REDUCIBILITY POINTS AND CHARACTERISTIC p LOCAL FIELDS I SIMPLE SUPERCUSPIDAL REPRESENTATIONS OF SYMPLECTIC GROUPS.

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Abstract. Let F be a non-Archimedean local field with odd characteristic p. Let N be a positive integer and $G = \mathrm{Sp}_{2N}(F)$. By work of Lomelí on γ-factors of pairs and converse theorems, a generic supercuspidal representation π of G has a transfer to a smooth irreducible representation Π_π of $\mathrm{GL}_{2N+1}(F)$. In turn the Weil–Deligne representation Σ_π associated to Π_π by the Langlands correspondence determines a Langlands parameter ϕ_π for π. This process produces a Langlands correspondence for generic cuspidal representations of G.

In this paper we take π to be simple in the sense of Gross and Reeder, and from the explicit construction of π we describe Π_π explicitly. The method we use is the same as in a previous paper, where we treated the case where F is a p-adic field. It relies on a criterion due to Mœglin on the reducibility of representations parabolically induced from $\mathrm{GL}_M(F) \times G$ for varying positive integers M. We extend this criterion to the case when F has *any* positive characteristic. The main new feature consists in relating reducibility to γ-factors for pairs.

1. Introduction

Let p be a prime number, and F a locally compact non-Archimedean field with residue characteristic p. Let N be a positive integer and G the locally pro-p group $\mathrm{Sp}_{2N}(F)$. All our representations of G and other reductive groups will be smooth representations on complex vector spaces, and a supercuspidal representation is for us irreducible. Let π be a supercuspidal representation of G.

When $\mathrm{char}(F) = 0$, work of Arthur gives a transfer of π to an irreducible representation Π_π of $\mathrm{GL}_{2N+1}(F)$, characterized via some endoscopic and twisted endoscopic character identities. For generic π the transfer can also be obtained by the method of Cogdell, Kim, Piatetski-Shapiro and Shahidi [5], using converse theorems and Langlands–Shahidi factors for pairs (see Appendix B in [1]). When $\mathrm{char}(F) = p$, Lomelí [16, 17] developed the Langlands–Shahidi method for pairs and also established the transfer for generic π.

Theorem 1.1 ([5, Chapter 7], [16, Proposition 9.3 and 9.4], [17, Theorem 7.2])**.** *Let π be a generic supercuspidal representation of $G = \mathrm{Sp}_{2N}(F)$. Then there is an irreducible generic*

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representation Π_π *of* $\mathrm{GL}_{2N+1}(F)$ *such that, for any non-trivial character* ψ *of* F*, any positive integer* M *and any supercuspidal representation* τ *of* $\mathrm{GL}_M(F)$*, we have*

$$\gamma(\pi \times \tau, s, \psi) = \gamma\left(\Pi_\pi \times \tau, s, \psi\right).$$

The representation Π_π *is unique up to isomorphism.*

The (class of) Π_π is called the transfer of π to $\mathrm{GL}_{2N+1}(F)$.

It is also known [5, Theorem 7.3],[16, Theorem 9.6] that Π_π is parabolically induced from a tensor product of unitary supercuspidal representations Π_i of $\mathrm{GL}_{N_i}(F)$, where the sum of the N_i's is $2N+1$, and the Π_i 's are self-dual, of orthogonal type (i.e. the Langlands–Shahidi factor $L(\Pi_i, \mathrm{Sym}^2, s)$ has a pole at $s = 0$), and non-isomorphic to one another. In particular Π_π is self-dual. It also follows from *loc.cit.* that $\Pi_\pi^\vee = \Pi_\pi$ is the transfer of $\pi^\vee$.

Let us now assume that p **is odd** and π is **simple supercuspidal** in the sense of Gross & Reeder [9]. A simple supercuspidal representation of G is generic (see [20, Propositions 5.2, 5.6]) so π has a transfer Π_π. There is an easy construction of π by compact induction (hence the word simple). In [4] we determined Π_π explicitly from π when $\mathrm{char}(F) = 0$. In fact we proved that Π_π is parabolically induced from the tensor product of an explicit quadratic character Π_1 of $\mathrm{GL}_1(\ F) = F^\times$ and an explicit simple supercuspidal representation Π_2 of $\mathrm{GL}_{2N}(F)$. We give a completely similar description here when $\mathrm{char}(F) = p$.

When $\mathrm{char}(F) = 0$ we used in [4] a criterion of Mœglin to determine the Π_i's. That criterion says that an irreducible unitary supercuspidal representation ρ of some $\mathrm{GL}_M(F)$ is (isomorphic to) one of the Π_i's if and only if the representation of $\mathrm{Sp}_{2M+2N}(F)$ parabolically induced from $\rho|\det|^s \otimes \pi$ reduces at some real number $s \geqslant 1$ (in fact at $s = 1$ since π is generic).

Our main contribution here is to show that this criterion is still valid when $\mathrm{char}(F) = p$ (whether or not p is odd). Indeed in [4, section 4] we determined the set of ρ satisfying the reducibility property above when p is odd, irrespective of the characteristic of F, so the reducibility criterion gives the same description of Π_π when $\mathrm{char}(F) = p$. We prove the criterion in section 2, and a precise statement (and proof) of our Theorem are in section 3.

Remark 1.2. Our result when $\mathrm{char}(F) = 0$ was not new, it had been obtained before by Adrian & Kaplan [2] and Oi [20] by different methods. We believe that when $\mathrm{char}(F) > 0$ the result is new. In section 4 we also comment on other approaches, when $\mathrm{char}(F) > 0$.

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2. MŒGLIN'S CRITERION AND L-FUNCTIONS FOR PAIRS

In this section, p **is any prime number,** π is a **generic** (for some choice of Whittaker datum) **supercuspidal** representation of $\mathrm{Sp}_{2N}(F)$ and, as in the introduction, the Π_i, for $i = 1, \dots, r$, are the supercuspidal representations of $\mathrm{GL}_{M_i}(F)$ appearing in the supercuspidal support of Π_π, so that $\Pi \simeq \mathrm{Ind}(\Pi_1 \otimes \cdots \otimes \Pi_r)$, where Ind denotes normalized parabolic induction. Mœglin's criterion to determine the Π_i is the following.

Criterion 2.1. *Let ρ be a unitary supercuspidal representation of some $\mathrm{GL}_M(F)$. Then the following conditions are equivalent:*

- (i) *ρ is isomorphic to one of the Π_i.*
- (ii) *The representation $\mathrm{I}(\rho \times \pi, s)$ of $\mathrm{Sp}_{2M+2N}(F)$ parabolically induced from $\rho |\det|^s \otimes \pi$ is reducible at some real number $s_0 \geqslant 1$.*

When $\mathrm{char}(F) = 0$, see [19, Théorème 3.2]. We prove below that criterion when $\mathrm{char}(F) > 0$.

Remark 2.2. We shall show that in (ii) reducibility occurs at $s_0 = 1$.

Remark 2.3. To be specific about condition (ii) we embed the product $\mathrm{GL}_M(F) \times \mathrm{Sp}_{2N}(F)$ as a Levi subgroup L of $H = \mathrm{Sp}_{2M+2N}(F)$ by sending (g, h) to the block-diagonal matrix with blocks[1] g, h and $(g^T)^{-1}$, and we choose the upper triangular parabolic subgroup P of H with Levi subgroup L. The parabolic induction we use is the normalized one.

Proof of Criterion 2.1. We write q for the cardinality of the residue field of F. Recall that Harish-Chandra's μ-function $\mu(s, \rho \otimes \pi, \psi)$ is the rational function in q^{-s} (also called the *Plancherel measure*) which is the meromorphic continuation of the composition of the standard intertwining operators between $\mathrm{I}(\rho \times \pi, s)$ and the representation induced from the opposite parabolic, for $\mathrm{Re}(s)$ sufficiently large (see [6, §6] for more explanation, including on its normalization).

Reducibility of $\mathrm{I}(\rho \times \pi, s)$ is governed by $\mu(s, \rho \otimes \pi, \psi)$, as recalled in Lemma 7.6 of [6], which gathers results of Harish-Chandra and Silberger valid for F of any characteristic. We deduce that $\mathrm{I}(\rho \times \pi, s)$ is irreducible for any real s if ρ is not self-dual, and that if ρ is self-dual, there is a unique $s_0 \geqslant 0$ where $\mathrm{I}(\rho \times \pi, s_0)$ reduces. Also $s_0 = 0$ if and only if $\mu(0, \rho \otimes \pi, \psi)$ is non-zero, and if $\mu(0, \rho \otimes \pi, \psi) = 0$, then $s_0 > 0$ is the only pole of $\mu(s, \rho \otimes \pi, \psi)$ for $s \geqslant 0$. If ρ is not self-dual, conditions (i) and (ii) are not satisfied, so we assume ρ self-dual.

Since we assume π generic, $\mu(s, \rho \otimes \pi, \psi)$ can be expressed as

$$\gamma(\rho \times \pi^\vee, s, \psi) \gamma\left(\rho^\vee \times \pi, -s, \bar{\psi}\right) \gamma\left(\rho, 2s, \Lambda^2, \psi\right) \gamma\left(\rho^\vee, -2s, \Lambda^2, \bar{\psi}\right)$$

(up to a non-zero constant), where the last two factors are Langlands–Shahidi factors for exterior square. To determine s_0 in terms of those γ-factors, there are two ways: either we

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[1] With the symplectic form used in [4] g^T is the transpose of g with respect to the antidiagonal; in general $g \mapsto g^T$ is the unique anti-involution of the matrix algebra $\mathbb{M}_M(F)$ such that this block-diagonal matrix belongs to H.

follow the pattern of Theorem 8.1 of [22], which, when applied to the classical group H with its Levi subgroup L implies the criterion when $\mathrm{char}(F)=0$, or as in [6] we take advantage of the work of Laurent or Vincent Lafforgue to write all γ-factors as factors attached to Weil–Deligne representations, and work on the Weil group side. We choose to follow Shahidi here, because Lomelí has already verified most of the ingredients Shahidi uses. More specifically, by [16, bottom of page 4314], Propositions 7.2 and 7.4, Lemma 7.5 and Corollary 7.6 of [22] are valid when $\mathrm{char}(F)>0$. In terms of the notation in *loc.cit.* $\gamma(\rho\times\pi^\vee,s,\psi)$ is Shahidi's $\gamma_1(\rho\otimes\pi,s,\psi)$, and $\gamma\left(\rho,2s,\Lambda^2,\psi\right)$ is Shahidi's $\gamma_2(\rho\otimes\pi,2s,\psi)$, which does not depend on π.

Write $\gamma(\rho\times\pi^\vee,s,\psi)=\varepsilon(\rho\times\pi^\vee,s,\psi)L(\rho^\vee\times\pi,1-s)/L(\rho\times\pi^\vee,s)$ where the L-factors are the Langlands–Shahidi factors (which do not depend on ψ). They are obtained by writing $\gamma(\rho\times\pi^\vee,s,\psi)$ as $e\left(q^{-s}\right)P\left(q^{-s}\right)/Q\left(q^{-s}\right)$, where P and Q are two coprime polynomials in one variable which take the value 1 at 0, and e is a monomial. Then one puts $L(\rho\times\pi^\vee,s)=1/P\left(q^{-s}\right)$ and $\varepsilon(\rho\times\pi^\vee,s,\psi)=e\left(q^{-s}\right)$.

Similarly write $\gamma\left(\rho,2s,\Lambda^2,\psi\right)=\varepsilon\left(\rho,2s,\Lambda^2,\psi\right)L\left(\rho^\vee,1-s,\Lambda^2\right)/L\left(\rho,\Lambda^2,s\right)$ by expressing $\gamma\left(\rho,2s,\Lambda^2,\psi\right)=e_2\left(q^{-s}\right)P_2\left(q^{-s}\right)/Q_2\left(q^{-s}\right)$ where P_2 and Q_2 are two coprime polynomials in one variable which take the value 1 at 0, and e_2 is a monomial. Proposition 7.3 of [22] says that all roots of P and P_2 have absolute value 1 (and consequently all roots of Q and Q_2 have absolute value q), and Corollary 7.6 of *loc.cit.* implies that the following conditions are equivalent:

a) $\mu(0,\rho\otimes\pi,\psi)=0$ (that is, $s_0>0$).
b) We have either $P(0)=0$ or $P_2(0)=0$ but not both.

Let us examine in turn the two conditions $P(0)=0$ and $P_2(0)=0$.

Since $\Pi_\pi^\vee$ is the transfer of $\pi^\vee$, we have

$$\gamma(\rho\times\pi^\vee,s,\psi)=\gamma\left(\rho\times\Pi_\pi^\vee,s,\psi\right)$$

and that is the product over i of the $\gamma\left(\rho\times(\Pi_i)^\vee,s,\psi\right)$, and similarly for the ε and L-factors. Moreover it is known that all those factors for $\rho\times(\Pi_i)^\vee$ can also be obtained by the Rankin–Selberg method [10]. In particular $L(\rho\times(\Pi_i)^\vee,s)$ has a pole at $s=0$ if and only if ρ is isomorphic to Π_i, so $\gamma(\rho\times\pi^\vee,0,\psi)$ is 0, equivalently $P(0)=0$, if and only if ρ is isomorphic to one of the Π_i's.

On the other hand $P_2(0)=0$ means that $L\left(\rho,\Lambda^2,s\right)$ has a pole at $s=0$, i.e. ρ is of symplectic type. Then $P(0)$ is not 0, so that ρ is isomorphic to none of the Π_i (which have orthogonal type), which in turn implies that the factor $\gamma(\rho\times\pi^\vee,s,\psi)$ has no zero nor pole for $s\geqslant 0$, and the same is true of the factor $\gamma\left(\rho^\vee\times\pi,-s,\bar\psi\right)$. On the other hand the only pole of $\gamma\left(\rho,2s,\Lambda^2,\psi\right)$ for $s\geqslant 0$ is at $s=1/2$, and the same is true of $\gamma\left(\rho^\vee,-2s,\Lambda^2,\bar\psi\right)$.

We deduce that if $s_0>0$, then either ρ is isomorphic to one of the Π_i, in which case $s_0=1$, or ρ is isomorphic to none of the Π_i, in which case $s_0=1/2$.

That proves the implication (ii) implies (i) in the criterion, and the fact that $s_0=1$ if (ii) is satisfied. On the other hand if (i) is satisfied, then clearly ρ is self-dual of orthogonal type (because the Π_i are) and in particular $P_2(0)$ is not zero since ρ cannot be of symplectic type, and the above analysis implies that (ii) holds with $s_0 = 1$. □

3. The Theorem

From now on (in this section and the next), we assume that p **is odd**.

To state the result properly we need more notation. We denote by $\mathfrak{o}_F$ the ring of integers of F, by $\mathfrak{p}_F$ its maximal ideal, and recall that q is the cardinality of the residual field $k_F = \mathfrak{o}_F/\mathfrak{p}_F$. We fix a character Ψ of F non-trivial on $\mathfrak{o}_F$ and trivial on $\mathfrak{p}_F$. For a given $\beta \in \mathbb{M}_{2N}(F)$ we denote by Ψ_β the function on $\mathrm{GL}_{2N}(F)$ defined by $\Psi_\beta(X) = \Psi \circ \mathrm{tr}(\beta(X-1))$. We use it on restriction to relevant subgroups of $\mathrm{GL}_{2N}(F)$, depending on β, on which it is a character.

We let δ be the (non-trivial) quadratic character of $\mathfrak{o}_F^\times$. The Gauss sum $\sum_{u\in k_F^\times} \delta(u)\Psi(u)$ is known to have modulus $q^{\frac{1}{2}}$ and to satisfy $\xi(\delta,\Psi)^2 = (-1)^{\frac{q-1}{2}}$ where we have put

$$\xi(\delta,\psi) = \frac{\sum_{u\in k_F^\times} \delta(u)\Psi(u)}{|\sum_{u\in k_F^\times} \delta(u)\Psi(u)|}, \text{ a fourth root of unity.} \tag{3.1}$$

We described in [4, §2 and §4.7] the simple supercuspidal representations of $G = \mathrm{Sp}_{2N}(F)$, according to the definition in [9], in our notation (a description already given in [2] and [20]). We recall this briefly.

We fix an Iwahori subgroup $\tilde{I}$ of $\mathrm{GL}_{2N}(F)$ which is stable under the involution whose fixed points are G, and the two first steps of its Moy-Prasad filtration $\tilde{I}(1)$ and $\tilde{I}(2)$. Intersecting with G we obtain an Iwahori subgroup I of G and the two first steps $I(1)$ and $I(2)$ of its Moy–Prasad filtration.

We let E be a totally ramified extension of F of degree $2N$ normalizing $I(1)$ and let β be the inverse of a uniformizer of E. Then Ψ_β is a character of $I(1)$ trivial on $I(2)$ and for any character χ of the center Z of G, isomorphic to $\{\pm 1\}$, the compactly induced representation $\text{c-Ind}_{ZI_{2N}(1)}^G \chi \otimes \Psi_\beta$ is irreducible hence supercuspidal. By definition [9], such a representation is called a simple supercuspidal representation. (See [4, §2], where $\mathrm{char}(F)$ is allowed to be positive, for more details on this.)

When F has characteristic 0, we proved in [4, Theorem 4.16] the Theorem that follows. Here we establish the same statement when $\mathrm{char}(F) = p$.

Theorem 3.2. *Let π be a simple supercuspidal representation of G, written as*

$$\pi = \text{c-Ind}_{ZI(1)}^G \chi \otimes \Psi_\beta,$$

where χ is a character of the center $Z \simeq \{\pm 1\}$ of G and β^{-1} is a uniformizer of a totally ramified extension E of F of degree $2N$ normalizing $I(1)$.
The supercuspidal support of Π_π is $(\mathrm{GL}_1(F) \times \mathrm{GL}_{2N}(F), \Pi_1 \otimes \Pi_2)$ where

- Π_1 *is the ramified quadratic character of* $F^\times$ *characterized by*
$$\Pi_1(N_{E/F}(\beta)) = (-1)^{(N+1)\frac{q-1}{2}};$$
- Π_2 *is the supercuspidal representation of* $\mathrm{GL}(2N,F)$ *defined by*
$$\Pi_2 = \text{c-Ind}_{E^\times \tilde{I}(1)}^{\mathrm{GL}(2N,F)} \tau_\beta \otimes \Psi_{2\beta}$$
where $(\tau_\beta)_{|\mathfrak{o}_E^\times}$ *is the quadratic character of* $\mathfrak{o}_E^\times$ *and*
$$\tau_\beta(\beta) = \chi(-1)\delta(2)\xi(\delta,\Psi).$$

Proof. In [4, Theorem 4.16] we proved that the representations Π_1 and Π_2 satisfy condition (ii) of Criterion 2.1, with reducibility at $s_0 = 1$, hence also condition (i). Since Π_1 is a representation of $\mathrm{GL}_1(F)$ and Π_2 a representation of $\mathrm{GL}_{2N}(F)$, while Π_π is a representation of $\mathrm{GL}_{2N+1}(F)$, we see that Π_1 and Π_2 are the only representations in the supercuspidal support of Π_π. □

Note that we only used the implication (ii) implies (i) in Criterion 2.1.

4. Other approaches and possible extensions

In [4, 5.2 and 5.3] we gave another proof of our main theorem when $\mathrm{char}(F) = 0$, which uses [3] but not the more involved computations of [4, §3 & §4]. But we needed the supplementary information given by a result of Lapid, saying that for a generic supercuspidal representation π of G with central character χ, we have $\varepsilon(\pi, 1/2, \psi) = \chi(-1)$. Lapid's paper [13] is written with the hypothesis that $\mathrm{char}(F) = 0$, and similarly for the more general result of Lapid & Rallis on non-generic representations, though they say that the hypothesis is only for convenience (Lapid–Rallis [14] when $\mathrm{char}(F) = 0$, see recent work of Kakuhama [11] when $\mathrm{char}(F) = p$). Taking that result for granted[2], the proof of [4, 5.2 and 5.3] goes through when $\mathrm{char}(F) = p$. On the other hand, by [4, 4.8], our Theorem implies in turn the identity $\varepsilon(\pi, 1/2, \psi) = \chi(-1)$, when π is simple supercuspidal.

As mentioned in the introduction, when $\mathrm{char}(F) = 0$, Adrian and Kaplan, and also Oi, had proved a result equivalent to our main Theorem, though in different notation.

Adrian and Kaplan [2] used [3], which determined Π_1 and Π_2 up to twist by an unramified character, together with some explicit computations of γ-factors $\gamma(\omega \times \pi, s, \psi)$ for quadratic characters ω of $F^\times = GL_1(F)$. Their paper is written with the hypothesis that $\mathrm{char}(F) = 0$, but we presume that their computations can be adapted when $\mathrm{char}(F) > 0$.

Oi [20] worked directly with the character identities of endoscopy and twisted endoscopy characterizing Arthur's transfer. The present state of the (twisted) trace formula does not allow to derive Arthur's results when $\mathrm{char}(F) > 0$ (see recent progress on the trace formula in [12] and [15]).

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[2] H. Kakuhama tells us that the main point would be to prove Lemma 13 of Lapid–Rallis when $\mathrm{char}(F) = p$.

But Ganapathy and Varma ([7], [8]) used Kazhdan's theory of close local fields to transport Arthur's transfer, for split classical groups, from $\mathrm{char}(F) = 0$ to $\mathrm{char}(F) > 0$. To deduce our Theorem one would need to show that both the construction of simple supercuspidals and the answer given by our Theorem are compatible with that transport. That should not be hard, but we do not embark on that, because our main point was to show that the arguments of [3], [4] are enough to give the result when $\mathrm{char}(F) > 0$. Note however that Lapid's result mentioned above can be easily transfered from $\mathrm{char}(F) = 0$ to $\mathrm{char}(F) > 0$ by the work of [7]. One can probably also prove in this way the extension (by Lapid and Rallis [14]) of Lapid's result to not necessarily generic representations, using that Kakuhama [11] has established the doubling method for $\mathrm{char}(F) > 0$, but the behaviour of factors obtained via the doubling method through close local fields has not been studied yet, and probably it is easier to work directly in Kakuhama's framework.

Simple supercuspidals exist for other (tame) classical groups, and when $\mathrm{char}(F) = 0$ their transfer has been described explicitly (see the references in [1], and also [23]). When $\mathrm{char}(F) > 0$ that remains to be done, although the use of close local fields, for example, seems promising.

On the other hand, when $\mathrm{char}(F) > 0$, Gan and Lomelí [6], using the work of Vincent Lafforgue, construct a transfer Π_π to $\mathrm{GL}_{2N+1}(F)$ for any supercuspidal representation π of $\mathrm{Sp}_{2N}(F)$, not necessarily generic. However they work under an assumption (working hypothesis), which they do not verify: when $\mathrm{char}(F) = 0$ the working hypothesis is true by work of Savin [21]. In future work we plan to prove that hypothesis when $\mathrm{char}(F) = p$, and consequently show that the results of [3] allow to compute the supercuspidal support of Π_π up to unramified character twists, and that similarly the results of [18] hold when $\mathrm{char}(F) = p$.

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