Mechanical Fault Detection in Induction Motors Using a Data-Driven Kalman Filter

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Abstract— In this paper, a data-driven algorithm to identify the eccentricity fault in induction motors is proposed. The algorithm is based on the Kalman Filter (KF) and utilizes experimental data collected from healthy and faulty three-phase stator currents at different speeds and load conditions. Additional data processing techniques including Discrete Wavelet Transform (DWT), Power Spectral Density (PSD), and cepstrum are used to extend the dataset. A feature extraction process involving a few statistical measures is applied to this dataset. For each feature, a State-Space Model (SSM) and a KF are formulated. By comparing the resulting output of the SSMs with the estimated output from KFs, a measure to identify an eccentricity fault is obtained. This method was tested on various operating modes of an induction motor, demonstrating its effectiveness in distinguishing healthy data from those indicating an eccentricity fault.

Keywords—Eccentricity fault, Kalman filter, data-driven state-space model, induction motor.

I. INTRODUCTION

Electric machines are a key element in all kinds of industries. Among all types of electrical motors, induction motors (IM) due to their reliability, robustness and low cost, are considered a first choice for several applications. Although these induction motors are highly reliable, they are susceptible to various types of faults. A machine fault can greatly impact the system performance, cause sudden interruptions and even provoke catastrophic failures. In order to improve the safety of the system, it is necessary to diagnose the faults. Faults in IMs can be categorized into electrical and mechanical faults. The common mechanical faults in IM are bearing failure, eccentricity and broken rotor bars. Approximately 40% of all failures arose due to these faults during the operation of industrial processes [1]. There are a large number of publications on mechanical fault detection and diagnostics.

Generally, detection methods can be classified into physics-based, data-driven and hybrid approaches [2]. Physics-based methods employ a mathematical model to describe the behaviour of failure is available. In this approach, the physical model with measurement results is combined to design model parameters and predict future states. Datadriven methods use previous information from training data to recognize the feature of the current failure state and predict the future states without using any specific physical model. Hybrid techniques are the combination of the two methods above mentioned which improve the prediction performance [3].

In this paper, eccentricity fault detection has taken attention due to its disastrous consequences. Several techniques have been developed regarding the diagnosis of eccentricity faults for induction motors using model-based and data-driven schemes. Model-based approaches are divided into MMF-permeance analysis [4,5], simulation analysis [6-8], instantaneous power analysis [9,10], auxiliary voltage injection [11], and sensor-based and observer-based methods [12,13]. The model-based method has advantages in forecasting long-term behaviour of failure. However, modelling of induction motors is difficult to work especially when the motor is used to power electronic converters and such models include many approximations and assumptions.

The data-driven fault detection approach covers a wide range of schemes in the literature. Some of the most important techniques are neural networks [14], fuzzy logic [15] and hybrid methods [16]. However, the performance of this method is dependent on adequate data; it has advantages in feature extraction and classification without complicated modelling and IM parameters [2]. In addition, advances in data acquisition technologies and sensor systems introduce a large number of raw data for numerous applications. In this paper, a Kalman filter (KF) was proposed for fault detection. This method can approximate the past, present and future states of a system through recursive equations, even when the measurement date is not precise enough [17]. It is have been used in various applications widely, especially fault detection and monitoring.

There are various successful cases of mechanical fault detection based on vibration signals. Although vibration signature analysis is an efficient method for finding mechanical failures, the sensors commonly used for vibration measurement are costly. Compared with other methods, the current signal has many advantages. The inverters can measure the current signal easily and it is an inexpensive approach and suitable for eccentricity fault detection [18].

In this paper, a fault detection algorithm is proposed using the three-phase stator current data of both healthy and faulty conditions. For this purpose, first, a feature set is derived based on both time and frequency signals such as the DWT, PSD and cepstrum. Corresponding to each feature, an SSM and a KF are formulated to predict the fault occurrence. The measure to detect the eccentricity fault is based on the residual vector, which is the difference between the outputs of SSMs and KFs.

The rest of the paper is organised as follows: In section II, the proposed methodology to identify eccentricity fault is described. Section III presents the experimental setup to gather the instantaneous three-phase stator currents. The feature extraction process is described in Section IV. In Section V, the mathematical procedure to obtain the state space model for the KF is presented. Section VI gives a background of the KF. Sections VII and VIII present the fault detection results and conclusion.

II. METHODOLOGY

The proposed approach to detect the eccentricity fault in the induction motor involves extending a KF based on a datadriven state-space model using a set of statistical features. The initial dataset is collected by measuring instantaneous threephase stator currents of both healthy and faulty motors at ad different speeds and load conditions. To incorporate both time and frequency contents of the measured data, both approximation and detail coefficients of Discrete Wavelet Transform (DWT), Power Spectral Density (PSD), and cepstrum signals are calculated from the stator currents. Then, a feature set is provided by computing five statistical features: mean, standard deviation, skewness, kurtosis, and crest factor. These features are extracted from both time and frequency signals.

To simplify the structure of the fault detection algorithm, a state-space model and its associated KF are extended corresponding to each feature. To detect the effects of the eccentricity fault, the state-space model and it's associated KF first are developed using only healthy features. Then, faulty features are applied as a part of the measurement to both model and KF. A significant difference between the output of the model and the estimated output using the KF is considered as a sign of fault.

The benefits of this methodology include its capacity to detect changes in system behaviour caused by the fault and evaluate the severity of the fault. Furthermore, using a datadriven approach allows for a more accurate representation of the system's behaviour and helps the system to adapt to changes over time. To consider the effects of measurement noise on the fault detection algorithm, a Gaussian distribution with zero mean and small variance is added to both state-space dynamics and measurement equation. The KF is the optimal estimator in the presence of noise. Figure 1 shows the structure of the proposed method from collecting the initial dataset to detect the eccentricity fault in the induction motor.



Fig. 1. Block diagram of the proposed fault detection algorithm

III. EXPERIMENTAL SETUP

In the proposed method, only three-phase stator currents are analysed to detect the eccentricity fault. Figure 2 shows the experimental setup involving two similar 4 KW induction motors coupled throughout a torque transducer, a power meter, and a data acquisition system. The instantaneous current signals are measured using three similar LEM LA 100-P current sensors. They operate based on the Hall effect and can measure both AC and DC currents up to 100 A. A target machine which is known as Speedgoat along with an FPGA Input/Output (I/O) module is used to collect data in the computer. The target machine's operating system is Simulink Real-Time, a powerful MATLAB software toolbox.



Fig. 2. Experimental setup to measure healthy and faulty stator currents

The specifications of the different components of the test rig are given in Table I.

TABLE I. SPECIFICATION OF TEST RIG COMPONENTS

Component Name	Specification
Induction motor	4 KW, 4 poles, 415V, star connection, 1500 rpm rated speed
Target machine	Speedgoat, Intel 2.0 GHz quad-core CPU, 4GB DDR3 RAM memory, 4 I/O slots
I/O module	Configurable FPGA, 13 differential I/O lines
Current sensors	Hall effect sensors, measure current up to 100 A
Toque transducer	200 Nm rated torque, less than 0.1% accuracy
Power meter	A three-phase device, up to 10 readings per second, 0.1 KHz to 100 KHz bandwidth

IV. FEATURE EXTRACTION

In various practical scenarios, the information gathered to identify faults has a high dimensionality, characterised by a wide range of factors that can potentially affect the monitored system. However, the fault detection task may not require all these variables. Indeed, some of them may cause unwanted interference or undesired results. Feature extraction helps to overcome this challenge by identifying and selecting only the most relevant features, thereby reducing the dimensionality. The goal is to detect faults by analysing the extracted features and identifying patterns and trends that indicate their presence in an induction motor. This improves the accuracy and efficiency of the process, requiring fewer computational resources to be used.

Techniques such as Principal Component Analysis (PCA), Independent Component Analysis (ICA), Wavelet Analysis and others can be used for feature extraction. These strategies look for patterns, trends and links in the data to help distinguish between normal and abnormal behaviour. The primary dataset involves three-phase stator currents as well as three different signal characteristics including approximation and detail wavelet coefficients, PSD and cepstrum. Five statistical measures involving mean, standard deviation, kurtosis, skewness, and crest factor are considered to form the final feature set.

The first feature is the mean (μ) , which represents the expectation of a dataset by finding its average value through the addition of all observations and dividing by their total number. As a reference value, the mean can be used to detect anomalies in the data. There may be a fault in the system if the mean value suddenly deviates significantly from its expected value. The standard deviation (σ) indicates the degree of dispersion in a dataset from its mean, calculated by taking the square root of the variance. It is an essential measure to describe the variability and uncertainty of data, with higher values indicating greater dispersion and unpredictability.

Skewness (s) is a measure of how asymmetric a dataset is around its mean, indicating whether it is skewed towards higher or lower values. Positive skewness shows an increase towards the right skewness, with a longer tail to higher values. Negative skewness, on the other hand, implies a higher skew to the left, with a longer tail towards lower values. Kurtosis (γ) is a statistical measure that compares a data set's peakiness or flatness to a normal distribution. It indicates whether the data has more or fewer extreme values than would normally be expected. Higher kurtosis values suggest more peakiness and a greater frequency of outliers, whereas lower kurtosis values indicate flatter distributions with a lower frequency of outliers. An increase in skewness or a decrease in kurtosis can be used to detect changes in the shape or distribution of the data. It may indicate a shift in the underlying process or system if the skewness or kurtosis values suddenly change.

The crest factor (c) calculates a signal's peak-to-average power ratio by comparing its peak amplitude to its RMS amplitude. This metric is commonly used in signal processing and electrical engineering. A high crest factor indicates increased variability since there are more peaks. Indeed, an increase in the crest factor may be indicative of a sudden increase in signal amplitude, which may be the result of a fault or anomaly in the system.

For a given dataset $\{x_1, \dots, x_n\}$, these features are calculated as follows [19]:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{1}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_{i-}-\mu)^2}{n}}$$
(2)

$$\gamma = \frac{\sum_{i=1}^{n} (x_{i-} - \mu)^4}{\sigma^4}$$
(3)

$$s = \frac{\sum_{i=1}^{n} (x_{i-} - \mu)^3}{\sigma^3}$$
(4)

$$c = \frac{\max_{i} x_{i}}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} |x_{i}|^{2}}}$$
(5)

Figure 3 shows the three-phase stator currents and their approximation wavelet coefficients, PSD and cepstrum, during a limited period of time. Figure 4 shows an example of extracted features from the primary dataset.



Fig. 3. An example of three-phase currents of the faulty motor, their DWT, PSD and cepstrum



Fig. 4. An example of the extracted features from the primary dataset

V. DATA-DRIVE STATE-SPACE MODEL

The present study investigates the application of KF for detecting eccentricity defects in induction motors. Extending the KF for fault prediction requires constructing of an accurate state-state model. The Auto-Regressive (AR) model can be used as the state-space model. AR models capture time dependence in a variable, which is crucial for analyzing time-varying data. The natural temporal order of many variables and the informative value of past values are taken into account. The idea behind the AR model is to calculate the present value of the time-varying series, X_t , by a function of p past values, X_{t-1} . X_{t-2} X_{t-p} , where p is the order of AR(p) model and can be determined by the Akaike Information Criterion (AIC) [20].

Moreover, increasing the order of the AR(p) model can be found the same precision as the Auto-Regressive Moving Average ARMA model [21]. To establish the estimated algorithm of fault detection using the KF, first, the AR model is built based on the historical data including the extracted features. An autoregressive model of order p is written as

$$X_{t} = \phi_{1}X_{t-1} + \phi_{2}X_{t-2} + \dots + \phi_{p}X_{t-p} + \varepsilon_{t}$$
(6)

where $\phi = (\phi_1, \phi_2, ..., \phi_p)$ is the vector of model coefficients, p is a non-negative integer, and $\varepsilon_t \sim N(0, \sigma^2)$ is the zero mean Gaussian white noise with the

variance σ^2 . There is a direct correlation between these coefficients and the covariance function of the process, and this correlation can be reversed to determine the coefficients from the autocorrelation function. This is done using the Yule–Walker equations []:

$$\gamma_m = \sum_{k=1}^p \phi_k \gamma_{m-k} + \sigma_{\varepsilon}^2 \delta_{m,0} \tag{7}$$

where, m = 0, ..., p, γ_m is the autocovariance function of X_t , σ_{ε} is the standard deviation of the input noise system and $\delta_{m.0}$ is the kronecker delta function. The equations can be solved for all coefficients { ϕ_m ; m = 1, ..., p} in matrix form:

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \\ \gamma_p \end{bmatrix} = \begin{bmatrix} \gamma_0 & \gamma_{-1} & \gamma_{-2} & \cdots \\ \gamma_1 & \gamma_0 & \gamma_{-1} & \cdots \\ \gamma_2 & \gamma_1 & \gamma_0 & \vdots \\ \vdots & \vdots & \vdots & \ddots \\ \gamma_{p-1} & \gamma_{p-2} & \gamma_{p-3} & \cdots \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_p \end{bmatrix}$$
(8)

Therefore, the procedure of fault detection based on KF can be summarized as follows:

- Consider the *n* values of a feature and construct the time series x(1).x(2)....x(n) from zero to the length of *n*.
- select the order *p* of the model and determine the autoregressive model with respect to historical data.
- Calculate the Yule–Walker equations to obtain the coefficients $(\phi_1, \phi_2, \dots, \phi_p)$, and establish the AR model.
- Based on the AR coefficients *p*, the matrices *A* and *C* of the state-space model are defined as

$$\begin{cases} A = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_{p-1} & \phi_p \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}_{p \times p} \\ C = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}_{1 \times p} \end{cases}$$
(9)

• The state space model of the system can be described below:

$$X(k + 1) = AX(k) + w(k)$$

y(k) = C X(k) + v(k) (10)

where, X(k) represents the system state at the time k, y(k) is the observed value, w(k) and v(k) are the white noises with zero mean and covariance **Q** and **R**, respectively.

• According to the obtained state space model, the KF equations to predict future states will be conducted.

To verify the performance of the AR model, the first 300 data points from a healthy feature are analyzed. The results are drawn in Figure 5. It can be observed that the initial data points are estimated by the proposed AR model and the remaining data have discrepancies between the original signal and the AR model.

VI. KALMAN FILTER- BASED FAULT DETECTION

The Kalman filter is a widely used mathematical algorithm in the field of estimation theory. It provides an efficient way to estimate the state of a system by processing measurements taken over time. In particular, it has proven effective for fault detection applications where it can detect deviations from expected behaviour or potential faults in systems.



Fig. 5. a) Simulation data points, b) Auro-Regressive model

The KF uses a model of the system's behaviour to predict the next state of the system based on past observations. The predicted state is then compared to the actual measurement, and the difference is used to modify the model and increase the accuracy of the next prediction [22]. The KF can be used in fault detection to monitor system behaviour and discover deviations from expected behaviour. The filter detects faults in the system by comparing the predicted state of the system to the actual measurement.

In this paper, a residual-based fault detection technique, in which the difference between the expected state and the actual measurement is employed to generate a residual signal, is proposed. This signal can then be analysed to discover system faults. If the residual signal, for example, reaches a specific threshold, it may indicate the presence of the eccentricity fault.

After the initialisation process including initial conditions of predicted states, and covariance matrix, The KF predicts the state of the system at the next time step using the following prediction equation:

$$\hat{x}(k+1,k) = A\hat{x}(k,k) + \eta(k)$$
(11)

where $\hat{\mathbf{x}}(\mathbf{k}, \mathbf{k})$ is the state vector at time k and $\eta(\mathbf{k})$ is the process noise with zero mean and covariance matrix Q_{η} . Then, KF corrects the predicted state based on new measurements using the following update equation:

$$\hat{x}(k,k) = \hat{x}(k,k-1) + K_k(z(k) - H\hat{x}(k,k-1))$$
(12)

where z(k) is the measurement vector at time k, H is the measurement matrix at time k, K_k is the Kalman gain at time k. The measurement vector is obtained as follows:

$$z(k) = Hx(k) \tag{13}$$

The error covariance, which is the uncertainty in the state estimate is updated below:

$$P(k,k) = (I - K_k H)P(k,k-1)(I - K_k H)^T + K_k R_k K_k^T (14)$$

where I is the unity matrix. The Kalman matrix gain is calculated as follows:

$$K_k(k) = P(k, k-1)H^T (HP(k, k-1)H^T + R_k)^{-1}$$
(15)

To detect the eccentricity fault, first, a sequence of data including healthy and faulty features are gathered and applied as the measurements to the KF. The residual (r_k) at time k, is the difference between the predicted measurement and the actual measurement, which is given below:

$$r_k(k) = z(k) - H\hat{x}(k,k) \tag{16}$$

The fault detection measure is comparing the norm of the residual vector through a limited period of time with a predefined threshold.

VII. RESULTS AND DISCUSSION

To evaluate the feasibility of the proposed method in this paper, the current data measurement is analysed. We use the five different features in order to predict the eccentricity fault based on the KF. These features are mean, standard deviation, kurtosis, skewness and crest factors. There are 1500 data points for each feature value achieved from the experimental test. The fault prediction curve for the mean feature is drawn in Fig. 6. It can be seen the estimated state has less ripple than the actual state. The occurrence of the failure can be detected at the 1400th point. The same simulation is done for the kurtosis feature. The predicted result is drawn in Fig. 6. It illustrates that the proposed method has effective tracking when the kurtosis feature from the 10th to 500th data points is changed.



Fig. 6. Prediction result of mean feature using the KF



Fig. 7. Prediction result of kurtosis feature using the KF

The prediction results based on the standard deviation, skewness and crest features are also simulated and shown in Figures 7, 8, 9 and 10. As shown in these figures, the prediction results at the beginning data points have the error, but after a bit, the estimated values coverage to the actual values. They have illustrated the failure starts at the 1400th point. In these simulations, all the 1500 data points are used to test the proposed method of fault prediction. The first 80 data points are employed to establish the autoregressive model with order p=12. The obtained prediction results of the different features compared to the true values indicate success in the tracking of the prediction method based on KF.



Fig. 8. Prediction result of standard deviation feature using the KF



Fig. 9. Prediction result of skewness feature using the KF



Fig. 10. Prediction result of crest factor feature using the KF

VIII. CONCLUSION

In this paper, eccentricity fault in the induction motor is detected using a data-driven Kalman filter. Based on experimental data including three-phase stator currents and their approximation and detail wavelet components, PSD and cepstrum, a primary dataset is established. Then, through a feature extraction process, a new feature set is obtained. Using AR models, state-space dynamics are developed corresponding to each feature. Then, using the prediction and update equations of the Kalman filter, a measure to detect the eccentricity fault is defined as the norm of the residual vector. The results demonstrated the efficiency of the proposed method for different prediction models. After applying the faulty data, the norm of the residual vector has a significant change indicating the presence of the eccentricity fault.

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