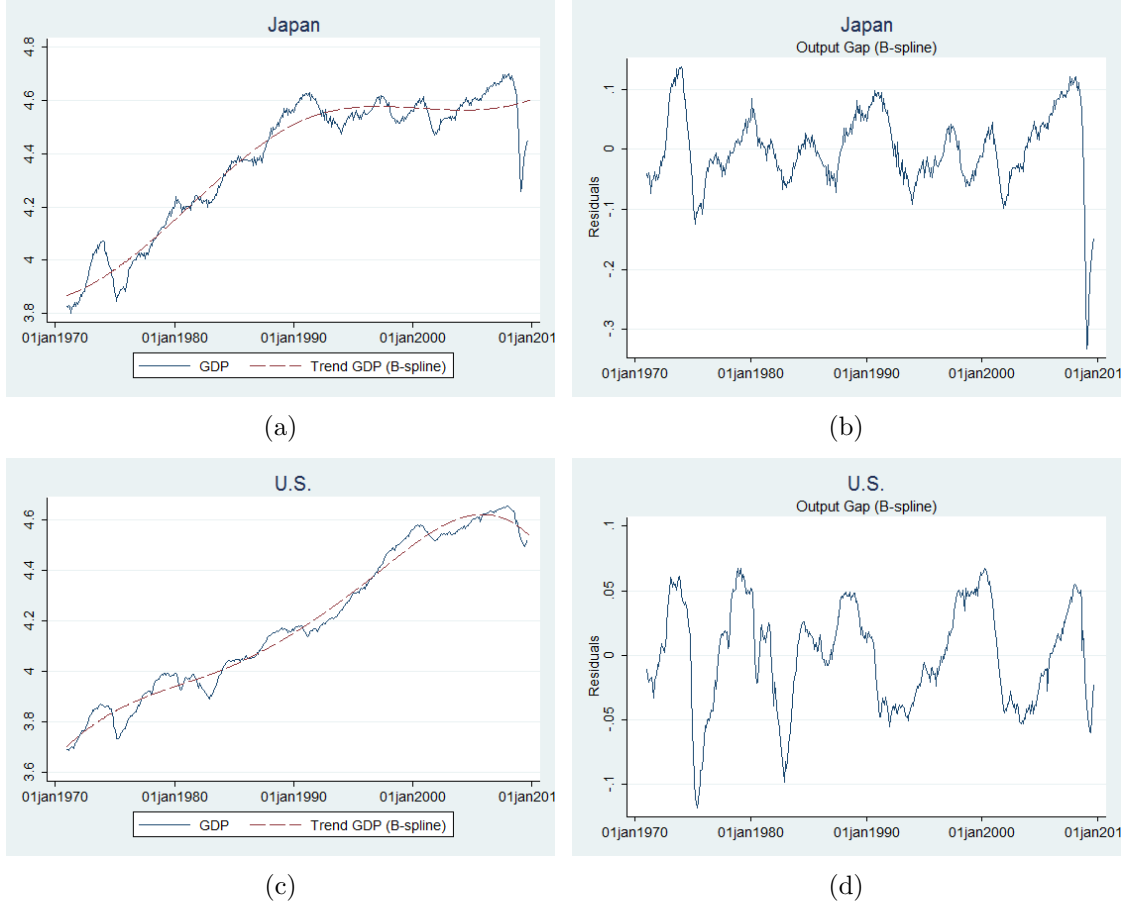


Figure 2.4: Output gap measurement (B-spline)

gap method. Meanwhile, despite the B-spline approach has its own benefits, it is still not perfect. Whether or not the trend line is appropriate is judged by observation alone, rather than by solid evidence or theoretical support. In future work it will be essential to compare the results of the B-spline approach with those of other economic models. For instance, so far the DSGE models have been commonly used to produce output gaps; this class of models believes that output gap is a driver of inflation, which implies that the path of inflation has an important bearing on the resulting output-gap path (Edge et al. 2007). If the DSGE's output gap estimates can be compared with those obtained by B-spline, and if the estimates are closely correlated to each other, it can be considered as strong proof that B-spline can be used as an output gap measure.

## 2.4 Taylor rule estimation

CGG demonstrate that three central banks in the developed countries (US, Germany and Japan) conducted Taylor rules in their monetary policy during the period from 1979 to 1998. CGG claim that central banks are insensitive to the monthly change in inflation rates, so that the expected inflation for next year is used in their specification ( $n = 12$ ). Rational expectations would suggest that the expected inflation rate at time  $t + 12$  would be equivalent to actual inflation at time  $t + 12$ . The baseline specification of a Taylor rule includes a lagged interest rate, inflation rate and output gap. CGG also allow the central banks at least some degree of autonomy in the monetary policy strategy, since the central banks may have independent objectives; for instance, stabilizing the foreign exchange market. Thus, money supply or exchange rates are tested in the alternative specifications. <sup>17</sup>The generalized method of moments (GMM) is used for the estimation. The reason GMM being used is that the regressors  $\pi_{t+n}$ ,  $x_t$  as well as  $i_{t-1}$  are not orthogonal to the error term  $\varepsilon_t$  in the specification, in other words, the regressors are not exogenous to the error term, or the regressors are endogenous. Thus, the Ordinary Least Square (OLS) can not be used, because it requires the orthogonality between error term and regressors.

In addition, CGG argue when the central bank chooses its interest rate, let  $u_t$  be the information set, the element of the information set include any previous variables that might help to forecast the future inflation rate and output, as well as any contemporaneous variables that are unrelated to the current interest rate

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<sup>17</sup>In CGG's estimation, the US Taylor rule does not react to the change in the real exchange rate since the dollar is the dominant currency in the world.

shock. Since the number of the elements in the information set, or we can say, the instrument set exceeds endogenous variables, then GMM rather than two stage least square (2SLS) should be used. Further, we need to test over-identifying restriction. Generally speaking, if the equation is exactly identified, the estimated parameters are efficient, otherwise, they are not. Hansen's J statistic is used to detect if our GMM estimations are over-identified. The null hypothesis is that the model is exactly identified. In the following discussion, we demonstrate most of the time the null hypothesis is satisfied in our cases. The instrument variables are the current and lagged interest rates, the current and lagged inflation rate and the current and lagged output gaps (in the alternative specifications, the current lagged variables of money supplies or real exchange rates are also instrument variables)<sup>18</sup>.

All our study follows CGG's work. The general specifications and estimation techniques are completely identical; however, our interest is not just in the Taylor rule estimations. We would also like to test whether the Taylor rules are consistent during a longer period of time for both countries, and if other output gap measures can improve the estimations. Japan and the US are taken as examples in our empirical study.

### *The case of Japan*

In order to test the consistency of the Taylor rules estimated for Japan and the US we need to collect the data for a longer period of time. As we mentioned in the introduction, the monthly data spans the period from January 1971 to December 2009, but only the data between January 1971 and December 2006 are used for Taylor rule estimations. In the middle of 2007 the global financial crisis

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<sup>18</sup>The lagged variables are from  $t - 1$  to  $t - 6$ ,  $t - 9$  and  $t - 12$ .

broke; it is believed there was a break in the monetary policy for both countries. Since the data from 2007 to 2009 are not enough for the GMM estimation, we drop them in the following sections. The whole sample between 1971 and 2006 can be divided into three subsamples, pre-1979, 1979-1998, and post-1998. If the parameters in the different periods of time vary significantly from each other then this provides evidence of inconsistency in the Taylor rules. In CGG's research only the Quadratic Time Trend is used to obtain output gap. In our specifications the HP-filter and the B-spline are also used.

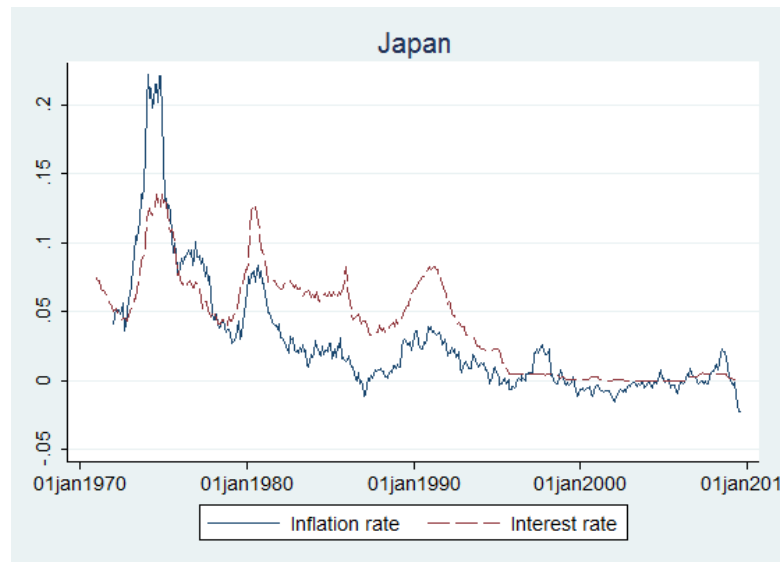


Figure 2.5: Inflation rate and Interest rate for Japan

Fig 2.5 shows the nominal interest rates and the inflation rates of Japan between January 1971 and December 2009. It can be clearly seen that Japanese interest rates dropped to around zero at the beginning of 1999, while the inflation rate in Japan also dropped to around zero. We would like to find out if there was a structural break in monetary policy strategy during the subsample periods of January 1971 to March 1979, April 1979 to December 1998 and January 1999 to December 2006. According to CGG, during April 1979 Japan experienced a significant financial market deregulation. The second time break was in January

1999 when the euro was born and Japan began to adopt an interest rate policy of around zero. By using Hodrick and Srivastava's (1984) test for structural change, we can find out whether there are structural breaks between subsamples 1 and 2, and between subsamples 2 and 3.

Table 2.1 displays the results of the estimation of the Taylor rule for Japan. The output gap is measured by three different techniques: the Quadratic Time Trend, the B-spline approach and the HP-filter. Under each technique the sample period is divided into three subsample periods, and the hypothesis of no structural change in any of the coefficients between subsample 1 and 2, and between subsample 2 and 3, are tested, the results being shown in the last row of the table. Generally speaking there is some common ground between the estimations in terms of different output gap measures. First of all, only the lagged interest rate played an important role during the period between January 1971 and March 1979, the other parameters not having significant effects on interest rate reaction function. This was possible because the Bank of Japan (BOJ) had not yet set an inflation target at that time. Secondly, there is strong evidence that the BOJ followed the Taylor rules between April 1979 and December 1998, there being evidence that all the coefficients were significant, and the real exchange rate had an influence on interest rate choices, but not for the money aggregates (not reported). The value of the coefficient of inflation rate was greater than one during this period, this being essential in order to identify whether or not the central bank is using the Taylor rules in their monetary policy decisions. The reason is that the rise in inflation will cause the central bank to raise nominal interest rates high enough to raise the real interest rate at the same time so that the reaction can compensate the effect of the inflation rate on the domestic economy. Thirdly, at the beginning of 1999, since the Bank of Japan fell into the liquidity trap, the nominal interest rate declined to zero, and monetary policy became inefficient.

	Quadratic Time Trend				B-spline				HP-filter	
	1971.1-1979.3	1979.4-1998.12	1999.1-2006.12	1971.1-1979.3	1979.4-1998.12	1999.1-2006.12	1971.1-1979.3	1979.4-1998.12	1999.1-2006.12	
$\gamma_\pi$	-1.0626 (2.6196)	1.4444** (0.2736)	-0.0918 (0.1087)	-6.2869 (352.92)	1.6602** (0.3520)	-0.1643 (0.2881)	2.2554 (2.5284)	2.3024** (0.4133)	0.0512 (0.094)	
$\gamma_x$	-3.1052 (5.1201)	0.2606** (0.0645)	0.0233* (0.0117)	-148.91 (8071.68)	0.2865** (0.1020)	0.0390* (0.0465)	3.2334 (3.0792)	0.6310* (0.3234)	0.0240* (0.022)	
$\rho$	1.0080** (0.1391)	0.9559** (0.0079)	0.9710* (0.0190)	1.0002** (0.0125)	0.9634** (0.0074)	0.9863** (0.0206)	0.9877** (0.0131)	0.9681** (0.0070)	0.9726** (0.0094)	
$\alpha$	-2.6124 (4.6713)	-0.2890 (0.0751)	-0.0277* (0.0263)	-0.8101 (62.514)	-0.2110* (0.0847)	-0.0565 (0.0919)	0.6184 (0.9853)	-0.1695 (0.0972)	-0.0202* (0.0242)	
$\gamma_q$	0.5665 (0.4445)	0.0652* (0.0085)	0.0061** (0.0057)	0.7076 (39.8987)	0.0482** (0.009)	0.0121** (0.010)	-0.1609 (0.1233)	0.0375* (0.0107)	0.0043** (0.0052)	
$J - stat$	36.565	28.675	21.164	37.042	29.478	18.718	33.750	31.773	23.827	
$P - value$	0.1902	0.5347	0.8827	0.1759	0.4926	0.9457	0.2909	0.3782	0.7798	
Structural Break	-	46.622** 0.000	60.4406** 0.000	-	35.4917** 0.0000	54.4157** 0.0000	-	53.605** 0.000	44.3689**	
$P - value$	-			-			- 0.000			

Table 2.1: Taylor rule estimation (The case of Japan)

Note: \* Indicates significance ( $p - value < 0.05$ ) \*\* Indicates strong significance ( $p - value < 0.01$ )  
All the P-values are the results for the one-tail test. It is expected that all the coefficients have positive signs

We do not have enough evidence that the BOJ still followed the Taylor rules during this period. Although we have evidence that the interest rate still reacted to the development of the output gap, lagged interest rate and real exchange rate, the coefficient of the inflation rate during this period was around zero and not significant in all cases. Thus, it seems that there has been a change in monetary policy-making since then. Hodrick and Srivastava's (1984) GMM test for structural change also shows significant evidence that there are structural breaks between different subsamples. Hansen's GMM tests (J-statistic) reject the over-identifying problem in all cases.

By comparison, it can be seen that the B-spline can produce results as good as those of the QTT. The coefficients obtained by the B-spline are significant and have the expected signs in most cases, since the results obtained by the QTT are significant as well; it is difficult to see which method is better at this stage. However, our estimation results are slightly less significant using the HP-filter. Thus, using B-spline to obtain output gap may not be a bad idea.

### *The case of the US*

CGG suggest that the Fed does not respond to the change in exchange rates since the dollar is the dominant currency throughout the world. Therefore, all the estimations below only measure the baseline case of the Taylor rule for the US. Table 2.2 shows the Taylor rule estimation for the US by using different output gap measures. Broadly speaking there is evidence of the structural changes between the different subsample periods. The Hensen's J statistic rejects the over-identification problem in all cases. All the results are consistent with the results of CGG and Mark (2009). Specifically, there is no evidence that the Fed used a Taylor rule during the 1970s since none of the coefficients obtained by

	Quadratic Time Trend				B-spline				HP-filter	
	1971.1-1979.3	1979.4-1998.12	1999.1-2006.12	1971.1-1979.3	1979.4-1998.12	1999.1-2006.12	1971.1-1979.3	1979.4-1998.12	1999.1-2006.12	
$\gamma_\pi$	0.6382** (0.1659)	1.6625** (0.4299)	-6.7179 (4.2103)	0.6385** (0.1717)	1.4676** (0.4105)	1.2131** (0.4862)	0.8953** (0.0845)	1.9820** (0.4976)	15.4931 (11.81)	
$\gamma_x$	0.2219* (0.1151)	0.2269* (0.1061)	0.4214** (0.1550)	0.2311* (0.1156)	0.2445* (0.1214)	0.5057** (0.0389)	0.2098 (0.1394)	1.9473** (0.7585)	10.2104 (7.8031)	
$\rho$	0.8567** (0.0275)	0.9530** (0.0146)	1.018** (0.0116)	0.8649** (0.0280)	0.9524** (0.0150)	0.9304** (0.018)	0.8336** (0.0239)	0.9490** (0.0166)	0.9931 (0.0053)	
$\alpha$	0.0021 (0.0016)	0.0105 (0.0127)	-0.003 (0.0007)	0.0026 (0.0017)	0.0007 (0.0007)	0.0002 (0.0009)	-0.0004 (0.0011)	-0.0002 (0.0008)	-0.0026 (0.0005)	
$J - stat$ $P - value$	26.258 0.2402	31.761 0.0816	34.094 0.0481	26.769 0.2201	33.363 0.0569	33.322 0.0575	29.064 0.1431	23.243 0.3881	30.301 0.1113	
Structural Break $P - value$	- -	47.437** 0.000	21.1122** 0.000	- -	28.9216** 0.000	14.143** 0.005	- -	37.2245** 0.000	11.8125* 0.025	

Table 2.2: Taylor rule estimation (The case of the US)

Note: \* Indicates significance ( $p - value < 0.05$ ) \*\* Indicates strong significance ( $p - value < 0.01$ )

All the  $p - values$  are the results for the one-tail test. It is expected that all the coefficients have positive signs

different output gap methods are significant, with the exception of the lagged interest rates. In the second subsample period we find strong evidence that the Taylor rule had been used. The coefficients of inflation and lagged interest rate are strongly significant under different output gap methods. The coefficients of output gaps are significant by using QTT and B-spline and strongly significant by using HP-filter. However, the coefficient of output gap being about eight times larger by using the HP-filter than those obtained by the other two methods, which indicates that this method tends to magnify the effect of the output gap in the interest reaction function. Since most studies demonstrate that the coefficient of output gap is around 0.5, the results obtained by the HP-filter may not be as appropriate as it would appear.

In the last subsample, the main difference is in the coefficient of inflation. The QTT gives us a negative and insignificant coefficient (column 4, row 3), but there is evidence that the Fed reacts to the inflation shown in B-spline and HP-filter. This little difference is essential. As we discuss above, the main drawback in the QTT is that it lacks flexibility. Both B-spline and HP-filter do not have this problem. If we compare Fig 2.1(c), 2.2(c) and 2.4(c), it is clear that QTT cannot capture the boom and recession in the 2000s, which is likely to cause QTT to be unable to capture the effect of inflation in the monetary policy-making during this period. Furthermore, the coefficients of inflation and output by HP-filter (the last column) are too high to be believable. Therefore, the B-spline method helps us to acquire the best estimations in this subsample.

It can be clearly seen that the results obtained by using B-spline are slightly more significant than those obtained by using Quadratic Time Trend, especially in the subsample of January 1999 to December 2006. There is no evidence that the Fed reacts to the development in inflation during this period by using QTT,

but the B-spline approach produces different results. The estimation made by HP-filter is the worst, because the coefficient of the inflation rate is not significant, and the value of the coefficient of the output gap is too large. Although the J-statistic indicates that there are overidentifying problems for the estimations by using QTT and B-spline, if the number of instruments are decreased, the problem can be solved (not reported), and the numerical value does not change significantly.

To sum up, there are some similarities in the Taylor rules estimated for both Japan and the US. First of all, Hodrick and Srivastava's (1984) GMM test demonstrates that there are structural breaks between different subsamples. Secondly, there is strong evidence that the Bank of Japan (BOJ) and the Fed followed Taylor rules between January 1979 and December 1998. All the coefficients obtained by different output gap methods are significant, and the real exchange rate has an influence on the BOJ's interest rate reaction function. Thirdly, the B-spline approach seems to perform better than the other two methods. The QTT method and HP-filter may give us better results sometimes, but not in all cases. Since there is strong evidence that both Japan and the US used Taylor rules in the period between 1979 and 1998, we apply the estimates of Taylor rules for both Japan and the US during 1979-1998 to the yen/dollar real exchange rate forecasting. This is also consistent with the time period chosen by CGG. We still use these three output gap measures to see which one can bring a better forecasting performance.

## 2.5 Exchange rate modelling and Taylor rules

One of the purposes of the chapter is to improve the out-of-sample predictability of the asset-pricing model of exchange rates incorporating Taylor rules. We follow the EW06's framework. The specific procedures are as follows: first of all, both the home and the foreign country are assumed to use Taylor rules in the sample period. We take Japan as the home country, the US as the foreign country, and the sample period as January 1971 to December 1998. In addition, the BOJ also reacts to the real exchange rate in the interest rate reaction function. In conjunction with 'Uncovered Interest Rate Parity' and 'Purchasing Power Parity', the real exchange rate can be determined by the sum of the current and discounted future fundamental variables. In the baseline case, both Japan and the US are assumed to use identical Taylor rules, so the parameters of inflation rates and output gap for both countries are the same. In the alternative specifications, this assumption is relaxed. We allow Japan's parameters are not equal to the US's parameters. The alternative specification can be expressed as:

$$\begin{aligned}
 q_t = (12 + \gamma_q)^{-1} \sum_{j=0}^{\infty} b^j E_t \{ & 12(E_t \pi_{t+j+1}^h - E_t \pi_{t+j+1}^f) - \gamma_{\pi}^h (p_{t+j+12}^h - p_{t+j}^h) \\
 & + \gamma_{\pi}^f (p_{t+j+12}^f - p_{t+j}^f) - \gamma_x^h x_t^h + \gamma_x^f x_t^f - \rho^h i_{t+j-1}^h \\
 & + \rho^f i_{t+j-1}^f - (u_{mt}^h - u_{mt}^f) \} \quad j = 0, 1, 2, 3, \dots \quad (2.25)
 \end{aligned}$$

Since the smoothing parameter lagged interest rate plays an important role in the Taylor rules, it would be interesting to learn if adding a lagged interest rate can make a difference in the exchange rate modelling. Three different output gap methods are used in the specifications. It is of interest to discover which one of them is the best fit to the model. In all, 12 different specifications are examined in this section.

An important technique in the forecasting process is to forecast the future fundamentals. Our method is similar to the one used in EW06, where the unrestricted vector autoregression is used for forecasting. Generally speaking, it is believed that inflation rate, output gap and interest rate are closely correlated with each other. Inflation affects output gaps and interest rates and visa versa. This situation is perfect for the VAR approach. EW06 made a complicated programme for VAR forecasting based on the software named “rats”.<sup>19</sup> In *Stata*, the one-step-ahead VAR forecast is easy to obtain by using the command “*fcst*”.

In the baseline case we followed EW06’s method to forecast the inflation and output gap differential based on a fourth-order VAR, which depends on a vector  $Z_{1t} = (\pi_t, x_t, i_t)$ <sup>20</sup>; in alternative specifications a range of different VARs are used for forecasting. However, if we look at Eq.(2.24) or (2.25), there is another fundamental variable that needs to be forecasted: the expected annual inflation for next year,  $p_{t+12} - p_t$ .<sup>21</sup>

To make it clear, in the baseline case, the vector  $Z_{1t}$  includes the Japanese/US monthly inflation rate differential, output gap differential and interest rate differential in order to obtain the one-step-ahead forecast of monthly inflation and output gap differential. In order to obtain the 12-step-ahead forecasts of annual inflation, monthly inflation rates are replaced by the annual one in the vector  $Z_{1t}$ , so that it becomes  $Z_{2t} = (p_t - p_{t-12}, x_t, i_t)$ . In alternative specifications, the Japanese and the US monthly inflation rate, output gap and annual inflation rate are forecast separately based on the vectors  $Z_{1t}^h = (\pi_t^h, x_t^h, i_t^h)$ ,  $Z_{1t}^f = (\pi_t^f, x_t^f, i_t^f)$

<sup>19</sup>The specific programming file can be found on Charles Engel’s homepage.

<sup>20</sup> $\pi_t$ ,  $x_t$  and  $i_t$  are the inflation rate differential, output gap differential and interest rate differential respectively between the home and the foreign country.

<sup>21</sup>The forecasting method we introduced is not accurate. The reason is that the exchange rate at time  $t$  is equal to the sum of current and discounted future variables far into the future, so it is important to forecast the future variables based on the data before time  $t$ ; in other words, several periods ahead rather than just a one-period-ahead forecast should be estimated.

and  $Z_{2t}^h = (p_t^h - p_{t-12}^h, x_t^h, i_t^h)$ ,  $Z_{2t}^f = (p_t^f - p_{t-12}^f, x_t^f, i_t^f)$ . The reason for making such changes is simple, EW06 argue that the differential variables are endogenous in general equilibrium, which indicates that the inflation rate differential between the home and the foreign country might affect output gap differential, and output gap differential might affect inflation rate differential. However, we believe that the domestic and foreign fundamental variables should be forecasted separately for each country. For instance, the past domestic interest rates can have an effect on the current domestic inflation, but it would be difficult to imagine that it could also have a simultaneous influence on current foreign inflation.

In order to forecast the fundamental variables for the whole sample period, a simple program is needed. The rough procedure is as follows: taking the baseline case as an example, the period of January 1971 to January 1972 was used as the initial period for estimating the fourth-order VAR model which relies on vector  $Z_{1t}$ . Then one-step-ahead monthly forecasts for inflation and output gap differential could be generated and the forecasts stored; for the next period's forecast the data between January 1971 and February 1972 were used, and the forecast for the time period March 1972 could be generated and stored. The procedures were repeated, 453 estimates for monthly inflation and output gap differential being obtained. The forecast of the expected annual inflation rate differential could be obtained in a similar way. The period of January 1971 to February 1973 was used as the initial period; since the first 12 observations were missing due to the fact that the annual inflation was generated by  $Z_{2t}$ . We were interested in the forecasting of inflation differentials for next year, the 12-step-ahead forecasts were generated by the VAR based on vector  $p_t - p_{t-12}$ , and then the last step forecast was stored. 441 estimates for annual inflation differential were generated in *Stata*<sup>22</sup>.

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<sup>22</sup>Although the fundamentals are forecasted between March 1972 and December 2009, we only

Panel A	Correlation			
	$(\hat{\pi}_{mt}, \pi_{mt})$	$(\hat{x}_t, x_t)$	$(\hat{\pi}_t, \pi_t)$	
QTT	0.0821	0.9795	0.2449	
B-spline	0.0892	0.9541	0.2071	
HP-filter	0.0563	0.6282	0.3283	
Panel B				
	Without $l.i$		With $l.i$	
	$\text{corr}(\hat{q}_t^1, q_t)$	RMSE	$\text{corr}(\hat{q}_t^2, q_t)$	RMSE
QTT	-0.1489	4.6971	-0.0090	4.6958
B-spline	-0.0264	4.6900	0.1134	4.6886
HP-filter	0.0303	4.7196	0.1861	4.7078

Table 2.3: The baseline case

Note:  $\pi_{mt}$ ,  $x_t$ ,  $i_t$ ,  $q_t$  and  $\pi_t$  are the monthly inflation rate differential, output gap differential, interest rate differential, the log of real exchange rate and annual inflation differential, respectively. “hat” denotes the estimates of variables. All the correlations are only for the time period of 1979.10 to 1998.12. Three different output gap methods are used in the model, which are the Quadratic Time Trend (QTT), B-spline and HP-filter.  $\hat{q}_t^1$  is the model-based exchange rate without adding lagged interest rate differential in the model,  $\hat{q}_t^2$  includes the lagged interest rates differential. The value of parameters for the annual inflation differential, the output gap and the lagged interest rate are as follows: 1.75, 0.25, 0.95.

Last but not least, in the last section the estimates of the weight on real exchange rate in the Taylor rule for Japan is roughly equal to 0.1, which indicates that the discount factor  $b$  is 0.99. In this case, the model-based real exchange rate depends on the discount fundamental variables far into the future. To make it clear, we specify the current and one to four periods-ahead discounted fundamental variables into Eq. (2.24) or (2.25).

Table 2.3 reports the results of the baseline forecasts of the yen/dollar exchange rates. Panel A shows the correlations between estimated fundamental variables by the VAR forecasts and actual data under different output gap measures. It is clear that the estimated output gaps and annual inflation rate differential use the data between April 1979 and December 1998 for exchange rate forecasting.

entials can imitate the actual data fairly well. However, the estimates of monthly inflation rate differentials do not match the actual data very well, regardless of the output gap methods used. In addition, with the increase in the flexibility of trend outputs, the correlations between the forecasted fundamentals and the observed values decrease.

Panel B displays the correlations and the Root Mean Square Error (RMSE) between model-based real exchange rates and the observed values. Columns 2 and 3 do not consider the lagged interest rate in the estimations. Columns 4 and 5 include the lagged interest rate. Column 2 shows that the correlation between the model-based real exchange rate and the actual data are negative in 2 out of 3 cases; the situation can be improved by adding lagged interest rate into the model, and Column 4 shows that the correlation is positive by using B-spline and HP-filter. However, the correlations are only 0.1134 and 0.18 respectively. In terms of the correlation, applying an HP-filtered output gap can provide the best results. Nevertheless, in terms of the RMSE, it seems that the B-spline approach can do a better job because it has the smallest RMSE in all cases.

In Table 2.4, Panel C describes the correlations between forecasted fundamental variables and the actual data. Columns 2 to 4 describe the case of Japan as the home country. The fifth to seventh columns show the correlations for the case of the US. Panel C reveals that, once we forecast the data for individual countries, the predictability can be improved. Specifically, by using the B-spline approach, the correlations between estimates and real data are higher than those obtained by the other two techniques. HP-filter provides the worst forecasts in 5 out of 6 cases. The results are similar to those obtained in Table 2.3.

Panel D shows the correlations and RMSEs between model-based real ex-

Panel C	Correlation						
	$(\hat{\pi}_{mt}^h, \pi_{mt}^h)$	$(\hat{x}_t^h, x_t^h)$	$(\hat{\pi}_t^h, \pi_t^h)$	$(\hat{\pi}_{mt}^f, \pi_{mt}^f)$	$(\hat{x}_t^f, x_t^f)$	$(\hat{\pi}_t^f, \pi_t^f)$	
QTT	0.3237	0.9694	0.7327	0.6538	0.9881	0.7508	
B-spline	0.3282	0.9553	0.7945	0.6631	0.9808	0.8140	
HP-filter	0.3172	0.6162	0.5533	0.6653	0.8553	0.5803	
Panel D							
		Without $l.i$		With $l.i$			
		$\text{corr}(\hat{q}_t^1, q_t)$		$\text{corr}(\hat{q}_t^2, q_t)$		RMSE	
	QTT	0.1883		0.2863		4.7456	
	B-spline	0.4055		0.4819		4.7413	
	HP-filter	0.6143		0.7208		4.7465	

Table 2.4: The alternative case

Note:  $\pi_{mt}, x_t, i_t, q_t$  and  $\pi_t$  are monthly inflation rate, output gap, interest rate, the log of real exchange rate and annual inflation rate, respectively. “ $l$ ” denotes home country (Japan). “ $f$ ” denotes foreign country (the US). “hat” denotes the estimates of variables. All the correlations are only for the time period of October 1979 to December 1998. Three different output gap methods are used in the model, which are the QTT, B-spline and HP-filter.  $\hat{q}_t^1$  is the model-based exchange rate without adding lagged interest rates in the model,  $\hat{q}_t^2$  includes lagged interest rates. The value of coefficients are as follows:  $\gamma_\pi^h = 2$ ,  $\gamma_x^h = 0.3$ ,  $\rho^h = 0.96$ ,  $\gamma_q = 0.1$ ,  $\gamma^f = 1.75$ ,  $\gamma_x^f = 0.25$ ,  $\rho^f = 0.95$ .

change rates and the actual data. The estimates of the real exchange rate are based on specification Eq. (2.25). Columns 2 and 3 do not consider the smoothing parameter, but Columns 4 and 5 include the lagged interest rate. There is a huge improvement in predictability in terms of the corrections between the estimates and the actual data. All the correlations are positive. If taking the lagged interest rate into account, by using B-spline, the correlation almost approaches a half. By using HP-filter, the correlation can approach three quarters. These results indicate that if the fundamental variables are forecasted on the basis of individual countries, using heterogeneous Taylor rule coefficients for both countries and adding smoothing parameters, the movements of real data can be better captured by the models. In addition, in terms of correlation, HP-filter provides the best results.

In terms of the RMSE, B-spline provides the best results, which indicates that B-spline can improve the accuracy of predictions more than the other two techniques. In addition, adding the interest rate deteriorates the RMSE in all cases, since adding new variables decreases the degree of freedom.

Comparing our results to those of EW06 and Mark (2009), mine are fairly significant. The correlation in Engel and West's specification is 0.32 and in Mark's (2009) is 0.26, but mine can achieve more than 0.5. However, Table 2.4 suggests that the results are not consistent. For instance, adding lagged interest rate could increase the correlation but also raise the RMSE. The B-spline approach can produce the smallest RMSE but HP-filter can have the largest correlation. The alternative specifications are unable to produce an optimal result at this stage.

## 2.6 Conclusions

This chapter investigated whether or not Japan and the US follow the Taylor rule throughout different subsample periods. The results confirm that both countries followed the Taylor rules during the period between 1979 and 1998. Out-of-sample exchange rate forecasting was conducted by combining the Taylor rule with a discounted present value model originating from Engel and West (2005, 2006). The discount factor and the coefficients of the model were imposed based on our estimations of Taylor rule for both countries. A variety of specifications were tested, and the model with heterogeneous coefficients and smoothing coefficients was demonstrated to be the best performer. Three output gap methods were used in the chapter, and there is strong evidence that B-spline and HP filter could provide better results in terms of Taylor rule estimations and the exchange rate forecasting than the Quadratic Time Trend method.

This chapter did not take into account the effect of unknown structural breaks on the exchange rate forecasting, or the possibility of nonlinearity of the model, or the unemployment as a replacement for the output gap measure. The forecasts of fundamentals were based on an unrestricted VAR. More advanced or more accurate methods such as the Kalman filter can be used in future.

## Chapter 3

Does the Fed follow a nonlinear  
Taylor rule?

## 3.1 Introduction

Macroeconomists are always interested in studying central bank behaviour. In the past two decades a simple interest rate reaction function developed by Taylor in 1993 which is known as the Taylor rule has been widely studied and demonstrated as being a rule to describe the monetary policy of central banks in the developed economies. In empirical study, one of the debates is the linearity of the Taylor rule. Several economists have managed to prove empirically that central banks follow linear Taylor rules (Clarida et al., 1998, 2000), whereas others have suggested that nonlinear or asymmetric Taylor rules are better at mimicking a central bank's behaviour (Dolado et al., 2005; Surico, 2007; Kenneth, 2007; Alex and Anton, 2008; Castro, 2011). This chapter is attempting to discuss this issue from a different angle. Taking the changes of economic structure into account, the Fed may follow varied linear or symmetric Taylor rules under different sub regimes instead of one nonlinear Taylor rule over time.

In order to understand whether or not a central bank's interest rate reaction function is symmetrical, a good starting point would be to consider the comments from the central bank authorities regarding whether or not the bank interventions are symmetric.

*Blinder (1997) argues that 'academic macroeconomists tend to use quadratic loss functions for reasons of mathematical convenience, thinking much about their substantive implications. The assumption is not innocuous...I believe that both practical central bankers and academics would benefit from more serious thinking about the functional form of the loss function'* Describing his experience as Fed Vice-Chairman, Blinder (1999) pushes the argument even further and claims *'in most situations the central bank will take far more political heat when it tightens*

*pre-emptively to avoid higher inflation than when it eases pre-emptively to avoid higher unemployment*', suggesting that political pressures can induce asymmetric central bank interventions.<sup>1</sup>

On the theoretical side, a number of recent studies suggest that the monetary authorities' responses to the business cycle are asymmetric. Persson and Tabellini (1999) shows that, under the assumption of imperfect information, politicians with career concerns are more likely to allow the monetary authorities to have a larger response to the poor economy. Gali et al. (2007) finds that the costs of output fluctuations for the US have been large and asymmetric.

If we assume that a central bank's interest rate reaction function is indeed asymmetric, in empirical study, the question is how to measure this phenomenon. In general, there are two directions: one is to assume that the function takes a nonlinear form, so that the nonlinear techniques can be used to estimate the reaction function; the other is to assume that the instability is caused by economic structural changes. One needs to identify the potential structural changes first, and then estimate the reaction function in each subsample period and test if these functions are linear or nonlinear. For the second method, it should be noted that it is possible the reaction function is still nonlinear, even in the subsample period, and thus nonlinearity should be tested even within the sub regimes.

The first type of direction has been well developed. Several papers argue that the nonlinear Taylor rule can do a better job in terms of mimicking the behaviour of a central bank. For instance, Carlin and Sosikice (2006) and Surico (2007) both find that the Fed behaved asymmetrically before 1979, with the interest rate reacting more aggressively to output contraction than expansion. In particular, the

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<sup>1</sup>Collected from Surico (2007).

Fed attaches a larger weight to output contractions than to output expansions of the same magnitude. Dolado et al. (2005), Kenneth (2007) and Alex and Anton (2008) also suggest that the Fed prefers to have a more aggressive response to the inflation when it is higher than the target. Castro (2011) finds evidence that the European Central Bank and the Bank of England are best described by a non-linear rule.<sup>2</sup>

The other direction has not been fully discussed. One of the issues is how to estimate the potential structural changes in the Taylor rule. One way to overcome this issue is to assume that certain events can cause the structural changes; for instance, changes in the central bank's chairmanship. Qin and Enders (2008) estimate the Taylor rule separately under each chairmanship of the Federal Reserve. They find evidence that the estimated Taylor rule differs between the pre-Greenspan and the Greenspan period. Nonetheless, we can not be sure that a change in the Fed's chairmanship can cause systematic changes in their monetary policies, which means that this assumption is lacking in solid empirical evidence. If there are unknown structural changes inside the sample period or false structural changes are assumed in the estimation it is possible that the estimates of both linear and nonlinear Taylor rules are biased. Bunzel and Enders (2010) view the Taylor rule as a threshold process but find only mild empirical support for their specifications. In the meantime, they also indicate that the threshold model is unstable.

Thus it is worthwhile investigating whether or not there are structural changes

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<sup>2</sup>Note the asymmetric Taylor rule in this chapter is different from the asymmetric exchange rate model incorporating Taylor rules in chapter 2. In chapter 2, when we derive the exchange rate model, if assuming that both the home and foreign country follow an identical Taylor rule, the exchange rate model derived from it is called a symmetric model. If we assume that the foreign country takes the real exchange rate into account, then there is a component of real exchange rate in the exchange rate model, so the model is called asymmetric model.

in the interest reaction function. The popular method for testing multiple structural changes is based on Bai and Perron (2003). This method, however, cannot be applied to the GMM framework. This chapter applies Andrews' method to find the potential economic structural changes. The sample is then divided into several subsamples based on these change points, and by re-estimating the Taylor rule under each subsample, whether or not the monetary authorities behave symmetrically might be revealed.

Another contribution of this chapter is to choose a more advanced estimator to estimate Taylor rules. In general one can choose between Least Square (LS), Instrument Variables (IV) and Generalized Method of Moments (GMM). In the early empirical studies a number of papers (Clarida et al., 1998, 2000; Mark, 2009) imply that GMM is a more consistent estimator. There are two reasons: first, the explanatory variables lagged interest rate, the expectation of inflation rate (or inflation rate) and output gap are all endogenous, and are correlated with the error term in the regression, which implies that the LS estimates are biased. Instead, Instrument Variables or GMM should be used. Second, GMM is superior to IV when heteroskedasticity is present in the regression. Clarida et al. (1998) suggest that there is evidence of heteroskedasticity in the error term, that IV is not asymptotically efficient, and that the GMM estimator should be a more appropriate choice than the other two techniques.

Despite the superiority of using GMM for the estimation, it is seldom applied to the non-linear estimation. The majority of papers that study the nonlinearity of the Taylor rule in general still use LS in principle. For instance, Qin and Enders (2008) and Alcidi et al. (2009) apply the smooth transition regression model (STR) to estimate the Federal Reserve's interest rate reaction function. Surico (2007), Kenneth (2007) and Bunzel and Enders (2010) employ the thresh-

old model for the same purpose. The difference between these two techniques is that the threshold model assumes that a model can switch between a higher regime and a lower regime, and that the switch between the two regimes is instant. The smooth transition model allows the speed of the adjustment to be slow, so to speak, that the change from a high to a low regime can be gradual, which is more plausible in reality. However, even if a Taylor rule follows a nonlinear function it is difficult to justify which technique is more appropriate, and sometimes the choices between them simply depend on the researchers' preferences. Up until now there have been few discussions regarding which method is better and how to demonstrate which method should be applied in various situations.

One of the reasons why the nonlinear form IV or GMM are rarely used in the Taylor rule estimation is that these methods have not been fully developed in statistical theory. Consequently the related software package is very rare as well. The latest development of the threshold model under a GMM framework is introduced by Caner and Hansen (2004) (CH04 hereafter). They considered one threshold model with endogenous variables but an exogenous threshold variable. They developed a two-stage least square estimator of the threshold parameter and a GMM estimator of the slope parameters. Zisimos and Jean-Francois (2009) apply CH04's method to a forward-looking Taylor rule where nonlinearity is introduced by inflation threshold. They demonstrate that the Bank of England did not follow a nonlinear Taylor rule during 1992-2003 period.

Regarding the threshold process of the Taylor rule, the threshold is always chosen between  $\pi_{t-d}$  and  $y_{t-d}$ , where  $\pi_{t-d}$  is the inflation rate at time  $t-d$ , and  $y_{t-d}$  is the output gap at time  $t-d$ ,  $d > 0$ . In this chapter we use Andrews' test to find out which variable should be treated as threshold, and then use Caner and Hansen's method to estimate the nonlinear Taylor rule.

The downside to using the threshold model is that, assuming a function follows a threshold model, most of the time, one can find a result. This does not mean, however, that the results are robust. In order to investigate whether or not the results are meaningful, further investigation is required. Caner and Hansen (2004) also develop a method to detect the existence of nonlinearity in the estimation, which can be very useful in investigating the nonlinearity or asymmetry of Taylor rule estimations in various subsample periods in order to find out whether unknown structural changes are the sole reason for the nonlinearity of the function in the long run.

The chapter is organized as follows: section 3.2 presents the linear and non-linear version of Taylor rules. In section 3.3 a GMM test for unknown structural breaks developed by Andrews (1993) is introduced and the stability of the Fed's monetary policy is investigated. In section 3.4 the linear Taylor rules in various sub-regimes for the US are estimated. In section 3.5 the non-linear form of the Taylor rule are estimated in each subsample based on the breaking points obtained in section 3.3 and compared with those of the linear form. Meanwhile, Ch04's test is used to identify whether or not the Taylor rule can be viewed as a threshold process, even in a short period of time. Section 3.6 concludes.

## **3.2 A linear and nonlinear Taylor rule**

The simple monetary policy rule described by Taylor (1993a) calls for changes in the federal fund rates in response to the change in the level of output and

inflation. This reaction function rule can be written as:

$$i_t^* = r^* + \pi_t + \alpha_\pi(\pi_t - \pi_*) + \alpha_x x_t \quad (3.1)$$

where  $\alpha_\pi > 0$ ,  $\alpha_x > 0$ . In the Taylor rule,  $i_t^*$  is the target for short term nominal interest rate,  $r^*$  is the long-run real interest rate,  $\pi_t$  is the inflation rate,  $\pi_*$  is the inflation target,  $x_t$  is the deviation of GDP from its trend, and  $\alpha_\pi$  and  $\alpha_x$  are the weights that the central bank put in the interest rate reaction function.

Clarida et al (1998)(CGG hereafter) develop a forward-looking version of the Taylor rule,

$$i_t = (1 - \rho)(\alpha + \gamma_\pi \pi_{t+n} + \gamma_x x_t) + \rho i_{t-1} + \epsilon_t \quad (3.2)$$

The details of the derivation of this Taylor rule can be found in section 2.2. Thus, rearrange equation 3.2, the linear Taylor rule can then be written as:

$$i_t = \alpha_0 + \alpha_1 \pi_{t+n} + \alpha_2 x_t + \rho i_{t-1} + \epsilon_t \quad (3.3)$$

where  $\alpha_0 = (1 - \rho)\alpha$ ,  $\alpha_1 = (1 - \rho)\alpha_\pi$  and  $\alpha_2 = (1 - \rho)\alpha_x$ .

The nonlinear Taylor rule can be written as:

$$i_t = (\alpha_0 + \alpha_1 E_t \pi_{t+12} + \alpha_2 x_t + \alpha_3 i_{t-1}) I_t + (1 - I_t)(\beta_0 + \beta_1 E_t \pi_{t+12} + \beta_2 x_t + \beta_3 i_{t-1}) + \epsilon_t \quad (3.4)$$

where  $I_t = 1$  if  $\delta_{t-d} \geq \tau$  and  $I_t = 0$  otherwise.  $\delta_{t-d}$  is the threshold variable in period  $t - d$  and  $\tau$  is the threshold.

### **3.3 A GMM test for structural breaks in the Taylor rule**

This section focuses on testing the stability of the Taylor rule of the Fed by applying Andrews' (1983) method. The rationale is that if the linear form of the interest rate reaction function of the Fed is unstable and there are structural breaks in the model, then it is appropriate to find the structural break points and study the properties of the Taylor rule of the Fed under each subsample. The merits of Andrews' method are: first, it is conducted under a GMM framework, which can keep the consistency of the analysis. There are other methods available for testing structural changes, but none of them can be applied under a GMM framework. Second, this method can be applied for testing a model or just the parameter of a variable in the model. It is particularly useful to have an idea whether or not an estimation of a model is stable and, in the mean time, find out which parameter of the model is the cause of the instability. Furthermore, the most volatile variable should be considered to be the threshold variable for the nonlinear estimation. This section also investigates whether or not different output gap measures would cause differences in the results of structural changes. Which method should be used in empirical study for measuring the output gap is always an issue. In this chapter three methods are applied: the Quadratic Time Trend (QTT) method, the B-spline and the HP filter.

#### **3.3.1 Methodology**

The mechanism of testing structural changes introduced by Andrews (1993) is as follows:

Considering a parametric model indexed by parameters  $(\beta_t, \delta_0)$  for  $t = 1, 2, \dots$ , the null hypothesis of this is the parameter  $\beta_t$  is stable:

$$H_0 : \beta_t = \beta_0 \quad \text{for all } t \geq 1$$

Regarding testing a pure structural change for the model,  $\delta_0$  would be assumed to be zero, as it only appears when testing the stability of a particular parameter in the model; in other words, in the test of a partial structural change. For instance, if one is only interested in the stability of the coefficient of the inflation rate, then this coefficient is  $\beta_t$ , and the constant term, the coefficients of the output gap and the lagged interest rate are all treated as the parameter  $\delta_0$ .

The alternative hypothesis has several forms. If we consider a one-time structural change with the change point  $\gamma \in (0, 1)$ , where  $T$  is the sample size,  $T\gamma$  is the change of time, then the alternative hypothesis with the one-time change point  $T\gamma$  is:

$$H_{1T}(\gamma) : \begin{cases} \beta_1(\gamma) & \text{for } t = 1, \dots, T\gamma \\ \beta_2(\gamma) & \text{for } t = T\gamma + 1, \dots \end{cases}$$

where  $\beta_1(\gamma) \neq \beta_2(\gamma)$ . Let us assume that  $T\gamma$  is known, then the GMM is used to estimate  $\beta_1(\gamma)$ ,  $\beta_2(\gamma)$ ,  $\hat{V}_1(\gamma)$  and  $\hat{V}_2(\gamma)$  and we can then form the Wald statistic:

$$W_T(\gamma) = (\hat{\beta}_1(\gamma) - \hat{\beta}_2(\gamma))' (\hat{V}_1(\gamma) + \hat{V}_2(\gamma))^{-1} (\hat{\beta}_1(\gamma) - \hat{\beta}_2(\gamma)) \quad (3.5)$$

where  $\hat{\beta}_1(\gamma)$  and  $\hat{\beta}_2(\gamma)$  are the coefficient parameters before and after the change point,  $\hat{V}_1(\gamma)$  and  $\hat{V}_2(\gamma)$  are the corresponding variance-covariance matrix of the estimators. The Wald statistic follows an asymptotic chi-square distribution

with  $k$  degree of the freedom;  $k$  is the number of the parameters in  $\beta_t$ . However, the change point  $T\gamma$  is always unknown, so that one can estimate all the Wald statistics for each time point and the one with the largest statistic is the potential change point. Thus, in order to find structural changes one needs to find the largest Wald statistic in the sample period:

$$\sup_{\gamma \in \Pi} W_T(\gamma)$$

If this statistic is greater than the related asymptotic critical value, there is evidence of rejecting the null hypothesis and implying a structural change or parameter instability in the model.

Note that  $\Pi$  is a pre-specified subset of  $[0,1]$ ,  $\Pi$  allows one to test for a structural change that is initiated by some political or institutional change that has occurred in a known time period. For example, we have the monthly data from 1971 to 2009, so that in order to test the chairman of the Federal Reserve effect in 1979 we can simply specify  $\Pi \in [0.19, 0.23]$ <sup>3</sup>. Even if we have no information regarding the change point, the choice of using the full sample set is not desirable as the statistical power of  $\sup_{\gamma \in \Pi} W_T(\gamma)$  is lower when the change occurs near the boundary of the sample. Thus Andrews suggests using the restricted interval  $\Pi \in [0.15, 0.85]$ .

The issue with Andrews' test is that it is designed to detect a single structural break in the model. However, the desired sample period can be defined by the researcher. For instance, the sample period in our case is between January 1971 and December 2009, and we can detect whether there are structural breaks between January 1980 and January 1981. In this way we can actually discuss

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<sup>3</sup>The sample section corresponds to the period between May 1978 and January 1980.

whether or not there are multiple structural breaks during the whole sample period. In the following work we calculate the Wald statistics for the whole sample period and plot them in the graph against time, thus providing a straightforward image that shows any multiple breaks in the model.

### **3.3.2 The Andrews' Wald test for structural changes in the Taylor rule of the US**

#### **3.3.2.1 Testing the Taylor rule**

In this section the stability of the Taylor rule for the US is tested. All parameters in the Taylor rule are considered as a group in an effort to check whether there are structural changes in the Taylor rule. This is estimated by using a GMM estimator. The Quadratic Time Trend (QTT) method is used in the estimation in sub-section 3.3.2.1. All the data are collected from International Financial Statistics (IFS) and Fed Reserve Bank of St. Louis Economic Dataset (FRED). The Federal fund rate is viewed as the nominal interest rate; the first difference of consumer price indices are used for monthly inflation rates; the industrial production indices (IPI) as proxies for GDP. The data range from January 1971 to December 2009, but only data from October 1978 to January 2004 are used for the test. The reason for this is that, according to the setting of the test, the interval of the observations for the test should be around  $\gamma \in [0.2, 0.85]$  so as to reduce the biased statistical power near the boundary of the sample. Fig 3.1 shows the Wald statistics of the Taylor rule model plotted against time. As we can see in Fig 3.1, the Wald statistics at the beginning and the end are significantly greater than those in the middle.<sup>4</sup> In other words, the evidence suggests huge structural

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<sup>4</sup>The asymptotic critical value of Andrews' Wald test is not a fixed number. It is greater at the beginning of the sample than at mid-term, and becomes even lower after the mid-point of the sample. Thus, the actual asymptotic critical line should be approximately a negative

changes at the beginning (pre-1981) and towards the end (post-2001). Furthermore, the policy was less stable between March 1981 and February 1993 since the value of Wald statistics are large and volatile during this period. The monetary policy was relatively stable throughout the rest of the 1990s. Specifically, the indicated change points are as follows: The first change point is not far from the majority of literature. Most of the studies assume that the first change point was in September 1979 after Volcker became the Chairman of the Fed, while the Wald statistic suggests that the change occurred one year later.

The second potential change point in the literature was in September 1987, after Greenspan replaced Volcker, as suggested by Qin and Enders (2008). This point is also captured by the Andrews' Wald test. The third change point based on the Wald statistic is a special case. The statistic suggests that the break occurs in 1993. After this change point the monetary policy becomes relatively stable as there is a downward trend in the Wald statistics. One historic event starting in 1990 was that the US government began to tackle the huge leftover deficits spawned by the Reagan years. In order to curb the deficit the Bush government began to cut spending and raise tax rates. In November 1990 the Omnibus Budget Reconciliation Act of 1990 was enacted to reduce the United States federal budget deficit. In the meantime the interest rate and inflation rate went down together. By the midyear of 1992 both interest and inflation rates reached their lowest level in years, but the unemployment rate reached 7.8 percent, the highest since 1984. After Bill Clinton entering the White House the government continued to curb the deficit and the economy experienced a rapid growth. Thus, one potential source of the change in monetary policy is the reverse stance of fiscal policy.

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sloped quadratic line rather than a straight horizontal line as indicated on the graph; however, Andrews did not provide the whole critical value table for all the sample periods so the true critical value line is not available.

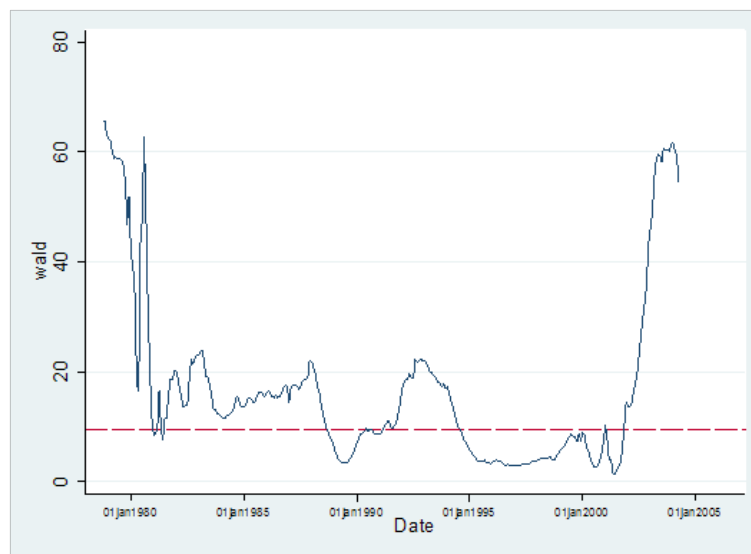


Figure 3.1: Andrews' Wald test for testing pure structural breaks in the US Taylor rule estimation

Note: the sold line is the Wald statistics obtained by using the Quadratic Time Trend. The Wald statistic starts to be calculated at sample period of  $0.2T$ , the 5% asymptotic critical value suggested by Andrews (1993) at this time point is 16.45 with 4 degree of freedom. The reference horizontal line ( $Wald_{qtt} = 9.49$ ) on the graph is the 5% asymptotic critical value of structural changes with 4 degree of freedom ( $df$ ) at time point  $0.5T$

The next potential change point was suggested by CGG to be in December 1998, after the euro was born. There is only mild evidence supporting this break point based on the Andrews' Wald statistic, however. The last change point suggested by the Wald statistic is around October 2001 and the monetary rule becomes quite volatile afterwards. In fact, it is reasonable to set the change point in October 2001. Due to the September 11 attacks and the various corporate scandals which undermined the economy the Greenspan-led Federal Reserve was afraid of the expectation of future recession so, in 2004, it initiated a series of interest cuts that brought down the Federal Funds rate to 1%. The real interest rate was negative during this period. The switch in monetary policy during this period had little to do with the interest rate reaction function; in fact, the inflation rate and output at that time were relatively stable. Another intriguing event which happened after 2001 was that the Bush government authorized wars

in Afghanistan and Iraq, which increased military spending enormously. Thus, the tightening fiscal policy during the Clinton years was loosening again. These fiscal policy changes were likely to contribute to the change in monetary policy as well.

### 3.3.2.2 The inflation rate

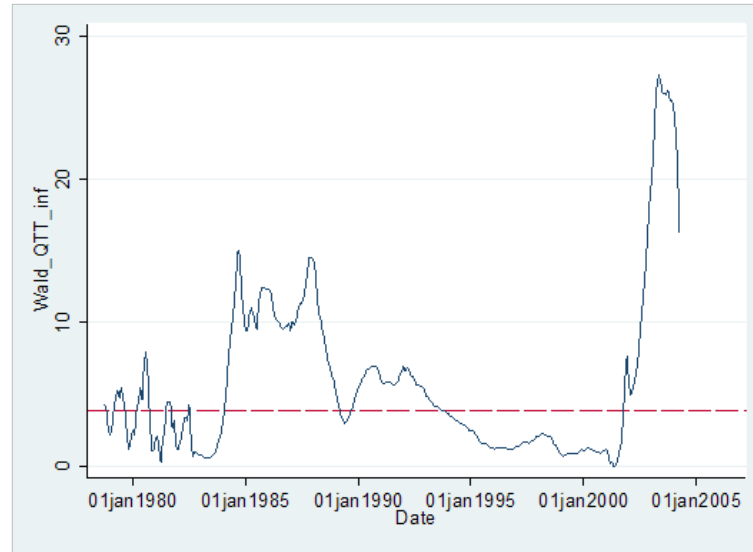


Figure 3.2: Andrews' Wald statistics for testing the partial structural break in the US Taylor rule (the case of inflation)

Note: the solid line is the Wald statistics obtained by using the Quadratic Time Trend. The Wald statistic starts to be calculated at sample period of  $0.2T$ , the 5% asymptotic critical value suggested by Andrews (1993) at this time point is 8.45 with 1  $df$ . The reference horizontal line ( $Wald_{qtt} = 3.84$ ) on the graph is the 5% asymptotic critical value of structural changes with 1  $df$  at time point  $0.5T$ .

Fig 3.1 shows the Wald statistics for the performance of all the coefficients as a group. It provides us with a general idea as to whether or not there are structural changes in the Taylor rule estimation over time. However, if one needs to find out the cause of these changes it is better to investigate the properties of each variable. By using Andrews' method one can find out which variable in the model has more structural breaks than the others, or in other words, which parameter of the variables is the most volatile. Testing the parameter of each

variable can assist us in discovering which variable leads to structural innovation and, thus, which variable should be set as the threshold.

Thus, let us discuss the coefficient parameters of the Taylor rule one by one. The assumption here is that the structural changes in the Taylor rule in the US may have various causes. They could be the result of instabilities in the weight parameter of inflation rate, output gap, or lagged interest rate. Alternatively, they might be caused by the changes in the inflation rate target. By checking the performance of the coefficients of different variables in the Taylor rule one can find out how the Federal Reserve has developed its monetary policy in the long run.

Fig 3.2 shows that the Wald statistics of the coefficient of expected inflation rate plotted against time. The changes in the coefficient parameter of inflation rate were relatively small between the end of 1970s and the start of 1980s. There were massive changes in the middle of 1980s and a significant downward trend at the end of 1980s, which imply that there were breaks in the mid-1980s. The statistics began to rebound at the beginning of the 1990; which suggests that the Federal Reserve altered the way of reacting to the inflation rate. Thus, there is evidence of the structural changes having started in the 1990s. The Wald statistics rose sharply and reached their peak in 2001, which suggests that there were significant changes of monetary policy regarding interest rate response to the inflation rate. The fluctuations in the Wald statistics in the 2000s indicate that the Fed's behaviour cannot be measured using the standard Taylor rule. The reason for this can be seen by comparing inflation rate and interest rate (Fig 3.3 and Fig 3.4). The Fed cut the interest rate constantly between 2001 and 2005 and raised it sharply afterwards, even though there were only mild fluctuations in the inflation rate. As we have suggested in the previous part, the abnormal

behaviour of the interest rate was due to the expected recession arising from the September 11, 2001 attack and the increase in budget spending.

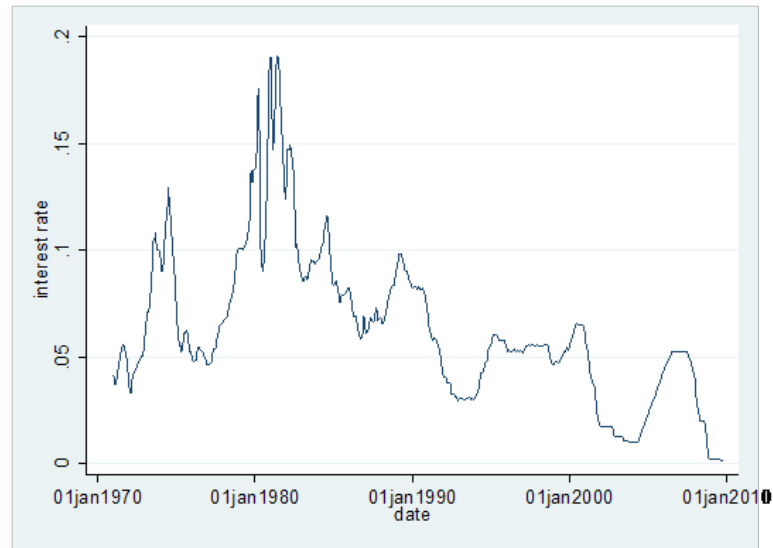


Figure 3.3: The nominal interest rate

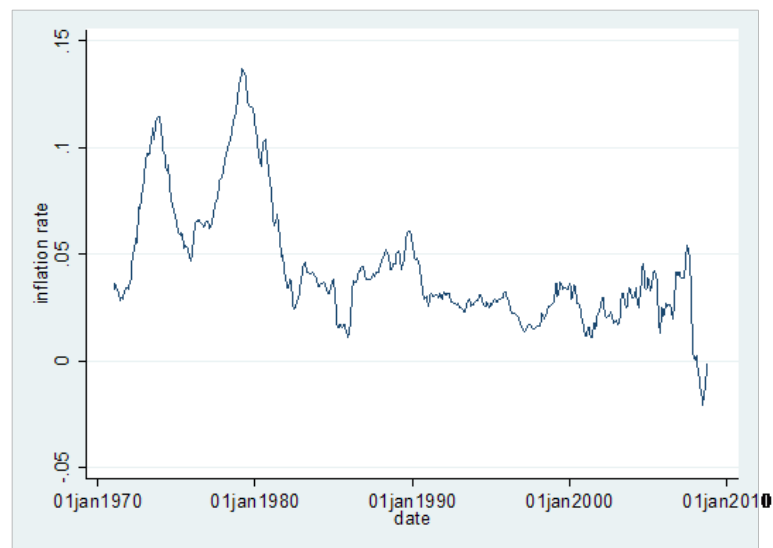


Figure 3.4: The inflation rate

### 3.3.2.3 The lagged interest rate

In terms of the lagged interest rate, the first conflict period was at the beginning of the sample, as shown in Fig 3.5. The reason for this could be that in order

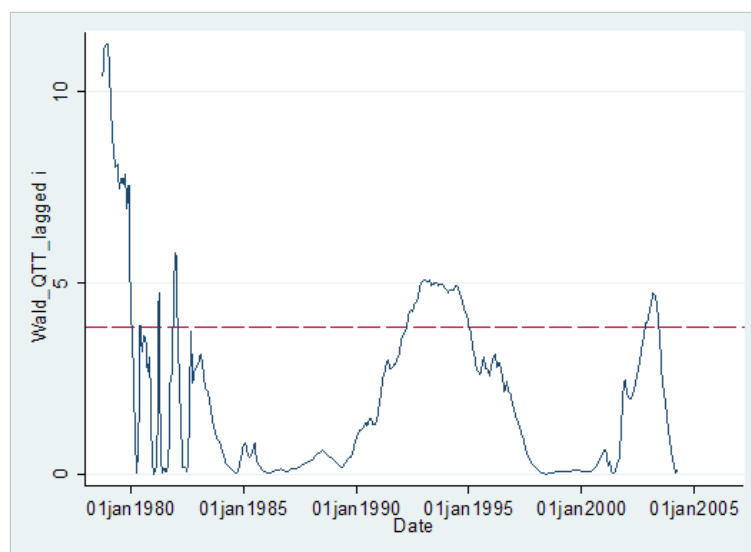


Figure 3.5: Andrews' Wald statistics for testing the partial structural breaks in the US Taylor rule estimation(the lagged interest rate)

to fight the high inflation rate during this period, the Fed had to raise the short term interest rate sharply in a short period of time, suggesting that the smooth parameter (the coefficient of the lagged interest rate) might not have been taken into consideration in the Fed's policy framework. There is one bump around 1993 and another one around 2003. The potential reasons for the change in 1993 can be seen in the interest rate graph. Firstly, the nominal interest rates in the 1980s were much higher than they were in the 1990s and 2000s. Secondly, after a significant cut in the interest rate began in 1990, the Fed adopted a cautious approach in adjusting the nominal interest rate, the Fed having intended to smooth the change in the interest rate in the 1990s. Thirdly, The break around 2003 is in line with the dramatic fall and rise in the nominal interest rate due to the budget spending increase and the expectation of recession.

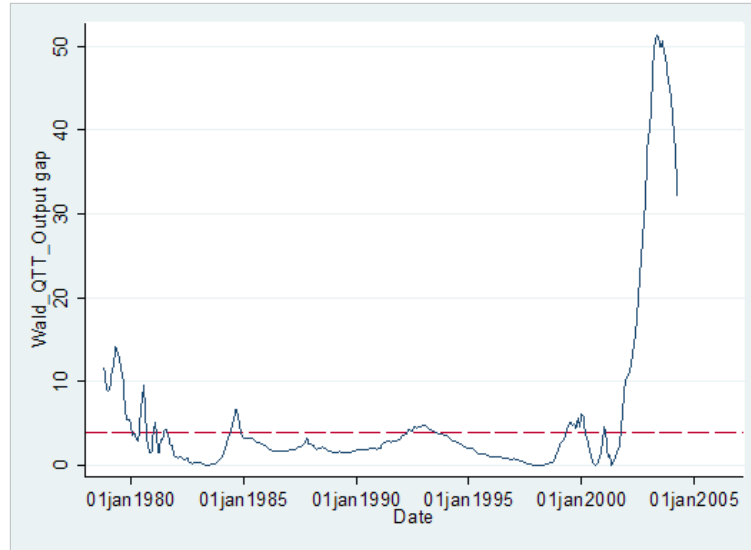


Figure 3.6: Andrews' Wald statistics for testing the partial structural break in the US Taylor rule estimation(the output gap)

#### 3.3.2.4 The output gap

For the output gap measured by the QTT, the first change point is at the beginning of the sample, which is in line with the event of stagflation in 1970s (see Fig 3.6). The second break point is in December 1984, which coincides with ending of the United States's stagflation. The third break point is around 1993, when the economy had emerged from the recession and experienced a high-speed growth. The major change is at the end. During that period, the Fed focused on the demand side of the economy, it is possible that the Fed did not want to upset economic growth. In other words, the Fed may put less weight on the positive output gap in the Taylor rule.

#### 3.3.2.5 The constant term

The constant term is unimportant in Clarida et al. (1998, 2000) because it should be around zero in theory, while the Wald statistics suggests that the constant ac-

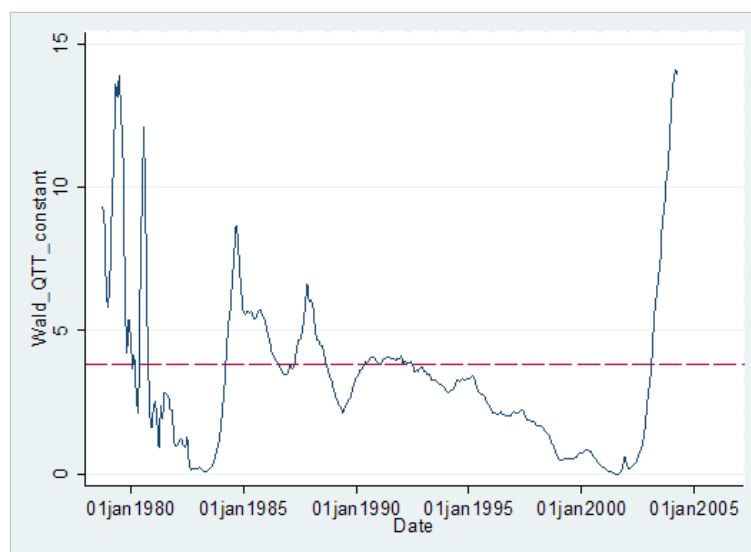


Figure 3.7: Andrews' Wald statistics for testing the partial structural break in the US Taylor rule estimation(the constant term)

tually evolved during the sample periods (Fig 3.7). The constant term includes interest rate target and inflation targets. One explanation for the change in the constant term is that there might have been little consensus in the Fed with regard to the appropriate target for these two variables, a variety of experiments having been conducted to find the right targets. Another one could be that the Fed began to conduct a more sophisticated Taylor rule. In fact, although there is no evidence that the Fed set a target in the 1970s and 1980s, the nominal interest rates were higher than 5% during most of this period. In the Greenspan (1987-2006) era, the inflation target was set implicitly at around 1.5% to 2%. Bernanke favours the explicit inflation target, and in 2006 the target was 1.5%. In October 2010, Bernanke announced that the new inflation target should be set to 2%. Thus, it seems that the target has been changing slightly over time.

To sum up, from the mid-1980s to the beginning of the 1990s the Taylor rule for the US was quite volatile, and in the 1990s the interest rate feedback rule

became consistent. Starting in the late 2001, however, the simple rule seemed to be failing in interpreting the policy. Thus a more sophisticated model might be needed.

### 3.3.3 A robust Andrews' Wald test for structural changes

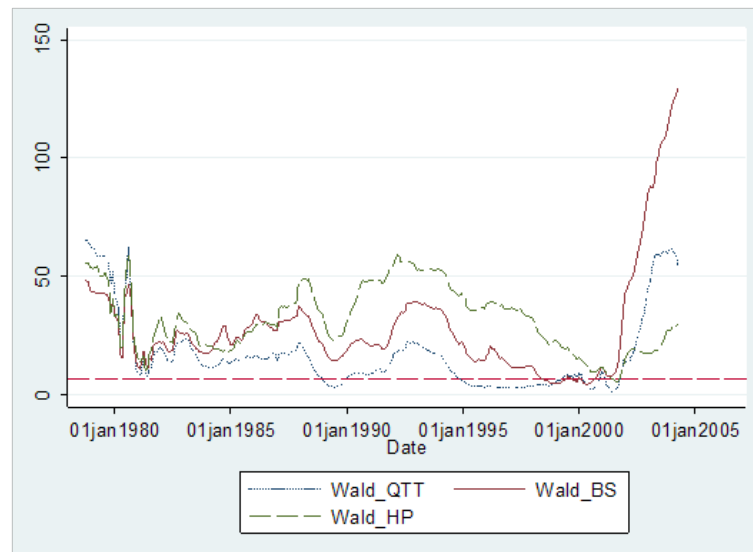


Figure 3.8: The robust Andrews' Wald test for pure structural changes in the US Taylor rule estimation

Note: the tight-dash line, solid line and long-dashed line are the Wald statistics obtained by using the Quadratic Time Trend, the B-spline and the HP-filter, respectively.

In the last section only the Quadratic Time Trend method is applied to measure the output gap. In this section, both the B-spline and the HP-filter are also used to investigate whether or not the output gap measures can lead to different results.

Fig 3.8 presents the test for structural changes in the Taylor rule by using different output gap measures. Apparently, by using B-spline and HP filter, the test statistics follow a similar trend to that obtained by QTT. However, the mag-

nitude of the statistic becomes greater with the increase in the flexibility of the output gap measures. As we know, the HP filter provides the smoothest filter of the three of them and, correspondingly, the leftover gap is the most volatile. It appears that the rise in the volatility of the output gap increases the instability of the Taylor rule estimates. In order to confirm this observation we discuss the properties of the coefficient of each variable in the follow part of this subsection in order to find out which variable results in the volatility of the Wald statistics.

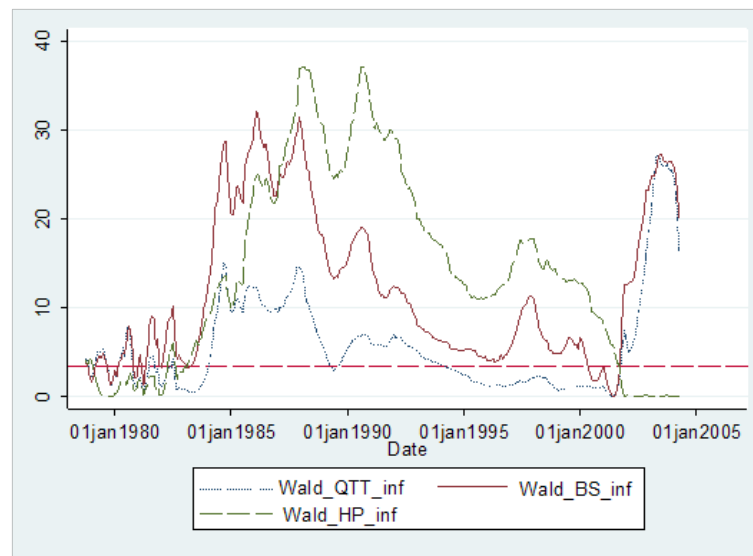


Figure 3.9: The robust Andrews' Wald test for partial structural changes in the US Taylor rule estimation.(the inflation rate)

Note: the tight-dash line, sold line and long-dashed line are the Wald statistics obtained by using the QTT, the B-spline and the HP-filter, respectively.

Fig 3.9 presents the test for the structural changes in the coefficient of the expected inflation rate. The results imply that the coefficient of the inflation rate in the Taylor rule estimation by using B-spline and HP filter are far more inconsistent than that obtained by using QTT. The coefficients are unstable throughout most of the sample period, which suggests that the Fed might not follow a consistent monetary policy in terms of responding to the expectation of inflation rate.

Although the results are not desirable, the test obtained by both B-spline and HP filter follow a similar trend to that obtained by QTT. There is a downward trend starting in 1992, which implies that the coefficient of the inflation rate is likely to be more stable than the rest of the sample period. Furthermore, there is a spike in the Wald statistics in 1997 using B-spline and HP filter that cannot be fully captured by those of QTT. This could be important because this period coincides with the Asian financial crisis. This might lead to a monetary policy inconsistency.

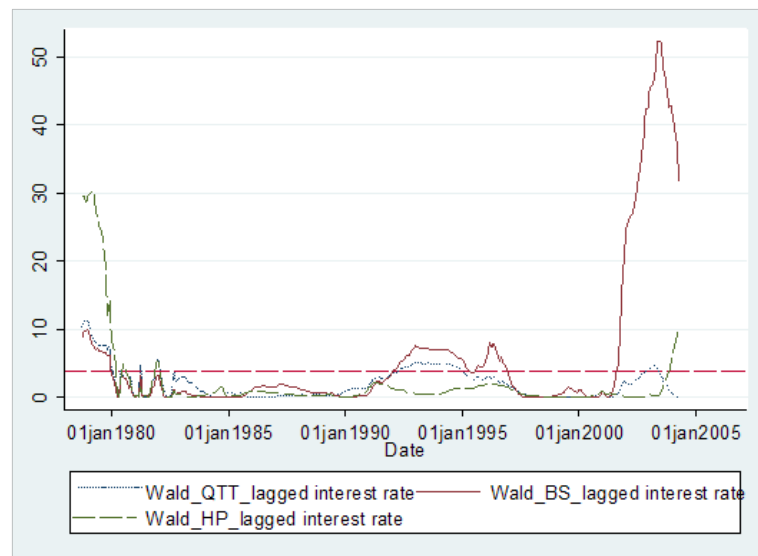


Figure 3.10: The robust Andrews' Wald test for partial structural changes in the US Taylor rule estimation.(the lagged interest rate)

Note: the tight-dash line, sold line and long-dashed line are the Wald statistics obtained by using the QTT, the B-spline and the HP-filter, respectively.

Fig 3.10 illustrates the structural changes in terms of the lagged interest rate. The change points are at the beginning of the sample, the time period during 1993, and post 2001. The B-spline provides the most volatile results and the HP filter provides the most tranquil ones. Of note is that the HP filter indicates that there was a structural break at the end of 2003, which is different from the

results obtained from QTT and B-spline, which suggest that the change could have happened in 2001.

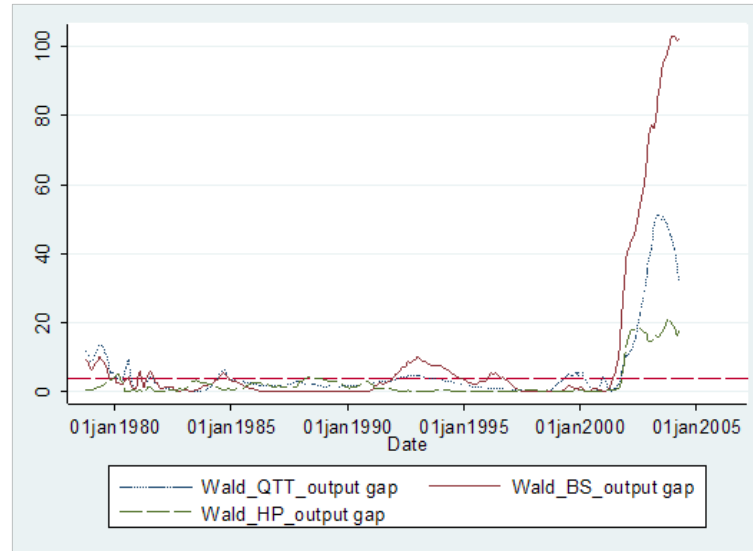


Figure 3.11: The robust Andrews' Wald test for partial structural changes in the US Taylor rule estimation (the output gap)

Note: the tight-dash line, sold line and long-dashed line are the Wald statistics obtained by using the QTT, the B-spline and the HP-filter, respectively.

In terms of output gap, the results vary (Fig 3.11). The HP filter provides the most consistent estimation of the coefficient of output gap and the B-spline provides the most volatile results. However, all of them suggest that the coefficient of the output gap does not develop significantly over time except at the beginning of the sample, in 1993 and again post-2001. The Fed's response to output gap is relatively consistent throughout the sample period.

The structural change test for the constant term is displayed in Fig 3.12. It is clear that the constant is not at a fixed level in the 1970s and 1980s by using B-spline and QTT, which is not the case for HP filter. In the 1990s all the estimates suggest that the constant term is relatively stable. There is also evidence

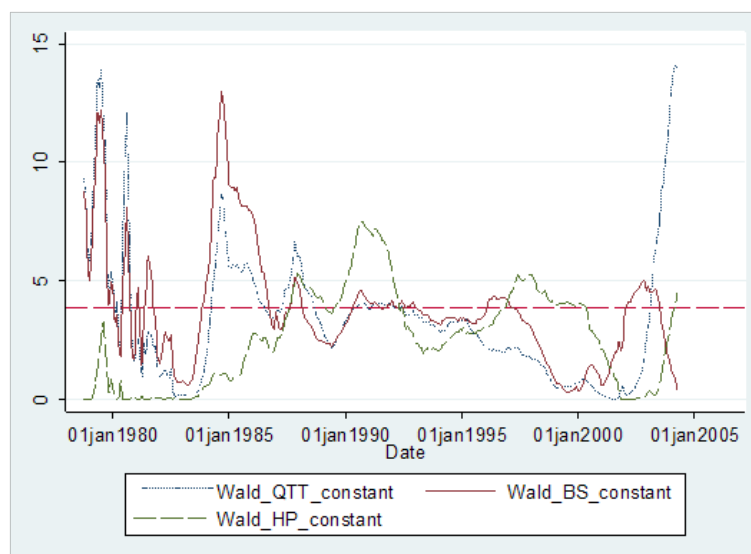


Figure 3.12: The robust Andrews' Wald test for partial structural changes in the US Taylor rule estimation (the constant)

Note: the tight-dash line, sold line and long-dashed line are the Wald statistics obtained by using the QTT, the B-spline and the HP-filter, respectively.

of structural changes after 2001.

To sum up, the estimation of the Taylor rule for the US suggests that the Fed's policy is not consistent over time, irrespective of the output gap measures. All test results follow a similar trend and suggest there might have been structural changes at the beginning of the 1980s, during 1993, and after 2001. The coefficients of inflation rate are more volatile than the other coefficients, and thus it is they that should be viewed as the threshold variable in the nonlinear Taylor rule analysis.

### 3.4 Linear Taylor rule

In this section we estimate the linear Taylor rules for the US from January 1971 to December 2006 in an effort to interpret the instability of the monetary policy. Both long-run and short-run Taylor rules are estimated, as the subsample periods have been identified in the last section. The first potential structural break date is March 1981, which is in line with Volcker becoming the chairman of the Fed, the break date also being confirmed by Andrews' Wald test. The second potential point is January 1988, which is close to the date when Greenspan became the Chairman and the 1987 financial crisis hit. The third one is January 1993, which is supported by the empirical test. The fourth one is July 1997, which coincides with the start of the Asian financial crisis. The last one is October 2001, after the 9/11 attack. GMM is used for the estimation, and three output gap measures are used for the estimation of the Taylor rule.

The results are displayed in the table 3.1-3.3. Firstly, there is evidence that the Fed has followed a Taylor rule in the long run if one looks at the results based on the estimations for the time periods of January 1971 to December 2006, March 1981 to December 2006, January 1971 to December 2001 or March 1981 to December 2001, irrespective of the output gap measures. Secondly, in the short run the results are not consistent over time; there is little evidence that the Fed employed a Taylor rule between January 1971 and March 1981 by applying any output measures. By using the QTT and the B-spline, there is no evidence of Taylor rules being employed during the periods of February 1988 to January 1993, July 1997 to September 2001, or October 2001 to December 2006. Thirdly, there is strong evidence of a Taylor rule being employed between March 1981 and January 1988. The coefficient of the expected inflation rate is around 5, which suggests that the Fed was very tough on the inflation rate at the time. The Fed

raised nominal interest rates by 5% when the inflation rate was 1% higher than the target. Thus, the real interest rate would have risen by around 4 percentage points. The high real interest rate would have harmed the aggregate demand, which would in turn have impaired the domestic economy and raised unemployment. This is consistent with the literature due to the fact that the Fed's priority was to contain high inflation and in turn sacrifice the domestic economy. There is significant evidence from using QTT and B-spline that the Fed followed a Taylor rule between January 1993 and July 1997, as the magnitude of the coefficient of inflation rate is much smaller than that of the 1980s. The coefficient is around 1.2. The reason for this might be that since inflation was not as high as in the 1980s the Fed did not need to conduct extreme means to curb it. Thus, if inflation was 1% higher than the target, then the real interest rate would have risen by around 0.2 percentage points and price level would be reined in gradually without hurting the health of the economy.

Fourth, it is important to be aware of the differences in estimation results caused by the choices of changing points. For instance, examining the results for the periods of January 1988 to October 2001 and February 1988 to December 2006, which fall in the Greenspan era, there is strong evidence that the Fed were following a standard Taylor rule. However, the results for the subsample periods of February 1988 to January 1993, July 1997 to September 2001, and October 2001 to December 2006 are contradictory. In addition, the performances of J-statistics differ under various time period selections. If there are no structural changes in the model then the  $p$ -value of the J-statistic is very high, which implies that the instruments are valid. Nevertheless, if the potential break point is involved in the regression the  $p$ -value decreases sharply. In certain cases the identification problems appear. These changes indicate that these potential breaks are likely to result in the misspecification of the model. Thus the J-statistic could

be another method of investigating structural changes, but further investigation does not be carried out for inclusion in this chapter due to limitations of space. Last but not least, if a GMM is used in the regression the traditional methods for selecting the suitable model does not seem to work. Akaike's information criterion (AIC), Bayesian information criterion (BIC) and the sum of squared residual (SSR) are the standard criteria for the model selection. Normally the lower the value of the criteria, the better the model fits the data. However, in this case, with more and more data being involved in the regression, both AIC and BIC become smaller, which implies that they are better models. In other words, the most appropriate specification of a Taylor rule is in the one for the period from January 1971 to December 2006. However, the SSR provides totally different results, it being smallest when the smallest amount of data is involved. Thus it is unclear which method should be considered the best benchmark for the selection of the model, and which periods should be considered following the Taylor rule. Nonetheless, given the structural breaks, there is mild evidence that the Fed was following the Taylor rule in the subsamples for the periods of March 1981 to January 1988 and January 1993 to July 1997.

Table 3.1: The Taylor rule estimation over time (QTT)

Start	End	$\alpha_0$ (stdev)	$\alpha_1$	$\alpha_2$	$\rho$	AIC	BIC	RSS	J- statistic (P-value)
1971.1	1981.3	0.003 (0.003)	0.085 (1.262)	0.981 (1.096)	0.956** (0.035)	-435.055	-424.715	0.011	28.115 (0.172)
	1988.1	0.005* (0.003)	-0.292 (0.637)	0.844 (0.381)	0.955** (0.018)	-748.878	-736.106	0.015	39.039 (0.014)
	1993.2	0.031 (0.002)	0.073 (0.817)	0.984* (0.432)	0.963** (0.015)	-1003.272	-989.333	0.015	38.004 (0.018)
	1997.7	0.001 (0.002)	1.112 (0.950)	0.564 (0.354)	0.969 (0.012)	-1221.808	-1207.073	0.015	37.899 (0.019)
	2001.10	-0.001 (0.001)	2.876* (1.418)	0.156 (0.131)	0.975** (0.012)	-1421.128	-1405.754	0.016	38.676 (0.015)
	2006.12	-0.002* (0.000)	3.563** (1.410)	0.118 (0.137)	0.978** (0.009)	-1667.765	-1651.730	0.016	37.130 (0.023)
1981.3	1988.1	-0.004 (0.002)	5.202** (2.039)	0.628* (0.353)	0.955** (0.021)	-447.818	-438.240	0.0036	25.627 (0.267)
	1993.1	-0.026 (0.002)	5.056* (2.507)	0.900 (0.699)	0.976** (0.014)	-780.459	-768.636	0.0039	36.170 (0.029)
	1997.7	0.001 (0.001)	1.858 (1.524)	0.968* (0.530)	0.974 (0.015)	-1066.168	-1053.076	0.0041	34.001 (0.0491)
	2001.10	-0.003** (0.001)	4.153** (0.816)	0.219** (0.068)	0.956 (0.016)	-1330.034	-1316.013	0.0043	42.918 (0.005)
	2006.12	-0.004** (0.001)	5.681** (1.326)	0.281** (0.101)	0.970** (0.011)	-1649.064	-1634.144	0.0046	45.344 (0.002)
1988.1	1993.1	0.007** (0.001)	-0.101 (0.329)	1.225** (0.169)	0.931** (0.108)	-510.786	-502.408	0.0001	22.353 (0.439)
	1997.7	0.002** (0.000)	1.519** (0.135)	0.452** (0.051)	0.002** (0.000)	-889.604	-878.695	0.0004	32.641 (0.067)
	2001.10	-0.002** (0.001)	2.423** (0.225)	0.210** (0.027)	0.911** (0.018)	-1165.809	1153.409	0.0008	48.828 (0.001)
	2006.12	-0.003** (0.001)	4.944** (0.770)	0.364** (0.077)	0.966 (0.008)	-1563.505	-1549.823	0.0010	57.666 (0.000)
1993.2	1997.7	0.002* (0.001)	1.156** (0.299)	0.360** (0.043)	0.914** (0.024)	-464.325	-456.521	0.0001	29.268 (0.1374)
	2001.10	-0.003** (0.001)	-4.809 (3.722)	0.200* (0.104)	1.020** (0.015)	-829.123	-818.583	0.0003	33.492 (0.0553)
	2006.12	-0.004** (0.001)	-1.427 (2.542)	0.288** (0.112)	1.019** (0.007)	-1268.785	-1256.361	0.0004	40.806 (0.0087)
1997.7	2001.10	-0.009 (0.017)	-1.400** (0.241)	0.298** (0.073)	1.146** (0.042)	-440.6842	-433.0361	0.0002	25.093 (0.2927)
	2006.12	-0.004** (0.001)	-3.440** (0.919)	0.402** (0.060)	1.045** (0.013)	-904.257	-893.383	0.0003	29.320 (0.1359)
2001.10	2006.12	-0.002** (0.001)	-1.427* (0.472)	0.282 (0.258)	1.040** (0.007)	-560.038	-551.595	0.0000	21.648 (0.4811)

Table 3.2: The Taylor rule estimation over time (B-spline)

Start	End	$\alpha_0$ (stdev)	$\alpha_1$	$\alpha_2$	$\rho$	AIC	BIC	RSS	J- statistic (P-value)
1971.1	1981.3	0.005 (0.004)	-0.180 (1.336)	1.023 (0.979)	0.949** (0.034)	-436.711	-426.371	0.0107	28.249 (0.168)
	1988.1	0.003 (0.002)	0.513 (0.503)	0.519* (0.193)	0.949** (0.019)	-752.228	-739.457	0.0146	40.148 (0.010)
	1993.2	0.001 (0.002)	1.121 (0.735)	0.568* (0.189)	0.958** (0.017)	-1006.382	-992.444	0.0149	39.333 (0.013)
	1997.7	0.001 (0.001)	1.220* (0.615)	0.476** (0.160)	0.957** (0.014)	-1225.750	-1211.015	0.0150	36.709 (0.026)
	2001.10	-0.001 (0.001)	2.085** (0.843)	0.370** (0.152)	0.966** (0.012)	-1428.966	-1413.592	0.0155	37.144 (0.020)
	2006.12	-0.011 (0.001)	2.251** (0.729)	0.331** (0.114)	0.965** (0.010)	-1679.811	-1663.775	0.0158	37.944 (0.019)
1981.3	1988.1	-0.005 (0.002)	6.224* (2.819)	0.624 (0.434)	0.963** (0.020)	-447.245	-437.667	0.0037	25.988 (0.252)
	1993.1	-0.004 (0.002)	5.623* (2.782)	0.712 (0.613)	0.977** (0.014)	-779.710	-767.887	0.0039	36.261 (0.029)
	1997.7	-0.002 (0.002)	3.275** (1.148)	0.329 (0.283)	0.960** (0.017)	-1063.484	-1050.392	0.0041	33.460 (0.056)
	2001.10	-0.004** (0.001)	3.922** (0.607)	0.210** (0.081)	0.939** (0.017)	-1313.717	-1299.695	0.0050	35.273 (0.036)
	2006.12	-0.005** (0.001)	4.663** (0.717)	0.317** (0.084)	0.946** (0.013)	-1619.814	-1604.893	0.0050	37.053 (0.023)
1988.1	1993.1	0.005** (0.001)	-0.164 (0.302)	0.953** (0.121)	0.923** (0.010)	-510.057	-501.678	0.0002	22.921 (0.406)
	1997.7	0.005** (0.001)	0.379* (0.216)	0.656** (0.070)	0.901** (0.012)	-904.653	-893.744	0.0002	27.345 (0.1983)
	2001.10	-0.000 (0.001)	1.886** (0.175)	0.283** (0.041)	0.883** (0.020)	-1159.636	-1147.236	0.0008	40.870 (0.008)
	2006.12	-0.002** (0.001)	2.658** (0.304)	0.411** (0.047)	0.926** (0.011)	-1548.915	-1535.233	0.0010	48.109 (0.001)
1993.2	1997.7	0.004** (0.001)	1.383** (0.166)	0.693** (0.049)	0.858** (0.021)	-470.625	-462.820	0.0001	25.667 (0.266)
	2001.10	-0.002** (0.001)	197.513 (6552.577)	0.786 (19.744)	0.999** (0.015)	-828.015	-828.015	0.0003	34.324 (0.046)
	2006.12	-0.004** (0.001)	-9.285 (6.709)	0.191 (0.235)	1.014** (0.011)	-1266.787	-1254.363	0.0004	42.727 (0.005)
1997.7	2001.10	-0.001 (0.002)	0.385 (0.702)	1.542** (0.058)	0.955** (0.029)	-451.993	-444.345	0.0001	26.753 (0.221)
	2006.12	-0.002* (0.001)	5.651 (7.664)	0.710* (0.333)	0.981** (0.023)	-912.693	-901.819	0.0003	36.202 (0.029)
2001.10	2006.12	0.004** (0.001)	0.467* (0.206)	0.948** (0.105)	0.897** (0.029)	-577.184	-568.741	0.0001	25.696 (0.265)

## 3.5 The nonlinear Taylor rule

### 3.5.1 The estimation of nonlinear Taylor rules

In this section there are two tasks: the first is to estimate nonlinear Taylor rules under each subsample period, this being determined in section 3.4; and the second is to test for whether or not the nonlinear form exists. The Taylor rule is viewed as a threshold process, the estimation method being based on the one introduced by Caner and Hansen (2004). The advantage of their method is that it allows the estimation to be conducted under the GMM framework so that the appropriate value of the threshold for the specification can be investigated. CH04 also propose a method to test the existence of the threshold process, which can help

Table 3.3: The Taylor rule estimation over time (HP-filter)

Start	End	$\alpha_0$ (stdev)	$\alpha_1$	$\alpha_2$	$\rho$	AIC	BIC	RSS	J- statistic (P-value)
1971.1	1981.3	-0.003 (0.002)	1.830* (0.871)	2.311 (1.989)	0.955** (0.033)	-438.751	-428.411	0.0105	29.608 (0.121)
	1988.1	0.002 (0.002)	0.683 (0.701)	2.426* (1.296)	0.965** (0.019)	-752.043	-739.272	0.0147	42.947 (0.005)
	1993.2	-0.001 (0.002)	2.557 (2.938)	4.962 (4.867)	0.985** (0.014)	-1002.831	-988.892	0.0151	42.096 (0.006)
	1997.7	-0.001 (0.001)	2.352 (1.736)	3.738 (2.370)	0.981** (0.012)	-1223.160	-1208.422	0.0152	39.862 (0.011)
	2001.10	-0.001 (0.001)	2.808 (2.128)	4.409 (3.081)	0.983** (0.012)	-1432.141	-1416.766	0.0154	40.580 (0.009)
	2006.12	-0.001 (0.001)	3.062* (1.619)	4.032* (2.088)	0.982** (0.009)	-1685.909	-1669.872	0.0156	37.584 (0.020)
1981.3	1988.1	-0.001 (0.002)	2.774* (1.268)	1.568** (0.613)	0.951** (0.019)	-448.760	-439.182	0.0036	24.908 (0.302)
	1993.1	-0.002 (0.002)	6.003 (5.043)	6.402 (7.216)	0.988** (0.014)	-775.436	-763.612	0.0040	29.280 (0.1371)
	1997.7	-0.002 (0.001)	3.323** (0.888)	2.163* (1.056)	0.962** (0.017)	-1064.183	-1051.091	0.0041	23.661 (0.365)
	2001.10	-0.002* (0.001)	3.640** (0.804)	2.329** (0.889)	0.961** (0.015)	-1335.221	-1321.200	0.0043	25.049 (0.295)
	2006.12	-0.002 (0.001)	4.493** (0.928)	2.695** (0.957)	0.967** (0.011)	-1656.012	-1641.091	0.0045	23.854 (0.355)
1988.1	1993.1	-0.003** (0.001)	3.102** (0.805)	3.730** (1.530)	0.968** (0.012)	-495.660	-487.282	0.0002	31.980 (0.078)
	1997.7	0.001** (0.001)	-3.998 (10.332)	11.137 (19.576)	0.992** (0.013)	-863.882	-852.973	0.0004	37.982 (0.018)
	2001.10	-0.000 (0.001)	2.022** (0.255)	1.680** (0.300)	0.931** (0.013)	-1206.136	-1193.737	0.0006	54.030 (0.000)
	2006.12	-0.003** (0.001)	4.683** (0.727)	2.870** (0.626)	0.966** (0.007)	-1576.697	-1563.014	0.0009	55.596 (0.000)
1993.2	1997.7	0.003* (0.000)	-0.644 (0.690)	3.505** (1.168)	0.967** (0.012)	-473.848	-466.043	0.0001	32.697 (0.066)
	2001.10	-0.001 (0.001)	2.902* (1.256)	2.774** (0.892)	0.968** (0.012)	-844.374	-833.836	0.0003	34.002 (0.049)
	2006.12	-0.003** (0.001)	-33.704 (52.653)	-19.736 (32.714)	1.003** (0.005)	-1289.797	-1277.373	0.0004	35.795 (0.032)
1997.7	2001.10	0.001 (0.002)	1.452* (0.627)	1.702** (0.428)	0.911** (0.038)	-447.450	-439.801	0.0001	20.848 (0.530)
	2006.12	-0.003 (0.001)	70.294 (228.777)	44.572 (41.701)	0.998** (0.005)	-925.864	-914.990	0.0002	26.834 (0.218)
2001.10	2006.12	-0.002** (0.000)	-1.851** (0.505)	-1.060* (0.452)	1.040** (0.008)	-568.891	-560.447	0.0001	24.699 (0.312)

us to identify whether or not there is a threshold process in various subsample periods.

Many papers suggest that the threshold variable should be chosen between the lagged inflation rate and the output gap. Based on the finding that the Andrews' Wald statistics of inflation rate are more volatile than those of the other variables, the inflation rate is considered to be the threshold variable. In the following section, the first-, second-, third- and sixth-lagged inflation rates are considered as the threshold variable, respectively. The reason for this arrangement is that the majority of studies choose the first- and second-lagged inflation rate as the threshold variable using quarterly data, but we use monthly data for the estimation, which corresponds to the third- and sixth-lagged monthly inflation rate. The difficulty is to find the optimum threshold  $\tau$ , since we assume that four different variables can be viewed as threshold variables. The estimation process is in two stages: the first stage is to find the value of the threshold, given the threshold variable, and the second stage is to compare the estimation results under each estimated threshold, and then determine the optimum threshold variable.

For the first stage, CH04 propose a method for finding the value of the threshold. According to their method the process is as follows: first, by programming, the order of the data for the inflation rate is reorganised, being sorted into the lowest to the highest. The lowest and highest 15% of the values are trimmed. The reason for this is that, in order to have enough observations for the following estimation, it is necessary to remove the extreme cases. The remaining data forms the threshold variable set ( $\Gamma$ ). Given any  $\tau \in \Gamma$ , the nonlinear Taylor rule

is estimated as follows:

$$i_t = (\alpha_0 + \alpha_1 E_t \pi_{t+12} + \alpha_2 x_t + \alpha_3 i_{t-1}) I(\pi_{t-d} \geq \tau) \\ + I(\pi_{t-d} < \tau) (\beta_0 + \beta_1 E_t \pi_{t+12} + \beta_2 x_t + \beta_3 i_{t-1}) + \epsilon_t \quad (3.6)$$

where  $I(\pi_{t-d} \geq \tau)$  is equal to 1 if  $\pi_{t-d} \geq \tau$  and zero otherwise. Then the sum of squared residual (SSR) is stored. The appropriate threshold is found by minimizing the SSR:

$$\tau = \min SSR = \min_{\tau \in \Gamma} \hat{\epsilon}(\tau)' \hat{\epsilon}(\tau) \quad (3.7)$$

Since four different lagged inflation rates are viewed as potential threshold variables, the optimum threshold needs to be determined by comparing the results of these four different threshold models. Since SSR is a key indicator of goodness of fit, as indicated by CH04, we still use SSR as the key parameter for selecting the optimum threshold. In brief, the one with lowest SSR is considered to be the optimum threshold.

At this stage we only use QTT to measure output gap for simplicity. One intriguing issue we would like to address is whether or not the nonlinear Taylor rule can outperform the linear Taylor rule, especially regarding the periods of 1981 to 1993 and 1993 to 2001. A number of studies demonstrate that the Fed's can be described as a nonlinear Taylor rule from 1980 to 2000. If that is the case then on dividing this sample period into two subsample periods one should still expect the existence of nonlinearity. As we have demonstrated in the last section, there is evidence of the linear Taylor rule being used during the time periods of 1981 to 1988 and 1993 to 1997. However, we failed to find evidence for the linear Taylor rule having been used in the period between 1988 and 1993.

If the data from 1993 to 2001 is used in the estimation there is no evidence of a linear Taylor rule either. In addition, there is no evidence of the use of the Taylor rule pre-1981 or between 2001 and 2006. Thus, the testing periods are pre-March 1981, March 1981 to January 1993, January 1993 to October 2001, and October 2001 to February 2006.

The table 3.4 shows the results if the Taylor rule for the US is viewed as a threshold process. The first sample period to be investigated is the period between January 1971 and March 1981. As we have demonstrated in the previous section, there is no evidence of a linear Taylor rule for the US being used during this period. Using a threshold model does not improve the performance. All the parameters of the variables are insignificant except for those of lagged interest rates.

The second sample period is March 1981 to January 1993. According to the Andrews' Wald statistics, the Taylor rule was very volatile at that time. Thus, although there is evidence of the linear Taylor rule being applied, especially as the parameter of expected inflation rate is very high, the nonlinear Taylor rule is worth investigating. The results confirm that the coefficients of expected inflation rate ( $\alpha_1$ ) above the thresholds are all positive, the magnitude being greater than 4. It is an important result as it proves the Fed's determination to curb the high inflation. However, there is no evidence that the Fed reacted to output gap or inflation when it was below the threshold. These results imply that the Fed's priority was to curb high inflation, and thus only reacted to a high inflation rate at that period of time. Moreover, the six-lagged inflation rate provides the lowest SSR, and thus should be considered as the optimum threshold variable.

The third sample period is from January 1993 to October 2001, this period

being distinguished from the 1970s and 1980s in that the US economy experienced steady growth and inflation was low. The Andrews' test suggests that the Fed might have employed a Taylor rule in general during this period as the Wald statistic line in Fig3.2 has a downward trend. It is expected that the Fed was likely to have followed a simple Taylor rule as much of the literature suggested, but we do not find significant evidence of linearity in section 3.4. On considering the Taylor rule model as a threshold model the result is not significant either. In general, none of the coefficients of output gap are significant, the inflation rate coefficient is significant in only 2 out of 8 cases, and these 2 cases do not have a positive sign. Hence the results contradict the Taylor principle, which indicates that the threshold process is unable to explain the Fed's behaviour at this time. Nonetheless, the parameter of the lagged interest rate is significant and its magnitude is greater than those in the previous periods. As we recall in section 3.3, the Andrews' Wald statistic suggests that there is a structural change in the lagged interest rate in the 1990s, which is confirmed by the threshold models. In addition, the threshold during this period is around 2.5%, which is much smaller than those in the first and second sample periods. This difference is likely to explain why there is a huge break in Taylor rule estimation around 1993. The first-lagged inflation rate is considered to be the optimum threshold model, since it provides the smallest SSR. This is also different from the results in the previous periods. After conducting the above analysis it is still not clear whether or not the Fed followed a Taylor rule from 1993 to 2001. It is worth exploring the rationale behind these results. As we explained in the last section, the Fed follows a Taylor rule in the long term, but might not do so in the short term. During this period there are 8 years of data available; there is evidence of the linear Taylor rule being used for the first 4 years, from 1993 to 1997. However, after the Asian financial crisis the Fed was unable to implement the same policies, as had been indicated by a simple Taylor rule, especially when inflation was higher

than the target and thus the Fed had to cut the real interest rate in order to prevent a potential economic recession, even if there was pressure on prices. This abnormal behaviour virtually caused the stock market bubble in 2000. Thus, the main reason for the failure of demonstrating the linearity of the Taylor rule during this period could have arisen through the influence of the Asian financial crisis.

The last sample period is from October 2001 to December 2006. There is no evidence of a linear Taylor rule being employed during this period. In fact, the coefficient of inflation rate is negative in the linear estimation, which deviates from the implication of a Taylor rule having been used. In terms of the nonlinear Taylor rule, the results are mixed. The coefficient of the inflation rate above the threshold is not significant in 3 out of 4 cases, and all of them are negative, which means that the Fed did not intend to curb inflation when the price pressure increased. When the inflation rate is less than the threshold, the coefficient becomes significant. If the first- or second-lagged inflation rates are taken as the threshold variable, the coefficient is positive, while it is negative when taking the third- or sixth-lagged inflation rates as the threshold variable. The positive coefficient of the inflation rate when it is below the threshold indicates that a fall in inflation drives the nominal and real interest rates down, which will provoke a rise in inflation back to the target; the value of negative coefficients calculated by using third- or sixth-lagged inflation rates is in fact less than 1, which indicates that the rise of nominal interest rates caused by inflation will not be sufficient to prevent the real interest rate from declining; in other words, the results are, in general, consistent, regardless of which lagged inflation rate is chosen. The first-lagged inflation rate, however, provides the lowest SSR, it should therefore be viewed as the optimum threshold variable.

Based on the estimated results obtained by using first-lagged inflation, the

coefficient of inflation rate is negative when the inflation is higher than the target; the coefficient turns to positive and the magnitude is greater than 1 when inflation is lower than the target. These results indicate that the Fed is willing to cut interest rates when inflation is lower than the target but reluctant to raise it when the inflation rate is higher. The coefficients of output gap are positive in all cases, and significant in 6 out of 8 cases. In addition, the magnitude of output gap is greater than 1 in 6 out of 8 cases. The results for the parameter of output gap is different from those in the previous period, which indicates that the Fed was more concerned about the stabilization of the business cycle. The coefficients of lagged interest rate are significant in all cases, and 4 of them are greater than 1. These results are important because it demonstrates that the Fed was more concerned about the output gap than the inflation rate, and the negative inflation gap rather than the positive one. Specifically, if the inflation rate was less than the threshold, the government was willing to lower the interest rate so as to provoke inflation; but if inflation was higher than the threshold the Fed did not have the motivation to push it down. This discovery confirms many criticisms of the Greenspan Fed. In an effort to tackle the potential recession that could be caused by the 9/11 attack and military spending increases, the interest rate was cut to 0.01% in 2004 so as to stimulate the economy, and this policy eventually caused the asset bubble and the financial crisis in 2007.

### **3.5.2 Testing for nonlinearity of the Taylor rule**

In the last section we have reviewed the Taylor rule regarding the threshold process in each sub period. There is mild evidence that the Taylor rule for the US is likely to have followed a threshold process during the periods of 1981 to 1993 and 2001 to 2006. In this section we would like to conduct a formal test to

Table 3.4: Nonlinear Taylor rule

Coef.	1971.1-1981.2						1981.3-1993.2						1993.3-2001.9						2001.10-2006.2					
$\tau$	$\pi_{-1}$	$\pi_{-2}$	$\pi_{-3}$	$\pi_{-6}$	$\pi_{-1}$	$\pi_{-2}$	$\pi_{-3}$	$\pi_{-6}$	$\pi_{-1}$	$\pi_{-2}$	$\pi_{-3}$	$\pi_{-6}$	$\pi_{-1}$	$\pi_{-2}$	$\pi_{-3}$	$\pi_{-6}$	$\pi_{-1}$	$\pi_{-2}$	$\pi_{-3}$	$\pi_{-6}$	$\pi_{-1}$	$\pi_{-2}$	$\pi_{-3}$	$\pi_{-6}$
$\alpha_0$	0.002 (0.012)	-0.001 (0.003)	-0.010 (0.016)	0.0045** (0.008)	-0.019** (0.005)	-0.0134** (0.005)	-0.006* (0.003)	-0.004* (0.002)	-0.004** (0.001)	-0.004** (0.001)	-0.005** (0.001)	-0.005** (0.001)	0.000 (0.001)	0.001** (0.001)	-0.001 (0.001)	-0.001 (0.001)	0.000 (0.001)	0.0265 (0.0265)	0.001** (0.001)	-0.001 (0.001)	0.0276 (0.0276)	0.0265 (0.0265)	0.0276 (0.0276)	0.0298 (0.0298)
$\alpha_1$	0.965 (0.734)	2.517 (1.965)	1.658 (1.032)	0.098 (0.156)	7.983** (1.986)	6.395** (1.708)	5.011** (2.119)	4.856* (2.619)	-32.585 (46.394)	267.497 (5443.324)	69.016 (457.892)	-6.617 (5.903)	-3.491* (1.811)	-0.206 (0.479)	-0.166 (0.785)	-2.429 (2.005)	-3.491* (1.811)	-0.206 (0.479)	-0.166 (0.785)	-2.429 (2.005)	-3.491* (1.811)	-0.206 (0.479)	-0.166 (0.785)	-2.429 (2.005)
$\alpha_2$	0.735 (0.219)	0.164 (0.388)	0.263 (0.283)	0.715** (0.096)	-1.159** (0.598)	-0.521 (0.521)	0.236 (0.311)	0.399 (0.419)	-0.097 (0.652)	0.743 (12.556)	0.944 (4.728)	0.150 (0.084)	1.257 (0.898)	1.601** (0.548)	7.752* (4.095)	2.727 (2.232)	1.257 (0.898)	1.601** (0.548)	7.752* (4.095)	2.727 (2.232)	1.257 (0.898)	1.601** (0.548)	7.752* (4.095)	2.727 (2.232)
$\alpha_3$	0.831** (0.084)	0.858** (0.095)	0.784** (0.101)	0.553** (0.072)	0.933** (0.022)	0.934** (0.017)	0.960** (0.017)	0.970** (0.016)	1.006** (0.016)	1.006** (0.016)	0.999 (0.015)	1.025** (0.021)	1.010** (0.008)	0.980** (0.008)	0.989** (0.004)	1.015** (0.011)	1.010** (0.008)	0.980** (0.008)	0.989** (0.004)	1.015** (0.011)	1.010** (0.008)	0.980** (0.008)	0.989** (0.004)	1.015** (0.011)
$\beta_0$	0.004** (0.002)	0.005** (0.002)	0.006** (0.002)	0.005* (0.002)	0.002 (0.001)	0.001 (0.002)	-0.003* (0.001)	-0.002 (0.002)	0.005** (0.002)	0.007* (0.001)	0.012 (0.001)	0.002* (0.001)	-0.002** (0.001)	-0.003** (0.001)	-0.002** (0.001)	-0.002** (0.001)	-0.002** (0.001)	-0.003** (0.001)	-0.002** (0.001)	-0.002** (0.001)	-0.002** (0.001)	-0.003** (0.001)	-0.002** (0.001)	-0.002** (0.001)
$\beta_1$	-2.034 (5.437)	3.180 (2.659)	-4.545 (11.465)	-0.153 (0.613)	5.456 (7.066)	2.091 (4.971)	-3.986 (3.714)	-15.051 (37.289)	-0.513 (0.369)	-0.209** (0.071)	-0.187** (0.055)	0.484 (0.526)	4.866** (1.162)	5.920** (1.457)	-0.871* (0.404)	-0.684** (0.171)	4.866** (1.162)	5.920** (1.457)	-0.871* (0.404)	-0.684** (0.171)	4.866** (1.162)	5.920** (1.457)	-0.871* (0.404)	-0.684** (0.171)
$\beta_2$	2.607 (5.149)	-2.373 (2.452)	4.464 (9.783)	1.002 (0.612)	-5.066 (6.913)	-4.050 (5.811)	-1.098 (0.904)	-3.452 (8.081)	-0.422* (0.174)	-0.007 (0.015)	0.027* (0.015)	-0.062 (0.078)	1.892** (0.663)	0.260* (1.111)	0.337* (0.164)	0.194* (0.113)	1.892** (0.663)	0.260* (1.111)	0.337* (0.164)	0.194* (0.113)	1.892** (0.663)	0.260* (1.111)	0.337* (0.164)	0.194* (0.113)
$\beta_3$	0.981** (0.036)	1.027** (0.029)	0.985** (0.031)	0.932** (0.032)	1.008** (0.010)	1.001** (0.111)	1.017** (0.010)	1.006** (0.012)	0.936 (0.027)	0.872** (0.021)	0.792** (0.026)	0.968** (0.016)	0.975** (0.004)	0.985** (0.005)	1.074** (0.204)	1.079** (0.081)	0.975** (0.004)	0.985** (0.005)	1.074** (0.204)	1.079** (0.081)	0.975** (0.004)	0.985** (0.005)	1.074** (0.204)	1.079** (0.081)
$J$ -statistic ( $\tau \geq \pi_t - d$ )	23.118 (0.395)	17.531 (0.733)	25.553 (0.254)	21.176 (0.510)	21.672 (0.479)	21.716 (0.477)	25.693 (0.2651)	23.768 (0.360)	21.680 (0.479)	21.640 (0.482)	26.542 (0.229)	21.194 (0.509)	20.861 (0.5293)	22.330 (0.4419)	26.120 (0.246)	16.696 (0.4055)	20.861 (0.5293)	22.330 (0.4419)	26.120 (0.246)	16.696 (0.4055)	20.861 (0.5293)	22.330 (0.4419)	26.120 (0.246)	16.696 (0.4055)
$J$ -statistic ( $\tau < \pi_t - d$ )	25.780 (0.261)	25.924 (0.255)	27.929 (0.178)	24.354 (0.329)	32.219 (0.073)	33.293 (0.058)	25.076 (0.294)	22.701 (0.418)	23.120 (0.395)	23.412 (0.379)	21.051 (0.518)	24.265 (0.334)	22.249 (0.445)	17.966 (0.708)	20.338 (0.562)	24.097 (0.342)	22.249 (0.445)	17.966 (0.708)	20.338 (0.562)	24.097 (0.342)	22.249 (0.445)	17.966 (0.708)	20.338 (0.562)	24.097 (0.342)
$SSR$	0.0096017 0.0099129 0.0096879 0.0084812 0.0036073 0.0035944 0.0036378 0.035837 0.0002410 0.0002412 0.0002453 0.0002518 0.0000529 0.000062 0.0000753 0.000087																							

demonstrate the existence of the threshold behaviour. The test is proposed by Caner and Hansen (2004), and the procedure is as follows:

The general form of a threshold model is:

$$\begin{aligned} y_i &= \theta_1' z_i + e_{1i}, & q_i &\leq \tau \\ y_i &= \theta_2' z_i + e_{2i}, & q_i &> \tau \end{aligned}$$

which may also be written in the form

$$y_i = \theta_1' z_i(q_i \leq \tau) + \theta_2' z_i(q_i > \tau) + e_i \quad (3.8)$$

where  $y_i$  is an dependent variable,  $z_i$  is an  $m$  vector, and  $q_i$  is the threshold variable, where  $q_i \in \Xi$ . The threshold parameter is  $\tau \in \Gamma$  where  $\Gamma$  and is a strict subset of  $\Xi$ .

The null hypothesis of the no threshold effect in the model is:

$$H_0 : \quad \theta_1 = \theta_2$$

The alternative hypothesis for the existence of the threshold effect is:

$$H_1 : \quad \theta_1 \neq \theta_2$$

To test  $H_0$ , CH04 recommend the extension of a Sup test developed by Davies (1977) to the GMM framework.

The statistic is formed as follows: First,  $\tau \in \Gamma$  is fixed at any value, and the model is estimated by the GMM estimator under the right moment conditions. Thus the parameter  $\hat{\theta}_1(\tau)$ ,  $\hat{\theta}_2(\tau)$  and the corresponding covariance matrices  $\hat{V}_1$  and  $\hat{V}_2$  can be obtained.

The Wald statistic for  $H_0$  is:

$$W_n = (\hat{\theta}_1(\tau) - \hat{\theta}_2(\tau))' (\hat{V}_1(\gamma) + \hat{V}_2(\gamma)) (\hat{\theta}_1(\tau) - \hat{\theta}_2(\tau)) \quad (3.9)$$

The calculation is repeated for all  $\tau \in \Gamma$ . The  $SupW$  statistic for  $H_0$  is then the largest value of these statistics:

$$SupW = SupW_n(\tau) \quad (3.10)$$

Since the parameter is not identified under the null hypothesis, the asymptotic distribution is not chi-square but can be written as the supremum of a chi-square process. In order to obtain the asymptotic distribution of  $SupW$ , Caner and Hansen (2004) suggest estimating the pseudo-dependent variable  $y_i^* = \hat{e}_i(\tau)\eta_i$ , where  $\hat{e}_i(\tau)$  is the estimated residual under the unrestricted model for each  $\tau$ , and  $\eta_i$  is simulated as independent and identically distributed (*i.i.d*)  $N(0, 1)$ . Then  $y_i^*$  is used in place of  $y_i$  to estimate the threshold model given the fixed regressor and each potential threshold. The Wald statistic is then calculated and  $SupW^*$  selected. The process is repeated 1,000 times; the resulting  $SupW^*$  having a asymptotic distribution of  $SupW$ .

The  $p$ -value of the  $SupW$  is used to demonstrate whether or not the null hypothesis is rejected. It is approximately equal to the number of  $SupW^*$  greater than  $SupW$  divided by the total number of  $SupW^*$ . Thus, if 20  $SupW^*$  is greater than  $SupW$ , the  $p$ -value =  $20/1000 = 0.02$ . If it is less than 0.05, we have evidence that the null is rejected. If it is less than 0.01, we have strong evidence that the null is rejected.

For the period from 1981 to 1993 the sixth-lagged inflation rate is viewed as

the threshold variable based on the results in the last section. The total number of observations is 142. The value of the threshold is 4%, which is also obtained in the last section, where 52 are greater than the threshold. The  $SupW$  is 45.82. The simulation is repeated 1,000 times, and the number of  $SupW^*$  greater than  $SupW$  is 470. Thus, the  $p$ -value is 0.478, which implies that we can reject the null hypothesis of linearity for a significance level greater than 47.8%. Given this result, the null hypothesis is not rejected at any conventional level. The test does not support the presence of the nonlinear Taylor rule during this period of time. Because the test suggests that the highly persistent structural changes in the 1980s are unable to be explained by a threshold model. Although the threshold model estimation is not significant for the 1980s and 1990s, one difference is that the indicated threshold of inflation rate was around 4% in the 1980s but decreased to 2% afterwards. Furthermore, Fig 3.4 shows clearly that the level of inflation was much higher in the 80s than for the rest of the samples, and thus, the change in the level of inflation might have been a potential cause of the structural changes, but it is inappropriate to interpret the evolution by using the threshold model. These findings are consistent with those obtained by Surico (2007).

For the period from 2001 to 2006 the first lagged inflation rate is viewed as the threshold variable, and the estimated threshold is 2.6%. There are 61 observations during this period, 34 of them having been greater than the threshold. Thus, there were enough data to perform the test. The  $SupW$  was 194.75. The simulation is repeated 1,000 times. The number of the  $SupW^*$  greater than  $SupW$  is 8. So the  $p$ -value is 0.008, which is considerably less than 0.05 and thus the null hypothesis can be rejected following any conventional level. In other words, there is evidence that the Fed was following a Taylor rule when the first lagged inflation rate was less than the threshold, but this was not the case when the lagged in-

flation was greater than the threshold, as the coefficient of the expected inflation rate was negative, which violates the Taylor principle. This result confirms the changes made in monetary policy by the Fed, from controlling high inflation to deliberately provoking inflation.

## 3.6 Conclusions

This chapter has discussed the stability of the forward-looking interest rate reaction function - the Taylor rule for the US by applying Andrews' Wald test, and attempted to determine whether threshold models can explain the instability of the Taylor rule under each subsample, given the structural breaks indicated by the Andrews test. The empirical results suggested that, although the Fed is likely to follow a linear Taylor rule in the long run, the monetary policy was very unstable between 1971 and 2006, and especially throughout the 1980s and post-2001, irrespective of any output gap measures. There is also evidence of a linear Taylor rule being used from March 1981 to January 1988 and again from January 1993 to July 1997. By employing Caner and Hansen(2004)'s method, the nonlinear Taylor rule was estimated for different sub-periods. There is no evidence of a threshold model being used in the 1980s, which indicated that the instability cannot be explained by a threshold process, so that it might be due to the fact that the level of inflation was substantially different at that time. There is evidence supporting the existence of a threshold process during the period between October 2001 and December 2006, although the process cannot be characterized as a classic nonlinear Taylor rule.

# Chapter 4

## Exchange rates and fundamentals

## 4.1 Introduction

Meese and Rogoff (1983b) demonstrate that there are no structural models that can outperform the naive random walk in terms of out-of-sample exchange rate forecasting performance. Cheung et al. (2005) use the data from the 1990s and more up-to-date structural models to investigate whether fundamentals have predictive power on the exchange rate, and the results confirm that no model can consistently do a better job than the naive random walk. However, there is literature that suggests that the monetary model may not be as bad as it appears. MacDonald and Taylor (1994) examine the sterling/dollar exchange rate and find evidence of the unrestricted monetary model outperforming the random walk using a multivariate co-integration technique; more recent research also find evidence that the exchange rate model incorporating the Taylor rule can increase the out-of-sample predictability. (Engel and West, 2004, 2005, 2006; Engel et al., 2007; Molodtsova et al., 2008; Molodtsova and Papell, 2009; Mark, 2009). It should be noted that the choice of the forecasting comparison methods (goodness-of-fit measures) and estimators for forecasting play a significant role in this type of literature. This chapter examines whether or not the out-of-sample forecasting performance of structural models can be improved when macro fundamentals, estimating methods, or goodness-of-fit measures vary. We include the latest developed model-the Taylor rule models-in the discussion, it having been demonstrated in recent studies that this model is superior to other traditional fundamental models.

The best choice of estimators varies according to the features of the fundamentals. The mainstream literature applies to an Error Correction Model (ECM) or regression in first differences model for exchange rate forecasts (Cheung et al., 2005; Mark, 1995). If all the fundamentals are  $I(1)$ , we can test for whether or

not the fundamentals are co-integrated with the exchange rate and, if so, the Error Correction Mechanism (ECM) should be used for the estimation. If not, all the variables should take first differences for the estimation. Both the specifications above imply that the exchange rate is endogenous and that the other variables are exogenous. Nonetheless, the macroeconomic theory implies that other fundamental variables such as the interest rate cannot be considered to be exogenous variables, since their behaviour is affected by fundamentals like the inflation rate or the money supply. On the other hand, the endogeneity of the exchange rate is also questionable. The assumption that it is only the exchange rate that is endogenous implies simultaneous bias. For instance, if the interest rate in Frenkel-Bilson's flexible approach is correlated to the error term, the estimator of the model would be biased. Engel and West (2005) also prove that exchange rates can granger cause macro fundamentals, implying the endogeneity of macro variables. If we consider both the non-stationarity of the variables and the endogeneity of fundamental variables in exchange rate models, then the Vector Error Correction Model (VECM) or the Vector Autoregressive Model (VAR) in first differences should be used, both specifications challenging the assumption that only the exchange rate is endogenous, all the variables having been treated symmetrically. If the fundamental are co-integrated, VECM should be used; if not, the VAR model should be used.

The choice of different goodness-of-fit measures can lead to different conclusions. Meese and Rogoff (1983a) choose Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) for the out-of-sample forecasting comparison. Cheung et al. (2005) use the Diebold Mariano and West statistic (the DMW statistic hereafter), the 'change of direction' test, and the 'consistency' test. Molodtsova et al. (2008); Molodtsova and Papell (2009) use a modified MSPE ratio test introduced by Clark and West (2006) (the CW statistic hereafter), so called CW

test. Since there are no unanimous comparison techniques in the literature, we use a variety of tests in the following sections.

Clark and West (2006) demonstrate that the DMW statistic tends to reject the hypothesis that fundamental models can outperform the naive random walk, whereas Rogoff and Stavrakeva (2008) imply that the CW statistic is not the minimum mean squared forecast error statistic. Since there is a debate on the performance of the DMW statistic and the CW statistic for testing nested models, this chapter performs a thorough investigation on both tests based on Monte-Carlo simulations, in order to find out which test is more appropriate for measuring the out-of-sample predictive power of exchange rate models. We explain why the DMW statistic is biased based on its mathematic function, and then describe the simulation designs for both statistics. The Q-Q plots are used to identify whether or not the statistic follows a standard normal distribution. At the end of this section the size and power of the CW statistic are discussed to examine whether the CW statistic has the appropriate size and power even if the rolling window or sample size varies. Our investigations are more thorough than those of Clark and West (2006); hence, we may be able to provide more insights on the issue.

This chapter is organised as follows: section 4.2 provides a description of the theoretical models which are applied to the following empirical study; section 4.3 explains the best choice of the estimators for the exchange rate forecasting based on the properties of the macro fundamentals; section 4.4 displays the out-of-sample comparison techniques; in section 4.5 the performances of the DMW and CW statistic for nested models based on the Monte-Carlo simulations are discussed; section 4.6 presents the relevant data of the UK and the US for empirical studies; section 4.7 tests whether the level and the first difference of fundamen-

tal variables and the differential between the home and the foreign country have unit roots; sections 4.8 and 4.9 test whether there are co-integrating vectors and weak exogeneity in the traditional monetary models; in section 4.10 the out-of-sample predictability of the exchange rate based on monetary models and Taylor rule models are shown. A variety of estimators are chosen to predict the sterling/dollar nominal exchange rates, and a list of goodness-of-fit measures are applied to show which model and estimator can provide the most robust results; section 4.11 concludes.

## 4.2 The theoretical models

**Model 1** is a naive random walk model, the movements of today's exchange rate are only related to yesterday's movements of exchange rate and an error term (or noise term). Under a RW framework, none of the structural model can be useful for forecasting.

$$s_t = s_{t-1} + u_t \quad (4.1)$$

where  $s_t$  is the log nominal exchange rate, the domestic currency price of a dollar. In other words, the exchange rate is the number of domestic currency units needed to buy one unit of dollar.  $u_t$  is the error term.

**Model 2** is the Purchasing Power Parity (PPP):

$$s_t = \alpha + \beta_1(p_t^h - p_t^f) + u_t \quad (4.2)$$

where  $p$  is the log price level, and  $h$  and  $f$  represent home and foreign country, respectively. The US is the foreign country in the sample. The model indicates

that the increase of  $s_t$  is the depreciation in domestic currency. If the domestic price is higher than the foreign price, the domestic currency depreciates. The model above is in fact the relative PPP condition, which allows a constant term. The relative version is examined because the price indices rather than price levels are used.

**Model 3** is the Frenkel-Bilson's flexible-price monetary model (FB model):

$$s_t = \alpha_0 + \alpha_1(m_t^h - m_t^f) + \alpha_2(y_t^h - y_t^f) + \alpha_3(i_t^h - i_t^f) + u_t \quad (4.3)$$

where  $m$  is the money supply,  $y$  is the output,  $i$  is the nominal interest rate. The flexible-price monetary model implies that the PPP and Uncovered Interest Rate (UIP) hold.  $\alpha_1 > 0$  means that given other things being equal, the increase in domestic money supply pushes up the demand for goods, raising domestic price level which, in turn, causes the domestic currency to depreciate.  $\alpha_2 < 0$  means that other things being equal, the rise in domestic nominal income increases money demand, which lowers the price level. The fall in price will require an appreciation in domestic currency.  $\alpha_3 > 0$  means that the rise in interest rate lowers the money demand, the excess money supply leads to a rise in the price level, so that the domestic currency depreciates.

**Model 4** is the Dornbusch -Frankel's sticky-price monetary model (DF model):

$$s_t = \alpha_0 + \alpha_1(m_t^h - m_t^f) + \alpha_2(y_t^h - y_t^f) + \alpha_3(i_t^h - i_t^f) + \alpha_4(\pi_t^h - \pi_t^f) + u_t \quad (4.4)$$

where  $\pi$  is the inflation rate. This model allows for slow domestic price adjustments and consequent deviations from PPP.  $\alpha_3 < 0$  means that the changes in the nominal interest rate reflect changes in the tightness of monetary policy.

“When the domestic interest rate rises relative to the foreign rate it is because there is a contraction in the domestic money supply... the higher interest rate at home than abroad attracts a capital inflow, which causes the domestic currency to appreciate instantly” (Frankel, 1979).  $\alpha_4 > 0$  because demand for currency falls when domestic inflation is higher relative to the foreign inflation rate, which causes a depreciation in domestic currency.

It is worth noting that the inflation rate in the model can be replaced by the price level; Engel and West (2005) use the price level instead of the inflation rate for investigating the relationship between the exchange rate and fundamentals. There is not much difference in theory between the price level and the inflation rate but, as we discuss later, they have quite different statistical characteristics.

**Model 5** is the Taylor rule model, different from the above in that, the first difference of log exchange rate is on the left-hand side of the equation. The detail on how to derive the model can be found in Appendix B.

$$\Delta s_t = \alpha_0 + \alpha_1(\pi_t^h - \pi_t^f) + \alpha_2(i_{t-1}^h - i_{t-1}^f) + \alpha_3(x_t^h - x_t^f) + \alpha_4 q_t + u_t \quad (4.5)$$

where  $q$  is the log real exchange rate and  $x$  is the output gap. If we believe that the UK central bank reacts to the Forex market then the real exchange rate should be included in the specification. In this case, the exchange rate model is known as an asymmetric Taylor rule model.<sup>1</sup> If not, it is called a symmetric Taylor rule model. Molodtsova and Papell (2009) apply Model 5 to examine the out-of-sample performance of the exchange rate model incorporating Taylor rules. They find strong evidence of predictability for 11 out of 12 currencies vis a vis the US dollar. Molodtsova et al. (2008) apply real-time data to forecast

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<sup>1</sup>This is different from the asymmetric Taylor rule which is introduced in chapter 3.

the dollar/Mark exchange rate and also find the strong predictability based on Taylor rule fundamentals.

One intriguing assumption in the Taylor rule model is to link the interest rate differential with the exchange rate forecast. Molodtsova and Papell (2009) believe that assuming Uncovered Interest rate Parity does not hold, so interest rate differential should cause an appreciation in the expected exchange rate ( $\alpha_2 < 0$ ). The argument here is that under the Uncovered Interest rate Parity conditions, the rise of domestic interest rate will cause immediate appreciation followed by forecasted depreciation. In this case, the rise in domestic inflation will cause an interest rate rise, followed by a forecast of exchange rate depreciation. However, there is evidence that the UIP does not hold in the short run. (Chinn, 2006; Eichenbaum and Evans, 1995). Gourinchas and Tornell (2004) provide the rationale behind this phenomenon. Their explanations are that the investors may systematically underestimate the persistence of interest rate shocks. For instance, if the Bank of England (BOE) raises the interest rate, then it will regain its equilibrium level gradually. If the investors know the path of interest rates, they will buy sterling against the dollar to a level equal to the interest rate differential. Then the future exchange rate would depreciate accordingly. This is called the ‘forward premium’ effect. If investors misperceive that the increase is transitory and will revert to its equilibrium quickly, the exchange rate will only appreciate moderately. In the following period these investors will find out that the rise in interest rate is in fact higher than they expected, leading them to revise their opinions about the persistence of the interest rate shock, thus causing further appreciation of sterling. This is called the ‘updating’ effect. If the updating effect dominates the forward premium effect then sterling will appreciate against the dollar. Gourinchas and Tornell’s (2004) explanation would seem more reasonable if a central bank smoothed interest rate changes. Because the initial changes in

the interest rate are smaller than those that have the maximum impact, a degree of under-prediction persists. The update effect will be stronger relative to the forward premium effect. Thus, the interest rate differential will cause an appreciation in exchange rates.

There are other classes of models, one of which includes the Balassa-Samuelson effect, adding productivity differential at the end of the second model. The other one is called a ‘composite’ model:

$$s_t = \alpha_0 + \alpha_1(p_t^h - p_t^f) + \alpha_2(\omega_t^h - \omega_t^f) + \alpha_3(i_t^h - i_t^f) + \alpha_4(gdebt_t^h - gdebt_t^f) + \alpha_5(tot_t^h - tot_t^f) + \alpha_6(nfa_t^h - nfa_t^f) + u_t \quad (4.6)$$

where  $\omega$  is the relative price of non-tradables, “*gdebt*” is the government debt to GDP ratio, “*tot*” is the log terms of trade and “*nfa*” is the net foreign asset. The specification closely resembles the Behavior Equilibrium Exchange Rate (BEER) model of Clark and MacDonald (1998).

Detailed discussion on Model 3, Model 4, Model 5 and the BEER model can be found in Appendix B. Although the BEER model is also popular in this field, it is quite difficult to get relevant monthly data. Thus, we do not apply the BEER model in this chapter.

### 4.3 The out-of-sample Forecasting Methodology

The conventional methods of estimation used in the forecast process in the empirical exchange rate modelling are ‘rolling regression’ and ‘recursive regression’.

With regard to rolling regression, given that the sample size is  $T$ , using the first  $P(P < T)$  period for initial estimation, one period out-of-sample forecast is produced. We then drop the first data point, add one additional data point at the end of the sample, and re-estimate the model. The procedure is repeated until all the out-of-sample observations are exhausted. The process has the potential advantage of alleviating parameter instability effects over time. Regarding recursive regression, the process is similar, except that after selecting the initial period and adding one additional data point at each step, we do not drop any of the earlier observations. The drawback of recursive regression is that the method puts more weight on the early sample period.<sup>2</sup>

Meese and Rogoff (1983a,b) use recursive regression<sup>3</sup> and choose the instrument variables (IV) technique as the benchmark estimator for estimating the structural model in order to deal with the endogeneity of the explanatory variables.

Four specifications are normally used for the estimation of exchange rate: an Error Correction Model (ECM), a regression in first differences specification, a VAR in first differences and a Vector Error Correction Model (VECM). All four specifications are based on the fact that most of the macro variables are non-stationary, so that the level regression is not appropriate for the estimation.

### **Regression in first differences model**

If all the variables are non-stationary, and there is no co-integration in the model, assuming only one variable ( $s_t$ ) is endogenous, regression in first differ-

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<sup>2</sup>For a detailed discussion on rolling regression, see section 4.5.

<sup>3</sup>In Meese and Rogoff (1983b), they claim that rolling regression is used, but in fact their rolling regression is viewed as recursive regression.

ences should be used to estimate the model. This refers to a regression model taking the first difference of variables.

$$\Delta s_t = \Delta Z_t \beta_t + u_t \quad (4.7)$$

where  $\Delta Z_t$  is an  $1 \times n$  vector of the first difference of fundamental variables, and  $\beta_t$  is an  $n \times 1$  vector of sloping parameters. In this type of specifications,  $Z_t$  is a vector of exogenous variables, and  $s_t$  is endogenous. In other words,  $Z_t$  has an impact on  $s_t$ , whereas  $s_t$  cannot influence  $Z_t$ .

### **Error Correction model (ECM)**

If all the variables are non-stationary, and there is co-integration in the model, assuming only one variable ( $s_t$ ) is endogenous, then the error correction model (ECM) should be used instead of the regression in first differences. ECM involves a two-step procedure. The first step is to identify the long-run co-integrating relationship using the Johansen procedure. In the second step, the estimated co-integrating vector ( $s_{t-k} - Z_{t-k} \hat{\Gamma}$ ) is incorporated into the error correction term, and the resulting equation

$$s_t - s_{t-k} = \delta_0 + \delta_1 (s_{t-k} - Z_{t-k} \hat{\Gamma}) + u_t \quad (4.8)$$

is estimated via OLS.  $k$  denotes the forecast step,  $Z_{t-k}$  is an  $1 \times n$  vector of the fundamental variables,  $\hat{\Gamma}$  is an  $n \times 1$  vector of sloping parameters. The term in brackets,  $s_{t-k} - Z_{t-k} \hat{\Gamma}$ , represents the co-integrating vectors in the model.

The differences between regression in first differences and ECM are that in the error correction specification, contemporaneous values of the right-hand side variables are not necessary in ECM, so it is true *ex ante* forecasts. On the other

hand, regression in the first differences specification has an informational advantage in forecasting.

Molodtsova and Papell (2009) use rolling regression and error correction specification introduced by Mark (1995), and do not find evidence that the traditional monetary models estimated by ECM have any significant predictive power on exchange rates. Cheung et al. (2005) use a different type of ECM for the forecast. The difference between Cheung et al and Mark's method is that the co-integrating vector is imposed *a priori* in Mark's case and he considers only money supply and price level to be the explanatory variables in his research, whereas Cheung et al. estimate the co-integrating vector before each forecasting step, which could improve the accuracy of the forecasts. Although Cheung's method is more accurate, it is also difficult to apply in practice. It involves complex programming for multi-step-ahead forecasts. Since the forecasting results based on the results which are provided by Cheung et al. are not robust, we do not use this method in this chapter.

### **Vector auto-regressive (VAR) model in first differences**

If all the variables are non-stationary and there is no co-integration in the model, assuming more than one variable is endogenous, then the Vector autoregressive model (VAR) in first differences should be used in the estimation. One of the advantages of VAR is that it allows more than one variable to be endogenous, so that it treats all variables symmetrically. That said, VAR model results are difficult to explain in economic terms.

The general VAR( $k$ ) framework is:

$$X_t = \sum_{i=1}^k X_{t-i} \beta_i + u_t \quad (4.9)$$

$X_t$  include the nominal exchange rate and the fundamentals. Since most of the fundamental variables are non-stationary, if there is no co-integration in the equations, one can take VAR in first differences form, so all the variables in the VAR are taken the first differences.

$$\Delta X_t = \sum_{i=1}^k \Delta X_{t-i} \beta_i + u_t \quad (4.10)$$

### Vector Error Correction Model (VECM)

In the presence of a unit root, using unrestricted VAR in levels or in first differences, as the estimator can be problematic (Phillips, 1991; Phillips and Loretan, 1991), Granger (1986) proposes the following vector error correction representation to solve the underlying problems above:

$$\Delta X_t = c + \sum_{i=1}^{k-1} \Pi_i \Delta X_{t-i} + \Pi X_{t-1} + \epsilon \quad (4.11)$$

where

$$\Pi = -(I - \sum_{i=1}^k A_i),$$

$$\Pi_i = -(I - A_1 - \dots - A_i), i = 1, \dots, k-1,$$

$X$  is an  $n \times 1$  vector,  $c$  is the deterministic drift term in  $\Delta X_t$ ,  $X_t$  and  $\epsilon$  are  $n \times 1$  vectors, and  $A_j$  is an  $n \times n$  matrix of parameters.

There are two main methods in terms of estimating the equation above. One is by ‘maximum likelihood’ procedures introduced by Johansen (1988) and the other is a generalized version of Engle and Granger (1987). Since Johansen’s method dominates the research, we focus on his method in the following discus-

sion.

When  $\Pi$  is full rank,  $n$ , all the elements in the model are stationary. It is appropriate to estimate the model in levels. When  $\Pi$  is zero rank,  $\Pi = 0$ , there is no co-integration in the model, which should be estimated in first differences. If, however,  $\Pi$  is of reduced rank,  $\Pi = r < n$ , then there will exist  $(r \times n)$  matrices  $\alpha$  and  $\beta$  such that  $\Pi = \alpha\beta'$  where  $\beta$  is the matrix of co-integrating parameters and the  $\alpha$  is viewed as the matrix of speed-of-adjustment parameters, indicating the speed with which the system responds to the last periods' deviation from equilibrium. In brief, the difference between VECM and VAR in first differences model depends on the existence of the co-integration.

We use the two tests for the co-integration which are proposed by Johansen. The first statistic test is the likelihood ratio (or trace) statistic test which investigates the null hypothesis that the number of distinct co-integrating vectors is less than or equal to  $r$  against a general alternative. The second test, the maximum eigenvalue statistic tests the null hypothesis that the number of co-integrating vectors is  $r$  against the alternative of  $r + 1$  co-integrating vectors. The critical values for both tests are generated by Osterwald-lenun (1992). Both are displayed below<sup>4</sup>:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i)$$

$$\lambda_{max}(r, r + 1) = -T \ln(1 - \hat{\lambda}_{i+1})$$

where  $\lambda_i$  is the estimated values of the characteristic roots (also called eigenvalues) obtained from the estimated  $\Pi$  matrix.  $T$  is the number of useable obser-

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<sup>4</sup>For detail, see Enders (2010): "Applied Econometrics Time Series, 3rd Edn.". pp 385-395.

uations.

If a co-integrating vector exists, then  $\Pi = \alpha\beta'$ . If any of the adjustment coefficients  $\alpha$  are significant, these variables are endogenous, whereas failing to find a significant  $\alpha$  indicates that the variables are weakly exogenous.

Weak exogeneity is an important concept in VECM. The differences between weak and strong exogeneity can be found in Johansen (1992a,b). Let us assume the vector  $\Delta X_t$  can be divided into two parts  $[\Delta X_{1t}, \Delta X_{2t}]$ . Johansen proves that for  $\Delta X_{2t}$  to be weakly exogenous,  $\Delta X_{2t}$  does not respond to any disequilibrium error term  $\beta X_{t-1}$ , and only responds to the lagged changes in vector  $\Delta X_t$ . Furthermore, a strong form of exogeneity of  $\Delta X_{2t}$  requires that it only respond to its own lagged changes, In other words,  $\Delta X_{1t}$  does not cause  $\Delta X_{2t}$ .

The knowledge that  $\Delta X_{2t}$  is weakly exogenous is important for determining the parsimonious estimator for the system. If some of the variables are exogenous, a partial system may be as efficient as a full system. For instance, let us assume that the exchange rate follows Purchasing Power Parity (PPP) and the exchange rate is endogenous, and the log of price levels for both the home and the foreign country are weakly exogenous, then one can use ECM instead of VECM as the estimator. However, if any of the price levels are endogenous, a VECM model should be used for the estimation. Another issue concerning weak exogeneity in terms of forecasts is that if a variable is weakly exogenous this means that it is unpredictable using the long run condition and, thus, VAR in first differences should be used as the estimator instead of VECM; a strong form of exogeneity means that a variable is unpredictable using other variables, and that the variable is only related to its own lags, that is, a autoregressive model (ARIMA) should be applied to the forecast.

The VECM can be considered to be a superior estimator compared to the other estimators in terms of the exchange rate forecasts. Similar to a VAR, a VECM treats all variables symmetrically. In addition, since most of the fundamental variables are non-stationary, it is plausible that the exchange rate and fundamentals have long run relationships. Taking both endogeneity and nonstationarity into account, the VECM framework is more appealing than a regression estimator or a VAR. Under the VECM, the co-integration between fundamental variables is considered, while also treating each fundamental symmetrically. An attractive feature of this method is that it also facilitates computing the short-run dynamic behaviour of the chosen exchange rates. In theory we can also consider the short-run dynamic behaviour in the ECM framework. It is difficult, however, to estimate the co-integrating vectors and short-run dynamics over time for forecasting purposes due to the limitations of econometrics techniques and related software packages. This might be partly the reason why only long run relationships between the exchange rate and fundamentals are discussed in the work of Cheung et al. (2005) and Molodtsova and Papell (2009). Since it is quite difficult to use the full function of the ECM specification, in this chapter we do not use ECM for forecasts but VECM is applied.

In the following part, the rank of the fundamental models are discussed first, and then display the forecast results using different models and techniques.

For the purpose of testing and forecasting the corresponding  $X_t$  for each model is displayed as follows:

$$\text{Model 2: } X_t = [p_t^h, p_t^f, s_t]'$$

$$\text{Model 3: } X_t = [m_t^h, m_t^f, i_t^h, i_t^f, y_t^h, y_t^f, s_t]'$$

$$\text{Model 4: } X_t = [m_t^h, m_t^f, i_t^h, i_t^f, y_t^h, y_t^f, \pi_t^h, \pi_t^f, s_t]'$$

The Taylor rule is not discussed under the VECM framework because we are unable to derive the VECM specifications using a Taylor rule model. The reason is that on the left-hand side of the equation the exchange rate has already taken in first difference. Thus, we cannot put in a lagged exchange rate to form a co-integrating vector on the right-hand side of the equation. In this case, we forecast the first difference of a sterling/dollar exchange rate using traditional models and the Taylor rule model in order to make our results comparable.

## 4.4 Forecast evaluation methods

At least six different methods are used to investigate the forecast accuracy. The first one is the Root Mean Square Error (RMSE), which compares the differences between the forecasts and the corresponding observed values, which are squared and then averaged over the sample. Then the square root of the average is taken. Since the errors are squared before they are averaged, the RMSE gives a relatively high weight to large errors. This means that the RMSE is most useful when large errors are particularly undesirable. It is a widely used measure of the differences between values predicted by a model and values produced by the actual series. The drawback of this method is that it would be an inappropriate criterion if exchange rates are governed by a non-normal stable process with an infinite variance.<sup>5</sup>

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<sup>5</sup>Non-normal stable distribution is also called ‘stable paretian’ distribution or ‘fat-tailed’ distribution. Exchange rates having ‘fat tails’ indicate the market is likely to move more extreme than would be predicted by the normal distribution.

The second method is the Mean Absolute Error (MAE). The MAE measures the average magnitude of the errors in a set of forecasts, without considering their direction. The MAE is the average over the verification sample of the absolute values of the differences between the forecasts and the corresponding observations. The MAE is a linear score which means that all the individual differences are weighted equally in the average. MAE is particularly useful for the case of exchange rates since they have fat tails.

Let  $k = 1, \dots, n$  denotes the forecast step, and  $N_k$  is the total number of forecasts in the projection period. The observed value  $A(t)$  is known, and  $F(t)$  is the forecast value:

$$RMSE = \left\{ \sum_{s=0}^{N_k-1} \frac{[F(t+s+k) - A(t+s+k)]^2}{N_k} \right\}^{\frac{1}{2}}$$

,

$$MAE = \sum_{s=0}^{N_k-1} \frac{|F(t+s+k) - A(t+s+k)|}{N_k}$$

### **The DMW statistic**

The third method is called the DMW statistic, proposed by Diebold and Mariano (1995) and West (1996). Cheung et al. (2005) use this method for forecasting comparison.

Specifically, given an actual series and two competing predictions, one may apply a loss differential, which is defined as the difference between the squared forecast error of the structural models and that of the random walk, and then calculate its standard error, which is constructed from a weighted sum of the sample autocovariances of the loss differential vector. The DMW statistic is the

ratio between the loss differential and its standard error.

### **The CW statistic**

Although the DMW statistic is valid for non-nested models, it has been demonstrated to be biased for two-nested models because it does not have a standard normal distribution when applied to forecasts from nested models.<sup>6</sup> Clark and West (2006) demonstrate analytically that the sample difference between the two MSPE's is biased downward from zero. Clark and West (2006) propose a correction, which results in the statistic being asymptotically normally distributed for rolling regression. The test is known as the CW statistic.

The detailed discussion on the performance of the DMW and CW tests can be found in the following section.

### **The Correlation**

The correlation between forecasts and corresponding observations is a simple way to detect the forecast performance. A positive correlation indicates that the forecasts and observed values move in the same direction, close to zero or negative correlation indicates bad forecasts. The test is used by Engel and West (2006).

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<sup>6</sup>Two models are nested if both of them contain the same terms and one has at least one additional term.

For example:

$$y = \alpha + \alpha_1 x_1 + \alpha_2 x_2 + \epsilon \quad (1)$$

$$y = \alpha + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \epsilon \quad (2)$$

Model (1) is nested within model (2). Model (1) is a reduced model and model (2) is the full model.

### The ‘direction of change’ test

The idea of the ‘direction of change’ test is straightforward. If the forecasts are insightful, the forecasts should move in the same direction as the observed values. In terms of the exchange rate, if we define  $\Delta e_{t+k}$  as the observed values, and  $\Delta \hat{e}_{t+k}$  as the  $k$  step-ahead forecasts, these two series evolve as follows:

$$\Delta e_t = \begin{cases} 1 & \text{if } s_{t+k} > s_t \\ -1 & \text{otherwise} \end{cases}$$

$$\Delta \hat{e}_t = \begin{cases} 1 & \text{if } E_t s_{t+k} > s_t \\ -1 & \text{otherwise} \end{cases}$$

Once the two series are calculated we can measure the performance of the forecast model in terms of correct direction changes (CDC):

$$CDC = \frac{1}{T} \sum_{t=1}^T \Phi\{\Delta e_t = \Delta \hat{e}_t\}$$

where  $\Phi$  takes the value 1 when the argument is true, that is,  $\Delta e_t = \Delta \hat{e}_t$ , and zero otherwise. The CDC above 0.5 or 50% indicates that the forecast has a better performance than a random walk randomly tossing a coin because the number of times that the forecasts agree with the observed values are more than one would expect by chance.

In this chapter all these methods are used for the out-of-sample forecast comparisons in order to find out whether or not using different goodness-of-fit measures lead to different conclusions on our exchange rate forecasting performance

based on monetary models.

## 4.5 The CW statistic VS the DMW statistic

In order to evaluate whether or not the series follows a martingale difference against the alternative that it is linearly predictable, Diebold and Mariano (1995) and West (1996) (DMW) suggest using out-of-sample mean squared prediction errors (MSPEs). The null hypothesis is that the series follows a martingale difference, and its MSPE should equal the MSPE calculated by its linear prediction; the alternative is that the series can be better estimated by a linear function.

Clark and West (2006) suggest that the DMW test is not normally distributed under the null; the Mean Square Prediction Error (MSPE) calculated by the martingale difference series is usually smaller than the MSPE arrived at by using the linear function if  $H_0$  is true. Thus, the DMW test tends to accept the null hypothesis.

Clark and West improve the DMW test and demonstrate that the CW test can do a better job. In the following process we try to reproduce the CW statistic and discuss whether or not the test is suitable for exchange rate forecasting.

The specification for both the DMW and the CW statistic are as follows:

$$\text{The null model : } y_t = \varepsilon_t \quad (4.12)$$

$$\text{The alternative model : } y_t = X_t' \beta + \varepsilon_t \quad (4.13)$$

$y_t$  and  $X_t'$  are assumed to be stationary processes ( $I(0)$ ). Under the null hy-

pothesis,  $\beta = 0$ ; under the alternative,  $\beta \neq 0$ . We are interested in comparing the mean square prediction errors (MSPE) from the two models. Under the null, one would assume that the MPSEs of both models are equal to each other so that the difference in MPSEs between the two is close to zero. Under the alternative, the linear function should have a smaller error term compared with that of the martingale difference. The reason is that the null model is nested in the alternative model, if the restriction ( $\beta = 0$ ) is not true, we would expect the alternative model to predict better than the null model. Thus, both DMW and CW tests are one-sided.

For simplicity, only the one-step-ahead forecast is considered for comparing the performance of the CW and DMW tests. The rolling regression is used to estimate  $\beta$  in each period. The full process of rolling regression is shown in Fig 4.1. One assumes the sample size is  $T + 1$ . The first  $R$  observations are used to estimate the model initially, and the reminding  $P$  observations are used for predication. Thus, one has  $T + 1 = R + P$ . The first step is to estimate the model using the first  $R$  observations, and then, using the estimated coefficients  $\hat{\beta}_R$  and  $X_{R+1}$ , one can forecast  $\hat{y}_{R+1}$ . (see step 1 in Fig 4.1)

After that, before the procedure is repeated, the sample is moved down or ‘rolled’ forward one observation, and the initial observation is removed from the estimation to keep the size fixed at  $R$  (step two in Fig 4.1). This process continues until all the out-of-sample observations are exhausted. Since the size of the observations is fixed and moving forward through the process, the fixed size  $R$  is called a moving window or rolling window.

The loss function or the sample forecast errors from the null and the alternative model are  $\hat{e}_{1,t+1} = y_{t+1}$  and  $\hat{e}_{2,t+1} = y_{t+1} - X'_{t+1}\hat{\beta}_t$ , respectively. The MSPEs

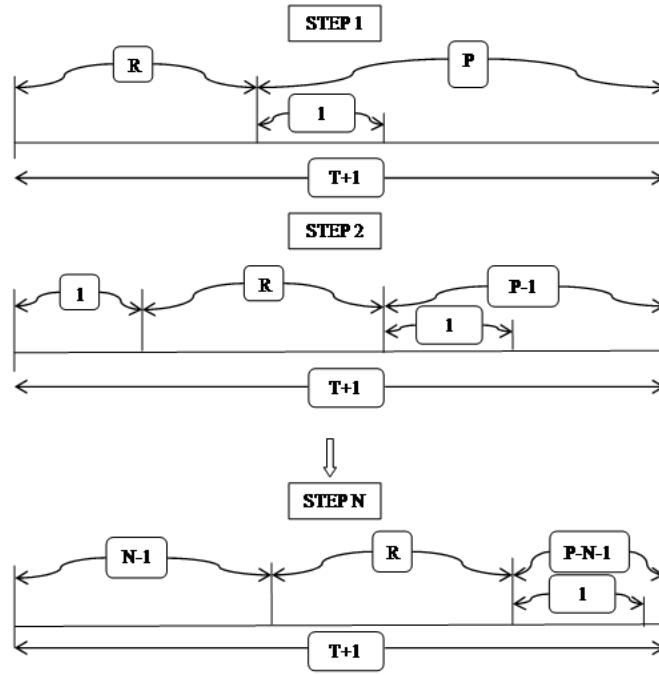


Figure 4.1: The procedure of the rolling regression

for both models are:

$$\hat{\sigma}_1^2 = \frac{\sum_{t=T-P+1}^T y_{t+1}^2}{P}$$

$$\hat{\sigma}_2^2 = \frac{\sum_{t=T-P+1}^T (y_{t+1} - X'_{t+1} \hat{\beta}_t)^2}{P}$$

### The DMW statistic

The null hypothesis of the DMW is equal in accuracy between the null and the alternative model, and the MSPE of both models should have equal predicative power,  $\hat{\sigma}_1^2 - \hat{\sigma}_2^2 \approx 0$ . The alternative hypothesis is that the linear fundamental model has a smaller MSPE. Thus, the DMW test is a one-tailed test. Using the following notations,

$$\hat{f}_{t+1} = \hat{e}_{1,t+1}^2 - \hat{e}_{2,t+1}^2 = y_{t+1}^2 - (y_{t+1} - X'_{t+1} \hat{\beta}_t)^2$$

$$\bar{f} = \frac{\sum_{t=T-P+1}^T \hat{f}_{t+1}}{P} = \hat{\sigma}_1^2 - \hat{\sigma}_2^2$$

$$\hat{V} = \frac{\sum_{t=T-P+1}^T (\hat{f}_{t+1} - \bar{f})^2}{P}$$

the DMW test can be computed in the following way:

$$DMW = \frac{\bar{f}}{\sqrt{P^{-1}\hat{V}}} \quad (4.14)$$

Although the DMW statistic is valid for non-nested models, it has been demonstrated to be biased for nested models because it does not have a standard normal distribution when applied to forecasts from nested models. Clark and West (2006) demonstrate analytically that the sample difference between the two MSPEs is biased downwards from zero:

$$\begin{aligned} \hat{\sigma}_1^2 - \hat{\sigma}_2^2 &= \frac{\sum_{t=T-P+1}^T y_{t+1}^2}{P} - \frac{\sum_{t=T-P+1}^T (y_{t+1} - X'_{t+1}\hat{\beta}_t)^2}{P} \\ &= \frac{2 \sum_{t=T-P+1}^T y_{t+1} X'_{t+1} \hat{\beta}_t}{P} - \frac{\sum_{t=T-P+1}^T (X'_{t+1} \hat{\beta}_t)^2}{P} \end{aligned} \quad (4.15)$$

Under the null, the first term is zero, whereas the second one is negative by construction. The reason for this is that  $y_{t+1} = \hat{e}_{1,t+1}$ ,  $E_t \hat{e}_{1,t+1} = 0$ . Therefore,  $E_t y_{t+1} X'_{t+1} \hat{\beta}_t = 0$ . The second term by construction is negative because when one simulates the statistic under the null, a non-zero slope estimate  $\hat{\beta}_t$  can always be generated by the regression, which is in fact not equal to zero (see Clark and West, 2006, p.160).

Based on the simulation, the density distribution of  $\hat{\sigma}_1^2 - \hat{\sigma}_2^2$  is displayed in Fig 4.2. It is clear that the distribution is negative skewed and most of the statistics are negative, which indicates that the statistic of the DMW is likely to be biased.

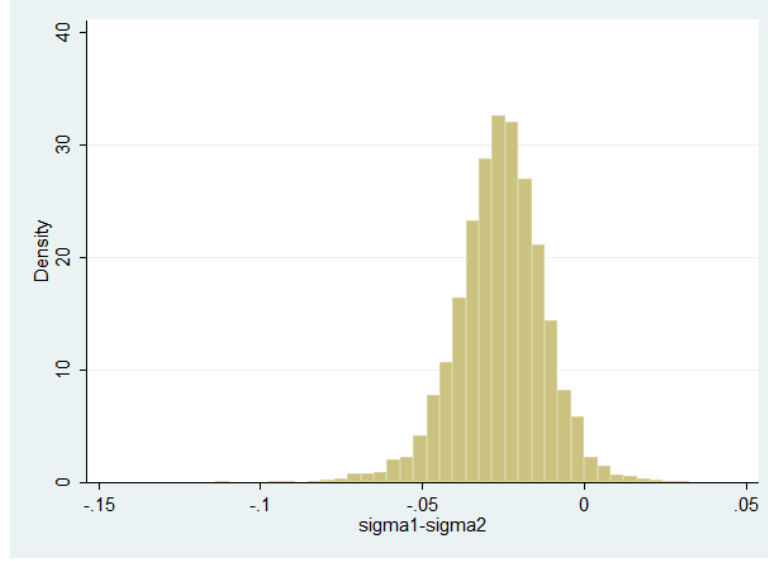


Figure 4.2: The density distribution

### The CW test

Clark and West (2006) propose a correction of the DMW statistic, which results in the following statistic to be asymptotically normally distributed for regression. Molodtsova and Papell (2009) and Engel et al. (2007) strongly recommend the CW statistic for the evaluation of exchange rate models and, by using this criterion, demonstrate that the Taylor rule fundamentals outperform a naive random walk.

$$\hat{\sigma}_1^2 - \left( \hat{\sigma}_2^2 - \frac{\sum_{t=T-P+1}^T (X'_{t+1} \hat{\beta}_t)^2}{P} \right) = \hat{\sigma}_1^2 - (\hat{\sigma}_2^2 - adj) = \bar{f}$$

To construct the point estimate of the statistic, we can define:

$$\hat{f}_{t+1} = y_{t+1}^2 - [(y_{t+1} - X'_{t+1} \hat{\beta}_t)^2 - (X'_{t+1} \hat{\beta}_t)^2]$$

$$\bar{f} = \frac{\sum_{t=T-P+1}^T \hat{f}_{t+1}}{P}$$

$$\hat{V} = \frac{\sum_{t=T-P+1}^T (\hat{f}_{t+1} - \bar{f})}{P}$$

The CW statistic is computed as:

$$CW = \frac{[\hat{\sigma}_1^2 - (\hat{\sigma}_2^2 - adj)]}{\sqrt{P^{-1}\hat{V}}} \quad (4.16)$$

#### 4.5.1 Simulation design

The baseline data-generating processes (DGPs) are similar to Clark and West (2006) as follows:

$$y_t = \beta x_t + e_t$$

$$x_t = 0.95x_{t-1} + \nu_t$$

where  $E_t e_t = 0$ ,  $E_t \nu_t = 0$ ,  $var(e_t) = 0.1$  and  $var(\nu_t) = \sigma_v^2 = 0.1^7$ .

The null hypothesis is set to  $\beta = 0$ , and the series becomes a martingale difference process. The alternative is to run a regression of  $y_t$  on  $x_t$ , giving a rolling window of  $R$  observations. For simplicity,  $\sigma_v^2$  is set to 0.1, and there is no correlation between  $e_t$  and  $\nu_t$ . 400 observations are generated to be the baseline case; the first 50 estimations are removed from the analysis to avoid the initial bias caused by the selection of first random number.

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<sup>7</sup>In CW06, the DGPs are  $y_t = \beta x_{t-1} + e_t$ , which might be more accurate considering we are interested in out-of-sample forecast. In addition,  $var(e_t) = 1$  in CW06, the difference is that the smaller the variance is in the series, the more concentration of the density of distribution and the more likely the power function will reach one as the slope coefficient increases, as we explain in the following section.

### 4.5.2 Simulation results

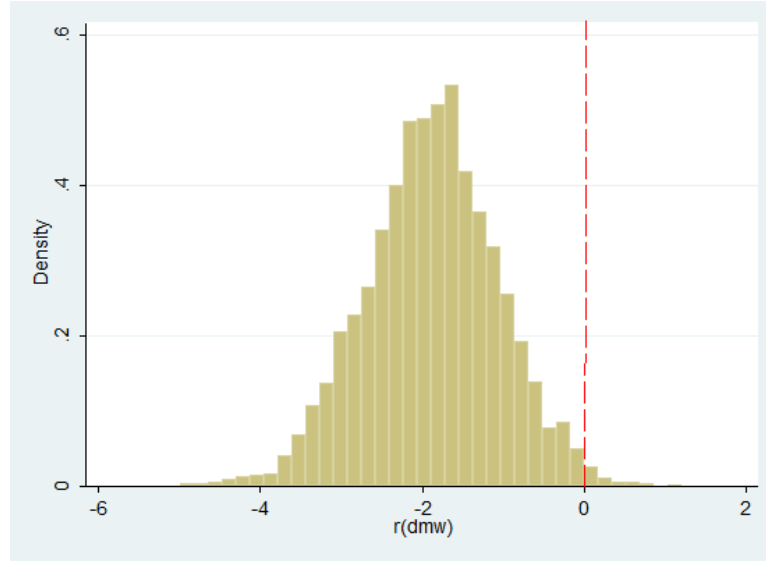


Figure 4.3: The density distribution of the DMW statistic

Based on the formula of the DMW test, we simulated the DMW statistic 5000 times in order to obtain the density graph of distribution (Fig 4.3). Although the shape of distribution is close to a normal distribution, the majority of the statistics are negative, which demonstrates that the DMW test tends to accept the null hypothesis of the series following a martingale difference.

After 5000 simulations, the summary of the statistic are shown in the table below. As we can see here, after the adjustment  $\hat{\sigma}_1^2 - (\hat{\sigma}_2^2 - adj)$ , the mean of the simulation is -0.1216, the standard deviation is 0.9705, the skewness is 0.0689 and the kurtosis is 2.8706. The four moments suggest that the distribution is close to a standard normal distribution, which means the CW adjustment can improve the test.

sum cw, detail

r(cw)

-----

	Percentiles	Smallest		
1%	-2.265666	-3.700092		
5%	-1.697813	-3.215853		
10%	-1.381155	-3.16073	Obs	5000
25%	-.7810394	-3.055263	Sum of Wgt.	5000
50%	-.1248715		Mean	-.121585
		Largest	Std. Dev.	.9705302
75%	.5272542	2.885763		
90%	1.13886	2.954384	Variance	.9419288
95%	1.536779	3.026332	Skewness	.0688597
99%	2.107144	3.139072	Kurtosis	2.87064

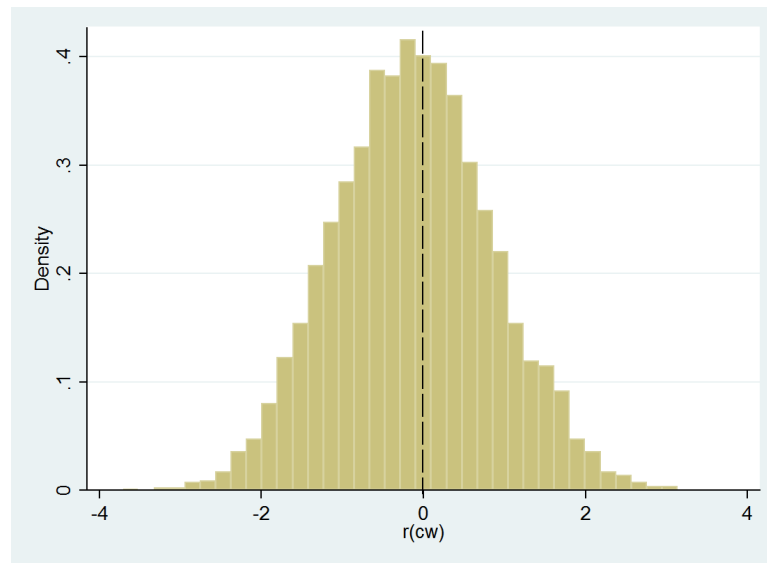


Figure 4.4: The density distribution of the CW statistic

Fig 4.4 show the density distribution graph of the CW statistic. The reference line  $x = 0$  indicates that the distribution has shifted slightly to the left, which is very close to the results obtained by CW06. The explanation from CW06 is that this small sample phenomenon cannot be captured by the asymptotic theory. This phenomenon can also explain why the statistic is undersized; the size of the CW statistic is 0.0386 at 5% significance level ( $p$ -value), which is smaller than the standard figure of 0.05.<sup>8</sup> As we demonstrate later, the change in the size of the sample and the rolling window will not systemically improve the statistical

<sup>8</sup>The  $p$ -value reports the rejection of the null using a one-sided test at 5% level. Our simulation results indicate that in 193 of the 5000 simulations the CW statistic was greater than 1.64.

performance, and there is no clear pattern either.

### Quantile-quantile plots (Q-Q plots)

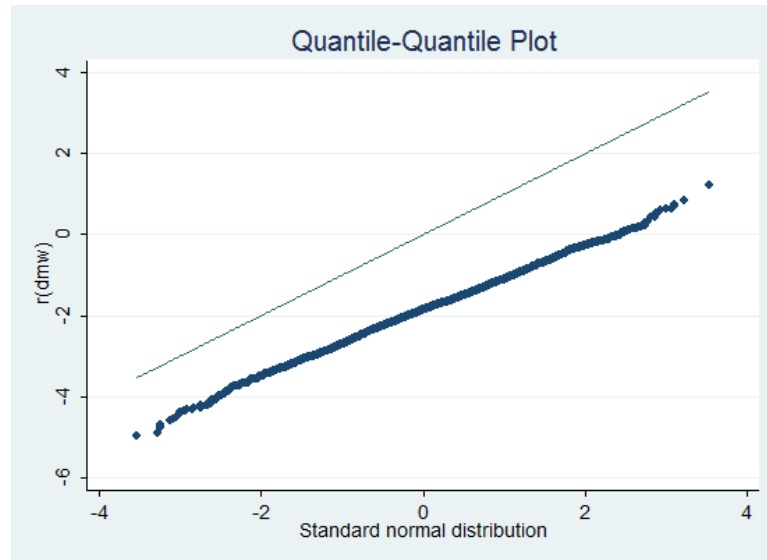


Figure 4.5: The Q-Q plots of the DMW statistic

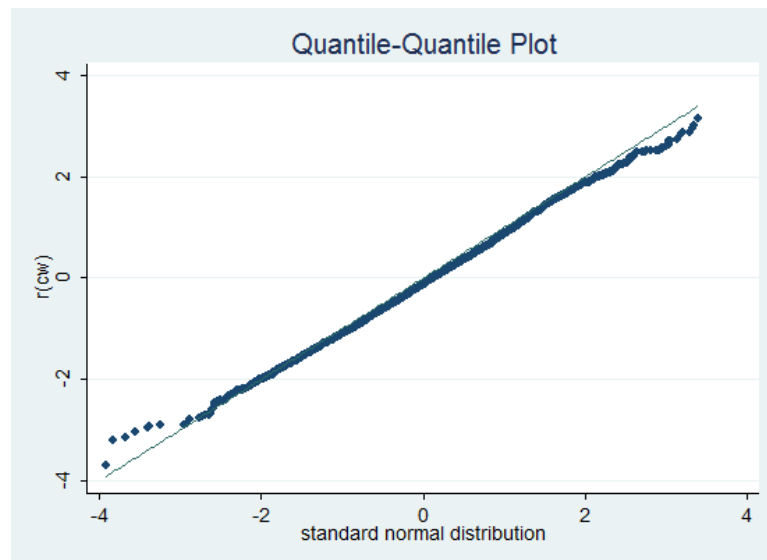


Figure 4.6: The Q-Q plots of the CW statistic

One can also use Q-Q plots (quantile-quantile plots) to investigate the density distribution of the statistics. Q-Q plots describe the quantiles of the statistics

against the quantiles of distribution we choose - in our case, a standard normal distribution. Fig 4.5 shows that the DMW statistic does not follow a standard normal distribution since all plots are parallel to the reference line representing the plots of standard normal distribution, whereas the CW statistic is very close to be standard normally distributed, since all the data are plotted along the reference line (Fig 4.6).

### 4.5.3 The size and the power of the CW statistic

In this section there are several questions that need to be answered: firstly, whether or not the increase of the sample size improves the CW test performance, and secondly, whether or not the choice of rolling windows has a significant effect on the CW statistic. In order to answer these questions we discuss the size and the power of the CW statistic.

In order to test whether the CW statistic has the correct size and power, two experiments are designed: in the first experiment we fix the rolling window at 50 observations and vary the sample size from 400, to 700, to 1000. In the second experiment, the sample size is fixed at 400, whereas the rolling window  $R = 50, 100, 120, 150$ . In order to examine the size of the statistic, the slope coefficient for both experiments is set to zero; to test the power of the statistic, the slope coefficient  $\beta$  is set to equal 0, 0.01, 0.03, ...0.35. Since it is a one-sided test, the normal size is 0.05, and the null is rejected if the CW statistic is greater than 1.64.

#### Size

Table 4.1 shows that the actual size of the CW statistic based on 5000 simu-

	400 obs	700 obs	1000 obs
Actual size	0.0386	0.0438	0.0378

Table 4.1: The actual size of the CW statistics, given that sample sizes vary.

lations. The rolling window is fixed at 50 observations. It is clear that the actual size of all three statistics are smaller than the 0.05 which is the correct size under the standard normal distribution. In addition, the size of the statistic does not increase or decrease as the sample size increases. Thus, the CW statistic is always undersized based on our simulations, and the increase in the sample size would not improve the size of the CW statistic.

	R=50	R=100	R=120	R=150
Actual size	0.0386	0.0362	0.0304	0.0176

Table 4.2: The actual size of the CW statistics, given that the rolling windows vary.

Table 4.2 reports the actual size of the CW statistic based on the simulations, allowing the rolling window to be different and the sample size to be fixed at 400 observations. The results show that the increase in the rolling window of a given sample size would lower the size of the statistic. It is worth noting that this exercise is different from those of CW06. In their experiment the sample size also increases as the rolling window increases (see Clark and West, 2006, p.173), and the prediction period  $P$  is 144 when  $R$  increases from 60 to 240, whereas in our experiment, the prediction period  $P$  will decrease as the rolling window moves forward.

## Power

The power function is used to check what would happen if the true statistic had a different value from the null hypothesis; in other words, the power function is

dealing with the situation when the alternative hypothesis is true.

The power function graph is displayed in Fig 4.7:

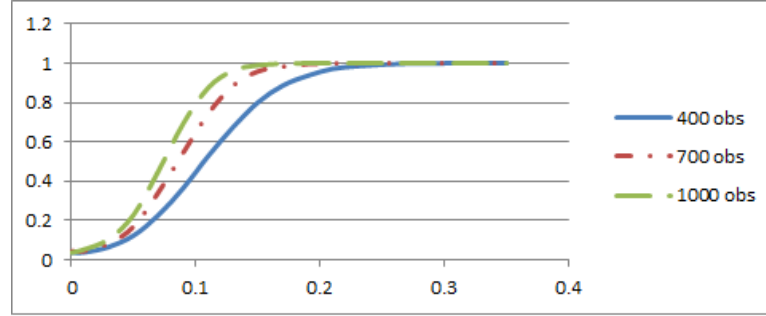


Figure 4.7: The Power function of the CW statistic,different sample size,  $R=50$

Fig 4.7 shows that the power function of the CW statistic under different sample sizes, given a rolling window of 50 observations. The probability of rejecting the null is increasing as the slope parameter rises. When the parameter is up to 0.3, the probability of rejecting null for all three functions is close to 1. The power function suggests that the CW test can work quite well.

Fig 4.7 also shows that the power rises with sample size as expected. Thus, the increase in the sample size can improve the power of the CW statistic.

Fig 4.8 reports the power functions of the CW statistic as the rolling window changes, the sample size being 400 observations. As the number of observations in the rolling window increases, it is not obvious that the power of the test increases accordingly. Between  $R = 100$  and  $R = 150$ , the results are not quantitatively different. Thus, the increase in  $R$  is unable to increase the power of the CW statistic.

It is worth noting the difference between our methods and those of CW06. In their work the power of the test is examined based on the assumption that

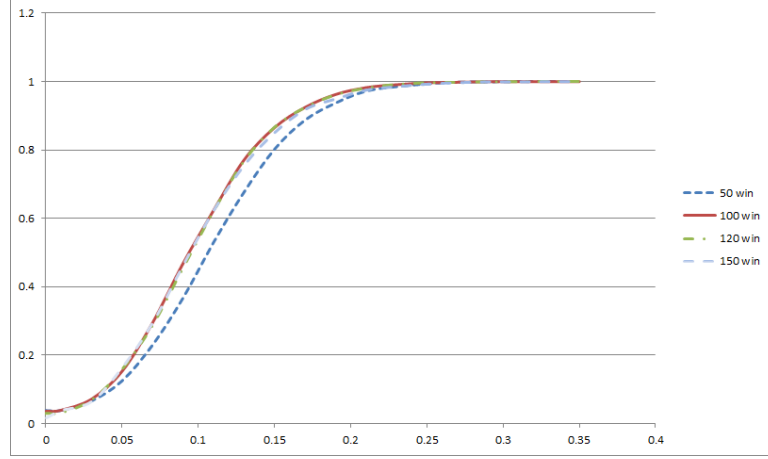


Figure 4.8: The Power function of the CW statistic,different window size, 400 observations

the slope coefficient is  $\beta=-2$  or  $0.365$ . and thus the actual size of the statistic is around  $0.04-0.06$ . In our case  $\beta$  is set to equal  $0, 0.01, 0.03, \dots 0.35$ , which is why we can draw a power function into the graph and our method is, to some extent, more thorough. Furthermore, our power function curve bends towards one much faster as the coefficient increases, considering that the actual size is close to one as the slope coefficient reaches  $0.3$ . Part of the reason is that we set a smaller variance for the error term of the DGPs. If we increase the variance, the curve would be more dispersed.

#### 4.5.4 Brief conclusions

The section studied the performance of the CW statistic based on the Monte-Carlo simulations. Our method was similar to those of Clark and West (2006). The section focused on the properties of the distribution of the CW statistic. By using graphic methods and Q-Q plots we demonstrated that the distribution of the CW statistic closely follows a standard normal distribution. In addition, our simulations imply that the statistic is undersized, regardless of the size of the observations and the choice of rolling windows. we also demonstrated that the

power of the CW test is very strong by using a power function.

## 4.6 Data description

We use monthly data from January 1975 to August 2010. The International Financial Statistics (IFS) is the main source for the UK and US data on the consumer price index, UK overnight interbank lending rate, Fed fund rate and Industrial Production Index (IPI). The monetary base is defined as  $m0$  for the UK and  $m1$  for the US for the purpose of comparison, we also collect  $m4$  for the UK and  $m2$  for the US as they are defined as the ‘broad money’, which includes monetary base and private-sector deposits, the data for  $m2_t^{US}$  and  $m4_t^{UK}$  start from July 1982. For interest rates, three-month Treasury bill rates are also collected, since for Models 3 and 4, short term bill rates are frequently used for forecasting purposes. The nominal exchange rate is the British sterling price per unit of US dollar; an increase indicates a depreciation in sterling. Both seasonally unadjusted and adjusted IPI data are collected as the proxy of output. All data except the interest rate are taking logarithms. Interest rates are divided by 100. The inflation rate is defined as the annual change in the log of the price level  $\ln P_t - \ln P_{t-12}$ . The output gap is calculated by the Quadratic Time Trend (QTT) method, B-Spline and HP-filter (the smooth parameter is 129,600).

## 4.7 Unit root testing

Before deciding which specifications should be implemented in the forecast, the first step is to investigate the properties of each of the macro fundamental variables in order to find out if all of them are  $I(1)$ . Only then it is reasonable to

verify further that there are co-integrating relationships in each model and which specifications should be included in the estimation. The test we applied here was the Dickey Fuller (DF) test and the Phillips-Perron test. Macro variables are discussed in both level and first differences in this section. Data from January 1975 to August 2010 are used,  $m2_t^{US}$  and  $m4_t^{UK}$  starting from July 1982 to October 2010.

### **Dickey-Fuller test**

Dickey-Fuller (or augmented Dickey-Fuller) (DF or ADF for short) test establishes whether or not a series follows a unit root process. The null hypothesis is that the variable contains a unit root, and the alternative is that the variable is generated by a stationary process. We may also select to exclude the drift, include a trend term or lagged values of the first differences of the variable in the regression. One weakness of the ADF test is how to select an appropriate number of lagged variables in the regression. In the following process we only apply the DF test for each fundamental and the test consists of three options: 1. The default option, which is the unit root with drift; 2. Unit root without drift; and 3. Unit root with trend.

Table 4.3 (Column 2 and 3) represents the DF statistics of fundamental variables in levels (Column 2) and in first differences (Column 3). Most of the tests include a constant in the DF regression. For testing price level we also include a time trend. The Dickey-Fuller test suggests that most of variables are non-stationary because the statistics are greater than 10% significant level, so that one cannot reject the null of unit root except in cases where the output gaps are measured by B-spline and HP filter, and the UK interest rate. Furthermore, the first difference of each variable is stationary. Thus, one can conclude that most

Dickey-Fuller test			Phillips-Perron Test	
	Level	The first difference	Level	The first difference
$s_t$	-2.078	-15.015***	-2.448	-14.894***
US				
$p_t$	-2.177	-14.730***	-0.950	-15.549***
$i_t$	-1.701	-16.457***	-2.400	-29.807***
$i_{3m}$	-1.271	-15.044***	-2.031	-18.422***
$m1_t$	-0.393	-27.320***	-4.635	-32.757***
$m2_t$	-1.714	-22.969***	-2.324	-24.541***
$\pi_t$	-1.744	-19.813***	-2.205	-19.984***
$y_t$	-2.714	-17.278***	-1.968	-18.174***
$x_t^{qtt}$	-1.243	-17.414***	-2.079	-18.286***
$x_t^{bs}$	-3.534***	-17.421***	-4.302***	-18.239***
$x_t^{hp}$	-3.581***	-18,182***	-4.788***	-18.933***
UK				
$p_t$	-1.066	-16.640***	-1.006	-17.834***
$i_t$	-3.607**	-28.532***	-2.400	-29.839***
$i_{3m}$	-2.279	-14.022***	-1.855	-16.081***
$m0_t$	-1.368	-28.572***	-2.010	-33.941***
$m4_t$	-2.420	-18.620***	-2.708*	-17.726***
$\pi_t$	-1.608	-17.119***	-2.650	-17.831***
$y_t$	-1.608	-17.119***	-2.119	-29.140***
$x_t^{qtt}$	-3.007**	-29.167***	-2.624*	-29.332***
$x_t^{bs}$	-5.259***	-29.257***	-5.010***	-29.477***
$x_t^{hp}$	-6.385***	-29.550***	-6.205***	-29.958***

Table 4.3: The unit root test

Note: The second and third columns show the DF statistics, and the fourth and fifth columns show the statistics which are calculated by Phillips-Perron test. The second column and fourth column show the statistics in level variables, the third and fifth column show the statistics in first differences of the variables.

\*\*\* indicates 1% significant level, \*\* indicates 5% significant level, \* indicates 10% significant level.

$p_t$  refers to the log of CPI,  $i_t$  is the Fed fund rate for the US and overnight lending rate for the UK,  $i_{3m}$  is the 3-month Treasury bill rate, and  $y_t$  is the seasonally unadjusted IPI. The adjusted IPI produces the similar results.  $x_t^{qtt}$ ,  $x_t^{bs}$  and  $x_t^{hp}$  represent the output gap measured by the QTT, B-spline and HP filter, respectively.

of the variables in the table follow unit root processes.

### **Phillips-Perron Test**

The Phillips and Perron (1988) (PPerron) test also investigates whether or not a variable has a unit root. The null hypothesis is that the variable contains a unit root, and the alternative is that the variable is stationary. The difference between PPerron and the ADF test is that PPerron uses Newey and West (1987) standard errors to account for serial correlations in the series whereas ADF uses additional lags of the first-differenced variable. Since it is difficult to justify the appropriate number of lags for the ADF test, the Phillips-Perron test can be considered as a better way to investigate a unit-root process.

Table 4.3 (Column 3 and 4) represents the PPerron statistics of fundamental variables in levels (Column 3) and in first differences (Column 4). The results obtained by the PPerron test are similar to those obtained by the DF test, the difference being that by using Phillips-Perron test the level of money supply, M4, for the UK is stationary at the 10% level. The level of the UK interest rate, UK output is  $I(1)$  regardless of the choice of critical values, whereas the DF test indicates that the interest rate is a stationary process at a 10% significance level. Furthermore, the output gap measured by the Quadratic Time Trend is less stationary by the Phillips-Perron criterion than the Dickey-Fuller test. Overall, it appears that the Phillips-Perron test has more power to accept the null hypothesis of unit root.

Since, in the following empirical study, the differential variables between the UK and the US are also used, it is necessary to check the features of these variables. Table 4.4 shows the results of the unit root for differential variables between the home and the foreign country. In Columns 2 and 3, the DF statistic

	Dickey Fuller test		Phillips-Perron Test	
	Level	The first difference	Level	The first difference
$p_t^{diff}$	-2.764	-21.181***	-2.411	-21.389***
$i_t^{diff}$	-4.688***	-24.345***	-4.356***	-24.931***
$i_{3m}^{diff}$	-3.826 ***	18.081***	-4.454***	-17.711***
$m1_t^{diff}$	-1.073	-33.281***	-1.200	-33.724***
$m2_t^{diff}$	-2.112	-19.607***	-2.041	-19.617***
$y_t^{diff}$	-0.707	-27.986***	-0.575	-28.206***
$\pi_t^{diff}$	-2.685*	-21.158***	-3.376**	-21.525***
$x_{qtt}^{diff}$	-3.360 **	-27.986***	-3.065**	-28.205***
$x_{bs}^{diff}$	-5.604***	-27.967***	-5.471***	-28.189***
$x_{hp}^{diff}$	-7.308***	-28.179***	-7.366***	-28.508***

Table 4.4: The unit root test (differential variables)

Note: all the variables in the first column are differential variables between the UK and the US.  $i_t^{diff} = i_t^{uk} - i_t^{us}$ ,  $i_{3m}^{diff} = i_{3m}^{uk} - i_{3m}^{us}$ ,  $m1_t^{diff} = m0_t^{uk} - m1_t^{us}$  and  $m2_t^{diff} = m4_t^{uk} - m2_t^{us}$ .

of differential variables in levels and in first differences are displayed. Columns 4 and 5 are those statistics calculated by the Phillips-Perron test. It is clear that both the interest rate differential and output gap measured by B-spline and HP filter are stationary. The inflation rate differential is stationary at 10% and 5% significance levels in the DF test and Phillips-Perron test respectively. The output gap measured by QTT is stationary at a 5% significance level. The other differential variables in levels are shown to be non-stationary by both the tests and, in addition, the first differences of these differential variables are stationary, which indicates that these non-stationary variables are I(1) processes.

## 4.8 Co-integration test

Before estimating the parameters of VECM models, we need to choose the number of lags in the underlying VAR, and the number of co-integrating equations. Since both AIC and SBIC indicate different numbers of lags for the period (not

reported), for simplicity, 4 lags were selected for estimating the VECM. In this section, we apply Johansen's trace statistic and maximum eigenvalue statistic to find the number of co-integrating equations in VECM models. Models 2, 3 and 4 are tested in this section.

Table 4.5 represents the trace and maximum eigenvalue statistics obtained using Johansen's method. The trace statistic tests the null that co-integrating vectors are less than or equal to  $r$  against a general alternative, and maximum eigenvalue statistic tests the null that the number of co-integrating vectors is  $r$  against the alternative of  $r + 1$  co-integrating vectors. Take the results in Model 3, for example, and one can reject the null of no co-integrating equations because the trace statistic at  $r=0$  of 164.79 exceeds its critical value of 124.24 (Row 6, Columns 2 and 3). In contrast, because the trace statistic at  $r=1$  of 92.25 is less than its critical value of 94.15 (Row 7, Columns 2 and 3), one cannot reject the null that there are one or fewer co-integrating equations. Thus, there is one co-integrating vector in Model 3 by the trace statistic. The maximum eigenvalue statistic comes to the same conclusion. The  $\lambda - max$  statistic at  $r = 0$  of 72.53 exceeds its critical value of 45.28, suggesting that the null can be rejected that  $r = 0$  in favour of  $r = 1$  (Row 6, Columns 5 and 6); however, one cannot reject the null that  $r = 1$  against  $r = 2$ , because the  $\lambda - max$  statistic at  $r = 1$  of 31.96 is less than its critical value of 39.37 (Row 7, Columns 5 and 6). The results in Table 4.5 suggest that at least one co-integrating vector is in the specifications of Models 2 to 4. Therefore, VECM or ECM should be considered as the estimator for these specifications.

Table 4.6 reports the results of testing co-integrating vectors for homogeneous VECM specifications. The results are less obvious than those in Table 4.5. The evidence suggests that the models are less likely to have at least one co-integrating

$\lambda - trace$ 5% critical value			$\lambda - max$ 5% critical value		
Model 2 ( $p_t^{us}, p_t^{uk}, s_t$ )					
$H_0 : r \leq 1$	39.54	29.68	$H_0 : r \leq 1$	35.27*	20.97
$H_0 : r \leq 2$	14.21*	15.41	$H_0 : r \leq 2$	13.28	14.07
Model 3 ( $i_t^{us}, i_t^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}, s_t$ )					
$H_0 : r \leq 0$	164.79	124.24	$H_0 : r \leq 0$	72.53*	45.28
$H_0 : r \leq 1$	92.25*	94.15	$H_0 : r \leq 1$	31.96	39.37
Model 3' ( $i_{3m}^{us}, i_{3m}^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}, s_t$ )					
$H_0 : r \leq 2$	69.0971	68.52	$H_0 : r \leq 1$	56.3314*	39.37
$H_0 : r \leq 3$	43.9201*	47.21	$H_0 : r \leq 2$	25.1770	33.46
Model 3'' ( $i_{3m}^{us}, i_{3m}^{uk}, m2_t^{us}, m4_t^{uk}, y_t^{us}, y_t^{uk}, s_t$ )					
$H_0 : r \leq 4$	31.1743	29.68	$H_0 : r \leq 2$	34.4383*	33.46
$H_0 : r \leq 5$	14.3841*	15.41	$H_0 : r \leq 3$	19.2110	27.07
Model 4 ( $i_t^{us}, i_t^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}, \pi_t^{us}, \pi_t^{uk}, s_t$ )					
$H_0 : r \leq 3$	99.71	94.15	$H_0 : r \leq 1$	65.91*	51.42
$H_0 : r \leq 4$	68.19*	68.52	$H_0 : r \leq 2$	41.55	45.28
Model 4' ( $i_{3m}^{us}, i_{3m}^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}, \pi_t^{us}, \pi_t^{uk}, s_t$ )					
$H_0 : r \leq 3$	94.2465	94.15	$H_0 : r \leq 2$	46.0806*	45.28
$H_0 : r \leq 4$	57.4595*	68.52	$H_0 : r \leq 3$	36.7870	39.37
Model 4'' ( $i_{3m}^{us}, i_{3m}^{uk}, m2_t^{us}, m4_t^{uk}, y_t^{us}, y_t^{uk}, \pi_t^{us}, \pi_t^{uk}, s_t$ )					
$H_0 : r \leq 3$	94.6112	94.15	$H_0 : r \leq 1$	54.1009*	51.42
$H_0 : r \leq 4$	65.0601*	68.52	$H_0 : r \leq 2$	42.0952	45.28

Table 4.5: Testing co-integrating vectors for heterogeneous VECM specifications

Note: Model 2-Mdoel 4 allow the parameter coefficients of the home and the foreign country to be different. The sample period is 1975:1- 1984:12. “ $r$ ” denotes the number of cointegrating vectors. The 5 % critical value of the  $\lambda - trace$  and  $\lambda - max$  are taken from Osterwald-Lenum (1990). The vector autoregressions includes a constant. An asterisk denotes significance at the 5 % level. 4 lags are used for estimating each of the VECM models.  $y_t$  is the seasonally unadjusted IPI. The adjusted IPI produces the similar results.

For Model 3, the baseline case uses  $i_t^{us}, i_t^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}$  as explanatory variables in the estimation. The alternative Model 3' uses the Treasury bill rate ( $i_{3m}^{us}, i_{3m}^{uk}$ ) instead of the Fed fund rate and UK market rate ( $i_t^{us}, i_t^{uk}$ ) as explanatory variables. The alternative Model 3'' uses the Treasury bill rates, broad money supply ( $m2_t^{us}, m4_t^{uk}$ ) and output as explanatory variables.

For Model 4, the baseline case uses  $i_t^{us}, i_t^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}, \pi_t^{us}, \pi_t^{uk}$  as explanatory variables in the estimation. The alternative Model 4' uses the Treasury bill rate ( $i_{3m}^{us}, i_{3m}^{uk}$ ) instead of the Fed fund rate and UK market rate ( $i_t^{us}, i_t^{uk}$ ) as explanatory variables. The alternative Model 4'' uses the Treasury bill rate, broad money supply ( $m2_t^{us}, m4_t^{uk}$ ), output and inflation rate as explanatory variables.

$\lambda - trace$ 5% critical value			$\lambda - max$ 5% critical value		
Model 2 ( $p_t^{diff}, s_t$ )					
$H_0 : r \leq 0$	24.25	15.41	$H_0 : r \leq 0$	24.16*	14.07
$H_0 : r \leq 1$	0.08*	3.76	$H_0 : r \leq 1$	0.08	3.76
Model 3 ( $i_t^{diff}, m1_t^{diff}, y_t^{diff}, s_t$ )					
$H_0 : r \leq 0$	26.9112*	29.68	$H_0 : r \leq 0$	16.5433	20.97
$H_0 : r \leq 1$	10.3678	15.41	$H_0 : r \leq 1$	8.5698	14.07
Model 3' ( $i_{3m}^{diff}, m1_t^{diff}, y_t^{diff}, s_t$ )					
$H_0 : r \leq 0$	31.9220	29.68	$H_0 : r \leq 0$	17.4021	20.97
$H_0 : r \leq 1$	14.5199*	15.41	$H_0 : r \leq 1$	10.0515	14.07
Model 3'' ( $i_{3m}^{diff}, m2_t^{diff}, y_t^{diff}, s_t$ )					
$H_0 : r \leq 0$	32.8794	29.68	$H_0 : r \leq 0$	26.4773*	20.97
$H_0 : r \leq 1$	6.4021*	15.41	$H_0 : r \leq 1$	6.2225	14.07
Model 4 ( $i_t^{diff}, m1_t^{diff}, y_t^{diff}, \pi_t^{diff}, s_t$ )					
$H_0 : r \leq 0$	72.43	68.52	$H_0 : r \leq 0$	20.2023	27.07
$H_0 : r \leq 1$	46.20*	47.21	$H_0 : r \leq 1$	13.3711	20.97
Model 4' ( $i_{3m}^{diff}, m1_t^{diff}, y_t^{diff}, \pi_t^{diff}, s_t$ )					
$H_0 : r \leq 0$	46.7712*	47.21	$H_0 : r \leq 0$	19.2170	27.07
$H_0 : r \leq 1$	27.5542	29.68	$H_0 : r \leq 1$	15.9234	20.97
Model 4'' ( $i_{3m}^{diff}, m2_t^{diff}, y_t^{diff}, \pi_t^{diff}, s_t$ )					
$H_0 : r \leq 3$	43.0272*	47.21	$H_0 : r \leq 1$	29.2760*	27.07
$H_0 : r \leq 4$	13.7511	29.68	$H_0 : r \leq 2$	7.1947	20.97

Table 4.6: Testing co-integrating vectors for homogeneous VECM specifications

Note: Model 2-Model 4 allow the parameter coefficients of the home and the foreign country to be the same. The sample period is January 1975- December 1984. “ $r$ ” denotes the number of cointegrating vectors. The 5 % critical value of the  $\lambda - trace$  and  $\lambda - max$  are taken from Osterwald-Lenum (1990). The vector autoregressions includes a constant. An asterisk denotes significant at the 5 % level. 4 lags are used for estimating each VECM models.  $y_t$  is the seasonally unadjusted IPI differential. The adjusted IPI produces the similar results.

For Model 3, the baseline case uses  $i_t^{diff}, m1_t^{diff}, y_t^{diff}$  as explanatory variables in the estimation. The alternative Model 3' uses the Treasury bill rate differential ( $i_{3m}^{diff} = i_{3m}^{uk} - i_{3m}^{us}$ ) instead of the Fed fund rate and UK market rate differential ( $i_t^{diff} = i_t^{uk} - i_t^{us}$ ) as explanatory variables. The alternative Model 3'' uses the Treasury bill rate differential, broad money supply differential ( $m2_t^{diff} = m4_t^{uk} - m2_t^{us}$ ), and output differential as explanatory variables. For Model 4, the baseline case uses  $i_t^{diff}, m1_t^{diff}, y_t^{diff}, \pi_t^{diff}$  as explanatory variables in the estimation. The alternative Model 4' uses the Treasury bill rate differential ( $i_{3m}^{diff} = i_{3m}^{uk} - i_{3m}^{us}$ ) instead of the Fed fund rate and UK market rate differential ( $i_t^{diff} = i_t^{uk} - i_t^{us}$ ) as explanatory variables. The alternative Model 4'' uses the Treasury bill rate differential, broad money supply differential ( $m2_t^{diff} = m4_t^{uk} - m2_t^{us}$ ), output differential and inflation rate differential as explanatory variables.

vector. It is worth noting that the Johansen's test is sensitive to the choice of the number of lags and number of observations. If we change the number of lags in the models or increase the observations in the estimation, disagreements appear, although we did not report those results here.

On the basis of these statistics we can reject the hypothesis that there are no co-integrating vectors for all fundamental models. The findings are consistent with those of Macdonald and Taylor (1994). It is of interest to note that the trace statistic and the  $\lambda - max$  statistic may indicate different results. For instance, in the process of testing Model 4, its trace statistics suggested the presence of four co-integrating vectors, whereas the  $\lambda - max$  statistic suggested only one co-integrating equation. In addition, the models that used the three-month Treasury bill rate for the UK and the US were performed better than those using the Fed fund rate and UK market rate. The use of the broad money supply instead of the monetary base did not have a significant effect on the performance of the tests. Although the evidence suggests that there is at least one co-integrating vector in the selected models, further tests are needed to find out which variables are endogenous so as to decide if VECM, ECM or other techniques should be applied to the out-of-sample forecast.

## 4.9 Testing weak exogeneity using VECM

In order to determine weak exogeneity we should focus on testing the adjustment coefficient  $\alpha$ , giving Mackinnon's critical values. If the absolute value of the  $t$ -statistic of  $\alpha$  is smaller than the absolute value of Mackinnon's critical value, there is no evidence of the significance of  $\alpha$ , which means that the corresponding variable does not react to the disequilibrium error, ergo, the variable is weakly

exogenous. Another issue regarding estimation is to set up the appropriate  $\beta$ . One can use theory to impose an exact structure for  $\beta$  and then estimate the adjusting parameter  $\alpha$ . Using the previous example, assuming the exchange rate follows PPP, the vector can be written as  $X = [s_t, p_t^h, p_t^f]$ , and  $\beta = [1, -1, 1]$  can be set based on PPP<sup>9</sup>.  $\beta$  can also be estimated by using Johansen's full information maximum likelihood technique. For simplicity, in the following discussion all the  $\beta$  of empirical specifications are estimated by using Johansen's method.

The model we are testing first is the Purchasing Power Parity; the data for the estimation is the monthly UK and US data from January 1975 to December 1984, which is the initial period of the in-sample estimation in the next section. All the variables are taken in log. For the estimation, the lags length of the VECM is 4, and a constant term is included in the estimation.

In *Stata*, one can obtain the information for  $\alpha$  and  $\beta$  by using the option “dforce” under command “vec”, respectively. The results are displayed below:

```
vec lnexchukus lncpi_uk lncpi_us, alpha lag(4), if t>228&t<385
(the Stata info on short run coefficients are not recorded due to limits of space)
```

Cointegrating equations

Equation	Parms	chi2	P>chi2
-----			
_ce1	2	24.82789	0.0000
-----			

Identification: beta is exactly identified

Johansen normalization restriction imposed

---

<sup>9</sup>In this study, the exchange rate is a direct quote, which means the unit of sterling per dollar. In this case, the long run equilibrium restriction in the equation 4.2 becomes  $s_t - p_t^h + p_t^f = 0$

	beta	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----							
_ce1							
lnexchukus		1	.	.	.	.	.
lnlncpi_us		25.41605	7.277848	3.49	0.000	11.15173	39.68037
lnlncpi_uk		-17.27474	5.264727	-3.28	0.001	-27.59341	-6.956059
_cons		-34.86249	.	.	.	.	.
-----							

#### Adjustment parameters

Equation	Parms	chi2	P>chi2
-----			
D_lnexchukus	1	.3278412	0.5669
D_lncpi_us	1	34.62182	0.0000
D_lncpi_uk	1	.729295	0.3931
-----			

alpha	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----						
D_lnexchukus						
_ce1						
L1.	-.0036022	.0062912	-0.57	0.567	-.0159326	.0087283
-----+-----						
D_lncpi_us						
_ce1						
L1.	-.0033513	.0005696	-5.88	0.000	-.0044676	-.002235
-----+-----						
D_lncpi_uk						
_ce1						
L1.	-.0011745	.0013754	-0.85	0.393	-.0038702	.0015211
-----						

The table above displays the key information of the VECM estimation on PPP. The co-integrating parameter  $\beta$  is [1, -17.27474, 25.41605], the parameters are significant and have the expected signs. The estimates of  $\alpha$  are displayed at the bottom of the table. Norrbin et al. (1997) argue that the critical value used for the  $t$ -statistics for  $\alpha$  are higher than usual because of the non-stationarity of the error correction variable. Therefore, the critical value from MacKinnon (1991) should be used. If the absolute value of the critical value is greater than the absolute value of the estimated  $t$ -statistic, there is no evidence that  $\alpha$  in the

equation is significant, and thus, the variable is weakly exogenous. The 5% absolute critical value from Mackinnon is 4.296. The spot rate and the UK price level are weakly exogenous because the absolute value of critical values are greater than those of  $t$ -statistics ( $|-0.57| < |-4.296|$  and  $|-0.85| < |-4.296|$ , indicating that the parameter  $\alpha$  is not significant. Thus the US price level is endogenous and the null hypothesis of  $\alpha = 0$  can be rejected.

It is a very interesting result, because the exchange rate is weakly exogenous, which means that neither ECM nor VECM should be used to forecast exchange rates, since they are not affected by the co-integrating vector  $\Pi X_{t-1}$ , these results being similar to those of Norrbin et al. (1997). In this case, as we explained before, VAR in first differences or autoregressive model should be applied, depending on whether or not the exchange rate is strongly exogenous.

A detailed analysis on the weak exogeneity of each variable in the empirical specifications used in the chapter is reported as follows:

The lagged error correction variable is called ECV ( $\beta X_{t-1}$ ) in Table 4.7, and Models 2 to 4 were tested. The VECMs had a lag of the deviation from the theoretical relationship ( $\beta X_{t-1}$ ), a constant, and 4 lags of the differenced variables as regressors. The number of lags of the differenced variables denotes the order of VAR. We also tested lower order or higher order VARs, which produced similar results. The testing period was from January 1975 to December 1984. The UK and US monthly data were applied.

Table 4.7 displays the estimations of  $\alpha$  in different specifications by VECM, the p-value being in brackets. The results in Table 4.7 show that the US price level in the PPP model, and UK interest rate in both the Frenkel-Bilson and

	$ECV_{t-1}(PPP)$	$ECV_{t-1}(FB)$	$ECV_{t-1}(DF)$
$\Delta s_t$	-0.0036022 (-0.57)	.0103563 (1.96)	.0024024 (0.22)
$\Delta p_t^{us}$	-0.0033513** (-5.88)	- -	- -
$\Delta p_t^{uk}$	-0.0011745 (-0.85 )	- -	- -
$\Delta m_t^{us}$	- -	0.0023722 (1.14)	0.0132243 (3.29)
$\Delta m_t^{uk}$	- -	0.0131936 (2.96)	0.0241792 (2.75)
$\Delta i_t^{us}$	- -	-0.0011012 (-0.68)	0.0079059 (2.59)
$\Delta i_t^{uk}$	- -	0.0157799** (5.64)	0.0295744 ** (5.46)
$\Delta y_t^{us}$	- -	-0.0020675 (-1.52)	-0.00074 (-0.28)
$\Delta y_t^{uk}$	- -	-0.0031004 (-1.10)	-0.0192928 (-3.58)
$\Delta \pi_t^{us}$	- -	- -	0.0052414 (4.04)
$\Delta \pi_t^{uk}$	- -	- -	0.0080171 (3.64)

Table 4.7: Testing weakly exogeneity on heterogeneous models using VECMs

Note: PPP refers to Purchasing Power Parity model (Model 2), FB is short for Frenkel-Bilson's model (Model 3) and DF is short for Dornbusch -Frankel's model (Model 4), we allow the parameter coefficients for the US and the UK to be different. In Model 3,  $i_t^{us}, i_t^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}, s_t$  are applied. In Model 4,  $i_t^{us}, i_t^{uk}, m1_t^{us}, m0_t^{uk}, y_t^{us}, y_t^{uk}, \pi_t^{us}, \pi_t^{uk}, s_t$  are applied. The value in the first line of each row is the value of  $\alpha$ . The value in parentheses are  $t$ -statistics, the critical values are from MacKinnon (1991). Significance is at the 5% critical value of 4.296. Significance is at the 1% critical value of 4.949.

Dornbusch-Frankel models are endogenous since the  $\alpha$  is significant. However, there was no evidence that the other variables in these models were endogenous. Therefore we have evidence that the spot rate is weakly exogenous in these three models during the testing period. We also tested the VECMs using different orders of lag, longer testing periods, homogeneous models and other alternative models, for instance, changing from the Fed fund rate and the UK market rate to the Treasury bill rates. The results were similar. Other fundamental variables rather than the spot exchange rate were found to be endogenous during the testing period, our results being similar to those of Norrbin et al. (1997). Results in this section also reveal why there is at least one co-integrating vector in the exchange rate models, this being caused by explanatory variables rather than the dependent variable. However, since the data we used for the test in the section are only a part of the whole sample, and we allow the parameter coefficients to vary over time, in the following sections we still use VECM as a method to forecast exchange rates.

## **4.10 Out-of-sample forecasting**

### **4.10.1 Out-of-sample forecasting using traditional monetary models**

In this section Models 2 to 4, introduced above, are applied to the out-of-sample exchange rate forecasting. The initial sample period of estimation was from January 1975 to December 1984, the forecasting period having lasted from January 1985 to August 2010, the window size being 120. This means that the first one-month-ahead forecast was generated from January 1985, then the sample was moved up one observation before the procedure was repeated. In the meantime

the first observation in the sample was removed from the estimation so that the number of data in each estimation remained fixed at 120. The process was continued until all the out-of-sample observations were exhausted. The first difference of the exchange rate was forecasted and compared with a naive random walk in order to make our results consistent with those obtained by the Taylor rules in the next section.

Three estimators of these theoretical models are examined: a Vector Error Correction Model, a VAR in the first differences and a rolling regression in first differences. These estimators have different implications for the relationships between the exchange rate and its economic determinants. As we have demonstrated in the previous section, most of the macro variables are  $I(1)$ , so that if the variables are co-integrated then ECM or VECM specifications should be implemented. The majority of the literature applies ECM to their forecasts. However, the drawbacks of ECM techniques are that they assume that only the exchange rate is endogenous, which is not the case, as we have demonstrated in section 4.9, and the two step estimating procedure for ECM is troublesome in practice. The VECM model can avoid both these drawbacks, given the fact that because VECM considers all the variables in the model symmetrically, and there is a written program for VECM forecasts in *Stata*, so that it is sensible to apply this technique.

Although in section 4.8 we have demonstrated that there is at least one co-integrating equation in Models 2 to 4, it is worth noting that the long run relationships between exchange rate and macro variables may vary over time. If the variables are not cointegrated, the VECM specification can lead to spurious results. Therefore the VAR in first differences and rolling regression in first differences are also considered.

In terms of forecasting performance comparison methods, we chose the MPSE ratio, MAE ratio, DMW statistic, CW statistic, correlation and change of direction statistic (or correct direction change test). The MPSE (or MAE) ratio is the fraction of competing model's MPSE (or MAE) divided by random walk's MPSE (or MAE). A ratio of less than one indicates that the competing model can outperform random walk.<sup>10</sup> The correlation between the observed values and the forecasts indicates the co-movement between the two series, and the higher the correlation, the better the forecasts. The DMW statistic, in essence, can still be clarified to be a version of the MPSE method, the difference being that DMW provides the extent of the significance of the comparison results. As we have demonstrated in section 4.5, the CW statistic can be considered as a modified technique of the DMW test. It is also of interest to observe how different the results of the CW statistic are from the DMW statistic. The correct direction change test (CDC) shows the extent to which the forecasts correctly reflect the direction of the observed values. This test is different from the traditional techniques since it does not involve the loss function. The reason we chose six different techniques is because there is no unanimous agreement on which method is the best indicator for the out-of-sample forecasting. In order to make our results for traditional models comparable to those of the Taylor rules model, we would like to compare  $\Delta s_t$  and  $\Delta \hat{s}_t$ . For each competing model we assume two scenarios: that the parameter coefficients of macro variables between the home and the foreign countries are either homogeneous or heterogeneous. Most of the literature concludes that the forecasts perform better when the coefficients are homogeneous. Nevertheless, allowing the heterogeneous coefficients is likely to

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<sup>10</sup>In terms of the out-of-sample forecast, the random walk model is not generated by lagged variables plus a random error term but is the true series of exchange rates. Therefore, outperforming a random walk means that a better forecast has to be provided than the actual series.

	Homogeneous coefficients				Heterogeneous coefficients			
	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences
MPSE	0.9903	1.0205		1.0128	1.0075	1.0337		1.0220
MAE	1.0022	1.0163		1.0153	1.0315	1.0240		1.0078
DMW	0.3473	-2.1856		-1.5617	-0.2412	-2.9040		-2.0186
CW	3.6979**	-1.1539		-0.5376	3.4132**	-1.7238		-0.7157
Correlation	0.2457	-0.0729		-0.0276	0.2284	-0.1146		-0.0348
CDC	0.5628	0.4951		0.4610	0.5889	0.4822		0.4935

Table 4.8: The out-of-sample forecasting performance of the PPP model

include more information in the forecasts, so it is interesting to investigate both cases.

In section 4.10.1, we consider the baseline case for competing models. This means the consumer price indices, Fed fund rate, UK market rate and monetary base ( $M1_{US}$ ,  $M0_{UK}$ ) are used for constructing differential variables and the forecasts. In Sections 4.8.2 to 4.8.4, other data are also considered. In all, 36 estimations and 216 tests were implemented in section 4.8.

### PPP model

The results in Table 4.8 suggest that the VECM model can outperform VAR in first differences and rolling regression in first differences model, especially with homogeneous coefficients. For the homogeneous VECM model, MPSE is less than 1, indicating that the MPSE (MAE) of PPP model is less than that of random walk, the CW statistic of 3.6979 is significant at a 5% level, and it is the most significant result of the six cases in this table. The correlation is 0.2457, suggesting that there is positive co-movement between actual exchange rate in first difference and the forecasts. The change of direction test is 0.5682, which is above 50%, so our forecasts using the homogeneous VECM PPP model are better than

	Homogeneous coefficients				Heterogeneous coefficients			
	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences
MPSE	1.0040	1.03157		1.0262	1.0322	1.0378		1.0181
MAE	1.0384	1.02335		1.0134	1.0642	1.0513		1.0293
DMW	0.3521	-2.0373		-3.5719	-0.8483	-2.3259		-2.1605
CW	3.7282**	-0.5180		-2.4805	4.0050**	-0.1030		-0.1248
Correlation	0.2350	-0.0295		-0.1507	0.2644	-0.0039		-0.0068
CDC	0.5566	0.4983		0.4481	0.5663	0.4692		0.5292

Table 4.9: the out-of-sample forecasting performance of the FB model

tossing a coin.

### Frenkel-Bilson's flexible-price monetary model

For Model 3, the traditional tests including the MPSE, MAE and DMW indicate that none of the specifications can outperform a random walk. The CW statistic suggests that the VECM model with heterogeneous coefficients performs the best. The CDC statistic suggests that the forecasts by the VECM and the rolling regression model with heterogeneous coefficients can predict the movement of the nominal exchange rate better than tossing a coin. The correlation test also shows that the VECM model can forecast at least 23%-26% of exchange rate movements. (see Fig 4.9)

The results in Table 4.10 are similar to those in Table 4.9. In terms of the MPSE and MAE ratios and DMW statistic all the specifications of the DF model were rejected. The CW statistic indicates that the VECM estimator is a better performer than a random walk and the other estimators, and the correlation and CDC also confirm that the VECM models have certain predictive powers. Comparing VECM with homogeneous coefficients to those with heterogeneous

	Homogeneous coefficients				Heterogeneous coefficients			
	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences
MPSE	1.0140	1.0278		1.0265	1.0337	1.0554		1.0072
MAE	1.0503	1.0366		1.0310	1.0969	1.0921		1.0149
DMW	-0.4621	-2.5122		-2.9115	-0.7955	-1.9686		-0.7652
CW	3.8708**	-0.7142		-1.6077	4.4809**	1.5630		1.4233
Correlation	0.2295	-0.0380		-0.1017	0.2991	0.0853		0.1048
CDC	0.5728	0.4886		0.4415	0.6019	0.4983		0.5422

Table 4.10: the out-of-sample forecasting performance of the DF model

coefficients, the MPSE, MAE and DMW statistics increase as the corresponding parameter coefficients between the home and the foreign countries are allowed to diverge, but the CW, the correlation and the CDC statistic all suggest that the heterogeneous VECM DF model performs better.

#### 4.10.2 A robust test on out-of-sample forecasting using traditional monetary models

In this section, certain new data replace the baseline case data to see if the choice of data would make any differences in terms of out-of-sample forecast performance. The 3-month Treasury bill rates, broad money supply (M2 for the US and M4 for the UK) and seasonally adjusted output are applied.

**Frenkel-Bilson's flexible-price monetary model(Treasury bill rate used)**

The results shown in Table 4.11 suggest that there is a significant improvement on overall predictive performance by using the data of 3-month Treasury bill rates. The MPSE, MAE suggest that rolling regression in first differences of

	Homogeneous coefficients			Heterogeneous coefficients		
	VECM	VAR in differ- ences	Regression in differ- ences	VECM	VAR in differ- ences	Regression in differ- ences
MPSE	0.9943	1.0243	1.0334	1.0178	1.0347	0.9820
MAE	1.0340	1.0175	1.0204	1.0376	1.0313	0.9749
DMW	0.1650	-1.9392	-1.4512	-0.3834	-2.1556	0.4986
CW	4.0102**	-0.5732	-0.4549	3.6387**	-0.0379	2.0382**
Correlation	0.2727	-0.0401	-0.0483	0.3012	0.0003	0.2352
CDC	0.5954	0.4951	0.5275	0.6245	0.5145	0.5728

Table 4.11: Robust tests on the out-of-sample forecasting performance of the FB model (Treasury bill rates)

the heterogeneous model perform better than the other specifications, as well as out-performing a random walk. The CW, correlation test and CDC also support this model. The second best performer in the table is the VECM model with homogeneous coefficients; MPSE, CW, the correlation test and CDC all support this specification. The VECM model with heterogeneous coefficients can predict the movement and direction of exchange rates the best; however, the model may involve too much information (or noise), which causes the MPSE and MAE to become greater than 1. Again, the VAR in first differences model does not have any predictive power.

#### **Dornbusch -Frankel's sticky-price monetary model (Treasury bill rate used)**

Table 4.12 shows that the rolling regression model with heterogeneous coefficients performs the best regarding the MPSE, MAE and CDC test. All the tests support this model except the DMW statistic. The CW statistic suggests that both homogeneous and heterogeneous VECMs can outperform a random walk, the correlation and CDC test also indicating that both of them have some ability to predict the movements of the sterling/dollar exchange rate. The VAR in first

	Homogeneous coefficients				Heterogeneous coefficients			
	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences	VECM	VAR differ- ences	in differ- ences	Regression in differ- ences
MPSE	1.0102	1.0301		1.0300	1.0302	1.0466		0.9687
MAE	1.0435	1.0309		1.0198	1.0678	1.0567		0.9579
DMW	-0.3174	-2.3418		-1.1825	-0.6022	-2.3142		0.7922
CW	4.0894**	-0.7941		-0.1043	3.8585**	0.1113		2.3861**
Correlation	0.2515	-0.0557		-0.0124	0.3172	0.0095		0.2859
CDC	0.5922	0.4725		0.5275	0.6214	0.5081		0.5922

Table 4.12: Robust tests on the out-of-sample forecasting performance of the DF model (Treasury bill rates)

differences models still does not work.

It is worth noting that the MPSE and MAE are smaller for the heterogeneous rolling regression model than the results in Table 4.12 (Column 7, Rows 3 and 4), as the inflation rate is introduced in the model. However, the results for VECMs do not improve. We do not have any explanation for these results at this stage.

### **Board money supply *VS* monetary base**

In this section We use M2 for the US and M4 for the UK as the money supply for the forecast, since the Treasury bill rates perform better than the base rates; We also keep using Treasury bill rates in the forecast process. The results do not show a significant improvement if board money is used as money supply variables for the models. We do not report the results here. Molodtsova and Papell (2009) obtain similar results.

### **Seasonally unadjusted IPI *VS* Seasonally adjusted IPI**

If seasonally adjusted series are used in the application of the exchange rate

	Homogeneous coefficients				Heterogeneous coefficients			
	VECM	VAR	in	Regression	VECM	VAR	in	Regression
		differ-		in differ-		differ-		in differ-
		ences		ences		ences		ences
MPSE	0.9638	1.013		1.0289	0.9655	1.0200		0.9822
MAE	0.9942	1.008		1.0083	1.0035	1.0166		0.9560
DMW	1.2903	-1.2415		-1.1190	0.9613	-1.5257		0.4618
CW	4.8147**	-0.0686		-0.3114	4.6352**	0.0609		1.4539
Correlation	0.3089	-0.0027		-0.0360	0.3404	0.0050		0.2161
CDC	0.5825	0.5081		0.5242	0.5987	0.5242		0.5598

Table 4.13: The out-of-sample forecasting performance of the interest rate model (Treasury bill rates)

forecasts the results would not change dramatically, and thus, they are not reported. The CDC and correlation become less significant if the latter are used. The MPSE and MAE, however, also become smaller. The result is not surprising, since the idea of seasonal adjustment is to smooth the month-to-month changes. The process would remove some information, leading to lower CDC and correlation, also removing the useless noise which lower the MPSE and MAE.

### Does interest rate matter?

In this last section we found out that changing interest rate data from the central bank base rates to the Treasury bill rates can improve the forecasting performance. Dick et al. (2012) apply survey data and find out that the interest rate plays a significant role in predicting the movement of exchange rates compared to other fundamental variables. In this section we only forecast the change of the spot exchange rate by using 3-month Treasury bill rates from both the UK and the US. Since there is no unanimous theory behind this estimation, this section is purely an empirical exercise. The main idea is to identify how important interest rates are in exchange rate forecasting.

The estimators were the same as in the previous section, and we also compared

the specification with homogeneous coefficients to those with heterogeneous coefficients. The results in Table 4.13 are similar to those in Tables 4.11 and 4.12, but the homogeneous VECM performed the best in this case in terms of the MPSE, MAE and CW statistic, and the correlation and CDC test also supported the model. The heterogeneous VECM also performed well in terms of the MPSE and CW, and also had the highest ranking in terms of the correlation and CDC statistic. The MPSE, MAE and CDC supported the heterogeneous rolling regression model in first differences, but not in terms of the CW. The DMW statistic still rejected all the specifications.

It is an interesting finding, since the CW, correlation and CDC statistic indicate that the forecasting performance of the interest rate on exchange rate is as good as those include other fundamental variables. Our finding suggests that, even by using realized data, Dick et al.'s results can be supported, and that the interest rate plays a more significant role than other fundamentals in predicting the movements of sterling/dollar exchange rates.

### **4.10.3 Discussion**

There are several intriguing findings in this section. First, the traditional performance comparison methods, the MPSE, MAE and DMW statistic tend to reject the predictability of fundamental models regardless of which estimator is chosen. This finding coincides with the majority of the literature. Second, the CW statistic tends to suggest that the VECM models can outperform a random walk. This finding indicates that the choice of performance comparison test can make a difference to the conclusion drawn as to whether or not fundamentals play a role in forecasting. Third, the CDC and correlation test indicate that the funda-

mental models have a certain power of predictability but the results are not very significant. Fourth, the interest rate is a more important fundamental variable than the other fundamentals in sterling/dollar exchange rate forecasting. Fifth, in terms of the choice of estimators, the VECM produces the most consistent results, the VAR in first differences is not a valid estimator for the forecasting, and the rolling regression in first differences works in some cases. Sixth, models with heterogeneous coefficients provide a good forecast of the directions of exchange rate but it may also lose the efficiency of the forecast by including too much information. Seventh, the choice of data plays a significant role in forecasting, and 3-month Treasury bill rates should be used for traditional models. The money supply does not have a significant impact on forecast performance. The seasonally unadjusted industrial production index can improve the prediction of the direction of the exchange rate, but the noise also lowers the efficiency of the forecast since the MPSE and MAE will increase.

#### **4.10.4 The Taylor rule model**

For the Taylor rule model we are unable to derive the VECM specification from this theoretical framework; thus we only used rolling regression on the first difference of exchange rate and the level of the fundamental variables such as inflation rate, output gap and interest rate. The process was identical to those in Molodtsova et al. (2008) and Molodtsova and Papell (2009), but the data had been updated and a variety of goodness-of-fit measures were applied.

For the forecast, several options were considered. First, we allowed the specification to have homogeneous or heterogeneous coefficients; second, whether or not the smoothing parameter (lagged interest rate) played a role in the forecasting is

	Homogeneous coefficients				Heterogeneous coefficients			
	Without s	With s	With s/q	With q Without s	Without s	With s	With s/q	With q Without s
MPSE ratio	1.0214	1.0434	1.0316	1.0165	1.0248	1.0447	1.0417	1.0219
MAE ratio	1.0135	1.02661	1.0183	1.0096	1.0233	1.0519	1.0608	1.0261
DMW	-1.3536	-2.2926	-1.8624	-1.0688	-1.1047	-1.6636	-2.0014	-1.2048
CW	0.4964	0.0912	0.4091	0.9684	1.9182**	1.9851**	1.3575	1.7846**
Correlation	0.0278	0.0211	0.0353	0.0513	0.1231	0.1222	0.0903	0.1131
CDC	0.5162	0.5162	0.5065	0.5162	0.5389	0.5317	0.5129	0.5324

Table 4.14: The out-of-sample forecasting performance of the Taylor rule model(QTT)

discussed; third, three different output measurements were used for the forecasts: they are Quadratic Time Trend (QTT), B-spline and HP filter. Fourth, whether or not the real exchange rate played a role in forecasting was tested. The specification with the real exchange rate as one of the explanatory variables is known as the asymmetric Taylor rule model, while a specification without it is called the symmetric Taylor rule model. The initial estimating sample period was from January 1975 to December 1984, while the forecast period lasted from January 1985 to August 2010, and the window size was 120. Six performance comparison tests are used for discussion. 24 forecasts are made and 144 tests are calculated in this section.

## Forecasting results

The data selected for this section are the overnight leading rates as the UK interest rate, Fed fund rate, seasonally adjusted IPI and the inflation rate. We have also tried the seasonally unadjusted IPI and the Treasury bill rates but the results are not significant so we do not report them in this chapter.

Table 4.14 shows the forecasting results obtained by using the QTT method. Our results are similar to those obtained by Molodtsova and Papell (2009). The models with heterogeneous coefficients perform better than those with homoge-

	Homogeneous coefficients				Heterogeneous coefficients			
	Without s	With s	With s/q	With q Without s	Without s	With s	With s/q	With q Without s
MPSE ratio	1.0121	1.0384	1.0430	1.0271	1.0142	1.0412	1.0455	1.0296
MAE ratio	1.0072	1.0195	1.0371	1.0194	1.0141	1.0461	1.0632	1.0433
DMW	-0.8441	-1.9864	-2.2337	-1.7792	-0.6358	-1.5297	-2.2536	-1.4580
CW	0.8804	0.3272	-0.0188	0.1727	2.0926***	2.0616****	1.3327	1.6253**
Correlation	0.0431	0.0245	-0.0159	0.0025	0.1467	0.1364	0.0905	0.1084
CDC	0.4902	0.5162	0.5092	0.4935	0.5422	0.5455	0.5259	0.5325

Table 4.15: The out-of-sample forecasting performance of the Taylor rule model (B-spline)

neous coefficients. The CW statistic confirms that, in the models with heterogeneous coefficients, the one without a smooth parameter and the one with an asymmetric parameter real exchange rate perform better than the others. The traditional methods MPSE, MAE and DMW statistic do not support the Taylor rule models. The correlation test and CDC test suggest that the models has some predicative power but it is considerably less compared to those of traditional monetary models in previous sections.

Table 4.15 show the results obtained by the B-spline method. The results are similar to those in Table 4.14. Models with heterogeneous coefficients perform better, in this type of models, those without smoothing and asymmetric parameters perform the best (Column 6). Our finding also suggest by using the B-spline method, the results are systematic improved by all the criteria, if we compare the best results in Table 4.15 (Column 6) with those in table 4.14 (Column 6).

The results in Table 4.16 are obtained by the HP filter. The results are consistent with those in Table 4.14 and 4.15. The best performer is the model with a heterogeneous coefficient, without smoothing parameter and asymmetric parameter (Column 6). Furthermore, the performance in Table 4.16 is less significant than those obtained in Table 4.15 in general, with a few exceptions, but better than those in Table 4.14, which indicates that the model measured by the B-

	Homogeneous coefficients				Heterogeneous coefficients			
	Without s	With s	With s/q	With q Without s	Without s	With s	With s/q	With q Without s
MPSE ratio	1.0139	1.0402	1.0409	1.0231	1.0117	1.0303	1.0440	1.0295
MAE ratio	1.0082	1.0160	1.0322	1.0232	1.0157	1.0413	1.0662	1.0497
DMW	-1.2944	-2.2073	-2.4580	-1.6103	-0.6135	-1.2593	-2.2991	-1.5118
CW	-0.3223	-0.4006	-0.3073	0.1641	2.0499***	1.8522***	0.9425	1.5363
Correlation	-0.0164	-0.0487	-0.0217	0.0211	0.1328	0.1227	0.0726	0.1051
CDC	0.5324	0.4870	0.5064	0.5097	0.5454	0.5227	0.5097	0.5292

Table 4.16: The out-of-sample forecasting performance of the Taylor rule model (HP filter)

spline is the best estimation in our findings.

In this section, Taylor rule models are discussed, our specification and estimator having been identical to those in Molodtsova and Papell (2009). Overall, our results are similar to their results, with a few new findings. The CW statistic tends to accept the model with heterogeneous coefficients regardless of the output gap measures; the model with heterogeneous coefficients consistently outperform those with homogeneous coefficients and HP filter can improve the forecasting performance. However, by using a variety of criteria, our results do not significantly support the models with smoothing and asymmetric parameters. The B-spline method produces the best forecasts in this section.

Although there is evidence that the Taylor rule model has some predictive power, the results are less significant than those of traditional monetary models measured by using VECM based on a variety of criteria. These results are particularly interesting because in Molodtsova and Papell's paper the forecasting results from the traditional model by using the ECM technique are worse than those of the Taylor rule model in comparison to the CW statistic. It appears that the choices of estimator can make a difference and that, by using the VECM technique, one can improve the forecasting capacity of the exchange rate. Also, although the CW statistic is significant for Taylor rule specification, if we use a

series of criteria it is not difficult to discover that the traditional models are not as useless as some literature indicates.

## 4.11 Conclusions

The chapter discussed the traditional monetary models and Taylor rule model on the forecasting performance of the first difference of sterling/dollar exchange rate with a random walk. Three types of estimators, VECM, and VAR in first differences and rolling regression in first differences were applied and a variety of performance comparison techniques were introduced. Our results suggested that the monetary model, and especially Frenkel-Bilson's flexible-price monetary model, had the power of predictability in exchange rate forecasting, and that the VECM estimator could improve the forecasting performance to some extent.

Since our simulations on CW statistic and DMW statistic suggested that the CW statistic could be a better goodness-of-fit measure than DMW, by focusing on the CW statistic, We found strong and consistent evidence that the traditional model could outperform a random walk. Although the CW statistic also supported the forecasts with Taylor rule fundamentals, the results were as strong as those obtained by traditional monetary models.

The out-of-sample forecasting comparison results also indicated that the selection of comparison techniques could induce different conclusions, and thus it might be a good idea to use a variety of techniques in exploring the matter of forecasting. It was found that the data selection could also induce different results. For instance, forecasts with traditional fundamentals work better if the three-month Treasury bill rate is considered as the nominal interest rate rather

than the overnight leading rate, so that it is important to choose appropriate data.

There are some limitations in the chapter. Firstly, this chapter only considered sterling/dollar exchange rates, and the findings in this chapter did not cover the features of other exchange rate series. This paper did not consider the microeconomic fundamentals such as order flow in dealing with exchange rate forecasting, which could be a fruitful direction to take. Evans and Lyons (2005) first propose a theoretical model demonstrating that the order flow can explain a large proportion of the exchange rate variation. Combining the conventional specification with the monetary model and the Evans and Lyons (2005) microstructure approach, Chinn and Moore (2011) propose a hybrid model of exchange rates. By searching out the private preference shocks which render the money demand unstable, the preference can be revealed through order flow. In their empirical study the hybrid model can make a significant improvement in forecasting for the yen/dollar and euro/dollar in terms of both in-sample and out-of-sample forecasting performance (see Bjornes and Rime, 2005, as well).

The realized data instead of real-time data were used in the chapter as there is an argument that the real-time data can improve forecasting power. This chapter did not consider the expectation of the effect of fundamental variables on exchange rate movements, which is also worth investigating (Engel and West, 2005, 2006). Structural breaks or nonlinear forms of specification were also left for future research.

# Chapter 5

## Conclusions

This thesis has carefully discussed the connection between Taylor rule fundamentals and the exchange rate. In chapter 2 we made a short summary on the development of the asset-pricing model of exchange rates incorporating Taylor rule fundamentals in order to set up a background for this type of the topics that follow. A few of the papers in this field suggest that the exchange rate should be considered as an asset price which is not only determined by current fundamentals, but also by discounted future fundamentals.

Based on this idea, our empirical study focused on the present-value model of yen/dollar exchange rates incorporating Taylor rule fundamentals, and in order to apply this type of model to forecasting we needed the current and future data of inflation rate, interest rate and output gap. The future data of the fundamentals were estimated based on an unrestricted VAR. The output gap was measured by three different techniques: the Quadratic Time Trend method, the B-spline, and the HP filter. The B-spline is a new technique for estimating output gap and has not yet been used widely in the research up until now. However, the output gap measured by the B-spline has performed well throughout our whole thesis, indicating that it can be considered as a output gap method.

The Taylor rule, for both Japan and the US, were estimated during three specific periods of time: January 1971 to September 1979, October 1979 to December 1998, and January 1999 to December 2006. The GMM estimator was used. We found strong evidence that both countries applied the Taylor rules from October 1979 to December 1998. Thus, the yen/dollar exchange rate was forecasted in that period of time, based on the present-value model. A variety of specifications were considered: the forecasts were based on different output gap measures; the corresponding coefficients of each fundamental between Japan and the US were allowed to be the same or different; whether or not the smoothing parameters

play an role was also discussed. The forecasting results suggested that the best performer was the specification with heterogeneous coefficients and smoothing parameters, and HP filter measured the output gap. The correlation between the model-based exchange rate and the observed value could approach 72%, which was much higher than the results obtained by EW06 and Mark (2009). In addition, the B-spline and HP filter could provide better results than the Quadratic Time Trend.

In chapter 3, our interests changed to the study of central bank behaviour. Since we demonstrated that the GMM should be considered to be the estimator for Taylor rule estimation in chapter 2, in chapter 3 we continued to use this technique. The US was the sample country for the study. In order to investigate whether or not the Fed's behaviour has been consistent over the last four decades, Andrews'(1993) method was used for testing potential structural changes. Andrews' method, in essence, is to split a model into two by assuming the potential structural break point, estimating the two sub models, and collecting the coefficients and corresponding variances to generate the Wald statistic. If the Wald statistic is greater than the critical value then we have evidence that there might be a structural break at the testing point. The results provided by Andrews' test suggested that the Fed's behaviour was not stable from the period of October 1978 to January 2004, and that this was especially true in the 1980s. Five structural break points were found: March 1981, January 1988, January 1993, July 1997, and October 2001. The robust tests based on different output gap measures provided similar results; however, the instability of the Taylor rule of the Fed was magnified by using the B-spline and the HP filter.

In section 3.4 a variety of linear Taylor rules were estimated based on different subsample periods and different output gap measures. The results suggested that

the Fed might follow a Taylor rule in the long term, but it might not be the case during specific subsample periods. There was evidence of a linear Taylor rule being used from March 1981 to January 1988 and again from January 1993 to July 1997.

In section 3.5, the Taylor rules in linear form under different subsample periods were estimated based on the break points which were obtained in section 3.3. we treated the Taylor rule as a threshold process, the estimating method being based on Caner and Hansen (2004). The advantage of their method is that it allows the estimation to be conducted under the GMM framework, enabling the estimation in the thesis to be kept consistent. CH04 also proposes a method to test the existence of the threshold process, which could help us to identify whether or not there is a threshold process during various subsample periods. The inflation rate were considered to be the threshold variable. There was no evidence of a threshold model being used in the 1980s, which indicated that the instability could not be explained by a threshold process, so that it might be due to the fact that the level of inflation was substantially different at that time. There is evidence supporting the existence of a threshold process during the period between October 2001 and December 2006, although the process could not be characterized as a classic nonlinear Taylor rule because the coefficients of inflation rates did not have the expected sign.

Chapter 4 tried to answer the important question we asked at the beginning of the thesis: “Can Taylor rule fundamentals outperform a naive random walk and the traditional monetary models?” The exchange rate model with Taylor rule fundamentals being used in this chapter was built based on Molodtsova et al. (2008) and Molodtsova and Papell (2009), who have conducted the most up-to-date research in this area, and provided convincing results that the Taylor rule models

could outperform other models. Three monetary models were also selected to compare with the Taylor rule models: PPP model, Frankel-Bilson's flexible-price monetary model, and Dornbusch-Frankel's sticky-price monetary model.

Unlike other literature on the subject, this chapter conducted detailed research into whether or not different estimators, goodness-of-fit measures, or selections of data could produce different forecasting performances. In an empirical study the first difference of the log sterling/dollar exchange rates from January 1975 to August 2010 was studied.

A detailed discussion on the best choice of estimators in terms of estimating exchange rates could be found in section 4.3. Three estimators were chosen for this empirical study: VECM, VAR in first differences and rolling regressions in first differences. Six goodness-of-fit measures were selected in the study: RMSE, MAE, DMW statistic, CW statistic, and the correlation and change of direction tests. There has been a debate on which will be the best performer between the CW and the DMW statistic when the testing models are nested. In section 4.5 a detailed comparison was made between the two based on Monte Carlo simulations. The density distribution and the Q-Q plots suggested that the CW statistic performed better than the DMW statistic. A further investigation also showed that the CW statistic was slightly undersized. Our results in section 4.5 are similar to those of Clark and West (2006).

The out-of-sample forecasting results of the sterling/dollar exchange rates provided some interesting findings: First, overall the VECM estimators performed better than the other two estimators, the best model being Frankel-Bilson's flexible-price model. Second, since our simulation of the CW statistic and the DMW statistic suggested that the CW statistic could be a better goodness-of-

fit measure than the DMW statistic; by focusing on the CW statistic we found strong and consistent evidence that the traditional model could outperform a random walk. Although the CW statistic also supported the forecasts with Taylor rule fundamentals, the results were not as strong as those obtained by traditional monetary models. Third, it was found that the data selection could indeed induce different results. For instance, forecasts with traditional fundamentals worked better if the three-month Treasury bill rate was considered as the nominal interest rate rather than the market rate, so that it was important to choose appropriate data.

Based on this thesis, we would like to make a short conclusion on whether or not Taylor rule fundamentals could predict exchange rates. In chapter 2 a present-value model of yen/dollar exchange rates incorporating Taylor rules was built, and there was evidence that the model had some predictive power. We did not use a variety of goodness-of-fit measures to investigate its performance because the model-based exchange rate had a much smaller mean and standard deviation than the observed exchange rate (not reported), which means that by using other criteria we would not be able to prove the forecasts were better than a naive random walk. In chapter 4 the models with Taylor rule fundamentals failed to outperform other monetary models or a naive random walk. Thus, it appears that we have failed to find solid evidence that the Taylor rule fundamentals have significant predictive power on the exchange rate.

However, there might be other reasons for these results, and these possible reasons suggest natural avenues for further research. First, in chapter 2 an important factor for forecasting exchange rates was the expectation of future fundamentals, which were generated by an unrestricted VAR in our case. This method was not quite appropriate because the forecasts were based on the interest rate, inflation

rate and output gap, and we omitted the effect of the real exchange rate on these variables because the aim was to forecast exchange rates. However, by using VAR, the forecasts for the fundamentals were less reasonable. A better technique for forecasting the expectations for the fundamentals might be to use better data in future.

Second, Molodtsova and Papell (2009) demonstrate strong evidence of the out-of-sample predictability of Taylor rule fundamentals for 12 out of 15 currencies against the dollar. Thus there is a possibility that our choices of exchange rate (yen/dollar and sterling/dollar) were not very representative for this type of models. Third, we did not fully consider the effect of structural breaks for the Taylor rule model in chapter 4. Chapter 3 showed that the Fed's behaviour had been very unstable over time, which might affect the market's perception on the monetary policy of the central bank and, consequently, affected the exceptions of exchange rate modelling. It could be the case that the Taylor rule model only worked for certain periods of time; however, it was difficult to model this effect in empirical study.

Fourth, the thesis did not consider the microeconomic fundamentals such as order flow in dealing with exchange rate forecasting, which could be a fruitful direction to take. Fifth and finally, the realized data instead of real-time data were used in the thesis, however, there is an argument that the real-time data can improve forecasting power. It might provide different conclusion if the real data is used in the thesis. This thesis did not consider the expectation of the effect of fundamental variables on exchange rate movements, which is also worth investigating. We leave all of these open questions to future researchers.

# Appendices

# Appendix A

## The generalized exchange rate modelling incorporating Taylor rules

$$Home : i_t^h = \gamma_q q_t + \gamma_\pi E_t \pi_{t+1}^h + \rho i_{t-1}^h + \gamma_x x_t^h + \gamma_{\Delta x} \Delta x_t^h + u_{mt}^h \quad (A.1)$$

$$Foreign : i_t^f = \gamma_\pi E_t \pi_{t+1}^f + \rho i_{t-1}^f + \gamma_x x_t^f + \gamma_{\Delta x} \Delta x_t^f + u_{mt}^f \quad (A.2)$$

where  $\gamma_q > 0, \gamma_\pi > 1, \gamma_x > 0, \gamma_{\Delta x} > 0, 0 < \rho < 1$ .

Take home country as an example, the domestic central bank sets the nominal interest rate  $i_t^h$ , to target the deviation of expected inflation from the central bank's target  $E_t \pi_{t+1}^h$ ; the output gap,  $x_t^h$ ; the lagged interest rate,  $i_{t-1}^h$ ; the difference between the output growth and its potential  $\Delta x_t^h$ ; the monetary policy shock  $u_{mt}^h$ ; and the real exchange rate  $q_t$ .  $q_t$  is measured in home currencies per unit of foreign currency. The increase in  $q_t$  means the home currency depreciates.

Note that the foreign country (i.e. the US) follows the similar monetary policy rule; however, "the foreign central bank is passive with respect to exchange rate

fluctuations”<sup>1</sup>. Here followed by EMW07, both the home country and the foreign country have the same policy parameters.  $(\gamma_\pi, \gamma_x, \gamma_{\Delta x}, \rho)$  are the weights on the relative variables in the interest rate rules. These definitions of parameters follow the board interpretation of Taylor rule.

Under the Uncovered interest parity (UIP):

$$i_t^h - i_t^f = E_t s_{t+1} - s_t + \omega \quad (\text{A.3})$$

Where  $E_t s_{t+1}$  is the expected value of next period’s nominal exchange rate.  $s_t$  is the nominal exchange rate in period  $t$ .  $\omega$  is the UIP deviation or risk premium.

EW05 assume that

$$q_t = s_t - \bar{s}_t \quad (\text{A.4})$$

$$\bar{s}_t = p_t^h - p_t^f \quad (\text{A.5})$$

where  $\bar{s}_t$  is a target for the exchange rate.  $p_t^h$  and  $p_t^f$  are the logged home and foreign price level, respectively. So the model also satisfies the purchasing power parity (PPP) according to Equation A.4 and A.5.

Eq A.1–A.2, we get:

$$i_t^h - i_t^f = \gamma_q q_t + \gamma_\pi (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) + \gamma_x (x_t^h - x_t^f) + \rho (i_{t-1}^h - i_{t-1}^f) + \gamma_{\Delta x} (\Delta x_t^h - \Delta x_t^f) + (u_{mt}^h - u_{mt}^f) \quad (\text{A.6})$$

Subtracting  $E_t \pi_{t+1}^h - E_t \pi_{t+1}^f$  on both sides of Eq A.3:

$$i_t^h - i_t^f - (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) = E_t s_{t+1} - s_t - (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) + \omega \quad (\text{A.7})$$

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<sup>1</sup>Engel and West (2006) state, in the empirical study of Clarida et al. (1998), the coefficient of real exchange rate is only statistically significant for Germany, not for the US. Thus, it is reasonable to assume the Fed does not take into account real exchange rate.

Since

$$E_t \pi_{t+1}^h = E_t p_{t+1}^h - p_t^h \quad (\text{A.8})$$

$$E_t \pi_{t+1}^f = E_t p_{t+1}^f - p_t^f \quad (\text{A.9})$$

Put Eq A.8 and A.9 into Eq A.7, and rearrange it, we get,

$$i_t^h - i_t^f - (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) = E_t s_{t+1} - (E_t p_{t+1}^h - E_t p_{t+1}^f) - [s_t - (p_t^h - p_t^f)] + \omega$$

According to Eq A.4 and A.5,

$$i_t^h - i_t^f - (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) = (E_t s_{t+1} - \bar{E}_t s_{t+1}) - (s_t - \bar{s}_t) + \omega$$

$$i_t^h - i_t^f = E_t q_{t+1} - q_t + (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) + \omega \quad (\text{A.10})$$

Put Eq. A.10 into A.6, we get,

$$E_t q_{t+1} - q_t + (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) + \omega = \gamma_q q_t + \gamma_\pi (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) + \gamma_x (x_t^h - x_t^f) + \rho(i_{t-1}^h - i_{t-1}^f) + \gamma_{\Delta x} (\Delta x_t^h - \Delta x_t^f) + (u_{mt}^h - u_{mt}^f)$$

Rearrange it,

$$(1 + \gamma_q) q_t = E_t q_{t+1} + (1 - \gamma_\pi) (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) - \gamma_x (x_t^h - x_t^f) - \rho(i_{t-1}^h - i_{t-1}^f) - \gamma_{\Delta x} (\Delta x_t^h - \Delta x_t^f) - (u_{mt}^h - u_{mt}^f - \omega)$$

Dividing  $1 + \gamma_q$  on both sides of the equation, define

$$b = \frac{1}{1 + \gamma_q},$$

$$Z = (1 - \gamma_q) (E_t \pi_{t+1}^h - E_t \pi_{t+1}^f) - \gamma_x (x_t^h - x_t^f) - \rho(i_{t-1}^h - i_{t-1}^f) - \gamma_{\Delta x} (\Delta x_t^h - \Delta x_t^f) - (u_{mt}^h - u_{mt}^f - \omega)$$

we get:

$$q_t = E_t q_{t+1} + E_t Z_t \quad (\text{A.11})$$

Here,

$$E_t q_{t+1} = b E_t q_{t+2} + b E_t Z_{t+1},$$

$$E_t q_{t+2} = b E_t q_{t+3} + b E_t Z_{t+2},$$

Put them into Eq A.11, we get:

$$q_t = b Z_t + b^2 E_t Z_{t+1} + b^3 E_t Z_{t+2} + \dots b^j E_t Z_{t+j} + b^j E_t q_{t+j+1}, \text{ where } j = 0, 1, 2, 3, \dots$$

Upon imposing the ‘no-bubbles’ condition,<sup>2</sup> that goes to zero as  $j \rightarrow \infty$ , I has the present-value relationship:

$$q_t = b \sum_{j=0}^{\infty} b^j E_t Z_{t+j}, \quad 0 < b < 1, \quad j = 0, 1, 2, 3, \dots \quad (\text{A.12})$$

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<sup>2</sup>“(Rational) bubbles represent a divergence from the equilibrium associated with the market fundamentals. Bubbles could be considered as one possible explanation of the observed volatility of exchange rates.”(Copeland, 2005, p.372) if there is bubble in the function, it can be written as:

$$q_t = b \sum_{j=0}^{\infty} E_t Z_{t+j} + B_t, \quad 0 < b < 1, \quad j = 0, 1, 2, 3, \dots$$

Where  $B_t$  is the bubble at time  $t$ . As long as it persists, the exchange rate deviates from its fundamental equilibrium. In other words, under the “no-bubbles” condition, the exchange rate is the level dictated by the fundamentals.

# Appendix B

## The detailed description of theoretical models

### Model 3: Frenkel-Bilson's flexible-price monetary model

The model is derived from a conventional money demand function and Purchasing Power Parity. The conventional money demand function for the home and the foreign country are:

$$\textbf{Home: } m_t^h - p_t^h = \alpha_1 y_t^h - \alpha_2 i_t^h \quad (\text{B.1})$$

$$\textbf{Foreign: } m_t^f - p_t^f = \alpha_1 y_t^f - \alpha_2 i_t^f \quad (\text{B.2})$$

where  $m_t$  is the log of money supply;  $p_t$  is the log of price level;  $y_t$  is the log of real income;  $i_t$  is the nominal interest rate.  $h, f$  denote home and foreign country, respectively. The money demand function expresses that the demand for real balances will increase with the volume of real incomes ( $\alpha_1 > 0$ ), but decrease with interest rate ( $\alpha_2 > 0$ ). The interest rate refers to the opportunity cost the economy has for holding money. The assumption here is that the agent chooses

between holding money and goods, and the rise in interest rate will increase the opportunity cost of holding money, which will create a temporary excess supply of money and an excess demand for goods, price levels will be driven up and consequently generate inflation.

By subtracting equation B.2 from equation B.1, and moving the money supply to the right-hand side of the equation, we have:

$$p_t^h - p_t^f = (m_t^h - m_t^f) - \alpha_1(y_t^h - y_t^f) + \alpha_2(i_t^h - i_t^f) \quad (\text{B.3})$$

Under the Purchasing Power Parity (PPP),

$$s_t = p_t^h - p_t^f \quad (\text{B.4})$$

In putting the PPP back into the equation B.3, we have

$$s_t = (m_t^h - m_t^f) - \alpha_1(y_t^h - y_t^f) + \alpha_2(i_t^h - i_t^f) \quad (\text{B.5})$$

In the empirical study, we can add a slope coefficient for money supply differential, a constant term, and an error term, so that the general specification of Frenkel-Bilson's flexible model can be written as:

$$s_t = \beta_0 + \beta_1(m_t^h - m_t^f) + \beta_2(y_t^h - y_t^f) + \beta_3(i_t^h - i_t^f) + u_t \quad (\text{B.6})$$

#### **Model 4: Dornbusch -Frankel's sticky-price monetary model**

The main assumption of this model is that the product markets adjust goods prices slowly, whereas the financial markets appear to adjust far more rapidly. The consequence of this assumption is that the financial markets have to over-

adjust to disturbances in order to compensate for the stickiness of prices in the goods markets. The reason is that, with goods prices initially fixed, the increase in money stock will create an instantaneous increase in demand for real balance. If the money market is clear rapidly, based on the demand function (equation B.1), the interest rate will fall and the nominal exchange rate will depreciate as well<sup>1</sup>. In the long run goods prices rise gradually and the change in real balance starts to reverse, driving aggregate demand, interest rate and exchange rate back towards their original level. The new nominal exchange rate only reflects the proportional change in the money supply, as in the Frankel-Bilson's flexible price model.

Dornbusch and Frankel assume that since the financial markets adjust instantaneously, the Uncovered Interest Rate Parity (UIRP) holds at all time. In other words, the fall of UK interest rate will raise the expectation of a future appreciation in sterling.

$$E_t s_{t+1} - s_t = i_t^h - i_t^f \quad (\text{B.7})$$

where  $E_t s_{t+1}$  is the current rate of expected exchange rate at time  $t + 1$ . The forward discount is also a function of the gap between the current spot rate and the equilibrium rate and of the expected long run inflation rate differential between domestic and foreign country.

$$E_t s_{t+1} - s_t = -\theta(s_t - \bar{s}) + E_t \pi_t^h - E \pi_t^f \quad (\text{B.8})$$

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<sup>1</sup>The reason the nominal exchange rate would depreciate after the rise in money supply in the short run can be explained as follows: let us assume that for some reason the UK interest rate falls, then from the UIRP condition we know that market participants become convinced that the pound will appreciate in coming months, in order to compensate for the low interest rate paid on sterling securities. In Dornbusch's world, the fall of interest rate will cause an immediate fall in the pound's value which is below to its long run level (equilibrium exchange rate) so as to generate the expectations of future sterling appreciation as it moves back towards its equilibrium.

where  $\bar{s}$  is the log of long run equilibrium exchange rate,  $E_t\pi_t^h$  and  $E_t\pi_t^f$  are the current rates of expected long run inflation rate.  $\theta$  is the exchange rate expectation elasticity. This equation assumes that in the short run the exchange rate is expected to return to its equilibrium level at a rate which is proportional to the current gap. In the long run, since  $s_t = \bar{s}$ , the change in exchange rate is only affected by the long run inflation rate differential.

Combining equation B.7 and B.8, we get

$$s_t - \bar{s} = -\frac{1}{\theta}[(i_t^h - E_t\pi_t^h) - (i_t^f - E_t\pi_t^f)] \quad (\text{B.9})$$

The expression inside the brackets is real interest rate differential. Assuming PPP holds in the long run,

$$\bar{s} = \bar{p}_t^h - \bar{p}_t^f \quad (\text{B.10})$$

Also, assuming the conventional money demand function holds (equation B.1 and B.2), combining B.10 and B.1, B.2, we get:

$$\bar{s} = \bar{p}_t^h - \bar{p}_t^f = (\bar{m}_t^h - \bar{m}_t^f) - \alpha_1(\bar{y}_t^h - \bar{y}_t^f) + \alpha_2(\bar{i}_t^h - \bar{i}_t^f) + u_t \quad (\text{B.11})$$

According to B.8, when  $s_t = \bar{s}$ ,  $i_t^h - i_t^f = E_t\pi_t^h - E_t\pi_t^f$ , then putting this equation into B.11,

$$\bar{s} = \bar{p}_t^h - \bar{p}_t^f = (\bar{m}_t^h - \bar{m}_t^f) - \alpha_1(\bar{y}_t^h - \bar{y}_t^f) + \alpha_2(E_t\pi_t^h - E_t\pi_t^f) + u_t \quad (\text{B.12})$$

Equation B.12 indicates that the domination of the long run equilibrium exchange rate. The rise in money supply differential raises the equilibrium exchange rate proportionally, and a rise in income or a fall in the expected inflation rate lowers the exchange rate.

Rearranging B.9 and substituting it into B.12, we get the spot rate determination:

$$s_t = (m_t^h - m_t^f) - \alpha_1(y_t^h - y_t^f) - \frac{1}{\theta}(i_t^h - i_t^f) + \left(\frac{1}{\theta} + \alpha_2\right)(E_t\pi_t^h - E_t\pi_t^f) + u_t \quad (\text{B.13})$$

The model for the empirical study can be reduced to:

$$s_t = \beta_0 + \beta_1(m_t^h - m_t^f) + \beta_2(y_t^h - y_t^f) + \beta_3(i_t^h - i_t^f) + \beta_4(E_t\pi_t^h - E_t\pi_t^f) + u_t \quad (\text{B.14})$$

This model allows for a slow domestic price adjustment and consequent deviation from PPP.  $\beta_1 > 0$ , the increase in domestic money supply pushes up the demand for goods, raising domestic price level which, in turn, causes the domestic currency to depreciate, and  $\beta_2 < 0$  because the rise in nominal income will increase money demand, which will lower the exchange rate. This model distinguishes itself from that of Frenkel-Bilson,  $\beta_3 < 0$ , because the change in the nominal interest rate reflects changes in the tightness of monetary policy. “When the domestic interest rate rises relative to the foreign rate it is because there is a contraction in the domestic money supply the higher interest rate at home than abroad attracts a capital inflow, which causes the domestic currency to appreciate instantly.” (Frankel, 1979). The sign of  $\beta_4$  can be quite different based on these assumptions. In Dornbusch (1976)’s overshooting model in terms of  $\beta_4 = 0$  because his theory indicates that the inflation rate differential is very stable, whereas Frenkel (1976) implies that the inflation differential can be very volatile, so that  $\beta_4 > 0$ , the expected inflation differential, captures the effect of the real interest rate on money demand, and thus on the exchange rate. An example of this is when the demand for currency falls when domestic inflation is high relative to the foreign inflation rate, which causes the depreciation of do-

mestic currency.

### **The Fundamental Equilibrium Exchange rate model (FEER model) and The Behavioural Equilibrium Exchange Rate Model (BEER model)**

The FEER and BEER model have been widely discussed in the literature and it is important to understand the differences between them. Clark and MacDonald (1998) have made a detailed comparison of these two models, which will be briefly summarized in this section.

In general, the FEER approach calculates the real exchange rate based on the assumptions that the current account is at full employment level and net capital flow is persistent. Thus, the function of the real exchange rate can be derived from the equation of ‘current account’ being equal to ‘capital account’. The BEER approach, which uses econometric methods to establish the behaviour link between the real exchange rate and relevant economic variables, makes no attempt to deal with internal or external balances.

### **The Fundamental Equilibrium Exchange rate model (FEER approach)**

The FEER concept, first introduced by Williamson (1985), is based on the internal and external balance in the economy in the medium term. Internal balance is identified as the level of output consistent with both full employment and a low and sustainable rate of inflation. External balance is described as the sustainable net capital flow between two countries when they are in internal balance. It abstracts itself from short term fluctuations and focuses on the economic fundamentals, which are likely to be persist over the medium term. The FEER

approach is designed to assess whether the current exchange rate is overvalued or undervalued.

The key function for the FEER approach is that the current account (CA) is equal to the negative of the capital account (KA):

$$CA = -KA \quad (\text{B.15})$$

Furthermore, assuming that the economy approaches both internal and external balance, the sustainable level of a current account is defined as:

$$\bar{CA} = b_0 + b_1 q_t + b_2 \bar{y}_h + b_3 \bar{y}_f = -\bar{KA} \quad (\text{B.16})$$

where  $b_1 < 0$ ,  $b_2 < 0$  and  $b_3 > 0$ .  $q_t$  is the real effective exchange rate, and  $\bar{y}_h$  and  $\bar{y}_f$  are defined as the home and foreign sustainable level of aggregate demand or output.  $\bar{KA}$ , which is the sustainable level of a net capital account, is normally replaced by the difference between desired aggregate saving ( $\bar{S}$ ) and investment at full employment ( $\bar{I}$ ). In rearranging equation B.16, the real effective exchange rate or fundamental equilibrium exchange rate is defined as:

$$FEER = q_t = \frac{-\bar{KA} - b_0 - b_2 \bar{y}_h - \bar{y}_f}{b_1} \quad (\text{B.17})$$

The FEER is consistent with medium-term macroeconomic equilibrium. It has the implicit assumption that the actual real exchange rate will converge into the FEER over time. The FEER approach only characterizes the equilibrium condition and the nature of adjustment is left unspecified.

**The Behavioural Equilibrium Exchange Rate Model (BEER approach)**

The BEER model, developed by Clark and MacDonald (1998), has been widely applied by central banks and financial researchers to assess the extent of misalignment between major world currencies. It can be considered as an alternative to the FEER approach, and uses a reduced form equation of fundamental variables to explain the behaviour of real exchange rate over time.

The starting point of the model is the risk-adjusted interest parity condition,

$$E_t \Delta s_{t+k} = (i_t^h - i_t^f) + \varpi_t \quad (\text{B.18})$$

where  $\varpi_t = \lambda_t + k$  is the risk premium that has a time-varying component,  $\lambda_t$ . Equation B.18 can be converted into a relationship between the real exchange rate and the real interest rate by subtracting the expected inflation differential from both sides of the equation:

$$E_t \Delta s_{t+k} - (E_t \pi_{t+k}^h - E_t \pi_{t+k}^f) = (i_t^h - i_t^f) - (E_t \pi_{t+k}^h - E_t \pi_{t+k}^f) + \varpi_t$$

$$E_t \Delta q_{t+k} = (r_t^h - r_t^f) + \varpi_t$$

$$q_t = E_t q_{t+k} - (r_t^h - r_t^f) - \varpi_t \quad (\text{B.19})$$

where  $r_t = i_t - E_t \pi_{t+k}$  is the *ex ante* real interest rate. Equation B.19 describes the real exchange rate, which is determined by the expected real exchange rate, the real interest rate differential and the risk premium. Clark and MacDonald (1998) assume that the time-varying component of the risk premium is a function of the relative supply of domestic to foreign government debt:

$$\lambda_t = f\left(\frac{gdebt_t^h}{gdebt_t^f}\right)$$

An increase in the domestic debt level relative to foreign debt will increase the risk premium, thus requiring a depreciation of domestic currency.

The expected exchange rate is considered as the long run equilibrium exchange rate ( $\bar{q}_t$ ) in Clark and MacDonalds specification, and three variables are considered to be the determination of the equilibrium exchange rate. The details of the selection of these variables are discussed in Faruquee (1995) and MacDonald (1998):

$$\bar{q}_t = E_t q_{t+k} = f(tot_t, tnt_t, nfa_t) \quad (\text{B.20})$$

where  $tot_t$  is the terms of trade,  $tnt_t$  is the Balassa-Samuelson effect, which refers to the relative price of non-traded to traded goods, and  $nfa_t$  is the net foreign assets. The increase in value of these variables will lead to an appreciation in home currency. These three determinants of exchange rate will be explained as follows:

**Terms of trade (TOT):** ‘Terms of trade’ refers to the price of exports divided by the price of imports. It indicates the quantity of import goods that can be purchased through the sale of exportable goods. The impact of TOT on real exchange rate is through the income and the substitution effect. In other words, the deterioration of TOT decreases in the level of TOT and generates a negative income effect through the decline of domestic purchasing power, offsetting the demand of non-traded domestic goods. This leads to a decline in overall price level and depreciation of the exchange rate. On the other hand, the weakening of TOT induces a positive substitution effect as imported goods appear relatively more expensive than non-traded domestic goods, and thus consumers

tend to purchase more domestic goods, which will raise the overall price level and a real appreciation in the home currency. The total effect of TOT depends on the strength of substitution and income effect. Recent studies suggest that the income effect is dominant and that, therefore, an improvement in TOT will lead to a real appreciation of the real exchange rate.

**Balassa-Samuelson effect (B-S effect):** The Balassa-Samuelson effect (or Balassa-Samuelson hypothesis), introduced independently by Balassa (1964) and Samuelson (1964), explains the main driving force behind the real appreciation of exchange rate. Their analysis is based on the observation that, at least in the industrialized world, productivity growth rises more rapidly in traded than in non-traded goods sectors. Thus, wages in the traded goods sector will tend to rise more rapidly than in the non-traded goods sector. If the labour market is integrated, wages in the non-tradable sector will tend to rise to the same level as in the tradable sector, even though their productivity has been slower. This means that the prices in the non-tradable sector will rise relative to those in the traded goods sector in order to maintain profitability. Thus, when productivity growth in one country is higher than in the other, inflation will be higher in the former. Hence the CPI-based real exchange rate is likely to appreciate in the long run. The Balassa-Samuelson effect also offers an explanation why consumer price levels in richer countries are systematically higher than in poorer ones. The B-S effect requires that PPP holds:

$$q_t = s_t - p_t^f + p_t^h \quad (\text{B.21})$$

where  $q_t$  denotes the real exchange rate, and  $s_t$  denotes the nominal spot exchange rate, defined as the foreign currency price of a unit of home currency. A similar relationship can be defined for the price of traded goods:

$$q_t^T = s_t^T - p_t^{fT} + p_t^{hT} \quad (\text{B.22})$$

where  $T$  indicates that the variables are defined for traded goods.

The general price level can be divided into traded and non-traded components:

$$p_t^h = (1 - \alpha_t^h)p_t^{hT} + \alpha_t p_t^{hNT} \quad (\text{B.23})$$

$$p_t^f = (1 - \alpha_t^f)p_t^{fT} + \alpha_t p_t^{fNT} \quad (\text{B.24})$$

where  $\alpha_t$  denotes the weights of non-traded goods in the economy, which are assumed to be time-varying, and  $NT$  denotes a non-traded good. Substituting B.23 and B.24 into B.21, we have:

$$q_t = s_t - (1 - \alpha_t^f)p_t^{fT} - \alpha_t p_t^{fNT} + (1 - \alpha_t^h)p_t^{hT} + \alpha_t p_t^{hNT}$$

And rearranging it:

$$q_t = (s_t - p_t^{hT} + p_t^{fT}) + \alpha_t^f(p_t^{fT} - p_t^{fNT}) - \alpha_t^h(p_t^{hT} - p_t^{hNT})$$

Substituting B.22 into the above equation:

$$q_t = (s_t - s_t^T + q_t^T) + \alpha_t^f(p_t^{fT} - p_t^{fNT}) - \alpha_t^h(p_t^{hT} - p_t^{hNT})$$

MacDonald (1998) assumes the long-run equilibrium real exchange rate,  $\bar{q}_t$ , may be defined as:

$$\bar{q}_t = q_t - s_t + s_t^T$$

Therefore,

$$\bar{q}_t = q_t^T + \alpha_t^f(p_t^{fT} - p_t^{fNT}) - \alpha_t^h(p_t^{hT} - p_t^{hNT}) \quad (\text{B.25})$$

Equation B.25 indicates that the long-run real equilibrium exchange rate is determined by three components: the real exchange rate for traded goods, which will arise if the traded goods are not perfect substitutes; movements in the relative prices of traded goods and non-traded goods between the home and the foreign country.

Productivity differences can induce a bias in the real exchange rate because productivity growth rises more rapidly in the tradable sector than in the non-tradable sector. If wages tend to be equalized in the market, and wages are linked to the price of goods, then the relative price of traded goods to non-traded goods tends to rise less rapidly for a country with relatively higher productivity. Assuming the law of one prices holds and the home country is a relatively fast growing country, equation B.22 will have a positive  $\alpha_t^f(p_t^{fT} - p_t^{fNT}) - \alpha_t^h(p_t^{hT} - p_t^{hNT})$  term, thereby, pushing  $\bar{q}_t$  above  $q_t^T$ . Therefore, the equations above demonstrate the rationale that the real exchange rates of fast growing countries are likely to appreciate over time.

**Net Foreign Assets (NFA):** Net foreign assets are the difference between total foreign assets and total foreign liabilities. These are important determinants of the real exchange rate and influence it through various channels (Faruquee, 1995; Lane and Milesi-Ferretti, 2001; Villavicencio and Bara, 2006). The portfolio-balances theory suggests that a deficit in the current account creates an increase in the net foreign debt of a country, given the interest rate, and that addressing the deficit can only be achieved by a depreciation in domestic currency. The

depreciation will help the country to increase the international competitiveness of its exports and improve its current account.

After addressing the determinants suggested by Faruquee (1995) and MacDonald (1998), according to the BEER approach, the real exchange rate is determined by:

$$BEER = f(r_t^h - r_t^f, \frac{gdebt_t^h}{gdebt_t^f}, tot_t, tnt_t, nfa_t) \quad (B.26)$$

Compared with the FEER approach, the BEER approach chooses to use the actual values of fundamental determinants of the real exchange rate instead of using domestic and foreign output at their potential level. Thus, the BEER approach can be considered as the best method to use for forecasting exchange rates.

### **Molodtsova and Papell's model**

Molodtsova and Papell (2009) examine out of sample exchange rate predictability using Taylor rule fundamentals.

Following Clarida et al. (1998), they declare that both the home country and the foreign country follow a similar Taylor rule.

$$i_t^h = (1 - \rho^h)(\gamma_\pi \pi_t^h + \gamma_x x_t^h) + \rho^h i_{t-1}^h + u_{mt}^h \quad (B.27)$$

$$i_t^f = (1 - \rho^f)(\gamma_q q_t + \gamma_\pi \pi_t^f + \gamma_x x_t^f) + \rho^f i_{t-1}^f + u_{mt}^f \quad (B.28)$$

The Taylor rule for the home country is subtracted from the rule for the foreign country, the equation having the interest rate differential on the left-hand side and a series of fundamentals on the right-hand side.

$$i_t^h - i_t^f = \alpha - \alpha_q q_t + \alpha_\pi^h \pi_t^h - \alpha_\pi^f \pi_t^f + \alpha_x^h x_t^h - \alpha_x^f x_t^f + \rho^h i_{t-1}^h - \rho^f i_{t-1}^f + \eta$$

where  $\alpha$  is a constant,  $\eta$  is the error term,  $\alpha_\pi = (1 - \rho)\gamma_\pi$ ,  $\alpha_x = (1 - \rho)\gamma_x$  and  $\alpha_q = (1 - \rho)\gamma_q$ .

Under the assumption of Uncovered Interest rate Parity (UIRP) and rational expectations, a rise in the domestic interest rate will cause immediate appreciation and a forecasted depreciation in the home currency. However, Molodtsova and Papell argue that UIRP is not held in the empirical study, so that in their model the rise in interest rate for the home country will cause a immediate appreciation and a forecasted appreciation<sup>2</sup>. Therefore, Molodtsova and Papell find a link between exchange rates and Taylor rule fundamentals.

$$\Delta s_{t+1} = \alpha_0 + \alpha_q q_t - \alpha_\pi^h \pi_t^h + \alpha_\pi^f \pi_t^f - \alpha_x^h x_t^h + \alpha_x^f x_t^f - \rho^h i_{t-1}^h + \rho^f i_{t-1}^f - \eta \quad (\text{B.29})$$

where  $s_t$  is the log of the nominal exchange rate which is determined by the home currency price of a unit of foreign currency, so that an increase in  $s_t$  is a depreciation of domestic currency.<sup>3</sup> This model describes how anything that causes a rise in domestic interest rate (or a fall in foreign interest rate) will cause both an immediate appreciation and a forecasted appreciation in the home currency.

One significant difference between Molodtsova and Papell's paper and those of others is that they consider a variety of specifications. Firstly, in the original Taylor rule model (Taylor, 1993a), the central bank sets their interest rate based on real interest rate, inflation rate, output gap, and inflation gap, which is the difference between inflation rate and potential inflation. If both the home and

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<sup>2</sup>This assumption is different from that of EW06, EMW07 and Mark(2009) who believe that an Uncovered Interest Rate Parity is held in the long run.

<sup>3</sup>In Molodtsova and Papell's model, US is the home country, UK is the foreign country.

foreign countries follow this rule, and the foreign country does not take the real exchange rate ( $\gamma_q$ ) into account, then the exchange rate model is a symmetric model. If the foreign country also considers that the real exchange rate, the model is asymmetric. Secondly, following CGG, Molodtsova and Papell consider the model with smoothing ( $\rho^h, \rho^f > 0$ ) and without smoothing ( $\rho^h = \rho^f = 0$ ), and the model can be symmetric or asymmetric. Thirdly, the two central banks can have the same coefficients in their interest rate reaction functions so that a homogeneous model is constructed where the fundamental differentials have the same coefficients ( $\alpha_\pi^h = \alpha_\pi^f, \alpha_x^h = \alpha_x^f$ ). If banks do not respond identically to changes in the inflation and output gap ( $\alpha_\pi^h \neq \alpha_\pi^f, \alpha_x^h \neq \alpha_x^f$ ), a heterogeneous model is derived. Both homogeneous and heterogeneous models can be symmetric and asymmetric. Fourth, if the two central banks have identical target inflation rate and real equilibrium interest rate then there is no constant in the exchange rate model ( $\alpha_0 = 0$ ); otherwise, a constant is included on the right-hand side of the model ( $\alpha_0 \neq 0$ ). Likewise the model, with or without a constant, can be symmetric or asymmetric.

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