Teacher Beliefs and the Didactic Contract on Visualization

Summary of the "Biza, I., Nardi, E., & Zachariades, T. (2009). Teacher beliefs and the didactic contract on visualisation. *For the Learning of Mathematics, 29*(3), 31-36."

What is it about?

We develop classroom scenarios that can be used to explore aspects of teachers' knowledge, beliefs and practices in context, rather than theoretically and out-of-context. In this paper, we look at teachers' *didactic contract* on visualisation. By *didactic contract we mean* the agreement (sometimes implicit) between a teacher and her/his students about what is expected of each other in the classroom. *In* this paper we focus on the didactic contract with regard to whether and how visual representations are an acceptable part of doing mathematics in the classroom: Can visual representations be used as mathematical proof or they can only act as tools for understanding and explanation? To answer this question, we developed the Tangent Task. This task addresses students' common beliefs about the tangent line: A line is tangent to a curve if there is one and only one common point between the line and that curve; and the tangent line keeps the curve in the one side of it. These beliefs are correct in some cases, but not in all: for example, in the case of a tangent at inflection point of a curve the tangent *crosses* the curve and splits in two parts. The Tangent Task brings the latter case to teachers' attention by asking for their feedback on two flawed students' responses to the question: Is y=2 a tangent to $f(x) = 3x^3+2$? The first student's answer is algebraic but it fails to offer adequate proof of why the line is tangent, as it is merely based on the fact that there is only one common point between the line and the curve. The second student's answer is based on a visual proof (graphing the curve and the line), and it incorrectly concludes that the line is not tangent because it "cuts across" the curve (p.32). We invited 91 pre- and in- service mathematics teachers in Greece to offer written responses to this Task. Then, we interviewed 11 of those teachers. Participating teachers' feedback on the students' answers reflect important aspects of their beliefs on visual representations as we summarise below.

Key results:

- 43 out of the 91 teachers incorrectly believed the line was not a tangent: 15 of whom thought this is the case because the line cuts through the curve and 10 of them did not state an argument to support their conclusion.
- Regarding the validity of a visual proof, there is a variation in teachers' beliefs. Some teachers will not accept a graph as a proof, some would accept it in some cases but not in others, while some believe visual proof is acceptable and even reflects a deeper understanding. "This variation of perspective on when a visual argument is acceptable and when it is not is far from alien in the world of mathematics. However, not addressing this variation explicitly in teaching is likely to have serious ramifications for the didactic contract offered to students with regard to mathematical reasoning and proof." (p. 35)
- A less vague *didactic contract* could see explicitly visualisation as a path to insight and algebraic proof as the way to establish the validity of insight. "In both cases there is a pedagogical opportunity for linking imagery with algebra and for embedding the algebra in the immediately graspable meaning in the image." (p. 35)

- Teachers make decisions regarding the use of visualisation in their teaching that reflect their roles as facilitators of students' learning; presenters of established mathematical practices; and mentors preparing students for examinations.
- Pursuing teachers' views on specific teaching situations through a task (similar to the one used in this paper) followed by an interview, does not only allow critical aspects of their views to emerge, but also opens opportunities for teachers' reflection and training.

  How to put these ideas into practice?

  - Why not share the task in this paper with your colleagues and discuss it with them? What different responses did you and your colleagues come up with? What is the role of visual representations in you teaching?
  - Can you think of similar examples?
  - Tell us your thoughts at @mathtask, https://www.uea.ac.uk/groups-and-centres/a-z/mathtask.