Incidence and Growth of Patent Thickets - The Impact of Technological Opportunities and Complexity

Georg von Graevenitz,[*] Stefan Wagner,[†] Dietmar Harhoff[‡]

July 26, 2012

Abstract

This paper analyzes incidence and evolution of patent thickets. The paper provides a modeling framework showing how competition for patent portfolios, complementarity of patented technologies and hold-up affect patenting incentives. It is shown that lower technological opportunity increases patenting in complex technologies, while reducing patenting in discrete technologies. Also, more competitors increase patenting in complex technologies and reduce it in discrete technologies. These predictions are tested using European patent data. A new measure of technological complexity is applied for the first time. The empirical analysis is based on a panel capturing patenting behavior of 2074 firms in 30 technology areas over 15 years. GMM estimation results confirm the predictions of the preferred theoretical model. The results show that patent thickets exist in 9 out of 30 technology areas. Also, decreased technological opportunity is found to be a surprisingly strong driver of patent thicket growth.

JEL: L13, L20, O34.$s$kip

Keywords: Patenting, Patent thickets, Patent portfolio races, Complexity, Technological Opportunities.

Acknowledgements: We thank Sven Feldmann, David Ulph, Tomaso Duso, Reiko Aoki, Sadao Nagaoka, Rene Belderbos, Dirk Czarnitzki, Georg Licht, Mark Schankerman, Bronwyn Hall and Joachim Winter for comments. Vincenzo Denicolò and Konrad Stahl provided detailed comments on the manuscript. Helmut Küchenhoff and Fabian Scheipl provided feedback on the model. Participants at the 64th ESEM, the 15th Panel Data Conference, the Knowledge for Growth Conference, the 2008 CRESSE Conference, the 3rd ZEW conference on the Economics of Innovation and Patenting, the 2008 workshop of the SFB TR/15 in Gummersbach and at seminars at NUS, HKUST, Melbourne Business School, ETH Zurich, EPFL, Tokyo, Leuven and Ecole des Mines provided feedback. Bronwyn Hall supplied code to consolidate applicant names. This paper was written with the support of the SFB Transregio 15 financed by the DFG. The usual disclaimer applies.

[*] Georg von Graevenitz, University of East Anglia, Norwich Business School, 102 Middlesex Street, London, E1 7EZ, g.graevenitz@uea.ac.uk

[†] Stefan Wagner, ESMT European School of Management and Technology, Schlossplatz 1, 10178 Berlin, stefan.wagner@esmt.org

[‡] Dietmar Harhoff, Ludwig Maximilians University, Munich School of Management, INNO-tec, Kaulbachstraße 45, D-80539, Munich, harhoff@lmu.de

1 Introduction

Strong increases in the level of patent applications have been observed at the United States Patent and Trademark Office (USPTO) (Kortum and Lerner, 1998, Hall, 2005a) and the European Patent Office (EPO) (von Graevenitz et al., 2007). These "patent explosions" pose serious challenges for existing patent systems and also for competition authorities.[1]

Existing explanations for this shift in patenting behavior focus on changes in the legal environment and management practices, the complexity of some technologies, greater technological opportunities and increased strategic behavior of firms. The existing literature shows that most of these factors play a role. Currently there are no models of patenting behavior integrating these determinants.[2] This paper sets out a modeling framework encompassing several of the explanations noted above and an empirical test of the main predictions.

Kortum and Lerner (1998, 1999) investigate the explosion of patenting at the USPTO which began around 1984 (Hall, 2005a). By a process of elimination they argue that increased patenting mainly results from changed management practices making R&D more applied and raising the yield of patents from R&D. Kortum and Lerner (1998, 1999) and Hall and Ziedonis (2001) also explore whether enhanced fertility of R&D led to an increase in patent filings, but cannot find systematic evidence for this. Hall and Ziedonis (2001) provide evidence that the patenting surge is a strategic response to an increased threat of hold-up in complex technologies in which products depend on the combination of large numbers of patents. Complexity of a technology implies that patents are complements, and therefore hold-up opportunities arise once patent ownership is dispersed (Shapiro, 2001, Ziedonis, 2004). Hall (2005a) shows that the patenting explosion at USPTO is driven by firms whose main technologies are complex.

The large volume of patenting in complex technologies and the notion of patent portfolio races (Hall and Ziedonis, 2001) suggests that a strategic complementarity between rival firms' patenting efforts exists in these technologies. This paper introduces a modeling framework for patenting that encompasses complex and discrete technologies to analyze how strategic complementarities arise. Within the framework we model competition for patents through two channels of strategic interaction: a legal channel capturing hold-up and a channel capturing technological complementarity. We confirm that in complex technologies patenting by one firm can induce strategic complementarity by increasing the marginal benefit of patenting for other firms. Not surprisingly, technological complementarity strengthens this effect further. Surprisingly, we also show that hold-up can undermine these strategic complementarities. Also, we show that the comparative statics of patenting in complex technologies are the opposite to those in discrete technologies: in a complex technology firms patent *less* in response to

[1] For extensive discussions of the policy questions surrounding current functioning of the patent systems in the United States and in Europe refer to National Research Council (2004), Federal Trade Commission (2003), Jaffe and Lerner (2004), von Graevenitz et al. (2007) and Bessen and Meurer (2008).

[2] Formal models of patenting abound, for a survey of this literature refer to Scotchmer (2005) or Gallini and Scotchmer (2002). Formal models of patenting in patent thickets do not attempt to span both complex and discrete technologies as we do here: Bessen (2004), Clark and Konrad (2008) and Siebert and von Graevenitz (2010). These models build on the patent race literature pioneered by Loury (1979), Lee and Wilde (1980), Reinganum (1989) and Beath et al. (1989).

greater technological opportunity and *more* if more other firms compete for patents.

The paper also provides an empirical analysis of patenting at the EPO. This is based on a comprehensive data set covering the years 1987 to 2002. The analysis reveals that restrictions derived from our preferred model of patenting cannot be rejected, while alternative models of patenting, that fall into our modeling framework, can be rejected. In undertaking the analysis the high persistence of patenting is taken into account and we employ system GMM estimators (Blundell and Bond, 1998, Arellano, 2003, Alvarez and Arellano, 2003) to deal with resulting problems of endogeneity. The results indicate that decreasing technological opportunities had a surprisingly strong effect on the rise of patent filings in Europe since the mid-1980s.

In the empirical analysis we apply a measure of complexity introduced by von Graevenitz et al. (2011). This is a count of the number of groups of three firms, within which each firm holds patents limiting new patents of each of the other two. They refer to such a group of firms as a *triple*. Below we explain why triples can be expected to arise more frequently in complex technologies. von Graevenitz et al. (2011) validate the measure by showing that triples arise much more frequently in technologies classified as complex by Cohen et al. (2000). Using the measure, we show in this paper, that patent thickets currently exist in 9 of the 30 technology areas making up the patent system.

The remainder of this paper is structured as follows. Section 2 sets out a theoretical modeling framework for the analysis of patenting strategies. We derive five empirically testable hypotheses from this modeling framework. In Section 3 we describe our data set and the variables we employ. Section 4 provides a descriptive analysis of cross industry patenting trends at the EPO. In Section 5 we discuss the empirical model and results. Section 6 concludes.

2 A Model of Patenting

Here we present a modeling framework to analyze patenting behavior. The framework allows us to test a number of models of patenting behavior that differ in the assumptions we make about the value and the costs of patenting. In each model we analyze how technological opportunity and complexity of technology affect the levels of patenting set by firms.

In this section we begin by motivating the modeling framework and discussing assumptions. Then, we provide a number of definitions. Next, we solve the main model and discuss results derived from alternative models. Then, we present several predictions. These underpin the empirical results presented in Sections 4 and 5 below.

2.1 Motivation

The patent system covers a multitude of different technology areas. Within these we posit distinct technological opportunities that derive from separate research efforts. Each technological opportunity consists of one or more patentable facets. Every facet corresponds to a potential patent. Technologically related facets are grouped together in technological opportunities because they derive from the same knowledge and science base.

The underlying model of R&D and of the patent office is kept as simple as possible: in each technology area firms select how many opportunities to research and how many facets of each to seek to patent. Facets and opportunities are chosen randomly by firms. Where more than one applicant applies for a facet, it is randomly assigned to one applicant. Patent allocation is the sole function of the patent office in the modeling framework. The main feature of this framework is competition of firms for granted patents. The model of technological opportunities and facets can be presented as a matrix of patents that firms compete for:

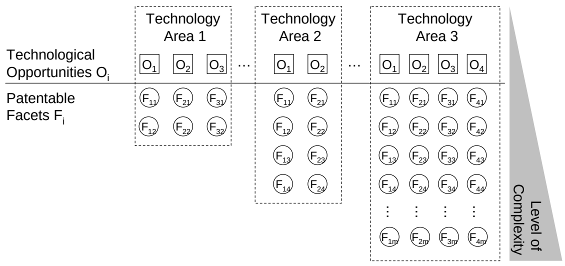

Figure 1: Complexity and the number of patentable facets per technological opportunity.

Figure 1 shows different matrices corresponding to technology areas with growing levels of complexity and varying levels of technological opportunity. Complexity increases with the number of facets. With higher complexity it is increasingly probable that ownership of patents in a technological opportunity becomes dispersed. Then the value of owning patents depends on the relative size and composition of firms' patent portfolios. Note, that here technological-opportunity and -complexity are assumed to be fixed in the short to medium term.[3]

Consider two polar examples: first, one patent generally suffices for the applicant to protect an ethical drug effectively against attempts to invent around the patent. This is the case of a discrete technology in which each patent covers one technological opportunity. Second, laser technology is used in a very wide range of applications such as eye surgery (e.g. LASIK) or pollution monitoring and forestry management (LIDAR). This is the case of a complex technology area within the field of optics.[4] Each application of laser technology can be thought of as a technological opportunity requiring a range of different patentable inventions that are combined in a functioning product.[5] Due to the complexity of the technology hold-up may

[3] In the long run technological opportunity may be affected by firms' patenting efforts. Unravelling this question will require a separate study with data on firms' R&D activities over a very long period.

[4] To further clarify the definitions of opportunities, facets and technology areas we discuss the example of LED technology at greater length in Appendix D.

[5] A product using laser technology will usually also embody some patents relating to different technological areas outside optics. We do not model this aspect of complexity to keep our model manageable.

arise: in the case of LASIK there was a string of court cases between VISX Inc. and Nidek Inc. after 1998 regarding infringement of VISX patents on LASIK. The companies finally settled their disputes world wide in April of 2003.

In a complex technology it is unlikely that any one firm holds all patents necessary for the commercialization of a product such as a laser for eye surgery. This implies that the value of firms' patent portfolios depends on the size and composition of rival firms' portfolios. Strategic interdependencies arise, which we model below. We model two channels for these interdependencies: a technological channel and a legal channel. To clarify how these two channels affect our results we present an encompassing model in the main text and three alternative models in the appendix. The alternative models isolate the channels we incorporate in the main model. All models are based on the same modeling framework and differ only in the functional relationships describing technological and legal interdependencies.

The technological channel is based on technological complementarity: firms owning patents on a complex technology benefit from cooperation with rivals. For instance, firms may jointly set a standard for a technology helping them to license this technology (Shapiro, 2001, Scotchmer, 2005, Lerner and Tirole, 2004). The technological channel is captured in the model by allowing the value of patenting to increase in all firms' patents. We find that under fragmentation[6] of patent ownership, firms' patenting choices are strategic complements.

Next, consider the legal channel: patenting firms often resort to litigation to resolve their disputes over the distribution of profits flowing from a complex technology. In such disputes, as in many cooperative settings, the size of the patent portfolio firms hold is an important determinant of the outcome of the dispute (Grindley and Teece, 1997, Shapiro, 2001, Federal Trade Commission, 2003). To capture this effect we also model strategic interaction through legal and bargaining costs. We allow for separate effects of the absolute and the relative size of firms' patent portfolios. The relative size of patent portfolios, per technological opportunity, is shown to determine whether or not strategic complementarity arises through legal costs. Surprisingly, we find that under realistic assumptions, competition for a larger share of patents undermines strategic complementarity.

The main model, set out below, contains both of these channels of strategic interdependence. In the analysis of this model we concentrate on identifying conditions leading to strategic complementarity. Only under strategic complementarity will all firms caught in a patent thicket have incentives to increase patenting efforts at the same time. This is the outcome described in the existing empirical literature on patent thickets.

2.2 Assumptions

We study a setting in which a technology area is characterized by (O) technological opportunities each of which consists of patentable facets. A technological opportunity is an independent source of profit to a firm and each facet is a separate patentable invention which is part of the

[6] Ziedonis (2004) provides empirical evidence for existence and strategic effects of fragmentation.

opportunity. The total number of patentable inventions (facets) offered by a technological opportunity is F. Thus a technology area is discrete if $F = 1$. We assume that:[7]

All technological opportunities in a technology area are symmetrical; they offer the same number of facets, and costs of R&D and of patenting are identical. (S)

The total set of patentable inventions in a technology consists of $\Omega = O \times F$ facets. As F grows the underlying technology grows more complex. If there is more technological opportunity, O grows. Variation in the two dimensions of the set of available patents Ω arises for different reasons. Current efforts in basic R&D open additional new opportunities in the future, raising O. The number of facets which are patentable on a given opportunity depends mainly on the nature of technology but also on institutional and legal factors.

Each technological opportunity is associated with a maximal total value $V(F)$ and an attained value $V(\tilde{F})$. The attained value depends on the number of facets of the opportunity patented by all firms $\tilde{F}$,[8] where $\tilde{F} \leq F$. To capture the complementarity of inventions in complex technologies we assume that the value of the technological opportunity increases in the number of facets patented by all firms: $\tilde{F}$.[9] While facets are complements, they are not perfect complements in this model: licenses to the set of patented facets will allow firms to assemble a viable product which has value $V(\tilde{F}) < V(F)$.[10] We impose the following conditions on the value function:

$$V(0) = 0, \quad \frac{\partial V}{\partial \tilde{F}} > 0 \quad \text{and} \quad \frac{\partial^2 V}{\partial \tilde{F}^2} > 0 \quad . \tag{VF}$$

Define the elasticity of the value of a technological opportunity (V) with respect to covered patents ($\tilde{F}$) as $\mu \equiv \frac{\partial V}{\partial \tilde{F}} \frac{\tilde{F}}{V}$.

There are $N + 1$ firms active in a given technology area. Each can apply for patent protection for all facets of a technological opportunity. A firm's strategy consists of the number of opportunities o_k ($o_k \in [0, O]$) it invests in and the number of facets f_k ($f_k \in [0, F]$) per opportunity which it seeks to patent. Subscripts index the firm. Each firm can only make one patent application per facet and it can only patent in technological opportunities which it has researched. The firm trades off patenting more facets per opportunity and patenting in more different technological opportunities. The share of granted patents per opportunity $\hat{s}_k$ that a firm obtains from the patent office, determines its share of the attained value $V(\tilde{F})$ per opportunity. The expected value (s_k) of this share depends on rivals' patenting efforts through the probability (p_k) of obtaining a facet: $s_k \equiv p_k f_k \big/ \tilde{F}$. We define and discuss the probability (p_k) in detail in the following section (Equation 4) and derive it formally in Appendix A.3.

[7]Note that this assumption rules out aspects of complexity that may be quite important in practice. Thus we rule out that some facets may belong to more than one technological opportunity, making patents on them particularly valuable blocking patents. We leave this aspect of complexity for future work.

[8]We define this value precisely in equation (2) in the following subsection.

[9]A similar assumption is made by Lerner and Tirole (2004). In Appendix B.4 we show that this assumption is one way of introducing supermodularity into the game we study. In sections B.2 and B.3 we analyze two models in which the value obtained by the firm depends only on patents granted to it: $V(p_k f_k)$.

[10]In contrast Clark and Konrad (2008) analyze a setting in which patents in complex technologies are perfect complements

While patenting facets is assumed to be costless,[11] a maintenance fee is payable (C_a) on granted patents. Additionally, firms must undertake costly R&D (C_o) on each technological opportunity they turn to. Finally, costs of coordinating separate research projects (C_c) are generally viewed as significant in the literature (Roberts, 2004). To summarize:

i *Per opportunity a firm invests in, it faces costs of R&D:* C_o*.*

ii *Per granted patent a firm faces costs of maintaining that patent:* C_a*.*

iii *The coordination of R&D on* different *technological opportunities imposes costs* $C_c(o_k)$*. Therefore, we assume that* $\frac{\partial C_c}{\partial o_k} > 0$*.* (FVC)

As the number of facets per technological opportunity grows, so does the probability that different firms own patents belonging to one opportunity. Hold-up becomes increasingly likely. Then, firms need to disentangle ownership rights, giving rise to legal costs (LC). These encompass the costs of monitoring, licensing, and negotiating settlements as well as court fees. In modeling legal costs we distinguish between the expected absolute size of a firm's patent portfolio ($\gamma_k \equiv f_k p_k$) and the firm's expected share of patents granted per opportunity s_k.

To capture costs of litigation associated with additional patents, we assume that legal costs increase in the absolute size of patent portfolios. However, the marginal cost of owning further patents is assumed to be decreasing in the share of patents owned by the firm, due to increased bargaining power. For bargaining what matters is the relative size of firms' patent portfolios. To capture the two mechanisms through which patenting affects legal costs we assume that:

$$L(\gamma_k, s_k), \text{ where } \frac{\partial L}{\partial \gamma_k} > 0, \frac{\partial^2 L}{\partial {\gamma_k}^2} \geq 0, \frac{\partial L}{\partial s_k} \leq 0, \frac{\partial^2 L}{\partial {s_k}^2} \geq 0, \frac{\partial^2 L}{\partial \gamma_k \partial s_k} = 0 \quad . \tag{LC}$$

These assumptions are further discussed in Appendix B.3.

Note that the modeling framework alluded to above subsumes all assumptions discussed here apart from the assumption on the value function (VF) and the legal costs function (LC). It also contains all definitions set out in the following section. This framework is the same for all models discussed below.

We assume throughout that the levels of N, O, F and V are known by all patenting firms.

2.3 Definitions

This subsection sets out a number of definitions that follow from our previous assumptions. Given that the number of firms N is common knowledge, firms can compute the expected number of rivals active within a technological opportunity, the expected number of facets on which patents are granted and the likelihood of obtaining a patent grant.

[11] We make this assumption in order to simplify the model, but it can be shown that it does not affect our results if patent filing costs are sufficiently low in comparison to the costs of maintenance. In practice, initial application and examination fees for patents are indeed much lower than post-grant translation and renewal fees, since most patent offices cross-subsidize the initial stages in order to encourage patent filing.

The expected number of rivals (N_O) competing for patents within a technological opportunity is derived in Appendix A.1. There we show that the expected number of rivals decreases in the level of technological opportunity (O) and increases in rivals' R&D investments (o_j):

$$\frac{\partial N_O}{\partial O} < 0 \quad \text{and} \quad \frac{\partial N_O}{\partial o_j} > 0 \quad . \tag{1}$$

To simplify notation we define the share of facets each firm k applies for per technological opportunity as $\phi_k \equiv f_k/F$. Given our simplified model of the patent application process the expected number of facets per technological opportunity on which patents are granted is:

$$\tilde{F}(f_k, \mathbf{f}_{\not{k}}, F, N_O(O, \mathbf{o}_{\not{k}}, N)) = F\left[1 - (1-\phi_k)\prod_{j\neq k, j=1}^{N_O}(1-\phi_j)\right] \quad , \tag{2}$$

where $\mathbf{f}_{\not{k}}, \mathbf{o}_{\not{k}}$ are vectors containing the choices of the number of facets and the number of opportunities to invest in, made by all rival firms ($_j$). This expression results from the assumptions that firms randomly choose facets and that the patent office randomly selects which application to grant. This model of patenting captures coordination failure and duplication of applications by firms. Here, the number of facets covered by at least one applicant is one minus the number of facets attracting no applications. In Appendix A.2 we show that the number of facets covered increases in the complexity of a technology, in the number of rivals investing in a technological opportunity and in the number of facets each firm invests in:

$$\frac{\partial \tilde{F}}{\partial F} > 0 \quad , \quad \frac{\partial \tilde{F}}{\partial N_O} > 0 \quad \text{and} \quad \frac{\partial \tilde{F}}{\partial f_k} > 0 \quad , \quad \frac{\partial \tilde{F}}{\partial f_j} > 0. \tag{3}$$

We also define and bound the elasticities $\epsilon_{\tilde{F}F} \equiv \frac{\partial \tilde{F}}{\partial F}\frac{F}{\tilde{F}}$ and $\epsilon_{\tilde{F}f_k} \equiv \frac{\partial \tilde{F}}{\partial f_k}\frac{f_k}{\tilde{F}}$ in Appendix A.2.

We assume that the patent office will grant each application for a patent on a facet with equal probability, but only grants one patent overall on the facet.Then the probability of patenting a facet depends on the expected number of rivals seeking to patent each facet and the probability with which the particular number of rivals occurs. In Appendix A.3 we show that the probability that firm $_k$ obtains a patent on a given facet is:

$$p_k(\mathbf{f}_{\not{k}}, F, N_O(O, \mathbf{o}_{\not{k}}, N)) = \sum_{i=0}^{N_O} \frac{1}{i+1}\binom{N_O}{i}\prod_{j=0}^{N_O-i}(1-\phi_j)\prod_{l=0}^{i}\phi_l \quad . \tag{4}$$

This expression shows that the probability of obtaining a patent on an application is a sum of weighted probabilities. Each element of the sum consists of the weighted probability of obtaining a patent $1/(1+i)$ given the number of rival firms also seeking a patent on the facet i. The weight captures the probability of observing a given number of rivals. In Appendix A.3 we show that the probability of obtaining a patent decreases in the level of facets rival firms seek to patent and in the number of rival firms per technological opportunity:

$$\frac{\partial p_k}{\partial \phi_j} < 0 \quad \text{and} \quad \frac{\partial p_k}{\partial N_O} < 0 \quad . \tag{5}$$

2.4 Results

In this section we set out a firm's objective function and the patenting game it is involved in. We analyze this game, show when it is supermodular and derive comparative statics results.

Given symmetry of technological opportunities (Assumption S) the expected value of patenting for firm k in a technology area is:

$$\pi_k(o_k, f_k) = o_k \left(V(\tilde{F}) s_k - L(\gamma_k, s_k) - C_o - f_k p_k C_a \right) - C_c(o_k) \quad . \tag{6}$$

Firms derive revenues from each technological opportunity and face costs of coordinating R&D across different technological opportunities (C_c). Profits per technological opportunity depend on the share of patents granted (s_k), legal costs (L) as well as costs of R&D on the technological opportunity (C_o) and costs of maintaining granted patents (C_a).

Define a game G in which:

- There are $N + 1$ firms.
- Each firm simultaneously chooses the number of technological opportunities $o_k \in [0, O]$ and the number of facets applied for per opportunity $f_k \in [0, F]$, to maximize the payoff function π_k. Firms' strategy sets S_n are elements of R^2.[12]
- Firms' payoff functions π_k, defined in equation (6), are twice continuously differentiable and depend only on rivals' aggregate strategies.
- Assumptions (VF) and (LC) describe how the expected value and the expected cost of patenting depend on the number of facets owned per opportunity.

Firms' payoffs depend on their rivals' aggregate strategies because the probability of obtaining a patent on a given facet is a function of all rivals' patent applications. Note that the game is symmetric as it is exchangeable in permutations of the players. This implies that symmetric equilibria exist, if the game can be shown to be supermodular (Vives, 2005).[13]

In this game firms compete for granted patents on a technological opportunity. They pick a certain number of technological opportunities and apply for patents on a share of the facets in each opportunity. As rival firms' applications increase, the probability of receiving a patent grant on each application decreases. In a discrete technology this reduces incentives to patent as the expected value and the expected costs of each patent are the same. In a complex technology the expected values and costs of marginal patents may change relative to one another

[12] We treat o_k and f_k as continuous real numbers in the paper. Both determine probabilities: that a firm will invest in specific technological opportunities in case of o_k or facets in case of f_k. These probabilities are defined in Appendix A.

[13] Note also that only symmetric equilibria exist as the strategy spaces of players are completely ordered.

as rivals patent more or less and this can induce strategic complementarity. We derive the conditions under which strategic complementarity arises in game G from Equations (33) and (34) in Appendix B.1. These are summarized in the following proposition:

Proposition 1

The game G, defined in particular by assumptions (VF) and (LC), is smooth supermodular if $\mu > \frac{\partial L}{\partial s_k}\frac{1}{V}$ and if ownership of the technology is expected to be fragmented.

Next to the assumptions cited, this proposition contains two conditions for supermodularity:

- $\mu > \frac{\partial L}{\partial s_k}\frac{1}{V}$ is a lower bound on the elasticity of the value function, implying the complementarity of facets within a technological opportunity must be sufficiently strong.
- Fragmentation arises if no firm is seeking to patent more than half of the facets per opportunity and if there are many patenting firms per opportunity.

Where game G is supermodular we characterize its comparative statics below. Otherwise, the comparative statics cannot be analyzed with the same degree of generality.[14] In the case of a discrete technology we need only slightly more restrictive assumptions, than those we make here. In that case there is only one facet $(F = 1)$ per technological opportunity so that firms only optimize over the number of opportunities. We characterize this important special case at the end of this section, as its comparative statics differ from those of game G.

To prove Proposition 1 we show in Appendix B.1 that firms' profit functions are supermodular (i) in their own actions and (ii) in every combination of their own actions with those of rival firms (Milgrom and Roberts, 1990, Vives, 1999, 2005, Amir, 2005). This is the case if the cross-partial derivatives between own as well as own and rival actions are positive, indicating that all of these actions are strategic complements.

Proposition 1 contains with a number of restrictions. It is interesting to consider why these arise. To do this we analyze a simpler game G' in Appendix B.2. This game is based on the same modeling framework as game G, but assumptions (VF) and (LC) are altered. In game G' we assume that the value and the costs of patenting are increasing functions of each firm's own patents only. We show that:

Lemma 1

A game G' in which the value of patenting and legal costs depend only on the expected number of patents per opportunity held by each firm ($V(\gamma_k), L(\gamma_k)$) is supermodular if the legal costs function is relatively more convex than the value function.

This result is important because it shows that complexity of the technology $(F > 1)$ and the curvature restriction are sufficient conditions to induce strategic complementarity of patenting incentives. To provide an example: the curvature condition is satisfied, if the value of patenting

[14] In the model we analyze simultaneous optimization over two parameters. In the absence of supermodularity a general characterization of comparative statics leads to the analysis of multiple implicit relations. We do not pursue this line of analysis as it will require a number of additional assumptions.

increases linearly in the number of patents a firm holds, while legal costs are strictly convex and the derivative of legal costs at zero is smaller than the derivative of the value function.

To see the intuition for Lemma 1, note that patenting by rival firms reduces the probability that a focal firm obtains a patent. This reduces the focal firm's legal costs and its share of value from the technological opportunity. In Appendix B.2 we show that firms' patenting efforts will be strategic complements because patenting by rivals reduces the expected costs of patenting more than the expected benefits under the curvature condition. The different rate of reduction in costs and benefits increases the marginal incentive to patent. This effect cannot arise in a discrete technology as the expected value and costs of patenting are fixed across opportunities.

This simple model illustrates why stricter enforcement of patent rights by the courts in the United States (Jaffe, 2000, Hall and Ziedonis, 2001) had such an important effect on patenting incentives: it increased the convexity of the legal costs function dramatically.

Game G' does not allow for the partial complementarity of patents in complex technologies nor for the logic of hold-up and patent portfolio races. Therefore, we also analyze a game G'' (Appendix B.3) in which we introduce the logic of bargaining based on the relative size of patent portfolios as discussed by Grindley and Teece (1997), Cohen et al. (2000) and Federal Trade Commission (2003). To do this we introduce assumption (LC) into game G':

Lemma 2

A game G'' in which there is legal interdependency of patenting efforts as in Assumption (LC) but no technological interdependency (i.e. $V(\gamma_k)$) is not supermodular.

While assumption (LC) requires that legal costs ($L(\gamma_k, s_k)$) be decreasing in the share of patents held by the firm (s_k) game G'' can only be supermodular if legal costs are increasing in the share of patents s_k. This would imply that holding constant the absolute number of patents in the portfolio, a firm would face lower legal costs, if its share of granted patents is lower. The assumption seems implausible in the context of the literature cited above, so we reject it.

Lemma 2 is surprising. The threat of hold-up and the reaction of increased patenting are an important part of the explanation delivered for patent thickets in the literature. Lemma 2 shows that where a larger relative share of patents held by the firm reduces legal costs, conditional on the absolute number of patents being constant, strategic complementarity fails. This is because firms are now competing for the largest share of patents and as one firm pulls ahead the marginal value of patenting for the followers decreases.

In contrast, if we allow for technological complementarities in the value function in game G', the resulting game is still supermodular:

Lemma 3

A game $\tilde{G}'$ in which there is no legal interdependency of patenting efforts (i.e. $L(\gamma_k)$) but technological interdependency, as in Assumption (VF), is supermodular, if $\mu > 1$ and the technology is fragmented.

This model is set out in Appendix B.4. As can be seen here the restrictions in Proposition

1 follow from assumption (VF). The only additional restriction in Proposition (1) comes from ensuring that the complementarity of facets is strong enough to overcome the strategic substitutability that derives from assumption (LC).

Finally, we show in Appendix B.5:

Proposition 2
An interior, supermodular equilibrium of game G exists, if Proposition 1 holds and the value of the marginal patent exceeds the sum of its administrative and direct legal costs.

In contrast, if the sum of administrative and legal costs exceeds the value of the marginal patent game G is at the corner solution with no patenting. For all positive sums of administrative and legal costs below this threshold, game G has an interior equilibrium.

There are two main implications that we can take away from these results. First, the modeling framework shows that simultaneous competition for patents on various technological opportunities is not necessarily characterized by strategic complementarities. Secondly, the conditions under which strategic complementarity arise in our modeling framework fit the current understanding of settings in which patent thickets arise very well. These are settings in which legal costs of patenting increase disproportionately as patent portfolios grow, in which technologies are highly complex, in which many firms patent and in which the combination of multiple parties' technologies yields better standards and products.

Comparative Statics of the Model

Now we provide comparative statics assuming that Proposition 1 holds. Throughout *patenting efforts* refers to the choice of f_k and o_k. All derivations are provided in Appendix B.

Corollary 1
If game G is supermodular, firms' patenting efforts increase in the number of competitors (N).

If firms' actions are strategic complements, then additional competitors raise the number of patents covered, increasing the expected value of all patents. At the same time the probability of success on any given patent application will fall. Both of these effects reinforce firms' patenting incentives and efforts. Additionally, we can show that:

Proposition 3
If game G is supermodular, firms' patenting efforts fall with technological opportunity (O).

If firms' actions are strategic complements, then greater technological opportunity reduces the number of patents granted per technological opportunity and the value of each opportunity while increasing the probability of success on any given patent application. Both of these effects reduce firms' patenting incentives and efforts. Finally:

Proposition 4
If game G is supermodular, greater complexity increases firms' patenting efforts.

Greater complexity of a technology has two effects. First, it increases the number of facets per technological opportunity, which makes it easier to patent. Second, it reduces the share of the value which a firm can secure with granted patents it already expects to hold. Both effects lead firms to step up their patenting efforts.

Discrete Technologies We turn now to the case of a discrete technology where - by definition - $F = 1$. Additionally, legal costs of defending and exploiting a patent right are not a function of the share of patents owned on a technological opportunity; this share is one by definition. Similarly V does not depend on the level of applications made: one granted patent application guarantees that a firm receives V. Then, firms' payoffs can be simplified to:

$$\pi_k = o_k V p_k - o_k L - o_k C_o - o_k p_k C_a - C_c(o_k) \quad . \tag{7}$$

Define game $G*$ with this payoff function. This game is no longer supermodular: firms' choices of the number of technological opportunities to invest in are strategic substitutes. Note that the number of opportunities to invest in is also the number of facets invested in, as $F = 1$. Therefore firms only have one choice variable here.

We can show that under the slightly stronger assumption that costs of coordinating technological opportunities ($C_c(o_k)$) are strictly convex in the number of opportunities firms invest in, we obtain a unique equilibrium for the game. We can demonstrate that:

Proposition 5
In a discrete technology, greater technological opportunity increases firms' patenting efforts.

In a discrete technology firms' choices of how many technological opportunities to invest in are strategic substitutes because the value of each opportunity is not a function of the overall level of patenting and because legal costs are constant. Then, greater technological opportunity reduces the costs of patenting by raising the probability of obtaining a granted patent. This increases patenting efforts. Notice that this result also implies that:

Corollary 2
In a discrete technology firms' patenting efforts decrease in the number of competitors (N).

In this section we have shown that there can be countervailing patenting incentives in complex and discrete technologies. In our modeling framework patenting efforts are strategic substitutes in a discrete technology whilst they become strategic complements in a complex technology. Strategic complementarity arises if there are sufficient numbers of competing firms, if complexity is high enough and if additional patented facets of a technological opportunity add value. We have shown that under strategic complementarity an increase of complexity raises patenting incentives, while increasing technological opportunity lowers them. In a discrete technology, where strategic complementarity is absent, greater technological opportunity leads to an increase in patenting activity.

3 Data set and Variables

In this section the data used in this study are introduced. The section also provides discussions of the measures for technological-opportunity, -complexity and fragmentation.

Our empirical analysis is based on the PATSTAT database ("EPO Worldwide Patent Statistical Database") provided by the EPO.[15] We extracted all patent applications filed at the EPO between 1980 and 2003: more than 1,5 million patent applications with about 4.5 million referenced documents. Patents are classified using the IPC classification, allowing us to analyze differences in patenting activities across different technologies. The categorization used is based on an updated version of the OST-INPI/FhG-ISI technology classification[16] which divides the domain of patentable technologies into 30 distinct technology areas.[17] We also classify all technology areas as discrete or complex as suggested by Cohen et al. (2000).

Below we discuss measures of patenting, technological opportunities and complexity. These are the most important variables needed to test the theoretical model. Additionally, we discuss variables that are used as covariates in the empirical model presented in Section 5.

Measures of Patenting, Complexity and Technological Opportunity

Number of Patent Applications We compute the number of patent applications A_{kat} filed by applicant k in year t separately for all of the 30 OST-INPI/FhG-ISI technology areas a. To aggregate patent applications to the firm level two challenges must be overcome: firm names provided in PATSTAT are occasionally misspelled, or different acronyms are used for parts of the firm names. Moreover, subsidiaries of larger firms are not identified in the data set. Therefore, we clean applicant names and consolidate ownership structures.[18] The aggregation of patent applications are based on these consolidated applicant identities. The variables discussed below are also based on this consolidation. Due to the skew distribution of patent applications as measured by A_{kat} we transform the variable logarithmically to derive a dependent variable for the empirical analysis.

Technological Opportunity In our model, we establish a clear relationship between firms' patenting levels in complex technologies and the extent of technological opportunities. Unfortunately, a direct (and time-variant) measure of technological opportunities does not exist. To fill this gap, we use a proxy measure that is based on the number of non-patent literature references in the search report of the patent. In the search report, the EPO examiner lists patent and non-patent references which allow her to assess the degree novelty and of inventive step of the invention described in the patent application. Non-patent literature consists largely

[15] We use the September 2006 version of PATSTAT.
[16] See OECD (1994), p.77.
[17] These are listed in Table 8 in the appendix.
[18] We would like to thank Bronwyn Hall for providing us with code for name consolidation. Ownership information was extracted from the Amadeus database and other sources. Detailed information on the cleaning and aggregation algorithms can be obtained from the authors upon request.

of scientific papers. A high number of such references reflects strong science-based research efforts, and a significant inflow of new technological opportunities, leading to a relatively high level of such opportunities for invention processes. The number of non-patent references can thus be used as a good proxy for the strength of the science link of a technology as a number of studies have pointed out (Meyer, 2000, Narin and Noma, 1985, Narin et al., 1997). Callaert et al. (2006) show that EPO patents contain a high proportion of scientific articles among non-patent references, making European patent data a good source for this measure of technological opportunity. We use the average number of non-patent references (NPR) per patent in a technology area as a proxy for the position of a technology area in the technology cycle and hence as a measure of technological opportunity.

In the theoretical model an increase in technological opportunity reduces competition for remaining facets in complex technologies. This has the effect of reducing the level of patenting. The measure of technological opportunity presented here will capture this effect as long as the number of patents that can be obtained from older technological opportunities does not change significantly and systematically in the opposite direction to the level of non-patent references. We are not aware of any reason to expect such systematic changes.[19]

Complexity of Technology Areas The distinction between discrete and complex technologies is widely accepted in the literature (Cohen et al., 2000, Kusonaki et al., 1998, Hall, 2005a). Discrete technologies are characterized by a relatively strong product-patent link (pharmaceuticals or chemistry), whereas in complex industries technology is modular. This means each technological component can be combined with different sets of additional components to make up separate products and generally each component is protected by patent(s).

Currently direct measures of technological complexity or indirect constructs related to complexity do not exist. Kusonaki et al. (1998) and Cohen et al. (2000) (footnote 44) classify industries as discrete or complex based on ISIC codes. These classifications are based on qualitative evidence. A major drawback of a classification based on industry codes is that is does not allow us to analyze the influence of different levels of complexity within technologies but only to distinguish between discrete and complex technologies.

An ideal measure of complexity would link patents to characteristics of products, showing how many patents are incorporated in each product and how frequently products incorporate patents of rival firms. The measure would also cover products that do not reach the market due to hold-up. The information necessary for such a measure is only very rarely available and not available consistently across technology areas and through time. However, it is possible to come close to this ideal by using information from patent data.

The examiners at the EPO determine and record the extent to which existing prior art limits patentability of an invention in a search report which is typically released 18 months after the

[19] In fact, the time-series graph of non-patent references in semiconductors closely mirrors, but anticipates, the time series graph of various measures of the speed of technological advance in semiconductors that are provided by Aizcorbe et al. (2008). This indicates that non-patent references are a reliable indicator of technological opportunity for this very important technology.

priority date of the patent application (Harhoff et al., 2006). *Critical* documents containing conflicting prior art are classified as X or Y references by the EPO patent examiner.[20] We use this information to identify which firms hold patents that limit some patents of a focal firm.

Using this information we measure how many other firms have patents that limit the application of some of the technology that a focal firm seeks to patent. An important question arises: does limiting of a new patent by prior art in a previous patent indicate that the technology in the new patent is a substitute or a complement to the technology in the older patent? Unless the two patents cover the same subject matter entirely, we read such a critical reference as an indication of a complementarity between technologies being patented by both firms.

Where a claim in an older patent overlaps with a claim in a newer patent the claims that overlap protect substitutable technologies. If the limited patent is not wholly blocked this indicates that the examiner believes the applicant also included technological advances in the patent application that are related to the blocked subject matter but not previously patented. This other subject matter is generally a complement to the subject matter conflicting with prior art. Similarly the patents containing limiting prior art cover technology that is a complement to the limiting claim. Thus technologies held by both firms are likely to be complements, if a critical reference connects two patents and the newer patent is not wholly blocked.

To classify two firms' technologies as complements we require that in the two years previous to and in the reference year each firm must have been cited as owning technology that limits a new patent applied for by the other firm (von Graevenitz et al., 2011). By relying on mutual critical references between firms' patent portfolios we obtain a strong signal of technological complementarity. Mutual critical references also arise in discrete technologies. There they can be resolved by contract between the affected parties or through litigation. The pernicious characteristic of patent thickets is that the modularity of the technology leads to very many overlapping claims. Firms in a patent thicket must simultaneously contract with many other partners, all of whom are also contracting with each other. This often leads firms to set standards or create patent pools (Shapiro, 2001). These contractual settings are far more complex and costly than those in discrete technologies (Federal Trade Commission, 2003).

To measure complexity we count how often three firms hold patents allowing each firm to limit use of the other two firms' patents. This is a *triple*[21] of firms. Often groups of firms are caught in many such triples and the more there are, the more complex and costly contracting becomes. Figure 2 illustrates the measurement of triples:

[20] A search report contains different types of references – not all of them are critical. Often, related patents which are not critical are also included in the search report in order to describe the general state of the art in the respective technology. These are then classified as A-type references. X-type references point to prior patents that on their own cast doubt on the patent's inventive step or novelty; Y-type references do the same, but only in conjunction with additional documents. We have found that for our purposes the distinction between X and Y references is not important and we aggregate them in our empirical analysis.

[21] Triples are one element of the triad census introduced by Holland and Leinhardt (1976), a widely used method of analyzing network structure. Patent thickets can be thought of as networks of firms where the edges are defined by the ability of firm A to limit firm B's use of its technology. Recently, Milo et al. (2002, 2004) define network motifs as "recurring, significant patterns of inter-connections" in network data. Some network motifs are elements of the triad census. Milo et al. (2002, 2004) show that triples characterize the network structure of domains on the World Wide Web and of three social networks better than all other measures in the triad census.

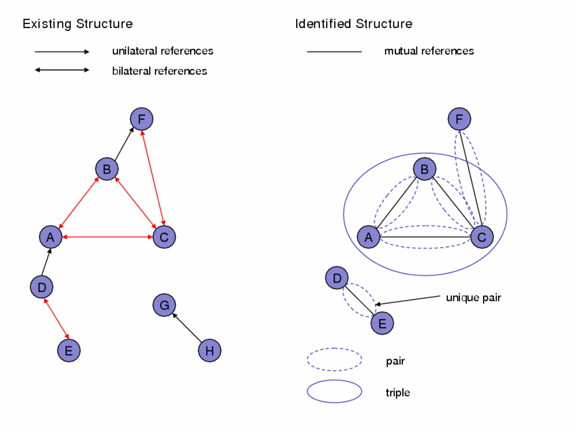

Figure 2: Identification of our measures of a technology field's complexity.

Note that this measure is established at the level of firms' patent portfolios.

The algorithm used to calculate triples is discussed in detail in von Graevenitz et al. (2011). They show that the level of triples in complex and discrete technologies as defined by Cohen et al. (2000) is not driven by the level of patenting. The measure is not distorted by the different rates of patenting that have previously been documented for complex and discrete technologies (Hall, 2005a, von Graevenitz et al., 2007). Note that this measure is very weakly correlated (0.044) with measures the Fragmentation index discussed next.

Fragmentation of Prior Art Ziedonis (2004) shows that semiconductor firms increase their patenting activities in situations where firms' patent portfolios are fragmented. Ziedonis' fragmentation index has predominantly been studied in complex industries (Ziedonis, 2004, Schankerman and Noel, 2006) where increasing fragmentation raises firms' patent applications. This is attributed to firms' efforts to reduce potential hold-up by opportunistic patentees owning critical or blocking patents – a situation associated with *patent thickets*.

We construct the index of fragmentation of patent ownership for each firm using the fragmentation index proposed by Ziedonis (2004):

$$Frag_{kat} = 1 - \sum_{j=1}^{n} s_{kjat}^2 \tag{8}$$

where s_{kjat} is firm k's share of critical references pointing to patents applications made by firm j in area a and year t. Following Ziedonis (2004), Hall (2005b) we correct the index for a bias arising if firms have few patents.

This index is based on the Herfindahl index of concentration. Small values of the fragmentation index indicate that prior art referenced in a firm's patent portfolio is concentrated among few rival firms and vice versa. For instance the measure takes the value zero, if all references of one firm point to just one other firm. If the references of a firm are many and highly dispersed, then the index approaches the value one. The more firms patent actively on the same technological opportunities the greater the index is likely to be. Therefore, the index proxies intensity of competition in a technology area (N in the theoretical model).

Unlike previous studies of patenting in complex technologies relying on USPTO patent data (Ziedonis, 2004, Schankerman and Noel, 2006, Siebert and von Graevenitz, 2010) we compute the fragmentation index solely from critical references which are classified as limiting the patentability of the invention to be patented (X and Y references). This distinction is not available in the USPTO data. Computing the fragmentation index based on critical references will yield a more precise measure of direct competition for similar technologies.

The fragmentation index is not as precise a measure of complexity as the triples measure. The triples measure combines information on actual blocking relationships within technological opportunities which the fragmentation index does not. The fragmentation index captures the number of potential rivals across all technological opportunities in a technology area. Therefore, the measures complement one another: triples capturing complexity, the fragmentation index capturing the intensity of competition.[22]

Covariates

Technological Diversity of R&D Activities A firm's reaction to changing technological or competitive characteristics in a given technology area might be influenced by its opportunities to strengthen its R&D activities in other fields. For example, if a firm is active in two technology areas it might react by a concentration of its activities in one area if competition in the other area is increasing. If a firm is active in only one technology area, it does not possess similar possibilities to react to increases in competitive pressure. In order to control for potential effects of opportunities to shift R&D resources we measure the total number of technology areas ($\text{Areas}_{k,t}$) with at least one patent application filed by firm k in year t.

Size Dummies. While we do not explicitly model the influence of firm size on patenting behavior, it seems reasonable to assume that the cost of obtaining and upholding a patent depends on the size of a firm. In particular, larger firms might face lower legal cost due to economies of scale, increased potential to source in legal services and accumulation of relevant knowledge which in turn might lead to a different patenting behavior than smaller firms. For instance Somaya et al. (2007), find that the size of internal patent departments positively influences firms' patenting propensity.

If the economies-of-scale argument holds, the cost of patenting should not be directly related to size characteristics such as a firm's number of employees, its total revenues or sales. Rather, the cost of patenting can be assumed to be a function of the total patents filed by a firm. Therefore, we include a 'size dummy' variable based on the number of patents filed by a firm in a technology area in a given year in our regressions. We distinguish between small and

[22] In unreported results we find that the number of firms patenting in a technology area has a strong positive correlation with the fragmentation index conditional on year and area fixed effects.

large patentees based on annual patent applications by area a. Firms belonging to the upper half of the distribution of patentees in a given year are coded as large firms.

4 Descriptive Analysis of Patenting in Europe

In this section we provide descriptive aggregate statistics on patenting trends at the EPO. We show that descriptive evidence on patenting supports the theoretical model. Also, the measure of complexity is validated by a comparison with existing measures. Figure 3 presents

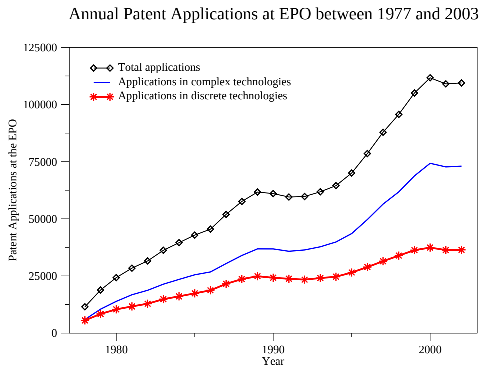

Figure 3: Annual number of patent applications filed at the EPO by priority year. Note: Black line (diamonds) indicates total patent applications. Blue line indicates patent applications in complex technology areas. Red line (starred) indicates patent applications in discrete technology areas.

annual patent applications filed at the EPO between 1978 and 2003. We distinguish applications filed in complex and discrete technology areas using the categorization of Cohen et al. (2000). Patenting grew strongly over the period we plot, with the main contribution coming from technology areas classified as complex. This development is comparable to trends at the USPTO. Hall (2005a) shows that the strong increase in patent applications at USPTO is is driven by firms patenting in the electrical, computing and instruments areas, all of which are complex technology areas by the classification of Cohen et al. (2000).

Now consider explanations for the strong growth in patenting. First, in a complex technology area fragmentation of patent rights is likely to raise firms' transactions costs as they compete with increasing numbers of rivals in patent portfolio races. Ziedonis (2004) and Schankerman and Noel (2006) show that increased fragmentation of patents leads to greater patenting efforts in the semiconductor and software industries respectively. Figure 4 provides annual averages of the fragmentation index at the EPO for the years 1980 to 2003. Two observations derived from Figure 4 are striking: First, fragmentation shows no clear upward or downward trend over the sample period. Second, the difference in the fragmentation index in complex and discrete technology areas is negligible.

Both observations raise the question whether the growth in patent applications can be attributed to fragmentation. Therefore, we now turn to our measure of technological complexity

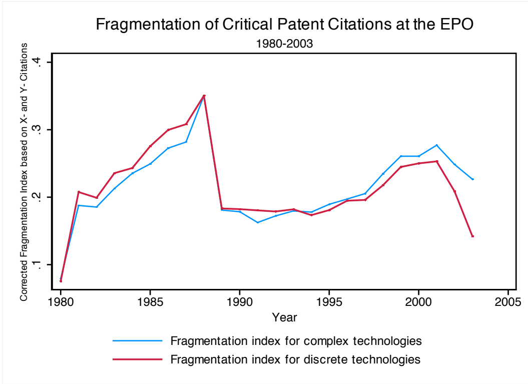

Figure 4: Average fragmentation index. Note: Blue line indicates average level of fragmentation index in complex technology areas. Red line indicates average level of fragmentation index in discrete technology areas.

and to a measure of technological opportunity.

First, the measure of technological complexity (triples) is presented in Figure 5. It contains annual averages of the number of triples in complex and in discrete areas.[23] We observe very different developments of the count of triples in these technology areas. The number of triples is stable at values well under 10 in discrete technology areas, while it increases strongly in complex technology areas. It is reassuring to see that our measure of complexity is greater in complex technologies as they were previously defined by Cohen et al. (2000).

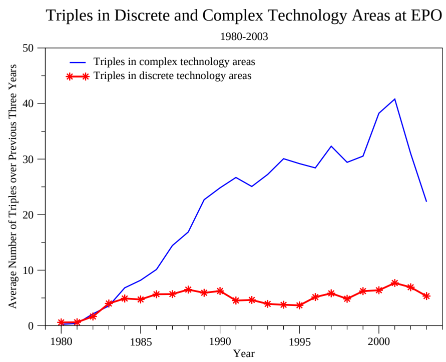

Figure 5: Average number of triples identified. Note: The blue line indicates average number of triples in complex technology areas. The red line (starred) indicates average number of triples in discrete technology areas. This figure is taken from von Graevenitz et al. (2011).

Table 1 provides additional information on the distribution of triples across the 30 technology areas. It shows the significant hold-up potential, measured by triples, within ICT technologies. There are between five and six times as many triples there as in other industries such as Handling, Printing which still exhibit significant complexity by this measure.

[23] We distinguish complex and discrete using the classification suggested by Cohen et al. (2000) here.

Table 1: The Distribution of Triples Between 1988 and 2002

Technology area	Mean	Median	Std. dev.	Minimum	Maximum
Electrical machinery, Electrical energy	24.65	20	8.80	10	42
Audiovisual technology	117.70	120	16.31	74	148
Telecommunications	102.65	93	37.41	27	166
Information technology	58.38	59	9.20	28	73
Semiconductors	62.39	63	18.43	26	91
Optics	57.66	58	11.92	42	77
Analysis, Measurement, Control	6.71	3	6.46	0	21
Medical technology	4.19	4	2.11	1	8
Nuclear engineering	0.94	1	1.16	0	4
Organic fine chemistry	3.51	2	3.71	0	15
Macromolecular chemistry, Polymers	15.68	14	7.91	4	32
Pharmaceuticals, Cosmetics	3.62	4	2.64	0	8
Biotechnology	0.00	0	0.00	0	0
Agriculture, Food chemistry	0.07	0	0.26	0	1
Chemical and Petrol industry	11.01	10	5.49	4	22
Chemical engineering	1.40	1	0.84	0	3
Surface technology, Coating	3.53	3	2.79	0	9
Materials, Metallurgy	2.46	2	2.14	0	6
Materials processing, Textiles, Paper	3.80	3	2.64	1	9
Handling, Printing	20.60	16	13.50	4	50
Agricultural and Food processing,	0.37	0	0.73	0	2
Environmental technology	3.37	0	4.79	0	15
Machine tools	1.93	1	1.58	0	5
Engines, Pumps and Turbines	22.39	15	21.30	3	69
Thermal processes and apparatus	0.38	0	0.62	0	2
Mechanical elements	2.37	2	2.17	0	7
Transport	17.07	14	12.17	2	50
Space technology, Weapons	0.00	0	0.00	0	0
Consumer goods	0.76	0	1.07	0	4
Civil engineering, Building, Mining	0.00	0	0.00	0	0

Second, consider the development of technological opportunities as an explanation of the overall patenting trends. Proposition 3 indicates greater technological opportunity in a complex technology should lower the pressure to patent. As noted in Section 3 we measure technological opportunity using changes in the rate of references to non patent literature within a technology area. This measure provides information about variation in technological opportunities between and across technology areas. The left panel of Figure 6 below shows a hump shaped pattern for technological opportunities in complex technology industries. In contrast, technological opportunities in discrete technologies also level off, but at a later date then in complex technologies. Note that technological opportunities in complex technology areas began to decline just after 1992, which coincides with the date at which the growth in patent applications at the EPO picked up as Figure 3 shows. The right panel of the Figure shows that average non patent references in complex technology areas mask considerable variation across and especially within technologies.

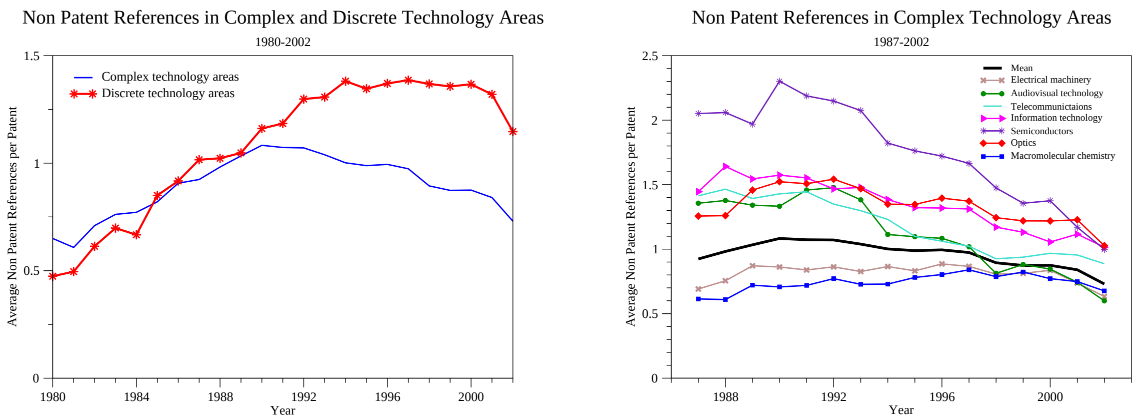

Figure 6: The left panel presents average non patent references per patent for complex (blue line) and discrete (red line, starred) technology areas. The right panel presents average non patent references per patent for several complex technology areas.

5 The Empirical Model and Results

In this section we set out empirical results. To begin with we provide a discussion of our empirical model and discuss descriptives for the sample used. Then we turn to the results from estimation and a discussion of their implications.

5.1 An Empirical Model of Patenting

Building on the results of Section 2 we estimate a reduced form model predicting the level of patent applications filed by a firm in a given year at the EPO. Patent applications are highly persistent as they reflect long term investments in R&D capacity. Therefore, we include a lagged dependent variable in our model. We estimate the following dynamic relationship:[24]

$$A_{i,t} = \beta_0 + \beta_A A_{i,t-1} + \beta_{AC} A_{i,t-1} C_{i,t} + \beta_O O_{i,t} + \beta_C C_{i,t} + \beta_{OC} O_{i,t} C_{i,t} + \beta_F F_{i,t} + \beta_{FC} F_{i,t} C_{i,t} + \boldsymbol{\beta}_X{}' \boldsymbol{X}_{i,t} + \Upsilon_i + \zeta_{i,t} \ , \qquad (9)$$

where:

$A_{i,t}$ – ln(Patent Applications) $\quad$ $O_{i,t}$ – Technological Opportunity: Non Patent References

$C_{i,t}$ – Complexity: Triples $\quad$ $F_{i,t}$ – Fragmentation index: Concentration

$\boldsymbol{X}_{i,t}$ – Control variables: Area count, Size

Υ_i – Firm area fixed effects $\quad$ $\zeta_{i,t}$ – Error term.

[24]Our model did not explicitly account for dynamic aspects of firms' strategic decisions. However, it seems appropriate to take the persistent nature of patenting decision into account when analyzing patenting over time.

With this specification we capture effects of technological opportunity β_O, complexity β_C and competition β_F as well as the effects of complexity and competition in complex technologies (β_{OC}, β_{FC}). We also allow the effect of the lagged dependent variable to differ in complex and discrete technology areas (β_{AC}).

In an extension of this basic specification we also include interaction terms that allow us to distinguish the patenting behavior of large and small firms in complex and discrete technologies. Our theoretical model indicates that firms' patenting behavior will depend on the share of patents they expect to receive on a given technological opportunity which may differ systematically between large and small firms.

Estimates of this specification provide a test of the following hypotheses. These reflect Propositions 3- 5 and Corollaries 1- 2:

H1 Greater complexity of technologies raises patent applications, $\beta_C > 0$ (Proposition 4);

H2 Competition raises patent applications in complex technologies, $\beta_{FC} > 0$ (Corollary 1);

H3 Technological opportunity reduces patent applications in complex technologies, $\beta_{OC} < 0$ (Proposition 3);

H4 Competition reduces patent applications in discrete technologies, $\beta_F < 0$ (Corollary 2);

H5 Technological opportunity raises patent applications in discrete technologies, $\beta_O > 0$ (Proposition 5).

Hypotheses 1-3 capture the effects of complexity, competition and technological opportunity in complex technologies. Proposition 1 shows that greater complexity of a technology is more likely to render firms' actions in a patenting game strategic complements. The reverse is true in a discrete technology, here firms actions are strategic substitutes and the comparative statics with respect to competition and technological opportunity are exactly reversed. By interacting complexity with the number of competing firms and our measure of technological opportunity in Hypotheses 2 and 3 we separate the two types of equilibria.

Notice, that if we fail to reject these hypotheses, we reject model G'', in which there is no technological complementarity of patents and in which legal costs depend on both the absolute number of patents and the relative share of patents held by each firm. If we cannot reject Hypothesis 1, then we reject model G' together with the restriction that each firm seeks to patent more than half the facets per technological opportunity.

5.2 Descriptive Statistics for the Sample

Our data set contains observations of patent applications by firms in specific technology areas and covers the period between 1978 when the EPO began operating and 2003. We intend to study patent applicants patenting over a prolonged period and possibly across several technology areas. Therefore, we excluded small patentees from the sample.

Table 2: Panel Descriptives for the Sample

Firm level (2074 firms)	**Mean**	**Median**	**SD**
Total patents	628.27	205	1944.94
Total patents (annual)	37.02	12	111.65
Technological areas (annual)	5.54	4	4.56
Area-Year level (650 area-year observations)	**Mean**	**Median**	**SD**
Total patents in area	2594.23	2310	1778.87
Total patents in area and sample	1449.35	1012	1695.86
Total firms in area	1077.62	893	668.14
Total firms in area and sample	266.84	263	253.71
Triples	14.67	2	27.69
Non Patent References	0.98	0.75	0.75
Fragmentation	0.05	0.05	0.03

Two criteria were used: first, we excluded all those patentees with fewer than 100 patent applications between 1980 and 2002. Second, we excluded patentees with fewer than three years of positive patent applications in a technology area in the fifteen years after 1987.

These criteria result in a sample containing 173,448 observations of patenting activity by a firm in a technology area. Table 2 shows that these patent applications are due to 2074 distinct firms. The average size of these firms' patent portfolios in 2002 was 628 patents resulting from an average of 37 patent applications per firm and year across all technology areas. 34% of observations in the data set contain a zero patent application count but only 0.05% of observations belong to firms that have no patent applications at all in a given year. The lower half of Table 2 shows that our sample covers on average 55.8% of the annual mean of 2594 patent applications filed within an average technology area. As the sample focuses on large patentees the share of firms we covered by the sample is smaller: on average 1077 firms patent per area per year and 24.8% of these are included in the sample.[25]

Firms operating in several technology areas are treated as distinct in each area. Hence, our panel structure is not defined over firms' total patent applications per year (firm-years) but over firms' annual patent applications within specific technology areas (firm-area-years). We do this to control for area specific patenting behavior of individual firms and its relation to area characteristics like complexity.[26] Where we use panel data, the panel is unbalanced due to entry and exit of firms into technology areas.

Table 3 presents descriptive statistics at the firm-area-year level. Most firms in the sample patent relative broadly across technology areas. While the number of patent applications within a given technology area is relatively low with 5.43 application per year firms are active

[25] We have experimented with alternative sample selection rules and found our results to be robust.

[26] We find that firms in more complex technologies are very slightly more likely to be active in more technology areas than the average firm, with a very weak positive correlation of 0.04.

Table 3: Descriptive Statistics for the Sample (1988-2002)

Variable	**Aggregation level**	**Mean**	**Median**	**Standard deviation**	**Mini-mum**	**Maxi-mum**
Patent applications per area	Firm	5.431	1.000	18.594	0.000	752.000
log Patent applications per area	Firm	1.051	0.693	1.052	0.000	6.624
Areas	Firm	8.751	7.000	6.027	0.000	30.000
Large dummy	Firm	0.504	1.000	-	0.000	1.000
Non Patent References	Area	1.151	0.894	0.827	0.174	4.532
Triples	Area	18.480	5.000	30.085	0.000	166.000
Fragmentation	Firm	0.210	0.000	0.427	0.000	1.961

Observations = 173,448

Sample Statistics for the Year 1992

Patent applications per area	Firm	4.235	1.000	14.024	0.000	387.000
log Patent applications p/a	Firm	0.923	0.693	0.990	0.000	5.961
Areas	Firm	7.746	6.000	5.563	0.000	27.000
Large dummy	Firm	0.438	0.000	-	0.000	1.000
Non Patent References	Area	1.205	0.970	0.747	0.290	3.554
Triples	Area	15.761	3.000	25.348	0.000	104.000
Fragmentation	Firm	0.175	0.000	0.389	0.000	1.935

Observations = 11,325

in 8 or 9 different technology areas. The average technology area contained about 18.5 triples in a given year – however the distribution is skew with a median of 5 and a maximum of 166 triples (observed in Telecommunications in 2000). The level of non patent references in the average technology area is 1.151. Table 3 also contains information about sample statistics for the year 1992, after which patent applications increased markedly as Figure 3 shows. A comparison of sample means (upper part of Table 3) and means for 1992 (lower part of 3) shows that firms patent in more areas, face more complexity (triples) and generate fewer non patent references after 1992 than in 1992. This confirms what we showed previously.[27]

5.3 Results

In this section we present results from estimation of the empirical model (Equation 9) using GMM. The lagged dependent variable and several explanatory variables which may be expected to be endogenous are instrumented. We show that the predictions of the main theo-

[27]To ensure that the possible break in aggregate patent applications at EPO in 1992 did not affect the results we report below, we also estimated our models for the period after 1992. We found no significant differences in our results. We have found some indication that dropping the most recent years (2000-2003) from the data weakens the effects of technological opportunity. Results are available from the authors upon request.

retical model presented in Section 2 cannot be rejected.

We use panel estimators to avoid misspecification of the empirical model arising from unobserved heterogeneity, such as variation in managerial ability. To capture persistence in patenting we introduce a lagged dependent variable into our models, which introduces an additional source of misspecification. This renders fixed and random effects estimators inconsistent in short panels such as ours (Arellano, 2003). Instead, we employ system GMM estimators which also allow us to address the potential endogeneity of some of our regressors.

We instrument potentially endogenous variables using lagged values. Exogeneity of these instruments is tested using difference in Hansen tests (Roodman, 2006). All models reported below contain the following explanatory variables: *non patent references*, *triples*, *fragmentation*, *area count*, *large dummy* and the lagged dependent variable as well as interactions of some of these variables. We consider *fragmentation* and *area count* to be endogenous as they reflect decisions about how widely and where to engage in research which may be contemporaneous with decisions determining the level of patent applications. The remaining variables are treated as predetermined variables since they depend in large part on the aggregated decisions of rival firms. Finally note that we include only year and area dummies in the levels equation as it is likely that the fixed effects are correlated with differences in the remaining explanatory variables. In all specifications we instrument predetermined variables with third order lags and endogenous variables with fourth order lags.

Instrument sets are collapsed[28] in order to reduce the number of instruments used. Throughout we rely on the Hansen test to determine whether the entires set of instruments used are exogenous. Where the statistic indicates that this is not the case we reject the models.

Table 4 presents results of system GMM estimators using forward deviations transformations (Blundell and Bond, 1998, Arellano and Bover, 1995, Alvarez and Arellano, 2003).[29] Reported standard errors are based on two step estimators using the correction suggested by Windmeijer (2005). Tests for first, second and third order serial correlation (m1-m3) indicate presence of first and second order serial correlation.

Specification SGMM A contains the lagged dependent variable, measures of technological opportunity (*non patent references (NPR)*), complexity (*triples*), the breadth of a firms' activities within the patent system (*areas*), a dummy for the size of a firms' patent portfolio (*large*) and dummies for year and main technology area.

Specification SGMM B adds the measure of fragmentation suggested by Ziedonis (2004). This is adjusted as proposed by Hall (2005b). In specification SGMM C we add interactions of the complexity measure (*triples*) with the measure of technological opportunity (*NPR*). Hansen tests for these simple specifications reject their validity, indicating that the instruments used are not exogenous.

In specification SGMM D *triples* are interacted with the lagged dependent variable, to

[28] Collapsing instrument sets reduces the number of moment conditions used for GMM (Roodman (2006)).

[29] All models were estimated with xtabond2 in Stata 9.2 . This package is described in Roodman (2006).

Table 4: GMM Models for Patent Applications

Variable	SGMM A	SGMM B	SGMM C	SGMM D	SGMM E
log Patentcount$_{t-1}$	0.720***	0.509***	0.426***	0.688***	0.749***
	(0.042)	(0.039)	(0.039)	(0.122)	(0.093)
log Patentcount$_{t-1}\times$ Triples				-0.018***	-0.017***
				(0.003)	(0.003)
Non Patent References (NPR)	0.226***	0.121***	-0.152*	1.700***	1.553***
	(0.030)	(0.032)	(0.061)	(0.344)	(0.254)
NPR $\times$ Triples			0.001	-0.043***	-0.036***
			(0.001)	(0.007)	(0.006)
NPR $\times$ Triples $\times$ Large					0.007***
					(0.002)
NPR $\times$ Large					-0.366***
					(0.081)
Fragmentation		0.641***	0.793***	-0.489*	-0.474**
		(0.065)	(0.064)	(0.213)	(0.170)
Fragmentation $\times$ Triples				0.010	0.006
				(0.006)	(0.006)
Triples	-0.000	0.000	-0.001	0.071***	0.055***
	(0.000)	(0.000)	(0.001)	(0.012)	(0.010)
Areas	0.067***	0.068***	0.073***	0.105***	0.096***
	(0.007)	(0.006)	(0.006)	(0.016)	(0.012)
Large	-0.079**	-0.117***	-0.183***	0.010	0.342**
	(0.026)	(0.025)	(0.025)	(0.078)	(0.117)
Year dummies	YES	YES	YES	YES	YES
Primary area dummies	YES	YES	YES	YES	YES
Constant	-0.403***	-0.221***	0.044	-1.700***	-1.443***
	(0.038)	(0.043)	(0.062)	(0.314)	(0.319)
N	173448	173448	173448	173448	173448
m1	-25.041	-23.908	-23.413	-7.934	-10.860
m2	18.356	13.590	10.836	3.131	4.739
m3	-1.707	-2.230	-2.285	1.606	.896
Hansen	525.187	412.714	456.374	19.221	10.988
p-value	9.1e-115	3.90e-89	2.08e-95	.004	.052
Degrees of freedom	2	3	6	6	5

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses
2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.
3. Hansen is a test of overidentifying restrictions. It is distributed as χ^2 under the null of instrument validity, with degrees of freedom reported below.
4. In all cases GMM instrument sets were collapsed and lags were limited.

capture the possibility that firms adjust their levels of patenting differently in complex and discrete technologies.

This specification performs better than SGMM A-C, the χ^2 statistic being significantly lower than for those specifications. Finally, specification SGMM E also includes interactions which test the effects of firm size on non patent references. This specification performs best, the Hansen test does not reject the model. We now focus on this model.

We find that greater technological opportunities (NPR) raise patenting levels showing that we cannot reject Hypothesis 5. The effect of technological opportunity is highly significant across almost all estimated specifications (see models (A) to (E) of Table 4). The inclusion of the interaction between our measure of complexity (triples) and technological opportunities shows that the effect differs in discrete - and complex technologies. In particular, if the number of triples in a technology area is larger than 39 (in specification (D)) or larger than 43 in specification (E) of Table 4, the overall effect from increasing technological opportunities is negative as $\beta_O + \beta_{OC} \times C_{i,t} < 0$. The negative coefficient on the interaction of complexity and non patent references supports Hypothesis 3: increasing technological opportunities reduce patenting efforts in more complex technology areas. Additionally, the significant positive coefficient on the effects of complexity alone supports Hypothesis 1. Table 1 shows the average number of triples for 5 technology areas in our sample is greater than 43. For Audiovisual technology and Optics triples are always above 43. This indicates that increased technological opportunities always or almost always reduce patenting efforts in these areas.

With regard to the effects of the number of competitors blocking a specific firm in technology space we fail to reject Hypothesis 4, i.e. more competition (greater fragmentation) reduces patenting efforts in discrete technologies. The coefficient on the interaction of fragmentation and complexity is not significant. However, the joint effect of fragmentation and complexity is significant. Thus we have weak evidence that increased competition raises patenting efforts in complex technologies (Hypothesis 2).

Finally, our results on the interaction of the lagged dependent variable with *triples* indicate that persistence of patenting decreases as technology areas become more complex. Persistence is entirely absent in very complex technologies. This shows that patentees are more responsive to their competitors' patenting behavior and to technological opportunity in complex technology areas than in discrete technology areas.

Table 5 below provides effects of changes in complexity (*triples*), technological opportunities (*non patent references*) and *fragmentation* for patenting rates in nine technology areas.[30] The table presents effects for small and large firms where appropriate and contains mean and median results. Five of the technology areas presented are highly likely complex as the mean and median levels of triples are clearly above 43 in these areas (viz. Table 1). They are Audiovisual Technology, Telecommunications, Information Technology, Semiconductors and Optics. We also present results for four additional areas. These are more likely discrete by

[30] These effects are calculated taking account of the logarithmic transformation of the dependent and the lagged dependent variable.

this measure: Medical Technology; Electrical Machinery; Analysis, Measurement, Control; and Pharmaceuticals.

Table 5 shows that in all discrete technologies an increase in technological opportunity raises patenting, while in all complex technology areas it lowers patenting. These results fit the predictions of Hypotheses 5 and 3 respectively. Most importantly the effects of a one standard deviation change in technological opportunity are comparatively large in the complex technologies. This is a surprising finding that indicates that technological opportunity is an important determinant of firms' patenting efforts.

Table 5: Mean and Median Percentage Changes in Patent Applications in Complex and Discrete Technologies

Technology area	**Applications growth 1990-2000**	**Triples**	**Triples SD change**		**Non patent references SD change**		**Fragmentation**	
			Small	**Large**	**Small**	**Large**	**Unit change**	**SD change**
Complex Technologies								
Audiovisual Technology	52%	118	-2.76%	10.13%	-51.59%	-45.21%	26.14%	10.65%
		120	6.29%	20.47%	-52.66%	-46.19%	27.89%	11.32%
Telecom-munications	253%	103	-22.38%	4.80%	-34.87%	-30.11%	15.25%	6.83%
		93	-11.40%	18.14%	-30.18%	-26.08%	8.76%	3.99%
Semiconductors	63%	62	-33.51%	-17.04%	-23.19%	-21.10%	-9.48%	-4.14%
		63	-31.09%	-13.51%	-23.83%	-21.62%	-9.15%	-3.99%
Information Technology	174%	58	-8.11%	0.04%	-9.75%	-9.03%	-11.63%	-4.99%
		59	-3.83%	4.69%	-10.12%	-9.33%	-11.31%	-4.85%
Optics	41%	58	-11.07%	-0.49%	-7.02%	-6.54%	-12.02%	-5.55%
		58	-6.16%	5.02%	-7.18%	-6.67%	-11.84%	-5.46%
Discrete Technologies								
Electrical Machinery	91%	25	7.97%	15.59%	4.28%	3.02%	-27.83%	-13.57%
		20	12.24%	20.27%	5.39%	3.90%	-29.81%	-14.64%
Analysis, Measurement, Control	75%	7	-1.48%	3.87%	10.07%	7.53%	-35.19%	-17.83%
		3	1.24%	6.63%	11.15%	8.38%	-36.62%	-17.87%
Pharmaceu-ticals	221%	4	-15.60%	-11.26%	55.06%	39.60%	-36.38%	-19.43%
		4	-16.29%	-11.93%	54.41%	39.13%	-36.24%	-19.49%
Medical Technology	148%	4	6.00%	6.54%	5.81%	4.38%	-36.16%	-17.99%
		4	6.62%	7.19%	5.84%	4.41%	-36.24%	-18.02%

This table reports **means** (upper row) and **medians** (lower row) for each technology area. We report changes in patent applications in response to standard deviation (**SD**) changes in each variable. For *Triples* and *Non patent references* we report effects for **small** and **large** firms.

Hypothesis 1 states that increases in the complexity of a technology will raise firms' levels of patenting in the technology is complex. Table 5 shows this result generally holds at the median and mean for large firms in complex technology areas apart from Semiconductors.[31]

Interestingly, Table 5 also shows that the effect of *fragmentation* on firms' patenting efforts in very complex technology areas is positive as predicted by Hypothesis 2. Also, *fragmentation* has a negative effect on patenting in discrete technology areas, as predicted in Hypothesis 4. The positive effects for complex technology areas support the findings of Ziedonis (2004), Schankerman and Noel (2006) who find that additional fragmentation of patent ownership increases patenting efforts in semiconductors and software in the United States. Note however, that *fragmentation* has small negative effects on patenting in the moderately complex technologies included in Table 5. In discrete technology areas fragmentation has a very strong negative effect so that overall we confirm the prediction that firms are more likely to patent more as fragmentation increases if technology areas are more complex.

5.4 Robustness of the Results

In a next step, we test the robustness of our results using alternative GMM estimators. Results from these tests are reported in Table 6. We vary the size of the instrument set and the estimator used. All models reported in Table 6 are estimated using forward deviations and reported standard errors corrected as previously noted. The models differ in the number of overidentifying restrictions employed as well as assumptions about the correlation of the explanatory variables with fixed effects. Hansen tests are used to determine which of the models are reliable. These show that only the first three models reported in the table are not rejected.

The four models reported in the central part of Table 6 allow for correlation between all explanatory variables with fixed effects. In two specifications on the right side of the table we assume subsets of the explanatory variables are uncorrelated with fixed effects. The number of observations in our data set implies that $T/N \to 0$. Therefore, a systems GMM estimator (Blundell and Bond, 1998) using forward deviations is asymptotically consistent (Alvarez and Arellano, 2003, Hayakawa, 2006). We employ this estimator as the patenting series are highly persistent in our sample: the coefficient on the lagged dependent variable in an AR1 model with time and primary area dummies is 0.92. Blundell and Bond (1998) note that difference GMM is affected by a weak instruments problem in this context which is not the case in the specification we report. However, the coefficient on the lagged dependent variable is somewhat above that reported for the comparable systems estimators. It is also significantly above the coefficients from the OLS regressions reported in Table 7. Therefore, we focus our analysis on the results from the system estimators.

In all models reported in Table 6 the instrument sets were collapsed and instrumenting lags were limited as described above. This was done as the Hansen test and difference in Hansen tests rejected the overall instrument sets as well as individual instruments where larger instrument sets were employed. Specification SGMM H illustrates how sensitive the Hansen

[31] The precise delineation of the areas for Information Technology and Semiconductors in the classification we use is not clear. In von Graevenitz et al. (2007) we find that a large proportion of patents from semiconductor firms are patented within the Information Technology area.

Table 6: Robustness Checks for Patent Applications Estimates

Variable	Allowing correlation with fixed effects SGMM F	SGMM E	DGMM G	SGMM H	Assuming no correlation with fixed effects SGMM I	SGMM J
log Patentcount$_{t-1}$	0.675***	0.749***	0.935***	0.617***	0.742***	0.879***
	(0.102)	(0.093)	(0.110)	(0.067)	(0.047)	(0.054)
log Patentcount$_{t-1}\times$ Triples	-0.021***	-0.017***	-0.014***	-0.011***	-0.009***	-0.011***
	(0.004)	(0.003)	(0.003)	(0.002)	(0.001)	(0.001)
Non Patent References (NPR)	1.880***	1.553***	1.502***	0.654***	0.475***	1.306***
	(0.361)	(0.254)	(0.258)	(0.112)	(0.039)	(0.173)
NPR $\times$ Triples	-0.042***	-0.036***	-0.035***	-0.019***	-0.016***	-0.031***
	(0.008)	(0.006)	(0.006)	(0.003)	(0.002)	(0.004)
NPR $\times$ Triples $\times$ Large	0.008***	0.007***	0.006***	0.006***	0.005***	0.004***
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
NPR $\times$ Large	-0.248*	-0.366***	-0.224***	-0.373***	-0.484***	-0.557***
	(0.125)	(0.081)	(0.040)	(0.057)	(0.036)	(0.049)
Fragmentation	-0.558**	-0.474**	-0.490**	-0.091	-0.007	-0.183
	(0.211)	(0.170)	(0.164)	(0.111)	(0.081)	(0.113)
Fragmentation $\times$ Triples	0.009	0.006	0.000	-0.004	-0.007*	-0.000
	(0.008)	(0.006)	(0.006)	(0.004)	(0.003)	(0.004)
Triples	0.056***	0.055***	0.052***	0.031***	0.034***	0.059***
	(0.013)	(0.010)	(0.008)	(0.006)	(0.004)	(0.007)
Areas	0.097***	0.096***	0.036*	0.098***	0.082***	0.090***
	(0.014)	(0.012)	(0.016)	(0.009)	(0.008)	(0.007)
Large	0.221	0.342**	0.256***	0.349***	0.562***	0.562***
	(0.165)	(0.117)	(0.065)	(0.083)	(0.060)	(0.080)
Year dummies	YES	YES	YES	YES	YES	YES
Primary area dummies	YES	YES	YES	YES	YES	YES
Constant	-1.311**	-1.443***		-0.703***	-0.886***	-2.000***
	(0.466)	(0.319)		(0.194)	(0.075)	(0.206)
N	173448	173448	171380	173448	173448	173448
m1	-9.878	-10.860	-8.276	-12.868	-19.934	-20.345
m2	2.651	4.739	4.881	6.166	11.269	14.863
m3	1.353	.896	.192	-1.051	-.951	.3055
Hansen	4.101	10.988	5.177	57.671	197.573	52.158
p-value	.129	.052	.270	3.76e-09	2.07e-38	5.43e-09
Degrees of freedom	2	5	4	9	8	7

* p<0.05, ** p<0.01, *** p<0.001

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses
2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.
3. Hansen is a test of overidentifying restrictions. It is distributed as $\chi 2$ under the null of instrument

validity, with degrees of freedom reported below.

4. In all cases GMM instrument sets were collapsed and lags were limited.

test is to the size of the instrument set here. This specification is identical to SGMM E, we just allow for an extra lag on the instrument sets for the endogenous variables in this specification. The specification is rejected by the Hansen test.

We estimate two models in which we treat *fragmentation* (SGMM J) and *non patent references* (SGMM I) as uncorrelated with fixed effects. Results from the Hansen tests for both specifications reported in Table 6 show that these models are clearly rejected.

Our preferred models are reported as SGMM F and SGMM E in Table 6. In SGMM F we restrict the number of instruments such that the model is just overidentified. Hayakawa (2006) argues that such a minimum instruments specification is unbiased in settings where T is fixed and $N \to \infty$. Specification SGMM E includes additional instruments for the endogenous variables. Results from these two specifications are statistically indistinguishable.

In addition to the GMM results reported here, Table 7 (Appendix C) provides results from OLS on the pooled sample and from fixed effects regressions. These results are known to be biased due to inclusion of the lagged dependent variable. However, they provide lower and upper bounds on the values of the lagged dependent variable for GMM (Bond (2002)). Once we take account of the interaction of the lagged dependent variable with triples we find that the coefficients on the lagged dependent variable are within the range provided by OLS and fixed effects estimates for technology areas of average complexity.

6 Conclusion

Patent applications have been increasing steeply at the USPTO and the EPO since 1984 and 1992 respectively. These increases have raised questions about the operations of the affected patent offices as well as effects of these trends on economic activity more generally (Federal Trade Commission, 2003, National Research Council, 2004, von Graevenitz et al., 2007, Bessen and Meurer, 2008). Our paper makes a number of contributions towards a systematic explanation of recent patenting trends. There is strong evidence by now that patenting has increased in response to evolution of the legal environment, specifically in the United States, to changes in the management of R&D and patenting, and to increasing complexity of technology and more strategic behavior of patent applicants (Kortum and Lerner, 1998, Hall and Ziedonis, 2001, Ziedonis, 2004). But the contribution of technological opportunity to current patenting trends and its interaction with other determinants has been less well understood.

This latter effect is central to our analysis. Our theoretical analysis is the first to consider the effect of complexity and of technological opportunity *jointly*. Moreover, while other studies have focused on selected industries, our model and the empirical test encompass discrete and complex technologies, providing predictions for patenting behavior in both types of technology. We provide a theoretical framework predicting that greater technological opportunity will raise patenting in discrete technologies but will lower it as technologies become increasingly complex. Additionally, we show that a higher number of patent applicants raises firms' patenting levels in complex technologies. The modelling framework also shows that increased legal costs due to litigation and licensing can have contributed to increased patenting, while competition for larger patent portfolios alone cannot explain why so many firms increased their patenting efforts simultaneously.

To test our predictions on technological opportunity and complexity empirically, we apply a new measure of technological complexity suggested by von Graevenitz et al. (2011). This measure exploits information on critical references to capture the density of patent thickets as they are reflected in European patent data. Using the measure we are able to confirm that patent thickets are a much more serious problem in technology areas previously identified as complex than in those previously identified as discrete.

The empirical results reported in this paper show that patenting behavior conforms to the predictions of our preferred theoretical model. As we test a reduced form model, we cannot exclude that alternative models may lead to similar restrictions. However, we note in the paper that certain restrictions that would arise from alternative modeling assumptions within our modeling framework are rejected by the data. The main limitation of our modeling framework is that it is static. In future work introducing multiple periods into the framework will provide an important test of the restrictions we have derived and tested in this paper.

Variation in technological opportunity affects firms' patenting levels to a surprising extent. Our data show that increased technological opportunity during the early 1990s counteracted the effects of growing complexity and retarded the onset of the patenting explosion observable after 1994. The patent explosion coincides with the decrease in technological opportunities after 1994. We also show - for the first time with European data - that greater fragmentation of patent ownership increases patenting in complex technologies (Ziedonis, 2004). We attribute this to a greater number of competing patent applicants as we control for the degree of hold-up potential with the triples measure of technological complexity.

Finally, our results show that as technology areas become more complex, firms' patenting activities increase. Since we use lagged values of complexity to instrument current complexity this finding is likely to reflect a causal mechanism - as firms encounter more complexity they respond by patenting more.

We find that patent thickets exist in nine out of thirty technology areas at the EPO. The data indicate that the extent of patent thickets at the EPO has been increasing in recent years. These increases are concentrated in complex technology areas. Resulting increases in transactions costs would therefore affect exactly those technologies that have been central to large productivity increases in the recent past (Jorgenson and Wessner, 2007). Extended "patent wars" may threaten this source of productivity gains in the long run. In future work we therefore intend to investigate whether strategic patenting has measurable effects on the productivity of firms' R&D investments and how the decision variables of patent offices (fees and administrative rules) might be used to influence patent filings.

Our findings on the effects of technological opportunity raise important questions about the relationship between patent breadth, the fecundity of research areas and firms' R&D investments. We find that the contest for patent rights becomes more intense as the level of technological opportunities decreases if a technology is complex. This raises the question how firms' incentives to patent more intensively interact with incentives to undertake basic research which might stem the reduced fecundity of these technologies. At a more fundamental level the findings indicate that research into the relationship between technological opportunities and R&D is important, if we are to understand the welfare implications of recent patenting trends better.

References

AIZCORBE, A., S. D. OLINER, AND D. E. SICHEL (2008): "Shifting Trends in Semiconductor Prices and the Pace of Technological Progress," *Business Economics*, 43, 2339.

ALVAREZ, J. AND M. ARELLANO (2003): "The Time Series and Cross-Section Asymptotics of Dynamic Panel Data Estimators," *Econometrica*, 71, 1121–1159.

AMIR, R. (2005): "Supermodularity and Complementarity in Economics: An Elementary Survey," *Southern Economic Journal*, 71, 636–660.

ARELLANO, M. (2003): *Panel Data Econometrics*, Advanced Texts in Econometrics, Oxford, GB: Oxford University Press.

ARELLANO, M. AND O. BOVER (1995): "Another Look at the Instrumental-Variable Estimation of Error-Components Models," *Journal of Econometrics*, 68, 29–51.

BEATH, J., Y. KATSOULACOS, AND D. ULPH (1989): "Strategic R&D Policy," *The Economic Journal*, 99, 74–83.

BESSEN, J. (2004): "Patent Thickets: Strategic Patenting of Complex Technologies," Tech. rep., Research on Innovation.

BESSEN, J. AND M. J. MEURER (2008): *Patent Failure*, Princeton University Press.

BLUNDELL, R. AND S. BOND (1998): "Initial Conditions and Moment Restrictions in Dynamic Panel Data Models," *Journal of Econometrics*, 115–143.

BOND, S. (2002): "Dynamic Panel Data Models: A Guide to Micro Data Methods and Practice," CEMMAP Working Paper CWP09/02, IFS.

CALLAERT, J., B. V. LOOY, A. VERBEEK, K. DEBACKERE, AND B. THIJS (2006): "Traces of Prior Art: An Analysis of Non-patent References Found in Patent Documents," *Scientometrics*, 69, 320.

CLARK, D. J. AND K. A. KONRAD (2008): "Fragmented Property Rights and Incentives for R&D," *Management Science*, 54, 969–981.

COHEN, W. M., R. R. NELSON, AND J. P. WALSH (2000): "Protecting their Intellectual Assets: Appropriability Conditions and Why U.S. Manufacturing Firms Patent (or Not)," Working Paper 7552, NBER.

FEDERAL TRADE COMMISSION (2003): "To Promote Innovation - The Proper Balance of Competition and Patent Law and Policy," Report, GPO.

GALLINI, N. T. AND S. SCOTCHMER (2002): "Intellectual Property: When Is It the Best Incentive System?" in *Innovation Policy and the Economy*, ed. by J. Lerner, S. Stern, and A. B. Jaffe, Cambridge, Mass.: MIT Press, vol. 2.

GRINDLEY, P. C. AND D. J. TEECE (1997): "Managing Intellectual Capital: Licensing and Cross-Licensing in Semiconductors and Electronics," *California Management Review*, 39, 8–41.

HALL, B. H. (2005a): "Exploring the Patent Explosion," *Journal of Technology Transfer*, 30, 35–48.

——— (2005b): "A Note on the Bias in Herfindahl-type Measures Based on Count Data," *Revue d' Économie Industrielle*, 110, 149156.

HALL, B. H. AND R. ZIEDONIS (2001): "The Patent Paradox Revisited: An Empirical Study of Patenting in the U.S. Semiconductor Industry, 1979-1995," *Rand Journal of Economics*, 32, 101–128.

HARHOFF, D., K. HOISL, AND C. WEBB (2006): "European Patent Citations - How to count and how to interpret them," .

HAYAKAWA, K. (2006): "The Asymptotic Properties of the System GMM Estimator in Dynamic Panel Data Models When Both N and T are Large," HiStat Discussion Paper Series 129, Hitotsubashi University.

HOLLAND, P. AND S. LEINHARDT (1976): "Local structure in social networks," *Sociological methodology*, 7.

HOLONYAK, N. AND S. F. BEVACQUA (1962): "Coherent (visible) Light Emission from Ga (AsP) Junctions," *Applied Physics Letters*, 1, 82.

JAFFE, A. B. (2000): "The U.S. Patent System in Transition: Policy Innovation and the Innovation Process," *Research Policy*, 29, 531–557.

JAFFE, A. B. AND J. LERNER (2004): *Innovation and Its Discontents: How Our Broken Patent System is Endangering Innovation and Progress, and What to Do About It.*, Princeton, N.J.: Princeton University Press.

JORGENSON, D. W. AND C. W. WESSNER, eds. (2007): *Enhancing Productivity Growth in the Information Age: Measuring and Sustaining the New Economy*, National Research Council.

KORTUM, S. AND J. LERNER (1998): "Stronger Protection or Technological Revolution: What is Behind the Recent Surge in Patenting?" in *Carnegie-Rochester Conference Series on Public Policy*, vol. 48, 247–304.

——— (1999): "What is behind the Recent Surge in Patenting?" *Research Policy*, 28, 1–22.

KUSONAKI, K., I. NONAKA, AND A. NAGATA (1998): "Organizational Capabilities in Product Development of Japanese Firms," *Organization Science*, 9, 699–718.

LEE, T. AND L. L. WILDE (1980): "Market Structure and Innovation: A Reformulation," *The Quarterly Journal of Economics*, 94, 429436.

LERNER, J. AND J. TIROLE (2004): "Efficient Patent Pools." *American Economic Review*, 94, 691 – 711.

LOURY, G. C. (1979): "Market Structure and Innovation," *The Quarterly Journal of Economics*, 93, 395–410.

MEYER, M. (2000): "Patent Citations in a Novel Field of Technology - What can they tell about Interactions Between Emerging Communities of Science and Technology?" *Scientometrics*, 48, 151–178.

MILGROM, P. AND J. ROBERTS (1990): "Rationalizability, Learning and Equilibrium in Games with Strategic Complementarities." *Econometrica*, 58, 1255–1278.

MILO, R., S. ITZKOVITZ, N. KASHTAN, R. LEVITT, S. SHEN-ORR, I. AYZENSHTAT, M. SHEFFER, AND U. ALON (2004): "Superfamilies of evolved and designed networks," *Science*, 303, 1538.

MILO, R., S. SHEN-ORR, S. ITZKOVITZ, N. KASHTAN, D. CHKLOVSKII, AND U. ALON (2002): "Network Motifs: Simple Building Blocks of Complex Networks," *Science*, 298, 824.

NARIN, F., K. HAMILTON, AND D. OLIVATRO (1997): "The Increasing Linkage Between U.S. Technology and Public Science," *Research Policy*, 26, 317–330.

NARIN, F. AND E. NOMA (1985): "Is Technology Becoming Science," *Scientometrics*, 7, 369–381.

NATIONAL RESEARCH COUNCIL (2004): *Patent System for the 21st Century, Report of the Board on Science, Technology, and Economic Policy*, Washington, D.C.: National Academies Press.

OECD (1994): *Using Patent Data as Science and Technological Indicators*, Patent Manual.

PALMER, J. AND K. KREUTZ-DELGADO (2003): "A general framework for component estimation," *Proceedings of the 4th International Symposium on Independent Component Analysis*.

REINGANUM, J. F. (1989): "The Timing of Innovation: Research, Development and Diffusion," in *Handbook of Industrial Organization*, ed. by R. Schmalensee and R. D. Willig, Amsterdam, Netherlands: Elsevier Science, vol. 1, 850–908.

ROBERTS, J. (2004): *The modern firm*, Oxford University Press.

ROODMAN, D. (2006): "How to Do xtabond2: An Introduction to "Difference" and "System" GMM in Stata," Working Paper 103, Center for Global Development, Washington.

SCHANKERMAN, M. AND M. NOEL (2006): "Strategic Patenting and Software Innovation," Discusion Paper 5701, CEPR.

SCOTCHMER, S. (2005): *Innovation and Incentives*, MIT Press.

SHAPIRO, C. (2001): "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard-Setting," in *Innovation Policy and the Economy*, ed. by A. B. Jaffe, J. Lerner, and S. Stern, Cambridge, Mass.: MIT Press, vol. 1, 119–150.

SIEBERT, R. AND G. VON GRAEVENITZ (2010): "Jostling for Advantage or Not: Choosing Between Patent Portfolio Races and Ex Ante Licensing," *Journal of Economic Behavior & Organization*, 73, 225–245.

SOMAYA, D., I. O. WILLIAMSON, AND X. ZHANG (2007): "Combining Patent Law Expertise with R&D for Patenting Performance," *Organization Science*.

VIVES, X. (1999): *Oligopoly Pricing, Old Ideas and New Tools*, Cambridge, Massachusetts and London, England: MIT Press.

——— (2005): "Complementarities and Games: New Developments," *Journal of Economic Literature*, XLIII, 437–479.

VON GRAEVENITZ, G., S. WAGNER, AND D. HARHOFF (2011): "How to measure patent thickets—A novel approach," *Economics Letters*, 111, 1–4.

VON GRAEVENITZ, G., S. WAGNER, K. HOISL, B. HALL, D. HARHOFF, P. GIURI, AND A. GAMBARDELLA (2007): *The Strategic Use of Patents and its Implications for Enterprise and Competition Policies*, Report for the European Commission.

WINDMEIJER, F. (2005): "A Finite Sample Correction for the Variance of Linear Efficient Two-step GMM Estimators," *Journal of Econometrics*, 26, 25–51.

YAM, F. K. AND Z. HASSAN (2005): "Innovative Advances in LED technology," *Microelectronics Journal*, 36, 129137.

ZIEDONIS, R. H. (2004): "Don't Fence Me In: Fragmented Markets for Technology and the Patent Acquisition Strategies of Firms," *Management Science*, 50, 804–820.

Appendix

A Technical Appendix for the Theoretical Model

In this section we derive several of the results which we make use of in deriving our theoretical predictions in Section 2. In particular we describe the functions describing the expected number of facets covered $\tilde{F}$ and the probability of patenting a facet p_k.

Note that below we also employ the following definitions:

$$\omega_k \equiv o_k/O \qquad\qquad \phi_k \equiv f_k/F \quad . \tag{10}$$

A.1 The Expected Number of Rival Investors

Here we derive the expected number of rival firms N_O that undertake R&D on the same technology opportunity as firm k. This expected number of rivals can be expressed as a sum of products. Each product gives the probability that a given number of rivals invest in the same technological opportunity. All of these probabilities are then summed to give the overall expected number of rival firms on a given technological opportunity:

$$\begin{aligned} N_O =& \binom{N}{1}\omega_j(1-\omega_j)^{N-1} + 2\binom{N}{2}\omega_j^2(1-\omega_j)^{N-2} + 3\binom{N}{3}\omega_j^3(1-\omega_j)^{N-3}... \\ =& \sum_{i=0}^{N} i\binom{N}{i}(1-\omega_j)^{(N-i)}\omega_l^i. \end{aligned} \tag{11}$$

It can be shown that N_O is increasing in ω. First rewrite N_O as a function of a firm m's choice (ω_m) and the choices of all other firms (ω_l):

$$N_O = N\omega_l^{(N-1)}\omega_m + \sum_{i=0}^{N-1}\left[(1-\omega_m)i + \omega_m(i+1)\right]\binom{N-1}{i}(1-\omega_l)^{(N-1-i)}\omega_l^i. \tag{12}$$

Next we take the derivative with respect to firm m's choice of level of opportunities:

$$\frac{\partial N_O}{\partial \omega_m} = N\omega_l^{(N-1)} + \sum_{i=0}^{N-1}\binom{N-1}{i}(1-\omega_l)^{(N-1-i)}\omega_l^i > 0. \tag{13}$$

An increase in the number of opportunities o_j which other firms invest in, increases the expected number of rivals patenting facets on the same technological opportunity.

A.2 The Expected Number of Facets Covered

The expected number of facets covered through the joint efforts of all firms investing in a technological opportunity is:[32]

$$\tilde{F} = F\Big[1 - (1-\phi_k)\prod_{j=1}^{N_O}(1-\phi_j)\Big] \tag{14}$$

As noted above, the derivatives of this expression with respect to F and f_k are important for the results there. Both of these can be shown to be positive:

$$\frac{\partial \tilde{F}}{\partial F} = 1 - (1-\phi_j)^{N_O}(1+\phi_j N_O) \geq 0\,, \quad \frac{\partial \tilde{F}}{\partial f_k} = \prod_{j=1}^{N_O}(1-\phi_j) > 0\,, \tag{15}$$

where we impose symmetry in the choice of f across firms in the derivative w.r.t. F. This derivative is used for comparative statics purposes, after first derivatives have been taken.

Finally note that the elasticities of $\tilde{F}$ with respect to F and $\tilde{f}_i$ are:

$$\epsilon_{\tilde{F}f_k} = \phi_k \frac{\Big[\prod_{j=1}^{N_O}(1-\phi_j)\Big]}{1-(1-\phi_k)\prod_{j=1}^{N_O}(1-\phi_j)} \quad , \tag{16}$$

$$\epsilon_{\tilde{F}F} = \frac{1-(1-\phi_j)^{N_O}(1+\phi_j N_O)}{1-(1-\phi_j)^{(N_O+1)}} \quad , \tag{17}$$

which shows that $1 \geq \epsilon_{\tilde{F}F} \geq 0$ as the denominator in the fraction is always greater than the numerator. It is useful to observe that the upper bound of the elasticity $\epsilon_{\tilde{F}f_k}$ is decreasing in N_O. To see this note that in equilibrium the elasticity is defined as:

$$\epsilon_{\tilde{F}f_k} = \phi_j \frac{(1-\phi_j)^{N_O}}{1-(1-\phi_j)^{N_O+1}} = \frac{(1-\phi_j)^{N_O}}{(N_o+1)\Big(1-\phi_j \frac{N_o}{2!} + \phi_j^2 \frac{N_o(N_O-1)}{3!} \ldots\Big)} \,. \tag{18}$$

The second expression above makes clear that the upper bound of the elasticity decreases in N_O: $\lim_{\phi_j \to 0} \epsilon_{\tilde{F}f_k} = 1\big/(N_O+1) \leq 1$. Here we make use of the binomial expansion of $(1-\phi_j)^{N_o+1}$. From this expression it is also clear that the lower bound of the elasticity is zero when $\phi_j = 1$.

A.3 The Probability of Patenting a Facet

Now turn to the probability of obtaining a patent on a facet given N_O:

$$p_k = \prod_{j=1}^{N_O}(1-\phi_j) + \frac{N_O}{2}\cdot \phi_j \prod_{j=1}^{N_O-1}(1-\phi_j) + \frac{(N_O)(N_O-1)}{6}\prod_{j=1}^{N_O-2}(1-\phi_j)\prod_{l=N_O-2}^{N_O}(\phi_l)\ldots\,,$$

[32] We are grateful for the help of Professor Helmut Küchenhoff and Mr. Fabian Scheipl in deriving this expression.

$$= \sum_{i=0}^{N_O} \frac{1}{i+1} \binom{N_O}{i} \prod_{j=0}^{N_O-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \tag{19}$$

The properties of this expression are not easily derived. Here we set out the derivative of p_k w.r.t. ϕ and we show that p_k decreases in N_O.

Consider first the effects of an increase in ϕ_m, i.e. an increase in the proportion of facets covered by firm m on the probability that firm k obtains a given facet. To investigate this we reexpress the probability of obtaining a facet as follows:

$$p_k = \left[\sum_{i=0}^{N_O-1} \left[(1-\phi_m)\frac{1}{i+1} + \phi_m \frac{1}{(i+2)} \right] \binom{N_O-1}{i} \prod_{j=0}^{N_O-1-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \right] \tag{20}$$

Then the derivative is:

$$\frac{\partial p_k}{\partial \phi_m} = \left[\sum_{i=0}^{N_O-1} \left[-\frac{1}{(i+1)(i+2)} \right] \binom{N_O-1}{i} \prod_{j=0}^{N_O-1-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \right] < 0 \quad . \tag{21}$$

Finally consider the effects of an increase in N_O on the probability of patenting a facet:

$$\begin{aligned} p_i(N_O+1) - p_i(N_O) = & \sum_{i=0}^{N_O} \frac{1}{i+1} \binom{N_O}{i} \prod_{j=1}^{N_O-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \\ & - \sum_{i=0}^{N_O-1} \frac{1}{i+1} \binom{N_O-1}{i} \prod_{j=1}^{N_O-1-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \\ = & \left[\sum_{i=0}^{N_O-1} (-\phi_m) \frac{1}{(i+1)(i+2)} \binom{N_O-1}{i} \prod_{j=1}^{N_O-1-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \right] \leq 0 \, . \end{aligned} \tag{22}$$

We also plot the function (Figure 7) , allowing ϕ and N_O to vary.

B Proofs

B.1 Proof of Proposition 1

Here we derive the first order conditions that determine the equilibrium number of facets ($\hat{f}_k$) and technological opportunities ($\hat{o}_k$).

$$\frac{\partial \pi_k}{\partial o_k} = V s_k - L(\gamma_k, s_k) - C_o - \gamma_k C_a - \frac{\partial C_c}{\partial o_k} = 0 \quad , \tag{23}$$

$$\frac{\partial \pi_k}{\partial f_k} = \frac{o_k p_k}{\tilde{F}} \left(\left[V \mu \epsilon_{\tilde{F} f_k} - \tilde{F} \left(\frac{\partial L}{\partial \gamma_k} + C_a \right) \right] + \left[V - \frac{\partial L}{\partial s_k} \right] (1 - \epsilon_{\tilde{F} f_k}) \right) = 0 \quad . \tag{24}$$

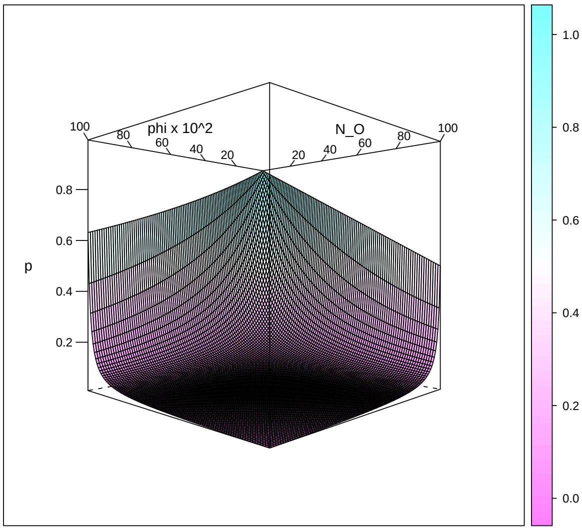

Figure 7: Simulation of p_i for $N_O \in 0, 100$ and $\phi \in 0, 1$.

Next, consider the cross-partial derivatives which must be positive if the game G is supermodular. First, we derive the cross partial derivative with respect to firms' own actions:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_k} = \frac{p_k}{\tilde{F}} \left(\left[V\mu\epsilon_{\tilde{F}f_k} - \tilde{F}\left(\frac{\partial L}{\partial \gamma_k} + C_a\right)\right] + \left[V - \frac{\partial L}{\partial s_k}\right](1-\epsilon_{\tilde{F}f_k})\right) = 0 \quad . \tag{25}$$

This expression corresponds to the first order condition (24) for the optimal number of facets.

Now consider effects of rivals' actions on firms' own actions:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial o_m} = \frac{\partial \tilde{F}}{\partial o_m}\frac{s_k}{\tilde{F}}\left[V(\mu-1) + \frac{\partial L}{\partial s_k}\right] + \frac{\partial p_k}{\partial o_m}\frac{f_k}{\tilde{F}}\left[\left(V - \frac{\partial L}{\partial s_k}\right) - \tilde{F}\left(\frac{\partial L}{\partial \gamma_k} + C_a\right)\right], \tag{26}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_m} = \frac{\partial \tilde{F}}{\partial f_m}\frac{s_k}{\tilde{F}}\left[V(\mu-1) + \frac{\partial L}{\partial s_k}\right] + \frac{\partial p_k}{\partial f_m}\frac{f_k}{\tilde{F}}\left[\left(V - \frac{\partial L}{\partial s_k}\right) - \tilde{F}\left(\frac{\partial L}{\partial \gamma_k} + C_a\right)\right], \tag{27}$$

$$\begin{aligned}\frac{\partial^2 \pi_k}{\partial f_k \partial o_m} &= \frac{\partial \tilde{F}}{\partial o_m}\left[\frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}f_k} - \frac{\partial L}{\partial \gamma_k} - C_a + \frac{\partial^2 L}{\partial {s_k}^2}\frac{s_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right] \\ &\quad - \frac{\partial \epsilon_{\tilde{F}f_k}}{\partial o_m}\left(V - \frac{\partial L}{\partial s_k} - \tilde{F}\frac{\partial V}{\partial \tilde{F}}\right) - \frac{\partial p_k}{\partial o_m}\left[\frac{\partial^2 L}{\partial {\gamma_k}^2}f_k + \frac{\partial^2 L}{\partial {s_k}^2}\frac{f_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right],\end{aligned} \tag{28}$$

$$\begin{aligned}\frac{\partial^2 \pi_k}{\partial f_k \partial f_m} &= \frac{\partial \tilde{F}}{\partial f_m}\left[\frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}f_k} - \frac{\partial L}{\partial \gamma_k} - C_a + \frac{\partial^2 L}{\partial {s_k}^2}\frac{s_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right] \\ &\quad - \frac{\partial \epsilon_{\tilde{F}f_k}}{\partial f_m}\left(V - \frac{\partial L}{\partial s_k} - \tilde{F}\frac{\partial V}{\partial \tilde{F}}\right) - \frac{\partial p_k}{\partial f_m}\left[\frac{\partial^2 L}{\partial {\gamma_k}^2}f_k + \frac{\partial^2 L}{\partial {s_k}^2}\frac{f_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right] \quad .\end{aligned} \tag{29}$$

The game is supermodular if the equations (26)-(29) are non-negative. The following results show that the conditions noted in Proposition 1 must hold simultaneously if the game is supermodular.

Using the first order condition (24), which will hold for any interior equilibrium, it can be shown that:

$$\left[\left(V - \frac{\partial L}{\partial s_k}\right) - \tilde{F}\left(\frac{\partial L}{\partial \gamma_k} + C_a\right)\right] = -\epsilon_{\tilde{F}f_k}\left(V(\mu-1) + \frac{\partial L}{\partial s_k}\right). \tag{30}$$

If $\left(V(\mu-1) + \frac{\partial L}{\partial s_k}\right) > 0$, then the second term in the cross-partial derivatives (26) and (27) is the product of two negative expressions, and then these cross-partial derivatives are positive.

Turning to equations (28) and (29) we can show that:

$$\frac{\partial \epsilon_{\tilde{F}f_k}}{\partial o_m} = \frac{\partial^2 \tilde{F}}{\partial f_k \partial o_m}\frac{f_k}{\tilde{F}} - \frac{\partial \tilde{F}}{\partial f_k}\frac{\partial \tilde{F}}{\partial o_m}\frac{f_k}{\tilde{F}^2} = -\tilde{F}^{-1}\frac{\partial \tilde{F}}{\partial o_m}\left(\frac{\phi_k}{1-\phi_k} + \epsilon_{\tilde{F}f_k}\right) \tag{31}$$

$$\frac{\partial \epsilon_{\tilde{F}f_k}}{\partial f_m} = \frac{\partial^2 \tilde{F}}{\partial f_k \partial f_m}\frac{f_k}{\tilde{F}} - \frac{\partial \tilde{F}}{\partial f_k}\frac{\partial \tilde{F}}{\partial f_m}\frac{f_k}{\tilde{F}^2} = -\tilde{F}^{-1}\frac{\partial \tilde{F}}{\partial f_m}\left(\frac{\phi_k}{1-\phi_k} + \epsilon_{\tilde{F}f_k}\right) \tag{32}$$

This result allows us to rewrite equations (28) and (29) as follows:

$$\begin{aligned}\frac{\partial^2 \pi_k}{\partial f_k \partial o_m} &= \frac{1}{\tilde{F}}\frac{\partial \tilde{F}}{\partial o_m}\left[\left(V(\mu-1) + \frac{\partial L}{\partial s_k}\right)\left(1 - 2\epsilon - \frac{\phi}{1-\phi}\right) + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}f_k} + \frac{\partial^2 L}{\partial {s_k}^2}\frac{s_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right] \\ &\quad - \frac{\partial p_k}{\partial o_m}\left[\frac{\partial^2 L}{\partial {\gamma_k}^2}f_k + \frac{\partial^2 L}{\partial {s_k}^2}\frac{f_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right],\end{aligned} \tag{33}$$

$$\begin{aligned}\frac{\partial^2 \pi_k}{\partial f_k \partial f_m} &= \frac{1}{\tilde{F}}\frac{\partial \tilde{F}}{\partial f_m}\left[\left(V(\mu-1) + \frac{\partial L}{\partial s_k}\right)\left(1 - 2\epsilon - \frac{\phi}{1-\phi}\right) + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}f_k} + \frac{\partial^2 L}{\partial {s_k}^2}\frac{s_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right] \\ &\quad - \frac{\partial p_k}{\partial f_m}\left[\frac{\partial^2 L}{\partial {\gamma_k}^2}f_k + \frac{\partial^2 L}{\partial {s_k}^2}\frac{f_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right].\end{aligned} \tag{34}$$

Given assumptions (VF) and (LC) these two equations will be positive if $\left(V(\mu-1) + \frac{\partial L}{\partial s_k}\right) > 0$ and $\left(1 - 2\epsilon - \frac{\phi}{1-\phi}\right) > 0$. We discuss each condition in turn here:

1. The term $\left(V(\mu-1) + \frac{\partial L}{\partial s_k}\right)$ arises in all the cross-partial derivatives (26)- (29) above, that we need to sign. The cross-partials (26) and (27) will only be positive if the term is positive. Given our assumptions about the legal cost function (LC) this requires that $V(\mu-1) > -\frac{\partial L}{\partial s_k}$, which will require at least that $\mu > 1$.

2. We can shown that $(1 - 2\epsilon_{\tilde{F}f_k}) - \frac{\phi_k}{1-\phi_k} > 0 \Leftrightarrow (1-2\phi) > (1-\phi)^{(N_O+1)}$. This holds for any $\phi < \frac{1}{2}$ and N sufficiently large. These conditions imply a setting in which the ownership of patents belonging to each opportunity is fragmented amongst many firms. It is more likely to arise if the technology is highly complex, otherwise the condition that $\phi < \frac{1}{2}$ is less likely to hold.

Thus we have now shown, that the game we analyze is supermodular if the value of the technology is increasing in the total number of patents granted per facet ($\mu > 1$), the technology is sufficiently complex and there are sufficiently large numbers of firms competing for patents on each technological opportunity.

B.2 Proof of Lemma 1

Define a game G', which differs from game G in that the following assumptions hold:

$$\gamma_k \equiv p_k f_k, \text{ and } V(\gamma_k), \text{ where } \frac{\partial V}{\partial \gamma_k} > 0; \qquad L(\gamma_k), \text{ where } \frac{\partial L}{\partial \gamma_k} > 0 \quad . \tag{35}$$

These assumptions capture the idea that firms benefit only from patents granted to them. However, these patents also raise firms' legal costs.

Objective function:

$$\pi_k(o_k, f_k) = o_k \left(V(\gamma_k) - L(\gamma_k) - C_o - \gamma_k C_a\right) - C_c(o_k) \quad . \tag{36}$$

First Order Conditions:

$$\frac{\partial \pi_k}{\partial o_k} = V(\gamma_k) - L(\gamma_k) - C_o - \gamma_k C_a - \frac{\partial C_c}{\partial o_k} = 0, \quad \frac{\partial \pi_k}{\partial f_k} = o_k p_k \left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a\right) = 0 \quad . \tag{37}$$

Second Order Conditions:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_k} = p_k \left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a\right) = 0 \tag{38}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial o_m} = f_k \frac{\partial p_k}{\partial o_m}\left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a\right) = 0 \tag{39}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_m} = f_k \frac{\partial p_k}{\partial f_m}\left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a\right) = 0 \tag{40}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial o_m} = o_k p_k f_k \frac{\partial p_k}{\partial o_m}\left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2}\right) \tag{41}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial f_m} = o_k p_k f_k \frac{\partial p_k}{\partial f_m}\left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2}\right) \quad . \tag{42}$$

Analysis of the first and second order conditions reveals that this simple model is supermodular under certain conditions. To see this note that the assumptions we impose here (35) and the first oder condition imply that $\frac{\partial V}{\partial \gamma_k} > \frac{\partial L}{\partial \gamma_k}$. Supermodularity requires that $\frac{\partial^2 V}{\partial {\gamma_k}^2} < \frac{\partial^2 L}{\partial {\gamma_k}^2}$ (Viz. equations 41 and 42). These conditions hold if the legal cost function is relatively more convex than the value function. Palmer and Kreutz-Delgado (2003) define greater relative convexity of a twice differentiable function $g(x)$ over a function $h(x)$ as: $g''/g' > h''/h'$. If both the above conditions hold, then the cost function is relatively more convex than the value function. The intuition for this condition is discussed below Lemma 1 in Section 2.4 above.

Apart from the curvature condition this model only requires that the number of facets per technological opportunity is greater than two, i.e. some technological complexity is a precondition for strategic complementarity.

B.3 Proof of Lemma 2

Define a game G'' in which the following assumptions hold:

$$V(\gamma_k), \text{ where } \frac{\partial V}{\partial \gamma_k} > 0; \text{ and assumption (LC).}$$

We impose the restriction on the cross-partial effect in assumption (LC) to keep the interpretation of the model simple. Note that $s_k(\gamma_k)$, so that an interaction of s_k and γ_k would be hard to interpret.

Assumption (LC) implies that legal costs are a function of the absolute number of patents held by a firm (γ_k) and the share of all patents on an opportunity which the firm holds (s_k). The direct effect of holding more patents is to raise costs as more litigation is likely to result. Understanding the effects of share of patents owned on an opportunity is more subtle. It is useful to focus on a setting in which all firms raise patent applications simultaneously. This can lead to an outcome in which the focal firm still obtains the same absolute number of granted patents as before, but holds a lower share of all patents on an opportunity. In this case assumption (LC) implies that the focal firm's bargaining power falls, raising legal costs. This happens although the absolute number of patents remains constant.

Objective Function:

$$\pi_k(o_k, f_k) = o_k \left(V(\gamma_k) - L(\gamma_k, s_k) - C_o - \gamma_k C_a \right) - C_c(o_k) \quad . \tag{43}$$

First Order Conditions:

$$\frac{\partial \pi_k}{\partial o_k} = V(\gamma_k) - L(\gamma_k, s_k) - C_o - \gamma_k C_a - \frac{\partial C_c}{\partial o_k} = 0, \tag{44}$$

$$\frac{\partial \pi_k}{\partial f_k} = o_k p_k \left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a \right) - o_k \frac{\partial L}{\partial s_k} \frac{\partial s_k}{\partial f_k} = 0 \quad . \tag{45}$$

Second Order Conditions:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_k} = p_k \left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a \right) - \frac{\partial L}{\partial s_k} \left(\frac{p_k}{\tilde{F}} - \frac{s_k}{\tilde{F}} \frac{\partial \tilde{F}}{\partial f_k} \right) = 0 \tag{46}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial o_m} = f_k \frac{\partial p_k}{\partial o_m} \left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a \right) - \frac{\partial L}{\partial s_k} \left(\frac{f_k}{\tilde{F}} \frac{\partial p_k}{\partial o_m} - \frac{s_k}{\tilde{F}} \frac{\partial \tilde{F}}{\partial o_m} \right) > 0 \tag{47}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_m} = f_k \frac{\partial p_k}{\partial f_m} \left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a \right) - \frac{\partial L}{\partial s_k} \left(\frac{f_k}{\tilde{F}} \frac{\partial p_k}{\partial f_m} - \frac{s_k}{\tilde{F}} \frac{\partial \tilde{F}}{\partial f_m} \right) > 0 \tag{48}$$

$$\begin{aligned} \frac{\partial^2 \pi_k}{\partial f_k \partial o_m} &= o_k \frac{\partial p_k}{\partial o_m} \left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a \right) + o_k p_k f_k \frac{\partial p_k}{\partial o_m} \left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2} \right) \\ &\quad - o_k \frac{\partial^2 L}{\partial {s_k}^2} \frac{\partial s_k}{\partial f_k} \frac{\partial s_k}{\partial o_m} - o_k \frac{\partial L}{\partial s_k} \frac{\partial^2 s_k}{\partial f_k \partial o_m} \end{aligned} \tag{49}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial f_m} = o_k \frac{\partial p_k}{\partial f_m}\left(\frac{\partial V}{\partial \gamma_k} - \frac{\partial L}{\partial \gamma_k} - C_a\right) + o_k p_k f_k \frac{\partial p_k}{\partial f_m}\left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2}\right)$$
$$- o_k \frac{\partial^2 L}{\partial {s_k}^2}\frac{\partial s_k}{\partial f_k}\frac{\partial s_k}{\partial f_m} - o_k \frac{\partial L}{\partial s_k}\frac{\partial^2 s_k}{\partial f_k \partial f_m} \tag{50}$$

Analyzing this model requires more additional calculations than previously. First, we use the first order condition (45) to substitute terms in the cross-partial equations (47,48):

$$\frac{\partial^2 \pi_k}{\partial o_k \partial o_m} = f_k \frac{\partial p_k}{\partial o_m}\left(\frac{1}{p_k}\frac{\partial L}{\partial s_k}\frac{\partial s_k}{\partial f_k}\right) - \frac{\partial L}{\partial s_k}\left(\frac{f_k}{\tilde{F}}\frac{\partial p_k}{\partial o_m} - \frac{s_k}{\tilde{F}}\frac{\partial \tilde{F}}{\partial o_m}\right) = \frac{\partial L}{\partial s_k}\left(\frac{f_k}{\tilde{F}}\frac{\partial p_k}{\partial o_m}(-\epsilon_{\tilde{F}f_k}) + \frac{s_k}{\tilde{F}}\frac{\partial \tilde{F}}{\partial o_m}\right)$$
$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_m} = f_k \frac{\partial p_k}{\partial f_m}\left(\frac{1}{p_k}\frac{\partial L}{\partial s_k}\frac{\partial s_k}{\partial f_k}\right) - \frac{\partial L}{\partial s_k}\left(\frac{f_k}{\tilde{F}}\frac{\partial p_k}{\partial f_m} - \frac{s_k}{\tilde{F}}\frac{\partial \tilde{F}}{\partial f_m}\right) = \frac{\partial L}{\partial s_k}\left(\frac{f_k}{\tilde{F}}\frac{\partial p_k}{\partial f_m}(-\epsilon_{\tilde{F}f_k}) + \frac{s_k}{\tilde{F}}\frac{\partial \tilde{F}}{\partial f_m}\right)$$

Both of the expressions are positive if $\frac{\partial L}{\partial s_k} > 0$ as $\epsilon_{\tilde{F}f_k} \geq 0$ and $\frac{\partial p_k}{\partial f_m} < 0$, $\frac{\partial p_k}{\partial o_m} < 0$.

Equations (49, 50) can be reorganized in the same way:

$$\frac{\partial^2 \pi_k}{\partial f_k \partial o_m} = o_k \frac{\partial p_k}{\partial o_m}\left(\frac{1}{p_k}\frac{\partial L}{\partial s_k}\frac{\partial s_k}{\partial f_k}\right) + o_k p_k f_k \frac{\partial p_k}{\partial o_m}\left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2}\right)$$
$$- o_k \frac{\partial^2 L}{\partial {s_k}^2}\frac{\partial s_k}{\partial f_k}\frac{\partial s_k}{\partial o_m} - o_k \frac{\partial L}{\partial s_k}\frac{\partial^2 s_k}{\partial f_k \partial o_m}$$
$$\Leftrightarrow \frac{o_k p_k}{\tilde{F}^2}\frac{\partial \tilde{F}}{\partial o_m}(1 - \epsilon_{\tilde{F}f_k})\left[1 + s_k \frac{\partial^2 L}{\partial {s_k}^2}\right] + o_k p_k f_k \frac{\partial p_k}{\partial o_m}\left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2} - \frac{1}{\tilde{F}^2}\frac{\partial^2 L}{\partial {s_k}^2}\right) \tag{51}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial f_m} = o_k \frac{\partial p_k}{\partial f_m}\left(\frac{1}{p_k}\frac{\partial L}{\partial s_k}\frac{\partial s_k}{\partial f_k}\right) + o_k p_k f_k \frac{\partial p_k}{\partial f_m}\left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2}\right)$$
$$- o_k \frac{\partial^2 L}{\partial {s_k}^2}\frac{\partial s_k}{\partial f_k}\frac{\partial s_k}{\partial f_m} - o_k \frac{\partial L}{\partial s_k}\frac{\partial^2 s_k}{\partial f_k \partial f_m}$$
$$\Leftrightarrow \frac{o_k p_k}{\tilde{F}^2}\frac{\partial \tilde{F}}{\partial f_m}(1 - \epsilon_{\tilde{F}f_k})\left[1 + s_k \frac{\partial^2 L}{\partial {s_k}^2}\right] + o_k p_k f_k \frac{\partial p_k}{\partial f_m}\left(\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2} - \frac{1}{\tilde{F}^2}\frac{\partial^2 L}{\partial {s_k}^2}\right) \tag{52}$$

The results in this section are interesting: we find that if the share of patents a firm holds on a given technological opportunity reduces legal costs directly $\frac{\partial L}{\partial s_k} < 0$, as assumed at (LC) above, then the game we analyze here is not supermodular. Similarly, if legal costs are concave in the share of patents and if the concavity of the legal cost function is very strong, then equations (51) and (52) are not positive and supermodularity will fail.

The cross-partial derivatives (47)- (50) are positive if legal costs are concave in the share of patents a firm holds and as long as $1 > -s_k \frac{\partial^2 L}{\partial {s_k}^2}$ and $\frac{\partial^2 V}{\partial {\gamma_k}^2} - \frac{\partial^2 L}{\partial {\gamma_k}^2} - \frac{1}{\tilde{F}^2}\frac{\partial^2 L}{\partial {s_k}^2} < 0$. This requires that the concavity of legal costs with respect to the share of patents held be sufficiently weak. Then, the game we analyze here is supermodular. Note, that the smaller the equilibrium share of patents or the higher the number of facets granted by the patent office, the more easily the model admits concavity of legal costs in the share of patents held by each firm.

While the assumption that legal costs are concave in the share of patents which a firm holds ensures supermodularity of the game, it implies that legal costs fall if the focal firm's share of patents decreases, holding constant the focal firm's absolute stock of granted patents. We do not believe this assumption reflects competition for patents in patent portfolio races.

B.4 Proof of Lemma 3

Define a game $\tilde{G}'$ in which the following assumptions hold:

$$\text{Assumption } (VF) \text{ and } L(\gamma_k), \text{ where } \frac{\partial L}{\partial \gamma_k} > 0 \quad .$$

Objective Function:

$$\pi_k(o_k, f_k) = o_k \left(V(\tilde{F}) s_k - L(\gamma_k) - C_o - \gamma_k C_a \right) - C_c(o_k) \quad . \tag{53}$$

First Order Conditions:

$$\frac{\partial \pi_k}{\partial o_k} = V(\tilde{F}) s_k - L(\gamma_k) - C_o - \gamma_k C_a - \frac{\partial C_c}{\partial o_k} = 0, \tag{54}$$

$$\frac{\partial \pi_k}{\partial f_k} = \frac{o_k p_k}{\tilde{F}} \left(V(\tilde{F})(1 - \epsilon_{\tilde{F} f_k}) + \tilde{F} \left(\frac{\partial V}{\partial \tilde{F}} \epsilon_{\tilde{F} f_k} - \frac{\partial L}{\partial \gamma_k} - C_a \right) \right) = 0 \quad . \tag{55}$$

Second Order Conditions:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_k} = \frac{p_k}{\tilde{F}} V(\tilde{F})(1 - \epsilon_{\tilde{F} f_k}(1 - \mu)) - p_k \left(\frac{\partial L}{\partial \gamma_k} + C_a \right) = 0 \tag{56}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial o_m} = \frac{f_k}{\tilde{F}} \left(\frac{p_k}{\tilde{F}} V(\tilde{F})(\mu - 1) \frac{\partial \tilde{F}}{\partial o_m} + \left[V - \tilde{F} \frac{\partial L}{\partial \gamma_k} - \tilde{F} C_a \right] \frac{\partial p_k}{\partial o_m} \right), \tag{57}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_m} = \frac{f_k}{\tilde{F}} \left(\frac{p_k}{\tilde{F}} V(\tilde{F})(\mu - 1) \frac{\partial \tilde{F}}{\partial f_m} + \left[V - \tilde{F} \frac{\partial L}{\partial \gamma_k} - \tilde{F} C_a \right] \frac{\partial p_k}{\partial f_m} \right), \tag{58}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial o_m} = \frac{V}{\tilde{F}} (\mu - 1)(1 - \epsilon_{\tilde{F} f_k}) \frac{\partial \tilde{F}}{\partial o_m} - \frac{\partial \epsilon_{\tilde{F} f_k}}{\partial o_m} V(1 - \mu) + \tilde{F} \left(\epsilon_{\tilde{F} f_k} \frac{\partial^2 V}{\partial \tilde{F}^2} \frac{\partial \tilde{F}}{\partial o_m} - f_k \frac{\partial^2 L}{\partial {\gamma_k}^2} \frac{\partial p_k}{\partial o_m} \right) \tag{59}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial f_m} = \frac{V}{\tilde{F}} (\mu - 1)(1 - \epsilon_{\tilde{F} f_k}) \frac{\partial \tilde{F}}{\partial f_m} - \frac{\partial \epsilon_{\tilde{F} f_k}}{\partial f_m} V(1 - \mu) + \tilde{F} \left(\epsilon_{\tilde{F} f_k} \frac{\partial^2 V}{\partial \tilde{F}^2} \frac{\partial \tilde{F}}{\partial f_m} - f_k \frac{\partial^2 L}{\partial {\gamma_k}^2} \frac{\partial p_k}{\partial f_m} \right) \quad . \tag{60}$$

From the first order condition it may be shown that the term in square brackets in equations (57 and 58) is negative. This implies that these two derivatives are positive if the elasticity of the value of patenting w.r.t. the number of covered facets (μ) is greater than one.

Turning to the remaining two second order conditions (59 and 60) we can show that these are also positive if $\mu > 1$, $\phi_k < \frac{1}{2}$ and $\frac{\partial^2 L}{\partial {\gamma_k}^2} \geq 0$.

Using the results we derived at (31) above we can then show that:

$$\frac{V}{\tilde{F}}(\mu-1)(1-\epsilon_{\tilde{F}f_k})\frac{\partial \tilde{F}}{\partial f_m}-\frac{\partial \epsilon_{\tilde{F}f_k}}{\partial f_m}V(1-\mu)=\frac{V}{\tilde{F}}(\mu-1)\frac{\partial \tilde{F}}{\partial f_m}\left((1-2\epsilon_{\tilde{F}f_k})-\frac{\phi_k}{1-\phi_k}\right)$$
$$\frac{V}{\tilde{F}}(\mu-1)(1-\epsilon_{\tilde{F}f_k})\frac{\partial \tilde{F}}{\partial o_m}-\frac{\partial \epsilon_{\tilde{F}f_k}}{\partial o_m}V(1-\mu)=\frac{V}{\tilde{F}}(\mu-1)\frac{\partial \tilde{F}}{\partial o_m}\left((1-2\epsilon_{\tilde{F}f_k})-\frac{\phi_k}{1-\phi_k}\right). \tag{61}$$

As discussed above at 2. under equations (33) and (34) the term in brackets here is positive if $\phi_k < \frac{1}{2}$ and N is sufficiently high. The remaining terms in equations (59) and (60) are positive if $\mu > 1$ and if $\frac{\partial^2 L}{\partial {\gamma_k}^2} \geq 0$.

B.5 Proof of Proposition 2

Rewriting the first order condition (24) we can show that:

$$\frac{V-\frac{\partial L}{\partial s_k}-\tilde{F}\left(\frac{\partial L}{\partial \gamma_k}+C_a\right)}{V-\frac{\partial L}{\partial s_k}-V\mu}=\epsilon_{\tilde{F}f_k} \tag{62}$$

Notice that numerator and denominator of the fraction above correspond to the terms in the square brackets in the cross-partials (26) and (27). Both terms must be negative if the equilibrium of the supermodular game is to exist. Equation (62) shows that one term is a multiple of the other. We have also noted that $1 \geq \epsilon_{\tilde{F}f_k} \geq 0$ (Appendix A.2). All this implies that:

$$\tilde{F}\left(\frac{\partial L}{\partial \gamma_k}+C_a\right)<\mu V(\tilde{F}) \Leftrightarrow \frac{\partial L}{\partial \gamma_k}+C_a<\frac{\partial V}{\partial \tilde{F}} \quad . \tag{63}$$

B.6 Proof of Corrolary 1

This result arises because a higher number of firms (N) increases the number of firms per opportunity $\frac{\partial N_O}{\partial N} > 0$. Also, from the definition of $\tilde{F}$ in equation (14) it can be seen that $\frac{\partial \tilde{F}}{\partial N_O} > 0$. The effect of more competitors on the probability of obtaining a patent on a facet $[\frac{\partial p_i}{\partial N_O} < 0]$ is derived in equation (22). Given these sign restrictions, we can show that:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial N}=\frac{f_k}{\tilde{F}}\frac{\partial N_O}{\partial N}\left(\frac{p_k}{\tilde{F}}\frac{\partial \tilde{F}}{\partial N_O}\left[V(\mu-1)+\frac{\partial L}{\partial s_k}\right]+\frac{\partial p_k}{\partial N_O}\left[\left(V-\frac{\partial L}{\partial s_k}\right)-\tilde{F}\left(\frac{\partial L}{\partial \gamma_k}+C_a\right)\right]\right)>0, \tag{64}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial N}=\frac{1}{\tilde{F}}\frac{\partial \tilde{F}}{\partial N_O}\frac{\partial N_O}{\partial N}\left[\left(V(\mu-1)+\frac{\partial L}{\partial s_k}\right)\left(1-2\epsilon-\frac{\phi}{1-\phi}\right)+\frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}f_k}+\frac{\partial^2 L}{\partial {s_k}^2}\frac{s_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right]$$
$$-\frac{\partial p_k}{\partial N_O}\frac{\partial N_O}{\partial N}\left[\frac{\partial^2 L}{\partial {\gamma_k}^2}f_k+\frac{\partial^2 L}{\partial {s_k}^2}\frac{f_k}{\tilde{F}}(1-\epsilon_{\tilde{F}f_k})\right]>0\,. \tag{65}$$

Note that equation (64) has the same structure as equation (26) while equation (65) has the same structure as equation (33). This implies that the terms in the square brackets of equations (64) and (65) are the same as in equations (26) and (33). Therefore these terms have the same signs. Also, the derivatives mutliplying these terms in brackets have the same signs as the derivatives in the proof for Proposition 1. For instance, $\frac{\partial p_k}{\partial o_m} < 0$ and $\frac{\partial p_k}{\partial N_O} < 0$ and these derivatives both multiply a term in brackets that we have shown is negative. As all of these expressions consist of sums of positive terms, this shows that the expressions above have the same signs as those in that proof even though the derivative terms are not identical. This shows that Corollary 1 holds if Proposition 1 holds.

B.7 Proof of Proposition 3

To determine the effects of an increase in technological opportunity O we investigate the following cross-partial derivatives:

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial O} \quad \text{and} \quad \frac{\partial^2 \tilde{\pi}_i}{\partial \tilde{f}_i \partial O} \quad . \tag{66}$$

If the game set out above is smooth supermodular, it follows from equations (26) and (33) that both cross-derivatives here are negative. To see this note that o_j and O only enter this model as a ratio: an increase in O is equivalent to a reduction in o_j.[33] Equations (26) and (33) are both positive if the game G is smooth supermodular. Their signs are determined by the derivatives $\frac{\partial \tilde{F}}{\partial o_j} > 0$ and $\frac{\partial p_i}{\partial o_j} < 0$. The derivatives $\frac{\partial \tilde{F}}{\partial O} < 0$ and $\frac{\partial p_i}{\partial O} > 0$ have exactly opposite signs, reversing the signs of the cross-partial derivatives above.

B.8 Proof of Proposition 4

Greater complexity of a technology reduces competition for individual patents and this increases patenting incentives:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial F} = \frac{\partial \tilde{F}}{\partial F} \frac{s_k}{\tilde{F}} \left[V(\mu - 1) + \frac{\partial L}{\partial s_k} \right] + \frac{\partial p_k}{\partial F} \frac{f_k}{\tilde{F}} \left[\left(V - \frac{\partial L}{\partial s_k} \right) - \tilde{F} \left(\frac{\partial L}{\partial \gamma_k} + C_a \right) \right] > 0, \tag{67}$$

$$\begin{aligned} \frac{\partial^2 \pi_k}{\partial f_k \partial F} = & \frac{1}{\tilde{F}} \frac{\partial \tilde{F}}{\partial F} \left[\left(V(\mu - 1) + \frac{\partial L}{\partial s_k} \right) \left(1 - \epsilon + \frac{\phi(N_O + 1 + \frac{F}{\tilde{F}})}{1 - \phi} \right) + \frac{\partial^2 V}{\partial \tilde{F}^2} \tilde{F} \epsilon_{\tilde{F} f_k} \right. \\ & \left. + \frac{\partial^2 L}{\partial {s_k}^2} \frac{s_k}{\tilde{F}} (1 - \epsilon_{\tilde{F} f_k}) \right] - \frac{\partial p_k}{\partial F} \left[\frac{\partial^2 L}{\partial {\gamma_k}^2} f_k + \frac{\partial^2 L}{\partial {s_k}^2} \frac{f_k}{\tilde{F}} (1 - \epsilon_{\tilde{F} f_k}) \right] > 0 \, . \end{aligned} \tag{68}$$

Note that equation (67) has the same structure as equation (26) while equation (68) has the same structure as equation (33). The terms in square brackest are again identical, apart from the term $\left(1 - \epsilon + \frac{\phi(N_O + 1 + \frac{F}{\tilde{F}})}{1 - \phi}\right)$ which is positive, just as the term it replaces. Now consider

[33] Compare the discussion of the expected number of rivals investing in the same technological opportunity (N_O) in Appendix A.1.

the derivatives in equations (67) and (68). In Section A.2 we show that $\frac{\partial \tilde{F}}{\partial F} > 0$. Next, the definition of p_k shows that this expression consists of products of ϕ and $(1-\phi)$. We have already shown that game G is only supermodular if $\phi < 1/2$. In this case the derivative of p_k with respect to F is negative by the definition of ϕ. This implies that the derivatives in both expressions above have the same signs as the corresponding derivatives in equations (26) and (33) and so both expressions above are positive whenever Proposition 1 holds.

B.9 Proof of Proposition 5

To see that this is true consider the first and second order derivatives of the payoff function with respect to technological opportunities invested in:

$$\frac{\partial \pi}{\partial o_k} = (V - L - C_a)p_k - \frac{\partial C_c}{\partial o_k} = 0 \qquad \frac{\partial^2 \tilde{\pi}}{\partial {o_k}^2} = -\frac{\partial^2 C_c}{\partial {o_k}^2} \quad . \tag{69}$$

If we assume that costs of coordinating technological opportunities are strictly convex: $\frac{\partial^2 C_c}{\partial {o_k}^2} > 0$, then Proposition 5 can be proved with the help of the implicit function theorem:

$$\frac{\partial o_k}{\partial O} = -\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial O} \Big/ \frac{\partial^2 \tilde{\pi}}{\partial {o_k}^2} > 0 \quad , \tag{70}$$

where $\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial O} = (V - L - C_a)\frac{\partial p}{\partial O} > 0$.

B.10 Proof of Corrolary 2

To see this is true note that $\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial N} = (V - L - C_a)\frac{\partial p_k}{\partial N_O}\frac{\partial N_O}{\partial N} < 0$. Then:

$$\frac{\partial o_k}{\partial N} = -\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial N} \Big/ \frac{\partial^2 \tilde{\pi}}{\partial {o_k}^2} < 0 \quad . \tag{71}$$

C Results from OLS and Fixed Effects Regressions

Table 7: Patent Applications Estimates using OLS and Fixed Effects

	OLS models			Fixed effects models		
Variable	OLS 1	OLS 2	OLS 3	FE 1	FE 2	FE 3
log Patentcount$_{t-1}$	0.586***	0.562***	0.560***	0.170***	0.155***	0.154***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
log Patentcount$_{t-1}\times$ Triples		0.001***	0.001***		0.001***	0.001***
		(0.000)	(0.000)		(0.000)	(0.000)
Non Patent References (NPR)	0.057***	0.067***	0.058***	0.004	0.012	-0.011
	(0.002)	(0.002)	(0.003)	(0.007)	(0.008)	(0.009)
NPR $\times$ Triples		-0.001***	-0.001***		0.000*	0.001**
		(0.000)	(0.000)		(0.000)	(0.000)
NPR $\times$ Triples $\times$ Large			-0.000***			-0.000*
			(0.000)			(0.000)
NPR $\times$ Large			0.022***			0.036***
			(0.003)			(0.006)
Fragmentation	0.569***	0.546***	0.544***	0.456***	0.432***	0.431***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Fragmentation $\times$ Triples		0.001***	0.001***		0.001***	0.001***
		(0.000)	(0.000)		(0.000)	(0.000)
Triples	0.000***	0.001***	0.001***	0.001***	-0.001**	-0.001**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Areas	0.018***	0.018***	0.018***	0.082***	0.082***	0.082***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Large	0.183***	0.189***	0.174***	0.259***	0.259***	0.226***
	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.008)
Year dummies	YES	YES	YES	YES	YES	YES
Primary area dummies	YES	YES	YES	YES	YES	YES
Constant	0.106***	0.105***	0.116***	0.031*	0.041**	0.066***
	(0.011)	(0.011)	(0.011)	(0.015)	(0.015)	(0.016)
R-squared	0.686	0.688	0.688	0.323	0.324	0.324
N	173448	173448	173448	173448	173448	173448

*p<0.05, ** p<0.01, *** p<0.001

D LED Technology

Light emitting diodes (LED) are based on physical principles that were discovered in the early 20^{th} century and were first introduced as a practical electronic component by Holonyak and Bevacqua (1962). LEDs consist of semiconducting material that has been impregnated with impurities to create so-called p-n-junctions that generate the physical characteristic of diodes, i.e., current is flowing only from the anode-side to the cathode-side, but not in the reverse direction. Depending on the materials used to impregnate the chip underlying the diode and the way of applying it to the supporting material, different wavelengths of light are emitted by LEDs. Historically, the first usable LEDs were infrared and red devices based on gallium arsenide.

Since the emergence of the first red LEDs, major research paths in LED-technology can be classified in two broad categories comprising (i) the identification of different materials to produce different colours and (ii) improvement of efficiency and operational parameters. The combination of the results from R&D in these two dimensions led to the gradual improvement of this technology.

The nature of the research conducted within the realm of LEDs is a good example of how we think about technology areas in terms of technology opportunity and patentable facets. First, the different materials that are used to impregnate semiconducting materials can be thought of as separate technological opportunities in the technology area of LEDs. Discovery of novel materials that can be used in the production of LEDs stems from basic research that can be conducted within firms or within universities.

Second, different materials require novel production techniques since efficient impregnation of the semiconducting base of LEDs largely depends on the characteristics of the material used (Yam and Hassan, 2005). Therefore, the emergence of novel materials opens up a certain number of patentable facets. Once a novel material has been discovered, firms have to adapt their production techniques to efficiently manufacture LEDs using that material and they have to invest in opportunity-specific R&D to do so. We model these specific R&D efforts as C_o in our theoretical model. Note that such opportunity specific R&D can also lead to more efficient LEDs over time.

Both novel manufacturing techniques as well as efficiency gains can be protected by patent rights and therefore can be considered as examples of patentable facets. If separate firms engage in R&D activities within opportunities it is likely that more than one firm obtains patents on crucial production steps. This might give rise to situations where firms need to access competitors property rights - which we consider to be a hallmark of a complex technology. In fact, patents are crucial in the LED industry and a high degree of cross-licensing and infringement law-suits among can be observed. A list of relevant deals and disputes can be found on http://www.ledsmagazine.com/features/1/8/21/1.

E Complex and Discrete Technologies

Table 8: Classification of technology areas according to OST-INPI/FhG-ISI

Area Code	Description	Classification
1	Electrical machinery, electrical energy	Complex
2	Audiovisual technology	Complex
3	Telecommunications	Complex
4	Information technology	Complex
5	Semiconductors	Complex
6	Optics	Complex
7	Analysis, measurement, control technology	Complex
8	Medical technology	Complex
9	Nuclear engineering	Complex
10	Organic fine chemistry	Discrete
11	Macromolecular chemistry, polymers	Discrete
12	Pharmaceuticals, cosmetics	Discrete
13	Biotechnology	Discrete
14	Agriculture, food chemistry	Discrete
15	Chemical and petrol industry, basic mat	Discrete
16	Chemical engineering	Discrete
17	Surface technology, coating	Discrete
18	Materials, metallurgy	Discrete
19	Materials processing, textiles paper	Discrete
20	Handling, printing	Discrete
21	Agricultural and food processing, machin	Discrete
22	Environmental technology	Complex
23	Machine tools	Complex
24	Engines, pumps and turbines	Complex
25	Thermal processes and apparatus	Complex
26	Mechanical elements	Complex
27	Transport	Complex
28	Space technology, weapons	Complex
29	Consumer goods and equipments	Complex
30	Civil engineering, building, mining	Complex

Description of the 30 technology areas contained in the OST-INPI/FhG-ISI technology nomenclature. We classified the 30 technology areas as complex or discrete attempting to replicate the classification of Cohen et al. (2000).