Structured light, transmission and scattering

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Numerous theoretical and experimental studies have established the principle that beams conveying orbital angular momentum offer a rich scope for information transfer. However, it is not clear how far it is practicable to operate such a concept at the single-photon level – especially when such a beam propagates through a system in which scattering can occur. In cases where scattering leads to photon deflection, it produces losses; however in terms of the retention of information content, there should be more concern over forward scattering. Based on a quantum electrodynamical formulation of theory, this paper aims to frame and resolve the key issues. A quantum amplitude is constructed for the representation of single and multiple scattering events in the propagation an individual photon, from a suitably structured beam. The analysis identifies potential limitations of principle, undermining complete fidelity of quantum information transmission.

Keywords: quantum communication, structured light, complex light, optical angular momentum, optical vortex, twisted beam, singular optics, quantum electrodynamics

1. INTRODUCTION

Some of the most widely vaunted potential applications of structured light concern the communication and retrieval of information through the production and detection of beams with a suitably tailored structure. It has long been known that it is possible to interpret the topological charge l of helically structured light[1] (the azimuthal quantum number, which is the integer number of twists in the wavefront per wavelength) by interferometric interrogation with counterpropagating or differently charged beams.[2,3] Notably, Leach *et al.*[4] showed that it is rather easily possible to distinguish between even and odd values of l at the single-quantum level. These and many other such studies have prompted wide-ranging propositions for methods that might deploy individual photons to encode information on beam structure – particularly orbital angular momentum – and these are schemes that promise a variety of new quantum communication and data handling applications.[5-8] However, it is not yet clear how far it is practicable to operate such concepts at the single-photon level; in most experimental measurements the perceived status of the observed field as comprising individual photons is a retrospective inference; the reported observations are not being made at the single-photon level. Of course, the experimental difficulties are compounded at low intensity levels – yet in many of the published schemes, operation at precisely this level of data communication is the aim.

Putting such technical difficulties aside there are several other, more fundamental, issues that need to be addressed to secure the necessary viability of principle. Foremost, any scheme for quantum informatics applications based on photon orbital angular momentum is necessarily limited by a well-characterized angle-angular momentum quantum uncertainty,[5,9] a feature convincingly supported by recent measurements of Einstein-Podolsky-Rosen correlations in down-conversion.[10] However the subject of the present analysis is another raft of issues, also signifying potential limitation. When photons from a structured beam propagate through a system in which scattering occurs, losses will obviously result from any process that produces photon deflection – not just because the emergent light may fail to reach the detector, but because there is in general a scrambling of modes when the geometric factors for the scattered light are recast by coordinate transformation, to correctly relate to the input beam. In quantum language, the emergent light is no longer in a state with a sharp value for the azimuthal quantum number. Even so, given that such photons will generally propagate out of the system and fail to reach the detector, it is usually only the associated lowering of efficiency that is a concern. As regards the effect of scattering interactions, the main issue for the viability of most quantum informatics proposals is therefore the degree of fidelity of information conveyance for those photons that continue onwards, through one or more forward scattering events. The analysis that follows slightly modifies the premises, and significantly develops the perspectives and conclusions of a preliminary study communicated in an earlier conference.[11]

2. THE SCATTERING OF PHOTONS PROPAGATING IN A TWISTED BEAM

In a photon-based protocol for information transmission, the only means of eliciting encoded information is by engagement with matter, and it is scattering that will concern us. In the course of a passage through any optical system, each photon within a structured beam will generally experience a sequence of scattering interactions, dominated by forward, elastic scattering; in the absence of absorption and emission, nothing else can change or interpret the state of the light at the level of low photon numbers. Potential implications for the fidelity of transmission thus demand attention not only in connection with optical transmission through or between material components, (forward elastic scattering being the mechanism by means of which refractive elements operate), but also where scattering events are separated by the distances associated with far-field propagation. Theoretical foundations based on quantum electrodynamics (QED) have previously been established and applied for various interactions of Laguerre-Gaussian (LG) light[12-15]; these afford a framework for an analysis that is well suited to identify the fundamental principles, fully accounting for the photonic nature of the interactions, and the orbital angular momentum character of the beam.

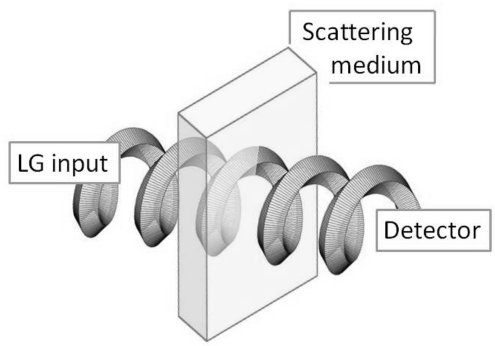

Fig. 1. By considering the successive scattering interactions of individual photons, the analysis aims to secure a better understanding of the degree of fidelity sustained in the passage of Laguerre-Gaussian light through a scattering medium.

The following development of theory primarily focuses on issues that arise first in the case of an isolated scattering event; the implications for a sequence of such events are considered in the subsequent Section. In the Power-Zienau-Woolley multipolar formulation of QED[16,17] all interactions of light with matter entail photon creation and/or annihilation. An isolated scattering event is thus represented as the annihilation of an input photon and the creation of an emergent photon, the corresponding quantum amplitude M_{FI} invoking second order perturbation theory;

$$M_{FI} = \sum_{R \notin \{I,F\}} \frac{\langle F|H_{\text{int}}|R\rangle\langle R|H_{\text{int}}|I\rangle}{(E_I - E_R)} \quad (1)$$

Here, H_{int} is the interaction operator, the bra and ket state designations for the initial, virtual intermediate and final states, $|I\rangle, |R\rangle, |F\rangle$ respectively, are those of the entire system comprising both matter and radiation field components; E_I denotes the initial (equally the final) system state energy, and E_R that of the intermediate state. For several reasons it is appropriate to adopt the electric dipole approximation, writing the interaction operator as $-\boldsymbol{\mu}\cdot\mathbf{e}^{\perp}(\mathbf{r})$, where $\boldsymbol{\mu}$ is the dipole operator and $\mathbf{e}^{\perp}(\mathbf{r})$ the electric field operator. In the single-photon interactions of structured light, orbital angular momentum exchange with matter occurs primarily in electric dipole coupling[18]: although a limited degree of orbital angular momentum can exchange with internal electronic transitions when higher order electric multipole interactions are entertained, each unit increment in multipolar order (in either the input photon annihilation or the output photon creation event) diminishes the associated quantum amplitude by a factor of typically 10^{-3}. Since the present analysis is concerned with the most prominent possible source of information loss, it is therefore reasonable to restrict considerations to electric dipole coupling. To address Laguerre-Gaussian beams with topological charge l and radial node count p, the electric field operator is cast as the following paraxial mode expansion:[12]

$$\mathbf{e}^{\perp}(\mathbf{r}) = \sum_{k,\eta,l,p} \mathrm{i}\left(\frac{\hbar ck}{\varepsilon_0 V}\right)^{1/2} \mathbf{e}^{(\eta)}(k\hat{\mathbf{z}})\, \mathrm{a}_{lp}^{(\eta)}(k\hat{\mathbf{z}})\, f_{lp}(r) \exp(\mathrm{i}kz - \mathrm{i}l\varphi) + h.c. \tag{2}$$

Here, $\mathrm{a}_{lp}^{(\lambda)}(k\hat{\mathbf{z}})$ and $\mathrm{a}_{lp}^{\dagger(\lambda)}(k\hat{\mathbf{z}})$ are the creation and annihilation operators respectively for a twisted mode with wave-vector $\mathbf{k} = k\,\hat{\mathbf{z}}$, polarization η and corresponding unit polarization vector $\mathbf{e}^{(\eta)}$; V is the quantization volume and $h.c.$ denotes Hermitian conjugate. For an LG beam of waist w_0, the radial distribution function is given by;

$$f_{lp}(r) = \frac{C_p^{|l|}}{w_0}\left[\frac{\sqrt{2}\,r}{w_0}\right]^{|l|} \exp\left(\frac{-r^2}{w_0^{\,2}}\right) \; L_p^{|l|}\left(\frac{2r^2}{w_0^{\,2}}\right). \tag{3}$$

From equation (1), results can be derived for the amplitude associated with scattering into any given optical mode. For the detailed calculation we shall consider a particle labeled A, in its ground electronic state, and with cylindrical position coordinates (z, r, ϕ), and displacement $\mathbf{R}_{\mathrm{A}}$ from an arbitrary origin, necessarily intercepted by the incident light (and therefore considered to be offset from the axial phase singularity). As regards the sum over states in (1) addresses both matter and radiation states, it is necessary to entertain two time-orderings for the two photon events: one where annihilation precedes emission, the other with the opposite sequence. The result that emerges for forward scattering, with identical input and scattering modes, is as follows:

$$M^{fi}(\mathbf{k},\mathbf{R}_{\mathrm{A}}) = -f_{lp}^{2}(r)\left(\frac{n\hbar ck}{2\varepsilon_o V}\right) e_i^{(\lambda)}(\mathbf{k})\,\bar{e}_j^{(\lambda)}(\mathbf{k})\,\alpha_{ij}^{\mathrm{A}}(\omega) \exp(\mathrm{i}\Delta l\phi), \tag{4}$$

Here. the angular momentum label λ denotes a state of specified spin and orbital angular momentum – the former determined by circularity of polarization and the latter, topological charge. As regards spin angular momentum, uniquely quantized in circular polarizations,[19] it emerges that accommodating the effects of local isotropy by effecting a rotational average[20] delivers a factor $e_i^{(\lambda)}(\mathbf{k})\bar{e}_j^{(\lambda')}(\mathbf{k})\delta_{ij}$, which vanishes unless the input and output have the same polarization circularity. The factor of n in equation (4) arises from the product of $n^{1/2}$ factors arising in photon annihilation and creation. In fact, for forward scattering with $\Delta l = (l' - l) \neq 0$, this factor would be reduced to $n^{1/2}$ delivered by the annihilation event, with unity for the creation. Again in (4), the dynamic polarizability is given by;

$$\alpha_{ij}^{\mathrm{A}}(-k;k) = \sum_{r_{\mathrm{A}}}\left\{\frac{\mu_i^{0r_{\mathrm{A}}}\mu_j^{r_{\mathrm{A}}0}}{E_{r0}^{\mathrm{A}} - \hbar ck} + \frac{\mu_j^{0r_{\mathrm{A}}}\mu_i^{r_{\mathrm{A}}0}}{E_{r0}^{\mathrm{A}} + \hbar ck}\right\} \equiv \frac{2}{\hbar}\sum_{r\in\{r_{\mathrm{A}}\}}\left\{\frac{\omega_{r0}}{\omega_{r0}^2 - \omega^2}\right\}\mu_i^{r0}\mu_j^{r0}, \tag{5}$$

in which the summation is performed over all electronic eigenstates r_{A} of the material, their energies relative to the ground state being designated $E_{r0} \equiv \hbar\omega_{r0}$. This result applies irrespective of the symmetry of the scatterer, about which no assumptions have been made, except the predominance of electric dipole interactions. In equation (5), the first expression for the polarizability is cast in the form of the separate contributions delivered by each of the contributory time-orderings; the given equivalence to the generally received form on the right is based on the assumption that the transition electric dipole moments are real (as is always possible, given a suitable choice of basis set for the electronic wavefunctions). For later reference, it is instructive to observe that the polarizability tensor is cast in terms of products of the time-conjugate transition electric dipole moments for 'up' and corresponding 'down' transitions, $0 \leftrightarrow$ r.

In passing, the same approach can be used to determine the quantum amplitude for scattering into any other optical mode, and in the present context the most interesting case is another LG mode differentiated by primes, $(\mathbf{k}',\eta',l',p')$. For elastic scattering, $|\mathbf{k}'| = |\mathbf{k}| = k$, and we find:

$$M^{fi}(\mathbf{k},\mathbf{R}_{\mathrm{A}}) = -f_{lp}(r) f_{l'p'}(r)\left(\frac{n^{1/2}\hbar ck}{2\varepsilon_o V}\right) e_i^{(\lambda)}(\mathbf{k})\bar{e}_j^{(\lambda')}(\mathbf{k}')\alpha_{ij}^{\mathrm{A}}(\omega)\exp\left[\mathrm{i}k(z_{\mathrm{A}} - z'_{\mathrm{A}})\right]\exp(\mathrm{i}\Delta \mathbf{l}\phi), \tag{6}$$

Here, the position vector of the scattering particle A is recast in a second cylindrical coordinate basis about an axis *z'* different from *z*, but such that $\mathbf{R}_{\mathrm{A}} = (z_{\mathrm{A}}, r_{\mathrm{A}}, \phi_{\mathrm{A}})_{\mathbf{k}} \equiv (z'_{\mathrm{A}}, r_{\mathrm{A}}, \phi_{\mathrm{A}})_{\mathbf{k}'}$; this is always possible if the two *z* axes are allowed to be non-coplanar.

Returning to the case of forward scattering, a prominent feature of the process is a generally high efficiency, owing to the constructive interference of scattering amplitudes from different locations. In the present context it is evident from equation (4) that, for $\Delta l \neq 0$, the amplitudes for scattering by particles disposed along the same azimuthal angle ϕ interfere constructively – although the effect clearly averages to zero in the bulk of any medium that has cylindrical symmetry. It is more significant that when $\Delta l = 0$, the matrix element loses its angle-dependent phase factor and hence there is no constraint on the spatial extent of constructive interference. Thus, when an LG beam engages in forward scattering, the interference of photon events at different locations are overwhelmingly destructive unless $\Delta l = 0$, serving to ensure orbital angular momentum conservation.

However there is no such guarantee for a single photon event. Indeed it is open to question whether, especially in scattering interceptions well away from the axis of singular phase, the probability amplitude is correctly delivered by integration over the whole of a perpendicular plane – which is the only case where the orthogonality of the Laguerre functions can assist in ensuring $\Delta l = 0$. Although it is evident that an optical input with given angular momentum – spin or orbital – can generate a forward propagating output with the same angular momentum character, it is necessary to exercise caution in drawing conservation-principle conclusions. The time reversibility of a particular quantum sequence does not necessarily denotes causality. A single photon trajectory, through a forward scattering that occurs at a particular location offset from the singular axis, might not accommodate the full character of the beam, and in particular its glide-cylindrical symmetry. It is salutary to recall the caution first advised by Allen *et al,*.[21] that quantized OAM is a concept whose validity holds primarily when it is determined across the full beam profile.[22,23]

3. CONSECUTIVE SCATTERING PROCESSES

There is a limit in the extent to which the above analysis of single-centre scattering can meaningfully inform on the passage of a structured-beam photon through a scattering medium; to account for photon propagation between successive scattering interactions we need to consider consecutive scattering interactions. It is important for several reasons to represent the course of electromagnetic propagation between two material interactions, neither of has the role of absorber or emitter. First, this separates propagation issues from other matters associated with photon creation or annihilation. Secondly, it elicits the detailed form of distance-dependence for such propagation. Thirdly, single- and double-scattering results together provide a clear basis for anticipating and comprehending the results of multiple scattering sequences, as will occur on passage through a solid material for example.

As noted earlier, at the photon level a fully retarded multipolar analysis requires each electromagnetic interaction to be mediated by the creation or annihilation of a photon. Hence for the representation of two consecutive scattering events, the quantum amplitudes entail a total of four photon interactions: input photon annihilation, virtual photon creation and annihilation (serving to connect the two scattering centers), and output photon creation. Here, the sites of virtual photon creation and annihilation will differ (or the calculation would deliver self-energy corrections). It is simplest to assume the sites of real photon annihilation and creation also differ; in the following it will be assumed that one input photon enters at A and one emerges from B. The position vectors of both A and B are considered to be displaced from the phase singular axis of the beam so that the pair displacement vector $\mathbf{R} \equiv \mathbf{R}_{\mathrm{B}} - \mathbf{R}_{\mathrm{A}}$ is approximately parallel to the wave-vector $\mathbf{k}$, not necessarily co-located with it. In calculating the fourth-order matrix element there are twenty-four individual, topologically distinct time-orderings that contribute, derived from the following expression:

$$M_{FI} = \sum_{R \notin \{I,F\}} \sum_{S \notin \{I,F\}} \sum_{T \notin \{I,F\}} \frac{\langle F | H_{\mathrm{int}} | T \rangle \langle T | H_{\mathrm{int}} | R \rangle \langle S | H_{\mathrm{int}} | R \rangle \langle R | H_{\mathrm{int}} | I \rangle}{(E_I - E_T)(E_I - E_S)(E_I - E_R)} \tag{7}$$

For the ensuing calculation, since the requisite virtual photon summations have to be effected over a complete basis set of radiation states, any suitably complete modal basis will suffice. However, a Laguerre-Gaussian mode set is calculationally awkward, and in fact ill-suited for deployment in the present context, particularly since the paraxial approximation imposes undue constraints on the local orientations of electric (and magnetic) field vectors, relative to the beam axis. This has considerable significance for near-field propagation because, as will be shown below, the near-field form of the coupling tensor accommodates prominent contributions from electric fields with finite longitudinal components. Therefore it is expedient, to preserve rather than lose generality, to utilize a standard plane wave basis for effecting the virtual photon summations over wave-vector (with arbitrary direction and magnitude) and polarization. Summing over all time-ordered contributions, and using calculational methods recently developed and described in detail elsewhere,[24,25] the matrix element duly emerges as follows:

$$\begin{aligned} M^{fi}(\mathbf{k},\mathbf{k}',\mathbf{R}_{\mathrm{A}},\mathbf{R}_{\mathrm{B}}) = f_{lp}(r) f_{l'p'}(r) \left(\frac{n^q \hbar c k}{2\varepsilon_o V}\right) e_i^{(\lambda)}(\mathbf{k}) \bar{e}_l^{(\lambda')}(\mathbf{k}') \alpha_{ij}^{\mathrm{A}}(\omega) V_{jk}(k, \mathbf{R}_B - \mathbf{R}_A) \alpha_{kl}^{\mathrm{B}}(\omega) \times \\ \exp\left[\mathrm{i}k(z_{\mathrm{A}} - z_{\mathrm{B}})\right] \exp(\mathrm{i}\Delta l \phi)\ . \end{aligned} \tag{8}$$

Focusing first on the two phase factors in this expression, it is clear that the z-position dependent factor $\exp\left[\mathrm{i}k(z_{\mathrm{A}} - z_{\mathrm{B}})\right]$ has a role in the quantum interference of different scattering pairs, but it has no bearing on angular momentum conservation; the other phase factor, $\exp(\mathrm{i}\Delta l \phi)$, operates in the same way as for single-centre scattering. Attention therefore turns to the electromagnetic coupling between A and B, which involves the retarded resonance electric dipole-electric dipole coupling interaction;[26-28]

$$V_{jk}(k,\mathbf{R}) = \frac{e^{\mathrm{i}kR}}{4\pi\varepsilon_0 R^3} \left\{ (1 - \mathrm{i}kR)\left(\delta_{jk} - 3\hat{R}_j \hat{R}_k\right) - (kR)^2 \left(\delta_{jk} - \hat{R}_j \hat{R}_k\right) \right\} \tag{9}$$

whose dissipative (imaginary) part also features in the dynamics of entanglement between the two centers[29]. The result accommodates both near-field and wave-zone limits. In the near-field; when kR □ 1, virtual features of the emergent photon are most apparent,[27] and the electric field has significant longitudinal components with respect to $\mathbf{R}$; transversality emerges with propagation into the wave-zone, kR □ 1, as forward propagation becomes dominated by contributions with wave-vector $\mathbf{k}$ closely parallel to $\mathbf{R}$. Moreover, the index-symmetric tensor V can be cast as a sum of two terms, one associated with angular momentum $j = 0$, the other $j = 2$, respectively represented by the two square-bracketed terms on the right in the following;[30-32]

$$V_{jk}(k,\mathbf{R}) = V_{jk}^{(0)}(k,\mathbf{R}) + V_{jk}^{(2)}(k,\mathbf{R}) \equiv \left[\frac{-k^2 e^{\mathrm{i}kR}}{6\pi\varepsilon_0 R}\delta_{jk}\right] + \left[\frac{e^{\mathrm{i}kR}}{12\pi\varepsilon_0 R^3}\left(3 - 3\mathrm{i}kR - k^2R^2\right)\left(\delta_{jk} - 3\hat{R}_j\hat{R}_k\right)\right] \tag{10}$$

In its tensor contraction with the polarizabilities of A and B, the coupling in equation (8) invokes the virtual transitions in each center, emerging in the form of a scattering tensor for the pair, expressible as a sum of terms in the form:

$$P_{il}\left(k,\mathbf{R}\right)\equiv\sum_{r_{\mathrm{A}}}\sum_{r_{\mathrm{B}}}\left[\mu_i^{0_{\mathrm{A}}r_{\mathrm{A}}}\mu_l^{r_{\mathrm{B}}0_{\mathrm{B}}}V_{jk}\left(k,\mathbf{R}\right)\right]\left(\frac{\mu_j^{r_{\mathrm{A}}0_{\mathrm{A}}}}{E_{r0}^{\mathrm{A}}\pm\hbar ck}\right)\left(\frac{\mu_k^{0_{\mathrm{B}}r_{\mathrm{B}}}}{E_{r0}^{\mathrm{B}}\pm\hbar ck}\right), \quad (11)$$

which can be similarly partitioned;

$$\begin{aligned}P_{il}\left(k,\mathbf{R}\right)&\equiv P_{il}^{(0)}\left(k,\mathbf{R}\right)+P_{il}^{(2)}\left(k,\mathbf{R}\right)\\ &\equiv\left[-\frac{k^2e^{\mathrm{i}kR}}{3\pi\varepsilon_0R}\sum_{r_{\mathrm{A}}}\sum_{r_{\mathrm{B}}}\frac{\mu_i^{r_{\mathrm{A}}0_{\mathrm{A}}}\mu_l^{r_{\mathrm{B}}0_{\mathrm{B}}}E_{r0}^{\mathrm{A}}E_{r0}^{\mathrm{B}}\left(\boldsymbol{\mu}^{r_{\mathrm{A}}0_{\mathrm{A}}}\cdot\boldsymbol{\mu}^{r_{\mathrm{B}}0_{\mathrm{B}}}\right)}{\left(\left(E_{r0}^{\mathrm{A}}\right)^2-\left(\hbar ck\right)^2\right)\left(\left(E_{r0}^{\mathrm{B}}\right)^2-\left(\hbar ck\right)^2\right)}\right]\\ &+\left[\frac{e^{\mathrm{i}kR}}{6\pi\varepsilon_0R^3}\left(3-3\mathrm{i}kR-k^2R^2\right)\sum_{r_{\mathrm{A}}}\sum_{r_{\mathrm{B}}}\frac{\mu_i^{r_{\mathrm{A}}0_{\mathrm{A}}}\mu_l^{r_{\mathrm{B}}0_{\mathrm{B}}}E_{r0}^{\mathrm{A}}E_{r0}^{\mathrm{B}}\left(\boldsymbol{\mu}^{r_{\mathrm{A}}0_{\mathrm{A}}}\cdot\boldsymbol{\mu}^{r_{\mathrm{B}}0_{\mathrm{B}}}-3\mu_z^{r_{\mathrm{A}}0_{\mathrm{A}}}\mu_z^{r_{\mathrm{B}}0_{\mathrm{B}}}\right)}{\left(\left(E_{r0}^{\mathrm{A}}\right)^2-\left(\hbar ck\right)^2\right)\left(\left(E_{r0}^{\mathrm{B}}\right)^2-\left(\hbar ck\right)^2\right)}\right].\end{aligned} \quad (12)$$

Full details of the derivation, including an analysis of the associated distance-dependence, are presented elsewhere.[33] It is evident that the form of $P^{(2)}$ alone engages longitudinal (z-)components of the transition moments. Clearly, $P^{(0)}$ is non-zero for every pairing of transition moments for A and B that are not specifically orthogonal in their mutual orientation; the equivalent exclusion condition for $P^{(2)}$ is where $\left(\boldsymbol{\mu}^{r_{\mathrm{A}}0_{\mathrm{A}}}\cdot\boldsymbol{\mu}^{r_{\mathrm{B}}0_{\mathrm{B}}}-3\mu_z^{r_{\mathrm{A}}0_{\mathrm{A}}}\mu_z^{r_{\mathrm{B}}0_{\mathrm{B}}}\right)=0$. In general, since $P^{(0)}$ and $P^{(2)}$ each accommodate state summations over every electronic state of both A and B, each term will deliver a significant contribution to the intervening optical propagation.

It is apparent from their constitutive definition, equation (12), that both $P^{(0)}$ and $P^{(2)}$ satisfy the rules of angular momentum coupling between electric dipole transitions. Specifically, when each term is coupled (by tensor index contraction) with the input polarization component e_i in equation (8), the associated relevant irreducible representation products are: for $e_iP_{il}^{(0)}\left(k,\mathbf{R}\right)$, $\mathscr{D}^{(1)}\otimes\mathscr{D}^{(0)}=\mathscr{D}^{(1)}$; for $e_iP_{il}^{(2)}\left(k,\mathbf{R}\right)$, $\mathscr{D}^{(1)}\otimes\mathscr{D}^{(2)}=\mathscr{D}^{(1)}\oplus\mathscr{D}^{(2)}\oplus\mathscr{D}^{(3)}$ where $\mathscr{D}^{(m)}$ is the representation of an irreducible tensor of weight m. Each case delivers the $\mathscr{D}^{(1)}$ contribution necessary for coupling to the other electric dipole transition, and no other; $\mathscr{D}^{(0)}$ does not feature in either result, and the $\mathscr{D}^{(2)}$ and $\mathscr{D}^{(3)}$ contributions vanish, because they produce a result whose angular momentum weight exceeds the product tensor rank. Physically, this result is therefore fully consistent with the intrinsic photon spin, and with the results of studies showing that orbital angular momentum cannot engage in such electric dipole transitions[18].

4. CONCLUSION

In more widely familiar areas of optical beam propagation, it is well established that constructive interference serves to enforce the axial propagation of individual photons, and the conservation of linear and angular momentum that is thereby entailed. One cannot simply assume the same principle applies for the propagation of orbital angular momentum – partly since that particular property is not necessarily intrinsic in photons – but it has nonetheless been shown by the above development of theory that this attribute, also, can be conserved in a similar manner. However there are a number of caveats, particularly necessary in any extrapolation to the behavior of single photons.

The present analysis makes plain the reasons why one cannot expect the topological charge l that quantizes orbital angular momentum to be preserved on deflective scattering, though that specific circumstance is not especially problematic; off-axis scattering would normally be regarded simply as a mechanism for signal loss. For the more important case of forward scattering, the analysis shows that the rules of angular momentum coupling certainly do support retention of orbital angular momentum. However, this principle falls somewhat short of what is desirable in the low-intensity limit that optimizes the potential efficiency of data transmission based on vortex radiation – the experimental conditions upon which some schemes for data transmission have been predicated. In optical systems where the passage and interrogation of individual photons is experimentally manageable, it has to be concluded that the admission of possibility for retaining topological charge cannot be identified with an enforcement of that retention, or elevated to a conservation principle. Primarily, this is because at such levels of intensity there is no longer any significant difference in weighting between the quantum amplitudes for forward elastic scattering into modes with a variety of l values. Secondly, one must also entertain reservations about the extent to which any individual photon can convey structural information that is, in its origin, specifically a beam property. Nonetheless, if any individual process of forward scattering were to produce an effective loss or gain of orbital angular momentum, this would necessarily produce a local torque, and the affected particle, acting as source for the onward propagation under the influence of such a torque, would measurably modify the field experienced by the second scatterer. Such a phenomenon would, indeed, be consistent with the effects of rotating optical elements.[34]

It is also interesting to reflect on certain similarities of feature in the resonance transfer of energy between spin-selected states in quantum dots,[35] a congruence that arises from the quantum amplitude for each process involving a term of identical mathematical structure to the factor cast in square brackets in equation (11). It has been shown that the quantum dot spin and energy transfer process provides for a complete fidelity of spin transmission only in cases where the initial electron spin is aligned, parallel or anti-parallel, with the pair displacement vector. In the present context, this correlates with the principle that scattering can preserve information on orbital angular momentum content when the input and output wave-vectors are parallel – though not necessarily collinear. The latter corollary represents a notable *difference* in behavior, a respect in which a scattering pair acts as if it were a single, extended entity – consistent with the tensor **P** in equation (12) acting as a scattering tensor for the pair. Thus, because the retarded resonance coupling **V** (which the structure of the tensor **P** subsumes) arises by summing over all possible directions of the virtual photon wave-vector, it becomes evident that in the near-field response, A and B can be differently offset with respect to the beam axis, as shown in Fig. 2.

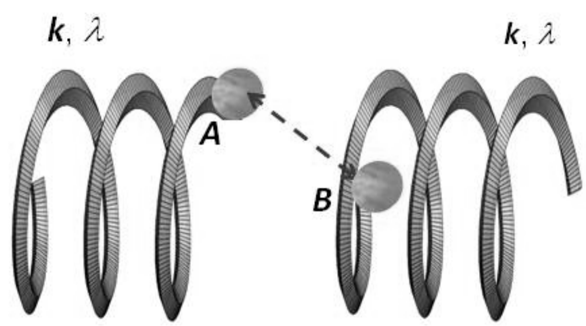

Fig. 2 Double scattering by particles A and B regenerates a photon of the same topological charge (azimuthal quantum number) as the input, even when the pair displacement vector is not aligned with the throughput direction.

To summarize, a raft of issues associated with the fidelity of photon orbital angular momentum transmission have been addressed by quantum electrodynamical analysis of individual and double photon scattering events. Interpreting the analysis has afforded new insights that extend in an obvious fashion to multiple scattering. The physical interpretation of the results is that the transmission of the photon, through a scattering medium, can conserve orbital angular momentum, and that it does so with increasing fidelity as the beam intensity increases. The principle thus confirms the essential viability of many data transmission schemes based on structured light, but it also casts some doubt on their viability at low levels of intensity. Under low-intensity conditions one cannot assume a one-to-one mapping between input and output topological charge, a conclusion that has significant implications for some of the sought developments in quantum informatics. Ultimately, a number of questions still remain over the extent to which one photon can convey all properties of twisted and similarly structured light – and these are issues that must await experimental resolution.

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