Computing the object colour solid using spherical sampling.

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Introduction

In [1, 2], the authors proposed that for the set of N colour systems the spectra located on the boundary of the object colour solid (optimal spectra) are the elementary step functions with the transition wavelengths at $\lambda_1, \ldots, \lambda_m$ if and only if the above set of transition wavelengths are the only zero-crossings of the following equation:

$$k_1 s_1(\lambda) + k_2 s_2(\lambda) + \cdots + k_n s_n(\lambda) = 0$$

where $k_1, k_2, \ldots, k_n$ are the set of arbitrary real numbers, where at least one of them is not equal to zero and $s_i(\lambda)$ are the colour system spectra which are the products of sensor spectral sensitivities $c_i(\lambda)$ and an illuminant spectrum $e(\lambda)$ i.e. $\mathbf{s}(\lambda) = \mathbf{c}(\lambda)e(\lambda)$.

Here, we observe that the components of vector **k** have the geometrical meaning i.e. they constitute the normal vector parametrising the surface of the object colour solid. Because the OCS is convex, in the direction **k**, we can, in closed form, find the unique system response which is maximum. And, from convexity it follows we can find all points on the OCS by extremizing all directions. Formally, we propose the parametric representation with respect to **k** of the surface of the object colour solid.

For a reflectance function $r(\lambda)$ the colour system responses are:

$$\phi_i(r) = \int_{\lambda_{min}}^{\lambda_{max}} r(\lambda) s_i(\lambda) \mathrm{d}\lambda, i = 1,2, \ldots, N$$

where λ_{min} and λ_{max}denote the limits of the visible spectrum. A reflectance spectrum $\mathrm{r}(\lambda)$ is a function with values between zero and one. The set of all possible colour system responses form a convex set M called the object color solid in R^N.

We project all colour system responses $\boldsymbol{\phi}(r)$=$(\phi_1(r), \phi_2(r), \ldots, \phi_N(r))$ onto a unit vector **k**. That is:

$$\mathbf{k} \cdot \boldsymbol{\phi}(r) = \int_{\lambda_{min}}^{\lambda_{max}} \mathrm{r}(\lambda) \mathbf{k} \cdot \mathbf{s}(\lambda) \mathrm{d}\lambda$$

It is clear that the maximum value of $\mathbf{k} \cdot \boldsymbol{\phi}(\rho)$ is obtained by:

$$r_{opt} = r(\lambda, \mathbf{k}) = \begin{cases} 0, & \mathbf{k} \cdot \mathbf{s}(\lambda) < 0 \\ 1, & \mathbf{k} \cdot \mathbf{s}(\lambda) \geq 0 \end{cases}$$

Computing the object colour solid.

In this Section, we will assume that the number of colour systems is N = 3, which corresponds to the dimensionality of the human visual system. This said, the algorithm is valid for any number of colour systems.

Let us generate a set of M normal vectors in R^3 using a spherical sampling method [3] and store them in the rows of M by N matrix $\mathbf{P}$. We store colour system spectra in N by q matrix $\mathbf{S}$. The

wavelength resolution is determined by q e.g. for 1nm resolution, $\lambda_{min} = 380$ and $\lambda_{max} = 730$, the colour system and reflectance spectra will have 351 components i.e. $q = 351$. A matrix resulting from multiplication of $\mathbf{P}$ by $\mathbf{S}$ is denoted as $\mathbf{A} = \mathbf{PS}$ and the signs of its elements determine the set of optimal spectra in matrix **R** as:

$$\mathbf{R}_{ij} = \begin{cases} 0, & \mathbf{A}_{ij} < 0 \\ 1, & \mathbf{A}_{ij} \geq 0 \end{cases}$$

The procedure described above requires no searching and is very rapid. In Figure 1, we can see 10,000 points on the surface of the object colour solid generated using our method and the convex hull created from these points. The wavelength resolution was 0.1nm and the illuminant spectrum we used was D65.

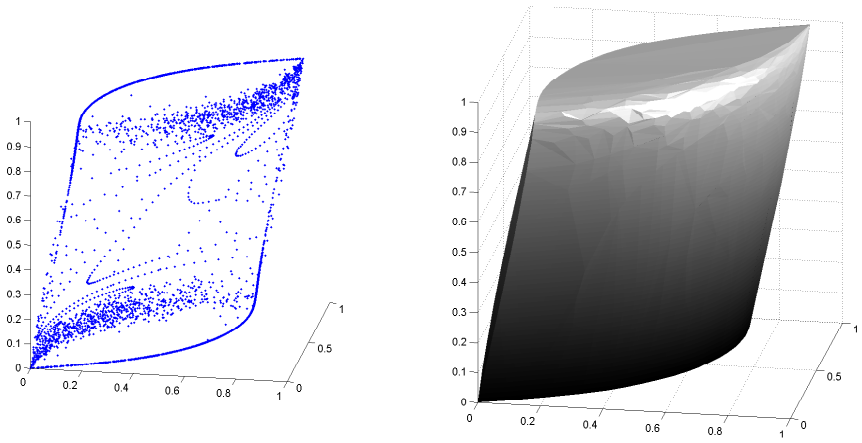

Figure 1. Ten thousand points obtained with our algorithm (left) and their convex hull (right).

Conclusions and Future work.

We admit that the fast generation of the object colour solid is generally not a significant problem. For example, for the set of three sensors, one could generate a number of sensor responses from the set of randomly generated elementary spectra with two transitions [4, 5]. However, our method allows for describing the solid with a small number of samples and it also allows for any higher number of transitions. Having said that, the main reason for this ongoing work is our intention to use this parametric representation together with the spherical sampling for efficient generation of metamer mismatch volumes, which will be the topic of our next publication.

References

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