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Essays on the Competitive Impact of Common Ownership

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Abstract

This thesis comprises three chapters on the competitive impact of common ownership. Chapter 1 is a survey of the literature which explains in detail the current approaches being taken, the methods being instigated, and the questions being asked. It explains that much of the current work in this area is overlooking the fundamental problems thwarting progress in our understanding of this topic and takes a vital step back to try and realign the questions being asked by this literature towards the fundamental issues in this topic. Ultimately, this chapter highlights that the most pressing issues are the lack of a theoretically grounded measure of common ownership which relates directly to a market outcome of interest, as well as a lack of variety in the data sources and economies being studied. Much of the literature has focussed on replicating a single dataset. Very few industries have been studied and evidence outside the US is lacking. The first of these issues is addressed in chapter 2 where we derive a theoretically grounded index for common ownership which directly relates the ownership matrix to output decisions by firms. We study this index under several assumptions and in a few special cases and provide theoretical evidence that common ownership can be anticompetitive. Next, chapter 3 addresses the other major gap in the literature identified and provides evidence on common ownership in three British industries: supermarkets, soft drinks and banking, over a 10-year period spanning 2011-2021. We analyse common ownerships impact on these industries using a variation of the index derived in chapter 2 and provide evidence that common ownership has reduced output in supermarkets and soft drinks, with the effect increasing in magnitude over the period, and remaining stable over the period, in the industries respectively. Evidence in banking is more mixed, but there is some evidence to support output may have been reduced by common ownership, but the trend is more unstable over the period.

Introduction

Ensuring that competition is effective and fair in marketplaces is one of the central pillars of a developed economy. Competition is important for everybody, to keep prices fair, to keep firms on a level playing field and to foster a culture of innovation, ensuring the best choice, quality, and prices. Ultimately, competition protects the welfare of all agents in an economy. Competition has been studied in product markets, and in financial markets. But how do these two sectors of the economy interact? How do the outcomes in financial markets impact competition in product markets? And do the outcomes of product markets influence decisions in financial markets? This is an important question because the two sectors do not exist separately and assuming that they work in isolation may miss many important interactions that may drive outcomes in both markets. These questions are multifaceted in how they could be approached, and every possible path for exploring it leads to a rich, detailed and expansive literature. For this reason, we focus on one branch of this question: How does ownership structure (an outcome of financial markets) impact competition in product markets?

It has been noted in recent years that in the US, the ownership of many large firms is becoming increasingly concentrated amongst a small set of investors. That firms are separate entities and will act to maximise their own profits without internalising the concerns of their rivals is a foundational assumption in many models of competition – but if the same entities own competing firms, are the firms truly separate? Would it not be in the interest of an investor holding shares in competing firms to maximise the profits of their portfolio and not the profits of any one firm in the market? If this is what happens, then competition may be jeopardised. This modern phenomenon of competing firms having shared investors is known as common ownership, and how common ownership impacts competition across firms, and ultimately within markets, is the subject of this thesis.

The first step in furthering the development of academic understanding on a topic is to collate what we already know, to identify what it is that we don't know, and to identify where it is we need to go on our journey for understanding. Existing literature on common ownership has addressed many important questions, and Chapter 1 of this thesis overviews the literature on common ownership, describing the current approaches and evidence and how they connect to each other to give an understanding of where the literature currently lies. It then goes on to explain the next big questions and considerations that should be taken into account in the literature. Specifically, Chapter 1 discusses that while the literature has asked important

questions about endogeneity issues in the measurement of common ownership, there are some more fundamental methodological questions which need to be asked before the benefits of resolving endogeneity issues can be fully realized. Moreover, it is discussed that a large portion of the empirical literature focuses on replicating a single dataset, and that while this work has contributed greatly to econometric aspects of the literature, wider evidence from a greater number of markets and data sources is an important next step before broader conclusions can be drawn on the impact of common ownership in a way that can be used by policy makers.

O'brien (2017) was the first to cast light on the key limitation in existing measures of common ownership. He outlines that much of the literature contests various econometric specifications and issues of endogeneity presented across papers in this field. While econometric specification and addressing endogeneity are key points in the robustness of economic analysis, they are not as important as asking ourselves if we are measuring the right thing to begin with. Much of the existing work using measures of common ownership which in fact, are not true measures of common ownership itself and don't have any clear theoretical relationship with measures of competition. How we can measure common ownership in a way that has a theoretically meaningful relationship with measures of interest is an important next step for the literature. Once we have a pure measure of common ownership that overcomes this issue, this can be used alongside the important econometric successes of existing literature to strengthen their analysis and results. Moreover, as mentioned earlier, a very high number of papers in the empirical literature on this topic focus on replicating a single dataset – initially presented by Azar et al (2018) in their seminal paper on the US airline industry. We need to look beyond this dataset and industry for further evidence on common ownership in different markets and in different economies before broader conclusions can be drawn.

Following on from Chapter 1's discussion of the logical next steps in the existing literature and having explained how we need to move forward, chapter 2 addresses what is outlined as the most pressing issue in the literature. Existing measures of common ownership are measures of concentration and they do not have any theoretically grounded relationship with measures of competition. This means that we cannot reliably interpret any results using existing measures, and no matter how perfect our econometric specifications, the interpretation of coefficients is, in O'Brien's words, 'meaningless' if the measure of common ownership used has no understood, and proven, relationship with our variable of interest. Chapter 2 develops a theoretical framework under a modified Cournot setting, which incorporates the ownership matrix of a market into firms profit maximising decisions to study how any configuration of

ownership (and degree of common ownership) impacts strategic behaviour and ultimately market outcomes. Thus, we provide a measure of common ownership which directly relates the ownership matrix to output decisions. The result is an index which provides a pure measure of common ownership's impact on both individual and aggregate quantities under the assumption of homogenous marginal costs. We then relax the assumption of symmetric marginal costs to obtain a version of the index which incorporates both the ownership matrix of a market and the cost asymmetries across firms to better understand how the two interact and what the impact might be on competition. We study the index under several special cases and provide evidence that common ownership can be anticompetitive.

Having addressed the issue of measuring common ownership in an economically meaningful way, chapter 3 addresses another gap identified in the literature. We need a wider variety of evidence on common ownership outside of the US and beyond the handful of markets that had so far been studied in order to assess the scope and scale of common ownership as an issue. Is it a wider issue and is it occurring in other markets and economies? And what are the implications, if any, for competition in these markets? Chapter 3 extends the index developed in chapter 2 to make it more empirically applicable and then applies it to three British industries: supermarkets, soft drinks and banking, using data from a Bloomberg terminal. We study what our index says about common ownership in each of these industries and look at how the index has changed in each industry over a 10-year period, spanning 2011-2021. We find evidence that common ownership has reduced competition in the supermarket industry, and that the effect is increasing over the time period. Common ownership also seems to have reduced competition in the soft drinks industry, but the magnitude of the effect was stable over the period. There is some evidence of reduced competition resulting from common ownership in the banking industry, but the index is more unstable and the trend less clear over the period in this industry. Having applied our own index to these markets, we also apply several existing measures of common ownership to the same datasets over the same period to study what these say about the trends and potential implications for competition. The purpose of this is to provide a benchmark analysis of these new industries in a way which is more comparable to existing studies to widen the evidence presented in this chapter and connect our work more directly to existing literature, and to compare the outcomes of our index with existing measures.

It should be noted upfront that Chapter 3 represents an initial descriptive piece of work towards the goal of broadening the scope of industries studied in this literature. The industries studied and methods applied were chosen based on the author's limited ability to obtain necessary data

and as a result do not represent the most theoretically ideal candidates for applying the index practically. Firstly, the index presented in this thesis is built on the assumption of a Cournot market, and that Cournot is an appropriate assumption for the banking industry is not an obvious conclusion. Moreover, due to the author's inability to access appropriate pricing data for the industries and time periods needed there is no empirical analysis conducted in this chapter. Thus, the work conducted in this paper represents only a first step and serves mainly to illustrate the application of the index presented in this thesis and demonstrate the sort of results one could obtain. It does show that with easily obtained data one can get clearly interpretable results using the index even in the absence of econometric analysis. This is certainly a useful (and user-friendly) feature of the measure, and policy makers or academics with access to superior data could potentially get very insightful results using the index. Moreover, the interpretation of the index is quite simple and intuitive, which is a strength in policy settings where communicating results simply is critical.

With respect to the potential for this work to be used by policy makers in the future, it is the authors view that the index presented in this work has both strengths and weaknesses. If policy makers are considering an industry that can be confidently assumed to operate under Cournot competition, then the index offers a way to study ownership impact on competitive outcomes in a way that is clearly grounded in theory. However, if the industry is more likely to operate under price competition, then alternative measures that relate to price-cost margins rather than aggregate output (such as the MHHI) might offer a more intuitive interpretation. The author takes the view that this index is another tool, alongside existing measures, in a policy makers arsenal with which to study common ownership. No perfect measure exists (at present) and having a variety of measures with different strengths expands the potential for useful studies.

Should somebody follow up this work, the next obvious steps would be to expand the model outlined in chapter 2 beyond the case of Cournot competition, as well as to introduce differentiated products. Further exploration of different methods of selecting the representative shareholder in the model would also be a natural development. Empirical analysis would also be very welcome, particularly if future researchers have access to pricing data for an industry and can expand the work of chapter 3 to include regressions to help us understand econometrically the external validity of the index presented in chapter 3.

Ultimately, the goal of this thesis is to explain what we know so far about common ownership impact on market competition, to highlight the key questions that should drive the direction of

the literature moving forwards, and to play a small part amongst the work aiming to move the path of this literature towards meaningful economic progress. This work outlines the key issues of this literature and provide new direction and evidence towards some of the issues facing researchers in this area. Nothing is ever perfect in academia, and the results are not conclusive, but I believe and hope that the work offered here provides a progressive steppingstone for future researchers to build upon and improve our understanding of this pivotal question.

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Chapter 1:

Common Ownership and Market Competition: The Story So Far

Abstract

The work outlines the key papers in the literature on common ownerships competitive impact at the time of writing. It discusses the early literature which led to the modern discussion, looks at different approaches to measuring common ownership, outlines empirical studies in this area and considers the balance of evidence in a discursive manner. Overall, it appears that the next steps in this work should address ways to measure common ownership in a manner that is grounded in a clear and direct theoretical relationship with the explanatory variable in question, and further evidence on a broader range of industries is needed in generalising our evidence. Much of the existing literature relates to the banking and airline industry, and an even greater proportion of the work relates specifically to American markets. Studies outside of these areas would prove useful.

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Disclaimer: The author of this chapter, Sabria Behilil, was employed as an intern by the CMA to help with the State of Competition II report. Sections of this literature review appear in the report and corresponding appendix.

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Introduction

This paper overviews some of the main literature on the impact of common ownership on market competition. We focus specifically on the theory of harm relating to how common ownership might impact pricing or quantity decisions in a market and do not consider entry/exit decisions or innovation implications. Common ownership is when a third-party investor holds shares in two or more competition firms within a market. The common ownership hypothesis states that where shareholders have diversified portfolios and when managers act in the interest of shareholders, firms might depart from own-profit maximisation behaviour and place a non-zero weighting on the profits of competitors – essentially softening competition. Since its introduction there has been a rich literature debating both sides of this question and that debate is what this paper aims to cover. We overview the main findings of the key papers at the time of writing and consider the merit of the arguments and the balance of evidence. We highlight some key gaps in the existing literature and motivate three main areas for future work to address. Firstly, more measures of common ownership that have clear theoretical groundings and relate directly to the market outcome against which they are being studied should be developed. Secondly, studies are needed on a broader range of industries and finally, studies conducted outside of American markets would be useful in broadening our understanding of how prevalent an issue common ownership is in modern times.

Other literature reviews on common ownership exist (For example, Schmalz, 2018; Schmalz, 2021; Schwalbe, 2018) however the literature is so vast that the potential mechanisms of harm so many, that each of these offers a unique angle on the state of the literature, focusing on different aspects. While there are inevitable overlaps in any discussion of common ownership, this work takes a different focal point than existing work. Schmalz (2018) for example considers governance mechanisms and data challenges in more detail, whereas this work focusses more exclusively on a single element of the literature: taking common ownership as given and investigating the evidence on what this means for competition, and so covers this particular element and the arguments of the corresponding papers in more detail. Furthermore, this work is to my knowledge, unique amongst literature reviews in this area in addressing in detail the portion of the literature relating to European markets. It is my view that all literature reviews in this area are complimentary, and each have their own strengths and weaknesses, depending on the interest of the reader. The goal of this work is to add to the understanding on the current state of the common ownership debate and to highlight some additional considerations for future research.

The paper proceeds as follows: Section 2 discusses the early literature in industrial organisation which inspired the modern literature on common ownership, section 3 covers current methods of measuring common ownership, section 4 covers the empirical literature, section 5 overviews studies relating to European markets and section 6 concludes.

Common Ownership vs Cross Ownership

Before discussing the literature, it is important to highlight two different types of ownership links that have been studied within this body of work. Common ownership is when a third-party investor holds shares in competing firms, and cross-ownership is when firms hold shares directly in each other. This thesis focusses on common ownership, but cross ownership is a parallel concept which will come up in the discussion of many papers. To be explicit, both common and cross ownership are hypothesized to potentially dampen competition. The key difference in practice is the mechanism through which this may occur. In cross ownership the channel of control is clearer in that if firms have direct ownership links in each other the agency problem is reduced. Contrastingly, in common ownership, there is a 'middleman' in the sense that firms have ownership links which occur via a (set of) common shareholder(s). In common ownership there is thus an obvious principle-agent problem as management may not necessarily want to act in the interest of the shareholders. For ownership links to affect competition materially two things are needed. Firstly, the incentive to reduce competition must exist, and secondly, the ability to make the firm act on this incentive must be present. Essentially, in cross ownership it is clearer to see how direct ownership across firms could translate into control, whereas in common ownership whether shareholders have any influence is not clear and is the subject of a vast body of literature (more on this later in chapter 2).

Both of these types of ownership links exist in the real world. Common ownership amongst institutional investors has been well documented – especially in the US. Institutional shareholders, such as Blackrock and Vanguard hold shares in almost all of the largest listed companies – and many of these companies compete with each other. For example, in the automobile industry, Ford and General Motors both list Vanguard, Blackrock and State Street as three of their largest shareholders, this is common ownership. An example of a cross ownership link in this industry is that Ford holds 2% of Tesla's shares, and General Motors owns 12.8% of Tesla's shares (Capital, 2023).

Having hopefully fixed the distinction between these two concepts, we now turn to the discussion of the main papers in this literature.

Early Industrial Organisation Literature

The modern literature on common ownership finds its ancestry in an earlier literature within industrial organisation (IO). Traditional economic theory presented that firms would each act to maximise their own profits. This is a fundamental concept in many of the models that comprise the core of competition economic thinking. However, the introduction of this objective of the firm led academics to ask the question: To what extent do firms stray from this objective in practice and what are the ramifications of this on market outcomes? Here we have what is essentially the central question of the common ownership literature when stripped down to its core. Although this question has been addressed across a wide realm of literatures within IO we focus, naturally, on the subset of literature concerned with how ownership structure influences whether (and to what extent) firms will depart from an objective of own-profit maximisation.

One way to look at papers in this literature is to consider the mechanism through which it is suggested that ownership structure influences firm behaviour and, as a result, market outcomes. Earlier work was more mixed and some papers laid out the narrative that partial ownership agreements can facilitate cartel formation and that it is anti-competitive through coordinated effects, while others suggested that the anticompetitive effects of common ownership come into fruition via unilateral effects that arise due to the features of a market characterised by overlapping ownership structures. The more modern literature, that will be discussed in the next section, tends to focus almost exclusively on unilateral effects. Earlier work tends to be purely theoretical whilst over time there has been a shift towards more empirical work as measures of common ownership arose in the literature. With this distinction in mind, let us consider the beginnings of the story.

Firstly, we look at the papers that present that common ownership can facilitate collusive actions and can be detrimental to competitive conduct through coordinated effects:

Rubenstein & Yaari (1983) is, to my knowledge, one of the earliest works considering whether a competitive stock market can act as a cartel maker. They lay out several examples to illustrate that investors who own separate firms have an incentive to acquires shares in each other's companies to reduce the ferocity of competition and move towards obtaining monopolistic power. Bernheim & Whinston (1985) show that where competing firms have common agency i.e. when they delegate decision making to a common third party (much like a third-party common shareholder), this serves as an indirect mechanism through which competing firms

can generate the collusive party. Notably though, they assume that this is a single party, and don't consider that it may be a small 'pool' of shared agents making the joint decisions for the firms.

Reitman (1994) investigates how partial ownership agreements affect the potential for collusion across rivals. They consider that firms can purchase claims to competitors profits as a commitment mechanism in weakening competitive ferocity. In this work it is assumed that firms have symmetric costs and technology so that the only remaining impact of partial ownership arrangements is to the firms profits – they use this benchmark to isolate the impact of ownership on profits, though it is obviously a simplification. They also assume that cross ownership of rivals confers no control on the rivals behaviour, just a claim to their profits. This makes their results interesting as it suggests anticompetitive effects can arise even in the absence of control. Ultimately, the paper shows that any partial ownership arrangement will result in lower aggregate output, higher prices, reduced consumer surplus and a deadweight loss to society. Gilo et al (2006) also considers whether cross ownership in rivals can facilitate collusion in the situation where cross ownership confers rights to profits but no control in decision making. Under Bertrand competition they show that a higher ownership share in a rival increases the range of discount factors at which tacit collusion can be sustained – and could thus be anticompetitive. However, they studied only the incentives and did not indicate any extent to which this may be occurring empirically.

Before discussing the next paper, let's quickly define the term 'keiretsu'. Put simply, the keiretsu refers to a set of companies that have intertwined shareholdings and represent a sort of informal "block" of companies. In many ways a keiretsu is reminiscent of a cartel, but in Japan the existence of the keiretsu is not illegal, and it is a prevalent business structure. Alley (1997) utilizes the existence of the keiretsu in Japanese markets to consider a comparison of the level of collusion that occurs in the Japanese automobile industry compared to the American automobile industry – given the presence of the keiretsu in Japan. The general premise of this paper is that if common ownership in the form of partial ownership agreements (POAs) does indeed facilitate collusion then, given the keiretsu, we should see higher collusion occurring in the Japanese market. On the contrary however, the results show that in fact the American industry has a higher degree of collusion. However, there are a few limitations to this work. To begin with, the level of POAs existing in the American market is not addressed. Essentially it is just assumed that the American market has a lower level of POAs than the Japanese market given the presence of the keiretsu. This may be true but providing no evidence

on this means any conclusions drawn rely heavily on this assumption and limits the robustness of results. Secondly, like is not compared with like - in estimating the impact of POAs on the PCM of the two markets (Japanese and American), the same equations are not estimated. The PCM of the Japanese market is estimated using an equation which includes both imports and exports, however export data was not available for the US market and so they estimated an equation with fewer terms for the American market. Comparing different econometric specifications doesn't provide a fair and unbiased comparison across the markets and so conclusions drawn on the differing levels of collusion across the two markets are not based on strong foundations and should be taken with a pinch of salt. One possible remedy could have been for the authors to have included an estimation of the Japanese equation without exports for a fairer comparison. However, nonetheless the paper does provide additional evidence that common ownership may not be anticompetitive.

Ono et al (2004) extends Alley's (1997) work by endogenising the determination of POAs to study why firms engage in joint ownership. For our purposes, the important point in this work is whether firms enter into POAs with anticompetitive motives or if they are driven by the desire to obtain cost saving synergies. This paper ultimately seems to suggest that the motives for POAs are more on the technology-sharing, cost-saving front rather than being an underhanded way for firms to collude. For this reason, it can be argued that common ownership agreements in the Japanese automobile industry could be subject to the efficiency defence and does not necessarily indicate anticompetitive behaviour. However, the authors did not look explicitly at how output levels changed given different levels of POAs, but I feel their result on the driving motivator behind POAs sheds a complimentary light onto our investigation of whether common ownership is anticompetitive.

Now, we discuss the papers who propose that ownership links influence competitive outcomes through unilateral effects:

Rotemberg (1984) produces the benchmark result that when identical shareholders are fully diversified competitive ferocity is lost and the monopolistic outcome prevails. This was the first instance in which it was argued that shareholders with heterogeneous incentives could coordinate in a way which reduced competitive forces purely by acting on their own unilateral incentives. This was a key result for motivating the following literature, but a key limitation to this result is to what extent identical shareholders with equally diversified portfolios can be thought to have truly heterogenous interests. Farrell (1985) produces a response to this result

and shows that it holds unless shareholders are consumers of the firm and have a consumption interest equal to their financial interest, in which case they would prefer that the firms compete. This aids the argument that a shareholder's financial interest cannot be considered in isolation in determining how they might behave. Reynolds & Snapp (1985) also offer specific conditions under which common ownership can generate anticompetitive outcomes. Using a modified Cournot model they study the trade-off between efficiency gains and anticompetitive incentives under common ownership and conclude that common ownership is anticompetitive in industries with high barriers to entry. This result is particularly interesting given the later work on the airline industry (which naturally has high entry barriers) which served as one of the seminal papers in the modern literature – more on this later. Similarly, Amundsen & Bergman (2002) look at power companies in Norway and Sweden (a market characterised by high entry barriers) and find using simulations under Cournot competition that partial ownership arrangements tend to lead to increased horizontal market power for firms and thus higher electricity prices.

Perhaps the most influential work in this early literature was the introduction of the modified Herfindahl-Hirschman Index (Bresnahan & Salop, 1986). This was the first attempt to quantify the anticompetitive effects of partial ownership agreements and was developed under the context of joint ventures. This work did not spring into the centre stage of the common ownership discussion until later when the model was generalised by O'Brien & Salop (2000) and the MHHI became the explanatory variable through which much of the most famous work in the empirical common ownership literature cited evidence of an anticompetitive impact. I discuss the origins and use of the MHHI in greater depth in section 3.

Farrell & Shapiro (1990) built on the work of Reynolds & Snapp (1985) and their results had implications for the interpretation of the MHHI introduced by Bresnahan & Salop (1986). Farrell & Shapiro (1990) study the effects of changes in ownership of productive assets in a concentrated Cournot industry. They look at three different types of changes in asset ownership, but of interest to our discussion is their third case, whereby they study the case in which a firm purchases a partial interest in a rival firm. They look at how this affects price, output and concentration. Specifically, they model a case in which firm 1 holds α in firm 2 and consider what happens if α increases. Reynolds & Snapp (1985) show that this kind of cross ownership increases price, but they don't consider welfare implications - Farrell & Shapiro build in this direction. Importantly, like Reynolds & Snapp (1985) they assume that ownership confers no control, and only a right to a share of profits. Their results indicate that when α

increases firm 1 reduces its output but all other firms increase their output as a response, so aggregate output increases (this is similar to the 'merger paradox' presented in Salant et al, 1983). A key result of this paper is that α and the HHI can move in the same direction i.e. an increase in ownership concentration can lead to an increase in the HHI. This happens because when α increases, firm 1 lowers its output, but all other firms increase their output, and so output is higher and concentrated more amongst fewer firms. The impact of an increase in α on the HHI depends on specifics of the model, but they show that in the cases where HHI is decreasing in α, then welfare is also decreasing in α. This means that higher cross ownership reduced concentration but also reduced welfare in their model. The authors propose that this indicates that Bresnahan & Salop's (1986) MHHI, which incorporates the HHI, is not suitable as a measure of welfare, it is only a measure of how concentrated ownership is.

Hansen & Lott (1996) introduce the notion that when shareholders have diverse portfolios managers will depart from own profit maximisation and instead will maximise the weighted average of their shareholders portfolio profits. We will see that this idea is used heavily in recent models used in measuring common ownership. Additionally, a key contribution of Hansen & Lott was to present one of the first empirical studies of the impact of common ownership on market outcomes. They show, both theoretically and empirically, that cross ownership has a large and significant (positive) impact on how much an acquiring firm pays for a target firm in a corporate takeover – indicating a weakening of competitive forces.

Flath's (1992) contribution is to introduce the potential importance of indirect ownership links and to study the impact of both direct and indirect shareholding links across competing firms on the prevailing competitive conduct of firms under Cournot competition. The results are that both direct and indirect shareholding links result in firms restricting production but that direct shareholding links across rivals have an impact of greater magnitude than indirect shareholdings. This work motivates the notion that the entire network of ownership should be considered. Dietzenbacher et al. (2000) extend this work, expanding an analysis of 3 firms to n firms and study the impact of common shareholding on price-cost margins in Cournot and Bertrand settings. The authors consider the case whereby investors have silent financial interests and find that under all circumstances competition is reduced as a result of shareholding links. Specifically, using the Dutch financial sector as a case study, they find that price-cost margins are 2% and 8% higher under Bertrand and Cournot respectively as a direct result of common ownership links. They note that empirical work in this area is rare (or it was at the time), but data limitations made quantifying the impact of common ownership seem

impossible. So, this was the first paper to try and directly quantify the impact of common ownership on price-cost margins in an empirical setting. Quantifying the impact of common ownership on market outcomes was a very important step in the development of the literature as this allows for a more objective understanding of the precise impact. Indeed, how to measure ownership and its impact on market outcomes is a central piece of the modern literature and we will discuss the various methods that have since been introduced in greater detail in 3.

Looking at the other side of the market, Bolle & Guth (1992) consider the case of common ownership within a market of (competing) sellers. They study the incentive effects of joint ownership amongst mutually dependent sellers and assume away the issue of corporate control by imposing that every firm will have a majority shareholder with a holding exceeding 50%. They present an oligopoly model and find that joint ownership will lead to higher prices and reduced consumer welfare. Equally, joint ownership is shown to provide strong incentives to shut down some firms in order to restrict market output in a way which mimics the behaviour of a competitive market. However, their results rely on the assumption of symmetric markets where firms all face identical marginal costs. This removes an important element of competition from the study and does mute the degree to which results can be extrapolated to more general cases of joint ownership.

Parker and Roller (1997) set out to answer a different question but their results feed into our literature. They capitalized on the deregulation of the American telecommunication industry, whereby the FCC created a duopoly which granted only two firms' licences to highly specific product and geographic markets. Their intention here was to study the extent to which duopolies resulted in competitive outcomes. Relevant to our interest, is that they note cross-ownership (alongside multi-market contact) as a key factor in explaining the prevalence of anti-competitive pricing. This supports the narrative that ownership links can facilitate anti-competitive behaviour.

So far, the papers discussed all present a unanimous perspective on the anti-competitive impact of common ownership. Though they differ in their methods and precise specifications, the overarching balance of this work is that joint ownership can facilitate anti-competitive behaviour. However, not all the early literature was in agreeance with this, and we now present some papers whose results present otherwise.

Malueg (1992) provides some of the first evidence suggesting that the relationship between common ownership and collusion is not as simple as much of the other work suggested. The

author outlines that in terms of the ability to sustain collusion, common ownership has two opposing influences: On one hand, a higher degree of common ownership will reduce the gains from cheating as a player is invested in the payoff of its rival, but on the other hand, higher common ownership means that because the rival firm is invested in the payoff of the player, cheating will yield a softer punishment. The first of these supports collusion, whilst the latter makes collusion less likely. Malueg extends on previous work in two main ways: Firstly, by extending analysis from static to dynamic games – specifically the author looks at repeated Cournot games, and secondly by considering several types of demand functions and not just a linear specification. The results are that in the repeated game the curvature of the demand curve proves critical in determining which of the two effects dominates and determining whether common ownership supports or inhibits collusion.

To be clear, the measure of common ownership is a term denoted v, which represents the ownership share of investor i in firm j, restricted to be between 0 and 50%. In this study it is assumed that there are only two firms and two investors, and so v captures the extent of common ownership. So essentially, for different elasticities of the demand curve, the author calculates the critical discount factor as a function of v to see how it varies in the common ownership term. From this analysis, the author finds that depending on the elasticity of demand, common ownership can either make collusion easier or more difficult to sustain. Thus, Malueg provides evidence that the impact of common ownership on competition demands on the structure of the demand curve. While his results do not dismiss the notion presented in other work that common ownership can make collusion easier to sustain, it provides evidence that in certain cases common ownership can in fact be pro-competitive. This is a stark contrast to other work in the literature and is thus an interesting contribution. The structure of the demand curve may thus be an important thing to note in empirical studies of common ownership.

Clayton & Jorgensen (2005) also analyse the motivation behind firms undertaking cross holdings and the consequences of this. In the spirit of Ono et al (2004) they endogenise the optimal cross-holding positions and argue that this (alongside their consideration of externalities) is what drives their striking results. Essentially, they derive an equilibrium in which they calculate the quantity and price of each firm as a function of their cross-ownership share – under the assumption of linear demand and constant, identical marginal costs. Having arrived at expressions for the output (and price) of the two firms as functions of their cross-ownership shares, the authors substitute zero into the cross-holding term to see what the equilibrium would be like in the absence of cross shareholdings (we will take a similar

approach in Chapter 3 when we derive our own index of quantity as a function of the ownership matrix). They find that when products are substitutes (competing firms) prices are lower, quantities are higher and consumer welfare is higher in the presence of cross ownership than its absence. This is in contrast to the evidence presented by most of the literature which suggests ownership links can be anticompetitive. However, in their proof of this assertion the authors make the very limiting assumption that the firms are symmetrical and the fact that $qi = qj$ is imperative in the proof. The assertion may hold for less restrictive cases, but this was not addressed and represents a limitation in the analysis when trying to extrapolate their evidence into implications for real-world policy. Nonetheless, their paper represents the strongest assertion that I can find presenting a unanimous argument that cross shareholdings are not anti-competitive. Importantly though, the distinction between cross and common ownership must be highlighted here. The literature as a whole considers multiple types of ownership links, some papers using cross ownership, some common ownership and some considering both types of links. Recall that cross ownership is when firms have direct shareholdings in each other, whilst common ownership is where firms have one or more shared third-party shareholders. The results of this paper are specific to the analysis of cross ownership and cannot necessarily be extrapolated to provide insight into the anti-competitive nature of common ownership.

From this earlier literature it seems to be that evidence generally suggests that common ownership is anticompetitive. The papers which argue this perspective argue it strongly, often presenting a unanimous conjecture that ownership links reduce competition. A limited number of caveats to this are presented in a few papers, for instance, it is suggested that entry barriers may play a role in driving the anti-competitiveness of common ownership but overall, it is argued that it facilitates collusion and leads to restrictions in output. However, it is worth noting that much of the work was theoretical and more empirical work would be valuable in strengthening the argument. While some papers stray from this storyline, the ones that do are not all agreed in their assertion of the pro-competitive nature of common ownership. For instance, Malueg (1992) shows that common ownership is pro-competitive under only some conditions but remains anti-competitive under others. For this reason, I do not feel that the early literature presents a strong case for arguing that common ownership is not anticompetitive. Thus, I feel that the balance of evidence in the early literature sets the scene nicely in motivating the continuation of research in this area. Specifically, very little work has been done in the way of quantifying the impact of common ownership, and this is what the next section will focus on.

An overview of the papers discussed in this section is provided in the following table:

Paper	Common or Cross ownership	Cooperative or unilateral effects	Theory or Empirical	Summary of results
Rubenstein & Yaari (1983)	Common	Cooperative	Theory	Investors who own separate firms have an incentive to acquires shares in each other's companies to reduce the ferocity of competition
Bernheim & Whinston (1985)	Common	Cooperative	Theory	Common agency (decision making by an external third party) provides an indirect mechanism through which competing firms can generate the collusive outcome.
Rotemberg (1984)	Common	Unilateral	Theory	When identical shareholders are fully diversified competitive ferocity is lost and the monopolistic outcome prevails
Farrell (1985)	Common	Unilateral	Theory	Rotemberg (1984) result holds unless shareholders are also consumers of the firms and certain conditions are met
Reynolds & Snapp (1985)	Common	Unilateral	Theory	Common ownership is anticompetitive in industries with high barriers to entry
Amundsen & Bergman (2002)	Cross	Unilateral	Mix (simulation built on theory but informed by data)	Partial ownership arrangements tend to lead to increased horizontal market power for firms and thus higher prices.
Reitman (1994)	Cross	Cooperative	Theory	Any partial ownership arrangement reduces output, raises prices, and reduces societal welfare and consumer surplus.

Gilo et al (2006)	Cross	Cooperative	Theory	Higher ownership share in a rival increases range of discount factors at which collusion can be sustained.
Bresnahan & Salop (1986)	Common	Unilateral	Theory	Introduced first version of MHHI and provides evidence that common ownership can be anticompetitive.
Farrell & Shapiro (1990)	Cross	Unilateral	Theory	Extend Reynolds & Snapp's analysis to consider welfare implications and show that it is possible for HHI to increase in cross ownership.
O'Brien & Salop	Common	Unilateral	Theory	Generalises MHHI introduced by Bresnahan & Salop (1986) to allow any ownership structure.
Hansen & Lott (1996)	Common	Unilateral	Theory and empirics	If shareholders have diversified portfolios managers will maximise weighted average of shareholders portfolio profits. They show, both theoretically and empirically, that cross ownership has a large and significant (positive) impact on competition.
Flath (1992)	Common	Unilateral	Theory	Shows that indirect (as well as direct) shareholding links reduce competition.
Dietzenbacher et al (2000)	Common	Unilateral	Theory & Empirics	Price-cost margins are 2% and 8% higher under Bertrand and Cournot respectively as a direct result of common ownership links.
Bolle & Guth (1992)	Common	Unilateral	Theory	Joint ownership will lead to higher prices and reduced consumer welfare.
Parker & Roller (1997)	Cross	Unilateral	Empirical	Cross ownership is a key factor in explaining the prevalence of anticompetitive pricing in a duopoly.
Malueg (1992)	Common	Cooperative	Theory	Common ownership has two opposing effects in collusion: higher common ownership reduces gains from cheating (supports collusion), higher common ownership softens punishment due to internalisation of rival's costs (supports cheating). Which effect dominates depends on the structure of the demand

				curve and common ownership can thus be anticompetitive or pro-competitive.
Alley (1997)	Common	Cooperative	Empirical	American automobile market has more collusion than Japanese automobile market despite the latter having high common ownership.
Ono et al (2004)	Common	Cooperative	Empirical	Motivation for entering partial ownership agreements (common ownership) is driven more by efficiency gains rather than incentive to collude.
Clayton & Jorgensen (2005)	Cross	Unilateral	Theory	When products are substitutes (competing firms) prices are lower, quantities are higher and consumer welfare is higher in the presence of cross ownership than its absence.
Model presented in Chapter 2 of this thesis	Common	Unilateral	Theory	Derives an index which links ownership matrix directly to aggregate output.

Measuring Common Ownership's Impact

Following on from the early work, the next major step in the literature followed in the footsteps of Bresnahan & Salop (1985) and focussed on how to measure common ownership. This is a central component to understanding how ownership influences competition as if you aren't measuring common ownership in a meaningful way any results that follow from the analysis do not capture a true picture of its impact.

The MHHI

I begin this section by introducing the MHHI, which is the measure of common ownership used in much of the key literature. An understanding of this will be helpful in following the upcoming discussion.

Introduction and Background of the MHHI

The MHHI (Modified Herfindahl-Hirschman Index) was first introduced by Bresnahan & Salop (1985) to measure quantitatively the degree to which joint ventures might impact competition in product markets. The authors note that the intention of this measure is purely to

quantify the anticompetitive effects arising from a joint venture and make clear that the intention is not to allow consideration of the trade-off between possible social benefits and reduced competition – they leave this to an analyst - but offer that their measure is a useful way to quantitatively identify the impact on competition. Moreover, they also explain that the MHHI does not take into consideration other channels through which a joint venture might impact competition – for example through the enablement of tacit collusion. The purpose of the MHHI is to quantify the anticompetitive effects arising as a direct result of a change to the objective function of the firm due to the change in ownership. The authors believe that this approach is complimentary to the literature studying the impact of joint ownership arrangements on the ease of instigating and sustaining tacit collusion. This original paper derived the MHHI under eight different corporate control structures and ranked them in order of competitiveness. However, the version of the MHHI which has become prolific in the modern literature on common ownership was a generalisation of this paper by O'Brien & Salop (2000), which expands the MHHI to allow application to a much wider set of corporate control structures. O'Brien & Salop (2000) highlight in their motivation for this generalisation that the impact of partial ownership on competition depends crucially on both financial interest and corporate control. One of their contributions is that they place emphasis on the distinction between these two elements in the study, as the distinction is absent from merger analysis where financial ownership is assumed to correlate directly with control, and so their approach captures a defining characteristic of partial ownerships influence on competition. To be clear, the authors outline the distinction as follows:

"Financial interest captures an entities right to a share of the firm's profits arising from firm activity, while corporate control refers to an entities right to make decisions on the firm's activities."

While, generally, a greater financial interest is associated by a higher degree of corporate control, the authors outline that in situations where there is no majority shareholder (a shareholder with holdings greater than 50% of the outstanding shares) then larger minority shareholders can have a disproportionate degree of control relative to their financial interest given their ability to potentially block votes or form voting coalitions.

They argue that the distinction between these two elements is important because in the case where one entity A acquires (through partial ownership agreements) a financial interest in another entity B, whether A has control over B is fundamental to the competitive impact of this

partial ownership agreement. In the case where A gains both financial interest and control in B, competition can be impacted. However, if A gains financial interest in B but has no control over the decision making of B, the authors argue that motivation for A to reduce competition between the firms will likely not be realised and so this may not lead to a reduction in competition.

Derivation of the MHHI

O'Brien & Salop (2000) derive the generalized MHHI under the assumption that there are N firms indexed j=1,…,N and M owners indexed i=1,…,M. The N firms compete under Cournot competition. The authors express that while in a standard oligopoly model without partial ownership interests, the firms act to maximise their own profits. They note that when partial ownership interests are introduced into the model the challenge turns to how to best determine the objective function of the manager given that owners now have heterogeneous interests. They propose that managers maximise a weighted sum of the shareholders returns, weighted by their corresponding control in the firm –such that greater influence of a shareholder corresponds with a higher weighting of that shareholders portfolio profits in the managers' objective function. They denote owner i's degree of control in firm j using the parameter γ_{ij}. Thus, a higher degree of influence by some shareholder i is captured by a higher γ_{ij}. The manager's objective function is then described as follows:

$$\max \Pi_j = \sum_i \gamma_{ij} \pi^i$$

Here, π^i represents the profit of owner i, and the authors express that

$$\pi^i = \sum_k \beta_{ik} \pi_k.$$

Where, β_{ij} denotes owner i's ownership share in firm j, and π_k represents the profit of firm k. So essentially, what we have here is that the profit of each owner is the sum of their ownership share in each firm multiplied by that firm's profit.

The first order condition of this maximisation problem for the manager's objective function is then given by:

$$\sum_i \gamma_{ij} \left\{ \sum_k \beta_{ik} P' x_k + \beta_{ij} [P - C'_j(x_j)] \right\} = 0$$

Multiply this through by $\frac{X}{X}$ and $\frac{1}{P}$ to obtain:

$$\sum_i \gamma_{ij} \sum_k \beta_{ik} \left(\frac{P'X}{P}\right) \frac{x_k}{X} + \sum_i \gamma_{ij}\, \beta_{ij} \frac{P - C'_j(x_j)}{P} = 0$$

Notice that $\frac{x_k}{X}$ is simply firm k's market share, which the authors denote s_k and that $\left(\frac{P'X}{P}\right)$ is the inverse of price elasticity of demand, denoted $\frac{1}{\eta}$.

Which re-arranges to yield:

$$\frac{P - C'_j(x_j)}{P} = \frac{1}{\eta} \sum_k \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij}\, \beta_{ij}}\right) s_k$$

They then multiply this by s_j and sum across all j to get:

$$\sum_j s_j \frac{P - C'_j(x_j)}{P} = \frac{1}{\eta} \left[\sum_k \sum_j \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij}\, \beta_{ij}}\right) s_k\, s_j \right]$$

From this, the bracketed term on the RHS of the equation is the expression for the MHHI:

$$MHHI = \sum_k \sum_j \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij}\, \beta_{ij}}\right) s_k\, s_j$$

Notice that in a Cournot game with no partial ownership $\sum_k \sum_j \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}}\right) s_k\, s_j$ would come out to be $\sum_j s_j^2$, which is the HHI.

You can then separate the term for which $k = j$ from the above equation to get:

$$MHHI = HHI + \sum_j \sum_{k \neq j} \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij}\, \beta_{ij}}\right) s_k\, s_j$$

And from this they obtain

$$MHHI\ Delta = \sum_j \sum_{k \neq j} \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij}\, \beta_{ij}}\right) s_k\, s_j$$

The authors propose that MHHI Delta captures the degree of concentration arising from common ownership in the industry, and that this can be used to measure the impact of common ownership on competition.

To summarise, what they have done is set up a Cournot oligopoly with the addition of partial ownership and assume an objective function for management which maximises the weighted profits of the owners, weighted by their respective influence on the firm captured by γ_{ij}. They solve the FOC for this maximisation problem and show that one can decompose $\sum_j s_j \frac{P-C'_j(x_j)}{P}$ into two terms: the HHI and the MHHI Delta, the latter of which they propose captures the additional competition effects of the partial ownership.

Empirical Studies

Two seminal papers have formed the foundation of the empirical literature on the anticompetitive effects of common ownership. Azar, Schmalz and Tecu (2018) study the airline industry and Azar, Raina & Schmalz (2016) studied the banking industry. A great deal of the rest of the literature has essentially stemmed from these two papers and pertains to one of these two industries as a direct response to these works. I thus divide the empirical literature into three branches of discussion: work relating to the airline industry, work addressing the banking industry and work addressing other industries.

The Airline Industry

Azar et al (2018) is one of the most well-known papers in this literature and really brought the common ownership story to the attention of academics and regulators. The key contribution of this paper is that it is the first work that empirically presents that ownership concentration has a direct impact on prices of portfolio firms' products. They use the MHHI to study the impact of common ownership on competition in the US airline industry. Specifically, their central research question is whether MHHI delta offers explanatory power for ticket prices after controlling for market concentration (HHI) and factors known to influence price. They first look at the incentives for firms to behave anticompetitively by calculating the MHHI delta of the industry and comparing the magnitude and increase in delta to the HHI merger guidelines. They then go on to test empirically whether this incentive is being acted upon by regressing the log of price on delta, HHI and some control variables. They study various specifications to support their results but present a general argument that there is a positive and statistically significant coefficient on MHHI delta. They present evidence to suggest that – based on HHI merger guidelines – common ownership as measured by MHHI delta yields increases in concentration ten times greater than those which antitrust authorities consider likely to reduce competition. However, it is not necessarily appropriate to apply HHI guidelines to analysis of

the MHHI or MHHI delta as the two measures don't follow the same scale. The HHI is bounded by 10,000 whereas there is no theoretical (upper) boundary for the MHHI and so a 1:1 comparison of values and impact is not necessarily meaningful. More research on how to analyse the magnitude of the components of MHHI together –that is the HHI and delta – would help in understanding exactly what these results imply for competition.

Dennis et al (2018)[1] produce a reply paper and argue that the results of AST arise only because of the precise specifications chosen by the authors and that the results do not prove robust to other assumptions and treatments. For this reason, they argue that common ownership does not have anticompetitive effects and that the observed positive correlation between airline prices and common ownership presented by AST arise due to variations in market shares and not ownership. DGS chiefly present four core arguments against the results of AST, which we will now outline.

The first argument that DGS present is the one which they believe to be the strongest criticism against the work of AST. They outline that AST's proposition that common ownership has anticompetitive effects stems from the positive coefficient on MHHI delta which they obtained when regressing it on the log of airline prices, and they outline two flaws to this regression analysis. Firstly, MHHI delta has two components, the first of which captures ownership and control and the second which represents market shares. Immediately, this creates the following question: Is it the component of delta which captures ownership or market shares which is driving the positive relationship between prices and delta observed by AST?
Following this, the second problem with AST's regression presented is that because MHHI delta contains a component which captures market share, the AST specification effectively regresses price on market shares twice. To see this, this is the regression equation presented by AST:

$$\log(p_{rjt}) = \alpha \sum_{j} \sum_{k \neq j} \left(\frac{\sum_i \gamma_{ijt} \beta_{ikt}}{\sum_i \gamma_{ijt} \beta_{ijt}} \right) s_{rjt} s_{rkt} + \eta \sum_{j} s_{rjt}^2 + \theta X_{rjt} + \alpha_t + v_{rj} + \varepsilon_{rjt}$$

Where $\sum_j \sum_{k \neq j} \left(\frac{\sum_i \gamma_{ijt} \beta_{ikt}}{\sum_i \gamma_{ijt} \beta_{ijt}} \right) s_{rjt} s_{rkt}$ is MHHI delta, and $\sum_j s_{rjt}^2$ is HHI. Notice that it is immediately clear that both the first and second term contain market shares, $s_{rjt} s_{rkt}$ in MHHI delta and s_{rjt}^2 in HHI. Thus, AST are regressing price on two functions of market shares.

[1] For simplicity, and following the convention of previous work, I will from the point forward refer to Dennis et al (2018) as DGS and Azar et al (2018) as AST

Moreover, the fact that AST show a positive relationship between HHI and prices makes one question whether it is the market share component of delta driving the positive relationship observed between price and delta. To study whether it is ownership or market shares driving AST's observed relationship between price and delta DGS conduct a placebo test, in which they replicate AST's regression but replace delta with two placebos (separately). The results of this analysis indicates that the positive relationship between delta and price observed by AST is driven by market shares and not common ownership.

The second argument DGS present against AST's results relies upon the format by which it is assumed that investors exert their control. This stems from the fact that while ownership can be objectively identified, it is not immediately clear how investors might display the control they have. DGS explain that the corporate governance literature outlines that investors use two main channels to exert control: voice and exit. The former refers to shareholders applying direct control over management through communication or voting, whilst the latter refers to investors ability to sell their shares if they are displeased with management. Essentially AST assume that investors behave exclusively through voice, whilst DGS show that if one considers investors also exert control through exit then the relationship between price and MHHI delta become statistically insignificant. Thus, AST's results rely on the assumption about how investors utilize control.

DGS also criticize AST's treatment of periods in which some carriers declare bankruptcy – which they identify as a significant issue in assessing results given that in half of the quarters included in the sample at least one of the major players operated under bankruptcy. Essentially, in these periods AST simply repeated the observation from the last period. This assumes that shareholders still maintain the same level of control even under bankruptcy, which is not a particularly reasonable assumption given that it is well acknowledged in financial economics that under bankruptcy it is creditors who yield the control rather than equity holders. DGS show that relaxing this assumption of shareholders maintaining control alters the results obtained by AST and so this as another limitation of the conclusion that common ownership is anticompetitive.

Finally, DGS show that AST's results depend on the use of passenger volume as regression weights in the estimated equation. Omitting these from the equation and re-estimating the regression lowers the magnitudes of the coefficients noticeably.

Azar et al (2018b) write a note responding to DGS's critique. The theme of the note is that DGS's results hinge wholly on their mistreatment of AST's ownership data which leads to a different sample with noticeably different summary statistics. They specifically address two of DGS's critiques – that the use of passenger volume as weights in the regression is central to results, and that AST's results are driven mainly by the top fifth percentile of markets. Essentially, they provide evidence to dispute these claims and cite DGS's mistreatment of data as the source of the problem. The placebo tests and the concern about which portion of MHHI delta is driving the positive correlation with price are not addressed, which is a limitation to the note as these posed a fundamental criticism and understanding of this point is central in the use of MHHI delta. Similarly, the critique that they are regressing on market shares twice is not addressed and so this criticism of their results still holds.

However, in 2022 AST produced another response to DGS (Azar et al, 2022) in which they address more adequately the specific critiques of their work presented by DGS. In particular, they argue that the main claim that the correlation between prices and common ownership was coming from the market share component of the MHHI delta is factually incorrect, and that the reason for DGS's results is a flaw in their model specification. In the placebo tests they used to isolate the ownership component of the MHHI delta DGS claim that the placebo behaves such as to keep market shares fixed. However, in this rebuttal AST show that the placebo used by DGS for this purpose is in fact negatively correlated with market shares, and they show that this arises because DGS's used an unbalanced panel. They correct this and implement their own method for isolating variation resulting from ownership from that resulting from market shares and show that ownership changes do predict changes in price, and that using this approach the impact is even higher than in their original paper.

Egland et al (2019) also write a response to AST and outline some limitations. Firstly, they identify that AST don't note the difference in financial incentives which arise between traditional asset managers and institutional investors. The argument here is essentially that AST assume an investors interest in a firm is equal to that investors fraction of holdings in that firm – both voting and non-voting. The authors argue that this is inappropriate in the case of asset managers who don't own the underlying assets and only manage the shares for clients – in essence making a principle-agent problem argument. Another argument the authors present is

that AST's estimations don't accommodate the changes in industry structure which occurred during the sample time period and impacted the relationship between price and covariates capturing demand and supply factors. Essentially, many aspects of the industry structure changed over the sample period e.g. bankruptcies, the 2008 financial crisis, and AST's regressions assume that such events had no impact on prices relationship with covariates, holding this constant across all estimates in the sample and thus damaging the accuracy of their estimated coefficients. Ultimately, Egland et al (2019) show that amending any one of these issues independently alters AST's results and renders the relationship between MHHI delta and price statistically insignificant. Thus, they argue that there is no evidence that common ownership is anticompetitive in the airline industry.

Gilje et al (2019) introduce a measure called GGL and replicate AST's regression replacing delta with it. GGL is a measure of common ownership which incorporates the fact that not all investors are attentive to managements actions and how they impact their payoffs and so this inattentiveness influences how managers make their decisions – in that they may only consider attentive investors. Essentially what they are proposing here is an alteration to the management objective function. It is assumed that an investors probability of being attentive is a positive function of how important that firm is in their portfolio, and this is taken to be proportional to their ownership stake in that firm.

GGL is comprised of the degree to which managers care about their shareholders preferences, the weight that shareholders place on the externality (competitors' profits) and the probability that the shareholder observes the impact of managerial actions on their private payoff. In the authors words GGL is best understood as a relative measure capturing the degree of importance managers place on common ownership per unit of the externality (internalisation of competitors profits). The key insight it provides is that common ownership is a necessary but not sufficient condition for shifting managerial incentives in a way which yields anticompetitive results, as management will only act on such incentives if shareholders are attentive and observe the payoff that such behaviour generates in their portfolios. If shareholders are inattentive, then management may not have any incentive to act on anticompetitive opportunities if they don't personally hold shares in the competitor. The payoff to management is indirect in that if shareholders 'win', they are more likely to support the manager. Attentiveness is proxied a la Iliev & Lowry (2015) by looking at whether shareholders follow the voting recommendation of the advisory firm ISS. It is assumed that

shareholders who follow the recommendation are inattentive and are free riding on the effort of ISS whereas attentive shareholders are more likely to stray from the recommendation and vote based on their own information and analysis.

Gilje et al (2019) find that when replacing delta with GGL that common ownership is not correlated with price in the airline industry. Interestingly, they perform a regression where they use GGL and adjust the parameters such that investors are all fully attentive and their finding of no relationship still holds. This indicates that the different results obtained when using delta vs GGL as the measure of common ownership in the regression don't stem from the difference in assumption about investor attentiveness. Instead, differences in results arise from other discrepancies between GGL and delta – such as the inclusion of market shares as an input in delta, which is in line with the argument of DGS.

Kennedy et al (2017)[2] perform a regression analysis to study the relationship between market outcomes and the profit weightings from O'Brien & Salop's (2000) model directly. They try to replicate the dataset of AST as closely as possible in order to perform a "robustness-check" on their evidence that common ownership is anti-competitive in the airline industry. The author's outline that what distinguishes their efforts is that they instrument for common ownership and estimate a structural oligopoly model which takes into account the interactions between the common ownership incentive terms and other market variables. They note that this is the first work to estimate a structural model of common ownership.

Essentially, what this work does is replicate AST's regression but replace the concentration-based measures of common ownership (MHHI) with the common ownership incentive terms in the regression. They instrument for potential endogeneity of these using Blackrock's 2009 acquisition of Barclays Global Investors because this acquisition affected common ownership but (likely) wasn't correlated with any demand or supply factors in the airline industry. The second instrument they included was constructed based on airlines inclusion in the Russell 1000 stock market index to capture any shareholdings in airlines which may have come about due to passive investment strategies.

The authors outline that their method is superior to that of AST's for two main reasons. Firstly, their estimation equations arise as a direct functional relationship obtained from theory, unlike

[2] We shall refer to this paper as KOSW

AST's use of the MHHI – see O'Brien (2017). Secondly, AST only instrumented the component of MHHI associated with common ownership – that is the MHHI Delta – but didn't instrument for the HHI component of MHHI. This work however instruments fully for the common ownership primitives.

The common ownership incentive term used in the regressions is taken directly from O'Brien & Salop's (2000) derivation of the MHHI. Specifically:

$$C_{jk} = \left(\frac{\sum_i \gamma_{ij}\beta_{ik}}{\sum_i \gamma_{ij}\,\beta_{ij}}\right)$$

C_{jk} is a common ownership incentive term which captures the degree of influence shareholders in firm j and k have over manager j.

The authors outline that theory suggests that equilibrium prices will be a function of cost and demand factors (X) and the common ownership matrix (C) which is a matrix of the common ownership incentive terms C_{jk}. They then estimate the following:

$$\ln p_{mt} = X_{mt}\theta + \lambda h(C_{mt}) + \epsilon_{mt}$$

Here C_{mt} is the common ownership incentive terms matrix in market m and time t, and $h(C_{mt})$ is an index capturing how common ownership impacts price. X_{mt} captures cost and demand factors in market m and time t. As noted earlier, this is essentially AST's specification with the common ownership incentives term matrix replacing the MHHI.

Under this methodology, KOSW find no evidence that common ownership leads to increased prices in the airline industry. They are able to closely construct AST's dataset and replicate most of their results, so the different outcomes are not driven by stark variations across datasets. Rather, KOSW present that the divergence across results arises as a consequence of the different methodology employed. Specifically, replacing MHHI delta with the common ownership primitives drives the different outcomes. KOSW cite O'Brien's (2017) critique of price-concentration analysis to argue that inferring a causal relationship between price and common ownership using MHHI is not meaningful and thus reject AST's results. Ultimately, they present that there is no evidence to support policy or regulatory intervention.

In a similar light, Park & Seo (2019) employ a structural model to estimate the competitive effects of common ownership on prices in the American airline industry. They also use the

common ownership incentive terms in place of MHHI delta, however they find a positive relationship between common ownership and price.

Azar, Schmalz & Tecu (2021) write another note, this time responding to the critiques of KOSW. They don't address the criticism of the underlying theoretical validity of using the MHHI for this purpose, and rather point the reader to a literature review by Schmalz in response to this critique. However, this literature review does little to remedy the concern (reasonably so, as it is not the purpose the paper). Azar et al (2021) focus instead on arguing KOSW's empirical critiques. The resounding story of the note is that they dismiss the rejection of AST and cite three central problems with KOSW's methodology which they argue inhibits the credibility of their claims. Firstly, they question the validity of the IV employed by KOSW as one of them negatively correlates with common ownership. Their final two points centre on the structural model estimated, in that KOSW use only 10% of the available data when estimating the model, and the model implies a negative relationship between distance of flights and marginal costs, which Azar et al (2021) suggest is economically implausible. They thus present an argument to say that KOSW's methodology is not, as they claimed it to be, superior to that of AST's. It is my view that the more interesting question here was not which empirical specification was correct, but rather which measure of common ownership is more appropriate as this seems to be the core difference between the papers – it doesn't matter how we specify an empirical model if we are measuring the wrong thing - and so I would encourage a direct comment on the use of common ownership primitives in place of the MHHI from future discussions on this topic. Moreover, the authors also cite the work of Park & Seo (2019) who they note as employing similar methodology to KOSW as evidence of the erroneous nature of KOSW's findings. However, they didn't comment on the empirical validity of Park & Seo's (2019) methodology. Looking at the two papers there are a couple of clear differences. Firstly, it's worth noting that Park & Seo suggest in their paper that whilst their work is most similar to KOSW in the literature, the differing results are likely due to differences in dataset, demand model and control variables. What is interesting for the discussion on whether their work support AST's results though is the difference in how the two papers define products/markets. KOSW define products as in AST and define a market to be a flight between two airports in the US, regardless of the direction of the flight. On the other hand, Park & Seo define a market in an alternative manner and follow Berry et al (2006) and Berry & Jia (2010) and define a market as a unidirectional pair of original and destination airports. Moreover, KOSW tried specifically to closely replicate AST's dataset and methodology – with the exception of

replacing delta with common ownership incentive terms - and viewed their results not as independent evidence but as a robustness check of AST's results. It is interesting then that AST use Park & Seo's work to support their initial findings when KOSW is much closer to what they did and Park & Seo defined a market in a different manner altogether.

Azar & Vives (2022) also build on the results of AST, however, in order to understand their results, we must first discuss the results of Azar & Vives (2021) whose theoretical framework underpins the empirical results of the 2022 paper. Azar & Vives (2021) note that common ownership is not an industry-wide phenomenon, but an economy-wide one and that this overlooked distinction is in fact crucial. They discuss that much of the rise in common ownership figures is due to the increase in 'universal owners' i.e. large institutional investors who hold shares in almost all major firms in an economy e.g. Blackrock, Vanguard, State Street. They introduce a general equilibrium model and show that economy-wide common ownership actually leads to lower markups in their framework because in a general equilibrium model the expansion of one industry generates positive externalities for firms in other industries. In this way, inter-industry common ownership creates incentives for firms to expand, which reduces their prices in that industry relative to the general price level. They show that in their standard model this effect outweighs the intra-industry effect that common ownership of firms in the same industry generates, and the overall effect is a reduction in markups. To summarise, investors are maximizing portfolio profits across the economy and not within a single industry and the externality effect across industries outweighs the anticompetitive effect within an industry. Azar & Vives (2022) follow on from this and incorporate intra-industry common ownership into AST's framework. Their primary result is that intra-industry common ownership is still associated with higher airline prices, but that as predicted by Azar & Vives (2021) inter-industry common ownership is associated with lower prices. The aggregate effect varies across routes, and this is driven by the fact that some shareholders portfolios are concentrated within the airline industry and so they do not benefit from across industry externalities and so the externality effect is not present for those shareholders. They further test their theory of 'universal shareholders' reducing markups by investigating the impact of common ownership by the big three: Blackrock, Vanguard and State Street, compared to common ownership by any other firms. They find that common ownership by the big 3 (universal investors) is associated with lower prices, whilst common ownership

outside of the big three is associated with increased prices. This lends further support to the theoretical results of Azar & Vives (2021).

Coming back to my point that the key question at the heart of this literature is whether we are measuring common ownership correctly, I now turn to O'Brien (2017). This presents an argument that price-concentration analysis does not have any grounding in economic theory and as a result the coefficients in price-concentration relationships have no clear interpretation. Given the MHHI measures the component of concentration arising due to common ownership, this argument is a direct rebuttal of the credibility of MHHI in measuring the impact of common ownership on price. The entire paper is essentially centred around one key point, and O'Brien provides many illustrations of. The chief argument is as follows:
O'Brien notes that in trying to measure the impact of common ownership on price, price-concentration analysis first estimates the relationship between price and concentration, and assumes concentration is a proxy for the impact of common ownership. However, if the relationship between price and concentration is ambiguous then concentration is not a good proxy for the impact of common ownership and the entire analysis loses meaning. O'Brien shows that there is indeed a spurious relationship between MHHI (concentration) and price. He shows that changes in MHHI that yield a higher price may increase or decrease concentration. Thus, regressions of price on MHHI don't capture any meaningful relationship between price and common ownership. He argues that the literature has not addressed this fundamental problem and has only attempted to address endogeneity issues through the use of IV's but resolving endogeneity issues doesn't resolve this critical lack of relationship between MHHI and price.

Fundamentally, O'Brien argues there is no functional relationship between price and concentration (MHHI), in that there is no one-to-one mapping and a given level of MHHI can be associated with two or more different prices. He provides an array of arguments under different scenarios to prove an impossibility result which states that no functional relationship between price and concentration exists under the relevant domain and implores that this is a vital flaw in this analysis.

Interestingly, O'Brien does mention that the method employed by KOSW – that is, replacing MHHI with the common ownership incentive terms – is a theoretically grounded approach that doesn't face his impossibility critique.

The Banking Industry

GHHI

The GHHI (Generalised Herfindahl-Hirschman Index) was first introduced by Azar, Raina & Schmalz (2016). The authors wanted to capture the impact of both common ownership and cross ownership on competition in the banking industry. They note that the MHHI was not suitable for this purpose as it only considered direct ownership i.e. who has a direct cash flow right (shareholding) to the profits of the firm. The GHHI is derived from the Cournot game in much the same way as the MHHI, and it takes the same behavioural assumption of managers, in that they act to maximise the weighted average of their shareholders returns. The formula for GHHI's computation appears the same as that of the MHHI, specifically GHHI can be computed as follows:

$$GHHI = \sum_k \sum_j \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}} \right) s_k \, s_j$$

As with the MHHI, this can be decomposed into:

$$GHHI = HHI + GHHI\ delta$$

So that the GHHI delta is the difference between GHHI and HHI, and thus captures the component of concentration arising as a result of common and cross ownership.

GHHI and MHHI differ in that where MHHI uses direct ownership as its beta's, the GHHI solves for ultimate ownership and uses this in place of direct ownership. Similarly, GHHI uses ultimate control in place of direct control as its gamma's. This allows GHHI to capture both common ownership and cross ownership. The authors note that in industries (such as the airline industry studied in Azar et. al, 2018) where there is no cross ownership and direct ownership is the same as ultimate ownership, the MHHI and the GHHI are the same.

The distinction between direct and ultimate ownership is as follows: Direct ownership refers to shareholders who have purchased shares in a firm and have a direct right to a proportion of that firms' profits. Ultimate ownership on the other hand refers to the indirect case, where some owner may own shares in a firm who has cross holdings in some other firm, and thus the owner has an indirect holding in the second firm. For a simple example, suppose shareholder 1 holds 20% of the shares in firm A, and firm A has a 10% holding in firm B. Shareholder 1 thus has a 2% holding in firm B (0.2*0.1) and is therefore an ultimate owner of firm B despite not having any direct holdings in that firm. In this case, the MHHI would take shareholder 1 as

being an owner in firm A but not firm B, whereas GHHI would consider shareholder 1 to also be an owner of firm B.

Interestingly, although the authors account for the occurrence of cross ownership in the banking industry, they find that it doesn't have a large impact on their empirical findings. So, whilst they conclude that the GHHI outperforms the HHI in explaining price differences across the banking industry, one cannot say whether it is superior to the MHHI or not – a potentially interesting and open question for proponents of concentration-based measures of competition.

GHHI in the Literature

The only paper I am aware of that specifically studies the empirical relevance of GHHI is the paper which introduced it by Azar, Raina & Schmalz (2016). Their central research question was whether the ownership of banks had an empirically material impact on their competitive behaviour. They explored this by studying whether the GHHI would outperform the HHI in explaining variation in the price of banking products – the idea being that because the only difference between the measures is that GHHI takes into consideration cross and common ownership, that if GHHI outperformed HHI, then they could infer that ownership was playing a role in determining the price of banking products.

Their findings were that GHHI was a more robust predictor of market outcomes than the HHI and they provided evidence to support that there might exist a causal relationship between prices of banking products and GHHI.

Gramlich and Grundl (2017) also study the banking industry and propose an alternative approach to the use of concentration-based measures of common ownership in assessing ownerships impact on market outcomes. The underlying point of this paper is that combining the weightings matrix with market shares in the way done in the derivation of MHHI and GHHI dilutes the interpretability of the results with respect to the impact of common ownership itself. For this reason, the authors suggest that one studies the ownership matrix itself to investigate the impact of ownership on market outcomes.

Specifically, the equations they estimate in the paper are as follows:

$$p_j = \theta_1 w_{jj} + \theta_2 \overline{w}_{kj} + \Theta_p X_j + \varepsilon_j$$

$$q_j = \theta_3 w_{jj} + \theta_4 \overline{w}_{kj} + \Theta_q X_j + \varepsilon_j$$

Where w_{jj} is the weight that firm j places on itself, $\overline{w}_{kj}$ is the average weight firm j's competitors place on firm j, and X_j includes fixed effects such as bank and market time.

They are testing the following null and alternative hypothesis:

$$H_0: \theta_1, \theta_2, \theta_3, \theta_4 = 0$$

$$H_1: \theta_1, \theta_2 > 0 \; and \; \theta_3, \theta_4 > 0$$

In plain English:

$$H_0: Common\ ownership\ does\ not\ impact\ competition$$

$$H_1: Common\ ownership\ is\ anti-competitive$$

They suggest that this approach of using the weightings directly offer several advantages over concentration-based measures.

Firstly, the weightings approach does not impose a particular model of competition i.e. Cournot or Bertrand, which is less restrictive than the Cournot specific approach of concentration based measures. Secondly, this approach requires only data on ownership and not on market shares, so it requires less data than MHHI or GHHI. Thirdly, the authors point out that MHHI and GHHI vary only at the market-time level whereas the weightings matrix W varies at the market-time-firm level which allows one to study hypothesis at a firm level and thus helps to avoid some of the endogeneity concerns of concentration-based measures. This final point is a bit less obvious in its merit, so let's expand on this: Essentially, what is being said is that MHHI and GHHI vary at a market level and so as long as aggregate outcomes are similar these measures will not change much even if there have been big changes at a firm level. For instance, ownership concentration can remain at a similar level aggregately but if there is a high rate of change in ownership then this may not capture an accurate picture on the level of competition. Consider the example presented by the authors: there are three competing firms such that A and B are commonly owned, and C is independently owned. In the next period, it might be that ownership changes so that C and B are now commonly owned, but A is now independently owned. The overall concentration of ownership in the market has not changed and so concentration-based measures of common ownership will not change much, however at a firm level ownership has changed quite substantially. The weightings matrix will capture this nuance and so offers a more accurate picture of what's happening at a firm level. This therefore

remedies some of the issues of endogeneity associated with concentration-based measures because we can see what is happening to ownership more precisely.

Specifically, the paper studies the banking industry. One interesting feature to note in this is that the prices they were using were essentially "reverse prices" because in this industry the banks (firms) are paying the consumers and so a higher price represents fiercer competition in this setting. They estimate the equations outlined earlier and the major outcome of their results show no clear evidence on the impact of common ownership on competitive conduct. They found mixed signs and low magnitudes on the coefficients – and in some cases even found evidence of (small) pro-competitive effects of common ownership on market outcomes – a stark contrast to the evidence presented by concentration-based measures.

Research into other Industries

We now turn to studies which consider industries outside of banking and airlines in assessing the impact of common ownership on competition.

Koch et al (2020) investigate whether common institutional ownership across firms in an industry has any impact on the prevailing level of product market competition. They investigate this using a variety of industry classifications, several measures of common ownership and profitability, as well as allowing for consideration of non-price competition. Essentially, the overarching theme of their results across specifications is that there is no significant relationship between common ownership and product market competition. Their evidence falls much more in line with the findings of DGS and KOSW. They look at the American manufacturing industry as well as the American airline industry in order to see if there is anything to AST's results.

Of particular interest to our discussion is what measures of common ownership the paper used. The authors employed 5 different measures. Specifically, they used the density of common ownership a la Azar (2011), the percentage of common funds, the percentage of common stocks, the MHHI and the common ownership incentive term 'C' as used by KOSW. From our discussion so far, we know that a test of common ownership is only as good as the measure of common ownership – there's no point looking for the impact of a measure if we are measuring the wrong thing. The first three measures used don't capture the incentives of common ownership and are just statistics of shared ownership which have no theoretically grounded relationship with product market outcomes. Thus, employing these is at most a test for correlation and doesn't offer any understanding behind the mechanisms involved nor does it

help in providing evidence of causation. There is nothing inherently wrong with this approach but it's important to be aware of the limited analysis we can draw from these measures. We've already discussed the drawbacks of the MHHI in detail and so I won't repeat this, but essentially the most promising of the employed measures is C – the common ownership incentive term. However, as stated earlier, none of the measures provided any statistical evidence in support of the common ownership hypothesis.

Llewellen & Lowry (2021) outline an important consideration which was overlooked by much of the previous literature. The main takeaway from the paper is that using mergers that occurred around the period of the 2008-2009 financial crisis as a source of exogenous variation in common ownership may mistakenly attribute improved performance which naturally follows a major recession with anticompetitive effects of common ownership. They specifically discuss the Blackrock-BGI merger (as used by AST) and show that when one uses only mergers which did not occur around the 2008-2009 period there is no evidence to suggest that common ownership leads to anticompetitive outcomes.

Nain & Wang (2016) investigate the relationship between partial equity ownership and product market competition using a relatively large cross industry sample of manufacturing firms in the US. They study this by looking at 774 completed acquisitions (of less than 50% of a firm's equity) in the manufacturing industry and looking at whether the retail producer price index (RPPI) and price-cost margins increase following an acquisition – an outcome consistent with reduced product market competition. They find that after a completed acquisition, RPPI is 2% higher and price-cost margins are 0.7% higher. Moreover, both results are significant. They later show that the anticompetitive effects of partial equity ownership are greater in industries with higher barriers to entry – in line with the findings of Reynolds & Snapp (1985). The paper thus provides the first large sample evidence in support of the common ownership hypothesis.

He & Huang (2017) claim to provide evidence of a causal relationship between common institutional ownership and reduced product market competition. They show causation using a difference-in-difference approach. The authors used five statistical measures of cross ownership, the main one of which they call *CrossDummy*, which is a dummy variable that takes the value of 1 if the firm is cross held by any of its competitors in one of the quarters of the given fiscal year, and 0 if it is not. They use market share as the main measure of product market competition. Right off the bat, we can see from our previous discussion that this approach is not grounded in any theoretical relationship and is simply a statistical measurement

of correlation. Nonetheless, the paper does provide an interesting narrative and offers evidence to support that common ownership is anticompetitive outside of the American airline industry. They look at firms with common stocks traded on NYSE, NASDAQ and AMEX between 1980-2014 and assign industry using 4-digit SIC codes. I think it's a real strength of their analysis that they considered several industries, as lack of diversity in industries being studied is a current weakness in the literature as a whole.

Backus, Conlon & Sinkinson (2021) introduce a new empirical method for investigating the common ownership hypothesis. They employ an exclusions restriction test (Berry & Haile, 2014) to alternate models of conduct to study whether the data best supports joint or own profit maximisation. They then use this approach in the ready-to-eat cereal industry to test the common ownership hypothesis. Evidence on the prevalence of reduced competition arising from ownership structure from a new industry is a valuable contribution to the literature because as we've seen from our earlier discussion, past work focuses almost exclusively on the banking and airline industries and before policy makers can have sufficient scope for introducing new legislation, evidence from industries across the economy are needed in order to understand the scope of the problem – if indeed there is a problem at all.

There are several aspects of the RTE cereal industry which make it a suitable candidate for this analysis. Firstly, it is an oligopolistic industry dominated chiefly by four major firms, and these firms all have considerable variation in ownership structure – which makes it an interesting industry for a study on the impact of ownership. The authors also note that there are many transactions in the ownership space which allows for the study of intertemporal variation in ownership on product market interactions across the firms. Finally, their choice was driven by the extensive past literature on this industry (Nevo, 2000, 2001) which outlines appropriate modelling choices for the empirical framework – at least for the timeline considered.

For comparison to previous literature the authors conduct a regression of product prices (at a firm level) against MHHI – though they note that such regressions have no meaningful interpretation in the context of differentiated products and that they conduct this analysis for the sole purpose of placing their findings in context with existing work. Nonetheless, they find that in fact MHHI delta is negatively associated with prices – suggesting that common ownership is pro-competitive in this setting. However, as the authors note, this is not a robust finding due to the spurious interpretation of delta when products are differentiated.

The main analysis of this paper utilizes a discrete choice multinomial logit demand system similar to that of Berry et al (1995) and Nevo (2000). They estimate diversion ratios and find that they are close to 35% which suggests that most consumers will switch brands if their favourite product becomes more expensive. This is a good sign for the competitive dynamics of the industry generally and suggests that common ownership would only be effective at raising prices if it could influence all (or at least most) of the main players. It would be no good raising your prices if consumers will just switch products, and for this reason 'coordination' across firms would need to involve many firms raising their prices for it to be profitable – at least in theory, however it could still be problematic if firms are diverting sales to another product which they own.

The paper considers four main models of conduct across firms: own price maximisation, perfect competition, a (hypothetical) monopolist comprising the top four major players and common ownership. They then conduct analysis to see which of these proposed behavioural strategies best captures the variation observed in the data and find strong evidence to suggest that firms behave in a way consistent with own price maximisation. Moreover, they go on to test own price maximisation against partial common ownership directly and once again find that their results are consistent with that of own price maximisation and repeatedly, they find evidence to reject tests of common ownership.

Overall, Backus et al (2021) find no evidence to suggest that pricing patterns which theory implies would occur as a result of common ownership are consistent with the pricing behaviour, they observe in the data for the RTE cereal industry.

European Studies

The astute reader might have noticed that so far, almost all of the papers discussed – certainly on the empirical side of the modern literature – are based on studies of American markets. There are currently not an extensive number of papers researching common ownerships competitive impact outside of the US, but in this section, I will cover some of the main work relating to common ownership in European markets.

The Joint Centre of Research [3](2016) carried out a study on common ownership levels across the EU – they state that it is to their knowledge the most widespread study of this topic

[3] JRC for short

concerning European markets. The report considered levels and trends in common ownership across 2007-2016 using data from the Orbis database. The general consensus of the report was that over recent years common ownership levels have increased in Europe – specifically they find that common shareholding with at least 5% ownership in more than one firm involved 67% of listed companies in 2016. Moreover, they state that the top shareholders held shares in up to 25% of firms, and there was a trend for these firms to be the biggest in their industry. This preference for of the largest shareholders for the largest firms increased in recent years and consequently, firms included in the largest portfolios represented over 90% of market capitalisation in all years across 2010-2016.

The JRC also studied the impact of the Blackrock-BGI merger on the Lerner Index of firms in the EU beverage industry. The merger was used as an endogenous shock to common ownership levels (representing an increase in common ownership). From this analysis, the JRC found a positive relationship between common ownership and market power, which provides some evidence that common ownership might be anticompetitive in this market. However, bear in mind that the criticism of using this specific merger as a source of endogenous variation in common ownership discussed above (Llewellen & Lowry, 2021) might explain these findings outside of common ownership having a truly anticompetitive outcome.

Burnside & Kidane (2020) also investigated common ownership from a European perspective after the European Commission cited common ownership as a decision variable in recent merger decisions – the Dow/DuPont and Mayer/Monsanto mergers[4]. The first question they addressed was whether common ownership levels in European markets were comparable to those documented in the US. Their findings show that they are not which led the authors to question the topics relevance to European markets. They suggested that European evidence is far too underdeveloped to be drawing policy implications from and suggested that reflection was needed and that policy makers shouldn't be too fast to jump on the bandwagon when too little is understood about the true implications of common ownership.

The Competition and Market Authority (CMA) conducted its State of Competition II report [5](2022) and included a section on adjusted concentration which addressed common ownership. The report provides evidence on the extent of common ownership more so than offering

[4] Links to these two merger reports in References section
[5] Disclaimer: The author of this paper, Sabria Behilil, was employed as an intern and worked on this report with the CMA. Sections of this literature review appear in the report and corresponding appendix.

evidence on its competitive impact due to some of the limitations in the data used. The dataset was constructed using data from Companies House and the Inter-Departmental Business Register (IDBR) to create a dataset which is "close to exhaustive in its coverage of the UK economy". Using this the CMA studies the extent of connectedness of ownership across British firms and used these to calculate adjusted measures of concentration to investigate possible consequences of this connectivity. The major limitation of this work is that the dataset only included common ownership where the ownership levels were at least 25% of ownership. Immediately, this means the dataset will not pick up most holdings by institutional investors – the main suspects in the common ownership story – as they almost always hold minority shareholdings. This is quite a significant drawback to the analysis and the CMA itself admit that this means the analysis will underestimate levels of common ownership in the UK. Moreover, the levels of common ownership above 25% could not be recorded precisely and were merely categorised into 3 levels: 25-50%, 50-75% and 75-100%. The overall results of the CMA's analysis were that concentrations measures (both the C10 and the HHI) fell when adjusted for the presence of common ownership. However, given the limitations of the data listed above, the results are subject to some scrutiny and should serve as a useful starting point for analysis of common ownership in UK markets.

From this discussion, it seems apparent that there is currently quite little evidence on the level and impact of common ownership in Europe. Current evidence seems very mixed in approach and there is a definite lack of consensus. Further work outside of US markets would be a valuable direction for future literature in determining whether common ownership is an American 'problem' or a more widespread phenomenon.

Conclusion

This outlined the main pieces in the literature on the impact of common ownership on product market competition at the time of writing. Early theoretical work generally provided evidence that common ownership would lead to higher prices and weakened competition. However, empirical work has provided evidence both ways and no clear consensus can yet be drawn from the literature. We have made progress though in that we are now asking some important questions – how do we measure common ownership and how should we test for it? I believe that O'Brien's evidence on the lack of a functional relationship between MHHI and price provides clear evidence that regressing ownership concentration measures on price can offer little in the way of meaningful insight into a causal relationship between ownership and

competition. However, whilst knowing how not to measure common ownership is an important piece of the story, new methods and further evidence on existing non-concentration measures are an obvious next step in the theoretical part of this story so that we can understand how to quantify the impact in a way that is appropriate for understanding causality and is based in a theoretical relationship between price and ownership.

It is also apparent that the bulk of the empirical work has focussed on the airline industry, and specifically on responding to AST. One implication of these papers being a direct response to AST is that they have tried to match their dataset as closely as possible, which means that a lot of the evidence we have either way is focussing on a (relatively) small piece of data when looking at the big picture. For this reason, studies that use other data and look at other industries using a variety of methods are vitally needed. Whilst other industries have been studied in more recent papers, a conclusion cannot be drawn from any single paper and so much more research must be conducted in this area if we are to draw policy implications.

Finally, almost all of the existing studies focus on American markets and there is very little evidence on the levels and impact of common ownership outside of the US. Further research in markets outside of the US, in Europe for example, would be a valuable addition to the literature.

One of the most exciting aspects of this literature – albeit one of the more challenging parts of writing a literature review on it – is that interest in this topic is reaching new highs in current times. It seems that everyday many new papers are coming out and producing new developments in this area. As a consequence of this, by the time this is read it will already be out of date. However, I hope that it provides a meaningful introduction to the topic and motivates some of the central questions I will attempt to address in the following chapters of my thesis.

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Chapter 2:

Quantifying Common Ownerships Impact on Competitive Outcomes

Abstract

This work outlines a method for investigating the relationship between the ownership structure of a market and the resulting competitive outcomes. The main result is the derivation of a new index which captures the direct relationship between the ownership matrix of an industry and the industry output. It provides theoretical evidence to suggest that common ownership can be anticompetitive and overcomes technical limitations associated with the MHHI – the measure of common ownership used most prolifically in the literature. Specifically, the MHHI is subject to O'Brien's (2017) impossibility critique, which shows that it has no functional (one-to-one) relationship with price, in that a single value of MHHI can be associated with both a high price and a low price. The index derived here overcomes this fundamental problem in the MHHI in that it has a functional relationship with the market outcome of interest – quantity, as we consider a Cournot setting. The model is considered under several different assumptions and some special cases are considered in more detail. All the variations of the model are presenting that common ownership and concentration are two related parts in the study of competition and that they can work together in influencing market outcomes.

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Introduction

This paper aims to investigate theoretically how common ownership – that is, having a third-party investor with holdings across competing firms - can influence the ferocity of competition within an industry under a modified Cournot setting. One of the fundamental assumptions in many models which study the interaction of competing firm is that every firm will act to maximise their own profits. However, if it is assumed that firms will act in the interest of their shareholders, and shareholders have diversified holdings across competing firms, the result might be that firms place a non-zero weighting on the profits of their competitors, resulting in a weakening of market competition (Backus et al, 2019). This effect is unilateral and doesn't require firms to coordinate. This concern has powerful implications for aggregate outcomes and welfare distributions and is called the common ownership hypothesis (COH). The question of whether common ownership is anti-competitive has never been more relevant given the startling increase of the phenomenon documented in recent times, specifically in US public firms. In the US, Azar (2016) presents that the probability of any two randomly selected S&P 1500 firms having a common shareholder with at least a 5% holding in both rose from 20% to 90% between 1999 and 2014. Moreover, Gilje et al (2017) study the percentage increase in various indicators of common ownership between 1980 and 2012 in United States public firms and document a substantial rise across indicators – varying from a 2300% to 330% increase in common ownership over the period, depending on which indicator is considered.

If the common ownership hypothesis does hold - to any extent - and common ownership has been increasing at such a substantial rate in recent times, then this could have very strong implications for market competition and consumer welfare. Several channels of impact have been discussed in the literature including the possibility of higher prices, reduced R&D and lower entry. This paper will consider the impact of common ownership on market output and the corresponding price level in a linear demand model.

In understanding common ownerships impact on market outcomes, the first and arguably most fundamental decision to be made is in how common ownership should be measured and quantified. The strongest measurement would have a clear theoretical relationship with market outcomes so that it could be interpreted in a meaningful way. As we have seen from Gilje et al's (2017) dramatic variance in changes to common ownership levels depending on which measure is chosen, the choice of measure is critical in determining the conclusions we draw on the impact of common ownership.

This paper aims to address this by providing a theoretically grounded way of relating a markets ownership structure directly to market outcomes. Using a modified Cournot setting in which shareholders have diversified holdings across the firms. We assume firms maximise the portfolio profit of a representative shareholder who, by design, will have holdings across some (or all) of the firms' competitors so that each firm is maximising the weighted profits of itself and its competitors, with the precise weighting's dependent on the portfolio of its representative shareholder. One of the key ideas our framework captures is that it's not the number of shares held by the shareholder that is influential to outcomes, but rather the ratio of their holdings across the firms. Using this maximisation problem, we derive the first order conditions and re-arrange these to obtain an expression which directly links ownership structure to market output levels. We do this under two main settings: the simplified case where firms have symmetric marginal cost, and the more realistic case where firms compete under asymmetric costs. In the first case, we derive a pure index of common ownership, which we call the Common Ownership Competition Index (COCI), and its direct impact on aggregate quantity. In the second case, we show how ownership structure and cost asymmetries work together to influence aggregate quantity and motivate that using measures of competition (such as concentration) that consider only cost asymmetries are missing half of the picture. In both cases, we provide evidence to suggest that common ownership of firms can lead to lower aggregate output and higher market prices in our setting.

This paper aims to contribute to the branch of literature concerned with developing tools used to quantify common ownerships impact on competitive outcomes.

Before discussing existing measures of common ownership, we first note that Hansen & Lott (1996) first introduced our key assumption that firms maximise portfolio values rather than individual profits. The paper produces a model which postulates that where common ownership exists shareholders do not want firms to focus on profit maximisation but rather wish them to act in a co-ordinated manner to maximise their portfolio values. They carry out a theoretical analysis of this concept, introducing a model and considering three cases: where firms have separate owners, where firms are commonly owned, and where shareholders are diversified across firms. The paper produces first a theoretical argument and then tests this using capital market data to investigate the impact of common ownership on merger prices. They present both theoretical and empirical evidence which supports the common ownership hypothesis. This chapter will also consider a theoretical model of common ownership which considers that firms act to maximise the portfolio profits of their investors. However, Hansen & Lott consider

only the case under common ownership whereby the owners hold equal shares in both firms, whereas this chapter will not place such restrictions on the degree of diversification of owners and will allow any combination of ownership to be studied.

We now turn to existing measures of common ownership.

The most used measure of common ownership at present is the modified Herfindahl-Hirschmann Index (MHHI), presented initially by O'Brien & Salop (2000). The MHHI comprises the HHI and another component the MHHI delta (MHHID) which captures the additional concentration arising because of common ownership. The MHHI has been the variable used to measure common ownership in most of the seminal empirical work in this area (Azar et al., 2018). However, the MHHI has been criticized and is not widely accepted as an appropriate measure of common ownership. Most notably for our purposes is the critique coming from O'Brien during the Federal Trade Committee hearing where he explains that the MHHI is a measure of concentration and not common ownership specifically, and so the MHHI is a poor proxy for the level of common ownership itself (Federal Trade Commission 2018). Therefore, a major critique of much of the existing work on the impact of common ownership is that it has been conducted using an inaccurate representation of ownership. O'Brien (2017) presents an argument that price-concentration analysis does not have any grounding in economic theory and as a result the coefficients in price-concentration relationships have no clear interpretation. Given the MHHI measures the component of concentration arising due to common ownership, this argument is a direct rebuttal of the credibility of MHHI in measuring the impact of common ownership on price. The entire paper is essentially centralised around one key criticism, and O'Brien provides many illustrations of this one underlying point. The chief argument is as follows: In trying to measure the impact of common ownership on price, price-concentration analysis first estimates the relationship between price and concentration, and assumes concentration is a proxy for the impact of common ownership. However, if the relationship between price and concentration is ambiguous then concentration is not a good proxy for the impact of common ownership and the entire analysis loses meaning. O'Brien shows that there is indeed a spurious relationship between MHHI (concentration) and price. He shows that changes in MHHI that yield a higher price may increase or decrease concentration. Thus, regressions of price on MHHI don't capture any meaningful relationship between price and common ownership. He argues that the literature has not addressed this fundamental problem and has only attempted to address endogeneity issues using IV's, but resolving endogeneity issues doesn't resolve this critical lack of relationship between MHHI and price.

Fundamentally, O'Brien argues there is no functional relationship between price and concentration (MHHI), in that there is no one-to-one mapping and a given level of MHHI can be associated with 2 or more different prices. He provides an array of arguments under different scenarios to prove an impossibility result which states that no functional relationship between price and concentration exists and implores that this is a vital flaw in this analysis. The model presented here relates the ownership matrix directly to market outcomes in a way which is directly grounded in economic theory and so we aim to provide a more explicit measurement of common ownership that overcomes the MHHI's lack of theoretical grounding.

Another variation of the HHI, The GHHI (Generalised Herfindahl-Hirschman Index) was first introduced by Azar, Raina & Schmalz (2016). The purpose of this measure was to capture the impact of both common ownership and cross ownership on competition (as opposed to the MHHI which only captures the impact of common ownership). The GHHI is derived from the Cournot game in much the same way as the MHHI, and it takes the same behavioural assumption of managers in that firms act to maximise the weighted average portfolio of shareholders. However, as with the MHHI, there is no demonstration of any theoretical relationship between GHHI and price levels because the GHHI is also a measure of concentration.

Gramlich & Grundl (2017) and Kennedy et al (2017) suggest using the weightings that firms place in each other profits instead of using concentration industries such as the MHHI to measure common ownership. However, once again, there is no theoretical model presented which directly explains how these weights are related to price and these papers instead focus on the results of regressing these weightings against price.

Gilje at al (2019) introduce a measure of common ownership called GGL, which postulates that managers only act in the interest of attentive shareholders i.e., managers will maximise the portfolio profits of only the attentive shareholders. Essentially, they recognise that not all shareholders are paying attention to management action and so how managers decisions impact these shareholders payoffs is immaterial to the manager. What they are proposing here is an alteration to the management objective function. It is assumed that an investors probability of being attentive is a positive function of how important that firm is in their portfolio, and this is taken to be proportional to their ownership stake in that firm. The authors develop a variation to a Cournot model which shows how investors probability of being attentive impacts the manager's objective function. GGL is made up of the extent to which managers value their

shareholders preferences, the weight that shareholders place on competitors' profits and the probability that the shareholder is attentive. The paper then regresses this measure against price in investigating how it relates to market outcomes. As with the other measures, GGL is not derived from a theoretical relationship with market outcomes.

As with all the measures of common ownership discussed, the model presented in this work has both its strengths and its weaknesses. The primary strength of this work is that it is derived from an explicit theoretical relationship in which the ownership matrix itself is linked directly to a market outcome of interest (output). However, the derivation of the model as presented in this work relies on the assumption of Cournot competition, which is not an appropriate assumption in all industries. Moreover, this work does not address potential agency problems that might occur between management and shareholders – we touch on this during the discussion section of the paper.

Thus, the overarching contribution of this work is to produce what is to provide a new way to study common ownership using a model which provides a clear theoretical demonstration of a direct link between the ownership matrix of an industry and the level of competition that will prevail.

The paper proceeds as follows:

Section 2 introduces the model and its setting under the assumption of symmetric marginal costs, section 3 outlines how the model can be used to obtain an index for market competition, section 4 relaxes the assumption of symmetric marginal costs, section 5 considers in greater detail the case of the model where there are only 2 firms and 2 shareholders, section 6 discusses the results and section 7 concludes.

The Model

The Setting

The setting for the model is as follows: We have m shareholders and n firms competing against each other a la Cournot in a single market. Initially, we consider the simple case under the assumption that firms have identical constant marginal cost of production c. Firms face a linear market demand and profit functions characterised as: $P=a-bQ$ and $\pi_i=(P-c)q_i$. Trivially, it must be true that $q_i \geq 0 \; \forall i \in n$.

The shareholders can each hold shares in any number of firms within 1,..,n (if they held shares in no firm they would not be a shareholder). Each shareholder has a weighting in their firms which denotes the proportion of the shares in that company held by that shareholder, for example w_{12} =0.2 would indicate that shareholder 1 holds 20% of firm 2's shares and so is entitled to 20% of firm 2's profits. For now, we assume that $w_{ij} \in \mathbb{R}$ where i=1,..,n and j=1,…,n. We consider that shareholders will aim to maximise their total portfolio value and not the total value of each firm independently. So, with respect to what influences firm's behaviour, it's not the number of shares that each shareholder holds, but rather the ratio of their holdings across the firms, as this will impact how much they care about that firms profits relative to the others. Simply put, if a shareholder holds 80% of the shares in firm i and only 2% of the shares in firm j, they benefit much more from the profit of firm i and so may wish that firm j does not compete aggressively to benefit firm i.

Given we have m shareholders, we assume that each firm will act in the interest of some representative shareholder who has a positive weighting in their firm. There are many possible avenues for how a firm might select a representative shareholder, however, addressing this quandary is a paper in itself and so for the purpose of deriving the model in this work, we simply assume that in each firm some shareholder has been assigned to be the representative owner. We thus obtain n representative shareholders – one for each of our n firms. In the discussion which follows we use the term shareholder i to refer to the representative shareholder of firm i, for i=1,…,n.

Shareholder Problem

Given that firms are acting in the interest of their representative shareholder, we need to know the maximisation problem of the representative shareholder as this will form the objective function of the firm. So, for instance, the shareholder problem for shareholder 1 will be to maximise their portfolio value π_1^P given their weighting in each of the n firms, and so firm 1 will behave according to the following maximisation strategy:

$$w_{11}\pi_1 + w_{12}\pi_2 + w_{1k}\pi_k + \cdots + w_{1n}\pi_n = \sum_{i=1}^{n} w_{1i}\pi_i = \pi_1^P$$

Similarly, firm n would act to maximise:

$$w_{n1}\pi_1 + w_{n2}\pi_2 + w_{nk}\pi_k + \cdots + w_{nn}\pi_n = \sum_{i=1}^{n} w_{ni}\pi_i = \pi_n^P$$

Thus, for the n firms we arrive at a system of n equations which can be summarised in matrix form as follows:

$$\begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nn} \end{bmatrix} \begin{bmatrix} \pi_1 \\ \vdots \\ \pi_n \end{bmatrix} = \begin{bmatrix} \pi_1^P \\ \vdots \\ \pi_n^P \end{bmatrix}$$

Where: $\pi_i = (a - b(q_1 + q_2 + \cdots + q_n) - c)q_i$

We can summarise this system as:

$$W\pi = P$$

Where W is the square matrix of order n containing all the weightings of each of the n shareholders in the n firms, π is the nx1 column vector containing the profits of each of the n firms, and P is the nx1 column vector containing the portfolio profits of each of the n representative shareholders.

Obtaining the Ownership Matrix

The next step towards obtaining our index is to normalise the weightings matrix **W** by dividing every element of the ith row by the ith diagonal to obtain a matrix in which element kj captures firm k's representative shareholders ratio of holdings in firm j relative to firm k. Thus, the elements of this matrix capture the weightings of each representative shareholder as proportions to indicate how much each shareholder values the profits of competing firms relative to the firm that they are the representative holder in. Trivially, every diagonal in this normalised matrix will equal 1 as the diagonal elements will be divided by themselves. We thus obtain:

$$\begin{bmatrix} \frac{w_{11}}{w_{11}} = r_{11} = 1 & \cdots & \frac{w_{1n}}{w_{11}} = r_{1n} \\ \vdots & \ddots & \vdots \\ \frac{w_{n1}}{w_{nn}} = r_{n1} & \cdots & \frac{w_{nn}}{w_{nn}} = r_{nn} = 1 \end{bmatrix}$$

With this normalisation in mind, we can now rewrite the maximisation problem of shareholder i as:

$$\max(\pi_i + r_{i1}\pi_1 + r_{i2}\pi_2 + r_{i3}\pi_3 + \cdots + r_{in}\pi_n)\forall i = 1,..,n$$

Thus, the problem for the representative shareholder of firm 1 will be:

$$\max(\pi_1 + r_{12}\pi_2 + r_{13}\pi_3 + \cdots + r_{1n}\pi_n)$$

Solving for the first order conditions (FOC's) we obtain:

$$\frac{\partial(\pi_1 + r_{12}\pi_2 + \cdots + r_{1n}\pi_n)}{\partial q_1} = 0$$

Yielding the following FOC for firm 1:

$$\frac{a-c}{2b} = q_1 + \frac{(1+r_{12})}{2}q_2 + \frac{(1+r_{13})}{2}q_3 + \cdots + \frac{(1+r_{1n})}{2}q_n$$

So generally, we see that the FOC for firm n is:

$$\frac{a-c}{2b} = q_n + \frac{(1+r_{n1})}{2}q_1 + \frac{(1+r_{n2})}{2}q_2 + \cdots + \frac{(1+r_{nn-1})}{2}q_{n-1}$$

And so, across our n firms we have the following system of equations capturing the FOC's:

$$\begin{bmatrix} 1 & \cdots & \frac{1+r_{1n}}{2} \\ \vdots & \ddots & \vdots \\ \frac{1+r_{n1}}{2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} \frac{a-c}{2b} \\ \vdots \\ \frac{a-c}{2b} \end{bmatrix}$$

We define the matrix R to be:

$$R = \begin{bmatrix} 1 & \cdots & \frac{1+r_{1n}}{2} \\ \vdots & \ddots & \vdots \\ \frac{1+r_{n1}}{2} & \cdots & 1 \end{bmatrix}$$

This is the matrix of ownership which we will use in our analysis. We can now state the following theorem.

Theorem 1: Supposing R has full rank, and $q_i \geq 0$ $\forall i \in n$, when firms have symmetric marginal costs, we can express market output as a function of common ownership within the industry as:

$$Q = \frac{a-c}{2b} e^T R^{-1} e$$

Where e is simply a nx1 vector of 1's: $e = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ *and* e^T *is the transpose of this vector.*

We can also find the individual quantity of each firm:

$$q = \frac{a-c}{2b} R^{-1} e$$

Proof in Appendix 1.

An Index for Market Competition

We now state:

Theorem 2: Supposing R has full rank, and $q_i \geq 0$ $\forall i \in n$, when firms have symmetric marginal costs, competition is increasing in $e^T R^{-1} e$, with monopoly occurring when $e^T R^{-1} e = 1$, perfect competition occurring when $e^T R^{-1} e = 2$ and Cournot occurring when $e^T R^{-1} e = \frac{2n}{n+1}$

Look again at our expression for market output presented in theorem 1:

$$Q = \frac{a - c}{2b} e^T R^{-1} e$$

Notice that $e^T R^{-1} e$ multiplies out to simply become a scalar which isolates the influence of the ownership matrix on quantity. We thus have an expression for the aggregate market quantity which is simply the product of two scalar – the monopoly quantity and $e^T R^{-1} e$. We can see from this that if $e^T R^{-1} e = 1$ that aggregate output is equal to the monopolistic quantity, and if $e^T R^{-1} e = 2$ we have the perfectly competitive market quantity. We have therefore uncovered a pure index for measuring the competitiveness of a market[6] which requires only knowledge of the ownership structure of the market. We call $e^T R^{-1} e$ the Common Ownership Competition Index (COCI). As COCI moves away from 2 and closer to 1 the market becomes less competitive. According to this framework, a reduction in the magnitude of COCI would indicate a weakening of competition by resulting in a reduction in output and a corresponding increase in price level.

Relaxing the Assumption of Symmetric Marginal Costs across Firms

Suppose now that we relax the assumption of symmetric marginal costs for the firms. This makes the model more interesting as it now allows for the fact that competition is no longer determined solely by the number of firms N in the market. Competition will now also depend on the degree of cost asymmetries across firms and allows us to consider what happens in the market when both cost asymmetries and common ownership can influence market outcomes together.

[6] Under our assumptions, including the assumption of homogenous technology across firms (constant marginal costs).

We can now derive a more general version of the index with this relaxation in place:

We now say that firm 1 has a marginal cost of c, and all other firms have a marginal cost of $\mu_i c$ where $0<\mu_i<1$. The maximisation problem would be the same as before but now the FOC will come out as:

$$\begin{bmatrix} 1 & \cdots & \frac{1+r_{1n}}{2} \\ \vdots & \ddots & \vdots \\ \frac{1+r_{n1}}{2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} \frac{a-c}{2b} \\ \vdots \\ \frac{a-\mu_n c}{2b} \end{bmatrix}$$

Denoting this in matrix form as:

$$Rq = C$$

We can now state:

Corollary 1: Supposing R has full rank, and $q_i \geq 0 \; \forall i \in n$, when firms have asymmetric marginal costs we can express market output as a function of common ownership within the industry as:

$$Q = e^T R^{-1} C$$

Where e is simply a nx1 vector of 1's: $e = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ and e^T is the transpose of this vector.

We can also find the individual quantity of each firm:

$$q = R^{-1} C$$

A Closer Look at the 2x2 Case

We now carry out some analysis of the simple case with two firms and two shareholders to illustrate the sort of intuition we can obtain on the influence of ownership structure from this analysis.

Before we begin, notice that because we only have two shareholders in each firm, the shareholder with the majority holding effectively holds control over that firm's behaviour (as they have the majority of votes, and the other shareholder cannot out-vote their decision) and so in this case we take the majority shareholder to be the representative shareholder for a firm.

To be clear, in the discussion that follows we will take common ownership as meaning that some owner has shares in both firms. So higher common ownership will mean a greater degree

of overlap in the ownership of firms. With respect to our weightings, a higher degree of common ownership means some owner has a higher weighting in both firms e.g. (0.6, 0.4) where (w_{11}, w_{12}) represents a higher degree of common ownership than (0.6, 0.2). Common ownership will be highest when one owner has the majority holding in both firms and will be lower when each firm has a different majority owner – lowering as the minority shareholder in a firm has a smaller and smaller weighting.

In the 2x2 case there will be two main cases. Firstly, we have the case whereby each firm has a different majority shareholder, and secondly the case whereby both firms have the same majority shareholder. We can summarise all possible types of distribution across ownership in the following table:

	$w_{11} > \frac{1}{2}$	$w_{11} = \frac{1}{2}$	$w_{11} < \frac{1}{2}$
$w_{22} < \frac{1}{2}$	Shareholder 1 owns majority in both firms	Shareholder 1 owns majority share in firm 2 Equal split in firm 1	Shareholder 2 owns majority in firm 1 Shareholder 1 owns majority in firm 2
$w_{11} = \frac{1}{2}$	Shareholder 1 owns majority share in firm 1 Equal split in firm 2	Equal split in both firms	Shareholder 2 owns majority in firm 1 Equal split in firm 2
$w_{22} > \frac{1}{2}$	Shareholder 1 owns majority in firm 1 Shareholder 2 owns majority in firm 2	Equal split in firm 1 Shareholder 2 has majority ownership of firm 2	Shareholder 2 has majority in both firms

In the analysis that follows we will primarily focus on the case such that shareholder 1 is the majority owner in firm 1 and shareholder 2 is the majority owner in firm 2. The outcome will be symmetrical in the opposite case such that shareholder 1 owns firm 2 and shareholder 2 owns firm 1, so it's complete to consider only one of these explicitly. We will also consider the case whereby one owner has the majority in both firms, again these cases will be symmetric. It's worth noting that this model doesn't precisely address the situation in which there is an equal split in ownership across both the firms and it's unclear what happens when there is a majority in one firm and an even split in the other as the model doesn't suggest who makes the

decisions in the case of a 50:50 split. In the case of there being a 50:50 split across both firms it could be argued that who makes the decision is irrelevant because both shareholders have identical portfolios and will thus agree on how the firm should behave under our framework.

Let us begin by considering the case whereby shareholder 1 holds the majority in firm 1 and shareholder 2 holds the majority in firm 2. We can now setup the FOCs as derived in the previous section:

$$\begin{bmatrix} 1 & \frac{1+r_{12}}{2} \\ \frac{1+r_{21}}{2} & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \pi_1^P \\ \pi_2^P \end{bmatrix}$$

$$\text{Where} \begin{bmatrix} 1 & \frac{1+r_{12}}{2} \\ \frac{1+r_{21}}{2} & 1 \end{bmatrix} = R$$

We can now find a general expression for our index $e^t R^{-1} e$ to see how it is related to the ratio of holdings held by each of the two shareholders. To begin, we calculate the inverse matrix:

$$R^{-1} = \begin{bmatrix} \frac{(1+r_{12})(1+r_{21})}{r_{12}+r_{21}+r_{12}r_{21}} & -\frac{1+r_{21}}{r_{12}+r_{21}+r_{12}r_{21}} \\ -\frac{1+r_{12}}{r_{12}+r_{21}+r_{12}r_{21}} & \frac{(1+r_{12})(1+r_{21})}{r_{12}+r_{21}+r_{12}r_{21}} \end{bmatrix}$$

And we can then quickly show:

$$e^t R^{-1} e = \frac{2(-2+r_{12}+r_{21})}{-3+r_{12}+r_{21}+r_{12}r_{21}}$$

In terms of aggregate output this would mean the following situation:

$$Q = \frac{a-c}{2b}\left(\frac{2(-2+r_{12}+r_{21})}{-3+r_{12}+r_{21}+r_{12}r_{21}}\right)$$

We can now see that $e^t R^{-1} e$ is decreasing, and thus competition is becoming weaker, as r_{12} and r_{21} grow larger. Given that we assumed at the start that shareholder 1 held the majority in firm 1 and shareholder 2 held the majority in firm 2, what we are seeing here is that market output is decreasing when shareholder 1 holds more shares in firm 2 and shareholder 2 holds more shares in firm 1. Essentially this is showing us that market output is decreasing when there is a greater degree of common ownership across the firms. Given the linear relationship between quantities and price in this framework, if higher common ownership leads to a fall in

output, we can see that this will yield a higher market price. This is consistent with higher common ownership resulting in a reduction in competition.

Essentially this is stating that quantity will decrease, and price will increase when firms place a greater weighting in their competitors' profits. This is in line with the argument that common ownership is anticompetitive and that if a shareholder has joint holdings in competing firms, they will maximise their portfolio profits rather than individual firm profits. This has implications for welfare as we know from standard economic theory that total societal welfare will be maximised when quantity is at the perfectly competitive level and will fall as output is restricted, with the most extreme plausible reduction of output being the monopoly level. Given we have assumed a standard setup for market output and price, this logic should apply to our case and so as output falls there is a deadweight loss to society and producer surplus increases at the cost of falling consumer surplus. As total output is seen to be decreasing in common ownership, we can conclude that total societal welfare and consumer surplus will be decreasing in the common ownership whereas producer surplus will be increasing. We summarise this in the following proposition:

Proposition 1: Under our framework in the case with two firms and two shareholders, market output, total societal welfare and consumer surplus are falling as the minority owners share in a firm increases, whilst producer surplus and price increase. This supports that common ownership is anticompetitive.

Let us now consider what would happen if some shareholder was the majority in both firms. Suppose now instead that some shareholder has the majority in both firms, this means that they will want to maximise the joint profits of the two firms (and hence the industry) subject to their weightings across the two firms. We can intuitively see that if there is only one shareholder making the decisions for the industry that they will choose to produce the monopoly outcome to earn maximal profits. There are three possible cases which could arise here:

- The majority shareholder has a greater weighting in firm 1 than firm 2 and so it will be optimal for them to only keep firm 1 active and shut down firm 2
- The majority shareholder has a greater weighting in firm 2 than firm 1 and so it will be optimal for them to only keep firm 2 active and shut down firm 1
- The majority shareholder has identical weightings in both firms and so will not necessarily want to shut down either firm. The majority shareholder is indifferent

between production in the two firms and so there are many possible equilibria where both firms are active.

For proof of these statements, see Appendix 2.

The majority shareholder will thus choose to shut down the firm in which it has a smaller holding. The case whereby it is the other firm that the majority owner has a greater shareholding in will simply be symmetrical. Now that we have clarified why this is the case, let us move on to show what this implies for aggregate outcomes.

Given the majority owner in both firms would choose to shut down one firm and produce the monopoly output in the firm in which they have a larger holding, in all three cases we have that, aggregately, the monopoly outcome prevails, and the industry is characterised by:

$$\begin{cases} Q = \dfrac{a-c}{2b} \\ P = \dfrac{a+c}{2} \end{cases}$$

Thus, we can see that aggregate market outcome is always the same in the case of a common majority owner and total welfare is always constant in this. The majority owner's weightings in the firms influence only the distribution of production across firms and thus the distribution of welfare, rather than the aggregate levels. Given monopoly is the least competitive outcome this supports that an ownership structure with one common majority owner is anti-competitive. We summarise this:

Proposition 2: Under our framework in the case with two firms and two shareholders, in the case that one shareholder is the majority owner in both firms, aggregate outcome will always be monopolistic and shareholdings across the firms influence only the distribution of output and welfare, and not the aggregate levels. This supports that common ownership is anticompetitive.

Isolating the Impact of Ownership Structure in the 2x2 case with asymmetric marginal costs

Here, we introduce a method for isolating the impact of common ownership on aggregate quantity from the impact of cost asymmetries in the 2x2 version of the model with asymmetric marginal costs.

Notice that $e^T R^{-1} C$ still multiplies out to be a scalar such that we have an expression for aggregate quantity. We can see however that we no longer have an expression which isolates the impact of the ownership matrix on quantity as multiplying this out gives an expression in which we cannot separate the common ownership terms from the cost terms. We thus need a new way to isolate the relationship between the ownership matrix and aggregate quantity which separates the impact of marginal cost asymmetries from ownership. To do this, we obtain the largest real eigenvalue of the inverse ownership matrix R^{-1}. This comes out to be a single number which can be shown to be an increasing function in the common ownership incentive in the 2x2 case. To illustrate this, the largest eigenvalue of the 2x2 matrix R^{-1} is given by:

$$\lambda_{max} = 1 + \frac{1}{2}\sqrt{1 + r_{12} + r_{21} + r_{12}r_{21}}$$

In the 2x2 case, r_{12} and r_{21} capture the degree of common ownership because they represent the relative interest of the two firms in the profits of the firm in which they are the minority owner, so as r_{12} and r_{21} increase each firm respectively is internalizing its rivals' profits more and thus competition is reduced. We can quickly see that:

$$\frac{\partial \lambda_{max}}{\partial r_{12}} > 0$$

$$\frac{\partial \lambda_{max}}{\partial r_{21}} > 0$$

Thus, we can easily see for the 2x2 case that as the largest eigenvalue of the inverse ownership matrix increases, common ownership increases and thus competition will be reduced. We summarise in the following corollary:

Corollary 2: When marginal costs are asymmetric, we can isolate the impact of common ownership on competition in the 2x2 case by looking at the largest eigenvalue of the inverse ownership matrix R^{-1}. A higher eigenvalue corresponds with a lower market output and thus indicates weaker competition.

The special case where $r_{12}=r_{21}$

In this brief section we show a special case of the model where $r_{12}=r_{21}$.

Considering again the case of asymmetric marginal costs in the 2x2 case, consider now the following decomposition. In what follows we assign shareholder 1 as the majority owner in firm 1 and shareholder 2 as the majority owner in firm 2.

We assume firm 1 has a marginal cost of c and firm 2 a marginal cost of λc, such that λ<1 and captures the cost asymmetry across firms. Using this, and following the same maximisation problem as before, our first order conditions for the firms then become:

$$\begin{bmatrix} 1 & \frac{1+r12}{2} \\ \frac{1+r21}{2} & 1 \end{bmatrix} \begin{bmatrix} q1 \\ q2 \end{bmatrix} = \begin{bmatrix} \frac{a-c}{2b} \\ \frac{a-\lambda c}{2b} \end{bmatrix}$$

Thus

$$Q = [1 \quad 1] \begin{bmatrix} 1 & \frac{1+r12}{2} \\ \frac{1+r21}{2} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{a-c}{2b} \\ \frac{a-\lambda c}{2b} \end{bmatrix}$$

In the special case where $r_{12}=r_{21}$ we can obtain an even cleaner multiplicative decomposition of quantity into two terms capturing the impact of cost asymmetries and common ownership separately.

Recalling:

$$Q = \frac{-2 + r_{12} + r_{21} + c(1 - r_{12} + \mu - r_{21}\mu)}{b(-3 + r_{12} + r_{21} + r_{12}r_{21})}$$

Now setting, $r_{12}=r_{21}=r$ and simplifying:

$$Q = \frac{2 - c(1+\mu)}{3b} \cdot \frac{3}{r+3}$$

Here, the first term captures the impact of cost asymmetries between firms and the second term captures the impact of the level of common ownership on quantity. We can once again see that quantity is falling in cost asymmetries μ and in common ownership r.

Choosing the representative shareholder

One of the key decisions in implementing the model derived in this work is to choose the representative shareholder for each firm in order to construct the ownership matrix. There are several ways in which one could do this, and this section will discuss a few of these.

Firstly, if there exists some majority shareholder who holds over 50% of the shares in a firm then it seems intuitive to assume that this shareholder will have control over the firm's behaviour and so their portfolio would be the representative one. However, in practice it is very rare to find a case whereby a publicly listed firm has a shareholder with over 50% and so we look to the literature to find alternative methods of choosing a representative shareholder when a majority owner is not available.

The next choice might be to consider that the largest shareholder would be the majority owner, and while this is an easy way to choose, it does not necessarily offer an intuitive reason as to why this owner might have effective control over the firm's behaviour. It could be that one owner has 42% and the rest are all very small shareholders and so in such a case it may be possible that the largest shareholder may effectively have control without holding a strict majority unless the others form some sort of coalition and act together against the wishes of the largest shareholder.

However, in practice with most publicly listed firms, it is commonly the case that the shareholders hold very small minority positions in the range of 1-2%, some larger institutional investors may hold up to 10% but it is usually the case that shareholdings are quite dispersed across many investors. Thus, we need some of way aggregating the preferences of these minority owners. The literature seems to offer two main ways to do this: using the median shareholder and the weighted average portfolio.

Some work in the corporate governance literature has suggested that firms may behave in the interest of the median shareholder. The idea of a pivotal voter in a voting system was first introduced by Condorcet (1785), however the median voter did not become prevalent explicitly in the literature until the work of Black (1958) who looked at group voting outcomes and showed that the median outcome will prevail where voters: have single peaked preferences, have transitive preference orderings and cast their votes in accordance with these, and where the winner is determined by simple majority. They show this occurs because the median is the only outcome which can get a simple majority over any alternative outcome (they are the Condorcet winner). A Condorcet winner is defined as the candidate who is able to win the majority of votes in every pairwise election against other candidates i.e. they are the most preferred candidate (Elkind et al, 2014). More recently, Buechel (2013) shows that if a Condorcet winner exists in a majority voting game with single peaked preferences, then the Condorcet winner corresponds with the median voter. However, in the common ownership

literature, the most common depiction of aggregating shareholder preferences is to assume that firms will maximise the weighted average portfolio of their shareholders (Hansen & Lott, 1996; O'Brien & Salop, 2000). The MHHI, the leading measure of common ownership in the literature makes this assumption in its calculations, as does the common ownership incentive term. Thus, to maximise comparability across indices it may be most appropriate to implement this method of aggregation in calculating the index and so the representative shareholder would be chosen as the weighted average shareholder when a majority shareholder is not available.

Ownership vs control

In the model presented, one assumption that is inherent in the derivation of the results is that ownership effectively translates into control. There is a principle-agent problem in the relationship between managers and shareholders, and how to best model this is an expansive literature. In the model, we implicitly assumed this away and present that firms behave in response to shareholders preferences, which essentially removes management from the question and does not consider any principle-agent issues. However, it is possible to incorporate this into the model. One could create another (nxn) matrix representing control weights, where each element ij represents shareholder i's 'control' over management in firm j and multiply this with the ownership matrix to get an 'adjusted' ownership matrix weighted by control. This idea is not novel, in that in the generalized version of the MHHI – the one presented by O'Brien & Salop (2000), ownership weights are also multiplied by control weights. However, a shareholder's degree of control is almost impossible to observe and so researchers are forced to make assumptions about how this would look. In practical applications, the most common approach in implementing the MHHI is to assume proportional control (Gramlich & Grundl, 2017; Azar et al, 2018; Azar et al, 2016, Backus et al, 2021), such that ownership=control. However, O'Brein & Waehrer (2017) point out that this assumption does not have any theoretical or empirical support – but equally, there does not appear to be an approach presented in the literature which does have theoretical and empirical support and so at present it is not clear how to best model this. In the original version of the MHHI, presented by Bresnahan & Salop (1986), eight different control structures were studied and ranked in order of relative competitiveness, but no justification was given for any choice of control structure. Other papers within the common ownership literature have placed different assumptions on the relationship between ownership and control. For instance, Gilje et al (2020) present in their model that managers place weight on a shareholder's preferences when that shareholder is attentive. They discuss different ways to model attentiveness, and in their work

attentiveness is proxied a la Iliev & Lowry (2015) by looking at whether shareholders follow the voting recommendation of the advisory firm ISS. It is assumed that those that follow the recommendation are inattentive and are free riding on the effort of ISS whereas attentive shareholders are more likely to stray from the recommendation and vote based on their own information and analysis.

Thus, the question of how to best model this problem is still very much unresolved, and so at present this modelling distinction adds another parameter upon which researchers must make assumptions, and as it is unobservable it is not clear that there is a correct way to model this. In this work, I present the simplified benchmark to avoid detracting from the main story of this paper with a tangential research question but note that this benchmark can be easily modified by researchers working specifically on the relationship between managers and shareholders to incorporate different control structures and stress that this would be a very welcome collaborative effort.

Discussion

We have now introduced several versions of a model which demonstrates a direct, theoretically grounded, relationship between the ownership structure of a market and the corresponding level of aggregate quantity – and thus competition. Analysing the 2x2 example in detail provides preliminary evidence to suggest that higher levels of common ownership will be anticompetitive. In the more general nxn version of the model we have shown clearly how the ownership matrix influences aggregate quantity but cannot with the information provided make broad claims about the impact of common ownership on competition. What we have provided those is a theoretically sound tool with which to study this question empirically. Measuring common ownership is the first step in determining its impact on any system and this method outlined here overcomes the lack of theoretical grounding in the existing measures.

In applying and analysing this model in real data, the main limitation would be that without access to precise cost data for the firms in the sample we would be unable to calculate the optimal level of common ownership from a welfare perspective. The model allows us to compare different ownership matrices by plugging them into the system and comparing relative values for aggregate quantity – which would allow for comparison across time for instance – but doesn't give us information on how good or bad this level of common ownership is for welfare. Of course, the exception to this is the case of the model with symmetric marginal costs,

whereby a higher value for the index means moving away from perfect competition and towards monopoly and so welfare can be clearly interpreted in this case, but symmetric marginal costs is an appropriate assumption in very few industries.

Besides studying common ownerships competitive impact, another possible application of the model presented here is in merger analysis. Currently, ownership structure of firms is not a key requirement in analysing the potential competitive implications of a merger. However, the evidence presented here suggests there is real scope for thinking that ownership has a direct impact on market behaviour, alongside concentration which is determined by cost asymmetries. For this reason, looking at how the ownership matrix would change before and after allowing a merger might provide novel insight into competitive dynamics and market outcomes previously unconsidered. The model outlined in this work is one way to study how the ex-post and ex-ante ownership matrix would influence aggregate outcomes alongside the cost asymmetries and concentration already central to existing merger policy. Further research on the intricacies of this would be needed before conclusions on the efficacy of this model to merger analysis could be determined, however it could be a fruitful direction for future work in merger policy.

Conclusion

This paper has produced a theoretical framework which relates the ownership structure of a market to the prevailing level of competition in a Cournot market. It considers the simple case under which firms have symmetric marginal costs, as well as the case of asymmetric marginal costs. Under symmetric marginal costs, the model obtains the COCI, a pure measure for competition which directly relates and isolates the impact of common ownership on aggregate quantity. In the case with asymmetric marginal costs, ownership structure and cost asymmetries work together to influence market outcomes. In this case we cannot generally isolate the impact of common ownership on competition, but we motivate that it is an influential factor in determining competitive dynamics and that it should be considered alongside concentration analysis (which depends on cost asymmetries) in assessing the competitive dynamics of a market. We study in greater detail the 2x2 case of the model and in this scenario provide evidence that common ownership can be anticompetitive. The fundamental contribution of this work is in providing a novel framework with which to measure common ownership in a way which is explicitly underpinned by a clear theoretical framework. This theoretical relationship

allows for clear interpretability of the measure and will hopefully serve as a useful steppingstone in our understanding of how common ownership influences markets.

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Chapter 3:

Common Ownership in Britain: The case of Supermarkets, Soft drinks, and Banking across 2011-2021

Abstract

The goal of this paper is to introduce and apply an index which measures the impact of common ownership on firms output levels within a market. Thereby, this work adds to the growing literature which studies the levels and impact of common ownership. Additionally, we add to the body of evidence considering the impact of common ownership in European markets, as most existing evidence is based on the US. Specifically, we look at three British industries: supermarkets, soft drinks, and banking over the period 2011-2021 using data from a Bloomberg Terminal. We introduce a new index, the common ownership output reduction index (COORI) which builds on the model outlined in Chapter 2 and implement this alongside existing measures of common ownership and concentration: the modified Herfindahl-Hirschman Index, the common ownership incentive term, and the Herfindahl-Hirschman Index, as well as the common ownership competition index (COCI) derived in chapter 2, to study the levels and trend of common ownership in these industries over the period and comment on what this information can offer in terms of evidence on the competitive impact of common ownership. We use several measures in order to see how the current tools available for analysing common ownership compare against each other, and against COORI, given the comparative strengths and weaknesses of each and consider the evidence in aggregate across measures to get an overview of how common ownership might be impacting these industries. Some of the advantages of COORI as a measure of common ownership are that it requires less data to calculate, it has a direct and intuitive interpretation, and it is derived from a clear theoretical relationship. All measures of common ownership considered indicated that competition across the supermarkets studied fell over the period. In soft drinks, COORI and COCI remained stable over the period, whilst the common ownership incentive term remained fairly stable but showed a mild reduction in competition, and MHHI-delta showed a fall in competitive conduct over the period. Results for banking were the least consistent across industries, with COORI and the MHHI delta showing a fall in competition over the period, whilst the common

ownership incentive term suggested increased competitive dynamics and the COCI jumped around and displayed no general trend in this industry.

Introduction

So far, the common ownership literature has very much focussed on the US, and it has been established that there is a deficiency of work looking at common ownership within Europe (Burnside & Kidane, 2020). Whether common ownership is detrimental to competition is of interest to regulators, and the competition and markets authority (CMA) have started looking into the issue in the UK (CMA, 2022). Before determining whether common ownership is harmful to competition, it is first necessary to understand to what extent, if any, it is occurring. Moreover, understanding how common ownership has evolved over time is important in helping to understand if the dramatic rise in common ownership documented in the US is a worldwide phenomenon or whether it is a feature unique to American markets. A recent paper which is relevant to this work is a paper by Banal-Estanol et al (2022). This paper compares the common ownership incentive term (COIT)7 in American and European banking sectors for the period spanning 2004-2015 and shows that common ownership rose more steeply in Europe but remained at a much lower level than in the US. Another paper, Banol-Estanol et al (2021), also uses the COIT to look at the impact of the 2007-2009 global financial crisis on common ownership patterns in the largest 25 publicly listed European banks for the period spanning 2003-2015. The results show that common ownership networks are becoming denser over the period, but that this saw a temporary fall during the financial crisis due to increased governmental support during this period. However, there was no comment on the competitive implications of these trends.

The goal of this work is two-fold. Firstly, we introduce a modification to the index derived in Chapter 2 to provide a new tool for analysing the competitive impact of common ownership. This measure offers several advantages over existing measures, in that it doesn't require pricing data, which is difficult to obtain, to offer insights into competitive conduct and it has a directly intuitive interpretation which allows for immediate information on how common ownership is affecting output. Secondly, we apply this theoretically driven model to data on British industries. The choice of industries was limited by the authors access to data and so the application to data conducted here serves as an illustration of the sort of results one can obtain using the indices derived in this thesis, and the results should be seen as illustrative examples of application and not empirical claims of observed industry conduct. Disclaimer aside, we look at the supermarkets, soft drinks and banking sector over the 10-year period spanning 2011 to 2021. Results for the banking sector are more mixed, but in supermarkets we see evidence of an increasing impact of common ownership on competitive conduct – in that competition

became less fierce over the period as a result of ownership links, and in soft drinks we see a stable impact over the period but there is evidence that output is being reduced as a result of common ownership.

In chapter 1 and 2 of this thesis, we discussed the limitations of existing measures of common ownership and introduced a new, theoretically underpinned, model of how common ownership is related to market competition. In this chapter, we develop this model further to introduce the COORI and apply this to data on three British industries over a 10-year period between 2011 and 2021. We also calculate, for the same datasets, several existing measures of common ownership in order to compare the trend across industries. The goal is to try and offer some of the first evidence on common ownership in Britain across several industries and to demonstrate how this might be affecting competition. In doing so this paper represents an advancement on the literature in several areas. Firstly, we provide an index which overcomes the lack of theoretical foundation that characterises the existing measures of common ownership. Secondly, we provide novel, and needed, evidence on British industries using a variety of measures and thus adds to the discussion on common ownership in Europe. We use several measures to see how the measures compare given their relative strengths and weaknesses and to consider a broader sphere of evidence when casting light on the British story of common ownership. Literature in the American industries has shown that choice of measure has profound impact on whether competition appears to be dampened or not by common ownership (Azar, 2018; Dennis et al, 2018) and so considering several measures seems appropriate in getting an idea of the bigger picture in the UK. In comparing the indices, we believe COORI offer advantages over existing measures in that it is derived clearly from theory, it has a direct and intuitive interpretation for the competitive impact of common ownership and requires less data to infer meaningful insights than existing measures, and so may provide an intuitive and practical tool for competition authorities.

We begin by introducing how one obtains COORI from the COCI derived in chapter 2 and then demonstrate its interpretation in section 2. Section 3 discusses our data and summary statistics. Section 4 outlines the methodology and shows how each index is calculated. Section 5 covers results for each index in each industry. Section 6 contains the discussion of the results and possible extensions to this work, and Section 7 concludes.

Theory

In chapter 2 we derived the following expression for market quantity under a modified Cournot model:

$$Q = e^T R^{-1} C$$

Where Q is aggregate quantity, R is our normalised ownership matrix, e is a vector of 1's and C is the vector capturing the differing marginal costs across firms.

In practice it is possible to observe the ownership structure of a market (R) if we are analysing the industry and given e is just a vector of 1's, the limiting factor to applying this model is that we cannot observe marginal costs and thus our analysis depends very much on how we choose to model C and the particular assumptions on this. However, we can do some manipulation to address this issue.

From our index we can see that (recalling $e^T q = Q$)

$$q = R^{-1} C$$

Multiply both sides by p:

$$pq = pR^{-1}C \quad \rightarrow \quad pC = Rpq$$

Assume now that E is the ownership matrix that represents sole ownership (no common ownership), such that:

$$E = \begin{bmatrix} 1 & \cdots & 0.5 \\ \vdots & \ddots & \vdots \\ 0.5 & \cdots & 1 \end{bmatrix}$$

We arrive at this because the ownership structure that is associated with sole ownership can be represented by the identity matrix, and plugging this into our model from chapter 2, the resulting ownership matrix is as above. For explanation of this procedure, see pages 50-51 in chapter 2.

We can then calculate:

$$\frac{e^T R^{-1} pC}{e^T E^{-1} pc} = \frac{e^T pq}{e^T E^{-1} Rpq} = \frac{Actual\ Revenue}{Revenue\ with\ Sole\ Ownership}$$

Notice now that since price is fixed at a given time, we need to observe R and pq, where pq is simply total revenue – which is directly observable from company accounts. Thus, we can now

estimate the index without needing to make limiting assumptions on C. Moreover, since price is fixed, we can say:

$$\frac{actual\ revenue}{revenue\ with\ sole\ ownership} = \frac{actual\ quantity\ (Q)}{quantity(Q)\ with\ sole\ ownership}$$

This allows us to now calculate an index which will tell us how much output is being restricted due to the prevailing level of common ownership in an industry, compared to the counterfactual where the firms were all owned by separate owners. This number is immediately interpretable in a clear way. For instance, suppose for example we found that under common ownership in some industry we obtained a quantity of 2000 units, whereas under the counterfactual of sole ownership we would have found a quantity of 2300 units. This index would thus be 0.8696. This indicates that output under common ownership is about 87% of what it would have been under sole ownership. Putting it differently, common ownership in this industry has resulted in a restriction of output of around 13%. This gives us an immediately interpretable proxy for the competitive impact of common ownership in an industry. We can see that if this index is below 1 then common ownership is having a negative impact on competition as proxied by market output, if equal to 1 then common ownership is not impacting competition, and if above 1 common ownership is increasing competition. We summarise this in the following Theorem.

Theorem 1: COORI is the ratio between output under observed ownership compared to output that would prevail under sole ownership.

$$COORI = \frac{e^T pq}{e^T E^{-1} Rpq}$$

The interpretation of COORI is intuitive in that:

If $\mathrm{COORI} < 1$ common ownership leads to a restriction of output relative to sole ownership.

If $\mathrm{COORI} = 1$ common ownership is not affecting output relative to sole ownership.

We show in appendix 3 that COORI cannot exceed 1.

One point worth discussing here is our use of price as a scalar in deriving COORI. We multiply our expression by price in order to convert quantity into revenue with the purpose of making the index easy to apply using directly (and easily obtained) observable data. Price is being used here as a scalar and not as a variable, which is an important distinction to bear in mind. This means that the price under common ownership is the same price being assumed in the case of

sole ownership. This is a simplification and means that the model is entirely static i.e., we take the market outcome under common ownership, effectively 'freeze' everything and change nothing but the ownership matrix in the counterfactual to see how this state of the world would compare before price has time to adjust to the demand curve when quantities change – this is our COORI. In reality, under sole ownership the prevailing price would likely not be the same as the one under common ownership. Under the assumptions of our model, common ownership increases firms market power and so it is likely that under sole ownership the prevailing market price would be lower and if we accounted for this that would thus decrease the denominator of COORI and lead to a higher level for the index. This means that COORI is underestimating the effect of common ownership in practice and is thus a conservative estimate for the impact of common ownership. Given that in our denominator we are using the level of price that would follow a reversion to sole ownership before any adjustment to the demand curve and quantity changes can occur, it is possible that COORI could be amended to have the counterfactual price be a function of the elasticity of demand so that we could capture to some degree the changes that would occur following the shift in quantities that would follow a change in the ownership matrix. Given this, if one can reasonably assume that the elasticity of demand in an industry is somewhat stable over time then the bias in our simplified COORI would also be steady over time and this would mean that looking at the trend of COORI over time can still provide meaningful insight, even if the level of the index is biased downwards – for this reason the trend in the index may be more informative that the level.

Another comment to be made here is that this index is comparing common ownership to the case of sole ownership and not to the benchmark of perfect competition. Whether sole ownership is an appropriate assumption in terms of welfare is an empirical question beyond the scope of this work. However, in terms of isolating the impact of common ownership on an industry, sole ownership is an intuitive counterfactual.

If one did want to consider how common ownership was impacting welfare more specifically, a modification to this index could be made as follows:

To begin, find the ownership matrix which maximises quantity – this could be sole ownership but in principle it could be something else, call this ownership matrix **S**. We then calculate:

$$\frac{e^T pq}{e^T S^{-1} R pq}$$

Again, what we need to observe is R and total revenue – since S is calculated theoretically. The neat thing about this index is that since S is the matrix which maximises quantity, this index is always between 0 and 1. In the same way as before, interpretation of this index is clear, the closer the value of the index is to 0, the more common ownership is leading to a restriction of output in the industry – which is associated with a reduction of competition, and this setting, can be said to be reducing welfare. We summarise this in the following corollary:

Corollary 1: We can calculate an index capturing the welfare impact resulting from common ownership by calculating the following index

$$\frac{e^T pq}{e^T S^{-1} Rpq}$$

Where S is the ownership matrix which maximises output (and thus consumer surplus).This index, naturally, always has a value between 0 and 1. A lower value indicates a greater reduction in consumer surplus resulting from common ownership.

It would also be possible to calculate this index replace S with a matrix T, which represented the matrix which would minimise quantity. We could then see how R relates to the two extremes to get an idea of where it falls on the welfare spectrum.

A note on industry choice

It should be disclosed upfront that what follows is a descriptive piece of work aimed at providing an illustration of an application of the models derived in this thesis. The purpose is to introduce the sort of results one can obtain using the indices we have so far derived. The industries analysed were chosen based on the author's limited ability to obtain necessary data and as a result do not represent the most theoretically ideal candidates for applying COORI and COCI practically. Fundamentally, COORI and COCI are derived based on the assumption of a Cournot market, and that Cournot is an appropriate assumption for the banking industry is not an obvious conclusion. Moreover, due to the author's inability to access appropriate pricing data for the industries and time periods needed there is no empirical analysis conducted in this work. Thus, the following represents a first step and serves mainly to illustrate the application of the index presented in this thesis and demonstrate the sort of results one could obtain. It does show that with easily obtained data one can get clearly interpretable results using the index even in the absence of econometric analysis, however where Cournot is not an appropriate assumption the interpretation of these results is no longer resting on a sound theoretic base.

Thus, the results should be seen as illustrative examples of application and not empirical claims of observed industry conduct.

Data

The analysis conducted in this work relies on ownership data of each of the firms studied. Ownership data was obtained from a Bloomberg Terminal, which is available to researchers affiliated with an institution with an active subscription to the platform. Ownership figures are presented as the number of outstanding shares held by each investor in a firm for each quarter, and a total number of outstanding shares is available for each firm in each quarter. We use this data to calculate the ownership percentage of each investor in each firm in each year – using quarterly data on the same date each year (31st December) to minimise potential seasonal effects that could arise from comparing a different date each year. Additionally, revenue data for each firm in each year studied was collected from the Bloomberg terminal. Again, the same date was considered across all years in the sample.

For this work, we considered firms operating in UK markets across three industries: supermarkets, soft drinks (specifically carbonated soft drinks) and banking over the last decade spanning 2011-2021. We considered only publicly listed firms as these are the ones with the potential to be commonly owned on the stock market and so are the sample of firms with the potential to be impacted by common ownership – this is also in line with other work in this literature (Banal-Estanol, 2021, 2022). In deciding which firms to include in an industry, the sample was directed chiefly by availability and suitability of data on the Bloomberg terminal. We considered the largest firms in each industry with respect to market cap (as listed on the Bloomberg terminal) and included firms who were publicly listed and who had adequate data available for each year in my sample. For instance, in banking Virgin Money was available on the terminal but only had ownership data available from 2015 onwards (as their IPO was 2015), and so was not included in the sample as data was missing for 2011-2014. The firms ultimately chosen for each industry do capture a substantial portion of the market share in recent years (discussed in the following sections) and so represent a significant portion of the industry and the competitive dynamics within it. What is important to note is that for each industry, the calculations we make which result in conclusions such as output is x% lower due to the presence of common ownership, are asserting these conclusions for the portion of the market considered in the sample. So, for supermarkets we are saying that these 5 firms produced x% less because of their common ownership links, we are not commenting on the production of

firms outside the sample. For this reason, we do not believe that the choice of sample restricts accuracy of the outcomes, it just affects which portion of the industry we can comment on. However, as the firms considered represent the main players in a market one could argue that this represents a substantial portion of the industry and so may reflect industry wide trends. We outline each industry's dataset more specifically now.

Supermarkets Data

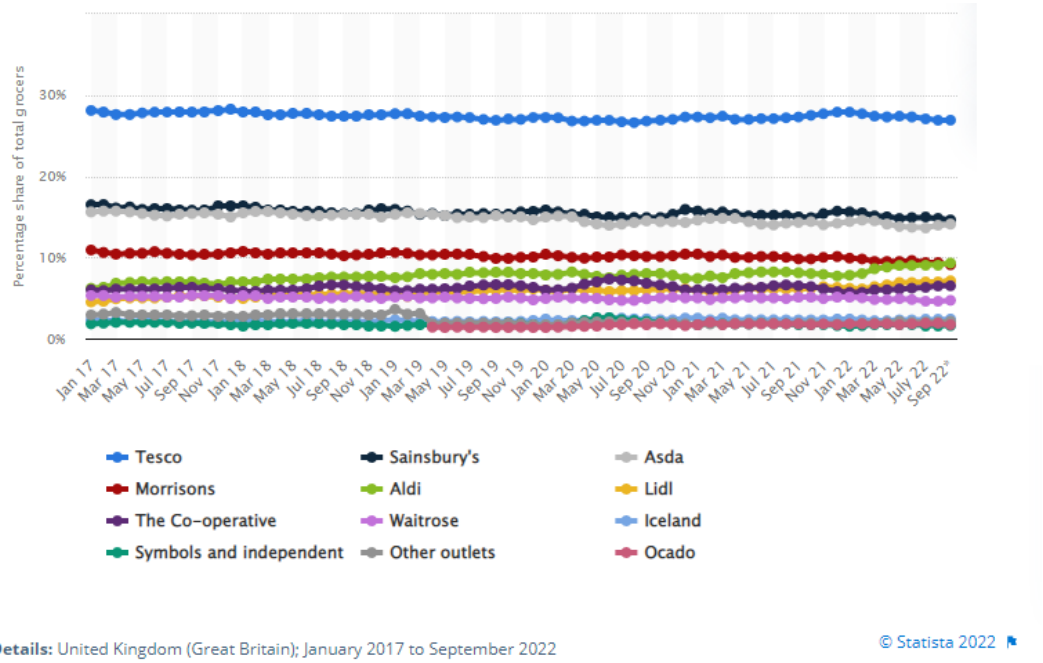
For supermarkets, the sample of firms is Tesco, Sainsburys, Asda, M&S and Morrisons. According to Kantar (2022), Tesco, Sainsburys, Asda and Morrisons accounted for 68.1% of the British supermarkets industry in 2021 (data on M&S was unavailable) and it looks as if market shares have been fairly stable since 2017 – see figure 2. This suggests that these firms do indeed represent a substantial component of the market. Moreover, other large players, such as Waitrose, Aldi and Lidl are not publicly listed and so do not have the potential to be influenced by common ownership in the way theorised by our model, and so their omission is natural in studying the impact of common ownership specifically. Considering only publicly listed firms has been done by other papers in this literature (Banal-Estanol, 2021, 2022).

For each of the five supermarkets in our sample, we have observations on the shareholding of 10,591 shareholders over a 10-year period from 2011-2021. Summary statistics are presented in figure 1. This shows that the average shareholding by a shareholder in one of the supermarkets over the period was 0.03% in 2011 and 0.04% in 2021, and the maximum shareholding observed by a shareholder was 52% in 2011 and 24% in 2021 – this was Walmart's share in Asda. The average is drawn down by the number of zero observations, but the standard deviation gives an idea of how spread around this average the dataset is over time – it appears common ownership levels became less spread in 2021 compared to 2011 based on the lower standard deviation.

Table 1: Summary Statistics for Supermarkets Data

Variable	Observations	Mean	Std. dev.	Min	Max
y2011	10591.000	0.032	0.620	0.000	52.880
y2013	10591.000	0.036	0.623	0.000	51.341
y2015	10591.000	0.041	0.578	0.000	45.154
y2017	10591.000	0.044	0.580	0.000	46.392
y2019	10591.000	0.044	0.584	0.000	46.392
y2021	10591.000	0.040	0.409	0.000	24.118

Figure 1: Supermarket market shares across 2017-2022 (Statistica, 2022)



Details: United Kingdom (Great Britain); January 2017 to September 2022 © Statista 2022

Soft drinks Data

For soft drinks, the sample of firms is Coca-Cola, Pepsi Co, AGBarr, Britvic and Dr Pepper. We consider energy drinks to be a separate market segment and so only consider non-energy drinks. Summary statistics are presented in figure 3. Across the 5 firms there are 15,109 observations of shareholdings. Market share data for soft drinks was not available. However, according to Nielsen data the top 10 soft (non-energy) drinks in the UK by sales volume were the same in 2021 and 2020, though the ranking alternated slightly. Table 1 summarises these products and their ownership. This shows that the sample of firms we consider own all 10 of the top selling non-energy soft drinks in the UK in 2020 and 2021 (Leader, 2021). This gives us some confidence that our sample is capturing the main players in the market and is a

meaningful portion of the industry. Unfortunately, market share data for these firms in the British economy is not available.

Table 2: Top 10 soft drinks by sales volume in the UK in 2020 and 2021

2021 ranking	2020 ranking	Product	Owning company
1	1	Coca-Cola	Coca-Cola
2	2	Pepsi	Pepsi
3	3	Fanta	Coca-Cola
4	5	Dr Pepper	Coca-Cola
5	4	Irn Bru	AG Barr
6	6	Schweppes	Coca-Cola
7	7	7UP	Pepsi
8	9	Tango	Britvic
9	8	Sprite	Coca-Cola
10	10	Barr	AG Barr

Table 3: Summary Statistics for Soft Drinks Data

Variable	Observations	Mean	Std. dev.	Min	Max
y2011	15109.000	0.027	0.307	0.000	13.257
y2013	15109.000	0.029	0.335	0.000	19.426
y2015	15109.000	0.029	0.340	0.000	21.096
y2017	15109.000	0.029	0.298	0.000	12.038
y2019	15,109	0.025632	0.299489	0	13.62369
y2021	15,109	0.02931	0.394096	0	33.35137

Banking Data

The sample of firms considered in banking are HSBC, Lloyds, Barclays, Natwest, Standard Chartered, Close Brothers, Secure Trust, Arbuthnot, and Santander. Market share data is not available for UK banks, but the ‘big four’ are known to control a large part of the UK banking market. The big four comprise HSBC, Lloyds, Barclays and Natwest (Megaw, 2020). As we have data on each of these banks, we can argue that we are covering the most important firms

in the market. Moreover, in 2021, the sample of banks we study comprised 77% of the lending market in the UK (Statistica, 2022). The summary statistics for each year in the banking sample are shown in figure 4.

Table 4: Summary Statistics for Banking Data

Variable	Observations	Mean	Std. dev.	Min	Max
y2011	8676.000	0.071	1.142	0.000	66.936
y2013	8676.000	0.076	1.289	0.000	67.005
y2015	8676.000	0.082	1.227	0.000	72.557
y2017	8676.000	0.087	1.139	0.000	70.496
y2019	8676.000	0.089	1.064	0.000	62.093
y2021	8676.000	0.094	1.021	0.000	56.087

Methodology

For each industry in our dataset, we will calculate the COORI developed in section 1, the COCI developed in chapter 2, the COIT, the modified Herfindahl-Hirschman index delta (MHHI delta), and the Herfindahl-Hirschman Index (HHI). To begin, we will briefly outline each index and explain its interpretation, before moving onto demonstrating with simple examples how each is practically calculated from the data we have.

The objective of this paper is to introduce COORI as a new tool in studying common ownerships impact and to see what this measure can tell us about some industries in Britain. For this reason, we begin by calculating COORI, but we also calculate the other indices to allow comparison and to see how COORI compares against existing measures of common ownership and competition. Existing literature has shown that the choice in measure of common ownership can dramatically alter the conclusions of the analysis on whether or not common ownership has anti-competitive effects. For instance, Azar et al (2018) show using MHHI delta that common ownership has an anti-competitive impact on prices in the airline industry, whereas Dennis et al (2018) replicate this study, replacing MHHI delta with the COIT and find that there is no significant influence on price. For this reason, we want to include several measures of common ownership in our study to offer a broad range of evidence on the impact of common ownership on competition in the British industries and to see how our index, COORI, compares against existing measures. As far as which indices we include, the MHHI

and COIT have been chosen because they represent the main measures of common ownerships impact present in the literature. Because MHHI delta contains a component of market shares, we also calculate HHI to consider if this captures a similar story to delta. If delta and HHI are moving together it gives more intuition that perhaps what we are observing is due to market share changes, whereas if delta is seeing movement and HHI is remaining stable it offers insight that perhaps the difference in movement is arising due to the component of MHHI derived from ownership. We now outline how each index is defined and calculated.

Calculating COORI

Recall, that in section 1 we defined COORI as being calculated as follows:

$$COORI = \frac{e^T pq}{e^T E^{-1} Rpq}$$

To be explicit, in appendix 4 we outline, using a simplified (fictitious) dataset, the step-by-step process in which one practically calculates COORI.

It is important to make clear the relationship between COORI and COCI, which was derived in chapter 2. Both are derived from the same underlying model, but the COCI requires the assumption that firms have symmetric marginal costs, which limits its applicability to real world markets. The COORI follows the same baseline model as the COCI and the two use the same construction of the ownership matrix (outlined in Appendix 1) but doesn't require the assumption of symmetric marginal costs.

Calculating the MHHI delta

The MHHI delta is currently the most well-known measure of common ownership in the literature. The MHHI delta is formally defined as:

$$MHHI\ delta = \sum_j \sum_{j \neq k} s_j s_k \left(\frac{\sum_i \beta_{ij} \beta_{ik}}{\sum_i \beta_{ij}^2} \right)$$

Where s_j is the market share of firm j, and β_{ij} is the ownership share of investor i in firm j.

Given we are unable to obtain data on the true market shares of these firms over a 10-year period, we calculate proxy market shares using our revenue data. Essentially, we assume that the sample we are looking at represents the market that we are studying and calculate the market share of each firm as that firms revenue in year t, divided by total revenue of the firms in our

sample in year t (total revenue is calculated for each industry separately). To be precise, the interpretation of our results is such that the total output of the sample studied will be x% lower (or possibly higher) due to common ownership. For this reason, in a simplified sense we are essentially treating our sample as representing the market and so this method of calculating market shares is not overly inappropriate for our purposes and interpretation of results. More precise measures of market share would be advantageous, but we did what was feasible with the data available. Note, that for practically and consistency, we implement a cut-off of 1% shareholding as inclusion criteria for our dataset – as in Banal-Estanol (2021, 2022).

To be clear, the MHHI delta captures the additional effective concentration arising due to common ownership and is thus a measure of the extent of common ownership. It doesn't directly say what common ownership is doing to competition, if anything. COORI, on the other hand, shows the output reduction that is arising due to the current level of common ownership. The two are capturing different, but related, aspects of the problem. This is important in comparing and interpreting the measures – they are not looking at exactly the same thing.

For a non-technical outline of how to practically calculate MHHI delta using ownership and market share data, see Lambert (2018).

Calculating the COIT

The COIT is the component of MHHI which doesn't include market shares. It has some advantages over the MHHI because it addresses O'Brien's (2017) impossibility critique (Kennedy et al, 2017) and it overcomes the issue with the MHHI identified in Dennis et al's (2018) critique of Azar et al's (2018) methodology, in that regressing price on the COIT in place of the MHHI delta removes the issue of effectively regressing on market shares twice. For these reasons, the COIT represents the most theoretically sound measure of common ownerships impact in existing literature, and so we include it as a comparison to COORI. The COIT is defined as follows:

$$COIT = \sum_{j} \sum_{j \neq k} \frac{\sum_i \beta_{ij} \beta_{ik}}{\sum_i \beta_{ij}^2}$$

Where β_{ij} is the ownership share of investor i in firm j. For a non-technical demonstration of how to calculate the COIT as part of the MHHI, see Lambert (2018). To be clear, technically the theoretical derivation of this index looks like this:

$$COIT = \sum_{j} \sum_{j \neq k} \frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}}$$

Where γ_{ij} represents the control rights that shareholder i has in firm j. However, because control rights are not observable, the accepted practice in the literature is to make the assumption of proportional control and set $\gamma_{ij} = \beta_{ij}$, which gives us the former expression for COIT. Thus, in order to calculate the COIT one must make an assumption on the relationship between γ_{ij} and β_{ij}, however there is no understanding on what the relationship between these should be or what drives it (Banal-Estanol, 2022), and so this is a weakening assumption necessary in implanting this measure.

In calculating COIT, we implement a cut-off of 1% ownership for consistency of comparison with COORI's results.

Calculating the HHI

The HHI is included simply as a benchmark against which to compare the MHHI delta, seeing as the MHHI is equal to delta plus the HHI. As in our calculation of MHHI we proxy market shares using calculations from our revenue data, again assuming our sample represents the market. It's worth noting that this will skew the HHI figures upwards, and so this should not be taken as a statement on the true concentration of the markets in question, but rather as a benchmark level against which to compare delta to consider the relative weighting of ownership vs market shares. For completeness, the HHI is defined as the sum of the squared market shares:

$$HHI = \sum_{i} s_i^2$$

Where s_i is the market share of firm i.

Results

In this section we will outline the trend of each index in each industry over the period 2011-2021 (for alternating years) and will discuss the interpretation of each result.

Supermarkets

We begin by reviewing the results of each index for supermarkets.

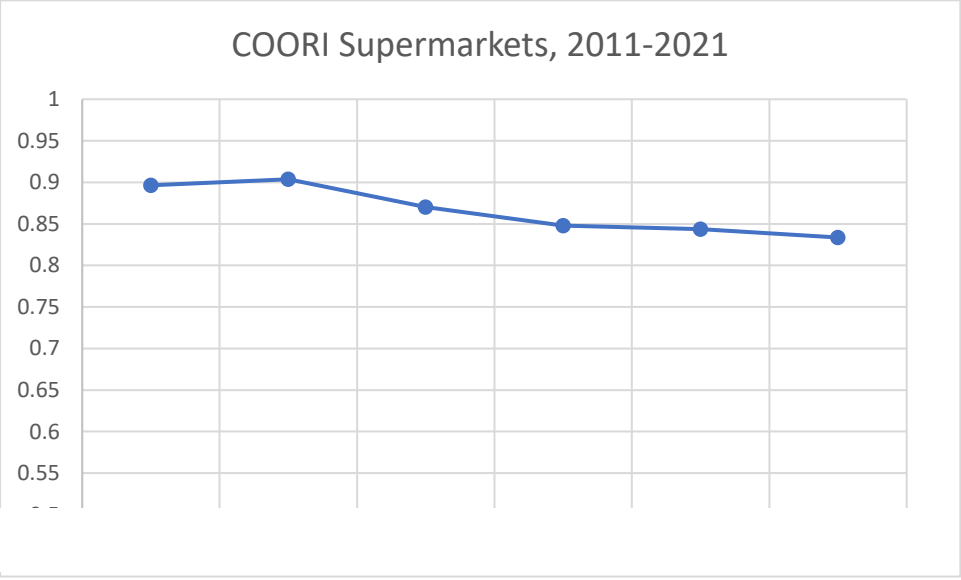
COORI

The trend for COORI over the period in this industry is captured in figure 2 and the precise values for each index are displayed in table 5.

Table 5: Index Values for Supermarkets 2011-2021

Year	COIT	MHHI delta	HHI	COCI	COORI
2011	12.19	4807	2942	1.53	0.90
2013	10.32	3686	2899	1.54	0.90
2015	10.72	4273	2905	1.50	0.87
2017	13.87	5297	2718	1.44	0.85
2019	15.57	5698	2834	1.43	0.84
2021	15.32	5963	2731	1.42	0.83

Figure 2: COORI Supermarkets 2011-2021



Recall that the interpretation of COORI is that it shows the quantity produced under common ownership as a fraction of the quantity that would have been produced under a counterfactual of sole ownership, holding other market characteristics the same (market shares as proxied by revenue). Thus, a value of COORI below 1 indicates that output has been restricted due to

common ownership in the sample of firms we are looking at and therefore suggests a reduction in the ferocity of market competition.

We can see that COORI declined slightly but steadily over the period, dropping from 0.8966 in 2011 to 0.8336 in 2021. This represents that output amongst these firms was between 10.34 and 16.64% lower due to the presence of common ownership links compared to the counterfactual of sole ownership. The results also suggest that ownership links have had an increasingly negative impact on competition across these 5 firms over the period studied.

COCI

The COCI is the index we derived in section 3 in chapter 2. It is calculated under the assumption of homogenous marginal costs across firms, which is a simplifying assumption, however we calculate it as benchmark. The interpretation is very clear in that a value of 1 indicates monopolistic market conditions, a value of 2 represents perfect competition and the value of the index that would prevail under Cournot competition for n firms, can be calculated as a value in between 1 and 2 which is dependent on n, specifically Cournot for the index occurs when:

$$e^T R^{-1} e = \frac{bn}{n+1}$$

Where n is the number of firms in the industry and b is the slope of the demand curve.

Given that the number of supermarkets in our sample is 5, the Cournot value for the index in this setting will be 1.666667.[7] The values for the index across the years in our sample are summarised in table 5 and illustrated graphically in figure 3. The trend in this index is quite clear across the period, in that there is a decrease over time. This indicates that competitive ferocity decreased in the industry over the last 10 years. Even the highest value of the index, recorded in 2011, was lower than the Cournot value, which indicates that competition may be somewhat dampened in this industry. However, the simplifying assumption of homogenous marginal costs across firms necessary in deriving these results is a limiting factor but it still

[7] Because you solve: Cournot Quantity=Index, which is:

$$\frac{n}{n+1}\frac{a-c}{b} = \frac{a-c}{2b} e^T R^{-1} e$$

When n=5

$$\frac{5}{6}\frac{a-c}{b} = \frac{a-c}{2b} e^T R^{-1} e$$

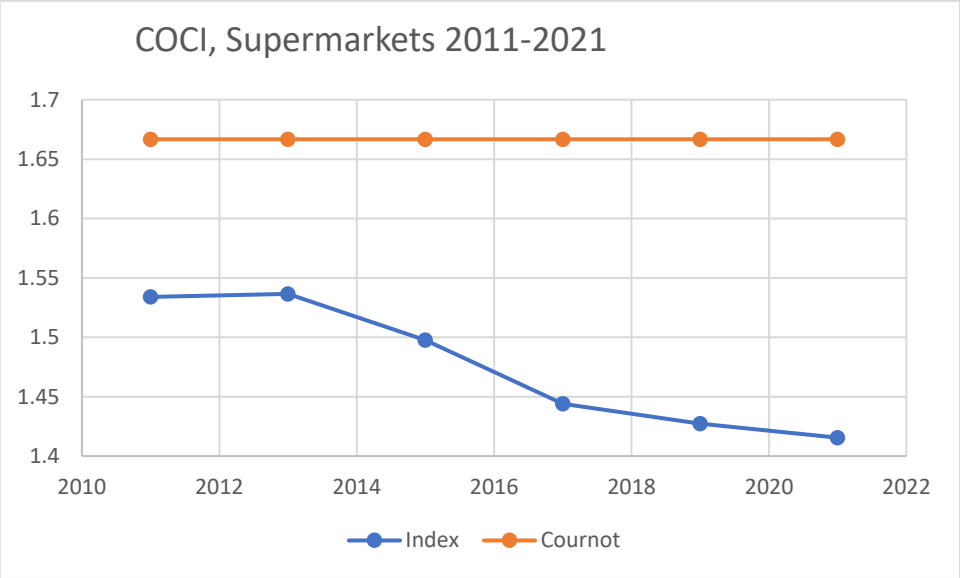
$$\frac{5}{6}\frac{2b(a-c)}{b(a-c)} = e^T R^{-1} e$$

Thus

$$e^T R^{-1} e = \frac{10}{6} = 1.66666667$$

serves as a benchmark analysis. The overall story told by this index is that competition between these 5 supermarkets has fallen due to common ownership in the period studied.

Figure 3: COCI against Cournot Value, Supermarkets 2011-2021

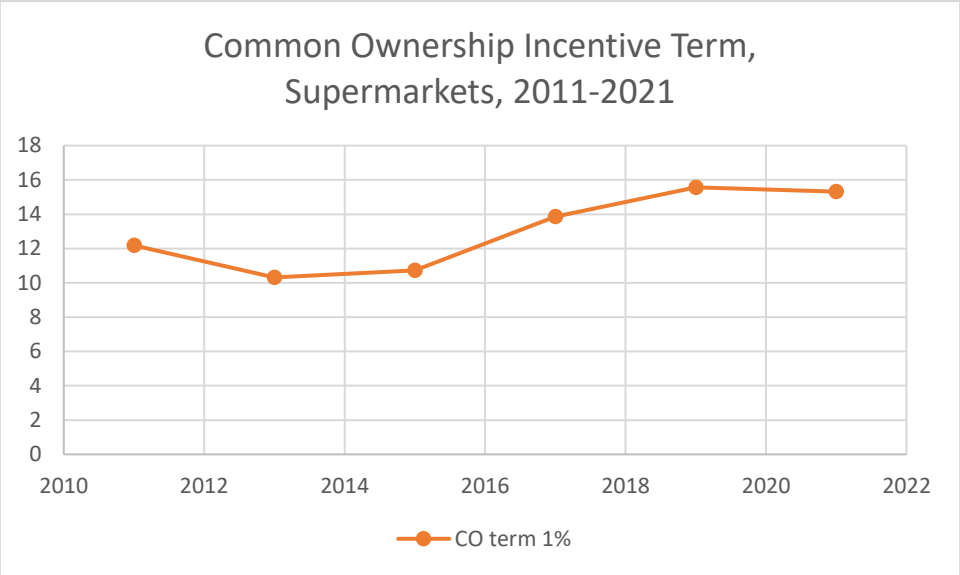


COIT

In analysing the COIT values for the industry, it is important to note that a higher COIT indicates an increased internalisation of competitors profits and thus indicates a reduction in market competition. Note that when comparing the graphs for COORI and the COIT that the two move in opposite directions for the same interpretation. Specifically, a fall in COORI indicates a reduction in the ferocity of competition, whereas a rise in the COIT indicates a reduction in market competition. Additionally, unlike COORI, the COIT does not have a direct interpretation as to the magnitude of the reduction in competition. Other papers have regressed it against measures of market competition, such as price to analyse its relationship with competition. Unfortunately, due to the unavailability of pricing data we were unable to conduct such analysis. Essentially, based on the movements of the COIT we are able to comment on the direction of movement and general trend in competitive ferocity suggested by the ownership links of the industry according to this model, but are unable to comment on the precise magnitude of the effects – as was the case in Banal-Estanol et al (2021). As a starting point for analysis of the impact of common ownership in British industries, this provides an initial narrative on the story using an index that has been generally accepted in the literature.

The precise values for the index are presented in table 5 and the trends are illustrated graphically in figure 4. We can see that initially the COIT falls in 2013 compared to 2011, but then shows a general upward trend. The interpretation of this is that competition has been reduced across these firms due to common ownership over the period studied. These results are in line with those captured by COORI, and the two indices seem to agree that these firms saw a slight reduction in competition over the period due to ownership links.

Figure 4: COIT, Supermarkets, 2011-2021



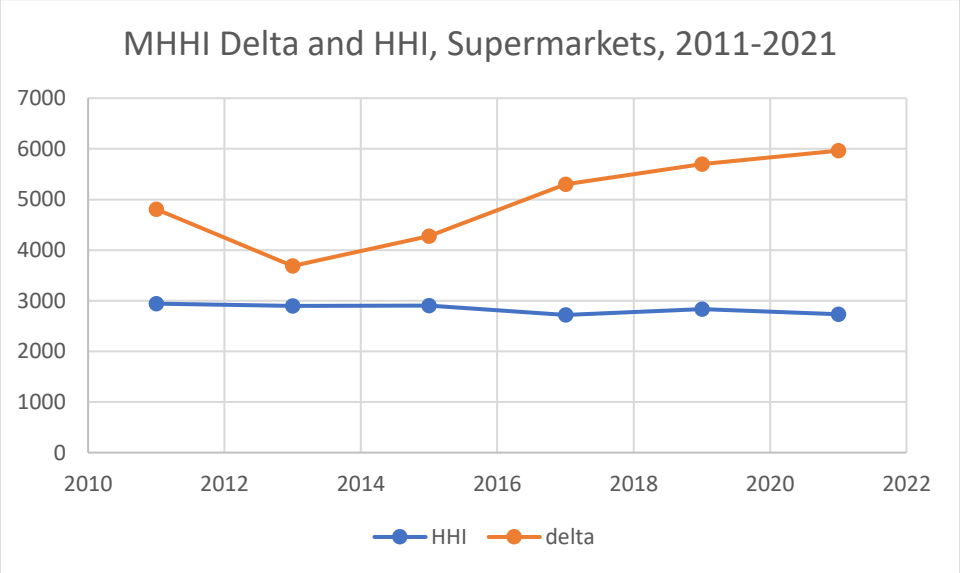
MHHI and HHI

The HHI is a measure of market concentration and doesn't capture common ownerships impact in its calculation but is included for completeness as a respected measure of market concentration and to use as a benchmark against which to consider MHHI delta. The values for the HHI across the period are captured in table 5 and the trend is illustrated in figure 5.

The HHI for this industry remains relatively stable over the period but does drop slightly, fluctuating around 2700-2900. A simplified interpretation of the HHI is that the inverse of the value as a decimal shows the number of firms effectively competing under the assumption that the firms are symmetric. Using this interpretation of the HHI this equates to effective competition of 3.4-3.7 firms, which is slightly low given our sample contains 5 firms and suggests a lack of competition in this industry may be increasing concentration – however the limitations of our market share proxies should be considered and so this should only be interpreted as a benchmark against which to compare delta. The MHHI delta is the component

of MHHI which captures the market concentration resulting from common ownership. The values for each year are captured in table 2 and the trend is illustrated in figure 8. Delta decreases in 2013 relative to 2011 but then increases year-on-year. In 2021 delta is 5963.57 compared to 4807.26 in 2011, which represents a 24% increase over the period. This suggests that competitive ferocity has remained constant over the period according to the HHI but has decreased over the period according to the MHHI delta, which suggests ownership links are driving reduced competition if one considers these results in isolation.

Figure 5: MHHI Delta and HHI, Supermarkets, 2011-2021



Soft drinks

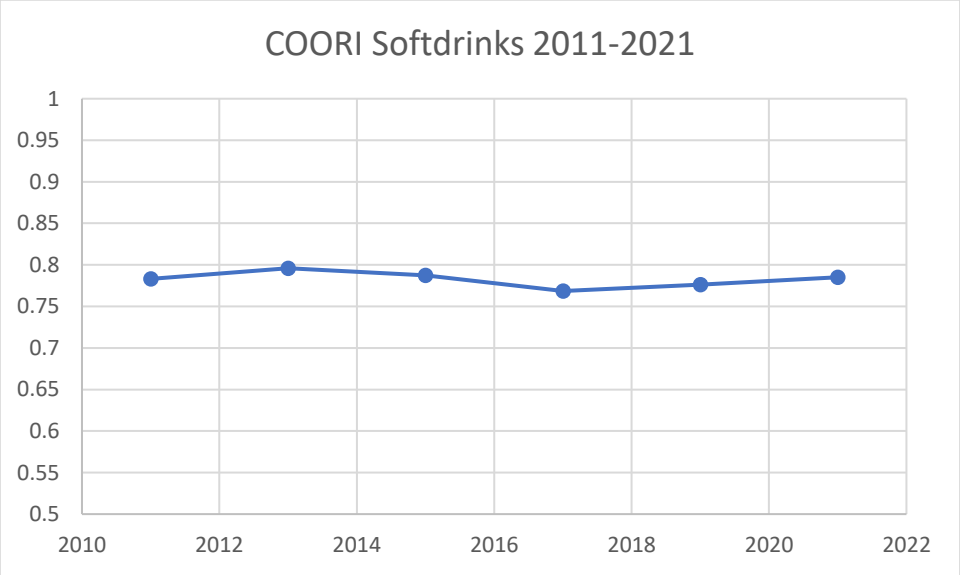
COORI

The values for COORI for this industry are summarised in table 6 and illustrated in figure 6. Quickly, we can see that the COORI for the soft drinks industry was very stable over the period and didn't move significantly in any direction. It fluctuates consistently around the 0.78 level, which indicates that in general over this period quantity was reduced by approximately 22% due to the presence of common ownership links compared to the counterfactual of sole ownership.

Table 6: Index Values for Soft Drinks 2011-2021

Year	COIT	MHHI-Delta	HHI	COCI	COORI
2011	9.92	7043	4499	1.44	0.78
2013	8.35	14329	4487	1.44	0.80
2015	8.12	26364	4443	1.42	0.79
2017	8.29	31018	4304	1.42	0.77
2019	11.37	39358	4338	1.39	0.78
2021	12.53	49103	4504	1.44	0.79

Figure 6: COORI, Soft drinks, 2011-2021

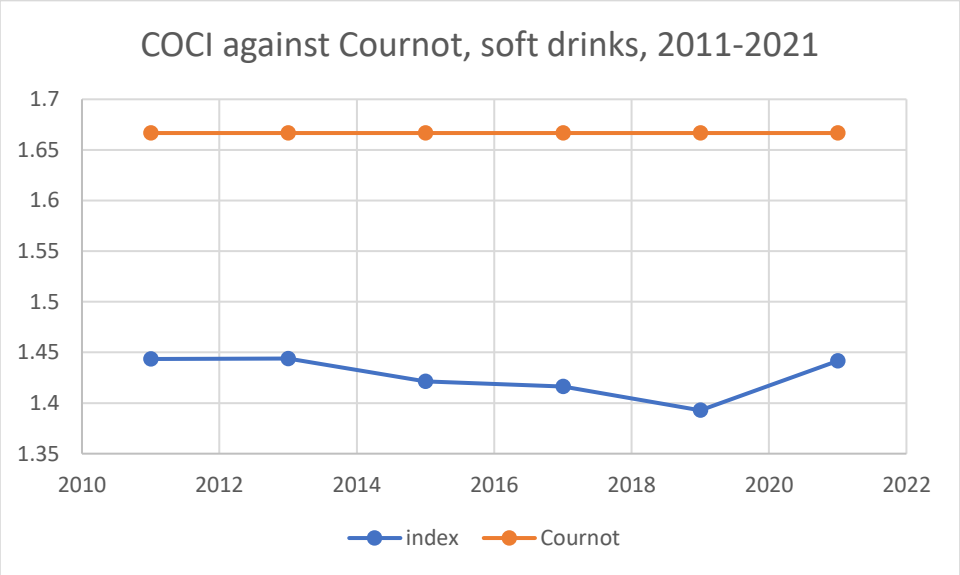


COCI

As we have 5 firms in our sample, the Cournot value for the COCI is again 1.666667. Values for the index are summarised in table 6 and figure 7. The value for the soft drinks industry is relatively static over the 10-year period, fluctuating around 1.44. It drops mildly each year across 2015 and 2019 but then rises back to 1.44 in 2021. This suggests that the competitive ferocity observed across the sample was relatively static, but that competition may have fallen slightly in between 2015 and 2019, before returning to 2011 levels in 2021. This level of

fluctuation doesn't necessarily indicate a material change in competition however, and overall, it appears competitive levels were stable over the period in our sample. However, the level of competition observed is weaker than Cournot according to this index.

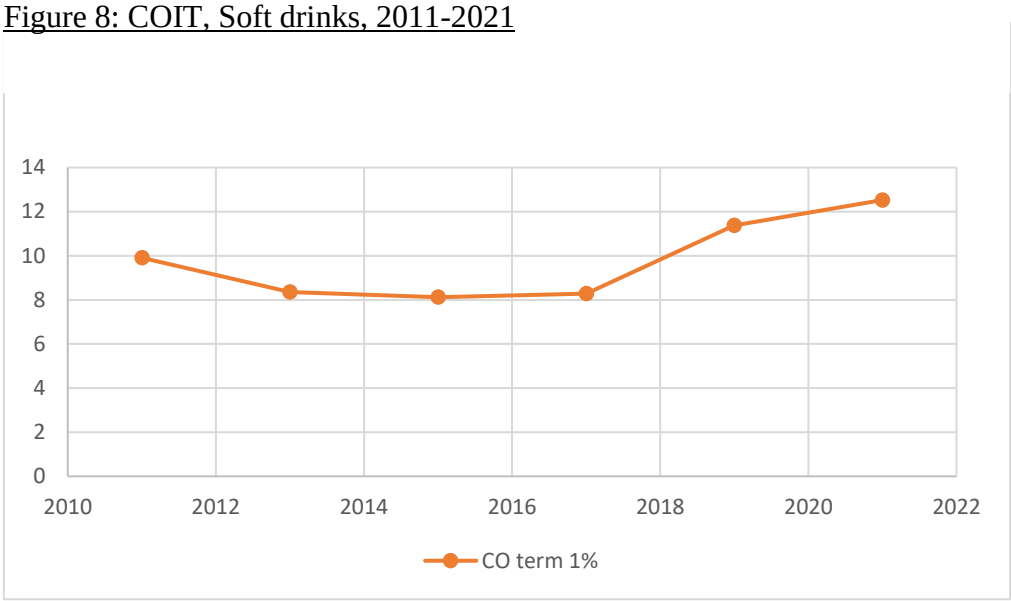
Figure 7: COCI against Cournot, Soft drinks, 2011-2021



COIT

The values for the COIT are summarised in table 6 and illustrated in figure 8. The values do not indicate a dramatic movement in competitive levels across the period. It appears that competition increases very slightly across 2011 to 2017, before decreasing in 2019 and 2021, ending at a level of competitive ferocity slightly weaker than at the start of the period. However, the magnitude of movement is very small, and one cannot confidently assert that this is anything more than a 'random' fluctuation in the index which doesn't capture a material change in competitive dynamics in the industry.

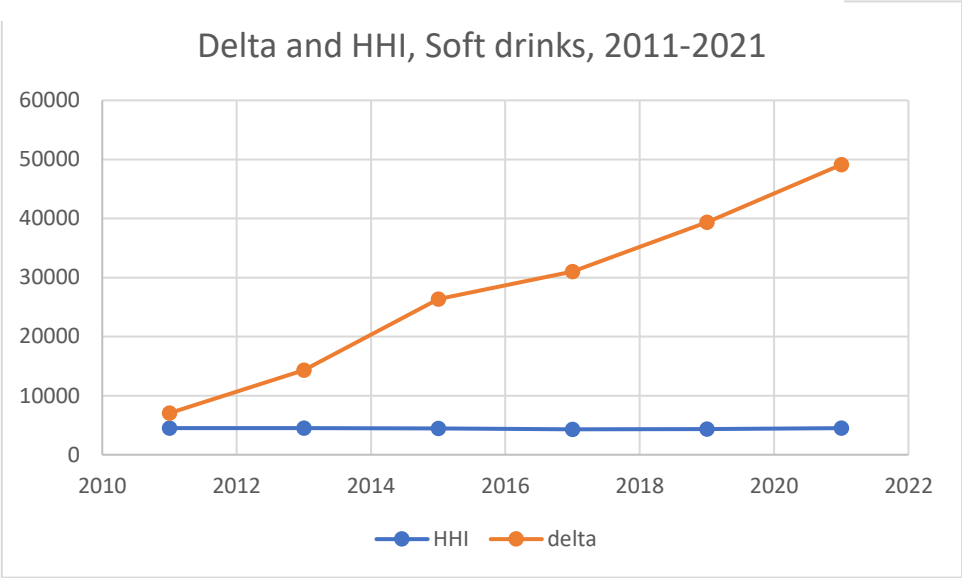
Figure 8: COIT, Soft drinks, 2011-2021



HHI and MHHI delta

The values for the HHI and MHHI are summarised in table 6 and figure 9. The HHI is fairly stable over the period, fluctuating around 4400-4500. This indicates a level of concentration higher than one would expect in an industry with 5 firms. Using the inverse interpretation of the HHI, this value is indicative of having around 2.25 symmetric firms effectively competing in the industry. The MHHI delta once again shows a much more dramatic increase in concentration and increases year-on-year across 2011-2021. Over the period it increases 6.97-fold, which would suggest a rise in concentration due to common ownership (and thus suggests a fall in competition).

Figure 9: Delta and HHI, Soft drinks, 2011-2021



Banking

COORI

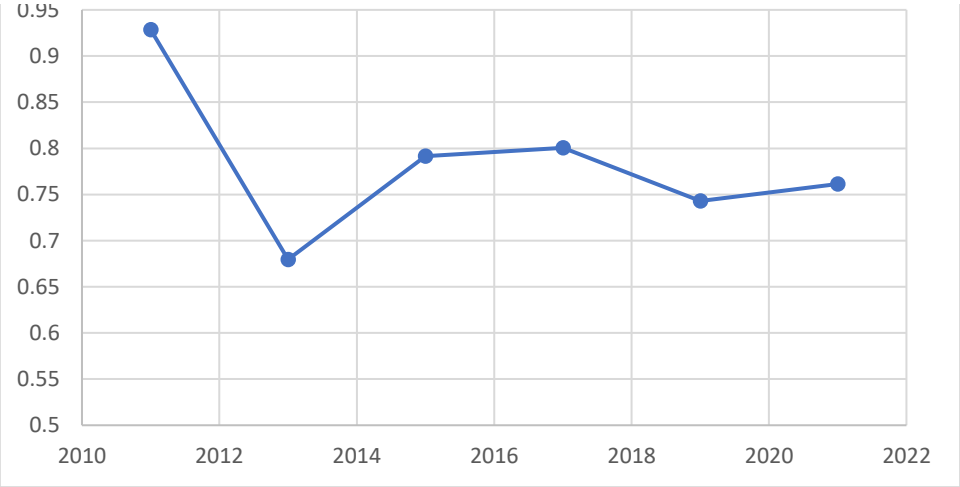
The values for the COORI across the period for banks are summarised in table 7 and figure 10.

The COORI moves markedly over the period and sees movement year-on-year. We can see that in 2011 COORI was 0.9285 and so this indicates that according to our model, the 5 firms in our sample restricted output by 7.15% compared to what they would have produced in the absence of any common ownership links. In 2013 COORI falls to 0.6796, which indicates a 32.04% reduction in output due to common ownership. This is quite a dramatic fall from the 2011 value. In the following years COORI becomes more stable and fluctuates around 0.75-0.8, which suggests a 20-25% reduction in output across these 9 banks due to the common ownership links. One explanation for why COORI is so much higher in 2011 compared to the following years could be taken from the discussion on how ownership patterns in European banks were affected by the 2007-2009 global financial crisis covered in Banal-Estanol (2021). This paper showed that the levels of common ownership saw a temporary fall around the time of the financial crisis due to increased government support but that common ownership levels increased again afterwards. It is possible that in 2011 banks had still been transitioning away from government support and that this artificially influenced the level of common ownership and thus the impact this would have on output (as measured by COORI).

Table 7: Index Values for Banks 2011-2021

Year	COIT	MHHI-Delta	HHI	COCI	COORI
2011	21.51	1963	1949	1.64	0.93
2013	34.92	12177	1955	1.14	0.68
2015	24.04	47908	2021	1.01	0.79
2017	28.36	59539	2085	1.64	0.80
2019	22.38	87892	2114	1.34	0.74
2021	16.73	94544	2100	1.59	0.76

Figure 10: COORI, Banks, 2011-2021

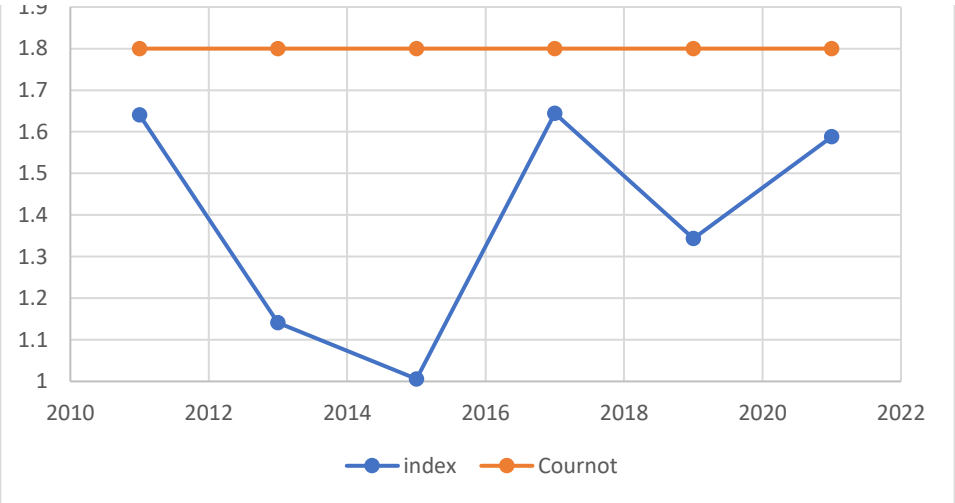


COCI

Our sample for banks contains 9 firms and so the Cournot value for the index is 1.8. The COCI sees a considerable amount of variation over the 10-year period that we study. The values and trend are summarised in table 7 and figure 11. In 2011 the index has a value of 1.6401, before falling to 1.1411 in 2013 and 1.0059 in 2015. The 2015 value is essentially the monopoly outcome, which is an extreme result. After this fall to monopolistic levels, the index rises rapidly to 1.6444 in 2017, before falling to 1.3433 in 2019 and then rising to 1.5881 in 2021. It is not clear why the 2015 value is such a striking outlier but it is likely due to some data abnormality captured in the representative shareholder matrix. The determinant of the R matrix in 2015 is very small and it might be that because it is so close to 0 that we see some abnormal movements in the index in this year. Overall, this index does not paint a clear picture of the

competitive conduct of the banking industry, but it does appear that the overall level of competition is weaker than Cournot. This might just be a reflection that the assumption of symmetric marginal costs implicit in this indices derivation is not an appropriate match for this dataset.

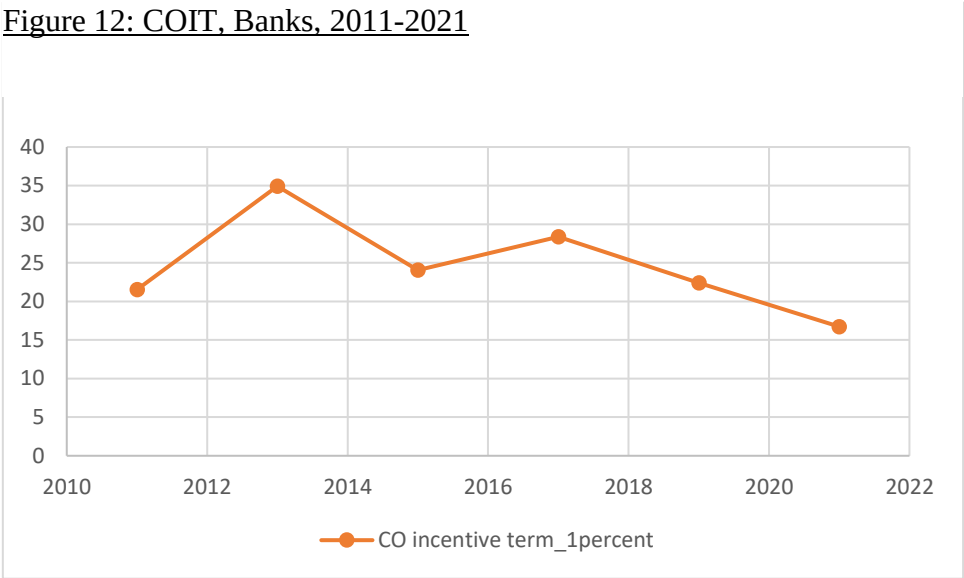
Figure 11: COCI against Cournot, Banks, 2011-2021



COIT

The COIT for banks is summarised in table 7 and figure 12. The COIT seems to indicate that competitive ferocity increased slightly over the period as the graphs show a subtle downward trend. Like COORI, we see a fall in competitive dynamics from 2011 to 2013, and then an increase between 2013 and 2015 – again, this is in line with the argument posed by Banal-Estanol (2021) that common ownership of European banks fell temporarily after the 2007-2009 financial crisis due to increased government support. However, we see a divergence in direction between the two measures in the two datapoints across 2015 and 2019. Interestingly, we see a different story across the COIT and COORI as the COIT seems to suggest that competition has subtly increased over the period, whilst COORI shows a fall in competition – primarily between 2011-2013 before remaining stable.

Figure 12: COIT, Banks, 2011-2021



HHI and MHHI Delta

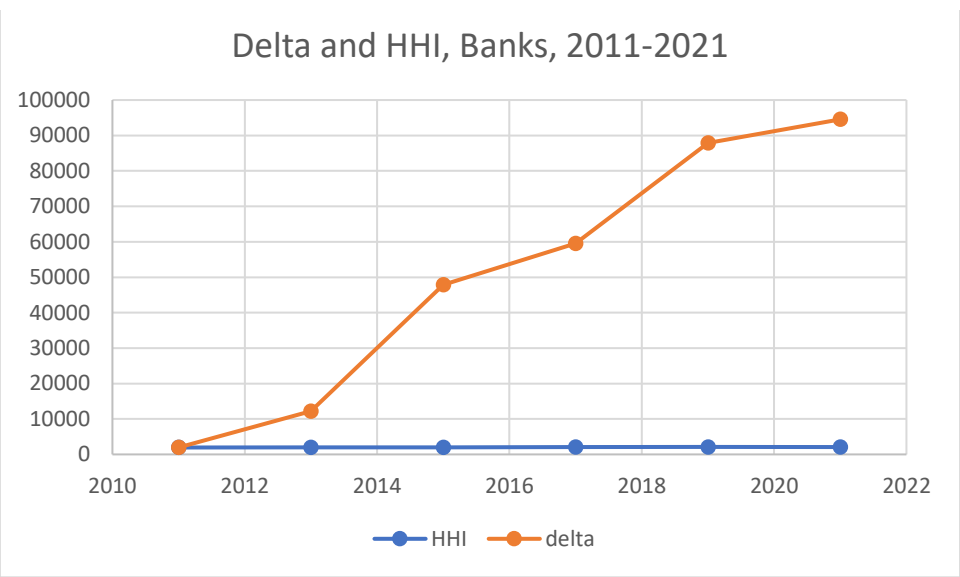
The figures for both of these indices are summarised in table 7 and illustrated in figure 13.

It can be seen that the HHI increases across every year in the period, apart from in the final year where it drops by 14 points. The HHI is a bounded index (with an upper limit of 10,000) and so we can comment on the magnitude of this change. A drop in 14 points is pretty insignificant and can be seen as reflecting a fluctuation around the same level. We can thus see that in terms of market concentration, this segment of the market saw approximately a 7.75% increase in concentration over the time period studied. Traditional interpretation suggests that this would indicate a reduction in competitive ferocity. The consistent upward trend is potentially indicative of increased consolidation but given the limitations of our market share data these results should be considered only in offering a comparison against other indices within our dataset and not taken as a signal of absolute impact on competition in the broad sense. Using the inverse interpretation of HHI, in 2011 there are effectively 5.13 symmetric firms in the market, and in 2021 there are effectively 4.76 symmetric firms in the market. Given the actual size of our sample is 9 firms, these values are very low.

MHHI delta increases every year, and the overall increase is dramatic. The value in 2021 represents a 47-fold increase compared to 2011. What is interesting about this is that in calculating the MHHI, one essentially calculates the COIT and then multiplies it by the cross-market share multiplications before summing up the elements. This dramatic increase is not apparent in the COIT, and it is also not apparent in the HHI – which is also a summation of

market share multiplications. This suggests that the increase is resulting due to an influence of the multiplication of cross-market shares and thus the dramatic change in suggested competitive conduct is unique to the MHHI delta. It is unclear why this is happening, but it is also apparent in other literature. For instance, Azar et al (2018) noted a large increase in common ownership arising due to MHHI delta, but Dennis et al (2018) found that this increase was not apparent when replacing delta with the COIT. It is possible that it is a feature in the calculation of delta which leads it to suggest amplified increases in concentration arising due to common ownership.

Figure 13: Delta and HHI, Banks, 2011-2021



Discussion

Supermarkets

Considering in aggregation the evidence across indices from our sample of supermarkets, it appears that common ownership may have negatively influenced competition over this period. All of the indices move in the direction which suggests reduced competitive ferocity, and the trend is consistent. COORI shows that across the period output by these firms was reduced by between 10.34 and 16.64% due to the presence of common ownership links, compared to the counterfactual where each of these firms were independently owned. Moreover, the COCI showed that competition was reduced over the period but was below the Cournot value for the entirety of the period. One way to interpret the COCI is to infer what the value of the index is in terms of equivalency to Cournot across n firms. For example, in 2011 for supermarkets the index had a value of 1.533918, which is equivalent to Cournot competition across 3.29 firms,

which is low considering the true sample size is 5 firms.[8] In 2021 the index fell to 1.415483, which is equivalent to Cournot competition across 2.42 firms. The COIT increased by 31.43% over the period. While we cannot comment on the exact impact this would have on competition without conducting a regression analysis, we know that a higher COIT is associated with firms increasing the extent to which they internalise their rivals' profits (Kennedy et al, 2018) and so we can infer that this would be associated with reduced competitive ferocity, but we can't comment on the scale of this effect. Interestingly, the HHI fell over the period, which suggests reduced concentration (associated with greater competition) but this is limited by our simplified definition of market shares. Moreover, the HHI does not consider the influence of ownership links. The MHHI however, increased over the period, which suggests that there was increased effective concentration due to ownership links over the period. As with the COIT we cannot comment directly on the impact on competition without a regression analysis, however the direction of movement is consistent with what the other indices indicate the influence on competition would be. Overall, the evidence seems to suggest that these firms saw an increased influence of common ownership on their behaviour over the period and might suggest that this is an industry which might be worth studying further if regulators have access to pricing data. The firms we consider account for over 60% of the market and so represent a substantial and significant portion of the wider market, so if competition is being dampened across these firms it is likely having negative consequences for consumers.

Soft drinks

Looking at the evidence we have collected on the soft drink industry it appears that on the whole, the influence of common ownership has remained relatively stable over the period. According to COORI, quantity across these firms is consistently around 22% lower due to common ownership. Similarly, the COCI is stable at a value of around 1.44 over the period, and this is consistently below the Cournot value for 5 firms, which suggests that competition is slightly dampened in this industry but that the extent of this has not changed over 2011-2021. Specifically, a value of 1.44 for the index of market competition is equivalent to Cournot

[8] To see this, recall that for Cournot

$$e^T R^{-1} e = \frac{2n}{n+1}$$

Solving for n:

$$\frac{e^T R^{-1} e}{2 - e^T R^{-1} e} = n$$

Since $e^T R^{-1} e$ is just a scalar, we substitute the value of the index into this and can solve for n to see what the value for the index is in terms of equivalency to Cournot across n firms.

competition across 2.57 firms, which is quite low given our sample contains 5 firms. The COIT remains relatively stable over the period but falls slightly in 2013 compared to 2011, remains at around the same level until 2019 and then rises above the initial value in 2021. However, the magnitude of these fluctuations is very small. It seems that the COIT tells much the same story as COORI in that the two measures did not see much variation over the period in the soft drinks industry. HHI sees a similar trend to the COIT, falling very slightly year on year until 2021 when it reaches its highest value over the period. However, again, these changes are of a very small magnitude, and as the HHI is a bounded index we can say that the change of 200 points is not a large-scale change in concentration. The MHHI delta rises year on year and the 2021 value represents an increase of 5.97 times the 2011 value – which implies that effective concentration was greatly increased over the period as a result of common ownership, but that general concentration (HHI) fell and then increased over the period. Our proxy market share data limits the reliability of these results as general conclusions on industry conduct, but it offers an interesting comparison against the other industries. On the whole though, the balance of evidence on soft drinks seems to suggest that common ownership may be dampening competition in this industry but that the extent of this effect has been consistent over the period studied.

Banking

The balance of evidence on banking is a lot more mixed than the other two industries we study. The COORI falls quite substantially in 2013 compared to 2011, and then fluctuates between 0.74 and 0.8 for the remainder of the period, which suggests that overall common ownership is reducing output across these firms compared to if they were each independently owned. Moreover, the COORI suggests that the magnitude of this effect has increased over the period. By 2021, the reduction in output due to common ownership is 16.6%. The COCI fluctuates dramatically but is consistently lower than Cournot. The 2015 value is basically showing a monopoly outcome and it can only be assumed that this is an outlier and that it represents some abnormality in the representative ownership matrix for that year. Aside from this outlier, the highest value for the index is equivalent to Cournot competition across 4.62 firms, which is low considering the true sample size is 9 firms. Generally, when put together these first two measures indicate that competition has been dampened somewhat by common ownership and that the level of this effect may have increased over the period. The COIT however shows a different story as it has an overall downward trend, which implies that firms are internalising each other's profits to a lower degree in 2021 compared to 2011, which should suggest

increased competition. It is interesting that the direction of suggested behaviour contrasts with the other two indices though. The HHI increases very mildly by around 200 points over the period, which suggests concentration increased slightly (associated with reduced competition) but this is quite a low level of change in the index. The MHHI however increased markedly, with the 2021 value representing a 47-fold increase on the 2011 value. This suggests dramatic increases in concentration arising due to ownership links. Overall, it appears difficult to draw a general conclusion on the data from the banking industry, but the COORI which has a direct interpretation, seem to suggest that common ownership may be reducing competition (and at an increasing rate) in this industry. However, further evidence would be needed to say anything with more certainty.

COORI and O'Brien's Impossibility Critique

Central to our discussion of existing methods of studying common ownerships impact on competition has been O'Brien's (2017) impossibility critique. This states that the coefficients of price-concentration regressions have no meaningful interpretation because the relationship between the dependent and independent variables are not grounded in any economic theory. Essentially, there is no one-to-one relationship between price and concentration measures such as the MHHI. We overview this issue and explain why COORI is not subject to this critique.

O'Brien (2017) denotes that C is some pure measure of common ownership, such that it depends on the ownership matrix. Equilibrium price (P) and quantity (Q) thus depend on C and some other exogenous factors such that P(C,X) and Q(C,X). Concentration (H) also depends on C and X, such that H(C,X). For price-concentration regressions to have meaningful coefficients it must be possible to invert H with respect to C such that C=g(H,X) so that one can substitute concentration for common ownership meaningfully and the relationship can be estimated as P(g(H,X), X). However, the impossibility result presented in the paper shows that concentration is not invertible in the relevant domain and so it is not possible to substitute C for g(H,X) in this way. Concentration based measures of common ownership, such as the MHHI, do exactly this and do not address this foundational instability in their meaning. Kennedy et al (2017) use the COIT in place of the MHHI to overcome this issue. This works because using COIT in place of MHHI delta means that one is no longer proxying common ownership with concentration and instead a measure which depends directly on the ownership matrix is being used, and thus the explanatory variable used in a regression against price is based on a relationship with sound theoretical foundations i.e. one is estimating P(C,X) and not P(H,X) and assuming H is a good proxy for C. COORI works in the same way in that it is a

pure measure of common ownership and thus using it in a regression against price allows one to study P(C,X). The advantage of COORI over COIT is that as well as overcoming O'Brien's critique, COORI is an index with an intuitively clear and immediately interpretable meaning. One can look at COORI directly and state that output is x% lower because of common ownership, before any concerns of designing econometric studies must be put in place. However, as COORI overcomes the impossibility critique it can also be used meaningfully in regression analysis if one had access to pricing data, and so it is a more flexible tool than existing measures.

O'Brien states in his conclusions that the key to overcoming this issue and meaningfully assessing the impact of changing ownership structure on market outcomes is to begin by estimating equations which are founded in a model that considers ownership structure, and to then compare this to a counterfactual analysis which considers an alternative state of the world. This is precisely what COORI does in that it uses a theoretical model to determine how common ownership impacts outputs and then compares it against the counterfactual state of the world in which there is no common ownership.

Additional Work

In an ideal world we would have been able to access pricing data across the period for each of the industries in question. This would have allowed us to complete regression analysis where we regress COORI, and the other measures, against price in order to compare directly the magnitude and direction of impact, as well as the statistical significance of each measure. It would have been useful to combine the datasets into one and add a variable indicating which industry the observation belonged to, so that we could have a larger sample to consider the relationship between the indices and price. We could then compare the impact, direction, and significance of these on the sample as a whole, as well as across industries separately to see if the stories differ. Unfortunately, pricing data was not obtainable. However, if future researchers have access to this, that would be the natural follow up to this work.

Common Ownership in Merger Analysis

As mentioned earlier, another natural progression to this work would be to make the counterfactual price in COORI a function of price elasticity of demand so that the price being used in the counterfactual of sole ownership is not assumed to be the same price existing under common ownership. Incorporating the price elasticity of demand here would better capture the changes in price that would result in response to the output adjustments that would follow the

change in ownership structure. Following this, given that COORI captures the reduction in output that arises due to common ownership, it would be interesting for future researchers to investigate if COORI could be converted into a measure of price increases resulting from common ownership if we had information on price elasticity. This could make it a parallel tool to the modified gross upward pricing pressure index (mGUPPI) in merger analysis. Indeed, this is an exciting next step for common ownership literature in general as a clear policy application would be to include it in merger analysis in the future. The mGUPPI is to my knowledge the only existing tool in this regard, and further work investigating how common ownership models and insights could enrich merger analysis would be an exciting development in the literature.

The traditional GUPPI (gross upward pricing pressure index) is an index which scores the unilateral incentives of a full merger in an industry where firms are engaging in price competition on differentiated products (Asoni & Sarafidis, 2017). The GUPPI is calculated from the perspective of each firm, in the sense that it independently scores each firm's post-merger incentive to increase prices (as these may not be symmetric). The score is calculated as follows (Azar and Tzanaki, 2021):

$$GUPPI_i = D_{ij} m_j \frac{p_j}{p_i}$$

Where D_{ij} is the diversion ratio from firm i to j, p_i is firm i's price and (p_j-c_j) is firm j's marginal cost.

Ultimately the GUPPI is a measure of the value of diverted sales, indexed to the pre-merger price. The GUPPI can be generalised to account for the ownership structure of the firms to obtain the mGUPPI, in the same way that the traditional HHI is generalised to become the MHHI (see chapter 1 for a discussion on this). The mGUPPI is a multiple of the traditional GUPPI where the multiplier depends on the degree of financial interest and control of shareholders (Asoni & Sarafidis, 2017). Azar and Tzanaki (2021) define the mGUPPI as:

$$mGUPPI_i = \sum_{j \geq 2} \lambda_{ij} \cdot D_{ij} m_j \frac{p_j}{p_i}$$

Here λ_{ij} captures the weight that firm i places on competitor j, D_{ij} is the diversion ratio from firm i to firm j, and m_j is firm j's percent margin. Azar & Tzanaki (2021) and Inderst & Thomas (2019) provide examples calculating and discussing the mGUPPI and what it can tell us about common ownership in merger analysis. It is possible that COORI could be used in a

similar way and offer a tool with a different portfolio of strengths and weaknesses to complement the existing tools. One can imagine a case whereby COORI could be calculated by replacing the counterfactual ownership matrix with the post-merger ownership matrix to get a ratio of output pre/post-merger. Exploring how this would look in practice, how to interpret it, and if it can be converted into a measure of pricing pressure would be a very interesting extension of this work.

Conclusion

This work studied the implications of common ownership on competitive conduct in three British industries. It represents some of the first work looking in detail at ownership links and their impact on competition in Britain. It introduces a new index, COORI, which builds on the model derived in chapter 2. COORI allows one to directly calculate the impact of common ownership links on firms output using data which is more easily available than pricing data required for other measures. COORI has an intuitive interpretation, and we believe it represents a potentially useful tool for competition authorities in getting quick insights on markets. COORI is also backed up by a clear theoretical relationship, which offers its interpretation validity. This is a strength as a lack of theoretical grounding has been discussed as the one of the major pitfalls to existing measures of common ownership and so in this way COORI is a step forward in the literature. Moreover, if pricing data is available, COORI can also be regressed against price in the way that other measures are, so it is perhaps a more flexible measure in terms of how it can be implemented.

We studied three British industries: supermarkets, soft drinks and banking. Evidence on the whole suggests that supermarkets have seen worsening competition over the period due to common ownership links – with COORI showing that output was 17% lower in 2021 because of common ownership. In the soft drinks industry, it appears that common ownership may have dampened competition but that the magnitude of the impact was quite stable over the period with COORI showing output was reduced by about 21% in 2021 due to common ownership. Evidence on banking was less coherent, but the COORI suggests that common ownership has weakened competition in this industry.

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Appendix 1: Proof of Theorem 1

We have shown that we can obtain:

$$\begin{bmatrix} 1 & \cdots & \frac{1+r_{1n}}{2} \\ \vdots & \ddots & \vdots \\ \frac{1+r_{n1}}{2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} \frac{a-c}{2b} \\ \vdots \\ \frac{a-c}{2b} \end{bmatrix}$$

Now, using our definition of R notice that we can rewrite this as follows:

$$Rq = \frac{a-c}{2b} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Now, notice that we can re-arrange this obtain the following:

$$e^T q = \frac{a-c}{2b} e^T R^{-1} e$$

Where e is simply a nx1 vector of 1's: $e = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ and e^T is the transpose of this vector.

We can see that:

$$e^T q = q_1 + q_2 + \cdots + q_n = \sum_{i=1}^{n} q_i = Q$$

Thus, we have:

$$Q = \frac{a-c}{2b} e^T R^{-1} e$$

We can also see quickly that:

$$q = \frac{a-c}{2b} R^{-1} e$$

Appendix 2: Proof that majority owner in both Firms will shut down the firm in which it has the lower holding

To see quickly why a shareholder with majority in both firms would want to shut down the firm in which they have a smaller holding consider the following simple example:

Suppose we have a shareholder who has α% of the shares in firm 1 and β% of the shares in firm 2, where $0.5<(\alpha, \beta)<1$ and $\alpha>\beta$. We know that as they are in control of the whole industry, they will choose to produce the monopoly quantity such that $q_1+q_2=Q_M$. However, recall that there is still some other shareholder who is entitled to some minority percentage of the profits in each firm. The portfolio profit of the majority shareholder in each of the three cases we discussed above would be as follows:

Action	Profit of majority shareholder
Produce Q_M in firm 1 and shut down firm 2	$\alpha\pi_M$
Produce Q_M in firm 2 and shut down firm 1	$\beta\pi_M$
Produce δQ_M in firm 1 and $(1-\delta)Q_M$ in firm 2, where $0<\delta<1$	$\alpha\delta\pi_M + \beta(1-\delta)\pi_M$

We can quickly see that $\alpha\pi_M > \beta\pi_M$ and that $\alpha\pi_M > \alpha\delta\pi_M + \beta(1-\delta)\pi_M$ iff:

$$\alpha\delta + \beta(1-\delta) < 1$$

Some quick re-arranging shows us this will be the case iff:

$$\delta(\alpha-\beta) < 1-\beta$$

Which we can see is true because $\alpha<1$ and so $1-\beta > (\alpha-\beta)$, and as $\delta < 1$ we know $\delta(\alpha-\beta) < (\alpha-\beta)$ and hence $\delta(\alpha-\beta) < 1-\beta$. We can then conclude that:

$$\alpha\pi_M > \alpha\delta\pi_M + \beta(1-\delta)\pi_M$$

In the third case where $\alpha=\beta$ the profit of the majority owner would be:

$$\alpha\delta\pi_M + \alpha(1-\delta)\pi_M = \alpha\pi_M$$

Thus, in the case where the majority owner has an equal holding in the two firms their portfolio profit is independent of the fraction of the monopoly quantity they choose to produce in the

two firms (δ) and so any possible combination of outputs across the firms that satisfies $q_1 + q_2 = Q_M$ could be optimal.

Appendix 3: Proof that COORI cannot exceed 1

From theorem 1:

$$COORI = \frac{e^T pq}{e^T E^{-1} Rpq}$$

Thus for

$$COORI > 1$$

It must be that:

$$e^T pq > e^T E^{-1} Rpq$$

We will show the proof for a matrix of dimension 3 to give a clear example, but the proof holds for any dimension of matrix.

Recalling that p is a constant, and the rest are vectors and matrices we can expand this out to be:

$$[1 \quad 1 \quad 1]\begin{bmatrix} pq_1 \\ pq_2 \\ pq_3 \end{bmatrix} > [1 \quad 1 \quad 1]\begin{bmatrix} 1.5 & 0.5 & 0.5 \\ 0.5 & 1.5 & 0.5 \\ 0.5 & 0.5 & 1.5 \end{bmatrix}\begin{bmatrix} 1 & r_1 & r_2 \\ r_3 & 1 & r_4 \\ r_5 & r_6 & 1 \end{bmatrix}\begin{bmatrix} pq_1 \\ pq_2 \\ pq_3 \end{bmatrix}$$

Where r_i is just some element of our R matrix.

This expression expands to yield that for COORI to exceed 1 it must be true that:

$$p(q_1 + q_2 + q_3) > p\left(\frac{1 + r_3 + r_5}{2} q_1 + \frac{1 + r_1 + r_6}{2} q_2 + \frac{1 + r_2 + r_4}{2} q_3\right)$$

Cancelling p:

$$(q_1 + q_2 + q_3) > \left(\frac{1 + r_3 + r_5}{2} q_1 + \frac{1 + r_1 + r_6}{2} q_2 + \frac{1 + r_2 + r_4}{2} q_3\right)$$

We know that the lowest value of shareholding an owner can have in a firm (assuming no short selling) is 0, and thus recalling that R is calculated as:

$$\begin{bmatrix} 1 & \cdots & \frac{1 + r_{1n}}{2} \\ \vdots & \ddots & \vdots \\ \frac{1 + r_{n1}}{2} & \cdots & 1 \end{bmatrix}$$

And the lowest value of r_{in} is 0, we know that the lowest possible value for all r_i is 0.5. Thus, in the case of sole ownership (the lowest level of common ownership), the expression would simplify to show that:

$$(q_1 + q_2 + q_3) = \left(\frac{1 + r_3 + r_5}{2} q_1 + \frac{1 + r_1 + r_6}{2} q_2 + \frac{1 + r_2 + r_4}{2} q_3\right)$$

As this is the lowest value of r_i possible it is thus true that the highest possible value of COORI is 1, when $r_i = 0.5 \; \forall i \in n$.

Appendix 4: Calculating COORI

To begin, the ownership dataset (for each year) looks like a table with firms along the columns and shareholders in the rows:

	Firm A	Firm B	Firm C
s1	8	12	6
s2	5	7	5.6
s3	3.2	3.4	1.2
s4	2.6	1.2	0
s5	1.4	1.6	0.6
s6	2.2	2.2	2.1
s7	0.2	0.3	1.3
s8	1.8	0.6	0.2
s9	1.5	0.84	0.8
s10	0.9	1.02	1.1

We will also have revenue data (for each year) for each firm, which we will use later, and this will be in the following format:

	Revenue (£m)
Firm A	32247
Firm B	18254
Firm C	29112

To be clear, in what follows, assume that the above fictitious ownership and revenue data is from the same year, so that they correspond to the same observation. The following procedure would need to be completed for each year in the sample.

In the ownership data, the elements represent the ownership share of each shareholder in each firm. For instance, we can see that shareholder 1 holds 8% of the shares in Firm A and shareholder 6 holds 2.2% of the shares in Firm B.

Once we have the dataset, the first step in calculating COORI is to calculate the representative shareholders portfolio for each firm. There are many ways in which one could choose a representative shareholder, as discussed in chapter 2, but we take this to be the weighted average portfolio across each firm's shareholders as this is what is considered standard in

calculating the MHHI and COIT in the literature, and so we make this choice to be consistent. Following this, the representative shareholder of firm A holds the weighted average portfolio of s1…s10 in firm A. In the real dataset, the number of observations for shareholdings is so large that we instigate a cut-off of 1% to make the calculations more manageable. This means that we assume only shareholders who hold 1% of more in a firm are considered in calculating the representative shareholder of a firm. Imposing a cut-off has been done in other papers (Azar, 2018; Banal-Estanol et al, 2022). We now outline how one finds the weighted average portfolio of each firm's shareholdings.

To begin, we will find the weighted average portfolio of the shareholders in Firm A. The first step is to observe which shareholders have holdings of 1% or above. We then write these in a new table, with the firms in the rows and the shareholders with above 1% in the columns and fill in the corresponding shareholdings. For Firm A we can see that shareholder's 7 and 10 have holdings below 1% and so are omitted from the calculation. We thus have:

	s1	s2	s3	s4	s5	s6	s8	s9
Firm A	8	5	3.2	2.6	1.4	2.2	1.8	1.5
Firm B	12	7	3.4	1.2	1.6	2.2	0.6	0.84
Firm C	6	5.6	1.2	0	0.6	2.1	0.2	0.8

Notice that the 1% cut-off only applies in selecting the shareholders which influence Firm A's decisions. We are choosing the shareholders based on their holdings in Firm A because we are calculating Firm A's representative shareholder. However, it is entirely possible that this shareholder may hold below 1% in the other firms. This is the case in calculating each firm's representative shareholder. In calculating firm B's average shareholder portfolio, we will only consider firms who have a 1% or above holding in B, but this shareholder might have a below 1% holding in A or C, etc. The idea here is that we are assuming that the largest shareholders are the ones with the most influence in determining firm behaviour. There could be other ways of determining influence, but that is not the purpose of this paper, and this route was chosen in this case for tractability.

Next, we sum up the shareholdings of the influential shareholders in Firm A and divide each position by this value to calculate the weighting of that shareholder. Essentially, this means we sum up all the elements in Row A and divide each element in row A by this total to get that columns weighting:

	s1	s2	s3	s4	s5	s6	s8	s9	sum
Firm A	8	5	3.2	2.6	1.4	2.2	1.8	1.5	25.7
Firm B	12	7	3.4	1.2	1.6	2.2	0.6	0.84	
Firm C	6	5.6	1.2	0	0.6	2.1	0.2	0.8	
weighting	0.3112	0.1946	0.1245	0.1012	0.05445	0.08560	0.0700	0.0584	

Here, 0.31128 is obtained by dividing 8 by 25.7, 0.194553 is obtained by dividing 5 by 25.7, etc. This shows us how much influence each of these shareholders has in Firm A's actions and so gives us the value of weight that Firm A will place on each shareholder. Trivially these sum to 1. Once we have obtained these weightings, we then multiply each column by its weighting to get the weighted shareholdings. Finally, we sum up each row to get the weighted average shareholding in each firm. This is the representative shareholders portfolio for Firm A.

	s1	s2	s3	s4	s5	s6	s8	s9	sum
Firm A	8	5	3.2	2.6	1.4	2.2	1.8	1.5	25.7
Firm B	12	7	3.4	1.2	1.6	2.2	0.6	0.84	
Firm C	6	5.6	1.2	0	0.6	2.1	0.2	0.8	
weighting	0.31	0.19	0.12	0.10	0.05	0.08	0.07	0.06	
	2.49	0.97	0.40	0.26	0.08	0.19	0.13	0.09	***4.60***
	3.74	1.36	0.42	0.12	0.09	0.19	0.04	0.05	***6.01***
	1.87	1.09	0.15	0	0.03	0.18	0.014	0.05	***3.38***

Thus, we can see that the representative shareholder of Firm A has a portfolio of 4.6% in Firm A, 6.0% in Firm B and 3.4% in Firm C.

We then repeat this process to find the representative shareholders portfolio for Firm B and Firm C. I will not outline the steps again, but here are the tables capturing the output of each step.

Firm B:

	s1	s2	s3	s4	s5	s6	s10	sum
Firm A	8	5	3.2	2.6	1.4	2.2	0.9	
Firm B	12	7	3.4	1.2	1.6	2.2	1.02	28.42
Firm C	6	5.6	1.2	0	0.6	2.1	1.1	
weighting	0.42	0.25	0.12	0.04	0.06	0.08	0.04	
	3.38	1.23	0.38	0.11	0.08	0.17	0.03	***5.38***
	5.07	1.72	0.41	0.05	0.09	0.17	0.04	***7.55***
	2.53	1.38	0.14	0	0.03	0.16	0.04	***4.29***

Firm C:

	s1	s2	s3	s6	s7	s10	sum
Firm A	8	5	3.2	2.2	0.2	0.9	
Firm B	12	7	3.4	2.2	0.3	1.02	
Firm C	6	5.6	1.2	2.1	1.3	1.1	17.3
weighting	0.35	0.32	0.07	0.127	0.08	0.06	
	2.77	1.62	0.22	0.27	0.02	0.06	***4.95***
	4.16	2.27	0.24	0.27	0.02	0.06	***7.02***
	2.08	1.81	0.08	0.25	0.10	0.07	***4.40***

So, the matrix of representative shareholders is as follows:

	sA	sB	sC
A	4.60	5.38	4.95
B	6.01	7.55	7.02
C	3.38	4.29	4.40

In strict matrix form:

$$\begin{bmatrix} 4.60 & 5.38 & 4.95 \\ 6.01 & 7.55 & 7.02 \\ 3.38 & 4.29 & 4.40 \end{bmatrix}$$

We calculate the COORI by taking this matrix and calculating R, as in chapter 2, before applying the equation from theorem 1 in section 2.

To calculate the R matrix, the first step is that we divide each row in the matrix by its corresponding diagonal, so we divide row 1 by the first element in row 1, we divide row 2 by the second element in row 2 and we divide row n by the n^{th} element in row n. This gives us:

$$\begin{bmatrix} 1 & 1.169625 & 1.076392 \\ 0.79632 & 1 & 0.930107 \\ 0.76823 & 0.97561 & 1 \end{bmatrix}$$

Next, we add 1 to each element and divide by 2 – see chapter 2 for an explanation of this procedure in deriving the R matrix. We thus arrive at:

$$R = \begin{bmatrix} 1 & 1.084813 & 1.038196 \\ 0.89816 & 1 & 0.965054 \\ 0.884115 & 0.987805 & 1 \end{bmatrix}$$

Now that we have the R matrix[9], we must calculate the counterfactual R matrix under the assumption of sole ownership (we call this the E matrix). To do this, we simply start with a matrix of sole ownership and treat this as we did the representative matrix above. Essentially, we start with:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We now divide each row by its corresponding diagonal, and then add 1 to each element and divide this by 2, as we just did before. This yields:

$$E = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$

The inverse of which is:

$$E^{-1} = \begin{bmatrix} 1.5 & -0.5 & -0.5 \\ -0.5 & 1.5 & -0.5 \\ -0.5 & -0.5 & 1.5 \end{bmatrix}$$

Using the total revenue figures listed at the beginning of this example, in matrix form, total revenue will be:

[9] Note: The COCI developed in chapter 2 is calculated simply by pre and post multiplying R by the vector of 1's (e). To be clear, this is the same R matrix, and I will not repeat its derivation to explain how to calculate the index of market competition. But the procedure is identical. The difference is how one proceeds to manipulate the R matrix.

$$pq = \begin{bmatrix} 32247 \\ 18254 \\ 29112 \end{bmatrix}$$

We now have everything we need to calculate COORI and must simply apply the equation from Theorem 1:

$$COORI = \frac{79613}{116619.1} = 0.6827$$

So, in this example, total production by Firms A, B and C is 31.73% lower due to the presence of common ownership compared to the counterfactual of sole ownership.