Online Maximum Torque per Ampere Control for Doubly-Fed Induction Machines

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***Abstract*— This paper introduces a control approach to optimize the ratio of the torque to the total input current in windings of the doubly fed induction machine (DFIM). Essentially, this strategy aims to share the currents more evenly between stator and rotor windings to achieve a specific torque level. Due to constraints imposed by the flux and the frame alignment, the angles of rotor and stator currents are interrelated. Consequently, a fundamental relationship is established between these angles, so the total current magnitude in terms of the rotor current angle is expressed. The optimal angle required to implement the maximum torque per total Ampere (MTPTA) control strategy is then determined using a numerical minimization process. Additionally, the maximum torque per inverter Ampere (MTPIA) strategy is proposed in this paper, which is demonstrated by minimizing the rotor current magnitude while maintaining a constant rotor current angle for a given torque level.**

Keywords—Doubly fed induction machine (DFIM), Maximum Torque per Inverter Ampere (MTPIA), Maximum Torque per Total Ampere (MTPTA), Rotor current angle.

I. Introduction

In principle, the primary goal of the maximum torque per ampere (MTPA) control approach is to produce torque while keeping the current magnitude as low as possible [1]. For electric machines that are singly fed, such as induction machines (IMs), and fully controllable, implementing MTPA strategies is straightforward, and numerous research papers have explored energy-saving techniques based on the MTPA control for singly fed machines [2]–[4]. However, when dealing with doubly fed machines (DFMs) with a partially rated converter, such as doubly fed induction machines (DFIMs), brushless doubly fed induction machines (BDFIMs), and brushless doubly-fed reluctance machines (BDFRMs), achieving the minimum total current for a given torque is challenging [5], and only limited research has been reported in the literature on implementing MTPA for DFMs.

In [6], the mathematical determination of the stator and rotor current command set is presented to minimize the copper losses in DFIMs. This calculation takes into account constraints related to stator and rotor currents and voltages, as well as the rotor flux. Additionally, a torque control method is introduced. Ademi *et al.* conducted a theoretical investigation on achieving maximum torque per inverter Ampere (MTPIA) in BDFRMs in [7]. Their study illustrates that MTPIA minimizes inverter loading while maintaining a constant torque.

An alternative control method is the maximum torque per total Ampere (MTPTA), which aims to distribute currents evenly between the rotor and stator windings to maximize torque for a given total input summation of stator and rotor currents. In [8], a search-based MTPTA control strategy is presented for a simulated BDFIM drive. This strategy involves incrementally increasing the d-axis current of the control winding (CW) until reaching the minimum current point, with continuous monitoring of the total stator current within a moving time window. Once the desired strategy is achieved, further increments in the d-axis current of CW are halted, and the BDFIM drive operates at a new operating point.

In [9], the MTPTA control strategy is adapted for the BDFIM drive by considering iron losses. The conditions for realizing MTPTA in the presence of iron losses are investigated through a theoretical approach. It is demonstrated that the proposed control strategy is achieved when the MTPTA criterion tracks zero as the reference signal. To address the low-efficiency situation of BDFIMs, an efficiency analysis method is proposed in [10]. The improved equivalent circuit, accounting for iron losses, is utilized to deduce the efficiency expression of BDFIMs. Furthermore, an optimal power winding (PW) frequency corresponding to maximum efficiency can be determined.

This paper presents a theoretical framework for implementing a model-based MTPTA strategy in the context of DFIMs. The strategy involves determining the minimum value of the total current magnitude (sum of stator and rotor currents) for a given torque, utilizing rotor current angles approximated by a suitable mathematical curve. By doing so, the proposed approach retains the advantages of a model-based control method, such as rapid response and high precision. Additionally, the paper introduces a parameter-free MTPIA control strategy aimed at minimizing the inverter rating by maintaining the rotor d-axis current at zero. Importantly, the realization criterion for the MTPIA strategy is unaffected by the torque level and frequency of DFIMs.

II. THE MODEL OF DFIMs

The mathematical model of the DFIM is written by [11]

$$\vec{V}_s = R_s\vec{I}_s + \frac{d}{dt}\vec{\lambda}_s + j\omega_s\vec{\lambda}_s \tag{1}$$

$$\vec{V}_r = R_r\vec{I}_r + \frac{d}{dt}\vec{\lambda}_r + j(\omega_r - \omega_s)\vec{\lambda}_r \tag{2}$$

$$\vec{\lambda}_s = (L_s + L_m)\vec{I}_s + L_m\vec{I}_r^* = L_{ss}\vec{I}_s + L_m\vec{I}_r^* \tag{3}$$

$$\vec{\lambda}_r = (L_r + L_m)\vec{I}_r + L_m\vec{I}_s^* = L_{rr}\vec{I}_r + L_m\vec{I}_s^* \tag{4}$$

where $\vec{V}_s$ is the stator voltage vector, $\vec{V}_r$ is the rotor voltage vector, $\vec{\lambda}_s$ is the stator flux linkage vector, $\vec{\lambda}_r$ is the rotor flux linkage vector, $\vec{I}_s$ is the stator current vector, $\vec{I}_r$ is the rotor current vector, R_s is the stator resistance, R_r is the rotor resistance, L_m is the magnetizing inductance, L_s is the stator leakage inductance, L_r is the rotor leakage inductance, L_{ss} is the stator self-inductance, L_{rr} is the rotor self-inductance, ω_s is the synchronous frequency, and ω_r is the rotor electrical frequency. In deriving the voltage equations, Eqs. (1) and (2) are referred to the reference frames rotating at ω_1 and ω_2, respectively. This interpretation of reference frames has been reported in [12].

The equation of electromagnetic torque can be expressed by

$$T_e = -\frac{3p}{4}L_m Im\left[\vec{I}_s^*\vec{I}_r^*\right] \tag{5}$$

where p is the number of poles. Eq. (5) is derived by substituting (1) and (2) into the space vector expression of the total three-phase active input power.

III. THE PROPOSED CONTROL STRATEGY

A. The MTPIA Control

The MTPIA control strategy is obtained by minimizing the rotor current magnitude under a constant torque. The torque equation can be expressed as follows:

$$T_e = \alpha\left(\lambda_{sd}i_{rq} + \lambda_{sq}i_{rd}\right) \tag{6}$$

where $\alpha = \frac{3p}{4}.\frac{L_m}{L_{ss}}$.

The stator flux orientation is achieved by aligning the d-axis of the synchronous reference frame with the stator flux vector. The resultant d and q-axis flux components are:

$$\lambda_{sd} = \left|\vec{\lambda}_s\right| \quad , \quad \lambda_{sq} = 0 \tag{7}$$

From Fig. 1, we can see that $i_{rq} = \left|\vec{I}_r\right|sin\alpha_r$, and thus, the MTPIA is written by

$$T_e/\left|\vec{I}_r\right| = \alpha\left|\vec{\lambda}_s\right|sin\alpha_r \tag{8}$$

$T_e/\left|\vec{I}_r\right|$ will be maximized when $\alpha_r = \pi/2$, i.e., to realize the MTPIA control, the rotor's d-axis current must be equal to zero. This criterion leads to a minimum inverter rating.

B. The MTPTA Control

A desirable control strategy for DFIMs is the maximum torque per total Amperes of the machine. In this strategy, it is necessary to derive an expression for the total summation current magnitude of the stator and rotor currents based on the stator and the rotor current angles. The basic expression of electromagnetic torque in (5) can be manipulated into a variety of forms. Another form of the torque expression is in terms of the two-axis components of the stator's flux and current as follows:

$$T_e = \beta\left(\lambda_{sd}i_{sq} - \lambda_{sq}i_{sd}\right) \tag{9}$$

where $\beta = \frac{3p}{4}$.

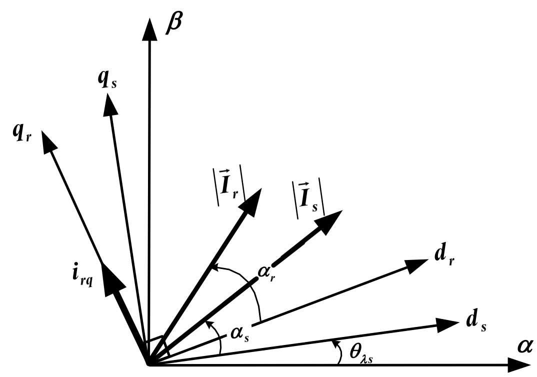

Fig. 1. The current vectors used in equations and their inter-relationships.

In the field-oriented control (FOC) scheme, the stator flux orientation has been selected, and Eqs. (6) and (9) can be rewritten as follows:

$$T_e = \alpha\left(\left|\vec{\lambda}_s\right|i_{rq}\right) \tag{10}$$

$$T_e = \beta\left(\left|\vec{\lambda}_s\right|i_{sq}\right) \tag{11}$$

Therefore, the torque can be controlled by the q-axis component of the rotor and stator currents, respectively. From Fig. 1, we have $i_{sq} = \left|\vec{I}_s\right|sin\alpha_s$ and $i_{rq} = \left|\vec{I}_r\right|sin\alpha_r$. Therefore, Eqs. (10) and (11) can be rewritten as follows:

$$T_e = \alpha\left|\vec{\lambda}_s\right|\left|\vec{I}_r\right|sin\alpha_r \tag{12}$$

$$T_e = \beta\left|\vec{\lambda}_s\right|\left|\vec{I}_s\right|sin\alpha_s \tag{13}$$

Substituting $\vec{I}_s = \left|\vec{I}_s\right|e^{j\alpha_s}$ and $\vec{I}_r = \left|\vec{I}_r\right|e^{j\alpha_r}$ into (5), and after a few manipulations, we can derive

$$T_e = \gamma\left|\vec{I}_s\right|\left|\vec{I}_r\right|sin(\alpha_s + \alpha_r) \tag{14}$$

where $\gamma = \beta L_m$.

Comparing (14) with (12), and (14) with (13), Eqs. (15) and (16) for the stator and rotor current magnitudes are obtained, respectively.

$$\left|\vec{I}_s\right| = \frac{\left|\vec{\lambda}_s\right|sin\alpha_r}{L_{ss}sin(\alpha_s + \alpha_r)} \tag{15}$$

$$|\vec{I}_r| = \frac{|\vec{\lambda}_s| sin\alpha_s}{L_m sin(\alpha_s + \alpha_r)} \tag{16}$$

To realize the MTPTA control, the sum of the stator and rotor current magnitudes, $|\vec{I}_T|$, should be minimized for a given torque. Considering (15) and (16), the total current magnitude of the stator and rotor currents is defined by

$$|\vec{I}_T| = |\vec{I}_s| + |\vec{I}_r| \tag{17}$$

The current angles are not independent due to the flux and the frame alignment conditions. To minimize (17), the relationship between α_s and α_r must be derived. In this regard, by (3) and (11), we have

$$\frac{i_{rq}}{i_{rd}} = tan\alpha_r = \frac{T_e L_{ss}}{\beta|\vec{\lambda}_s|\left(|\vec{\lambda}_s| - L_{ss} i_{sd}\right)} \tag{18}$$

which can be simplified to obtain

$$i_{sd} = \frac{\beta|\vec{\lambda}_s|^2 tan\alpha_r - T_e L_{ss}}{\beta|\vec{\lambda}_s| L_{ss} tan\alpha_r} \tag{19}$$

Likewise, from (11) and (19), we have

$$\frac{i_{sq}}{i_{sd}} = tan\theta_s = \frac{T_e L_{ss} tan\alpha_r}{\beta|\vec{\lambda}_s|^2 tan\alpha_r - T_e L_{ss}} \tag{20}$$

Eq. (20) is the fundamental relationship between the current angles, which allows us to convert the expression of $|\vec{I}_T|$ as follows:

$$|\vec{I}_T| = \frac{\dfrac{|\vec{\lambda}_s| L_m tan\alpha_r}{cos\alpha_s} + \dfrac{|\vec{\lambda}_s| L_{ss} tan\alpha_s}{cos\alpha_r}}{L_m L_{ss}(tan\alpha_s + tan\alpha_r)} \tag{21}$$

By knowing $cos\alpha_i = 1/\left(\sqrt{1 + tan^2\alpha_i}\right), (i = 1,2)$ and substituting (20) into (21), the expression of $|\vec{I}_T|$ in terms of α_r can be obtained as follows:

$$|\vec{I}_T| = \frac{\sqrt{\left(\beta|\vec{\lambda}_s|^2 tan\alpha_r - L_{ss}T_e\right)^2 + (L_{ss}T_e tan\alpha_r)^2}}{\beta L_{ss}|\vec{\lambda}_s| tan\alpha_r} + \frac{L_{ss}T_e\sqrt{1 + tan^2\alpha_r}}{\beta L_m|\vec{\lambda}_s| tan\alpha_r} \tag{22}$$

To find the minimum value of $|\vec{I}_T|$ for a given torque, the derivative of equation (22) with respect to $tan\alpha_r$ needs to be set to zero. However, due to the complexity of the derivative equation, analytical solutions for this problem are not available. Consequently, the optimal value of α_r is determined using a numerical minimization method, specifically by employing the "fminsearch" function in MATLAB. The output of this function provides the phase angle of the rotor current. Table I illustrates variations in the phase angle of the rotor current for different torque values. The relationship between the d- and q-axis components of the rotor current is determined accordingly.

$$tan\,\alpha_r = i_{rq}/i_{rd} \Rightarrow i_{rd}\,tan\,\alpha_r - i_{rq} = 0 \tag{23}$$

According to (23), the control strategy is realized when $i_{rd} tan\alpha_r - i_{rq}$ tracks zero as command.

TABLE I. PHASE ANGLES OF THE ROTOR CURRENT (α_R) FOR VARIOUS TORQUE VALUES UNDER THE MTPTA CONTROL STRATEGY

	Torque (pu)								
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
α_r°	28.7	32.9	34.2	36.1	39.3	42.5	46.7	49.7	52.3

IV. SIMULATION RESULTS

The performance of the proposed control strategy is validated through simulations using MATLAB/Simulink. The overall block diagram of the proposed drive system is shown in Fig. 2.

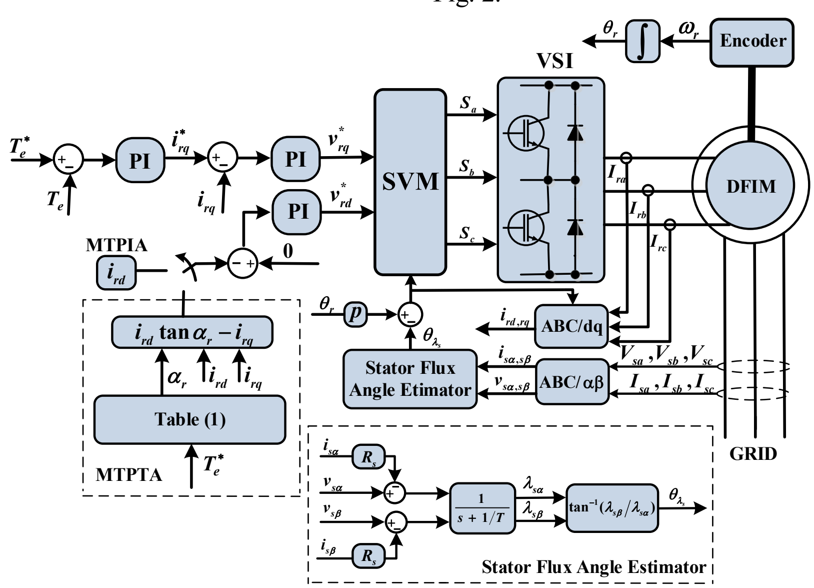

Fig. 2. The block diagram of DFIM-based drive system.

Table II shows the specifications of the DFIM used for simulations. Firstly, the proposed MTPIA control strategy is validated by Fig. 3, where the two control objectives, the torque and MTPIA strategy realization are satisfied. The rotor's d-axis current component always fluctuates around the zero, which indicates that the MTPIA control strategy is realized. The performance of the MTPTA control strategy for DFIMs is evaluated by Fig. 4(b), showing a proper tracking of the strategy realization criterion. In Fig. 4(a), the torque tracks the repeating sequence of the reference torque between 0.4 p.u. and 1 p.u. Figs. 4(c) and 4(d) show that the stator and rotor current magnitudes change with respect to the torque command.

TABLE II: DFIM PARAMETERS [13]

Parameter	Value
Rated power (kVA)	7.5
Rated frequency (Hz)	50
Rated stator voltage (v)	220
Stator resistance (Ω)	0.462
Rotor resistance (Ω)	0.473
Stator leakage inductance (mH)	3.93
Rotor leakage inductance (mH)	3.93
Magnetizing inductance (mH)	103.4
Pole pairs	2

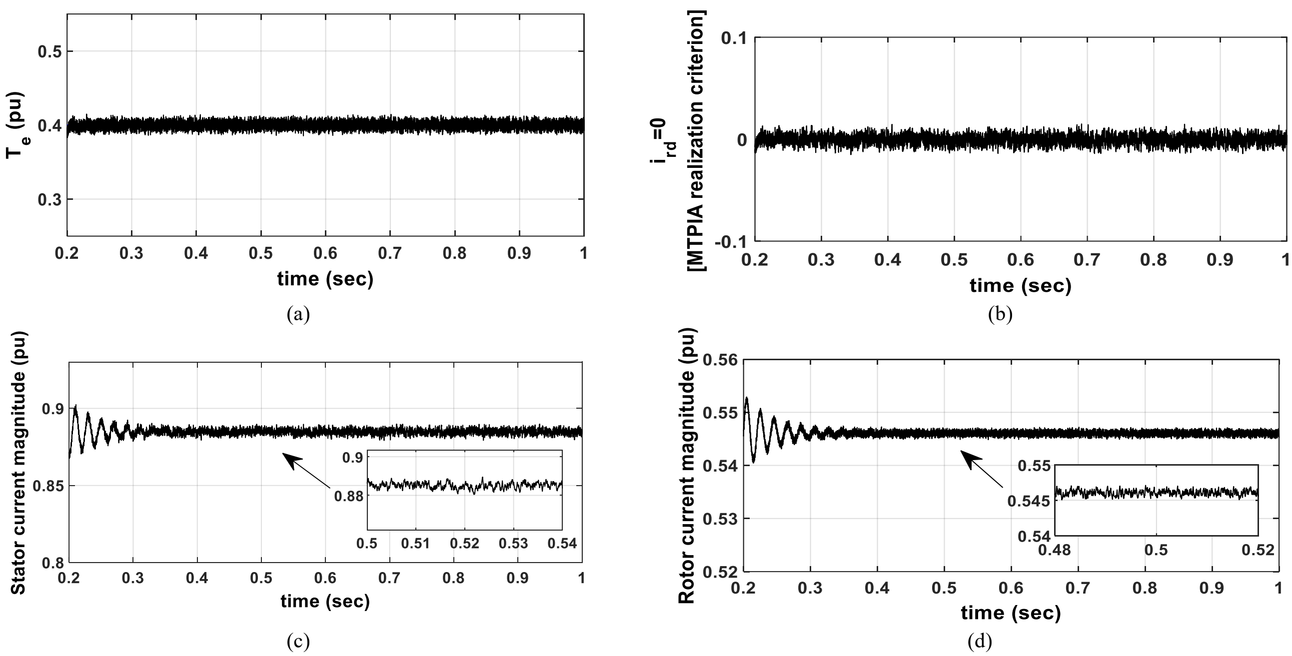

Fig. 3. Simulation results of the MTPIA control strategy (steady state)

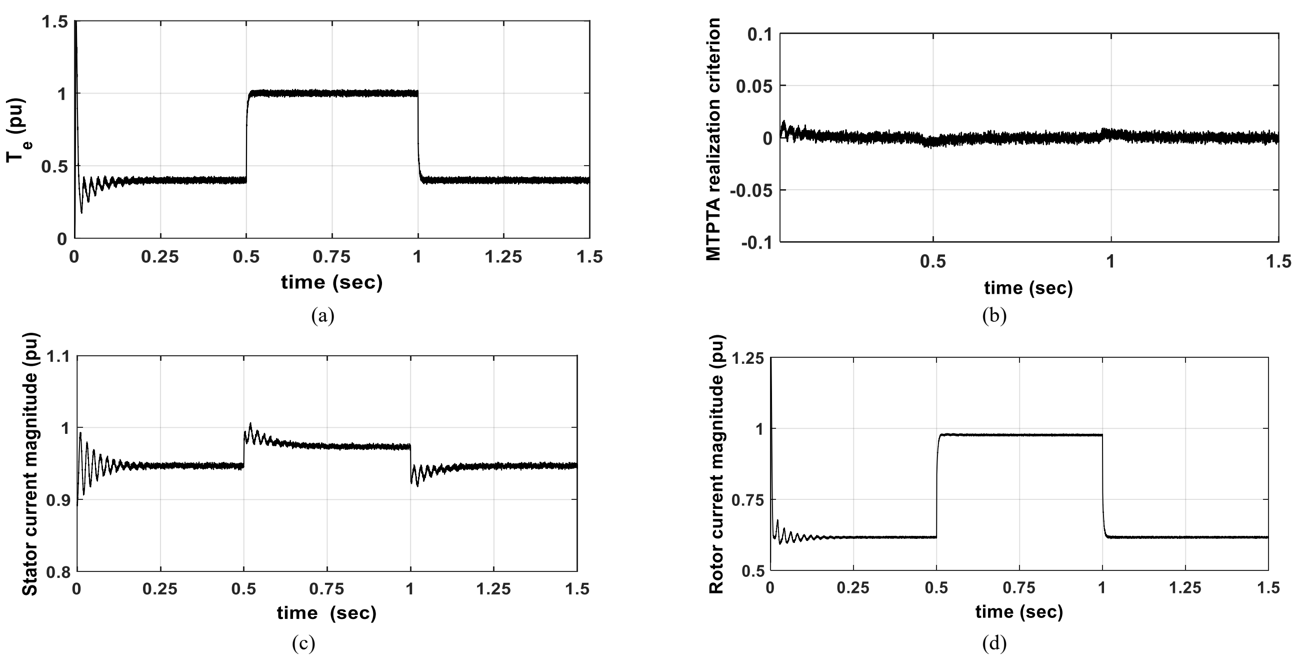

Fig. 4. Simulation results of the MTPTA control strategy (transient).

By realizing the MTPIA control strategy, the rotor current magnitude is less than that under the MTPTA control. As shown in Figs. 3(d) and 4(d), for $T_e = 0.4\, p.u.$, the rotor current magnitudes for MTPIA and MTPTA control strategies are approximately $0.54\, pu$ and $0.6\, pu$, respectively. Since, the MTPIA is only applied to the rotor, the stator current increases with the rotor current's decrease. For a given torque, the magnitude of the total current is higher than that under the MTPTA strategy, in which the total current is decreased.

V. Conclusion

In this paper, two control methods, known as MTPTA and MTPIA control are proposed for doubly fed induction machine drives. The study demonstrated that the proposed MTPTA control strategy is achieved by ensuring the MTPTA criterion follows a zero-reference signal for a fair distribution of currents among the stator and rotor windings and to minimize the total current (including stator and rotor currents) for a given torque. Moreover, the MTPIA strategy was proposed and it was proven to minimize the rotor current magnitude for a given torque, the rotor current angle should be always constant.

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