MICRO-EVOLUTION OF DOCUMENTATIONAL WORK IN THE TEACHING OF THE VOLUME OF REVOLUTION

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In this paper, we draw on the documentational approach to analyse the evolution of one experienced secondary teacher's work towards the teaching of the topic of "volume of revolution". He used a range of paper and computer based resources including the software Autograph. Data were collected in observations of three lessons on this topic taught to two different groups of 16-18 years old students and a follow up interview with the teacher where he was asked to reflect on his choices in these lessons. The findings illustrate teacher's documentational work with the used resources, and his schemes of use – aims, rules of actions, operational invariants and inferences – and identify the micro-evolution, namely the small changes and the rationale behind these changes, of these schemes across the lessons.

INTRODUCTION

Teaching is a complex profession that requires teachers to interact with, and promptly respond to, a range of factors in their teaching environment. As a result, teachers' practices are not merely a reflection of their plans and beliefs. Other factors also come into play, such as teachers' and students' personalities and epistemologies, institutional constraints, unexpected circumstances, time issues and available materials (Nardi, Biza, & Zachariades, 2012). These factors should be taken into account when studying teachers' practices (Herbst & Chazan, 2003). Indeed, Lerman (2013) suggested that research should avoid "implied telos about 'good teaching' [… and] study what happens in practice and offer multiple stories of that practice" (p. 623). In this paper, we report findings from Kayali's PhD study that investigates mathematics teachers' ways of tuning the different elements in their working environment, especially when using mathematics-education software (i.e. software designed for mathematics teaching and learning purposes). Specifically, we look at teachers' 'live' practices within specific contexts and examine consistencies and potential gaps between intended and actual practices (Kayali & Biza, 2017). Here, we draw on the documentational approach (Gueudet & Trouche, 2009) to analyse data of three video-recorded lesson observations of one teacher's work on the topic of "the volume of revolution", and a follow up audio-recorded interview with him where incidents from the observation were discussed in order to respond the research question: "How does teacher's documentational work change across lessons, if it changes, and why?".

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THE DOCUMENTATIONAL APPROACH

The *documentational approach* (Gueudet & Trouche, 2009) explores teachers' work with *resources*. The term resource here has a wider definition; it can be an artefact, a teaching material, a social interaction or anything that influences a teacher's activity (ibid.). This approach, also, refers to Adler's definition of resource "as the verb re-source, to source again or differently" (2000, p. 207). According to the documentational approach, teachers while interacting with resources develop *schemes of use*. A scheme of use adopts a set of resources to be used across different situations according to specific procedures (Gueudet, 2017). It consists of the *aim of the teaching activity* (e.g. to teach about the volume of revolution); *rules of action*, which represent teacher actions (e.g. solving past-exam questions on the volume of revolution); *operational invariants*, which are the reasons adopted by a teacher to justify her stable actions in a range of similar situations (e.g. it is useful to use Autograph and the textbook to introduce the formula); and, *inferences* (e.g. it would work better if I present the image from the textbook first). A teacher develops a *document* when she associates a set of resources with the scheme of use of these resources (ibid.). Document can be "thought of as the verb *document*: to support something (here the teacher's professional activity) with documents" (Gueudet & Trouche, 2009, p.205, italics in original). A teacher's documentational work includes the set of resources encountered, collected, amended or developed by that teacher for a specific goal (ibid.). The documentational approach offers lenses for exploring the evolution of a teacher's documents, which in turn "contributes to the study of her professional evolution. Naturally, such a study must not be limited to the material aspect of documents, but must also investigate the evolution of usages […] and operational invariants" (ibid., p. 211). In this study, we aim to explore the characteristics of one teacher's document by investigating his set of resources and schemes of use during the teaching of three lessons on the volume of revolution, taking into account the justifications he made during the lessons and in the follow-up interview.

METHODOLOGY

This paper reports outcomes from a PhD project conducted in the UK by the first author. The study looks at upper secondary mathematics teachers' documentational work, specifically schemes of use that also concern mathematics-education software. It employs qualitative analysis based on an interpretative research methodology (Stake, 2010). In this paper, we discuss three video-recorded lesson observations and the follow-up interview of one participant, George. At the time of the data collection, George had 15 years of teaching experience mostly in upper secondary education (ages 16-19). The follow up interview was conducted after the initial analysis of the three video-recorded observations. The interview questions focused on the teacher's main steps and choices that were identified during this initial analysis. In the interview, George was invited to reflect and comment on these specific choices (e.g. the use of Autograph). The follow up analysis of George's responses in the interview and actions during the lessons was performed by using the documentational approach. Specifi-

cally, the analysis identified the used resources as well as the schemes of use: aims of the teaching activity, rules of actions, operational invariants and inferences in the context of the observed lesson and summarised them in a documentational work table, similar to the one used by Gueudet (2017) in her analysis of university teachers' work. Here, a simplified version of this table was produced (Table 1), summarizing the rules of actions and operational invariants related to the aims of teaching about volume of revolution and preparing students for exams, in the first lesson. Any changes to the rules of actions or operational invariants in the second or third lessons are discussed in the data analysis section, where we also address the used resources and inferences.

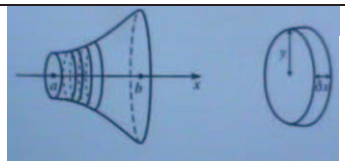
THE VOLUME OF REVOLUTION: THE THREE LESSONS

The data presented here are from three lesson observations. Each lesson was 50-minute long and taught to two mixed gender groups (here G1 and G2) of Year 13 students (17-18 years old). George was teaching the same topic, "volume of revolution", to G1 (first and third lessons) and G2 (second lesson). In the first lesson to G1, George started by asking the students about the formula for the area of a circle. He, then, used Autograph (a dynamic environment with visualising graphs affordances, see http://www.autograph-maths.com) to show the graph of $y=x(x-3)$, which he had pre-prepared. George rotated the graph to show the students that it was done in 3D mode. After that, he applied trapezium rule (which the students had seen before) on the area between the graph and the x-axis. He used a small number of divisions to show how the trapezium rule gives an underestimate of the area. Then, he used Simpson's rule (not known to the students at that time) to shade the area between the graph and the x-axis. He commented that this rule was more accurate and that the students were going to learn more about it in the next lessons. Afterwards, he rotated the shaded area around the x-axis, and he got a shape which he described as a "pointy sphere", a "Pacman", or a "smarty". Then, George opened another graph, also pre-prepared, this time of $y=\sqrt{x}$, and showed the students the rotation of the area between this graph and the x-axis, around the x-axis. After that, he tried to use Autograph to show the students slices of the solid on the screen and to lead them to the formula of the volume of revolution. After trying a few commands in the software, the demonstration was not clear and George did not seem satisfied, but he still kept trying to illustrate how the formula can be explained by using the graph on Autograph. Then, he moved to talking about who came up with the integration notations and explained that integration is like "sum", that was why the symbol for integration (∫) is like an (s) shape. His next step was to invite students to practice on questions from the textbook (Wiseman & Searle, 2005) when he spotted an illustration of the formula (Figure 1, ibid, p. 108). He asked his students to look at this illustration and he explained the formula again by using the image. Having done that, he started solving textbook questions on the board, explaining that he was starting with an "easy example" (Figure 2). Then, he displayed the formula sheet on the interactive whiteboard, which seemed to have reminded him that he had not explained the formula of the volume of revolution for rotations around the y-axis. So, he went quickly through this formula by advising the students to replace y by x and the dx by dy

in the initial formula. Next, he showed the students some past-exam questions and started solving one on the board. He also answered students' questions on the topic. At the end of the lesson, George gave the students paper copies of the formula sheet.

During the second lesson, to G2 this time, George followed similar steps to introduce the volume of revolution. However, in this lesson, instead of using Autograph to explain the formula for the volume of revolution, he did so by displaying on the board a pre- scanned copy of the textbook illustration (Figure 1). Another difference is that, in this lesson, he solved two examples from the textbook, the one he solved in the first lesson and another one. As a result, he did not have the time to solve past exam questions within the lesson, although he mentioned that students should solve some of these. A third difference was the additional example of $y = \sin(ax+b)+c$ he presented on Autograph. With this example, he used a, b and c to transform the graph; the rotation of which gave different shapes that seemed very impressive to the students. During this lesson, George recalled two questions asked by students in the first lesson and answered them. Towards the end of the lesson, he pointed out some questions in the textbook which were too difficult and exceeded exam requirement.

The third lesson was again for G1 where George devoted some time to quickly review the idea and formula of volume of revolution. He showed the same example used in the previous two lessons $y=x(x-3)$, and, then, used the textbook illustration to explain how the formula was deduced. He also used cards to remind the students of the formulae. He mentioned that there were two types of questions: "easy ones" (Figure 2) and "more difficult" ones (Figure 3). Then, he proceeded with a past-exam question solution and a presentation of its mark scheme. After that, he gave the students some time to solve questions independently until the end of the lesson.



$$\int_a^b \pi y^2 dx$$

Figure 1: Textbook illustration (Wiseman & Searle, 2005, p.108) and the formula for volume of revolution.

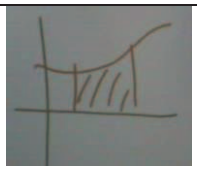


Figure 2: An "easy" question.

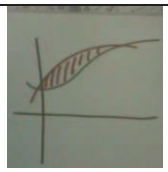


Figure 3: A "more difficult" question.

In an interview conducted four months later, George was invited to reflect on his way of teaching the volume of revolution. He said that he found the textbook diagram better than anything he could do on Autograph. He added that he liked using both the software and the textbook. He said that the software enabled him to show different shapes and added "fun" to the lessons. When using Autograph, he mentioned that he used familiar functions to reinforce students' previous knowledge. Specifically, he used $y = \sin(ax+b)+c$ to reinforce students' previous knowledge about transformations. He added that the use of past exam questions came in response to students' requests and needs to practice for the exam.

DATA ANALYSIS

The resources George used in these three lessons were: interactive white board; board, curriculum of year 13; textbooks; past exam questions and mark schemes; past teaching-experience; students' previous knowledge; calculators; notebooks; Autograph and pre-prepared graphs; formulae sheet; personal website; school website; and, formulae cards. Although the formula cards were on display next to the board all the time and shown to the students only in the second and third lessons while the formulae sheet was used in the first and second lessons, we would say that the resources stayed almost the same across the three lessons.

The schemes of use George followed during all three lessons had the same aims: a specific aim "teach students about the volume of revolution", and a more general one "to prepare students for the exams". In Table 1, we have summarised two elements of his scheme of use during the first lesson: rules of actions (numbered R1, R2… in the first column following the order of events during the first lesson), and operational invariant (numbered O1, O2… in the second column not in chronological order). In the second and third lessons, during which George introduced the volume of revolution to G2 and continued working on the topic with G1, the operational invariants stayed the same. Although, most of the rules of actions remained the same, we observed some differences in their appearance in George's teaching and in their order. In the rest of our analysis we focus on these differences in the rules of actions by making references to the R1-R20 in Table 1. We also discuss the inferences in George's scheme of use as those were identified in the three observations and the follow-up interview.

In the second lesson, George started by R15 (Table 1): "Use the formula sheet to show the formula", and then showed the textbook diagram (Figure 1) on the interactive whiteboard. After that, he followed R2-R8 by showing graphs of function and rotations of areas on Autograph, in a way similar to the way he followed in the first lesson. Then, he proceeded with R12: "Use the textbook diagram to explain the formula" without attempting to do R9 "Introduce the formula for volume of revolution using Autograph". Next, he followed R13-R19. In the third lesson, with G1, George continued working on the volume of revolution topic by quickly going though R2-R7, then moved to R12 followed by R19. In the last two lessons, R9 "Introduce the formula for volume of revolution using Autograph" was not a rule of action. In the interview, George commented on this by saying that the textbook diagram "show[ed] it cut up a little bit easier […] and [was] better than anything [he] could do on Autograph" (O5). As a result, George's first inference from the three lessons is that he found *the textbook diagram more helpful in explaining the formulae*. He added that he found it useful to have "both Autograph and the textbook". This leads us to his other inference: *it is useful to use both Autograph and the textbook as resources.* Another inference is related to *the functions entered on Autograph and how these were chosen to expand and build on students' previous knowledge* (O3).

Rules of action	Operational invariants
R1. Remind students of the formula of circle's area R2. Use Autograph to show the students the graph of $y=x(x-3)$ drawn in 3D R3. Connect new ideas with students' previous knowledge R4. Use trapezium rule and Simpson's rule on Autograph R5. Explain that Simpson's rule is more accurate than trapezium rule in this case R6. Rotate the shaded area around the x-axis R7. Show the students different positions and rotations of the shaded area R8. Show another example prepared previously for the graph of $y=\sqrt{x}$ R9. Introduce the formula for volume of revolution using Autograph R10. Give an idea about the history of integration notation R11. Explain why integration is used to find the volume of revolution R12. Use the textbook diagram to explain the formula R13. Solve an example from the textbook R14. Start with an "easy" question R15. Use the formula sheet to show the formula R16. Explain the formula for rotations around the y-axis R17. Give tips to the students R18. Answer students' questions R19. Use past exam questions to give students idea about how they are tested on the formula of revolution R20. Give hard copies of the formula sheet	O1. Autograph helps students visualise the volume of revolution in 3D O2. With Autograph we "can make weird shapes and have fun" O3. Using a familiar graph helps "reinforce previous knowledge" O4. Using pre-prepared graphs helps in focusing on the new topic and saves time O5. The textbook diagram works better than Autograph in terms of explaining the formula O6. The use of exam-style questions is in response to students' needs, and to give them some practice for the exam

Table 1: Rules of actions and operational invariants for the first lesson's scheme

George commented during the interview on his choice of functions to graph on Autograph, and specifically his use of the sine function during the second lesson:

> Partly from using that in previous lessons. So, knowing that that is going to give an interesting shape, and from playing around with sine graphs and things like that in previous lessons. So, using functions that they were aware of […] So, a transformation of the sine curve I think, we were doing that with it. What I'm also doing there I am also reinforcing or going back over making sure that they know about their transformations. So, I'm kind of

> teaching two topics at once. So, although we are doing this volume of revolution, I am also reminding them of what they do when they do their transformations because I know they are going to get asked about that one

George also thought that using the sine function is "more interesting than using polynomials". However, he only showed the sine function in the second lesson and not in the first or third. It was not clear from the data collected whether he showed the sine function to the G1 in a different lesson, or whether he chose not to show it to them for any reason. However, in the interview he admitted explicitly that what he did with the sine function on Autograph was a good choice for the volume of revolution topic. In terms of the use of past-exam questions, George used these after solving one example from the textbook in the first lesson (R19). In the second lesson, he mentioned he was going to solve past-exam questions, but the lesson finished before he did. In the third lesson, he solved a past-exam question and explained its mark-scheme on the board. When asked about these choices, George mentioned that it was in response to students' needs that he now used past-exam questions frequently (O6). He added that students felt that not all textbook questions were exam-style questions, and some were even "more difficult" than exam questions (which is something he pointed to in the second lesson). It was also because he wanted to give his students some practice for the exam. As a result, he chose to use past-exam questions for every topic he taught. Finally, we noted that George did not have the time to solve past exam questions in the second lesson, maybe because he chose to solve two textbook examples although this was not evident in the data, which do not indicate the warrant of this choice.

DISCUSSION

The resources George used stayed the same throughout the three lessons. However, we noticed differences in the way these resources were used. George's experience with Autograph in the first lesson, led him to amend the way he used it in the next lessons by deciding to use it for visualization of the concept of volume of revolution, but not to explain the formula. Hence, based on the experience of the first lesson which became a resource for George in the following lessons, we noticed his inference in relation to the textbook diagram (Figure 1) being preferred for the purpose of explaining the formula for the volume of revolution. Also, we observed some variation in the order of the rules of actions between the three lessons, reflecting the interplay between Autograph and the textbook. In terms of the use of past-exam questions, George considered these an important resource for every topic. During the interview, it was not possible to focus on every change or difference from one lesson to another (e.g. not using the sine function in the first and third lessons) because the interview was done a few months after the observations and this is one of the limitations of this study. In general, from the data collected and by using the documentational approach we explored how George's practices evolved, how he reflected on that, how he re-sourced his experiences, and what inferences he adopted during and after teaching these three lessons. Findings from our analysis demonstrate the potencies of the documentational approach in our insight into teachers' 'live' work by capturing also the dynamic nature of this

work. Observations of lessons supported by further evidence from interviews and reflections from the teachers can explore the micro-evolution, namely the small changes and the rationale behind these changes, of teachers' documents from one lesson to another. We consider this micro-evolution in this instance as re-scheming from one lesson to another, implying that the teacher was scheming "again or differently" (Adler, 2000, p. 207) or recycling his scheme from one lesson to another.

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