Information processing on equity prices and exchange rate for cross-listed stocks

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Abstract

I propose a novel structural setting to investigate the dynamics of information processing on equity prices and the exchange rate for cross-listed stocks. Using high-frequency data on Brazilian cross-listed firms, I disentangle the effects on firm value of the exchange rate from the other determinants of a firm's cash flow. In general, the results suggest that the U.S. is faster than the home market and that there is a net positive relationship between the value of the domestic currency and the firm's value. This result is linked to the likely partially segmented market characteristic of the home market. Robustness checks confirm the results.

JEL classification: G15, G12, G14, G32, C32, F31

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1 Introduction

Cross-listed foreign firms have become more popular among investors. In the United States, 130 billion shares of American depositary receipts (ADRs) were traded in 2011, up from 38 billion shares in 2005, an astonishing increase of 240%. Raising capital through depositary receipts has also increased (178% from 2013 to 2014) primarily through initial public offerings.[1] Non-arbitrage among markets implies that ADR prices adjusted by the exchange rate should not deviate from their analogous shares traded on the home market for too long. In turn, the geographical price discovery of both equity and the exchange rate has become of great interest (e.g., Eun and Sabherwal 2003, for instance).

In this paper, I methodologically expand the standard price discovery measures to a structural framework. The two most prominent measures of price discovery are the information share (IS) of Hasbrouck (1995) and the component share (CS), as based on the work of Gonzalo and Granger (1995). These methodologies and their numerous variations have been broadly applied to different markets [Eun and Sabherwal (2003) with Toronto and U.S. exchanges], assets [Fernandes and Scherrer (2018) with common and preferred stocks] and financial instruments [Chakravarty et al. (2004) for stocks and options]. However, the methodologies above are static measures that do not allow for an analysis of the speed and dynamics of market adjustment or for a distinction between informational and frictional innovations (Lehmann, 2002; and Yan and Zivot, 2010). Specifically, as emphasized by Lehmann (2002), to measure price discovery, one would be interested in examining the effects of the uncorrelated shocks that drive efficient prices.

Yan and Zivot (2010) move in the direction of a structural methodology by introducing a dynamic measure of price discovery. They adopt a modification of Gonzalo and Ng (2001) and introduce a structural measure of price discovery in the context of one efficient price (one common factor). In a multivariate context of two or more efficient prices, such as the exchange rate and firm value, the literature has yet to provide a strategy to identify the

[1] Market Fragmentation: Does it Really Matter?, transaction services, Citi.

impact of the orthogonal shocks on each efficient price. I propose a strategy that allows for the identification of the structural shocks (informational innovations) that drive efficient prices. Differently from Yan and Zivot (2010), this strategy lifts zero restrictions from the correlation between the two efficient prices. Instead, it imposes a normalization restriction on the variances of the permanent and transitory innovations. This strategy exactly identifies the structural parameter matrices and impulse response functions for structural innovations in that it is possible to quantify how permanent innovations are impounded on prices and exchange rate instantaneously and over time. Finally, a structural price discovery model helps to illustrate the understanding of such a method.

I use a two-year dataset of cross-listed firms from B3 (the Brazilian stock exchange), NYSE, and NYSE Arca (ARCA) and investigate how information processing takes place once accounting for the exchange rate. The results show that the U.S. market impounds information on fundamental values of equities more quickly than the Brazilian market and that U.S. prices adjust instantaneously to changes in the exchange rate. Insights into the size of and liquidity dominance in U.S. stock markets when compared to emerging markets may be derived from this evidence. There is a substantial difference in the initial impact of a latent price shock (the home market captures on average 45% of the total effect and 56% of the foreign market) and the total impact realized, meaning that investors are assimilating and processing the new information in a dynamic fashion. Moreover, most information processing takes place in the first minute. The results also suggest a net positive relationship between the value of the domestic currency and the firm's value that may be linked to the emerging market nature of Brazil, the high correlation between its currency and its equity market, and its likely partially segmented market characteristic.

The use of Brazilian firms is of interest because these firms show significant activity in the cross-listing equity market. In fact, some Brazilian firms are among the top ten most liquid ADR programs in terms of volume and value movers, showing a 20% increase in investor positions from 2008 to 2010. A number of companies have even more intense trading activity

in the U.S. market than in the Brazilian market. Brazilian and U.S. data have the additional advantage of offering large overlapping trading hours, which allows much more information to be gathered compared to the overlap of European and U.S. markets.

The remainder of the paper proceeds as follows. In Section 2, I introduce the price process for cross-listed stocks. In Section 3, I present the estimation procedure and discuss the identification strategy. In Section 4, I present the primary data features, discuss the empirical results, and describe the robustness exercises. Concluding remarks are given in Section 5. The technical results are in Appendix A and in Appendix B I discuss a Monte Carlo study that addresses the performance of the estimation methodology.

2 A simple price discovery model for cross-listed firms

Cross-listing has become a popular way of raising equity capital at a lower cost while boosting liquidity. For instance, on the NYSE, 498 foreign companies from 46 different countries were listed on June 30, 2016. Therefore, geographical price discovery has attracted ample interest.

When a firm cross-lists its shares in a foreign market, prices in the home and foreign markets should not drift apart because they reflect the value of the same security. Because shares are traded in different currencies, two fundamental values link these prices: the firm's fundamental value (the efficient price) and the fundamental link between the two currencies (the efficient exchange rate). Although the prices of the same asset traded at different venues should not drift apart, they may not be equal to the efficient prices at every point in time because markets may process information differently. Liquidity issues and asymmetric information cause transaction prices to adjust to the efficient prices at various speeds (e.g., Harris et al., 1995; Hasbrouck, 1993; and de Jong and Schotman, 2010).

The most natural class of structural time series models for market microstructure and price discovery in fragmented markets is the partial adjustment model (e.g., Amihud and

Mendelson, 1987; Hasbrouck and Ho, 1987; and Yan and Zivot, 2010). To formally address how cross-listed stock prices adjust to their efficient price and exchange rate, I extend the partial price adjustment model to accommodate the efficient exchange rate and its interactions with the observed share prices. Essentially, the model now includes the observed prices as a function of the fundamental value of the asset, the efficient exchange rate, their lagged values, and transitory terms. This structural setting allows the analysis of how transaction prices in different markets are affected by changes in the firm's efficient price and the exchange rate over time. As a by-product, the partial adjustment model shows how exchange rate movements have a permanent impact on the firm's intrinsic value.

Assume that the firm's efficient price (m_t) and the efficient exchange rate (e_t) are expressed in logarithmic terms and modeled as random walk processes. They are latent prices driven by two uncorrelated innovations: one associated with the firm's efficient price (η_t^m) and another with the efficient exchange rate (η_t^e). In this context, η_t^m summarizes all of the information affecting the present value of the firm's future cash flows, except for the one contained in η_t^e. Accordingly, m_t and e_t are as follows:

$$m_t = m_{t-1} + \eta_t^m + \rho\eta_t^e, \tag{1}$$

$$e_t = e_{t-1} + \lambda\eta_t^m + \eta_t^e, \tag{2}$$

where e_t is defined in terms of the home currency (home over foreign currency) and η_t^m and η_t^e are assumed to be serially and mutually uncorrelated with diagonal covariance matrix $diag\left(\varsigma_m^2, \varsigma_e^2\right)$ and $\mathbb{E}\left(\eta_t^m\right) = \mathbb{E}\left(\eta_t^e\right) = 0$. Notably, the structural innovations η_t^m and η_t^e are labeled permanent innovations so that $\eta_t^P = (\eta_t^m, \eta_t^e)'$.

The novelty of (1) is that it accommodates effects from the exchange rate, as ρ accounts for the exchange rate effect on firm value. This feature incorporates the exchange rate effect that is well documented in low-frequency data in a high-frequency structural setting. Accordingly, the results in Subsection 4.2.2 highly support the exchange rate effect in (1).

Additionally, in (2) λ is allowed to be different from zero. This is in line with the literature that suggests that changes in equity prices in emerging countries are not independent from exchange rate movements (Bekaert et al. 2011).

Let P_t now be a k-dimensional vector containing the observed log prices for the same asset traded at different markets (home and foreign markets) and the exchange rate. Without loss of generality, fix $k = 4$, meaning that $P_t = \left(p_{1,t}, w_t, p^*_{3,t}, p^*_{4,t}\right)'$ consists of the logarithm of the transaction price on the home market ($p_{1,t}$), the logarithm of the observed exchange rate (w_t) defined as the home currency over the foreign currency, and the logarithm of transaction prices in two foreign markets expressed in foreign currency ($p^*_{3,t}$ and $p^*_{4,t}$).[2] Denote γ_i and $\dot{\gamma}_i$, with $i = 1, 2, 3, 4$, the partial adjustment coefficients from changes in m_t and e_t, respectively, and η^T_t the 2×1 vector of transitory innovations.[3] Specifically, transitory innovations are assumed to be serially and mutually uncorrelated white noise processes and reflect the presence of trading frictions. Differently from the permanent innovations η^P_t, η^T_t does not affect the efficient price and exchange rate. From Gonzalo and Ng (2001), the defining characteristics of η^P_t and η^T_t are rather different. Specifically, η^P_t has a non-zero long-run effect on the expected price levels, while η^T_t is defined such that there is no long-run impact on the observed transaction prices:

$$\lim_{h\to\infty} \partial \mathbb{E}_t(P_{t+h}/\partial \eta^{P\prime}_t) \neq 0, \quad \text{and} \quad \lim_{h\to\infty} \partial \mathbb{E}_t(P_{t+h}/\partial \eta^{T\prime}_t) = 0, \tag{3}$$

where $\mathbb{E}_t$ denotes the conditional expectation in relation to past information up to time t.

[2] $p^*_{3,t}$ and $p^*_{4,t}$ entail prices in foreign currencies (the currencies in which they are actually traded), whereas $p_{3,t}$ and $p_{4,t}$ are expressed in the home currency. The relationship between them is given as $p^*_{i,t} = p_{i,t} - w_t$ with $i = 3, 4$.

[3] For identification purposes, the number of structural shocks (permanent and transitory) must be equal to the number of observed prices.

It follows that the partial price adjustment model is as follows:

$$p_{1,t} = p_{1,t-1} + \gamma_1 (m_t - p_{1,t-1}) + \dot{\gamma}_1 (e_t - w_{t-1}) + b_1 \eta_t^T, \tag{4}$$

$$w_t = w_{t-1} + \gamma_2 (m_t - m_{t-1}) + \dot{\gamma}_2 (e_t - w_{t-1}) + b_2 \eta_t^T, \tag{5}$$

$$p^*_{3,t} = p^*_{3,t-1} + \gamma_3 (m_t - p_{3,t-1}) + \dot{\gamma}_3 (e_t - w_{t-1}) + b_3 \eta_t^T, \tag{6}$$

$$p^*_{4,t} = p^*_{4,t-1} + \gamma_4 (m_t - p_{4,t-1}) + \dot{\gamma}_4 (e_t - w_{t-1}) + b_4 \eta_t^T, \tag{7}$$

where b_1, b_2, b_3, and b_4 are 1×2 parameter vectors. The partial adjustment parameters $\dot{\gamma}_i$ and γ_i capture the speed of information processing (price discovery). Solving for ΔP_t allows expressing market returns as a function of current and lagged values of the permanent and transitory innovations, thus giving the structural infinite vector moving average (VMA) representation:

$$\Delta P_t = \breve{d}_0 \eta_t + \breve{d}_1 \eta_{t-1} + \breve{d}_2 \eta_{t-2} + ... = \sum_{i=0}^{\infty} \breve{d}_i \eta_{t-i} = \breve{D}(L) \eta_t, \tag{8}$$

where $\Delta P_t = (\Delta p_{1,t}, \Delta w_t, \Delta p^*_{3,t}, \Delta p^*_{4,t})'$, $\eta_t = (\eta_t^P, \eta_t^T)'$, and $\breve{D}(L) = \left(\breve{d}_0 + \breve{d}_1 L + \breve{d}_2 L^2 ...\right)$ is an infinite order lag operator with $\breve{d}_0, \breve{d}_1, \breve{d}_2, ...$ denoting the 4×4 parameter matrices and L being the usual lag operator. As advocated by Lehmann (2002), the structural VMA representation in (8) enables a clear interpretation of price discovery, because the sources of shocks are identified (i.e., the permanent innovations driving the fundamental price and exchange rate). Specifically, three price discovery metrics emerge from (8): first, the instantaneous (initial) impact, $\breve{d}_0$; second, the impulse response analysis showing the responses of market prices to permanent innovations; third, the long-run impact matrix $\breve{D}(1)$ that corresponds to the total response of market prices to changes in the permanent innovations, and enables inference on the exchange rate effects on firm value. Using equations (1)-(2) and (4)-(7), the solutions for $\breve{d}_0$ and $\breve{D}(1)$ are summarized in Proposition 1.

Proposition 1 *Let the partial price adjustment model defined in equations (4)-(7) and the*

efficient prices defined in equations (1)-(2) hold. The initial impact, $\breve{d}_0$, and the long-run impact, $\breve{D}(1)$, matrices are given by:

$$\breve{d}_0 = \begin{pmatrix} \gamma_1 + \dot{\gamma}_1\lambda & \dot{\gamma}_1 + \gamma_1\rho & b_1 \\ \gamma_2 + \dot{\gamma}_2\lambda & \dot{\gamma}_2 + \gamma_2\rho & b_2 \\ \gamma_3 + \dot{\gamma}_3\lambda & \dot{\gamma}_3 + \gamma_3\rho & b_3 \\ \gamma_4 + \dot{\gamma}_4\lambda & \dot{\gamma}_4 + \gamma_4\rho & b_4 \end{pmatrix}, \qquad \breve{D}(1) = \begin{pmatrix} 1 & \rho & 0_{1\times 2} \\ \lambda & 1 & 0_{1\times 2} \\ 1-\lambda & \rho - 1 & 0_{1\times 2} \\ 1-\lambda & \rho - 1 & 0_{1\times 2} \end{pmatrix}. \tag{9}$$

The proof of Proposition 1 appears in Appendix A.

The first and second columns of $\breve{d}_0$ account for the instantaneous responses to η_t^m and η_t^e, respectively, and are thus seen as a structural price discovery measure. Furthermore, $\breve{d}_0$ depends on the partial adjustment coefficients, meaning that the instantaneous response to shocks on the permanent innovations may be larger or smaller than the long-run impact matrix $\breve{D}(1)$, depending on the sign and magnitude of γ_i and $\dot{\gamma}_i$ with $i = 1, 2, 3, 4$. Notably, $\breve{d}_0$ markedly differs from the reduced form based Hasbrouck's (1995) IS type of measure, as the former is related to the uncorrelated innovations, while the latter is constructed using market innovations, which tend to be highly contemporaneously correlated even at high frequencies (Dias et al. 2020).

The total effect of innovations in η_t to observed prices is given by $\breve{D}(1)$. Specifically, the first column collects the long-run effect of an innovation in η_t^m, while the second column identifies the total exchange rate effect on firm value (i.e., the total response to a shock in η_t^e). It follows that the partial price adjustment model implies that innovations to the efficient exchange rate have a total effect on the home and foreign prices equal to ρ and $(\rho - 1)$, respectively; hence, ρ accounts for the net effect on firm value. Note that $\breve{D}(1)$ is solely a combination of parameters from (1) and (2). This follows because the transitory innovations are assumed to have no long-term effect on observed prices, which implies that parameters loading on these transitory innovations, γ_i and $\dot{\gamma}_i$ with $i = 1, 2, 3, 4$, should also have no long-term effect on prices.

3 Econometric framework

This section shows how to estimate instantaneous and total effects in a structural setting, such that indirect inference can be performed to uncover the values of ρ and λ.

3.1 Price discovery measures in reduced and structural forms

Consider the market setting where a single asset trades in the home and foreign markets and the exchange rate. Let P_t be a $k \times 1$ vector collecting market prices and the exchange rate. Prices at different markets should not drift apart much, oscillating around the (latent) efficient price, as they refer to the same asset. In econometric terms, P_t is integrated of order one, $I(1)$, and the price changes ΔP_t are integrated of order zero, $I(0)$. Furthermore, the prices of cross-listed assets are expected to cointegrate. Because P_t consists of market prices and the observed exchange rate, there are $k-2$ cointegrating relationships, with log prices sharing the asset's efficient price and the efficient exchange rate as their common stochastic trends. The dynamics of the first differences of P_t can be represented by the vector error correction (VEC) models:

$$\Delta P_t = \alpha\beta' P_{t-1} + \sum_{\ell=1}^{p} \Gamma_\ell \Delta P_{t-\ell} + u_t, \tag{10}$$

where α is a $k \times r$ error correction matrix; β is a $k \times r$ cointegrating matrix; r is the number of cointegrating vectors; Γ_ℓ with $\ell = 1, 2, ..., p$ are the $k \times k$ autoregressive matrix coefficients; and u_t is a zero-mean white noise process with a $k \times k$ non-diagonal covariance matrix Ω. The VEC model in (10) has a reduced-form VMA representation as follows:

$$\Delta P_t = u_t + \psi_1 u_{t-1} + \psi_2 u_{t-2} + ... = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j} = \Psi(L) u_t, \tag{11}$$

where $\Psi(L) = (I_k + \Psi_1 L + \Psi_2 L^2, ...)$ is an infinite order lag polynomial and Ψ_j with $j = 1, 2, ...$ are non-linear functions of α, β, and Γ_ℓ, with $\ell = 1, 2, ..., p$ in (10).

Reduced-form measures of price discovery, namely, any variant of Hasbrouck's (1995) IS and the CS measures (e.g., Booth et al., 1999; and de Jong, 2002) are based on the estimates of the common row of $\Psi(1)$ and the covariance matrix of the market innovations Ω.[4] However, as pointed out by Yan and Zivot (2010), inference on the price discovery mechanism should be gauged from the innovations on the efficient prices, rather than market innovations, as the latter are contaminated by frictional information that ultimately confounds the price discovery analysis.

A natural way of addressing the abovementioned issues is to write a VMA in its structural form, i.e., as a function of orthogonalized permanent and transitory innovations that satisfy the conditions expressed in (3). Gonzalo and Ng (2001) show that because P_t is $I(1)$, there must exist $k-r$ innovations that have permanent effects on the levels of P_t. It then follows that ΔP_t admits the following structural VMA:

$$\Delta P_t = d_0\eta_t + d_1\eta_{t-1} + d_2\eta_{t-2} + ... = \sum_{i=0}^{\infty} d_i\eta_{t-i} = D(L)\eta_t = \begin{pmatrix} D(L)_{11} & D(L)_{12} \\ D(L)_{21} & D(L)_{22} \end{pmatrix} \begin{pmatrix} \eta_t^P \\ \eta_t^T \end{pmatrix}, \quad (12)$$

where $\eta_t = (\eta_t^{P\prime}, \eta_t^{T\prime})'$ is a $k\times 1$ vector, η_t^P is a $(k-r)\times 1$ vector denoting the permanent innovations, η_t^T is a $r\times 1$ vector collecting the transitory innovations, $D(L) = (d_0 + d_1L + d_2L^2 + ...)$ is an infinite order lag operator, and $D(L)_{11}$, $D(L)_{12}$, $D(L)_{21}$ and $D(L)_{22}$ are blocks of $D(L)$ with coefficient matrices of dimensions $(k-r)\times(k-r)$, $(k-r)\times r$, $r\times(k-r)$, and $r\times r$, respectively. As highlighted by Gonzalo and Ng (2001), the number of structural shocks must equal the number of variables in the system, so that $D(L)$ is invertible. The difference in the economic interpretation between (11) and (12) is the same as the well-known case

[4] More precisely, Hasbrouck's (1995) IS measure for market m is defined as $IS_m = \left([\psi'Q]_{(m)}\right)^2 / \psi'\Omega\psi$ with $m = 1, .., k$, where ψ is the common row of $\Psi(1)$, and Q is any factorization (usually the Cholesky decomposition) of Ω, such that $QQ' = \Omega$, $[\psi'Q]_{(m)}$ is the m^{th} element of the row matrix $[\psi'Q]$. Variants of the IS measure that cover the setting of cross-listed stocks are presented in Grammig et al. (2005) and Fernandes and Scherrer (2018). The CS measure for market m is defined as $CS_m = \alpha_{\perp,(m)}$ with $m = 1, .., k$, where $\alpha_{\perp,(m)}$ denotes the m element of $\alpha_\perp$ and $\alpha_\perp$ is the orthogonal complement of the speed-of-adjustment parameter α, such that $\alpha_\perp'\alpha = 0$ and $\sum_{m=1}^{k} \alpha_{\perp,(m)} = 1$.

of the structural versus reduced-form vector autoregressive models, with the former being widely used for policy analysis in macroeconomics contexts (see Rubio-Ramírez et al. 2010 for a detailed discussion).

In the price discovery context, the structural VMA representation in (12) relates directly to the solution of the partial price adjustment model in (8). First, because d_0 is not an identity matrix, the first $(k-r)$ columns of d_0 account for the instantaneous responses to permanent shocks on prices and are therefore seen as measures of price discovery. Second, a dynamic measure of price discovery is obtained by computing the accumulated responses over n periods to an impulse in η_t^P. It answers how quickly permanent information is impounded in the different markets and is computed as the first $(k-r)$ columns of $\sum_{i=0}^{n} d_i$, where n is arbitrarily fixed. Finally, as a by-product, $D(1) = \sum_{i=0}^{\infty} d_i$ gives the total accumulated effects of impulses into the permanent and transitory innovations. Specifically, the first $(k-r)$ columns of $D(1)$ reflect the long-run responses to shocks on the permanent innovations, whereas the remaining r columns are expected to equal zero, because transitory innovations should have no long-run impact on price levels. In that, the first $(k-r)$ columns are useful for inference on the structural parameters ρ and λ.

3.2 Identification strategy

The goal in this subsection is to recover η_t, d_0, and $D(1)$ from the parameters of the reduced-form VEC model in (10). Moving from the reduced-form VMA to its structural counterpart involves a series of identification restrictions that usually require prior knowledge of the importance of each market, which might be difficult or perhaps questionable. One way to partly overcome this issue is to consider assumptions regarding permanent and transitory innovations (Gonzalo and Granger, 1995; and Gonzalo and Ng, 2001), as this strategy does not require any prior judgement about the market's importance. To this end, I adopt a modification of the two-step orthogonalization procedure discussed in Gonzalo and Ng (2001).

The first step consists of identifying "unorthogonalized" permanent and transitory inno-

vations from the reduced-form VMA representation of the VEC model in (11). Essentially, the goal is to define a matrix that rotates u_t and expresses ΔP_t as function of $(k-r)$ permanent and r transitory innovations, rather than the market specific innovations. This step is based on Gonzalo and Ng's (2001) Proposition 1 (the P-T decomposition) and rotates u_t so that it can be decomposed into permanent, ε_t^P, and transitory, ε_t^T, innovations. Notably, these shocks are still in their reduced form, meaning that they are not mutually uncorrelated. In turn, there exists a $k \times k$ matrix G, such that $G\epsilon_t = \varepsilon_t = (\varepsilon_t^{P\prime}, \varepsilon_t^{T\prime})'$, where G must be chosen such that ε_t^P has a permanent effect on P_t, whereas the reduced-form transitory innovations ε_t^T have no long-term effects on the level or first difference of P_t. Gonzalo and Ng (2001) achieve the P-T decomposition by defining the G matrix as $G = (\alpha_\perp, \beta)'$, where $\alpha_\perp$ denotes the $k \times (k-r)$ orthogonal complement of α, such that $\alpha_\perp'\alpha = 0$. Choosing $G = (\alpha_\perp, \beta)'$ follows directly from the implications of the Granger representation theorem, as $\alpha_\perp$ and β concern the non-stationary (permanent) and stationary (transitory) directions of the process, respectively.[5] There is, however, a caveat associated with Gonzalo and Ng's (2001) choice of matrix G when the number of cointegrating vectors exceeds one. If $r > 1$, the way the cointegrating matrix β is normalized (e.g., triangular representation), plays a role, meaning that G becomes order-variant and, consequently, the subsequent estimates of d_0 and $D(1)$ also become order-variant. To overcome this issue, I adopt Warne's (1993) alternative specification of the $r \times k$ lower block of matrix G that is used to identify the reduced-form transitory innovations. Specifically, replace β' by $\alpha'\Omega^{-1}$ and define $G^* = (\alpha_\perp,\ \Omega^{-1}\alpha)'$. Likewise β, α also "knocks out" the long-run effect to identify the transitory innovations in their reduced form, meaning that the long-run effect of the reduced-form transitory innovations is zero. The Monte Carlo exercises in the Appendix B show the advantages of using G^* instead

[5] The Granger representation theorem decomposes the process P_t into $I(1)$ and $I(0)$ components. It reads as follows: $P_t = \beta_\perp \left[\alpha_\perp' \left(I_k - \sum_{\ell=1}^{p} \Gamma_\ell\right) \beta_\perp\right]^{-1} \alpha_\perp' \sum_{i=1}^{t} u_i + B(L)u_t + P_0$, where $B(L)u_t = \sum_{i=0}^{\infty} B_i \epsilon_{t-i}$ is an $I(0)$ process and P_0 contains initial values.

of G. It then follows that rotating the reduced-form VMA yields:

$$\Delta P_t = \Psi(L)u_t = \Psi(L)G^{*-1}G^*u_t = \Upsilon(L)\varepsilon_t = \begin{pmatrix} \Upsilon(L)_{11} & \Upsilon(L)_{12} \\ \Upsilon(L)_{21} & \Upsilon(L)_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_t^P \\ \varepsilon_t^T \end{pmatrix}, \tag{13}$$

where $\Upsilon(L) = (\Upsilon_0 + \Upsilon_1 L + \Upsilon_2 L^2 + ...)$ is an infinite lag operator with $\Upsilon(L) = \Psi(L)G^{*-1}$, $\varepsilon_t = G^*u_t$, with $\varepsilon_t = (\varepsilon_t^{P\prime}, \varepsilon_t^{T\prime})'$, and ε_t^P and ε_t^T have dimensions $(k-r)\times 1$ and $r \times 1$, respectively. As a direct implication of the P-T decomposition in (13), the long-run effect of an impulse in ε_t^T must be zero, meaning that $\Upsilon(1)_{12} = 0_{(k-r)\times r}$ and $\Upsilon(1)_{22} = 0_{r\times r}$.[6] Proposition 2 formalizes this result.

Proposition 2 *(The P-T decomposition). Let P_t be a $k\times 1$ vector of $I(1)$ variables that satisfy a VEC(p) representation as in equation (10) with r cointegrating vectors and a VMA(∞) representation as in equation (11). Let* $G^* = \begin{pmatrix} \alpha_\perp' \\ \alpha'\Omega^{-1} \end{pmatrix}$. *Then,* $\Upsilon(1) = \begin{pmatrix} \Upsilon(1)_{(k-r)}, \Upsilon(1)_r \end{pmatrix} = \begin{pmatrix} \beta_\perp (\alpha_\perp'\Gamma(1)\beta_\perp)^{-1}, & 0_{k\times r} \end{pmatrix}$, $\varepsilon_t^P = \alpha_\perp' u_t$, *and* $\varepsilon_t^T = \alpha'\Omega^{-1}u_t$, *where* $\Upsilon(1)_{(k-r)} = \begin{pmatrix} \Upsilon(1)_{11} \\ \Upsilon(1)_{21} \end{pmatrix}$ *and* $\Upsilon(1)_r = \begin{pmatrix} \Upsilon(1)_{12} \\ \Upsilon(1)_{22} \end{pmatrix}$ *are $k\times(k-r)$ and $k\times r$ matrices, respectively, and* $\Gamma(1) = I_k - \sum_{\ell=1}^p \Gamma_\ell$.

The proof of Proposition 2 is given in Appendix A. Furthermore, it is possible to express $\Upsilon(1)_{(k-r)}$, the matrix collecting the total effect of an impulse in ε_t^P to market prices, as a function of the orthogonal complement of the cointegrating matrix β. Denote the $k\times(k-r)$ orthogonal complement of β as $\beta_\perp$ and note that $\beta_\perp$ satisfies $\beta'\beta_\perp = 0_{r\times(k-r)}$. Because $\alpha_\perp$ and $\beta_\perp$ are not unique, they remain orthogonal to α and β, respectively, up to any rotation. It then follows that $(\alpha_\perp'\Gamma(1)\beta_\perp)^{-1}\alpha_\perp'$ remains orthogonal to α, as $(\alpha_\perp'\Gamma(1)\beta_\perp)^{-1}$

[6] It is important to note that, differently from $\beta'P_t$, $\alpha'\Omega^{-1}P_t$ is not a stationary process. Proposition 2 shows that the stationarity of $\alpha'\Omega^{-1}P_t$ is not needed and setting $\varepsilon_t^T = \alpha'\Omega^{-1}u_t$ yields a useful definition of the reduced-form transitory shock as it guarantees $\Upsilon(1)_{12} = 0_{(k-r)\times r}$ and $\Upsilon(1)_{22} = 0_{r\times r}$ via $\Psi(1)\alpha = 0$.

simply rotates $\alpha_\perp$. In view of the partial price adjustment model discussed in Section 2, the two natural cointegrating vectors are $\beta_1 = (1, -1, 0, -1)'$ and $\beta_2 = (0, 0, 1, -1)'$, with $\beta = (\beta_1, \beta_2)$. These cointegrating vectors determine a 4×2 orthogonal complement given by:

$$\beta_\perp = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & -1 \end{pmatrix}. \tag{14}$$

This means that an estimate of $\alpha_\perp$ can be obtained from the the first two rows of $\Psi(1)'$, which in turn can be easily computed by dynamic simulation, (see, for instance, Hamilton 1994, pp. 318-323). The total-effect matrix reads:

$$\Upsilon(1) = \begin{pmatrix} \Upsilon(1)_{11} & \Upsilon(1)_{12} \\ \Upsilon(1)_{21} & \Upsilon(1)_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}. \tag{15}$$

Implementing the P-T decomposition via the G^* matrix brings an additional advantage: it imposes a block structure on the variance of ε_t, meaning that $Cov\left(\varepsilon^P_{\jmath,t}, \varepsilon^T_{\jmath',t}\right) = 0$ for all $\jmath = 1, .., (k-r)$ and $\jmath' = 1, .., r$. Specifically, the variance of ε_t is as follows:

$$\Xi = \mathbb{E}\left(\varepsilon_t \varepsilon_t'\right) = G^* \Omega G^{*\prime} = \begin{pmatrix} \Xi_{11} & 0_{(k-r)\times r} \\ 0_{r\times(k-r)} & \Xi_{22} \end{pmatrix}, \tag{16}$$

where $\Xi_{11} = \mathbb{E}\left(\varepsilon^P_t \varepsilon^{P\prime}_t\right)$ denotes a $(k-r)\times(k-r)$ covariance matrix of permanent innovations in their reduced form and $\Xi_{22} = \mathbb{E}\left(\varepsilon^T_t \varepsilon^{T\prime}_t\right)$ is the $r \times r$ covariance matrix of the reduced-form transitory innovations. Because Ξ_{11} and Ξ_{22} are not diagonal matrices, a second step is necessary to uncover a structural VMA process as in (12): the P-P and T-T decompositions.

This second step seeks a transformation from ε_t and $\Upsilon(L)$ to η_t and $D(L)$, respectively. It

fundamentally means to define a $k \times k$ matrix H, such that $D(L) = \Upsilon(L)H$ and $\eta_t = H^{-1}\varepsilon_t$. As highlighted in the Gonzalo and Ng's (2001) practical rule, this is essentially tantamount to solving the following equality:

$$\Xi = H\Sigma_\eta H', \tag{17}$$

where $\Sigma_\eta = \mathbb{E}(\eta_t\eta_t')$ is the $k \times k$ covariance (symmetric) matrix of the structural innovations and H is a $k \times k$ matrix of unknowns. From (17), it is immediate that Ξ renders $k(k+1)/2$ equations to solve for $k^2 + k(k+1)/2$ unknowns, meaning that is necessary to impose k^2 further restrictions to achieve an exact identification. For illustrative purposes, consider the baseline partial price adjustment model introduced in Section 2 that consists of an asset traded on the home market, the exchange rate, and the same asset traded on two foreign markets (i.e., $k = 4$). In this case, Ξ contains $k(k+1)/2 = 10$ equations to solve for $k^2 + k(k+1)/2 = 26$ unknowns in $H\Sigma_\eta H'$, which implies that $k^2 = 16$ restrictions must be imposed on H and Σ_η to achieve an exact identification. Gonzalo and Ng (2001) achieve this goal by assuming that Σ_η equals an identity matrix ($k(k+1)/2$ restrictions) and $H = F$ ($k(k-1)/2$ restrictions), where F is the Cholesky decomposition of Ξ, such that $\Xi = FF'$. Again, considering the baseline partial price adjustment model with $k = 4$, setting $\Sigma_\eta = I_{k=4}$ and $H = F$ add $k(k+1)/2 = 10$ and $k(k-1)/2 = 6$ restrictions, respectively, which amounts to the required 16 restrictions. It then follows that $\Xi = FI_kF'$, meaning that:

$$\Delta P_t = \Upsilon(L)FF^{-1}\varepsilon_t = \begin{pmatrix} D(L)_{11} & D(L)_{12} \\ D(L)_{21} & D(L)_{22} \end{pmatrix} \begin{pmatrix} \eta_t^P \\ \eta_t^T \end{pmatrix}, \tag{18}$$

$\eta_t = F^{-1}\varepsilon_t$, $D(L) = \Upsilon(L)F$, and $D(1) = \Upsilon(1)F$, with $D(1)_{12} = 0_{(k-r)\times r}$ and $D(1)_{22} = 0_{r\times r}$.

There is a caveat associated with choosing F as a lower triangular matrix. Apart from the usual limitation that the ordering of the variables matters, setting F as a lower triangular matrix also restricts the correlation between the common factors. To see this limitation more clearly, assume $k = 4$ and $r = 2$, i.e., the price system is driven by two stochastic trends

(common factors). From Proposition 1, the off-diagonal elements of the 2×2 upper left-hand block of $\breve{D}(1)$ give the structural parameters ρ and λ and hence the correlation between the common factors. Using the Gonzalo and Ng's (2001) identification strategy, $D(1) = \Upsilon(1)F$ reads as follows:

$$D(1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} F_{(1,1)} & 0 & 0 & 0 \\ F_{(2,1)} & F_{(2,2)} & 0 & 0 \\ F_{(3,1)} & F_{(3,2)} & F_{(3,3)} & 0 \\ F_{(4,1)} & F_{(4,2)} & F_{(4,3)} & F_{(4,4)} \end{pmatrix} = \begin{pmatrix} F_{(1,1)} & 0 & 0 & 0 \\ F_{(2,1)} & F_{(2,2)} & 0 & 0 \\ F_{(1,1)} - F_{(2,1)} & -F_{(2,2)} & 0 & 0 \\ F_{(1,1)} - F_{(2,1)} & -F_{(2,2)} & 0 & 0 \end{pmatrix}. \quad (19)$$

It is possible to further rotate F so that the diagonal elements of $D(1)_{11}$ are equal to ones and, hence, matches $\breve{D}(1)$ in Proposition 1. Define Φ_F as a diagonal matrix containing the diagonal entries of F. It then follows that $\Delta P_t = D(L)\Phi_F^{-1}\Phi_F\eta_t$, with $\breve{D}(1) = D(L)\Phi_F^{-1}$ and $\breve{\eta}_t = \Phi_F\eta_t$.[7] The long-run impact matrix $\breve{D}(1)$ is then:

$$\breve{D}(1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \xi_{21}/\xi_{11} & 1 & 0 & 0 \\ 1 - \xi_{21}/\xi_{11} & -1 & 0 & 0 \\ 1 - \xi_{21}/\xi_{11} & -1 & 0 & 0 \end{pmatrix}, \quad (20)$$

where ξ_{21} is the off-diagonal element of Ξ_{11}, i.e., the covariance between the reduced-form permanent innovations, $\text{Cov}\left(\varepsilon_{1,t}^P, \varepsilon_{2,t}^P\right)$, and ξ_{11} is the first diagonal element of Ξ_{11}, namely $\text{Var}\left(\varepsilon_{1,t}\right)$. Finally, if F is a lower triangular matrix, then $F_{(1,2)} = 0$, which amounts to a zero restriction on the dynamics between the common factors: $D(1)_{(1,2)} = 0$ and hence $\rho = 0$. Restricting either λ or ρ to zero is an inevitable outcome when applying the Cholesky decomposition (or any other lower triangular matrix). This result remains true regardless of

[7] Setting $F\Phi_F^{-1}$ coincides with the factorization adopted in Yan and Zivot (2010). Specifically, they factorize Ξ as $\Xi = BCB'$, where B plays the role of H and it is defined as a lower triangular matrix with ones in its main diagonal ($k(k+1)/2$ restrictions), and $\Sigma_\eta = C$, where C is a diagonal covariance matrix with unknown positive entries ($k(k-1)/2$ restrictions). As B is a lower triangular matrix, this factorization imposes the same restrictions to those in Gonzalo and Ng (2001).

the ordering of variables.

To address this issue, I propose an alternative identification strategy that replaces the usual Cholesky decomposition with the more flexible spectral decomposition. The use of the spectral decomposition is not new in the price discovery literature, with Fernandes and Scherrer (2018) making use of this decomposition to achieve order-invariant IS measures. However they do not attempt to identify the structural innovations and simply apply the spectral decomposition directly to Ω. In this paper, the spectral decomposition is used on Gonzalo and Ng's (2001) second step; hence its main motivation is to uncover the orthogonalized permanent and transitory innovations. Moreover, the spectral decomposition identifies $D(L)$ and η_t without restricting the correlation between the common factors to zero, enabling inference on the impact of the exchange rate on firm value.

The spectral decomposition usually places restrictions that are not obvious or of easy economic interpretation. Fortunately, that is different in the cross-listed price discovery setting, meaning that these restrictions are neatly and easily traceable. Specifically, this novel identification procedure lifts zero restrictions from the $(k-r)\times(k-r)$ top-left block of H that relates to the permanent innovations. Considering my price discovery setting, it places only a cross-element restriction based on the off-diagonal elements and a restriction on the diagonal elements of this block, while all the remaining restrictions are placed on the elements of H that concern the transitory innovations.

The identification procedure works as follows. Let $\widetilde{S}$ be the spectral decomposition of $\widetilde{\Xi} = \Xi\Theta^{-1}$, such that $\widetilde{\Xi} = \widetilde{S}\widetilde{S}$ and Θ is a diagonal matrix containing the diagonal entries of Ξ.[8] From the properties of $\widetilde{S}$, it follows that one can rewrite Ξ as:[9]

$$\Xi = \widetilde{S}\widetilde{S}\Theta = \widetilde{S}\Theta\widetilde{S}'. \tag{21}$$

It is evident that the last equality in (21) resembles the elements in (17), meaning that $\widetilde{S}$ and

[8] $\widetilde{S}$ is defined as $\widetilde{S} = V\Lambda^{1/2}V^{-1}$, where Λ is a diagonal matrix with the eigenvalues of $\widetilde{\Xi}$ and the columns of the $k\times k$ matrix V are the corresponding eigenvectors.

[9] The proof of this equality is presented in Appendix A.3.

Θ play the role of H and Σ_η, respectively. Because $\widetilde{S}$ is an exactly identified factorization of $\widetilde{\Xi}$ and $\widetilde{\Xi}$ has the same number of linearly independent equations as Ξ ($k(k+1)/2$ equations), it follows that $\widetilde{S}$ contains $k(k+1)/2$ unknowns and $k(k-1)/2$ restrictions.

This first set of restrictions simply impose the desirable property that transitory innovations should have no permanent effect on P_t (i.e., the last r columns of $D(1)$ are equal to zero). Because Ξ is a block diagonal matrix, $\widetilde{\Xi}$ remains block diagonal. It then follows that the spectral decomposition places $r(k-r)$ zero restrictions on the off-diagonal upper block of $\widetilde{S}$,

$$\widetilde{S} = \begin{pmatrix} \widetilde{S}_{11} & 0_{(k-r)\times r} \\ \widetilde{S}_{21} & \widetilde{S}_{22} \end{pmatrix}, \tag{22}$$

where $\widetilde{S}_{11}$ is a $(k-r)\times(k-r)$ block associated with the permanent innovations, $\widetilde{S}_{22}$ is a $r\times r$ block that relates to the transitory innovations, and $\widetilde{S}_{21}$ is a $r\times(k-r)$ matrix summarizing the dynamics between permanent and transitory innovations. Setting $\widetilde{S}_{12} = 0_{(k-r)\times r}$ also matches Gonzalo and Ng's (2001) requirement that H must be lower *block* triangular.[10]

It is still necessary to unveil $(k-r)(k-r-1)/2$ restrictions on $\widetilde{S}_{11}$ and $r(r-1)/2$ restrictions on $\widetilde{S}_{22}$.[11] In the case of two stochastic trends (efficient price and exchange rate), $r = k-2$, meaning that the upper block $\widetilde{S}_{11}$ is a 2×2 matrix and $(k-r)(k-r-1)/2 = 1$. The spectral decomposition sets $\widetilde{S}_{11\,(1,1)} = \widetilde{S}_{11\,(2,2)}$. In turn, the identification strategy put forward in this section removes a zero restriction from the relationship between permanent innovations in their reduced form to a normalization assumption on the diagonal elements. Finally, the remaining $r(r-1)/2$ restrictions are placed on the $\widetilde{S}_{22}$ block, which are due to the transitory innovations and hence do not carry significance importance to our analysis, as the last r columns of $D(1)$ are restricted to be zero (transitory shocks cannot have permanent effects on P_t). Again, in the case of $k=4$ with $r=2$, then $r(r-1)/2 = 1$, which forces

[10] See detailed derivation of $\widetilde{S}$ in Appendix A.3.

[11] As noted in Gonzalo and Ng's (2001) the $(k-r)(k-r-1)/2$ restrictions on $\widetilde{S}_{11}$ are necessary to orthogonalize ε_t^P.

$\widetilde{S}_{22\,(1,1)} = \widetilde{S}_{22(2,2)}$. In sum, the $D(1) = \Upsilon(1)\widetilde{S}$ reads as follows:

$$D(1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \widetilde{S}_{(1,1)} & \widetilde{S}_{(1,2)} & 0 & 0 \\ \widetilde{S}_{(2,1)} & \widetilde{S}_{(1,1)} & 0 & 0 \\ \widetilde{S}_{(3,1)} & \widetilde{S}_{(3,2)} & \widetilde{S}_{(3,3)} & \widetilde{S}_{(3,4)} \\ \widetilde{S}_{(4,1)} & \widetilde{S}_{(4,2)} & \widetilde{S}_{(4,3)} & \widetilde{S}_{(3,3)} \end{pmatrix},$$

$$D(1) = \begin{pmatrix} \widetilde{S}_{(1,1)} & \widetilde{S}_{(1,2)} & 0 & 0 \\ \widetilde{S}_{(2,1)} & \widetilde{S}_{(1,1)} & 0 & 0 \\ \widetilde{S}_{(1,1)} - \widetilde{S}_{(2,1)} & \widetilde{S}_{(1,2)} - \widetilde{S}_{(1,1)} & 0 & 0 \\ \widetilde{S}_{(1,1)} - \widetilde{S}_{(2,1)} & \widetilde{S}_{(1,2)} - \widetilde{S}_{(1,1)} & 0 & 0 \end{pmatrix}. \tag{23}$$

It is still possible to move one step further and express $\widetilde{S}_{(2,1)}$ and $\widetilde{S}_{(1,2)}$ in terms of the elements of $\Xi_{11} = \mathbb{E}\left(\varepsilon_t^P \varepsilon_t^{P\prime}\right)$. By manipulating the elements of $\widetilde{S}_{11}$ in (A.26) (Appendix A.3), it follows that:

$$\widetilde{S}_{(2,1)} = \frac{\xi_{21}}{\xi_{11}} \left(2\widetilde{S}_{(1,1)}\right)^{-1} \quad \text{and} \quad \widetilde{S}_{(1,2)} = \frac{\xi_{21}}{\xi_{22}} \left(2\widetilde{S}_{(1,1)}\right)^{-1}, \tag{24}$$

where ξ_{21} is the off-diagonal element of Ξ_{11}, i.e., $\text{Cov}\left(\varepsilon_{1,t}^P, \varepsilon_{2,t}^P\right)$; and ξ_{11} and ξ_{22} are the diagonal elements of Ξ_{11}, i.e., $\text{Var}\left(\varepsilon_{1,t}^P\right)$ and $\text{Var}\left(\varepsilon_{2,t}^P\right)$, respectively. In turn, the spectral-based identification strategy neatly divides the covariance between the reduced-form permanent innovations over the structural shocks. This can ultimately be seen as a cross-element restriction that rules out either $\widetilde{S}_{(2,1)}$ or $\widetilde{S}_{(1,2)}$ being equal to zero. In fact, $\widetilde{S}_{(2,1)} = \widetilde{S}_{(1,2)} = 0$ only if $\xi_{21} = 0$. Differently, the Cholesky-based identification sets $F_{(1,2)}$ to zero and assigns the covariance between the reduced-form permanent innovations to only $F_{(2,1)}$ (scaled by the variance of $\varepsilon_{1,t}^P$).[12]

[12] I thank the anonymous referee for highlighting this point and pointing out that the restrictions imposed by the spectral decomposition are reminiscent of an alternative definition of Hasbrouck's (1995) IS measure with one permanent component only. This IS variant equally distributes the covariance between the VEC

The complete two-step identification procedure (the P-T and P-P and T-T decompositions) that gives the relationship between (11) and (12) can be summarized as follows:

$$\Delta P_t = \Psi(L)u_t = \Psi(L)G^{*-1}\widetilde{S}\widetilde{S}^{-1}G^{*}u_t = \Upsilon(L)\widetilde{S}\widetilde{S}^{-1}\varepsilon_t = D(L)\eta_t, \tag{25}$$

where $D(L) = \Psi(L)G^{*-1}\widetilde{S}$, $d_0 = G^{*-1}\widetilde{S}$, $D(1) = \Psi(1)G^{*-1}\widetilde{S}$, and $\eta_t = \widetilde{S}^{-1}\varepsilon_t = \widetilde{S}^{-1}G^{*}u_t$. In view of Proposition 1, it is possible to directly infer ρ and λ from the estimates of $D(1)$ in (23). It is straightforward to see that one can further rotate $\widetilde{S}$ so that the diagonal elements of the rotated $D(1)_{11}$ are equal to ones and, hence, matches $\breve{D}(1)$ in Proposition 1. First, note from (25) that $\Delta P_t = D(L)\eta_t$. Define $\Phi_{\tilde{s}}$ as a diagonal matrix containing the diagonal entries of the top-left block $\widetilde{S}_{11}$ and ones on the remaining r elements. It then follows that $\Delta P_t = D(L)\Phi_{\tilde{S}}^{-1}\Phi_{\tilde{s}}\eta_t$, such that:

$$\breve{D}(1) = \Upsilon(1)\widetilde{S}\Phi_{\tilde{S}}^{-1} = D(1)\Phi_{\tilde{S}}^{-1}, \tag{26}$$

with $\rho = \breve{D}(1)_{(1,2)}$ and $\lambda = \breve{D}(1)_{(2,1)}$. Finally, using a Monte Carlo simulation, I show that the identification strategy put forward in this section achieves the best finite sample results (see the discussion in the Appendix B).

To conclude, the novel identification strategy allows innovations from a given common factor to have a permanent impact on any other common factor, as opposed to the identification strategies used in Gonzalo and Ng (2001) and Yan and Zivot (2010), which rule out this possibility. Furthermore, the estimation method proposed here provides an exact identification for matrices d_0 and $D(1)$.

innovations over the markets. Specifically, for $k = 2$, it reads as follows:

$$IS_m = \frac{\psi_m^2\omega_1^2 + \psi_1\psi_2\omega_{12}}{\psi'\Omega\psi},$$

where ψ_m and ω_m^2 denote the m^{th} element of the 1×2 vector ψ and the m^{th} diagonal element of Ω, respectively, ω_{12} is the off-diagonal element of Ω, and $m = 1, 2$.

4 Brazilian cross-listing: exchange rate and price discovery

4.1 Institutional background

B3 - Brasil Bolsa Balcão, formerly BM&FBovespa, is the Brazilian stock exchange and the leading exchange in Latin America. B3 is one of the world's largest financial market infrastructure companies, providing trading services in an exchange and OTC environment. Brazilian cross-listed companies are traded and listed in B3 and in the U.S. market through the ADR program. I use a 23-month tick-by-tick dataset of Brazilian blue-chip companies spanning from December 2007 to November 2009. The dataset consists of transaction prices recorded at three trading venues, namely, B3, NYSE, and ARCA, and the exchange rate. Among the Brazilian firms cross-listed in the U.S., I chose those that are very liquid in the markets considered in this study to avoid losing information during the aggregation process between a very liquid venue and an illiquid one. Additionally, these firms are from a variety of industries, so the results are not sector- or industry-specific. The firms are Ambev (beverage), BR Telecom (telecommunication), Bradesco (finance), Gerdau (steel), Vale (mining), and Petrobras (oil). Apart from BR Telecom, they are all part of the B3 benchmark market index (IBOVESPA). Preferred shares of Vale and Petrobras are the most heavily traded shares on the Brazilian exchange, with Gerdau and Bradesco among the top 15.

In view of the price discovery literature, the use of high-frequency data from Brazil provides two distinct advantages. First, in stark contrast to the European markets, the trading hours of the U.S. and Brazilian exchanges overlap for six and a half hours during most of the year. Hence, very little information is left out of the analysis, which considerably strengthens the price discovery analysis. Second, Brazilian companies are very liquid in the U.S. market, sometimes exhibiting more trading activity in the U.S. than they do in Brazil. In fact, some Brazilian firms are among the top 10 most liquid ADR programs and are top

volume and value movers. Furthermore, there was an increase of 20% in investors' positions in ADRs from Brazilian companies from 2008 to 2010 (which includes the sample period). As of February 2018, the ADRs on Vale and Petrobras are ranked 9th and 15th, respectively, as the NYSE's most active stocks.

The use of a high-frequency dataset is crucial to the price discovery analysis because it provides a timely incorporation of new information in each market (see, for instance, the discussion in Grammig et al. 2005 and references therein). However, two important preliminary steps must be implemented when handling a tick-by-tick database. The first step consists of cleaning the data from entries that do not correspond to plausible market activity. As a cleaning filter, I deploy the algorithm proposed by Brownlees and Gallo (2006).[13] The second step relates to aggregating the non-synchronous high-frequency price series. The rise of algorithmic trading, along with the fact that markets operate at fast time frames (see, among others, O'Hara 2015; and Hasbrouck 2019), I sample at the 5-second frequency for all stocks but Ambev and BR Telecom, which are fixed to 15- and 30-second sampling intervals, respectively. This choice is driven by a tradeoff between sampling at a high enough frequency to estimate the instantaneous effects, d_0, and the impulse response functions, but low enough to have sufficiently many transactions to avoid biasing these estimates due to a high number of consecutive zero returns. As a robustness check, I also present results for alternative frequencies: 15-, 30-, 60-, and 120-second sampling intervals.

4.2 Discussion of Results

Currently, many firms list their shares on more than one exchange. Studies on the topic focus mainly on understanding the effects of cross-listing (see, among others, Stulz 1999; Doidge et al. 2004; and Doidge 2004). With cross-listing as a typical strategy for companies, geographical price discovery is of major interest to stock exchanges and firms (Eun and Sabherwal 2003). Therefore, the main research question encompasses the geographi-

[13] See Table S.1 in the Online Appendix for the cleaning details.

cal price discovery of both stock prices and the exchange rate. Specifically, two important questions are relevant in this context: What market moves first and, hence, whether the domestic market is more important than any foreign market, and how fast innovations in the efficient exchange rate and firm value are impounded in market prices. Additionally, as a by-product of the flexible identification strategy discussed in the previous section, assessment on the net feedback effect of exchange rate innovations on firm value is possible. It is important to highlight that the usual reduced-form price discovery measures (the IS and CS measures) are unable to clearly answer these questions, as both measures are proportional to the markets' instantaneous responses to the structural transitory innovations (see detailed discussion in Yan and Zivot (2010)). In contrast, the structural setting neatly answers all of these questions, as the identification strategy discussed in Subsection 3.2 successfully isolates the permanent innovations from the transitory.

The identification of two permanent innovations (the efficient exchange rate and firm value) requires at least three time series that cointegrate. I consider transaction prices from four markets, $k = 4$: the exchange rate (Brazilian reais/U.S. dollars), shares traded on the Brazilian market (B3), NYSE, and ARCA.[14] Shares traded on the B3 are quoted in Brazilian reais (R$), while shares traded on the NYSE and ARCA are expressed in U.S. dollars (USD). Prices on the B3, NYSE, and ARCA and the exchange rate (R$/USD) are expected to cointegrate, and the results from Johansen's tests indeed confirm the existence of two cointegrating vectors, $r = 2$. The two cointegrating vectors yield two common factors: the efficient exchange rate and the efficient price of the firm. In turn, two linear combinations of the price series should render stationary processes (deviations from the long-run equilibrium), and estimates of the cointegrating vector depict the expected pattern: $(0, 0, NYSE, -ARCA)$ and $(B3, -ExRate, 0, -ARCA)$.[15] Setting $r = 2$, the free parameters of the VEC model are estimated individually for each firm using the full-information maximum likelihood (FIML).[16]

[14]For Vale, there are three markets.

[15]The Online Appendix contains the results of the maximum eigenvalue and trace tests. It also contains the full set of estimates for the cointegrating vectors.

[16]See Johansen (1988, 1991) and Hamilton (1994). Details on model specifications and diagnostic tests

Based on the parameter estimates of the reduced-form VEC model, estimates of d_0, the impulse response function $\sum_{i=0}^{n} d_i$, and $D(1)$ are computed using the identification strategy discussed in Section 3.2, and their corresponding standard errors are computed using the parametric bootstrap.

4.2.1 Markets' importance

Table 1 displays the estimates of the contemporaneous and total responses to an impulse in the permanent innovation associated with the efficient price, η_t^m. Specifically, Panel A presents the estimates of the column of d_0 that relate to the instantaneous response to η_t^m, whereas Panel B reports the estimates of $D(1)$ associated with η_t^m (first column of $D(1)$). Overall, there is strong evidence that the instantaneous responses are not only different across trading venues, but also different from the estimates of $D(1)$. The latter finding confirms the role played by the partial adjustment parameters $\dot{\gamma}_i$ and γ_i in the structural model, as these parameters essentially account for the speed of the price discovery process. Generally, the NYSE instantaneously assimilates a greater proportion of an innovation in the efficient price. Considering the highly liquid stocks that are sampled at the 5-second interval, I find that for a 1% change in the efficient price, the B3 and NYSE respond by absorbing 0.81% and 0.90% of this change within the 5-second interval, respectively (average across stocks). The size of the bootstrap standard errors confirms the statistical significance of these estimates. Regarding the less liquid Ambev and BR Telecom firms, the home market appears to respond to changes in the efficient price more quickly, as B3 instantaneously assimilates a higher proportion of a price change in the efficient price. Therefore, the results suggest that the U.S. market is the most important market for the price discovery processes of highly liquid cross-listed Brazilian firms. A variety of reasons (e.g., the type of platform, variety in the group of investors, the supply of other assets, and transaction fees) may explain

are in the Online Appendix. In particular, results of the Breusch Godfrey Lagrange multiplier test for the autocorrelation of the residuals indicates that the model is well specified, as we cannot reject the null of no serial correlation in all instances.

the greater importance attributed to the U.S. market, but the main reason is its dominance as one of the largest and most liquid global markets. In contrast, Grammig et al. (2005) find the home market to be more important than the U.S. market for German cross-listed stocks. Eun and Sabherwal (2003) document that U.S. market prices generally adjust more to the Canadian stock exchange than vice versa, although the U.S. market is also dominant for many firms in their sample. Brazil's status as a less-developed country (compared to Germany and Canada) may contribute to the differences in these results. Pulatkonak and Sofianos (1999) indeed show that being a developed market reduces the U.S. market share of cross-listed stocks by 30% when compared to that in emerging markets.

The second price discovery measure consists of evaluating the accumulated impulse response functions from impulses to the structural innovations. Accumulated impulse response functions are used to assess the speed with which markets impound information from the efficient prices. This price discovery metric stems directly from the structural approach, as constructing such an analysis using non-orthogonal market innovations would result in impulse response functions that are also functions of transitory innovations. Figure 1 presents the results for Gerdau (the results for the remaining firms appear in the Online Appendix). Most information processing takes place in the first minute, although there are minor adjustments afterwards. This result is robust across the highly liquid companies, with very small variations. Specifically, the B3 and NYSE incorporate 79% and 89%, respectively, of a unit shock in η_t^m within the first 15 seconds. The proportion of information impounded into market prices increases to 90% and 98% for B3 and NYSE, respectively, at the 45-second interval. This result suggests that while markets operate at increasing time frames, it still takes up to a minute for changes in the permanent innovations be fully incorporated into market prices.

Finally, the upper right panel of Table 1 displays the estimates of $D(1)$ related to the total effect of the efficient price innovation on transaction prices. The total impact is higher than the initial shock (average across stocks of 0.07%), reflecting an eventual feedback effect

between the exchange rate and the efficient price of the firm.

Taking Proposition 1 and equation (26) in Subsection 3.2 into consideration, an estimate of λ can be directly inferred from rotating $D(1)$: $\widehat{\rho} = \left[\widehat{D}(1)\widehat{\Phi}_{\widetilde{s}}^{-1}\right]_{(2,1)}$, where $\widehat{D}(1)$ denotes the estimate of $D(1)$ and $\widehat{\Phi}_{\widetilde{s}}$ is a diagonal matrix containing the diagonal entries of $\widehat{D}(1)_{11}$ and ones in the remaining two elements. First, the results suggest that treating λ as a free parameter is supported by the data, as its estimates are significantly different from zero across firms. Second, estimates of λ are negative and fairly stable,ranging from -0.07 to -0.10, suggesting that λ may reflect a systematic component of η_t^m. A negative λ indicates that positive innovations on η_t^m cause an appreciation in the exchange rate. This finding is consistent with the idea that upward movements in the Brazilian stock market (coming from overall good news in the economy) lead to a significant inflow of foreign currency, which increases the exchange rate. This is also consistent with the Bekaert et al.'s (2011) idea that emerging markets present a higher degree of market segmentation and investment in equities that is not independent from local currency movements, mainly because many of these countries' economies rely on commodities.

4.2.2 Exchange rate price discovery

The methodology in this paper delivers three measures regarding the exchange rate. The first one is the instantaneous adjustment of equity prices, given an exchange rate shock. This is an immediate attempt to keep the link between domestic and foreign prices and the exchange rate and therefore is a measure of the price discovery of the exchange rate. The second one captures the speed of adjustment (impulse response function), while the third is related to the net feedback effect of exchange rate innovations on the intrinsic value of the firm.

With respect to the price discovery of the exchange rate, as the results in the previous section show, the U.S. market is the fastest in incorporating news on equity value, so a similar pattern for the exchange rate can be expected. The results in Panel C of Table 1 display

higher parameters in absolute value for ARCA and NYSE when compared to B3. Considering the innovation in the exchange rate (R$/USD), prices at the NYSE usually receive the most adjustment. Therefore, the results indeed show that U.S. prices adjust instantaneously to a change in the exchange rate to maintain the equilibrium. I also document an instantaneous overshooting of the observed exchange rate once an efficient exchange rate innovation occurs. The observed exchange rate has an average instantaneous impact that is 78% higher than the shock, and this behavior occurs for all stocks. Intuitively, the overshooting could signal the existence of herd behaviour during turbulent periods. Indeed, the Brazilian currency depreciated 49% over 90 days in mid-July 2008 and early October 2008 and partially recovered a few months later. Analyzing the accumulated impulse response functions (Figure 1), similar to the efficient price case, markets take approximately one minute to incorporate all the information from the exchange rate. Interestingly, B3 presents a rather different dynamic, with the accumulated effect starting positive and then turning negative after approximately 30 seconds. It finally reaches 98% of the total effect after one minute from the initial shock. Regarding NYSE and ARCA, they incorporate 70% and 42% of the total effect from a unit impulse in η_t^e, respectively, within the first 15 seconds. These proportions increase to 86% and 51% for NYSE and ARCA, respectively, for a 30-second interval.

Panel D in Table 1 displays the estimates of the second column of matrix $D(1)$, regarding the total effect of the exchange rate innovation on transaction prices. As a direct implication from the identification strategy discussed in Subsection 3.2, the elements of $D(1)$ that relate to the structural parameter ρ in the theoretical model in (1) are now treated as free parameters in the structural VEC model. Thus, it becomes possible to assess the exchange rate effect to the efficient price. All parameters are negative for the Brazilian and U.S. markets (for the U.S., it is approximately the same value as the Brazilian value plus one unit (all negative), where -1 is the result of the non-arbitrage adjustment). These results suggest that a depreciation in the home currency is associated with a significant net decrease in firm value. For instance, for Gerdau, a 1% innovation in the exchange rate indicates an effect

of -0.51% on the home asset price and an effect of -1.49% on the foreign market (-1% from the non-arbitrage adjustment). This finding holds for all companies, regardless of how liquid their stocks are. In Proposition 1 and equation (26), an estimate of ρ can be directly inferred from rotating $D(1)$: $\widehat{\rho} = \left[\widehat{D}(1)\widehat{\Phi}_{\widetilde{S}}^{-1}\right]_{(1,2)}$. These estimates of ρ are highly significant and range from -0.20 to -0.52.

The absence of a total integrated market may explain such behavior. Bekaert and Harvey (1995), for instance, model gradual changes in market integration and find that many emerging markets present time-varying integration with world equity markets. In the context where countries present a certain degree of market segmentation, local factors may play a role in equity price determination. Bekaert et al. (2011) quantify market segmentation by constructing a measure that is the difference between portfolio yields of industries valued locally and globally. As the difference in yields increases, market segmentation also increases. Interestingly, the authors show that the level of equity market segmentation is still meaningful for many emerging markets. In particular, Brazil presents an average degree of segmentation of 5% between 2001 and 2005, above the emerging market average of 4.3%. Bekaert and Harvey (2017) show that equity returns and the Brazilian currency changes (USD/R$) correlate at 0.5, which is well above the average for the emerging market sample they consider (0.3). Considering these factors, the suggestion of a decrease in equity prices may be a sign that investors require a higher risk premium to invest in the Brazilian equity market.

4.3 Robustness

Data synchronization and the choice of aggregation frequency are typically a concern when dealing with multivariate high-frequency price series. My sampling frequency must account for market liquidity and the presence of microstructure noise. While sampling at lower frequencies balances out the effect of market microstructure noise, it may also exclude valuable information from the analysis. In this subsection, I examine the robustness

of the instantaneous and total effects results presented in subsection 4.2. Therefore, the first robustness exercise consists of estimating the parameters with alternative frequencies. Because estimates of $D(1)$ account for the total effect of the structural innovations on the observed returns, they should be stable across the different intra-day sampling frequencies. By contrast, the estimates of d_0 (instantaneous effects) should reflect changes that occur within the sampling interval and should thus differ across the different frequencies. However, because information processing occurs within a couple of minutes, one should expect the estimates of d_0 to approach those estimates of $D(1)$ as the sampling interval increases. Tables 2 and 3 provide the results for the estimates of d_0 and $D(1)$, respectively.

Regarding the estimates of $D(1)$, they are indeed remarkably stable across the different sampling frequencies, indicating that alternative intra-day sampling frequencies do not play a significant role when estimating the total effect of the permanent innovations on the market prices; therefore, these results are robust to alternative intra-day sampling frequencies. As predicted, the estimates of d_0 are not constant across the different sampling frequencies, but the gap between d_0 and $D(1)$ estimates becomes narrower as the sampling intervals increase. This finding reinforces the dependence of price discovery measures on the sampling frequency and, in turn, the importance of using high-frequency data for inference on the instantaneous reaction of markets to permanent innovations. Finally, note that the estimates of d_0 at the alternative sampling frequencies match the accumulated impulse response functions from my baseline sampling interval.

The last robustness exercise consists of implementing the Cholesky-based identification strategy discussed in Subsection 3.2. Recall that the Cholesky decomposition sets $F_{(1,2)} = 0$, meaning that the long-run effect of an impulse on the first permanent innovation to the first variable of the $k \times 1$ vector of market prices is restricted to zero. In turn, the estimation results are now order variant, which implies that the ordering of the variables in the P_t vector is important. The Cholesky-based approach sets either λ or ρ to zero in (9). Choosing $\lambda = 0$ may be the least stringent restriction between the two alternatives. In order to set

$\lambda = 0$, the ordering of variables in the P_t vector should change to $P_t = \left(w_t, p_{1,t}, p^*_{3,t}, p^*_{4,t}\right)'$. Consequently, the two natural cointegrating vectors also need to change, and they now read $\beta_1 = (-1, 1, 0, -1)'$ and $\beta_2 = (0, 0, 1, -1)'$, with $\beta = (\beta_1, \beta_2)$. In turn, the orthogonal complement of β and the total-effect matrix $\breve{D}(1)$ obtained with the Cholesky decomposition read as follows:

$$\beta_\perp = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ -1 & 1 \end{pmatrix} \quad \text{and} \quad \breve{D}(1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ F_{(2,1)}/F_{(1,1)} & 1 & 0 & 0 \\ F_{(2,1)}/F_{(1,1)} - 1 & 1 & 0 & 0 \\ F_{(2,1)}/F_{(1,1)} - 1 & 1 & 0 & 0 \end{pmatrix}, \tag{27}$$

where $F_{(1,1)}$ and $F_{(2,1)}$ are elements of the Cholesky decomposition of Ξ that is now computed from prices ordered as $P_t = \left(w_t, p_{1,t}, p^*_{3,t}, p^*_{4,t}\right)'$.[17] Table 4 reports the results for both instantaneous and total-effect matrices. Notably, I rearrange the entries in Table 4 so the layout is comparable to Table 1. Overall, there is a significant difference between instantaneous and total responses to impulses in the permanent innovations, confirming the role played by the partial adjustment parameters in the structural model; the U.S. market remains generally the fastest in incorporating news, but the B3 gains more importance compared to the main set of results; estimates of the total-effect of an impulse in η^m_t to observed asset prices and the exchange rate are restricted as in (27), opposite to those results in Table 1; estimates of ρ are larger in magnitude than the ones obtained with the spectral decomposition, as they now range from -0.42 to -1.00 compared to -0.20 to -0.52 in the main set of results displayed in Table 1; and, most importantly, the net positive relationship between the value of the domestic currency and the firm's value remains valid.

Finally, it is also informative to decompose the long-run variance of market prices and the observed exchange rate into components accounted for by innovations in the efficient

[17] It is important to note that $F_{(1,1)}$ and $F_{(2,1)}$ are not the same numbers as those in (19), as the elements in Ξ adjust to the changes in the ordering of the variables in P_t. Furthermore, it is noteworthy that $\beta_\perp$ also changes to reflect the change in the cointegrating vectors, which stem from the alternative ordering of the variables in P_t.

price and efficient exchange rate. For instance, the long-run variance decomposition of share prices traded on B3 is defined as the long-run variance proportions of B3 accounted for by η_t^m and η_t^e innovations. Notably, the long-run variance proportions of B3 accounted for by the transitory innovations must be zero, which implies that long-run variance decompositions for η_t^m and η_t^e must sum to one. Table 5 displays the long-run variance decomposition of the market prices and exchange rate system with relative contributions of the permanent shocks η_t^m and η_t^e. Additionally, the last rows in each of the panels report the estimates of the standard deviation, denoted by ς, η_t^m, and η_t^e. The upper and lower panels refer to spectral- and Cholesky-based identification schemes, respectively. First, the standard deviation of η_t^e is the same under the two identification strategies. Furthermore, the difference between the estimates of the standard deviation of η_t^m are usually small and not significant. These results indicate that any significant difference in terms of the long-run variance decomposition of market prices follows from the estimates of $\breve{D}(L)$. Second, in line with the larger estimates of ρ obtained from the Cholesky-based identification strategy, it is reassuring to observe that the proportions of the total variance of market prices that are due to the innovations in the efficient exchange rate η_t^e are also larger in this identification setting than those from the spectral-based identification. In that, by restricting $\lambda = 0$, the Cholesky-based decomposition strategy assigns the covariance between the reduced form permanent innovations to ρ, which ultimately increases the relative importance of η_t^e into the long-run variance of market prices.

5 Conclusion

I investigate the price discovery of equity and the exchange rate for Brazilian cross-listed companies. A structural VEC framework is used to disentangle informational from frictional innovations so that dynamic price discovery measures can be constructed solely as a function of the innovations on the efficient price and exchange rate. I methodologically propose a novel

identification strategy that allows correlation among common factors (i.e, the efficient price and efficient exchange rate).

I show that information processing may take time because there is a significant difference in what home and foreign markets can assimilate instantaneously and one minute later. I document that the U.S. market is, in general, the most efficient market for instantaneously incorporating shocks as compared as to B3, which might be linked to Brazil's emerging market position.

Additionally, the results suggest that a depreciation/appreciation of the home currency is associated with a decrease/increase in firm value. These results are homogenous across firms and may reflect a certain level of segmentation in the Brazilian equity market. Future research should analyze whether these results hold for other emerging countries and/or segmented equity markets.

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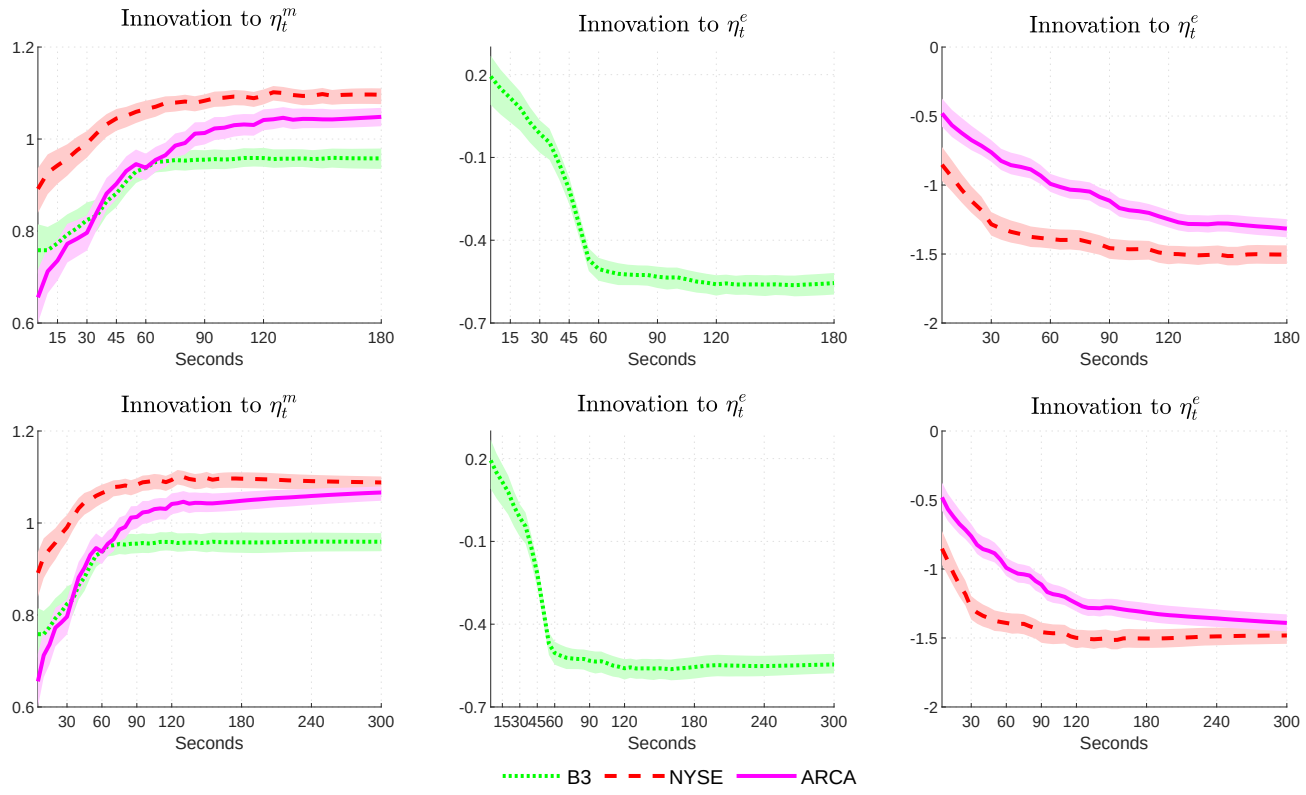
Figure 1: Gerdau: IRF



The first column displays the cumulative impulse response functions showing the effect in B3, ARCA and NYSE of an innovation on the firms' efficient price over 180 and 300 seconds (upper and lower graphs, respectively). The second and third columns display cumulative impulse response functions showing the effect of an innovation on the efficient exchange rate over 180 (upper graphs) and 300 (lower graphs) seconds in B3 (left graph) and in ARCA and NYSE (right graph). The sample period is from December 2007 to November 2009. The empirical 95% confidence intervals are obtained using the bootstrap standard errors.

Table 1: Instantaneous and total effects

	Panel A - Efficient price: Instantaneous effect						Panel B - Efficient price: total effect					
	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale
B3	0.76	0.86	0.72	1.07	1.25	0.90	0.98	0.98	0.98	0.99	0.99	0.97
	(0.026)	(0.020)	(0.025)	(0.023)	(0.019)	(0.027)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
ExRate	0.08	0.28	0.15	0.04	0.00	0.18	−0.08	−0.10	−0.09	−0.07	−0.07	−0.10
	(0.005)	(0.007)	(0.006)	(0.007)	(0.005)	(0.004)	(0.002)	(0.004)	(0.003)	(0.004)	(0.004)	(0.002)
NYSE	0.89	0.90	0.94	0.59	0.61	0.87	1.06	1.08	1.08	1.07	1.07	1.08
	(0.027)	(0.016)	(0.023)	(0.047)	(0.034)	(0.036)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.002)
ARCA	0.66	0.65	0.63	0.61	0.68	-	1.06	1.08	1.08	1.07	1.06	-
	(0.034)	(0.036)	(0.037)	(0.044)	(0.031)		(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	

	Panel C - Efficient exchange rate: instantaneous effect						Panel D - Efficient exchange rate: total effect					
	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale
B3	0.19	−0.07	0.25	0.07	−0.11	−0.28	−0.51	−0.46	−0.43	−0.20	−0.34	−0.51
	(0.036)	(0.037)	(0.028)	(0.018)	(0.038)	(0.036)	(0.017)	(0.029)	(0.019)	(0.012)	(0.019)	(0.022)
ExRate	2.02	1.79	1.93	1.56	1.40	1.98	0.98	0.98	0.98	0.99	0.99	0.97
	(0.014)	(0.033)	(0.023)	(0.010)	(0.013)	(0.018)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
NYSE	−0.85	−1.33	−0.92	−0.49	−0.22	−0.81	−1.49	−1.43	−1.41	−1.19	−1.33	−1.48
	(0.051)	(0.044)	(0.042)	(0.046)	(0.062)	(0.062)	(0.017)	(0.027)	(0.018)	(0.011)	(0.018)	(0.021)
ARCA	−0.48	−0.57	−0.71	−0.46	−0.21	-	−1.49	−1.43	−1.41	−1.19	−1.33	-
	(0.042)	(0.042)	(0.043)	(0.038)	(0.052)		(0.017)	(0.027)	(0.018)	(0.011)	(0.018)	

The upper panels report the instantaneous and total effect (Panels A and B, respectively) of an impulse in the efficient price of the underlying security. The lower panels report the instantaneous and total effect (Panels C and D, respectively) of an impulse in the efficient exchange rate. The instantaneous effects of an impulse in the efficient price of the underlying security and efficient exchange rate correspond to the first and second columns of the estimates of d_0, respectively. The total effects of an impulse in the efficient price of the underlying security and efficient exchange rate correspond to the first and second columns of the estimates of $D(1)$, respectively. The exchange rate is in R$ per US dollar. The lag length in the VEC model is determined through Schwarz criterion. Prices are sampled at a 5-second sampling interval for Gerdau ($T = 2,128,453$), Petrobras ($T = 2,120,549$), Bradesco ($T = 2,128,463$) and Vale ($T = 2,114,419$). Prices are sampled at a 15-second frequency for Ambev ($T = 709,591$) and a 30-second frequency for BR Telecom ($T = 353,300$). Vale does not trade at ARCA. The sample period is from December 2007 to November 2009. The bootstrap standard errors are in parentheses.

Table 2: Instantaneous effect

Panel A - Efficient price

	Gerdau		Petrobras		Bradesco		Ambev		BR Telecom		Vale	
	15 sec.	30 sec.	15 sec.	30 sec.	15 sec.	30 sec.	30 sec.	60 sec.	60 sec.	120 sec.	15 sec.	30 sec.
B3	0.79	0.87	0.84	0.88	0.78	0.88	1.07	1.09	1.20	1.13	0.84	0.85
	(0.027)	(0.024)	(0.017)	(0.021)	(0.026)	(0.022)	(0.022)	(0.022)	(0.020)	(0.021)	(0.028)	(0.026)
ExRate	0.07	0.08	0.21	0.17	0.14	0.11	0.05	0.09	0.02	0.01	0.16	0.14
	(0.006)	(0.008)	(0.006)	(0.006)	(0.007)	(0.009)	(0.010)	(0.008)	(0.004)	(0.006)	(0.006)	(0.007)
NYSE	0.92	0.89	0.98	1.03	0.96	0.91	0.61	0.65	0.69	0.76	0.92	1.01
	(0.025)	(0.023)	(0.012)	(0.010)	(0.022)	(0.019)	(0.046)	(0.043)	(0.036)	(0.036)	(0.029)	(0.019)
ARCA	0.68	0.67	0.79	0.90	0.74	0.78	0.60	0.63	0.70	0.76	-	-
	(0.033)	(0.037)	(0.025)	(0.030)	(0.043)	(0.029)	(0.040)	(0.039)	(0.032)	(0.036)		

Panel B - Efficient exchange rate

	Gerdau		Petrobras		Bradesco		Ambev		BR Telecom		Vale	
	15 sec.	30 sec.	15 sec.	30 sec.	15 sec.	30 sec.	30 sec.	60 sec.	60 sec.	120 sec.	15 sec.	30 sec.
B3	0.09	−0.34	−0.12	−0.18	0.16	−0.11	−0.11	−0.18	−0.20	−0.24	−0.32	−0.31
	(0.046)	(0.049)	(0.031)	(0.038)	(0.036)	(0.037)	(0.030)	(0.024)	(0.039)	(0.049)	(0.040)	(0.046)
ExRate	1.50	1.26	1.27	1.06	1.43	1.17	1.35	1.18	1.24	1.13	1.46	1.21
	(0.018)	(0.017)	(0.023)	(0.026)	(0.021)	(0.020)	(0.014)	(0.015)	(0.013)	(0.015)	(0.018)	(0.022)
NYSE	−1.17	−1.47	−1.40	−1.46	−1.27	−1.46	−0.72	−0.96	−0.40	−0.66	−1.18	−1.45
	(0.062)	(0.064)	(0.033)	(0.036)	(0.050)	(0.048)	(0.060)	(0.071)	(0.058)	(0.077)	(0.060)	(0.055)
ARCA	−0.63	−0.80	−0.85	−1.19	−0.97	−1.23	−0.58	−0.71	−0.29	−0.54	-	-
	(0.054)	(0.072)	(0.048)	(0.048)	(0.058)	(0.060)	(0.051)	(0.069)	(0.060)	(0.081)		

The upper panel reports the instantaneous effect of an impulse in the efficient price of the underlying security, which corresponds to the first column of the estimates of d_0. The lower panel reports the instantaneous effect of an impulse in the efficient exchange rate, which corresponds to the second column of the estimates of d_0. Results are reported for alternative sampling intervals: 15- and 30-second intervals for Gerdau, Petrobras, Bradesco, and Vale; 30- and 60-second intervals for Ambev; and 60- and 120-second intervals for BR Telecom. Exchange rate is in R$ per U.S. dollars. Lag length in the VEC model is determined through Schwarz criterion. Vale does not trade at ARCA. The bootstrap standard errors are in parenthesis.

Table 3: Total effect

Panel A - Efficient price

	Gerdau		Petrobras		Bradesco		Ambev		BR Telecom		vale	
	15 sec.	30 sec.	15 sec.	30 sec.	15 sec.	30 sec.	30 sec.	60 sec.	60 sec.	120 sec.	15 sec.	30 sec.
B3	0.97	0.97	0.97	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.97	0.97
	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
ExRate	−0.09	−0.09	−0.10	−0.11	−0.10	−0.10	−0.08	−0.09	−0.07	−0.07	−0.11	−0.11
	(0.003)	(0.005)	(0.003)	(0.004)	(0.004)	(0.005)	(0.006)	(0.006)	(0.004)	(0.005)	(0.003)	(0.005)
NYSE	1.07	1.07	1.08	1.08	1.08	1.08	1.07	1.08	1.07	1.07	1.08	1.08
	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.005)	(0.003)	(0.003)	(0.002)	(0.003)
ARCA	1.07	1.07	1.08	1.08	1.08	1.08	1.07	1.08	1.07	1.07	-	-
	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.005)	(0.003)	(0.003)		

Panel B - Efficient exchange rate

	Gerdau		Petrobras		Bradesco		Ambev		BR Telecom		vale	
	15 sec.	30 sec.	15 sec.	30 sec.	15 sec.	30 sec.	30 sec.	60 sec.	60 sec.	120 sec.	15 sec.	30 sec.
B3	−0.58	−0.60	−0.47	−0.49	−0.45	−0.45	−0.24	−0.24	−0.34	−0.38	−0.54	−0.56
	(0.028)	(0.033)	(0.028)	(0.032)	(0.027)	(0.029)	(0.020)	(0.022)	(0.022)	(0.027)	(0.026)	(0.029)
ExRate	0.97	0.97	0.97	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.97	0.97
	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)
NYSE	−1.55	−1.57	−1.45	−1.47	−1.43	−1.44	−1.23	−1.23	−1.34	−1.37	−1.52	−1.53
	(0.027)	(0.031)	(0.026)	(0.030)	(0.025)	(0.027)	(0.019)	(0.021)	(0.021)	(0.026)	(0.024)	(0.027)
ARCA	−1.55	−1.57	−1.45	−1.47	−1.43	−1.43	−1.23	−1.23	−1.34	−1.37	-	-
	(0.027)	(0.031)	(0.026)	(0.030)	(0.025)	(0.027)	(0.019)	(0.021)	(0.021)	(0.026)		

The upper panel reports the total effect of an impulse in the efficient price of the underlying security, which corresponds to the first column of the estimates of $D(1)$. The lower panel reports the total effect of an impulse in the efficient exchange rate, which corresponds to the second column of the estimates of $D(1)$. Results are reported for alternative sampling intervals: 15- and 30-second intervals for Gerdau, Petrobras, Bradesco, and Vale; 30- and 60-second intervals for Ambev; and 60- and 120-second intervals for BR Telecom. Exchange rate is in R$ per US dollars. Lag length in the VEC model is determined through Schwarz criterion. Vale does not trade at ARCA. The bootstrap standard errors are in parenthesis.

Table 4: Instantaneous and total effects: Cholesky-based identification, $\lambda = 0$

	Panel A - Efficient price: Instantaneous effect						Panel B - Efficient price: total effect					
	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale
B3	0.82 (0.026)	0.92 (0.020)	0.79 (0.029)	1.09 (0.021)	1.29 (0.020)	0.95 (0.024)	1.00	1.00	1.00	1.00	1.00	1.00
ExRate	0.25 (0.007)	0.49 (0.008)	0.35 (0.011)	0.16 (0.009)	0.10 (0.006)	0.42 (0.005)	0.00	0.00	0.00	0.00	0.00	0.00
NYSE	0.88 (0.026)	0.82 (0.016)	0.91 (0.027)	0.57 (0.045)	0.61 (0.036)	0.85 (0.031)	1.00	1.00	1.00	1.00	1.00	1.00
ARCA	0.66 (0.033)	0.63 (0.038)	0.60 (0.039)	0.58 (0.045)	0.69 (0.031)	-	1.00	1.00	1.00	1.00	1.00	-

	Panel C - Efficient exchange rate: instantaneous effect						Panel D - Efficient exchange rate: total effect					
	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale
B3	−0.20 (0.041)	−0.46 (0.052)	−0.06 (0.032)	−0.16 (0.028)	−0.53 (0.051)	−0.73 (0.044)	−1.00 (0.037)	−0.89 (0.048)	−0.84 (0.039)	−0.42 (0.022)	−0.67 (0.036)	−0.99 (0.041)
ExRate	1.94 (0.020)	1.62 (0.038)	1.83 (0.028)	1.56 (0.015)	1.39 (0.014)	1.83 (0.025)	1.00	1.00	1.00	1.00	1.00	1.00
NYSE	−1.29 (0.082)	−1.71 (0.058)	−1.30 (0.066)	−0.64 (0.053)	−0.43 (0.062)	−1.23 (0.078)	−2.00 (0.037)	−1.89 (0.048)	−1.85 (0.039)	−1.42 (0.022)	−1.68 (0.036)	−1.99 (0.041)
ARCA	−0.81 (0.064)	−0.85 (0.056)	−0.97 (0.046)	−0.59 (0.054)	−0.44 (0.055)	-	−2.00 (0.037)	−1.89 (0.048)	−1.85 (0.039)	−1.42 (0.022)	−1.68 (0.036)	-

The upper panels report the instantaneous and total effect (left- and right-panels, respectively) of an impulse in the efficient price of the underlying security. The lower panels report the instantaneous and total effect (left- and right-panels, respectively) of an impulse in the efficient exchange rate. The exchange rate is in R$ per US dollar. The lag length in the VEC model is determined through Schwarz criterion. Identification of the structural shocks is obtained from the Cholesky decomposition as in Subsection 3.2. The ordering of the variables is changed such that the long-run effect of an impulse in the efficient price to the exchange rate is restricted to zero. Prices are sampled at a 5-second sampling interval for Gerdau ($T = 2,128,453$), Petrobras ($T = 2,120,549$), Bradesco ($T = 2,128,463$) and Vale ($T = 2,114,419$). Prices are sampled at a 15-second frequency for Ambev ($T = 709,591$) and a 30-second frequency for BR Telecom ($T = 353,300$). Vale does not trade on the ARCA exchange. The sample period is from December 2007 to November 2009. The bootstrap standard errors are in parentheses.

Table 5: Long-run variance decomposition: spectral- and Cholesky-based identification schemes

Spectral-based identification

	Panel A - Efficient price						Panel B - Efficient exchange rate					
	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale
B3	0.96	0.95	0.96	0.98	0.98	0.94	0.04	0.05	0.04	0.02	0.02	0.06
	(0.004)	(0.003)	(0.005)	(0.002)	(0.003)	(0.004)	(0.004)	(0.003)	(0.005)	(0.001)	(0.002)	(0.004)
ExRate	0.04	0.05	0.04	0.02	0.02	0.06	0.96	0.95	0.96	0.98	0.98	0.94
	(0.004)	(0.003)	(0.005)	(0.001)	(0.002)	(0.004)	(0.004)	(0.003)	(0.005)	(0.001)	(0.002)	(0.004)
NYSE	0.77	0.72	0.73	0.69	0.77	0.72	0.23	0.28	0.27	0.31	0.23	0.28
	(0.005)	(0.005)	(0.005)	(0.006)	(0.008)	(0.009)	(0.005)	(0.005)	(0.005)	(0.006)	(0.008)	(0.009)
ARCA	0.77	0.72	0.73	0.69	0.77	–	0.23	0.28	0.27	0.31	0.23	–
	(0.005)	(0.005)	(0.005)	(0.006)	(0.007)	(0.000)	(0.005)	(0.005)	(0.005)	(0.006)	(0.008)	(0.000)
$\varsigma \times 10^3$	0.59	0.50	0.50	0.65	1.22	0.52	0.23	0.23	0.23	0.39	0.54	0.23
	(0.027)	(0.034)	(0.038)	(0.014)	(0.020)	(0.025)	(0.013)	(0.008)	(0.007)	(0.007)	(0.013)	(0.021)

Cholesky-based identification: $\lambda = 0$

	Panel C - Efficient price						Panel D - Efficient exchange rate					
	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale	Gerdau	Petrobras	Bradesco	Ambev	Br Telecom	Vale
B3	0.85	0.83	0.85	0.94	0.91	0.77	0.15	0.17	0.15	0.06	0.09	0.23
	(0.014)	(0.011)	(0.020)	(0.006)	(0.010)	(0.015)	(0.013)	(0.011)	(0.019)	(0.005)	(0.009)	(0.015)
ExRate	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
NYSE	0.59	0.51	0.54	0.57	0.63	0.48	0.41	0.49	0.46	0.43	0.37	0.52
	(0.013)	(0.010)	(0.015)	(0.006)	(0.014)	(0.018)	(0.014)	(0.010)	(0.015)	(0.006)	(0.014)	(0.018)
ARCA	0.59	0.51	0.54	0.57	0.63	–	0.41	0.49	0.46	0.43	0.37	–
	(0.013)	(0.010)	(0.015)	(0.006)	(0.014)	(0.000)	(0.013)	(0.010)	(0.015)	(0.006)	(0.014)	(0.000)
$\varsigma \times 10^3$	0.54	0.45	0.46	0.63	1.16	0.47	0.23	0.23	0.23	0.39	0.54	0.23
	(0.023)	(0.028)	(0.026)	(0.012)	(0.018)	(0.017)	(0.013)	(0.008)	(0.007)	(0.007)	(0.013)	(0.021)

The upper and lower panels report long-run variance decomposition of the market prices and observed exchange rate system based on the spectral and Cholesky identification schemes, respectively, with relative contributions of the permanent shocks η_t^m and η_t^e. Specifically, the upper- and lower-left panels display the proportion of the long-run variance of B3, exchange rate, NYSE, and ARCA, accounted for by the permanent innovation associated with the efficient price. The upper- and lower-right panels display the proportion of the long-run forecast error variance of B3, ExRate, NYSE, and ARCA, accounted for by the permanent innovation associated with the efficient exchange rate. Finally, the last rows ($\varsigma \times 10^3$) on the left-hand panels account for the standard deviation of η_t^m multiplied by 10^3, whereas the last rows of the right-hand side panels are the standard deviation of η_t^e multiplied by 10^3. The exchange rate is in R$ per US dollar. The lag length in the VEC model is determined through Schwarz criterion. Prices are sampled at a 5-second sampling interval for Gerdau ($T = 2,128,453$), Petrobras ($T = 2,120,549$), Bradesco ($T = 2,128,463$) and Vale ($T = 2,114,419$). Prices are sampled at a 15-second frequency for Ambev ($T = 709,591$) and a 30-second frequency for BR Telecom ($T = 353,300$). Vale does not trade at ARCA. The sample period is from December 2007 to November 2009. The bootstrap standard errors are in parentheses.

Appendix

A Technical appendix

A.1 Proof of Proposition 1

Start by rewriting the observed exchange rate as a function of the structural parameters λ, ρ, $\dot{\gamma}_i$, and γ_i, with $i = 1, 2, 3, 4$. To this end, subtract w_{t-1} from both sides of (5) and collect the terms, such that:

$$\begin{aligned} w_t - w_{t-1} &= w_{t-1} - w_{1,t-2} + \dot{\gamma}_2 \left(\Delta e_t - \Delta w_{t-1}\right) + \gamma_2 \left(\Delta m_t - \Delta m_{t-1}\right) + b_2 \left(\eta_t^T - \eta_{t-1}^T\right), \\ \left(1 - L + L\dot{\gamma}_2\right) \Delta w_t &= \dot{\gamma}_2 \Delta e_t + \gamma_2 \left(\Delta m_t - L\Delta m_t\right) + b_2 \left(\eta_t^T - L\eta_t^T\right), \\ \left(1 - L + L\dot{\gamma}_2\right) \Delta w_t &= \dot{\gamma}_2 \left(\eta_t^e + \lambda \eta_t^m\right) + \gamma_2 \left(\eta_t^m + \rho \eta_t^e - L\Delta m_t\right) + b_2 \left(\eta_t^T - L\eta_t^T\right). \end{aligned} \tag{A.1}$$

Setting $L = 0$ in (A.1) gives the instantaneous effect of an impulse in η_t on Δw_t, i.e., the second row of $\breve{d}_0$. It then reads as follows:

$$\breve{d}_{0_{(2,1)}} = \gamma_2 + \dot{\gamma}_2 \lambda, \quad \breve{d}_{0_{(2,2)}} = \dot{\gamma}_2 + \gamma_2 \rho, \quad \text{and} \quad \breve{d}_{0_{(2,3)}} = b_2, \tag{A.2}$$

where $\breve{d}_{0_{(2,1)}}$, $\breve{d}_{0_{(2,2)}}$, and $\breve{d}_{0_{(2,3)}}$ denote the first, second, and third elements of the second row of $\breve{d}_0$, respectively. Next, the total response of Δw_t to an impulse in η_t is obtained by setting $L = 1$ in (A.1) and noting that $\Delta m_t = \eta_t^m + \rho \eta_t^e$. The elements of the second row of $\breve{D}(1)$ then read as follows:

$$\breve{D}(1)_{(2,1)} = \lambda, \quad \breve{D}(1)_{(2,2)} = 1, \quad \text{and} \quad \breve{D}(1)_{(2,3)} = 0_{1\times 2}. \tag{A.3}$$

It readily follows from setting $L = 1$ that $\Delta w_t = \Delta e_t$, meaning that in the long run, changes in the efficient exchange rate are fully incorporated into Δw_t.

Regarding the first row of $\breve{d}_0$ and $\breve{D}(1)$, subtract $p_{1,t-1}$ from both sides of (4), and collect the terms, such that:

$$\begin{aligned}
p_{1,t} - p_{1,t-1} = & p_{1,t-1} - p_{1,t-2} + \gamma_1 \left(m_t - m_{t-1} - p_{1,t-1} - p_{1,t-2}\right) \\
& + \dot{\gamma}_1 \left(e_t - e_{t-1} - (w_{t-1} - w_{t-2})\right) + b_1 \left(\eta_t^T - \eta_{t-1}^T\right), \\
\Delta p_{1,t} - \Delta p_{1,t-1} = & \gamma_1 \left(\Delta m_t - \Delta p_{1,t-1}\right) + \dot{\gamma}_1 \left(\Delta e_t - \Delta w_{t-1}\right) + b_1 \left(\eta_t^T - L\eta_t^T\right), \\
\left(1 - L + L\gamma_1\right) \Delta p_{1,t} = & \gamma_1 \Delta m_t + \dot{\gamma}_1 \left(\Delta e_t - L\Delta w_t\right) + b_1 \left(\eta_t^T - L\eta_t^T\right), \\
\left(1 - L + L\gamma_1\right) \Delta p_{1,t} = & \gamma_1 \left(\eta_t^m + \rho\eta_t^e\right) + \dot{\gamma}_1 \left(\lambda\eta_t^m + \eta_t^e - L\Delta w_t\right) + b_1 \left(\eta_t^T - L\eta_t^T\right). \qquad \text{(A.4)}
\end{aligned}$$

As in the exchange rate case discussed in equations (A.1)-(A.3), the instantaneous and total effect responses of an impulse in η_t on $\Delta p_{1,t}$ are computed by making $L = 0$ and $L = 1$, respectively, and using the result that $\Delta e_t = \Delta w_t$ when $L = 1$. In that, the second row of d_0 and $D(1)$ becomes:

$$\breve{d}_{0_{(1,1)}} = \gamma_1 + \dot{\gamma}_1\lambda, \quad \breve{d}_{0_{(1,2)}} = \dot{\gamma}_1 + \gamma_1\rho, \quad \text{and} \quad \breve{d}_{0_{(1,3)}} = b_1, \qquad \text{(A.5)}$$

$$\breve{D}(1)_{(2,1)} = 1, \quad \breve{D}(1)_{(2,2)} = \rho, \quad \text{and} \quad \breve{D}(1)_{(2,3)} = 0_{1\times 2}. \qquad \text{(A.6)}$$

Similar steps are implemented to obtain the remaining rows of $\breve{d}_0$ and $\breve{D}$. By subtracting lagged values of the observed prices in the foreign markets (6) and (7), it then follows that these prices can be expressed as functions of the permanent and transitory shocks:

$$\left(1 - L + L\gamma_3\right) \Delta p_{3,t}^* = \gamma_3 \left(\eta_t^m + \rho\eta_t^e - L\Delta w_t\right) + \dot{\gamma}_3 \left(\eta_t^e + \lambda\eta_t^m - L\Delta w_t\right) + b_3 \left(\eta_t^T - L\eta_t^T\right), \qquad \text{(A.7)}$$

$$\left(1 - L + L\gamma_4\right) \Delta p_{4,t}^* = \gamma_4 \left(\eta_t^m + \rho\eta_t^e - L\Delta w_t\right) + \dot{\gamma}_4 \left(\eta_t^e + \lambda\eta_t^m - L\Delta w_t\right) + b_4 \left(\eta_t^T - L\eta_t^T\right), \qquad \text{(A.8)}$$

where $\Delta p_{3,t}^* = \Delta p_{3,t} - \Delta w_t$ and $\Delta p_{4,t}^* = \Delta p_{4,t} - \Delta w_t$. Setting $L = 0$, the third and fourth

rows of d_0 are then:

$$\breve{d}_{0_{(3,1)}} = \gamma_3 + \dot{\gamma}_3\lambda, \quad \breve{d}_{0_{(3,2)}} = \dot{\gamma}_3 + \gamma_3\rho, \quad \text{and} \quad \breve{d}_{0_{(3,3)}} = b_3, \tag{A.9}$$

$$\breve{d}_{0_{(4,1)}} = \gamma_4 + \dot{\gamma}_4\lambda, \quad \breve{d}_{0_{(4,2)}} = \dot{\gamma}_4 + \gamma_4\rho, \quad \text{and} \quad \breve{d}_{0_{(4,3)}} = b_4. \tag{A.10}$$

Finally, the last two rows of $D(1)$ follow directly from setting $L = 1$ in (A.7) and (A.8), and noting that $\Delta w_t = \Delta e_t$ holds if $L = 1$:

$$\breve{D}(1)_{(3,1)} = 1 - \lambda, \quad \breve{D}(1)_{(3,2)} = \rho - 1, \quad \text{and} \quad \breve{D}(1)_{(3,3)} = 0_{1\times 2}, \tag{A.11}$$

$$\breve{D}(1)_{(4,1)} = 1 - \lambda, \quad \breve{D}(1)_{(4,2)} = \rho - 1, \quad \text{and} \quad \breve{D}(1)_{(4,3)} = 0_{1\times 2}, \tag{A.12}$$

which concludes the proof. ■

A.2 Proof of Proposition 2

Start by manipulating the VMA(∞) representation in (11), such that:

$$\Delta P_t = \Psi(L)u_t = \Psi(L)G^{*-1}G^*u_t = \Psi(L)G^{*-1}\varepsilon_t = \Upsilon(L)\varepsilon_t. \tag{A.13}$$

Note that $\varepsilon_t = G^*u_t$, meaning that $\varepsilon_t^P = \alpha_\perp' u_t$ and $\varepsilon_t^T = \alpha'\Omega^{-1}u_t$. Next, it suffices to show that the long-run effect of an impulse in ε_t^P and ε_t^P are nonzero and zero, respectively, meaning $\Upsilon(1)_{11} \neq 0_{(k-r)\times(k-r)}$, $\Upsilon(1)_{21} \neq 0_{r\times(k-r)}$, $\Upsilon(1)_{12} = 0_{(k-r)\times r}$, and $\Upsilon(1)_{22} = 0_{r\times r}$. First, partition the inverse of G^* as:

$$G^{*-1} = \left(G^*_{+,(k-r)}, \quad G^*_{+,r}\right), \tag{A.14}$$

where $G^*_{+,(k-r)}$ and $G^*_{+,(r)}$ have dimensions $k \times (k-r)$ and $k \times r$, respectively. It then follows that G^{*-1} must satisfy:

$$G^* G^{*-1} = \begin{pmatrix} \alpha'_\perp G^*_{+,(k-r)} & \alpha'_\perp G^*_{+,r} \\ \alpha' \Omega^{-1} G^*_{+,(k-r)} & \alpha' \Omega^{-1} G^*_{+,r} \end{pmatrix} = I_k, \tag{A.15}$$

which implies $G^*_{+,(k-r)} = \Omega\alpha_\perp \left(\alpha'_\perp \Omega \alpha_\perp\right)^{-1}$ and $G^*_{+,r} = \alpha \left(\alpha' \Omega^{-1} \alpha\right)^{-1}$. Recall that $\Upsilon(L) = \Psi(L) G^{*-1}$, which implies that the matrices of long-run effect of an impulse in the reduced-form permanent and transitory innovations are given by:

$$\begin{aligned} \Upsilon(1) = \Psi(1) G^{*-1} &= \left(\Psi(1) G^*_{+,(k-r)}, \quad \Psi(1) G^*_{+,r}\right) \\ &= \left(\Psi(1)\Omega\alpha_\perp \left(\alpha'_\perp \Omega \alpha_\perp\right)^{-1}, \quad \Psi(1)\alpha \left(\alpha' \Omega^{-1} \alpha\right)^{-1}\right), \end{aligned} \tag{A.16}$$

such that:

$$\Psi(1) G^*_{+,(k-r)} = \begin{pmatrix} \Upsilon(1)_{11} \\ \Upsilon(1)_{21} \end{pmatrix} = \Psi(1)\Omega\alpha_\perp \left(\alpha'_\perp \Omega \alpha_\perp\right)^{-1} \quad \text{and} \tag{A.17}$$

$$\Psi(1) G^*_{+,r} = \begin{pmatrix} \Upsilon(1)_{12} \\ \Upsilon(1)_{22} \end{pmatrix} = \Psi(1)\alpha \left(\alpha' \Omega^{-1} \alpha\right)^{-1}. \tag{A.18}$$

Replace $\Psi(1)$ in (A.17) and (A.18) by the Johansen factorization $\Psi(1) = \beta_\perp \left(\alpha'_\perp \Gamma(1) \beta_\perp\right)^{-1} \alpha'_\perp$ and use the identity $\alpha'_\perp \alpha = 0$. It then follows that:

$$\begin{pmatrix} \Upsilon(1)_{11} \\ \Upsilon(1)_{21} \end{pmatrix} = \beta_\perp \left(\alpha'_\perp \Gamma(1) \beta_\perp\right)^{-1} \neq 0_{k \times (k-r)} \quad \text{and} \tag{A.19}$$

$$\begin{pmatrix} \Upsilon(1)_{12} \\ \Upsilon(1)_{22} \end{pmatrix} = \beta_\perp \left(\alpha'_\perp \Gamma(1) \beta_\perp\right)^{-1} \alpha'_\perp \alpha \left(\alpha' \Omega^{-1} \alpha\right)^{-1} = 0_{k \times r}, \tag{A.20}$$

which finalizes the proof. ■

A.3 Normalization proof: $\widetilde{S}\widetilde{S}\Theta = \widetilde{S}\Theta\widetilde{S}' = \Xi$

As $\widetilde{\Xi}$ is block diagonal, the property of block matrix determinants implies that:

$$\Lambda = \begin{pmatrix} \Lambda_{11} & 0_{(k-r)\times r} \\ 0_{r\times(k-r)} & \Lambda_{22} \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} V_{11} & 0_{(k-r)\times r} \\ 0_{r\times(k-r)} & V_{22} \end{pmatrix}, \tag{A.21}$$

where Λ_{11} and Λ_{22} are diagonal matrices containing the eigenvalues of $\widetilde{\Xi}_{11}$ and $\widetilde{\Xi}_{22}$, respectively, the columns of V_{11} contain the eigenvectors of $\widetilde{\Xi}_{11}$ associated with Λ_{11}, and V_{22} is the matrix of eigenvectors of $\widetilde{\Xi}_{22}$ associated with Λ_{22}. It then follows that:

$$\Xi = \widetilde{S}\widetilde{S}\Theta = \begin{pmatrix} V_{11}\lambda_{11}^{1/2}V_{11}^{-1} & 0_{(k-r)\times r} \\ 0_{r\times(k-r)} & V_{22}\lambda_{22}^{1/2}V_{22}^{-1} \end{pmatrix} \Theta = \begin{pmatrix} \widetilde{S}_{11}\widetilde{S}_{11}\Theta_1 & 0_{(k-r)\times r} \\ 0_{r\times(k-r)} & \widetilde{S}_{22}\widetilde{S}_{22}\Theta_2 \end{pmatrix}, \tag{A.22}$$

where Θ_1 and Θ_2 are the $(k-r)\times(k-r)$ and $r\times r$ upper left- and lower right-diagonal blocks of Θ, respectively. Therefore, it suffices to show that $\widetilde{S}_{11}\widetilde{S}_{11}\Theta_1 = \widetilde{S}_{11}\Theta_1\widetilde{S}'_{11}$ and $\widetilde{S}_{22}\widetilde{S}_{22}\Theta_2 = \widetilde{S}_{22}\Theta_2\widetilde{S}'_{22}$. Assume the baseline price discovery model, such that $k = 4$, $r = 2$, and Ξ_{11} and Ξ_{22} are a 2×2 matrices. Define Ξ_{11} and Θ_1 as follows:

$$\Xi_{11} = \begin{pmatrix} \xi_{11} & \xi_{21} \\ \xi_{21} & \xi_{22} \end{pmatrix} \quad \text{and} \quad \Theta_1 = \begin{pmatrix} \xi_{11} & 0 \\ 0 & \xi_{22} \end{pmatrix}. \tag{A.23}$$

It then follows that $\widetilde{\Xi}$ reads as:

$$\widetilde{\Xi}_{11} = \Xi_{11}\Theta_1^{-1} = \begin{pmatrix} 1 & \frac{\xi_{21}}{\xi_{22}} \\ \frac{\xi_{21}}{\xi_{11}} & 1 \end{pmatrix}, \tag{A.24}$$

with eigenvalues and eigenvectors given by:

$$\Lambda_{11} = \begin{pmatrix} \frac{\sqrt{\xi_{11}\xi_{22}}-\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}} & 0 \\ 0 & \frac{\xi_{21}+\sqrt{\xi_{11}\xi_{22}}}{\sqrt{\xi_{11}\xi_{22}}} \end{pmatrix} \quad V_{11} = \begin{pmatrix} -\frac{\sqrt{\xi_{11}}}{\sqrt{\xi_{22}}} & \frac{\sqrt{\xi_{11}}}{\sqrt{\xi_{22}}} \\ 1 & 1 \end{pmatrix}. \tag{A.25}$$

The spectral decomposition of Ξ_{11} then becomes:

$$\begin{aligned}\widetilde{S}_{11} &= V_{11}\Lambda_{11}^{1/2}V_{11}^{-1} \\ &= \begin{pmatrix} \frac{1}{2}\left[\left(\frac{\sqrt{\xi_{11}\xi_{22}}-\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}+\left(\frac{\xi_{21}+\sqrt{\xi_{11}\xi_{22}}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}\right] & \frac{\sqrt{\xi_{11}}}{2\sqrt{\xi_{22}}}\left[\left(\frac{\xi_{21}+\sqrt{\xi_{11}\xi_{22}}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}-\left(\frac{\sqrt{\xi_{11}\xi_{22}}-\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}\right] \\ \frac{\sqrt{\xi_{22}}}{2\sqrt{\xi_{11}}}\left[\left(\frac{\xi_{21}+\sqrt{\xi_{11}\xi_{22}}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}-\left(\frac{\sqrt{\xi_{11}\xi_{22}}-\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}\right] & \frac{1}{2}\left[\left(\frac{\sqrt{\xi_{11}\xi_{22}}-\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}+\left(\frac{\xi_{21}+\sqrt{\xi_{11}\xi_{22}}}{\sqrt{\xi_{11}\xi_{22}}}\right)^{1/2}\right] \end{pmatrix}. \end{aligned} \tag{A.26}$$

Combining (A.26) with Θ_1, it readily follows that $\widetilde{S}_{11}\widetilde{S}_{11}\Theta_1 = \widetilde{S}_{11}\Theta_1\widetilde{S}_{11}' = \Xi_{11}$,

$$\begin{aligned}\widetilde{S}\Theta\widetilde{S}' &= \begin{pmatrix} \frac{\xi_{11}}{2}\left[\left(1-\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)+\left(\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}+1\right)\right] & \frac{\sqrt{\xi_{11}\xi_{22}}}{2}\left[\left(\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}+1\right)-\left(1-\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)\right] \\ \frac{\sqrt{\xi_{11}\xi_{22}}}{2}\left[\left(\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}+1\right)-\left(1-\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)\right] & \frac{\xi_{22}}{2}\left[\left(1-\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}\right)+\left(\frac{\xi_{21}}{\sqrt{\xi_{11}\xi_{22}}}+1\right)\right] \end{pmatrix}, \\ &= \begin{pmatrix} \xi_{11} & \xi_{21} \\ \xi_{21} & \xi_{22} \end{pmatrix} = \Xi_{11}. \end{aligned} \tag{A.27}$$

Using the same steps, it is straightforward to show that $\Xi_{22} = \widetilde{S}_{22}\Theta_2\widetilde{S}_{22}'$, implying that $\widetilde{S}\widetilde{S}\Theta = \widetilde{S}\Theta\widetilde{S}' = \Xi$ holds.

B Simulations

In this section, I illustrate the proposed estimation methodology by comparing it with existing methodologies in the literature. Using the identification strategy discussed in Subsection 3.2, it is possible to isolate the relative performance of the two methodological changes implemented in this paper, namely, the computation of matrix G^* using $\alpha'\Omega^{-1}$ instead of β' and the use of spectral decomposition rather than the usual Cholesky or BCB' decompositions.[18]

The model used for data generation in this set of simulations is consistent with other partial adjustment models (see Amihud and Mendelson, 1987; Hasbrouck and Ho, 1987; and Yan and Zivot, 2010). I work with two common factors, but the extension to the case with

[18] Yan and Zivot (2010) adopt the BCB' decomposition as an alternative to the Gonzalo and Ng's (2001) Cholesky choice. Essentially, the BCB' decomposition factorizes Ξ, such that $\Xi = BCB'$, where B is a lower triangular matrix with ones in its main diagonal and C is a diagonal matrix with unknown positive entries.

more common factors is straightforward. The data generation process is given by:

$$m_t = m_{t-1} + \eta_t^m,$$

$$e_t = e_{t-1} + \eta_t^e,$$

where e_t is the efficient exchange rate and m_t is the asset efficient price. The structural innovations η_t^e and η_t^m are random normal processes that are generated with a diagonal covariance matrix. The transitory innovations η_t^T are also normally distributed. The observed prices are given by:

$$\Delta p_{1,t} = \gamma_1 \left(m_t - p_{1,t-1}\right) + b_1 \eta_t^T,$$

$$\Delta e_t = \eta_t^e,$$

$$\Delta p_{2,t}^* = \gamma_2 \left(m_t - p_{2,t-1}\right) - \dot{\gamma}_2 \left(e_t - e_{t-1}\right) + b_2 \eta_t^T,$$

$$\Delta p_{3,t}^* = \gamma_3 \left(m_t - p_{3,t-1}\right) - \dot{\gamma}_3 \left(e_t - e_{t-1}\right) + b_3 \eta_t^T,$$

where $p_{1,t}$ are the transaction prices observed in the domestic market; e_t is exogenous; $p_{2,t}^*$ and $p_{3,t}^*$ are prices observed in the foreign market and expressed in foreign currency; and the 1×2 vectors $b_i = (b_{1i}, b_{2i})$ with $i = 1, 2, 3$ have the parameters that accompany the transitory innovations.

The elements of d_0 are the parameters that give the partial adjustment between efficient and observed prices, as shown below:

$$d_0 = \begin{pmatrix} \gamma_1 & 0 & b_{1i} & b_{2i} \\ 0 & 1 & 0 & 0 \\ \gamma_2 & \dot{\gamma}_2 & b_{1i} & b_{2i} \\ \gamma_3 & \dot{\gamma}_3 & b_{1i} & b_{2i} \end{pmatrix}. \tag{B.1}$$

Table B.1 reports the results based on the four comparisons. First, I seek to measure the

benefit of computing d_0 using the matrix G, which is constructed with $\alpha'\Omega^{-1}$. Therefore, I compare $\widetilde{d}_0$ with $\dot{d}_0$, where $\dot{d}_0$ represents d_0 computed using $\alpha'\Omega^{-1}$ and decomposed with BCB', and $\widetilde{d}_0$ represents d_0 calculated with the matrix G computed using β' and the BCB' decomposition. In the second comparison, I assess the benefit of using only the spectral decomposition. Hence, I compute two estimates of d_0: the first one uses the $\alpha'\Omega^{-1}$ expression in G and the spectral decomposition (denoted as $\widehat{d}_0$), whereas the second measure uses $\alpha'\Omega^{-1}$ and the BCB' decomposition (denoted as $\dot{d}_0$). In the third comparison, I address the benefits of combining the two methodological changes discussed in this work. I compute d_0 using both the $\alpha'\Omega^{-1}$ and the spectral decomposition (denoted as $\widehat{d}_0$), and I denote $\widetilde{d}_0$ as the estimates computed using β' and the BCB' decomposition. Finally, I also compare $\widehat{d}_0$ with the methodology suggested by Gonzalo and Ng (2001) (computing with β' and using the Cholesky decomposition). I denote the latter as $\overline{d}_0$.

I report the results in terms of the mean, relative mean squared errors (RMSE), and relative root mean squared errors (RRMSE). I also display a ratio that offers information on how these two measures are computed. For instance, the ratio $\widehat{d}_0/\overline{d}_0$ implies that the relative measures in columns (9) and (13) in Table B.1 are computed with $\overline{d}_0$ in the denominator and $\widehat{d}_0$ in the numerator. Thus, relative measures that are smaller than one in columns (9) and (13) indicate that the $\widehat{d}_0$ outperforms $\overline{d}_0$.

The results show that $\widetilde{d}_0$ is biased for systems with more than one cointegrating vector (computations of $\widetilde{d}_0$ for a smaller system with only one cointegrating vector eliminate the bias). Using $\alpha'\Omega^{-1}$ to construct matrix G eliminates the finite sample bias, even when the BCB' decomposition is adopted (see the results of $\dot{d}_0$). Hence, $\dot{d}_0$, $\overline{d}_0$, and $\widehat{d}_0$ are not biased. By analyzing the relative measures, I show that compared to the $\widetilde{d}_0$ measures, $\widehat{d}_0$ presents massive gains. Similar results are obtained when $\dot{d}_0$ is compared to $\widetilde{d}_0$, showing that the use of $\alpha'\Omega^{-1}$ instead of β' considerably improves the estimates of the d_0 matrix. In summary, the proposed measure outperforms all competitors.

Table B.1: Monte Carlo simulations

True value	Mean				RMSE				RRMSE			
	$\beta_{BCB'}$	$\alpha_{BCB'}$	β_C	α_S	$\frac{\alpha_{BCB'}}{\beta_{BCB'}}$	$\frac{\alpha_S}{\alpha_{BCB'}}$	$\frac{\alpha_S}{\beta_{BCB'}}$	$\frac{\alpha_S}{\beta_C}$	$\frac{\alpha_{BCB'}}{\beta_{BCB'}}$	$\frac{\alpha_S}{\alpha_{BCB'}}$	$\frac{\alpha_S}{\beta_{BCB'}}$	$\frac{\alpha_S}{\beta_C}$
d_0	$\widetilde{d}_0$	$\dot{d}_0$	$\overline{d}_0$	$\widehat{d}_0$	$\dot{d}_0/\widetilde{d}_0$	$\widehat{d}_0/\dot{d}_0$	$\widehat{d}_0/\widetilde{d}_0$	$\widehat{d}_0/\overline{d}_0$	$\dot{d}_0/\widetilde{d}_0$	$\widehat{d}_0/\dot{d}_0$	$\widehat{d}_0/\widetilde{d}_0$	$\widehat{d}_0/\overline{d}_0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
$d_{0,(11)} = 0.8$	0.87	0.79	0.79	0.79	0.01	1.00	0.01	0.99	0.09	1.00	0.09	0.99
$d_{0,(21)} = 0.0$	0.01	0.00	0.00	0.00	0.20	0.70	0.10	0.51	0.45	0.50	0.32	0.71
$d_{0,(31)} = 0.2$	0.45	0.20	0.20	0.20	0.00	0.96	0.00	0.93	0.04	0.93	0.04	0.96
$d_{0,(41)} = 0.5$	0.54	0.50	0.49	0.50	0.01	0.93	0.01	0.86	0.10	0.87	0.10	0.93
$d_{0,(12)} = 0.0$	0.46	0.00	0.00	0.00	0.00	0.80	0.00	0.65	0.03	0.64	0.02	0.81
$d_{0,(22)} = 1.0$	1.03	0.99	0.99	0.99	0.05	1.00	0.05	0.92	0.22	1.00	0.22	0.96
$d_{0,(32)} = 0.2$	1.15	0.20	0.20	0.20	0.00	0.99	0.00	0.97	0.01	0.99	0.01	0.99
$d_{0,(42)} = 0.5$	0.95	0.50	0.50	0.50	0.00	0.95	0.00	0.87	0.05	0.91	0.05	0.93

Results are expressed in terms of relative mean squared error (RMSE) and relative root mean squared error (RRMSE). Sample size and replication number are fixed at 10,000 and 1,000, respectively. The variable $d_{0,(i,j)}$ denotes the ij^{th} element of the d_0 matrix. Finally, columns are labelled from (1) to (13).