‘Warrant’ Revisited: Integrating Mathematics Teachers’ Pedagogical and Epistemological Considerations into Toulmin’s Model for Argumentation

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What is it about?

In previous studies (Biza, Nardi & Zachariades, 2009) we have found that teachers make decisions regarding the use of visualisation in their teaching that reflect their many and varied roles in the classroom: as facilitators of students’ learning; as presenters of established mathematical practices; and as mentors preparing students for examinations. In this paper we use Toulmin’s Model for Argumentation (Toulmin, 1958)[1] to investigate further how teachers form their arguments when they discuss the decisions they make in the classroom. Especially, we are interested in the dependence of the warrant deployed in an argument on the field of the decision to which this argument relates. What are teachers’ warrants when they say, for example, *Yes, I would accept a justification based on a graph in my class*?

We argue that deeper understanding of teachers’ arguments can be developed if these arguments are considered in the context of *their* priorities and considerations. For this purpose we develop a set of tasks for teachers to reflect on. The Tangent Task is the one we use in this paper. This task addresses students’ common beliefs about the tangent line: A line is tangent to a curve if there is one and only one common point between the line and that curve; and the tangent line keeps the curve in the one side of it. These beliefs are correct in some cases, but not in all: for example, in the case of a tangent at inflection point of a curve the tangent *crosses* the curve and splits the curve in two parts. The Tangent Task [Link to TASK Tangent] brings the latter case to teachers’ attention by asking for their feedback on two flawed students’ responses to the question: Is $y=2$ a tangent to $f(x) = 3x^3+2$? The first student’s answer is algebraic but it fails to offer adequate proof of why the line is tangent, as it is merely based on the fact that there is only one common point between the line and the curve. The second student’s answer is based on a visual argument (graphing the curve and the line), and it incorrectly concludes that the line is not tangent because it “cuts across” the curve (p.171). We invited 91 pre- and in-service mathematics teachers in Greece to offer written responses to this Task. Then, we interviewed 11 of those teachers. Teachers’ arguments in this paper are those they use to solve the mathematical problem in the Tangent Task [Link to TASK Tangent], evaluate students’ responses to it, and describe the feedback they would give to the students. Our analysis of teachers’ arguments showed the great diversity of warrants used by teachers. We illustrate this in our summary of results below.

[1] Toulmin, S. (1958). *The uses of argument.* Cambridge, UK: Cambridge University Press.

Key results:

- Teachers base their arguments on a range of warrants:
  - An *a priori warrant* can be: *a priori–epistemological* based on a mathematical theorem or concept; or, *a priori–pedagogical* based on a pedagogical principle.
  - An *institutional warrant* can be: *institutional–curricular* based on what is recommended by textbooks; or, *institutional–epistemological* based on the standard practices of the mathematics community.
  - An *empirical warrant* can be: *empirical–professional* based on the arguer's teaching experiences; or, *empirical–personal* based on personal learning experiences in mathematics.
  - An *evaluative warrant* can be based on justifications of a pedagogical choice on the grounds of personal beliefs.

  "Our point is relatively simple: teachers' acceptance, scepticism or rejection of students' mathematical utterances—as expressed in their evaluation of these utterances and their feedback to the students—does not have exclusively mathematical (epistemological) grounding. Their grounding is broader and includes a variety of other influences, most notably of a pedagogical, curricular, professional and personal nature" (p.161)
- The warrants for some teachers' arguments were backed by another argument, which is in turn warranted by being backed by another argument, and so on...
- "[The] strength of conviction with which teachers put forward their arguments is certainly germane to the stability and stealth of the ways in which they are processing prior experience, policy guidelines, professional development and training" (p.170).
- "[There] is often an overt discrepancy between theoretically and out-of-context expressed teacher beliefs about mathematics and pedagogy and actual practice. Therefore, teacher knowledge is likely to be better explored in situation-specific contexts" (p.162). We argue the tangent task offered us insights into what shapes teachers' arguments. More generally, using such tasks affords better understanding of teachers' knowledge and beliefs.

How to put these ideas into practice?

- Why not share the task in this paper with your colleagues and discuss it with them? What different responses did you and your colleagues come up with? What choices would you make in your classroom? What are your warrants for these choices?
- Can you think of similar examples?
- Tell us your thoughts at @mathtask, https://www.uea.ac.uk/groups-and-centres/a-z/mathtask.