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The uniform Schanuel conjecture over the real numbers. (English summary)
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Schanuel's conjecture is the following well-known statement:

Conjecture 1. Suppose that $a_1, \dots, a_n$ are real numbers such that the transcendental degree over the rationals of the field $\mathbf{Q}(a_1, \dots, a_n, e^{a_1}, \dots, e^{a_n})$ is $< n$. Then there exist rational integers $m_1, \dots, m_n$, not all zero, such that

$$\sum_{i=1}^{n} m_i a_i = 0.$$

The so-called uniform Schanuel conjecture is the following statement:

Conjecture 2. Let $V \subseteq \mathbf{R}^{2n}$ be an algebraic variety over $\mathbf{Q}$ of dimension $< n$. Then there exists an integer N such that if $(a_1, \dots , a_n, e^{a_1}, \dots, e^{a_n}) \in V$, then there exist rational integers $m_1, \dots, m_n$, not all zero, such that

$$\sum_{i=1}^{n} m_i a_i = 0$$

with $|m_i| \le N$ for $i = 1, 2, \dots, n$.

Clearly, Conjecture 2 implies Conjecture 1. The authors prove the converse.

Reviewed by *Maurice Mignotte*

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.