Utilising 3D Printing Technology Tools and Learning-as-Making Activities with In-service Teachers to Explore and Impact Teaching Identities Towards Mathematics and Mathematising

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Abstract

This thesis explores the teaching identities towards mathematics and mathematising of in-service teachers through their engagement with 3D printing technology tools (3D printing pens, Tinkercad and a Makerbot 3D printer) in learning-as-making activities. The six in-service teachers' subject specialties span Key Stages 1-3 and the early years framework of the English national curriculum and include design and technology, cooking and nutrition, computing, and early years. To understand in-service teachers' teaching identities towards mathematics, I examine their creative acts when interacting with 3D printing technology tools and their conversations about mathematics during learning-as-making activities. Mathematising starts when the in-service teachers make shapes with the movements of their hands while holding a 3D printing pen to construct a 3D skeleton cube. In this research, mathematising is a physical manifestation and a conscious act. The learning-as-making activities engaged participants in using 3D printing pens to construct 2D (i.e., circle, square, rectangle) and 3D (i.e., triangular prism, cube) skeleton shapes on a silicone mat or flat surface, manipulating 3D models with an easy-to-use modelling software called Tinkercad, and discussing how a 3D printer could be employed in their classrooms and with the mathematical content of the English national curriculum.

Communities of practice (CoP) is the overarching theoretical framework applied in this study. In addition, I draw on shape perception theory (Pizlo, 2008) to give insight into 2D and 3D representation and inclusive materialism (De Freitas and Sinclair, 2014) to understand the role of materials in mathematical activities. I also apply diffraction (Barad, 2007) and narrative inquiry (Clandinin and Connelly, 2006; Clandinin, 2022) as methodological approaches to understanding how bodies, materials, and conversation produce meaningful interactions in mathematics. Data for this study include raw transcripts of participants' utterances during the learning-as-making activities, video and audio recordings of participants' engagement in the activity, and field notes. To analyse the data, I draw upon diffractive analysis (Mengis and Nicolini, 2021; Mazze, 2014), such as using clips of the video recordings to observe participant engagement with the 3D printing technology tools, and thematic analysis (Riessman, 2008) to discern the participants' mathematical meaning through their conversations during the activities.

Findings suggest that in-service teachers' engagement with 3D printing technology tools enables them to mathematise the mathematics used in their respective disciplines to construct meaningful mathematical models. Their creative acts in learning-as-making activities provide insight into their teaching identities (dialogues that consist of talking about mathematics), maker identities (construction of 2D and 3D shapes), and performative identities (hand movements and gestures using the 3D printing technology tools). The combination of employing the theoretical framework of CoP, shape perception theory, and inclusive materialism enables this thesis to contribute to the scholarship addressing the use of materials and 3D printing technology in the practice of early years and primary education. This contribution could allow us to envision how the different technology tools can be employed in learning-as-making activities in the early years and primary sectors to make mathematics more transparent to in-service teachers and students.

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Chapter One: Introduction

1.1 Purpose of this study and research questions

Research on employing 3D printing technology in schools has increased over the past decade. Recent studies discuss the implementation of 3D printing technology such as 3D printing pens (Ng and Chan, 2021; Ng and Ferrara, 2020; Ng and Sinclair, 2018), a 3D modelling software called Tinkercad (O'Reilly and Barry, 2023; Bhaduri et al., 2021; Piesticker et al., 2021), and 3D printers (Anđić et al., 2024; Anđić et al., 2023; Anđić et al., 2022; Asempapa and Love, 2021) in classroom activities. The literature indicates how 3D printing technology tools in mathematical activities can impact the teaching and learning of mathematics at the primary and early years levels. This study was driven by this recent scholarship and by research on in-service teachers' competence in using 3D technology in the classroom.

The participants in this study are in-service teachers whose specialisations are not related to mathematics. They encounter mathematical applications in their everyday curricula, though as non-specialists in mathematics. Several research studies conducted in England have focused on identities of in-service teachers who teach mathematics applications in their classrooms without specialising in mathematics (Crisan and Hobbs, 2019; Crisan and Rodd, 2017; Crisan and Rodd, 2014; Crisan and Rodd, 2011). The participants' backgrounds in this study include primary education specialities such as design and technology, nutrition and computing, along with early years education. To date, only a limited number of studies have investigated in-service teachers' identities as they engage with learning-as-making activities.

When the in-service teachers in this study discuss using 3D printing pens to construct 2D and 3D shapes, they do so in reference to their own classrooms. Their teaching identities start taking shape as the mathematical conversation is sparked through their mathematising. When in-service teachers engage in learning-as-making activities, they are mathematising. The term 'mathematising' refers to 'a material act of creation by which the virtual is actualised' (Ng and Ferrara, 2020, p. 942). Mathematising begins the moment in-service teachers pick up their 3D printing pens, constructing a 3D skeleton cube through the movements of their hands while holding the pen. When in-service teachers use 3D printing pens as an apparatus that extrudes filaments (materials) to construct 2D and 3D skeleton shapes, they are mathematising. As in-service teachers interact with the 3D modelling software Tinkercad (dragging 3D models on the Tinkercad plane and manipulating 3D shapes), they are mathematising. In-service teachers discussing ways that 3D printers such as the Makerbot can be employed in their classroom activities to show 3D geometrical shapes are also mathematising. When in-service teachers mathematise, they simultaneously unfold their identities as non-specialists in mathematics.

In-service teachers mathematising using 3D printing technology is captured through the methodological approaches of diffraction (Barad, 2007) and narrative inquiry (Clandinin, 2022; Connelly and Clandinin, 2006). Using Barad's (2007) explanation, diffraction is understood as an ocean wave passing through a barrier, where the wave is considered the 'diffracted' and the barrier is the 'diffracted apparatus'. Concerning the learning-as-making activities in this study, the 3D printing pen is the diffracted apparatus, 'which produces effects that help us see how mathematical meanings are entangled with the physical/pen' (Ng and Ferra, 2020, p. 933). When I use the term 'diffractive reading', I am referring to 'the participants and their intra-active responding, their choice of activity and location, their bodily discomfort, their growing confidence' (Mengis and Nicolini, 2021). I use diffractive reading as an analytical tool to observe hand movements, gestures, and posture through video recordings of the in-service teachers using 3D printing pens. Diffractive analysis is 'respectful of the entanglement of ideas and other materials in ways that reflexive methodologies are not' (Barad, 2007, pp. 29-30). I employ Clandinin's (2022) 'story to live by' to analyse in-service teachers' dialogues during the learning-as-making activities, which could reveal their sense of becoming in their teaching identities and mathematising.

To understand in-service teachers' formation of identities, I employ three theoretical lenses. These include Wenger's (1998) CoP, Pizlo's (2008) shape perception, and Sinclair and De Freitas's (2014) inclusive materialism. The six in-service teachers are engaging in a practice that involves interacting with 3D printing pens. The community in this research includes six in-service teachers from the primary level (design and technology, nutrition, computing), and early years. Wenger (1998) uses the three modes of belonging – engagement, imagination, and alignment – to discuss the formation of identities. To understand the role of imagination in in-service teachers' identities, I transition to Pizlo's (2008) shape perception to give further insight into in-service teachers' construction of 2D and 3D shapes. Moving from in-service teachers' imagination to engaging with materials in learning-as-making activities, De Freitas and Sinclair's (2014) inclusive material theories provide insight into how in-service teachers become engaged with materials in mathematical activities.

Data for this study include audio-video recordings of each participant's engagement in the learning-as-making activities, raw transcripts from these audio-video recordings, and field notes. Only three in-service teachers participated in all of the learning-as-making activities, while the other three in-service teachers participated in specific activities due to time limitations. In addition, three in-service teachers filled out the optional survey after the learning-as-making activities. To analyse the data, I draw upon diffractive analysis (Mengis and Nicolini, 2021; Mazze, 2014), using clips of the video recordings to observe participant engagement with the 3D printing technology tools. I also employ thematic analysis (Riessman, 2008) to discern the participants' mathematical meanings through their utterances during the activity. The research questions this study aims to answer include:

> R.Q.1: How do in-service teachers mathematise using 3D printing technology tools to connect with the mathematical potential of their activities?
>
> R.Q.2: How do in-service teachers' mathematising in learning-as-making activities impact their teaching identities?
>
> R.Q.3: What does a diffractive reading of in-service teachers' work with 3D printing pens tell us about their mathematising?

In the next section, I discuss the overarching structure of this thesis.

1.2 Structure of the thesis

The theoretical framework of this study is discussed in chapter two. I start with discussing the theories of Wenger's (1998) communities of practice, beginning with how communities of practice came about through Lave and Wenger's (1991) concept of legitimate peripheral participation, continuing to Wenger's (1998) three modes of belonging. Afterward, I discuss Pizlo's (2008) shape perception theory and its relation to understanding representation and visualisation of 2D and 3D shapes. I conclude the chapter by giving insight into Sinclair and De Freitas's (2014) inclusive materialism, focusing on the role of materials in mathematical activities.

Chapter three consists of my literature review. I discuss in more depth the literature from which the theoretical framework of chapter two evolved. I introduce the concept of legitimate peripheral participation in makerspace research. Then, I review the recent mathematics education literature that employs communities of practice theory. Following, I review the literature review in teachers' education that uses community of practice theory. Afterwards, I discuss making, maker, and tinkering literature as it pertains to understanding the concept of learning-as-making activities. Also, literature on visualisation, representation and spatial thinking of geometrical shapes. I end the chapter with a discussion of the literature on 3D printing technology in mathematics education.

In chapter four, I introduce the methodologies used to conduct my research. Two methodological approaches in particular structured my research: narrative inquiry (Connelly and Clandinin, 2006) and diffraction (Barad, 2007). I discuss narrative inquiry through research on teacher practices and the concept of story-to-live by, focusing on three dimenisons: temporality, sociality, and place. Next, I examine mathematical identity and identity work. Following this I introduce my own story-to-live by, recounting my experience of collecting data in unprecedent times and offering an example of how I employed a story-to-live by data collection in this study. I then transition to discussing Barad's (2007) diffraction methodology along with literature that employs performativity and diffraction in mathematics education. I give a concrete example of how diffractive reading is employed in this thesis. Afterward, I discuss the six participants in this study (Beyonce, Tracy, Joseph, Taylor, Emily, and Elton) and my methods of data collection. Following, I discuss diffractive analysis (Mengis and Nicolini, 2021; Mazze, 2014) and thematic analysis (Riessman, 2008). Afterwards, I discuss the trustworthiness of the study. I end the chapter with telling my story of collecting data and conducting fieldwork in unprecedented times, the role of the researcher, the limitations of my methodology, and ethical considerations.

In chapter five, entitled mathematising, I present the data collection in episodes that consist of dialogues (i.e., story-to-live by) and diffraction readings of participants in each school (Gryffindor, Ravenclaw, and Hufflepuff). The chapter consists primarily of dialogues and video screen shots of in-service teachers mathematising using 3D printing technology tools such as 3D printing pens and Tinkercad. I end each episode by re-telling the story of the participants in each school engaging in the learning-as-making activities. A discussion section concludes the chapter, drawing upon themes abstracted from the episodes. It describes the six-in-service teachers mathematising, and unfolding their teaching identities towards mathematics.

Chapter six presents my conclusions. I review the research questions and answer them through the findings in chapter five, along with connections these findings have with the current literature. In addition, I discuss my contribution to the three theoretical frameworks (communities of practice, shape perception, and inclusive materialism) and to the practice of early years and primary education. I conclude the chapter by proposing ideas for future research and reflecting on my four-year journey as a post-graduate researcher in mathematics education.

Chapter Two: Theoretical Framework

2.1 Introduction

In this section, I discuss the three theoretical frameworks that I employed in this study. The first and overarching theoretical lens is called Communities of Practice (CoP). I build on Wenger's (1998) comprehensive exploration of CoP theory in section 2.2 by discussing Lave and Wenger's (1991) *Situated Learning: Legitimate Peripheral Participation*, and then introducing what Wenger (1998) sees as the defining three characteristics of a community of practice. This discussion leads to an examination of Wenger's (1998) ideas about how identity is formed in communities of practice. Wenger (1998) examines the modes of belonging in forming an identity, and presents the mode of imagination as essential in shaping how one 'images of the world' (Wenger, 1998, p. 173). This imagination is not limited to visualising the world, but it also pertains to how a person imagines figures and shapes. Exploring this idea leads directly to my second theoretical framework, Pizlo's (2008) shape perception theory, which looks at the role of imagination in manipulating, designing, visualising and representing 2D and 3D shapes (section 2.3). Pizlo (2008) contends that imagination is not only limited to one's mind, but it also includes the way a person can engage and align their imagination to the real-world using materials. Pizlo's (2008) work informs De Freitas and Sinclair's (2014) theory of inclusive materialism, my third theoretical frame for this study, which offers insight on utilising gesture, dialogue, and devices to explore how students engage with materials in mathematical activities (section 2.4). Lastly, I discuss the potency of connecting these three theoretical lenses through Wenger's (1998) modes of belonging in relation to my study (section 2.5).

2.2 Communities of Practice

Communities of Practice (CoP) theory is most often associated with the work of educational theorist Etienne Wenger, who published *Communities of Practice: Learning, Meaning, and Identity* in 1998. Wenger defines a 'community of practice' as a group of people who learn by interacting. That is to say, in a community of practice, group members share a common interest, concern, or passion about which they develop a better understanding through collaborative learning and interaction. The concept of CoP originated with the work of both Wenger and cognitive anthropologist Jean Lave. Together they wrote *Situated Learning: Legitimate Peripheral Participation* (1991) in which they discuss the concepts of learning, participation, and identity. The fundamental notion they put forth is that learning is a social process, as opposed to something that occurs solely on one's own terms and in one's head. Learning is 'situated' in interactive, overlapping circles of people who together advance knowledge.

2.2.1 **Situated Learning: Legitimate Peripheral Participation**

Lave and Wenger's (1991) situated learning theory introduces the term legitimate peripheral participation, which informs the later development of Wenger's (1998) communities of practice theory. To understand this term, in this section I elaborate on several terms that Lave and Wenger (1991) frequently use, such as 'newcomer', 'old-timer', 'identity', 'communities of practice' as well as 'legitimate peripheral participation'. Lave and Wenger (1991) distinguish between ‘newcomers’, who are people new to the CoP and need to develop competencies, and ‘old-timers’, who participate fully in the CoP. They use the term ‘legitimate peripheral participation’ to describe how the newcomers participate in the community. Legitimate peripheral participation allows newcomers to gain the experience and competence needed to become full participants in the CoP (Lave and Wenger, 1991). Lave and Wenger (1991) state that legitimate peripheral participation ‘provides a way to speak about the relations between newcomers and old-timers, and about activities, identities, artifacts, and communities of knowledge and practice’ (p. 29).

To learn something legitimately in a community, Lave and Wenger (1991) explain, means to learn according to the rules of engagement that are acceptable within that community. In addition, ‘a person's intentions to learn [must be] engaged.’ If they are, then ‘the meaning of learning is configured through the process of becoming a full participant in a sociocultural practice’ (p. 29). Lave and Wenger (1991) define learning as ‘an integral and inseparable aspect of social practice’ (p. 31). Their definition describes the social context in which learning takes place and the role of participation in a social context. Lave and Wenger (1991) emphasise that learning happens through engaging with activity in a community. Whereas knowledge, they explain, is specific to a practice. Lave and Wenger (1991) state that ‘the generality of any form of knowledge always lies in the power to renegotiate the meaning of the past and future in constructing the meaning of the present circumstance’ (Lave and Wenger, 1991, p. 34). They also define knowledge as ‘a set of perspectives and practices shared by a community’ (Lave and Wenger, 1991, p. 34).

Lave and Wenger (1991) discuss how a person’s identity can be constructed through learning. As learning is part of the social practice, it also involves a person’s engagement in activities and in forming new understanding. Lave and Wenger (1991) discuss that ‘learning thus implies becoming a different person with respect to the possibilities enabled by these systems of relations’ (p. 53). Lave and Wenger (1991) also elaborate on the term ‘sociocultural’, explaining that learning as social practice 'implies not only a relation to specific activities but a relation to social communities … becoming a full participant, a member, a kind of person' (p. 53). As Lave and Wenger (1991) describe ways to become a full participant in social practice, they refer to ‘engaging with technologies of everyday practices, as well as participating in the social relations, production processes, and other activities of communities of practice' (Lave and Wenger, 1991, p. 101). When learning involves activity and social components, identities begin to develop.

The term 'identity' is defined by Lave and Wenger (1991) as the way in which a 'person understands and views himself, and is viewed by others, a perception of self which is fairly constant' (p. 81). This term is described in greater detail in Wenger's (1998) later work in which he introduces a comprehensive CoP theory. Another term that Lave and Wenger (1991) introduce is ‘apprenticeship’. According to Lave and Wenger (1991), legitimate peripheral participation gives rise to an apprenticeship as newcomers and old-timers engage with each other in a practice. Furthermore, ‘apprenticeship opportunities for learning are, more often than not, given structure by work practices instead of by strongly asymmetrical master-apprentice relations’ (Lave and Wenger, 1991, p. 93). In other words, apprenticeship is embedded in a community of practice through work practices which involve a group of people working together. Lave and Wenger (1991) lay out the foundation of communities of practice in stating:

> From a broadly peripheral perspective, apprentices gradually assemble a general idea of what constitutes the practice of the community. This uneven sketch of the enterprise (available if there is legitimate access) might include who is involved; what they do; what everyday life is like; how masters talk, walk, work, and generally conduct their lives; how people who are not part of the community of practice interact with it; what other learners are doing; and what learners need to learn to become full practitioners. It includes an increased understanding of how, when, and about what old-timers collaborate, collude, and collide, and what they enjoy, dislike, respect, and admire. (Lave and Wenger, 1991, p. 95)

A 'community of practice', then, is a 'set of relations among persons, activity, and world, over time, and in relation with other tangential and overlapping communities of practice' (1991, p. 98). In the next section, I will discuss the three primary characteristics of a community of practice.

2.2.2 **Characteristic components of Communities of Practice**

Wenger (1998) introduces the concept of communities of practice by discussing how people come together as a group and how that group shares a common goal. As 'we interact with each other and with the world and we tune our relations with each other and with the world, accordingly, in other words, we learn' (p. 73). In a group with shared goals, people are mutually engaging with one another for a common purpose. 'This collective learning results in practices that reflect both the pursuit of our enterprises and the attendant social relations' (Wenger, 1998, p. 73). Eventually, collective learning leads to a joint enterprise, which gives meaning to a group's practice as they mutually engage with one another. The three components of communities of practice explained in this section are mutual engagement, joint enterprise, and shared repertoire.

Within a community of practice, an important element to observe is how a person is negotiating meaning in a practice. The development of a meaningful experience within a community of practice, Wenger explains, starts with a collaborative practice. He explains that negotiation of meaning refers to how 'we experience the world and our engagement in it as meaningful' (Wenger, 1998, p. 53). The negotiation of meaning starts with a person's participation in a practice. When a person is mutually engaging in a practice, as in collaborating with other members in a practice, their actions unfold how they are developing a meaningful experience. Wenger suggests that the negotiation of meaning can also lie in the process of reification. The concept of reification, Wenger notes, 'refers to the process of giving form to our experience by producing objects that congeal this experience into thingness' (Wenger, 1998, p. 58). Wenger claims that reification is central to a practice as 'any community of practice produces abstraction, tools, symbols, stories, terms, and concepts that reify something of that practice in a congealed form' (Wenger, 1998, p. 59). Moreover, in a community of practice, Wenger explains, one learns to negotiate their own meaning, learning, and identity in relation to that practice.

As a person displays their negotiation of meaning in a practice, we can identify where meaning is being developed in the three components of communities of practice. The mutual engagement component is where a person demonstrates their action and interaction in a practice, as in 'how people treat each other, and how to work together' (Wenger, 1998, p. 152). Membership becomes visible as a person mutually engages with others in a practice. Wenger explains that 'practice resides in a community of people and the relations of mutual engagement by which they can do whatever they do' (Wenger, 1998, p. 152). Membership in a practice does not require someone to be a specialist in the subject, but only a member who is working together with others toward a common goal. Mutual engagement involves not only the competency of an individual subject but also the competency of a collective group.

As individuals mutually engage with one another, they are establishing a joint enterprise by 'creating relations of mutual accountability that become an integral part of the practice' (Wenger, 1998, p. 79). When a person is establishing a joint enterprise in a practice, they are mutually engaging with their group members in what they find important in the practice. If the task in the practice involves a group engaging with artifacts, this is where Wenger suggests that in the joint enterprise phase, the group will determine if 'artifacts are good enough and when they need improvement or refinement' (Wenger, 1998, p. 81). When Wenger refers to artifacts, he means tools that can be used as vehicles to solve problems in the community of practice. An example of an artifact in mathematics education is a traditional graphical calculator which 'help[s] students develop a better intuitive understanding of calculus' (Kidron, 2020). When studying or working with 3D printing in mathematics education, an artifact may include 3D printing pens (Dilling and Witzke, 2020; Ng and Ferrara, 2020; Ng and Sinclair, 2018) that enable students to solve problems related to visualising, touching, and holding 3D printed models of graphical function representation. Once members of a community utilise tools in their community of practice, those tools are now part of what Wenger calls a shared repertoire. Wenger (1998) builds from his description of the three characteristic components of communities of practice - mutual engagement, joint enterprise, and shared repertoire – to discuss the formation of identities.

2.2.3 **Identities in Communities of Practice: Wenger's three modes of belonging**

To help explain identities in Communities of Practice (CoP), Wenger (1998) states that 'building an identity consists of negotiating the meaning of our experience of membership in social communities. The concept of identity serves as a pivot between the social and the individual, so that each can be talked about in terms of the other' (Wenger, 1998, p.145). Through Wenger's lens, identity in practice has the following six characteristics: lived, negotiated experience, community membership, learning trajectory, nexus of multi-membership, and relation between the local and the global. The term negotiated experience refers to the participation and reification in the practice. Wenger (1998) focuses on the 'lived experience of engagement in practice' (Wenger, 1998, p. 151), meaning that he looks at not only how individuals describe themselves, but their role in the practice. Wenger (1998) suggests that 'identity in practice is defined socially, not merely…because it is produced as a lived experience of participation in specific communities. What narratives, categories, roles, and positions come to mean as an experience of participation is something that must be worked out in a practice' (Wenger, 1998, p. 151). Wenger's (1998) three properties of communities of practice (mutual engagement, joint enterprise, and shared repertoire) 'become dimensions of identity' (p. 152). Community members in a community of practice form an identity 'relating to the world … We experience and manifest ourselves by what we recognize and what we don't … In practice, we know who we are by what is familiar, understandable, usable, negotiable; we know who we are not by what is foreign, opaque, unwieldy, unproductive' (Wenger, 1998, p. 153). Wenger's (1998) learning trajectory characteristics 'give meaning to [a person's] engagement in practice in terms of the identity they are developing' (Wenger, 1998, p. 155). The nexus of multi-membership involves identity as multi-membership and reconciliation. By including reconciliation, Wenger (1998) suggests 'that the maintenance of an identity across boundaries requires work and, moreover, that the work of integrating our various forms of participation is not just a secondary process' (Wenger, 1998, p. 160). In addition, 'identity is neither narrowly local to activities nor abstractly global' (Wenger, 1998, p. 163).

The six characteristics of identity that Wenger (1998) proposed were a part of describing identity formation as modes of belonging. The modes of belonging have two categories, called identification and negotiability, that consist of identities of participation and non-participation. Wenger's (1998) three distinct modes of belonging include:

Engagement: 'active involvement in mutual processes of negotiation of meaning' (Wenger, 1998, p. 173).

Imagination: 'creating images of the world and seeing connections through time and space by extrapolating from our own experience' (Wenger, 1998, p. 173).

Alignment: 'coordinating our energy and activities in order to fit within broader structures and contribute to broader enterprises' (Wenger, 1998, p. 174).

These three distinct modes of belonging can inform the perception of shapes that Pizlo's (2008) work describes. Engaging with our imagination to picture and define a shape involves various negotiations of what we know about a shape. The process of 'creating imagines of the world', 'coordinating our energy' and 'negotiat[ing] …meaning' can be seen in Pilzo's (2008) shape perception theory through redefining, understanding and conceptualising shapes, which I discuss in the next section.

2.3 Distinct modes of belonging in the lens of Shape Perception Theory

The three distinct modes of belonging from Wenger's (1998) CoP theory can connect to Pizlo's shape perception theory as a way to provide further insight into the role that imagination plays in manipulating, designing, visualising and representing 2D and 3D shapes. I use Pizlo's (2008) work on shape perception to complement Wenger's (1998) discussion of the mode of imagination. Wenger (1998) does not talk about shape constancy or figure-ground organization, while Pizlo (2008) does. Pizlo's (2008) terms portray how one might conceptualise 2D and 3D orientation.

In explicating shape perception theory, Pizlo defines perception as 'becoming aware of the external world through the action of the senses' (Pizlo, 2008, p. 1). Senses aid our vision in determining the type of shapes that our eyes might perceive. Pizlo (2008) uses the term shape to mean

> geometrical characteristics of a specific three-dimensional (3D) object that make it possible to perceive the object veridically from many different viewing directions, that is, to perceive it as it actually is in the world 'out there'. Understanding how the

human visual system accomplishes this is essential for understanding the mechanisms underlying shape perception. Understanding this is also essential if we want to build machines that can see shapes as humans do. (Pizlo, 2008, p. 1)

In an earlier study, Pizlo and Salach-Golyska (1995) explain that the 'perception of shapes of objects involves making inferences about the three-dimensional (3-D) shape of an object on the basis of its 2-D retinal image (or several images)' (Pizlo and Salach-Golyska, p. 692). When we look at a 3D object from different angles – take, for example, a cube – its size, and shape, stay consistent. However, our retina images change when looking at different angles of the cube. This is an example of shape constancy. Pizlo (2008) suggests that shape constancy 'refers to the fact that the perception of the shape of a given object remains constant despite changes in the shape of the object's retinal image' (Pizlo, 2008, p. 3). To give an example of the perception of shape, Pizlo (2008) discusses the different perceptions of an ellipse (a) and a rectangle (b), as seen in Figure 1.

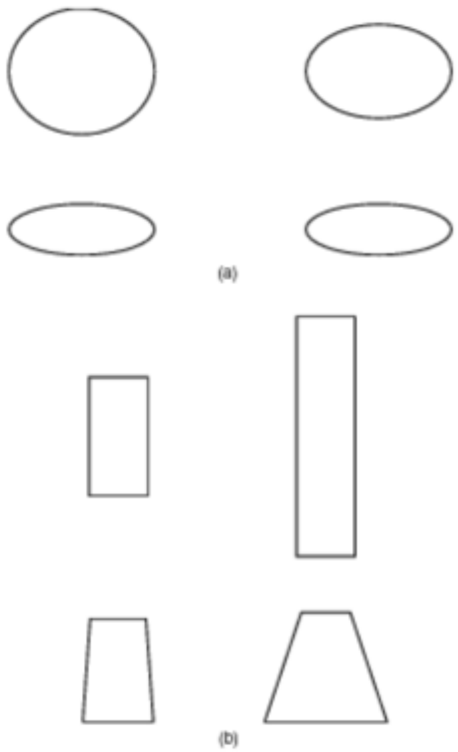


Figure 1: Shape Perception of Ellipse (a) and Rectangle (b) (Pizlo, 2008, p. 7)

In the given example, Pizlo (2008) explains:

(a) Ellipses with different shapes (top) can produce identical retinal images (bottom). The ellipse on the top left was slanted around the horizontal axis more than the ellipse on the top right. As a result, their retinal images (bottom) are identical. (b) Rectangles with different shapes cannot produce identical retinal images. The rectangle on the top right was slanted around the horizontal axis more than the rectangle on the top left. As a result, the heights of their retinal images (bottom) are identical, but their shapes are not. (Pizlo, 2008, p. 7)

Ellipses are not the only shape that have retinal images that are identical; triangular shapes also pose the same retinal image. Both ellipses and triangle shapes have similar viewing orientations due to the limited complexity of their shape depth. To understand these differences in shape theory, Pizlo (2008) compared shape theory through the lens of both Descartes and Helmholtz. Descartes suggests that our perception comes from prior knowledge of geometrical shapes, whereas Helmholtz suggests that the perception of shape comes from sensation. According to Bennett (2024), who investigated shape appearance and shape constancy through a coin which represents an ellipses shape, 'our sense of shifting, changing shape results from detecting/representing the latter, relational-projective shape properties' (Bennett, 2024, p. 488). In addition, Pizlo (2008) stated that 'to solve the shape constancy problem, the observer must recognize that these different retinal images can be produced by the same figure' (Pizlo, 2008, p. 19). Pizlo (2008) later suggested that 'studying shape constancy requires manipulating the viewing direction, which changes the shape of test stimuli on the retina. One cannot claim to be studying shape or shape constancy when the viewing direction and the retinal shape of the stimuli are kept constant' (Pizlo, 2008, p. 21).

Pizlo's (2008) work is in line with Helmholtz's perception of shape, such that 'the memory of a 3D shape (its mental representation) involves a collection of 2D images of the shape (plus Tactile sensations) obtained from different viewing directions' (Pizlo, 2008, p. 16). Pizlo (2008) developed shape perception theory based on earlier work by Gestalt, who introduced the law of organisation. Pizlo (2008) claims that the most essential law of organisation is figure-ground organization. The term figure-ground organization means 'determining which contours and which regions of an image correspond to a single object' (Pizlo, 2008, p. 3). For example, Li et al. (2009) investigated a computational model that recovers the 3D shape of an object from a single 2D retinal representation. In their study, they used a demonstration of a 2D object to recover a 3D shape, as seen below (Figure 2):

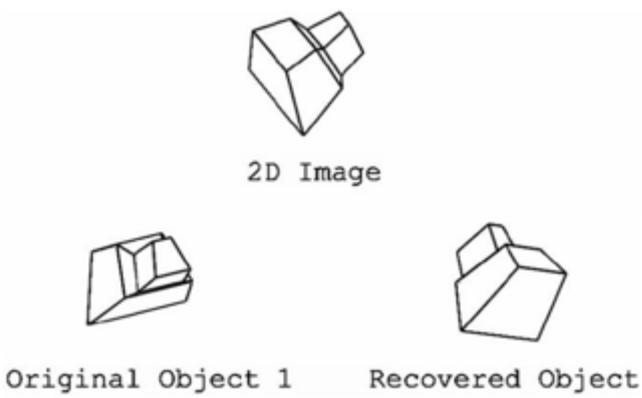


Figure 2: Li et al. (2009, p. 980) demonstration of using a 2D image to recover a 3D object

Li et al. (2009) noted that 'the 3D shape of the Recovered Object is almost identical to the shape of the Original Object despite the fact that it is seen from a different viewing direction. This means that the model can achieve shape constancy' (Li et al., 2009, p. 980). Pizlo (2008) uses Koffa's (1935) perception of shapes to talk about the perception of 3D shapes from 2D shapes. Koffa (1935) explains that 'three-dimensional shapes are matters of organization in the same way as two-dimensional ones, depending on the same kind of law' (Koffa, 1935, p. 161). According to Pizlo (2008), 'shape must be based on the perception of depth because the perception of a 3D surface is derived from depth cues' (Pizlo, 2008, p. 89). The term depth cues 'refers to the visual information that can be used to compute the spatially local properties of surfaces, namely, the distances from the observer, relative distances along the depth direction, and 3D orientation' (Pizlo, 2008, p. 89).

Our visual information that is produced through our action of sense is a form of our initial engagement in a practice. Wenger (1998) suggests that 'the process of identity formation can remain largely transparent because our identities can develop by being engaged in action without being themselves the focus of attention' (Wenger, 1998, p. 193). When one is picturing their memory of 3D shapes with a collection of 2D images, they are using their imagination. The imagination mode of identity formation is a 'kind of picture of the world and of ourselves we can build' (Wenger, 1998, p.194). In this case, we are imagining a picture of a 3D orientation based on our perception and memory. When one is producing mental images, one has ownership of these images leading them to alignment. Wenger (1998) suggests that alignment is the distinct mode that 'requires the coordination of actions and therefore the encounter of various perspectives and meaning' (Wenger, 1998, p. 205).

Connecting Pizlo's (2008) work on the engagement, imagination, and alignment of shapes through our perception and Wenger's (1998) three distinct modes of belonging and applying them in a physical sense, leads to a discussion in the next section of De Freitas and Sinclair's (2014) theory on inclusive materialism. This discussion brings together the role of 3D orientation and sense of belonging with the use of handheld devices (i.e. pencil, touchpad, 3D printing pen).

2.4 Inclusive Materialism

In this section, I introduce De Freitas and Sinclair's (2014) theory on inclusive materialism. De Freitas and Sinclair (2014) are interested in the process by which the body is becoming in relation to mathematical activities and materials. Their work combines literature from diffraction methodology (Barad, 2007), wherein Barad (2007) refers to an ocean wave passing through a barrier, and the scholarship of Châtelet (1993, 2000, 2006), which focuses on gesture and diagrams. To understand making and mathematical thinking, materialism plays an essential role in learning-as-making activities. De Freitas and Sinclair (2014) 'study the body more in terms of its becoming than its being,' and argue that a 'body is a set of material relations that seems to structure the other material relations around it' (De Freitas and Sinclair, 2014, p. 34). Materialism is linked to 'various kinds of empiricist epistemologies that centre the sense and sensation in our coming to know' (De Freitas and Sinclair, 2014, p. 39). I used the theory of inclusive materialism from De Freitas and Sinclair (2014) to understand the role of materials in mathematical activities. The examples that are shown in this section are chosen to give insight on ways learning-as-making activities can be utilised in this study.

De Freitas and Sinclair's (2014) interest in inclusive materialism centres around a term called 'boundary-making practice'. They argue that 'boundary-making practice[s] … are involved in the emergence of mathematical concepts' (De Freitas and Sinclair, 2014, p. 50). The concept of boundary-making practice refers to a 'device that reconfigures the world rather than simply representing it or coding it' (De Freitas and Sinclair, 2014, p. 50). The making process of drawing and using materials can have an impact on mathematical learning. To give an example, De Freitas and Sinclair analyse a drawing by a young child in grade seven. The drawing is of a square and four trees located at each corner on the page. The authors explain,

> The trees are not only iconic representations of 'real' trees; they are also part of a diagrammatic device that effects the individuation of 'real' trees, recombining the material world with the new material square and simultaneously engendering the meaning of square. This device begins to perform the meaning of square and, as the children engage with it, the apparatus enacts this splitting. Thus, the diagramming is not merely an act of meaning-making, but rather it engages with and reconfigures the imbrication of meaning and matter. In other words, meaning and matter are intricately recombined through this simple sketch. (De Freitas and Sinclair, 2014, p. 51)

The drawing device plays a role in the seventh-grade student's ability to produce mathematical meaning, which the student does by contrasting a square and drawing four trees at each corner. The study of this drawing device leads De Freitas and Sinclair (2014) to use the term 'pedagogy of concept' (De Freitas and Sinclair, 2014, p. 55). De Freitas and Sinclair's (2014) pedagogy of concept focuses on the role and impact of materiality in and on mathematical knowledge. To understand the pedagogy of concept, they used the work of Culter and MacKenzie (2011), whose research centers on the bodies of learning. Culter and Mackenzie (2011) proposed,

> Perhaps the real challenge to Cartesianism lies not in reconfiguring how we know what we know but in dethroning the epistemological project itself by considering the body of knowledge on the same material plane of existence as the lived and the physical bodies? Perhaps the challenge is to treat learning as an ontological rather than an epistemological problem? (Culter and Mackenzie, 2011, p. 63)

De Freitas and Sinclair (2014) are interested in how a device used in an activity influences the way our gestures, hand-movements, and creation of diagrams intertwine with one another to create mathematical meaning. The drawing devices in the learning-as-making activities consist of 3D printing pens, a 3D modelling interface on the computer, and a 3D printer.

To understand gesture in mathematical terms, De Freitas and Sinclair (2014) build on the work of Gilles Châtelet (1993, 2000, 2006) who was interested in how gesture relates to constructing diagrams. According to De Freitas and Sinclair (2014), 'gestures give rise to the very possibility of diagramming, and diagrams give rise to new possibilities for gesturing' (De Freitas and Sinclair, 2014, p. 64). For example, the 'making of a circle, for instance, is a simple gesture that particular assemblages and bodies are inclined to produce without being told or instructed, and frequently without any prescribed verbal equivalent or operating program' (De Freitas and Sinclair, 2014, p. 66). The simple hand gesture of drawing a circle can be performed in a variety of ways, and then redrawn. And circles are similar to diagrams, De Freitas and Sinclair (2014) suggest, in that a 'diagram may be reactivated through our engagement' (De Freitas and Sinclair, 2014, p. 66).

In understanding gesture and diagram in a virtual world, De Freitas and Sinclair (2014) state that

> the virtual is the mobility that is presupposed by an apparently static figure, a generalised mobility that was central to its creation in the first place. In other words, the virtual or potentiality of a diagram consists of all the gestures and future alterations that are in some fashion 'contained' in it. (De Freitas and Sinclair, 2014, pp. 68-69)

De Freitas and Sinclair (2014) expand on this concept by experimenting with five students who were asked to 'show with diagrams how the circle moves from being concave up to concave down' (De Freitas and Sinclair, 2014, p. 75). The activity consisted of the five students using a pencil and paper to carry out the task. All five students that De Freitas and Sinclair (2014) reported on had different drawings. However, all five students exhibited common gestures in the activity. De Freitas and Sinclair (2014) describe their movements, writing:

> they start with arms held above their heads, fingertips touching, and then separate their hands and circle their arms out until they reach a horizontal, straight position before curving them back towards each other, finally touching them together at stomach height. (De Freitas and Sinclair, 2014, p. 82)

In their activity, the goal is to show 'how the mathematical subject comes into being (is always becoming) as an assemblage of material encounters' (De Freitas and Sinclair, 2014, p. 85). Later, De Freitas and Sinclair coined the term 'mathematical inventiveness', explaining that this 'exists in the dance between the gesturing and drawing hand, which expresses and captures the temporal and dynamic moment when the new or the original comes into (in-venire) the world at hand' (De Freitas and Sinclair, 2014, p. 88). Mathematical inventiveness occurs in mathematical activities that consist of engagement with the virtual and actual world. According to De Freitas and Sinclair (2014), the 'boundary between the virtual and the actual is constantly shifting and being remade in mathematics classrooms through material interactions' (De Freitas and Sinclair, 2014, p. 88). In addition, the concept of mathematical inventiveness in the classroom means the 'relationship between learners/teachers and the material world' (De Freitas and Sinclair, 2014, p. 88). De Freitas and Sinclair (2014) suggest four characteristics of a creative act: it 'introduces or catalyses the new, is unusual, is unexpected or unscripted, [and] is without given content' (De Freitas and Sinclair, 2014, p. 89). In their lens, the assemblage process starts with 'material actions (gestures and diagrams) that constitute inventive moments (processes of actualization)' (De Freitas and Sinclair, 2014, p. 90). In the same way, in-service teachers engage in the learning-as-making activity assemblage process, starting with the material action of holding and moving the 3D printing pen to extrude material filament to produce an object.

To induce mathematical inventiveness in an activity, De Freitas and Sinclair (2014) are interested in expressive technologies that 'provide tools that enable learners to construct mathematical objects and explore relationships among them' (De Freitas and Sinclair, 2014,

p. 90). In their earlier example of the seventh-grade student drawing a square and four trees, they saw evidence of 'potential inventive moments in which the human-technology assemblage gives rise to new ways of thinking and moving' (De Freitas and Sinclair, 2014, p. 90). This led to their interest in digital technology, in particular 'dynamic geometry environments (DGEs) and motion detectors' (De Freitas and Sinclair, 2014, p. 90). Motion detectors play a role in how bodies are interacting with a device to create mathematical meaning. De Freitas and Sinclair (2014) explain:

> New ways of thinking are offered through the experience of this sensorimotor feedback (for example, you will move your hand faster when you anticipate steeper graphs; you will imagine, draw and gesture new diagrams as generated by particular motions you have not performed before). (De Freitas and Sinclair, 2014, p. 91)

Let's look back at the example of a 3D printing pen. The way a student can hold a 3D printing pen and change the speed of the filament being extruded can lead to new diagrams, or in this case, shapes, being produced (Ng and Sinclair, 2018). In terms of a dynamic geometry environment, the Tinkercad interface allows students to engage with various types of 3D models on the computer. The dynamic geometry environment that De Freitas and Sinclair (2014) examined they called 'the geometer's sketchpad' (p. 92). In one classroom situation with year-one students, Sinclair is the instructor who 'is facing both the children and the digital projector' (De Freitas and Sinclair, 2014, p. 92). The lesson that Sinclair conducted was about the intersection of two lines. The children had not yet received a lesson regarding perpendicular or parallel lines. The lesson not only prompted students' gestures, it also sparked their imaginations through their discussion. De Freitas and Sinclair (2014) presented the dialogue among the students in response to Sinclair's instruction to '"use your imaginations" to decide whether they intersect' (De Freitas and Sinclair, 2014, p. 93). This comment sparked a discussion among the children about the possibility of the two lines intersecting on the screen. The children provide reasoning along with hand gestures to determine if the line will intersect. At the end of the lesson, Sinclair introduced the 'word "parallel" to describe two lines that are never going to intersect' (De Freitas and Sinclair, 2014, p. 96).

This example of Sinclair's classroom experience is in line with the four characteristics of a creative act. De Freitas and Sinclair (2014) summarize their experience, stating,

> The technology plays a central role in affording this material act of creation. In addition, the children perform creative acts in gestures that literally make manifest the convergence and intersection of the lines… the collective actions (both movement and discourse) by which the plane is extended and the point of intersection is created can be considered unusual, because such actions involve the non-visible and the potential… The creative acts are genuinely unexpected and unscripted in the sense that the teacher is experimenting with a new technology, as well as with ideas that are not usually part of the grade one curriculum. More importantly, the creative acts are also unexpected for the children…The unfolding path of the lines on the screen, as well as the 'uncovering' of a hidden intersection, provoke gestures amongst the children that actualize infinitely extending lines and their invisible points of intersection. (De Freitas and Sinclair, 2014, pp. 98-99)

In another example, De Freitas and Sinclair (2014) discuss a lesson on different models of motion using a software entitled 'Motion Visualizer DV (MV)' (De Freitas and Sinclair, 2014, p. 99). In this lesson, 'the children create a new space where they can reason about the graph of the vertical line: a gestural space not physically possible, but mathematically actualizable' (De Freitas and Sinclair, 2014, p. 107). The use of digital technology produces an experience known as 'performative identities' (De Freitas and Sinclair, 2014, p. 108). According to De Freitas and Sinclair (2014), 'performative identities refers to the extension and transformation of our identities in cyberspaces' (De Freitas and Sinclair, 2014, p. 108).

Reflecting on the activity used to talk about intersecting lines with the children, De Freitas and Sinclair (2014) write,

> The children soon join this new world, using their bodies, arms and hands to conjure more lines, thereby extending and transforming their own spaces beyond that of the visible (e.g., 'it's going to connect somewhere over here'; 'it's always going to slant because right there'; 'it might intersect somewhere far, far away'). And while they do not interact directly with the mouse, or even the points and lines (the teacher does the 'dragging'), their bodily involvement is acute, as can be seen in the 'dynasties of gestures', to use Châtelet's phrase, they produce. (De Freitas and Sinclair, 2014, p. 108)

To understand the materialist approach in the mathematics classroom, De Freitas and Sinclair (2014) looked at the mathematical conversations being created with material engagements. According to De Freitas and Sinclair (2014), 'students speak mathematics with material and physical verbs – often conjugated in diverse and sometimes contradictory ways – while written mathematics shows little to no trace of these material verbs' (De Freitas and Sinclair, 2014, p. 113). De Freitas and Sinclair use Barad's (2007) work on diffraction methodology to suggest that

> Materiality is a discursive performance of the world, but discursive practices are not reducible to human-based actions. Matter does not serve as a mere support for discourse, nor is it merely the end product of human-based citational practices. Rather, discursive practices are specific material configurations/(re)configuring's of the world through which local determinations of boundaries, properties, and meanings are differentially enacted. That is, discursive practices are ongoing agential intra-actions of the world. (Barad, 2008, p. 173)

De Freitas and Sinclair (2014) refer to language as a 'material expression' (De Freitas and Sinclair, 2014, p. 116). They see 'language as part of the material assemblage, rather than as representing or coding it' (De Freitas and Sinclair, 2014, p. 116). According to De Freitas and Sinclair (2014),

> In educational research, we comb over transcripts for evidence of mastery and misconception, and we look for how students and teachers use words to carry out identity work, but we also need to look at how speech is operating outside of a regime of signal caution – outside or alongside its capacity to refer, and even to communicate. (De Freitas and Sinclair, 2014, p. 117)

To De Freitas and Sinclair (2014), 'speech erupts like a spark or singularity, migrating across the material network and recoupling with gestures, objects and patterns, so that mattering and meaning are conjoined processes' (De Freitas and Sinclair, 2014, p. 117). This also includes looking at 'speech as part of material expression' (De Freitas and Sinclair, 2014, p. 118).

To give an example of these ideas, De Freitas and Sinclair (2014) used a seventh-grade classroom in California that implemented a mathematical task showing a 10 by 10 square on the projector. The authors used a video recording to analyse the activity. They focused in particular on a student named Colin who demonstrated confidence in explaining how the square shown is a 10 by 10 through the use of a pointer. The pointer, which is normally located on the teacher's desk, is considered an apparatus because Colin is using the pointer to explain his reasoning. De Freitas and Sinclair (2014) notice that Colin '(his hands at the projector, and the shadow of his hands on the screen) produces another fold in the growing assemblage, another dimension in the multiplicity of his ever-changing embodiment' (De Freitas and Sinclair, 2014, p. 119). In analysing the video recording, De Freitas and Sinclair (2014) observed the dialogue, gestures, and apparatus (pointer). Through reorganising their transcript to show all three features of Colin's reasoning and hand movement using the pointer, they attempted to understand key words, especially transition words such as 'overlapping', 'because', and 'then' (De Freitas and Sinclair, 2014, p. 124). These words being employed in the dialogue is part of boundary-making practice. According to De Freitas and Sinclair (2014), 'the movement of the pointer acts with equal force to the words that are spoken, co-generating the meaning of the words and investing the explanation with validity' (De Freitas and Sinclair, 2014, p. 124).

To De Freitas and Sinclair (2014), the human voice plays an essential role in understanding the connection between gesture, dialogue, and use of apparatus in sustaining our participation. Using the example of Colin, De Freitas and Sinclair (2014) explain:

> We see this assemblage of meaning as being essentially composed of human voice and chalk taps, in addition to the diagram, the moving hand and its shadows on the screen, as well as the many other aspects that are not captured in the video. The sound of the chalk announces that the talk and gestures are being inscribed on the blackboard, creating a new material configuration in which the teacher translates and legitimizes the 'method'. (De Freitas and Sinclair, 2014, p. 137)

Next, De Freitas and Sinclair (2014) turn their attention to mathematics and human sense. In their perspective, human sense 'refer[s] to the sensory organs (eyes, ears, skin, nose and tongue) that we normally associate with our sensing of the external world (hearing, seeing, touching, smelling and tasting)' (De Freitas and Sinclair, 2014, p. 140). In discussing human sense, they are interested in how human sense engagement with technology, such as computer software, impacts the way we learn mathematics. According to De Freitas and Sinclair (2014),

> The mathematics classroom, especially at the elementary school level, has long employed a variety of material resources to support student learning. These 'manipulatives' are often used to provide concrete forms of interaction, so that children can actually touch three-dimensional shapes or work physically and visually with blocks, abaci or geoboards. (De Freitas and Sinclair, 2014, p. 149)

To give an example of material resources, De Freitas and Sinclair (2014) focus on an interaction of a kindergarten student named Katy using a software called *TouchCounts* on the iPad. De Freitas and Sinclair (2014) observed how Katy moved her finger across the iPad to touch the number with the colour circle, when the number is called from the audio. De Freitas and Sinclair (2014) are interested in the 'rhythm that operates beneath concept-producing judgements', and the impression that 'our attention is drawn to microlevel relational responses' (De Freitas and Sinclair, 2014, p. 153). This observation led to the authors noticing Katy's sensations and perceptions. According to De Freitas and Sinclair (2014), 'an object is perceived when it has been assigned to the synthesized parts of a spatial-temporally apprehended multiplicity' (De Freitas and Sinclair, 2014, p. 156).

In understanding the virtuality of mathematical concepts, De Freitas and Sinclair (2014) explain that 'mathematical entities are…material objects with virtual and actual dimensions' (De Freitas and Sinclair, 2014, p. 202). De Freitas and Sinclair (2014) use an example of a triangle to describe virtual and actual dimension. According to the authors,

> It may be helpful to think of the virtual triangle that takes on a multitude of locations, sizes and shapes that cannot be fixed in space or time, only one of which ever gets cut out, at any given time, into a triangle diagram. In this way of thinking, the rigid, inert, paper-based triangle as we know it – along with its necessary and sufficient definition – emerges from a boundary-making practice that stops motion. (De Freitas and Sinclair, 2014, p. 205)

In thinking of Wenger's (1998) three distinct modes of belonging, utilising drawing devices and making demonstrates negotiability through engagement. Wenger (1998) states that 'learning depends on our ability to contribute to the collective production of meaning because it is by this process that experience and competence pull each other' (Wenger, 1998, p. 202). The conversations that are happening through our hand gestures and interactions with computer interfaces are part of negotiability through imagination. Wenger (1998) discusses that 'stories can transport our experience into the situations they relate and involve us in producing the meanings of those events as though we were participants' (Wenger, 199, p. 203). When interacting with a computer software such as *TouchCounts* or Tinkercad, this is considered a distinct mode of alignment. Wenger (1998) notes that 'a computer program is an extreme example of a reification designed to generate alignment without negotiability' (Wenger, 1998, p. 206). In the next section, I discuss the ways that the three theoretical lenses employed in this study - communities of practice, shape perception, and inclusive materialism – intertwine and influence one another.

2.5 The potency of connecting the three theoretical frameworks

Wenger's (1998) three distinct modes of belonging (engagement, imagination, and alignment) connect to Pizlo's (2008) shape perception theory and De Freitas and Sinclair's (2014) inclusive materialist lens. Wenger's (1998) communities of practice lens serves as the overarching framework for this study. The community observed for this study consists of six in-service teachers engaging in a practice that involves the use of 3D printing technology tools and learning-as-making activities. Investigating in-service teachers' sense of belonging in a practice, Wenger's (1998) community of practice theory affords an understanding of how in-service teachers could interact in a social lens sparking their mutual engagement, joint enterprise, and shared repertoire. Since participants in this study are engaged in designing 2D and 3D shapes using a 3D printing pen, it is essential to consider their shape perceptions. Pizlo's (2008) shape perception theory can be employed to provide insight into in-service teachers' present sense of belonging as they engage with the mental image of the shapes and recall the 2D orientations. In-service teachers' imaginations allow them to create a mental picture of a 3D shape in relation to their view of geometrical shapes. This demonstrates their alignment to their broader understanding of shapes. When projecting their 2D and 3D shapes in their minds in order to actually construct them and make them a reality, De Freitas and Sinclair's (2014) inclusive materialist lens is employed to follow in-service teachers' sense of belonging when using devices that produce a material construction (i.e., 2D and 3D skeleton shapes). When in-service teachers engage with the devices, such as the 3D printing pen or 3D modelling software Tinkercad, they are demonstrating their imagination of 2D and 3D shapes through creating a virtual or physical construction with their device. Their alignment in the distinct mode of belonging is potentially shown using their gestures and hand movements in creating their 2D and 3D shapes. In the next section, I review the literature that employs theories of CoP, shape perception, and inclusive materialism in relation to mathematics and making.

Chapter Three: Literature Review

3.1 Introduction

In this section, I present the literature relevant to learning-as-making activities. I first review the literature on makerspace research that utilises Lave and Wenger's (1991) concept of legitimate peripheral participation (Section 3.2). Then I introduce Wenger's (1998) communities of practice theory as it relates to mathematics education (Section 3.3) and teacher education (Section 3.4). Following this, I discuss the scholarship on making, makers, and tinkering to provide insight into the definition of the making process in this research (Section 3.5). I also review the literature focusing on the representation, visualisation, and spatial reasoning of 2D and 3D shapes (Section 3.6). Lastly, I discuss the literature on 3D printing in mathematics education, including scholarship that utilises 3D printers, 3D modelling software, and 3D printing pens in the classroom (Section 3.7).

3.2 The concept of legitimate peripheral participation in makerspace research

Lave and Wenger's (1991) concept of legitimate peripheral participation has been adopted by researchers over the past three decades. Most recently, the concept has been used in research studies to understand the use of makerspaces (Becker and Jacobsen, 2023; Bonnette and Crowley, 2020; Willet, 2016). A makerspace is known to be a situated learning space (Becker and Jacobsen, 2023; Forest et al., 2014), where situated learning is defined as 'learning that takes place in the same context in which it is applied, through participation in the life and activities of the maker community' (Forest et al., 2014, p. 23). A makerspace is an environment that enables people to come together to engage in activity with tools physically or digitally presented in the space. Digital tools can be used just as socially and interactively as physical tools, as individuals enter a makerspace within an online platform and perform activities that allow them to interact and share ideas with others in that online space (Hui and Gerber, 2017). Researchers have employed the concept of legitimate peripheral participation in a makerspace environment to investigate how learning and identities unfold (Becker and Jacobsen, 2023; Bonnette and Crowley, 2020).

For example, Becker and Jacobsen (2023) studied a group of Canadian elementary school students and their teacher, examining the teacher's identity in a makerspace environment. Their study provides an example of how a teacher engages with the concept of legitimate peripheral participation. They contend that 'the learning curriculum in the makerspace evolves with the needs of the learner' (p. 3). The use of legitimate peripheral participation in their research helped them observe how the elementary school teacher engaged with her students and also enabled them to explore the students' learning and identities as they unfolded in a makerspace environment. Bonnette and Crowley (2020), like Hui and Gerber (2017) mentioned above, examined learning in both physical and digital makerspaces, showing that people engage not only with physical tools but also with digital tools presented through computer-aided programmes. In Bonnette and Crowley's (2020) study, they used the concept of legitimate peripheral participation to investigate an emerging adult learner – one who grew up in foster care – learning in a makerspace environment. They were interested in how legitimate peripheral participation provides insight into emerging adult learning and identity formation in a makerspace environment through ‘maker community engagement’, ‘maker skills and knowledge’, and ‘maker community-member identity’.

Research focused on legitimate peripheral participation provides insight into how situated learning in a makerspace environment can affect people’s identities through engagement in hands-on activities. Makerspace environments have not only been used to explore the concept of legitimate peripheral participation but have also been examined as environments that form communities of practice (Mbaezue et al., 2020; Doorman et al., 2019; Sheridan et al., 2014). While the fieldwork research upon which my study is based did not examine the concept of legitimate peripheral participation per se, it is nonetheless essential to understand how the concept has been used by other researchers to employ the three components of communities of practice: domain, practice, and communities. In the next section, I discuss how the communities of practice theory has been employed in mathematics education research.

3.3 Communities of Practice theory in mathematics education research

In mathematics education, a community of practice could be formed by a group of in-service teachers (Crisan and Rodd, 2017; Akkoc et al., 2016), prospective teachers (Solomon et al., 2017; Cavanagh et al., 2012), or undergraduate mathematics students (Biza et al., 2014; Solomon, 2007), among other possibilities. Biza et al. (2014), who researched communities of practice at the university level, suggested that:

> when teachers and students are participating in the same practice to conceptualise the practice… Dialogue in engagement contributes to reification of concepts as part of participation. Teacher and students play different, but highly interactive, roles and develop identities through their engagement, use of imagination, and alignment with the norms and expectations in the setting. (Biza et al., 2014, p. 171)

Solomon (2007), who applied the communities of practice framework to twelve undergraduate mathematics students at a United Kingdom university, found that students’ identities can reveal multiple communities of practice. In her study, undergraduate mathematics students engaged with interview questions regarding mathematics and learning experiences at the secondary or university level. In investigating the three modes of belonging among undergraduate mathematics students, Solomon (2007) found that ‘identities are differentially experienced within multiple communities of practice’ (Solomon, 2007, p. 93).

In understanding how communities of practice are used to explore in-service teachers’ identities, Crisan and Rodd (2017) employed communities of practice theory on non-mathematics teaching specialists (NSTM) in the United Kingdom who were teaching mathematics and also enrolled in a mathematics developmental programme course. In their study, they applied Wenger’s (1998) modes of belonging to better understand in-service teachers’ identity work:

> *Identification with school mathematics* refers to how the NSTMs constructed identities as learners of mathematics during their in-service course. Identification through *engagement*, *imagination*, and *alignment* refers to how the NSTMs invested

> themselves in learning about and doing school mathematics topics, how they constructed images about how pupils learn mathematics and how their views converged towards an increasing connection with how the mathematics teaching community views mathematics as a practice. *Negotiability in mathematics teaching* through *engagement*, *imagination*, and *alignment* refers to how the NSTMs negotiated their ways in the mathematics teaching community, how the NSTMs constructed images of themselves as potential specialist mathematics teachers and how their views converged towards an increasing connection with the mathematics teaching community. (Crisan and Rodd, 2017, p. 109)

Akkoc et al. (2016) investigated eight pre-service mathematics teachers' perceptions of mathematics teaching through the communities of practice lens in Turkey. They used the three modes of belonging as follows:

> *Engagement* refers to instances where participants themselves engage in activities in school… *Observing* engagement refers to instances where participants observe the way in which members of the school community engage in activities of daily practice… *Alignment* represents cases where participants adapt to the school culture or challenge it…*Observing* refers to instances where participants observe the way in which members of a school community adapt to school culture or challenge it…[and] *Imagination* is concerned with how participants reflect on their observations to build their perception of the mathematics teacher and their selves as mathematics teachers. (Akkoc et al., 2016, pp. 42-43)

Incorporating Wenger's (1998) concept of three modes of belonging and literature in mathematics teacher education, I define the three modes of belonging in my study as follows: *Engagement* refers to how the non-mathematics in-service teachers utilise 3D printing technology tools in a learning-as-making activity to mathematise. *Alignment* refers to how the in-service teachers adapt to the activity and challenges posed by the 3D printing technology tools (i.e., 3D printing pen malfunction during their construction of 2D or 3D shapes). Lastly, *Imagination* relates to how in-service teachers are mathematising, as in the direction chosen to construct their 2D and 3D models using the 3D printing pen. To better understand Wenger's (1998) modes of belonging, in the next section, I discuss communities of practice theory employed in the literature on teacher education.

3.4 Communities of Practice theory in teachers' education

A community of practice is 'a group of people who share a concern, a set of problems, a passion about a topic and who deepen their knowledge and expertise in that area by interacting on an ongoing basis' (Wenger et al., 2002, p. 4). There is 'a direct link between learning and performance because the same people participate in communities of practice and in teams and business units' (Wenger, 2015, p. 5). In the literature on teacher education, a lens grounded in the communities of practice theory has been employed to understand how teachers develop a practice and community (Kaschak and Letwinsky, 2015; MacPhail et al., 2014; Parker et al., 2012; Jimenez-Silva and Olson, 2012). Kaschak and Letwinsky's (2015) research observed the initial development of a community of practice consisting of pre-service teachers enrolled in an undergraduate middle education programme who were collaborating on a project-based activity. In discussing the CoP component of shared repertoire, Kaschak and Letwinsky (2015) focused on the 'collaborative development of a public service announcement on ecological footprints. Preservice teachers pointed to the use of discussions, electronic modes of communication, and the process of filming and editing as tools that facilitated their work on the project' (Kaschak and Letwinsky, 2015, pp.152-153). Kaschak and Letwinsky (2015) noted that the pre-service teachers' 'discussion and communication emerged as particularly dynamic tools characterizing a shared repertoire' (Kaschak and Letwinsky, 2015, p.153). MacPhail et al.'s (2014) research investigated professional learning through communities of practice and suggested that 'CoP are places and spaces where teacher educators have the opportunity to engage in worthwhile conversations and actions about the nature and direction of their work with teacher candidates' (MacPhail et al., 2014, p. 42). Parker et al.'s (2012) research focused on the landscape in which communities of practice are developed by looking at the professional training of physical education teachers in Ireland. They argued that physical education teachers found their

> purpose changing, from a collection of teachers whose intention is to gain new ideas to a working CoP with a clearly articulated internal purpose. Success in these groups varies also, ranging from achieving their intended purpose to achieving the purpose in addition to teachers' gaining empowerment and ownership. (Parker et al., 2012, p. 323)

Jimenez-Silva and Olson's (2012) research investigated communities of practice in teacher education focusing on teacher-learner communities. In their study, they 'arranged the desks or tables in the classroom in a manner that encouraged pre-service teachers to look at and listen to each other as they discuss sensitive ideological and political issues related to cultural and linguistic diversity in education' (Jimenez-Silva and Olson, 2012, p. 339). Through arranging the desks or tables, they were able to promote the development of teacher-learner communities.

Incorporating the literature in teacher education on developing communities of practice through professional development, in my study the learning-as-making activities utilised a shared repertoire of 3D printing pens that could allow for the emergence of discussion and communication of in-service teachers engaging with mathematics. The landscape of the learning-as-making activities took place within in-service teachers' classrooms. In this way, they could provide insight into how the activities relate to their own classroom needs and gain new ideas for implementing learning-as-making activities. For in-service teachers who worked in groups, the situated design and technology classroom allowed them to communicate ideas and observe each other's 3D model creations. In the next section, I discuss the literature around making, makers, and tinkering.

3.5 Making, makers, and tinkering

The fundamental approach to learning-as-making activities requires the process of making, whether it is engaging with fabrication, modelling software, or pencil and paper. Honey (2013), who researches the design, play, and engage approach, describes making as a way 'to build or adapt objects by hand, for the simple personal pleasure of figuring out how things work' (Honey, 2013, p. 4). Hsu et al.'s (2017) research looks at making and maker education and finds that 'making encourages students to share ideas and projects, and show off their creations' (Hsu et al., 2017, p. 590). Martin (2017) has researched the maker movement in education and defines 'making as a class of activities focused on designing, building, modifying, and/or repurposing material objects, for playful or useful ends, oriented toward making a "product" of some sort that can be used, interacted with, or demonstrated' (Martin, 2017, p. 31).

Whenever engaging in a making activity, one becomes a maker. Indeed, makers are seen as assuming a particular identity (Halverson and Sheridan, 2017, p. 496). Davis and Mason (2017), who research maker identity, suggest that

> a maker does not consist merely of making a thing, but rather [displays] a conglomeration of multiple behaviors, perhaps repeatedly, over time, such as tinkering, … [or] hands-on technology crafting, and [in addition] describing oneself as a maker. (Davis and Mason, 2017, p. 178)

Toombs et al. (2014) looked at maker identity in a hackspace and found that the tools used in making impact maker identities. To explain this, the authors used the term 'tools sensibility', which suggests that the one using the tools holds 'a deep understanding of existing tools and how they are used, an ability to judge which tools are the most appropriate for the task at hand, and a sophistication concerning the materials and medium available to the makers' (Toombs, 2014, p. 4). To be a maker, one becomes engaged with a broad range of technological tools. As such, 'learning through making embraces constructionist, multiliteracies, and embodied and distributed cognition perspectives to arrive at three core principles: designing as learning, creating and sharing artifacts, and attending to process and product as outcomes' (Halverson and Peppler, 2018, p. 291).

When one is engaging in making activities, one is also tinkering (Resnick and Rosenbaum, 2013; Wilkinson and Petrich, 2014). In Wilkinson and Petrich's (2014) book entitled *Art of Tinkering,* the authors describe tinkering as engaging

> directly with phenomena, tools, and materials. It's thinking with your hands and learning through doing…It's also about making something…Because when you tinker, you're not following a step-by-step set of directions that leads to a tidy end result. Instead, you're questioning your assumptions about the way something works, and you're investigating it on your own terms. (Wilkinson and Petrich, 2014, p. 13)

To better understand tinkering, Bevan et al. (2015) developed a tinkering learning dimension framework that includes *engagement*, *initiative and intentionality, social scaffolding,* and *development of understanding*. Making, makers, and tinkering are intertwined in the concept of makerspace. In the next section, I discuss the representation, visualisation, and spatial thinking of 2D and 3D shapes.

3.6 Visualisation, representation, and spatial thinking of geometrical shapes

In relating shape perception to the teaching and learning of mathematics, mathematical thinking enables us to understand geometrical reasoning through three components: visualisation, representation, and spatial thinking. To describe the visualisation and representation of 2D and 3D geometrical shapes, I use the work of Duval (2017, 2014, 2006, 1999) to understand the cognitive process of learning mathematics. Pizlo's (2008) shape perception theory, which I rely on primarily for this study, does not necessarily agree with Duval's work, but it is important to observe how others are using the concept of shape perceptions in their own work.

In an early work, Duval (1999) introduced two types of cognitive representation:

> Those that are intentionally produced by using any semiotic system: sentences, graphs, diagrams, drawings... Their production can be either mental or external. And there are those which are causally and automatically produced either by an organic system (dream or memory visual images) or by a physical device (reflections, photographs). (Duval, 1999, p. 4)

In a later study, Duval (2006) stated that 'to do any mathematical activity, semiotic representation must necessarily be used even if there is the choice of the kind of semiotic representation' (Duval, 2006, p. 107). Unlike Pizlo (2008), who finds that shape perception relies on sensation, Duval's representation of mathematical objects relies on mathematical knowledge, such as one having an understanding of the geometrical shape in mind. In looking at or seeing geometrical figures through a mathematical lens, Duval (2006) introduces three terms: 'shapes, basic geometric figures and figural units' (p. 9). He describes each term as follows:

> The *shapes* are closed outlines that stand out against a set of plotted or colored elements and that we recognize at first glance as a whole, either surface or a solid…The *basic geometrical figures* are typical shapes that can be constructed with specific tools (ruler, compasses or software) and which have some notable intrinsic properties…circle, square, triangles…when introduced at the primary school, their properties are the early contents of teaching geometry. (Duval, 2006, p. 9)

In describing the term figural units, Duval (2006) explains that 'these elements are not characterized by their shape but *by their shape dimensions* [0D,1D, 2D, or 3D]' (Duval, 2006, p. 10). Duval (2006) uses an example of deconstructing shape recognition to illustrate 'that a *mathematical property cannot be visualized by a single figural unit, but only by the visual relation between two figural units*, from the same number of dimensions, or not.' (Duval, 2006, p. 11). This goes back to Duval's (1999) earlier work describing vision with two cognition functions – an epistemological function and a synoptic function. In describing the two functions, Duval (1999) explained the epistemological function by saying that 'vision is the opposite of representation, even of the "mental images", because representation is something which stands instead of something else' (Duval, 1999, p. 12). As for the synoptic function, Duval (1999) writes that 'vision is the opposite of discourse, of deduction, which requires a sequence of focusing acts on a string of statements' (Duval, 1999, p. 12).

In modern times, the computer plays a major role in influencing our vision of 2D and 3D representations. Duval's (2017) work discussed the phenomenological mode of production of representation in relation to computer monitors. If we look at a 3D modelling program such as Tinkercad, it is an easy-to-use program that allows users to manipulate 3D models by rotating, duplicating, or changing their dimensions with the click of a button. As seen through Duval's (2017) lens, this 'allows [us] to meet a new epistemological function that the other modes of production cannot do: exploration by simulation' (Duval, 2017, p. 100). When exploring by simulating 3D objects, we tap into our spatial thinking ability.

Spatial thinking is when one visualises, analyses, and reasons with 2D and 3D objects (Küçük et al., 2023; Gagnier et al., 2022; Bhaduri et al., 2021; Newcombe, 2010). According to Küçük et al. (2023), who researched the effect of creating 3D objects with block code using spatial and computational thinking skills, 'spatial thinking involves a variety of cognitive skills that enable individuals to organize, reason about, and mentally manipulate both real and imaginary spaces' (Küçük et al., 2023, p. 3). Gagnier et al. (2022), who investigated the spatial thinking of elementary teachers in STEM (science, technology, engineering, and mathematics), describe spatial thinking as

> a collection of cognitive skills that allow us to organize, reason about, and mentally manipulate spaces that are both real and imagined… Spatial thinking skills are critical in daily life as we search for specific locations (e.g., where is the movie theater in relation to the supermarket), use representation of space such as navigating with a map or assembling furniture from a diagram, or imagining what our living room furniture might look like in an alternative arrangement. (Gagnier, 2022, p. 97)

For example, Bhaduri et al. (2021) investigated the spatial thinking of seventh and eighth-grade students in the United States designing and engaging with Tinkercad. Their findings show that 'if students are given the opportunity to iterate on their designs, over time making multiple sketches of their designs and then creating 3D models, we can allow for better development of 3D mental models. It can support the development of their spatial thinking skills around 2D to 3D translation and vice-versa' (Bhaduri et al., 2021, p. 265). In the next section, I discuss the literature on 3D printing pens, Tinkercad, and MakerBot 3D printing in mathematics education.

3.7 3D printing technology tools in mathematics education

There have been numerous publications in mathematics education that look at 3D printing pens (Ng and Chan, 2021; Ng and Ferrara, 2020; Ng and Sinclair, 2018), Tinkercad (O'Reilly and Barry, 2023; Bhaduri et al., 2021; Piesticker et al., 2021), and 3D printers (Anđić et al., 2024; Anđić et al., 2022a; Anđić et al., 2022b; Asempapa and Love, 2021). Through video recordings, Ng and Chan (2021) observed in-service teachers utilising 3D printing pens in a primary classroom in Hong Kong. Their research gauged whether utilising the 3D printing pen in a primary classroom could prompt new teaching methods in introducing concepts of geometry in 3D. In an earlier study, Ng and Ferrara (2020) investigated learning-as-making through utilising 3D printing pens with primary students in Hong Kong. This study focused on the material aspects of the activity, such as using 3D printing pens to draw pre-made geometrical shapes. Additionally, Ng and Sinclair (2018) introduced 3D printing pens to activities that involve drawing 'derivative functions' (p. 306). When I refer to derivative functions, I mean drawing a quadratic function (i.e., $y = ax^2 + bx + c$) or a linear equation (i.e., $y = mx + c$).

In a virtual interface, Tinkercad is software employed in the classroom to enable students to interact with 3D shapes. O'Reilly and Barry (2023) investigated the effect that modelling software such as Tinkercad has on students aged 8 to 11 years old. In their study, the way students talked about using different shapes to create their models showed the students' confidence in mathematics. For example, they displayed their ability to name and describe the shapes being used. Piesticker et al. (2021) employed Tinkercad at the fourth-grade elementary level (ages 9-10) to see how students transferred their mathematical knowledge of shapes from the classroom to Tinkercad and to analyse their creation of 3D models online.

Increasingly, and to achieve a variety of educational goals, 3D printers have been employed in classroom activities to produce 3D shapes. For example, Asempapa and Love (2021) hosted a professional development seminar that focused on mathematical modelling using 3D printing for teachers of the K-12 curriculum in the United States. Their findings show that 'the process of designing and testing 3D printed parts allows additional opportunities for students to apply mathematical modeling concepts and also provides a more rigorous STEM learning experience' (Asempapa and Love, 2021, p. 87). Anđić et al. (2024) conducted a longitudinal study that observed secondary school teachers in Montenegro utilising 3D modelling and 3D printing in the classroom. In their study, Anđić et al. (2024) had mathematics teachers discuss their 3D modelling experiences with students. They found that '3D printing can be used as a teaching tool, as a bridge between the ideas developed in the digital world and the physical world' (Anđić et al., 2024, p. 27). In an earlier study focusing on teachers' conceptions of 3D modelling and 3D printing, Anđić et al. (2022) found that 'teachers considered 3DMP [3D modelling and printing] as a tool that connects students with different professions—allowing them to understand the importance of transdisciplinarity, critical thinking, and creativity as crucial twenty-first-century skills for their future occupations' (Anđić et al., 2022, p. 51). In another study investigating 3D modelling and 3D printing usability among primary school teachers, Anđić et al. (2023) stated that 'teachers agreed [that] the successful implementation of 3D modelling and printing in STEAM education requires at least basic knowledge from mathematics such as geometry' (Anđić et al., 2022, p. 3).

3D printing technology tools in the literature on mathematics education can be seen as a shared repertoire in a community of practice. Having 3D printing tools in the classroom develops a kind of makerspace that in-service teachers can interact with and develop a community around. This community of practice reflects how in-service teachers see themselves when using technology in relation to their teaching subjects. In other words, the community can reveal their teaching identities towards mathematics. The in-service teachers' gestures and hand movements as they use the 3D printing technology can offer insight into their perceptions of shape and the influence materialism may have on mathematical activity. Following this literature review and my discussion in Chapter Two of the three theoretical frameworks employed in this study, I now transition to a discussion of my methodological approach in the next chapter.

Chapter Four: Methodology

4.1 Introduction

In this chapter, I introduce the two methodological approaches deployed in the study: narrative inquiry (section 4.2) and diffraction (section 4.3). I give an example of how I employed narrative inquiry through a 3D printing pen learning-as-making activity with six teacher trainees in the United Kingdom (section 4.2.4). Following this, I give an example of diffraction reading through the 3D printing pen activity in the Calculus for Business module at a UK university (section 4.4). I then introduce the data collection and participants, describing the learning-as-making activities they completed (section 4.5) and offering a description of the method of data analysis (section 4.6). I discuss the role of the researcher in this study (section 4.7), as well as my experience of collecting data and conducting fieldwork in unprecedented times (section 4.8). Following this, I talk about trustworthiness of the study overall (section 4.9). Lastly, I discuss the methodological limitations of this study (section 4.10), and the ethical procedures (section 4.11).

4.2 Narrative Inquiry Approach

4.2.1 **Narrative inquiry as a research method to study teachers' practices and professional experiences**

Narrative inquiry has long been used as a research methodology in the study of teacher practices, knowledge, preparation, and professional development (Clandinin, 2022; Clandinin, 2019; Clandinin and Rosiek, 2007; Connelly and Clandinin, 2006; Clandinin, 2006; Clandinin and Connelly, 2000; Connelly et al., 1997; Connelly and Clandinin, 1988). Connelly and Clandinin introduced the concept of 'personal practice knowledge', calling it

> a term designed to capture the idea of experience in a way that allows us to talk about teachers as knowledgeable and knowing persons. Personal practical knowledge is in the teacher's past experience, in the teacher's present mind and body, and in the future plans and actions. Personal practical knowledge is found in the teacher's practice. It is, for any one teacher, a particular way of reconstructing the past and the intentions of the future to deal with the exigencies of a present situation. (Connelly and Clandinin, 1988, p. 25)

In addition to personal practice knowledge, teachers have professional practice knowledge, the two differentiated by the particular environment of the practice. Clandinin's (2019) work explains that teachers demonstrate professional practice knowledge through their own classroom practices, showing how they perform in the classroom through 'routines and rhythms' (Clandinin, 2019, p. 34). To help explain teachers' professional practice knowledge, Clandinin and Connelly coined the term Teachers Professional Knowledge Landscape (Clandinin and Connelly, 1995; Clandinin and Connelly, 1996; Clandinin, 2018). The word landscape is used to describe the social context of a teacher's professional knowledge, as in whether one's story is taking place inside of school or outside of school. Clandinin and Connelly (1995) explain that

> living within the professional knowledge landscape creates epistemological dilemmas for teachers and others, such as school administrators, who also inhabit the landscape. These epistemological dilemmas are understood narratively in terms of sacred, secret, and cover stories. We described 'the uneasy state of teachers' professional lives as part and parcel of moral and epistemological dilemmas associated with living in, and repeatedly crossing, back and forth between, two epistemologically different places on the landscape'. (Clandinin and Connelly, 1995, p. 5)

When Clandinin and Connelly discuss teachers' professional practice knowledge, they often use the term landscape as 'it captures the exceedingly complex intellectual, personal and physical environment for teachers' work' (Connelly, 1997, p. 673). According to Clandinin and Connelly's (1996) work on Teacher's Professional Knowledge Landscapes, the 'professional knowledge landscape [is] composed of relationships among people, places, and things, [and is] … both an intellectual and a moral lands' (Clandinin and Connelly, 1996, p. 25). In addition, 'teachers on landscapes learn how to act and think in appropriate ways, ways that are sanctioned by others positioned in the conduit' (Clandinin and Connelly, 1995, p. 158). In a later study, Schaefer et al. (2014) stated, 'It is in personal and professional knowledge landscapes, and in the spaces between the two, that teachers attempt to live out their imagined stories of who they are, and are becoming' (Schaefer et al., 2014, p. 14). To understand teachers' professional and personal knowledge landscapes, we must understand their stories to live by, which refers to the importance of their stories as forms of narrative.

The concept of story-to-live by relates to teachers' personal and professional knowledge landscapes because a story-to-live by occurs both inside and outside of school time. According to Clandinin (2019), a teacher's identity is considered a story-to-live by, as a story-to-live by is 'a narrative concept of identity' (Clandinin, 2019, p. 9). The story-to-live by informs 'how knowledge, context, and identity are linked and can be understood narratively' (Connelly and Clandinin, 1999, p. 4). In addition, Clandinin (2022) noted:

> A narrative way of thinking about identity speaks to the nexus of a person's personal practical knowledge, and the landscapes, past and present, on which a person lives and works. A concept of stories to live by allows us to speak of the stories that each of us lives out and tells of who we are and are becoming. This highlights the multiplicity of each of our lives—lives composed, lived out and told around multiple plotlines, over time, in different relationships and in different landscapes. (Clandinin, 2022, p. 32)

In this study, the concept of story-to-live by is demonstrated through the participants' dialogue, transcribed in the data collection, which tells a story of the in-service teachers mathematising using 3D printing technology tools. In the next section, I will define narrative inquiry and explain how a three-dimensional inquiry space approach is employed to aid the understanding of a person's story-to-live by.

4.2.2 **Story-to-live by: Temporality, sociality, and place**

The concept of 'story-to-live by' stems from narrative inquiry. When a person tells a story, the action shows that this 'person is, at once, engaged in living, telling, retelling, and reliving stories' (Connelly and Clandinin, 1999, p. 4). In evaluating individual experience, narrative –

also called storytelling – can be employed to understand people's experiences at a deeper level. Clandinin and Connelly (2000) define narrative as a 'form of representation that describes human experience as it unfolds through time' (p. 6). As narrative can be a representation of individual stories, it reveals 'a portal through which a person enters the world and by which their experience of the world is interpreted and made personally meaningful' (Connelly and Clandinin, 2006, p. 375). In addition, Connelly and Clandinin (2006) suggest that

> people shape their daily lives by stories of who they and others are and as they interpret their past in terms of these stories. Story, in the current idiom, is a portal through which a person enters the world and by which their experience of the world is interpreted and made personally meaningfully. (Connelly and Clandinin, 2006, p. 479)

To understand how experiences are formed and interpreted, we must first become inquirers. That is, we must observe 'an act within a stream of experiences that generates new [responses] that then becomes a part of future experience' (Clandinin and Connelly, 2000, p. 7). Inquiries can be 'a series of choices, inspired by purposes that are shaped by past experiences, undertaken through time, and [that] trace the consequences of these choices in the whole of an individual or community's lived experience' (Clandinin and Rosiek, 2007, p. 15). In Clandinin's (2022) recent works, she elaborates, explaining that

> as inquirers, we are part of the present landscape and the past landscape, and we acknowledge that we helped to make the world in which we find ourselves. As narrative inquirers, we become part of participants' lives and they part of ours. Therefore, our lives, and who we are and are becoming in our and participants' landscapes, are also under study. (Clandinin, 2022, p. 12)

In defining narrative inquiry, Connelly and Clandinin (2006) wrote that 'the study of experience as story, then, is first and foremost a way of thinking about experience. Narrative inquiry as methodology entails this view of the phenomenon. To use narrative inquiry methodology is to adopt a particular view of experience as a phenomenon under study' (Connelly and Clandinin, 2006, p. 479). The view of experience as Connelly and Clandinin (2006) mention is not only focused on individual experience, but also on other factors that form one's experience. For example, Clandinin and Rosiek (2006) expand on narrative inquiry as 'an exploration of the social, cultural and institutional narrative within which an individual's experiences are constituted, shaped, expressed and enacted – but in a way that begins and ends that inquiry in the storied lives of the people involved' (Clandinin and Rosiek, 2006, p. 42). In a later study, Clandinin (2022) explained that the 'narrative inquirer stud[ies] the individual's experience in the world, an experience that is storied both in the living and telling and that can be studied by listening, observing, living alongside one another, and writing and interpreting texts' (Clandinin, 2022, p. 7). To observe one's experience as living and telling, Clandinin (2022) emphasises the importance of identifying the three commonplaces that form the overarching framework of narrative inquiry.

These three commonplaces include temporality, sociality, and place. The temporality commonplace directs 'inquirers toward the past, present, and future of people, places, things, and events under study' (Clandinin, 2022, p. 23). The sociality commonplace focuses on the personal and social conditions of a person. A personal condition includes a person's 'feelings, hopes, desires, aesthetic reactions and moral dispositions' (Connelly and Clandinin, 2006, p. 480). A social condition, on the other hand, is 'understood, in part, in terms of cultural, social, institutional, familial, and linguistic narratives' (Clandinin, 2022, p. 24). Through the sociality commonplace, we look at the experience of a person's personal condition, such as emotions, and understand it in the social context of what is happening in that specific event. Lastly, place commonplace is known as 'the specific concrete, physical, and topological boundaries of place or sequences of places where the inquiry and events take place' (Clandinin, 2006, p. 480).

When observing the three commonplaces in one experience, it is important to employ Clandinin and Connelly's (2000) three-dimensional narrative inquiry space approach as people's experiences are always changing and the commonplaces are not always fixed. In describing the three-dimensional narrative inquiry space approach, Clandinin and Connelly (2000) explain:

> Our terms are personal and social (interaction); past, present, and future (continuity); combined with the notion of place (situation). This set of terms creates a metaphorical three-dimensional narrative inquiry space, with temporality along one dimension, the personal and social along a second dimension, and place along the third. Using this set of terms, any particular inquiry is defined by this three-dimensional space; studies have temporal dimensions and address temporal matters: they focus on the personal and the social in a balance appropriate to the inquiry; and they occur in specific places or sequences of places. (Clandinin and Connelly, 2000, p. 50)

In this study, the concept of the three commonplaces (temporality, sociality, place) is used to unpack in-service teachers' experience through their stories to live by. In the next section, I will introduce the terms narrative identity and identity work to gain insight into how narrative plays a role in unfolding one's identity.

4.2.3 **Narrative identity, mathematical identity and identity work**

The practice of narrative and identity are intertwined (McAdam, 2019; Sfard and Prusak, 2005; Watson, 2006). According to McAdam (2019), who is interested in reconstructing a person's past through narrative, 'narrative identity derives from storytelling, and storytelling derives ultimately from human sociality. Stories are inherently social in two fundamental ways' (McAdam, 2019, p. 2). Through McAdam's lens, narrative identity consists of stories being told and these stories normally take place in a social setting. When examining identity as narrative, Sfard and Prusak (2005), who studied the ways that students developed their identities as a tool for investigating the learning of 17-year-old migrant students in a grade 11 programme, wrote that

> the reifying, significant narratives about a person can be split into two subsets: actual identity, consisting of stories about the actual state of affairs, and designated identity, consisting of narratives presenting a state of affairs which, for one reason or another, is expected to be the case, if not now then in the future. (Sfard and Prusak, 2005, p. 18)

To differentiate between actual and designated identity, Sfrad and Prisak (2005) describe actual identity as stories ‘usually told in present tense and … formulated as factual assertions’, while designated identity ‘can be recognized by their use of the future tense or of words that express wish, commitment, obligation, or necessity, such as should, ought, have to, must, want, can, cannot, and so forth. Narratives such as “I want to be a doctor” or “I have to be a better person” are typical of designated identities’ (Sfrad and Prusak, 2005, p. 18). In their work, Sfrad and Prusak used the stories of the 17-year-old students, which they gathered from the students talking about themselves or sharing stories with their teachers from time to time. Watson (2006), interested in the construction of identity in teachers, stated:

> Teachers’ stories provide a means by which they are able to integrate knowledge, practice and context within prevailing educational discourses. Telling stories involves reflection on, selection of and arrangement of events in an artful manner which contains meaning for the teller and seeks to persuade the listener of their significance. (Watson, 2006, p. 525)

To understand identity in a mathematical lens, Lutovac and Kaasila (2011) investigated pre-service teachers’ mathematical identity at a training school in a university in Finland. They suggest that ‘one’s mathematical identity is manifested when telling or writing stories about one’s relationship to mathematics, its learning and teaching’ (Lutovac and Kaasila, 2011, p. 227). Kaasila et al. (2011) studied the mathematical identity of five pre-service teachers in elementary teacher education in Finland and found that mathematical identity is ‘something that people use to justify, explain, and make sense of themselves in relation to mathematics and to other people acting in mathematical communities’ (Kaasila et al., 2011, p. 2011). Transitioning to the mathematical identity of mathematics teachers, Darragh (2021) used previous scholarship on mathematical learners’ identity to help define mathematics teacher identity as

> A socially produced way of being, as enacted and recognized in relation to learning mathematics. It involves stories, discourses and actions, decisions, and affiliations that people use to construct who they are in relation to mathematics, but also in interaction with multiple other simultaneously lived identities. This incorporates how they are treated and seen by others, how the local practice is defined and what social discourses are drawn upon regarding mathematics and the self. (Darragh and Radovic, 2018, p. 1)

Mathematical identity and narrative identity are both used in this study to describe mathematics related teacher identity. Mathematics related teacher identity originally derives from Lutovac and Kaasila’s (2018) work on future directions in research on mathematics teacher identity. They observed two cohorts of teachers – mathematics teachers and elementary school teachers who were non-specialists in mathematics. For this study, I use teaching identities towards mathematics to refer to in-service teachers who are non-specialists in mathematics talking about mathematics while engaging and interacting with 3D printing technology tools in a learning-as-making activity. Boaler (2002), who investigated mathematical identities in school mathematics classes, stated that ‘different pedagogies are not just vehicles for more or less knowledge, they shape the nature of the knowledge produced and define the identities students develop as mathematics learners through the practices in which they engage' (Boaler, 2002, p. 132). I am interested in the in-service teacher identities towards mathematics that unfold in this study through engagement with 3D printing technology tools.

When a teacher is constructing their stories, they are essentially doing 'identity work' (Brown and Toyoki, 2013; Watson, 2008). According to Watson (2008), who investigated managing identities, identity work

> involves the mutually constitutive processes whereby people strive to shape a relatively coherent and distinctive notion of personal self-identity and struggle to come to terms with and, within limits, to influence the various social-identities which pertain to them in the various milieux in which they live their lives. (Watson, 2008, p. 129)

The process of identity work has been studied often in mathematics education research (Graven and Heyd-Metzuyanim, 2019; Chronaki and Kollosche, 2019; Lutovac and Kaasila, 2018b; Lutovac and Kaasila, 2014). Mathematical identity work is the construction of narrative in relation to mathematical experiences. Lutovac and Kaasila (2014), who studied preservice teachers' future oriented identity work in mathematics education courses in Finland and Slovania, 'understand mathematical identities as storytelling' (Lutovac and Kaasila, 2014, p. 133). Their work found that when preservice teachers are doing mathematical identity work, the work leads them to establish a mathematical identity. Lutovac and Kaasila (2014) used narrative interviews of preservice teachers to investigate their mathematical identity work.

In their later work, Lutovac and Kaasila (2018b) investigated the narrative identities of fourteen preservice teachers in Finland. They employed biography to understand the identity work of one preservice teacher named Vesa. Lutovac and Kaasila's (2018b) study showed that Vesa's identity work expanded beyond mathematics, however the content of mathematics was a factor that shaped her narrative. This is considered identity work.

Chronaki and Heyd-Metzyvanim (2019) used discourse theory to examine the identity work of a 15-year-old German secondary school student named Anja, highlighting Anja's rejection of mathematics, or their dislike of mathematics as a subject. Although their study did not employ a narrative inquiry approach, their research provided a glimpse into an aspect of identity work that includes 'a bodily active and collaborative individual ready to espouse an alternative discursive materiality' (Chronaki and Kollosche, 2019, p. 466). In the next section, I show an example of how I employed narrative inquiry with teacher trainees using 3D printing pens.

4.2.4 **Story-to-live by: Teacher trainees mathematising using 3D printing pen**

In this section, I provide an example of 'story-to-live by' through observing six teachers training in a teacher training programme. To indicate how I constructed Chapter Five, I describe the stories-to-live-by as episodes. The goal of these episodes is to show teachers' mathematical conversations being formed as they engage with 3D printing pens in the learning-as-making activities. Below, I reveal the dialogue exchange among the six teachers (Paul, Ringo, Lisa, Hilary, Michelle and Madonna).

Episode I: Using 3D printing pen in learning-as-making activities

Figure I: Paul using the 3D printing pen

Matt:	How do you feel with [using] the [3D printing] pen?
Paul:	Yeah, it feels alright. It feels quite chunky, but it's all right
Matt:	How can you see this, use this in a classroom?
Paul:	In a classroom? I guess we talk about perimeter, I guess, so you can visually see, yeah, the perimeter of a shape, and then if you do 3D, which I haven't done yet, I guess you could see the 3D models
Matt:	Okay
Paul:	That's useful
Matt:	How is that useful?
Paul:	Well, if there's a cube, you can count how many faces are on the cube to work out the surface area and things like that
Matt:	Okay
Paul:	That's one thing that kids struggle with, working out that there's six faces. Some of them forget to do all six for cube
Matt:	How would you draw a cube?
Paul:	How do I draw a cube on this? I haven't. We'll get there

Turning to the classmate next to Paul, I asked about the shape he was constructing with the 3D printing pen.

Figure II: Ringo's rectangle shape

Matt:	And this one here, what is that, by the way?
Ringo:	I have no idea. I like this. It's a rectangle, but I've seen a lot of videos of these, and to build structures they're going to do cross-sections

I then walked across the room to Madonna to observe her engagement with the 3D printing pen.

Figure III: Madonna's attempted circle

Matt:	What are you up to?
Madonna:	I was trying to draw a circle, but I found it hard to control the speed of the pen. It was either too fast or too slow

Afterward, I walked to the next station where Hilary and Michelle were using the 3D printing pen.

Hilary:	I did a triangle
Matt:	How do you feel about using [a 3D printing pen], by the way?
Hilary:	It's pretty hard. I'm struggling to get it to stop, like stick at the end. Just showing that, that happened the first time
Matt:	Did you draw this using the table or the [silicone mat]?
Hilary:	The mat. Yeah, I couldn't do it on the table. It wouldn't stick
Matt:	Okay
Hilary:	I just found it makes it flow on top of [mat]

I returned to Ringo as I saw him using a 3D printing pen for fun to draw his classmate's face on the table.

Matt:	Who are you drawing?
Paul:	That's me
Ringo:	He requested it
Matt:	What other [subjects], besides from mathematics, how can 3D pens relate to other curricula, other topics, out of curiosity?
Ringo:	Can imagine them using it a lot in DT, various building things
Matt:	Are these [3D printing pens] easy to use?
Ringo:	No

Paul: When you get the hang of it
Matt: Okay. Now, is there time in the curriculum to get the hang of it, though?
Paul: It depends how long it's going to take us to get the hang of it. I'm improving
Ringo: We haven't been at it long, have we?

Figure IV: Ringo draws Paul's face using 3D printing pen

Paul: I've done the glasses
Ringo: You need to take a photo of that

Constructing from 2D to 3D using 3D printing pen

Hilary asked Michelle about what she is making using the 3D printing pen.

Michelle: I'm just going to make four triangles and then I'm going to stick them all together
Hilary: What do you call that?
Michelle: The triangle-based pyramid. It has a name, doesn't it?
Hilary: Yeah. I don't know. I can't remember

In the last ten minutes of the session, I ask the class again how they like working with the 3D printing pens.

Matt: How do you feel using 3D [printing] pens?
Ringo: They're fun. I think there's ease for frustration if you're struggling to get it to work. They're quite complicated to start with, and once you get them... I can imagine a lot of students get frustrated
Matt: Would the frustration hamper their learning? Or would that matter?
Ringo: I think it depends on the students. I think if you've got a driving passion for math, frustration's almost motivating to go through it, but then some kids would get frustrated and then they'd be like, "No"
Paul: Some would struggle to see, I guess, the relevance to... I don't know. It depends on the teacher's sell. If you didn't sell it well, then they probably

	will wonder why they're doing it, particularly if they are struggling or frustrated
Matt:	You guys see 3D pens in your future class?
Paul:	No
Ringo:	I don't see how it could be feasible and practical… they all need time to learn to use it before we can even get on to the math. There's quite a few like embarrassed
Matt:	What about after school activity?
Ringo:	More likely, I think, if there's a passion for this sort of thing. And then I think you can link that with other subjects
Paul:	I think for an extracurricular day, the drop-down days, the kids would like it. I couldn't see it regularly being used in math
Matt:	Okay
Paul:	I think they'd love it. I think some kids would really love it, your creative sorts
Ringo:	It's amazing
Matt:	What do you love about it?
Paul:	The creativity
Matt:	Creativity
Paul:	Yeah, just playing with it, like to see what you think of it
Matt:	Okay, what have you made?
Paul:	I've made quite a few things
Ringo:	See how well people have done, and then that gives an idea, because personally, you're skilful at it

Discussion

This episode could provide information on the value of 3D printing pens for teaching mathematics applications. Analysing these six teachers as they train with 3D printing pens to prepare for secondary mathematics instruction in the United Kingdom might also offer valuable feedback on the relation between 3D pen functionalities, mathematising, and the mathematical content of the English national curriculum. Paul made a connection between the 3D printing pen and the curriculum when he talked about how students might discuss the perimeter of shapes or create in three dimensions. He saw the potential learning opportunity of transitioning from drawing a 2D shape using the 3D printing pen into creating an entire 3D model. Ringo made an interdisciplinary connection seeing that 3D printing pens can be employed in a design and technology classroom. Hilary mathematised using the 3D printing pen to construct four triangular shapes to make a triangular based pyramid. Her classmate Michelle had to remind Hilary about the name of the shape that she constructed. Ringo claimed that he does not see how 3D printing pens can be implemented in mathematics classrooms as students would need time to learn how to use the 3D printing pen. However, Paul does see the creative value of implementing 3D printing pens for extracurricular activities.

In this observation, the teacher trainees were situated in groups of two sitting throughout the room. It was a challenge to observe how each teacher trainee was constructing their shape from scratch. The teacher trainees' episode informs us about how they mathematised, and through their dialogue we see the various functionalities of the 3D printing pens. However, their stories and dialogue alone, though excellent analytical tools, do not do justice to their making processes and the mathematising that occurred while creating with the 3D printing pens. Below are pictures of the 2D and 3D shapes constructed by the group (Figures V a-c), which help us visualise their stories and conversations.

Figure V-a

Figure V-b

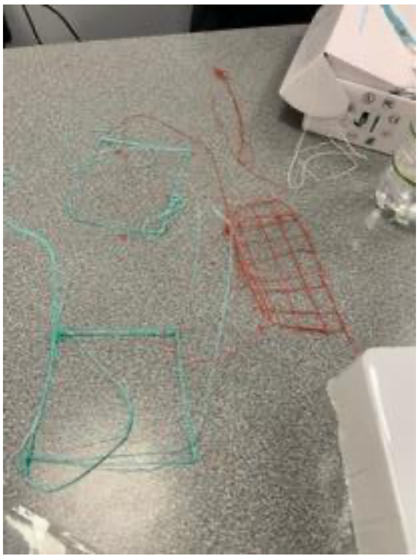


Figure V-c

These images are examples of how attention to detail is needed in order to fully assess the ways the teacher trainees mathematised and the mathematical thinking processes behind their making practice. To understand further how in-service teachers in this study are mathematising and making using 3D printing pens, I discuss diffraction as a methodological approach in the next section.

4.3 Diffraction as a Methodological Approach

4.3.1 **The metaphor of diffraction**

In this study, I adopt the concept of diffraction as a methodological approach for understanding the complex relationships between data, materials, ideas, people, and performative processes of creating. Diffraction is commonly recognized in classical physics as the bending of waves as they move around a corner or through the opening of an obstacle. Theoretical physicist and feminist scholar Karen Barad (2007) applied this concept beyond the natural sciences with significant implications for the social sciences in *Meeting the universe halfway: Quantum physics and the entanglement of matter and meaning*. In the previous section, I discussed how I employ a narrative inquiry approach to understand how individuals reveal meaning in their lives through narrative. The concept of diffraction builds on this methodology, allowing for 'reading insight through one another' (Barad, 2007, p. 25). Diffraction methodology focuses on engaging with material objects in their interaction with the world. The concept of diffraction can be seen in the early works of feminist-materialist social theorists such as Donna Haraway, who in *Modest_Witness@Second_Millenium* (1997) comments that

> diffraction can be a metaphor for another kind of critical consciousness…one committed to making a difference…Diffraction is a narrative, graphic, psychological, spiritual, and political technology for making consequential meanings. (Haraway, 1997, p. 16)

Barad (2007) built on Haraway's idea, fully evolving the concept of diffraction from the sole standpoint of a physical phenomenon. In the social sciences, diffraction methodology is known as a 'critical practice of engagement' (Barad, 2007, p. 90). Barad (2007) uses this concept to understand how engaging with material objects has an impact on knowing-making practices. Barad (2007) provides the example of ocean waves passing through a barrier and suggests that 'diffraction will be an object of investigation and at other times it will serve as an apparatus of investigation' (p. 73). As the ocean wave hits the object, the wave is called the 'diffracted', while the object the wave has hit serves as the 'diffracted apparatus' for interfering with the ocean wave. To give another example, Figure VI below represents river waves hitting a rock in Yosemite National Park in California. In this example, the river waves are considered the diffracted and the rock is the diffracted apparatus.

Figure VI: River waves hitting a rock in Yosemite National Park

In mathematical-physics, the term diffraction can be traced back to the mid 1600s (Hecht, 2017; Hall, 1990). Diffraction, which at the time did not have a specific term, was first observed by mathematician and physicist Francesco Maria Grimaldi, when he recognised 'bands of light within the shadow of a rod illuminated by a small source' (Hecht, 2017). During the 1600s, there were four scientists who practiced diffraction: Issac Newton, Francesco Maria Grimaldi, Lyons Jesuit Honore Fabri, and Robert Hooke. According to Hall (1990), there was no recollection of who used the term diffraction as 'Grimaldi's term [was] diffractio, but Hooke did not use this word either, while Fabri mostly spoke of deflectio' (p. 18). There has been much scholarly debate about who actually coined the term diffraction. For the purpose of this study, I deploy the concept of diffraction as a metaphor. For this, I am inspired by the idea of diffraction when a wave hits an object. Barad's (2007) work is my starting point.

The diffraction phenomenon, according to Barad (2007), focuses on how material objects are inter-acting and interfering with the world. Ng and Ferrara (2020), for example, looked at 3D printing pens as diffracted apparatus in order to more clearly see how 3D printing pens are intertwined with the production of mathematical meaning. Barad (2007) states that 'knowledge practices have material consequences', but the more important point is 'that *practices of knowing are specific material engagements that participate in (re)configuring the world*' (Barad, 2007, p. 91).

As material objects interact with the world, objects can produce patterns. Barad (2007) uses an example of a light wave that hits a screen to produce various colours. The term diffraction means 'to break apart in different directions' (Barad, 2014, p. 168). The example of a light wave hitting a screen can be argued to be a reflection. Haraway and Barad use the concept of diffraction as a counter argument to reflection. To them, while the term reflection 'reflects the themes of mirroring and sameness, diffraction is marked by pattern of difference' (Barad, 2007, p. 71). When various colours were produced in the light wave example, 'diffraction effects are attentive to fine details' (Barad, 2007, p. 91). Diffraction is 'a mapping of interference, not of replication, reflection, or reproduction. A diffraction pattern does not map where differences appear, but rather maps where the effects of difference appear' (Haraway, 1992, p. 300). In understanding diffraction patterns, Barad (2007) notes that 'we can understand diffraction patterns – as patterns of difference that make a difference – to be the fundamental constituents that make up the world' (Barad, 2007, p. 72). A diffracted apparatus is used to create diffraction patterns, as seen in the example of the ocean wave and barrier earlier. In addition, 'diffraction gratings can be used to exhibit some of the smallest details of nature' (Barad, 2007, p. 91).

Diffraction is a methodological approach that Barad (2007) situated within agential realist ontology, and it is continuously formed through material entanglements. Agential realist ontology, a concept developed by Barad (2003) in earlier work, is described as an

> account of technoscientific practices, the 'knower' does not stand in a relation of absolute externality to the natural world being investigated—there is no such exterior observational point. It is therefore not absolute exteriority that is the condition of

> possibility for objectivity but rather agential separability—exteriority within phenomena. 'We' are not outside observers of the world. Nor are we simply located at particular places in the world; rather, we are part of the world in its ongoing intra-activity. (Barad, 2003, p. 828)

Barad (2007) contends that 'agential realist account scientific practices do not reveal what is already there; rather what is "disclosed" is the effect of the intra-active engagements of our participation with/in and as part of the world's differential becoming' (Barad, 2007, p. 361).

Barad (2007) discusses another important component of diffraction methodology as well, known as performativity. Barad (2007) introduces the concept of performativity to challenge the idea of representationalism, which means that 'representations and the objects (subjects, events, or states of affairs) they purport to represent are independent of one another' (Barad, 2007, p. 28). In terms of representationalism, there are pre-existent stories that can be told between a person and an object in a practice. Barad (2007) explains that

> representation raised to the nth power does not disrupt the geometry that holds object and subject at a distance as the very condition for knowledge's possibility. Mirrors upon mirrors, reflectivity entails the same old geometrical optics of reflection. (Barad, 2007, p. 88)

Indeed these ideas of representation and reflectivity contrast with the views of Barad and Haraway such that 'knowing does not come from standing at a distance and representing but rather [from] *a direct material engagement with the world*' (Barad, 2007, p. 49). To go beyond just reflection and representationalism, performativity is used to understand how someone's observation and thinking process with the world is part of the practice of engagement. Barad (2007) introduces a post-humanist performative approach that examines the 'matters of practice, doing, and actions' (Barad, 2007, p. 134). In relation to understanding diffractive methodology through practices of engagement, diffraction investigates 'the entanglement of bodies, texts, relationships, data, language, and theory' (Mazzei, 2013, p. 745).

In addition, diffraction methodology gives insight into how the body interacts with materials in the real world to produce something meaningful. In material-discursive phenomena, the body plays a critical role in discursive practices. Barad (2007) describes discursive practice in relation to agential realist accounts such that they 'are specific material (re)configurings of the world through which the determination of boundaries, properties, and meanings are differentially enacted' (Barad, 2007, p. 148). The activities that involve materials and bodies interacting to produce something meaningful is known as intra-action. The concept of intra-action consists of interactions between the material and the body. Intra-action allows us to understand the body movement and what constitutes meaning between the body and materials. Barad (2007) explains that 'the world is a dynamic process of intra-activity and materialization in the enactment of determinate causal structures with determinate boundaries, properties, meanings, and patterns of marks on bodies' (Barad, 2007, p. 140). To understand the concept of intra-action through text, Barad (2010) draws on diffraction as "reading texts intra-actively through one another" (Barad, 2010, p. 243). Using diffraction, one can become a diffractive reader who 'zoom[s] in on how texts, artefacts and human subjects interpellate or affect each other' (Van der Tuin, 2018, p. 100). In understanding the intra-activity concept of materials and body through text, I introduce the concept of performativity and how it can give insight into one's identity relating to mathematics.

4.3.2 The concept of performativity in mathematics education

In the lens of Barad (2007), performativity 'is precisely a contestation of the excessive power granted to language to determine what is real…performativity is properly understood as a contestation of the unexamined habits of mind that grant language and other forms of representation more power in determining our ontologies than they deserve' (Barad, 2007, p. 133). Many mathematics education researchers have used and defined performativity in various ways as it relates to identity (Gholson and Martin, 2019; Darragh 2015; Chronaki, 2011). To precisely understand the term performativity, I will build on Ball's (2001, 2003) use of performativity in his research on teachers. Ball (2003) stated that

> performativity is a technology, a culture and a mode of regulation that employs judgements, comparisons and displays as means of incentive, control, attrition and change based on rewards and sanctions (both material and symbolic). The performances (of individual subjects or organizations) serve as measures of productivity or output, or displays of 'quality', or 'moments' of promotion or inspection. (Ball, 2003, p. 216)

Ball (2003) discusses performativity by focusing on several different categories such as technology, the culture of competitiveness, and fabrication. These categories arise from one of his earlier works (2001) that investigated performativities and fabrication in education. Specifically, Ball (2001) looked at performative text and considered what it tells us about associating meaning. For example, Ball (2001) refers to fabrication not in the material sense but as textual, such that it provides '"new modes of description" and "new possibilities for action"; thus creating new social identities - what it means to be educated; what it means "to be a teacher" or a researcher' (Ball, 2001, p. 2). In addition, Ball (2003) noted that 'fabrications themselves become embedded in and are reproduced by systems of recording and reporting on practice' (Ball, 2003, p. 225). Similarly, Butler (1988), who investigated performative act and gender constitution, classified gender identity as a performative that claims 'the body is a historical situation, as Beauvoir has claimed, and its manner of doing, dramatizing, and reproducing a historical situation' (Butler, 1988, p. 521). Both Ball (2003) and Butler (1988) suggest that the concept of performativity is continuous. In employing the concept of performativity to understand textual language, I will give examples of how it is used in mathematics education to give insight into individual identity.

Chronaki (2011), who uses the concept of performativity in mathematics to investigate student identities within school mathematics, suggested that 'performativity refers to an embodied culturally scripted character of identity where its main focus is to expose hegemonic conceptions of identity as fictions generated by power through repeated reproductions of norms' (Chronaki, 2011, p. 209). Chronaki's (2011) study investigated performativity through a primary school arithmetic teaching experiment involving an 11-year-old student acting as an instructor, teaching his peers arithmetic. In Chronaki's (2011) study, she observed the children's worksheets and their actions in the classroom, such as writing on the chalkboard. The findings of her study show that performativity suggests that ‘identity-work becomes captured by what is already done in the past and what can be imagined for the future’ (Chronaki, 2011, p. 223).

Darragh (2015) employs a performative lens to investigate the identity of students ages 13 to 14 who are transitioning from primary to secondary school. Darragh’s study is interested in identifying how a student is good at mathematics, suggesting that ‘we become a mathematics learner in a performative manner, and it is the repetition of “performances” in mathematics learning contexts that generates our recognition of ourselves in certain ways as learners of mathematics’ (Darragh, 2015, p. 85). The term performative manner in Darragh’s work refers to how the students respond to mathematics in the classroom, such as when they raise their hands, and the content of their written work. In addition, Darragh uses the term performative scripts to discuss the action of performance in the dialogue of the students’ responses to the question about being good at mathematics. In the study, Darragh found that the students were using the agency of performance to construct their identity toward mathematics.

Lastly, Gholson and Marti (2019) employ the concept of performativity in gender and race, focusing on a black student named Cameryn who is a middle school student in the United States, age 13. Their study constructed performativity themes using the fabrication of text from the audio and video recording to understand Cameryn’s identity towards mathematics. In the dialogue text, they describe how Cameryn mentions that mathematics is difficult and she hated learning the topic. Ghoslson and Marti (2019) also followed Cameryn into the classroom and observed her body language and actions. Their results show that performativity is significant not only in the student’s response, but in her everyday actions in the classroom, both of which inform her identity. In the next section, I will continue with the concept of performativity using a diffractive reading lens, looking at how our body language and actions come about through text in mathematics education.

4.3.3 **Diffractive reading in mathematics education**

To understand through a mathematical lens how the body interacts with materials, diffractive reading presents a way to examine the learning of mathematics through artefacts such as drawing, written work, and textbook resources (Burnard et al., 2019; Ng and Ferrara, 2020; Palmer and Lenz Taguchi, 2013; Palmer, 2011). Burnard et al. (2019) employed diffractive reading as an analytical tool in observing the mathematics artwork of South African students from grade 8 to 12. Out of 500 drawings, 113 were selected for review in their chapter. Of these, they used only three drawings to present how diffractive reading was employed. The objective of the study was to understand how art informs STEAM in regards to knowing, being and doing. With each drawing, the associated artist gave a written statement about their drawing, reflecting on how it relates to mathematical applications such as the number line, shapes, and measurements. From their study, Burnard et al. (2019) found that employing diffractive reading does give insight into how matter and meaning are intertwined, and this informed their study of how mathematics and arts are intertwined.

Ng and Ferrara (2020), whose study looks at 3D printing pens in school mathematics of students aged 10 to 12 in Hong Kong, use diffractive analysis in their work to understand how students using 3D printing pens create new ideas relating to mathematics. They use diffractive reading to analyse the video of twenty-five students using 3D printing pens as an apparatus to create 3D shapes. In one episode of their study, a student is shown using a 3D printing pen to create a trapezoid; he discusses how he made it bigger. Diffractive analysis enabled Ng and Ferrara (2020) to explore learning-as-making through four themes on making: co-construction of meaning, mathematising, assemblage with technology, and inventing.

Palmer (2011) employs diffractive readings on stories that were written by 75 students in a childhood teacher education maths course in Germany from 2005 to 2007 and by 105 teachers in a general early childhood education programme in 2006. In her analysis, she regards the written stories as concepts of performativity and narrative stories as textual production. Palmer introduces the story of one of her participants, Ella, in which Ella talks about feeling happy to have mathematics class in the morning. Within the story, Ella mentions that she uses a fill-in-the-book to write her mathematics problems. According to Palmer, this mathematics book is an artefact that is an active performative agent. In addition, Palmer also considers the pencil, bookbag, and other school materials as active performative agents. Palmer's study uses multiple methodological strategies, from performative to diffraction, in understanding how the material-discursive plays a role in understanding mathematical learning.

In a later study, Palmer and Lenz Taguchi (2013) employ diffractive analysis in observing written stories of two schoolgirls' (grade 7 and 10) wellbeing within a school environment in Sweden. In investigating one girl's story, Palmer and Lenz Taguchi (2013) suggest that 'concepts such as mathematics become productive of difference that will come to matter in the experiences of the girl in the story, as bodily sickness and increased anxiety' (p. 677).

Similar to Palmer (2011), Palmer and Lenz (2013) looked at various active performative agents such as place, emotions, and body language in understanding the girls' development and in their study of the performative enactments of the two schoolgirls. In the four examples given in this section, I hypothesise that stories and artefacts play a role as active performative agents in understanding mathematical learning from a material-discursive lens. The four examples shown in this section are studies that employed diffractive reading and appear prominently in the literature on mathematical learning. I hypothesise that diffractive reading applied to the study of mathematics can provide insight into how to understand artefacts such as drawing and text.

4.3.4 **Diffractive reading as a tool to analyse mathematising using 3D printing pens**

In this section, I explain how diffractive reading is used in my study as an analytical tool to investigate in-service teachers mathematising while using 3D printing pens. Mengis and Nicolini (2021), who researched practicing diffraction in video-based research, suggested that diffractive reading 'involves also the participants and their intra-active responding, their choice of activity and location, their bodily discomfort, their growing confidence' (Mengis and Nicolini, 2021). Mazzei (2014) who investigated diffractive analysis saw diffractive reading as 'a moment of plugging in, or reading-the-date-while-thinking-the-theory, of entering the assemblage, of making new connectives (Mazze, 2014, p. 743). Ng and Ferrara (2020) studied learning-as-making in primary school mathematics and used diffractive reading of video clips of students creating using 3D printing pens. I use diffractive reading of video clips as well, capturing step-by-step moments of in-service teachers constructing 2D and 3D shapes using a 3D printing pen. Unlike Ng and Ferrara (2020), I am interested in learning through my research how in-service teachers are mathematising using 3D printing pens by capturing their hand movements working back and forth in multiple dimensions (0D, 1D, 2D, and 3D) to construct their 3D shapes. What follows is an example of how I employ diffractive reading in this study.

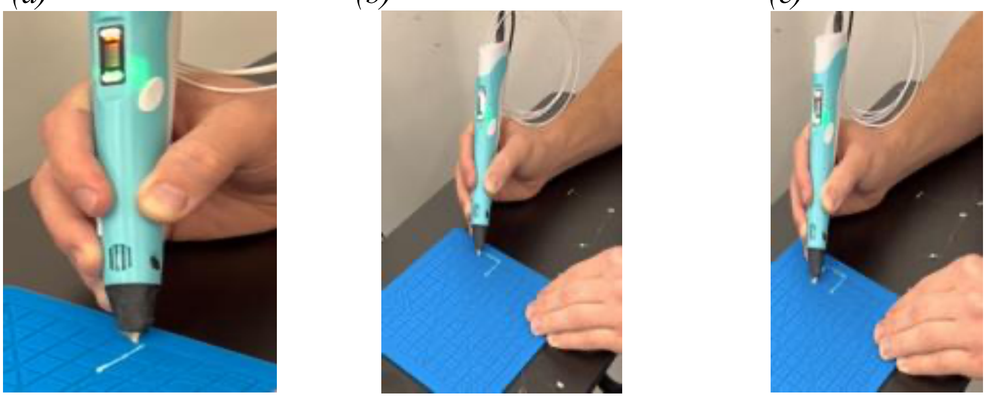
4.4 Employing diffractive reading of undergraduate students mathematising while using 3D printing pens

I now give an example of using diffractive reading as an analytical tool through my observation of two undergraduate students who are non-specialists in mathematics enrolled in a Calculus for Business module at an American university in the United Kingdom. The diffractive reading discussed in this section is of students utilising 3D printing pens to explore 2D and 3D representations of basic shapes through diffraction patterns. The term diffraction pattern in this case refers to when the 3D printing pen produces filament on the silicone mat or flat surface to construct the 2D and 3D visual representation (i.e., Figures VIII a-g). Similar to Ng and Ferrara (2020), I conducted a diffraction reading of a video recording of the two students' engagement with their 3D printing pens as they constructed 3D skeleton objects. The initial rationale for employing 3D printing pens in a Calculus for Business mathematical activity was the university's initiative for more interactive teaching. In addition, imagining basic geometrical shapes, such as a sphere, cylinder, or cube, is an important activity as students learn to differentiate 3D shape volumes in calculus word problems regarding related rates. I was able to perform the activity in question with 134 undergraduate students due to my access as a module leader and the head of mathematics at the university.

Of the 134 students, two undergraduates volunteered to be recorded. I focused on two students named Chandler and Rachel working together to construct a 3D skeleton cube. 3D printing pen activities were offered on an experiential learning day for one classroom session in Calculus for Business. During this session, both Chandler and Rachel played and engaged with the technology. However, they only constructed 2D shapes. Chandler had experience using 3D printing pens, though not in a mathematics classroom, and Rachel had never used one before the experiential learning day.

Chandler Constructing a Square Using a 3D Printing Pen

(a) *(b)* *(c)*



(d)

(e)

(f)

(g)

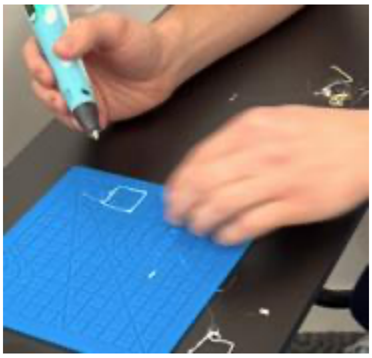
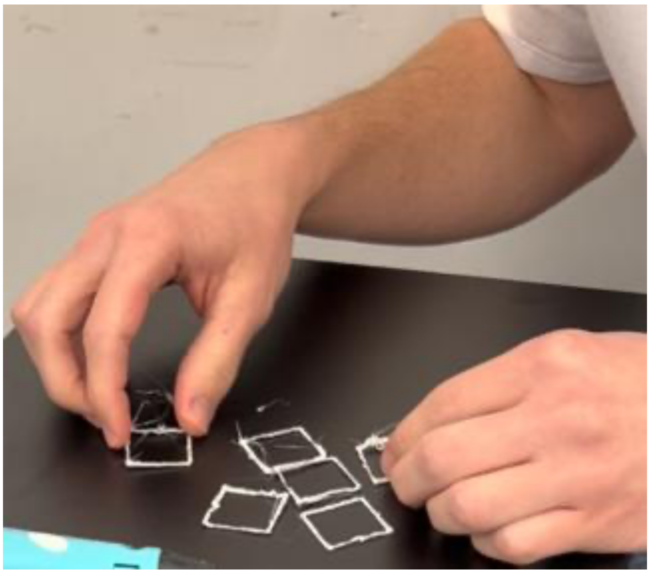


Figure VII (a-g) represents Chandler constructing his square using the silicone mat

Chandler was not instructed to use the silicone mat nor was he told about the dimensions of the square he would construct. He was asked to construct a 3D skeleton cube, and his first action was to create a square. Figures VII (a-c) show how Chandler smoothly moved through the silicone mat to construct his square. When Chandler was completing the construction of the last leg of the square (Figure VII-d), he noticed that the filament was sticking on to the 3D printing pen. As he pushed the filament back onto the silicone mat, Chandler raised his 3D printing pen to see how the square looked (Figure VII-e). Using the 3D printing pen as a glue gun, he connected the missing side (Figure VII-f). Figure VII-g represents Chandler's first completed square. Chandler mirrors the process of making a square another five times, as seen in Figure VIII.



FigureVIII: Chandler's six squares

In Figure VIII, there are six squares that Chandler created using the 3D printing pens. According to Koffa (1935), 'three-dimensional shapes are matters of organization in the same way as two-dimensional ones, depending on the same kind of laws' (Koffa, 1935, p. 161). Chandler will use the six squares in the next episode to form a 3D skeleton cube with the help of Rachel.

Chandler and Rachel Collaborating Together to Construct a 3D Skeleton Cube

Chandler used the six squares in the previous episode to form a skeleton 3D cube. In Figures IX (a-d) below, he attempts to combine two squares together.

(a) *(b)* *(c)*

(d)

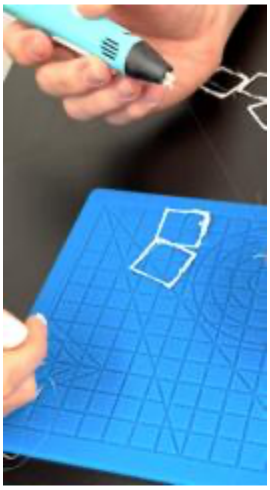


Figure IX (a-d): Chandler attempting to construct his 3D skeleton cube

In Figure IX-c, Chandler is attempting to use the 3D printing pen as a glue gun to trace over the filament to stick the two squares together. Chandler realised it was a struggle to stick two squares together just on his own (as seen in Figure IX-d). At this point, Rachel told Chandler to let her hold one square to enable Chandler to use the 3D printing pen to stick the squares together more easily, as seen in Figure X below.

Figure X: Rachel holds Chandler's square

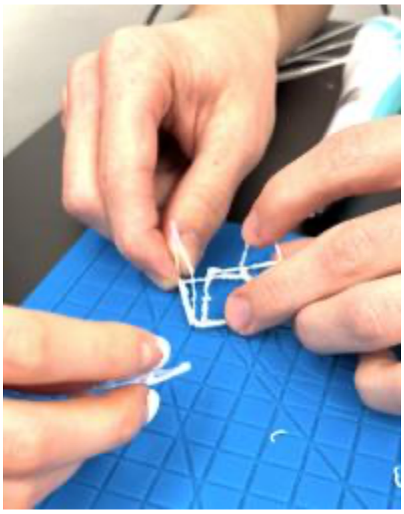
From this point, Chandler begins to use the 3D printing pen as a glue gun to complete the 3D skeleton cube, as seen in Figure XI.

Figure XI: Rachel holding the 3D skeleton cube

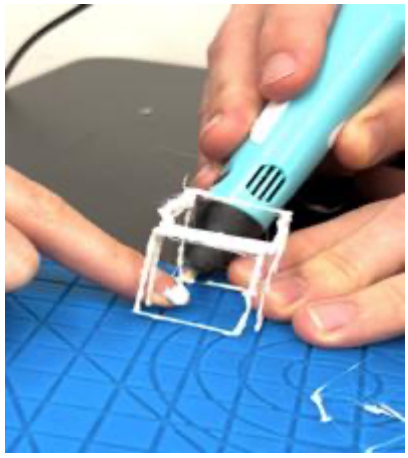
Both Chandler and Rachel continue this process to complete the 3D skeleton cube, as seen in Figures XII (a-g).

(a)

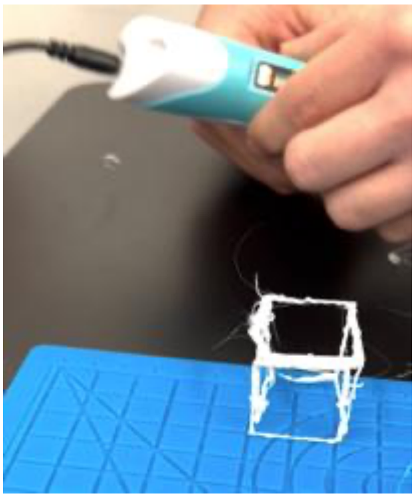
(b)

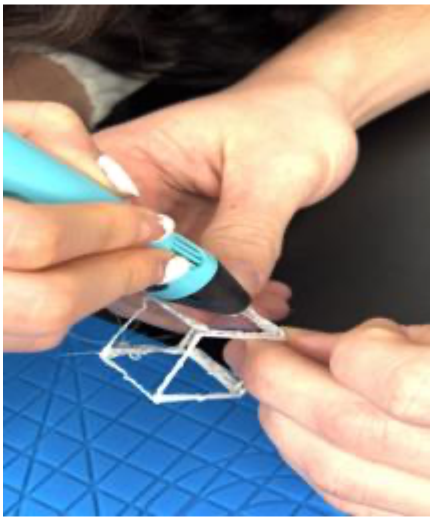


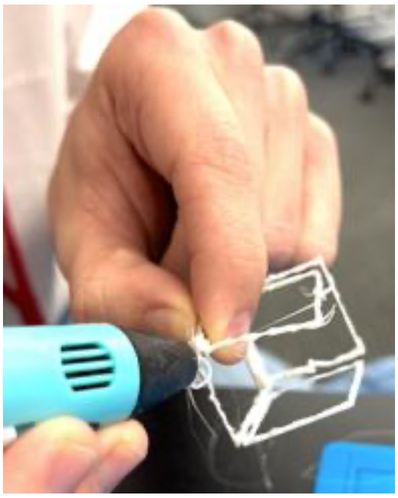
(c)



(d) *(e)* *(f)*







(g)

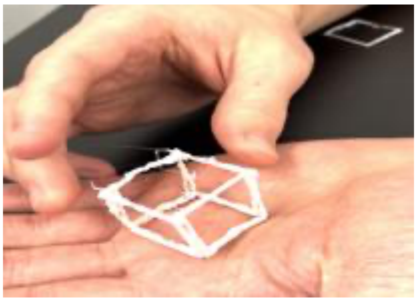


Figure XII (a-g): The process of Chandler and Rachel constructing their 3D skeleton cube

Rachel Constructing 3D Skeleton Shapes Using a 3D Printing Pen

Rachel attempted to construct a square using the silicone mat but found it a challenge to trace with the 3D printing pen on the mat. It was her first time using the silicone mat as she did not use it when she was playing and engaging during the experiential learning class time. Instead, she used the flat surface of the desk, as seen in Figures XIII (a-c) below, to construct her 3D skeleton cube.

(a) *(b)* *(c)*

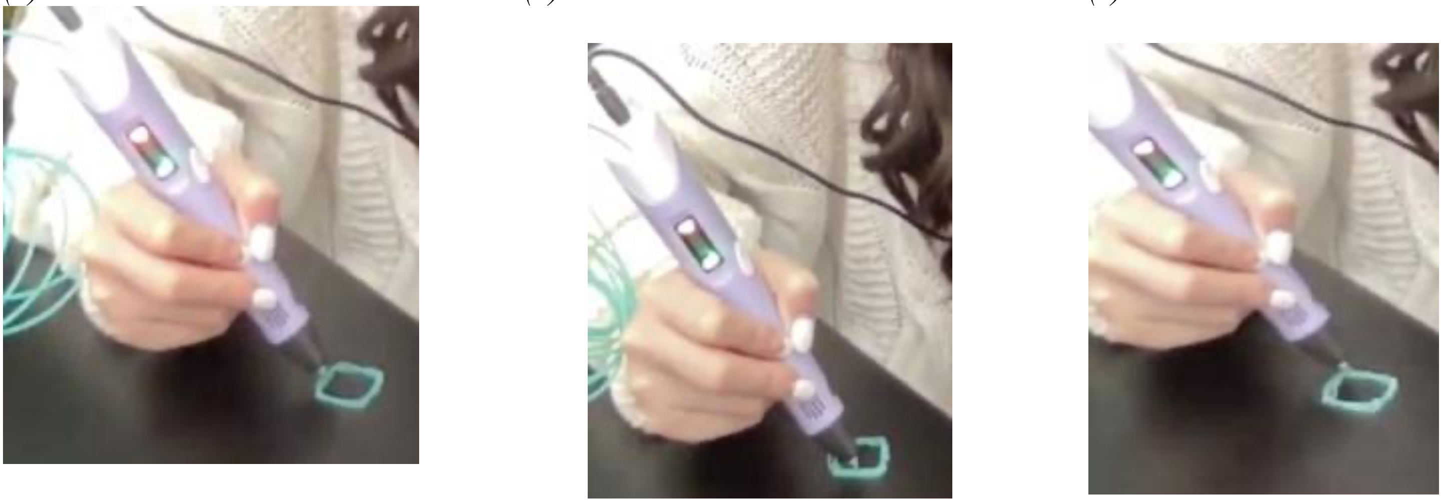


Figure XIII (a-c): Rachel constructing a 3D skeleton cube starting from a square

Rachel continues this process for approximately three minutes, as seen in Figure XIV (a-c), tracing the square in the same manner as a MakerBot 3D printer does.

(a) *(b)* *(c)*

Figure XIV (a-c): Rachel constructing a skeleton cube

Rachel called her 3D skeleton shape a ‘failure cube’ because there are two sides of the constructed model that are ‘open’ and four sides that are ‘closed’. She then thought about using one of the open sides to construct a pyramid skeleton shape, as seen in Figures XV (a-c).

(a) *(b)* *(c)*

Figure XV (a-c): Rachel attempting to construct a skeleton pyramid

In Figure XV-c, Rachel realises that her constructed 3D skeleton was not like a pyramid. She tries to think of a polygon name. This is where Chandler took over to call it a trapezoid, and Rachel agreed.

Discussion

Chandler and Rachel both had different shape perceptions of mathematising a 3D skeleton cube. According to Francis and Whiteley (2014), who investigated the interaction between three- and two-dimensional shapes,

> Recognition of the 3D object from its 2D representation cannot be assumed, and spatial reasoning about 3D objects requires an unambiguous reconstruction of the object (at least mentally). Moving back and forth between 2D representations and 3D objects requires acculturation to conventions used, and developed practice of when and how to move between 3D and 2D. Fluency with moving between 2D and 3D space is essential for reasoning and connecting representations of scientific concepts. (Francis and Whiteley, 2014, p. 134)

In addition, working together allowed Rachel and Chandler to collaborate for a common purpose of completing the task of constructing a 3D skeleton cube through holding, touching, and feeling the object as they pieced the squares together to form the cube. Their mutual engagement through hand gesture movements is captured through the video clip images, which is the focus of this section. Notice how Rachel used the term 'failure cube' and attempted to construct different types of skeleton shapes. Here, Rachel's shape perception was continuously changing as she investigated her skeleton shape each time she paused using the 3D printing pen. In addition, she was influenced by Chandler and named her final shape a trapezoid. Dialogue is not a focus of this section but will be part of the episodes in chapter seven, where I discuss how dialogue informs participant mathematisation. In the main study, I employ diffraction reading as an analytical tool through taking screenshots of video recordings, showing how in-service teachers are mathematising using 3D printing pens. I conjecture that diffraction reading can offer valuable insight into in-service teacher mathematisation and I deploy this reading in the data and analyses that I present in chapters five and six.

4.5 Data Collection, Participants, and Learning-as-Making Activities

Data collection for the principal research of this dissertation includes audio and video recording of participants' engagement utilising both 3D printing pens and Tinkercad in a learning-as-making activity that was situated in the classrooms of participants' respective schools. The audio and video recordings also capture the researcher's engagement in the study through discussions that occurred with participants while they were involved in the activity. I focused on specific dialogue with participants that was situated in the activity. For example, I collected participants' conversations on their interaction with the 3D printing pens, I observed their actions to notice how they were using the 3D printing pens to construct their models, and I listened to their storytelling conversation about their professional and personal experiences. Six hours and forty-five minutes of transcript was gathered during the data collection process. Chapter five on mathematising presents episodes of key conversations and diffractive reading that could provide insight into participants' engagement in using 3D printing technology tools in the learning-as-making activity. In addition, audio and video recordings were made in both Beyoncé's design and technology after school programme and Taylor's reception classroom. Dialogue of the students in Beyoncé's classroom was collected and used in this thesis as well, with the consent of the students. Fieldnotes were taken and collected throughout the fieldwork. An optional survey questionnaire was also filled out by three of the participants.

This study observed six in-service primary teachers and one teaching assistant engaging with 3D printing technology tools in a mathematical activity. To help facilitate understanding of how the in-service teachers and teaching assistant mathematise in the study, I have organised them according to their respective schools, listed below in Table 1 with name changes for anonymity. Table 1 contains each school's name, participant names, school position titles, and hours dedicated to the mathematical activity. In addition, I added a table in the descriptive section for individual participants entitled 'data collected'.

School Name	Participant Name	School Position Title	Hours Dedicated
School of Gryffindor	Beyoncé	Head of Design and Technology	2 hours and 30 minutes
	Tracy	Culinary Teacher	1 hour and 30 minutes
	Joseph	Teaching Assistant in Design and Technology	1 hour
School of Ravenclaw	Taylor	Reception Teacher	1 hour and 30 minutes
	Emily	Reception Teacher	30 minutes
School of Hufflepuff	Elton	Primary School Teacher	1 hour and 15 minutes

Table 1: Participants' schools, positions, time allotted to the study

A description of each participant follows.

Beyoncé

Data Collected	Audio
	Video
	Classroom Observation
	Professional Development
	Survey

Table 2: Beyoncé data collected

Beyoncé is the head of design and technology in the School of Gryffindor. Beyoncé teaches years 7 and 8 design and technology topics. In addition, she runs a design and technology after school programme for years 7 and 8 students. In this programme, students design, play with, and construct objects using tools such as glue guns, cardboard, and wood available in the design and technology classroom. In her classroom, Beyoncé has two flashforge 3D printers that she used for her Tinkercad activity with years 7 and 8 students.

Beyoncé's academic background consists of English literature, architecture design education and creative practice research. Beyoncé chose to become a design and technology primary school teacher due to her architecture skills. As a design and technology teacher, she is not aware of the mathematics learning outcome for Key Stages 1-4 of the English National Curriculum. However, the activity she implements in her year 7 and 8 design and technology classroom does have application links to mathematical topics such as identifying and defining 2D and 3D shapes. For example, she employs Tinkercad for activities in which her students design and construct their 3D shape to create key rings or small buildings that are then printed using a 3D printer. Prior to this study, Beyoncé had never utilised a 3D printing pen in her design and technology classroom activities.

Tracy

Data Collected	Audio
	Video

Table 3: Tracy data collected

Tracy is a culinary primary teacher in the School of Gryffindor. Tracy trained for her culinary certifications in New York prior to joining the school. She is a mother of two young children and approached the mathematical activity through the lens of her own children's eyes, wondering how they would feel about playing with 3D printing pens. Tracy has no academic background in mathematics, and it was the first time she had used a 3D printing pen and the Tinkercad software.

Joseph

Data Collected	Audio
	Video

Table 4: Joseph data collected

Joseph is a teaching assistant under Beyoncé in the design and technology classroom. Joseph has a background in engineering and became a teaching assistant in the School of Gryffindor because his grandson attended the school. In his day-to-day activity, Joseph is in charge of setting up the design and technology classroom, which includes maintenance of the flashforge 3D printers and other hand-held tools, such as 3D glue guns, that are located in the room. He does not have any experience using a 3D printing pen in the classroom.

Taylor

Data Collected	Audio
	Video
	Classroom Observation
	Survey

Table 5: Taylor data collected

Taylor is a reception teacher for the School of Ravenclaw. Taylor pursued a career in reception teaching after she had home-schooled her two young children during the pandemic, at the time ages two and four. The two main skills that she learned during her pandemic home-school experience were patience and time management. She recognises these as important skills to have to become a reception teacher. During her PGCE (Post Graduate Certification in Education) training, she experienced working with early years and year 2 students. Based on this experience, Taylor felt working with early years students connected with her teaching style, which is creative and enthusiastic. Before pursuing a PGCE, Taylor got an undergraduate degree in Business Information Systems and a Masters in Business Information Systems. Taylor has no experience using 3D printing technology tools in the classroom. However, she follows Development Matters, which is a non-statutory curriculum guidance for the early years foundation stage (EYFS). This allows Taylor to understand the mathematics learning outcome that her students need to achieve by the end of the school year.

Emily

Data Collected	Audio
	Video

Table 6: Emily data collected

Like Taylor, Emily is a reception teacher for the School of Ravenclaw. Emily is in her tenth year of reception teaching. Emily has no experience using 3D printing technology tools in her classroom. She was recruited a year after Taylor, when she saw Taylor using a 3D printing pen to construct her skeleton cube. Due to her tight schedule, Emily was only available to participate in the first part of the learning-as-making activities. In addition, Emily is familiar with the mathematical content of the early years foundation stage (EYFS) framework.

Elton

Data Collected	Audio
	Video
	Classroom Observation

Table 7: Elton data collected

Elton is a year 5 primary teacher for the School of Hufflepuff. Elton teaches primary mathematics topics such as place values and timetables. He is familiar with Key Stages 1-4 of the mathematical content that is required learning for all students. In addition, Elton uses the curriculum from the National Centre for Computing Education (NCCE) where he employs activities on Tinkercad, such as creating name tags. Prior to our mathematical activity, Elton was aware of 3D technology tools such as 3D printing pens, Tinkercad and the MakerBot 3D printer. In addition, before pursuing a PGCE programme, Elton worked for a publishing company and specialized in electronic information. Our activity was the first time Elton had used a 3D printing pen to construct 3D skeleton models.

Learning-as-making activities content

The learning-as-making activities were designed using mathematical content from Development Matters – non-statutory curriculum guidance for the early years foundation stage (EYFS) and Key Stages 1-2 of the English national curriculum. Specifically, the mathematical content in the learning-as-making activities included:

1. *Spatial reasoning skills…such as shape, space, and measurements.* (EYFS, DfE, 2023)

2. *For ages 3 and 4, talking about and exploring 2D and 3D shapes (for example, circles, rectangles, triangles and cuboids) using informal and mathematical language, such as 'sides', 'corners', 'straight', 'flat', and 'round'.* (Development Matters, DfE, 2023)

3. *Recognising and naming common 2-D and 3-D shapes.* (Key Stages 1-2, DfE, 2023)

4. *Comparing and sorting common 2-D and 3-D shapes and everyday objects.* (Key Stages 1-2, DfE, 2023)

5. *Drawing 2-D shapes and making 3-D shapes using modelling materials; recognising 3-D shapes in different orientations and describing them.* (Key Stages 1-2, DfE, 2023)

Utilising 3D printing pens in learning-as-making activities

3D printing pens are hand-held devices that enable users to construct 2D and 3D skeleton shapes. The pens are connected to an outlet plug. Similar to a MakerBot 3D printer, a 3D printing pen has an extruder at the tip of the device that heats up to 180 degrees Celsius. To construct 2D and 3D skeletons, the user must first hold the 3D printing pen at a 60-degree angle. When the 3D printing pen reaches 180 degrees Celsius – this usually takes about one to two minutes – users can input the filament into the hole located at the top of the device. To construct 2D and 3D shapes, users can employ a 3D printing pen on a flat surface such as a desk, or they can use a 3D silicone mat.

There are several ways to construct 2D and 3D shapes using a 3D printing pen device. For example, let's think about drawing a triangle – which is a 2D shape – using the 3D printing pen. A user can think about the property of a triangle that includes three sides of equal length. To start, a user can draw a line with equal length three times. Then the user can construct the triangle through rearranging the three lines into a form of a triangle. Since the 3D printing pen produces extremely hot filaments, the device can serve as a superglue gun. This allows the user to produce filament at the endpoints of each line to connect the three lines and form a triangle. A simpler way to construct a triangle using a 3D printing pen is by drawing on the 3D silicone mat. The silicone mat features illustrations of a line, triangle, and square that users can easily trace while drawing. Most importantly, the silicone mat has measurements of centimetres and inches that allow the user to construct a shape with precise dimensions. When the user constructs a 2D shape of a triangle using the silicone mat, once the drawing of the shape is completed, the user can take the shape off of the silicone mat.

The description above is an example of constructing a 2D shape of a triangle. But what about constructing a 3D shape of a triangular prism? A triangular prism is formed by constructing three congruent rectangles, and two congruent triangles. For accuracy and precision, the 3D silicone mat would be ideal for a user to construct a triangular prism. The same methods apply for constructing a 2D shape of triangle, however, a rectangle consists of parallel sides congruent to each other. The user can construct three rectangles and two triangles using the silicone mat and rearrange the shapes together to form a triangular prism. In order for the shape to hold as the user rearranges it, the 3D printing pen can once again act as a glue gun. Constructing 3D shapes are not limited to the silicone mat or the flat surface. Users can also construct 3D shapes in the 'air'.

In this study, 3D printing pens were employed for participants to construct 2D and 3D skeleton shapes. Participants had the freedom to design any shape that came to mind but were given examples of shapes that could be created, including a circle, triangle, square, rectangle and skeleton cube. In addition, each participant was told how to hold the 3D printing pen and offered safety procedures, such as reminders not to hold the 3D printing pen from the extruder due to its extreme heat. This part of the activity lasted between 30 minutes to one hour.

This focus is appropriate for students in Key Stages 1-3. According to the Design and Technology Key Stages 1 to 3 curriculum, the objectives for students are as follows:

1. ‘Select from and use specialist tools, techniques, processes, equipment and machinery precisely, including computer-aided manufacture’ (DfE, 2024, Key Stage 3).

2. ‘Develop and communicate design ideas using annotated sketches, detailed plans, 3-D and mathematical modelling, oral and digital presentations and computer-based tools’ (DfE, 2024, Key Stage 3).

3. ‘Understand and use the properties of materials and the performance of structural elements to achieve functioning solutions’ (DfE, 2024, Key Stage 3).

Utilising Tinkercad in a learning-as-making activity

Tinkercad is an AutoCad programme that is free and has an easy-to-use interface. In this activity, each participant was given a nickname and assigned to their designated schools using the ‘join a class’ function on the Tinkercad website, as seen in Figure a.

Figure a: The user interface of Tinkercad in the Classrooms Webpage (Autodesk, 2023)

When participants were ready to ‘join a class’, they were brought to a webpage to enter the class code I had provided them, as seen in Figure b.

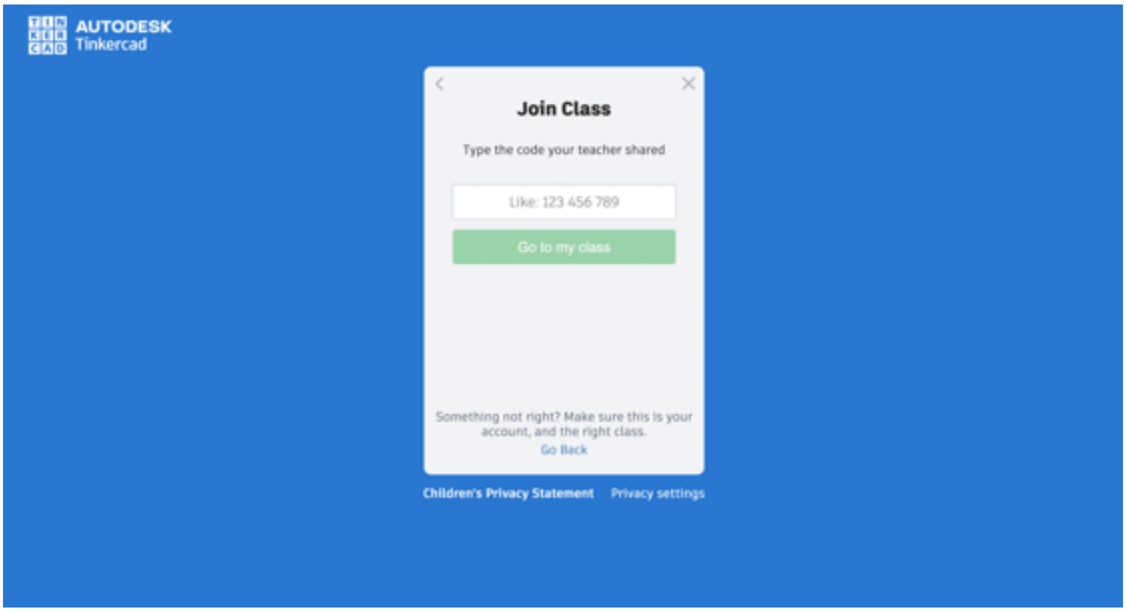


Figure b: Tinkercad user interface to enter class code (Autodesk, 2023)

From a researcher's perspective, I was acting like the participants' classroom teacher, showing them how to enter the class code or providing them a URL to type into their browser, as seen in Figure c.

Figure c: Tinkercad user interface Share link webpage (Autodesk, 2023)

When participants entered their classroom code, they were directed to their Tinkercad design webpage. Using the interface diagram from Pielsticker et al. (2021), seen in Figure d, I will explain the functionalities of Tinkercad.

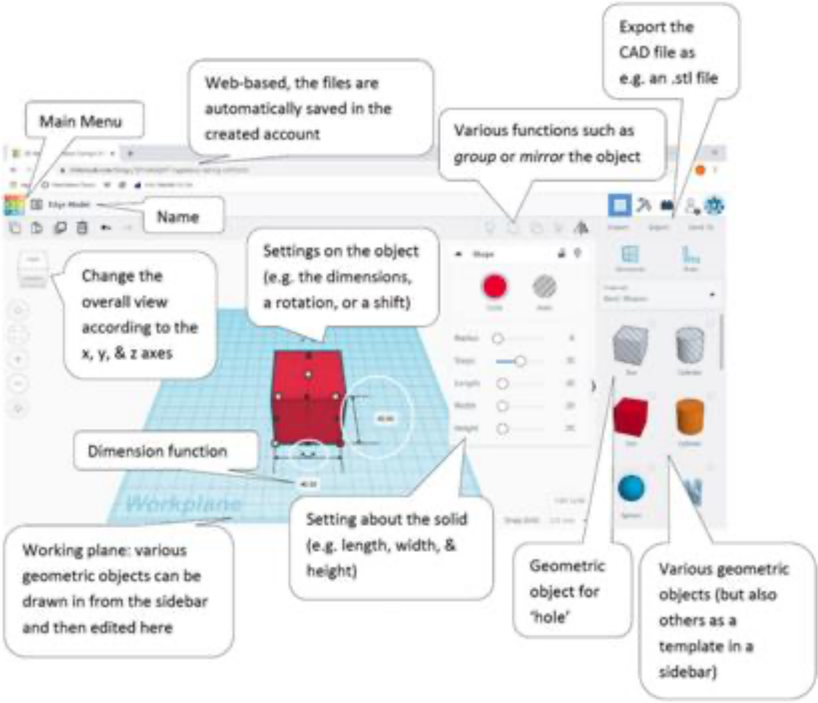


Figure d: The user interface of Tinkercad as seen in Pielsticker et al. (2021)

Once participants entered the Tinkercad design page interface, they were able to engage with Tinkercad functionalities. For a Tinkercad user to place an object on the plane, they first must use the sidebar to find the 3D object, then click and hold to drag it to the workplane. Once the object is placed on the plane, the user is able to see its dimensions in millimetres. Tinkercad functionalities provide the object dimensions so that users can adjust the measurements for the length, width and height. In addition, users can change the colour of an object or the hole function. The hole function on Tinkercad is only used to group objects that will make a hole or remove material in another 3D object, as seen in Figures e, f, g, h and i.

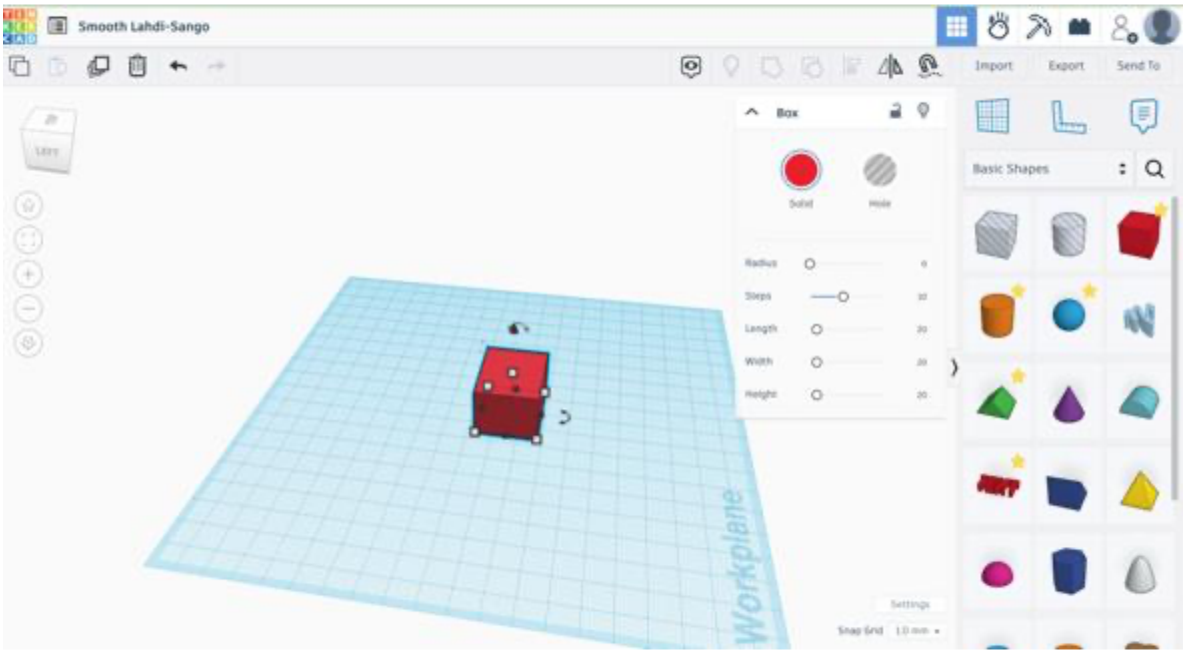


Figure e: A box is placed on the workplane (Autodesk, 2023)

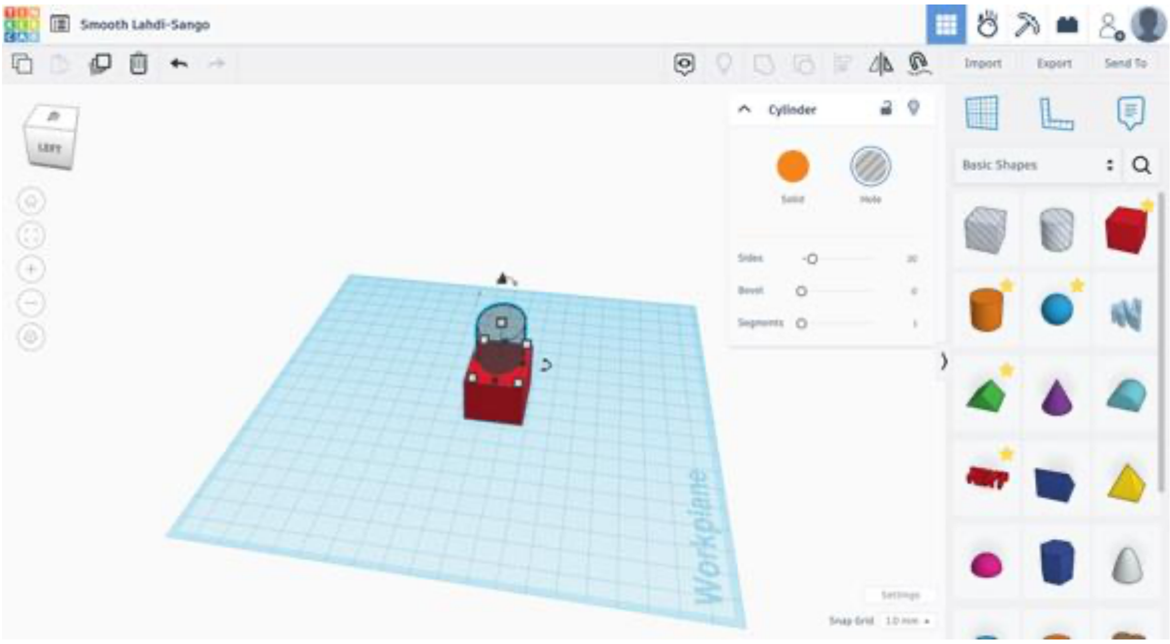


Figure f: A cylinder that is selected using the hole function is placed in the centre of the box (Autodesk, 2023)

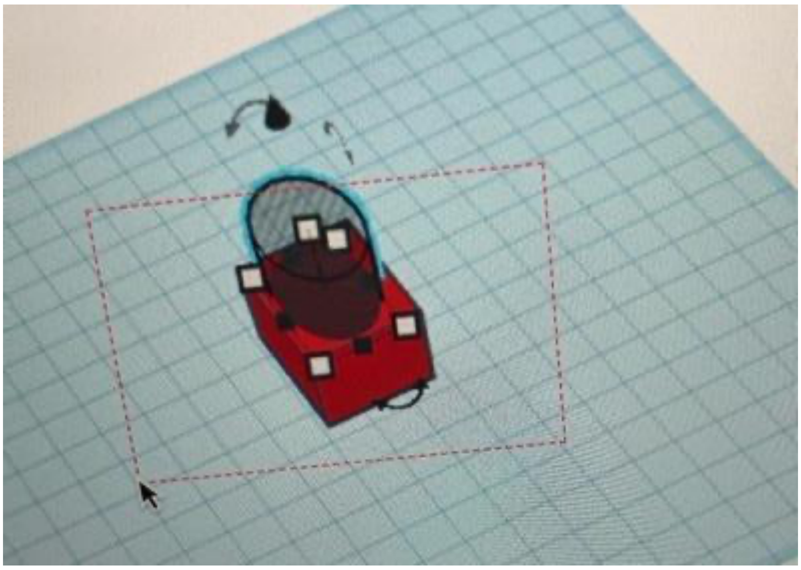


Figure g: Selecting the area on the plane that has both the cylinder hole and box (Autodesk, 2023)

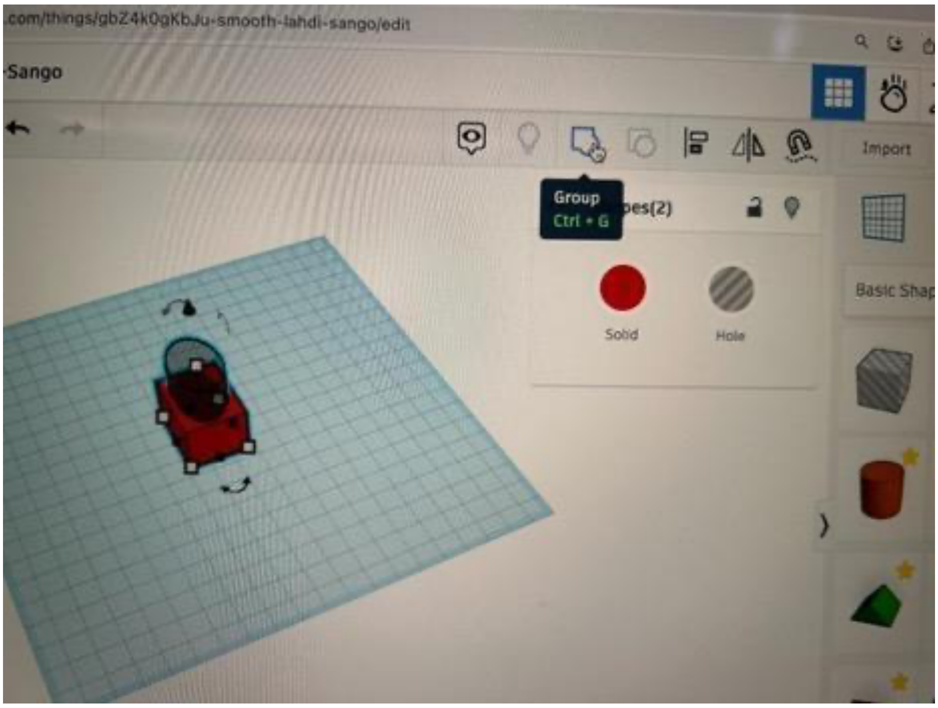


Figure h: Click on the group function to create a hole in the box (Autodesk, 2023)

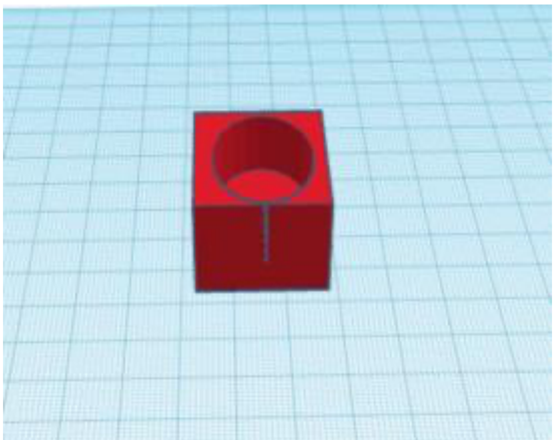


Figure i: The box with a hole of a cylinder shape (Autodesk, 2023)

While this feature is a useful tool for 3D printing, the activity in my study did not need the hole function of Tinkercad because my research focused on how participants manipulate 3D shapes. This focus is appropriate for students in Key Stages 1-2. According to the Computing Key Stages 1 to 2 curriculum, the objectives for students are as follows:

1. 'use technology purposefully to create, organise, store, manipulate and retrieve digital content' (Computing Key Stages 1 to 2, DfE, 2013).

2. 'recognise common uses of information technology beyond school' (Computing Key Stages 1 to 2, DfE, 2013).

3. 'understand computer networks including the internet; how they can provide multiple services, such as the world wide web; and the opportunities they offer for communication and collaboration' (Computing Key Stages 1 to 2, DfE, 2013).

Combining the objective for the mathematical content and computing content for Key Stages 1 to 2, this study utilised Tinkercad to observe how primary and reception teachers might employ Tinkercad in their own classrooms. The shapes that the participants were told to engage with on Tinkercad included box, sphere, cone, and roof. This part of the activity lasted between 30 minutes to one hour.

Utilising Makerbot 3D Printer in learning-as-making activities

The purpose of the Makerbot 3D printer in the learning-as-making activities is to ask in-service teachers how they see the 3D printing pens and Tinkercad connecting with a 3D printer, such as the Makerbot, in their classroom. Solid objects, such as a solid cube, were shown to the in-service teachers, who were told that the objects were printed using the Makerbot 3D printer. I am interested in hearing how 3D printers, such as the Makerbot 3D printer, could make an impact on learning in regard to in-service teacher specialisations and mathematical content.

4.6 Data Analysis

In this study, I collected data from six participants, using video recordings of their involvement in a learning-as-making activity that employed 3D printing pens. Detailed transcriptions of their dialogue (Story to Live By) and close observation of video clips showing their engagement in the activity (diffraction reading) tells a story of participants' mathematising. I extracted specific dialogues from the raw transcripts of the six participants to create a theme for each of the episodes, shared in the next chapter. To assist in analysing the dialogues, I look at the associated video recordings and share screenshots to provide visual representation of the participants' experience. There are two types of analysis that I employ in this study. These include diffractive analysis (Mengis and Nicolini, 2021; Mazze, 2014) and thematic analysis (Riessman, 2008). Below I explain the rationale for using diffractive analysis and thematic analysis to answer my research questions. I have three research questions that I explore throughout the study, and these are:

R.Q.1: How do in-service teachers mathematise using 3D printing technology tools to connect with the mathematical potential of their activities?

R.Q.2: How do in-service teachers' mathematising in learning-as-making activities impact their teaching identities?

R.Q.3: What does a diffractive reading of in-service teachers' work with 3D printing pens tell us about their mathematising?

My interest in diffraction analysis began when I studied undergraduate students using 3D printing pens to construct 3D skeleton cubes. Diffractive analysis can be used to closely look at the interaction among participants using the 3D printing pens. Conversations are observed, but equally important is the way in which participants move their hands in response to others, interact with their device, and position their bodies. Diffraction analysis could provide insight into all three research questions, aided by screen shots of the video recording of participants mathematising using their 3D printing pens. Once I had taken numerous screen shots from my fieldwork recordings, I arranged them in order from the beginning of the participants' 3D skeleton construction to the end, showing a story of how participants mathematised.

Thematic analysis is used in this study to attend to the mathematical meaning created by participants as they mathematise when designing with their 3D pens. This is done by studying the raw transcripts to find themes and create episodes around these themes. The transcriptions include both parts of the mathematical activity – the interaction with the 3D printing pen and the interaction with Tinkercad. Themes were produced based on the conversations that were taking place during the activities. These themes develop a story. According to Riessman (2005), 'narratives do not speak for themselves or have unanalysed merit; they require interpretation when used as data in social science' (p. 2). Pre-and-post interviews were not conducted in this study. However, I asked open-ended questions during the activity as participants were engaging with 3D printing pens. This approach is similar to that of Ng and Chan (2021), using thematic analysis to gain a deeper interpretation and understanding of the significance of 3D printing pens in the teaching and learning of mathematics. Using thematic analysis enabled me to investigate the transcripts more deeply and organize my findings more effectively to reveal the participants' mathematising while using 3D printing pens. This approach can help me answer the first two research questions (R.Q.1 and R.Q.2).

4.7 The Role of the Researcher

In this section, I discuss my role as the researcher in this study. The six participants in this study were working as classroom teachers in primary and early years schools. I am familiar with teaching and the situation of classroom teachers as I was a lecturer of mathematics in the United States and am currently a head of mathematics at a teaching university in the United Kingdom. These experiences allowed me to have a meaningful and deep connection with the participants as we could relate to one another as teachers.

When implementing the mathematical activity for the study, I did not interfere with participant mathematisation as they worked with the 3D tools and engaged in making 2D and 3D shapes with their printing pens. I did ask questions about their mathematisation as they worked, such as inquiring how they were constructing their models, what they thought about the 3D printing technologies as they used them, and what their journeys as classroom teachers had been. I did not provide any feedback on their construction of 3D shapes, nor about their journeys in becoming classroom teachers. When it came to implementing the activity with young children, I had to remind the participants to talk about the activity with their parents beforehand, and I had to walk around the room during the activity to ensure that the children were using the 3D printing pens safely. While having a background in teaching mathematics was beneficial to build a rapport with participants, potentially helping them feel that I understood their day to day life as teachers, I nonetheless took precautions to ensure that my presence did not influence the participants' decisions throughout the study. I discuss these ethical considerations in section 4.11.

4.8 Trustworthiness

Since this thesis is interested in the stories formed by participants' mathematisation, in this section I will discuss the trustworthiness of my study. Connelly and Clandinin (1990), who investigated stories of experience and narrative inquiry, advised that 'like other qualitative methods, narrative relies on criteria other than validity, reliability, and generalizability. It is important not to squeeze the language of narrative criteria into a language created for other forms of research' (Connelly and Clandinin, 1990, p. 7). There are several strategies that can help ensure the trustworthiness and credibility of results (Lincoln and Guba, 1985; Creswell and Miller, 2000). Below, I explain four of the strategies that I used and that best relate to this study:

1. **Data Triangulation**: I employ data triangulation in this study by using various materials to analyse the dialogue and participants' engagement with 3D printing pens. These include raw transcription of participant dialogue during the activity, multiple screenshots of videos of participant hand gestures and body positions, and images of participants' 3D constructed models.

2. **Researcher Reflexivity**: I establish a clear description of my role as a researcher in section 4.9 in relation to analysing and collecting the data in this study.

3. **Peer Debriefing**: 'A peer review or debriefing is the review of the data and research process by someone who is familiar with the research, or the phenomenon being explored' (Creswell and Miller, 2000, p. 129). I have discussed at length with my supervisor and members of the Research in Mathematics Education (RME) group ways to reduce bias in the analysis of my data. In addition, I have discussed findings of the data at regional conferences, including the BSLRM (British Society of Learning and Researching in Mathematics).

4. **Thick, Rich Description:** I provide a thick and rich description of data in chapter five of this thesis. The analysis chapter (Chapter 5) includes participants' dialogues during the activity, images of their hand gestures and sometimes body positions as they designed with their 3D pens, and images of their 3D constructed models. The reader has the freedom to decide the applicability of the data interpretation (Creswell and Miller, 2000).

4.9 Collecting data and conducting fieldwork in unprecedented times

The narrative account I present in this section tells a story of how I recruited my six research participants. Before the idea of using 3D printing pens had come about, I entered the doctoral programme in 2019 with a thesis proposal to utilise Tinkercad and a Makerbot 3D printer in a PGCE primary programme. Little did I know, our lives – the way we lived, researched, and learned – would change with the COVID-19 pandemic beginning in March 2020. My study involved in-person interaction with participants to allow them to engage, design, and play with 3D printing technology tools, but this kind of research work suddenly became prohibited because of the pandemic.

In the 2020-2021 academic year, e-learning (or online learning) became a major method of delivering school content to post graduate students at home (Naidoo, 2020; Singh-Pillay and Naidoo, 2020). Naidoo (2020), for example, conducted research on students in a post graduate mathematics education course in South Africa during the pandemic. They found that e-learning had benefits, such as the freedom to upload and post materials that were easily accessible to students. However, post graduate students faced challenges in adapting to e-learning, such as learning in a new platform and equity concerns regarding 'the availability of technology-based tools to engage with digital platforms when learning' (Naidoo, 2020, p. 7). In Singh-Pillay and Naidoo's (2020) study, they observed three lecturers in a teacher training university in South Africa and studied how they conducted an e-learning classroom. Singh-Pillay and Nadoo (2020) found that the lecturers faced challenges in understanding their students' competency in learning remotely as there was a 'lack of interpersonal interaction and engagement during online teaching' (Singh-Pillay and Nadoo, 2020, p. 1134).

In my own experience, I found that it was a challenge to implement my study in a PGCE programme in the United Kingdom. Course leaders in PGCE programmes were adapting to a new way of delivering content to teacher training students. When asked to participate in my study, PGCE programme leaders did not want to commit to the study as PGCE students were pressed for time due to their own in-school placements and the hours dedicated to their modules. I do believe this was a blessing in disguise, however, as it was in March 2022 when my supervisor allowed me and another colleague to host the Research in Mathematics Education seminar.

This was a turning point in my PhD study as it was in this research seminar that I came up with my focus on utilising 3D printing pens in a mathematical activity. At this moment in time, I had already sent hundreds of emails to PGCE course leaders in various institutions in the United Kingdom. The article that I presented in the research group was entitled *Drawing in space: Doing mathematics with 3D pens* by Ng and Sinclair (2018). I found this article to be a turning point as I fell in love with the idea of using 3D printing pens in mathematics education. While reading the article, I realised I could expand my study to include a mathematical activity incorporating both 3D printing pens and Tinkercad. In addition, because Ng and Sinclair's (2018) study focuses on primary student participants, I realised that I could use 3D printing pens in primary schools, which would greatly expand my participant base. In the spring of 2022, I reached out to primary school mathematics teachers to invite them to participate in my study. Similar to the responses received from PGCE course leaders, most of the primary mathematics teachers did not feel they had enough time to participate in the study due to teaching and administrative duties at their schools. In addition, I found that some primary mathematics teachers did not recognize the mathematical component in the study. The mathematics integral to the proposed activities was not transparent to them. The in-service teachers who participated in my pilot study also felt this way and I will elaborate on this in the section where I discuss the pilot study.

A key issue here is the transparency of the mathematics in the study. I turned my focus to recruiting teachers at the primary and early years levels as they utilise mathematics content in their own disciplines, such as design and technology, and in the reception classroom. There were two types of schools I targeted to recruit participants – state schools and private schools in the Greater London area. I chose London as my location due to the large number of schools in the area compared to other places in the United Kingdom such as Norwich, Cambridge, and Liverpool. When reaching out to primary and early years teachers, I made one promise: '*I will bring the 3D printing pens and Makerbot 3D printer to your classroom for you to utilise in the study'*. In Figure XVI, you can see the Makerbot 3D printer, which I purchased with a university grant, occupying space on the Tube.

Figure XVI: Travelling with Makerbot 3D printer in the London Metropolitan Underground Line

Behind my promise was a motive to equalise opportunities for the school participants in my study. None of the three participating schools (pseudonyms Gryffindor, Ravenclaw, and Hufflepuff) owned 3D printing pens. The school of Gryffindor is a private school that has a surplus in school funding. For example, they have a design and technology classroom with 3D printers and glue guns, and a culinary room that teaches children how to cook at a young age. In the schools of Ravenclaw and Hufflepuff, on the other hand, funding was tight. These are low-income state schools that struggle to get teachers basic supplies such as markers or notebooks. In these low-income state schools, the teachers are not only dealing with the lack of funding, but also teaching students from low-income families who potentially battle mental health issues (Adegbove et al., 2021; Kurian et al., 2022; Brännlund et al., 2017). Brännlund et al. (2017), who investigated poor mental health in upper secondary schools in Sweden, found that ‘failures in school could lead to mental-health problems’. Similarly, Adgebove et al. (2021) observed primary school students who come from low-income families in the United Kingdom and found mental health challenges relating to the pandemic. They argued that there was an increase in anxiety in school due to the students’ parents’ financial struggles and resulting mental health issues. The idea of delivering educational 3D printing technology tools into classrooms was exciting to me as it would enable me to potentially bring enjoyment to students’ learning. In the next section, I discuss the limitations of this study.

4.10 Limitation of Methodology

In this section, I discuss the limitations of this study in employing narrative inquiry and diffraction as methodological tools on a small participant pool of six in-service teachers. This study took place in the midst of the COVID-19 pandemic and involved participants engaging directly with physical technology tools (such as 3D printing pens) that posed a risk of transmitting the disease during that time. Certainly, the small sample size findings of this study should not be generalisable to a larger data set. According to Creswell (2012), ‘small samples may not be representative of the larger population, and findings from the study may not apply to others’ (Creswell, 2012, p. 186). This study focuses on in-service teachers who are non-mathematics specialists and the findings may not represent in-service teachers at the primary and early years levels who could have a specialisation in mathematics.

Another limitation of this study is the number of hours participants dedicated to the study. Narrative inquiry usually involves in-depth analysis of participants’ pre-and-post interviews to tell a story. This study did not have pre-and-post interviews given the time constraints that the participants had due to their obligations as classroom instructors. All participants in this study were involved in adapting to teaching in a post-pandemic world, which entailed new methods of conducting almost every aspect of their courses. However, intertwining narrative inquiry and diffraction allowed me to form a story of participant mathematising using 3D printing technology tools. By exploring participants’ interaction with 3D printing pens as diffractor apparatuses and analysing their conversation and engagement during the activity, this study aims to inform how in-service teachers who are non-mathematics specialists see themselves in relation to mathematics when they still have to teach the mathematical application at primary and early year levels.

There have been instances during the study where the 3D printing pens were malfunctioning due to overheating or because the power adapters were not located in a convenient space in the classroom, making it difficult to plug in the 3D printing pens to construct 2D and 3D shapes. These cases slowed down participants' processes of mathematisation and potentially changed their hand movements in constructing their shapes. These instances, however, did not hinder the data collection process as participants continued to engage with 3D printing pens to complete the learning-as-making activities.

4.11 Ethical Considerations

Once I received ethical approval from the EDU REC, I reached out to many PGCE programmes, secondary schools, and primary schools in the United Kingdom to gather interest in participating in the study. Six in-service teachers volunteered to be part of the study from three different schools. Each was provided a consent form, a participant statement form, and a description of the data that would be collected and used in this thesis. I also informed participants of potential future journal publications (as seen in Appendix 9.1). In addition, the participants were given safety guidelines, such as how to properly hold the 3D printing pens and how to keep their fingers away from the heat of the pens (Appendix 9.2).

Participants were reassured that they could drop out of the study at any time, in which case their data would be destroyed. The undergraduate participants were assured that the data analysis would take place after the completion of the assessment process and the submission of student grades. They would have the opportunity to exit the study if they chose without any effect on their marks and learning experience. All participants were informed that they did not need to participate in all parts of the activity. Two participants who were in-service teachers did not participate in the second part of the activity which entailed engagement with Tinkercad. Three participants – two undergraduates and one primary student – who partook in a trial study, only engaged in the 3D printing pen part of the activity.

To ensure ethics of care was taken into further consideration, I did not influence their construction of 2D and 3D skeleton shapes nor the conversation about classroom implications of using 3D printing pens in a lesson. The relationship developed with the participants was only through the learning-as-making activities and stories being shared as a teacher myself back in New York and current head of disciplines. The stories of my teaching in such a wide array of levels were not meant to influence the study. Rather, they were meant to establish a relationship with the participants as a narrative inquiry researcher.

Only I and my supervisory team have access to the data. This includes raw transcripts, field notes, audio-and-video recordings, and consent forms. That data will be uploaded to a university approved hard drive (e.g., Microsoft's OneDrive) and stored in a secure location. All measures will be taken to ensure the confidentiality of the information participants provided in this intervention. Sensitive information includes, for example, the audio and video recordings of the participants' engagement in the mathematical activity. The recordings reveal participant names and show their faces, therefore measures will be taken to protect participant anonymity. Facial recognition in the video recordings will be impeded by blurring faces, and pseudonyms will be used throughout, including university and school names. No payment was made to participants.

I have presented parts of the data collection in the Research in Mathematics Education (RME) group meetings at the university. The data analyses of this thesis have been presented at several regional conferences in the United Kingdom, including the British Society for Research into Learning Mathematics (BSRLM) and the Complete Mathematics Conference. Lastly, data analysis was presented to a teaching inquiry fellow group at a university in the United States.

Chapter 5: Mathematising

5.1 Introduction

In this chapter, I present the findings from each school (Gryffindor (5.2), Ravenclaw (5.3), and Hufflepuff (5.4)) as independent episodes. Each episode explores the way participants were mathematising in the learning-as-making activities. The episodes give details about the participants' narratives, such as their descriptions of using 3D printing pens, comments on ways the activities relate to their curriculum needs, stories about being a teacher, communication about drawing 2D and 3D skeleton shapes, and ideas about tinkering on Tinkercad. In addition, the episodes give insight into participants' construction of their 2D and 3D skeleton shapes through diffractive readings of screenshots of their hand gestures when using a 3D printing pen as a diffractive apparatus (Ng and Ferrara, 2020). Lastly, each episode retells the story of the group or individual in-service teacher's engagement in the study. The combination of participants' narratives and diffraction reading pulls forth themes within and across episodes and aims to unfold the various ways to mathematise using 3D printing technology tools in a learning-as-making activity (5.5).

5.2 Episode 1: Mathematising in the School of Gryffindor

5.2.1 Background

The episode takes place in a design and technology classroom at the School of Gryffindor as seen in figure 1. Beyonce, Joseph, and Tracy are situated at a rectangular table that consists of outlet plugs and design tools (i.e. sawdust, machinery) located around the table. Members of the group are colleagues and they teach different disciplines (Design and Technology, and Nutrition). Noone in the group has ever seen a 3D printing pen before nor used one in their own classroom activities. However, Beyonce was confident enough after the first session using the 3D printing pen to employ it in her after school activity with her students and then return several months after the first visit to see how well she remembered how to construct a skeleton Cube. In addition, I observed how both Beyonce and Tracy explore tinkering using Tinkercad functionalities to model their creation.

figure 1

5.2.2 Engage, play, and design with 3D printing pens in learning-as-making activities

Beyonce, Joseph, and Tracy engaged in the learning-as-making activities by using the 3D printing pen to construct 2D and 3D skeleton shapes. They were encouraged and allowed to explore the functionality of the 3D printing pen (i.e. Joseph holding a 3D printing pen to see the temperature rising (figure 2-c), Tracy helping Beyonce set up her 3D printing pen (figure 2-b), and Tracy holding the 3D printing pen on the mat (figure 2-a)) as the session began. All participants played around with the pen, as in moving it in different directions, changing the speed of the filaments being produced, and understanding how to hold the 3D printing pen, as seen in figure 2a-c.

(a) *(b)* *(c)*

figure 2 (a-c)

When participants felt comfortable using the 3D printing pen, they began to design their 2D or 3D skeleton shapes. Tracy decided to design a shape that she called a 'caged heart', as seen in figure 3(a-c).

Matt:	What are you making there?
Tracy:	I'm doing a caged heart
Matt:	What 3D object reflects the caged heart?
Tracy:	I'm doing a cube

Beyoncé, sitting next to Tracy, entered the conversation by asking:

Beyoncé:	So do you think everybody, that to make 3D, you have to do it in 2D and then join the 2D bits?
Tracy:	That's what I'm doing, yeah. But you probably don't have to if you can manage it. I just don't know how you would get it to stick together
Beyoncé:	Yeah

(a) *(b)* *(c)*

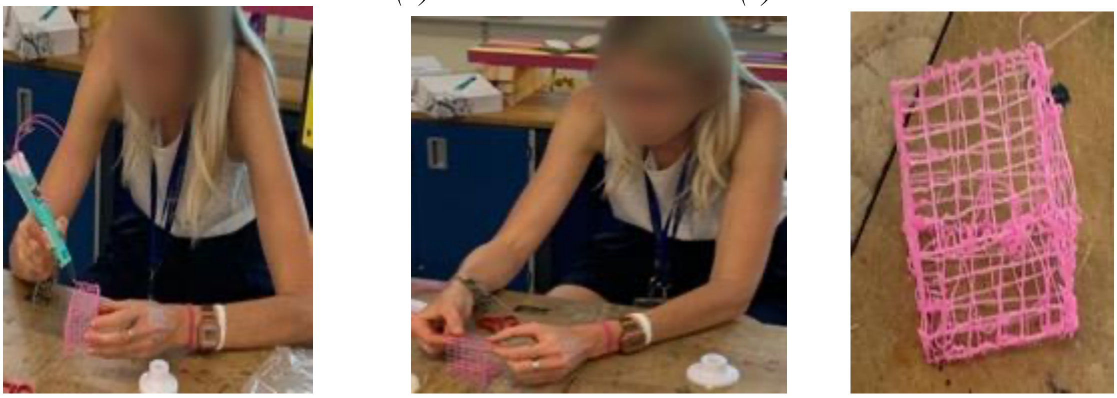


figure 3 (a-c)

Joseph decides to construct a 3D skeleton shape that he called a 'triangular prism', as seen in figure 3.

Joseph: I've done my prism
Matt: So, how did you do it?
Joseph: Just created a triangle…The more you use [3D pen], the better you get
Beyoncé: Did you do it on the mat?
Joseph: On the mat, yes. And the glue gun, will have to stick it together. The more you use it, the better you get
Tracy: Yeah, Of course

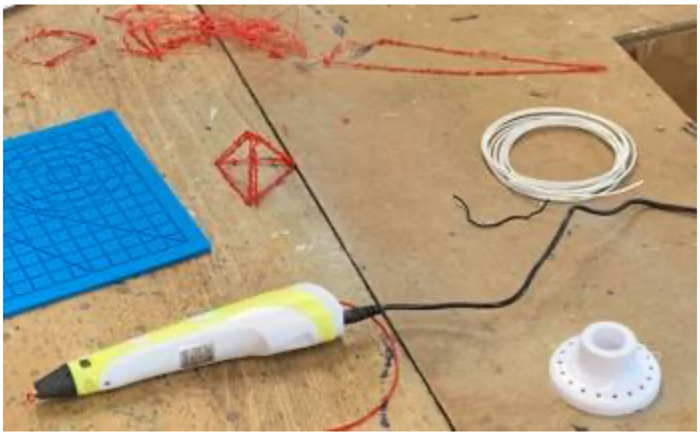


figure 4

Beyonce decided to design various 2D shapes using the 3D printing pen on the silicone mat and flat surface as seen in figure 5(a-d).

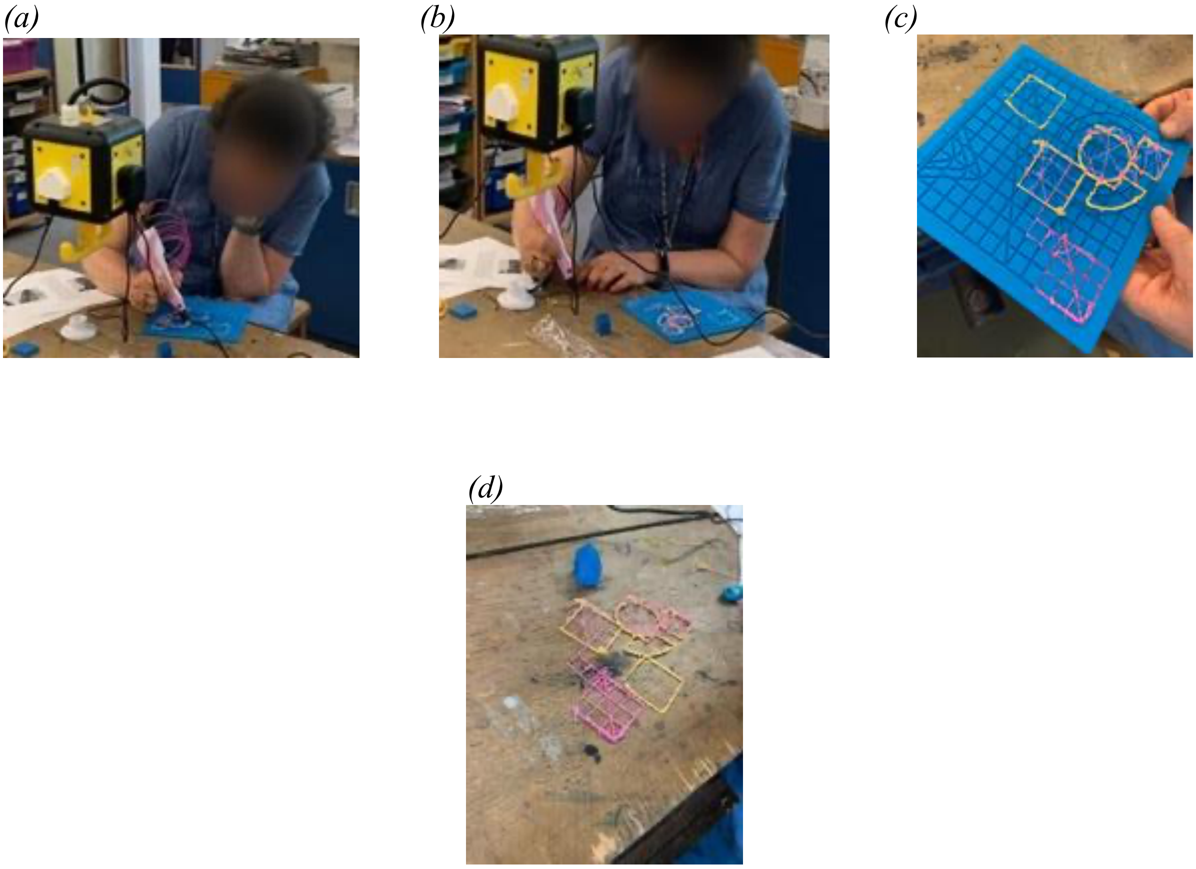


figure 5 (a-d)

On the first day of the learning-as-making activity, Beyoncé is seen using the 3D printing pen and silicone mat to construct 2D shapes. Similar to the diffractive reading of Tracy's first use of 3D printing pens above, the above (Figures 5 a-d) are short images of Beyoncé mathematising using the 3D printing pen in a group setting.

Beyoncé starts to mathematise using the shapes that are presented on the silicone mat. Figure 5-a very clearly shows Beyoncé in the making process of drawing various 2D shapes such as a square. In figure 5-b, she is seen using the 3D printing pen on the flat surface to construct a circle. Figure 5-c shows Beyoncé's final production of 2D shapes on the silicone mat, which she then places all together on the flat surface, as seen in figure 33-d. Beyoncé's mathematising process was influenced by the silicone mat as she is seen following the lines on the mat. In addition, there are multiple lines (1D) that connect each side of the square and circle.

Five months after the first session of the learning-as-making activity, she decided to implement the learning-as-making activity using 3D printing pens with her year seven students in an after-school design and technology programme. Beyoncé conducted a quick demonstration for her after-school class of drawing on the silicone mat with a 3D printing pen, as seen in figures 6 a-d.

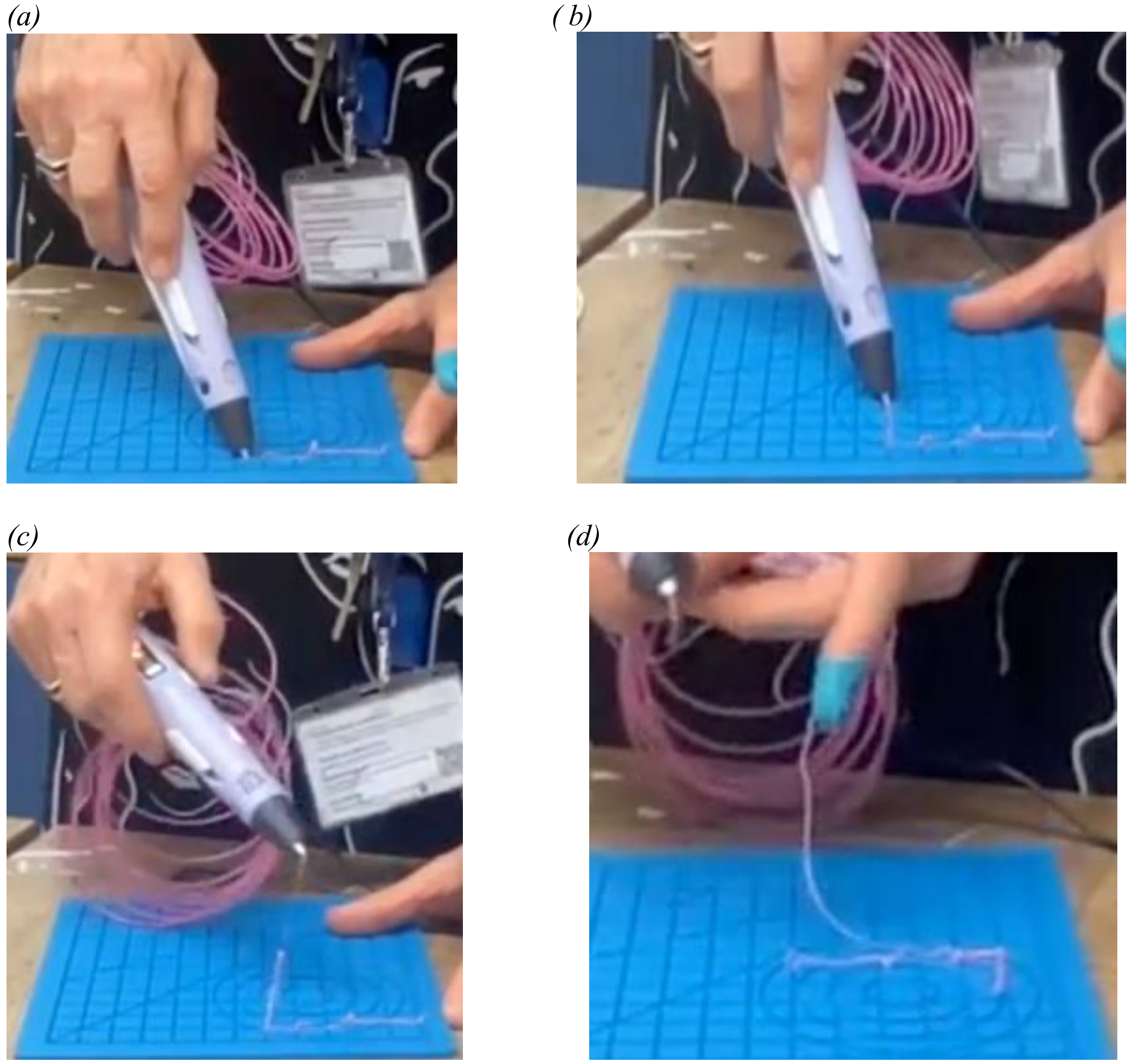


figure 6 (a-d)

Below I share Beyoncé's dialogue with her student named Lewis, who created a square using the 3D printing pen, seen in figure 7.

Lewis:	So I'm making a 4 x 4 square with a 3D pen
Beyoncé:	And how are you going to start?
Lewis:	I would start on a corner and then do the edges
Beyoncé:	How fast do you go?
Lewis:	You would want to go not too fast
Beyoncé:	Could you just hold it up for everybody? That's quite a beautiful square, isn't it?

figure 7

Reflecting on how Beyoncé was motivated to conduct the learning-as-making activities with her own students, I decided to invite her to a one-to-one session to observe if she could construct a 3D skeleton cube. This time I paid special attention to the detail of her hand gestures when using the 3D pen. Beyoncé's hand gestures are seen in figure 8(a-e).

Beyoncé was able to draw single lines (1D) by tracing them on the silicone mat, however, this work still did not start her process of mathematising a 3D skeleton cube. She proceeded to think about using the flat surface instead of the silicone mat to see if this would be easier, as seen below in figures 8(a-e).

(a) *(b)* *(c)*

(d) *(e)*

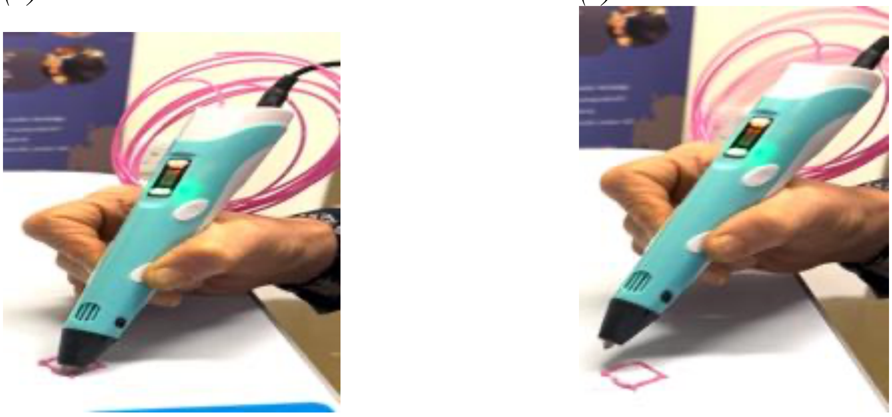
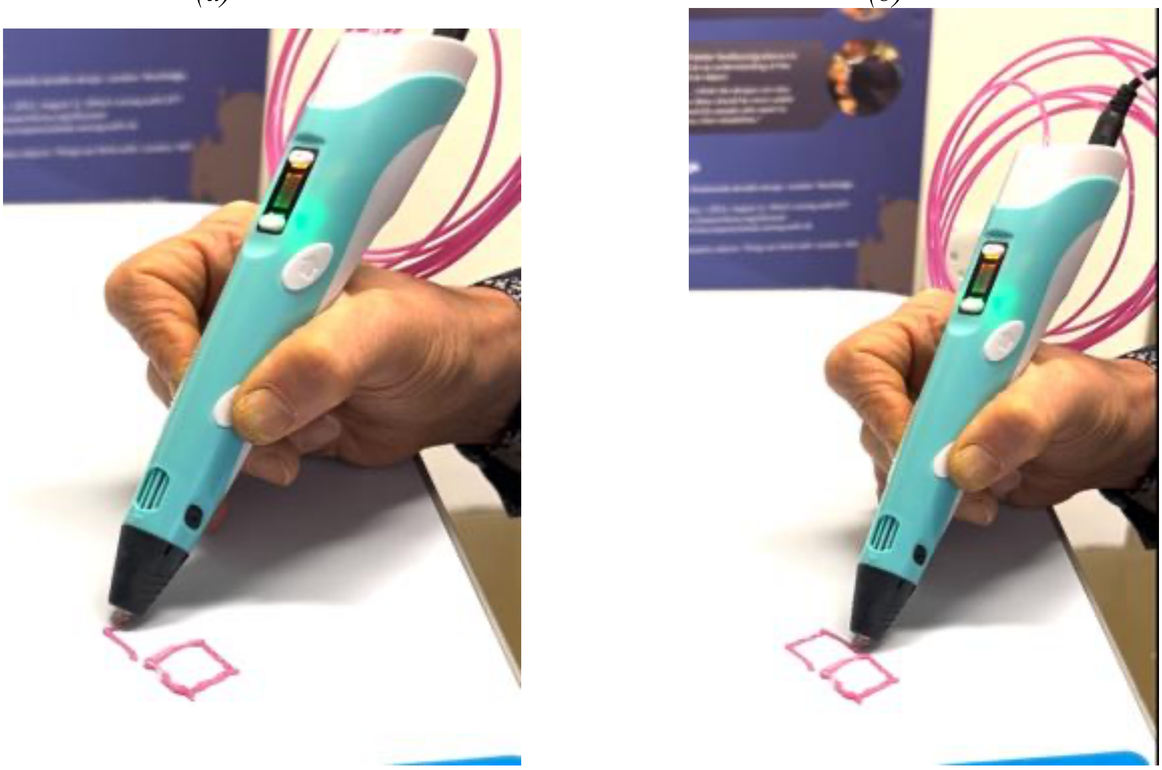


figure 8 (a-e)

Indeed, Beyoncé was able to mathematise a square (2D) using the 3D printing pen on the flat surface. She then began to construct another square side-by-side the first, as seen in figures 9(a-d).

(a) *(b)*



(c)  *(d)* 

figure 9 (a-d)

Beyoncé used the 3D printing pen as a glue gun to trace over the side of the first square to connect the two squares. Though I had asked her to construct a 3D skeleton cube, her making process took a turn (see figures 10 (a-c)) and her next 2D shape was a circle instead of a square.

(a)

(b)

(c)

figure 10 (a-c)

Using the same method of drawing shown in figures 10 (a-d), Beyoncé decided to draw a third square on the same row of the two squares already created (figure 11).

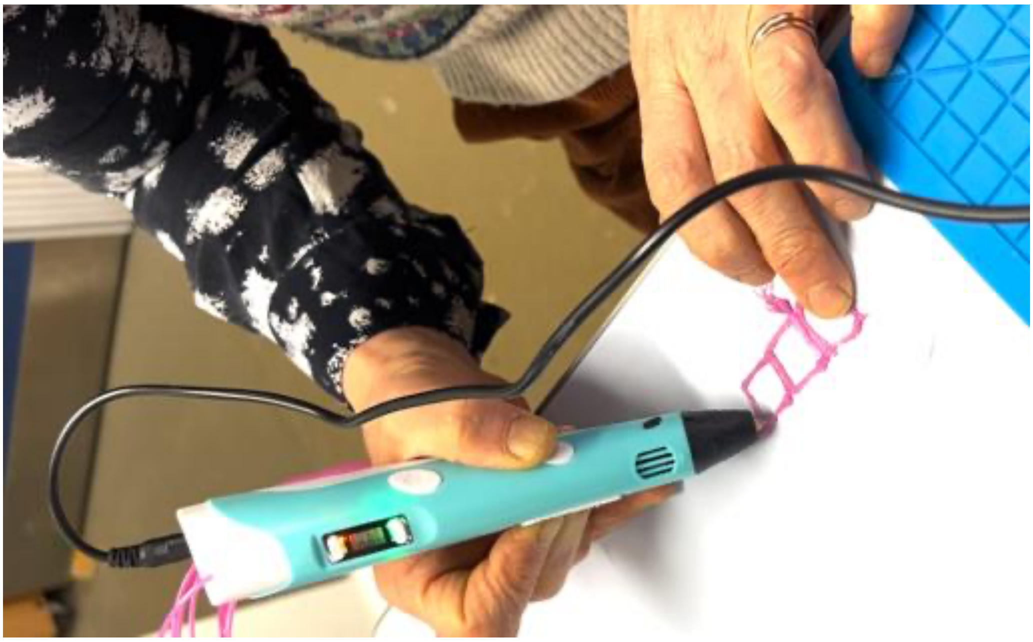


figure 11

Having already three squares and a circle drawn, Beyoncé squeezed the shapes together to form the 3D shape pictured below in figures 12 a-c.

(a)

(b)

(c)

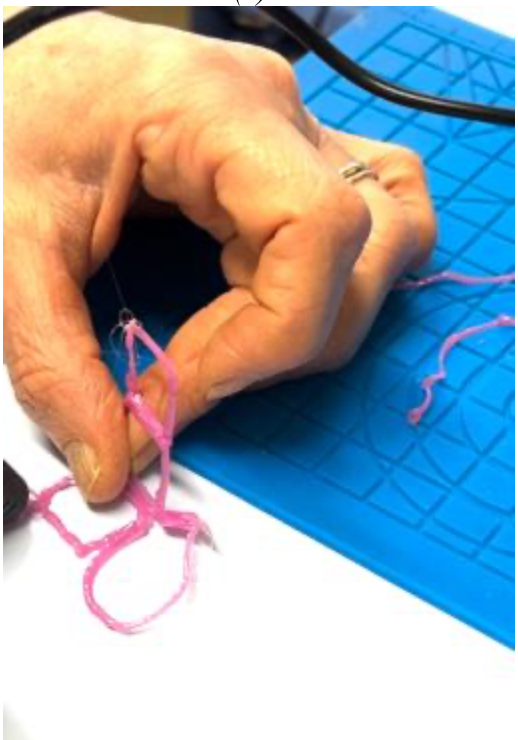


figure 12 (a-c)

By squeezing the shapes together, a 3D skeleton model began to take shape (figure 12-c). Beyoncé then decided to use the square as a base and draw multiple single lines (1D) as seen in figures 13 (a-d).

(a)

(b)

(c) *(d)*

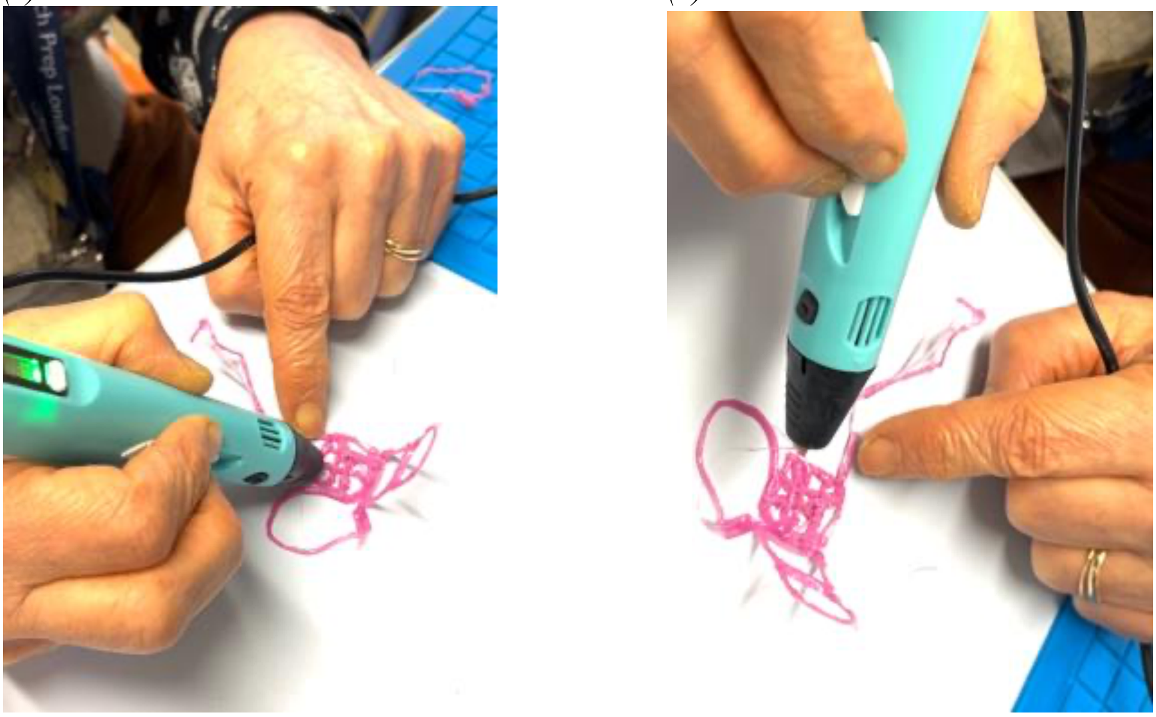


figure 13 (a-d)

Beyoncé continued to fill the other two squares by drawing straight lines (1D) to connect one side of the square to another, as seen below (Figure 14).

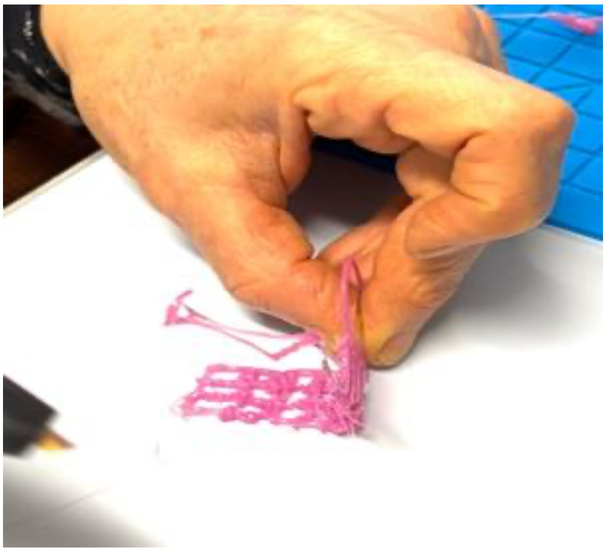


figure 14

Due to a colleague's need for Beyoncé to attend another class, she was not able to complete her 3D skeleton of a cube. Even so, Beyoncé final 3D skeleton model can be seen below (figure 15).

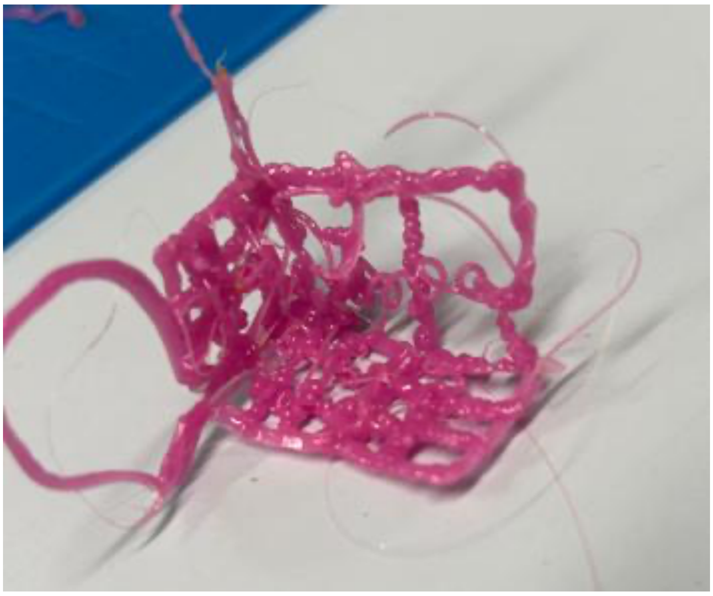


figure 15

Technical Difficulties and Challenges of Using 3D Printing Pens

The three teachers experienced technical difficulties and other challenges in using the 3D printing pens. One challenge was that the filament being produced through the 3D printing pens was not sticking to the silicone mats. As Tracy started to use her 3D printing pen, she noticed that her 3D skeleton model of a line was not precise. She asked, 'How do you prevent it from squiggling?' In addition, Tracy also wondered, 'What is 60 degrees?' as she held the 3D printing pen and moved her hand to position the pen to what in her view was 60 degrees.

figure 16: Tracy holding 3D printing pen at 60 degrees

Another difficulty early on was the inability of the 3D printing pens to heat up to the temperature of 180 Celsius. Beyoncé noticed this first, having seen in the instructions that, to function properly, the pens had to heat up to 180 degrees. We discovered that the reason for this difficulty was not because there was a defect in the 3D printing pens, but because the electrical outlet we were using was faulty.

figure 17: Beyoncé's technical difficulty using 3D printing pen

One challenge in using the 3D pens was their physical size and weight. Both Tracy and Joseph noted that the pens are lightweight.

Joseph:	It's too light
Tracy:	Oh, too maybe it'll fall over. But, remember their [the students'] hands are much smaller. I think that's a benefit actually, that it's light. But I hear you. It feels a bit flimsy when you're standing it up

figure 18: Tracy feeling the weight of 3D printing pen

Joseph:	A bit of weight would have been better
Tracy:	Yeah
Joseph:	It would keep it more steady as well

figure 19: Joseph feeling the weight of 3D printing pen

From a classroom perspective, Beyoncé feels that students might get frustrated using the 3D printing pens due to the technical difficulties.

Beyoncé:	One of the things is what I am doing now. Getting impatient
Tracy:	Because you've got an idea and you can't [do it], or it's not quite working
Beyoncé:	You don't want to wait for it to cool down and you want to be able to do things, so you get frustrated with it not being easy to join things together

Real-life Application of Mathematics through Utilising 3D Printing Pens

Matt:	What mathematical components could you see here? If you know any, I know you're not math [people], but what do you think or how would you use [3D printing pens] in a math classroom?
Tracy:	You could definitely use it for all kinds of shapes and dimensions. But I would imagine that this would still be [suited to the] design and technology domain because in maths, if you are going to spend a whole lesson doing this, I can't imagine they want to give up that time
Beyoncé:	Expect they do really nice things with pattern in maths. [Pattern] that sometimes they use [in art or architecture], you [see] on the tiles in Asia, like in a Mosque or something
Tracy:	Yeah. Moroccan, exactly
Beyoncé:	And it would be a nice sort of slightly different way to introduce pattern. And I mean, we haven't worked out how to do 3D yet, but that'll be a very nice thing for structure and forces

5.2.3 Tinkering in learning-as-making activities

In the second part of the learning-as-making activities, Beyoncé and Tracy engaged with Tinkercad to explore the different functionalities and played with the objects provided in the interface. Beyoncé is familiar with using Tinkercad in her classroom, whereas Tracy does not use it in her curriculum. Below I share a dialogue between myself and Beyoncé, asking about her creation using Tinkercad, with related images in figure 20(a-b).

Matt: What shapes do you see there?

Beyoncé: Well, I'm actually just using what you put down here, boxes, spheres, roofs and cones. And when you said, "You must keep count of the numbers"

Matt: Yeah

Beyoncé: So rather than keeping count. I've done all the same [shapes]

Matt: Okay

Beyoncé: I've just tried to follow your instructions. And now I'm trying to just remember how you can align them because I'd like them to go in lines

Matt: Do you think when Tinkercad calls a box... They call a cube a box, and a triangular prism, a... Do you think that'll cause a problem in maths lesson?

Beyoncé: Will it cause a problem in the classroom?

Matt: If we're teaching cube. This is a cube, can Tinkercad say, "Oh, this is a box?" Is it okay for primary teachers and secondary teachers to say, "Oh, by the way, we're going to do a box today?"

Beyoncé: Yeah, are they the same? Would the boys understand? Shall I tell you what I think? I think in design and technology, which primary teachers have to teach as well, you are... I think I've written it on your sheet as well, that you're very often using it to represent models. We use it quite a lot to represent buildings. That is a very obvious shape for a roof

Matt: Okay

Beyoncé: And also in D&T, we use boxes a lot for packaging, and they're not always cubes. So if you're saying cube, but you can stretch these cubes

Matt: Got it

Beyoncé: Boxes in a way, box and roof, for us is very useful. Because when we make little bird houses and things, we use those shapes. For us, yeah, it's quite a good plan. And also, we've used Tinkercad to think about how we might make chocolate molds and then to put... We would cast these, you know?

Matt: Yeah

(a) *(b)*

figure 20 (a-b)

In addition, I discussed with Tracy about her creation (see figure 21(a-b)) on Tinkercad, as recounted in the dialogue below:

Matt:	Do you find it challenging that [Tinkercad] named a box cube and roof a triangular prism?
Tracy:	Yeah
Matt:	Do you think that will pose a challenge for students learning?
Tracy:	No, as long as they know that's what you are using?
Matt:	Okay
Tracy:	No I don't think so. I've never used Tinkercad actually, so this is all new to me
Matt:	How does it feel then?
Tracy:	It's great. Am I allowed to ask you… How do I put a shape on top of another shape?
Matt:	If you click on another shape, you could just drag it to the plane and there should be arrows pointing up…What shapes are you using?
Tracy:	As in the names, or how am I choosing?
Matt:	Yeah
Tracy:	I just want things that look fun
Matt:	Okay
Tracy:	It's going to look like [an] ice cream, just lollies, sweets
Matt:	What mathematical shapes do you see?
Tracy:	So a cone, a sphere, a half sphere. I don't know what this... What is this called? A ring? I don't actually know what that is called in 3D shape. What did I use here? This was a polygon. There you go. Does that have a name? A tube. There you go, a tube
Matt:	Tube? If it's called a tube, do you know what is the mathematical term for it?
Tracy:	No
Matt:	How do you think this could be used in a maths classroom?
Tracy:	Oh, definitely. Well, first of all learning all the shapes, seeing how they work together in terms of relation ratio, what could fit on top... If you have half a sphere, that could go with that, for example, and form something else with a cone. Oh no, I think it would be super useful
Matt:	What do you teach here [again], I'm sorry, art?
Tracy:	Food tech, so food, cookery
Matt:	Okay
Tracy:	Hence, my very poor knowledge of maths shapes. Oh, it's very cool. Okay, we'll do a few more

(a) (b)

figure 21 (a-b)

5.2.4 Retelling a story: The group interaction in the learning-as-making activities

Making

Making is an essential component of the design and technology key stages 1-3. Throughout the learning-as-making activities, each in-service teacher demonstrated the ability to make using the 3D printing pen. Whether it was a skeleton caged heart (Tracy), a triangular based pyramid (Joseph), or tracings of various 2D shapes on the silicone mat (Beyoncé), all of the in-service teachers participated actively in learning the new skill. Beyoncé volunteered to fill out a six-question survey. One question asked:

> Tell me a story about your experience taking part in the study. Start when you signed up for the study without having seen a 3D printing pen, Tinkercad, or Makerbot 3D printer. Then discuss your participation over the summer as you engaged with the 3D printing technology with the researcher. Finally, discuss your experience in the classroom using the 3D printing pen and your students' experience using it in the after-school programme.

Beyoncé responded:

> I was initially intrigued and slightly nervous about using 3D pens, worried about whether they would be hot, how you could construct, whether I would be any good at using them. I was nervous that as a DT teacher, I should be skilled at the processes and didn't want to be embarrassed in front of my colleagues. Working with the researcher in an informal CPD session where we could chat and try things out was fun and reassuring, it was very interesting to see what other people did with the technology. The students really enjoyed experimenting with the 3D pens and we have put their work on display in the corridor for the school to see. There has been quite a lot of interest. The boys are getting confident using Tinkercad now and I am starting to see how easy the Makerbot is to use, am still concerned about not being able to fix it if it goes wrong. I can see the value of the early comfortable and relaxed sessions with staff.

During the learning-as-making activity on the first day, the ‘slightly nervous’ comment could suggest that the reason Beyoncé did not want to construct a 3D skeleton cube was due to being ‘embarrassed in front of colleagues’. But this did not stop Beyoncé from mathematising. The diffractive reading of the first day of the learning-as-making activities shows that Beyoncé constructed 2D shapes successfully on the silicone mat. However, the diffractive reading also tells us that there were times when Beyoncé struggled to use the 3D printing pen to mathematise 2D and 3D shapes. For example, Beyoncé demonstrated how to use the 3D printing pen in front of her students, but did not completely finish drawing a shape on the silicone mat. There are two ways to interpret this situation: (1) Beyoncé took a design, play and engage approach where she wanted her students to freely use the 3D printing pens to construct shapes; or (2) Beyoncé quickly moved on due to forgetting how to construct a 2D shape using the silicone mat. Regardless, Beyoncé claims ‘students really enjoyed experimenting with the 3D pens and we have put their work on display in the corridor for the school to see’. Figure 22(a-d) shows a few images of Beyonce’s students’ 3D modelling work from the after-school programme.

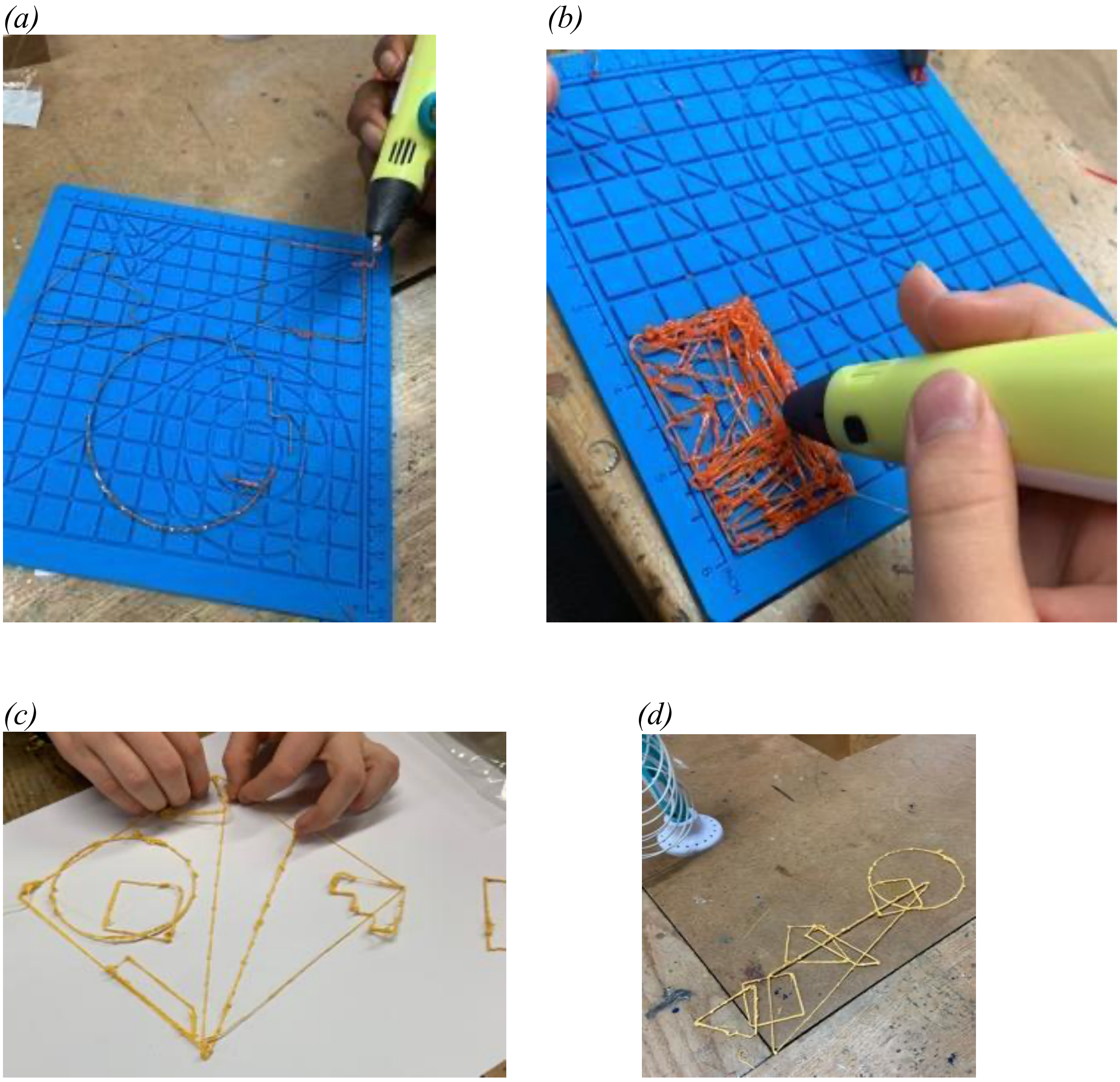


figure 22 (a-d)

Thanks to her continued professional development in setting up her school's new MakerBot 3D printers, Beyoncé is able to say that she 'is starting to see how easy the Makerbot is to use'. Before taking part in the learning-as-making activities, Beyoncé had experience using Tinkercad and performing classroom activities with Tinkercad. In the learning-as-making activities, she used an iPad, where the interface with Tinkercad is different than with a regular desktop or laptop computer. Although this prevented her from manipulating the 3D models, Beyoncé's making process on the iPad demonstrated the easy-to-use interface of Tinkercad, wherein the user simply clicks and drops models on the plane.

Before discussing how in-service teachers mathematise from the researcher's point of view, I will give insight on Tracy and Joseph's making process. Making is part of the everyday professional practice for both Tracy (as the school culinary teacher) and Joseph (as the teaching assistant for Beyoncé). Tracy as a culinary teacher is used to making with food, now she has explored making with materials. She was quickly able to adapt using the 3D printing pen to make a skeleton caged heart without any guidance from the silicone mat or her colleagues. Similarly, Joseph's experience as teaching assistant and his former background as an engineer allowed him to make a 3D skeleton triangular based prism.

Mathematising

When Beyoncé was asked in her six-question survey, 'As a design and technology teacher at the primary level, are you familiar with the mathematical learning outcome for students at the primary level (key stages 1-3)?' she replied, 'not really'. Although Beyoncé is not familiar with the mathematical content in key stages 1-3, the design and technology content of key stages 1-3 does include mathematical modelling. This is the rationale for implementing mathematical activity in a design and technology classroom.

Through the learning-as-making activity, each in-service teacher mathematised through their own making process. For example, Tracy gave attention to detail when creating her skeleton cage. This included the hand movements of the 3D printing pen from a 2D shape (square) to straight line (1D) to connect each end of the square to building a skeleton cage that became a 3D model. Tracy moved from 1D to 2D and 2D to 3D as she was making and mathematising. Likewise Joseph went from creating three triangles of the same height in 2D to using the 3D printing pen as a glue gun to construct a 3D triangular based pyramid.

Beyoncé's mathematising process happened in three different periods, as revealed in the diffractive reading. The period of mathematising that I want to bring to attention is Beyoncé's attempt to construct a 3D skeleton cube. She started with her making process of creating two identical squares. At first, I was interested to see if Beyoncé would take the same direction as Chandler (the undergraduate student in the calculus for business class), who constructed six identical squares to form a skeleton cube. Beyoncé decided to make three identical squares and a circle before squeezing the parts together to attempt to build a skeleton cube. This process shows not only the way Beyoncé is thinking about making a 3D skeleton cube, but also the way she is mathematising by using different shapes to construct the cube, driven by her shape perception more than her mathematical knowledge of shapes. Through making in this learning-as-making activity, each in-service teacher experienced mathematising using 3D printing pens to construct shapes from 1D (straight line) to 2D (squares) and 2D (squares) to 3D (cube).

5.3 Episode 2: Mathematising in the School of Ravenclaw

5.3.1 Background

This episode takes place in an early years classroom that consists of everyday teaching objects (i.e. whiteboards, markers, smartboards, desks, and a sitting area for early years students). Taylor and Emily have no experience in using a 3D printing pen prior to taking part in this study. Taylor engaged, played, and designed with 3D printing pens and Tinkercad during the first visit. Five months after this first visit, she was interested in using the 3D printing pen in her early year classroom to demonstrate 2D shapes to her students. Similar to episode one with Beyonce, I asked Taylor if she could demonstrate constructing a 3D skeleton cube using the 3D printing pen. When Taylor was demonstrating how to construct a 3D skeleton cube, Emily was in the classroom and interested in the device. As a result, I observed Emily on another visit to the school as she used a 3D printing pen to construct 2D and 3D skeleton shapes.

5.3.2 Engage, play, and design with 3D printing pens in learning-as-making activities

Taylor started engaging with the 3D printing pen through using the desk instead of the silicone mat. She drew a 3D skeleton shape in the air (Ng and Sinclair, 2018). Below is a dialogue of our conversation during Taylor's construction of her 3D skeleton cube, and images of her work can be seen in figure 23(a-c).

Matt:	Okay. Is it easier to draw in here [silicone mat] or there [table surface]?
Taylor:	I think it might be easier on the table. Yeah. It is easier on the table
Matt:	Do you know how to draw a cube?
Taylor:	A cube?
Matt:	Yeah. Like a box?
Taylor:	A cube? So a cube. 3D shape?
Matt:	3D shape, yes
Taylor:	And I can make the cube
Matt:	Where is this idea coming from?
Taylor:	I've seen that happen. The simplest way I can draw a cube
Matt:	You saw this from where? Other worksheet?
Taylor:	I would say maybe school. Draw a square first and then
Matt:	Okay. This is a cube because it's a box
Taylor:	This is a cube
Matt:	So you're picturing yourself drawing this on the board, correct?
Taylor:	Yeah
Matt:	Now I get you. No, this is cool. This is cool because what I ask participants, I need you to draw it in the air. But you decided to draw it on the surface
Taylor:	Yes
Matt:	Which is nice
Taylor:	So they would draw a cube for you in the air?
Matt:	Yeah. Do you want to try drawing a cube in the air?

Taylor: Just thinking
Matt: But I can't say this but …
Taylor: Using the pen or without the pen?
Matt: Using the pen
Taylor: In the air?
Matt: But can you draw this using another cube like basically two of these and two of those?
Taylor: Oh, I'll draw it. Okay
Matt: Gotcha, all right. I see. Smart. Do you see this being used in your classroom?
Taylor: How do I see this?
Matt: Sorry, do you see this being used in your classroom?
Taylor: Probably. Maybe when teaching the difference between maybe 2D and 3D shapes
Matt: Okay
Taylor: I guess it's another way of kind of using pen, one way. Or maybe for older children so maybe not reception

Taylor's Discussion on Implementing 3D Printing Pens in the Classroom

Matt: Is there an after-school program here?
Taylor: They do after-school class, yeah
Matt: Okay. Is there a math one or a science one?
Taylor: Actually, there isn't, no. And to be fair, I would not think of using this in a math class. I would think of using this in a DT
Matt: DT, so design and technology. Okay. If you had to show a student this, what can you talk about a cube, what they're talking about?
Taylor: I would say the sides, the faces, maybe right angles. If it was for older children, maybe I would say okay, so if I was to work up this side and maybe to about 180 and what is the side or maybe in that side. Talk about the difference between this being maybe a 2D shape, 3D shape. I'd think okay with that. But then again, I heard that this is 3D because you can see that it is actually
Matt: Yeah, okay
Taylor: So if it's on the board then it's 2D. If you can feel it, physically feel if it's flat on my hand it's still 2D
Matt: Okay.
Taylor: So yeah
Matt: So touching it and laying between touch and hold gives it something meaningful or …
Taylor: Yes
Matt: Why is that?
Taylor: Because seeing it on the board is not as effective as touching it and feeling something and kind of interacting with it
Matt: Yeah
Taylor: So I think it's different when they're interacting with it. If it's just on the board you might not retain as much information'.

Matt: Okay. And the pen costs £29.99. Do you think the school will provide it or not?
Taylor: £29.99?
Matt: Yeah, like do they have funds for that or …
Taylor: I don't know, but I would've thought it was like £299
Matt: No. Yeah, so these only cost £29.99. Comes with the filament and comes with not the plug but this here
Taylor: Interesting. But it's hot'
Matt: Correct. So the black part, the extruder is what they call it
Taylor: This part would be hot?
Matt: Hot when you press it, yes
Taylor: Okay. I would say maybe, but I would say it's difficult for reception children because of their safety
Matt: Yeah, so that I understand
Taylor: I would maybe say even six children
Matt: Okay
Taylor: Maybe six or five
Matt: So this would still be not in a primary school? What do you think? Or should it be secondary or high school students?
Taylor: I think primary school. It's good to get them using different tools to access different ... Because I just thought, "You know what, this I would use in DT and I could use it in maths lesson as well." So it's showing that you can use tools for different purposes or different parts or even maybe accessing the curriculum, different parts of the curriculum using a device. I would say this is still DT, but yes it's maths because I made a cube and they connect in size at different sides
Matt: If you have a 3D printer, you'd print this so it's more like this entices the students, do you think?
Taylor: If I was to ...
Matt: 3D prints like a cube
Taylor: Yeah
Matt: And you showed this, would this be more ... How can you use this in the classroom?
Taylor: I would say that's a cube
Matt: Okay
Taylor: Again, I guess with this one you would just say side because this has faces
Matt: Okay
Taylor: Yeah, I guess having different shapes like cubes would really get
Matt: Can you draw a sphere, do you think, with a 3D pen?
Taylor: Ooh, I can try
Matt: If it's possible. No worries. I'm putting you on the far end right there
Taylor: Okay. Do you have to …
Matt: Oh, so just click it there. Go to 180. It's just when you're not using it for a little bit, it just stops, so it cools down
Taylor: Okay
Matt: Are you teaching here in the system?
Taylor: In the school? Yes
Matt: Okay

(a) *(b)* *(c)*

figure 23 (a-c)

Due to the similarity in colour of the yellow desk and peach filament, the snapshots below, which were taken from video footage, may not be as clear as others. Taylor started mathematising a skeleton cube by drawing a square (2D) using a 3D printing pen on a flat surface (Figure 11-a). To construct a 3D skeleton cube, she continues to draw separate squares on the flat surface and connect each one using the 3D printing pen as a glue gun (Figure 11 b-c). Note that Taylor only drew four squares to form her 3D skeleton cube and she did not use the 3D silicone mat.

After the first session of utilising 3D printing pens to construct a skeleton cube, Taylor saw potential in using the learning-as-making activity in her own classroom to demonstrate to early years students how to identify and name 2D shapes (see figure 11(a-d)). The conversation with her students went as follows:

Taylor:	Okay. We are going to be starting. We're going to be looking at shapes. From next week, we're going to be moving from numbers. Who can count from one to five? Let's see. One, two, three, four, five
Class:	One, two, three, four, five
Taylor:	What comes after five? Six, seven, eight, nine, ten
Class:	Six, seven, eight, nine, ten
Taylor:	We know all of our numbers. Next week, we're going to be starting our shapes. What are the different shapes that you can name? Let me ask Kiara. What shape can you make, Kiara?
Kiara:	A triangle
Taylor:	Triangle. That's one shape. Ashely, can you give me another? Ashely, another shape?
Ashely:	Circle
Taylor:	Circle. Lauren, another shape
Lauren:	A heart
Taylor:	A heart. That is a shape, isn't it. Hilary, can you name another one?
Hilary:	Triangle
Taylor:	Triangle. Madison
Madison:	A rhombus
Taylor:	A rhombus, well done. Who else? Let's have one more. Garry?
Garry:	A heart

Taylor: A heart. What other shapes? One more. Something different. Rhianna?
Rhianna: A triangular prism
Taylor: You need to be looking
Class: Kerching
Taylor: Okay. Miss Taylor now is going to be using a 3D pen. Miss Taylor has a pen on her desk. When I draw on paper, you can see it on paper. But this one is very different. It's a 3D pen, so when I draw, you can actually see the object. You can see the shape. Miss Taylor is going to do that… I'm just waiting for my pen to be ready, okay? Then I'm going to be drawing my shapes on my card, on my mat, okay? Miss Taylor's just waiting for this pen to be ready. When it's ready, I'm going to be drawing my shapes on my blue mat'
Taylor: Okay, what do you think Miss Taylor is going to be making? What's the first shape?
Madison: It's a circle
Taylor: Madison thinks it's going to be a circle
Hilary: I think, it's a triangle
Taylor: Oh, someone else said they think it's going to be a triangle
Kiara: I think she made a heart
Taylor: You think it's going to be a heart?
Kiara: Yeah
Taylor: Before I show you, what do you think? Any more guesses?
Hilary: Rectangle
Taylor: You think rectangle. Let's see. Are you right? What shape is this?
Class: Rectangle
Taylor: It is a rectangle. Well done. It is a rectangle. Right. Miss Taylor is going to think of another shape now. Miss Taylor is going to think of another shape.

Taylor: In a minute, Miss Taylor's going to ask you, I'm going to ask the children that are sitting very nicely to come and see my rectangle. Not everyone is going to have a chance, but I'm going to try and choose the children that are really sensible, that are sitting really nicely. Miss Taylor's going to think of another shape now. Are you ready?
Madison: Yeah
Taylor: Okay
Zelda: This is dry [referring to the 3D skeleton rectangle]
Taylor: Have you guessed what shape Miss is doing?
Kiara: A heart
Ashley: A heart
Taylor: Anyone else? What other shape could Miss make?
Madison: I think it's a rectangle
Kiara: A square
Taylor: A square? You think a square?
Madison: I think it's going to be a star
Taylor: Right, let me see. I heard star. I heard heart, who else?
Madison: A star. A star
Zelda: A star
Class: Star

Taylor:	Right, are we ready to see what shape? A heart. Yeah, let's see what it is. Oh, what shape is that?
Madison:	A square
Taylor:	A square. Shout it out to everyone. What shape is this?
Class:	A square
Taylor:	A square. It is a square. Now, it's orange
Madison:	Why is it orange?
Taylor:	Why is it orange? Because I was able to use a different colour. This time I've got a square. Miss Taylor has a rectangle. Can you see my rectangle?
Class:	Yeah
Taylor:	And now Miss Taylor has a square. Do you see my square?
Zelda:	Yes
Taylor:	Miss Taylor’s going to be thinking of another shape now
Zelda:	It's a different colour. It's a different colour now
Taylor:	Yes
Zelda:	Because you have this colour. Was it an orange colour, Miss? Right, Miss?
Taylor:	Yes, you can. Zelda, out the way, please. We can't see
Zelda:	I think it's a triangle
Taylor:	You think? You're not sure yet. Yeah?
Madison:	Is it a star?
Kiara:	Heart
Madison:	It's a star
Taylor:	Who else?
Ashley:	Rhombus
Zelda:	I knew it. It is a triangle. I knew it
Madison:	Diamond
Class:	Diamond
Taylor:	What shape is this?
Class:	Diamond
Taylor:	It is a diamond. This time, I have a diamond. How amazing is this pen? It's a very, very special pen. Is it like the pen that Miss Ouma uses on the white board?
Class:	No
Taylor:	It's not because every time I think of a shape, I can actually hold it and I can feel it
Zelda:	Miss, it's not going to break now?
Taylor:	Let me see. I'm thinking of one shape. Actually, I can do maybe two more shapes because I can see, on my mat, it has shapes on here that I haven't yet used, okay? Maybe, I can think of something else I can do. Let Miss Ouma try two more shapes and then...
Zelda:	No. I think it's a triangle. Yeah
Taylor:	This time, Miss Taylor is using the blue mat to go over the shapes that are already on here
Zelda:	Triangle
Madison:	Triangle
Kiara:	Heart
Zelda:	Triangle
Madison:	It is a triangle

Kiara:	Heart
Zelda:	It's a triangle. It's a triangle. Double time. I'm thinking triangle
Taylor:	Right, Miss Taylor. Oh, one, two, three
Class:	Triangle
Taylor:	A triangle, and one more
Zelda:	I knew it
Taylor:	One more
Class:	Circle
Taylor:	A circle. This is amazing. I've got a circle. I've got a triangle. I've got a...
Class:	Square!
Taylor:	... square, and I also have a rectangle
Class:	Rectangle
Madison:	A diamond, diamond, diamond
Taylor:	And a diamond. Yes, that's right. And my diamond. How many shapes?
Zelda:	I can't reach them all
Taylor:	Five shapes. Miss Taylor made five shapes
Zelda:	They can't break. They can't break
Taylor:	You think it can't break?
Zelda:	No. It can't break anymore

Taylor repeated the process of drawing three additional shapes (square, diamond, and circle) and had back and forth dialogue with her students. In figure 24(a-d), Taylor is seen drawing the 2D shapes using the silicone mat while seated in front of the class.

(a)

(b)

(c)

(d)

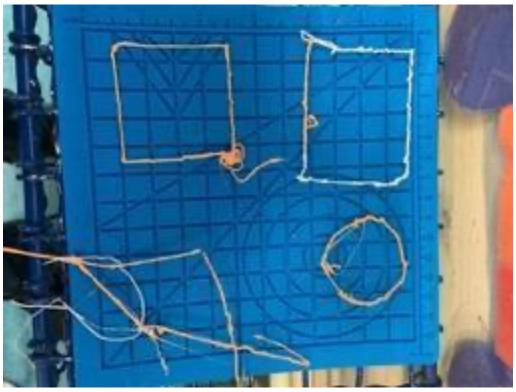
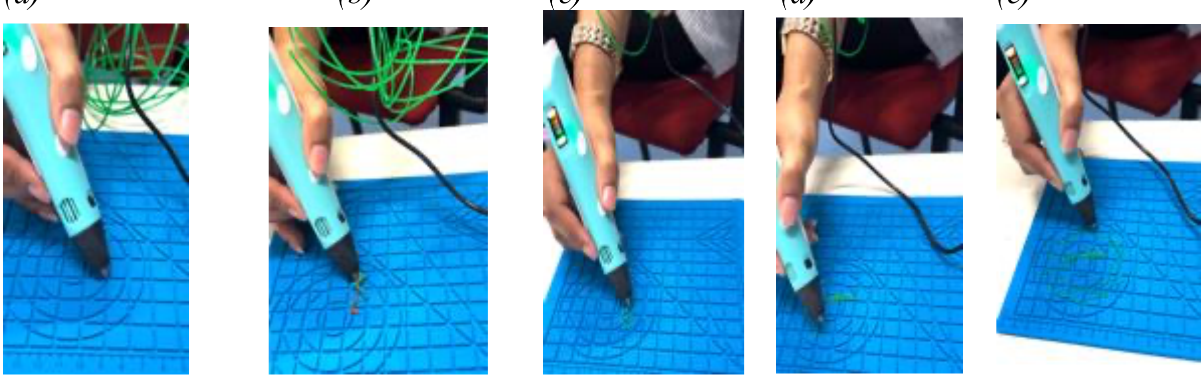


figure 24 (a-d)

After the second visit, I visited Taylor for a third time (one year from the first visit) as I was interested to see if she remembered how she constructed a 3D skeleton cube and curious if she would construct the cube differently this time around. I paid attention to detail in her hand gestures when using the 3D printing pen. Taylor does not have a 3D printing pen herself, nor does the school use 3D printing pens for any activities. This diffractive reading unfolds Taylor remembering how to employ a 3D printing pen in learning-as-making activities. First, I asked Taylor to use the 3D printing pen to draw 2D shapes on the silicone mat. Each of her shapes – circle (Figures 25 a-e), triangle (Figures 26 a-c), and rectangle (Figures 27 a-e) – can be seen below.

Circle

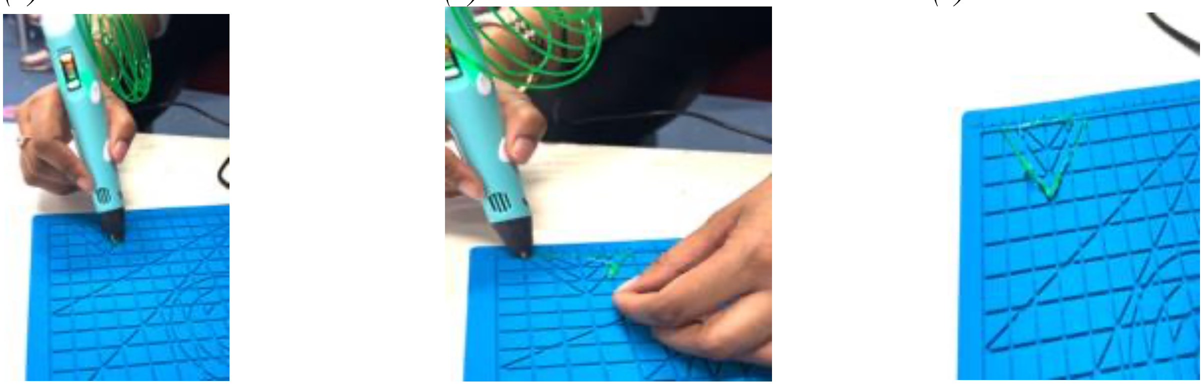
(a) *(b)* *(c)* *(d)* *(e)*



figures 25 (a-e)

Triangle

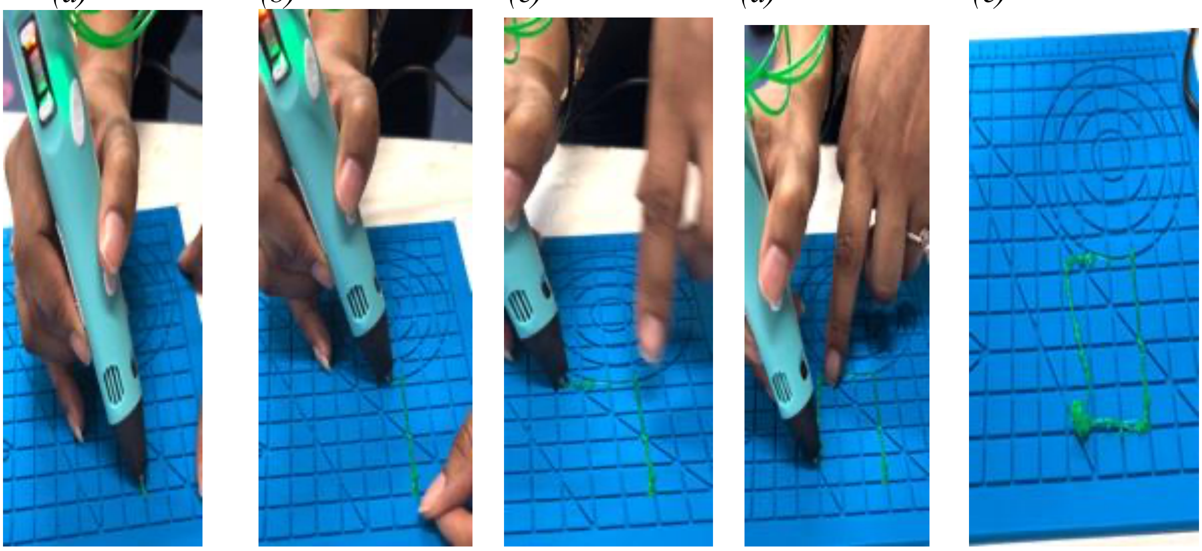
(a) *(b)* *(c)*



figures 26 (a-c)

Rectangle

(a) *(b)* *(c)* *(d)* *(e)*

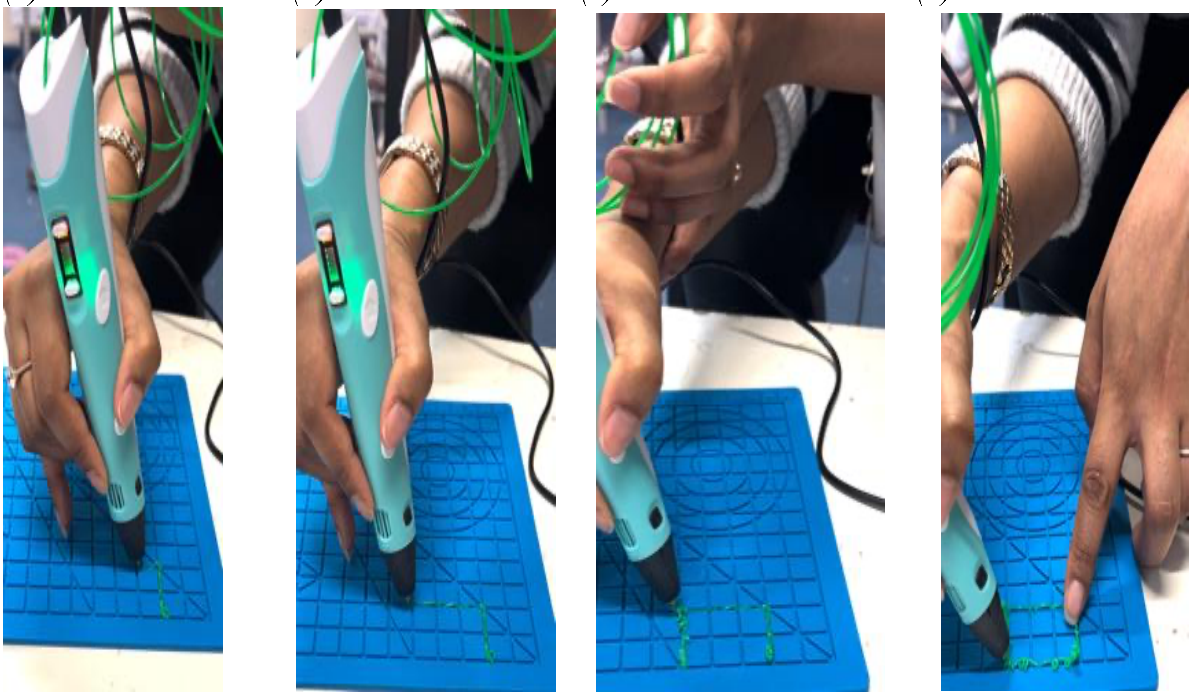


figures 27 (a-e)

After Taylor had drawn her circle, triangle, and rectangle, I asked her to construct a 3D skeleton cube with no guidance, using the silicone mat or a flat surface. The images below show her creations of a cuboid (Figure 28 a-d and Figure 29 a-d) and cube (Figure 30 a-d).

Cuboid

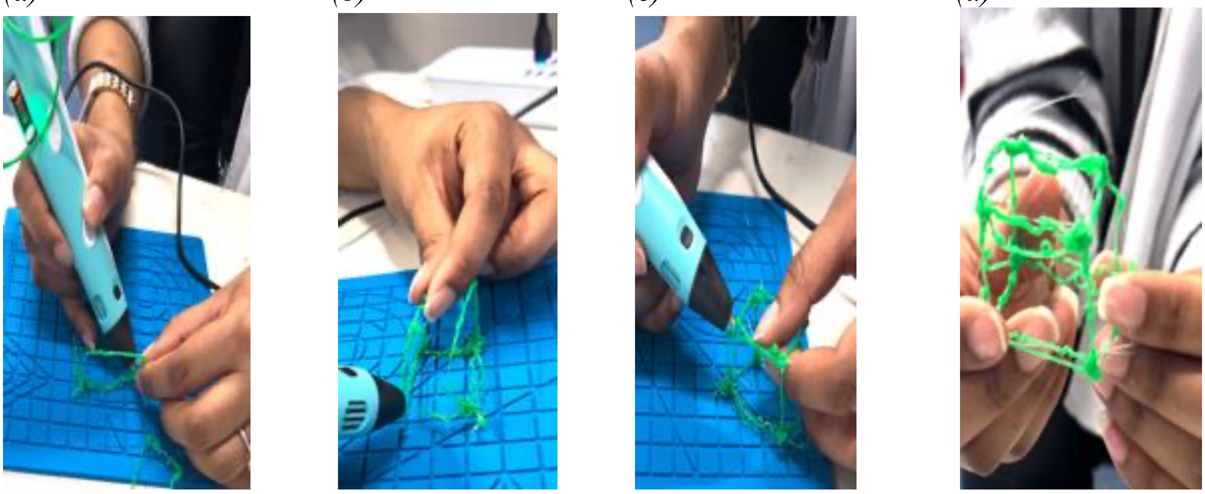
(a) *(b)* *(c)* *(d)*



figures 28 (a-d)

Taylor repeated the same process four times, constructing the same dimensions of the rectangle, to form a cuboid.

(a) *(b)* *(c)* *(d)*



figures 29 (a-d)

From here, Taylor recognises for herself that she constructed a cuboid and I asked her if there was a way to construct a cube using the 3D printing pen. She demonstrated the construction of a 3D skeleton cube (Figure 30 a-i).

Cube

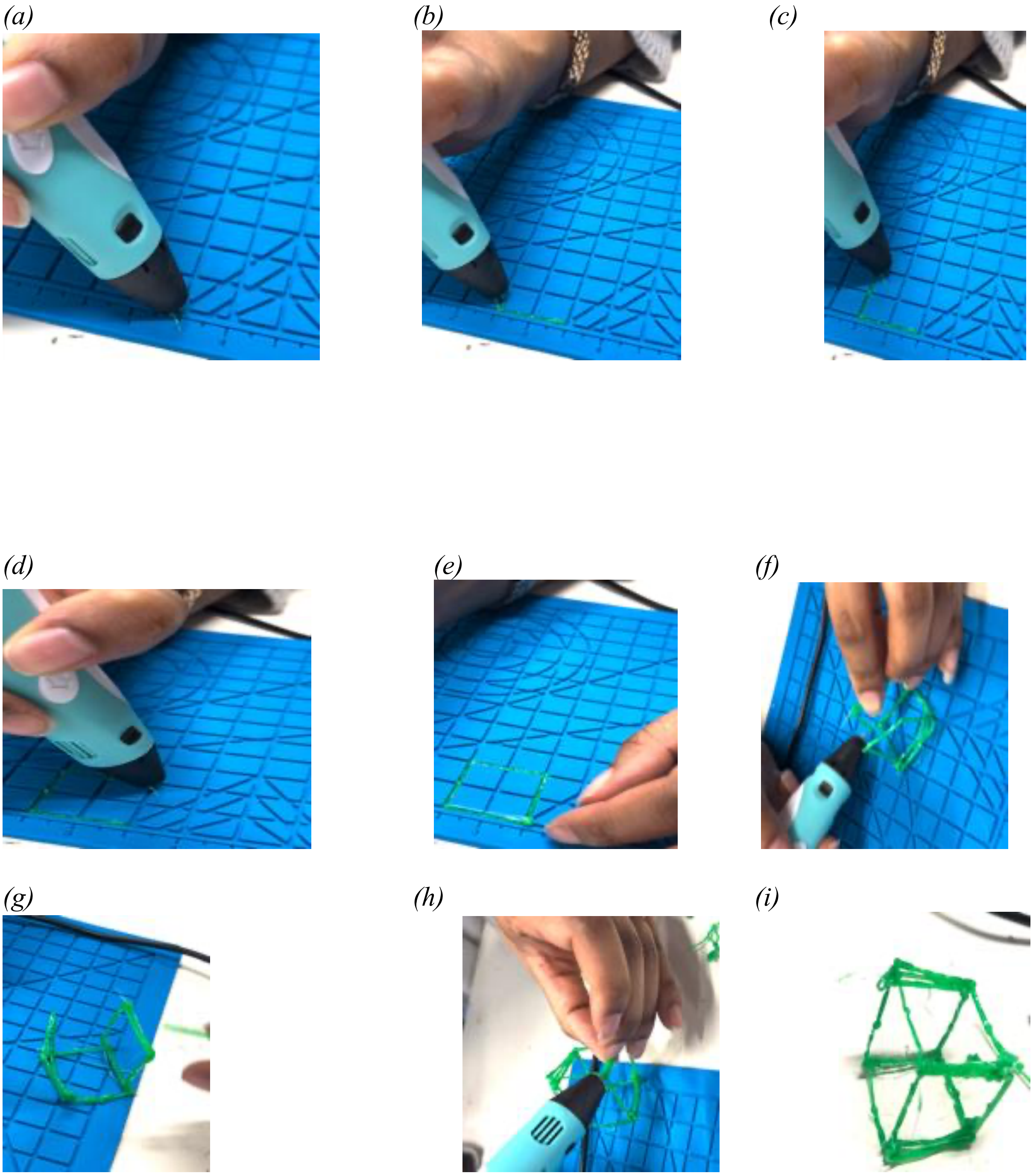


figure 30 (a-i)

Emily was Taylor's colleague who was interested in trying the 3D printing pen and learning-as-making activity. I visited Emily at the end of the school day and observed her hand gestures when using the 3D printing pen to construct a 3D skeleton shape. Images of Emily's hands working and her final product are in figure 35 (a-t).

Emily:	What do I press?
Matt:	Right here. That's it
Emily:	This one?
Matt:	Yes
Emily:	Okay. Is it not coming out? Matt, am I doing something wrong?
Matt:	No, I think
Emily:	Oh, it's coming
Matt:	Double-click to stop. Yeah, you're okay. There we go
Emily:	Okay. I'm going to have to do it again
Matt:	No, you're all right
Emily:	That's not a circle
Matt:	You never know. Let's see
Emily:	Okay. Circle?
Matt:	Yeah
Emily:	It's got stuck there. Oh, no. This is even worse
Matt:	That's okay. You want to try a square?
Emily:	I do, yeah. 'Cause the circle's quite hard. Okay. Still not quite a square, but I can...
Matt:	Yeah
Emily:	Do this and make it more of a square
Matt:	You want to try a triangle?
Emily:	Yeah. I'm rubbish at this
Matt:	How do you think this could be used in early years?
Emily:	This could be used with many subjects. Say maths. We could use it as part of physical development
Matt:	Physical development?
Emily:	Their fine motor skills
Matt:	Okay
Emily:	Because obviously they're using their pinchy fingers to hold the tool
Matt:	Got you
Emily:	And they need quite a bit of pressure. They need that strength in their fingers
Matt:	And now, rectangle
Emily:	Yep
Matt:	Go for it. What made you want to be a teacher, or early year teacher?
Emily:	It's my passion to work with the younger children. I feel like we can get... I like that we build the foundations down here
Matt:	Yeah?
Emily:	It's my passion to work with the younger children. I feel like we can get... I like that we build the foundations down here and we set them up for the future. So they learn everything with us, and they take all of that knowledge to the upper years. So I like being that first..
Matt:	You want to try a cube?

Emily: Yeah. A what?
Matt: A cube. You know?
Emily: Are you joking me? How do I do that?
Matt: What do you know about a cube?
Emily: I know it has six faces
Matt: Okay. What is the face?
Emily: A square. Six squares? On top of each other?
Matt: Mm-hmm. Or?
Emily: Taylor, help me out?
Taylor: Go on
Emily: I've got to do cube. Why are looking at me like that? Oh, I know
Matt: You could jump in [Taylor]
Taylor: You're a professional
Emily: So, yeah, jump in. So, do I do this square?
Taylor: Yeah
Emily: Don't laugh at my shapes, yeah?
Taylor: See how you do it, yeah
Emily: And then I am build it up now. But it's not going to stay up. Do I let it dry?
Matt: So you want to build it up, and then what?
Emily: Now I want to go across, but how's it going to stay? Do I need to hold it?
Matt: Careful
Emily: No
Matt: If you want to draw it up, what can you do? How do you hold a table up? You call this a desk
Emily: Mm-hmm
Matt: What does it have?
Emily: Legs
Matt: Okay
Emily: So it needs legs now
Matt: Okay
Emily: So then, do I just then go across now?
Matt: Can you?
Emily: No, because it's going to flop down
Matt: Okay. Can you think about another way?
Emily: Why aren't you helping me?
Matt: Or you want to pitch, tell her another way?
Taylor: Because I need to see your potential. Remember, it's your creativity
Emily: Oh, my gosh. This is too tricky. Okay, so then I'm just going to lay it flat now
Matt: Okay
Emily: Oh. But it's not going to look very nice now because I've made it dry and it's gone out of shape
Taylor: Remember, it does need to dry. Oh, that's nice
Emily: Yeah
Matt: Okay
Taylor: Oh, nice
Emily: Really?
Taylor: Yeah

Emily: Thank you
Matt: Okay. What are you going to do from here?
Emily: I'm going to now pick it up, and I'm going to turn it around again. And I'm going to do the next part, which is going to be really tricky
Matt: Okay
Emily: This bit here
Taylor: Nice
Emily: Go like that? I don't know if that's right, but let's see. Shall we? Okay
Matt: Okay. Now what are you thinking?
Emily: Now I'm thinking that I've got to do this part
Matt: Okay. I think you're folding
Emily: Am I not allowed to?
Matt: No, you can. Think about making a cube. Anything you want
Emily: I can, yeah. If I put one here, it'll go up. And then one here, it'll go up. Okay
Matt: You do art with your students, or no?
Emily: Do I do what?
Matt: Art. Drawing?
Emily: No. Never
Matt: Okay
Emily: I'm very good at displays though, I must say. Classroom displays. Give me a big wall and I can do something on there. Oh, gosh. How do I stop? Double-click?
Matt: Double-click, yeah
Emily: Yeah. Just wait for that to dry a bit. Okay. My only issue is it's not sticking together. What have I done wrong?
Matt: Do you think of a way …
Emily: Can I add some … Yeah, can I add some more to it to make it stick?
Matt: Add more what?
Emily: Of this? Yeah?
Matt: Yeah, go for it
Emily: Okay
Matt: What did that remind you of? As you're sticking everything together? What are you using, that's being pronounced?
Emily: It's like paper and glue, isn't it? Is that making it out of paper?
Matt: Your [own] kids are in early years as well, right?
Emily: Yeah
Matt: How do you find it?
Emily: As in my own child?
Matt: Yeah
Emily: Yeah. So I've got a boy in early years
Matt: Wow
Emily: Yeah. No, he loves it. He's very boisterous, my son. But yeah. No, he's really enjoying it. He's not in this school, though
Matt: My dad is a maths lecturer
Emily: Oh, wow
Matt: I was going to say. So yeah, so math has had to be a choice. But I actually liked history. My background was American history and math

Emily:	Ah. This is not gone to plan now. Okay, that's better. Stick that together. Now we need to get this part together. I don't know where this extra part has come from here. I think you can just... Oh, there we go
Taylor:	There you go
Matt:	See how you feel using it
Emily:	Yeah
Matt:	Is it fun?
Emily:	Yeah. It takes practice. So, yeah. Definitely you need to have a practice with it before you say to a child, "Make this," or you won't get the perfect result
Matt:	For early years, they won't be able to use it because…
Emily:	It's too hot. Yeah
Matt:	How long you been teaching for?
Emily:	I've been in this school for 10 years. I've been teaching for eight
Matt:	Oh
Emily:	Yes. Done. No, it's not great but it's all right

Diffractive reading of Emily using 3D printing pen

I unfold how Emily constructed her 2D shapes (circle, square, triangle, and rectangle) and 3D skeleton cube. It is important to note that Emily did not use the silicone mat. The reason I did not give Emily the option of using the silicone mat was because before the learning-as-making activity began, she made a comment about being quite busy due to a lesson she had to prepare. Each of her shapes – circle (figures 31 a-i), square (figures 32 a-e), triangle (figures 33 a-e), and rectangle (figures 34 a-e) – can be seen below.

Circle - Emily's first attempt

(a)

(b)

(c)

(d)

Circle – Emily's second attempt

(e)

(f)

(g)

(h)

(i)

figures 31(a-i)

Square

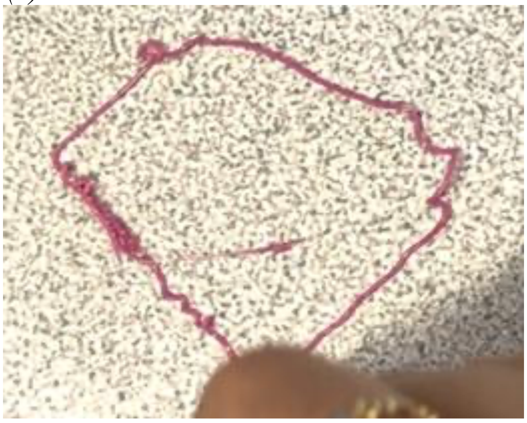
(a)

(b)

(c)

(d)

(e)



figures 32 (a-e)

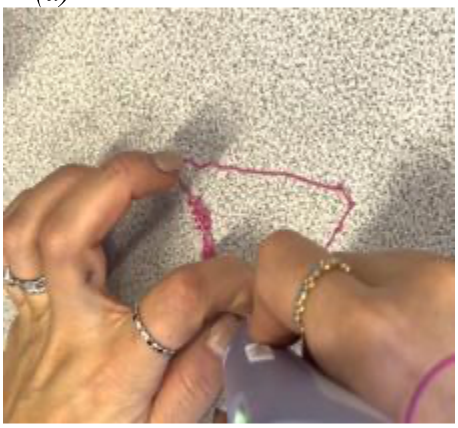
Triangle

(a)

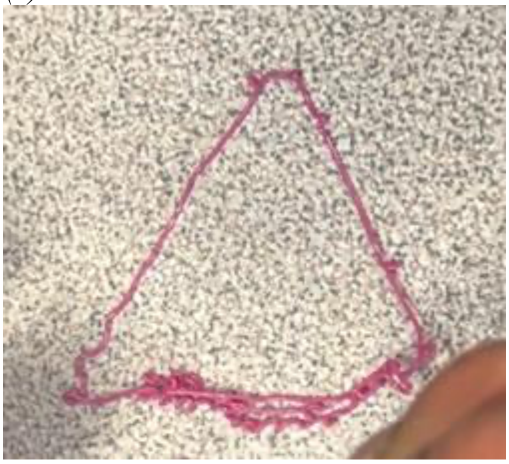
(b)

(c)

(d)



(e)

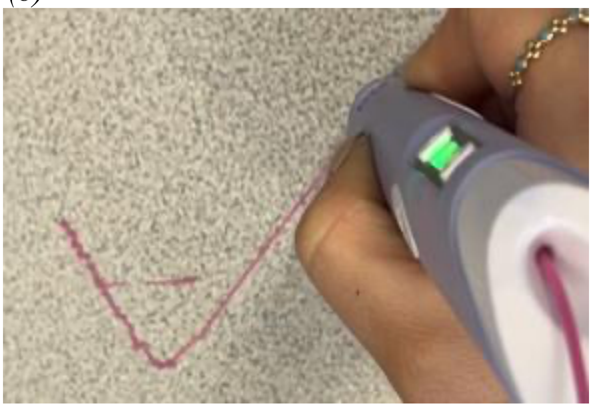


figures 33 (a-e)

Rectangle

(a)

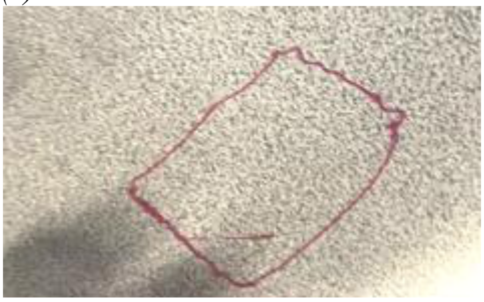
(b)



(c)

(d)

(e)



figures 34 (a-e)

I asked Emily to make a cube, and Emily turned to Taylor for help. However, Taylor did not help Emily in the making process as she wanted to see her creativity. Below is Emily's construction of a Skelton cube (Figure 35 a-t)

(a)

(b)

In the two figures above (Figures 35 a-b), Emily starts her skeleton cube making process by drawing a square (2D) as the base. Next, she thinks about drawing lines (1D) in the air. Emily tried to make a line by dragging the 3D printing pen across the square. However, she realises that the line will collapse flat, as seen below (figure 35-c).

(c)

figure 35 (a-c)

Instead, Emily decided to use the straight line (1D) that is a leg on the square to draw another square (2D), as seen below (figures 35 d-e).

(d) *(e)*

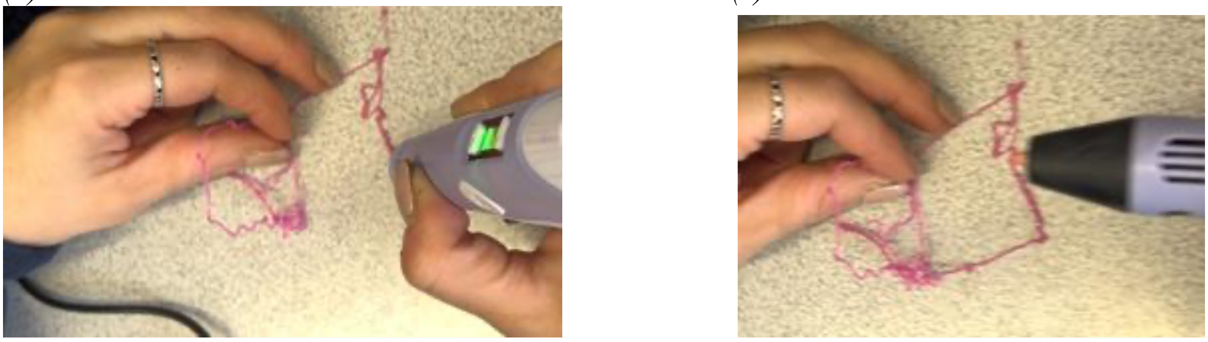
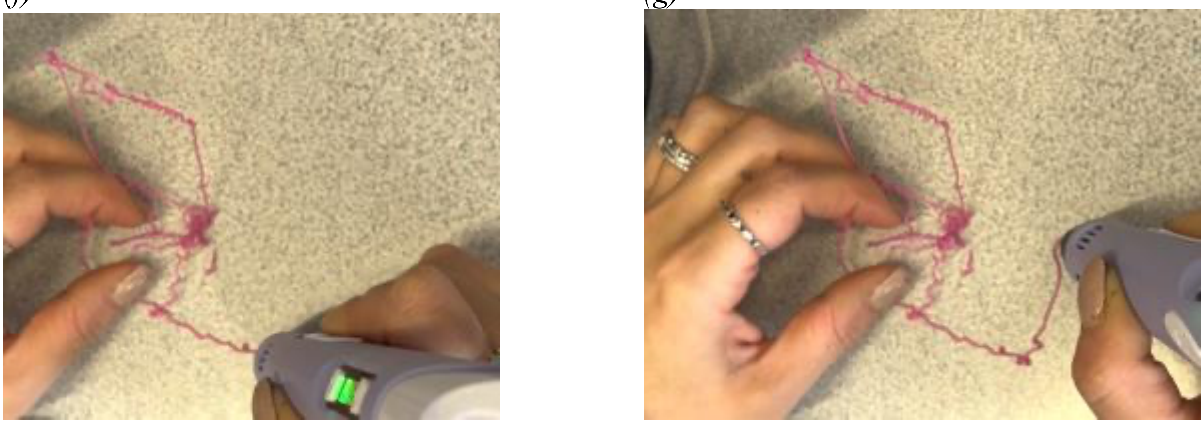


figure 35 (d-e)

Afterward, Emily used one square that was constructed to make another square, or face of the skeleton cube, as seen below (figures 35 f-h).

(f) *(g)*



(h)

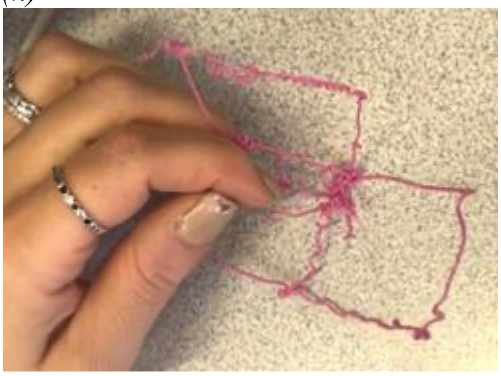


figure 35 (f-h)

She continues the same process one more time.

(i) *(j)* *(k)*

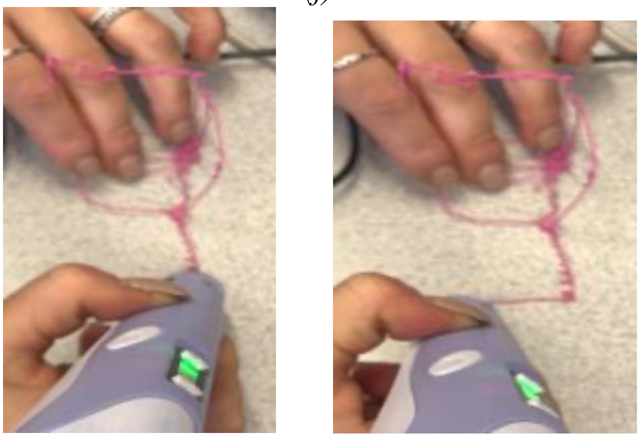


figure 35 (i-k)

From this moment, Emily decided to use her hand to squeeze the squares to form a skeleton cube, as seen in figures 35 (l-n).

(l) *(m)* *(n)*

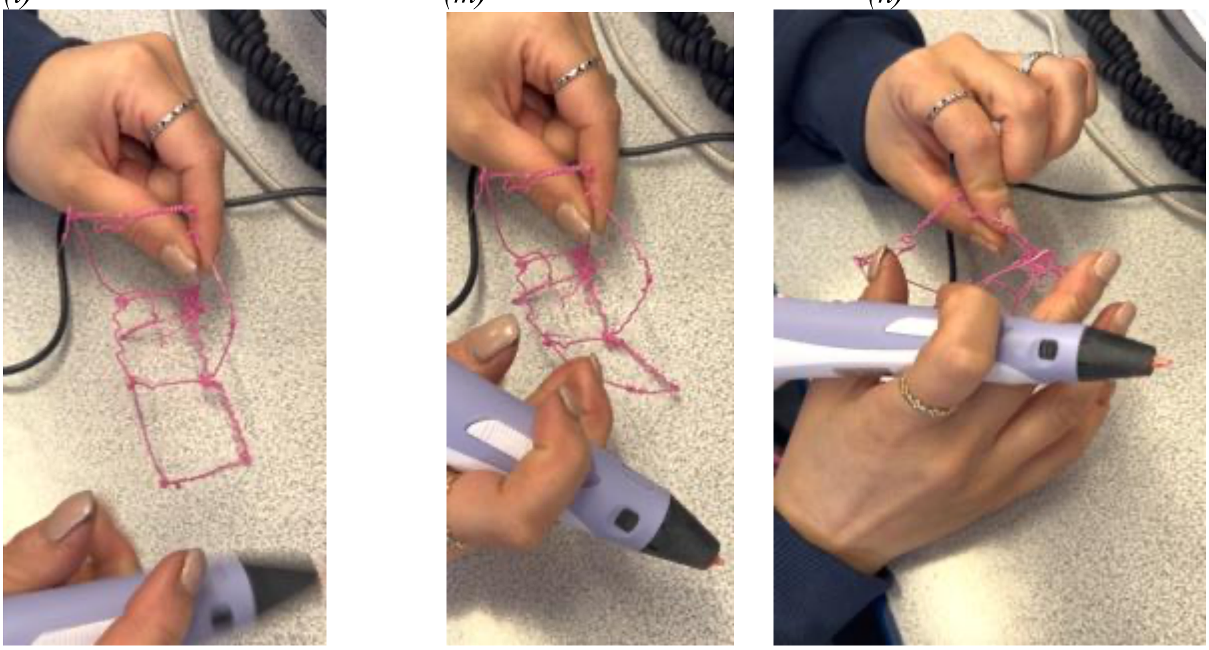
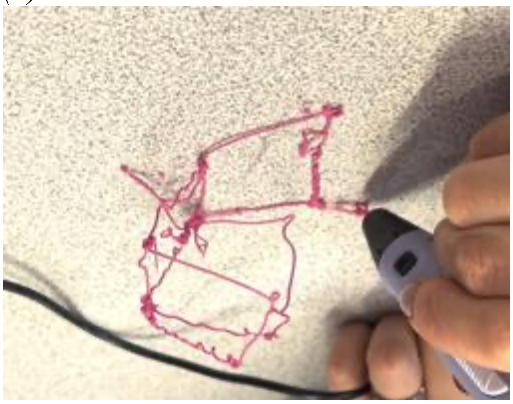


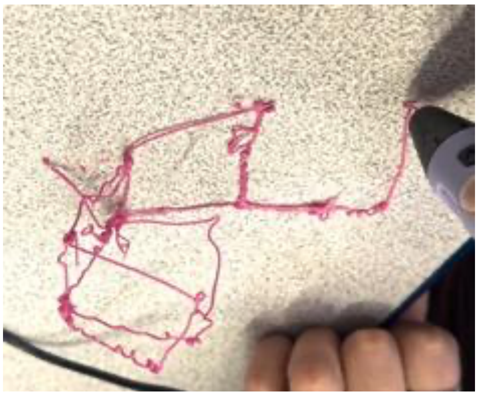
figure 35 (l-n)

From squeezing the parts of the skeleton cube together, Emily realised that she needs another side, as seen in figure 35 o-q.

(o)



(p)



(q)

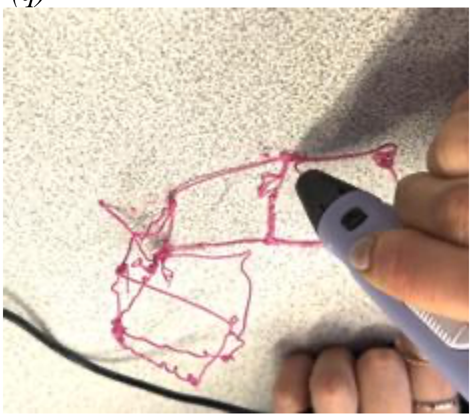


figure 35 (o-q)

To complete her 3D skeleton cube, Emily folded the 3D skeleton cube together and used the 3D printing pen as a glue gun, mentioned in her dialogue. See figures 35 (r-s).

(r)

(s)

figure 35 (r-s)

Emily's completely constructed 3D skeleton cube:

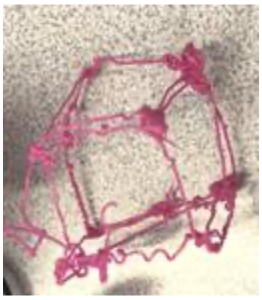


figure 35 (t)

5.3.3 Retelling a story: Emily's engagement with 3D printing pens

Emily was introduced to a 3D printing pen when she saw Taylor using one a year prior in my learning-as-making activity. She thought it was a cool technology tool, so I asked Emily if she would like to participate in the research study. She agreed. Emily is familiar with the mathematical learning outcomes for students in the early years level. When asked in my written survey, *Tell me a story about your experience in the 3D printing pen part of the learning-as-making experience*, Emily replied:

> To begin with, I was worried of how the outcome would be as my first attempt I found slightly tricky. But after 2-3 attempts I surely got the hang of it!

While Emily did not have an opportunity to employ 3D printing pens in her classroom, I asked her through the survey, *How do you see 3D printing pens being employed in early year classroom?* She replied:

> Through creative activities… a teacher models this. Learning shapes and colours.

During the activity, she looked at Taylor for advice in constructing her 3D skeleton cube. However, Taylor took a teaching approach, wanting to see her colleague's own imagination and design in making the skeleton cube. Emily talks about her knowledge of a cube as she begins constructing her skeleton cube. In the survey, I asked Emily, *Why did you pursue a career in early years teaching? Was there a special person that made an impact on your career path?* She replied:

> My passion is early years. I've always wanted to build the foundations and be a child's first teacher and be part of that first learning journey of a child's.

In the next section, I talk about Taylor's participation in the Tinkercad component of the learning-as-making activity.

5.3.4 Tinkering in learning-as-making activities

Taylor engaged with the Tinkercad functionality to create using Tinkercad's four primary shapes: a sphere, a cube (box), a triangle prism (roof), and a cone (see figure 36(a-c)). Below is our conversation as Taylor creates her models on Tinkercad.

Matt:	All I want you to do is to use the four shapes, a sphere, a cube, a triangle prism, and a cone.
Taylor:	So this is the triangle prism?
Matt:	So notice they call triangle prism a roof. Do you call a roof, a triangle prism a roof?
Taylor:	Right
Matt:	And a cube a box. Okay. Do you think this poses a problem when teaching math?
Taylor:	Maybe better with staging sometimes
Matt:	Okay
Taylor:	So I've got a cube, triangle prism, cone as well, right? Cone, I would say children like ice cream cone
Matt:	Okay
Taylor:	Maybe if it relates to something they see daily, it helps them remember
Matt:	How can this programme be employed in the classroom, in a math classroom?
Taylor:	I guess in terms of what can we use to bring the shapes home like for maths. When they can ... Let's say when you're making something or building or constructing something, what materials would you use and then just maybe what shapes would you use
Matt:	Gotcha. Is this easy to use?
Taylor:	I don't know because you telling me that I can manipulate [shapes]... Reception I don't see them using [Tinkercad]
Matt:	Would students be interested in playing around with the software, do you think? If you tell them to build a house?
Taylor:	Maybe, yes. Again, a purpose
Matt:	What purpose do you see?
Taylor:	Well, building a house. Let's say we're building a school.
Matt:	If we had to print this here, the students made something, how do you think printing it makes them feel?
Taylor:	So once they make it and then they print it, it's like, "I made this"
Matt:	Okay
Taylor:	Happy with their creation
Matt:	Why do you think they're happy with their creation?
Taylor:	Because it's their hard work, their idea
Matt:	Gotcha
Taylor:	You've just given them like, "Okay, we need to make a cup. We need to make a house, make a school. How would you build your school?" And then they'd build it using the shapes. They have no particular names on the shapes, but you would be like, "Okay, so when you talk to me about it and you show the class what you made, I want you to tell me the shapes you used, name them." So not just, "I used this. I made this," but link it again to a lesson. That's the lesson I made. I made the roof out of this shape or I made the body of the house out of this shape

Matt:	Right. I understand. So a MakerBot 3D printer costs 3,000 pounds. Do you see a primary school purchasing that or that's too much money?
Taylor:	No, I think that's too much. I guess it depends on the budget because schools have very tight budgets. They have a low budget. I wouldn't see it kind of. If I was to print this, it would come out a 3D shape?
Matt:	Okay
Taylor:	It would be amazing for the children, but I don't think it's something the school would maybe think of purchasing
Matt:	Okay. And do you see how this makes a connection to a real-life occupation?
Taylor:	Definitely, yeah especially when they see people doing this a little bit. Okay, let's think about your house. What shapes do you see? What shapes is the roof? What shape is this? What shape is your bed? Is it square or rectangle? Why is it rectangle? Why is it a square?
Matt:	Also, which one was easier to use, the 3D pen or this [Tinkercad]?
Taylor:	I think the 3D pen because it's like you're using it
Matt:	Okay
Taylor:	To be fair, it's hands on. But this one. Yeah, it is easier. It depends how you look at it
Matt:	Okay. So easier would be the pen, and then this one [physical 3D printed cube] would be more meaningful?
Taylor:	Yeah
Matt:	Okay. Would you employ [3D printing pen], if you had an opportunity in the classroom?
Taylor:	Yes. And I would think about the curriculum where I would place it in curriculum

(a) *(b)* *(c)*

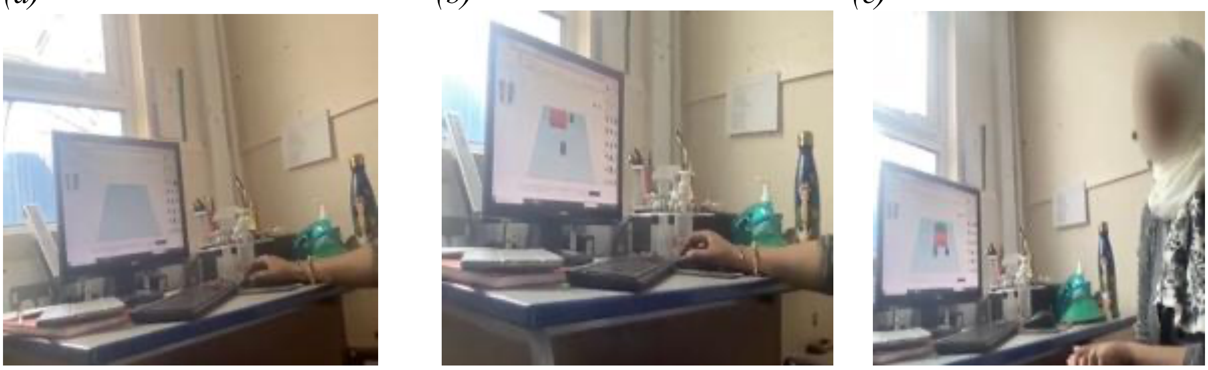


figure 36 (a-c)

5.3.5 Retelling a story: Taylor’s participation in this study

I provide a short dialogue that occurred on the first day of the learning-as-making activity at the school of Ravenclaw, and I share the survey question that Taylor filled out after the activity.

Matt:	What made you become a teacher?
Taylor:	I think my children in lockdown
Matt:	What were you doing before?
Taylor:	Working at the job centre

The survey question that Taylor filled out after taking part in the learning-as-making activity asked, ‘What influenced you to become an early years teacher?’ Taylor responded as follows:

> I have always wanted to become a teacher throughout my educational journey but never had the opportunity to pursue a career in education. During the pandemic, I was home-schooling my children, aged 2 and 4, when I found that I had the patience, planning skills and enthusiasm needed to become a teacher in the early years. I decided to apply and start my PGCE. During my training year, I was able to experience working with early years and year 2. I found that I was able to be adventurous, creative, and enthusiastic when teaching reception compared to year 2.

After observing Taylor’s first day of the learning-as-making activities and watching her employ 3D printing pens to identify 2D shapes with her early year classroom, I gave her another survey question asking her to:

> Tell me a story about your experience taking part in the study. Start when you signed up for the study without having seen a 3D printing pen, Tinkercad, or Makerbot 3D printer. Then discuss your participation over the summer as you engaged with the 3D printing technology with the researcher. Finally, discuss your experience in the classroom using the 3D printing pen and your students' experience seeing it in action.

Taylor responded:

> I was still a trainee in the summer of 2022 when I signed up for the experiment. I had my own class then because I was also working as an unqualified teacher. I was not aware of what the 3D printing pen was, the Tinkercad or the Makerbot 3D printer. Firstly, after trying the pen and seeing what it did, I was able to start thinking about the different ways it can be used in the school with the children in my class. I enjoyed working with the researcher as he made me think outside the box. His questions were open-ended, making sure I was able to think outside the box and we discussed the different approaches, lessons and lesson plans I could use the 3D pen in my class. When Matthew came into the class, the children were excited to see what they could do with the 3D pen and were fascinated with being able to touch and feel the shapes after I created them using the pen. I was able to make it into a game asking the children to guess what shape I was going to make next and what the shapes were after I had shown them. We were able to add different mathematical lessons into the shapes

> session as they used their numbers knowledge to count the sides and edges. It was an amazing experience for myself, the staff and the children in the class.

On the first day of the learning-as-making activity, Taylor was able to construct a 3D skeleton cube using the flat surface to construct 2D shapes of a square. Given that Taylor had used the 3D printing pens three times throughout the study, she was able to plan her classroom activity herself and I did not know what type of activity she was going to run with her students. In the second visit, she employed the 3D printing pen in a lesson that 'talk[ed] about and explore[d] 2D and 3D shapes' (DfE, 2023, p. 89). During this lesson, Taylor sat in front of the class using the 3D printing pen with the silicone mat on her lap to construct 2D shapes. From a teaching perspective, students were engaged and she had a good interaction with the students as seen in the story-to-live by dialogue. What is not seen in the dialogue is the time it took to set up the device and classroom management of the students at reception level. To mathematise 2D shapes, Taylor not only needed the 3D printing pen to function, but she also needed the cooperation of the class to conduct her lesson effectively.

To confirm that Taylor was aware of the mathematics content of the early year framework, I asked in my survey, *As an early-year teacher, are you familiar with the mathematical learning outcomes for students at the early-year level? (i.e., Are you aware of the EYFS [early years foundation stage]?)* She responded:

> Early years follow the development matters framework. As an early-year teacher, I am aware of the mathematical learning and follow the guidance to understand what the children need to learn by the end of the year. I am also aware of the outcomes that the children need to have in order to see if they have met the early learning goal by the end of reception.

To understand how Taylor is mathematising using the 3D printing pen, I will take a look at her making process using a diffractive reading of her construction of the 3D skeleton cube and 2D shapes. On the first day of the learning-as-making activity, she constructed a 3D skeleton shape by using the flat surface to create four squares (2D) to assemble together to then create her 3D skeleton shape. A year later after she had conducted her in-class activity, Taylor's diffractive reading shows that she struggled to stay in the groove tracing with the 3D pen on the silicone mat compared to the lesson with her class. This could be a result of having a different hand gesture or of the speed at which the 3D printing pen produced filament. The main observation here is noticing that she used the silicone mat to construct her 3D skeleton cuboid and 3D skeleton cube. Most importantly, Taylor kept constant the length and width of each side of the cuboid – 3 centimetres long and 2 centimetres wide. In addition, the four squares of the 3D skeleton cube were 1 centimetre long and 1 centimetre wide. Here, not only is Taylor's shape perception being displayed, but also her mathematical knowledge of the properties of a cube. As mentioned in her survey response, she is aware of the mathematical learning required in the development framework. Basic knowledge of shape is a part of the framework for early years.

Taylor also demonstrates mathematising when she uses Tinkercad and discusses the impact that a Makerbot 3D printer could have on students' learning. She imagines a hypothetical teaching situation using Tinkercad in which she suggests to her students,

> “We need to make a cup. We need to make a house, make a school. How would you build your school?" And then they'd build it using the shapes. They have no particular names on the shapes, but you would be like, "Okay, so when you talk to me about it and you show the class what you made, I want you to tell me the shapes you used, name them.”

Taylor imagining that she initiates an activity in which her students use Tinkercad to make an object such as school or cup and then having them relate it to the geometrical object shows that she is mathematising in terms of relating real-life objects to mathematics. In addition, she acknowledges that creating a shape on Tinkercad and producing it on the Makerbot 3D printer can give a purpose to students. She explained that ‘once they make it and then they print it, it's like, "I made this”’. Throughout the learning-as-making activities for Taylor in all three visits, she was able to not only demonstrate her making process, but also mathematise using materials.

5.4 Episode 3: Mathematising in the School of Hufflepuff

5.4.1 Background

This episode is situated in Elton’s primary classroom. Elton is an in-service teacher who teaches computer science and mathematics in the primary level. He has experience teaching his students the basic functions of Tinkercad using computing content of the English National Curriculum. However, this was Elton’s first time using the 3D printing pen. He engaged, played, and designed a skeleton 3D cube in the learning-as-making activity. Then, for the second part of the activity, he described his experience using Tinkercad.

5.4.2 Engage, play, and design with 3D printing pens in learning-as-making activities

Elton started engaging with the 3D printing pen using the silicone mat. He drew a cube on a piece of paper and then proceeded to use a 3D printing pen to construct a skeleton cube, as seen in figure 37(a-f). Below are dialogues between Elton and myself in the first part of the learning-as-making activity:

Elton’s Experience Using 3D Technology Tools

Elton:	I have used a 3D pen before. We have just started using some 3D modeling as part of the computing curriculum we have here. We follow the NCCE. So, we are literally within sort of couple of weeks of having started work on Tinkercad
Matt:	Okay
Elton:	But that is my first introduction to anything like this
Matt:	Wow, okay
Elton:	So, you've kind of come at a good time, really
Matt:	Okay. So you guys use Tinkercad for math, as well?
Elton:	Not for maths. As far as I know, it's not used elsewhere down the school. It appears in the NCCE curriculum in year six; it's one of the modules in year six. That's the first time that it appears

Matt:	Okay. So, if you want, so first, to use it, you just hold it in a 60 degree angle and you press this button to heat it up. So, it has to go to 200 Celsius, so, 190, okay?
Elton:	Okay
Matt:	Do you think it'd be a problem, if it's too hot for students, or do you think they'll follow direction?
Elton:	Which bit is hot?
Matt:	The black one. If they touch the black one, at the end
Elton:	The black head, that would be hot?
Matt:	Yeah
Elton:	I think maybe at this age, possibly yes. I mean, it's difficult to say. But I can currently, this year, I could foresee issues, if you're telling me that that black thing there, there's nothing between a kid's hand or something and the heat. I'm not going to touch it, by the way, I'm just feeling the heat coming off it
Matt:	Yeah, no worry
Elton:	I could see... does that always have to be exposed like that? Is there not a way that it could be covered in something?
Matt:	No, it has to be exposed like this. So, what they do is they have this here
Elton:	Okay

Elton's Engagement with Functionalities of the 3D Printing Pen

Matt:	So, it's 3D Doodle Pen. So, you put it in here and that's the way to hold it, but it's always exposed outside
Elton:	Okay
Matt:	Let's see. Is it heated? Yeah, it's perfect. Okay, so it's 190. So, I'm just going to ask you to insert the filament here. And so you put it in the tube hole and then you just press the button down. There we go
Elton:	Just the one, yeah?
Matt:	Yeah. And the slide, up and down, the slide here, is the speed, okay?
Elton:	Yeah, that's fine
Matt:	So, you click it once to keep going. And then you double-click again to stop. There you go
Elton:	Okay
Matt:	So, it will come out eventually. I'll give you a little example here. Takes a little while
Elton:	Yeah
Matt:	There we go. See how it came out?
Elton:	Oh, yeah. I see
Matt:	So, perfect. Let's go get you the... I believe I have the tongs in here. But once it comes out, it's not too hot. Yeah
Elton:	So, it's kind of like a glue gun
Matt:	And then you use this as a way to take it out, if you want. But it cools down in a moment
Elton:	Okay. So, I use this sort of scraping …
Matt:	Well, it's like you put it on your finger, that kind of thing. You see this, right here? The holes, or this here

Elton:	Oh, put that on my finger?
Matt:	Yeah, mm-hmm. I know. It's really kids... yeah, your pinkie
Elton:	And then?
Matt:	And then wipe it off
Elton:	Wipe it off. I got you. You've lucked out here, because I am absolutely the most uncoordinated person
Matt:	No worries
Elton:	And I don't actually understand, because that's not working at all
Matt:	Yeah, the other way. Like, if you want, take it off and then rub it out. Sometimes it gets stuck in there. There we go. But in case it gets stuck, what happens is that, when you print again, it will just come out
Elton:	Okay
Matt:	So now, hold it like a pen, like you're using a pen. And then try and draw a 2D shape. Like, a circle, square, whichever one you like
Elton:	On?
Matt:	On the surface. And then, it won't stick to the surface. Don't worry
Elton:	Okay. So, without pressing anything else, I just draw a 2D shape, yeah?
Matt:	Yeah. So, once you press it, you have to click the button. That one
Elton:	That one?
Matt:	Yeah, mm-hmm. And then, once it comes up, then you can go ahead
Elton:	Okay, gotcha
Matt:	How do you see this being helpful for primary students for 2D shape, out of curiosity?
Elton:	In a way that we're not doing it already. We're not, obviously, because we have access to whiteboards, pens, pencils. I have to be honest, I can't see anything at the moment. I can't see at this stage, how this replaces the whiteboard and whiteboard pen, or the ruler and the pencil, and the book. Very nice, very interesting
Matt:	Gotcha. So, try doing that on the pad now. Draw it, like, the same circle and the square
Elton:	Within the …
Matt:	With the mat, yeah
Elton:	Within the mat. Okay. Got you. I suppose the other thing, as well, of course, is you can see, without a ruler or without anything, again, you can't draw straight. Okay, well, apart from that. That's slightly wrong. So, I can see the mat, obviously, with the grooves, help with the accuracy. I'll do one more. Doing that side. Okay
Matt:	So, it helps with the accuracy. What else can the mat help with, out of curiosity?
Elton:	Well, now you've got it on the mat, you can start to look at things like angles, you can see things like area. You can compare the circumferences of the circles or the diameter as well. You can start to see that. I don't know. That's it, I would say. You can obviously measure in centimetres as well. So, obviously a lot more accuracy. And more detail
Matt:	Was it easier to draw on the mat or the surface?
Elton:	You'd think it'd be easier on this, wouldn't you? But this has got, there's more extra bits …
Matt:	Bits, filament. Yeah. I got you

Elton:	Filament than there is on there
Matt:	Because you said earlier you couldn't measure the measurement on the table. Yeah
Elton:	Yeah, on these parts it's definitely easier to keep it in the groove. Maybe these bits here are because I'm not too experienced about how to switch it off and switch it on again
Matt:	Gotcha
Elton:	Maybe it's that. So, yeah. This was easier to draw a circle

Elton's Story of Becoming a Primary School Teacher

Matt:	What made you become a primary school teacher?
Elton:	Yeah. Working in publishing before, and then electronic information
Matt:	Wow
Elton:	Worked for a company called ProQuest. Have you heard of ProQuest?
Matt:	No
Elton:	They do a lot of online …
Matt:	ProQuest? Yeah, yeah. Wait …
Elton:	Online journals
Matt:	Yes, yeah
Elton:	Online information. I worked for ProQuest on the ... God, it gets better when you get into a groove, doesn't it? On the sales and marketing side. And then realized that I'd done that and there was always an educational element to, obviously, what we do. We're very much working in education. Except I was usually called in on places like university libraries and universities and research establishments. And it's always something that I thought about. And I wasn't getting any younger. Who is? And so, decided that I'd make the jump. So, I think this one's a bit of a young man's game or a young person's game, because you're traveling around the world. There's foreign travel. And by that stage, I had a family, some children. There's a lot of things that seemed quite nice. "Hey, you've got the sales conference for a week in Las Vegas." Which, when you're younger, are great. They kind of wear off a bit. Don't know if they wear off completely, but they wear off a little bit. So, and various sort of things happened. I mean, I think it's long-term it just wasn't something I saw myself doing. So, about ten years ago, nine years ago, I stopped. I just did some sort of more local work to get some money in, and applied for PGCE at a university in London and was accepted on that. And then spent a year on PGCE. And then a year as what used to be called a newly qualified teacher, which was announcing that now you're an early qualified ECT. That's a two-year thing. Yeah, did two years as a newly qualified teacher. And that was about nine years ago. I've been here now about seven years
Matt:	Oh, wow
Elton:	Yeah
Matt:	Yeah, I'm twenty-eight. I mean, I'm twenty-nine in August, but I moved here three and a half years ago. I was a teacher. In New York it's so hard, because I wanted to be a university, community college lecturer, but you

need a PhD. A PhD program costs fifty grand in America. And it wasn't worth it. It didn't make any sense. And education over there is very different. It's no standards. Every state is different. So, if you want to go to New Jersey, I'm from New York, you have to take the exam again

Elton: Yeah, I've picked up on that, actually. Because I'm a member of a few teaching groups on Facebook. And there's a few teachers there, obviously, from the States that are on there sharing information and knowledge and what have you. And you get the idea that it's very much a state system. Whereas here, of course, we've got the national ... well, within England, Scotland, Wales. They do their own thing. But pretty much, the national curriculum is the national curriculum

Matt: Exactly

Elton: And pretty much you should be able to teach anywhere in England

Matt: Which is nice

Elton: Yeah, it's nice. It's nice to know that you've got a skill that you could, if you want to, go and teach somewhere else in the country

Matt: So, your undergrad's in journalism, or … ?

Elton: No. My degree is in politics

Matt: Oh, interesting

Elton: But that's going back a while. But I did some marketing qualifications as well at one point. So, I am actually the school's computing lead. So, in no way am I technically qualified at all. But I'm from that background of using technology, using computers. So, it is just kind of a thing for me. We're trying to get things going at this school, really. Because we have so many of the kind of competing things going on all the time. And obviously, we serve quite a deprived area, really. So, this is an education gem. But also things like computing technology. This is a big thing for the kids here, because they're not seeing it or getting it anywhere else. I mean, they're getting iPads at home. They're getting devices at home. But not really, I don't think, any ideas about how this could be a career, or you could do things. Apart from being a YouTuber, of course

Matt: A TikToker

Elton: Yeah, or TikTok

Matt: Do you have TikTok?

Elton: Yeah

Matt: I do, too

Elton: But yeah, so the idea that this is a career or there's other things …

Matt: Like STEM?

Elton: … related to, I don't think that's too well-known. So, all this kind of stuff is great

Elton: You got your cube. A couple poor circles

Matt: Okay. So, the cube, right?

Elton: Yep

Matt: So, can you draw it in the air for me?

Elton: Can I draw it in the air?

Matt: So, this here is a ... so, right now …

Elton: I see what you mean. I've not drawn a cube. I've drawn a square. Okay

Matt: Okay

Elton:	Yes, yeah
Matt:	Can you draw me a cube, though? How do you think you would do it?
Elton:	Well, if I was going to draw the cube, I would go over all these, all the, what do you call them? Grid lines, grooves, whatever you call them. I'd go over all of those. Well, actually, I wouldn't maybe. I'd start smaller. So, let's say I did a three-by-three here. So, I've drawn it up there. Or a two-by-two. So, I've drawn it up there. I do the bottom. And then I'd just slowly, I guess, build up the layer around the outside
Matt:	So, you're thinking more of like a 3D printer
Elton:	I guess so. But that's the only way that I could see me doing a cube. I'm not deliberately trying to do it as a 3D printer. But you're asking me to draw a cube in 3D
Matt:	Okay. So, you're thinking of a, I'm going to use the word, like a solid cube, correct?
Elton:	Yeah
Matt:	What about no solid cube? Just a thin one. Can you draw that one using a 3D pen?
Elton:	You mean just on paper, like, can I draw?
Matt:	Yeah

From here, Elton attempted to construct a skeleton cube using the 3D printing pen and silicone mat.

Elton:	I wouldn't say that's a cube, but
Matt:	That's okay
Elton:	But that was the... I would say that one, I did that the first time. I thought, "Oh, actually, this is going to work. This is great." And then from there, it seemed not to... I don't know what happened. If it's something I've done wrong, or it's just the thing, the heat build-up or whatever it is. But I seemed unable to get another sort of upright like that, in any of the other corners
Matt:	Okay. No, don't worry about it. I got you. I got you. Does this school have funding? When you purchase stuff, does the school get funding from the government or how does that work?
Elton:	Well, we get funding. We get funding basis of per child, I believe. But you also get pupil. There's a number of different funding streams. One of them is Pupil Premium Funding. So, that's based upon the number of children who are receiving free school meals. That's additional funding for schools for children in challenging circumstances. And then there's the main funding that all schools get. I've got to say here that we don't have things like Parent Teacher Association that do fundraising. So, a lot of schools will have a PTA, some fair, they'll do things like that. We don't have one of those. In fact, some fair that we're doing next weekend, that's the teachers coming in on Saturday to do that
Matt:	Wow
Elton:	Because we have got various reasons. We don't have that. So, it means you've got a very uneven playing field. Because the school down the road will have a PTA with lots of kind of involved parents and others, and

people with skills and connections and ways of making money. I mean, for example, one round, a school near where I live, last year or the year before, had various prizes that had been donated by the local shops in the area. Raffle prizes and things like that. So, one prize would be a beautiful morning of yoga at the yoga school. Or it'd be a tennis lesson with someone who's, not a pro, but someone who's a person in tennis

Matt: I hear you

Elton: Or it might be this or it might be that. So, you've got parents who've got those connections and know how to exploit those connections or use those connections. Whereas we don't really have access to any of that, really. So, it's the difference between a school, say when they do a fundraising, a difference between making a few hundred pounds, £300, £400, £500, and a school making several thousand pounds, or more than that

Matt: So, you were placed in this school? Or did you select this school?

Elton: No. You mean who comes here?

Matt: I'm not sure how the job works. Like, if you apply for the …

Elton: Oh, I applied to come here

Matt: Oh, okay

Elton: Yes, sorry. I thought you meant the children. No, I applied to come here. Yeah

Matt: Oh, okay. You didn't want to go to Prep or anything like that, or?

Elton: Well, Prep's a little too far away for me to commute. No, I think it's …

Matt: They're a good school, right? That's what I heard

Elton: Oh, yeah. They are. Yeah. Quite a few famous people have been, their children. I think, along with a few others. No, because I think it's important to go where you can make a difference

Matt: There you go

Elton: And I mean, all teaching is hard. All teaching is hard

Matt: It is

Elton: Whatever you're teaching

Matt: Of course. I hear you on that. I hear you on that

Elton: So, we're not going to go down this route and say, Oh, my job's harder

Matt: No

Elton: All teaching is hard. And sometimes the teaching that goes on in more affluent schools have different challenges. You're dealing with parents more. You've got high expectations. There are more politics around dealing with parents and these sorts of things

Matt: Oh, no. I know

Elton: So, different schools, different challenges

Matt: I used to work in a private school in New York. And the parents are like, "I'm paying a lot of money. How come my student didn't get an A?"

Elton: Yeah, exactly. So.

Matt: And the schools boost the grade for no reason

Elton: Yeah, exactly

Matt: It's crazy

Elton: So, if they give a lot of money to the school, you have influence. I get it. So, everything, all schools are difficult. All schools are a challenge. But I was looking for a new school. I came here. I liked the atmosphere. I liked the head teacher at the time, obviously liked me. And I was offered the job. It's not too far from where I live, but far enough. And came. I've been here for, it's seven years in May

Matt: Wow

Elton: But the people are good here. We've got a good team of people that work here. Everyone's always very supportive. They're the reason, I guess, we keep coming. You come for the kids, but it's the people around you. If they're not supporting you, then what chance have you got?

Matt: I hear you on that

Elton: Yeah, I must like something about it. Because I'm still here seven years later, which is not the longest of time, because you do hear of some teachers who are in a school for years, and years, and years

Matt: You've got another fifty years here

Elton: But it's a decent amount of time. It's a decent amount of time for me to go, "Well, this is what I did." So yeah, so there you have it

Matt: Awesome. Now, do you think primary school students could do this? Or is this better for secondary school kids?

Elton: I feel this is a secondary school activity. I think that for a couple of reasons. So, the fine motor skills that are involved, for one thing. Even if we had more time and I could perfect it. So, there's that. And just the concentration, just the focus that's going to be required, or the modelling that you're going to have to do. It's almost like you'd need... I mean, I have no idea if I'm doing this the correct way or not. But let's say this is one of the correct ways to do it, all right?

Matt: Okay

Elton: There's no right way or wrong, but let's say this is one of the ways that I could do it. I would have to model, in a class of thirty children, how to start. And every time that we stop, everyone's heat level's going to drop. So, people are going to be working at different times. So, people are going to be on red when some people are on green. Some people aren't going to do it at all. Some people aren't going to understand about …

Matt: So, right now it's on SPL. Do you see right there? So, it changed

Elton: Right

Matt: So, you have to put it back to PLA. So, sometimes you have to change it back

Elton: Right

Matt: Yeah

Elton: They'll need to remember that. So, I think that for most of primary, and I think I'm going to include year six as well, I think this is, at least this is, in my experience, from where I am, a little bit too beyond at the moment

Matt: I hear you

Elton: But I can see how things like this in secondary school, as part of design technology and those things, I can see that. Well, because they're doing those things already. They're making things. They're making things with plastic. They're making things with metal. They're looking at design

Matt: Does this school have design and technology, or?

Elton: We don't use it. Well, design and technology, somewhere it appears now and again. It's not a timetable subject that every week we're teaching D&T.

We teach English, math, reading, science. We teach history, we teach geography. We teach computing, and PE. And that's the timetable. You mix in what we call RH-2, relationship and health education. And probably we'll squeeze in some art at some point. But that may not be every week

Matt: Understand

Elton: Plus fifty minutes quiet reading, too. It's quiet reading every afternoon. Plus we have assemblies, plus actual. It's a very, very full timetable. But if teachers want to find time to put D&T in as part of the work that they're doing, they would find time. That also said, we would need... I don't know about elsewhere. But because we have a class of thirty, how would you work with a class of thirty? Sorry. I'm thinking out loud. How would you work with a class of thirty doing this lesson? Does everybody get the same …

Matt: I was going to ask you that. Would it be easier for everyone to get the same instrument? Or do you work in groups of two? In pairs?

Elton: You're probably better working in groups of two. Your other issue is, in that case, you are going to need fifteen sockets. Or you're going to need some of these scattered around the classroom. So, we've got that going on, too. These are not insurmountable problems. But for a private school, there's just things that make this all…

Matt: So, this would be cost efficient, is it? Cost effective. Sorry

Elton: Yeah, I can't see it being cost effective at the moment. Who knows. You could see …

Matt: But each one of these costs 29.99. £30

Elton: Okay. So, £30. If I told you I'm still trying to get Air Server installed on the six or eight PCs that allow us to take an iPad and put what's on the iPad onto the screen of the thing, and the subscription for that was about £80 for the eight or nine or ten PCs, that's where we are with budgeting for us at the moment. That's not to say that other schools out there, for whom £30 for fifteen of these... yeah

Matt: Yeah, I see that

Elton: That's, "Yeah, we'll have one each." But we are not. Yeah, we're not one of those schools where this could even be something... yeah, I just can't see it. I just cannot. And I hate to say it, because it sounds like I'm being …

Matt: No

Elton: Oh, this'll never work. But I'm trying to put my practical teacher's hat on and kind of see how this would work in a class environment for us

Matt: Got it

Elton: And I don't see it working for us

Matt: All right

Matt: Is there time in the curriculum to put this in, or no?

Elton: Yeah, no. I would say not. I would say this would fit that... I know you can use it as part of maths. I know you can use it as part of D&T. I know you can use it, you can argue, as part of computing. It's 3D printing. And 3D printing is included in computing. But if you did, that's still only one topic within that lesson

Matt: Got it

Elton:	You know what I mean? I wouldn't see us revisiting this every week, doing something. I just don't think we'd have the time. We're so pushed as it is for squeezing everything in already, that I just don't see it. Which is not to say that we shouldn't find some time. It's not to say that we shouldn't be doing this stuff. I just think for a primary setting, for us certainly, it's a little bit... we're not quite there yet with it

Diffractive reading of Elton mathematising a 3D skeleton cube

To begin mathematising a cube, Elton drew a square as a base for the skeleton cube on the silicone mat. Then, he started drawing straight lines (1D) in the air with the 3D printing pen, as seen in figures 37 (a-f).

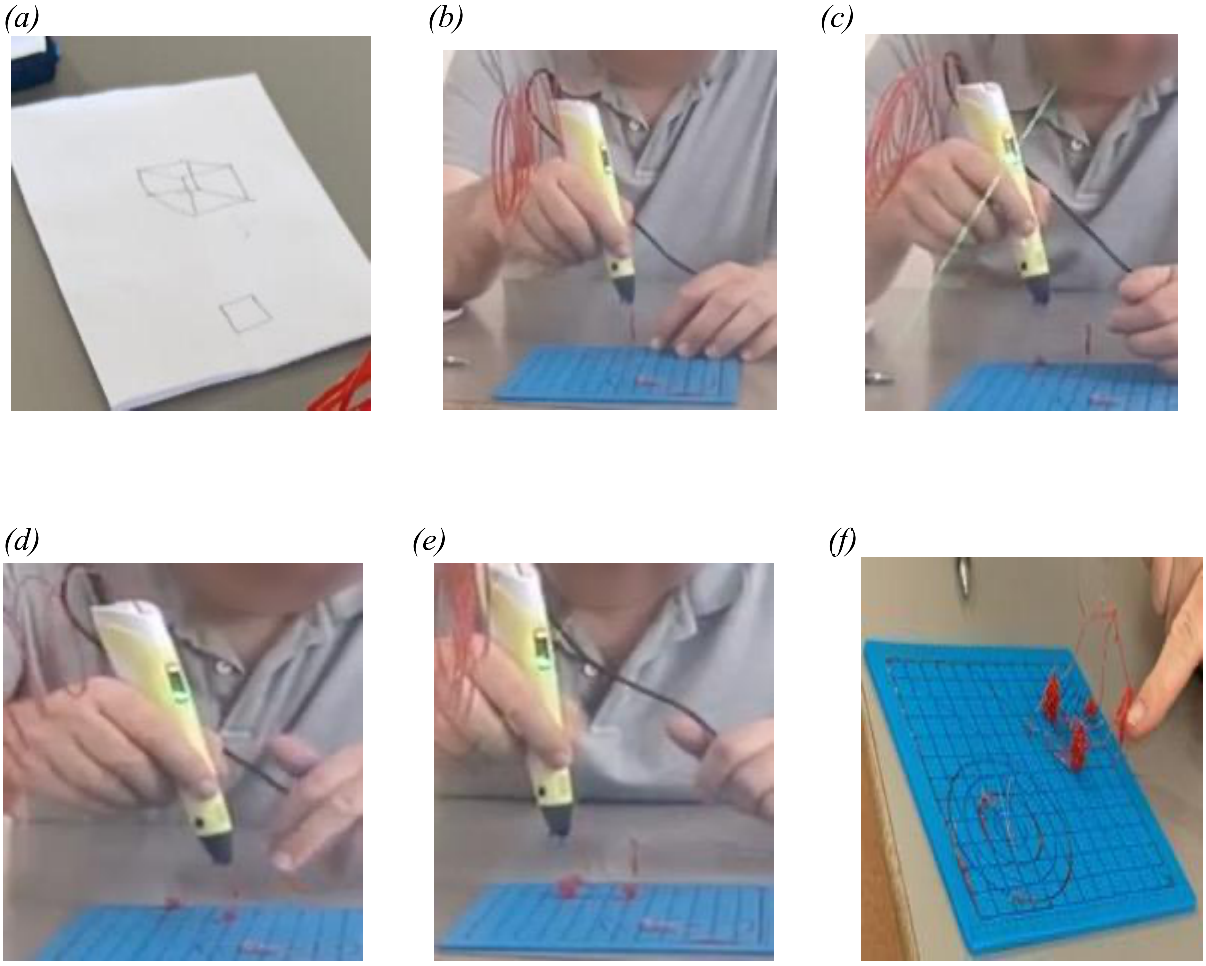


figure 37 (a-f)

5.4.3 Tinkering in learning-as-making activities

In the second part of the learning-as-making activities, Elton engaged, designed, and played with the functionalities of Tinkercad (figure 38(a-f)). Below is a dialogue of our conversation:

Matt:	So, what you could do is log into 'join class'. So …
Elton:	What do I mark for this?
Matt:	So, if you go on the browser, just backspace. So, right here
Elton:	Oh, sorry
Matt:	Yep. Click it and then click “join class.” Type in “join class” in the browser
Elton:	In here?
Matt:	Yeah
Elton:	Okay
Matt:	And then class code is this…what I did was, on Tinkercad, when you make your own educator account, you have your own classroom and you could have the students do this here, what I'm going to show you here, as well
Elton:	That I have done, actually. We just started doing this. So, I have done that
Matt:	Join with nickname. So, I gave you your first name. Lowercase. Elton
Elton:	Okay
Matt:	Yep. And create new design. Cool. And usually, the task, what I gave is, using four 3D shapes, a box, a sphere, a roof, and a cone, create anything you'd like
Elton:	A box, a sphere, a roof, and…
Matt:	So, you notice how Tinkercad uses this term “box”?
Elton:	Yeah
Matt:	Would that cause a problem in the math classroom?
Elton:	It would be... maybe not. Why do they call it “box” and not “cube,” I wonder?
Matt:	Why do you think so?
Elton:	So, a box, a roof, a sphere, and cone. Let me just get those out. Box, a sphere, a roof, and cone. Any shape that I want. Okay
Matt:	Are you teaching them a lesson on cube by any chance?
Elton:	We've only just really got so far. And this is Tinkercad. Pull shape, change, adjust the work plane, make shapes bigger or smaller. I'm doing this and now I've got all sorts of
Matt:	How can you see Tinkercad being used to teach in a math classroom, out of curiosity?
Elton:	I can see it at the end of the term, or end of the unit, when we've been doing shapes. But you wouldn't need... even these six. We don't implement half of these shapes. We use cubes, we use cuboids, we use spheres. See, I can't do this now. Okay, that's ambitious. Should I move this
Matt:	Would there be a challenge using Tinkercad with primary school students?
Elton:	I think it is a challenge, but again, the unit we're on, we get to spend, I think about six weeks on this
Matt:	Okay
Elton:	And so, lesson one is switch on, get your account, make sure you've logged on, find a cube, put it on the work plane. That's basically the first lesson. Second lesson might be put one more shape on. The next lesson is something else. Even I'm struggling here. I seem to have forgotten everything that I've, I've learned. Why is that ...
Matt:	Do you introduce the term of 3D printing to the students, or is it just 3D modelling?

Elton:	It has appeared in the first lesson. So, not for anything they actually do. It just says 3D printing. So, once you've done the actual Tinkercad bit, there's a few slides that explain what 3D printing is. In fact, I was saying to my kids the first week that we did this, which was pretty much the first time that I had done anything. I played around on Tinkercad a very small amount
Matt:	Okay
Elton:	And I said that 3D printing was going to be... I've forgotten half of what I went to do. I've forgotten how to draw these shapes again. There's something I'm missing. But I don't know how to do it
Matt:	Okay. So, you could move them top and bottom. You could combine them together
Elton:	I was trying to put this on top
Matt:	And you can change the view, okay? There's also a ruler function, if you want to explore that as well. Have you seen the ruler function?
Elton:	That one
Matt:	Oh, no. So, if you click here and put it on the plane, you're going to see something here. Yeah, it tells you the measurements on the plane
Elton:	I've come to the conclusion I can't work in 3D. I find 3D world very confusing. Right. That's what I want to do. I want to drag it up on top. I want to move those together
Matt:	So, are the kids still using Tinkercad right now? Or is it finished?
Elton:	Yeah. I think all we've done, we're aiming to do a name badge with their name on it. So, we're aiming to, I guess, do one of these. Put that there. Have one of these, but thin. And then, put their name onto it here. But we haven't really got... I need to read the next part, because I've literally learned this from scratch. I'm literally picking this up from not doing it before, to having to teach how to do this
Matt:	Would you guys be interested, would you want me to bring my MakerBot to the classroom another day? Would that be possible?
Elton:	Oh, that would be interesting
Matt:	You would see it, the kids would see it. Literally, I'll bring my printer to this classroom and they see a model being printed
Elton:	That would be interesting. Yeah
Matt:	Of one of their things being printed
Elton:	That would be interesting. Yes. So, yeah, it's coming back to me now. So, we are literally at the stage of taking the text and dragging onto this. And kind of that's where we are. That's all we'll probably get done this term. That's like, introduction to 3D printing
Matt:	Got it
Elton:	In fact, I don't think I saved it. But I can show you the lesson. What we, instead of computing, all NCCE, National Centre of Computing Education
Matt:	Yeah, we're there
Elton:	So, lesson plan. Size. I'll just show you something quickly. So, this is one of the units. Year six, here's the lesson plan. That's talking about 2D shapes, 3D shapes
Matt:	Is this public, by the way, or?
Elton:	It's NCCE. National Centre for Computing Education
Matt:	And that's year six, right?

Elton: Yeah. And, okay. So, somewhere, it's not on here and I don't know why. Maybe it's on the next set of lessons. But there was a set of extra information. Maybe it was a separate hand-out. I can't remember. But it was an extra hand-out that explains to children what 3D printing is. And I did say to them that it's a really exciting area to do things. Because I think on the news the same week, somebody had printed a school. In Africa, they 3D printed a school, or a classroom. And I said I'd be looking for someone to 3D print glasses for me. Because I can never get glasses... I bought these recently, they were desperation more than anything. I can't get glasses to fit me. Because the ratio between my head width and my nose width and my whatever is weird. And so, I can't get glasses to fit me

Elton: However, 3D printed glasses, where your head and all your dimensions are measured perfectly, where they're going to fit exactly. So, we had a little chat about that. So, my kids are aware of it. And we started doing this. But this is their first introduction to it. Their very first time of coming across any kind of... and it's mine, as well. I'm aware that I'm doing it, and the amount of skills that are now required to teach computing at the primary level, just as a normal class teacher are just like, "Well, that's crazy. That's crazy." Because until this point, you taught Word, Excel, getting on the internet, maybe Scratch. Filler bits and pieces. Now it's Data log in year four. It's 3D printing in year six. It's something else in year five. And it's just for teaching an hour a week, you've got to become really skilled, really know your stuff

Matt: And did you try that in your PGCE programme or no?

Elton: Well, again, we got a small number of computing lessons. Nothing. Almost nothing, really. Compared to what you need. We got a couple. Literally a couple of lessons, so say, "Well, here's one thing you could do with a lesson. Here's a PaintBox application. You do pictures or whatever, that's one. Or how about this, you could take everyone's shoe sizes and do a graph, or a pie chart." That was literally our computing lessons at PGCE level

Matt: So, barely a level

Elton: Yeah, so the requirement for what you need to know and do, compared to where folks, ordinary teachers are, is just horrific. I mean, I've joined [a school] the local secondary school. They're kind of like a hub. And I've just been speaking to them to say, "Yeah, I want to come to your hub meetings. Because I need to …

Matt: Which hub is that one, by the way?

Elton: [A community school]. So, I don't know what the name is. But it's based in …

Elton: Probably about half my kids will go there in a few weeks time. But they're also in …

Matt: A Community School, right?

Elton: Yeah. But they're a computing school. They're a CAS hub for the area. That's where the hub is. Can't remember the guy's name. It will come back to me. I can let you have it if it comes to me. I've got his email somewhere. And I've just been trying to reach out to him, saying, "I want to come to meets and tell me stuff so I can pass it back down here." Because other people are frightened of technology, but there's an awful lot to take on board, as I've demonstrated. There's an awful lot to take on board in a relatively short space of time

figure 38 (a-g)

5.4.4 Retelling a story: Elton's engagement in the learning-as-making activities

Elton's participation in the learning-as-making activities could reveal not only mathematising with 3D printing technology, but also his teaching identities towards mathematics. In mathematising with the 3D printing pen, I asked him to construct a cube. However, he interpreted my request to mean that he should 'create a 2D representation of a 3D object'. Here, Elton is showing his mathematical knowledge – knowledge that is required to construct a 3D model of a cube. In addition, he talked about using the square on the silicone mat as his base, from which he would build up to construct the cube. Examining closely how he ultimately made his 3D cube, Elton did not build up using the 3D printing pen as planned. Instead, Elton took the approach of drawing a cube in the air by using a square as a base and drawing four individual lines and connecting the top through the air. His cube drawn using the 3D printing pen is not the same as his drawing using a pencil and paper. However, Elton is still taking part in mathematising because his intention was to construct a 3D skeleton cube.

I was able to get insight on Elton's professional perspective on using 3D printing pens in the classroom. Below, he talks about the challenge of employing the 3D printing pen.

> But let's say this is one of the ways that I could do it. I would have to model, in a class of thirty children, how to start. And every time that we stop, everyone's heat level's going to drop. So, people are going to be working at different times. So, people are going to be on red when some people are on green. Some people aren't going to do it at all. Some people aren't going to understand about [it].

In addition, Elton talked about the significance of school funding. He noted that his particular school could not afford to purchase 3D printing technologies compared to other schools in his area. Also, he commented that,

> All teaching is hard. And sometimes the teaching that goes on in more affluent schools have different challenges. You're dealing with parents more. You've got high expectations. There are more politics around dealing with parents and these sorts of things.

From these two examples, I could better understand his role and identity as a primary teacher.

As for the Tinkercad component of our learning-as-making activities, Elton has familiarity with using Tinkercad in his own classroom. Elton informed me that he already employs lessons on Tinkercad that allow students to use the basic functions. He explained a bit about the order and pace of these lessons:

> Lesson one is switch on, get your account, make sure you've logged on, find a cube, put it on the work plane. That's basically the first lesson. Second lesson might be put one more shape on. The next lesson is something else. Even I'm struggling here. I seem to have forgotten everything that I've, I've learned.

This comment shows not only that Elton has taught his students to use Tinkercad, but also that he himself 'seem[s] to have forgotten everything that' he has learned in regards to Tinkercad functionalities. In the dialogue, Elton reveals that in his PGCE training, there was no training for the use of technology, Elton learned about Tinkercad during his time as an in-service teacher. During the Tinkercad part of our learning-as-making activities, I pointed out to Elton a few functions of Tinkercad that he did not know, such as how to rotate shapes, that there is a ruler function, and so on. I decided to tell Elton this information due to his acknowledgment of forgetting what he had originally learned about Tinkercad. However, this does not impact the way he was mathematising using the shapes on the interface. As he engaged and dragged shapes onto the Tinkercad plane, he had full control in manipulating the shapes. Mathematising in this part of the activity is playing and engaging with 3D shapes on a computer interface. Elton admits that he 'find[s] [the] 3D world very confusing'. However, he is still engaging with the Tinkercad interface knowing that he is doing a lesson with his students on creating name badges. The name badges consist of letters and a plank to stand the letters, as seen in figure 15(a-f). Intentionally, Elton is mathematising when creating a name badge because there is engagement with the Tinkercad functions of rotation, measurement, and geometrical shapes.

5.5 Discussion

In this section, I abstract from the data from each school to discuss four themes. These themes tell a story that draws on the literature review and is underpinned by the study's theoretical framework to talk about in-service teachers' mathematising and teaching identities (especially in relation to mathematics) through utilising 3D printing technology tools.

Theme 1: In-service teachers mathematising using 3D printing pens to construct 2D and 3D skeleton shapes

All six in-service teachers demonstrated mathematising using 3D printing pens in the learning-as-making activities. When in-service teachers pick up the 3D printing pens to construct 2D and 3D skeleton shapes, it is the start of their mathematising. To understand their journey in mathematising using 3D printing pens in the learning-as-making activity, below I describe the full experience of each school – Gryffindor, Ravenclaw, and Hufflepuff.

The School of Gryffindor

Beyoncé, Taylor, and Joseph were situated in a Design and Technology room where they engaged with printing pens in a learning-as-making activity. They are part of a community of primary school teachers that is entering a practice in this study that involves their engagement with 3D printing pens. Even before they begin mathematising, this group is participating in a community of practice. According to MacPhail et al. (2014), communities of practice are 'places and spaces where teacher educators have the opportunity to engage in worthwhile conversations and action about the nature and direction of their work' (p. 42). As seen in figure 1, the desks are arranged in the design and technology room in a way for in-service teachers to discuss and engage with 3D printing pens. This promotes a sense of a teacher-learner community (Jimenez-Silva and Olson, 2012). Most importantly, this arrangement promotes the start of their mathematising journey as a community.

When the group starts using 3D printing pens to construct their skeleton 2D and 3D models, three things are taking place: hand gestures, dialogue among the group, and the use of an apparatus (3D printing pens). To observe the journey of the group mathematising, I take into account De Freitas and Sinclair's (2014) suggestion to listen to human voices and Clandinin's (2022) narrative inquiry approach in understanding their mathematising process. It was through her voice that Tracy revealed that her 3D skeleton design is called a cube and through Joseph's voice that he revealed his creation of a triangular prism. Both Joseph and Tracy began their mathematising by embracing the 'design of learning, creating… and product outcome' (Halverson and Peppler, 2018, p. 291). While Joseph and Tracy only participated for one day engaging with a 3D printing pen, Beyoncé's journey was longer, beginning with mathematising through playing, engaging, and designing with the 3D printing pen using the silicone mat (figure 5(a-c)). In episode one, Beyoncé later used the 3D printing pen activity with her students in her after school program. During the afterschool program, Beyoncé calls on Lewis to talk about his 4 by 4 square model that he constructed using the 3D printing pen. This is an example of a boundary-making practice similar to De Freitas and Sinclair's (2014) example of a seventh-grade student explaining how the square is a 10 by 10 using a pointer. The key words that Lewis used to describe the square, such as 'corner' and 'edges', demonstrate his reasoning and hand gestures in creating the 2D shape. While

Beyoncé watched her students mathematise using 3D printing pens, I further investigated their journey of mathematising through a diffractive reading lens.

In the school of Gryffindor, I was focused on the diffractive reading of Taylor and Beyoncé work as they create 3D skeleton shapes. The diffraction reading of video snapshots of Taylor and Beyoncé indicates that there is more to analyse than only their shape constancy, as in their perception of objects, which Pizlo (2008) mentioned 'remains constant despite changes in the shape of the object's retinal image' (Pizlo, 2008, p. 3). Also significant in relation to Taylor and Beyoncé's creative processes is Duval's (1998, 2006) work in understanding the cognitive process of working back and forth in shape dimensions [0D, 1D, 2D, 3D]. Beyoncé worked from 2D to 3D and Tracy worked from 2D to 1D and 2D to 3D. Both Beyoncé and Tracy had a different orientation in configuring their 3D skeleton cubes. Beyoncé meshed up the 2D shapes to construct a partially made skeleton cube, and Tracy constructed four squares (2D) with multiple single lines (1D) inside the square to form a 3D skeleton caged model of a cube. The diffractive reading in these examples is supported by the reasoning of Beyoncé and Tracy as they create. They are both engaging with 3D printing as it relates to their own disciplines, but simultaneously reflecting on the mathematical application of what they are doing, similar to the diffractive reading that Burnard et al. (2019) employed on the mathematics artwork of South African students from grade 8 to 12. Through what I interpreted as diffractive reading and dialogue between them, Taylor and Beyoncé mathematised using 3D printing pens.

Tracy

Beyoncé

The School of Ravenclaw

Taylor and Emily were situated in their own classroom to take part in engaging with 3D printing pens in learning-as-making activities. I interpreted Taylor's first engagement using the 3D printing pen to make a cube through Barad's (2007) concept of boundary-making practice. The 3D printing pen (apparatus) is employed as she remembers to draw a square as the base of the cube and then combine multiple squares in making a cube (figure 10(a-c)). Taylor is mathematising in her first attempt as she recalls how to construct a cube from her past as a student while using the 3D printing pen to produce a material that becomes a cube. In her first attempt, it is important to keep in mind her gestures and hand movements in creating the skeleton cube. Similar to Ng and Ferrara's (2018) study, Taylor used the 3D printing pen to draw the cube in the 'air', as seen figure 11-c.

Taylor

In addition, Taylor saw the opportunity to embed the 3D printing and learning-as-making activity in her naming and identifying 2D shape lesson with her early years' students. I saw in Taylor's using the 3D printing pen on the silicone mat to draw 2D shapes (rectangle, square, triangle, diamond) an example of De Freitas and Sinclair's (2014) pedagogy of concept. Taylor was mathematising through coming up with the name of 2D shapes to draw using the 3D printing pen in front of her students. When Taylor's mathematical content knowledge is demonstrated as she draws the 2D shapes with the 3D printing pen, what I see is how the production material in the 2D shape fabrication is intertwined with mathematical knowledge. I also see in Taylor showing her students the different 2D shapes she constructed using the 3D printing pens – as well as her choice to use different colours to help associate the name of the shape with the object itself – Barad's (2007) concept of intra-action. Materials and body-movement are being intertwined in using the 3D printing pen to construct these meaningful 2D shapes to be used for mathematical learning at early year level. In addition, in Taylor's work – as Taylor engages with the 3D printing pen in the classroom for a mathematics lesson through material interactions – I see manifestation of De Freitas and Sinclair's (2014) idea of mathematical inventiveness. Mathematising is happening when Taylor is engaging with her body, the materials and a 3D printing pen (apparatus) in the classroom.

Interested in Barad's (2007) post-humanist performative approach, I visited Taylor for a third time to observe if she would change her approach in constructing a 3D skeleton cube. As Barad (2007) explains, the post-humanist performative approach involves 'matter of practice, doing, and actions' (p.134). The term mathematising in this thesis expands on the post-humanist performative approach with in-service teachers using 3D printing pens to extrude filament (materials) to construct 2D and 3D shapes. In observing Taylor's third use of a 3D printing pen, I saw that she was comfortable in using the apparatus. Taylor continues to work from 2D to 3D in constructing her 3D skeleton cube. The difference is she was mathematising through the lens of a maker, showing how she can construct a 3D skeleton cube. This is similar to the examples of the work from the two calculus for business students in chapter four. Like them, Taylor draws six separate squares and uses the 3D printing pen as a glue gun to stick the squares together to form a cube (figure 39). After three visits, Taylor clearly demonstrated how she mathematises during her classroom activity and in one to one situations.

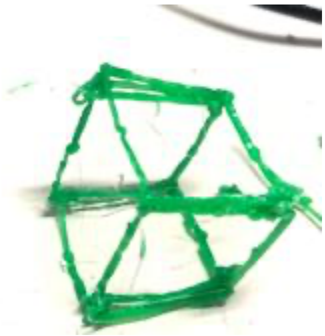


figure 39

Taylor’s colleague Emily had a different way of mathematising in the study. She demonstrated mathematising as she talked about how 3D printing pens could be employed in many subjects, such as physical development, referring to early year students’ motor skills. In observing how Emily mathematised, I used Duval’s (1999, 2006) work on shape dimensions, watching Emily work from 1D to 2D, 2D to 1D, back to 1D to 2D and 2D to 3D to form the 3D skeleton cube. Taylor was in the classroom at the time and did not help Emily, as Emily wanted her to show off her creativity. Emily’s engagement with the 3D printing pen to construct the skeleton cube also shows mathematical inventiveness, as per De Freitas and Sinclair (2014). This also demonstrates Emily’s mathematising. The learning-as-making activity consisted of an early year and primary mathematical component of constructing a skeleton cube using a 3D printing pen. Through Emily working with different dimensions in constructing her skeleton cube, she was also developing a potential relationship between herself (as an in-service) teacher and the material world that is the constructed 3D shape. By observing Emily via a diffraction reading of video clips of her hand gestures while using the 3D printing pen to construct a 3D skeleton shape, we see that Emily is mathematising (figure 40).

figure 40

The School of Hufflepuff

Elton was situated in his own classroom to take part in the learning as making activities, engaging with a 3D printing pen. Before constructing a 3D skeleton cube, Elton decided to draw a cube using a pencil and paper (figure 16-a). By drawing on the paper, he demonstrates his perception of shapes (Pizlo, 2008), mathematical knowledge of a cube (Duval, 2006), and De Freitas and Sinclair’s (2014) concept of mathematical inventiveness. When Elton moved to construct the 3D skeleton cube using the 3D printing pen and silicone mat, he noticed that the silicone mat helped in bringing in mathematical concepts such as angles, circumference and diameter of a circle, and measurements such as centimetres. When Elton attempted to construct his 3D skeleton cube, he took a drawing in the air approach (Ng and Ferrara, 2008). In understanding how Elton constructed his 3D skeleton shape, I used Duval's (1999, 2006) work on shape dimensions and observed that Elton worked from 2D to 1D and 1D to 2D to form a 3D skeleton shape (figure 41). Elton demonstrated mathematising through using the 3D printing pen and communicating mathematically his knowledge of constructing a 3D skeleton cube. In addition, Elton's mathematising as he constructed the skeleton cube in the air was driven by the use of the 3D printing pen.

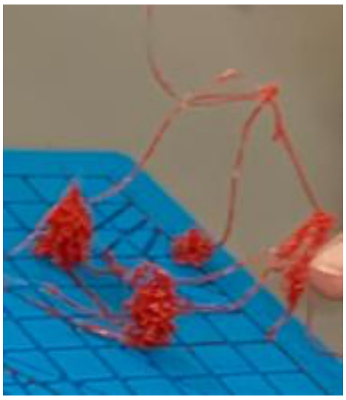


figure 41

Fieldwork Observation

Based on the fieldwork observation, in-service teachers mathematise in the learning-as-making activities through the use of 3D printing pens to engage with materials and with their mental imaginations, as in their perception of 2D and 3D shapes. The in-service teachers were told to construct a 3D skeleton cube, but were not given an example to replicate. Elton provided insight into his shape perception by drawing a cube on a pencil and paper. He knew what type of model to produce using a 3D printing pen. Tracy revealed her shape perception by modelling the 3D skeleton cube as a caged heart. Both Beyoncé and Taylor did not verbally talk about their shape perception, however, their mathematical thinking informed the way they constructed their 3D skeleton cubes. When I refer to the teachers' mathematical thinking, I mean their basic knowledge of geometrical shapes similar to Duval (1999). The four in-service teachers each had a different orientation in configuring their 3D skeleton cube. Elton constructed his skeleton cube in the air, Beyoncé meshed up the 2D shapes to construct a partially made skeleton cube, Taylor constructed four simple squares (2D) to glue them together using the 3D printing pen to form the cube. Taylor took the same approach one year after first using a 3D printing pen in her classroom. Lastly, Tracy constructed four squares (2D) with multiple single lines (1D) inside the square to form a 3D skeleton caged model of a cube. As seen in these images of in-service teacher's 3D skeleton models, materials play a role in the creation of 3D skeleton cubes. When I talk about materials, I am referring to 3D printing pen filament and how the extruder from the 3D printing pen produces a plastic that forms a shape. As the in-service teachers are mathematising their 3D skeleton cubes, they are using the plastic filament that is produced from the 3D printing pen to form their shape. In addition, the in-service teachers were allowed to choose the colour of their filament, which caused them to engage with materials at the beginning of their making process. In the learning-as-making activity, mathematising was shown through the six in-service teachers' engagement with materials and 3D printing technology

Theme 2: In-service teachers tinkering in relation to their classroom

Beyonce, Tracy, Taylor, and Elton engaged with Tinkercad through exploring the functionalities in their own ways. As Wilkinson and Petrich (2014) mentioned, tinkering is not about following procedures to get to a particular result; rather, tinkering questions how something works in relation to one's own understanding. When in-service teachers are engaging with Tinkcercad, they are engaging with their performative identities, as explored by De Freitas and Sinclair's (2014) inclusive materialism. As the four in-service teachers play, engage, and design in Tinkercad, they are also engaging with their identities in cyberspace. When in-service teachers are constructing their designs on Tinkercad, they are engaged in creative acts, as they were not given guidance in creating their designs. They are seen engaging with their imaginations and the functionalities of Tinkercad to construct their 3D models. To understand how the four in-service teachers are tinkering, I will describe each experience as follows:

Beyoncé

In reflecting on Beyoncé's engagement with Tinkercad, I recall De Freitas and Sinclair's (2014) inclusive materialism example of the young child named Katy using TouchCount on the iPad to interact with shapes and numbers. In the authors' example, Katy touches the shape associated with the number that is being called out in the audio. De Freitas and Sinclair (2014) go on to notice Katy's perception and sensation from the activity. Comparing this example to my observation of Beyoncé's engagement with Tinkercad, there are several similarities. Beyoncé used the iPad to engage with Tinkercad because it enabled her to use it as a touch screen to hold and drag objects on the plane. Beyoncé knew she had to select a box, cylinder, cone, and roof. However, she was not told how to manipulate the objects nor the number of objects to put on the plane. As seen in figure 42, Beyoncé kept the box, cylinder, cone, and roof all the same sizes.

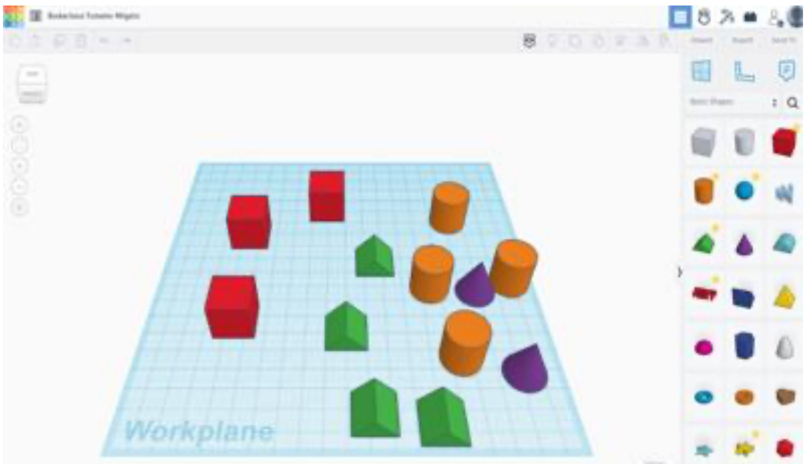


figure 42

In addition, Beyoncé mentions in episode one that 'you must keep count of the numbers' and mentions that Tinkercad is used in her classroom to construct birdhouses and chocolate molds. To see how Beyoncé was tinkering in the activity, I present a dialogue below that is relevant in understanding how Beyoncé is connecting Tinkercad to her classroom needs.

Matt:	Can you tell me what shapes you use in there, by any chance? The shapes that you have in there?
Beyoncé:	For this?
Matt:	Yeah
Beyoncé:	What shapes? Yes, very good idea. Well, I think he's used... This isn't my design, but it's a student's design. Yeah, he's used a lot of what you call those roof shapes and triangles. Yeah, I don't know how he's done it actually. There's that one, and this one is quite straightforward though, isn't it, because he's used the roof and the box and then it's a smaller... It's more of a rectangular shape
Matt:	Yeah
Beyoncé:	This one's very interesting. You're right, actually, especially from the bottom
Matt:	When I saw it, it's like, this is unusual
Beyoncé:	It is unusual. And you know what, I hadn't looked at it
Matt:	This here, there was a base part. This had to be printed separately
Beyoncé:	Oh, I see. Yes, yes. Because it wasn't joined
Matt:	When you sent the file, this was floating in the air
Beyoncé:	Yeah
Matt:	When you have that physical model, what experience does this have to students? What does it give to the students? Do you see it being valuable to print the 3D objects for the students?
Beyoncé:	Shall I show you what we've done in the past?
Matt:	Yeah, I'd be happy
Beyoncé:	We've made lots of models of chairs
Matt:	Okay, interesting
Beyoncé:	So you can actually see... We can take it out because it's just on a stand, that's it. They designed them. We looked at eco modelling and looked at trees and all sorts of things, and then we looked at these. That's different. But we made models of chairs to see what they looked like
Matt:	Wow
Beyoncé:	That's how, in D&T, we could use it. And then we made this in our first year of the 3D printer. We made models of structures, of buildings. And what was so amazing is they made them in wood first, and then they made them with a 3D printer and it came out really, really accurate

Beyoncé's engagement with Tinkercad was motivated from her previous experience of seeing her design and technology class using Tinkercad to construct 3D models (i.e., chairs, trees). In figure 43(a-c), Beyoncé shows a 3D model produced by one of her students and several 3D printed chairs also made by her students. Her classroom experiences give a glimpse into how Beyoncé envisions tinkering.

(a) (b) (c)

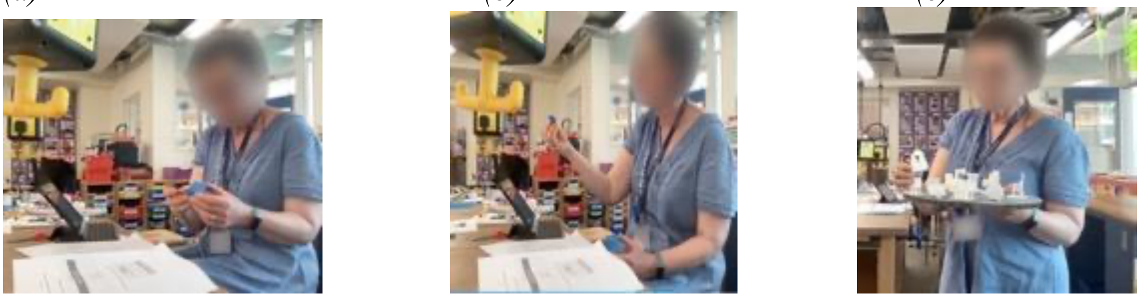


figure 43 (a-c)

Tracy

In reflecting on Tracy's work with Tinkercad, she demonstrated a design, play, and engage approach. As seen in figure 44, Tracy engaged with many functionalities of Tinkercad, including the drawing tools, typing tools and several premade objects such as a star.

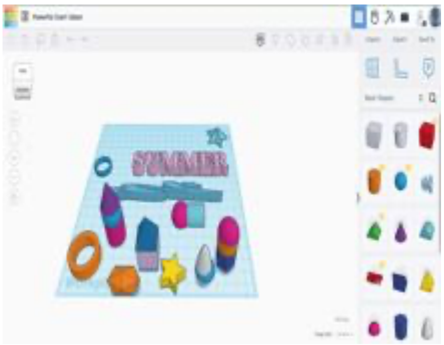


figure 44

Tracy talks about making 'things that look fun', in alignment with her role as an in-service nutrition teacher. Martin (2017) mentioned that making involves designing and building towards the purpose of being playful and interacting with objects. Tracy was looking at her 3D models and associating them with looking 'like [an] ice cream, just lollies, sweets'. When Tracy is making 3D models on Tinkercad, she is also tinkering (Resnick and Rosenbaum, 2013; Wilkinson and Petrich, 2014). When Tracy had the opportunity to connect her creation to mathematics, she said it was about 'learning all the shapes, seeing how they work together in terms of relation ratio'. This relates to the idea of De Freitas and Sinclair's (2014) called the virtuality of mathematical concept. This concept allows Tracy to envision the mathematical entities of the 3D objects on Tinkercad. It allows her to think about the 3D models that Tinkercad used, such as roof or box, and relate them to proper mathematical terminology, such as triangular prism and cube. In addition, Tracy's intentional drawing with the computer mouse on Tinkercad also demonstrates the rise of new possibilities of gestures, noted both by the work of Gilles Châtelet (1993, 2000) and that of De Freitas and Sinclair (2014). Through Tracy's design, play, and engagement with Tinkercad and its connection towards her own vision as an in-service nutrition teacher, Tracy is demonstrating the notion of tinkering.

Taylor

This was Taylor's first time engaging with Tinkercad as a 3D modelling program. In reflecting on Taylor's engagement and creation on Tinkercad (figure 44), I recall the literature of O'Reilly and Barry (2023), which investigates the effect Tinkercad has on students ages 8 to 11 years old. In the second episode, Taylor hinted that Tinkercad could have an impact on students' confidence by suggesting that students engaging with Tinkercad to build 3D models could lead them to be 'happy with their creation' and their 'hard work, and idea'. In addition, Taylor linked Tinkercad activity to making by thinking how she would use Tinkercad in her classroom. Taylor talked about asking students the questions about the type of shapes that could be used to create a cup, school, or house on Tinkercad. By thinking about having this discussion with her students, she is envisioning how she is encouraging her students to become makers by sharing ideas (Hsu et al., 2017). In addition, Taylor is intentionally allowing students of a young age at the early year level to start speaking about mathematics through 'material objects with virtual and actual dimensions' (De Freitas and Sinclair, 2014, p. 202). The way Taylor describes how she would deliver a lesson to her students on Tinkercad demonstrates how she is tinkering.

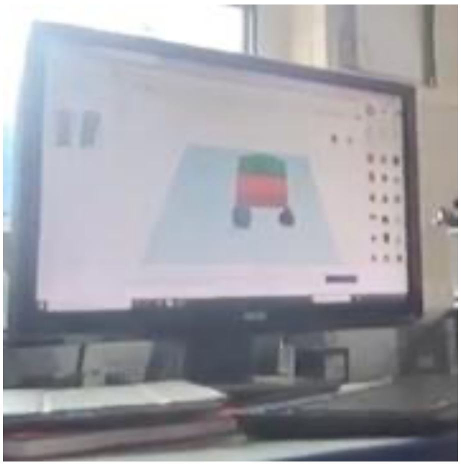


figure 45

Elton

In reflecting on Elton's engagement using Tinkercad, he had prior knowledge of Tinkercad through his classroom activity of creating name badges with students. Elton's conversation demonstrates how Tinkercad could prompt De Freitas and Sinclair's (2014) virtuality of mathematical concept through introducing shapes in a mathematics lesson in which he would use cubes, cuboids, and spheres at the end of a unit. When Elton was asked to construct a 3D model of a house using Tinkercad (figure 46), this was the start of his tinkering. He was not following a step-by-step guide, but instead following his own assumptions to construct his 3D model. This is similar to Wilkinson and Petrich's (2014) notion of tinkering. Elton's construction of a 3D model house and his previous discussions on creating name badges with his students demonstrates how he is tinkering.

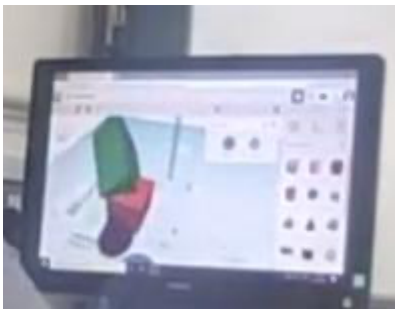


figure 46

Theme 3: The role of 3D printers at the primary and early years level

In this study, the Makerbot 3D printer was present in the room as the in-service teachers engaged with the learning-as-making activities. The rational behind of in-service teachers observing the Makerbot 3D printer is to determine the application it has to their classroom needs. Elton allowed me to use some of his classtime to demonstrate to his students how a Makerbot 3D printer function. In a ten-minutes demonstration, he wanted to show how a tiny 3D cube is printed. During the lesson, Elton asked his students about the type of object that could be produced using a 3D printer and his students did response with real-life application. Due to Elton request not to have video recording in the classroom, I was still able to observe students interaction with Elton. Similar to Anđić et al. (2022) study, Elton was prompting students to think about ways 3D printing application relate to other disciplines. The making component of the 3D printer constructing a cube produced excitement in Elton classroom. The Makerbot 3D printer in elton classroom activity was used as a teaching tool that is mentioned by Anđić et al. (2024) to produce conversation around constructing 3D printing objects. Elton saw the makerbot 3D printer is not affordable to their state school as there are not enough funding to purchase a 3D printer. Similarly, Taylor and Emily school does not have the funding either but saw the potential impact it can have on students learning.

Beyoncé and Joseph school have the funding to purchase 3D printer to be used in their design and technology classroom. Before taking part in the study, Beyoncé is familiar teaching primary students about 3D printing and use a 3D printer such as a Prusa 3D printer in the classroom. In figure 17 (a-b), she shows one of her student 3D printed object. She ask her students to construct a 3D model on Tinkercad and allowed me to print out the 3D object. Taylor was able to envision how a 3D printer could be used in other discipline beside Design and Technology (i.e. mathematics) similar to Anđić et al. (2022) study. After taking part in the learning-as-making activities on the first visit, she purchased two Makerbot 3D printers as seen in figure 22 (a-b). Her rational of purchasing a Makerbot 3D printer is she found it easier to use compared to Prusa 3D printer. In this study, all in-service teachers notice impact of using a 3D printer as educational tool and the potential to relate activities to other disciplines.

Theme 4: The formation of teaching identities towards mathematics

In describing the formation of teaching identities of the six inservice teachers in this study, I start with reflecting on their teaching identities through using Barad (2007) concept of post-humanist performative approach to understanding in-service teaching identities towards mathematics. Post-humanist performative in Barad (2007) lens includes 'matters of pratice, doing, and actions' (Barad, 2007, p. 134). During the first part of the learning-as-making activity using 3D printing pen, all in-service teachers demonstrated practicing using the 3D printing pen to construct 2D and 3D shapes. To understand their identity in a performative lens, I used the literature in mathematics education of Ball (2003) to observe their social identities being displayed such as '"new modes of description" and "new possibilities for action" (Ball, 2001, p. 2). In the dialogue of each schools, each in-service teachers demonstrate a new modes of description constructing a 3D skeleton cube. New modes of description refers to their conversation in constructing their 3D skeleton cube using the 3D printing pen. Their new possibilities for action involves how each in-service teachers see 3D printing pens as an affordance tool to be used in their daily classroom activity. When in-service teachers are performing these actions using a 3D printing pen, they are performing De Fretias and Sinclair (2014) creative act. In addition, performative approach capture their identity work in the study as each in-service teachers discuss how they would use the 3D printing pen in their future classroom. Both Beyoncé and Taylor demonstrated their identity-work throughout the learning-as-making activities due to they have used the 3D printing pen over three times. As Chronaki's (2011) mentioned 'identity-work becomes captured by what is already done in the past and what can be imagined for the future' (Chronaki, 2011, p. 223).

Their identity-work was captured through their narrative and diffractive reading of screen shot of in-service teachers mathematising using 3D printing pen and Tinkercad. The four in-service teachers (Elton, Taylor, Tracy, and Beyoncé) who used Tinkercad, protrary their performative identities through the use interacting with 3D shapes in a virtual world. This is known as virtuality of mathematical concept, which De Fretias and Sinclair (2014) mention about interacting with mathematical entities in virtual dimension. When in-service teachers are interacting with 3D shape in virtual dimension such as Tinkercad, they are portraying their identities as a maker.

Through their making process, the in-service teachers reveal their teaching identities towards mathematics. The learning-as-making activities were designed for in-service teachers to engage with the mathematical content of key stage 1-3 in the English national curriculum. However, conversations within the stories-to-live by of the in-service teachers could show that one can use these activities in non-mathematics classrooms and relate to the content of mathematics, such as patterns or shapes. For example, both Beyoncé and Taylor used the activities for their own purposes in relation to mathematics. Taylor used 3D printing pens to conduct a lesson on identifying and naming 2D shapes with her early years classroom. Beyoncé used the 3D printing pens in her after-school design and technology classroom to design, play, and engage in making. In Beyoncé's classroom, the conversation about making turned into a student named Lewis talking about his construction of a square. Talking about the measurement of the square, and identifying and naming a 2D shape such as a square, places Beyoncé as a non-specialist teacher of mathematics. Even despite this conversation, the design and technology content of the English national curriculum asks that students 'be taught how to develop and communicate design ideas using…3-D and mathematical modelling' (DfE, 2023, key stage 3). This too places Beyoncé as a non-specialist teacher of mathematics. Elton and Taylor also began to reveal their teaching identities towards mathematics when they used Tinkercad. Both Elton and Taylor acknowledged that Tinkercad can be used to introduce shapes to students. Elton uses Tinkercad to explore other applications such as designing a name badge. But when asked the question about how to employ Tinkercad in a mathematics context, his episode shows him imagining a situation where he is employing this activity as a teacher of mathematics.

In the dialogues, the in-service teachers notice that making on Tinkercad could prompt students to share ideas. This is also discussed in Hsu et al.'s (2017) study, and it creates a bridge to connecting their discipline to mathematics. The way four in-service teachers play, engage, and design using Tinkercad shows their creative action through using the laptop mouse to interact with the Tinkercad interface. The action of using the mouse to interact with Tinkercad is a form of mathematising in a virtual world. Bringing the virtual world of 3D modeling to reality, the Makerbot 3D printer allows them to talk about how they imagine a 3D printer being used in their discipline in the future.

In-service teachers' engagement in the learning-as-making activities provides insight into their community of practice. As they engage in their practice, each of the in-service teachers display multiple identities in relation to their discipline and mathematics. They use 3D printing and Tinkercad as shared repertoire in the practice. While these in-service teachers are already part of a community that includes primary or early years teachers, there are two new communities that they are entering into as indicated by their stories-to-live by and diffractive readings. These communities are maker communities and teachers of mathematics communities. When in-service teachers are designing, playing, and engaging with the 3D printing pen, they are considered makers. Through their making process, different designs of a 3D skeleton cube and 2D shapes were formed. In addition, designing and engaging with shapes on Tinkercad allows them to become makers. The in-service teachers are making within a 3D software, and their creations could then be printed using a Makerbot 3D printer, such as Beyoncé physically printing her students' 3D models. The availability of 3D printing technology is influencing these teachers to become makers.

To fully understand their teaching identities towards mathematics, I now reflect on Wenger's (1998) three modes of belonging: engagement, imagination, and alignment. All in-service teachers' engagement shows how they came together in a practice that involved the use of materiality and 3D printing technology (i.e., 3D printing pens, Tinkercad, and a Makerbot 3D printer) to construct 2D and 3D shapes. When the in-service teachers construct their 3D skeleton models or 3D models on Tinkercad, their imaginations are taking form in the sense of their perceptions of shape being employed and by demonstrating a mathematical inventiveness through gestures and drawings of shapes. Their imaginations are coming to life physically and virtually through the use of 3D printing technology. Wenger (1998) discusses computer programs as a ratification that produces alignments. From this perspective, 3D printing pens and Makerbot 3D printers could be said to generate alignment in the practice. In-service teachers' formation of teaching identities towards mathematics included their performative approach and identity-work that was actively being formed in the practice. Also, understanding their engagement with materials and 3D printing technology offers insight into their teaching identities towards mathematics, especially as they discuss how the learning-as-making activities relate to their classroom needs through mathematical applications.

Chapter 6: Conclusion

6.1 Introduction

In this chapter, I answer the three research questions driving this study (section 6.2). These discussions are based on the findings of my fieldwork, presented in chapter five. I then discuss the contributions my work makes to the practice of early years and primary education (section 6.3.1). Also discussed are the contributions my work makes to the theories of communities of practice, shape perception, and inclusive materialism (6.3.2). Following this, I talk about the limitations that impacted my overall study (section 6.4), and I propose ideas for future research beyond the doctoral thesis (section 6.5). Lastly, I reflect on my four-year journey as a postgraduate researcher in mathematics education (section 6.6).

6.2 Answering the Research Questions

This study observes six in-service teachers mathematising using 3D printing technology tools in a learning-as-making activity to explore their teaching identities towards mathematics and mathematising. In this section, I use the episodes and themes presented in chapter five to answer the three research questions that were presented in chapter one.

R.Q.1: How do in-service teachers mathematise using 3D printing technology tools to connect with the mathematical potential of their activities?

R.Q.2: How do in-service teachers' mathematising in learning-as-making activities impact their teaching identities?

R.Q.3: What does a diffractive reading of in-service teachers' work with 3D printing pens tell us about their mathematising?

In section (6.2.1), I discuss the findings from the three episodes in chapter five to examine how in-service teachers are using 3D printing technology tools to make the connection with the mathematical content of their classrooms and the English national curriculum. In addition, I analyse the English national curriculum Key Stage 1-3 of the design and technology and cooking and nutrition learning requirements to justify this connection. Afterward, I use the dialogue and diffraction reading of chapter five to explore how in-service teachers' mathematising in learning-as-making activities impacts their teaching identities (section 6.2.2). Lastly, I use the diffractive reading of the data from participants in each episode in chapter five to make the connection between in-service teachers' work with 3D printing pens and their mathematising (section 6.2.3).

6.2.1 **(R.Q.1) How do in-service teachers mathematise using 3D printing technology tools to connect with the mathematical potential of their activities?**

In reflecting on the episodes and themes presented in chapter five, the learning-as-making activities encourage the six in-service teachers to play, engage, and design with 3D printing pens to construct 2D and 3D skeleton shapes. The moment when each in-service teacher initiates creating their 2D and 3D skeleton shape using the 3D printing pen, they are committing to a 'material act of creation' known as mathematising (Ng and Ferrara, 2020, p.

942). Mathematising is captured through diffractive readings (Ng and Ferrara, 2020) of video snapshots of in-service teachers' hand movements, gestures and uses of the apparatus (i.e., 3D printing pens). In the lens of inclusive materialism, in-service teachers' sense of engagement with technology (i.e., 3D printing pens, Tinkercad, Makerbot 3D printer) potentially reveals ways they learn mathematics. In addition, in using the 3D printing pen, mathematising is seen when in-service teachers are tinkering with 3D objects (i.e., box, roof, sphere, and cone). When in-service teachers are tinkering, they are mathematising as they use manipulatives on Tinkercad that allow them to virtually touch and change dimensions using a mouse or touchpad. When in-service teachers envision ways a MakerBot 3D printer can be used to print physical 3D objects such as a cube, they are mathematising. As these in-service teachers are non-specialists in mathematics, they are seeing the 'importance of transdisciplinarity' that Anđić et al. (2022, p. 51) mentioned. To further answer this research question (R.Q.1), I organised the six in-service teachers into their subject specialties: Design and Technology (Beyoncé and Joseph), Cooking and Nutrition (Tracy), Early Years (Taylor and Emily), and Computing (Elton).

Design and Technology

According to the design and technology Key Stage 3 framework, students are expected to be taught to 'develop and communicate design ideas using annotated sketches, detailed plans, 3-D and mathematical modelling, oral and digital presentations and computer-based tools' (DfE, 2013, p. 3). In the school of Gryffindor, Beyoncé and Joseph both created 3D designs using the 3D printing pens in their first meeting. Joseph constructed a prism, just by using three triangles and gluing them together. As he explained what he was doing, he associated the word prism with the 2D shape of a triangle. This association acknowledges the mathematical content of Key Stages 1-3, which says that students should identify and describe 2D shape properties.

Beyoncé sketched various 2D shapes using a 3D printing pen on the silicone mat. During my second visit to Beyoncé's school, I observed her implement the same activity with her students in an afterschool design and technology programme. She called on a student named Lewis and demonstrated mathematical reasoning by using terms with Lewis such as 'corner and edges', which are parts of constructing a 2D object. She not only connected her classroom to the mathematical content of Key Stages 1-3, but at the same time she ensured that her students were learning to make using 3D printing pens. The design and technology Key Stage 3 framework requires students to 'select from and use specialist tools, techniques, process, equipment and machinery precisely, including computer-aided manufacture' (DfE, 2013, p. 3). In this case, the 3D printing pens acted as the specialist tool such that students learned to assemble with the 3D printing pen. This included learning to hold the 3D printing pens to draw, plus functionalities specific to the pens such as the need to heat the extruder to 180 degrees Celsius in order to start producing filament to construct models.

When I visited Beyoncé a year later to see if she could produce a 3D skeleton cube again, she introduced me to a maker manual entitled Doodling Baubles. Produced by Daan Veerman from Project Store, this maker manual uses 3D printing pens on plastic products, which allows makers to become aware of material products in everyday life as well as geometrical shapes and sizes. Doodling with 3D printing pens and materials is nothing new, however. Several researchers have employed this method in making (Bernard and Mendez, 2020; Ng and Ferrara, 2020; Takahashi and Kim, 2019). While Beyoncé did not incorporate this into her own classroom, she is connecting the mathematical content to her own classroom through the design component of Key Stage 3 that states ‘use research and exploration, such as the study of different cultures, to identify and understand user needs’ (DfE, 2013, p. 3).

3D printing pens are not the only 3D printing tool that Beyoncé used to mathematise. She also used Tinkercad and the MakerBot 3D printer. Before taking part in the learning-as-making activity, she asked two of her students to use Tinkercad to construct model houses, which I physically printed through a 3D printing company. Her activity links to the literature on makings as part of the curriculum through the work of Osborne (2019), who was interested in building a maker mindset using ‘3-D printed houses to teach students how setting is an essential element of narrative writing’ (Osborne, 2019, p. 50). As Beyoncé was following the instructions to use Tinkercad and work with certain shapes, she placed the shapes ‘box, sphere, roof, and cylinder’ on the plane. In addition, Beyoncé was mathematising using the Tinkercad programme on an iPad. To fulfil the activity requirement of using all four shapes, she simply clicked and dragged the shapes to the Tinkercad plane. This resulted in having the same size for each shape due to Tinkercad’s default shape size. During this learning-as-making activity using Tinkercad, Beyoncé made a connection with mathematical content when she asked whether students would be confused about the terms for certain shapes. For example, would calling a triangular prism a roof, or a cube a box, confuse students about mathematical terminology? At one point Beyoncé stated that ‘primary teachers have to teach as well…very often using [shapes] to represent models. We use it quite a lot to represent buildings. That is a very obvious shape for a roof.’ In this moment, she was connecting with both the mathematical and design and technology content of Key Stage 3 of the English nation curriculum.

Cooking and Nutrition

Tracy was the only member of the school of Gryffindor who used the 3D printing pen to construct a 3D skeleton model. She called her model a caged heart. She acknowledges that a caged heart is a model of a cube. Tracy connects to the mathematical content of the English national curriculum by ‘recognising and naming common 2-D and 3-D shapes’ (Key Stages 1-2, DfE, 2023). In the cooking and nutrition content of the national curriculum, there is no mention of any mathematical content such as measurements or weights that could be useful terminology in a cooking lesson. However, cooking and nutrition is generally taught through the design and technology Key Stages 1-3 framework. Using the 3D printing pen could be considered the ‘electrical equipment’ that students in cooking and nutrition class in Key Stage 3 need to be proficient in using.

In addition, Tracy engaged with the Tinkercad component in the learning-as-making activity. In referring to her design, she mentions that ‘it’s going to look like [an] ice cream, just lollies, sweets’. Tracy is relating the mathematical content of shapes with sweets items in her cooking and nutrition class. When connecting to the mathematical content of the English national curriculum, she is comparing 3D shapes to everyday objects. She talks about ‘learning all the shapes, seeing how they work together in terms of ration, what could fit on top…If you have half a sphere, that could go with, for example, and form something else with a cone.’ Manipulating more than one shape to create a 3D object could relate to the mathematical content of the English national curriculum through building up children’s competence (O'Reilly and Barry, 2023) and improving their spatial thinking (Bhaduri et al., 2021).

Early Years

On the first day of the learning-as-making activity, Taylor observed that the 3D printing pens could be used effectively in the early years students' counting numbers lesson. The school of Ravenclaw website puts a heavy emphasis on teaching early years students to count numbers by using different techniques such as objects and imagery. Something to note about Taylor's comment is that 3D printing pens are acceptable to use starting at the age of six. In addition, when asked directly how she saw this learning-as-making activity being employed in her classroom, Taylor replied that she saw it used best 'when teaching the difference between 2D and 3D shapes'. Chen et al.'s (2021) research using 3D printing pens in primary education notes that 'children use simple shapes to practice 3D printing pens' (Chen et al., 2021, p. 7). Chen et al.'s (2021) simple shape refers to 2D shapes of 'straight lines, rectangles, squares, circles, ellipses, and arcs' (Chen et al., 2021, p. 7). In addition, Taylor associated engaging with 3D printing pens with the word fun. This choice of words ties directly to the developmental framework of Key Stages 1-3 as the curriculum states that early years students are expected to 'enjoy drawing freely…[and] explore colour and colour mixing' (DfE, 2023).

In the second visit to the school of Ravenclaw, I observed how Taylor created a learning-as-making activity in her classroom by showing students different drawings of 2D shapes using a 3D printing pen. This activity is reminiscent of Ng and Tsang's (2023) research on 3D printing pens in mathematics classrooms at the primary level, where they reflect that in using 3D printing pens, 'a 2D diagram that would have stayed dormant on the page when drawn with paper-and-pencil thereby becomes a physical object that can be held, moved, and turned when drawn with the [3D printing] Pen' (Ng and Tsang, 2023, p. 333). In the lesson, Taylor was facing the students in front of the classroom using the 3D printing pen to draw a circle, rectangle, square, and diamond on the silicone mat. Each time Taylor created a shape, she would hold it up for the class to see and ask the students to name the shape. This engagement with the class could promote 'opportunities for children to develop their spatial reasoning skills across all areas of mathematics including shape, space and measures' (EYFS, DfE, 2023).

6.2.2 **(R.Q.2) How do in-service teachers' mathematising in learning-as-making activities impact their teaching identities?**

In reflecting on the communities of practice overarching framework, in-service teachers engaging in learning-as-making activities demonstrate Wenger's (1998) three modes of belonging: engagement, imagination, and alignment. When the six in-service teachers engage with the 3D printing pens, they are giving a glimpse into how they would use the 3D printing pens in relation to their practice to show mathematical application of 2D and 3D shapes. When they are imagining their 2D and 3D shape designs in their minds, they are engaging with their perception of shape that Pizlo and Salach-Golyska (1995) discussed. In-service teachers' perceptions of shape give a glimpse into their understanding of 2D and 3D shapes. While Pilzo's (2008) work was not intended to be used mathematically, these in-service teachers used their perception of shape in relation to mathematical application. As in-service teachers use 3D printing pens to construct skeleton 2D and 3D shapes, they are speaking ‘mathematics with material and physical verbs’ (De Freitas and Sinclair, 2014, p. 113). When in-service teachers are speaking about mathematics when engaging with 3D printing pens or Tinkercad, they are intentionally revealing their teaching identities towards mathematics. To further answer this research question (R.Q.2), I bring back the literature review on maker identity and teaching identity towards mathematics that I observed during the study.

Maker Identity

Making in chapter five refers to ‘a set of activities that can be designed with a variety of learning goals in mind’ (Halverson and Sheridan, 2014, p. 501). Each in-service teacher in chapter five unfolds their identity as a maker in the learning-as-making activity as follows:

Beyoncé – In chapter five, Beyoncé told a story of a time where she taught a design and technology course in a classroom intended for art lessons. She discusses employing a design and technology activity without having the necessary equipment. Beyoncé talked about how she taught that lesson by deciding ‘to look at mapping’. She explained:

> And so, we looked at maps for different type of scales and then [students] did a personal map of their area and we used acrylic so that they could draw some to scale and some not to scale. And they looked at the roads and things and then they started to put important buildings using Tinkercad. So, designing it and putting it on the map.

The makerspace in Beyoncé’s story would be the art classroom (Halverson and Sheridan, 2014). In addition, Beyoncé created various kinds of 2D shapes using the 3D printing pen, manipulated 3D models on Tinkercad, and, a year after taking part in the learning-as-making activity of my research, mathematised a 3D skeleton cube using a 3D printing pen.

Tracy – On the first day of the learning-as-making activity, Tracy used the 3D printing pens to construct a 3D skeleton model of a caged heart. In addition, Tracy used Tinkercad to create various 3D models that represent ‘ice cream, just lollies, sweets’.

Joseph – Joseph used the 3D printing pen to construct a 3D skeleton prism.

Emily – Emily used the 3D printing pen to construct 2D shapes (circle, triangle, square) and 3D skeleton cube.

Taylor – Taylor used the 3D printing pen on the first day of the learning-as-making activity to construct 2D shapes and attempt to create a 3D skeleton cube. On my second visit, I observed her employing a 3D printing pen in her early year classroom to teach a lesson on naming and identifying 2D shapes. Finally, like Beyoncé, a year after taking part in the learning-as-making activity, Taylor mathematised using a 3D printing pen to construct a 3D skeleton cube.

Elton – In chapter five, Elton talks about how he is implementing a name tag design activity for his students to create using Tinkercad. He used the 3D printing pen to construct a 3D skeleton cube, played and designed using Tinkercad, and showcased the Makerbot 3D printer to his classroom by printing out a small cube.

When in-service teachers are making using 3D printing pens, Tinkercad or the Makerbot 3D printer, they are revealing their identities as makers. Making is not only designing and playing with materials, however. In-service teachers are also using 'mathematics as a form of making' (Ng and Sinclair, 2020, p. 501). In addition, Ng and Sinclair (2020) state, 'making is mathematising' (p. 925) and in-service teachers are mathematising using 3D printing technology tools in the learning-as-making activities.

Teaching Identity towards Mathematics

The learning-as-making activities promoted mathematical conversations about constructing 2D and 3D shapes among the six in-service teachers. According to Wenger (1998),

> We engage in different practices in each of the communities of practice to which we belong. We often behave rather differently in each of them, construct different aspects of ourselves and gain different perspectives. (Wenger, 1998, p. 159)

The six participants were engaging in the learning-as-making activity through the lens of a teacher of mathematics. For example, Beyoncé and Taylor both volunteered to use the learning-as-making activity for their own classroom purposes. Beyoncé employed the 3D printing pen activity with her design and technology students to mathematise through constructing 2D and 3D skeleton shapes. In this activity, she asked one of her students to walk her through how to construct a 2D shape of a square. Taylor used the 3D printing pen activity in her own classroom curriculum to teach students about 2D shapes. Neither in-service teacher was told how to implement the learning-as-making activity for their own purposes. Talking to students about identifying and naming 2D shapes can be considered a teaching identities towards mathematics as the in-service teacher demonstrates 'practices of doing mathematics (Identification with school mathematics)' (Crisan and Rodd, 2017, p. 109).

In addition, Beyoncé, Taylor, Tracy, Emily and Elton further engaged their identity work in relation to mathematics. Beyoncé discussed employing 3D printing pens in a maths class to teach primary students about patterns. Taylor saw 3D printing pens being useful in the early year curriculum to teach young children how to count numbers. Tracy envisioned using Tinkercad in mathematics classrooms as ideal for learning shapes and ratios. Elton agreed that Tinkercad could be used to introduce shapes, but thought that there was not enough time for students to play and engage with Tinkercad in the classroom. In describing how Tinkercad or 3D printing pens can be employed in a mathematics classroom, Beyoncé, Taylor, Tracy, Emily and Elton are engaging in their identity work by thinking of themselves as teachers of mathematics (Neumayer-Depiper, 2013). In this learning-as-making activity, it is important to note the role that technology - whether the 3D printing pens or the Tinkercad software - could play in influencing the identity work of the in-service teachers (Chronaki and Matos, 2014).

All six participants engaged in a performative manner using 3D printing pens to mathematise 2D and 3D skeleton shapes. Beyoncé had mathematised using 3D printing pens to produce 2D shapes, and a year later produced a partial completed skeleton cube. Tracy produced a skeleton cage using the shape of a cube as guidance. Elton constructed a cube using pencil and paper, then eventually constructed a cube using the 3D printing pen. Joseph designed a skeleton triangular-based prism using the knowledge of combining three triangles together. Taylor mathematised using a 3D printing pen to construct a skeleton cube on the first day of the learning-as-making activity, constructing 2D shapes with her class and constructing a 3D skeleton cube a year later. Also, Emily constructed 2D and 3D skeleton shapes using a 3D printing pen in the first part of the learning-as-making activity. These examples provide insight into the in-service teachers' participatory identity towards mathematics. As Darragh (2015) suggested,

> We become a mathematics learner in a performative manner, and it is the repetition of performances in mathematics learning contexts that generates our recognition of ourselves in certain ways as learners of mathematics. (Darragh, 2015, p. 85)

In this study, I explore in-service teachers' teaching identities in relation to maker identity, teachers' identities, identity work, and participative identity. Because the in-service teachers are engaging in a learning-as-making activity as non-specialist mathematics teachers, the identities discussed in this section are known as teaching identities towards mathematics.

6.2.3 **(R.Q.3) What does a diffractive reading of in-service teachers' work with 3D printing pens tell us about their mathematising?**

Reflecting on Barad's (2007) methodological framework on diffraction, as the six in-service teachers use the 3D printing pens that extrude a filament in the direction that their hand movements are intending in the learning-as-making activity, they are demonstrating the concept of intra-action. In addition, when in-service teachers are utilising the 3D printing pens, they are conducting a post-humanist performative approach that examines the 'matters of practice, doing, and actions' (Barad, 2007, p. 134). In chapter five, each episode reveals a dialogue along with a diffractive reading. The episodes show dialogues of in-service teachers using 3D printing pens in relation to mathematical applications. This is an example of how diffractive reading does give insight into how matter and meaning are intertwined, similar to Burnard et al. (2019) regarding mathematics artwork. To further answer this research question (R.Q.3), I analyse how each of the six in-service teachers began the process of mathematising using a 3D printing pen in the study's learning-as-making activity.

Their individual mathematising processes can be observed closely from the screenshots of the video recordings and pictures of fieldwork that are presented in chapter five. Each in-service teacher used a 3D printing pen to construct a 2D shape. Beyoncé constructed various 2D shapes with different filament colours, Joseph constructed a triangle on the flat surface, Tracy constructed a square on the flat surface, Elton constructed a square on the silicone mat and Taylor used the silicone mat in her class to construct four 2D shapes (circle, square, rectangle, and diamond). According to Ng et al. (2018), 'The very act of drawing, even in 2D, involves a certain movement of the hand that can become associated with certain ways of thinking and communicating mathematics' (Ng et al., 2018, p. 566).

The way the in-service teachers construct 2D shapes can be part of their senses (Pizlo, 2008). To elaborate, in-service teachers can be seen changing the speed of their pens, adjusting hand movements, and evolving their thinking processes while using the 3D printing pen to construct their 2D shapes. In addition, the in-service teachers intentionally took into account recognition of figural units when drawing their 2D shapes (Duval, 2014; Duval, 2017; Ng and Ye, 2022). For example, Tracy's hand movements while holding the 3D printing pen demonstrated that her construction of a caged heart happened first by constructing a square (2D), then by creating multiple lines (1D) within the square which connected one side of the square to the other. Similarly, Beyoncé used the 3D printing pen on a silicone mat to construct a square (2D) and used the grid inside the square to construct straight lines (1D) that connected one side of the square to another. Her hand movements and gestures were guided by the silicone mat, as it already contained the geometrical shapes of a circle, triangle, and square. Compared to pencil and paper, it is important to note that these designs that Beyoncé and Tracy created are influenced by the way the 3D printing pen filament is extruded onto the flat surface or silicone mat (Ng and Ferrara, 2020).

This can be seen as the in-service teachers mathematised using the 3D printing pen to construct a 3D skeleton cube. For example, Elton first used a paper and pencil to draw a 3D model of a cube. He then transitions to using the 3D printing pen where he draws a square (2D) as a base for the 3D skeleton cube. This is similar to Ng and Ye's (2022) work, where the primary students in the study constructed a 2D shape to be the foundation of their 3D shape. Elton then used the square base on the silicone mat to attempt to transform the square (2D) into a skeleton cube (3D) by drawing in the air (Ng and Sinclair, 2018). From Elton's construction of a 3D skeleton cube, there are two different representations – one from creating with the 3D printing pen and the other using a pencil and paper. While the two constructions are very different, his perception is consistent as he calls the creation he made using the 3D printing pen a 'cube' and the one on paper a 'square' (Pizlo, 2008). Tracy continues to replicate squares to make the sides of her cage. She uses her 3D printing pen on the flat surface to create a total of six sides. She decided to use the 3D printing pen as a glue gun to connect all six sizes to form her 3D skeleton caged heart. Similarly, Joseph created three individual triangles on the flat surface and used the 3D printing pen as a glue gun to form a triangular based pyramid. Elton, Tracy, and Joseph demonstrate how their perceptual recognition of shapes moves from 2D to 3D (Duval, 2014).

To see how in-service teachers would mathematise again using a 3D printing pen a year later, I visited Beyoncé and Taylor. Previously, Beyoncé did not attempt a 3D skeleton cube. During my second visit, however, she did. This was the first time she used the 3D printing pen to demonstrate a 3D skeleton drawing. In the first meeting, she demonstrated her competence in using the 3D printing pen to draw 2D shapes on the silicone mat. In this meeting a year later, she used the flat surface by drawing a square base to construct the skeleton cube. The reason for using the flat surface this time was that the 3D printing pen filament was not sticking on the silicone mat. This can be due to the potentially different speeds being used, or to the way Beyoncé is holding the 3D printing pen. In this attempt, Beyoncé drew two squares and a circle to construct her skeleton cube. She then squeezes the filament together to continue constructing the 3D skeleton cube. This time, Beyoncé used the circle as a base and inscribed squares in that circle. While Beyoncé did not complete the 3D skeleton cube, she was mathematising the shape of the cube differently compared to Elton and Tracy.

On the other hand, Taylor used the 3D printing pen proficiently a year later to construct a 3D skeleton cuboid and cube. She constructed the skeleton cuboid creating four rectangles of the same size using the silicone mat. Taylor then used the 3D printing pen as a glue gun to connect the filament together to form the 3D skeleton cuboid. She decided to construct a cube in the same way by creating six squares of the same size using the silicone mat and glueing them together using a 3D printing pen. Taylor was not guided or told how to construct her skeleton models. Similarly, Emily, who participated in the first part of the learning-as-making activities, was not guided in constructing her 2D and 3D shapes. She asked Taylor for help, but Taylor reinforced that she had to construct the shape on her own. From this diffractive reading, we can see how Taylor and Emily are mathematising in using the 3D printing pen to transition from a 2D shape to a 3D shape based on their prior mathematical knowledge of basic shapes (Duval 2006). The diffractive reading of each of the six in-service teachers demonstrates how they mathematise differently to construct their 2D and 3D shapes using a 3D printing pen in the learning-as-making activity. Their hand movements and gestures do have an influence on the way they mathematise to construct their 2D and 3D shapes.

6.3 Contribution to Theory and Practice of Early Years and Primary Education

6.3.1 **Contribution to the practice using 3D printing technology tools in early years and primary education**

This thesis contributes to the stated need for greater preparation of future early years and primary teachers in the use of 3D printing technology in the classroom. According to results from the Department for Education's (2013) own 3D printer project, 'schools need to consider challenges faced in introducing the technology, including those relating to developing teaching, teacher training and technical support' (DfE, 2013, p. 23). Although the Department for Education's project took place eleven years ago, recent literature still suggests that teachers are not competent in using 3D printing technology. According to Ulbrich et al. (2023), 'Teachers might not have experience with [3D printing] and they can hesitate to engage in modelling tasks with their students' (Ulbrich et al., 2023, p. 167). In my study, we see that all six in-service teachers in this research did not have any experience using 3D printing pens in front of their students nor in training as teachers. For example, Beyoncé did not demonstrate in her design and technology after school programme how to make a 3D object using a 3D printing pen to her students. Instead, she allowed her students to play and engage with the 3D printing pens to explore making.

Several scholars have observed how primary mathematics students utilise 3D printing pens (Ng and Ye, 2022; Ng and Ferrara, 2020), Tinkercad (O'Reilly and Barry, 2023; Pielsticker et al., 2021) and 3D printers (Khurma and Khine, 2023; Hu and Liu, 2022) for learning. However, there is a limited amount of research on teacher training programmes and continued professional development preparing teachers to use 3D printing technology for mathematics or other disciplines. The use of 3D printing technology can be a crossroad between mathematics and other subjects such as design and technology, arts, and science. Below are ways that this thesis contributes to the existing literature and the practice of early years and primary education:

1. **Teachers' Continuing Professional Development (CPD)**

Continuing professional development (CPD) for primary and early years teachers presents an opportunity to engage with 3D printing technology tools. In this study, I delivered the 3D printing pens and MakerBot 3D printer to each school. To make 3D printing pens and the MakerBot 3D printer accessible for CPD sessions, the organiser may have to provide these teachers with the 3D printing technologies. Elton, Emily, and Taylor mention in the study that their schools could not afford 3D printing technology. Beyonce, Tracy, and Joseph's school already had programmes for primary students to use a 3D printer and Tinkercad. For continuing professional development organisations to provide 3D printing technology activities, they must be able to obtain government grants to purchase 3D printing technologies to be used with the in-service teachers. Providing the 3D printing technology tools allows for equal opportunity for in-service teachers to be trained in the practice.

2. **Teacher Education Programmes**

Teacher Education programmes present an opportunity for prospective teachers entering the early years and primary level to play, engage, and design with 3D printing technology tools. In chapter four, I presented an episode of six prospective secondary teachers engaging with 3D printing pens in a learning-as-making activity. This is a potential opportunity for teacher education programmes to embed summative projects that allows them to take a term (i.e Autumn or Spring) to learn and engage using 3D printing technologies in their future classrooms. For example, the summative assessment in a teacher education programme could consist of three parts: (1) creating a fictional task where prospective teachers envision themselves as the role of the teacher communicating to their future students using 3D printing pens in a lesson; (2) prospective teachers give a presentation to the entire cohort about their lesson plan using 3D printing pens in their future classroom; and (3) prospective teachers write a reflection about their experience, writing a fictional task and presenting a lesson on 3D printing pens.

Regarding opportunities for prospective teachers to engage with Tinkercad or a MakerBot 3D printer in a teacher education programme, I recommend creating summative assessments that enable prospective teachers to use Tinkercad as well as print their creations using a MakerBot 3D printer. The summative assessment mentioned earlier could be used in this case. However, the teacher education programme would need access to a 3D printer to allow prospective teachers to print out their object from Tinkercad.

6.3.2 **Contribution to the theories of Communities of Practice, Shape Perception, and Inclusive Materialism**

In this section, I discuss the contribution this research makes toward the three theoretical lenses employed in this study. In-service teachers utilising 3D printing pens in the learning-as-making activities contribute to Wenger's (1998) communities of practice. The 3D printing pens act as a shared repertoire in the learning-as-making activities. All in-service teachers are engaging with the 3D printing pens to construct 2D and 3D skeleton shapes. Their discussion and conversation during the study reveal how they are aligning and imagining the learning-as-making activities in relation to their classrooms. All in-service teachers are seeing the importance of using 3D printing pens in their own fields, which provides a glimpse into the early formation of new communities of practice being developed. The communities of practice being developed in the study also reveal how teaching identities towards mathematics are arising through using a 3D printing pen in the learning-as-making activities.

The use of 3D printing technology with in-service teachers is a good example of Wenger's (1998) mention of resources and tools in a community of practice.

Furthermore, in-service teachers using 3D printing technology tools in a practice contributes to Pizlo's (2008) shape perception theory. The teachers' imagination and mental picture of constructing a skeleton shape or using a modelling software to manipulate shapes is prompted by the use of technology. 3D printing pens, 3D modelling software, and 3D printers are technology tools that enable in-service teachers to create visual and mental representations of their possible creations. This contributes to the theory of shape perception in that it suggests that shape perception is more than just a sensation (Pilzo, 2008) or mathematical knowledge (Duval, 2017, 2014, 2006, 1999); rather, it is the curiosity of our imagination of shapes that is influencing our use of 3D printing technology.

Lastly, De Freitas and Sinclair (2014) discuss in their work that the creative act is one that 'introduces or catalyses the new, is unusual, is unexpected or unscripted, [and] is without given content' (De Freitas and Sinclair, 2014, p. 89). The contribution of this study towards inclusive materialism is that it reveals how the creative act takes form through the conversation and diffractive reading of in-service teachers utilising 3D printing pens in the learning-as-making activities. As the in-service teachers hold the 3D printing pens that produce materials, their hand movements and gestures are showing us their creative act of constructing 2D and 3D skeleton shapes. Through these diffractive readings of their creative acts, they are also having conversations about how they see 3D printing pens being employed in their classroom. This study reveals how the powerful combination of in-service teachers' discussion, hand movements, and gestures plays an essential role in understanding the formation of a creative act.

6.4 Limitations of this Study

In this section, I will discuss the limitations of the overall study. In chapter five, I shared the dialogue and diffractive reading of three schools. The presentation of chapter five itself is due to one of the study's limitations, which is its small sample size of six participants. The data shown in the previous chapter does not represent a larger population of non-mathematics specialist in-service teachers, much less the mathematising processes and mathematical related identities of such teachers.

The first and arguably most notable limitation of this study was the lack of access to a pool of early years and in-service teachers during the time 2020 to 2022. In a life without COVID-19, conducting in-person activity for this project could have taken place in the 2020-2021 academic year, with in-person teaching being the norm and no need for social distancing. Unfortunately, however, my research took place during the worst of the COVID-19 pandemic. In-service teachers were dealing with the challenge of adapting to a hybrid way of teaching and in-person teaching was entirely halted until the latter half of 2021.

As I mention in section (4.2.4), the original target audience for collecting fieldwork was preservice teachers who are enrolled in Post Graduate Certificate in Education (PGCE) programmes preparing to teach at primary level. Aiming to reach this particular audience led to the second limitation of my study which is the course modification policy of the UK university system. As I became a UK academic, I learned that in the United Kingdom there needs to be course modification approval just to implement a one-day activity with PGCE students. At the time that I needed such approval (the 2021-2022 academic year), I was a new post graduate researcher who did not know about these UK rules and regulations, which are much different than those of the United States educational system. These required guidelines delayed my work and led me into the winter of 2022 sending hundreds of emails throughout maths hub networks and private schools seeking participants for my study. One memorable experience resulting from these efforts was my eyebrow-raising commute to London with the Makerbot 3D printer and a duffle bag of 3D printing pens (see section 4.2.4).

The third limitation that I encountered in my study came from the classroom environments in which the fieldwork took place. In the previous chapter I explained that I implemented the learning-as-making activity inside the classrooms of the participating schools. Only one of those classrooms, however, was fully suitable for a learning-as-making activity. The lack of sufficient and accessible electrical outlets, for example, potentially could have limited the stories-to-live by shared by Elton and Taylor, whose classroom was in multiple ways not fit for a 3D learning-as-making activity. According to Clandinin and Rosiek (2007),

the focus of narrative inquiry is not only on individuals' experience but also on the social, cultural, and institutional narratives within which individuals' experiences are constituted, shaped, expressed, and enacted. (Clandinin and Rosiek, 2007, p. 42)

This limitation not only potentially impacted the participants' stories, but it also affected the ability to set up the 3D printing equipment. There were some malfunctions with the 3D printing pens in the classrooms like Taylor's and Elton's. When some electrical outlets in the classroom were not functioning, for example, the participants used their desktop USB ports to plug in the 3D printing pens and start their drawing of 2D and 3D shapes. Moreover, some classrooms did not have a secure area for the Makerbot 3D printer. As such, for safety reasons, I had to commute with the Makerbot 3D printer for several trips.

A fourth limitation relates to the 3D printing assembly process. In chapter five, I shared the dialogue between participants in each school using Tinkercad to mathematise in constructing 3D models of neighbourhoods. I was unable to print my participants' physical objects using the Makerbot 3D printer because some objects were still floating in the air and not on the flat surface of the Tinkercad plane. In addition, the 3D printing process includes the time it takes to print an object. Because the in-service teachers were given the freedom to create any size 3D model on Tinkercad, this created severe limitations due to the number of days some of the models would have taken to print.

An alternative to using my Makerbot 3D printer was to have the in-service teachers' models printed using a 3D printing company. However, the high costs of doing this became the fifth limitation of my study. Before the day of the learning-as-making activity at Beyoncé's school, I used a 3D printing company to print two 3D models that Beyoncé's student had created in her classroom through Tinkercad. The expense, 28 pounds, was determined by the size, depth, and filament used to create the object. As many of the objects designed by research participants were even larger and more complicated than Beyoncé's student's, it became clear that printing participants' Tinkercad creations through a commercial printing facility was cost-prohibitive. As a result, models that were created during the fieldwork activities were not physically printed using my Makerbot 3D printer (due to time) nor a commercial 3D printer (due to costs). The ability for participants to hold the actual objects they had designed was a goal I had originally, and I think would be a highly effective way to teach 3D shapes. To pursue this research fully, a larger budget will be necessary.

6.5 Considerations for Future Research

In this section, I discuss possible future research ideas that emerged in the course of this doctoral study. This study focused on in-service teachers who are non-mathematics specialists at early year and primary levels. To explore their mathematical related identities, I observed the teachers mathematising using 3D printing technology tools. As seen in chapter five, all participants had different ways of mathematising using the 3D printing pens to construct a skeleton model cube. Below I discuss four areas where I see a need for future research and will focus on upon graduation: early years education, primary education, university education, and elementary teacher education in the United States.

Early Years Education

Taylor used the 3D printing pen in her early year classroom to demonstrate the construction of 2D shapes. In this 45-minute classroom meeting, she was able to purposefully engage with her students showcasing her 2D shapes and asking the students to identify each shape verbally. To expand on this pedagogical method, I would like to obtain a cohort of twenty early year teachers who would be willing to use one class time to demonstrate to early year students the creation of 2D shapes. This research agenda is in line with the EYFS (Early Years Foundation Stage) development matter framework (DfE, 2023). The contents of this developmental framework include:

- ‘Talk about and explore 2D and 3D shapes (for example, circles, rectangles, triangles and cuboids) using informal and mathematical language’ (DfE, 2023, p. 89).

- ‘Select, rotate and manipulate shapes to develop spatial reasoning skills’ (DfE, 2023, p. 98).

3D printing pens are not suitable for early years children to hold on their own. However, at this stage a child can still play and engage with 3D printing materials with an early teacher supervisor. For example, during the learning-as-making activities, Taylor discussed using 3D printing pens to introduce numbers to early years students. Sinclair and De Freitas’s (2014) inclusive materialism theoretical lens can be employed to understand how early years students’ hand movements over the 3D printed numbers created by their teachers aid their learning of whole numbers. According to the early years foundation stage statutory framework (EYFS, 2024), children should be exposed to the materials in an arts and design classroom. Early year children are expected to ‘explore a variety of materials, tools and techniques, experimenting with colour, design, texture, form and function’ (EYFS, 2024, p. 15). The purpose of this future research is to observe early years teachers mathematising using 3D printing pens in order to investigate how 3D printing technology impacts the way early years instructors teach through manipulatives in the classroom.

Primary Education

Beyoncé, Elton, Tracy, and Joseph all engaged with 3D printing technology from the primary education perspective. One of the common backgrounds that all four primary participants shared was having a non-mathematics specialisation. To expand beyond my doctoral study, I am interested in a future research project that will have a cohort including both primary non-mathematics specialist teachers and primary mathematics teachers. According to Lutovac and Kaasila (2018), who investigated linking research on mathematics teachers with elementary teachers,

there is a need to compare both cohorts and connect findings from both fields in a way that can inform and benefit both elementary and mathematics teachers. Indeed, their differences would be viable starting points for investigating and supporting the development of identities for all mathematics teachers. With these differences in mind, we need to acknowledge that research on mathematics-related teacher identity will always be as diverse as the teachers who teach mathematics. (Lutovac and Kaasila, 2018, p. 10)

In my future research projects, I will implement a learning-as-making activity that consists of mathematical content from Key Stages 1-3 of the English national curriculum. The learning-as-making activity will be used with both primary non-mathematics and mathematics specialist teachers. The goal of the research will be to observe how the primary non-mathematics specialists and primary mathematics specialists employ learning-as-making activities in their classrooms. I conjecture that this research will offer a glimpse into mathematics related teacher identity from both cohorts. I also conjecture that I will gain insight into which group, the non-mathematics or mathematics specialists, can best envision the use of 3D printing pens for mathematics applications.

University Education

In this study, I demonstrated a diffractive reading of two Calculus for Business students mathematising to construct 3D skeleton shapes. To further this research at the university level, implementing maker fairs twice a year can allow students from all parts of the university to come to an area to play, design and engage with 3D printing technology tools. For example, my current academic position enabled me to collaborate in a university wide maker fair in Autumn 2023 that incorporated the use of 3D printing pens, 3D modelling software and the original PRUSA MK4 3D printer (see Figure B). One creation from that event is pictured in Figure A.

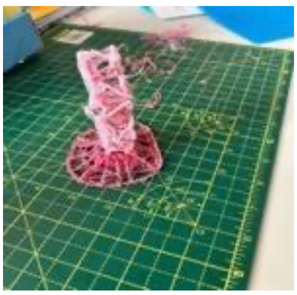


Figure A: An undergraduate student creation using a 3D printing pen to create a skeleton glass sculpture.

Figure B: The original PRUSA MK4 3D printer

The term maker fair refers to ‘a large science and engineering fair’ (Blikstein and Krannich, 2013, p. 614). The research objective in hosting a bi-annual maker fair will be to observe students at the fair who are from STEM (Science, Technology, Engineering, and Mathematics) and non-STEM backgrounds to see how they utilise 3D printing technologies to engage with STEM applications. After students have played, designed, and engaged with 3D printing technology tools, I will invite them back for an interview to talk about their experience. I conjecture that the students’ narratives could give insight into how university students from STEM and non-STEM backgrounds utilise 3D printing technology tools to mathematise.

Elementary Teacher Education in the United States

What allowed me to establish a strong working relationship with the participants in this study was my story of being a mathematics teacher in New York City. As my current workplace position has ties to Boston, I would like to further my research with in-service teachers at the elementary school level in Boston. Elementary school in the United States begins with Pre-Kindergarten and continues to Grade 4 (ages 3 to 10). I will implement a learning-as-making activity with elementary school teachers to investigate how they mathematise using 3D printing pens to teach 2D and 3D shapes to their students. By conducting an international examination of in-service teachers, I am also comparing the English national curriculum with the Massachusetts Curriculum Frameworks for PreK-12. I conjecture that conducting research on elementary in-service teachers in the United States could give insight into global similarities and differences of in-service teachers’ teaching identities towards mathematics.

6.6 A reflection on my journey as a post-graduate researcher in mathematics education

In this section, I present my reflections on being a post-graduate researcher in mathematics education. The story begins in October 2019, when I enrolled in a PhD programme as a new post graduate researcher with no background in research or extensive writing experience. My primary supervisor asked me at the end of that first month to produce a proposal for the innovation research grant offered by the university. When writing this proposal, I came across the communities of practice theory and knew that this was the perfect theoretical frame for my work. In November 2019, I was awarded the innovation grant for £3,199.00. This was one of my major accomplishments during my probationary period as it enabled me to purchase a Makerbot 3D printer which I would subsequently use for my fieldwork, as seen in section (4.2.4).

From November 2019 to March 2020, I focused on reading literature and developing a probationary paper. At first, this was a challenging period. I had never written a probationary paper in either my bachelor's or master's degree programmes in the United States. Coming into the PhD programme, I knew I would have challenges compared to students who received their educations in the United Kingdom or Europe where research is prioritised at the master's programme level. The one thing that motivated me was my supervisory team; they did not give up on me. When COVID-19 hit in March 2020 enforcing a stay-at-home order, my supervisor gave me one more chance to produce a quality probationary paper. I had spent endless nights revising, reading, and developing vocabulary to prove that I was at the level of post graduate researcher. In May 2020, I passed, and successfully moved on as a full-fledged post graduate researcher.

In the 2020-2021 academic year, I focused on developing my writing abilities. My supervisor suggested that I write a series of 1000-word papers on specific areas related to my subject (i.e., mathematical identity, teacher identity, etc). Doing these exercises not only exposed me to a vast amount of scholarship, but it also pointed me in the direction of a concentration on mathematics-related identity. During this time, I was not able to conduct any field work as all of academia was adapting to new ways of teaching and learning remotely. This was not too much of a concern in my mind because at the time I still thought that my research would include only a computer software entitled Tinkercad and my new MakerBot 3D printer. Little did I know that this was going to change significantly in the following year.

In the 2021-2022 academic year, I experienced my first major hurdle as a post-graduate researcher. It was time to collect participants for the fieldwork portion of my study. I had thought that this would be a smooth process, using the PGCE cohort at the university as my participant base. Unfortunately, the cohort did not respond to my advertising pitch to join the study. So I decided to seek only six participants from the Secondary PGCE cohort and conduct a pilot study to refine my research goals. I set up a research activity using Tinkercad and my Makerbot 3D printer. While the six pilot participants were intrigued, they could not see how the activity related to mathematics. I had to go back to the drawing board and develop a new research activity.

My search took me to the Research in Mathematics Education (RME) group meeting for ideas. My supervisor was on sabbatical at the time, and she allowed me to host one RME meeting in March 2022 alongside another colleague. One of the skills that I learned at this time was how to develop a meeting agenda and manage time for each topic during the meeting. More importantly, at this RME group meeting, I introduced an article by Ng and Sinclair (2018) entitled *Drawing in space: Doing mathematics with 3D pens*. The article and our discussion of it helped me find my research direction. At the end of March, I applied and won an SSF Associate Dean innovation grant for £799.70. With this money I purchased twenty 3D printing pens and twenty silicone mats.

Throughout the summer of 2022, I conducted my field work in three different primary schools in London. Doing so, I not only accomplished my field work but also got to see how mathematics is taught at the primary level in the United Kingdom. In addition, I enjoyed a glimpse of the day-to-day life of primary school teachers and got the chance to learn about differences between private and state schools in the UK. One major difference is the availability of funding. The two teachers whom I worked with in state schools struggled to get school supplies, whereas the private school teachers could purchase expensive equipment. In fact, the main private school teacher I worked with ordered two Makerbot 3D printers for her design and technology classroom after participating in my fieldwork activity.

In the 2022-2023 academic year, I started analysing my data. What I quickly realised was that there was so much more to the video recordings than mere dialogue. While participants' mathematical conversations yielded highly interesting material, I noticed that hand and body movements, as well as facial expressions, also demonstrated how participants were mathematising as they used 3D printing pens and that this was a crucial component to explore. Striving to contextualise this primary data in previous literature, I came across Barad's (2007) diffraction methodological approach and began to understand how our body movements and the materials around us intertwine to create something meaningful. At the same time, I was appointed as assistant professor and associate head of mathematics at a different university. This was a challenging time as I was learning to be a teaching academic again while writing up my thesis. It was also an exciting time because I found ways to implement my research in my new classes and amongst the other mathematics professors. Most importantly, I learned about the structure of higher education in the United Kingdom, including course moderation and other policies that are more strictly enforced than in the higher education of the United States.

Meanwhile, I had stayed involved in the RME group meeting and through this enjoyed another theoretical breakthrough. Dr Bryan Maddox, a member of the RME group and part of a research team in France interested in digital mathematics assessment, introduced me to Pizlo's (2008) shape perception theory and Duval's (1999) framework of mathematical thinking. I realised that both mathematical thinking and shape perception played a role in my study (see Section 2.2). This insight helped me to create a bridge from communities of practice theory to shape perception theory to materialism and enabled me to begin to tie all the parts of my analysis together. As the past four and half years have gone by, I have found my true passion in research. I look to continue this journey in my current academic position, which I hope will lead me to a home university that has an education department.

Adegboye, D., Williams, F., Collishaw, S., Shelton, K., Langley, K., Hobson, C., Burley, D. and van Goozen, S. (2021). Understanding why the COVID-19 pandemic-related lockdown increases mental health difficulties in vulnerable young children. *JCPP Advances*, 1(1), p. e12005.

Akkoç, H., Balkanlioglu, M.A. and Yesildere-Imre, S. (2016). Exploring preservice mathematics teachers' perception of the mathematics teacher through communities of practice. *Mathematics Teacher Education and Development*, 18(1), pp.7-51.

Anđić, B., Lavicza, Z., Ulbrich, E., Cvjetićanin, S., Petrović, F. and Maričić, M. (2022). Contribution of 3D modelling and printing to learning in primary schools: A case study with visually impaired students from an inclusive biology classroom. *Journal of Biological Education*, pp.1-17. doi: 10.1080/00219266.2022.2118352.

Anđić, B., Maričić, M., Weinhandl, R., Mumcu, F., Schmidthaler, E. and Lavicza, Z. (2024). Metaphorical evolution: A longitudinal study of secondary school teachers’ concepts of 3D modelling and printing in education. *Education and Information Technologies*, pp.1-36. doi: 10.1007/s10639-023-12408-x.

Anđić, B., Ulbrich, E., Dana-Picard, T., Cvjetićanin, S., Petrović, F., Lavicza, Z. and Maričić, M. (2023). A phenomenography study of STEM teachers’ conceptions of using three-dimensional modeling and printing (3DMP) in teaching. *Journal of Science Education and Technology*, 32(1), pp.45-60.

Asempapa, R.S. and Love, T.S. (2021). Teaching math modeling through 3d-printing: Examining the influence of an integrative professional development. *School Science and Mathematics*, 121(2), pp.85-95.

Autodesk (2023). *From mind to design in minutes.* Available at: https://www.tinkercad.com/ (Accessed: 21 October 2023).

Ball, S.J. (2001). Performativities and fabrications in the education economy: Towards the performative society, in Gleeson, D. and Husbands, C. (eds.) *The performing school: Managing teaching and learning in a performance culture*. London: Routledge Falmer, pp. 143-155.

Ball, S.J. (2003). The teacher's soul and the terrors of performativity. *Journal of Education Policy*, 18(2), pp.215-228.

Barad, K. (2014). Diffracting diffraction: Cutting together-apart. *Parallax*, 20(3), pp.168-187.

Barad, K. (2007). Meeting the universe halfway. *Quantum physics and the entanglement of matter and meaning*. Durham: Duke University Press.

Barad, K. (2010). Quantum entanglements and hauntological relations of inheritance: Dis/continuities, spacetime enfoldings, and justice-to-come. *Derrida Today*, 3(2), pp.240-268.

Becker, S. and Jacobsen, M. (2023). A year at the improv: The evolution of teacher and student identity in an elementary school makerspace. *Teaching Education*, 34(1), pp.1-18.

Bernard, P. and Mendez, J.D. (2020). Drawing in 3D: Using 3D printer pens to draw chemical models. *Biochemistry and Molecular Biology Education*, 48(3), pp.253-258.

Bevan, B., Gutwill, J.P., Petrich, M. and Wilkinson, K. (2015). Learning through STEM-rich tinkering: Findings from a jointly negotiated research project taken up in practice. *Science Education*, 99(1), pp.98-120.

Bhaduri, S., Biddy, Q.L., Bush, J., Suresh, A. and Sumner, T. (2021, June). 3DnST: A framework towards understanding children's interaction with Tinkercad and enhancing spatial thinking skills. *Interaction Design and Children*, pp.257-267. doi: 10.1145/3459990.3460717.

Blikstein, P. and Krannich, D. (2013, June). The makers' movement and FabLabs in education: Experiences, technologies, and research. In *Proceedings of the 12th international conference on interaction design and children*, pp.613-616.

Bonnette, R.N. and Crowley, K. (2020). Legitimate peripheral participation in a makerspace for emancipated emerging adults. *Emerging Adulthood*, 8(2), pp.144-158.

Brännlund, A., Strandh, M. and Nilsson, K. (2017). Mental-health and educational achievement: The link between poor mental health and upper secondary school completion and grades. *Journal of Mental Health,* 26(4), pp.318-325.

Brown, A.D. and Toyoki, S. (2013). Identity work and legitimacy. *Organization Studies*, 34(7), pp.875-896.

Burnard, P., Sinha, P., Steyn, C., Fenyvesi, K., Brownell, C., Werner, O. and Lavicza, Z. (2019). Reconfiguring STEAM through material enactments of mathematics and arts: A diffractive reading of young people's intradisciplinary math-artworks, in Burnard, P. and Colucci-Gray, L. (eds.) *Why Science and Art Creativities Matter*. Brill, pp.171-199.

Butler, E., Prieto, E., Osborn, J.A., Howley, P., Lloyd, A., Kepert, A. and Roberts, M. (2019). Learning across discipline boundaries through narrative inquiry: A study of a collaboration to improve mathematics teacher education. *Mathematics Teacher Education and Development*, 21(2), pp.87-106.

Butler, Judith. (1988). Performative acts and gender constitution: An essay in phenomenology and feminist theory. *Theatre Journal*, 40(4), pp.519–531.

Cavanagh, M.S. and Garvey, T. (2012). A professional experience learning community for pre-service secondary mathematics teachers. *Australian Journal of Teacher Education*, 37(12), pp.57-75.

Chen, J., Xiang, S., Yuan, Y. and Zeng, Y. (2021, August). The exploration and practice of 3D printing pens in primary school education. In *2021 International Conference on Diversified Education and Social Development (DESD 2021)*. Atlantis Press, pp.6-11.

Chronaki, A. and Matos, A. (2014). Technology use and mathematics teaching: Teacher change as discursive identity work. *Learning, Media and Technology*, 39(1), pp.107-125.

Chronaki, A., and Kollosche, D. (2019). Refusing mathematics: A discourse theory approach on the politics of identity work. *ZDM Mathematics Education*, 51, pp.457-468.

Chronaki, A. (2011). '"Troubling" essentialist identities: Performative mathematics and the politics of possibility', in Kontopodis, M., Wulf, C. and Fichtner, B. (eds.) *Children, Development and Education: Cultural, Historical, Anthropological Perspectives*. Springer, Dordrecht, pp.207-226.

Clandinin, D.J. and Rosiek, J. (2007). 'Mapping a landscape of narrative inquiry: Borderland spaces and tensions', in Clandinin, D.J. (ed.) *Handbook of narrative inquiry: Mapping a methodology.* Sage Publications, pp.35–76.

Clandinin, D.J. and Connelly, F.M. (1995). *Teachers' professional knowledge landscapes*. Teachers College Press.

Clandinin, D.J. and Connelly, F.M. (2000). *Narrative inquiry: experience and story in qualitative research*. San Francisco: Jossey-Bass Publishers.

Clandinin, D.J. and Rosiek, J. (2007). 'Mapping a landscape of narrative inquiry: Borderland spaces and tensions', in Clandinin, D.J. (ed.) *Handbook of narrative inquiry: mapping a methodology.* Sage Publications, pp.35–75.

Clandinin, D.J. and Connelly, F.M. (1996). Teachers' professional knowledge landscapes: Teacher stories—stories of teachers—school stories—stories of schools. *Educational Researcher*, 25(3), pp.24-30.

Clandinin, D.J. (ed.) (2006). *Handbook of narrative inquiry: mapping a methodology*. Sage Publications.

Clandinin, D.J. (2019). *Journeys in narrative inquiry: the selected works of D. Jean Clandinin*. Routledge.

Clandinin, D.J. (2022). *Engaging in narrative inquiry*. Routledge.

Connelly, F.M. and Clandinin, D.J. (1988). *Teachers as curriculum planners. Narratives of experience*. Teachers College Press, New York.

Connelly, F.M. and Clandinin, D.J. (1990). Stories of experience and narrative inquiry. *Educational Researcher*, 19(5), pp.2-14.

Connelly, F.M. and Clandinin, D.J. (2006). 'Narrative inquiry', in Green, J.L., Camilli, G., and Elmore, P.B. (eds). *Handbook of complementary methods in education research*. Washington, DC: Lawrence Erlbaum Associates, pp.477-487.

Connelly, F.M., Clandinin, D.J. and He, M.F. (1997). Teachers' personal practical knowledge on the professional knowledge landscape. *Teaching and Teacher Education,* 13(7), pp.665-674.

Creswell, J.W. (2012). *Educational research: planning, conducting, and evaluating quantitative and qualitative research*. 4th ed. Boston, MA: Pearson.

Creswell, J.W. and Miller, D.L. (2000). Determining validity in qualitative inquiry. *Theory Into Practice*, 39(3), pp.124-130.

Crisan, C. and Hobbs, L. (2019). 'Subject-specific demands of teaching: Implications for out-of-field teachers', in Hobbs, L. and Torner, G. (eds.) *Examining the Phenomenon of "Teaching Out-of-field" International Perspectives on Teaching as a Non-specialist*. Springer: Singapore, pp.151-178.

Crisan, C. and Rodd, M. (2017). Learning mathematics for teaching mathematics: Non-specialist teachers' mathematics teacher identity. *Mathematics Teacher Education and Development*, 19(2), pp.104-122.

Crisan, C. and Rodd, M. (2014). Talking the talk… but walking the walk? How do non-specialist mathematics teachers come to see themselves as mathematics teachers? *Proceedings and Agenda for Research and Action from the 1st Teaching Across Specialisations (TAS) Collective Symposium, August 2014*, 1, pp.25-26.

Crisan, C. and Rodd, M. (2011). 'Teachers of mathematics to mathematics teachers', in Smith, C. (ed). *Proceedings of the British Society for Research into Learning Mathematics*, 31(3), pp.29-34.

Darragh, L. (2015). Recognising 'good at mathematics': Using a performative lens for identity. *Mathematics Education Research Journal,* 27(1), pp.83–102.

Davis, D. and Mason, L.L. (2017). A behavioral phenomenological inquiry of maker identity. *Behavior Analysis: Research and Practice*, 17(2), p.174-196.

De Freitas, E. and Sinclair, N. (2014). *Mathematics and the body: material entanglements in the classroom*. Cambridge University Press.

Department for Education (2013). *Computing programmes of study - key stages 1 and 2.* Available at: https://assets.publishing.service.g ov.uk/media/5a7da548ed915d2ac884cb07/PRIMARY_national_curriculum_-_Mathematics_220714.pdf (Accessed: 21 Oct 2023).

Department for Education (2013). *Design and technology programmes of study: key stage 3*. Available at: https://assets.publishing.service.gov.uk/media/5a7c99ebed915d6969f46087/SECONDARY_national_curriculum_-_Design_and_technology.pdf (Accessed: 14 March 2024).

Department for Education (2013). *National curriculum in England: design and technology programmes of study - key stages 1 and 2*. Available at: https://assets.publishing.service.gov.uk/media/5a7ca43640f0b6629523adc1/PRIMARY_national_curriculum_-_Design_and_technology.pdf (Accessed: 14 March 2024).

Department for Education (2013). *The national curriculum in England: mathematics programmes of study - key stages 1 and 2*. Available at: https://assets.publishing.service.gov.uk/media/5a7da548ed915d2ac884cb07/PRIMARY_national_curriculum_-_Mathematics_220714.pdf (Accessed: 14 March 2024).

Department for Education (2023). *The national curriculum in England: development Matters-non-statutory curriculum guidance for the early years foundation stage*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1180056/DfE_Development_Matters_Report_Sep2023.pdf (Accessed: 14 March 2024).

Department for Education (2024). *Statutory framework for the early years foundation stage*. Available at: https://assets.publishing.service.gov.uk/media/65aa5e42ed27ca001327b2c7/EYFS_statutory_framework_for_group_and_school_based_providers.pdf (Accessed: 14 March 2024).

Dilling, F. and Witzke, I. (2020). The use of 3D-printing technology in calculus education: Concept formation processes of the concept of derivative with printed graphs of functions. *Digital Experiences in Mathematics Education*, 6, pp.320-339.

Doorman, M., Bos, R., de Haan, D., Jonker, V., Mol, A. and Wijers, M. (2019). Making and implementing a mathematics day challenge as a makerspace for teams of students. *International Journal of Science and Mathematics Education*, 17, pp.149-165.

Duval, R. (1999). Representation, vision and visualization: Cognitive functions in mathematical thinking. *Basic issues for learning. Proceedings of PME 23*, 1, pp.3–26.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1-2), pp.103-131.

Duval, R. (2014). Commentary: Linking epistemology and semi-cognitive modeling in visualization. *ZDM*, 46(1), pp.159-170.

Duval, R. (2017). *Understanding the mathematical way of thinking: the registers of semiotic representations*. Cham: Springer International Publishing.

Forest, C.R., Moore, R.A., Jariwala, A.S., Fasse, B.B., Linsey, J., Newstetter, W., Ngo, P. and Quintero, C. (2014). The invention studio: A university maker space and culture. *Advances in Engineering Education*, 4(2), pp.1-32.

Francis, K. and Whiteley, W. (2015). 'Interactions between three dimensions and two dimensions', in *Spatial Reasoning in the Early Years.* Routledge, pp.121-136.

Gagnier, K.M., Holochwost, S.J. and Fisher, K.R. (2022). Spatial thinking in science, technology, engineering, and mathematics: Elementary teachers' beliefs, perceptions, and self-efficacy. *Journal of Research in Science Teaching*, 59(1), pp.95-126.

Gholson, M.L. and Martin, D.B. (2019). Blackgirl face: Racialized and gendered performativity in mathematical contexts. *ZDM*, 51, pp.391-404.

Graven, M. and Heyd-Metzuyanim, E. (2019). Mathematics identity research: The state of the art and future directions: Review and introduction to ZDM special issue on identity in mathematics education. *ZDM*, 51(3), pp.361-377.

Hall, A.R. (1990). Beyond the fringe: Diffraction as seen by Grimaldi, Fabri, Hooke and Newton. *Notes and Records of the Royal Society of London*, 44(1), pp.13-23.

Halverson, E.R. and Sheridan, K. (2014). The maker movement in education. *Harvard Educational Review*, 84(4), pp.495-504.

Halverson, E. and Peppler, K. (2018). 'The maker movement and learning', in *International handbook of the learning sciences*. Routledge, pp.285-294.

Haraway, D. (1997). Modest_Witness@Second_Millenium. FemaleMan©_Meets_Onco Mouse™. *Feminism and Technoscience*. New York: Routledge.

Hecht, E. (2017). *Optics*. Boston: Pearson.

Honey, M. (ed.) (2013). *Design, Make, Play: Growing the Next Generation of STEM Innovators*. Routledge.

Hsu, Y.C., Baldwin, S. and Ching, Y.H. (2017). Learning through making and maker education. *TechTrends*, 61, pp.589-594.

Hui, J.S. and Gerber, E.M. (2017, February). Developing makerspaces as sites of entrepreneurship. In *Proceedings of the 2017 ACM Conference on Computer Supported Cooperative Work and Social Computing*, pp.2023-2038.

Jimenez-Silva, M. and Olson, K. (2012). A community of practice in teacher education: Insights and perceptions. *International Journal of Teaching and Learning in Higher Education*, 24(3), pp.335-348.

Kaschak, J.C. and Letwinsky, K.M. (2015). Service-learning and emergent communities of practice: A teacher education case study. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, 88(5), pp.150-154.

Khurma, O.A., Ali, N. and Khine, M.S. (2023). Exploring the impact of 3D printing integration on STEM attitudes in elementary schools. *Contemporary Educational Technology*, 15(4), p.ep458.

Koffka K. (1935). *Principles of gestalt psychology*. New York: Harcourt, Brace and Company.

Küçük, M., Talan, T. and Demirbilek, M. (2024). The Effect of Creating 3D Objects with Block Codes on Spatial and Computational Thinking Skills. *Informatics in Education*, *23*(1), pp.125-143.

Kurian, J., Murray, D.W., Kuhn, L. and LaForett, D.R. (2022). Examining frequency and modality of parent engagement in an elementary school mental health intervention. *Journal of Applied School Psychology*, 38(1), pp.74-93.

Lave, J. and Wenger, E. (1991). *Situated learning: legitimate peripheral participation*. Cambridge: Cambridge University Press.

Leinonen, T., Virnes, M., Hietala, I. and Brinck, J. (2020). 3D printing in the wild: Adopting digital fabrication in elementary school education. *International Journal of Art & Design Education*, 39(3), pp.600-615.

Lenz Taguchi, H. and Palmer, A. (2013). A more 'livable' school? A diffractive analysis of the performative enactments of girls' ill-/well-being with (in) school environments. *Gender and Education*, 25(6), pp.671-687.

Li, Y., Pizlo, Z. and Steinman, R.M. (2009). A computational model that recovers the 3D shape of an object from a single 2D retinal representation. *Vision research*, 49(9), pp.979-991.

Lincoln, Y.S. and Guba, E.G. (1985). *Naturalistic inquiry*. Sage.

Lutovac, S. and Kaasila, R. (2009). Using narratives as innovative tools in mathematics education courses in Finnish teacher education. In *Proceedings of the Conference "Development of Competencies in the World of Work and Education*, pp.355-359.

Lutovac, S. and Kaasila, R. (2018). An elementary teacher's narrative identity work at two points in time two decades apart. *Educational Studies in Mathematics*, 98, pp.253-267.

Lutovac, S. and Kaasila, R. (2014). Pre-service teachers' future-oriented mathematical identity work. *Educational Studies in Mathematics,* 85(1), pp.129–142.

Lutovac, S. and Kaasila, R. (2018b). An elementary teacher's narrative identity work at two points in time two decades apart. *Educational Studies in Mathematics,* 98(3), pp.253–267.

McAdams, D.P. (2019). “First we invented stories, then they changed us”: The evolution of narrative identity. *Evolutionary Studies in Imaginative Culture*, 3(1), pp.1-18.

MacPhail, A., Patton, K., Parker, M. and Tannehill, D. (2014). Leading by example: Teacher educators' professional learning through communities of practice. *Quest*, 66(1), pp.39-56.

Martin, L. (2015). The Promise of the Maker Movement for Education. *Journal of Pre-College Engineering Education Research (J-PEER)*, 5(1), Article 4. https://doi.org/10.7771/2157-9288.1099

Martinie, S.L., Kim, J.H. and Abernathy, D. (2016). “Better to be a pessimist”: A narrative inquiry into mathematics teachers' experience of the transition to the Common Core. *The Journal of Educational Research*, 109(6), pp.658-665.

Mathieu-Soucy, S., Corriveau, C. and Hardy, N. (2018, April). Exploration of new post-secondary mathematics teachers' experiences: Preliminary results of a narrative inquiry. *INDRUM 2018*. https://hal.science/hal-01849930.

Mazzei, L.A. (2013). Materialist mappings of knowing in being: Researchers constituted in the production of knowledge. *Gender and Education*, 25(6), pp.776-785.

Mazzei, L.A. (2014). Beyond an easy sense: A diffractive analysis. *Qualitative Inquiry*, 20(6), pp.742-746.

Mbaezue, C., Brubaker, E.R. and Sheppard, S. (2020, June). Understanding a Maker Space as a Community of Practice. In *2020 ASEE Virtual Annual Conference Content Access*.

Mengis, J. and Nicolini, D. (2021). ‘Practising diffraction in video-based research’, in Grosjean, S. and Matte, F. (eds.) *Organizational video-ethnography revisited: making visible material, embodied and sensory practices*. Palgrave Macmillan, pp.79-97.

Naidoo, J. (2020). Postgraduate mathematics education students' experiences of using digital platforms for learning within the COVID-19 pandemic era. *Pythagoras*, 41(1), p.568.

Neumayer-Depiper, J. (2013). Teacher identity work in mathematics teacher education. *For the Learning of Mathematics*, 33(1), pp.9-15.

Newcombe, N.S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator*, 34(2), pp.29-43.

Ng, O.L. and Chan, T. (2021). In-service mathematics teachers’ video-based noticing of 3D printing pens “in action”. *British Journal of Educational Technology*, 52(2), pp.751-767.

Ng, O.L. and Ferrara, F. (2020). Towards a materialist vision of ‘learning as making’: The case of 3D printing pens in school mathematics. *International Journal of Science and Mathematics Education*, 18, pp.925-944.

Ng, O.L. and Sinclair, N. (2018). 'Drawing in space: Doing mathematics with 3D pens' in Ball, L., Drijvers, P., Ladel, S., Siller, H.S., Tabach, M., and Vale, C. (eds.) *Uses of technology in primary and secondary mathematics education: tools, topics and trends*. Springer, Cham, pp.301-313.

Ng, O.L. and Tsang, W.K. (2023). Constructionist learning in school mathematics: Implications for education in the fourth industrial revolution. *ECNU Review of Education*, 6(2), pp.328-339.

Ng, O.L. and Ye, H. (2022). Mathematics learning as embodied making: Primary students' investigation of 3D geometry with handheld 3D printing technology. *Asia Pacific Education Review*, 23(2), pp.311-323.

Ng, O.L., Sinclair, N. and Davis, B. (2018). Drawing off the page: How new 3D technologies provide insight into cognitive and pedagogical assumptions about mathematics. *The Mathematics Enthusiast*, 15(3), pp.563-578.

O'Reilly, J. and Barry, B. (2023). The effect of the use of computer-aided design (CAD) and a 3D printer on the child's competence in mathematics. *Irish Educational Studies*, 42(2), pp. 233-256.

Osborne, A.T. (2019). Create in 3-D: Building a maker mindset one print at a time. *Knowledge Quest*, 48(1), pp.48-53.

Parker, M., Patton, K. and Tannehill, D. (2012). Mapping the landscape of communities of practice as professional development in Irish physical education. *Irish Educational Studies*, 31(3), pp.311-327.

Palmer, A. (2011). "How many sums can I do"?: Performative strategies and diffractive thinking as methodological tools for rethinking mathematical subjectivity. *Reconceptualizing Educational Research Methodology (RERM)*, 1(1), pp.3-18.

Pielsticker, F., Witzke, I. and Vogler, A. (2021). Edge models with the CAD software: Creating a new context for mathematics in elementary school. *Digital Experiences in Mathematics Education*, 7, pp.339-360.

Pizlo, Z. (2008). *3D shape: its unique place in visual perception*. MIT Press.

Pizlo, Z. and Salach-Golyska, M. (1995). 3-D shape perception. *Perception & Psychophysics*, 57, pp.692-714.

Resnick, M. and Rosenbaum, E. (2013). 'Designing for tinkerability', in *Design, make, play*. Routledge, pp.163-181.

Rho, H., Lee, H.S., Kim, Y.H., Lee, K.H. and Chang, W.H. (2017). Therapeutic potential of 3D printing pen in stroke rehabilitation. *Brain & Neurorehabilitation*, 10(2), pp.1-6.

Schaefer, L., Downey, C.A. and Clandinin, D.J. (2014). Shifting from Stories to Live By to

Stories to Leave By: Early career teacher attrition. *Teacher Education Quarterly,* 41(1), pp.9-27.

Sfard, A. and Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, 34(4), pp.14-22.

Sheridan, K., Halverson, E., Litts, B., Brahms, L., Jacobs-Priebe, L. and Owens, T. (2014). Learning in the making: A comparative case study of three makerspaces. *Harvard Educational Review,* 84(4), pp.505–531.

Singh-Pillay, A. and Naidoo, J. (2020). Context matters: Science, technology and mathematics education lecturers' reflections on online teaching and learning during the COVID-19 pandemic. *Journal of Baltic Science Education,* 19(n6A), pp.1125-1136.

Solomon, Y., Eriksen, E., Smestad, B., Rodal, C. and Bjerke, A.H. (2017). Prospective teachers navigating intersecting communities of practice: Early school placement. *Journal of Mathematics Teacher Education*, 20, pp.141-158.

Sun, Y. (2023). Action-based embodied design: Spatial-mathematical learning experiences with Tinkercad 3D modeling for elementary students. *Digital Experiences in Mathematics Education*, 9(3), pp.492-507.

Taguchi, H.L. (2012). A diffractive and deleuzian approach to analysing interview data. *Feminist Theory*, 13(3), pp.265-281.

Takahashi, H. and Kim, J. (2019, May). 3D pen+ 3D printer: Exploring the role of humans and fabrication machines in creative making. In *Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems*, pp.1-12.

Toombs, A., Bardzell, S. and Bardzell, J. (2014). Becoming makers: Hackerspace member habits, values, and identities. *Journal of Peer Production*, 5(2014), pp.1-8.

Ulbrich, E., Andjic, B. and Lavicza, Z. (2023, February). 'Possibilities for STEAM teachers using 3D modelling and 3D printing', in *Learning mathematics in the context of 3D printing: proceedings of the international symposium on 3D printing in mathematics education*. Wiesbaden: Springer Fachmedien Wiesbaden, pp.163-185.

Van der Tuin, I. (2018). 'Diffraction', in Braidotti, R. and Hlavajova, M. (eds.) *Posthuman Glossary*. London: Bloomsbury Academic, pp.99–101.

Watson, C. (2006). Narratives of practice and the construction of identity in teaching. *Teachers and Teaching*, 12(5), pp.509–526. doi: 10.1080/13540600600832213.

Watson, T.J. (2008). Managing identity: Identity work, personal predicaments and structural circumstances. *Organization*, 15(1), pp.121-143.

Wenger, E. (1998). *Communities of practice: learning, meaning, and identity*. Cambridge: Cambridge University Press.

Wenger, E., McDermott, R. and Snyder, W.M. (2002). Seven principles for cultivating communities of practice. *Cultivating Communities of Practice: a guide to managing knowledge*, 4, pp.1-19.

Wenger-Trayner, E. and Wenger-Trayner, B. (2015). Introduction to communities of practice: a brief overview of the concept and its uses. Available at: https://www.wenger-trayner.com/introduction-to-communities-of-practice. (Accessed: 10 October 2023).

Wilkinson, K. and Petrich, M. (2014). *The Art of tinkering: meet 150+ makers working at the intersection of art, science & technology*. Weldon Owen International.

Willett, R. (2016). Making, makers, and makerspaces: A discourse analysis of professional journal articles and blog posts about makerspaces in public libraries. *The Library Quarterly*, 86(3), pp.313-329.

Chapter Eight: Appendices

APPENDIX I

Matthew Rudy Meangru
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Utilising 3D Printing Technology Tools and Learning-as-Making Activities with In-service Teachers to Explore and Impact their Mathematical Identities and Mathematising

PARTICIPANT INFORMATION STATEMENT

(1) What is this study about?

The purpose of this study is to employ 3D technology tools (3D pens, 3D modelling software, and 3D printing) and mathematical modelling activities to explore and impact prospective teachers' mathematical affect, identities, and competence through a Communities of Practice (CoP) lens. The study begins with teachers, working on a 3D technology and mathematical modelling activity to design, play, and engage with mathematical models in three dimensions**.** The design of this and all research activities is based on key stages 1 and 2 mathematics content from the English National Curriculum. The activities will provide an opportunity for trainees to recall this content and engage with it in new ways.

You are invited to participate in this research study because you are part of the Angles Math Hub network. This participant information statement gives you an overview of the research. Please read this sheet carefully and ask questions about anything that you don't understand or want to know more about. Participation in this research study is voluntary. By giving consent to take part in this study, you are telling us that you:

- ✓ Understand what you have read.
- ✓ Agree to take part in the research study as outlined below.
- ✓ Agree to the use of your personal information as described.
- ✓ Have received your own copy of this Participant Information Statement.

(2) Who is running the study?
The study is being carried out by Matthew Rudy Meangru, who is conducting research for his doctoral thesis as a part of the Doctor of Philosophy (PhD) degree at the University of East Anglia (UEA). This study will take place under the supervision of Professor Elena Nardi at UEA.

(3) What will the study involve for me?
Your participation to the workshop involves engagement with 3D technology tools and mathematical modelling activities at your home institution. These activities include: (1) Using 3D pens to design, play and engage with 3D mathematical models (e.g. cubes or spheres). (2) Creating designs of 3D mathematical models on Tinkercad (i.e. software for 3D modelling). (3) Learning about a 3D printer's functionalities and printing a 3D mathematical model. The workshop will be audio and video recorded. You will be able to review the transcript of your workshop if you wish to ensure they are an accurate reflection of our discussion. Also, responding to the questionnaire form is optional.

(4) How much of my time will the study take?
The workshop involving the 3D technology tools and mathematical modelling activities will require one meeting, which will last two hours. The workshop is intended to take place at your home institution outside the teaching sessions. All of this will add up to approximately two hours.

(5) Do I have to be in the study? Can I withdraw from the study once I've started?
Being in this study is completely voluntary and you do not have to take part. Your decision whether to participate will not affect your current or future relationship with the researchers or anyone else at the University of East Anglia or other universities in the UK. Also, your decision whether to participate will not affect your marks or your learning experience in

the course you attend (e.g., Media, Culture and Learning). I reassure you the analysis of the data will take place after the completion of the assessment process of the module and the submission of your grades. If you decide to take part in the study and then change your mind later, you are free to withdraw at any time. You can do this by letting me know by email (m.meangru@uea.ac.uk). You are free to stop your workshop observation including video and audio recording at any time as well. Unless you say that you want me to keep them, both the audio and video portions of any workshop/observation recordings will be erased and the information you have provided will not be included in the study results. You may also refuse to answer any questions that you do not wish to answer during the workshop. If you decide at a later time to withdraw from the study, your information will be removed from the records and will not be included in any results. The only point at which you cannot extract yourself and your data (all anonymized) from the study is after I have already analysed and published the results.

(6) Are there any risks or costs associated with being in the study?
Beyond your time, there is no risk associated with this study.

(7) Are there any benefits associated with being in the study?
I would hope that by talking about your experiences in the study, it will allow you to reflect on those aspects of the study that have helped your journey in mathematics teaching and learning, as well as those areas that might need additional support. The benefit of being in this study is that you will be able to discover new learning instruments, such as 3D pens, 3D modelling software and 3D printing, in mathematical modelling activities that can be utilized in your classroom as primary school teacher in the future. Another potential benefit is that you might experience a positive change in your feelings about mathematics and, as a result, your engagement with, and enjoyment in, teaching mathematics!

(8) What will happen to information about me that is collected during the study?
By providing your consent, you are agreeing to have personal information collected by me (Matthew Rudy Meangru) for the purposes of this research study. Your information will only be used for the purposes outlined in this Participant Information Statement. Data management will follow the 2018 General Data Protection Regulation Act and the University of East Anglia Research Data Management Policy (2019). Your information will be stored securely, and your identity/information will only be disclosed with your permission, except as required by law. Study findings may be published, but you will not be identified in these publications. In this instance, data will be stored for a period of 10 years and then destroyed.

(9) What if I would like further information about the study?
When you have read this information, I will be available to discuss it with you further and answer any questions you may have. You can contact me through email at m.meangru@uea.ac.uk.

(10) Will I be told the results of the study?
You have a right to receive feedback about the overall results of this study. You can tell me that you wish to receive feedback by providing a contact detail on the consent section of this information sheet. This feedback will be in the form of a one-page lay summary of the findings. You will receive this feedback after the study is finished.

(11) What if I have a complaint or any concerns about the study?
The ethical aspects of this study have been approved under the regulations of the University of East Anglia's School of Education and Lifelong Learning Research Ethics Committee.

If there is a problem, please let me know. You can contact me via the University at the following address:
Matthew Rudy Meangru
School of Education and Lifelong Learning
University of East Anglia
NORWICH NR4 7TJ
m.meangru@uea.ac.uk

If you would like to speak to someone else on the project team, please contact my supervisor:
Professor Elena Nardi
School of Education and Lifelong Learning
University of East Anglia
NORWICH NR4 7TJ
Tel: +44 (0)1603 592631
E.Nardi@uea.ac.uk

If you are concerned about the way this study is being conducted or you wish to make a complaint to someone independent from the study, please contact the Head of the School of Education and Lifelong Learning, Professor Yann Lebeau at y.lebeau@uea.ac.uk.

(12) OK, I want to take part – what do I do next?

You will need to fill out one copy of the consent form (attached). Once completed, you have the option to send it to me through email or arrange a time and place for me to pick it up in person. Due to COVID-19, it may not be possible to meet in person. In this case, please email the consent form to my email address (m.meangru@uea.ac.uk). Please keep the letter, this information sheet, and the 2nd copy of the consent form for your information.

This information sheet is for you to keep.

APPENDIX II

Consent form for six in-service teachers

PARTICIPANT CONSENT FORM (1st Copy to Researcher)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researchers or anyone else at the University of East Anglia now or in the future.

✓ I understand that I can withdraw from the study at any time.

✓ I understand that I may stop the interview at any time if I do not wish to continue, and that unless I indicate otherwise any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.

✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my name or any identifiable information about me.

- **Audio-recording of activity** YES □ NO □
- **Video-recording of activity** YES □ NO □
- **Reviewing activity transcripts** YES □ NO □
- **Reviewing Questionnaire Responses** YES □ NO □
- **Would you like to receive feedback about the overall results of this study?** YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

…………………………………………………………… …………………………………………………………… …………………………

Signature **PRINT name** **Date**

PARTICIPANT CONSENT FORM (2nd Copy to Researcher)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researchers or anyone else at the University of East Anglia now or in the future.

✓ I understand that I can withdraw from the study at any time.

✓ I understand that I may stop the interview at any time if I do not wish to continue, and that unless I indicate otherwise any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.

✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my name or any identifiable information about me.

- **Audio-recording of activity** YES □ NO □
- **Video-recording of activity** YES □ NO □
- **Reviewing activity transcripts** YES □ NO □
- **Reviewing Questionnaire Responses** YES □ NO □
- **Would you like to receive feedback about the overall results of this study?** YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

……………………………………………………………… ……………………… ………………………………………… …………

Signature **PRINT name** **Date**

APPENDIX III

Consent form for Calculus for Business Students

PARTICIPANT CONSENT FORM (1st Copy to Researcher)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researchers or anyone else at the Northeastern University-London and University of East Anglia now or in the future.

✓ I understand that data analysis will take place after the completion of the assessment process of the module Calculus for Business of my grades. My decision whether to participate will not affect my marks or my learning experience.

✓ I understand that I can withdraw from the study at any time.

✓ I understand that I may stop the interview at any time if I do not wish to continue, and that unless I indicate otherwise any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.

✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my name or any identifiable information about me.

I consent to:

- **Audio-recording of activity** YES □ NO □
- **Video-recording of activity** YES □ NO □

- **Reviewing activity transcripts** YES □ NO □
- **Would you like to receive feedback about the overall results of this study?** YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

…………………………………………………………… …………………………………………………………… ……………………….

Signature **PRINT name** **Date**

PARTICIPANT CONSENT FORM (2nd Copy to Participant)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researchers or anyone else at the Northeastern University-London and University of East Anglia now or in the future.

✓ I understand that data analysis will take place after the completion of the assessment process of the module Calculus for Business of my grades. My decision whether to participate will not affect my marks or my learning experience.

✓ I understand that I can withdraw from the study at any time.

✓ I understand that I may stop the interview at any time if I do not wish to continue, and that unless I indicate otherwise any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.

✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my name or any identifiable information about me.

I consent to:

- **Audio-recording of activity** YES □ NO □
- **Video-recording of activity** YES □ NO □
- **Reviewing activity transcripts** YES □ NO □
- **Would you like to receive feedback about the overall results of this study?**
  YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

....

Signature **PRINT name** **Date**

APPENDIX IV

Consent form for the Teacher Trainees

PARTICIPANT CONSENT FORM (1st Copy to Researcher)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researchers or anyone else at the University of East Anglia now or in the future.

✓ I understand that I can withdraw from the study at any time.

✓ I understand that I may stop the interview at any time if I do not wish to continue, and that unless I indicate otherwise any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.

✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my name or any identifiable information about me.

I consent to:

- **Audio-recording of workshop** YES □ NO □
- **Video-recording of workshop** YES □ NO □
- **Photography** YES □ NO □
- **Reviewing workshop transcripts** YES □ NO □
- **Reviewing Questionnaire responses** YES □ NO □

- **Would you like to receive feedback about the overall results of this study?**

YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

……………………………………………………………… ……………………………………………………………….. ………………………..

Signature **PRINT name** **Date**

PARTICIPANT CONSENT FORM (2nd Copy to Participant)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researchers or anyone else at the University of East Anglia now or in the future.

✓ I understand that I can withdraw from the study at any time.

✓ I understand that I may stop the interview at any time if I do not wish to continue, and that unless I indicate otherwise any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.

✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my name or any identifiable information about me.

I consent to:

- **Audio-recording of workshop** YES □ NO □
- **Video-recording of workshop** YES □ NO □
- **Photography** YES □ NO □
- **Reviewing workshop transcripts** YES □ NO □
- **Reviewing Questionnaire responses** YES □ NO □
- **Would you like to receive feedback about the overall results of this study?**
  YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

....

Signature **PRINT name** **Date**

APPENDIX V

PARENT INFORMATION STATEMENT (Opt-in form for parents)

Matthew Rudy Meangru
PhD Researcher
Date: TBD

Faculty of Social Sciences
School of Education

University of East Anglia
Norwich Research Park
Norwich NR4 7TJ
United Kingdom

Email: m.meangru@uea.ac.uk

Utilising 3D Printing Technology Tools and Learning-as-Making Activities with In-service Teachers to Explore and Impact Teaching Identities Towards Mathematics and Mathematising

PARENT INFORMATION STATEMENT

(1) What is this study about?

The purpose of this study is to employ 3D technology tools (3D pens, 3D modelling software, and 3D printing) and mathematical modelling activities to explore and impact prospective teachers' mathematical affect, identities, and competence through a Communities of Practice (CoP) lens. The study begins with teachers working on a 3D technology and mathematical modelling activity to design, play, and engage with mathematical models in three dimensions. Afterwards, teachers will carry out the same activities with their current students to see how they engage with the 3D technology and mathematical modelling activity. The design of this and all research activities is based on key stages 1 and 2 mathematics content from the English National Curriculum. The activities will provide an opportunity for trainees to recall this content and engage with it in new ways.

Your child's teacher is participating in this research study and will use 3D technology tools and mathematical modelling activities with their students. Please read this information statement carefully and ask questions about anything that you don't understand or want to know more about. Participation in this research study is voluntary. By giving consent to take part in this study, you are telling us that you:

- ✓ Understand what you have read.
- ✓ Agree for your child to take part in the research study as outlined below.
- ✓ Agree to the use of your child's personal information as described.
- ✓ Have received a copy of this Parent Information Statement.

(2) Who is running the study?

The study is being carried out by Matthew Rudy Meangru, who is conducting research for his doctoral thesis as a part of the Doctor of Philosophy (PhD) degree at the University of East Anglia (UEA). This study will take place under the supervision of Professor Elena Nardi at UEA.

(3) What will the study involve for my child?

Your child's participation includes engaging in 3D printing technology and mathematical modelling activities for only one classroom meeting. I will be present during this class meeting, and the teacher will run their class as normal. The mathematical modelling activities will include: (1) using 3D pens to design, play and engage with 3D mathematical models (e.g. cubes or spheres); (2) creating designs of 3D mathematical models on Tinkercad, a software for 3D modelling; and (3) learning about a 3D printer's functionalities as well as printing a 3D mathematical model. You will be able to review a transcript of the classroom observation if you wish to ensure it is an accurate reflection of our discussion.

[If the teacher agreed to audio-recording] With your permission I would also like to audio-record the lesson so that I can recall specific dialogues between the teacher and the students if necessary. My notes will only pertain to how the teacher engages with your child and I will not focus on your child's engagement with the 3D technology. If you do not wish for your child to be audio-recorded and you prefer a video recording instead, please indicate this on the consent form. I will consider your preference and if necessary, I will only use video recording. If you do not want your child to be recorded as part of the classroom observation, I will move them to a position in the classroom where they will not be recorded.

[If the teacher agreed to video-recording] With your permission I would also like to video-record the lesson so that I can recall specific dialogues between the teacher and the students if necessary. My notes will only pertain to how the teacher engages with your child and I will not focus on your child's engagement with the 3D technology. If you do not wish for your child to be video-recorded and you prefer an audio recording instead, please indicate this on the consent form. I will consider your preference and if necessary, I will only use audio-recording. If you do not want your child to be recorded as part of the classroom observation, I will move them to a position in the classroom where they will not be recorded.

(4) How much of my child's time will the study take?
This study involves one of your child's class times and will be part of their normal everyday activities. As such, no additional time is required.

(5) Does my child have to be in the study? Can they withdraw from the study once they started?
Being in this study is entirely voluntary, and your child does not have to participate. Your child's participation will not affect their current or future relationship with the researchers or anyone else at the University of East Anglia, your current school, or classroom teacher. If you want your child to participate in the study and then change your mind later, your child is free to withdraw at any time. You can do this by letting me know by email (m.meangru@uea.ac.uk). Your child is free to stop classroom observation, including video and audio recording, at any time. Unless your child says that you want me to keep them in the classroom, both the audio and video portions of any observation recordings will be erased, and the information your child has provided will not be included in the study results. Your child may also refuse to answer any questions he or she does not wish to answer during the observation. If your child decides to withdraw from the study later, their information will be removed from the records and will not be included in any results.

(6) Are there any risks or costs associated with my child being in the study?
Your child will be engaging with 3D technology tools such as 3D pens and a 3D printer that could potentially be harmful if the child is unattended. During this study, your child will be monitored by the classroom teacher and myself to ensure they are using the 3D pen safely. The 3D printer will be placed a safe distance away from the students in the classroom, such that your child will be able to see an object being printed but not touch the actual printer. There is no harm possible when using the 3D modelling software Tinkercad, as this part of the activity is done using a computer. Lastly, there is no cost for taking part in this study and your child's time will not be affected as this study will take place during regular class time.

(7) Are there any benefits associated with my child being in the study?
By allowing your child to participate in the study, they will play, engage, and design using 3D technology instruments such as 3D pens, 3D modelling software and a 3D printer in mathematical modelling activities. The experience may potentially impact the way students learn mathematics and the way teachers teach their students' mathematics. The potential interaction between your child and the classroom teacher could allow the teacher to reflect on aspects of the study that have helped their journey in mathematics teaching and learning.

(8) What will happen to information about me that is collected during the study?
By providing your consent, you are agreeing to have personal information collected by the researcher (Matthew Rudy Meangru) for the purposes of this research study. Your child's information will only be used for the purposes outlined in this Participant Information Statement. Information about your child will not be revealed in the write-up of research as I am only focusing on how the teacher engages and interacts with their students in the classroom.

Data management will follow the 2018 General Data Protection Regulation Act and the University of East Anglia Research Data Management Policy (2019). Your information will be stored securely, and your identity/information will only be disclosed with your permission, except as required by law. Study findings may be published, but your child will not be identified in these publications. Data will be stored for a period of 10 years and then destroyed.

(9) What if we would like further information about the study?
When you have read this information, I will be available to discuss it with you further and answer any questions you may have. You can contact me through email at m.meangru@uea.ac.uk.

(10) Will I be told the results of the study?

You and your child have the right to receive feedback about the overall results of this study. You can tell me that you wish to receive feedback by providing contact information on the consent section of this information sheet. This feedback will be in the form of a one-page lay summary of the findings. You will receive this feedback after the study is finished.

(11) What if we have a complaint or any concerns about the study?

The ethical aspects of this study have been approved under the regulations of the University of East Anglia's School of Education and Lifelong Learning Research Ethics Committee.

If there is a problem, please let me know. You can contact me via the University at the following address:
Matthew Rudy Meangru
School of Education and Lifelong Learning

University of East Anglia
NORWICH NR4 7TJ
m.meangru@uea.ac.uk

If you would like to speak to someone else on the project team, please contact my supervisor:

Professor Elena Nardi
School of Education and Lifelong Learning
University of East Anglia
NORWICH NR4 7TJ
Tel: +44 (0)1603 592631
E.Nardi@uea.ac.uk

If you are concerned about the way this study is being conducted or you wish to make a complaint to someone independent from the study, please contact the Head of the School of Education and Lifelong Learning, Professor Yann Lebeau at y.lebeau@uea.ac.uk.

(12) OK, I'm happy for my child to take part– what do I do next?

You will need to fill out one copy of the consent form (attached) and ask your child to return this to [name of the teacher] by [date]. Please keep the letter, this information sheet, and the 2nd copy of the consent form for your information.

This information sheet is for you to keep.

APPENDIX VI

PARENT/CARETAKER CONSENT FORM (1st Copy to Researcher)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what my child will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my child's involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and my child does not have to take part. My decision whether to let my child be in the study will not affect our relationship with the researchers or anyone else at the University of East Anglia, the classroom teacher, or our primary/secondary school, now or in the future.

✓ I understand that my child can withdraw from the study at any time.

✓ I understand that my child may stop participating in an observation at any time if they do not wish to continue.

✓ I understand that personal information about my child that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about my child will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my child's name or any identifiable information about me.

I consent to:

- **Audio recording of my child**'s **classroom** YES □ NO □
- **Video recording of my child's classroom** YES □ NO □
- **Photography of my child's engagement with the classroom teacher** YES □ NO □
- **Would you like to receive feedback about the overall results of this study?**
  YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

....
Signature **PRINT name** **Date**

PARENT/CARETAKER CONSENT FORM (2nd Copy to Parent/Caretaker)

I, ... [PRINT NAME], agree to take part in this research study.

In giving my consent I state that:

✓ I understand the purpose of the study, what my child will be asked to do, and any risks/benefits involved.

✓ I have read the Participant Information Statement and have been able to discuss my child's involvement in the study with the researchers if I wished to do so.

✓ The researchers have answered any questions that I had about the study, and I am happy with the answers.

✓ I understand that being in this study is completely voluntary and my child does not have to take part. My decision whether to let my child be in the study will not affect our relationship with the researchers or anyone else at the University of East Anglia, the classroom teacher, or our primary/secondary school, now or in the future.

✓ I understand that my child can withdraw from the study at any time.

✓ I understand that my child may stop participating in an observation at any time if they do not wish to continue.

✓ I understand that personal information about my child that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about my child will only be told to others with my permission, except as required by law.

✓ I understand that the results of this study may be published, but these publications will not contain my child's name or any identifiable information about me.

I consent to:

- **Audio recording of my child's classroom** YES □ NO □
- **Video recording of my child's classroom** YES □ NO □
- **Photography of my child's engagement with the classroom teacher** YES □ NO □
- **Would you like to receive feedback about the overall results of this study?**
  YES □ NO □

If you answered **YES**, please indicate your preferred form of feedback and address:

□ Postal: ____

□ Email: ____

....
Signature **PRINT name** **Date**

APPENDIX VII

PARTICIPANT INFORMATION STATEMENT (Students of the Classroom Teacher)

Matthew Rudy Meangru
PhD Researcher
Date: TBD

Faculty of Social Sciences
School of Education

University of East Anglia
Norwich Research Park
Norwich NR4 7TJ
United Kingdom

Email: m.meangru@uea.ac.uk

Utilising 3D Printing Technology Tools and Learning-as-Making Activities with In-service Teachers to Explore and Impact Teaching Identities Towards Mathematics and Mathematising

STUDY INFORMATION SHEET FOR CLASSROOM OBSERVATION

Hiya! My name is Matthew Meangru and I am conducting a research study to find out ways your teachers can employ 3D technologies tools such as 3D pens, 3D modelling software, and 3D printer in mathematical classroom activities. Your teacher's participation allows me to explore how 3D technology tools can potentially impact the way we teach and learn mathematics. Part of my study requires that I conduct classroom observation of your teacher's interaction with you as you design, play, and engage with this cool 3D technology.

I would like to invite you to be a part of the classroom observation because you are a student in the class.

You can decide if you would like to be a part of my classroom observation or not. You don't have to – it is up to you.

This study information sheet tells you what I will ask you to do, if you do decide to take part in this classroom observation. Please read it carefully so that you can make the decision that is best for you!

If you decide you want to be in the classroom observation and then you change your mind later, that is ok. All you need to do is let me or your classroom teacher know.

If you have any questions, you can ask me, your classroom teacher, or your caretaker. If you want to, please feel free to call me at **[Private Telephone number was shown].**

(1) What will happen if I say that I want to be part of the classroom observation?

If you decide that you want to be part of the classroom observation, I will ask you to:

- Allow me to come and observe your classroom for one session and make notes of your classroom teacher's interaction and engagement with you and fellow classmates. The notes will include things you discuss with your teacher during the activities. If you agree, I may also want to take some video of your lesson so I can look at them later on to remind myself of you and your teacher's interactions. [if the classroom teacher does not agree to video recording, this sentence will be removed].

- Be yourself and do what you normally do in your classroom.

[Insert-if applicable to the study]

[**Audio-Recording**]: If you say ‘Yes’, I will record your discussion with your teacher with an audio recorder.

[**Video-Recording**]: If you say ‘Yes’, I will video-record you in the classroom.

(2) Will anyone else know what I say in the classroom observation?

I won’t tell anyone else what you discuss with your teacher. The one exception to this is if you talk about harming yourself or others. Then we might need to tell someone to keep you and others safe.

All of the information that I have about you from the classroom observation will be stored in a safe place. I will write a report about the classroom observation which will only be reviewed by your teacher and my supervisory team. I will not use your name in the report, and no one will know that you were in the classroom observation and a part of this study.

(3) How long will this study take?

This study will take one classroom visit on [**date to be determine**]. Your classroom teacher and I will be present in the room.

(4) Are there any good things about being in the classroom observation?

You will not receive anything for taking part in this classroom observation. However, you will get to design, play, and engage with cool 3D technology tools. Also, your participation will greatly help my study.

(5) Are there any bad things about being in the classroom observation?

This study will take place during your regular classroom time. It will not cost you any money or time.

(6) Will you tell me what you learnt in the classroom observation in the end?

No. This classroom observation only pertains to the teacher’s engagement and conversation with you. It is about their performance using 3D technology in the classroom. For security reasons, this information is only shared between the teacher and researcher.

(7) What if I am not happy with the study or the people conducting the classroom observation?

If you are not happy with how I am conducting the classroom observation, then you or your parents or caretaker can:

- Call the university at +44 01603 59 1451
- Write an email to e.nardi@uea.ac.uk

(8) Ok, I would like to take part in this classroom observation – what do I need to do next?

Please fill in the two forms on the two pages included. Give the first one back to your teacher at your next class; I will get the form from your classroom teacher. You can keep this sheet and the second form to remind you about the classroom observation.

This sheet is for you to keep.

APPENDIX VIII

Student of the Classroom Teacher Consent Form

STUDENT CONSENT FORM (1st Copy to Researcher)

If you are happy to be in the study, please

- **write** your **name** in the space below
- **sign** your **name** at the bottom of the next page
- put the **date** at the bottom of the next page

You should only say 'yes' to being in the study if you know what it is about, and you want to be in it. If you don't want to be in the study, don't sign the form.

I, ... [PRINT NAME], am happy to take part in this research study.

In saying yes to being in the study, I am aware that:

✓ I know what the study is about.

✓ I know what I will be asked to do.

✓ My parent/caretaker or teacher talked to me about the study.

✓ My questions about the study have been answered.

✓ I know that I don't have to be in the study if I don't want to.

✓ I know that I can pull out of the study at any time if I don't want to do it anymore.

✓ I know that I don't have to answer any questions that I don't want to answer.

✓ I know that the researchers won't tell anyone what I say when we talk to each other, unless I talk about being hurt by someone or hurting myself or someone else.

Now we are going to ask you if you are happy to do a few other things in the study. Please circle

'Yes' or 'No' to tell me what you would like.

- **Are you happy for me to audio record your voice while you are working in the classroom?**
  YES NO

- **Are you happy for me to video record your actions in the classroom?**
  YES NO

- **Are you happy for me to take a photograph of you in the classroom?**
  YES NO

- **Are you happy for me to observe you in the classroom?**
  YES NO

- **Would you like me to tell you what I learnt in the study?**
  YES NO

....	....	....
Signature	**PRINT name**	**Date**

STUDENT CONSENT FORM (2nd Copy to Student)

If you are happy to be in the study, please

- **write** your **name** in the space below
- **sign** your **name** at the bottom of the next page
- put the **date** at the bottom of the next page

You should only say 'yes' to being in the study if you know what it is about, and you want to be in it. If you don't want to be in the study, don't sign the form.

I, ... [PRINT NAME], am happy to take part in this research study.

In saying yes to being in the study, I am aware that:

✓ I know what the study is about.

✓ I know what I will be asked to do.

✓ My parent/caretaker or teacher talked to me about the study.

✓ My questions about the study have been answered.

✓ I know that I don't have to be in the study if I don't want to.

✓ I know that I can pull out of the study at any time if I don't want to do it anymore.

✓ I know that I don't have to answer any questions that I don't want to answer.

✓ I know that the researchers won't tell anyone what I say when we talk to each other, unless I talk about being hurt by someone or hurting myself or someone else.

Now we are going to ask you if you are happy to do a few other things in the study. Please circle

'Yes' or 'No' to tell me what you would like.

- **Are you happy for me to audio record your voice while you are working in the classroom?**
  YES NO

- **Are you happy for me to video record your actions in the classroom?**
  YES NO

- **Are you happy for me to take a photograph of you in the classroom?**
  YES NO

- **Are you happy for me to observe you in the classroom?**
  YES NO

- **Would you like me to tell you what I learnt in the study?**
  YES NO

....

Signature **PRINT name** **Date**