# Uncertainty and the Business Cycle: Theory and Empirics

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## Abstract

Recessions create uncertain economic environments which agents must navigate when making costly decisions. Across four chapters, this thesis adds to the current understanding of how time-varying economic uncertainty can be measured and how it affects real economic activity, especially during times of economic crisis.

The first two chapters introduce a new framework for examining the response of investment to changes in economic regime. Both the price of a firm's output good and its production function depend on a continuous time Markov chain which switches between an expansionary regime and a recessionary regime. The latter is characterised by higher uncertainty about the output price and lower productivity in the production process.

Chapter one models an investor's decision to acquire and sell this firm. Switches between regimes produce various patterns of acquisitions and sales depending on the differences in uncertainty and productivity between the regimes. The model provides a mechanism for explaining the wave like pattern of acquisitions over the business cycle. Chapter two focuses on the active firm's capital accumulation policy. Here, the model can generate lumpy patterns of investment following switches between the regimes, and demonstrates how entering a persistent recessionary regime with high uncertainty and low productivity can lead to extended periods of low investment.

Chapter three estimates the effect of a firm's idiosyncratic uncertainty, as measured by the volatility its stock returns, on its investment rate using almost 30 years of U.S. data. Consistent with the predictions of the models in the first two chapters, the relationship between the variables is negative. The increase in uncertainty during the Great Recession also played a large role in causing the observed fall in investment. Furthermore, there is evidence that uncertainty has been more of a drag on investment after the Great Recession; firms with growth opportunities sufficient to neutralise its effect before the recession are sensitive to its variation in the years after.

Despite widespread use in the literature, the volatility of stock returns is not an ideal measure of economic uncertainty. Using an instrumental variable SVAR model, chapter four set identifies shocks to uncertainty and stock market volatility and demonstrates that the latter can vary even

when there is no change in economic uncertainty. The identified shocks also produce different impulse response functions for several key macroeconomic indicators. The price of gold around events that make the future harder to predict is used as an instrument for uncertainty shocks while exogenous changes in the spread between Baa-rated corporate bonds and the 10-year treasury bond rate is used as a proxy for shocks to stock market volatility.

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# Introduction

The slow recovery from the Great Recession of 2008 led to a surge in literature examining the causes of protracted recessions (Ball, 2014; Blanchard et al., 2015; Cerra et al., 2023). Economists pointed to a range of factors such as debt overhang, austerity measures, and liquidity traps as sources of the decline and weak recovery in key macroeconomic indicators (Jermann & Quadrini, 2012; DeLong & Summers, 2012; Eggertsson & Krugman, 2012). In their review of the channels through which the Great Recession affected the U.S. economy, Stock & Watson (2012) highlighted the role of the financial upheaval in the aftermath of the subprime mortgage crisis. However, they also concluded that one of the main drivers of the decline in activity was heightened economic uncertainty[1]. In an influential paper which aims to create an empirical measure of uncertainty, Jurado et al. (2015) define this concept as the conditional volatility of a disturbance that is unforecastable from the perspective of economic agents[2]. Essentially, uncertainty reflects how difficult it is for agents to forecast the future.

Since Bernanke (1983) and Dixit & Pindyck (1994), most of the models explaining the relationship between uncertainty and economic activity have been based on the theory of 'real options'. Any agent making a decision which incurs an unrecoverable cost must decide when they should act given they can continue waiting for more information about the future. This is analogous to an investor holding a financial option giving the right but not the obligation to purchase or sell an asset for a fixed price. While classical economic theory states an action should be taken when the

[1]The authors put the weak recovery after the recession primarily down to declining labour force participation, while recognising the magnitude of the shocks to other macroeconomic variables also played a role in delaying the recovery.

[2]Early work by Frank Knight (1921) distinguished uncertainty from risk on the basis that the former is unobservable while the latter can be measured. The literature has moved away from this definition and towards that of Jurado et al. (2015), which is also adopted in this thesis.

marginal benefit of that action outweighs the marginal cost, real option theory appreciates that waiting for more information has real economic value for the agent. Hence, the optimal time to act is modified such that the benefit outweighs the monetary cost plus the value of waiting, which is higher when uncertainty about the future is higher. Consequently, with all else constant, economic activity declines in uncertain periods because the optimal decision rule tells agents to hold back from making irreversible decisions. Note that if the decision is fully reversible there is nothing to be gained by delaying the action until more is known about the future.

Uncertainty has remained a key concern for economists since the Great Recession. This is in part because of the frequency of uncertainty shocks in the succeeding years. Examples include the results of the U.S. presidential election and Brexit referendum in 2016, the outbreak of the Coronavirus pandemic in 2019, and the Russian invasion of Ukraine in 2022. Davis (2019) also points out a gradual increase in economic policy uncertainty due to a breakdown in international trade relations. According to the World Uncertainty Index of Ahir et al. (2022) such a sequence of uncertainty shocks in a changing global environment has resulted in the period since 2012 being the most uncertain in the past 60 years. Policy makers will be alarmed by this observation given the predictions of the real options theory and Stock & Watson's (2012) conclusion about uncertainty's key role in propagating the Great Recession. Indeed, current managing director of the IMF Christalina Georgieva (2020) cited increasing uncertainty as the key theme of the next decade.

Clearly, agents make decisions in an environment not just characterised by uncertainty, but by *time-varying* uncertainty. Moreover, uncertainty moves counter cyclically, periods of relative prosperity are associated with lower uncertainty while periods of economic distress make it difficult to forecast the future (Bloom, 2009; Baker et al., 2016; Jurado et al., 2015). Early real options models tended not take this fact into consideration when examining the optimal timing of actions[3]. Its counter-cyclical property also means that periods of high uncertainty tend to arrive simultaneously with declines in other economic variables. In fact, Bloom et al. (2018) show that recessions are characterised by a decrease in the level and increase in the variance of plant-level total factor productivity (TFP) and sales growth.

[3]Two exceptions are Bloom et al. (2007) who model uncertainty as a stochastic mean-reverting process and Guo et al. (2005) who consider switches between higher-uncertainty and lower-uncertainty regimes.

This thesis contributes to both the theoretical and empirical literature examining the consequences of uncertainty for economic activity on the microeconomic and macroeconomic level[4]. The first two chapters are concerned with the optimal timing of investment decisions[5] in an environment which includes both time-varying uncertainty and productivity. While Bloom et al. (2018) considers these factors in a general equilibrium setting to examine the impact of exogenous shocks, I model them in a regime switching framework representing transitions between periods of prosperity and recession to examine the optimal timing of investment decisions.

A perfectly-competitive firm produces an output good whose price follows a geometric Brownian motion with drift and volatility parameters dependent on a continuous time Markov chain switching between an expansionary and a recessionary regime. The volatility parameter is higher in the recessionary regime, which introduces time-varying uncertainty into the model. The productivity parameter of the firm's Cobb-Douglas production function depends on the same Markov chain, and is lower in the recessionary regime. The persistence of the two regimes is determined by the transition probabilities of the Markov chain in a given time interval.

This more closely replicates the changes in economic conditions faced by investors and firms compared to previous models in the literature. Chapter one considers a representative investor making a decision to purchase an infinitely-lived firm while chapter two considers an active firm deciding when to adjust its capital stock. In both chapters, the solution to the decision problem involves finding the threshold values of the stochastic process underlying the value of the firm which justify a change in action. I make the decision problem more realistic by allowing some of the initial cost of the investment to be recovered in the future, rather than assuming total irreversibility. Finding these solutions involves solving complex systems of simultaneous non-linear equations, which will generally be done numerically.

After solving, the models make a novel set of predictions about the dynamics of investment when transitioning to and from recessionary regimes. The investor in chapter one is making a decision

[4]While related, each chapter in this thesis could stand alone and has its own introduction which will discuss the motivation and findings more thoroughly.

[5]Most of the real options literature has focused on adjustments in fixed capital rather than labour. Furthermore, Bloom (2009) finds much weaker evidence for the kinds of irreversible costs required for the application of real options theory in hiring and firing compared to capital stock adjustments. The first three chapters follow this tendency and focus on investment activity.

about whether to acquire a representative firm outright, which she can sell at a later date for a fraction of its purchase price. Corporate acquisitions are known to display 'wave' like behaviour which follows the business cycle and are influenced by technology shocks (Harford, 2005). The model provides a mechanism to explain this behaviour. Regime switches can trigger immediate decisions to acquire or sell the firm if the price of the output good lies above or below the required threshold.

When the recessionary regime is characterised by a large decrease in productivity compared to the expansionary regime, it is very likely that transitioning to the recessionary regime will result in the investor immediately selling the firm, if she owned it, and the heightened uncertainty will result in a much lower chance that she will buy the firm again while in the recessionary regime due to the real options effect. However, transitioning back to the expansionary regime is more likely to trigger an immediate acquisition. For many investors and firms, this implies a new wave of acquisitions. On the other hand, if the recessionary regime has similar productivity levels to the expansionary regime, transitioning to the former will not cause an immediate change in the investor's decision to own the firm or continue holding the option to invest, but she will still be less willing to change her current action while in the recessionary regime.

Chapter two uses the same regime-switching stochastic process as chapter one, so there are parallels between the conclusions it produces for firm capital accumulation and those made about the one-shot investment decision in chapter one. These conclusions build upon the literature describing firm-level investment patterns as 'lumpy', meaning capital adjustments tend to be clustered in discrete packages (Doms & Dunne, 1998; Cooper & Haltiwanger, 2006; Bloom et al., 2007; Fiori, 2012). For large productivity differences between the regimes, transitioning to the recessionary regime *always* triggers a large discrete decrease in the capital stock while returning to the expansionary regime implies an analogous increase in the capital stock. Investment is very responsive to regime changes in this case. In contrast, if the only difference between the regimes is higher uncertainty in the recessionary regime, transitioning to this regime has no immediate effect on the capital stock, but a relatively higher (lower) value of the marginal unit of capital is still required to justify investment (disinvestment). These dynamics are also dependent on the persistence of the regimes, the firm is less willing to build its capital stock, and more willing to decrease it, in both regimes when the recessionary regime is more persistent. Thus, transitioning to a persistent recessionary regime featuring higher uncertainty and lower productivity will feature much less investment for an extended period.

Clearly, the model provides an explanation for why investment recovery might be weak in the years following a recession, thereby highlighting a channel through which recessions cause persistent declines in economic activity. Weak investment can have a long-run impact on growth rates due to a lack of capital accumulation relative to previous trends. By equipping workers with more or better machinery, investment in the capital stock provides a base from which future innovation is driven (De Long & Summers, 1991). Policy makers such as Yellen (2016) note the importance of this issue for the future of macroeconomic policy. If recessions cast shadows over future growth rates, there is greater justification for government intervention to hasten recoveries.

Motivated by the models studied in the first two chapters and the large decline in investment between 2008 and 2009, chapter three estimates the empirical relationship between uncertainty and firm-level investment over almost 30 years of annual data and examines how this relationship changed in the aftermath of the Great Recession. The econometric model is grounded in the theory outlined in Hayashi (1982) which relates investment to a measure of a firm's growth opportunities determined by the market value of its capital stock relative to its replacement cost (Tobin's Q). This is the empirical analogue of the marginal value of capital, which determines the optimal investment rule of the representative firm in chapter two.

Previous work in this area has not focused on the implications of the model for investment during recessions, or whether there is evidence that the estimated coefficients change after such periods of economic upheaval. In line with previous studies, the model also controls for the firm's financing structure (leverage and cash flows) and the first lag of investment. Interaction terms explore whether the relationships between the explanatory variables and the investment rate are contingent on each other, and whether the Great Recession had a lasting impact on these relationships. An instrumental variable generalised method of moments estimator recovers the coefficients capturing the relationship between the investment rate and the explanatory variables, and the results are shown to be robust to various relevant changes in specification.

As expected, uncertainty is found to have a negative impact on the investment rate and appears to be a significant driver of the decline in investment during the Great Recession. Despite being significantly correlated with investment over the whole sample, there is little evidence to suggest changes in cash flows or leverage during the Great Recession were responsible for the decline. The primary driver, however, seems to be the decrease in growth opportunities, as measured by Tobin's Q. The negative impact of uncertainty on investment is attenuated by higher growth opportunities, which makes sense in light of the model in chapter two, where uncertainty increased the value of the marginal product of capital that justified investment. Given Tobin's Q is the empirical analogue of the marginal value of capital, the strength of uncertainty's effect on investment decreases when it is very high. However, there is evidence that this attenuating effect of higher Q was weaker in the years after the Great Recession, so uncertainty was more damaging to investment in the years after 2008 for a given value of Q.

Despite the stock of literature examining the effect of uncertainty on economic activity, measuring uncertainty remains a difficult task. Chapter three, expanding on other widely-cited studies, uses the conditional volatility of firms' idiosyncratic stock returns. However, stock returns can become more volatile even when there is no change in how easy it is for agents to forecast the future. While Jurado et al. (2015) and Aït-Sahalia et al. (2021) pointed this out, they only provided cursory evidence as to how uncertainty and stock market volatility differ. A more formal examination is given in Arnold & Vrugt (2008), who find little evidence that stock market volatility causes increases in economic uncertainty. However, they do not attempt to identify structural shocks to the variables in their vector autoregression (VAR) model, which is necessary if one wants to understand how exogenous innovations in the variables affect the real economy or estimate their contemporaneous relationship.

Building on this hypothesis, chapter four separates structural shocks to uncertainty from structural shocks to stock market volatility in an instrumental variable structural VAR model and examines the impact of these shocks on several key U.S. macroeconomic indicators. The proxy for uncertainty shocks is constructed from the variation in the price of gold around events expected to increase uncertainty about future economic conditions while the proxy for stock market volatility shocks comes from innovations in the credit spread between Baa rated corporate bonds and the 10-year treasury bond rate, which captures exogenous changes in investor sentiment and credit supply conditions. Set identification is used because these proxies are both correlated with the shock they do not target, though this correlation is weaker than with the shock they do target.

In keeping with previous studies, uncertainty shocks cause a decline in economic activity which persists for several months. This is matched with a persistent increase in stock market volatility. On the other hand, the identified shocks to stock market volatility do not significantly increase economic uncertainty. These shocks also have a much weaker effect on real economic activity but account for a larger proportion of the variation in wages, inflation, and the federal funds rate. This confirms the hypothesis that increases in stock market volatility do not necessarily imply an increase in economic uncertainty, and that any measures of uncertainty based solely on stock markets will come to erroneous conclusions.

The first three chapters focus on the investment decision of individual agents and the variable capturing uncertainty in each case is assumed to be exogenously determined. In chapter four, however, it is shown that uncertainty is partly dependent on variation in other macroeconomic variables. Ludvigson et al. (2021) provide a general overview of which studies assume uncertainty is endogenous and which assume it is exogenous. Notably, the real options literature tends to assume uncertainty is an exogenous driver of the business cycle, as do the models of Bloom (2009) and Bloom et al. (2018), where uncertainty comes from the process underlying growth. As for models where uncertainty is endogenous, Bachmann & Moscarini (2011) suggest that uncertainty increases during recessions because of an increase in risk-seeking behaviour which raises the observed cross-sectional dispersion in macroeconomic time series. Alternatively, Fajgelbaum et al. (2017) suggest endogenous uncertainty is the result of a slowdown in information flows, which consequently makes the future harder to predict. Their model produces 'uncertainty traps', where an initial increase in uncertainty discourages investment, which reduces activity and causes a further increase in uncertainty because agents are less able to learn from one another. This mechanism further discourages activity and amplifies the effect of recessions.

Ludvigson et al. (2021) aimed to determine whether uncertainty was an endogenous response or exogenous driver of the business cycle. They find that uncertainty about financial markets is more likely to be the driver of output fluctuations, while the increase in macroeconomic uncertainty around recessions arises endogenously as a result of shocks to output. The authors also acknowledge that uncertainty increases the severity of other shocks during recessions. These results are in contrast to those in Carriero et al. (2018), who found that financial uncertainty is endogenous and macroeconomic uncertainty is exogenous when considering their effect on the U.S. economy. Angelini et al. (2019) also concluded that uncertainty was primarily an exogenous driver of the business cycle and that the effects of uncertainty shocks are larger in recessionary periods.

It is worth mentioning that there are theories other than real options explaining the relationship between uncertainty and economic activity, and not all of them expect the relationship to be negative. While Gilchrist et al. (2014) recognise the real option effect of uncertainty, they suggest that it primarily affects the real economy by increasing the required rate of return on corporate bonds, which makes debt financing more expensive. Alternatively, Fernández-Villaverde et al. (2011) and Basu & Bundick (2017) explain the negative relationship between uncertainty and activity through a precautionary savings effect. A positive relationship between uncertainty and activity can be produced in models where the agent's value function is convex in the variable which generates the uncertainty, as in Hartman (1972) and Abel (1983). Fernández-Villaverde & Guerrón-Quintana (2020) show that uncertainty shocks can be expansionary in a macroeconomic model without nominal rigidities or financial frictions.

# Chapter 1

# A Regime-Switching Model of Corporate Acquisitions

## 1.1 Introduction

The economy often transitions into periods characterised by a slowdown in economic activity and high uncertainty which investors must navigate when considering whether to acquire a new firm. Furthermore, disruptions to the supply side of the economy or the market for loanable funds may reduce the economy's productive potential and trigger periods of persistently lower output levels relative to previous trends, consequently reducing the expected future value of proposed projects (Ball, 2014; Blanchard et al., 2015; Cerra et al., 2023). While acquisitions can be very profitable and provide growth opportunities in terms of size, market share, and innovation, they also have a high failure rate and often do not yield their expected benefit when they do succeed (Bonaime et al., 2018; Renneboog & Vansteenkiste, 2019). Buying at the wrong time could result in the failure of an otherwise promising project and, from a policy perspective, when promising start-ups are acquired by larger firms only to be dropped because of inadequate returns, the potential loss in innovation is a drag on economic growth (Fons-Rosen et al., 2022).

While the effects of uncertainty on the decision to undertake a one-off investment project are well described by the real options literature (Bernanke, 1983; Dixit & Pindyck, 1994), these models tend to assume that uncertainty and productivity are constant over time. Furthermore, investment decisions are usually assumed to be fully irreversible, so the investor cannot recover any of the initial acquisition cost. At least some of the acquired firm's assets will have resale value, so some of the losses from acquisitions which do not yield their expected benefit can be recovered, making the assumption of irreversibility unrealistic (Officer, 2007). Therefore, the predictions that current models make about acquisitions over the business cycle are limited. This motivates a more comprehensive framework which can account for the observed characteristics of low-productivity, high-uncertainty regimes where acquiring agents know they can recover at least part of the cost of their investment.

To incorporate these factors into current models evaluating optimal investment timing, this chapter considers a representative investor's decision to acquire an infinitely-lived firm whose output price and production function are dependent on a regime-switching process. For clarity, I will call the acquiring party the 'investor' and the acquired party the 'firm' throughout. Also, for simplicity, I ignore any other assets the investor might own and just focus on the benefits accrued by ownership of the infinitely-lived firm.

One of the two regimes features some or all of the following characteristics relative to the other regime; lower productivity, lower growth in the output price, and higher uncertainty about the future output price. Switching to this regime represents an economic downturn, so I call it the 'recessionary regime' and call the other the 'expansionary regime'. The persistence of the regimes is determined by the probability of switching regime within a given interval of time, a low probability of switching means the regime is persistent and the firm's productivity remains below its previous level for a long period, while the output price remains lower and more volatile. The model therefore generates changes in the economic environment along the lines of Bloom et al. (2018), where economic shocks cause changes in both the levels and volatility of key parameters affecting the decision problem.

The flow of profit from ownership of the firm must be sufficiently high or low to justify purchasing or selling it. Because profits change stochastically with the output price, solving the decision problem involves finding the critical values of the output price which justify a change in the investor's current ownership position (whether she holds the firm itself or the option to acquire the firm). Crucially, the investor's acquisition of the firm is at least partially irreversible, meaning the full cost cannot be recovered. The degree of irreversibility is controlled by a sale price which can either be zero, the fully irreversible case, or some constant value less than the purchase price.

When the investor does not own the firm, she still holds the option to purchase it at a later date. This option has real economic value and needs to be considered when evaluating the costs and benefits of an investment decision. Similarly, she holds an option to sell the firm at a later date when she currently owns it, so the full benefit of ownership is the sum of the value of the firm and the option value of selling it at a later date. Uncertainty primarily affects investment decisions by increasing the value of waiting for more information before changing the current ownership position. In other words, higher uncertainty increases the project's option value. Meanwhile, changes in productivity affect the investment decision by changing the expected future value of the project. Intuitively, the critical value of the output price justifying the acquisition of a firm is lower when the productivity of the firm is higher, because more productive firms are still profitable at lower output prices. Likewise, the output price must be relatively low to justify selling a more productive firm at a later date.

In this set up, I make three main contributions to the literature. The first is including the partial irreversibility of acquisitions in a regime switching context. Initially, I suppose the investor cannot sell the firm once it has been acquired. I demonstrate that this fully-irreversible case produces a very similar solution to the model of capital accumulation by Guo et al. (2005), characterised by two critical values of the output price which justify investment in each regime. In the partially reversible case, there are four critical values justifying a change in the investor's current policy, two justifying acquisition and sale in regime one and two justifying acquisition and sale in regime two. Partial reversibility is, in general, a more realistic representation of the investor's decision, as there is usually some potential resale value in an acquisition, whether in the form of fixed assets or intangible assets such as intellectual property rights.

The second is the inclusion of regime-dependent productivity levels which cause a persistent fall in the firm's output from its original level. When combined with a low probability of switching back to the higher-productivity state, the model replicates the persistence of recessions pointed out by Cerra & Saxena (2008), Ball (2014), and Blanchard et al. (2015). Therefore, unlike previous models, this chapter embeds the investor's acquisition decision in a framework which more closely replicates the business cycle. Third, I show that the immediate effect of a regime switch on the investor's ownership position depends on which parameters change and the magnitude of the changes. For many investors and firms, an immediate change in ownership position following a regime switch implies a wave like pattern of acquisitions related to the business cycle, which is observed in empirical data (Mitchell & Mulherin, 1996; Harford, 2005). Immediate changes are more likely when productivity differences between the regimes are large.

The key results are as follows. Compared to the expansionary regime, the investor delays her decision to acquire the firm in the recessionary regime if she currently does not own it, and likewise delays her decision to sell the firm if she does currently own it. Simultaneous productivity declines and uncertainty increases exaggerate the delay when buying the firm because a less productive firm will only generate a sufficiently high expected profit flow once the output price reaches a relatively higher level. With very persistent recessionary regimes, the output price required to make new acquisitions becomes higher still. These findings agree with empirical literature suggesting that the number of acquisitions falls in the aftermath of negative economic shocks (Mitchell & Mulherin, 1996; Maksimovic & Phillips, 2001; Harford, 2005). However, there is ambiguity about the direction of the regime change on the investor's desire to sell the firm. While uncertainty favours delaying the sale, lower productivity favours selling the firm at higher values of the output price compared to the expansionary regime because the expected future profit flow generated from ownership of the firm does not justifying hanging onto it despite relatively high output prices.

If productivity declines are large enough, there is a region of the output price for which the investor will always change her current ownership position after a regime switch. This creates an aggressive business-cycle effect on acquisitions in this region of the output price where the investor always sells the firm after switching to the recessionary regime and always buys it back when the economy returns to the expansionary regime. This finding predicts a large uptake in acquisitions when periods of depressed economic activity are resolved. Without very large productivity shocks, switching to the recessionary regime will not result in an immediate change in the investor's current ownership position and the decline in the number of acquisitions in the recessionary regime comes solely through the real options effect of waiting for more information before making a partially irreversible decision.

The rest of this chapter is structured as followed. Section 1.2 discusses some relevant previous studies in this field. Sections 1.3 and 1.4 set up the model and introduces the solution methods. Section 1.5 solves the model in the fully irreversible case when the sale price of the firm is zero and 1.6 does the same for the partially reversible case. A conclusion summarises the key findings of the chapter.

## 1.2 Related Literature

This chapter is linked to the literature examining how the business cycle influences the frequency of mergers & acquisitions in the economy. Since the model examines the decision to make a one-off purchase of an infinitely-lived project providing an indefinite profit flow, it is more applicable to acquisitions than mergers. Maksimovic & Phillips (2001) found that the frequency of corporate asset purchases is strongly pro-cyclical, with 7% of plants changing owners in expansion years compared to an annual average of 3.89% in the whole sample. Both Mitchell & Mulherin (1996) and Harford (2005) show that corporate mergers, acquisitions, and takeovers tend to come in waves which are related to regulatory and technology shocks. Furthermore, Nguyen & Phan (2017) and Bonaime et al. (2018) find that higher uncertainty about future economic policy can negatively affect the number of mergers & acquisitions. Consistent with real option theories, the decrease is larger when the costs are less recoverable. These findings motivate a formal examination of how the number of acquisitions can be affected by economic downturns, periods which are known to coincide with heightened uncertainty (Jurado et al., 2015). Successful acquisitions stoke innovation and benefit firms through synergy effects (Bonaime et al., 2018; Jones & Miskell, 2007), so understanding the response of acquisitions to economic shocks is also interesting for policy makers.

Lukas et al. (2019) also modelled acquisitions under uncertainty, however, their focus was on how uncertainty changes a firm's acquisition strategy rather than on how economic shocks change the incentive to undertake acquisitions. They find that firms planning on making acquisitions will prefer to target several smaller firms than make one large purchase when uncertainty is high. Ebina et al. (2022) use a real options model to examine the conditions under which a firm will bid for a takeover and what type of amalgamation will occur. Hostile takeovers occur when directors do not wish the firm to change ownership but a third party acquires a sufficient stake to take control regardless. Friendly takeovers occur when all parties agree to this course of action. In Ebina et al.'s model, firms consider the option value of waiting and the fear of a pre-emptive hostile takeover by other firms in their decision problem, and the authors show that higher volatility of future cash flows tends to result in more hostile takeovers. This chapter does not consider the nature of the takeover, whether hostile or friendly, and only considers the benefits for the acquiring party.

The foundational model for this chapter is found in Dixit & Pindyck (1994, pp.215-229). An investor has the option to make a lump-sum purchase of a project whose value depends on an output price following a geometric Brownian motion (GBM) with constant drift and volatility parameters. The investor can sell the project for a constant sum less than the purchase price at a later date, so acquiring the project also means acquiring this option[1]. Dixit & Pindyck show that as the volatility of the output price increases, a greater wedge is placed between the thresholds justifying purchase and sale of the project because the option value of waiting for more information before making a costly decision is increasing in volatility. Therefore, their model predicts that higher uncertainty causes lower activity. Equating increasing volatility with increasing uncertainty is common to all the theoretical models reviewed in this chapter but is not technically accurate. Volatility measures the spread of a distribution around a measure of central tendency, usually the mean, while uncertainty is about the difficulty estimating the parameters of a distribution, including the mean (Aït-Sahalia et al., 2021). I address this issue in more detail in chapter four and for now continue to equate volatility with uncertainty.

One significant limitation of Dixit & Pindyck's model is the assumption of constant uncertainty. In reality, uncertainty moves counter-cyclically and is higher during economic downturns (Bloom, 2009; Jurado et al., 2015). Guo et al. (2005) introduce time-varying volatility by making the stochastic GBM underlying the value of the firm dependent on a continuous-time Markov chain (CTMC) which switches between two regimes, with the average time until a switch occurs governed by a Poisson distribution. They consider a firm who adjusts their capital stock upwards and assume downward adjustments are impossible, so the decision problem features total irreversibility. As one would expect, the threshold justifying investment is higher in the high-volatility regime.

[1] Real options theory borrows directly from the theory of pricing financial options. The option to buy the project resembles an American call option and the option to sell resembles an American put option. Hence, the value of the option to buy is increasing in the output price and the option to sell is decreasing in the output price.

Additionally, it is also higher when the persistence of the regimes are lower because the option value of waiting increases when switches between regimes are very frequent.

An important feature of their model implies that an investor's ownership position may or may not change immediately after a regime switch. Suppose the economy is currently in the high-uncertainty regime. If the GBM underlying the value of the firm (the price of the output good in this chapter) is such that the investor would not acquire the firm in the high-uncertainty regime (the GBM is below the threshold value necessary for adjustment in this regime) but would do so in the low-uncertainty regime (the GBM is above the threshold value necessary for adjustment in the other regime), then a regime switch to the low-uncertainty regime will cause the investor to immediately purchase the firm. Guo et al. describes the region of values of the GBM for which a change in regime brings about an immediate change in policy as a *transient region*. Such regions will feature heavily in this chapter and the next.

Guo et al. (2005) focus on capital accumulation, which is the subject of the next chapter. Here, rather than acquiring an arbitrary project as in the Dixit & Pindyck (1994, pp.215-229) case, the investor acquires a firm which produces with a Cobb-Douglas production function along the lines of Abel (1983). I let the same CTMC which affects the regime of the output price also affect the production function through an exogenous productivity parameter, so switching to the low-productivity regime also causes a persistent fall in output. Bloom et al. (2018) shows the substantial negative impact that first-moment and second-moment shocks to firm-level productivity has on macroeconomic variables. Their model highlights one of the key observations from real business cycle (RBC) theory, that economic shocks affect the productive potential of the economy and therefore persist much longer than transitory demand-side shocks. Indeed, some recessions appear to cause very persistent or even permanent shortfalls of output from previous trends (Cerra & Saxena, 2008; Ball, 2014; Blanchard et al., 2015). As mentioned, there is empirical evidence that these shocks affect the frequency of acquisitions (Mitchell & Mulherin, 1996; Harford, 2005).

Bloom et al. (2018) suggest that declines in productivity during high uncertainty periods is due to the slow down in hiring and investment preventing the efficient allocation of resources from low productivity firms to high productivity firms. Furceri et al. (2021) finds evidence in a sample of 18 advanced economies that the misallocation of resources during recessions leads to a fall in the level of TFP. Additionally, Blanchard et al. (2015) find that over half of recessions are followed not only by a period of lower output levels but also with lower output growth relative to the pre-recessional trend, suggesting that economic shocks usually do have an impact on the productivity and technology process underlying growth. Other potential causes of drops in productivity are disruptions in supply chains caused by geo-political crises such as the Russo-Ukraine war or the oil crisis of the 1970s, disruptions in productions lines such as those experienced during the Coronavirus pandemic, or difficulties obtaining financial capital due to an increase in borrowing constraints like during the Great Recession of 2008.

In the model, the investor knows the values of the parameters in both regimes and the probability of a regime switch occurring in a given time interval. Therefore, while the exact time of an economic downturn is unknown, how bad the downturn will be is known in advance. This characterisation of business cycles, as switches between periods relative prosperity and depression, is in contrast to the model of Bloom et al. (2018), where downturns are triggered by random exogenous shocks. Hamilton (1989) first showed that the growth in the trend of GNP could be described by a two-regime Markov chain. Cerra & Saxena (2005*a*) and Cerra & Saxena (2005*b*) show that switching to a regime with negative growth rates will cause permanent shortfalls in output relative to previous trends if there is no 'recovery' regime where growth is faster than the normal growth regime. They later found that permanent shortfalls actually appear to be the empirical norm (Cerra & Saxena, 2008). For simplicity, regime switches in this chapter cause changes in the level of productivity rather than growth, so the level of productivity remains depressed for as long as the recessionary regime lasts.

## 1.3 Description of the Firm

### 1.3.1 Geometric Brownian Motion for Output Price

The price of the output good, $P_t$, follows a GBM with a known drift rate $\alpha_i$, a known volatility $\sigma_i$, and $W_t$ being a standard Brownian motion

$$dP_t = \alpha_i P_t dt + \sigma_i P_t dW_t. \tag{1.3.1}$$

The subscript $i$ indicates that the parameter depends on the observable CTMC $\varepsilon_t \in \{1, 2\}$ so $i = 1$ whenever $\varepsilon_t = 1$. The rate of leaving regime $i$ and switching to regime $j$ is given by $\lambda_{ij}$. In other words, the time between switches in $\varepsilon_t$ follows an exponential distribution with an average event time (the time it takes to leave state $i$) of $1/\lambda_{ij}$. If $\tau_i$ is the time at which the process leaves regime $i$, then for the interval $\Delta t$

$$Pr\left(\tau_i > \Delta t\right) = e^{-\lambda_{ij}\Delta t} \approx 1 - \lambda_{ij}\Delta t$$

is the probability that $\varepsilon_t$ will switch to regime $j$ after a period of time longer than $\Delta t$ and

$$Pr\left(\tau_i < \Delta t\right) = 1 - e^{-\lambda_{ij}\Delta t} \approx \lambda_{ij}\Delta t$$

is the probability that $\varepsilon_t$ will switch to regime $j$ after a period of time shorter than $\Delta t$. Therefore, the following transition matrix describes the regime switching behaviour of $\varepsilon_t$ in the interval $\Delta t$

$$\begin{bmatrix} (1 - \lambda_{12}\Delta t) & \lambda_{12}\Delta t \\ \lambda_{21}\Delta t & (1 - \lambda_{21}\Delta t) \end{bmatrix}.$$

Figure 1.3.1 shows a discrete time approximation of the regime switching GBM (orange line) in comparison to two standard GBM processes which have the same drift and volatility parameters as the two regimes but no regime switching. The red line is the result of always being in regime two and the blue line is the result of always being in regime one. The shaded area represents periods

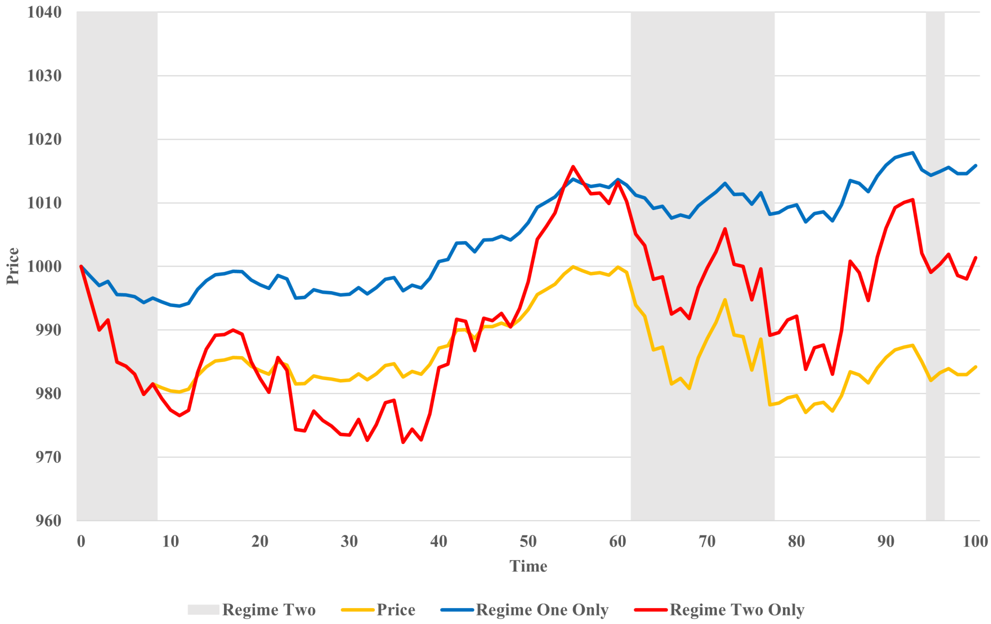



Figure 1.3.1: Discrete Time Approximation of Price Process

In the figure, $\alpha_1 = 0.04$, $\alpha_2 = -0.04$, $\sigma_1 = 0.05$, $\sigma_2 = 0.15$, $\lambda_{12} = 0.05$, and $\lambda_{21} = 0.25$. The time interval is $\Delta t = \frac{1}{365}$, so the $\alpha_i$ and $\sigma_i$ parameters can be interpreted as the annualised drift and volatility parameters and the time increment is one day. The Brownian Motion is approximated as $\sqrt{\Delta t}$ multiplied by a random draw from the standard normal distribution. The parameter values were chosen purely in the interest of making the graph easy to interpret.

where $\varepsilon_t = 2$, so the unshaded regions represent periods where $\varepsilon_t = 1$. $\lambda_{12} < \lambda_{21}$ implies that the economy spends more time in regime one over the sample period. Because $\alpha_1 > 0$ and $\alpha_2 < 0$ in this example, price will tend to grow over time in regime one and fall over time in regime two. Higher volatility in regime two $(0.05 < 0.15)$ means the time path of $P_t$ is more jagged because there is a larger distribution of observations around the mean growth path. I maintain the assumption that $\alpha_1 > \alpha_2$ and $\sigma_1 < \sigma_2$ throughout and refer to regime two as the 'recessionary regime'.

### 1.3.2 Production and Costs

A representative investor possesses the option to acquire an infinitely-lived firm producing with a Cobb-Douglas production function

$$F(L_t, K_t, \varepsilon_t) = \omega_i L_t^a K_t^b \tag{1.3.2}$$

where $F$ is the output of the firm, $K_t$ is capital stock used in production, $L_t$ is quantity labour, and $\omega_i$ is a regime-switching productivity parameter dependent on the same CTMC $\varepsilon_t$. Let $\omega_1 \geq \omega_2$ reflect the fact that economic downturns are often associated with dips in aggregate and firm-level productivity, as discussed in section 1.2. This parameter will affect the *level* of output after regime switches in the next two chapters, which requires recessionary regimes to affect the *level* of productivity. In this chapter and the next, I interpret the $\omega_i$ parameter as firm-specific but switches in $\varepsilon_t$ affecting the whole economy. In the representative agent model, all firms use the production function in equation 1.3.2 but may have different values of $\omega_i$.

The market for the output good is perfectly competitive. The production function exhibits decreasing returns to scale, so the elasticity of output with respect to labour and capital, $a$ and $b$ respectively, are both less than one. This assumption permits a positive profit flow given the firm produces the output good using the optimal levels of capital and labour. These optimal levels are determined by the parameters $a$ and $b$, and a linear cost function comprised of the wages paid to labour, $w$, and the price of capital goods, $r$. All factors adjust instantaneously, so the firm can always adjust its stock of labour and capital to the optimal level in the interval $\Delta t$.

### 1.3.3 Profits

The profit function is derived by assuming the firm always minimises costs and solving the constrained optimisation problem given by the Lagrangian

$$\min_{K_t, L_t} \mathcal{L} = [wL_t + rK_t] + \mu \left[F - \omega_i K_t^a L_t^b\right]. \tag{1.3.3}$$

Differentiating the minimised cost function with respect to $F$ and setting equal to $P_t$ (marginal cost equals marginal revenue) gives the profit maximising level of output, which can then be substituted into the revenue ($P_t F$) and cost functions to obtain the maximised profit function given by equation 1.3.4

$$\Pi\left(P_t, \varepsilon_t\right) = [1 - (a+b)] \left[\omega_i P_t \left(\frac{a}{w}\right)^a \left(\frac{b}{r}\right)^b\right]^{\frac{1}{1-(a+b)}}. \tag{1.3.4}$$

Because $\Pi\left(P_t, \varepsilon_t\right)$ is a function of $P_t$ and $P_t$ follows a GBM, $\Pi\left(P_t, \varepsilon_t\right)$ also follows a GBM, only with modified drift and volatility parameters compared to equation 1.3.1.

$$d\Pi\left(P_t, \varepsilon_t\right) = \hat{\alpha}_i \Pi dt + \hat{\sigma}_i \Pi dW_t \tag{1.3.5}$$

Where $\hat{\alpha}_i = \left[\alpha_i \left[1 - (a+b)\right] + (a+b)\frac{1}{2}\sigma_i^2\right]$ and $\hat{\sigma}_i = \sigma_i \left[1 - (a+b)\right]$. The same transition matrix governs the probability of switching between regimes in the interval $\Delta t$. Figure 1.3.2 shows a discrete time approximation of the profit process relative to the price process where the parameters remain as they were in figure 1.3.1. The volatility of the profit flow is lower compared to the price process, as evident from the expression defining $\hat{\sigma}_i$ and the fact that $0 < [1 - (a+b)] < 1$.

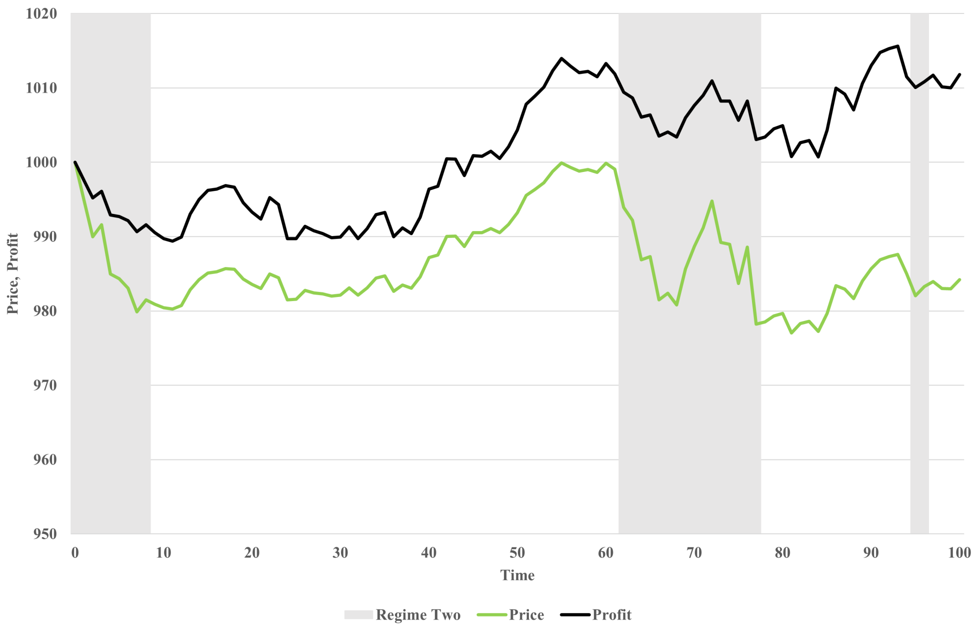



Figure 1.3.2: Discrete Time Approximation of Profit Process

Having specified the variables and constants in the model, the following sections drop time subscripts and arguments of functions to keep the equations concise. As stated in (Dixit & Pindyck, 1994, p.107), the fact that the firm is infinitely lived and the profit function and parameters of the GBM are not explicit functions of time means the value of the firm is also not a function of time insofar as the calendar date is irrelevant in determining its value. The initial conditions do matter but from there on the problem looks the same no matter the value of $t$.

## 1.4 Value of the Firm

### 1.4.1 The Fundamental Value

The value of the firm, $V_i$ is the net present value of the flow of all future profits in regime $i$ discounted at a constant rate $\rho$. This means $V_i$ is also dependent on the regime of the GBM in equation 1.3.1. Over the interval $dt$, the firm gives a profit flow of $\Pi_i dt$ and has an expected value over the rest of its infinite lifespan. The sum of the immediate profit flow and the expected present value gives the model's Bellman equation.

Since the profit flow can be in one of two regimes over the interval $dt$, the Bellman equation must also reflect the change in the value of the firm caused by a regime switch. In the interval $\Delta t$, the profit flow is $(1 - \lambda_{ij}\Delta t)(\Pi_i \Delta t) + \lambda_{ij}\Delta t\,(\Pi_j \Delta t)$. All terms of order $(\Delta t)^2$ go to zero much faster than those of $\Delta t$ and so should be ignored in the limit as $\Delta t \to 0$. Thus, only the term $\Pi_i \Delta t$ survives. It is clearer to present the differential equations below for the value of the firm and the option value as functions of the price of the output good rather than the profit flow. Hence, let

$$h = [1 - (a+b)] \left[ \left(\frac{a}{w}\right)^a \left(\frac{b}{r}\right)^b \right]^{\frac{1}{1-(a+b)}}$$

and $\nu = \frac{1}{1-(a+b)}$ so the profit flow is

$$\Pi_i = h\omega_i^{\nu} P^{\nu}.$$

The Bellman equation is

$$V_i = \Pi_i dt + e^{-\rho dt}\,\mathbb{E}[(1 - \lambda_{ij}dt)(V_i + dV_i) + \lambda_{ij}dt(V_j + dV_j)]. \tag{1.4.1}$$

Expanding $V_i$ using Ito's lemma, ignoring all differential terms of order greater than $dt$ (such as $dt^2$) and rearranging yields the following second-order differential equation describing the dynamics of $V_i$ over the interval $dt$

$$\frac{1}{2}\sigma_i^2 P^2 V_i^{''} + \alpha_i P V_i^{'} - (\rho + \lambda_{ij})V_i + h\omega_i^{\nu} P^{\nu} + \lambda_{ij} V_j = 0. \tag{1.4.2}$$

It is normal in this field to assume there are no 'speculative bubbles' regarding the value of the technology (Dixit & Pindyck, 1994, p.180). Mathematically, this means ignoring the homogeneous part of the solution to the differential equation. Economically, it means the investor does not consider the option value of selling the firm. This assumption is necessary when considering the case of irreversible acquisitions. It also means the non-homogeneous solution of equation 1.4.2 has an economically intuitive interpretation as the expected net present value of the technology, taking into account the drift and volatility parameters of the profit flow as well as the probability of switching to the other regime

$$V_i = \mathbb{E}_{\varepsilon_t} \int_0^\infty e^{-\rho t} \left[ \Pi_i \mid \varepsilon = i \right] dt. \tag{1.4.3}$$

Direct substitution into 1.4.2 shows that $V_i = \theta_i h P^\nu$, where $\theta_i$ is some constant to be determined, solves the differential equation if

$$\theta_i = \frac{\omega_i^\nu + \lambda_{ij}\omega_j^\nu \theta_j}{(\rho + \lambda_{ij}) - \alpha_i \nu - \frac{1}{2}\sigma_i^2 \nu \left(\nu - 1\right)}.$$

Now let

$$\eta_i(x) = (\rho + \lambda_{ij}) - \alpha_i(x) - \frac{1}{2}\sigma_i^2 x \left(x - 1\right) \tag{1.4.4}$$

then the value of the firm is given in equation 1.4.5, where the expression for $\theta_j$ is substituted into $\theta_i$

$$V_i = \frac{\eta_j(\nu)\omega_i^\nu + \lambda_{ij}\omega_j^\nu}{\eta_i(\nu)\eta_j(\nu) - \lambda_{ij}\lambda_{ji}} h P^\nu = \theta_i h P^\nu. \tag{1.4.5}$$

In order for $\theta_i$ to be positive, $\eta_i(\nu) > 0$ must hold. Given that $\rho > \alpha_i$ is assumed to ensure waiting for more information is not always the best policy, this condition implies that either $\rho$ should not be too small or that $\sigma_i$ and $\nu$ should not be too large. $\nu$ is relatively small when the returns to scale of the production function are sufficiently decreasing, a similar condition exists for the model in (Dixit & Pindyck, 1994, p.365). I ensure this by keeping $\nu$ less than the positive root of $\eta_i(z)$ and $H(z)$ defined below. Volatility is under the control of the modeller and in most studies of this kind it is kept around 0.2 (Dixit & Pindyck, 1994, p.156; Guo et al., 2005). Therefore, I primarily use $\rho$ to ensure the discounted present value of the firm is positive and that there is some finite value of $P$ which justifies exercising the option.

### 1.4.2 The Option Value

The value of the option to invest in the technology, $\Phi_i$, can also be solved as a function of the price of the output good. The Bellman equation for the option to invest in the technology is

$$\Phi_i = e^{-\rho dt}\,\mathbb{E}[(1-\lambda_{ij}dt)(\Phi_i + d\Phi_i) + \lambda_{ij}dt(\Phi_j + d\Phi_j)] \tag{1.4.6}$$

which, after expansion using Ito's lemma, yields the homogeneous linear differential equation 1.4.7. As required, the value of the option in state $i$ is dependent on the dynamics of the option in state $j$

$$\tfrac{1}{2}\sigma_i^2 P^2 \Phi_i^{''} + \alpha_i P \Phi_i^{'} - (\rho + \lambda_{ij})\Phi_i + \lambda_{ij}\Phi_j = 0. \tag{1.4.7}$$

Again, direct substitution of

$$\Phi_1 = \sum_{j=1}^{4} A_j P^{z_j} \quad \text{and} \quad \Phi_2 = \sum_{j=1}^{4} B_j P^{z_j} \tag{1.4.8}$$

into 1.4.7 solves the differential equation if there four real roots to the following quartic equation

$$H(z) = \eta_1(z)\eta_2(z) - \lambda_{12}\lambda_{21} = 0 \tag{1.4.9}$$

in which case $\frac{\eta_1(z_j)}{\lambda_{12}}A_j = B_j$ and the relationship between $A_j$ and $B_j$ is fixed. Appendix 1.A.1 demonstrates that two of these roots are greater than one, call them $z_1$ and $z_2$, and two of them are negative, $z_3$ and $z_4$. Notice that every constant of integration is multiplied by $P$ raised to the power of $z_j$ and that the subscript of the constant matches the subscript of the associated $z_j$.

More constants of integration will be needed later in this chapter as the solutions to the differential equations change when considering transient regions and when the acquisition of the firm is allowed to be partially reversible. For future reference, table 1.4.1 gives an overview of the notation adopted for these constants. It only includes the constants which need to be identified, so it does not include the $B_j$ constants above because these are known as soon as the $A_j$ are found. At present, I have only introduced constants $A_j$ and $B_j$ with $j \in \{1, 2, 3, 4\}$ and stated that their subscripts match those on the roots of the characteristic equation which they are associated with. The use of the other constants and the meanings of the terms in the column headers will become clear throughout the chapter.

| Constants | Regime | Associated Root Sign | Region | $\Phi/V$ | Configuration Nested | Configuration Separated |
|---|---|---|---|---|---|---|
| $A_1, A_2$ | 1 | $+, +$ | Base | $\Phi_1$ | ✓ | ✓ |
| $A_3, A_4$ | 1 | $-, -$ | Base | $V_1$ | ✓ | ✓ |
| $C_1, C_2$ | 2 | $+, -$ | Transient | $\Phi_2^T$ | ✓ | ✓ |
| $C_3, C_4$ | 2 | $+, -$ | Transient | $V_2^T$ | ✓ | ✗ |
| $C_5, C_6$ | 1 | $+, -$ | Transient | $V_1^T$ | ✗ | ✓ |
| $D_1, D_2, D_3, D_4$ | 1 | $+, +, -, -$ | Linking | $V_1^L$ | ✗ | ✓ |

Table 1.4.1: Naming Conventions for the Constants of Integration

## 1.5 Irreversible Investment

### 1.5.1 Option Value in a Transient Region

In the irreversible case, the investor purchases the firm for a constant one-time purchase price $I$ and cannot sell it at a later date. Terms in $\Phi_i$ associated with a negative power imply that as the price of the output good goes to zero, the value of the option will go to infinity. This is inconsistent with the logic that the value of the option to invest in the firm should go to zero as $P$ goes to zero. Hence, set A3 = A4 = B3 = B4 = 0. I show that the solution to the investor's problem in this framework is similar to that of a firm deciding when to adjust its capital stock upwards under total irreversibility, as in Guo et al. (2005). I borrow the terminology used in their study and refer to regions of $P$ for which a switch in regime brings an immediate change in the investor's ownership position as *transient regions*.

Equation 1.4.7 assumes that the option to invest in the firm will not be exercised in regime $j$ following a switch in the CTMC. But even though the output price may not be sufficient to justify exercising the option in regime $i$, it could be sufficient for regime $j$. In this case, a switch in the

CTMC will mean the investor immediately acquires the firm itself (minus the purchase price $I$) rather than the option to invest in the firm. For exposition, assume that the CTMC is currently in regime two and $P$ is such that the investor should not exercise her option to invest in the firm. However, if there is a switch to regime one, $P$ is such that exercising the option is justified. The differential equation needs to be modified accordingly, with the option value in state one being replaced by the payoff for exercising the option to invest; $V_1 - I$. As will be seen, the assumption that regime two represents an economic downturn implies $\Phi_2$ is in a transient region for some range of values of the output price, but the exact same reasoning could be applied to $\Phi_1$ if no constraints were placed on the relative values of the parameters, so the result is fully generalisable. For clarity, let $\Phi_2^T$ be the value of $\Phi_2$ when it is in the transient region. Then the differential equation describing the dynamics of the option value is

$$\frac{1}{2}\sigma_2^2 P^2 \left(\Phi_2^T\right)'' + \alpha_2 P \left(\Phi_2^T\right)' - (\rho + \lambda_{21}) \left(\Phi_2^T\right) + \lambda_{21}[\theta_1 h P^\nu - I] = 0. \tag{1.5.1}$$

The solution to 1.5.1 consists of a homogeneous and non-homogeneous part, the former reflecting the option to invest in the firm and the change in the option value that occurs at the boundary of the transient region, and the latter reflecting the probability weighted value acquired due to a switch in the CTMC to regime two (Guo et al., 2005). Because the transient region is bounded above and below by the investment thresholds in the two regimes, it does not make sense to think of the case where $P$ goes to zero, so there is no argument to eliminate either of the constants of integration from the homogeneous part of the solution. Let $\gamma_1 > 1$ and $\gamma_2 < 0$ be the roots of $\eta_2(\gamma)$ and note that $\frac{\lambda_{21}\theta_1}{\eta_2(\nu)} = \left(\theta_2 - \frac{\omega_2^\nu}{\eta_2(\nu)}\right)$, then 1.5.2 is the solution to 1.5.1

$$\Phi_2^T = C_1 P^{\gamma_1} + C_2 P^{\gamma_2} + \left(\theta_2 - \frac{\omega_2^\nu}{\eta_2(\nu)}\right) h P^\nu - \tfrac{\lambda_{21} I}{\rho + \lambda_{21}}. \tag{1.5.2}$$

The model predicts that if the economy is currently in the recessionary regime and $P > P_1^*$, the investor will immediately purchase the firm if there is a regime switch to the expansionary regime. Therefore, there could be a sudden increase in the number of acquisitions when switching to an expansionary regime. This requires the output price be relatively high in the recessionary regime which is less likely due to the lower value of $\alpha_2$. In economic terms; if the profitability of the firm does not slip too far during the recession, there may be an burst in activity after the return to the expansionary regime.

### 1.5.2 Boundary Conditions

With $\Phi_2$ in a transient region over some range of values of the output price, there are six unknowns in the model; four constants of integration $A_1, A_2, C_1$ and $C_2$, and two threshold values of $P$, $P_1^*$ and $P_2^*$, justifying investment in the firm in regime one and two respectively. This requires six equations. Four of these are provided by the *value matching* and *smooth pasting* conditions at the threshold values of $P$, which ensure that exercising the option is indeed the investor's optimal strategy at these values. Value matching sets the option values equal to the value of the project at the boundary, which is intuitive as the boundary can be seen as a point of indifference between holding the option and holding the project, and smooth pasting ensures these equations meet tangentially at the boundary[2]

$$\sum_{j=1}^{2} A_j \left(P_1^*\right)^{z_j} = \theta_1 h \left(P_1^*\right)^{\nu} - I \tag{1.5.3}$$

$$\sum_{j=1}^{2} z_j A_j \left(P_1^*\right)^{z_j - 1} = \nu\theta_1 h \left(P_1^*\right)^{\nu-1} \tag{1.5.4}$$

$$\sum_{j=1}^{2} C_j \left(P_2^*\right)^{\gamma_j} + \left(\theta_2 - \frac{\omega_2^{\nu}}{\eta_2(\nu)}\right) h \left(P_2^*\right)^{\nu} - \frac{\lambda_{21} I}{\rho + \lambda_{21}} = \theta_2 h \left(P_2^*\right)^{\nu} - I \tag{1.5.5}$$

$$\sum_{j=1}^{2} \gamma_j C_j \left(P_2^*\right)^{\gamma_j - 1} + \nu \left(\theta_2 - \frac{\omega_2^{\nu}}{\eta_2(\nu)}\right) h \left(P_2^*\right)^{\nu-1} = \nu\theta_2 h \left(P_2^*\right)^{\nu-1}. \tag{1.5.6}$$

[2]See (Dixit & Pindyck, 1994, pp.130-131) for an intuitive explanation of the value matching and smooth pasting conditions in the context of an optimal stopping problem.

The final two conditions ensure than $\Phi_2$ and $\Phi_2^T$ and their first derivatives are equal at the threshold value $P_1^*$

$$\sum_{j=1}^{2} C_j \left(P_1^*\right)^{\gamma_j} + \left(\theta_2 - \frac{\omega_2^\nu}{\eta_2(\nu)}\right) h \left(P_1^*\right)^\nu - \frac{\lambda_{21} I}{\rho + \lambda_{21}} = \sum_{j=1}^{2} \frac{\eta_1(z_j)}{\lambda_{12}} A_j \left(P_1^*\right)^{z_j} \tag{1.5.7}$$

$$\sum_{j=1}^{2} \gamma_j C_j \left(P_1^*\right)^{\gamma_j - 1} + \nu \left(\theta_2 - \frac{\omega_2^\nu}{\eta_2(\nu)}\right) h \left(P_1^*\right)^{\nu - 1} = \sum_{j=1}^{2} z_j \frac{\eta_1(z_j)}{\lambda_{12}} A_j \left(P_1^*\right)^{z_j - 1} \tag{1.5.8}$$

Further intuition behind these conditions is given in (Dixit, 1993, pp.30-31).

To solve this system, first find the constants of integration which solve the first four boundary conditions then substitute these expressions into equations 1.5.7 and 1.5.8 and derive a non-linear equation in $R = \frac{P_1^*}{P_2^*}$. Finding a unique root $R < 1$ to this equation means identifying a pair of thresholds which ensure $P_1^* < P_2^*$. The solution has a very similar form to a model of capital accumulation by Guo et al. (2005).

### 1.5.3 Constants of Integration

The constants of integration which solve equations 1.5.3 to 1.5.6 are given by:

$$A_1 = -\frac{1}{\left(z_2 - z_1\right)\left(P_1^*\right)^{z_1}} \left[\left(\nu - z_2\right)\left(\theta_1 h \left(P_1^*\right)^\nu\right) + z_2 I\right] \tag{1.5.9}$$

$$A_2 = -\frac{1}{\left(z_1 - z_2\right)\left(P_1^*\right)^{z_2}} \left[\left(\nu - z_1\right)\left(\theta_1 h \left(P_1^*\right)^\nu\right) + z_1 I\right] \tag{1.5.10}$$

$$C_1 = -\frac{1}{\left(\gamma_2 - \gamma_1\right)\left(P_2^*\right)^{\gamma_1}} \left[\left(\nu - \gamma_2\right)\left(\frac{\omega_2^\nu}{\eta_2(\nu)} h \left(P_2^*\right)^\nu\right) + \gamma_2 \frac{\rho I}{\rho + \lambda_{21}}\right] \tag{1.5.11}$$

$$C_2 = -\frac{1}{\left(\gamma_1 - \gamma_2\right)\left(P_2^*\right)^{\gamma_2}} \left[\left(\nu - \gamma_1\right)\left(\frac{\omega_2^\nu}{\eta_2(\nu)} h \left(P_2^*\right)^\nu\right) + \gamma_1 \frac{\rho I}{\rho + \lambda_{21}}\right]. \tag{1.5.12}$$

Their values are such that $V_1 = \Phi_1$ at $P_1^*$, $\Phi_2^T = V_2$ at $P_2^*$, and $\Phi_2^T = \Phi_2$ at $P_1^*$. $A_1$ and $A_2$ together indicate the value of the option to acquire the firm in regime one. Their sum will be positive because the expected discounted profit flow generated by the firm is positive. Likewise, $C_1$ and $C_2$ together reflect the option value of acquiring the firm in the transient region of regime two and their sum will also be positive. I will devote more time to discussing the constants of integration in the case of partially irreversible acquisitions.

### 1.5.4 Non-linear Equation in $R$

Substituting the constants of integration into equations 1.5.7 and 1.5.8 and rearranging gives two non-linear equations which define the value of $h\left(P_2^*\right)^\nu$,

$$h\left(P_2^*\right)^\nu = \frac{I\left(\frac{\gamma_2 R^{\gamma_1}-\gamma_1 R^{\gamma_2}}{\gamma_2-\gamma_1}\frac{\rho}{\rho+\lambda_{21}}+\frac{\eta_1(z_2)z_1-\eta_1(z_1)z_2}{\lambda_{12}(z_2-z_1)}+\frac{\lambda_{21}}{\rho+\lambda_{21}}\right)}{R^\nu\left(\theta_2+\frac{(\nu-z_2)\eta_1(z_1)-(\nu-z_1)\eta_1(z_2)}{\lambda_{12}(z_2-z_1)}\theta_1-\frac{\omega_2^\nu}{\eta_2(\nu)}\right)+\frac{(\nu-\gamma_1)R^{\gamma_2}-(\nu-\gamma_2)R^{\gamma_1}}{\gamma_2-\gamma_1}\frac{\omega_2^\nu}{\eta_2(\nu)}} \tag{1.5.13}$$

$$h\left(P_2^*\right)^\nu = \frac{I\left(\frac{\gamma_1\gamma_2(R^{\gamma_1}-R^{\gamma_2})}{\gamma_2-\gamma_1}\frac{\rho}{\rho+\lambda_{21}}+\frac{z_1 z_2(\eta_1(z_2)-\eta_1(z_1))}{\lambda_{12}(z_2-z_1)}\right)}{R^\nu\left(\nu\theta_2+\frac{z_1(\nu-z_2)\eta_1(z_1)-z_2(\nu-z_1)\eta_1(z_2)}{\lambda_{12}(z_2-z_1)}\theta_1-\frac{\nu\omega_2^\nu}{\eta_2(\nu)}\right)+\frac{\gamma_2(\nu-\gamma_1)R^{\gamma_2}-\gamma_1(\nu-\gamma_2)R^{\gamma_1}}{\gamma_2-\gamma_1}\frac{\omega_2^\nu}{\eta_2(\nu)}}. \tag{1.5.14}$$

These two equations must be equal, so divide 1.5.13 by 1.5.14 to get one equation and one unknown, $R$. I find the root of this expression numerically, using reasonable values for the parameters in the model based on Guo et al. (2005) and real-world data. Substituting $R$ into 1.5.13 gives $h\left(P_2^*\right)^\nu$, which after rearranging and solving for $P_2^*$ and using the fact that $P_1^* = RP_2^*$ gives the threshold price level in regime one. With this found, the threshold justifying investment could also be expressed in terms of $\Pi_i^*$ by multiplying the expressions by $\omega_i$. Importantly, $R$ will tend towards one as $\alpha_1 \to \alpha_2$, $\sigma_1 \to \sigma_2$, and $\omega_1 \to \omega_2$ meaning that switching between two identical processes reduces the model to one without switching where there is only one critical value of $P$ justifying investment.

### 1.5.5 Numerical Solutions to the Non-linear equation

Table 1.5.2 shows how the thresholds values of $P$ in the two regimes change as the parameters of the model change by one percentage point. I calibrate the model so that regime two represents an economic downturn and demonstrate how the investment thresholds change when the disparity between the two regimes becomes greater. Thus, the table shows the effect of making the downturn more severe; with higher uncertainty, lower price growth, and lower productivity. Likewise, it shows the effect of making the other regime better for investors; with higher price growth, lower uncertainty, and higher productivity. In both cases, I show the effect of increasing the persistence of the regimes, which means decreasing the probability of switching to the other regime in the interval $dt$. Table 1.5.1 gives some base values for the parameters in the model; unless stated otherwise, these are the values the parameters take in the simulations.

| $a$ | $b$ | $\alpha_1$ | $\alpha_2$ | $\sigma_1$ | $\sigma_2$ | $\omega_1$ | $\omega_2$ | $\rho$ | $\lambda_{12}$ | $\lambda_{21}$ | $I$ | $r$ | $w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 0.02 | 0.01 | 0.2 | 0.25 | 1 | 1 | 0.1 | 0.05 | 0.25 | 10 | 1 | 1 |

Table 1.5.1: Parameter Values

The base rate of transition from an expansion to a recession is chosen based on the average number of quarters an expansion has lasted since 1950 according to the NBER's method of identifying turning points in the business cycle. Since $1/\lambda_{ij}$ is the expected time elapsed before a switch, and the average time between recessions is 20.5 quarters, $1/\lambda_{12} = 20.5$ so $\lambda_{12} \approx 0.05$. Likewise, the transition rate from a recession to an expansion is the inverse of the average number of quarters recessions have lasted since 1950, which is just above 4, yielding $\lambda_{21} = 0.25$. The discount rate used by the investor is set at a base rate of 10%. This is much higher than the average interest rate on three-month treasury bills since 1950 (4%), however, is more in line with recent estimates by Gormsen & Huber (2023) from 2500 firms across 20 countries which suggests an average required return on investment projects of 16%. Mechanically, $\rho$ must be sufficiently large to ensure the positive roots of $\eta_2(z)$ and $H(z)$ are greater than $\nu$, otherwise the discounted value of profits would be negative. The base rate of nominal growth in the price of the output good and its volatility is in a range consistent with the models of Guo et al. (2005) and (Dixit & Pindyck, 1994, p.153) and should be interpreted as quarterly rates. In the base case, $\omega_i = 1$ so the regime change does not have an effect on productivity. The value of $I$ does not play a significant role in the model, it just scales the values of the thresholds up or down.

With these values, the base value of $P_1^*$ is 3.1774 and the base value of $P_2^*$ is 3.4517. Expressed in terms of profit flow, the values are 1.6587 and 1.8670 respectively. Therefore, the model predicts that switching to a high-uncertainty regime with lower price growth will cause the price justifying the acquisition of a new firm to increase, suggesting a fall in acquisitions during downturns. This is consistent with the results of Nguyen & Phan (2017) and Bonaime et al. (2018) discussed in section 1.2 as well as the models of investment under uncertainty in Dixit & Pindyck (1994). For completeness, the constants of integration are $A_1 = 0.002$, $A_2 = 1.33$, $C_1 = 0.02$, and $C_2 = 17.85$.

The parameters affect both the value of the firm and the value of the option to invest, the former through the constants $\theta_i$ and $\theta_2 - (\omega_2^\nu/\eta_2(\nu))$, and the latter through the roots of the characteristic polynomials and the constants of integration. Any parameter which increases the characteristic roots

| Parameter | $\Delta$ | $\Delta P_1^*$ | $\Delta P_2^*$ |
|---|---|---|---|
| $\alpha_1$ | $+$ | -0.083 | -0.047 |
| $\alpha_2$ | $-$ | 0.0103 | 0.0575 |
| $\sigma_1$ | $-$ | -0.0583 | -0.0138 |
| $\sigma_2$ | $+$ | 0.0036 | 0.0526 |
| $\lambda_{12}$ | $-$ | -0.0049 | -0.0023 |
| $\lambda_{21}$ | $-$ | 0.0005 | 0.0029 |
| $\rho$ | $+$ | 0.1733 | 0.1805 |
| $\omega_1$ | $+$ | -0.0292 | -0.0096 |
| $\omega_2$ | $-$ | 0.0023 | 0.0251 |

Table 1.5.2: Simulation Results

$z_1$, $z_2$, and $\gamma_1$ will decrease the investment threshold by decreasing the option value of waiting. The comparative statics of the roots are discussed in appendix 1.A.2. Any parameter which increases $\theta_i$ will also decrease the investment threshold by increasing the expected present value of the firm, thus making purchase more attractive even at relatively lower output prices.

The first key result from table 1.5.2 is that the threshold justifying investment is higher in high uncertainty regimes and increases as uncertainty in that regime increases, as shown from the effect of a one percentage point increase in $\sigma_2$. Investors adopt a wait-and-see mentality which dampens the amount of acquisitions in regime two. It also causes a slight increase in the threshold in regime one because the decision to purchase the firm in regime one has to be conditioned on the option value in regime two, which is now higher. Note that the convexity of the labour-optimised profit function means increases in $\sigma_2$ will increase the expected discounted value of the firm for the investor. Hence, the option value of waiting is sufficiently large to overcome the expected increase in future profits generated by the higher volatility parameter, which lowers the investment threshold. This is a consequence of the restriction that $\nu$ is less than the roots of $\eta_i(z)$ and $H(z)$.

Higher persistence of the recessionary regime (lower $\lambda_{21}$) results in a higher investment threshold, though the effect of a one percentage point change is notably lower than it is for the other parameters in the model. A lower $\lambda_{21}$ lowers the positive root $\gamma_1$ which will tend to increase the investment threshold. The intuition is that as the high-uncertainty regime becomes more persistent, the value of waiting for more information is higher (in both regimes). For the base parameters chosen, lower $\lambda_{21}$ also reduces the expected fundamental value of the firm in both regimes, which also causes the investment threshold to increase because the opportunity cost of *not* purchasing the project is lower.

In contrast, the investment threshold decreases when the low-uncertainty regime becomes more persistent. Given the initial parameters, lower $\lambda_{12}$ will decrease the larger positive root ($z_1$), and increase the smaller positive root ($z_2$) so there is ambiguity about the effect on the investment threshold. Additionally, the fundamental value of the firm increases when $\lambda_{12}$ is lower which will favour earlier exercise. Together, table 1.5.2 shows these effects cause a fall in the investment threshold. Economically, as the low-volatility regime becomes more persistent, the option value of waiting for more information falls. Acquiring the firm makes more sense when relatively prosperous periods are expected to last longer.

Changes in $\omega_i$ affect the thresholds in both regimes by influencing the expected present value of future profit flows in both regimes. Higher $\omega_1$ means the investor will purchase the technology earlier because the profit flow it generates will outweigh the cost of purchase plus the option value of waiting at a lower value of $P$. It also means the probability-weighted value of a regime switch from regime one to regime two is higher, i.e. $\lambda_{21}[\theta_1 h P^{\nu} - I]$ is higher, which will favour earlier exercise in regime two. Likewise, lower $\omega_2$ means the investor will wait longer before exercising her option because the expected profit flow is lower. This is a mechanism through which low-productivity regimes can reduce the amount of acquisitions and can explain their business cycle dynamics pointed out by Maksimovic & Phillips (2001).

Overall, this analysis reveals that highly-persistent economic regimes which cause both high uncertainty and low productivity will significantly dampen the number of acquisitions. The effects are not just limited to the recessionary regime because investors take both regimes into consideration when solving their decision problem.

## 1.6 Partially Reversible Investment

### 1.6.1 Value of the Firm with Reversibility

If the firm can be sold at a later date for $U < I$, part of the homogeneous solution to 1.4.2 should be kept alive to reflect the option value of selling it at a later date. The investor is also able to buy the firm back after they have sold it for the same purchase price $I$. Officer (2007) notes that selling subsidiaries is a common method for cash-strapped firms to overcome liquidity issues. The homogeneous solution to 1.4.2 will have exactly the same form as 1.4.8. In this instance, the constants of integration associated with terms featuring positive powers of $z$ ($z_1$ and $z_2$) should be set equal to zero, because the option value of selling the project will be worth very little for very large values of $P$. Hence, $A_1 = A_2 = B_1 = B_2 = 0$ and the value of the firm is

$$V_1 = \theta_1 h P^{\nu} + A_3 P^{z_3} + A_4 P^{z_4} \tag{1.6.1}$$

$$V_2 = \theta_2 h P^{\nu} + \frac{\eta_1(z_3)}{\lambda_{12}} A_3 P^{z_3} + \frac{\eta_1(z_4)}{\lambda_{12}} A_4 P^{z_4}. \tag{1.6.2}$$

As before, the investor will purchase the project when the option value is equal to the value of the firm minus the purchase cost. However, if the price of the output good is sufficiently low the investor will sell the project for a lump sum $U$ and acquire the option to repurchase it at a later date. This means there are four critical values to locate in the model; two values of $P$ justifying investment and disinvestment in regime one, $P_1^I$ and $P_1^U$, and two corresponding thresholds in regime two $P_2^I$ and $P_2^U$. Importantly, the investor always owns the firm in regime $i$ when price is above the upper threshold in regime $i$ and likewise always holds the option to purchase it when price is below the lower threshold in regime $i$. Whenever $P_i^U < P < P_i^I$, the investor could hold either the firm or the option.

### 1.6.2 Transient Regions with Reversibility

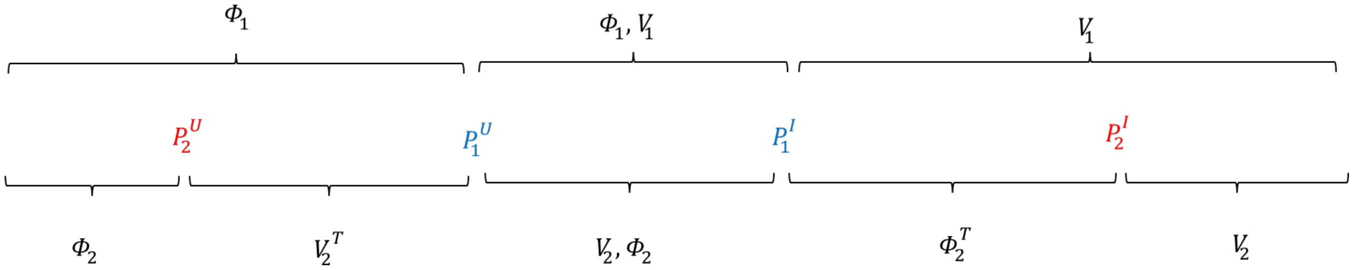
The locations of the transient regions depend on the configurations of the investment and disinvestment thresholds. By configuration, I mean the location of the thresholds in regime one relative to the thresholds in regime two. In this chapter, I consider two configurations; a ‘nested configuration’ where the thresholds in one regime are bounded by those of the other regime, and a ‘separated configuration’ where the sale threshold in one regime is higher than the purchase threshold in the other regime[3]. There is also an intermediate case where the thresholds overlap, so the sale threshold in one of the regimes is bounded by the purchase and sale thresholds in the other regime. This chapter only focuses on the two extreme cases with the understanding that the 'overlapping configuration' is produced by parameter values lying somewhere in between. Chapter two focuses on capital accumulation rather than a one-time purchase and includes a solution in the overlapping configuration. Figure 1.6.1 shows the locations of the transient regions in the two configurations.

Notice that in figure 1.6.1, $P_2^I$ is always the rightmost threshold, implying that it is the highest threshold value of $P$. Likewise, $P_2^U$ is the leftmost threshold in the nested configuration and is higher than $P_1^I$ in the separated configuration. This is a consequence of the assumptions which made regime two the recessionary regime. $\sigma_2 > \sigma_1$, $\alpha_2 < \alpha_1$ and $\omega_2 \leq \omega_1$ all imply the investment threshold in regime two will always lie above the investment threshold in regime one. Hence, a nested configuration must have $P_2^U$ as the lowest threshold value of $P$ and a separated configuration must have $P_2^U > P_1^I$.

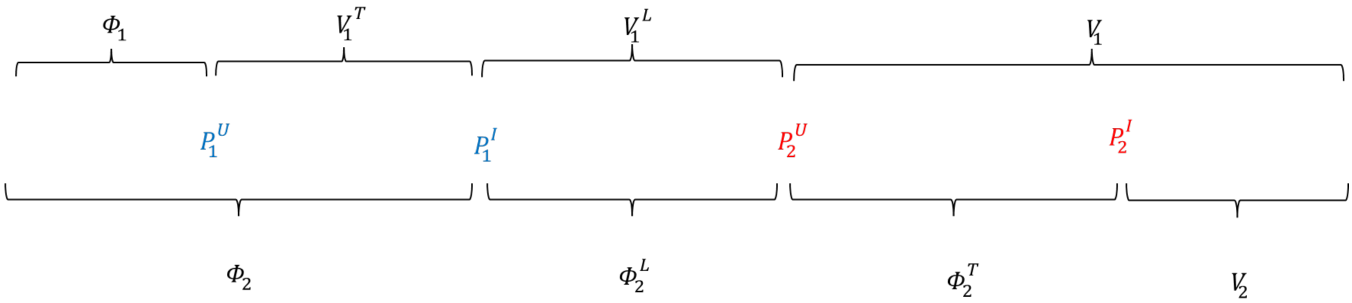
Of course, I could have chosen regime one to be the recessionary regime, in which case the subscripts in figure 1.6.1 should all be swapped. The key point is that the results in this section are fully generalisable. The restrictions making regime two the recessionary regime means this chapter is only concerned with the configurations in figure 1.6.1, however, if I swapped the subscripts of all the parameter values it would produce the same configuration but would also swap the subscripts of the threshold values of $P$.

Remember that transient regions occur whenever a regime switch causes an immediate change in the investor's policy. For example, in panel 1.6.1a if $P_2^U < P < P_1^U$ and the investor currently owns the firm in regime two, a switch to regime one will cause the investor to immediately sell the project and acquire the option to invest in regime one because the price of the output good is below the critical value justifying sale in regime one. If the CTMC is in regime one and the investor possessed the option in the region $P_2^U < P < P_1^U$, the regime switch would not cause a change in the ownership position because the output price is not sufficiently high to justify purchase in

[3]A separated configuration is only possible if $I$, $U$, or $F$ changes when the regime changes. I will show why this is the case in chapter two. In the knife-edge case where $P_2^U = P_1^U$ in the nested configuration, the boundary conditions will give 10 equations but 9 unknowns, so there is no solution. The same is true in the knife-edge case for the separated configuration. It would be very difficult to find parameter values which cause these knife-edge cases though so this chapter and the next are not concerned with them.




(a) Nested Configuration




(b) Separated Configuration

Figure 1.6.1: Configurations of the Boundaries

regime two. Similarly, if $P_1^I < P < P_2^I$ and the investor currently holds the option to invest in regime two, a switch to regime one would justify the immediate purchase of the firm. If the investor currently owned the firm in this region, however, a switch to regime one would not cause a change in ownership position because $P$ is far above the threshold required to sell the firm in regime one. Notice that transient regions only occur for regime two in the nested configuration. Hence, $V_2^T$ denotes the value of the firm in regime two within the transient region $P_2^U < P < P_1^U$ while $\Phi_2^T$ denotes the value of the option to invest in regime two within the transient region $P_1^I < P < P_2^I$.

The same logic applies when locating the transient regions in panel 1.6.1b. For example, if the investor owns the firm in the region $P_1^U < P < P_1^I$, a switch from regime one to regime two justifies selling the firm and acquiring the option. If the investor instead held the option in regime one, the regime switch would not cause a change in ownership position. Therefore, the value of the firm is in a transient region between $P_1^U < P < P_1^I$. By the same argument, the option value in regime two is transient in the region $P_2^U < P < P_2^I$. The differential equations and in the transient regions of figure 1.6.1 are

$$\frac{1}{2}\sigma_2^2 P^2 \left(\Phi_2^T\right)'' + \alpha_2 P \left(\Phi_2^T\right)' - (\rho + \lambda_{21}) \left(\Phi_2^T\right) + \lambda_{21} \left[\theta_1 h P^{\nu} + A_3 P^{z_3} + A_4 P^{z_4} - I\right] = 0, \tag{1.6.3}$$

$$\frac{1}{2}\sigma_2^2 P^2 \left(V_2^T\right)'' + \alpha_2 P \left(V_2^T\right)' - (\rho + \lambda_{21}) \left(V_2^T\right) + h\omega_2^{\nu} P^{\nu} + \lambda_{21} \left[A_1 P^{z_1} + A_2 P^{z_2} + U\right] = 0, \tag{1.6.4}$$

and,

$$\frac{1}{2}\sigma_1^2 P^2 \left(V_1^T\right)'' + \alpha_1 P \left(V_1^T\right)' - (\rho + \lambda_{12}) \left(V_1^T\right) + h\omega_1^{\nu} P^{\nu} + \lambda_{12} \left[\tfrac{\eta_1(z_1)}{\lambda_{12}} A_1 P^{z_1} + \tfrac{\eta_1(z_2)}{\lambda_{12}} A_2 P^{z_2} + U\right] = 0. \tag{1.6.5}$$

Equation 1.6.3 states that if the investor holds the option in regime two, a switch to regime one means she immediately acquires the firm minus the purchase cost. Likewise, a switch in regime causes the investor to immediately sell the firm she currently owns in equations 1.6.4 and 1.6.5. Remembering that $\left(\theta_2 - \frac{\omega_2^{\nu}}{\eta_2(\nu)}\right) = \lambda_{21}\frac{\theta_1}{\eta_2(\nu)}$ and from the quartic equation $\frac{\eta_2(z_j)}{\lambda_{21}} = \frac{\lambda_{12}}{\eta_1(z_j)}$, the solutions to these three equations are:

$$\Phi_2^T = C_1 P^{\gamma_1} + C_2 P^{\gamma_2} + \left(\theta_2 - \frac{\omega_2^{\nu}}{\eta_2(\nu)}\right) h P^{\nu} - \tfrac{\lambda_{21} I}{\rho + \lambda_{21}} + \tfrac{\eta_1(z_1)}{\lambda_{12}} A_3 P^{z_3} + \tfrac{\eta_1(z_4)}{\lambda_{12}} A_4 P^{z_4}, \tag{1.6.6}$$

$$V_2^T = C_3 P^{\gamma_1} + C_4 P^{\gamma_2} + \tfrac{\lambda_{12}}{\eta_1(\nu)} h\omega_1^{\nu} P^{\nu} + \tfrac{\lambda_{21} U}{\rho + \lambda_{21}} + \tfrac{\eta_1(z_1)}{\lambda_{12}} A_1 P^{z_1} + \tfrac{\eta_1(z_2)}{\lambda_{12}} A_2 P^{z_2}, \tag{1.6.7}$$

and,

$$V_1^T = C_5 P^{\beta_1} + C_6 P^{\beta_2} + \tfrac{\lambda_{12}}{\eta_1(\nu)} h\omega_1^{\nu} P^{\nu} + \tfrac{\lambda_{12} U}{\rho + \lambda_{12}} + A_1 P^{z_1} + A_2 P^{z_2}. \tag{1.6.8}$$

Where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of $\eta_1(z)$. Table 1.4.1 helps keep track of the notation used for the constants of integration.

### 1.6.3 Linking Region

The separated configuration contains a region where the value of the project in regime one and the option value in regime two are simultaneously transient in the region between $P_1^I$ and $P_2^U$, forming another system of coupled differential equations. A regime switch in this region always causes a change in the investor's ownership position; she will own the firm in regime one and hold the option in regime two. I call this the linking region because it bridges the gap between the lower threshold in regime two and the upper threshold in regime one. Let $V_1^L$ be the value of the firm in regime one and $\Phi_2^L$ be the option value in regime two in the linking region.

The system of coupled differential equations is

$$\frac{1}{2}\sigma_1^2 P^2 \left(V_1^L\right)'' + \alpha_1 P \left(V_1^L\right)' - (\rho + \lambda_{12}) \left(V_1^L\right) + h\omega_1^\nu P^\nu + \lambda_{12} \left(\Phi_2^L + U\right) = 0 \tag{1.6.9}$$

$$\frac{1}{2}\sigma_2^2 P^2 \left(\Phi_2^L\right)'' + \alpha_2 P \left(\Phi_2^L\right)' - (\rho + \lambda_{21}) \left(\Phi_2^L\right) + \lambda_{21} \left[V_1^L - I\right] = 0 \tag{1.6.10}$$

which has the solution

$$V_1^L = \theta_3 h P^\nu + \vartheta_1 + \sum_{j=1}^{4} D_j P^{z_j} \tag{1.6.11}$$

$$\Phi_2^L = \theta_4 h P^\nu + \vartheta_2 + \sum_{j=1}^{4} \frac{\eta_1(z_j)}{\lambda_{12}} D_j P^{z_j} \tag{1.6.12}$$

where

$$\theta_3 = \frac{\omega_1^\nu \eta_2(\nu)}{\eta_1(\nu)\eta_2(\nu) - \lambda_{12}\lambda_{21}}, \quad \theta_4 = \frac{\lambda_{21}\omega_1^\nu}{\eta_1(\nu)\eta_2(\nu) - \lambda_{12}\lambda_{21}},$$

$$\vartheta_1 = -\frac{\lambda_{12}\lambda_{21} I - (\rho + \lambda_{21})\lambda_{12} U}{(\rho + \lambda_{12})(\rho + \lambda_{21}) - \lambda_{12}\lambda_{21}}, \text{ and } \vartheta_2 = \frac{\lambda_{12}\lambda_{21} U - (\rho + \lambda_{12})\lambda_{21} I}{(\rho + \lambda_{12})(\rho + \lambda_{21}) - \lambda_{12}\lambda_{21}}.$$

As before, in the limit as the transition rates tend towards zero, these expressions would reduce to their equivalents in a model without regime switching. The $\vartheta_i$ terms account for the probability weighted value of a regime switch when the output price is inside the linking region. The constants of integration $D_j$ reflect the option values of buying ($D_1$ and $D_2$) and selling ($D_3$ and $D_4$) the firm in the linking region.

### 1.6.4 Nested Configuration Boundary Conditions

There are twelve equations and twelve unknowns in the nested configuration; four threshold values justifying investment and disinvestment in the two regimes and eight constants of integration. Table 1.4.1 will help keep track of the constants of integration used in the following sections. Regime one has $A_1$ and $A_2$ associated with the option value of investment, as well as $A_3$ and $A_4$ associated with the option value of selling the firm. The set of value matching and smooth pasting conditions given below pin down the unknowns at the threshold values of $P$ in regime one. For consistency with the graphical representation of the solutions in the following sections, I write the boundary conditions such that the right-hand side is always a constant and $V_i$ terms always appear first on the left-hand side

$$V_1(P_1^I) - \Phi_1(P_1^I) = I \qquad V_1(P_1^U) - \Phi_1(P_1^U) = U \tag{1.6.13}$$

$$V_1'(P_1^I) - \Phi_1'(P_1^I) = 0 \qquad V_1'(P_1^U) - \Phi_1'(P_1^U) = 0. \tag{1.6.14}$$

Regime two has $C_3$ and $C_4$ associated with the option to sell the firm in the transient region between $P_2^U$ and $P_1^U$, and $C_1$ and $C_2$ associated with the option to purchase the firm in the transient region between $P_1^I$ and $P_2^I$. As always, the value of the firm (including the option to sell it later) must be equal to the option of purchasing it at the boundaries justifying a change in policy. Additionally, $V_2$ and $V_2^T$ must meet tangentially at the $P_1^U$ boundary and likewise for $\Phi_2$ and $\Phi_2^T$ at the $P_1^I$ boundary, hence,

$$V_2(P_2^I) - \Phi_2^T(P_2^I) = I \qquad V_2^T(P_2^U) - \Phi_2(P_2^U) = U \tag{1.6.15}$$

$$V_2'(P_2^I) - \left(\Phi_2^T\right)'(P_2^I) = 0 \qquad \left(V_2^T\right)'(P_2^U) - \Phi_2'(P_2^U) = 0 \tag{1.6.16}$$

$$\Phi_2(P_1^I) - \Phi_2^T(P_1^I) = 0 \qquad\qquad V_2(P_1^U) - V_2^T(P_1^U) = 0 \tag{1.6.17}$$

$$\Phi_2'(P_1^I) - \left(\Phi_2^T\right)'(P_1^I) = 0 \qquad\qquad V_2'(P_1^U) - \left(V_2^T\right)'(P_1^U) = 0. \tag{1.6.18}$$

### 1.6.5 Nested Configuration Solution

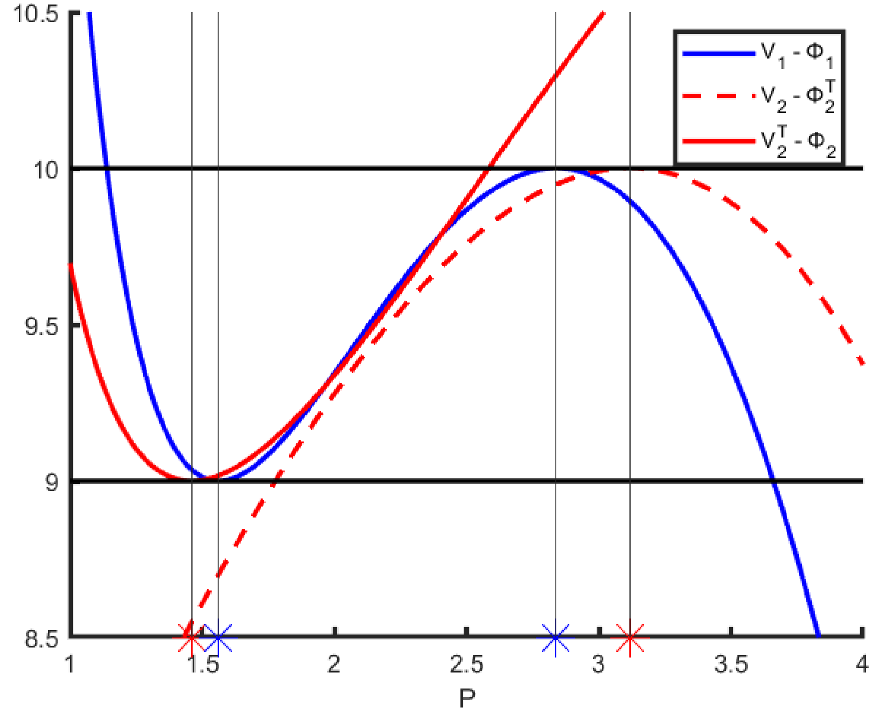
The same base parameter values are used as in 1.5.5 with the addition of $U = 9$. The nested configuration is characteristic of regime switches which affect uncertainty but have only a modest affect on the drift of the output price or on the productivity parameter. An example of such an event is the result of the 2016 U.S. presidential election or the UK's decision to leave the European Union. These events created high uncertainty over future economic policy and of potential access to global markets but had only moderate immediate supply-chain or production line effects for the majority of economic sectors (Aït-Sahalia et al., 2021).

| $P_1^I$ | $P_1^U$ | $P_2^I$ | $P_2^U$ |
|---|---|---|---|
| 2.8393 | 1.5626 | 3.1169 | 1.4609 |
| $A_1$ | $A_2$ | $A_3$ | $A_4$ |
| 0.011745 | 1.5133 | 4.9478 | 2.4439 |
| $C_1$ | $C_2$ | $C_3$ | $C_4$ |
| 0.061596 | 0.40661 | -0.03155 | 2.2679 |

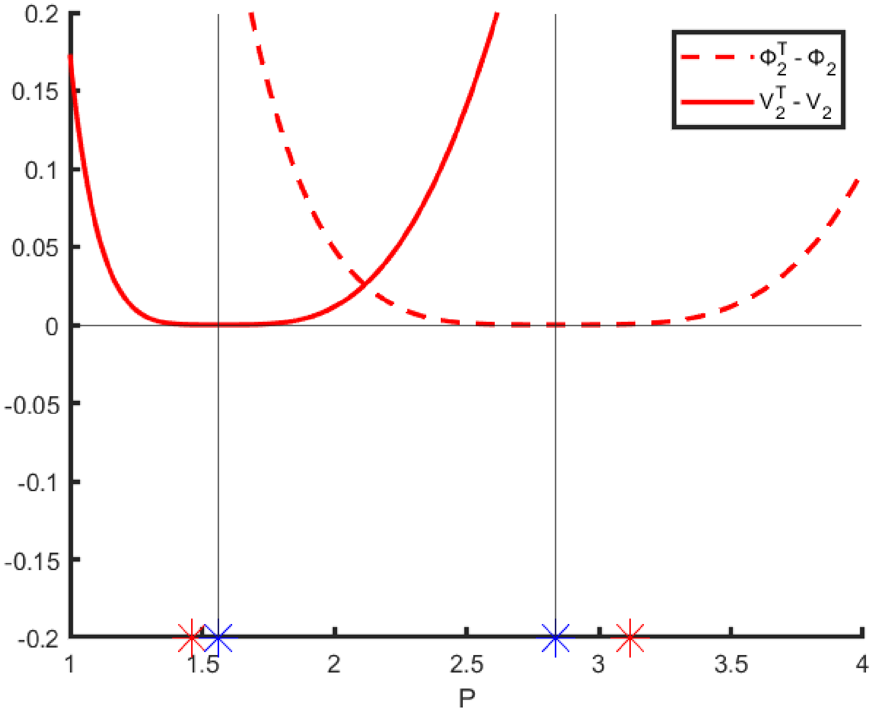
Table 1.6.1: Nested Configuration Solution

Table 1.6.1 gives the values of the thresholds and constants of integration in this configuration using the base parameters from table 1.5.1 but with $\sigma_2 = 0.3$ to make the difference between the thresholds easier to see when they are graphed in figure 1.6.2. The first panel shows the tangency points between $I$, $U$, and the function $V_1(P) - \Phi_1(P)$ while the second panel confirms that the variables in the transient regions smooth paste with $V_2$ and $\Phi_2$ at the regime one thresholds. The stars on the horizontal axis represent the threshold values of price, blue in regime one and red in regime two.

The shape of the blue function in figure 1.6.2 is explained in (Dixit & Pindyck, 1994, p.220) and




(a) Thresholds in the Nested Configuration




(b) Transient Regions in the Nested Configuration

Figure 1.6.2: Nested Configuration

gives some intuition about the signs of the constants of integration. The full expression is

$$V_1(P) - \Phi_1(P) = \theta_1 h P^{\nu} + \sum_{j=3}^{2} A_j P^{z_j} - \sum_{j=1}^{2} A_j P^{z_j}$$

where all the $A_j$ terms are positive in the numerical solution. Given $z_3 < 0$ and $z_4 < 0$ the terms containing these powers dominate for small values of $P$, which explains why the function is initially decreasing. This allows the function to have a local (in the range $[P_2^U, P_2^I]$) minimum where $V_1(P) - \Phi_1(P) = U$. $A_1$ and $A_2$ are positive but there is a minus sign in front of them in $V_1(P) - \Phi_1(P)$. Because $z_1 > z_2 > \nu$, the slope of the blue curve is diminishing after the local minimum. Eventually, the terms including $A_1$ and $A_2$ will dominate and the function must slope downwards again. The values of the constants of integration are such that $V_1(P) - \Phi_1(P)$ has its local maximum at $P_I^1$ where it is tangent to $I$.

The same logic can be applied to the transient regions. Given the equation for the dashed red line,

$$V_2(P) - \Phi_2^T(P) = -C_1 P^{\gamma_1} - C_2 P^{\gamma_2} + \tfrac{\omega_2^{\nu}}{\eta_2(\nu)} h P^{\nu} + \tfrac{\lambda_{21} I}{(\rho + \lambda_{21})}$$

with $C_1 = 0.06$ and $C_2 = 0.41$, the function is initially upward sloping. However, it will also be diminishing because the restriction that $\gamma_1 > \nu$ means the term $C_1 P^{\gamma_1}$ eventually dominates, allowing the function to have a local maximum at $P_2^I$, as required. So it must be the case that $C_1 > 0$ in this model. The value of $C_2$ depends on how far $P_1^I$ is from $P_2^I$. Remember that $\Phi_2^T(P)$ must also smooth paste with $\Phi_2(P)$ at $P_1^I$. If the thresholds are far apart, the slope of $V_2(P) - \Phi_2^T(P)$ after $P_1^I$ must be relatively shallow, while if they are close together the slope must be steep. With $\gamma_2 < 0$ and a minus sign in front of the $C_2$, higher values of $C_2$ mean a steeper slope and that the thresholds are relatively close together.

The solid red line in 1.6.2 has a very similar form;

$$V_2^T(P) - \Phi_2(P) = C_3 P^{\gamma_1} + C_4 P^{\gamma_2} + \tfrac{\omega_2^{\nu}}{\eta_2(\nu)} h P^{\nu} + \tfrac{\lambda_{21} U}{(\rho + \lambda_{21})}$$

with $C_3 = -0.03155$ and $C_4 = 2.2679$. $C_4$ must be positive to ensure the function is tangent to $U$ at $P_2^U$. $C_3$ depends on the distance between $P_2^U$ and $P_1^U$. The slope of the function gets steeper as

$C_3$ increases, meaning the thresholds must lie relatively close together.

The investment thresholds in table 1.6.1 are lower than those in section 1.5.5, even with a higher value of $\sigma_2$ in this section. This reflects the fact that the acquisition is now partially reversible. When some of the cost is recoverable, the investor is willing to acquire the firm even at lower values of $P$. Models which do not take into account the partial reversibility of investment decisions will tend to overestimate the effect of uncertainty on the number of acquisitions.

As expected based on the results of table 1.5.2, higher uncertainty in the recessionary regime increases the wedge between the two investment thresholds and the two disinvestment thresholds. It also increases the wedge between $P_i^U$ and $P_i^I$. Figure 1.6.3 shows the effect increasing $\sigma_2$ has on the thresholds. The scales on the vertical axis reveal that the changes are smaller in regime one. In the high-uncertainty regime the investor requires a higher price to justify purchasing the firm, implying fewer acquisitions. Furthermore, if she currently owns the firm, an investor requires a lower output price to justify selling it. Acquisitions which are not profitable enough to maintain in expansionary regimes are kept alive in high-uncertainty regimes. Holding onto unprofitable projects while waiting for more information is itself a potential driving force for the misallocation of resources responsible for creating low productivity in high uncertainty regimes, as described in Bloom et al. (2018).

While the model predicts less activity in the recessionary regime, switching to regime two will not cause an immediate change in the investors ownership position. If the investor owned the firm in regime one she will continue to hold it immediately after the regime switch and likewise if she held the option. There will not be a wave of acquisitions or sales to mark the transition to the recessionary regime. In fact, the ownership position is only immediately affected when switching back to the expansionary regime. If the output price is above $P_1^I$, the return to the expansionary regime is accompanied by an immediate acquisition. If it is below $P_1^U$, there will be an immediate sale.

Notice that the transient region between $P_1^I$ and $P_2^I$ is larger than the one between $P_1^U$ and $P_2^U$. This can be explained by the fact that the lower $\alpha_2$ in the recessionary regime will tend to increase the investment threshold *and* the disinvestment threshold. Higher uncertainty and lower price growth thus have competing effects on the disinvestment threshold and tend to make the transient region between $P_1^U$ and $P_2^U$ smaller than the one between $P_1^I$ and $P_2^I$. So on the one hand,

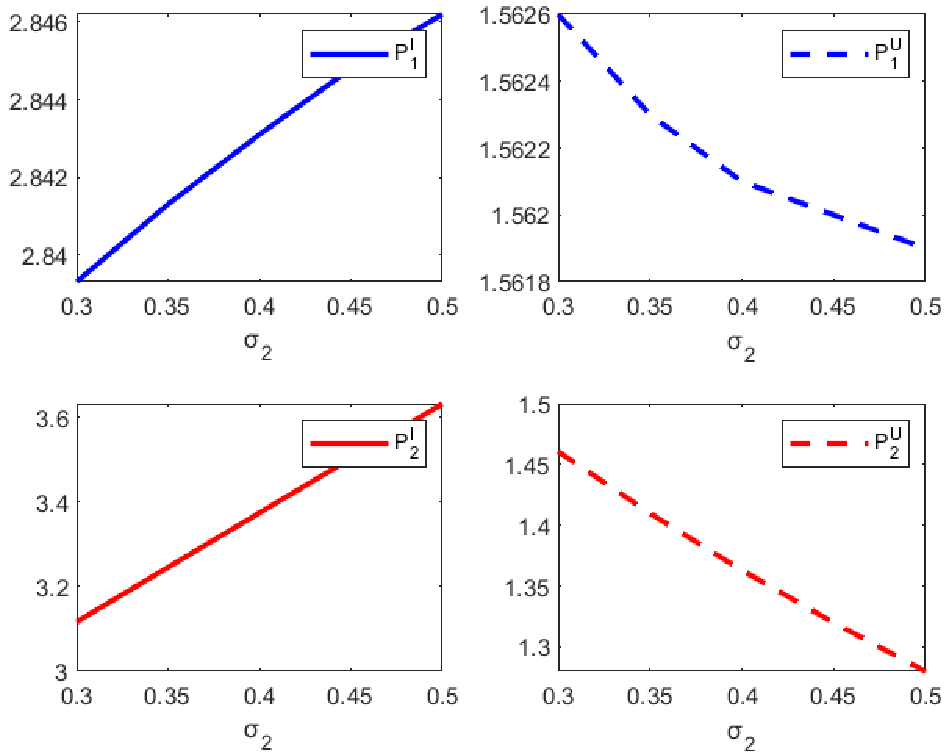



Figure 1.6.3: Nested Configuration Thresholds with Increasing Uncertainty

by decreasing the size of the transient region, lower $\alpha_2$ makes it less likely the output price will be such that there is a wave of sales after switching back to the expansionary regime. But on the other hand, if $\alpha_2$ is negative, the output price will tend to drift downwards over time, making it unlikely that $P$ will be such that $V_2^T$ is transient and the switch to an expansionary regime is followed by an increase in acquisitions. Succinctly, recessionary regimes characterised by higher uncertainty but little change in productivity or price growth will have lower acquisitions and sales compared to the expansionary regime, but will not begin with a change in ownership position and will only end with one under quite specific conditions.

### 1.6.6 Separated Configuration Boundary Conditions

The additional linking region means there are sixteen equations and sixteen unknowns in the separated configuration. These are mostly the same as in the nested configuration except $C_5$ and $C_6$ replace $C_3$ and $C_4$ because the value of the firm in regime one is now in a transient region between $P_1^U$ and $P_1^I$ while the value of the firm in regime two is never in a transient region. There are also the constants $\{D_j\}_{j=1}^4$ associated with the option values to buy and sell the firm in the linking region (see figure 1.6.1b, and table 1.4.1 to keep track of the constants). The following eight equations establish the values of all the $C_j$ terms in the model as well as the investment and disinvestment thresholds

$$V_1^T(P_1^I) - \Phi_1(P_1^I) = I \qquad V_1^T(P_1^U) - \Phi_1(P_1^U) = U \tag{1.6.19}$$

$$\left(V_1^T\right)'(P_1^I) - \Phi_1'(P_1^I) = 0 \qquad \left(V_1^T\right)'(P_1^U) - \Phi_1'(P_1^U) = 0 \tag{1.6.20}$$

$$V_2(P_2^I) - \Phi_2^T(P_2^I) = I \qquad V_2(P_2^U) - \Phi_2^T(P_2^U) = U \tag{1.6.21}$$

$$V_2'(P_2^I) - \left(\Phi_2^T\right)'(P_2^I) = 0 \qquad V_2'(P_2^U) - \left(\Phi_2^T\right)'(P_2^U) = 0. \tag{1.6.22}$$

The values of $C_j$ can be found algebraically following the method of Abel & Eberly (1996). The thresholds are then determined by two non-linear equations as functions of the ratio of the investment and disinvestment thresholds in regime one and two respectively. I have relegated these solutions to the appendix because chapter two examines a similar model which does not have a linking region and consequently does not require the additional step of identifying the constants of integration in this region. The expressions in the appendix fully characterise the solution without the presence of the linking region. In the present case, the constants of integration $\{D_j\}_{j=1}^4$ as well as $\{A_j\}_{j=1}^4$ must also be found. This is achieved by the following eight equations

$$V_1^L(P_1^I) - V_1^T(P_1^I) = 0 \qquad V_1^L(P_2^U) - V_1^T(P_2^U) = 0 \tag{1.6.23}$$

$$\left(V_1^L\right)'(P_1^I) - \left(V_1^T\right)'(P_1^I) = 0 \qquad \left(V_1^L\right)'(P_2^U) - \left(V_1^T\right)'(P_2^U) = 0 \tag{1.6.24}$$

$$\Phi_2^L(P_1^I) - \Phi_2(P_1^I) = 0 \qquad \Phi_2^L(P_2^U) - \Phi_2^T(P_2^U) = 0 \tag{1.6.25}$$

$$\left(\Phi_2^L\right)'(P_1^I) - \Phi_2'(P_1^I) = 0 \qquad \left(\Phi_2^L\right)'(P_2^U) - \left(\Phi_2^T\right)'(P_2^U) = 0. \tag{1.6.26}$$

### 1.6.7 Separated Configuration Solution

With the base parameters from 1.5.1, I set $\omega_2 = 0.3$ to produce a separated configuration. The difference in between $\omega_2$ and $\omega_1$ must be sufficiently large to push the sale threshold in regime two above the purchase threshold in regime one. Smaller differences in $\omega_i$ are required when $\sigma_i$ in the two regimes are relatively similar, and when $\alpha_2$ is relatively low, because the lower boundary in regime two will already be reasonably close to $P_1^I$. As mentioned, the first eight boundary conditions of the separated configuration are enough to pin down the thresholds. For the parameter specifications in 1.5.1 but with $\omega_2 = 0.3$, figure 1.6.4a graphs the numerical solution for the separated configuration. The values of the constants of integration are given in table 1.6.2.

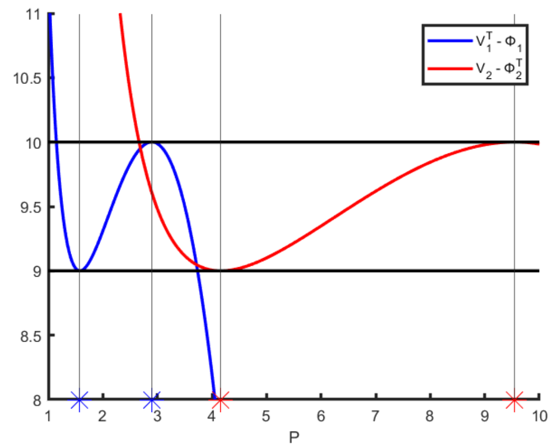
| $P_1^I$ | $P_1^U$ | $P_2^I$ | $P_2^U$ |
|---|---|---|---|
| 2.897 | 1.5798 | 9.5471 | 4.1635 |
| $A_1$ | $A_2$ | $A_3$ | $A_4$ |
| 0.0057 | 1.1505 | 13.177 | -17.484 |
| $C_1$ | $C_2$ | $C_5$ | $C_6$ |
| 0.0004 | -41.486 | -0.360 | 5.811 |
| $D_1$ | $D_2$ | $D_3$ | $D_4$ |
| $-2 \times 10^5$ | 0.0198 | 6.1856 | 0.4532 |

Table 1.6.2: Constants of Integration in the Separated Configuration

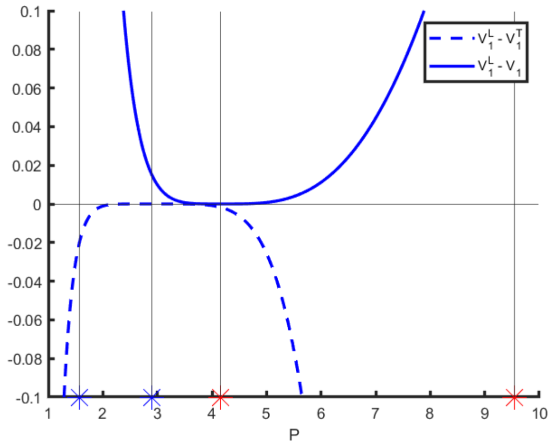
Notice that $C_1$ and $C_5$ have opposite signs, as do $C_2$ and $C_6$. This is because I always write the boundary conditions in the form $V_i - \Phi_i$, which means the terms including $C_1$ and $C_2$ have the been moved to the left hand side of the equation, thus changing their sign. In the separated configuration, the signs of these constants of integration are certain. The blue and red curves in figure 1.6.4a have the same shape as that of the entry and exit model by (Dixit & Pindyck, 1994, p.220), only the function in regime two is elongated which produces the key result $P_1^I < P_2^U$. Figure 1.6.4b shows that $V_1^L$ smooth pastes with the $V_1$ at $P_2^L$, and with $V_1^T$ at $P_1^I$. Analogously, the second panel demonstrates that $\Phi_2^L$ smooth pastes with the $\Phi_2^T$ at $P_2^L$ and with $\Phi_2$ at $P_1^I$.

Unlike higher uncertainty, lower productivity causes an increase in the threshold justifying the sale of the firm in regime two. Because the expected stream of payments from the firm is now lower for any given output price, the investor is willing to sell it for the one-time sale price $U$ at a higher output price compared to the case where $\omega_1 = \omega_2$. This effect is compounded if the persistence of the recessionary regime also increases. Figure 1.6.5 shows the change in the thresholds as $\omega_2$ decreases for $\lambda_{21} = 0.25$ and $\lambda_{21} = 0.1$. The insight is that more persistent low-productivity regimes cause investors to abandon their previous acquisitions sooner. As was the case without productivity differences, the price required to justify acquiring a firm is higher in recessionary regimes, and increases slightly further when the regime is more persistent.

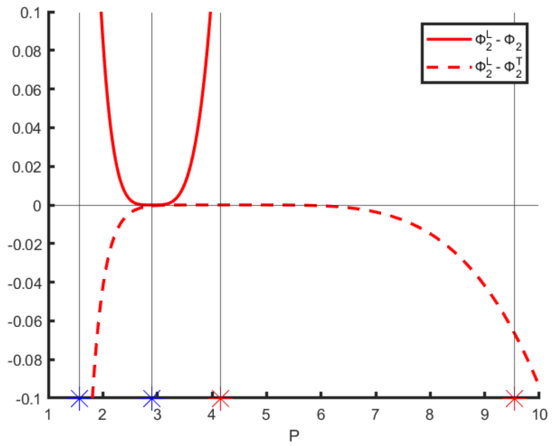
The separated configuration implies very aggressive business cycle dynamics. In the linking region, a regime switch will always cause investors to change their ownership position between the option and the firm. Switching from regime one to regime two in the linking region causes an abandonment of firms which are no longer profitable enough to keep. If regime two is very persistent and displays higher volatility, fewer investors will undertake new acquisitions because the threshold justifying investment is higher. However, contrary to the results in the nested configuration, the separated configuration also predicts a degree of creative destruction. Acquisitions which would be maintained in regime one are abandoned at a much higher price in regime two, so only the most productive acquisitions survive during the recession. If $P$ is above the threshold justifying investment in regime one when the recession ends and the economy switches back to the expansionary regime, the investor immediately acquires the firm. Again, this is another source of the waves of corporate takeovers pointed out in Mitchell & Mulherin (1996), Maksimovic & Phillips (2001), and Harford (2005).




(a) Thresholds in the Separated Configuration




(b) Transient Region and Linking Region in Regime One




(c) Transient Regions and Linking Regions in Regime Two

Figure 1.6.4: Separated Configuration

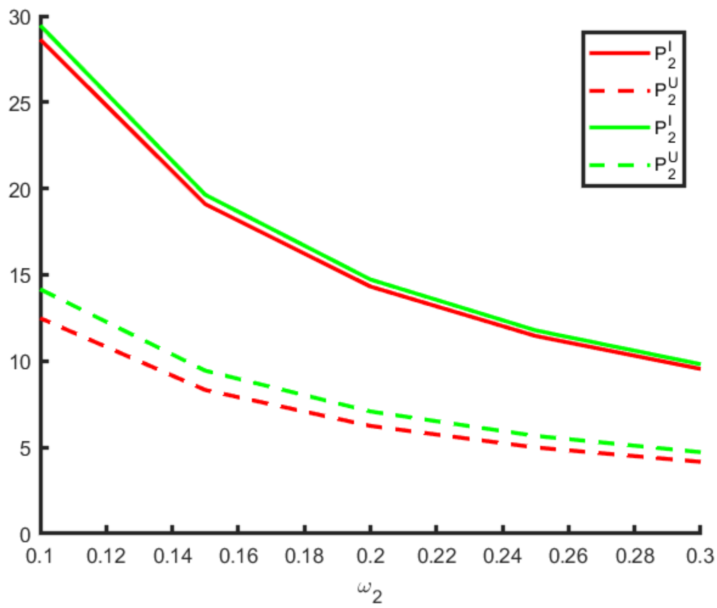



Figure 1.6.5: Effect on Productivity Shocks

This figure uses the base parameters from table 1.5.1. The red lines are for $\lambda_{21} = 0.25$ and the green lines are for $\lambda_{21} = 0.1$. Higher persistence (lower $\lambda_{21}$) compounds the effect of productivity shocks.

## 1.7 Conclusion

This chapter modelled a representative investor's decision to acquire a perfectly-competitive firm given the stochastic price of the output good and the production function followed a CTMC switching between recessionary and expansionary regimes. It solved the investor's problem both in a fully irreversible case where the cost of the acquisition could not be recovered and in a partially reversible case where at least some of the initial cost could be recovered. This framework is an improvement over previous models because it allows for both time-varying uncertainty and productivity, both of which are key characteristics of business cycles. Hence, a wider range of predictions about investor behaviour in different economic regimes was possible. Solving the problem involved finding the value of the firm and the value of the option to invest in the firm as functions of the output price and then applying a set of boundary conditions to identify the threshold values of price which triggered a change in the ownership position.

The solution in the irreversible case turned out to be very similar to the model of capital accumulation in Guo et al. (2005) and had closed-form solutions once the root of a non-linear equation in the ratio of the investment thresholds was known. The standard result from the real options literature holds; the higher uncertainty regime has a wider region of inaction where the investor does not change her current ownership position. As uncertainty increases, the region of inaction increases in both regimes. In the irreversible case, this means there will be fewer acquisitions. If the economy is in the recessionary regime but the output price is such that investment is justified in the expansionary regime, a regime switch causes the representative investor to immediately purchase the firm. If the productivity of the firm is also lower during the recessionary regime and the regime is persistent, the threshold justifying investment in both regimes is even higher. The sudden increase in acquisitions after the recession then becomes less likely given the output price would have to be that much higher in the recessionary regime.

With partial reversibility, the systems of equations formed by the boundary conditions are too complex to permit a closed-form solution but can be solved numerically. The configurations of the investment and disinvestment thresholds determine how the investor will respond to regime changes. These configurations are produced by the difference in parameter values between the regimes. A nested configuration is produced when uncertainty in the recessionary regime is higher than the expansionary regime but there are only modest differences in productivity and price growth. Changes in ownership position are less likely in the recessionary regime, so there is a stagnation of acquisition activity. An investor holding the option to invest will require high values of the output price to purchase the firm and an investor in ownership of the firm will require very low output prices to justify selling the firm. There is a range of values of the output price for which a switch to the expansionary regime causes an immediate change in ownership position, an immediate sale if the output price is between the sale thresholds of the two regimes and an immediate purchase if the output price is between the purchase threshold of the two regimes.

Immediate changes in ownership position are more likely in a separated configuration. Such configurations are the result of large declines in productivity. The investment and disinvestment thresholds are both relatively high in the recessionary regime, implying fewer acquisitions and more sales for many investors and firms. Regime switches in this configuration will likely trigger waves of acquisitions and sales. The model opens several future avenues for empirical research explaining the response of acquisitions to the business cycle. Most notably, attention should be paid to whether the dynamic changes in the number of acquisitions after regime switches in uncertainty and total factor productivity match the predictions seen in the nested and separated configurations. While this chapter simplified the problem by ignoring other assets owned by the investor, future studies could consider an investor who owns several firms and potential synergy effects between the acquired firm and others owned by the investor.

# Appendix 1.A

## 1.A.1 Proving the of Existence of Quartic Roots

**Theorem 1.** *The quartic equation* $H(z) = \eta_1(z)\eta_2(z) - \lambda_{12}\lambda_{21}$ *has two real roots greater than one and two real roots less than zero.*

Figure 1.A.1 shows an example of the quartic equation, its roots, and its turning points ($z_n$ and $z_p$). The proof follows from some discernable facts about the two quadratics $\eta_1(z)$ and $\eta_2(z)$ (defined in equation 1.4.4), the value of the quartic at zero and one, and the intermediate value theorem.

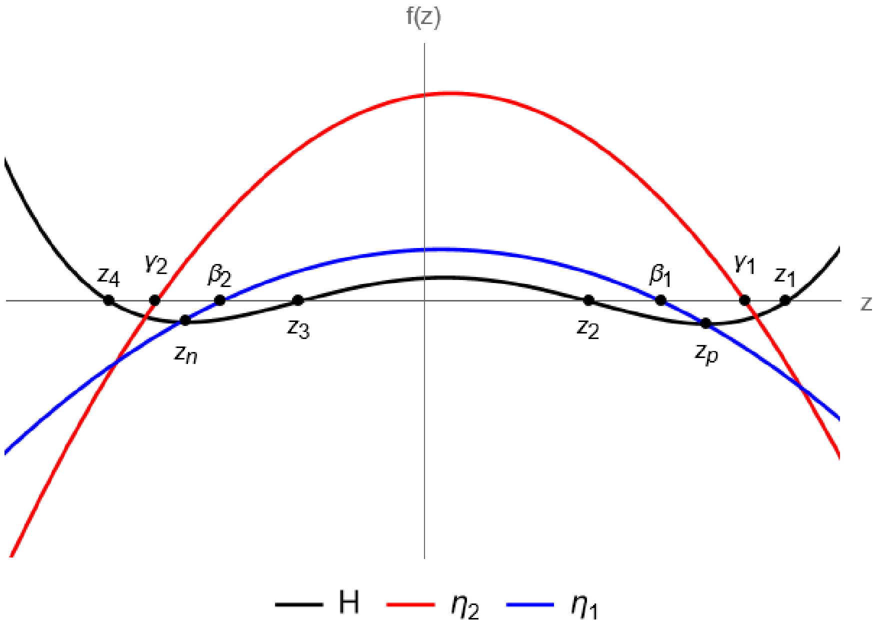



Figure 1.A.1: Example Quartic

*Proof.* First, $H(0) = \rho^2 + \rho(\lambda_{12} + \lambda_{21}) > 0$ and $H(1) = [\lambda_{12} + (\rho - \alpha_1)][\lambda_{21} + (\rho - \alpha_2)] - \lambda_{12}\lambda_{21} > 0$ given $\rho > \alpha_i$. Furthermore, the quadratics, $\eta_1(z)$ and $\eta_2(z)$, have one root greater than one if $(\rho - \alpha_i) + \lambda_{ij} > 0$ and one root less than zero if $2\sigma_i^2(\rho + \lambda_{ij}) > 0$. The first is true under the assumption that $\rho > \alpha_i$ and the second is true because all the terms are positive. Both quadratics are also positive at zero because $\eta_i(0) = (\rho + \lambda_{ij}) > 0$. Thus, both quadratics are concave with one root greater than one and one root less than zero. As in the main body of the chapter, let $\beta_1 > 1$

and $\beta_2 < 0$ be the roots of $\eta_1(z)$ and $\gamma_1 > 1$ and $\gamma_2 < 0$ be the roots of $\eta_2(z)$. For convenience and without loss of generality, let $\beta_1 < \gamma_1$ and $\beta_2 > \gamma_2$.

When $z$ is equal to the smallest positive quadratic root ($\beta_1$), the quartic must be negative. Given $H(1) > 0$ and $H(\beta_1) = -\lambda_{12}\lambda_{21} < 0$ then there must be a root of the quartic between one and $\beta_1$ by the intermediate value theorem. Eventually, at some higher value of $z$ both quadratics will be negative, so their product must be positive. Given $\lambda_{12}\lambda_{21}$ is just some finite constant greater than zero, the product $\eta_1(z)\eta_2(z)$ with $z > \gamma_1$ must eventually be larger than $\lambda_{12}\lambda_{21}$ because $\eta_1(z)\eta_2(z) \to \infty$ as $z \to \infty$. The quartic thus has another root greater than $\gamma_1$.

The same argument works for the two negative roots. When $z$ is equal to the largest negative root ($\beta_2$), the quartic must be negative. But because $H(0) > 0$, there must be a root between zero and $\beta_2$ by the intermediate value theorem. Eventually, at some value of $z < \gamma_2$ both quadratics will be negative, so their product must be positive and eventually will be greater than $\lambda_{12}\lambda_{21}$. Therefore, the quartic will again intercept the horizontal axis which gives the final root. □

### 1.A.2 Comparative Static for the Quartic Roots

Totally differentiating the quartic and evaluating at the two positive roots gives more insight into the dynamics of the model as the parameters of the regime-switching GBM change. First, an increase in $\sigma_i$ will decrease $z_1$ and $z_2$. This result is consistent with that of Dixit & Pindyck (1994, p.144) in a model without regime switching. Their model is also simple enough to algebraically show that the threshold justifying investment will be higher when the root of the characteristic equation of the homogeneous part of the solution to 1.4.2 decreases, however, the complexity of equations 1.5.13 and 1.5.14 makes this additional step impractical in the regime switching model. Instead, numerical solutions showed that increases in $\sigma_i$ raised the threshold justifying investment.

Totally differentiating equation 1.4.9 with respect to $\sigma_i$ gives

$$\frac{\partial H}{\partial z}\frac{\partial z}{\partial \sigma_i} + \frac{\partial H}{\partial \sigma_i} = 0.$$

Calculate

$$\left.\frac{\partial H}{\partial z}\right|_{z=z_1} = -\left[\left(\alpha_1 - \frac{1}{2}\sigma_1^2\right) + \sigma_1^2 z_1\right]\eta_2(z_1) - \left[\left(\alpha_2 - \frac{1}{2}\sigma_2^2\right) + \sigma_2^2 z_1\right]\eta_1(z_1) > 0$$

and

$$\left.\frac{\partial H}{\partial z}\right|_{z=z_2} = -\left[\left(\alpha_1 - \frac{1}{2}\sigma_1^2\right) + \sigma_1^2 z_2\right]\eta_2(z_2) - \left[\left(\alpha_2 - \frac{1}{2}\sigma_2^2\right) + \sigma_2^2 z_2\right]\eta_1(z_2) < 0.$$

The terms in square brackets are positive as long as $\frac{\alpha_i}{\sigma_i} + z_i > \frac{1}{2}$, which is guaranteed given $\nu > 1$ and the restriction that $z_i > \nu$. Also,

$$\left.\frac{\partial H}{\partial \sigma_i}\right|_{z=z_1} = -\left[\sigma_i z_1 \left(z_1 - 1\right)\right]\eta_j(z_1) > 0$$

and

$$\left.\frac{\partial H}{\partial \sigma_i}\right|_{z=z_2} = -\left[\sigma_i z_2 \left(z_2 - 1\right)\right]\eta_j(z_2) < 0.$$

Hence, it must be the case that

$$\left.\frac{\partial z}{\partial \sigma_i}\right|_{z=z_1} < 0 \text{ and } \left.\frac{\partial z}{\partial \sigma_i}\right|_{z=z_2} < 0$$

in order for the total differential formula to balance. The sign of this derivative is analogous to the simple model without regime switching. The same method can be applied to the other parameters in the model. A full summary of the sign of the derivative of the positive roots with respect to each of the parameters can be found in table 1.A.1 using the parameters from table 1.5.1. Note that the derivative of the quartic with respect to $\lambda_{ij}$ is

$$\rho - \alpha_j z - \frac{1}{2}\sigma_j z(z-1).$$

When evaluated at the largest quartic roots in absolute value, this quadratic is definitely negative. However, it is ambiguous as to whether it is positive or negative for the quartic roots with the smallest absolute value. Hence, the effect of the change in $\lambda_{ij}$ on the two smaller roots is ambiguous.

| Parameter | $\Delta z_1$ | $\Delta z_2$ | $\Delta z_3$ | $\Delta z_4$ |
|---|---|---|---|---|
| $\sigma_i$ | $-$ | $-$ | $+$ | $+$ |
| $\alpha_i$ | $-$ | $-$ | $-$ | $-$ |
| $\rho$ | $+$ | $+$ | $-$ | $-$ |
| $\lambda_{12}$ | $+$ | $-$ | $+$ | $-$ |
| $\lambda_{21}$ | $+$ | $+$ | $-$ | $-$ |

Table 1.A.1: Response of Roots to Marginal Increase in Parameters

Increases in $\sigma_i$ will lower the positive roots, consistent with Dixit & Pindyck (1994). However, the sign of the derivative with respect to $\rho$ is not consistent with their model, which argues that the relationship between $\rho$ and the positive roots should be negative. Figure 1.A.1 can again help resolve this apparent inconsistency. Dixit & Pindyck (1994) assume a fixed relationship between $\rho$ and $\alpha_i$ given by $\xi_i = \rho - \alpha_i > 0$. Any change in $\rho$ will now have an effect on $\alpha_i$ if $\xi_i$ is to remain fixed. In this case, given the positive roots of the two quartics are both greater than one, $\eta_i(z)$ will shift downwards when $\rho$ increases and given the derivative of the quadratics are negative at $\beta_1$ and $\gamma_1$ this must mean that the positive root decreases in $\rho$. This chapter assumes no such fixed relationship between $\rho$ and $\alpha_i$ which means changes in $\rho$ shift $\eta_i(z)$ upwards and the positive root increases in $\rho$. A change in $\rho$ holding $\alpha_i$ constant means $\xi_i$ must increase. The latter can be thought of as the dividend payments which accrue from holding the firm, in other words, it reflects the opportunity cost of holding the option to invest in the firm rather than the firm itself (Dixit & Pindyck, 1994, p.149). When this increases, the investor will choose to exercise the option earlier. But table 1.5.2 shows that the threshold justifying investment is increasing in $\rho$, which indicates that the decrease in the discounted value of the firm outweighs the increase in the opportunity cost of *not* exercising the option to invest.

### 1.A.3 Algebraic Solutions for Separated Configuration

This solution will be seen again in chapter two, therefore, I merely present it here and defer discussion about it until then. First, define the function

$$\psi(x;\kappa_1,\kappa_2) = \frac{x^{\kappa_1} - x^{\nu}}{x^{\kappa_1} - x^{\kappa_2}}. \tag{1.A.1}$$

The values of the constants of integration which satisfy the boundary conditions 6.19 - 6.22 are

$$C_5 = -\frac{\nu}{\beta_1}\frac{\omega_1^\nu}{\eta_1(\nu)}\left[1-\psi(G_1;\beta_1,\beta_2)\right] h\left(P_1^L\right)^{\nu-\beta_1} < 0 \tag{1.A.2}$$

$$C_6 = -\frac{\nu}{\beta_2}\frac{\omega_1^\nu}{\eta_1(\nu)}\left[\psi(G_1;\beta_1,\beta_2)\right] h\left(P_1^L\right)^{\nu-\beta_2} > 0 \tag{1.A.3}$$

$$C_1 = \frac{\nu}{\gamma_1}\frac{\omega_2^\nu}{\eta_2(\nu)}\left[1-\psi(G_2;\gamma_1,\gamma_2)\right] h\left(P_2^L\right)^{\nu-\gamma_1} < 0 \tag{1.A.4}$$

$$C_2 = \frac{\nu}{\gamma_2}\frac{\omega_2^\nu}{\eta_2(\nu)}\left[\psi(G_2;\gamma_1,\gamma_2)\right] h\left(P_2^L\right)^{\nu-\gamma_2} > 0 \tag{1.A.5}$$

To simplify notation, let

$$\Omega_1(G_1) = \frac{\omega_1^\nu}{\eta_1(\nu)}\left[1-\frac{\nu\left[1-\psi(G_1;\beta_1,\beta_2)\right]}{\beta_1}-\frac{\nu\left[\psi(G_1;\beta_1,\beta_2)\right]}{\beta_2}\right]$$

and

$$\Omega_2(G_2) = \frac{\omega_2^\nu}{\eta_2(\nu)}\left[1-\frac{\nu\left[1-\psi(G_2;\gamma_1,\gamma_2)\right]}{\gamma_1}-\frac{\nu\left[\psi(G_2;\gamma_1,\gamma_2)\right]}{\gamma_2}\right].$$

These equations are what is left over after substituting the constants of integration into the value matching conditions and factoring out the common term. Collecting all terms containing $I$ or $U$ to the right-hand side of the equation and dividing through by $\Omega_i(G_i)$ gives four equations defining $hP^\nu$ at the boundaries

$$h\left(P_1^I\right)^\nu = \frac{(\rho+\lambda_{12})\,I-\lambda_{12}U}{\Omega_1(G_1^{-1})\,(\rho+\lambda_{12})} \tag{1.A.6}$$

$$h\left(P_1^U\right)^\nu = \frac{\rho U}{\Omega_1(G_1)\,(\rho+\lambda_{12})} \tag{1.A.7}$$

$$h\left(P_2^I\right)^\nu = \frac{\rho I}{\Omega_2(G_2^{-1})\,(\rho+\lambda_{21})} \tag{1.A.8}$$

$$h\left(P_2^U\right)^\nu = \frac{(\rho+\lambda_{21})\,U-\lambda_{21}I}{\Omega_2(G_2)\,(\rho+\lambda_{21})}. \tag{1.A.9}$$

Now divide 1.A.6 by 1.A.7 and 1.A.8 by 1.A.9 to obtain two non-linear equations in $G_1$ and $G_2$. Using the result in Abel & Eberly (1996) there is a unique $G_1 > 1$ and $G_2 > 1$ which satisfies the following two non-linear equations 1.A.10 and 1.A.11 respectively as long as $I > U$ and $(\rho + \lambda_{21})\,U - \lambda_{21} I > 0$, so the wedge between the purchase and sale price is not too large

$$\Omega_1(G_1^{-1})G_1^{\nu} - \left(\frac{(\rho + \lambda_{12})\,I - \lambda_{12} U}{\rho U}\right)\Omega_1(G_1) = 0 \tag{1.A.10}$$

$$\Omega_2(G_2^{-1})G_2^{\nu} - \left(\frac{\rho I}{(\rho + \lambda_{21})\,U - \lambda_{21} I}\right)\Omega_2(G_2) = 0. \tag{1.A.11}$$

### 1.A.4 Convergence to a Model without Regime Switching

Table 1.A.2 shows that the model converges to a modified form of the one presented in Abel & Eberly (1996) when the transition probabilities go towards zero. The last four rows of the table show the values of the constants of integration obtained by running two models which have the same parameter values as the two regimes but no regime switching. In Abel & Eberly's model, there are two constants of integration and two threshold values determining investment and disinvestment. In this section, let $NS_i^+$ (NS for 'no switching') be the constant of integration associated with the positive root in regime $i$ and $NS_i^-$ do the same for the constant of integration associated with the negative root. I set $\lambda_{12} = \lambda_{21} = 0.0001$ in the simulations and use the baseline parameters in 1.5.1. It is easy to see that the thresholds converge.

Recall that $\frac{\eta_1(z_j)}{\lambda_{12}} A_j = B_j$. Furthermore, it can be verified from the quartic equation; $z_1 \to \beta_1$, $z_2 \to \gamma_1$, $z_3 \to \gamma_2$, and $z_4 \to \beta_2$ as the transition probabilities go to zero. Table 1.A.2 reveals that as $\lambda_{12}, \lambda_{21} \to 0$, $A_1 + A_2 \to NS_1^+$, $A_3 + A_4 \to NS_1^-$, $B_1 + B_2 \to NS_2^+$, and $B_3 + B_4 \to NS_2^-$. This result has an analogue in the fully irreversible case, as the transition probabilities go to zero the sum of $A_1$ and $A_2$ in section 1.5.3 converge towards the single constant obtained in an irreversible investment model without regime switching as found in Dixit & Pindyck (1994, pp.136-147).

Transient regions were defined over values of $P$ where switching brings about an immediate change in ownership position. The fact this occurred over a finite range of values of $P$ meant no limiting arguments were applied to eliminate constants of integration. Without switching, this

| Unknown | Nested Configuration $\lambda_{12} = \lambda_{21} \approx 0$ | Nested Configuration No Switching | Separated Configuration $\lambda_{12} = \lambda_{21} \approx 0$ | Separated Configuration No Switching |
|---|---|---|---|---|
| $P_1^I$ | 2.8332 | 2.8332 | 2.8333 | 2.8332 |
| $P_1^U$ | 1.5623 | 1.5623 | 1.5623 | 1.5623 |
| $P_2^I$ | 3.0037 | 3.0037 | 10.0123 | 10.0125 |
| $P_2^U$ | 1.5201 | 1.5201 | 5.0668 | 5.0671 |
| $A_1$ | 1.3916 | - | 1.4062 | - |
| $A_2$ | 0.0203 | - | 0.0018 | - |
| $A_3$ | 0.0119 | - | 0.0707 | - |
| $A_4$ | 7.2142 | - | 7.1219 | - |
| $B_1$ | -0.0159 | - | -0.016 | - |
| $B_2$ | 1.3444 | - | 0.1184 | - |
| $B_3$ | 6.6866 | - | 39.7423 | - |
| $B_4$ | -0.0069 | - | -0.0069 | - |
| $C_1$ | 1.3242 | - | 0.0981 | - |
| $C_2$ | -6.6726 | - | -39.7284 | - |
| $C_3$ | -1.3241 | - | - | - |
| $C_4$ | 6.6744 | - | - | - |
| $C_5$ | - | - | -1.4076 | - |
| $C_6$ | - | - | 7.2211 | - |
| $\frac{\eta_1(z_1)}{\lambda_{12}} D_1$ | - | - | $1 \times 10^{-5}$ | - |
| $\frac{\eta_1(z_2)}{\lambda_{12}} D_2$ | - | - | 0.0981 | - |
| $D_3$ | - | - | $2 \times 10^{-5}$ | - |
| $D_4$ | - | - | 7.2211 | - |
| $NS_1^+$ | - | 1.4124 | - | 1.4124 |
| $NS_1^-$ | - | 7.2255 | - | 7.2255 |
| $NS_2^+$ | - | 1.3280 | - | 0.0985 |
| $NS_2^-$ | - | 6.6801 | - | 39.7287 |

Table 1.A.2: Convergence to a Model without Regime Switching as $\lambda_{12} = \lambda_{21} \to 0$.

argument is no longer meaningful and one of the constants in each of the transient regions will have to be removed to get back to a model without regime switching. Focus on the nested configuration in table 1.A.2. It shows that $C_1$ converges to $NS_2^+$ and $C_4$ converges to $NS_2^-$. Because $C_2$ appears in the equation defining the value of the option (see table 1.4.1), which should go to zero as $P \to 0$, and is associated with the negative root $\gamma_2$, it should be removed without regime switching. Likewise, $C_3$ should be removed when considering the equation defining the value of the firm. Then, all that is left are the constants of integration in regime two. This logic applies to the separated configuration as well.

For the linking region, two of the constants tend towards zero as the transition probabilities tend towards zero. Section 1.6.3 also pointed out that the expressions defining the value of the firm and the option value in this region reduced to their equivalent without regime switching when this occurs. What remains is just the option value given by the parameters used in regime two, $\frac{\eta_1(z_2)}{\lambda_{12}} D_2 P^{z_2}$, and the value of the firm given the parameters used in regime one, $D_4 P^{z_4} + \frac{\omega_1^{\nu}}{\eta_1(\nu)} h P^{\nu}$, if there was no regime switching.

# Chapter 2

# Partially Irreversible Investment in a Regime Switching Economy

## 2.1 Introduction

A firm deciding when to adjust its capital stock does so in an economic environment characterised by switches between periods of higher and lower uncertainty. The optimal timing of investment can be very different in these two regimes. High uncertainty regimes also tend to coincide with periods of declining productivity, which also affects the incentive to undertake partially irreversible investment decisions. These observations have become particularly salient given recent empirical findings that recessions, periods of low economic activity usually accompanied by heightened uncertainty, tend to have a persistent negative effect on economic activity long after the initial shock has passed (Ball, 2014; Blanchard et al., 2015; Cerra et al., 2023). Investment decisions today not only form a component of GDP but also lay the foundations of production and innovation in future periods, which suggests its behaviour in different regimes could play a key role in depressing economic activity. However, no current model of firm-level investment examines the impact of switching to a regime capturing the empirical characteristics of recessions, with both heightened uncertainty and depressed productivity. This motivates a formal investigation of a profit-maximising firm's decisions under these conditions to properly inform policy makers and managers who plan their optimal responses after regime changes.

This chapter models a perfectly-competitive representative firm's decision to adjust its capital stock given that its output price follows a geometric Brownian motion (GBM) whose drift and volatility parameters follow a continuous time Markov chain (CTMC) switching between two regimes and its production function contains a productivity parameter dependent on the same CTMC. Within this more comprehensive model of business cycle dynamics compared to previous literature, I show how investment decisions will respond to changes in uncertainty and productivity.

Just like in chapter one, one regime is characterised by higher uncertainty and lower productivity, which puts the firm's decision in a framework which reflects Bloom et al.'s 2018 observation that recessions produce both increases in firm-level uncertainty and decreases in productivity. Also following from chapter one, the firm produces with a Cobb-Douglas production function with decreasing returns to scale. Unlike previous models, both upward and downward adjustments in the capital stock are permitted, which is an analogue to the investor being able to buy and sell the firm in chapter one. Bidirectional adjustment of the capital stock is more representative of a firm's actual decision problem compared to the case where investment decisions are totally irreversible and downward adjustments are not permitted.

I derive an expression for the marginal value of capital from the Bellman equation representing the dynamics of the firm's value. This expression is just the numerator of what the literature calls *Tobin's marginal q*. It includes both the fundamental value of the next unit of capital based on the present expected discounted value of future profit flows and the real option value associated with adjusting the capital stock upwards and downwards. Using a set of value matching and smooth pasting conditions, which demand the marginal value of capital should meet tangentially with its marginal cost whenever it is optimal to adjust the capital stock up or down, I pin down the threshold values of the marginal value of capital which determine changes in the capital stock in both regimes.

The model generates the familiar result that increasing uncertainty widens the firm's *region of inaction*, meaning it waits longer before making capital stock adjustments when the economy enters a higher volatility regime. Capital stock adjustments come in discrete bursts whenever the marginal value of capital reaches the required thresholds. The quantity of capital installed is just sufficient to return to the inaction region. Such dynamics are consistent with plant-level evidence which suggests adjustments in the capital stock are 'lumpy', meaning a period of inactivity is followed by a large adjustment and subsequent return to inactivity.

The introduction of regime switching productivity exaggerates the lumpiness by directly affecting the marginal productivity of capital. Economies experiencing episodes of high uncertainty coupled with large declines in productivity will immediately see decreases in firms' capital stocks followed by periods of depressed investment where little activity takes place. Conversely, when the recession ends and the economy switches back to the expansionary regime, there is a sudden large positive adjustment in firms' capital stocks. Investment thus becomes very volatile over the business cycle when the difference between productivity levels between the regimes is large.

The persistence of the recessionary regime is determined entirely by a constant transition rate. When this is low, the recessionary regime persists for a long time and activity will remain depressed relative to the expansionary regime. Weak investment recovery is a potential drag on economic growth, so explaining the mechanisms through which this occurs is vital for policy makers (Yellen, 2016). On the other hand, high transition rates imply low persistence and quick recoveries, so investment is only depressed for a short time.

The rest of this chapter is organised as follows. Section 2.2 gives an overview of the previous literature on firm-level investment theory and its response to changes in the economic environment. Section 2.3 sets up the firm's problem and defines its optimal investment rule while 1.4 solves its dynamic programming problem to find an expression for Tobin's marginal q. All remaining unknowns are determined in section 2.5. A brief conclusion summarises the results.

## 2.2 Related Literature

Neoclassical economic theory states that a firm should accumulate capital such that it maintains the expected present discounted value of the marginal revenue product of capital ($q$) equal to its user cost (Jorgenson, 1963; Tobin, 1969). If the firm's production function, $F$, satisfies $\partial F/\partial K \to 0$ as $K \to \infty$ and $\partial F/\partial K \to \infty$ as $K \to 0$, this means purchasing capital when $q$ is above the user cost and selling capital when it is below the marginal cost. The equivalence of $q$ and user cost remains central to the theory of capital accumulation, however, the introduction of partial irreversibility, uncertainty, and non-linear adjustment costs to the neoclassical model improved its explanatory power for firm-level investment patterns (Dixit & Pindyck, 1994; Fiori, 2012; Gilchrist et al., 2014). This chapter integrates these concepts together and puts them in a regime switching context to examine the response of firm-level investment to changes in the economic environment consistent with the findings in Bloom et al. (2018).

Under the canonical theory, uncertainty impacts investment through an 'option value' effect which is only relevant when there is a benefit of waiting for more information. If all decisions are fully reversible, uncertainty does not create an option value of delaying decisions (Dixit & Pindyck, 1994, p.6). Uncertainty can still impact investment decisions through its effect on the fundamental part of $q$ (the part of $q$ not based on the option value of waiting). The sign of this relationship depends on the curvature of the marginal revenue product function in terms of the variable generating the uncertainty. If it is convex, uncertainty will increase $q$, if it is concave, uncertainty will decrease $q$ (Abel, 1983).

Uncertainty in investment theory is generally modelled as the volatility of a stochastic process such as a GBM which affects the firm's profit function. If the firm is in a competitive market, a natural candidate for the stochastic variable is the price of the output good. This is adopted by Abel (1983) and Abel & Eberly (1997). Alternatively, the stochastic variable could represent a random component of the demand function faced by the firm, with an additional deterministic part being a function of the firm's output. This is suitable if the firm has a degree of market power and is used by Bertola (1988) and Abel & Eberly (1996). As in chapter one, this chapter assumes a perfectly competitive firm and thus follows the approach of Abel (1983). This is the first study to solve a perfectly competitive firm's decision problem in a partially reversible setting.

For a firm seeking to choose investment such that it maintains its optimal capital stock, the easiest way to introduce irreversibility is to assume that once the firm has made the decision to invest in the marginal unit it cannot be uninstalled at a later date. Good examples of this assumption are found in (Dixit & Pindyck, 1994, pp.357-367) and Guo et al. (2005). Abel & Eberly (1996) introduce partial reversibility by allowing the firm to sell a marginal unit of installed capital for a fraction of the price it paid for the good. This makes sense if capital goods have a degree of specialisation which makes them difficult to repurpose, if there are additional costs associated with bringing used capital goods to the market, or if there are asymmetric information effects in second-hand markets for capital goods (Abel & Eberly, 1994). While more realistic, partial reversibility makes finding closed-form solutions much more difficult, Abel & Eberly (1996) themselves can only give a local approximation of a solution to their model when the ratio of the purchase and resale price of capital is close to unity.

Firm-level investment is described as 'lumpy', occasional periods of inactivity are broken up by large adjustments in the capital stock which show far lower persistence than suggested by earlier work which assumed the cost of capital was convex, reflecting a time-to-build effect penalising large adjustments in the capital stock in small windows of time (Doms & Dunne, 1998; Cooper & Haltiwanger, 2006). Lumpy investment patterns can be reproduced in models which incorporate the (partial) irreversibility of investment decisions[1]. Abel & Eberly (1996) is a classic example. A 'wedge' between the purchase and resale prices of capital generates a region of the model's state space where $q$ is neither high enough to justify investment in the capital stock, nor low enough to justify disinvestment. This is known as the 'region of inaction'. Investment only becomes different from zero if the stochastic process underlying $q$ reaches some critical value which brings a tangency point between the $q$ and the price of capital. Their model falls into the category of (S, s) control policies introduced by Arrow et al. (1951) where a controller makes a discrete adjustment to the quantity of goods in an inventory once the process describing the flows into that inventory reaches some critical value. The lower case 's' represents the critical value or boundary and the upper case 'S' represents the size of the adjustment.

The aim of these control policies is always to return to a state of inaction and solving the model requires finding the critical values which trigger action. Dixit (1991) highlights two types of adjustments in (S, s) models which are important in this chapter; barrier control and impulse control. Barrier control applies at the boundaries of the model and involves small instantaneous discrete adjustments in the inventory to prevent it from crossing the boundary. Impulse control applies whenever the quantity of goods in the inventory is beyond its boundary and involves the controller making a discrete adjustment sufficient to return to a state of inaction. Usually, the

[1]Another possibility is to introduce a fixed cost of adjusting the capital stock. There is empirical evidence to suggest these costs exist (Bloom, 2009) but, as explained by (Dixit & Pindyck, 1994, p.386) including them in the continuous time model considered in this chapter will mean incurring a cost at every instant of time the adjustment takes place, which over an interval $\Delta t$ would be infinitely costly.

inventory will only be above the boundary because of the starting values, after the firm returns to the inaction region, barrier control takes over and the controller always acts to keep the inventory below its critical value (Harrison & Taksar, 1983).

Given some level of irreversibility in the model and the volatility of a stochastic process, Bernanke (1983), Abel (1983), and Dixit & Pindyck (1994) applied the concept of the 'option value' from financial literature to the case of investment decisions. The classical investment rule stating firms should invest when $q$ is greater than the marginal cost of investment is wrong under this framework because it does not take into account the uncertainty of the future value of $q$. In general, firms should wait longer before making the decision to invest in highly uncertain environments. Studies by Bloom et al. (2007), Bloom (2009), Bachmann et al. (2013), Gilchrist et al. (2014), and Bloom et al. (2018) have shown empirically that uncertainty shocks do indeed have persistent effects on real economic activity.

Another important observation in Bloom (2009) and Bloom et al. (2018) is that uncertainty shocks tend to occur alongside dips in aggregate and firm-level productivity. Bloom (2009) suggests that the dip in investment and hiring reduces reallocation of resources from low productivity to high productivity firms, which is a primary source of productivity in the economy. Since Hamilton (1989), regime switching has been a popular method of modelling the dynamics of economic variables over the business cycle. A good example in the context of firm-level investment is Guo et al. (2005). They consider the case of totally irreversible investment in a two regime model and demonstrate that increasing uncertainty in one of the regimes drives a wedge between the regime-specific thresholds justifying investment. Guo et al. introduced her regime switching process in (1999) where the drift and volatility parameters of a GBM depend on a CTMC with the rate of leaving a given regime following an exponential distribution.

Bloom et al. (2018) find that investment drops by as much as 15% after an uncertainty shock and GDP overall drops sharply by around 2.5% before bouncing back quickly and continuing a sluggish growth path afterwards. Importantly, investment also remains depressed for 12 quarters after the shock while the weaker irreversibilities in the labour market means labour demand returns to its pre-shock growth rate relatively quickly. Their characterisation of a recession is an event which causes a negative first-moment shock and a positive second-moment shock to the productivity of the firm. Based on the evidence in Blanchard et al. (2015), shocks originating from the energy or finance sectors exhibit more persistence than a typical economic shock so the transition rate from the recessionary regime will be lower. Jurado et al. (2015) find the largest spikes in economic uncertainty during these protracted recessions. There are no productivity parameters embedded in the regime switching model of Guo et al. (2005), so a typical recession is marked solely through changes in demand by a transition to a state which has a lower drift and a higher volatility compared to the regime representing 'normal' economic circumstances. This chapter augments their model to consider changes in productivity between regimes.

## 2.3 Description of the Firm

### 2.3.1 Regime Switching Price Process

The regime switching stochastic process is the same as in chapter one. $P_t$, follows a GBM, where $\alpha_i$ is the drift parameter, $\sigma_i$ is the volatility parameter and $W_t$ is a standard Brownian motion. The parameters $\alpha_i$ and $\sigma_i$ depend on the regime of $\varepsilon_t \in \{1, 2\}$, which follows a CTMC independent of $W_t$ . The probability of switching between regimes in the interval $\Delta t$ is described by the following transition matrix

$$\begin{bmatrix} (1-\lambda_{12}\Delta t) & \lambda_{12}\Delta t \\ \lambda_{21}\Delta t & (1-\lambda_{21}\Delta t) \end{bmatrix}. \tag{2.3.1}$$

Equation 2.3.2 describes the resulting GBM for the output price

$$P_t = \alpha_i P_t dt + \sigma_i P_t dW_t. \tag{2.3.2}$$

As in chapter one, the economy switches to a recessionary regime when $\varepsilon_t = 2$, so let $\sigma_1 < \sigma_2$. This reflects Bloom's (2009) observation that recessions (and even less severe disruptions in economic conditions) are associated with periods of high volatility on the stock market, a proxy measure for economic uncertainty[2]. The model does not account for equity markets, hence, the volatility of

[2]Finding appropriate measures of uncertainty is a difficult task which ideally needs to determine how difficult it

output price proxies for uncertainty faced by the firm. If $\alpha_1 > \alpha_2$ and $\lambda_{12} < \lambda_{21}$, then a transition to regime two represents a typical recession; there is a general fall in the expected growth of prices and increase in uncertainty which shows some persistence but much less than the regime representing normal economic conditions.

### 2.3.2 Inputs and Production

The firm's production function

$$F(\varepsilon_t, K_t, L_t) = \omega_i L_t^a K_t^b \tag{2.3.3}$$

depends on two inputs, capital ($K_t$) and labour ($L_t$) with $a + b < 1$ ensuring decreasing returns to scale. Decreasing returns allows competitive firms to make positive profits and turns out to be a necessary assumption in (S, s) models of capital accumulation. If returns to scale were not decreasing, a threshold value of $P_t$ justifying the installation of a marginal unit of capital would also justify the installation of all succeeding marginal units of capital (Dixit & Pindyck, 1994, p.365).

The wage paid to labour is $w$ and labour is fully adjustable, so the firm can find its optimal level by solving an instantaneous maximisation problem. This assumption has empirical justification from Bloom (2009), who finds that ignoring labour adjustment costs does not prevent his general equilibrium model from achieving a good fit for real-world data. On the other hand, capital adjustment costs are important for achieving a good fit, so there is weaker evidence that labour adjustment costs play a significant role over the business cycle. It also simplifies the model because operating profits ($\Pi$), the stream of payments received from selling output on the market minus labour costs, can be expressed solely as a function of $K_t$ and $P_t$.

Capital cannot be adjusted immediately, it is fixed in any interval of time $\Delta t$. For this reason, capital costs will enter into the definition of operating profit in section 2.3.4. What can change over this interval is gross investment; the flow of new capital goods into the firm. Let $I_t > 0$ represent the total cumulation of all purchases of capital until time $t$ (installing capital) and let $U_t > 0$ represent the total cumulation of all sales of capital until time $t$ (uninstalling capital). Installed capital

is for agents to forecast the innovations in all relevant economic variables for their decision problems. Some widely cited attempts come from Bachmann et al. (2013), Jurado et al. (2015), and Baker et al. (2016). See also the more recent global measure by Ahir et al. (2022). Chapters three and four will deal with this in more detail depreciates at a constant rate determined by $\delta$. Therefore, in the infinitesimally small interval $dt$, net additions to the capital stock are given by

$$dK_t = dI_t - dU_t - \delta K_t dt. \tag{2.3.4}$$

The capital accumulation constraint is defined in differential terms, using $dI_t$ and $dU_t$, because the firm's optimal investment policy is such that $K_t$ is not technically differentiable with respect to time as investment occurs in discrete bursts (Bagliano & Bertola, 2004, pp.86-89). The notation in equation 2.3.4 is similar to that of Abel & Eberly (1996).

$\omega_i$ represents a regime-dependent constant exogenous technology multiplier. It depends on the same CTMC $\varepsilon_t$. Let $\omega_2 \leq \omega_1$ reflect the fact that periods of high uncertainty are often associated with dips in aggregate and firm-level productivity. As in chapter one, I assume this productivity parameter is specific to the firm.

### 2.3.3 Investment Costs

Capital costs $\phi_I$ to buy and can be sold for $\phi_U$, with $\phi_I > \phi_U$, so the firm cannot make infinite profits by purchasing capital and then selling it for a higher price at a later date. This captures the partial irreversibility of investment decisions. Equation 2.3.5 shows the cost of purchasing and selling capital goods

$$\phi_k = \begin{cases} \phi_I & \text{if} \quad I_t > 0 \\ \\ \phi_U & \text{if} \quad U_t > 0. \end{cases} \tag{2.3.5}$$

For notational convenience, let $R \equiv \frac{\phi_I}{\phi_U} > 1$ be the ratio of the purchase and sale prices of capital. As simple economic reasoning would suggest, a higher $R$ will mean the range of values of the stochastic variable for which the firm does not adjust its capital stock in either direction increases.

### 2.3.4 Operating Profit and Cash Flows

Operating profit is total revenue minus total production costs as shown in equation 2.3.6. It does not take into account investment costs, which come under the definition of cash flows ($\Psi$); the stream of payments accrued from selling output on the market after accounting for labour costs *and* the cost of adjusting the capital stock. Using these definitions and the fact that labour is fully adjustable at every instant of time means that profits can be written independently of the level of labour. Maximising operating profits with respect to labour and substituting the derived optimal $L_t$ into 2.3.6 gives operating profit as a function of output price and the capital stock where $h = (1-a)\left(\frac{a}{w}\right)^{\frac{a}{1-a}}$. These expressions are identical to those in Abel (1983) except for the exogenous productivity parameter. Maximising labour before choosing the capital stock makes operating profits a convex function of price given the marginal product of labour is diminishing. Furthermore, the operating profit function is concave in the capital stock because of the assumption that $a + b < 1$.

$$\Pi(P_t, \varepsilon_t, K_t, L_t) = \omega_i P_t L_t^a K_t^b - W_t L_t \tag{2.3.6}$$

$$\Pi(P_t, \varepsilon_t, K_t) = h\omega_i^{\frac{1}{1-a}} P_t^{\frac{1}{1-a}} K_t^{\frac{b}{1-a}} \tag{2.3.7}$$

$$\Psi\left(P_t, \varepsilon_t, K_t, I_t, U_t\right) = \Pi(P_t, K_t) - \phi_I dI_t + \phi_U dU_t \tag{2.3.8}$$

The first derivative of 2.3.7 with respect to $K_t$ plays an important role in the model because it determines the fundamental value of the marginal unit of capital, that is, the value of the marginal unit of capital without considering the speculative value added by expected future price movements. It is the marginal revenue product of capital given that labour has already been maximised. This is easy to see by noting that 2.3.9 can also be derived by first calculating the marginal revenue product of capital $P_t F_{K_t} = b\omega_i P_t L_t^a K_t^{b-1}$ and then substituting in the optimal level of labour

$L_t = \left(\frac{a}{w}\right)^{\frac{1}{1-a}} P_t^{\frac{1}{1-a}} K_t^{\frac{1}{1-a}}$

$$\Pi_{K_t} = \frac{bh}{1-a}\omega_i^{\frac{1}{1-a}} P_t^{\frac{1}{1-a}} K_t^{\frac{a+b-1}{1-a}} = \tilde{h}\omega_i^{\nu} P^{\nu} K^{(a+b-1)\nu}. \tag{2.3.9}$$

Defining $\tilde{h} = b\left(\frac{a}{w}\right)^{\frac{a}{1-a}}$ and $\nu = \frac{1}{1-a}$ makes the notation in the rest of the chapter less cumbersome.

### 2.3.5 The Firm's Objective

The firm's objective is to choose a flow of capital goods which maximises the present discounted sum of all future cash flows, where $\rho$ is the constant discount factor used by the firm. Since the expected present discounted sum of all future cash flows is just the value of the firm, this is equivalent to saying the firm aims to maximise its value, $V$. It does this through its investment policy. The variables $I_t$ and $U_t$ are not differentiable due to the optimal investment policy in (S, s) models allowing for discrete jumps in the capital stock. In technical terms, they are not *absolutely continuous* and so cannot be integrated by standard Riemann integration[3] (Harrison & Taksar, 1983; Abel & Eberly, 1996). Therefore, $dI_t$ and $dU_t$ must be interpreted as Riemann-Stieltjes integrals, which do not require the function to be absolutely continuous, if their expected value is to be defined in equation 2.3.10

$$V(P_t, \varepsilon_t, K_t) \equiv \max_{I_t, U_t, K_t} \mathbb{E} \int_0^{\infty} \left[\Pi(P_t, \varepsilon_t, K_t)dt - \phi_I dI_t + \phi_U dU_t\right] e^{-\rho t} dt \tag{2.3.10}$$
$$s.t.\ \ dK_t - dI_t + dU_t + \delta K_t = 0.$$

Intuitively, the firm should invest in a marginal unit of capital when the marginal product of that unit is greater than its cost of installation. Likewise, the firm should sell a marginal unit of capital when the marginal product of that unit is less than the sale price of capital. Because the sale and purchase prices are not equal in this model, there is a region of inaction, where the firm neither invests nor divests.

[3]The Cantor function or 'devil's staircase' is a classic example of a continuous function that is not absolutely continuous.

### 2.3.6 Investment Rule

Optimal investment is defined at the point where the marginal contribution of a unit of capital to the value of the firm is equal to the marginal cost of investment in that unit. Let $V_{K_t}(P_t, \varepsilon_t, K_t)$ denote the change in the value of the firm given a marginal change in capital. Then the optimal investment rule for the firm is defined by equation 2.3.11, where $V_{K_t}(P_t, \varepsilon_t, K_t)$ is equated with the marginal cost of purchasing or selling capital goods

$$V_{K_t}(P_t, \varepsilon_t, K_t) \equiv q_t(P_t, \varepsilon_t, K_t) = \phi_k. \tag{2.3.11}$$

The value gained by a marginal change in capital must be greater than the marginal cost of capital goods, $\phi_I$, for investment to take place, and less than the resale price, $\phi_U$, for disinvestment to take place. For $\phi_U \leq V_{K_t}(P_t, \varepsilon_t, K_t) \leq \phi_I$, investment is zero. $V_{K_t}(P_t, \varepsilon_t, K_t)$ is the numerator of what is traditionally called Tobin's marginal q, the ratio of the marginal value of installed capital to its cost of installation (Hayashi, 1982). It is simpler in this model to interpret $q$ in absolute terms, not scaled by the price of capital, so let $V_{K_t}(P_t, \varepsilon_t, K_t) \equiv q(P_t, \varepsilon_t, K_t)$ and call it 'absolute q' (Abel & Eberly, 1996). The firm's investment policy is to maintain $q(P_t, \varepsilon_t, K_t) = \phi_k$.

Now imagine $q(P_t, \varepsilon_t, K_t) > \phi_I$. This will occur due to increases in $P_t$ and the depreciation of $K_t$, both of which drive the marginal revenue product of capital upwards. This means the marginal value of capital is greater than its cost so the firm should invest. Because the production function exhibits decreasing returns to scale, increasing the capital stock leads to a decrease in the marginal value of capital. Therefore, when $q(P_t, \varepsilon_t, K_t) > \phi_I$ the firm instantaneously invests an amount of capital such that it immediately returns to its optimality condition $q(P_t, \varepsilon_t, K_t) = \phi_I$. This policy is known as 'barrier control' (Dixit & Pindyck, 1994, p.362) and is represented in figure 2.4.1. A similar argument holds in the case of disinvestment. As mentioned, for value of $q(P_t, \varepsilon_t, K_t)$ between $\phi_I$ and $\phi_U$, the firm should maintain a policy of zero investment. For this reason, the region between $\phi_I$ and $\phi_U$ is called the 'inaction region'. The analysis focuses on the firm's problem in this region since its investment policy means it immediately returns to this region whenever it adjusts its capital stock. The fact that $q(P_t, \varepsilon_t, K_t) = \phi_k$ everywhere outside of the inaction region means $V(P_t, \varepsilon_t, K_t)$ is known everywhere outside the inaction region.

## 2.4 Capital Accumulation

### 2.4.1 Bellman Equation

Dynamic programming finds $V(P_t, K_t, \varepsilon_t)$ as a function of price and capital by splitting the firm's problem up into two parts; the immediate flow of operating profits in the interval $dt$, and the expected present value gained from adjusting the capital stock. The Bellman equation describing the value of the firm is the sum of these two parts and after manipulation yields a differential equation for the value of the firm. The firm's decision problem takes place over an infinite horizon and $V(P_t, K_t, \varepsilon_t)$ does not explicitly depend on time, so time is not relevant for the value of the firm beyond the values of the variables at $t = 0$ (Dixit & Pindyck, 1994, p.107). To avoid the notation in this section becoming too cluttered, I will stop writing subscripts denoting time and arguments of functions from this point on.

As in chapter one, the value of the firm is regime dependent and the firm must account for the fact that $\varepsilon$ can suddenly switch to a new regime. The subscript $i \in \{1, 2\}$ denotes the value of the firm in regime of one and two respectively. Because switches in regime also affect the production function, $\Pi_i$ denotes the profit flow in regime $i$. In the interval $\Delta t$, the profit flow is $(1 - \lambda_{ij}\Delta t)(\Pi_i \Delta t) + \lambda_{ij}\Delta t (\Pi_j \Delta t)$. All terms of order $(\Delta t)^2$ go to zero much faster those of $\Delta t$ and so should be ignored in further derivations. Thus, only the term $\Pi_i \Delta t$ survives. The Bellman is defined over the infinitesimal interval $dt$ in 2.4.1

$$V_i = (\Pi_i dt - \phi_I dI + \phi_U dU) + e^{-\rho dt}\left[\mathbb{E}\{(1 - \lambda_{ij}dt)(V_i + dV_i) + \lambda_{ij}dt(V_j + dV_j)\}\right]. \qquad (2.4.1)$$

Applying Ito's lemma, dividing through by $dt$, and remembering that $V_{i,K} \equiv q_i$ gives the following differential equation describing the value of the firm in both regimes

$$\frac{1}{2}\sigma_i^2 P^2 (V_i)_{PP} + \alpha_1 P (V_i)_P - (\rho + \lambda_{ij})V_i - \delta K q_i + \Pi_i + \lambda_{ij} V_j = 0. \qquad (2.4.2)$$

The two terms $(q_i - \phi_I)\, dI$ and $(q_i - \phi_U)\, dU$ are missing from equation 2.4.2 because $dI > 0$ only if $q = \phi_I$ and $dU > 0$ only if $q = \phi_U$. Hence, these two expressions are always zero and 2.4.2 holds for both inactive and active investment policies (Abel & Eberly, 1996).

Because $q_i \equiv (V_i)_K$, the first derivative of equation 2.4.2 gives a differential equation governing the dynamics of $q_i$

$$\frac{1}{2}\sigma_1^2 P^2 (q_i)_{PP} + \alpha_1 P (q_i)_P - (\rho + \delta + \lambda_{ij}) q_i - \delta K (q_i)_K + (\Pi_i)_K + \lambda_{ij} q_j = 0. \tag{2.4.3}$$

Although equation 2.4.3 is a PDE, the fact that every term is homogeneous in the state variables $P$ and $K$ means that finding a solution is still possible using the conventional techniques from chapter one. However, a simple transformation can turn this partial differential equation into an ordinary differential equation (ODE), which will simplify the steps and notation in the succeeding sections.

Let $y = P/K^{1-a-b}$. This is a stochastic variable with the same volatility and rate of switching as the process in 2.3.2 but an adjusted drift parameter to account for the depreciation in capital, given by $\mu_i = \alpha_i + (1-a-b)\,\delta$. Now $q_i$ is a function of this stochastic variable so rewrite equation 2.4.3 as an ODE in $y$

$$\frac{1}{2}\sigma_i^2 y^2 q_i'' + \mu_i y q_i' - (\rho + \delta + \lambda_{ij}) q_i + \tilde{h}\omega_i^{\nu} y^{\nu} + \lambda_{ij} q_j = 0. \tag{2.4.4}$$

The succeeding sections will work in terms of this transformed variable.

### 2.4.2 Transient Regions

Equation 2.4.4 assumes that a change in regime will not cause the firm to immediately change its investment policy from inactive to active. In this case, a switch from regime $i$ to regime $j$ means the firm acquires the option to make a marginal adjustment in the capital stock in regime $j$, which has a value $q_j$. However, the parameters in the model can be such that although it is not worth adjusting the capital stock in regime $i$, it is doing so in regime $j$. Hence, a switch to regime $j$ leads to an immediate adjustment of the capital stock sufficient to bring the value of $q_j$ into the inaction region for regime $j$. This is known as 'impulse control' and is demonstrated alongside barrier control in 2.4.1 which plots the threshold values of output price justifying investment ($P_i^H$) and disinvestment ($P_i^L$) in regime $i$ and regime $j$ as functions of $K$. The thresholds are first found in terms of $y = P/K^{1-a-b}$, which means the thresholds expressed in terms of $P$ must be concave functions of $K$.

For demonstration, imagine a fixed level of $K$. Impulse control occurs in the regions $\left[P_j^H, P_i^H\right]$ and $\left[P_i^L, P_j^L\right]$ following a switch from regime $i$ to regime $j$. In this diagram, these regions can only be reached when $\varepsilon = i$ because the barrier control policy prevents $q_j$ from going beyond the $P_j^H$ and $P_j^L$ boundaries when $\varepsilon = j$. Other configurations of the thresholds are possible and will be explained in greater detail below.

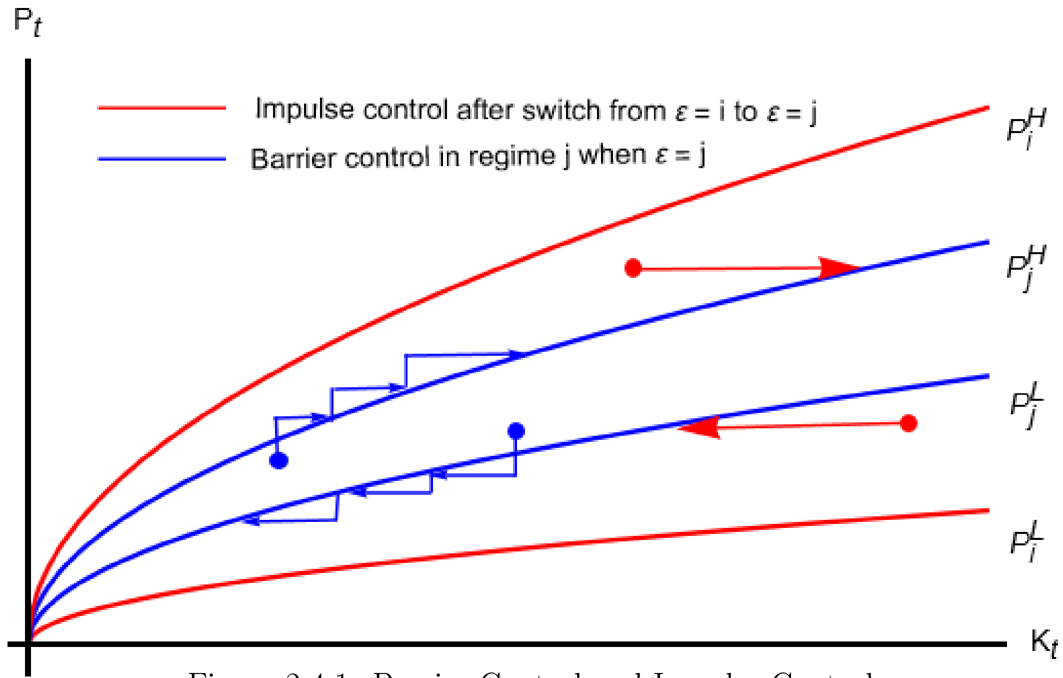



Figure 2.4.1: Barrier Control and Impulse Control
In the figure, $q_i$ is in a transient region for a fixed $K$ whenever $q_i \in \left[P_j^H, P_i^H\right]$ and $q_i \in \left[P_i^L, P_j^L\right]$ because a switch to regime $j$ means immediately paying $\phi_I$ or $\phi_U$ in order to move to the inaction region for regime $j$ between $\left[P_j^L, P_j^H\right]$.

Given the firm's investment rule in equation 2.3.11, the value of $q_j$ is known outside the inaction region; it is just equal to the marginal cost of capital. For concreteness, suppose $\varepsilon = i$ and $\phi_U < q_i < \phi_I$, so the firm is inactive. Now suppose that if $\varepsilon$ switches to regime $j$ then $q_j > \phi_I$ so the firm will immediately invest to restore $q_j = \phi_I$. In this case, equation 2.4.4 needs to be modified to equate $q_j$ with $\phi_I$ in the event of a regime switch. The same argument applies for a downward adjustment of the capital stock; although disinvestment may not currently be worthwhile in regime $i$, the GBM can switch at a point where it is optimal to sell capital in regime $j$. Equation 2.4.5 describes the dynamics of $q_i$ when in a 'transient' state, denoted $q_i^T$

$$\frac{1}{2}\sigma_i^2 y^2 \left(q_i^T\right)'' + \mu_i y \left(q_i^T\right)' - (\rho + \delta + \lambda_{ij}) \left(q_i^T\right) + \tilde{h}\omega_i^\nu y^\nu + \lambda_{ij}\phi_k = 0. \tag{2.4.5}$$

### 2.4.3 Fundamental Value of $q$

Suppose that $q_1 = \theta_1 \tilde{h}\omega_1^\nu y^\nu$ and $q_2 = \theta_2 \tilde{h}\omega_2^\nu y^\nu$. Additionally, define the function

$$\eta_i(x) = (\rho + \delta + \lambda_{ij}) - \mu_i x - \tfrac{1}{2}\sigma_i^2 x (x-1). \tag{2.4.6}$$

Taking the first and second derivatives of these trial solutions and substituting them into the differential equations shows that they solve the differential equation if the constants are given by[4]

$$\theta_i = \frac{\eta_i(\nu) + \lambda_{ji}\left(\frac{\omega_j}{\omega_i}\right)^\nu}{\eta_i(\nu)\eta_j(\nu) - \lambda_{ij}\lambda_{ji}}.$$

Reassuringly, if $\lambda_{12} = \lambda_{21} = 0$ these constants take on values consistent with a model without regime switching. The non-homogeneous solution is

$$q_i = \theta_i \tilde{h}\omega_i^\nu y^\nu. \tag{2.4.7}$$

It is clear from 2.3.9 and the definition of $y$ that $\tilde{h}\omega_i^\nu y^\nu$ is the marginal product of capital. Equation 2.4.7 is the expected present discounted value of the marginal unit of capital taking into consideration the dynamics of $P$, $K$, and $\varepsilon$ if the firm maintains an inactive investment policy in all future periods. As discussed in Guo et al. (2005), this result can also be obtained by application of the Feynman-Kac formula,

$$\theta_i \left(\Pi_i\right)_K = \mathbb{E}_{P,\varepsilon}\left[\int_0^\infty \left(\left(\Pi_i\right)_K e^{-(\rho+\delta)t} \mid \varepsilon = i\right) dt\right]. \tag{2.4.8}$$

[4]There has been a subtle change in the definition of $\theta_i$ from chapter one in that an $\omega_i^\nu$ term has been factored out. This was done to keep the notation in this chapter closer to Abel & Eberly (1996). The equations in chapter one were more concise without making this factorisation but the solutions in this chapter are mostly numerical, so there is no loss of clarity by factoring out the $\omega_i^\nu$ here.

$\eta_1(\nu) > 0$ and $\eta_2(\nu) > 0$ must hold for the integral in 2.4.8 to converge. Notice also that if this were not the case $q_i$ could be negative. This is guaranteed in the models of (Abel & Eberly, 1996) and (Guo et al., 2005) because the operating profit function is concave in the stochastic variable. Convergence is assumed in this model and ensured by the chosen parameters in all the simulations below. Given the assumption from Dixit & Pindyck (1994) that $\rho > \alpha_i$, this is achieved by preventing $\sigma_i$ being too large and keeping $\nu$ less than the roots of $\eta_i(z)$ and $H(z)$ below.

The non-homogeneous solution to 2.4.5 is

$$q_i^T = \begin{cases} \theta_i^T \tilde{h}\omega_i^{\nu} y^{\nu} + \dfrac{\lambda_{ij}\phi_I}{\rho+\delta+\lambda_{ij}} & \text{if} \quad q_j = \phi_I \\ \\ \theta_i^T \tilde{h}\omega_i^{\nu} y^{\nu} + \dfrac{\lambda_{ij}\phi_U}{\rho+\delta+\lambda_{ij}} & \text{if} \quad q_j = \phi_U \end{cases} \tag{2.4.9}$$

where $\theta_i^T = \frac{1}{\eta_i(\nu)}$. The first term in these expressions is the value generated from the marginal product of capital and the second in the probability weighted value gained from a regime switch.

### 2.4.4 Option Value of $q$

For the homogeneous solution to the equations for $q_1$ and $q_2$, let $q_1 = \sum_{j=1}^{4} A_j y^{z_j}$ and $q_2 = \sum_{j=1}^{4} B_j y^{z_j}$. Again, substitution into the differential equations in the system defined by 2.4.4 gives

$$-\sum_{j=1}^{4} \eta_1(z_j) A_j y^{z_j} = -\lambda_{12} \sum_{j=1}^{4} B_j y^{z_j} \tag{2.4.10}$$

$$-\sum_{j=1}^{4} \eta_2(z_j) B_j y^{z_j} = -\lambda_{21} \sum_{j=1}^{4} A_j y^{z_j}. \tag{2.4.11}$$

The method of undetermined coefficients yields $\frac{\eta_1(z_j)}{\lambda_{12}} A_j = B_j$ and $\frac{\eta_2(z_j)}{\lambda_{21}} B_j = A_j$. Therefore, the following quartic equation must hold for the variable $z$, which will have two roots greater than one and two negative less than zero

$$H(z) = \eta_1(z)\eta_2(z) - \lambda_{12}\lambda_{21} = 0. \tag{2.4.12}$$

Chapter one proved the existence and locations of these roots. Then, the homogeneous solution is given by

$$q_1 = \sum_{j=1}^{4} A_j y^{z_j} \tag{2.4.13}$$

$$q_2 = \sum_{j=1}^{4} \frac{\eta_1(z_j)}{\lambda_{12}} A_j y^{z_j}. \tag{2.4.14}$$

If the non-homogeneous solution is the value of $q_i$ given the firm remains in the inaction region, the homogeneous solution accounts for the option value of adjusting the capital stock. Terms with negative powers in 2.4.13 and 2.4.14 are the option values associated with disinvestment and the terms with positive powers are associated with investment. Again, the solutions take a different form in transient regions. Let $\gamma_1 > 1$ and $\gamma_2 < 0$ be the roots of $\eta_2(z)$ and $\beta_1 > 1$ and $\beta_2 < 0$ be the roots of $\eta_1(z)$. Then the homogeneous solutions in the transient regions are given below

$$q_1^T = \begin{cases} D_1 y^{\beta_1} + D_2 y^{\beta_2} & \text{if} \quad q_2 = \phi_I \\ \\ D_3 y^{\beta_1} + D_4 y^{\beta_2} & \text{if} \quad q_2 = \phi_U \end{cases} \tag{2.4.15}$$

$$q_2^T = \begin{cases} C_1 y^{\gamma_1} + C_2 y^{\gamma_2} & \text{if} \quad q_1 = \phi_I \\ \\ C_3 y^{\gamma_1} + C_4 y^{\gamma_2} & \text{if} \quad q_1 = \phi_U. \end{cases} \tag{2.4.16}$$

I have used $C_i$ for the constants of integration when $q_2^T$ is in a transient region to maintain consistency with chapter one. Table 2.4.1 should help keep track of the names of the constants of integration used in this chapter.

### 2.4.5 Boundary Conditions

The marginal purchase and resale prices of capital form the boundaries of the inaction region, therefore, $q_i = \phi_I$ at the upper boundary and $q_i = \phi_U$ at the lower boundary. Call the values of the stochastic variable $y$ at these boundaries $y_i^H$ and $y_i^L$ respectively (H for high and L for low). These will be referred to as the threshold values of $y$. Therefore,

$$q_i\left(y_i^H\right) = \phi_I \quad \text{and} \quad q_i\left(y_i^L\right) = \phi_U \tag{2.4.17}$$

at the boundaries. These are *value matching conditions.* The *smooth pasting conditions* ensure that $q_i$ and $\phi_k$ meet at a tangency point by equating the derivatives of the expressions on both sides of 2.4.17

$$q_i'\left(y_i^H\right) = 0 \quad \text{and} \quad q_i'\left(y_i^L\right) = 0. \tag{2.4.18}$$

If $q_j$ is in a transient region of $[K, P]$ space both above the $y_i^H$ threshold and below the $y_i^L$ threshold, then the value matching and smooth pasting conditions also apply to $q_j^T$

$$q_j^T\left(y_j^H\right) = \phi_I \quad \text{and} \quad q_j^T\left(y_j^L\right) = \phi_U \tag{2.4.19}$$

$$\left(q_j^T\right)'\left(y_j^H\right) = 0 \quad \text{and} \quad \left(q_j^T\right)'\left(y_j^L\right) = 0. \tag{2.4.20}$$

Finally $q_j$ and $q_j^T$ should smooth paste together at the $y_i^H$ and $y_i^L$ boundaries (see Guo et al. (2005) and Dixit (1993, p.31)) as in the conditions below

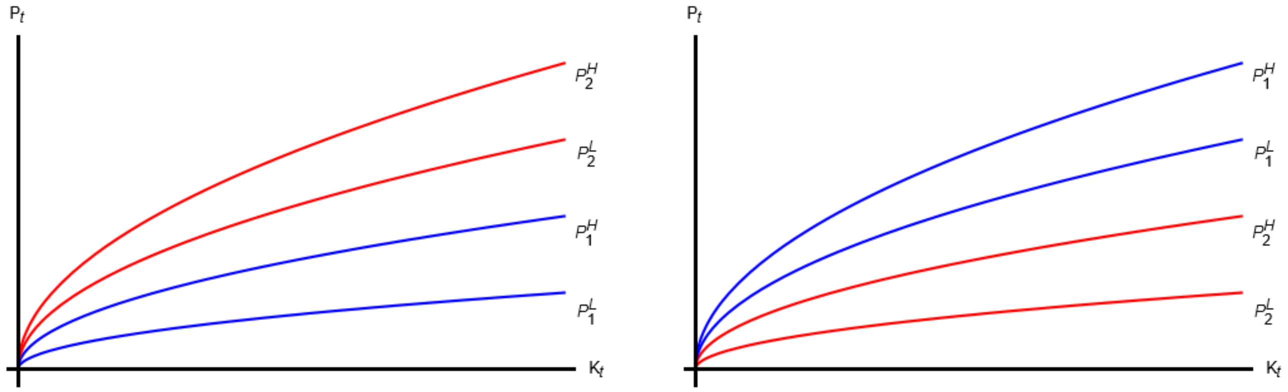
$$\lim_{y\uparrow y_i^H} q_j = \lim_{y\downarrow y_i^H} q_j^T \quad \text{and} \quad \lim_{y\downarrow y_i^L} q_j = \lim_{y\uparrow y_i^L} q_j^T \tag{2.4.21}$$

$$\lim_{y\uparrow y_i^H} q_j' = \lim_{y\downarrow y_i^H} \left(q_j^T\right)' \quad \text{and} \quad \lim_{y\downarrow y_i^L} q_j' = \lim_{y\uparrow y_i^L} \left(q_j^T\right)'. \tag{2.4.22}$$

Conditions 2.4.21 and 2.4.22 change depending on the region of the state space where $q_i$ and $q_j$ are transient. There are six possible configurations of the boundaries, as shown in figure 2.4.2 below. A 'nested configuration' occurs when the boundaries of one regime lie entirely within the boundaries of another regime. An 'overlapping configuration' occurs when the investment or disinvestment boundary of regime $i$ lies within the interval $\left[P_j^L, P_j^H\right]$ for a given $K$. Finally, a 'separated configuration' occurs when the interval $\left[P_i^L, P_i^H\right]$ lies entirely above $\left[P_j^L, P_j^H\right]$. The separated configuration requires $P_i^L > P_j^H$. This is only possible if the regime switching affects the marginal revenue product of capital or the purchase and resale prices of capital. Section 2.5.1 explains this in more detail. Conditions 2.4.21 and 2.4.22 do not apply when in the separated configuration because $q_i$ is always in a transient region.

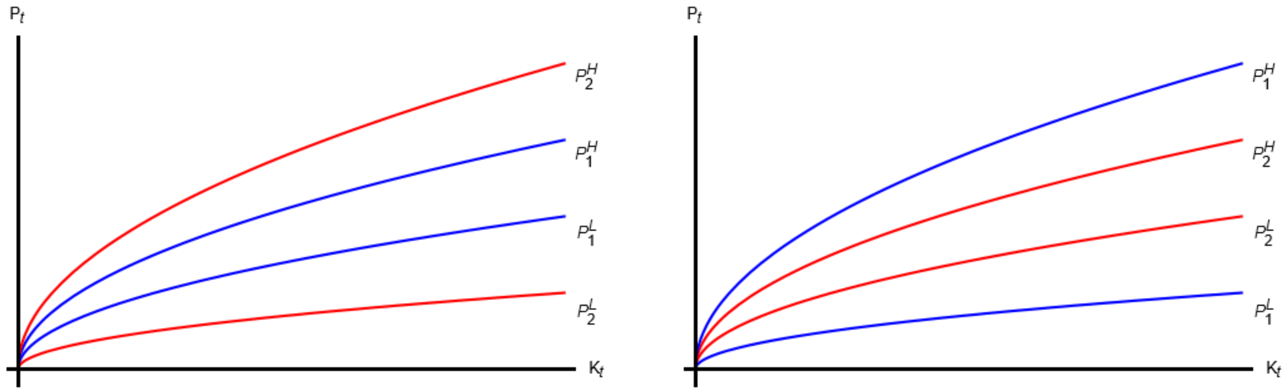
In chapter one, there was a 'linking region' which bridged the gap between $P_i^L$ and $P_j^H$ in the separated configuration. Because of the barrier control policy, $q_i = \phi_k$ everywhere outside the inaction region and the firm *immediately* installs an amount of capital sufficient to return it to the inaction region whenever $q_i$ hits a boundary (Dixit & Pindyck, 1994, p.362). Therefore, there is no linking region in this model because $q_i$ will never spend time between $P_i^L$ and $P_j^H$. Despite this, there will still be several constants of integration to keep track of. Table 2.4.1 should make this easier. The constants $C_1$ and $C_2$ will be required in every configuration. This is a consequence of $P_2^H$ always being the uppermost boundary in the left-hand side of figure 2.4.2. Analogously, $D_1$ and $D_2$ will never be required because $P_1^L$ is always the lowermost boundary. The $q_j$ column gives the value of $q_j$ in the other regime.

It is clear from figure 2.4.2 that there are actually three pairs of configurations which mirror each other insofar as swapping the subscripts of the left-hand panels gives the configurations in the right-hand panels. Given the assumption that regime two represents a recessionary regime, the left-hand panels are the ones this chapter is interested in. After finding the solutions to the model for the left-hand panels, the solutions to the right-hand panels can easily be derived by swapping all regime one parameters with those from regime two and vice-versa. Therefore, the solutions in the next section focus on the separated configuration in panel 2.4.2a, the nested configuration in panel 2.4.2c, and the overlapping configuration in panel 2.4.2e.



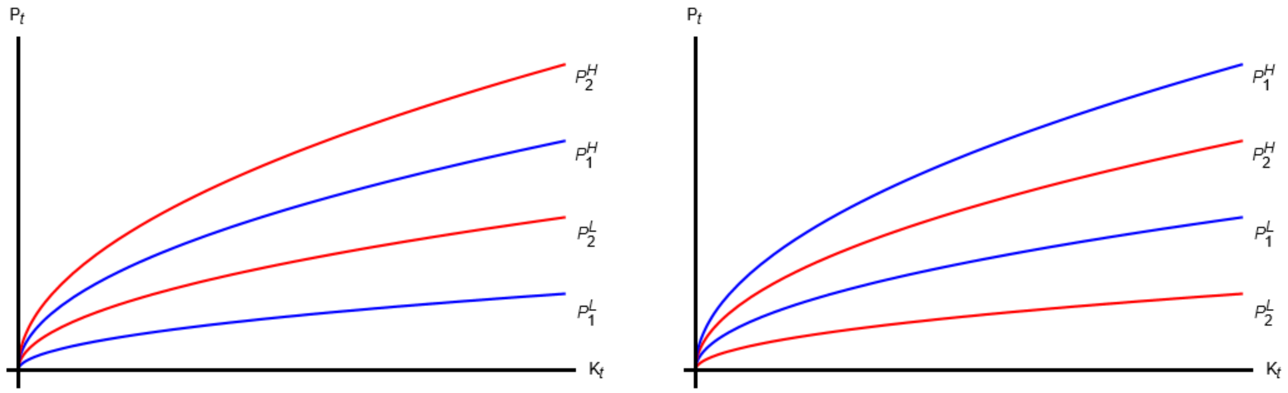

(a) (b)

Separated Configuration




(c) (d)

Nested Configuration




(e) (f)

Overlapping Configuration

Figure 2.4.2: Configurations of the Boundaries

| Constants | Regime | Associated Root Sign | Region | $q_j$ | Configuration Separated | Nested | Overlapping |
|---|---|---|---|---|---|---|---|
| $A_1, A_2$ | 1 | $+, +$ | Base | $q_j$ | ✗ | ✓ | ✓ |
| $A_3, A_4$ | 1 | $-, -$ | Base | $q_j$ | ✗ | ✓ | ✓ |
| $C_1, C_2$ | 2 | $+, -$ | Transient | $\phi_I$ | ✓ | ✓ | ✓ |
| $C_3, C_4$ | 2 | $+, -$ | Transient | $\phi_U$ | ✗ | ✓ | ✗ |
| $D_1, D_2$ | 1 | $+, -$ | Transient | $\phi_I$ | ✗ | ✗ | ✗ |
| $D_3, D_4$ | 1 | $+, -$ | Transient | $\phi_U$ | ✓ | ✗ | ✓ |

Table 2.4.1: Naming Conventions for the Constants of Integration Given I Focus on the Left-Hand Side of Figure 2.4.2

### 2.4.6 Value of the Firm

Before solving the model, there is one more convergence condition to be aware of. It must be possible to get back to $V_i$ from $q_i$ by taking the anti-derivative of $q_i$ with respect to $K$. If the constants of integration in the homogeneous solution for $q_i$ represent the value of a marginal adjustment of the capital stock at the boundaries, then in the expression for the value of the firm they represent the expected option value of all future increases and decreases in the capital stock. Solutions for the constants of integration will be given in the next section. As show by Dixit & Pindyck (1994, p.365), the terms associated with increases in the capital stock in equation 2.4.14 should be integrated between he initial capital stock and infinity to reflect the option value of all future increases in the capital stock. Similarly, the terms associated with decreases in the capital stock should be integrated between the initial capital stock and zero to reflect the option value of all future decreases in the capital stock. This leads to the following expressions, remembering that

$y = P/K^{1-a-b}$, $z_1 > 1$, $z_3 < 0$, $\beta_1 > 1$, and $\beta_2 < 0$

$$\int_K^\infty A_1 P^{z_1} K^{z_1(a+b-1)} dK = \tfrac{1}{1-z_1(1-a-b)} A_1 P^{z_1} K^{1-z_1(1-a-b)} \text{ iff } z_1 > \tfrac{1}{1-a-b}$$

$$\int_0^K A_3 P^{z_3} K^{z_3(a+b-1)} dK = \tfrac{1}{1-z_3(1-a-b)} A_3 P^{z_3} K^{1-z_3(1-a-b)} \text{ iff } z_3 < \tfrac{1}{1-a-b}$$

$$\int_K^\infty C_1 P^{\beta_1} K^{\beta_1(a+b-1)} dK = \tfrac{1}{1-\beta_1(1-a-b)} A_1 P^{z_1} K^{1-\beta_1(1-a-b)} \text{ iff } \beta_1 > \tfrac{1}{1-a-b}$$

$$\int_0^K C_2 P^{\beta_2} K^{\beta_2(a+b-1)} dK = \tfrac{1}{1-\beta_2(1-a-b)} C_2 P^{\beta_2} K^{1-\beta_2(1-a-b)} \text{ iff } \beta_2 < \tfrac{1}{1-a-b}.$$

The same process can be carried out for all the other constants of integration in the model. More generally, all the positive roots in the model must be greater than $\frac{1}{1-a-b}$ and all negative roots must be less than $\frac{1}{1-a-b}$. The latter condition is guaranteed by the fact that $\frac{1}{1-a-b} > 0$ but the former must be assumed in this model and is achieved by ensuring that $\frac{1}{a-a-b}$ is less than the positive roots of $\eta_i(z)$ and $H(z)$. A similar assumption exists in (Dixit & Pindyck, 1994, p.365). Again in the models of Abel & Eberly (1996) and Guo et al. (2005) this is guaranteed by the marginal revenue product of capital being concave in the stochastic variable.

## 2.5 Solution and Discussion

This section provides solutions for the remaining unknowns in the model, and discusses the implications of the results for firm-level capital accumulation. In the overlapping and nested configurations shown in figure 2.4.2, the remaining unknowns are found as part of a system of twelve equations and twelve unknowns. These non-linear systems are too complex to solve algebraically but numerical solutions are available. However, finding a solution to the separated configuration requires solving a simpler system of eight equations and eight unknowns, which are actually two separate systems of four equations and four unknowns, due to the fact that $q_1$ and $q_2$ are always inside a transient region, so conditions 2.4.21 and 2.4.22 do not apply. This means the switching parameters in one regime have no effect on decisions in the other regime. The solution in this case turns out to be a modified form of the solution in Abel & Eberly (1996) where the system of eight equations is characterised by just two non-linear equations whose roots are the ratios of the investment and disinvestment thresholds in regime one and regime two.

I start by showing the algebraic solution in the separated configuration and then provide numerical solutions to the other two configurations. The same baseline parameter values from chapter one will apply to numerical solutions in this chapter as well. For convenience, table 2.5.1 lists them again, along with the new parameters $\delta = 0.025$, $\phi_I = 10$, and $\phi_U = 9$. The value of $\delta$ is an estimate of the quarterly depreciation rate for U.S. firms taken from Gilchrist et al. (2014). The transition rates, $\lambda_{ij}$ are set to reflect the proportion of quarters since 1950 the U.S. has spent in a recession and expansion. All other parameters are chosen based on Dixit & Pindyck (1994), Guo et al. (2005), and Abel & Eberly (1996).

| $a$ | $b$ | $\alpha_1$ | $\alpha_2$ | $\sigma_1$ | $\sigma_2$ | $\omega_1$ | $\omega_2$ | $\rho$ | $\delta$ | $\lambda_{12}$ | $\lambda_{21}$ | $\phi_I$ | $\phi_U$ | $r$ | $w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 0.02 | 0.01 | 0.2 | 0.25 | 1 | 1 | 0.1 | 0.025 | 0.05 | 0.25 | 10 | 9 | 1 | 1 |

Table 2.5.1: Parameter Values

### 2.5.1 Separated Configuration

In the separated configuration 2.4.2a, $q_i$ is always transient in both regimes because switching to the other regime immediately triggers an active investment or disinvestment policy in the form of impulse control. Here, regime two parameters have no effect on the option values in regime one and vice-versa. There are only four constants of integration in the model and conditions 2.4.21 and 2.4.22 do not apply. The general solution for $q_i$ in this region is

$$q_i^T = \begin{cases} \theta_1^T \tilde{h} \omega_1^\nu y^\nu + \dfrac{\lambda_{12}\phi_U}{\rho + \delta + \lambda_{12}} + D_3 y^{\beta_1} + D_4 y^{\beta_2} & \text{if} \quad \varepsilon = 1 \\ \\ \theta_2^T \tilde{h} \omega_2^\nu y^\nu + \dfrac{\lambda_{21}\phi_I}{\rho + \delta + \lambda_{21}} + C_1 y^{\gamma_1} + C_2 y^{\gamma_2} & \text{if} \quad \varepsilon = 2 \end{cases} . \tag{2.5.1}$$

To solve, let $G_1 = y_1^H / y_1^L$ and $G_2 = y_2^H / y_2^L$. Next, define the function

$$\psi\left(x;\kappa_1,\kappa_2\right) = \frac{x^{\kappa_1} - x^{\nu}}{x^{\kappa_1} - x^{\kappa_2}}. \tag{2.5.2}$$

An important feature of this function is that

$$\psi\left(G_1;\beta_1,\beta_2\right) x^{\beta_2-\nu} = \psi\left(G_1^{-1};\beta_1,\beta_2\right),$$

and likewise for $\psi\left(G_2;\gamma_1,\gamma_2\right)$. Abel & Eberly (1996) use this function to solve their (S,s) investment model with partial irreversibility but without regime switching. Prior to their paper Bertola (1988) believed it was impossible to find an analytical solution to this system, despite numerical solutions suggesting that their was a unique solution.

The value matching and smooth pasting conditions in equations 2.4.19 and 2.4.20 ensure $q_1^T$ and $q_2^T$ are equal to the purchase and sale prices of capital at the boundaries. (Again, because the firm's investment policy ensures $q_i^T$ is always inside the inaction region, conditions 2.4.21 and 2.4.22 do not apply.) The values of the constants of integration which satisfy these boundary conditions are

$$D_3 = -\frac{\nu}{\beta_1}\left[1-\psi(G_1;\beta_1,\beta_2)\right]\theta_1^T\tilde{h}\omega_1^{\nu}\left(y_1^L\right)^{\nu-\beta_1} < 0 \tag{2.5.3}$$

$$D_4 = -\frac{\nu}{\beta_2}\left[\psi(G_1;\beta_1,\beta_2)\right]\theta_1^T\tilde{h}\omega_1^{\nu}\left(y_1^L\right)^{\nu-\beta_2} > 0 \tag{2.5.4}$$

$$C_1 = -\frac{\nu}{\gamma_1}\left[1-\psi(G_2;\gamma_1,\gamma_2)\right]\theta_2^T\tilde{h}\omega_2^{\nu}\left(y_2^L\right)^{\nu-\gamma_1} < 0 \tag{2.5.5}$$

$$C_2 = -\frac{\nu}{\gamma_2}\left[\psi(G_2;\gamma_1,\gamma_2)\right]\theta_2^T\tilde{h}\omega_2^{\nu}\left(y_2^L\right)^{\nu-\gamma_2} > 0 \tag{2.5.6}$$

as long as equations 2.5.11 and 2.5.12 have unique solutions greater than one. $D_3$ and $C_1$ reflect the option value of investment in regime one and regime two respectively. They are negative because increasing the capital stock by a marginal unit causes $q_i^T$ to fall by the assumption of diminishing marginal product of capital. For the same reason, $D_4$ and $C_2$ are positive because they reflect the option value of selling the marginal unit of capital. To simplify notation, let

$$\Omega_1(G_1) = \theta_1^T\left[1 - \frac{\nu\left[1-\psi(G_1;\beta_1,\beta_2)\right]}{\beta_1} - \frac{\nu\left[\psi(G_1;\beta_1,\beta_2)\right]}{\beta_2}\right]$$

and

$$\Omega_2(G_2) = \theta_2^T \left[ 1 - \frac{\nu \left[1 - \psi(G_2; \gamma_1, \gamma_2)\right]}{\gamma_1} - \frac{\nu \left[\psi(G_2; \gamma_1, \gamma_2)\right]}{\gamma_2} \right].$$

These equations are what is left over after substituting the constants of integration into the boundary conditions 2.4.17 and 2.4.18 and factoring out the common term, which is the marginal revenue product of capital at the boundaries. Collecting all terms containing $\phi_k$ to the right-hand side of the equation and dividing through by $\Omega_i(G_i)$ gives four equations defining marginal revenue product of capital at the boundaries

$$\tilde{h}\omega_1^\nu \left(y_1^H\right)^\nu = \frac{(\rho + \delta + \lambda_{12})\,\phi_I - \lambda_{12}\phi_U}{\Omega_1(G_1^{-1})\,(\rho + \delta + \lambda_{12})} \tag{2.5.7}$$

$$\tilde{h}\omega_1^\nu \left(y_1^L\right)^\nu = \frac{(\rho + \delta)\,\phi_U}{\Omega_1(G_1)\,(\rho + \delta + \lambda_{12})} \tag{2.5.8}$$

$$\tilde{h}\omega_2^\nu \left(y_2^H\right)^\nu = \frac{(\rho + \delta)\,\phi_I}{\Omega_2(G_2^{-1})\,(\rho + \delta + \lambda_{21})} \tag{2.5.9}$$

$$\tilde{h}\omega_2^\nu \left(y_2^L\right)^\nu = \frac{(\rho + \delta + \lambda_{21})\,\phi_U - \lambda_{21}\phi_I}{\Omega_2(G_2)\,(\rho + \delta + \lambda_{21})}. \tag{2.5.10}$$

Now divide 2.5.7 by 2.5.8 and 2.5.9 by 2.5.10 to obtain two non-linear equations in $G_1$ and $G_2$. Using the result in Abel & Eberly (1996) there is a unique $G_1 > 1$ and $G_2 > 1$ which satisfies the following two non-linear equations 2.5.11 and 2.5.12 respectively as long as $\phi_I > \phi_U$ and $(\rho + \delta + \lambda_{21})\,\phi_U - \lambda_{21}\phi_I > 0$, so the wedge between the capital prices in not too large[5]

[5]Otherwise the fundamental component of the marginal product of capital at the disinvestment boundary in regime two could be negative. The marginal resale value of capital must be larger than the probability-weighted value of the marginal unit of capital in the event of a switch from regime two to regime one.

$$\Omega_1(G_1^{-1})G_1^{\nu} - \left(\frac{(\rho+\delta+\lambda_{12})\,\phi_I - \lambda_{12}\phi_U}{(\rho+\delta)\,\phi_U}\right)\Omega_1(G_1) = 0 \tag{2.5.11}$$

$$\Omega_2(G_2^{-1})G_2^{\nu} - \left(\frac{(\rho+\delta)\,\phi_I}{(\rho+\delta+\lambda_{21})\,\phi_U - \lambda_{21}\phi_I}\right)\Omega_2(G_2) = 0. \tag{2.5.12}$$

As is obvious from figure 2.4.2c, $y_1^H < y_2^L$ must hold for this solution to be correct. This can never be the case if regime switching does not affect the marginal revenue product of capital or the marginal price of capital. The reason is that the model converges to having just one price of capital when the ratio of the thresholds goes to unity, which establishes a maximum disinvestment threshold and a minimum investment threshold determined by the user cost of capital in Jorgenson (1963). It could be the case that either $\phi_I \to \phi_U$ from above or $\phi_U \to \phi_I$ from below. Because the minimum investment threshold is higher than the maximum disinvestment threshold, a disinvestment threshold can never sit above an investment threshold in $[K, P]$ space without regime switching productivity.

To see why, first notice that by the application of L'Hôpital's rule

$$\lim_{x\to 1}\psi\,(x;\kappa_1,\kappa_2) = \frac{\kappa_1-\nu}{\kappa_1-\kappa_2}.$$

Also, it can be verified that

$$1 - \frac{\nu\mu_1}{\rho+\delta+\lambda_{12}} - \frac{\frac{1}{2}\sigma_1^2\nu(\nu-1)}{\rho+\delta+\lambda_{12}} = 1 - \left(\frac{\nu-\beta_2}{\beta_1-\beta_2}\right)\frac{\nu}{\beta_1} - \left(\frac{\beta_1-\nu}{\beta_1-\beta_2}\right)\frac{\nu}{\beta_2}$$

$$1 - \frac{\nu\mu_2}{\rho+\delta+\lambda_{21}} - \frac{\frac{1}{2}\sigma_2^2\nu(\nu-1)}{\rho+\delta+\lambda_{21}} = 1 - \left(\frac{\nu-\gamma_2}{\gamma_1-\gamma_2}\right)\frac{\nu}{\gamma_1} - \left(\frac{\gamma_1-\nu}{\gamma_1-\gamma_2}\right)\frac{\nu}{\gamma_2}.$$

The expressions on the right hand side of the last two equations are the result of substituting the limit of $\psi\,(G_1;\beta_1,\beta_2)$ as $G_1 \to 1$ and $\psi\,(G_2;\gamma_1,\gamma_2)$ as $G_2 \to 1$ into $\Omega_1(G_1)$ and $\Omega_2(G_2)$ respectively.

Now, note that

$$\eta_i(\nu) \times c = \frac{1}{\rho + \delta + \lambda_{ij}} \quad \text{if} \quad c = 1 - \frac{\nu\mu_i}{\rho + \delta + \lambda_{ij}} - \frac{\frac{1}{2}\sigma_i^2 \nu(\nu - 1)}{\rho + \delta + \lambda_{ij}}.$$

Then it must be the case that $\Omega_i(1) = \frac{1}{\rho+\delta+\lambda_{ij}}$, which in turn can be substituted into equations 2.5.11 and 2.5.12 along with $G_i = 1$ to yield $\phi_I = \phi_U$. This finding makes economic sense. It states that if there is no wedge between the thresholds justifying investment and disinvestment, capital adjustments must be fully reversible and there can only be one price of capital in the model. Now suppose that $\phi_U \to \phi_I$ so $\phi_I$ is the only price of capital in the model and that $\omega_1 = \omega_2 = 1$ so there are no differences in productivity between the regimes. Equations 2.5.7 and 2.5.9 immediately reveal that

$$y_1^H = y_2^H = \left( \frac{(\rho + \delta)\, \phi_I}{\tilde{h}} \right)^{\frac{1}{\nu}}.$$

This condition gives the minimum investment threshold in the model. Likewise if $\phi_I \to \phi_U$ so $\phi_U$ is the only price of capital in the model, then

$$y_1^L = y_2^L = \left( \frac{(\rho + \delta)\, \phi_U}{\tilde{h}} \right)^{\frac{1}{\nu}}$$

which is the maximum disinvestment threshold.

Given $\phi_I > \phi_U$, the minimum investment threshold must always be greater than the maximum disinvestment threshold. There is no combination of $\alpha_i$, $\sigma_i$, and $\lambda_{ij}$ which can push the disinvestment (investment) threshold above the maximum (minimum) determined by the Jorgensonian user cost. Figure 2.A.1 and in the appendix demonstrates this in $[K, P]$ space. As $\alpha_2$ decreases or $\sigma_2$ increases, $P_2^L$ converges to the Jorgensonian user cost given by the lower black curve[6].

It is also clear why regime switching technology allows $y_1^H < y_2^L$. The minimum investment and maximum disinvestment thresholds in regime one and two can be different when $\omega_1 \neq \omega_2$. The

[6]Note that decreases in $\lambda_{21}$ also bring $P_2^L$ closer to its maximum possible value but $\lambda_{21} = 0.001$ in these simulations so its effect on $P_2^L$ is already negligibly small.

minimum investment boundary in regime one becomes

$$y_1^H = \frac{1}{\omega_1}\left(\frac{(\rho+\delta)\,\phi_I}{\tilde{h}}\right)^{\frac{1}{\nu}}$$

and the maximum disinvestment boundary in regime two becomes

$$y_2^L = \frac{1}{\omega_2}\left(\frac{(\rho+\delta)\,\phi_U}{\tilde{h}}\right)^{\frac{1}{\nu}}.$$

Clearly, for a small enough $\omega_2$, $y_1^H < y_2^L$ . In economic terms, this means that the difference in productivity between the regimes must be sufficiently large to produce a separated configuration.

The key characteristic of the separated configuration is that switches to regimes characterised by higher uncertainty and lower productivity always cause an immediate decrease in the firm's capital stock followed by a period of increased inaction, regardless of how close to its adjustment threshold the firm was before the regime switch. The initial disinvestment is due to impulse control following the regime shift while the latter is due to the greater wedge between the investment and disinvestment thresholds in regime two. When transitioning from the recessionary regime, the firm installs a lump of capital and is then willing to invest at a lower price of the output good. Therefore, differences in productivity large enough to produce a separated configuration will lead to very aggressive response of capital accumulation to the business cycle. Firms with very different productivity levels between the two regimes will adjust their capital stocks in large lumps whenever there is a change in regime. The higher $y_2^L$, the larger the discrete jump in the capital stock necessary to move to the inaction region of regime two following a switch from regime one.

The first row of table 2.5.2 shows a numerical solution to the separated configuration using the baseline parameters. The second and third rows show the effect of decreasing the productivity level in regime two by 5 percentage points and increasing uncertainty by 5 percentage points respectively. Lower productivity in regime two increases both the investment and disinvestment thresholds. A higher price of the output good is now required for the firm to add to its capital stock and is also sufficient to justify subtracting from its capital stock. Overall, recessions will be associated with less investment and more disinvestment compared to the expansionary regime.

| | $y_2^H$ | $y_2^L$ | $G_2$ | $C_1$ | $C_2$ |
|---|---|---|---|---|---|
| $\omega_2 = 0.3$ | 27.70 | 11.94 | 2.32 | $-1 \times 10^{-5}$ | 2473.59 |
| $\omega_2 = 0.25$ | 33.24 | 14.32 | 2.32 | $-6 \times 10^{-6}$ | 4601.61 |
| $\sigma_2 = 0.3$ | 29 | 11.49 | 2.52 | $-5 \times 10^{-5}$ | 456.53 |
| $\sigma_2 = 0.3$ & $\omega_2 = 0.25$ | 34.80 | 13.79 | 2.52 | $-3 \times 10^{-5}$ | 746.73 |

Table 2.5.2: Separated Configuration

As mentioned, regime two parameters have no effect on the unknowns in regime one in a separated configuration. This comes from the fact that switching to the other regime always gives the known payoff $\phi_k$ and triggers a discrete adjustment in the capital stock. The unknowns in regime one are $y_1^H = 8.38$, $y_1^L = 4.48$, $G_1 = 1.87$, $D_1 = -0.02$, and $D_4 = 2.32$ for all specifications in table 2.5.2. $C_1$ is very close to zero and $C_2$ is very large relative to the constants of integration in regime one. This is because the disinvestment threshold is so high. It means there is a relatively large increase in $q_2$ if the firm removes the marginal unit of capital. Likewise, there is a relatively small decrease in $q_2$ if the firm installs the marginal unit of capital. When the capital stock is relatively unproductive in the recessionary regime, the marginal value of the next unit of capital is greatly increased by reducing the capital stock.

While uncertainty also causes upward movements in the investment threshold, it decreases the disinvestment threshold. The model therefore predicts some ambiguity about whether there will be more or less disinvestment following a switch to a recessionary regime with lower productivity and higher uncertainty. This is intriguing in light of the fact that downward adjustments in the capital stock are known to be less frequent in firm-level data (Doms & Dunne, 1998; Cooper & Haltiwanger, 2006). The third column of table 2.5.2 shows that in this case the upward effect from productivity dominates and the disinvestment threshold increases. Notice also that only changes in uncertainty alter the ratio between the investment and disinvestment thresholds.

### 2.5.2 Nested Configuration

A switch from regime one to regime two in the region $\left[y_1^L, y_1^H\right]$ means acquiring the expected present discounted value of the marginal unit of capital plus the options to install and uninstall the next marginal unit. The regions between $\left[y_1^H, y_2^H\right]$ and $\left[y_2^L, y_1^L\right]$ can only be reached in regime two.

Switching in the region $\left[y_1^H, y_2^H\right]$ triggers immediate investment in the form of impulse control, so $q_2$ is in a transient region with $q_1 = \phi_I$. Likewise, switching in the region $\left[y_2^L, y_1^L\right]$ triggers immediate disinvestment, so $q_2$ is in a transient region with $q_1 = \phi_U$. The general solution to $q_1$ is

$$q_1 = \theta_1 \tilde{h} \omega_1^\nu y^\nu + \sum_{j=1}^{4} A_j y^{z_j} \tag{2.5.13}$$

and the general solution for $q_2$ is

$$q_2 = \begin{cases} \theta_2 \tilde{h} \omega_2^\nu y^\nu + \sum_{j=1}^{4} \frac{\eta_1(z_j)}{\lambda_{12}} A_j y^{z_j} & \text{if} \quad q_2 \in \left[y_1^L, y_1^H\right] \\ \\ \theta_2^T \tilde{h} \omega_2^\nu y^\nu + \frac{\lambda_{21}\phi_I}{\rho + \delta + \lambda_{21}} + \sum_{j=1}^{2} C_j y^{\gamma_j} & \text{if} \quad q_2 \in \left[y_1^H, y_2^H\right] \\ \\ \theta_2^T \tilde{h} \omega_2^\nu y^\nu + \frac{\lambda_{21}\phi_U}{\rho + \delta + \lambda_{21}} + \sum_{j=3}^{2} C_j y^{\gamma_j} & \text{if} \quad q_2 \in \left[y_2^L, y_1^L\right]. \end{cases} \tag{2.5.14}$$

The boundary conditions are exactly as specified in 2.4.5 where $i = 1$ and $j = 2$. Evaluating these four equations at the boundaries gives a system of twelve equations and twelve unknowns which is not solvable using the techniques of Abel & Eberly (1996) and Guo et al. (2005). However, numerical solutions are available whose validity can be checked by comparison to a model without regime switching. As $\lambda_{12} \to 0$ and $\lambda_{21} \to 0$, the model converges to a modified form of Abel & Eberly (1996), which does have closed-form solutions once the ratio of capital prices is known. I present solutions in appendix 2.A.2 which show this convergence by setting $\lambda_{12} = \lambda_{21} = 0.0001$ .

Nested configurations are characterised by one regime having higher uncertainty about the output price than the other. This is because higher uncertainty in one regime will push the investment threshold upwards and disinvestment threshold downwards, allowing the thresholds in the other regime to lie between those in the high-uncertainty regime. As oppose to the separated configuration, switching to a recessionary regime in a nested configuration never results in a lump adjustment in the capital stock. Such adjustments only occur when switching *from* a recessionary regime. If the switch to the recessionary regime occurs when $P$ and $K$ are such that $q_2^T$ is in the region $[y_1^H, y_2^H]$, there is an immediate lump increase in the capital stock. Likewise, there will be a lump decrease in the capital stock when $\varepsilon$ switches from regime 1 to regime 2 and $q_2^T$ is in the region $[y_1^L, y_2^L]$. The dynamics of investment after switching from a high-uncertainty regime are therefore ambiguous, if $P$ was relatively high a firm who was delaying the decision to invest will suddenly install a lump of capital, while if $P$ is relatively low a firm who was delaying the decision to disinvest will sell a lump of capital.

As shown in the first column of table 2.5.3, the baseline parameters produce a nested configuration. It is first interesting to note that under the assumption of no downward adjustment in the capital stock, as in Guo et al. (2005), and using the baselines parameter values in this chapter, the investment threshold in regime one is 8.78 and the investment threshold in regime two is 9.53. For completeness, there are four constants of integration in the fully irreversible model, given the baseline parameters, they have values $3 \times 10^{-5}$, -0.116, -0.001 and -524.65. Comparison with $y_1^H$ and $y_2^H$ in table 2.5.3 reveals that if the firm has the option to adjust the capital stock downwards as well as upwards, they will invest at lower prices of the output good. This makes sense because the decision to purchase capital is now only partially irreversible, so the firm can recover some of the cost of investment at a later date. The regime switching model in Guo et al. (2005) thus overestimates the price required for investment to take place for a given set of starting parameters.

As $\sigma_2$ increases to 0.4 in the second column, $y_2^H$ increases and $y_2^L$ decreases but notice also the slight changes in the regime one thresholds as well. Higher uncertainty in the recessionary regime has also caused a change in the optimal decision in the other regime, specifically, the region of inaction has increased. Given the values of the other parameters, regime one's investment threshold has been affected more than its disinvestment threshold. The third column resets $\sigma_2 = 0.25$ and shows the effect of making the expansionary regime less persistent (higher $\lambda_{12}$), meaning it is more likely for $\varepsilon$ to switch to the recessionary regime. This leads to increases in all of the thresholds in the model, relatively small increases in regime two and larger increases in regime one. If recessions are expected to occur more frequently, a higher value of the output price for a given $K$ will be required to justify investment and firms are willing to decrease their capital stocks at higher prices of the output good.

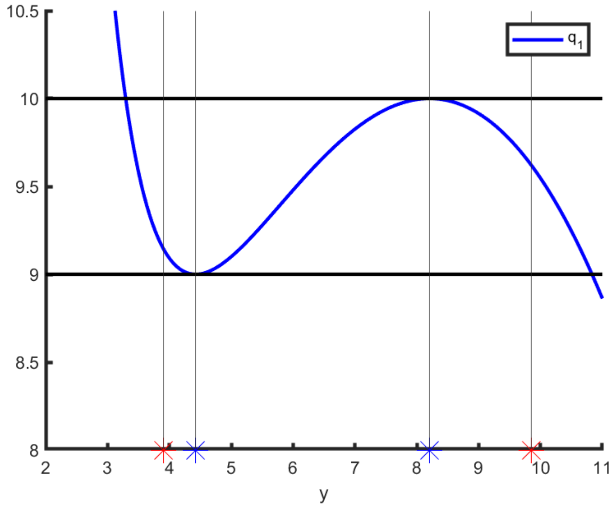
| Unknown | $\sigma_2 = 0.25$ | $\sigma_2 = 0.4$ | $\lambda_{12} = 0.15$ |
|---|---|---|---|
| $y_1^H$ | 8.1853 | 8.2073 | 8.2087 |
| $y_1^L$ | 4.4254 | 4.425 | 4.4302 |
| $y_2^H$ | 8.6436 | 9.8542 | 8.6476 |
| $y_2^L$ | 4.3067 | 3.899 | 4.3089 |
| $A_1$ | $-3 \times 10^{-5}$ | -0.0021 | $-1 \times 10^{-5}$ |
| $A_2$ | -0.1196 | -0.1604 | -0.1171 |
| $A_3$ | 76.851 | 11.046 | 61.442 |
| $A_4$ | 122.9 | 198.07 | 298.29 |
| $C_1$ | -0.0008 | -0.0081 | -0.0008 |
| $C_2$ | -67.971 | 1.9308 | -69.467 |
| $C_3$ | -0.0002 | -0.0049 | -0.0002 |
| $C_4$ | 104.39 | 13.229 | 104.5 |

Table 2.5.3: Nested Configuration

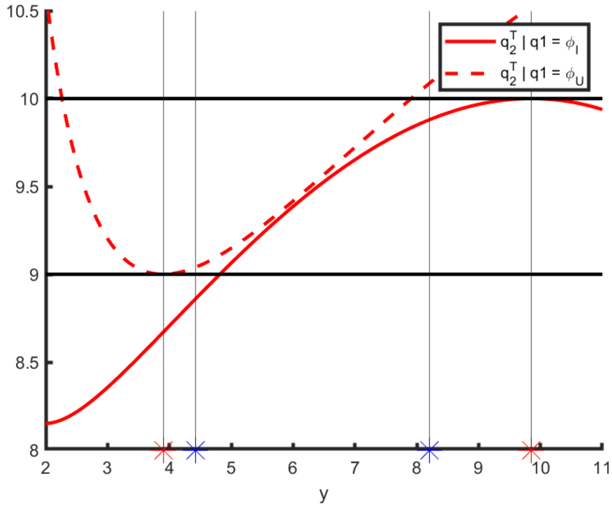
The constants of integration must simultaneously ensure that it is optimal to adjust the capital stock when $y$ is at a threshold value and that $q_2^T$ is tangent to $q_2$ at the investment and disinvestment thresholds in regime one. Figure 2.5.1a shows the graph for $q_1$ from the second results columns of table 2.5.3. It has the same shape as the functions encountered in chapter one when outside of the transient regions. The threshold values of $y$ are marked by stars on the horizontal axis, blue in regime one and red in regime two.

Given chapter one covered this in some detail, I will only comment that $A_3 > 0$ and $A_4 > 0$ in table 2.5.3 means $q_1$ is decreasing for small values of $y$ but these terms become less influential as $y$ increases. There is always the fundamental part of $q_i$ which is convex in $y$, in this case given by $\theta_1 \tilde{h} \omega_1^\nu y^\nu$. This means the function starts increasing after the fundamental part starts to dominate the terms containing $A_3$ and $A_4$, which produces a local (in the range $[y_1^L, y_2^H]$) minimum at $y_1^L$ where $q_1 = \phi_U$, after which the function slopes upwards. Then, $A_1 < 0$ and $A_2 < 0$, as well as the restriction that $\nu < z_1 < z_2$, means the function is diminishing before reaching a local maximum at $y_1^H$ where $q_1 = \phi_I$.

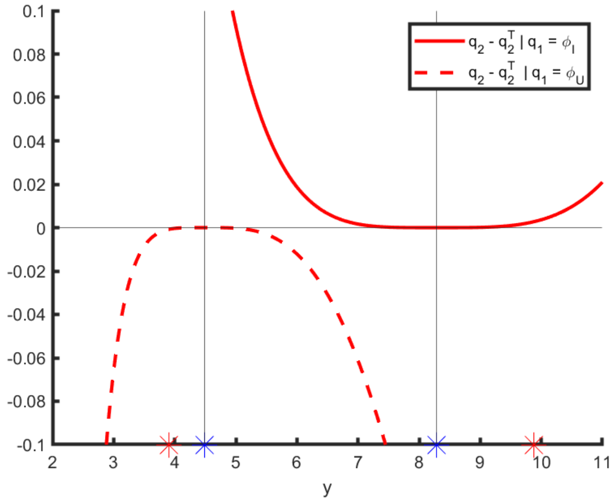
Figure 2.5.1b graphs $q_2^T$ in the cases where $q_1 = \phi_I$ and $q_1 = \phi_U$. I have omitted the $q_2$ function so the panel is not too cluttered. Panel 2.5.1c shows that $q_2$ is indeed tangent to $q_2^T$ at regime one's threshold values, as required. Returning to 2.5.1b, given $C_4 > 0$, $q_2^T|_{q_1=\phi_U}$ (the dashed line) slopes downwards for small values of $y$, before reaching a local minimum at $y_2^L$ where $q_2^T = \phi_U$. $C_3$ is negative in this case and indicates how steep the function has to be after the local minimum to




(a) $q_1$




(b) $q_2^T$




(c) $q_2 - q_2^T$

Figure 2.5.1: $q_i$ at the Boundaries in the Nested Configuration

smooth paste with $q_2$ at $y_1^L$. The lower (more negative) it is, the more gradual the slope and the further away $y_1^L$ is from $y_2^L$.

For $q_2^T|_{q_1=\phi_I}$ (the solid line), $C_1 < 0$ and $\gamma_1 > \nu$ allows this function to have a maximum at $y_2^H$ because the term $C_1 y^{\gamma_1}$ dominates $\theta_2^T \tilde{h}\omega_2^\nu y^\nu$ and means the slope of $q_2^T$ is diminishing between $y_1^H$ and $y_2^H$. $C_2$ takes on both positive and negative values in table 2.5.3. If $y_1^H$ is relatively close to $y_2^H$ the function $q_2^T$ must be steeper in this interval, so $C_2$ can be negative to ensure this. Otherwise, when $y_1^H$ is relatively far away from $y_2^H$, higher values of $C_2$ allow the slope to be more gradual.

### 2.5.3 Overlapping Configuration

From figure 2.4.2e, $q_2$ is in a transient region in the interval $\left[y_1^H, y_2^H\right]$ with $q_1 = \phi_I$, however, $q_1$ is in a transient region for $\left[y_1^L, y_2^L\right]$ with $q_2 = \phi_U$. The general solution for $q_i$ is now

$$q_1 = \begin{cases} \theta_1 \tilde{h}\omega_1^\nu y^\nu + \sum_{j=1}^{4} A_j y^{z_j} & \text{if} \quad q_1 \in \left[y_2^L, y_1^H\right] \\ \\ \theta_1^T \tilde{h}\omega_1^\nu y^\nu + \dfrac{\lambda_{12}\phi_U}{\rho + \delta + \lambda_{12}} + \sum_{j=3}^{2} D_j y^{\gamma_j} & \text{if} \quad q_1 \in \left[y_1^L, y_2^L\right] \end{cases} \tag{2.5.15}$$

and the general solution for $q_2$ is

$$q_2 = \begin{cases} \theta_2 \tilde{h}\omega_2^\nu y^\nu + \sum_{j=1}^{4} \dfrac{\eta_1(z_j)}{\lambda_{12}} A_j y^{z_j} & \text{if} \quad q_2 \in \left[y_2^L, y_1^H\right] \\ \\ \theta_2^T \tilde{h}\omega_2^\nu y^\nu + \dfrac{\lambda_{21}\phi_I}{\rho + \delta + \lambda_{21}} + \sum_{j=1}^{2} C_j y^{\gamma_j} & \text{if} \quad q_2 \in \left[y_1^H, y_2^H\right] \end{cases} \tag{2.5.16}$$

The boundary conditions 2.4.21 and 2.4.22 are slightly modified in the overlapping configuration because both $q_1$ and $q_2$ are in a transient region over some range of $[K, P]$ space. In this section, they are given by

$$\lim_{y \uparrow y_1^H} q_2 = \lim_{y \downarrow y_1^H} q_2^T \quad \text{and} \quad \lim_{y \downarrow y_2^L} q_1 = \lim_{y \uparrow y_2^L} q_1^T \tag{2.5.17}$$

$$\lim_{y \uparrow y_1^H} q_2' = \lim_{y \downarrow y_1^H} \left(q_2^T\right)' \quad \text{and} \quad \lim_{y \downarrow y_2^L} q_1' = \lim_{y \uparrow y_2^L} \left(q_1^T\right)'. \tag{2.5.18}$$

As with the nested configuration substituting the general solutions for $q_i$ into the relevant boundary conditions produces a system of twelve equations and twelve unknowns which can be solved numerically. The overlapping configuration in figure 2.4.2e tends to occur when regime two has a higher volatility and a much lower drift or moderately lower productivity parameter compared to regime one. It can therefore be viewed as and intermediate case; the result of switching to a more severe recession than the nested configuration but not as severe as the separated configuration.

Between $y_1^H$ and $y_2^H$ where $q_2$ is transient, switching to regime one will have the same effect as in the nested configuration, the firm will immediately install a lump of capital. As in the other configurations, switching *from* a recessionary regime results in a wave of investment. However, with $q_1$ transient between $y_1^L$ and $y_2^L$, switching *to* the recessionary regime in this region will result in a lump decrease in the capital stock. This decrease will be smaller than in the separated configuration because $y_2^L$ is lower than $y_1^H$. It is also possible that $\varepsilon$ switches from regime one to regime two while $q_1$ is not in the transient region. In this case, the switch will have no immediate effect on the capital stock but the firm will be closer to its disinvestment threshold now that the economy is in the recessionary regime (see figure 2.4.2e).

| Unknown | $\alpha_2 = -0.04$ | $\omega_2 = 0.8$ | $\omega_2 = 0.8$ & $\lambda_{21} = 0.15$ |
|---|---|---|---|
| $y_1^H$ | 8.2072 | 8.2642 | 8.2738 |
| $y_1^L$ | 4.4376 | 4.458 | 4.4608 |
| $y_2^H$ | 8.9847 | 10.53 | 10.651 |
| $y_2^L$ | 4.4916 | 5.0695 | 5.176 |
| $A_1$ | $4 \times 10^{-7}$ | $-7 \times 10^{-5}$ | -0.0004 |
| $A_2$ | -0.0795 | -0.1119 | -0.1068 |
| $A_3$ | 37.063 | 89.047 | 64.877 |
| $A_4$ | 166.05 | 68.576 | 90.828 |
| $C_1$ | -0.0001 | -0.0004 | -0.0019 |
| $C_2$ | -5.8639 | 15.322 | 69.837 |
| $D_3$ | -0.0227 | -0.02295 | -0.02299 |
| $D_4$ | 206.65 | 208.65 | 208.92 |

Table 2.5.4: Overlapping Configuration

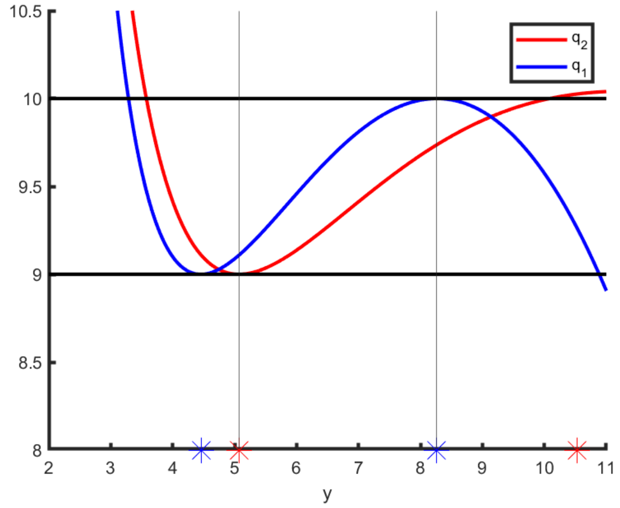
Table 2.5.4 shows some parameter combinations which produce an overlapping configuration. In the first column, the drift parameter in regime two is $-0.04$, so the output price tends to decline over time when in this regime. As required, $y_2^L$ now lies above $y_1^L$. Relative to the first column of table 2.5.3, all of the thresholds have increased, reflecting the fact with the much lower growth in the output price in regime two, installing capital is fundamentally less valuable for the firm. A similar, but larger, effect is found in the second column of table 2.5.4 after a decline in the productivity parameter in regime two relative to the baseline case. As shown in the third column, increasing the persistence of the recessionary regime (decreasing $\lambda_{21}$) causes further increases in the thresholds. Installing capital is less valuable when recessions are more persistent and selling the marginal unit is justified at a higher price of the output good.

Figure 2.5.2 shows the solutions in the third results column of table 2.5.4 graphically. The first panel shows that $q_2$ smooth pastes with $\phi_U$ at $y_2^L$ and $q_2$ smooth pastes with $\phi_I$ at $y_1^H$. Because both regimes now feature a transient region over some range of $y$, neither $q_i$ function smooth pastes with both $\phi_I$ and $\phi_U$. Instead, the second panel shows how $q_1^T$ smooth pastes with $\phi_U$ at $y_1^L$ and $q_2^T$ smooth pastes with $\phi_I$ at $y_2^H$. The final panel shows the equivalence of $q_i$ and $q_i^T$ at the boundaries defining the transient regions.

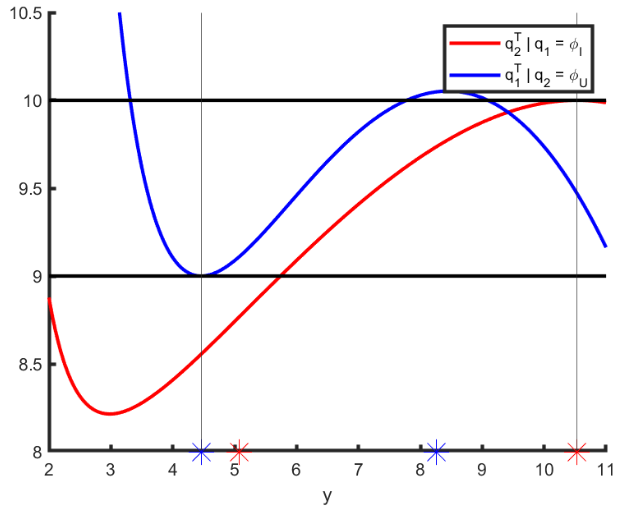
Again, $C_2$ changes sign in the second and third results columns of table 2.5.4, reflecting the fact that $y_2^H$ is relatively far away from $y_1^H$ and the function must increase gradually over this region to ensure the boundary conditions are satisfied. The slight decrease in $D_3$ between the first and second columns likewise accounts for the fact that $y_2^L$ is further away from $y_1^L$.

## 2.6 Conclusion

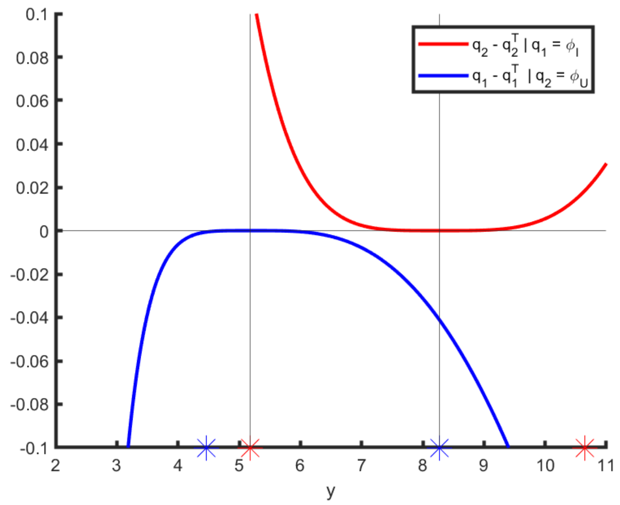
By incorporating regime switching uncertainty and productivity into a model of optimal capital accumulation, this chapter has demonstrated how switching between expansionary and recessionary regimes produces lumpy adjustments in a representative firm's capital stock, followed by periods of enhanced or depressed investment activity. Previous related studies have tended to assume both uncertainty and productivity are time invariant, which limits the predictions they can make about firms' optimal investment policy over the business cycle. In contrast, this chapter shows that there are multiple possible responses of firm-level investment to regime changes where the only financial friction is a wedge between the purchase and sale prices of capital.




(a) $q_1$ and $q_2$




(b) $q_1^T$ and $q_2^T$




(c) $q_1 - q_1^T$ and $q_2 - q_2^T$

Figure 2.5.2: $q_i$ at the Boundaries in the Overlapping Configuration

The price of the perfectly competitive firm's output good followed a GBM whose drift and volatility parameters were dependent on a CTMC switching between two regimes representing expansions and contractions. Time-varying uncertainty was introduced through this regime switching volatility parameter. The productivity parameter in the firm's Cobb-Douglas production function was also dependent on the CTMC, and introduced time-vary productivity into the model. Thus the model captures two observed characteristics of recessions; higher uncertainty and lower productivity. The firm's decision problem was solved by finding the threshold values of the stochastic variable underlying the value of the marginal unit of capital which triggered a change in investment policy. With two regimes and partial reversibility of investment regimes, there were four such thresholds to locate, and their relative positions generated a set of predictions about the response of investment to regime changes.

The most radical patters of investment activity were produced in the separated configuration, where switching to the recessionary regime always resulted in a lump decrease in the capital stock and switching to the expansionary regime always resulted in a lump increase in the capital stock. Higher uncertainty in the recessionary regime also meant investment activity would be reduced. This required very large differences in productivity between the two regimes and suggests this kind of investment pattern will only be observed in firms or plants whose productivity level is highly dependent on the state of the economy.

A nested configuration, where the thresholds in the expansionary regime lied within those of the recessionary regime, was produced when the latter regime displayed higher uncertainty but the differences in the productivity and drift parameters were relatively small. Here, transitioning into the recessionary regime does not produce a lump adjustment in the capital stock but does result in lower investment activity because the firm's region of inaction widens. Upon switching back to the expansionary regime, the firm may remain inactive, install capital, or sell capital depending on whether the value of the marginal unit of capital is above or below the investment threshold in the expansionary regime when the regime switch occurs. Hence, coming out of periods of high uncertainty can be marked by lump changes in capital stocks.

The overlapping configuration, when the disinvestment threshold in regime one was lower than that in regime two, represented an intermediate case where the recessionary regime displayed higher uncertainty but also notably lower productivity or growth in the output price. Lump adjustments in the capital stock are now possible, but not certain, in both regimes. For a given $K$, switching to an expansionary regime may be marked by a lump increase in the capital stock if the output price is high enough and switching to a recessionary regime may be marked by a lump decrease in the capital stock if the output price is low enough. Because higher uncertainty decreases the disinvestment threshold but lower drift and productivity have the opposite effect, there is ambiguity about how simultaneous changes in these variables will affect the sale of capital.

Overall, the model makes testable predictions about the response of investment activity to regime changes. Future work could identify whether periods of high uncertainty and low productivity produce patterns of firm level investment which match these predictions. This would give policy makers forecasting future investment additional insight into how changes in the economic environment might affect firms' optimal investment decisions. One shortcoming of the model is that productivity only changes with $\varepsilon$. In reality, firm-level productivity would also be better modelled as a stochastic process where regime changes affected its growth and its volatility, rather than leading to persistent falls in its level. This would complicate the model but is another future avenue of research.

# Appendix 2.A

## 2.A.1 Convergence to Minimum and Maximum Boundaries in the Separated Configuration

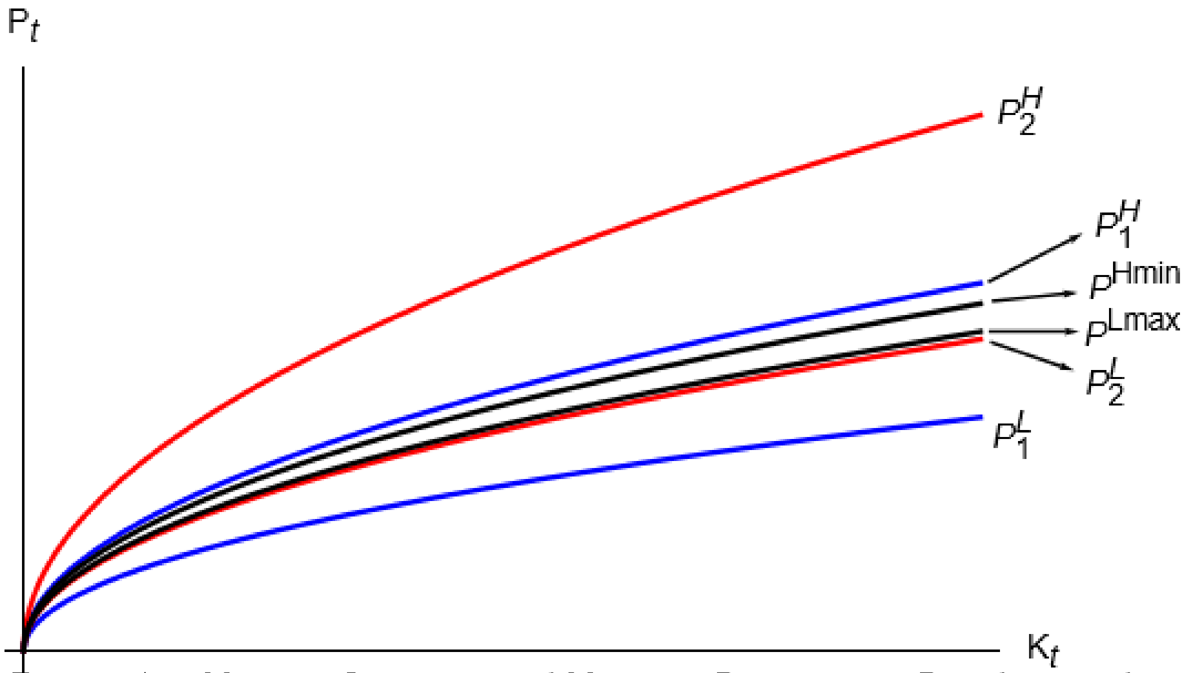



Figure 2.A.1: Minimum Investment and Maximum Disinvestment Boundaries without Regime-Switching Technology

Here, $\alpha_1 = 0.04$, $\alpha_2 = -0.1$, $\sigma_1 = \sigma_2 = 0.07$, and $\lambda_{12} = \lambda_{21} = 0.001$. These parameters are extreme and demonstrate that the existence of the maximum disinvestment and minimum investment thresholds which are shown by the black curves.

## 2.A.2 Convergence to a Model without Regime Switching in the Nested and Overlapping Configurations

Abel & Eberly's (1996) model has four unknowns, the two threshold values of the stochastic variable triggering adjustments in the capital stock and two constants of integration giving the change in the value of $q$ at these boundaries. As the regime switching parameters go to zero, both regimes should converge to Abel & Eberly's model, given the drift and uncertainty of the stochastic GBM process and the parameters in the production function. In this section, let $AE_i^H$ be the constant of integration associated with the value of $q$ in regime $i$ at the investment boundary and $AE_i^L$ do the same for the disinvestment boundary. Table 2.A.1 shows the convergence in the nested and overlapping configurations.

| Unknown | Nested Configuration $\lambda_{12} = \lambda_{21} \approx 0$ | Nested Configuration No Switching | Overlapping Configuration $\lambda_{12} = \lambda_{21} \approx 0$ | Overlapping Configuration No Switching |
|---|---|---|---|---|
| $y_1^H$ | 8.1721 | 8.1721 | 8.1722 | 8.1721 |
| $y_1^L$ | 4.4223 | 4.4223 | 4.4223 | 4.4223 |
| $y_2^H$ | 8.6992 | 8.6993 | 9.1314 | 9.1315 |
| $y_2^L$ | 4.3214 | 4.3214 | 4.5668 | 4.5669 |
| $A_1$ | -0.119 | - | $4 \times 10^{-6}$ | - |
| $A_2$ | -0.0028 | - | -0.1215 | - |
| $A_3$ | 0.0527 | - | 0.0244 | - |
| $A_4$ | 148.21 | - | 148.25 | - |
| $B_1$ | 0.0024 | - | -0.0047 | - |
| $B_2$ | -0.1133 | - | -0.0001 | - |
| $B_3$ | 44.1155 | - | 27.6982 | - |
| $B_4$ | -0.091 | - | -0.0476 | - |
| $C_1$ | -0.1105 | - | -0.0047 | - |
| $C_2$ | 44.048 | - | 27.668 | - |
| $C_3$ | -0.1105 | - | - | - |
| $C_4$ | 44.062 | - | - | - |
| $D_3$ | - | - | -0.1214 | - |
| $D_4$ | - | - | 148.3 | - |
| $AE_1^H$ | - | -0.1218 | - | -0.1218 |
| $AE_1^L$ | - | 148.199 | - | 148.199 |
| $AE_2^H$ | - | -0.1109 | - | -0.0047 |
| $AE_2^L$ | - | 44.0512 | - | 27.6731 |

Table 2.A.1: Convergence to a Model without Regime Switching as $\lambda_{12} = \lambda_{21} \to 0$.

One difference from chapter one is worth mentioning. The constants of integration in the transient regions now both converge to those in the model without regime switching, whereas chapter one argued that one from each pair needed to be eliminated to get back to the standard model. This is because $q_i$ is always bounded by the investment and disinvestment thresholds in this chapter, so there is no argument to eliminate the constants as $y$ gets very large or very small. The constants in the transient regions all show the required convergence to reflect this.

# Chapter 3

# Determinants of Corporate Investment and the Impact of the Great Recession: The Role of Balance Sheets and Uncertainty

## 3.1 Introduction

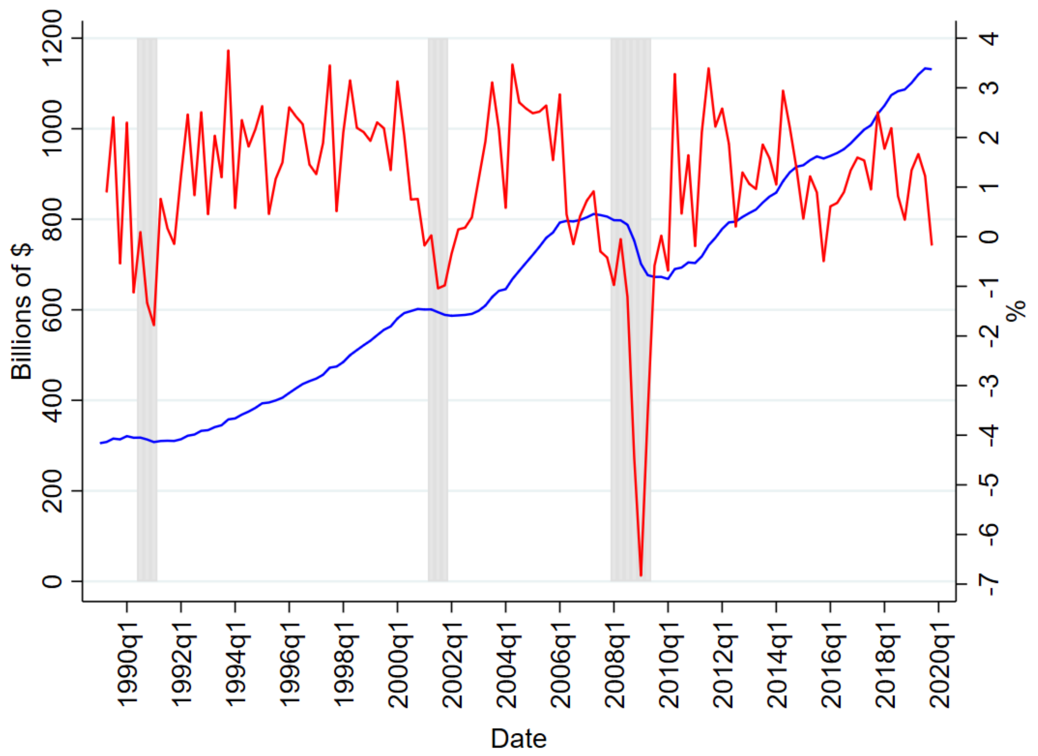
In the aftermath of the Great Recession (GR) of 2008-2009, U.S. Gross Fixed Capital Formation (GFCF) showed a dramatic decline. Including the minor fall in the two quarters before the GR officially hit (according to the NBER's method of identifying turning points in the business cycle), quarterly GFCF growth was negative for over two years during this period. GFCF did not recover to its pre-recessional level until the last quarter of 2012. Similar to the observations Cerra & Saxena (2008) make about GDP, GFCF does not display a sustained period of above average growth after a large recession to return to previous trends but instead remains depressed for a few quarters after the recession before returning to its normal growth path. Relative to GDP, the decline and delayed recovery appears more dramatic. Between 1989 and 2019, the average GFCF to GDP ratio was 20.6%, a rate not seen until 9 years after the recession. These dynamics are shown in figure 3.1.1b. The shaded areas are the NBER recessionary periods. Large declines in investment during recessions coupled with delayed recoveries concern policy makers because investment decisions today affect the capital stock available to drive growth and innovation in future periods. Former head of the Federal

Reserve Janet Yellen (2016) noted that the justification for expansionary policies after recessions is greater if they are expected to leave scars on the economy which affect growth in future periods.

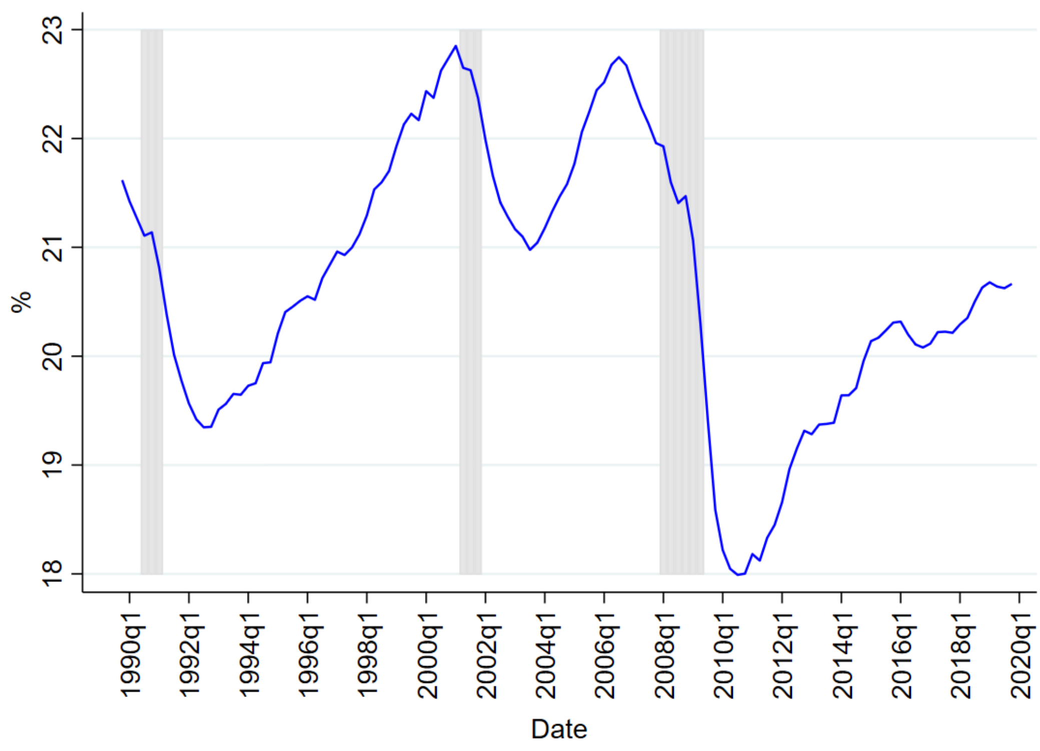
The aim of this chapter is to identify the variables that affect the investment rate over the business cycle and examine their role in causing its decline after the GR. Furthermore, in light of the scale of the GR and the weak performance of investment relative to GDP, I also examine whether these relationships changed in the years after the crisis. To accomplish this, I use a representative sample of 1658 listed U.S. firms' company accounts data and stock price information to understand the factors underlying corporate investment. Building on chapter two of this thesis, the econometric model is based on the q-theory of investment outlined in Hayashi (1982), which relates investment rates (additions to fixed assets divided by the capital stock) to a measure of firms' financial growth opportunities which is empirically known as Tobin's average Q[1]. This model is augmented with other variables that are also found to be significant predictors of the investment rate; net sales, uncertainty, leverage, cash flows, size, and the lag of the investment rate itself. Other models examining the determinants of corporate investment in the existing literature tend not to include all of these variables, so the regression coefficients in this chapter are robust to the inclusion of a wider range of relevant predictors.

The coefficient on the lagged dependent variable is known to be biased if estimated with a fixed effects model, which removes unobserved time-invariant factors by subtracting the individual-specific mean from each observation in the sample (Nickell, 1981). Therefore, the generalised method of moments (GMM) estimator by Arellano & Bond (1991) recovers the coefficients capturing the relationships between the variables and the investment rate. Here, the bias introduced by including the lagged dependent variable is dealt with by using past lags as instruments in a transformed model. Instead of the conventional first difference transformation used in these models, a forward orthogonal deviations transformation (FOD) removes the time-invariant error term, which also creates a source of bias in the estimated coefficients because it is likely correlated with the explanatory variables. The FOD transformation subtracts the mean of all future values from the current observation, so any

[1]Tobin's marginal q was described in chapter two as the marginal value of the next unit of capital. Outside of (S,s) investment models, it is usually defined as the ratio of this variable to the marginal cost of capital. Average Q is its empirical analogue, most simply defined as the market value of the capital stock divided by its replacement cost. Hayashi (1982) gives the necessary conditions for these concepts to be identical. I use a capitalised Q in this chapter to keep the concepts distinct.




(a) Gross Fixed Capital Formation (blue line) and its Growth (red line)




(b) U.S. Gross Fixed Capital Formation as a Proportion of GDP

Figure 3.1.1: Investment Time Series

factors which do not change over time (observed or unobserved) are removed. It has an advantage over first differencing because it does not mean sacrificing observations early in the sample.

Most of the variables have a precedent in previous literature and are readily available from company book data. However, measuring uncertainty is a difficult task because there can be no forecastable information contained in the measure and it must be forward looking. Following Leahy & Whited (1996) and Gilchrist et al. (2014), I base my measure on the idiosyncratic volatility of firms' stock returns. Unlike these studies, I examine whether there are significant changes in the estimated relationship between uncertainty and investment when uncertainty is measured by the simple volatility (standard deviation) of returns compared to a measure constructed after first purging returns of their forecastable information in a four-factor asset pricing model. Stock markets can move even when there is no change in uncertainty, which is a source of potential contamination in the measure. Indeed, section 3.4.2 suggests that the results do not change whether using simple volatility or volatility conditional on common risk factors. I examine in chapter four whether changes in uncertainty and stock market volatility have different effects on the real economy.

The key result of this chapter is that the decline in growth opportunities (Q) and rise in uncertainty were the key determining factors in causing the decline in firms' investment rates during the GR. Over the business cycle, these variables also show the most significant interaction; there is evidence that firms with the lowest growth opportunities are more affected by increasing uncertainty. This makes sense given the model studied in chapter two, where uncertainty raised the marginal value of capital required to justify investment. Firms with high enough growth opportunities will not be affected by increases in uncertainty. However, there is evidence that high growth opportunities were less effective at attenuating the negative relationship between uncertainty and investment in the years after the GR. It also appears that firms were less able to make use of their growth opportunities after the crisis because the coefficient on Q falls in the years after the GR compared to those before.

The results support the theory that the real options effect plays a big role in determining investment over the business cycle and especially during crises, when uncertainty is very high (Bloom, 2009; Jurado et al., 2015). In contrast, there is little evidence to suggest that firms in the U.S. were hampered by financing issues during or immediately after the GR. Despite these variables having a statistically significant relationship with the investment rate, neither the average cash flows nor leverage ratios held by firms greatly changed during this period compared to uncertainty and growth opportunities. This corroborates with earlier findings in Banerjee et al. (2015). Overall, the results highlight that the combination of declining growth opportunities and high uncertainty mattered most for determining the fall in investment during the GR. Recessions come from different sources, but after recessions originating in financial markets this chapter suggests sending clear signals to the market about future policy in order to reduce uncertainty may placate the effect of the recession on corporate investment

The rest of this chapter is organised as follows. Section 3.2 first reviews the literature on the determinants of firm-level investment. Information on the sample and the econometric specification is outlined in 3.3. The results and robustness checks are presented and discussed in 3.4 before a brief conclusion summarises the chapter.

## 3.2 Related Literature

### 3.2.1 Balance Sheets and Investment

The Modigliani & Miller (1958) hypothesis states that in the absence of financial frictions, a firm's capital structure should not matter for investment decisions. This has since been challenged on numerous fronts. In a relatively simple model of a firm's decision to expand capacity, Myers (1977) shows that debt financing can prevent firms undertaking profitable investment. The additional liability creates a wedge between the cost of acquiring the new asset and the increase in the value of the firm once is is installed, so the assets that would be purchased if funded through internal funds are avoided under debt financing. Firms with higher debt to equity ratios also pose a higher risk of default and will therefore pay higher interest rates on their debt, which in turn leads to credit rationing and cuts leveraged firms off from a primary channel of funding investment (Merton, 1974; Stiglitz & Weiss, 1981). Firm's who cannot use debt to finance otherwise profitable investments are said to suffer from 'debt overhang'. Lang et al. (1996) noted that leverage reduces investment for firms with low growth prospects, as measured by Tobin's Q, but does not have a significant effect for firms with higher growth prospects. Their finding implies that firms with sufficiently high growth prospects can accumulate higher debt before they face binding credit constraints.

In the absence of access to loanable funds, cash flows may become a more important determinant of corporate investment. As a measure of the liquid assets coming into a firm in a given period, cash flows provide an indication of the internal funds available for investment projects (Gilchrist & Himmelberg, 1995). Early empirical work by Fazzari et al. (1988) found evidence that firms with cash flow issues and those facing borrowing constraints have lower investment spending compared to more profitable firms. This is especially true for smaller and younger firms who represent a riskier prospect for financial intermediaries and investors. However, in contrast to the prediction of Fazzari et al. (1988), Lang et al. (1996) found that lower leverage had a larger positive impact on investment compared with higher cash flows.

Bernanke et al. (1996) outline how these financial market imperfections help propagate macroeconomic shocks. When the economy stalls, some firms become credit constrained, perhaps because lenders call in their debts or because drops in equity prices or increases in risk premiums increase leverage ratios. Firms must delever before undertaking more investment financed through the debt channel. Furthermore, the fall in consumer spending reduces cash flows which further cripples the firms' capability to fund new projects. This 'financial accelerator' mechanism is similar to the 'sudden stop' literature explained by Mendoza (2010) where inflows of foreign capital dry up after a shock to the financial system. This mechanism was examined as a key component in propagating the impact of the 1990s Asian crisis on the real economy of South-East Asian nations (Coulibaly & Millar, 2011; Dagher, 2014). Like in Fazzari et al. (1988), smaller firms were particularly at risk of facing binding borrowing constraints when highly levered.

A large unexpected shock with origins in the financial sector, like the GR, will intensify firms' balance sheet deterioration and by the financial accelerator and sudden stop theories could lead to a protracted decline in investment expenditure. Gebauer et al. (2018) and Kalemli-Özcan et al. (2022) study the effects of debt overhang on European firms after the GR. The former study suggests a debt-to-asset ratio over 80% distorts investment spending through higher default risks and costs of debt financing. The latter uses a unique firm-level dataset which links individual firms with their primary bank and estimate that 40% of the drop in aggregate investment after the GR was caused by debt overhang.

Fewer studies have examined the relationship between non-financial firms' balance sheets and investment in the U.S. after the GR, with more attention given to household and investment bank debt-to-equity ratios (Gertler & Gilchrist, 2018). One exception is Giroud & Mueller (2016). Their key finding is that more levered firms cut back on employment, but they also find that, just like in Europe, firms with higher leverage ratios find it harder to raise capital and cut back more on investment. It is worth noting that when considering cross-country differences, the organisation of the financial sector is important. Market-based systems, like the U.S., are more likely to rely on internally generated funds compared to bank-based systems, hence, cash flows may be more important in determining investment (Bond et al., 2003).

On aggregate, Banerjee et al. (2015) note that the issuance of debt and equity remained relatively strong in the U.S. after the recession, meaning there was no substantial drop in the supply of funds from these sources. Additionally, compared to a firm with the same characteristics in the 1990s, U.S. firms after the crisis held 10% more cash after the Great Recession (Pinkowitz et al., 2013). Bliss et al. (2015) show that dividend payments and share repurchases both fell after the crisis. This trend was most notable for highly levered firms, who can then use these funds to maintain cash balances and also potentially fund investment. The key point is that for high-growth firms with lower leverage, access to lines of credit may not have changed after the recession.

### 3.2.2 Uncertainty and Investment

As outlined chapter two, a firm's real option to expand capacity is more valuable during times of uncertainty. In that model, the required added value of the marginal unit of capital which justifies expansion of the capital stock is higher in higher uncertainty regimes. Most studies seeking to test this hypothesis on the firm-or-establishment level base their measure of uncertainty on the volatility of firms' idiosyncratic stock returns. In theory, a firm's stock price reflects the expected present discounted value of all its future profit flows, taking into account all relevant information available at the time (Malkiel, 2003). A representative sample indexing the value of U.S. equities should reveal investors' estimate of future profit flows. The average deviation of changes in stock prices around the mean gives an indication of how difficult it is for investors to predict the future profitability of firms, taking into account both idiosyncratic information about the firm and information about the macroeconomy. Hence, stock market volatility is in theory a good measure of economic uncertainty (Leahy & Whited, 1996).

However, previous studies recognise that some of the variation in stock prices is forecastable and, therefore, not uncertain. While Bloom et al. (2007) and Baum et al. (2008) use the simple standard volatility of daily stock returns each year to measure uncertainty, Bulan (2005), Panousi & Papanikolaou (2012) and Gilchrist et al. (2014) use the residuals from an asset pricing model. This chapter builds on Gilchrist et al.'s method because they use the four factor Fama-French-Carhart four factor model rather than a single-factor model including only the market portfolio. One potential drawback of their approach was already pointed out by Leahy & Whited (1996) who note that uncertainty should be based on future expectations rather than ex post data. They construct forecasts of firms' idiosyncratic uncertainty and their exposure to market uncertainty using a panel-data VAR model. As mentioned, stock prices today should reflect the markets best estimation of a firm's future value so in this sense they already contain information based on expectations. None of these studies consider the volatility of errors based on the difference between predicted returns and actual returns, which in theory is closer to the idea of uncertainty because it captures the difference between the expected change in the value of the firm (predicted returns) and the change actually observed. This chapter considers this extension.

The findings of these studies have been broadly consistent; higher idiosyncratic uncertainty has a negative relationship with investment rates, consistent with the real options literature in Dixit & Pindyck (1994). Once idiosyncratic volatility is accounted for, there is not a significant relationship between investment rates and a firms' exposure to aggregate market uncertainty (Leahy & Whited, 1996; Bulan, 2005). In Leahy & Whited (1996) the negative relationship disappeared after controlling for Tobin's Q, which the authors interpret as evidence for the fact that uncertainty affects investment by changing Q. In contrast, Bulan (2005) found uncertainty affects investment even after controlling for Q and the marginal product of capital. Her results were also robust to including cash flows. In this chapter, uncertainty is significant even after controlling for Q but a statistically significant interaction term between the two reveals the effect of uncertainty on investment is neutralised by high values of Q, as the real option models in Abel & Eberly (1996) and (Dixit & Pindyck, 1994, pp.357-367) suggest. Rather than controlling for firm leverage in the regression model, both Leahy & Whited (1996) and Bulan (2005) scale firms' stock volatility by their debt-to-equity ratio based on the observed positive relationship between stock volatility and leverage (Aït-Sahalia et al., 2013).

After controlling for the credit spread between the yield on individual corporate bonds and a risk-free asset, Gilchrist et al. (2014) found that the significance of the estimated relationship between uncertainty and the investment rate was reduced. Credit spreads are a measure of how difficult it is for firms to access credit or the 'tightness' of financial conditions. When credit spreads are high it signals that lenders believe there is a higher risk that firms will default on their loans. Risk and uncertainty are related but distinct concepts, the outcome of a gamble can be uncertain but if the downside loss is relatively low then it should not be classed as high risk. Nonetheless, as the average deviation of economic variables from their expected historical mean increases, as they will in times of increasing uncertainty, the riskiness of investments will tend to increase. In this chapter, the ease of accessing credit is controlled for by the leverage ratio but it is worth noting that there is still a potential source of bias in the uncertainty-investment relationship if credit spreads are correlated with uncertainty after conditioning on leverage[2].

## 3.3 Data and Methodology

### 3.3.1 Sample

The sample is an unbalanced panel of 1658 U.S. firms gathered from Thomson Reuters' Worldscope database[3] between 1992 and 2019. I restricted the sample to focus on firms listed on a stock exchange because firms who are traded over the counter (OTC) usually publish relatively little book data

[2] Corporate bond data does not have sufficient coverage in the Worldscope database to include a firm-level credit spread measure in this chapter.

[3] It is more common to use Compustat when focusing on a sample of U.S. firms. Ulbricht & Weiner (2005) notes that the two databases lead to comparable results, with similar coverage of firms' accounts. Because most cited studies use Compustat, it is interesting to examine whether a different sample leads to different conclusions. The baseline models 3.4.1 suggest estimated coefficients are similar in sign and magnitude compared to previous literature, so there is no evidence that very different conclusions will be drawn from the different datasets.

and therefore display a large number of missing values in their series. Nofsinger & Varma (2014) also note that they do not have the same level of regulatory oversight and trade in their equity is dominated by individual investors trading on speculation rather than the fundamental value of the firm. As discussed, using stock prices as a measure of uncertainty makes the assumption that a firm's market value reflects investor's best estimate of the future discounted profit flows of the firm.

Banks, financial service firms, insurers and real estate investment funds were not sampled on the basis that firms in these sectors are likely to have different optimal investment strategies, are likely to operate under investment insurance schemes often unavailable to other firms, and are subject to different regulations in terms of capital requirements (Medina, 2012). While some studies such as Leahy & Whited (1996) restrict their focus to manufacturing firms on the basis that they should be the driving force of fixed investment, this is not followed in this paper and service firms are included in the analysis. Gilchrist et al. (2014) and Kalemli-Özcan et al. (2022) also include service firms. Indeed, the results suggest there is no statistically significant difference in investment rates between service firms and manufacturing firms. There may still be some concern than service sector and manufacturing firms face different regulations that are not controlled for in this study which potentially affects the results. However, estimated coefficients were similar when service sector firms were removed from the sample.

The dependent variable is the natural log of the investment rate: $\ln\left(\frac{I_{it}}{K_{i,t-1}}\right) \equiv \psi_{it}$ where $I_{it}$ is annual expenditures on capital goods and $K_{i,t-1}$ is last year's stock of property plant and equipment, where $i \in \{1, ..., 1658\}$ and $t \in \{1, ..., 28\}$. A large number of firms have some missing values for the dependent variable. This is largely due to the increasing coverage of the Worldscope database throughout the 1990s. Figure 3.3.1 shows how the number of firms in the sample grows until the year 1999 before stabilising between 1150 and 1200. The introduction of new firms into the sample could affect estimates if those firms have characteristics which affect investment that cannot be controlled for in the database. This is a form of sample selection bias. For example, managers of firms that have just been listed may have a stronger preference for expansion compared to more established firms. I test for this by comparing the results from the full sample to those of a restricted sample including only those firms who are present from the start. A similar restriction includes only firms present from the year 2000 onwards. Neither significantly alters the results.

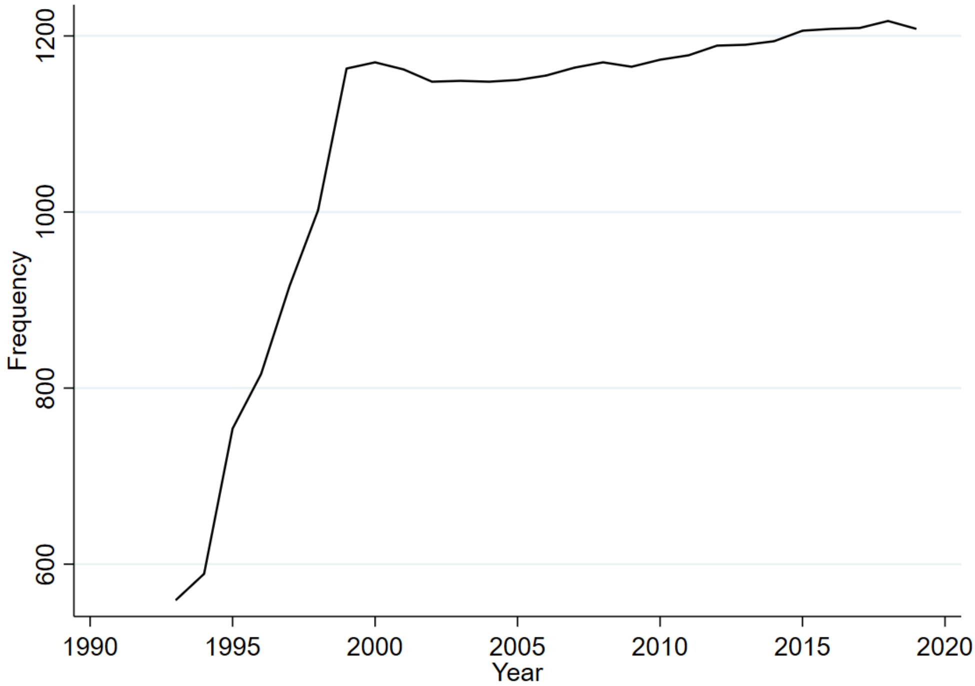



Figure 3.3.1: Number of Firms in the Sample over Time

The histogram in figure 3.3.2a shows the distribution of missing values for the dependent variable. As depicted, 42.76% of firms (709) have fewer than 10% of their observations missing, while 57.06% (931) have fewer than 20% missing. Firms with a very high number of missing values either arrive in the sample late or drop out soon after entering. I remove any firm with fewer than 10 observations to prevent these fluctuations in sample size from affecting the results. There is also a concern about the quality of book-keeping for firms who drop out of the sample and re-enter it at a later date. These firms might have lower regulatory standards or may have been temporarily removed from a listed stock exchange. Figure 3.3.2b shows the distribution of runs of missing values. Zero runs of missing values naturally means the firm has no missing values. All firms have at least one missing value because the dependent variable uses the lag of the capital stock, $K_{i,t-1}$. One run of missing values means once the firm entered the sample it was present in all subsequent years. Values greater than one are the number of times the firm left the sample. I remove any firm which leaves the sample more than twice.

Applying these restrictions to the dependent variable leaves 1058 firms. These firms have reasonably complete series for the other variables discussed in the following sections as well, suggesting the restrictions have removed firms with poor record keeping. One exception is cash flows, where about half of firms have roughly 50% of their values missing. Taking cash flows out of the baseline regressions does not change the signs of the coefficients and their magnitude remains similar. The standard errors decrease though, because the sample size increases by around 50%. Cash flows are highly significant in the regression models and correlated with other explanatory variables, so leaving it out biases the coefficients on other explanatory variables it is correlated with and creates correlation between the explanatory variables and the error term. Hence, I keep it in the model and note here that its inclusion inflates the standard errors.

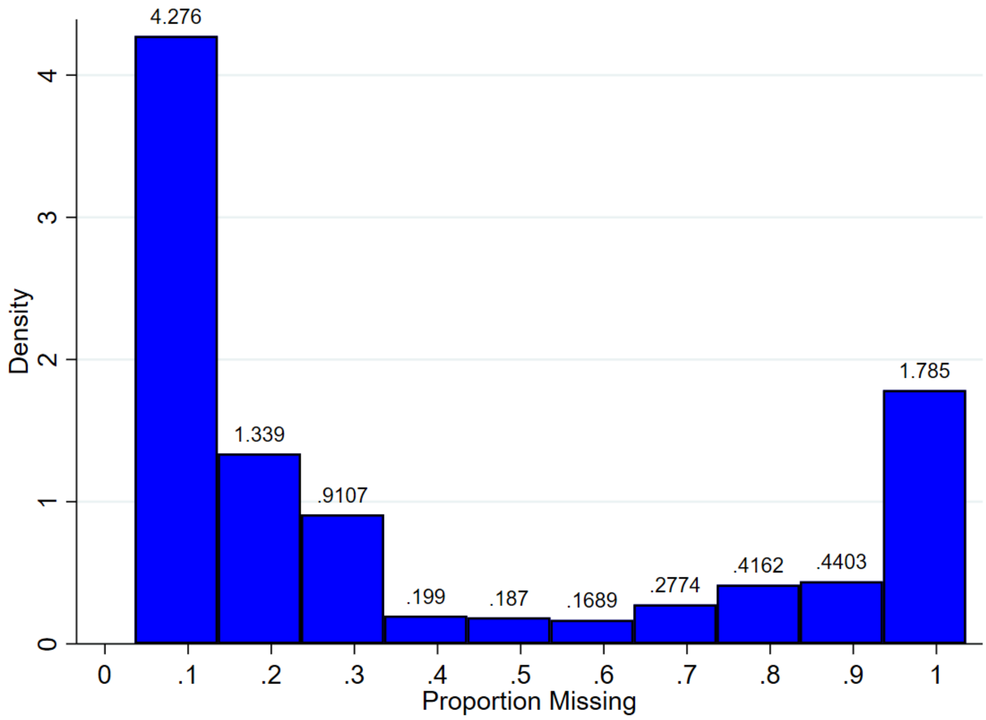
### 3.3.2 Book Variables

Book variables which influence investment rates other than average Q ($\tilde{Q}_{it}$)[4] are leverage ($LEV_{it}$), cash flows relative to the capital stock ($CF_{it}$), net sales relative to the capital stock ($Y_{it}$), and size ($\tilde{L}_{it}$) (the number of employees). Average Q is defined as in Chortareas et al. (2021) as the ratio of total liabilities plus market capitalization to common shareholder equity plus total liabilities. Leverage is total liabilities divided by total assets as in Kalemli-Özcan et al. (2022); total debt divided by common shareholder equity, another common measure of leverage (seeBulan (2005)) did not have a significant effect on the investment rate after controlling for the other variables. The sales to capital ratio is an indication of the marginal product of capital[5]. Although it does not take into account the difference between sales and production Gilchrist & Himmelberg (1998) found that there is a 0.99 correlation between firms' production levels (which tend to only be available for a few firms) and their sales. They note that $Y_{it}$ can be used as a measure of a firm's growth opportunities beyond $\tilde{Q}_{it}$ and call it 'fundamental Q'. To avoid confusion with the more conventional Tobin's Q, I will refer to it as capturing 'real growth opportunities' as opposed to the financial growth opportunities captured by $\tilde{Q}_{it}$.

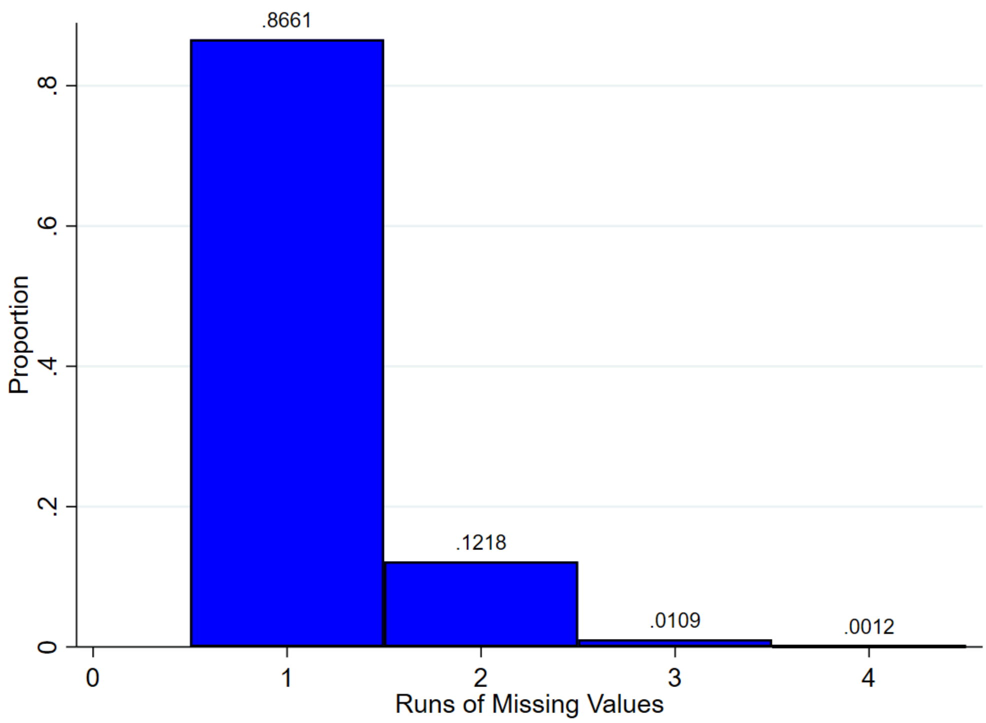
To reduce the influence of outliers, all variables were winsorised at the 0.5% level, setting any values above the $99.5^{th}$ or below the $0.5^{th}$ percentile equal to the value at that percentile. Win-

[4]Average Q and size are marked with a tilde because later I will take logs of these variables and work with logs for the rest of the chapter, the notation will be less cumbersome if $\ln\left(\tilde{Q}_{it}\right) = Q_{it}$.

[5]Suppose a firm produces with a Cobb-Douglas production function with inputs of capital and labour, $Y = K^{\alpha}L^{1-\alpha}$, then $Y/K$ is the marginal product of capital as a proportion of capital's constant share in income.




(a) Distribution of Missing Values by Firm for the Dependent Variable




(b) Runs of Missing Values for the Dependent Variable

Figure 3.3.2: Missing Values in the Dependent Variable

sorising is common in the literature and the choice of percentile affects fewer observations than others in the field (Bulan, 2005; Kalemli-Özcan et al., 2022). It is more appropriate than trimming the distribution in this case because there is no reason to believe these outliers are mistakes, for instance, service-sector firms with very small capital stocks will naturally generate very large sales to output ratios if they are successful. However some observations are very extreme and will have a significant influence on coefficient estimates (Chatterjee & Hadi, 1986). Even after winsorising, the distributions display a large positive skew and high kurtosis, so observations in the tails of the distributions might still have a large effect on the results. Taking the natural log of the variables produces distributions closer to normality. Table 1 gives some summary statistics of the book variables used in the model before taking the log. Note that the minimum value of $CF_{it}$ is zero so I add one to all observations before taking the log (Gilchrist et al., 2014).

| | N | Mean | Standard Deviation | Min | Max |
|---|---|---|---|---|---|
| $Y_{it}$ | 25997 | 1336.536 | 2747.955 | 19.39105 | 24469.05 |
| $\tilde{Q}_{it}$ | 26827 | 2.146946 | 1.696368 | .5286462 | 12.81989 |
| $CF_{it}$ | 17613 | 230.4786 | 703.2919 | 0 | 7064.167 |
| $LEV_{it}$ | 27139 | .5006233 | .2429175 | .0165959 | 1.529492 |
| $\tilde{L}_{it}$ | 26273 | 16587.11 | 42225.37 | 19 | 350000 |

Table 3.3.1: Summary Statistics for the Book Variables

### 3.3.3 Uncertainty

Uncertainty is measured by the idiosyncratic volatility of firms' stock returns. As discussed in section 3.2, the measure of uncertainty should be forward looking and free from forecastable variation. In theory, equity prices should already be forward looking because they represent the market's best estimate of the present discounted value of a firm. I use the four factor asset pricing model of Fama & French (1993) and Carhart (1997) to remove forecastable variation from stock returns. Gilchrist et al. (2014) use the standard deviation of the error term from this regression as their measure of firm-specific uncertainty. Formally, if $r_{it_d}$ represents the log difference in a firm's stock price on day

$t_d$

$$r_{it_d} - RF_{t_d} = \gamma_0 + \gamma_1 MKT_{t_d} + \gamma_2 SMB_{t_d} + \gamma_3 HML_{t_d} + \gamma_4 MOM_{t_d} + \upsilon_{it_d} \tag{3.3.1}$$

$$\sigma_{it}^{FFC} = \sqrt{\frac{\sum_{t_d=1}^{T_d} \left(\upsilon_{it_d} - \bar{\upsilon}_{it}\right)^2}{t_d - 1}}. \tag{3.3.2}$$

These regressions are run for each firm in each year in the sample, so $T_d$ is approximately 252, the number of trading days in the year. I have subscripted the terms in the first equation with $t_d$ to show that they change daily. As with the book variables in the previous section, $\sigma_{it}^{FFC}$ varies annually. $RF_{t_d}$ is the risk-free rate which means $\gamma_0$ reflects the risk-premium on the firm's stock when all other risk factors are zero. Notice the lack of an $i$ subscript because the risk-free rate varies over time but not across firms. The next four variables are the risk factors that explain variation in risk-free returns, available at the daily frequency from Kenneth French's website. $MKT_{t_d}$ is the risk-free return on the market portfolio, an equally weighted portfolio of all stocks on the NYSE, AMEX, and NASDAQ stock exchanges. $SMB_{t_d}$ is the difference in returns between a portfolio formed of firms with low market capitalisation and high market capitalisation. Analogously, $HML_{t_d}$ is the difference in returns between firms with a high book-to-market value and those with a low book-to-market value. In both cases the thresholds used to determine what is big (high) and small (low) are based on the percentiles of the data. $MOM_{t_d}$ reflects Carhart's (1997) observation that stocks with higher returns in the recent past tend to display higher returns in the near future.

Leahy & Whited (1996) criticised the use of ex post data to construct measures of uncertainty because uncertainty is about expectations and not actual outcomes. I use the parameters estimated in equation 3.3.1 to compute the error variance from the Fama-French-Carhart model (FEV) each day between the firm's expected return and their actual returns the year after the estimation period. This generates the difference between the firm's best estimate of their risk-free returns on that date, given they have previously estimated their stock's loadings on the risk factors, and what is actually observed. The model is trained on the previous three years of data. For example, the errors for the year 2000 are generated based on estimating 3.3.1 between the years 1997 to 1999 and then calculating the difference between predicted and actual returns in the year 2000. I chose a window of three years to strike a balance between using the data to produce reliable parameter estimates and losing years from the sample which must be kept back in training data. This method assumes all firms in the sample use the asset pricing model in 3.3.1 to estimate their loadings on the risk factors and update the model annually. Let this measure of uncertainty be denoted by $\sigma_{it}^{FEV}$. Because it addresses the issues raised by Gilchrist et al. (2014) and Leahy & Whited (1996), it will be the primary measure of uncertainty in the chapter.

I compare the results of this estimate to a forecast of next year's volatility, again based on three years of training data, assuming the variance of return errors in 3.3.1 follow a GARCH(1, 1) process. Hansen & Lunde (2005) found that including higher autoregressive and moving average terms generally does not improve GARCH estimates. If $\mathbf{X}_{it}$ generically denotes the explanatory variables,

$$\upsilon_{it_d}|\mathbf{X}_{it} \sim \mathcal{N}\left(0, \eta_{it_d}^2\right) \tag{3.3.3}$$

$$\eta_{it_d}^2 = \theta_0 + \theta_1 \upsilon_{i,t_d-1}^2 + \theta_2 \eta_{i,t_d-1}^2. \tag{3.3.4}$$

Annual uncertainty in this case is the mean of the forecasted daily GARCH volatility for the whole year[6]

$$\sigma_{it}^G = \frac{1}{T_d} \sum_{t_d=1}^{T_d} \eta_{it_d}. \tag{3.3.5}$$

Unlike previous studies, I examine whether the relationship between uncertainty and investment based on these measures of uncertainty differs from a measure based on the annual standard deviation of risk-adjusted returns. If the other measure produces results close to this baseline ($\sigma_{it}^B$), then there is evidence that the uncertainty measures are tainted by variation in stock markets not due to uncertainty (Jurado et al., 2015). Figure 3.3.3 shows the annual median idiosyncratic volatility across firms in the sample. The large increase in volatility during the late 1990s reflects dot-com

[6]GARCH is often initialised by setting the conditional variance on the first day equal to the unconditional variance for the whole sample. Once the volatility on the first day has been determined, 3.3.4 can be estimated from the model error and the initial value of $\upsilon_{it_d}^2$.

bubble's formation and burst throughout 2000-2001. This bubble highlights Jurado et al.'s criticism of equity market-based-measures of uncertainty, while the formation of the bubble may partly be due to the uncertainty about the future profitability of the new wave of technology start-ups, it also reflects investor's over-reaction to news and the influence of noise traders (De Bondt & Thaler, 1987; De Long et al., 1991). It is therefore important to examine how removing this event from the sample changes the results in the next section.

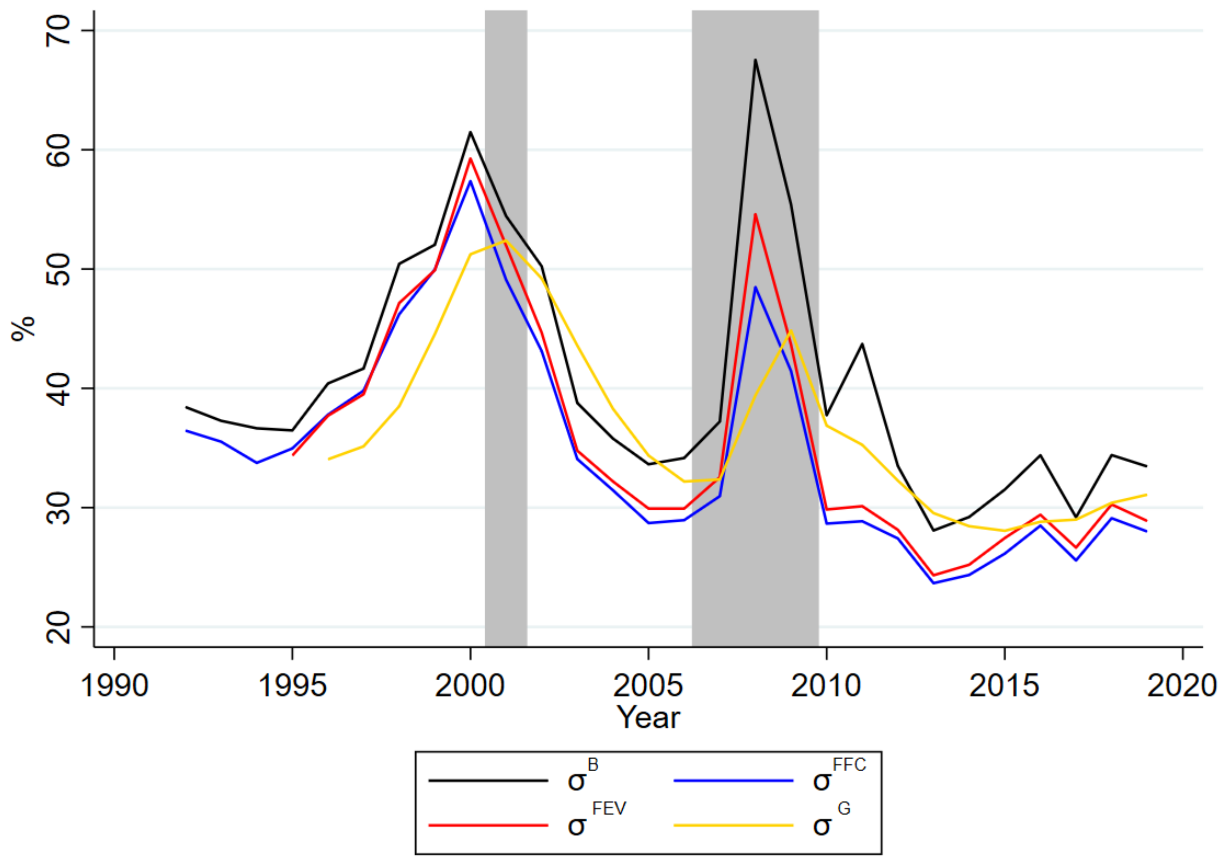



Figure 3.3.3: Uncertainty Measures

### 3.3.4 Econometric Specification

Equation 3.3.6 gives the full econometric specification of the log of the investment rate ($\psi_{it}$), where the lower case letters or the absence of a tilde in the explanatory variables indicates that they are in logs. $D_j$ are dummy variables for each year in the sample controlling for time variant shocks common to all firms, which can bias estimates and create correlation in the errors across firms in the sample

$$\psi_{it} = \beta_0 + \beta_1 y_{i,t-1} + \beta_2 Q_{i,t-1} + \beta_3 \sigma_{it} + \beta_4 lev_{i,t-1} + \beta_5 cf_{i,t-1} + \beta_6 L_{i,t-1} + \alpha_1 \psi_{i,t-1} + \sum_{j=1}^{24} \delta_j D_j + u_i + \epsilon_{it}. \quad (3.3.6)$$

There are 24 time dummies for the years 1996 to 2019. Three years from the sample were held back in training data for the measures of uncertainty. Notice also that the explanatory variables are all lagged one year in 3.3.6, so the end of year investment rate in year $t$ is affected by the variables as they were in year $t-1$. This reflects the fact that investment projects take time to plan, finance, and ultimately install (Fazzari et al., 1988; Bulan, 2005; Tori & Onaran, 2018). The only variable which has a contemporaneous affect on the investment rate is uncertainty, which is a measure of how hard it is for a manager to predict economic conditions over the next year. All variables enter the equation in logs to reduce the positive skew of the distributions and mitigate the influence of observations in the tails. This also means the coefficients can be interpreted as the elasticity of the investment rate with respect to the explanatory variables.

$u_i + \epsilon_{it}$ is the model's error term; with $u_i$ representing time-invariant factors not included in the model and $\epsilon_{it}$ doing the same for time-variant factors. The latter is assumed to be uncorrelated with the explanatory variables but I relax this assumption in one of the models considered in section 3.4.3. But it is very likely that some of the explanatory variables are correlated with $u_i$, for example, one component of $u_i$ might be the firm's risk aversion, more risk-averse firms will have lower leverage on average and may also have lower investment rates since adding to the capital stock means incurring a partially irreversible cost. This implies a fixed effects estimator is more appropriate because of the potential bias introduced by the correlation between the explanatory variables and error term. The fixed effects estimator works by subtracting the sample mean from all variables for each firm, hence, removing any effects which do not change over time[7]. Indeed, for a model without the lagged dependent variable, the estimated correlation between the time invariant error and the explanatory variables is -0.61 and a Hausman (1978) test suggests that there is a systematic difference between the fixed effects and random effects coefficients ($\chi^2(32) = 695.94$, p-value $= 0$). Clustered standard errors are used in light of the possible heteroskedasticity in the time varying error term.

Bloom et al. (2007), Bulan (2005), and Gilchrist et al. (2014) recognise the importance of

[7] I add back the mean across observations and over time in order to estimate $\beta_0$.

the first lag of the investment rate in explaining current rates. Eberly et al. (2012) argue that lagged investment rates predict current rates so well because managers who are undecided about what investment budget to set tend to revert to last year's value as a reference point (Bloom et al., 2012). However, dynamic panel data models come with further challenges because of the correlation between $u_i$ and $\psi_{i,t-1}$. Even after removing the time-invariant error by the fixed-effects transformation, Nickell (1981) pointed out that the mean of the dependent variable will contain information correlated with the mean of the error term, which introduces an invariably negative bias to the parameter $\alpha_1$ in equation 3.3.6.

First differencing the model, as oppose to time demeaning, also removes unobserved time-invariant heterogeneity but because $\Delta\psi_{i,t-1} = \psi_{i,t-1} - \psi_{i,t-2}$ and $\Delta\epsilon_{it} = \epsilon_{it} - \epsilon_{i,t-1}$ there is still a source of endogeneity because by equation 3.3.6 $\psi_{i,t-1}$ was generated using the error $\epsilon_{i,t-1}$. First differencing also magnifies gaps in unbalanced panels and throws out potentially useful information. Arellano & Bover (1995) suggest using a forward-orthogonal deviations (FOD) transformation instead, which subtracts the forward mean from each observation within each panel. Formally, for a generic variable $x_{it}$, the FOD transformation $x_{it}^{\perp}$ is:

$$x_{it}^{\perp} = x_{it} - \frac{1}{T-t}\sum_{s=t+1}^{T-t} x_{is} \tag{3.3.7}$$

so the FOD-transformed error is:

$$\epsilon_{it}^{\perp} = \epsilon_{it} - \frac{1}{T-t}\sum_{s=t+1}^{T-t} \epsilon_{is} \tag{3.3.8}$$

and for lagged dependent variable, this can be written as

$$\psi_{i,t-1}^{\perp} = \psi_{i,t-1} - \frac{1}{T-t+1}\psi_{it} - \frac{1}{T-t+1}\sum_{s=t+1}^{T-t} \psi_{is} \tag{3.3.9}$$

I will refer to GMM models using a FOD transformation as 'deviations GMM'.

Anderson & Hsiao (1982) proposed using further lags of the dependent variable as instruments for the lag of the first difference ($\Delta\psi_{i,t-1}$). Although $\Delta\psi_{it}$ is correlated with $\Delta\epsilon_{it}$ through $\psi_{i,t-1}$, the next lag, $\psi_{i,t-2}$, should be correlated with $\Delta\psi_{it}$ and under the assumption that the errors are not serially correlated it will also be exogenous. With the FOD transformation, there is a source of endogeneity between $\psi^{\perp}_{i,t-1}$ and $\epsilon^{\perp}_{it}$ through the term $\frac{1}{T-t+1}\psi_{it}$ in equation 3.3.9. However, the first lag of $\psi_{it}$ should now be available as an instrument because $\epsilon^{\perp}_{it}$ contains no terms of order $t-1$. The availability of one extra lag is another advantage of the FOD transformation.

The efficiency of this instrumental variable approach can be improved by re-imagining the assumed exogeneity of the instruments as moment conditions and allowing the number of lags available as instruments to grow with the sample. For exposition, if $t=5$ the moment conditions between the instrument matrix $\mathbf{Z}_{it}$ and the FOD transformed error, $\epsilon^{\perp}_{it}$ are

$$\mathbb{E}\left[\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \psi_{i1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_{i2} & \psi_{i1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_{i3} & \psi_{i2} & \psi_{i1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_{i4} & \psi_{i3} & \psi_{i2} & \psi_{i1} \end{pmatrix}' \begin{pmatrix} \epsilon^{\perp}_{i1} \\ \epsilon^{\perp}_{i2} \\ \epsilon^{\perp}_{i3} \\ \epsilon^{\perp}_{i4} \\ \epsilon^{\perp}_{i5} \end{pmatrix}\right] = 0. \tag{3.3.10}$$

Notice that the $\mathbf{Z}_{it}$ matrix has to be transposed before multiplication. There is a $10 \times 5$ matrix and a $5 \times 1$ vector inside the expectation, so their product will yield a $10 \times 1$ column vector. The first row contains all zeros because there are no moment conditions to specify at $t=1$. Multiplying the matrices inside the expectation reveals that each moment condition for every lag at every time period is expressed individually, so the vector of moment conditions is

$$\left[\mathbb{E}[\psi_{i1}\epsilon^{\perp}_{i2}]\ \mathbb{E}[\psi_{i2}\epsilon^{\perp}_{i3}]\ \mathbb{E}[\psi_{i1}\epsilon^{\perp}_{i3}]\ \mathbb{E}[\psi_{i3}\epsilon^{\perp}_{i4}]\ \mathbb{E}[\psi_{i2}\epsilon^{\perp}_{i4}]\ \mathbb{E}[\psi_{i1}\epsilon^{\perp}_{i4}]\ \mathbb{E}[\psi_{i4}\epsilon^{\perp}_{i5}]\ \mathbb{E}[\psi_{i3}\epsilon^{\perp}_{i5}]\ \mathbb{E}[\psi_{i2}\epsilon^{\perp}_{i5}]\ \mathbb{E}[\psi_{i1}\epsilon^{\perp}_{i5}]\right]' = 0.$$

This means the number of moment conditions will almost always be greater than the number of explanatory variables and the model will be overidentified. GMM estimation is used in this instance to minimise the magnitude of the matrix of moment conditions, essentially choosing the GMM-estimated coefficients such that $\mathbb{E}[\mathbf{Z}'_{it}\epsilon^{\perp}_{it}]$ is as close to zero as possible. The algebraic derivation of this estimator, and discussion of its asymptotic and small sample efficiency, can be found in Roodman (2009*b*). This chapter uses the two-step estimator outlined therein, which generally improves efficiency compared to one-step GMM, and makes the small sample correction proposed by Windmeijer (2005).

Explanatory variables suspected of being endogenous can also be included in $\mathbf{Z}_{it}$ and instrumented for using their lagged values. To be clear, an endogenous variable in this context is one which is correlated with the contemporaneous time-varying error term $\epsilon_{it}$. Insofar as these lagged variables affect the investment rate, they also help achieve a more efficient estimate of $\alpha_1$. It is common in the GMM-literature to distinguish endogenous regressors from predetermined regressors. The latter are not correlated with the contemporaneous error but are correlated with past errors, so $\mathbb{E}[x_{it}\epsilon_{i,t-1}] \neq 0$. The first column of table 3.4.3 allows the regressors to be correlated with the time varying error term. For the models in table 3.4.1, the assumption that the explanatory variables are uncorrelated with $\epsilon_{it}$ rules out reverse causality between uncertainty and the investment rate. Gilchrist & Himmelberg (1998) and Gilchrist et al. (2005) are examples of studies using panel-data VAR models in the corporate investment literature.

If the error term is not autocorrelated or correlated across individuals, the GMM estimation method of Arellano & Bond (1991) reduces the bias and variance of the estimated $\hat{\alpha}_1$. Arellano & Bover (1995) and Blundell & Bond (1998) then developed a system GMM approach, where both the levels and differenced (or FOD) forms of equation 3.3.6 are used to recover $\alpha_1$. They use lagged differences of the dependent variable as instruments in the level equation and lagged levels as instruments in the differenced equation. This method is even more efficient but requires the additional assumption that the differenced instruments used in the level equation are uncorrelated with the time-invariant error-term $u_i$ (Roodman, 2009*b*).

While these models offer a convenient way to estimate dynamic relationships, they are also complex and impose non-trivial assumptions on the data. Arellano & Bond (1991) developed a test for serial correlation in the residuals, which will violate the exogeneity assumption of the instruments if detected. Unlike single instruments in two-stage-least-squares, a Hansen test is available to check whether there is evidence to suggest the instruments are truly exogenous. Under the null that the moment conditions are valid and the instruments are exogenous, the Hansen statistic will follow a $\chi^2$ distribution with degrees of freedom equal to the number of instruments minus the number of estimated coefficients (this is just the number of overidentifying restrictions). Roodman

(2009*a*) warns that this test is weakened by the number of instruments in the model, which grows quadratically with the number of time periods available in the Arellano & Bond (1991) estimator and quartically in the system GMM approach. There are essentially two ways to limit the number of instruments in the model. The first is to choose a maximum lag length for the instruments in $\mathbf{Z}_{it}$ and the second is to 'collapse' the matrix so the moment conditions are written as one equation for each time period rather than expressing each moment condition individually. Continuing the previous example where $t = 5$, the moment conditions become

$$\mathbb{E}\left[\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \psi_{i1} & 0 & 0 & 0 & 0 \\ \psi_{i2} & \psi_{i1} & 0 & 0 & 0 \\ \psi_{i3} & \psi_{i2} & \psi_{i1} & 0 & 0 \\ \psi_{i4} & \psi_{i3} & \psi_{i2} & \psi_{i1} & 0 \end{pmatrix}' \begin{pmatrix} \epsilon_{i1}^{\perp} \\ \epsilon_{i2}^{\perp} \\ \epsilon_{i3}^{\perp} \\ \epsilon_{i4}^{\perp} \\ \epsilon_{i5}^{\perp} \end{pmatrix}\right] = 0. \tag{3.3.11}$$

## 3.4 Results

### 3.4.1 Comparison of Models

Table 3.4.1 compares the estimated coefficients in equation 3.3.6 generated by five linear estimators, all of which maintain the assumption that the regressors are uncorrelated with the time varying error term $\epsilon_{it}$. All the variables in the model are in logs, so the coefficients should be interpreted as the percentage change in the investment rate caused by a 1% change in the given variable, holding all other explanatory variables and the time-invariant error term constant. The reported $\chi^2$ statistic with $J - r$ degrees of freedom is the test statistic for the Hansen test of the over-identification restrictions, where $J$ is the number of instruments and $r$ is the number of estimated coefficients. The A-bond statistic is for the Arellano & Bond (1991) test for serial correlation in the residuals. Time dummies are included in all regressions and are always jointly significant at the 1% level. All results tables in this section show t-statistics in parentheses under the estimated coefficients and the stars indicate statistical significance at the 1%, 5%, and 10% levels.

The first column shows the results from a fixed effects model without including the lagged dependent variable. Contrasting these results with the third column demonstrates the changes on the coefficients due to the omitted variable bias. Although the fixed effects estimator in column three is also expected to be biased because of the correlation between the lagged dependent variable and $\epsilon_{it}$ discussed in Nickell (1981), the signs and significance of the coefficients that it generates are supported by the results of the instrumental variable GMM estimators.

Columns two and three set bounds for the expected value of the coefficient on $\psi_{i,t-1}$. The OLS estimator is known to be biased in panel-data models where the regressors are correlated with the time invariant error $u_i$. In the case of the lagged dependent variable, this bias is expected to be positive. $u_i$ can be thought of as the average unexplained investment rate over the sample. A negative exogenous shock to the investment rate at time $t$ will cause a negative deviation of the unexplained investment rate from the sample average, so $u_i$ will appear to be lower over the whole sample. However, at $t+1$ both $u_i$ and the lagged dependent variable will be lower. Hence, there is an expected positive correlation between $u_i$ and $\psi_{i,t-1}$ so the coefficient $\hat{\alpha}_1$ should be upward biased. Roodman (2009*b*) notes that the expected positive bias of the OLS estimate for the lagged dependent variable and the invariably negative bias of the fixed effect estimator provide convenient thresholds for determining an acceptable range of $\alpha_1$. This is important because the estimates from the GMM models in columns four and five of table 3.4.1 are sensitive the researchers assumptions.

I now focus on interpreting the coefficients of the deviations GMM model in column four, which are consistent as long as the explanatory variables are uncorrelated with the time varying error term. In theory, growth opportunities alone should be all that is required to predict investment rates, with the rest of the variation being the result of exogenous shocks. The model has been further augmented by several other variables which previous empirical and theoretical work suggests is correlated with investment rates.

In this model, which is more comprehensive in its range of explanatory variables than others in the field, the acceptable range for the coefficient of the lagged dependent variable is between 0.506 and 0.254. The coefficient on $\psi_{i,t-1}$ is 0.282, and therefore falls within this range. This estimate is 11% greater than the one obtained from fixed effects. The bias of the coefficient on the lagged dependent variable is known to diminish as the time-dimension of the panel increases. Judson & Owen (1999) showed with Monte-Carlo simulations that with 30 periods the bias falls to around

20% of the true parameter value. Therefore, it is reasonable to expect in this sample that the magnitude of the bias is quite small, and the fixed effect estimates will lie close to GMM.

As in Gilchrist & Himmelberg (1998), Bulan (2005), and Gulen & Ion (2016), both real and financial growth opportunities have a significant positive effect on the log of the investment rate. A 1% increase in net sales to capital stock ratio in year $t-1$ causes a 0.26% increase in the investment rate, holding other explanatory variables and individual heterogeneity constant. Financial growth opportunities have a similar effect, with the 1% increase in Tobin's average $Q$ causing a 0.34% increase in the investment rate. This is consistent with the q-theory literature since Hayashi (1982) but the magnitude of the coefficient is notably lower than the 0.645 found by Gilchrist et al. (2014) in a model which only controls for uncertainty.

Cash flows and size both have their expected signs, but have a much weaker effect on the investment rate compared to growth opportunities. The positive relationship between cash flows and investment rates is well-established, but it is interesting that it remains significant even after controlling for financial and real growth opportunities as well as leverage. It suggests that even after controlling for access to credit markets and financial performance, cash-strapped firms will still invest less than those with a healthy flow of liquid assets. Smaller firms tend to invest a higher proportion of their capital stock partly because expansion is easier at smaller scales but also because growth is an important part of survival for small firms in competitive markets (Evans, 1987). A 1% increase in the number of employees causes a 0.06% fall in the investment rate.

The effects of leverage and uncertainty are both negative and significant at the one percent level. The coefficient on uncertainty is about 0.03 percentage points greater than that found by Gilchrist et al. (2014) in a model with only Tobin's $Q$ as a control variable. A 1% increase in either of these variables causes a 0.12% fall in the investment rate, so the strength of the effect of a 1% change is not as strong as for growth opportunities or the lagged dependent variable, but is greater than that of size and cash flows. Given leverage is statistically significant, there is evidence that U.S. firms in financial distress struggle to access credit markets to fund investment. This agrees with the results in Lang et al. (1996), who used a smaller dataset of U.S. firms (640 firms sampled over 19 years).

The decline in the predicted investment rate due to uncertainty occurs even when controlling for a firm's access to credit markets through leverage and available liquid assets through cash flows.

Gilchrist et al. (2014) found that controlling for credit spreads between corporate bonds and a proxy for the risk free interest rate, which they also interpret as a measure of credit tightness, rendered uncertainty's effect on investment statistically insignificant. This difference in results may be because credit spreads are a better indication of the willingness of investors to lend to distressed firms or because it captures other factors which affect investment and also correlate with uncertainty, such as risk preferences.

| | Fixed Effects | OLS | Fixed Effects II | Deviations GMM | System GMM |
|---|---|---|---|---|---|
| $y_{i,t-1}$ | 0.407*** | 0.0801*** | 0.264*** | 0.256*** | 0.237*** |
| | (15.73) | (8.81) | (11.52) | (10.46) | (9.43) |
| $Q_{i,t-1}$ | 0.389*** | 0.225*** | 0.339*** | 0.334*** | 0.343*** |
| | (15.54) | (15.08) | (15.54) | (15.92) | (15.76) |
| $\sigma_{it}^{FEV}$ | -0.125*** | -0.0360** | -0.128*** | -0.118*** | -0.127*** |
| | (-4.49) | (-2.20) | (-5.42) | (-5.15) | (-5.06) |
| $lev_{i,t-1}$ | -0.154*** | -0.0160 | -0.110*** | -0.118*** | -0.112*** |
| | (-6.15) | (-1.06) | (-5.39) | (-5.66) | (-5.24) |
| $cf_{i,t-1}$ | 0.0500*** | 0.0368*** | 0.0444*** | 0.0449*** | 0.0424*** |
| | (5.51) | (6.32) | (5.62) | (5.76) | (5.37) |
| $L_{i,t-1}$ | -0.0210 | 0.00653 | -0.0552*** | -0.0620*** | -0.0414*** |
| | (-1.08) | (1.23) | (-3.52) | (-4.05) | (-2.95) |
| $\psi_{i,t-1}$ | | 0.506*** | 0.254*** | 0.282*** | 0.303*** |
| | | (33.67) | (15.76) | (12.85) | (12.59) |
| $\beta_0$ | 1.464*** | 0.959*** | 1.684*** | | 1.619*** |
| | (5.81) | (8.53) | (8.22) | | (7.69) |
| $N$ | 15501 | 15497 | 15497 | 14444 | 15497 |
| $R^2$ | 0.261 | 0.491 | 0.311 | | |
| $RMSE$ | 0.565 | 0.612 | 0.546 | | |
| $J$ | | | | 55 | 57 |
| $\chi^2$ | | | | 23.58 | 43.92 |
| p-value | | | | 0.486 | 0.0111 |
| A-Bond | | | | 1.038 | 1.165 |
| p-value | | | | 0.299 | 0.244 |

Table 3.4.1: Regression Results

Under the assumption that the regressors other than the lagged dependent variable are unrelated to the time-varying error, the deviations GMM model passes the basic diagnostic tests. The number of instruments was limited by collapsing the $\mathbf{Z}_{it}$ matrix as demonstrated in 3.3.11. The Hansen-test $\chi^2$ statistic with 24 degrees of freedom is 23.58 and the associated p-value is 0.486, thus failing to reject the null hypothesis that the moment conditions are valid and the instruments are exogenous from the FOD-transformed error. Roodman (2009*a*) warned that p-values over 0.25 may be a sign that the test statistic has been weakened by the number of instruments, however, with only 55 instruments and 1011 panels in the sample, it seems unlikely that instrument proliferation is an issue in this case. The test for serial correlation devised by Arellano & Bond (1991) suggests there is no evidence that the residuals are autocorrelated at the second lag.

When the coefficient estimate on the lagged dependent variable is close to the fixed effect estimate, it is good practice to compare the conclusions of deviations GMM to the system GMM estimator outlined in Blundell & Bond (1998). As mentioned in section 3.3.4, this estimator requires the additional assumption that lagged differences of the dependent variable are not correlated with the untransformed time varying error $\epsilon_{it}$ or the fixed effects $u_i$, so they are available as instruments for $\psi_{i,t-1}$ in the levels equation. Although there is still no evidence of serial correlation in the residuals, the Hansen test rejects the null hypothesis that the instruments are valid at the 5% level, implying the lagged differences of the investment rate are correlated with the composite error term. The coefficient on $\psi_{i,t-1}$ has increased by 0.021 percentage points but the estimate is not valid.

### 3.4.2 Uncertainty Measures

Table 3.4.2 presents the results of running the same models as in table 3.4.1 but with the other measures of uncertainty constructed from firm-level stock price data in section 3.3.3. All measures have a negative relationship with investment, which is significant at the 1% level in all instances except under the biased OLS. The coefficient on $\sigma^G_{it}$ is notably smaller in magnitude compared to the other measures.

The results in 3.4.2 highlight an important point which has generally been overlooked in the uncertainty literature so far, that the various methods of purging stock returns of their forecastable component and ensuring the measure is forward-looking does not lead to significantly different results compared to using the simple standard deviation of stock returns. Table 3.4.2 could be

| | Fixed Effects | OLS | Fixed Effects II | Deviations GMM | System GMM |
|---|---|---|---|---|---|
| $\sigma_{it}^{B}$ | -0.111*** | -0.0303* | -0.122*** | -0.116*** | -0.127*** |
| | (-3.86) | (-1.74) | (-5.08) | (-4.99) | (-5.10) |
| $N$ | 15766 | 15762 | 15762 | 14706 | 15762 |
| $\sigma_{it}^{FFC}$ | -0.109*** | -0.0373** | -0.117*** | -0.107*** | -0.117*** |
| | (-4.09) | (-2.32) | (-5.11) | (-4.83) | (-4.85) |
| $N$ | 15766 | 15762 | 15762 | 14706 | 15762 |
| $\sigma_{it}^{G}$ | -0.0608*** | -0.00594 | -0.0521*** | -0.0456*** | -0.0529** |
| | (-3.14) | (-0.45) | (-3.20) | (-2.87) | (-3.20) |
| $N$ | 13776 | 13772 | 13772 | 12719 | 13772 |

Table 3.4.2: Uncertainty Measures

taken as evidence that the efficient market hypothesis holds and equity markets reflect investors' best estimates of the future profitability of firms, so the standard deviation of stock returns in a given year is a good reflection of the uncertainty faced by economic agents. Alternatively, it could reflect the fact that the methods implemented to partial out the forecastable component of stock returns did not eliminate the influence of other factors that affect equity market volatility independently of uncertainty. In chapter four, I try to separate these effects at the aggregate level. Presently, I interpret the the coefficient on $\sigma_{it}^{FEV}$ with caution, knowing it may contain variation related to factors other than uncertainty.

### 3.4.3 Further Robustness Checks

The models in table 3.4.3 show that the results from the deviations GMM model in table 3.4.1 are robust to some salient sample restrictions and assumptions about the model. The first column allows the explanatory variables to be correlated with the time-varying error term $\epsilon_{it}$. Here, the coefficients on the other explanatory variables are also estimated using all their available lags. Even when collapsing the instrument matrix as in 3.3.11, the number of instruments still grows to 201. The Hansen statistic does not reject the null hypothesis that the identification restrictions are valid, furthermore, it is not large enough to suggest that the larger number of instruments has weakened the power of the test (as mentioned, Roodman (2009*a*) suggests a p-value greater than 0.25 might be cause for suspicion when the number of instruments is large). There is also no evidence of serial correlation in the residuals based in the Arellano-Bond test. While the parameters have generally increased in absolute value, notice especially that a 1% increase in $L_{i,t-1}$ now causes a 0.2% decrease in the investment rate holding the other variables and unobserved firm heterogeneity constant, the coefficient on the lagged dependent variable has fallen below its expected range based on the OLS and fixed effects models in columns two and three of table 3.4.1. Hence, there is evidence that this coefficient is downward biased when assuming the explanatory variables are contemporaneously correlated with the time varying error term.

| | Endogenous Regressors | After 2000 | Manufacturing Firms | Permanent Firms |
|---|---|---|---|---|
| $y_{i,t-1}$ | 0.231*** | 0.287*** | 0.264*** | 0.225*** |
| | (5.12) | (9.44) | (8.42) | (8.09) |
| $Q_{i,t-1}$ | 0.456*** | 0.316*** | 0.314*** | 0.318*** |
| | (11.78) | (12.71) | (12.87) | (12.91) |
| $\sigma_{it}^{FEV}$ | -0.153*** | -0.127*** | -0.107*** | -0.0572** |
| | (-2.79) | (-5.02) | (-3.59) | (-2.40) |
| $lev_{i,t-1}$ | -0.149*** | -0.121*** | -0.114*** | -0.0909*** |
| | (-3.87) | (-4.84) | (-4.68) | (-3.45) |
| $cf_{i,t-1}$ | 0.0444*** | 0.0483*** | 0.0506*** | 0.0323*** |
| | (3.96) | (5.63) | (4.68) | (3.83) |
| $L_{i,t-1}$ | -0.201*** | -0.0720*** | -0.0410** | -0.0689*** |
| | (-3.95) | (-3.62) | (-2.11) | (-3.77) |
| $\psi_{i,t-1}$ | 0.246*** | 0.255*** | 0.287*** | 0.393*** |
| | (10.76) | (9.66) | (9.85) | (15.83) |
| $N$ | 14444 | 12387 | 9256 | 7805 |
| $J$ | 201 | 49 | 55 | 55 |
| $\chi^2$ | 192.8 | 21.57 | 26.43 | 25.01 |
| p-value | 0.111 | 0.605 | 0.332 | 0.405 |
| A-Bond | 0.800 | 0.852 | 1.081 | 1.046 |
| p-value | 0.424 | 0.394 | 0.280 | 0.295 |

Table 3.4.3: Robustness Checks of Deviations GMM Model

Section 3.3.1 noted that the number of firms in the sample increases dramatically between 1995 and 2000. If the new firms have characteristics correlated with investment rates but not accounted for in the model, it will bias the regression coefficients. Column two of table 3.4.3 suggests that this is not the case. The signs and statistical significance of the variables remain the same as in table 3.4.1. Running this model with an OLS estimator and then a fixed effects estimator, the expected range of the coefficient on the dependent variable is between 0.51 and 0.22, which does indeed bound the coefficient on $\psi_{i,t-1}$ in column two. This finding is also true for the next two columns of table 3.4.3.

Previous studies like Leahy & Whited (1996) and Bulan (2005) restricted attention to manufacturing firms, yet the results in this chapter suggest that there is no statistically significant difference in the effects of the explanatory variables on investment rates in manufacturing, service, or energy and mining sectors. Restricting the sample to include only manufacturing firms does not change any of the conclusions from the baseline deviations GMM model. Interaction terms between an indicator variable capturing the three sectors and the explanatory variables indicate whether the effect of the latter is conditional on a firm's sector. These interaction terms are all insignificant, with two exceptions. Table 3.4.4 shows that the investment rate for firms in the energy and mining sectors is less sensitive to changes in real growth opportunities ($y_{i,t-1}$) compared to manufacturing firms. A 1% increase in $y_{i,t-1}$ leads to a 0.129% increase in $\psi_{i,t-1}$ in the energy and mining sector compared to a 0.273% increase in the manufacturing sector. Furthermore, investment rates are much more sensitive to financial growth opportunities ($Q_{i,t-1}$) in the energy and mining sector, the coefficient being more than twice as large than for manufacturing firms. Notice the interaction terms are not significant for the service sector, so the effect of growth opportunities on investment rates in these firms is statistically indistinguishable from that of manufacturing firms.

| | Interaction with Sector: | | |
|---|---|---|---|
| | Manufacturing | Services | Energy and Mining |
| $y_{i,t-1}$ | 0.273*** | -0.0149 | -0.144*** |
| | (9.55) | (-0.30) | (-3.22) |
| $Q_{i,t-1}$ | 0.330*** | -0.0319 | 0.365*** |
| | (13.99) | (-0.68) | (5.47) |
| $N$ | $J$ | $\chi^2(24)$ | A-Bond |
| 14301 | 59 | 23.25 | 1.110 |

Table 3.4.4: Effect of Growth Opportunities on $\psi_{it}$ when Interacted with an Indicator Variable Capturing Economic Sector

If the sample is restricted to include only those firms with complete time-series for the dependent variable, the coefficient on $\sigma_{it}^{FEV}$ is about half the size of its value in the baseline model, and only significant at the 5% level. This might suggest that well-established firms are less affected by idiosyncratic uncertainty. From a real options perspective, this would occur if investment decisions for these firms are easier to reverse. Well-established firms may have developed practices through past experience which makes them better at hedging the costs of investment projects[8]. The coefficient on the lagged dependent variable is also larger for these firms, suggesting they are more likely base their investment decisions this year on their decisions in the previous year (Eberly et al., 2012). Aside from these observations, the coefficients are relatively similar to the baseline model after making this restriction.

### 3.4.4 Interactions with Growth Opportunities

Lang et al. (1996) previously noted that high leverage is more of a drag on investment rates for firms with smaller values of Q. Firms viewed as having strong future prospects can invest more aggressively with higher leverage ratios because they can attract capital through equity markets and maintain lower premiums on debt, thereby mitigating the financial burden of that debt. A similar argument also means that the effect of cash flows on investment might be dependent on growth opportunities. I test whether the effects the explanatory variables have on the investment rate is dependent on growth opportunities by interacting the explanatory variables with $Q_{i,t-1}$. I also test for significant interaction terms between $y_{i,t-1}$ and the explanatory variables, appreciating the observation by Gilchrist & Himmelberg (1998) that net sales capture real growth opportunities.

Bloom et al. (2007) previously conditioned uncertainty's effect on investment on the net sales to capital ratio, while Gulen & Ion (2016) did the same for financial growth opportunities. Given Evans's (1987) finding that smaller firms face different incentives and have different optimal strategies compared to larger firms, I also tested for interaction effects between the other explanatory variables and $L_{i,t-1}$, however, none of these interactions were statistically significant after controlling for interactions with growth opportunities.

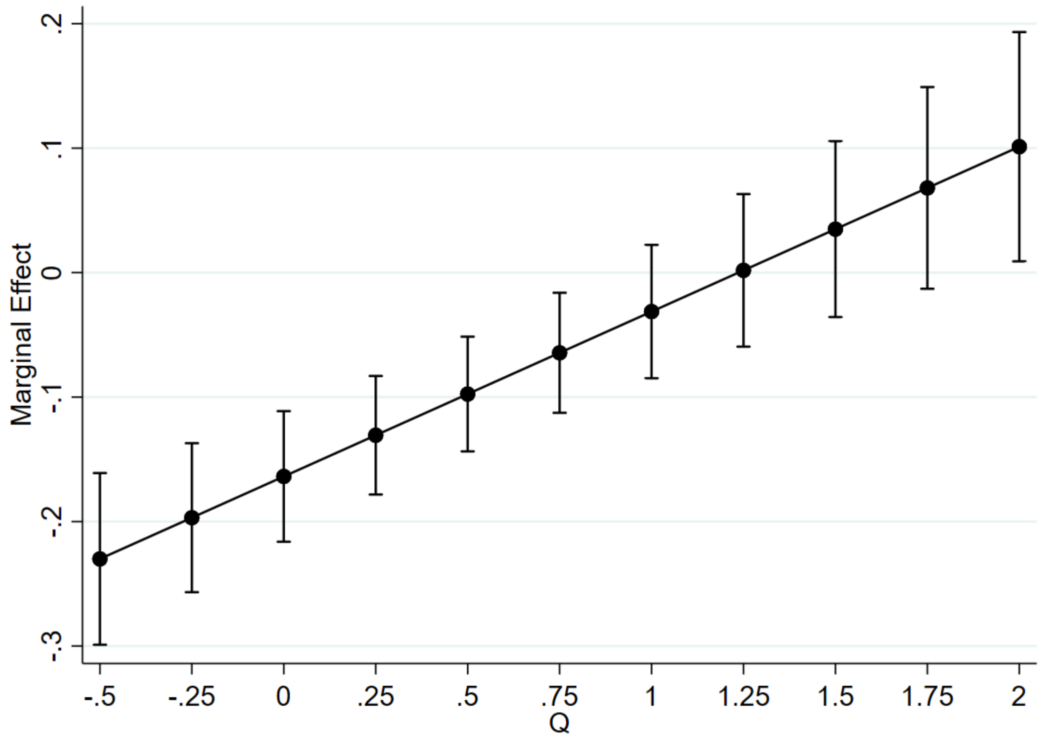
In addition to the interactions between the variables and growth opportunities, uncertainty, Tobin's Q, and cash flows showed statistically significant quadratic relationships with the investment rate (at the 5% significance level or lower). It is generally easier to interpret non-linear relationships in the model if they are presented graphically. Therefore, I have relegated the full table of coefficients

[8] See Panousi & Papanikolaou (2012) for discussion of measuring the irreversibility of investment projects.

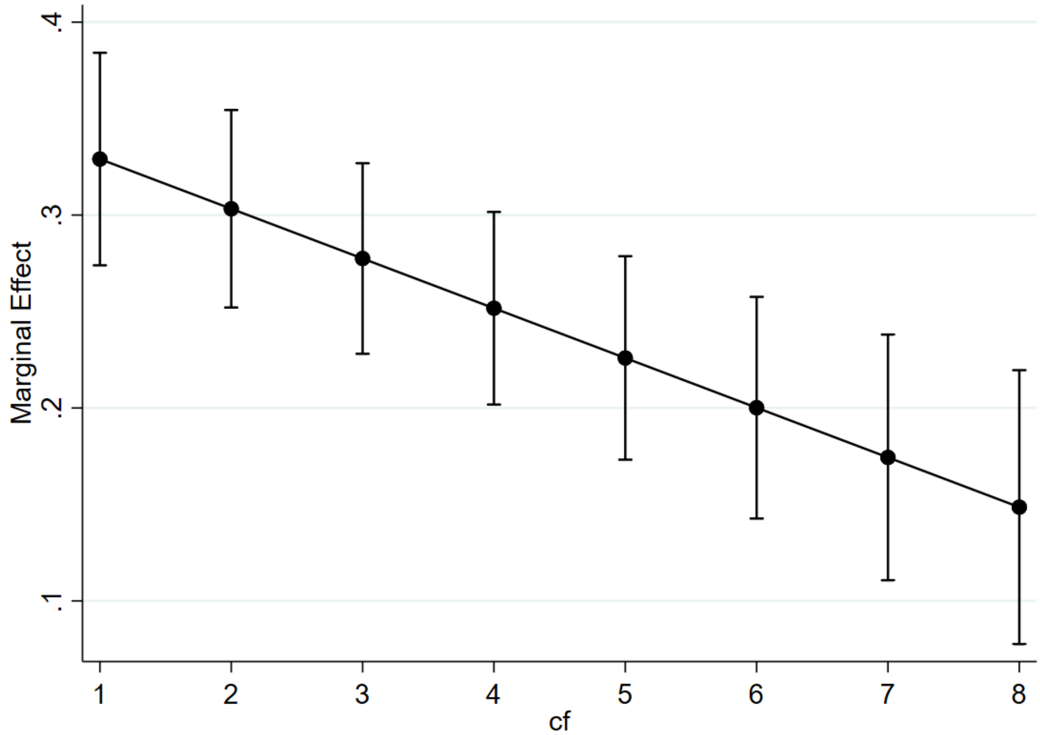
to appendix 3.A.1. Figure 3.A.1 in the appendix plots the predicted value of the investment rate inside its 95% confidence interval over the range of the variables found in the sample. Despite their significance, the figures reveal that for the range of values observed in the sample, the non-linearity is quite weak, so the linear specification in 3.3.6 is a good approximation of the relationship.

Two interaction terms between the other variables and growth opportunities were statistically significant. The first was between $Q_{i,t-1}$ and $\sigma_{it}^{FEV}$. This relationship is displayed in figure 3.4.1a, which shows the estimated marginal effect of uncertainty on the investment rate for increasing values of $Q_{i,t-1}$, holding the other variables constant at their medians. The negative effect of uncertainty on the investment rate is mitigated for firms with higher growth opportunities. For firms with $Q_{i,t-1}$ greater than unity, uncertainty actually has a negligible effect on the investment rate. The estimates from $Q_{i,t-1} = 1$ to $Q_{i,t-1} = 1.8$ are not statistically distinguishable from zero. In contrast, for firms with a value of $Q_{i,t-1}$ close to zero, a 1% increase in uncertainty will cause a 0.23% decrease in the investment rate if all variables are held at their median values. This is in line with the model of capital accumulation in chapter two. There, uncertainty created a region of inaction where the firm delayed its decision to undertake partially irreversible investment until the marginal value of the next unit of capital was sufficiently high. Firms with larger values of Q will add to their capital stock even in times of heightened uncertainty because the expected value of expansion is sufficiently high to overcome the option value of waiting until uncertainty declines (Dixit & Pindyck, 1994; Abel & Eberly, 1996).

The effect of uncertainty on the investment rate is not dependent on $y_{i,t-1}$ after controlling for $Q_{i,t-1}$, so higher net sales will not mitigate the negative effect of uncertainty on investment. However, figure 3.4.1b shows that the marginal effect of $y_{i,t-1}$ on the investment rate is attenuated by higher cash flows. Higher net sales relative to the capital stock always has a positive effect on the investment rate, but the effect of a 1% increase in net sales is much smaller as long as the firm has a healthy flow of liquid capital. This is partly explained by the fact that high cash flows make firms less dependent on revenue generated through sales to fund investment spending. It also implies that investment declines due to low real growth opportunities can be offset if significant cash flows can be raised from other sources.




(a) Marginal Effect of Uncertainty on the Investment Rate




(b) Marginal Effect of Net Sales on the Investment Rate

Figure 3.4.1: Significant Interaction Terms

### 3.4.5 Impact of the Great Recession

The dynamic aspect of the model introduced by the lagged dependent variable means the changes in the variables the year the recession hit will propagate through the system. The dashed lines in figure 3.4.2 shows this in the case where each of the explanatory variables from column four of table 3.4.1 are set equal to their mean change across firms in 2008, which is just the mean growth rate given the variables are all in logs. The effect of this change in each of the variables on the change in the investment rate is shown holding the other variables constant, i.e. assuming they did not change in 2008. The initial change in the investment rate of zero is also assumed to be zero and any exogenous shocks are ignored. For example, denote the mean change in $Q_{i,t-1}$ in 2008 as $\Delta\bar{Q}_0$, then:

$$\begin{aligned}
\Delta\bar{\psi}_1 &= \hat{\beta}_2\Delta\bar{Q}_0 \\
\Delta\bar{\psi}_2 &= \hat{\alpha}_1\hat{\beta}_2\Delta\bar{Q}_0 \\
\Delta\bar{\psi}_3 &= \hat{\alpha}_1^2\hat{\beta}_2\Delta\bar{Q}_0 \\
&\vdots \\
\Delta\bar{\psi}_n &= \hat{\alpha}_1^{n-1}\hat{\beta}_2\Delta\bar{Q}_0.
\end{aligned}$$

The effect of the change diminishes over time as long as $\hat{\alpha}_1 < 1$.

In figure 3.4.2, time zero shows the initial impact of the change in a given variable on the investment rate, holding other variables constant at their means (so they are assumed not to change). Further time periods show the effect of the change in the following years given the dynamic nature of the model. The changes brought about by Tobin's Q and uncertainty are so much larger that they are plotted on a separate panel for clarity. This highlights the key role of declining growth opportunities and higher uncertainty in determining the initial fall in investment during GR.

There are some changes in these dynamics when the interaction terms are considered, as displayed by the solid lines in figure 3.4.2. For example, the effect of the average change in uncertainty

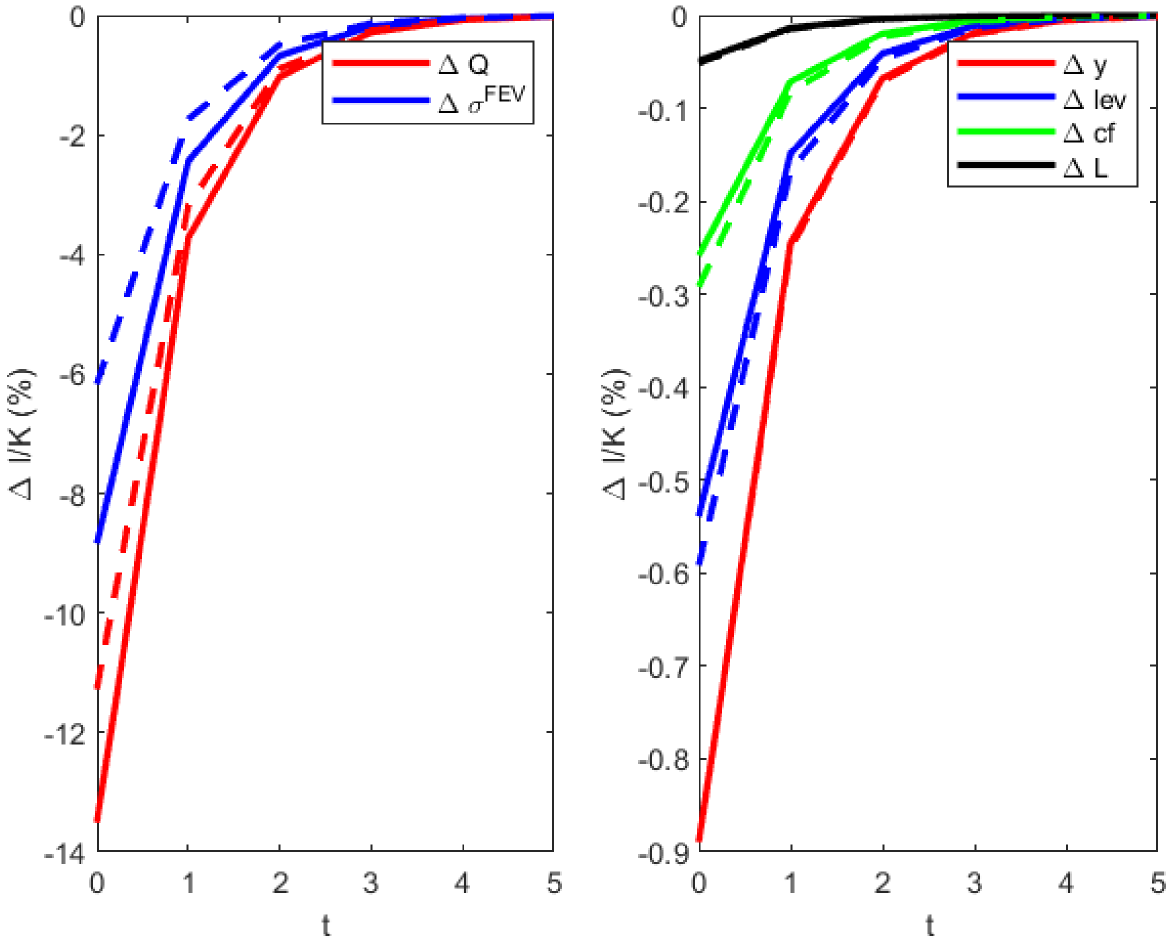



Figure 3.4.2: Change in the Investment Rate due to Change in the Explanatory Variables in 2008

holding other variables constant is

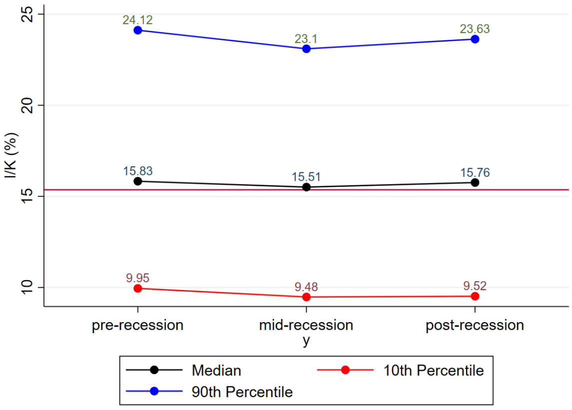
$$\beta_3 \left(\Delta\overline{\sigma_t^{FEV}}\right) + \gamma_1 \Delta \left(\overline{\sigma_t^{FEV}}\right)^2 + \gamma_2 \left(\Delta\overline{\sigma_t^{FEV}} \times \overline{Q_{t-1}}\right).$$

When the interaction between $Q_{i,t-1}$ and $\sigma_{it}^{FEV}$ is considered, the change in these variables leads to an even larger fall in the investment rate. However, there are only minor differences in the impact of the other variables after controlling for the interaction terms. Hence, the evidence suggests that the immediate fall in the investment rate after the GR was mediated largely through changes in uncertainty and Tobin's Q. Notice that even after two years the impact of the changes in these variables in 2008 is still larger than the initial fall caused by changes in variables such as cash flows and leverage.

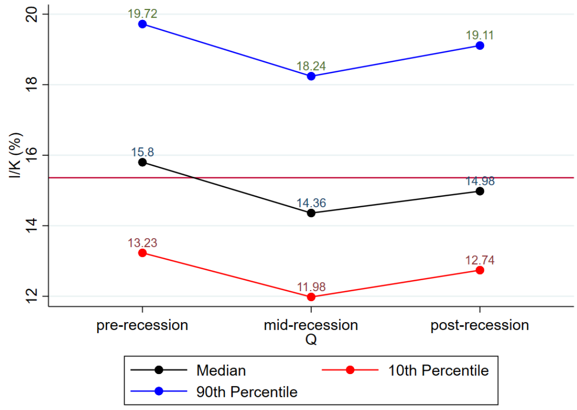
Figure 3.4.2 gives some idea of the initial impact and propagation of the change in the variables after the crisis but obscures the full dynamics during the recessionary and the recovery periods. Figure 3.4.3 sheds light on this by plotting the predicted investment rate over these periods using the model including statistically significant interaction terms and non-linear effects. The horizontal line in each of the panels represents the predicted investment rate holding all variables at their full-sample median (15.36%). The black curves show the investment rate when the specified variable is set equal to its median value in the two years before (pre-recession), the two years during (mid-recession), and the two years after (post-recession) the GR with all other variables held at their full-sample median. The blue and red curves repeat this process for the 10th percentile and the 90th percentile of the specified variable, again *holding all other variables at their full-sample medians.*

For reference, figure 3.A.2 in the appendix shows how the distributions of the explanatory variables change between pre-recession, mid-recession, and the post-recession periods. For example, the median level of $\sigma_{it}^{FEV}$ increased by 0.46 percentage points (13.37% of its pre-recession value) between the pre-recession and mid-recession periods, which is a relatively large change compared to the other variables. The 90th percentile increased by even more, 0.53 percentage points (13.89% of its pre-recession value), suggesting that the increase in uncertainty was concentrated in the upper-end of the distribution. This large increase is not surprising in light of the finding by Bloom (2009) and Jurado et al. (2015) that uncertainty is strongly counter-cyclical. In contrast, median leverage remained fairly stable across the three periods, echoing the observation in Banerjee et al. (2015) and Kalemli-Özcan et al. (2022) that U.S. firms typically did not display the significant increases in debt witnessed in European firms after the GR.

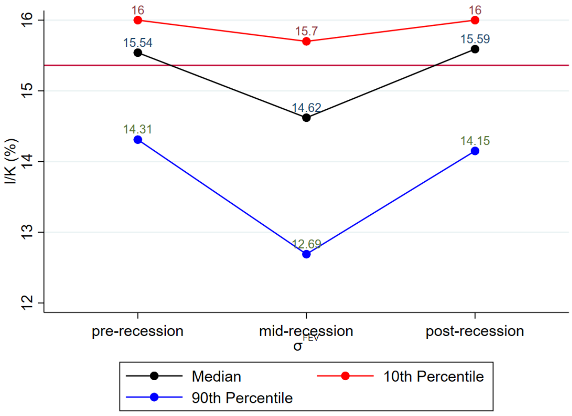
The variation in the predicted investment rate within each period is captured by the vertical distance between the curves in figure 3.4.3. Changes between the periods show the predicted response of the investment rate to the change in the specified variable if all other variables are held constant in that period. For example, setting $\sigma_{it}^{FEV}$ equal to its median value in the pre-recession period while holding all other variables at their full-sample median gives a predicted investment rate of 15.54%, slightly above the rate predicted if $\sigma_{it}^{FEV}$ was also at its full-sample median. This reflects that fact that uncertainty was lower in the pre-recession period relative to the full sample. The positive coefficient on the (see figure 3.4.4d) interaction term between uncertainty and Tobin's



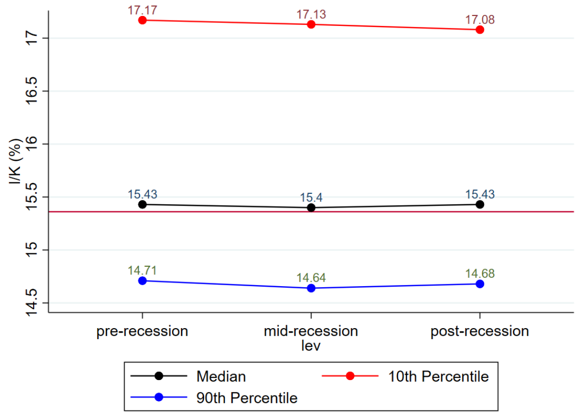

(a) Net Sales



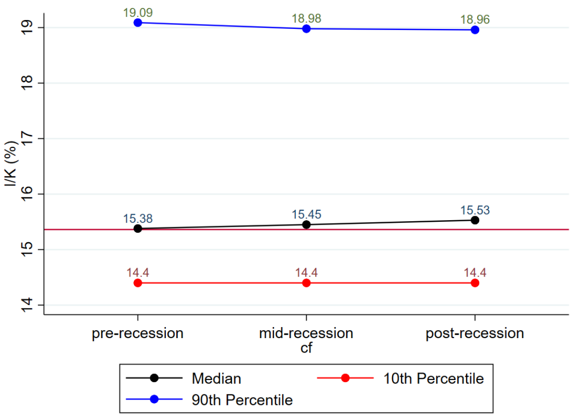

(b) Q



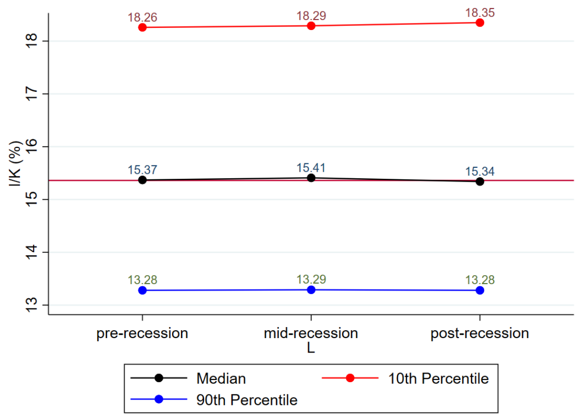

(c) Uncertainty




(d) Leverage




(e) Cash Flows




(f) Size

Figure 3.4.3: Predicted Values of the Investment Rate

Q means all the values in panel 3.4.3c are larger that they would be if the interaction term was ignored. If $\sigma_{it}^{FEV}$ is set to its value at the 10th percentile, which of course is a relatively lower level of uncertainty, while holding all other variables at their full-sample median, the predicted investment rate is 16%.

As expected from the previous discussion, changes in uncertainty and Tobin's Q had the largest effect on the investment rate during the recession, holding other variables constant. The change in the median value of $Q_{i,t-1}$ between the pre-recession and mid-recession periods is predicted to cause a 1.44 percentage point decline in the investment rate. Meanwhile, the change in the median of $\sigma_{it}^{FEV}$ between these periods causes a 0.92 percentage point decline. Notably, the median value of $Q_{i,t-1}$ did not sufficiently recover to bring the investment rate back to its pre-recessional level in the two years after the crisis. $Q_{i,t-1}$ was therefore a drag on the recovery of investment after the GR.

Notice the change brought about by the increase in uncertainty during the recession was much larger at the 90th percentile of the distribution. This is due to the influence of the negative coefficient on the quadratic term $\left(\sigma_{it}^{FEV}\right)^2$ (see figure 3.A.1b in the appendix). Higher uncertainty decreases the investment rate at an increasing rate. By contrast, the change in the predicted investment rate is much lower as uncertainty changes from its value at the 10th percentile in the pre-recession period to its value at the 10th percentile in the mid-recession period. Note also that the variance of the predicted investment rate within each period caused by adjusting the value of $\sigma_{it}^{FEV}$ is lower compared to Tobin's Q. For example, in the mid-recession period, the predicted investment rate ranges between 11.98% and 18.24% for $Q_{i,t-1}$ but the corresponding values for uncertainty range between 12.69% and 15.7%.

Other than decreases caused by lower $y_{i,t-1}$ at the 90th percentile of the distribution, there is little evidence to suggest that any of the other variables changed substantially enough to cause notable changes in the investment rate over the six year period. Despite this, the variation in the predicted investment rate for some of the variables within each period is large. The lowest predicted investment rate in the figure is obtained by setting $y_{i,t-1}$ equal to its value at the 10th percentile. Thus, when looking at the effects of variables in isolation, firms with the lowest sales to capital ratios will display the lowest investment rates in each period.

Interacting the variables on an indicator variable encoding the pre-recession, mid-recession, and post-recession periods yields no significant coefficients. Hence, there is no evidence to suggest that the estimated coefficients changed during these periods. Despite this, there is evidence to suggest that firms were less able to make use of their growth opportunities and that uncertainty was a greater drag on investment in the post-crisis period. There are several mechanisms through which an event such as the GR could cause the estimated coefficients in 3.4.1 to change in its aftermath. For example, the negative effect of uncertainty on investment may have been larger during and in the years after the GR if the option value of waiting before making irreversible decisions was valued higher by managers who were now more wary of the prospect of an economic shock. Changes in financial regulations and investor preferences could also mean leverage and cash flows had different effects on investment in the post-crisis period.

I test for changes in the relationship between the investment rate and the explanatory variables after the GR by interacting them with an indicator variable taking a value of zero between 1996 and 2007 and a value of one for all years after. The indicator variable itself does not enter the regression due to perfect collinearity between it and the time dummies. The results are presented in table 3.4.5. I have used the model without non-linear and interaction terms in this table because it is difficult to interpret their coefficients. I show their post-crisis changes graphically in figure 3.4.4. In table 3.4.5, note that the Hansen test does not reject the null hypothesis that the over-identification assumptions are valid and there is no evidence that the errors display serial correlation based on the test developed by Arellano & Bond (1991).

The only coefficient that appeared to significantly change after the GR is the one on $Q_{i,t-1}$. Before the GR, a 1% increase in $Q_{i,t-1}$ caused a 0.379% increase in the investment rate but after the GR it only caused a 0.275% increase. Economically, this means that firms on average responded less aggressively to their growth opportunities after the recession, which is consistent with the idea that a greater level of caution was exercised by managers after the GR. Despite this, there is only weak evidence that uncertainty had a stronger negative effect on investment. At the 5% significant level, the decrease in the investment rate caused by a 1% increase in uncertainty was -0.09 percentage points larger after the recession compared to before, so there is some evidence that managers valued their options to delay before making investment decisions more highly after the GR. There is also

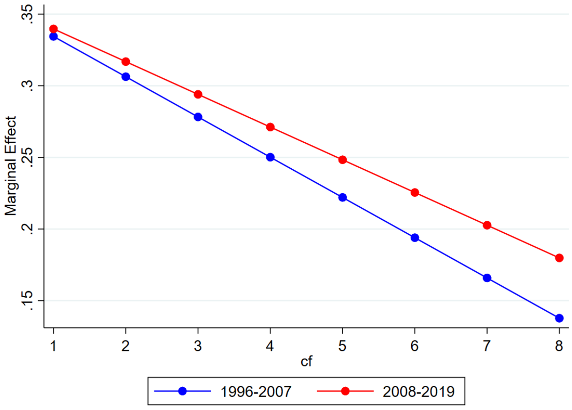
| | Parameter | Interaction |
|---|---|---|
| $\psi_{i,t-1}$ | 0.222*** | 0.0529** |
| | (9.93) | (2.17) |
| $y_{i,t-1}$ | 0.276*** | 0.0151 |
| | (10.42) | (0.91) |
| $Q_{i,t-1}$ | 0.379*** | -0.104*** |
| | (14.90) | (-3.40) |
| $\sigma_{it}^{FEV}$ | -0.0829** | -0.0857** |
| | (-2.51) | (-2.47) |
| $lev_{i,t-1}$ | -0.119*** | 0.00511 |
| | (-4.52) | (0.18) |
| $cf_{i,t-1}$ | 0.0561*** | -0.0196* |
| | (5.43) | (-1.88) |
| $L_{i,t-1}$ | -0.0498*** | -0.0108 |
| | (-3.07) | (-1.18) |
| $N$ | $\chi^2(63)$ | A-Bond |
| 14444 | 26.25 | 0.964 |

Table 3.4.5: Interactions with GR Indicator

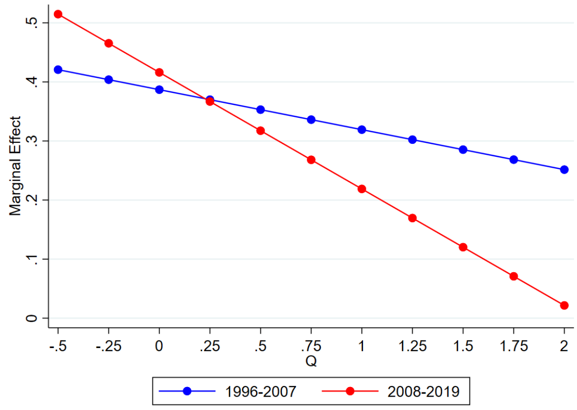
evidence at the 5% significance level that the auto-regressive effect of the investment rate was 0.05 percentage points higher after the GR, implying changes in the explanatory variables will cause more persistent effects on the investment rate.

Figure 3.4.4 demonstrates the change in the coefficients after the GR when including the interaction and non-linear terms in the model. The blue lines represent their value in the period 1996-2007 and the red lines represent the period 2008-2019. All the changes in the marginal effects presented are significant at the 1% level except for in panel 3.4.4c, so while there was weak evidence that the linear coefficient on uncertainty was more negative after the crisis, this significance disappears when considering all interaction terms and non-linear effects. The lines in panel 3.4.4c slope downwards because the quadratic term on uncertainty is negative, so the negative effect of higher uncertainty on the investment rate is increasing in the level of uncertainty.

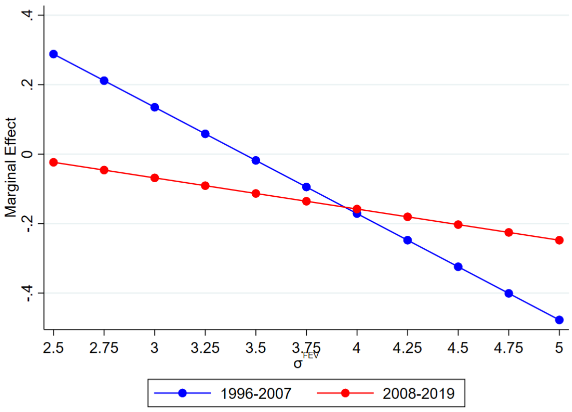
To make the interpretation of the graphs clear, before the GR a 1% increase in Tobin's Q when $Q_{i,t-1} = 1$ caused a 0.31% increase in the investment rate holding other variables constant. After



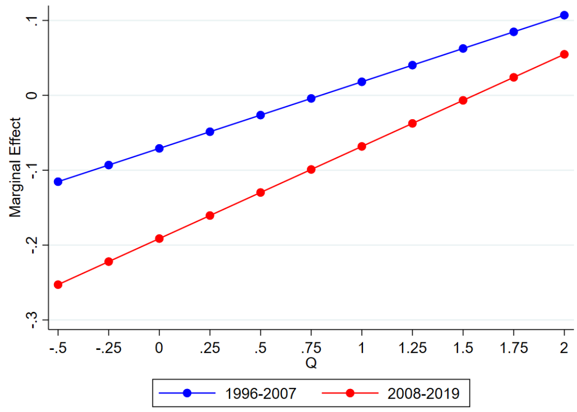

(a) Net Sales and Cash Flows



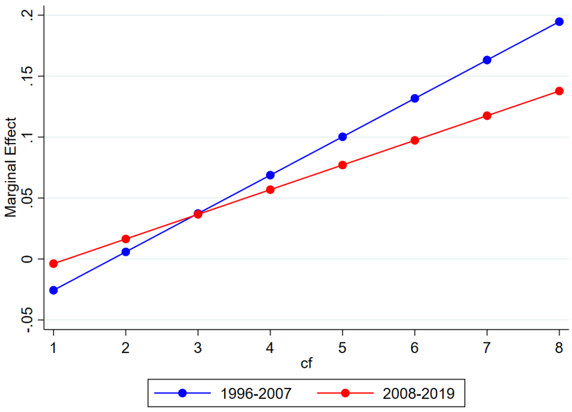

(b) Q




(c) Uncertainty




(d) Uncertainty and Q




(e) Cash Flows

Figure 3.4.4: Marginal Effects of Variables on the Investment Rate

the GR, a 1% increase if $Q_{i,t-1} = 1$ caused just a 0.22% increase. This change is statistically significant, which means the effect of increasing $Q_{i,t-1}$ on the investment rate diminished faster after the recession. Again, firms with high growth opportunities were less able or less willing to invest after the crisis. Note however that the marginal effect is always positive over the range of $Q_{i,t-1}$ observed in the sample.

Figure 3.4.1b previously showed that the positive effect of real growth opportunities on the investment rate was dampened by higher cash flows, suggesting that net sales matter less for investment when there is a healthy flow of liquid assets coming into the firm. After the crisis, this relationship still exists but net sales appear to matter more regardless of cash flows, as evident by the fact that the red line in panel 3.4.4a lies above the blue line. Furthermore, from panel 3.4.4e, the positive marginal effect of cash flows diminishes faster after the crisis. In sum, these results seem to imply that investment was less sensitive to cash flows after the GR.

The lower sensitivity of investment to cash flows and growth opportunities after the GR suggests firms were more hesitant about making irreversible decisions. While there was weak evidence in the linear model of table 3.4.5 that uncertainty had a more negative effect on investment after the GR, there was no evidence of this in model including interaction terms and non-linear effects. However, panel 3.4.4d shows that high growth opportunities were less effective at mitigating the negative impact of uncertainty on the investment rate after the GR. Whereas before the recession a value of $Q_{i,t-1} = 0.75$ would mean the marginal effect of uncertainty on the investment rate was very close to zero, after the recession a 1% increase in $\sigma_{it}^{FEV}$ leads to a 0.1% decline in the investment rate.

This is consistent with the idea that firms' attached greater value to their option to wait before making irreversible decisions after the GR. A higher value of Tobin's Q is required after the GR to neutralise the effect of higher uncertainty on the investment rate, just as in the models of Abel & Eberly (1996) and Guo et al. (2005) a higher marginal q is required to justify investment when the real option is more valuable. Overall, the evidence in this section points to higher uncertainty and firms being less responsive to their growth opportunities as the main covariates driving the dynamics of corporate investment in the aftermath of the GR.

## 3.5 Conclusion

This chapter used an instrumental variable GMM estimator to examine the determinates of corporate investment rate over the business cycle in a dynamic panel data model based on the q-theory of investment. The model was more comprehensive than previous studies in the literature and allowed for non-linear relationships and interaction terms between the variables. The net-sales to capital ratio, Tobin's Q, uncertainty, leverage, cash flows, and firm size were all found to be statistically significant predictors of the investment rate over the business cycle. I ensured the results were robust to various sample restrictions and assumptions. Most notably, there is no evidence that the results change when restricting the full sample to include only manufacturing firms. Studies which exclude service sector firms should have strong justification as to why the restriction is necessary.

Two interaction terms between the explanatory were significant. Higher cash flows appear to mitigate the positive effect of the net-sales to capital ratio on the investment rate, which suggests the revenues generated from sales are less important in determining investment when firms have a healthy flow of liquid capital. The other significant interaction term was between uncertainty and Tobin's Q. Higher values of Q neutralised the negative effect of uncertainty on the investment rate. This is in line with the real-options literature arguing that uncertainty drives a wedge between the marginal benefit of installing capital and its user cost. For high enough values of Q, expanding the capital stock is worthwhile despite high uncertainty about the future profitability of investment.

Within this set-up, the fall in growth opportunities and increase in uncertainty during the GR were shown to cause a large fall in the predicted investment rate relative to the other variables in the model. I used the dynamic nature of the model to show how these changes would propagate through the system in the succeeding years. The data suggests that any financing issues firms had during the crisis were relatively minor and soon resolved. Interaction terms with an indicator variable capturing the years before and after the GR revealed that firms were less willing or less able to make use of their growth opportunities in the years after the crisis. There was some evidence that uncertainty may have had a more negative effect in the years after the crisis, suggesting firms attached higher value to their options of waiting for more information before making investment decisions. This finding was corroborated by the fact that higher Q was not as effective at mitigating the negative impact of uncertainty after the recession compared to before. In other words, Q had to be relatively higher after the recession to justify investment given the level of uncertainty.

These findings present a problem for policy makers because it is harder to find policy instruments to aid low growth opportunities and high uncertainty compared to assisting firms in mending their balance sheets. Policy makers should send clear signals to the market to try and reduce uncertainty in the aftermath of recessions. Finally, note the measure of uncertainty used in this paper is likely contaminated by variation in stock markets independent of uncertainty. The different macroeconomic effects of changes in uncertainty and changes in stock market volatility will be examined in the next chapter.

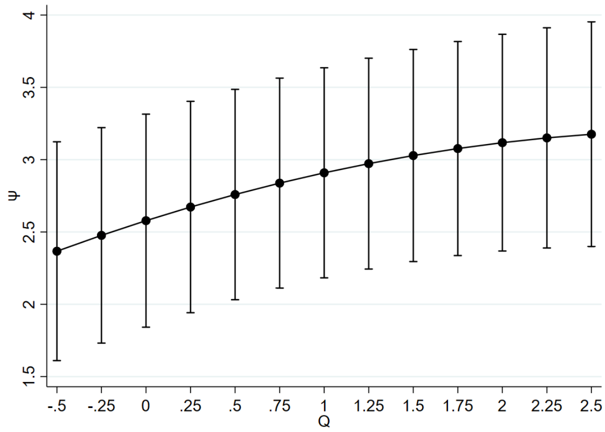
# Appendix 3.A

## 3.A.1 Model with Interaction Terms

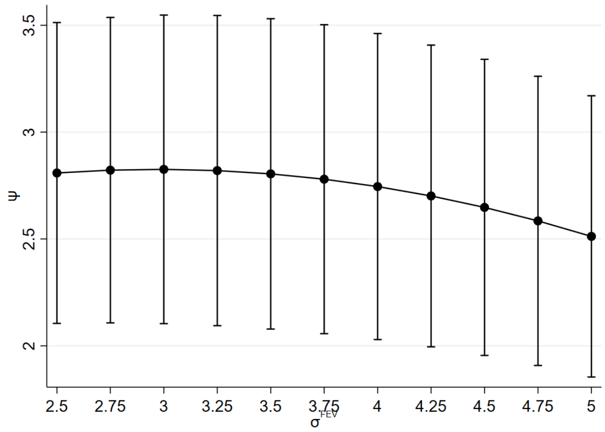
| | Deviations GMM |
|---|---|
| $\psi_{i,t-1}$ | 0.276*** |
| | (12.53) |
| $y_{i,t-1}$ | 0.355*** |
| | (11.46) |
| $Q_{i,t-1}$ | -0.0697 |
| | (-0.68) |
| $\sigma_{it}^{FEV}$ | 0.368*** |
| | (2.65) |
| $lev_{i,t-1}$ | -0.107 |
| | (-1.55) |
| $cf_{i,t-1}$ | 0.126*** |
| | (4.08) |
| $L_{i,t-1}$ | -0.0589*** |
| | (-3.35) |
| $y_{i,t-1} \times cf_{i,t-1}$ | -0.0258*** |
| | (-4.67) |
| $Q_{i,t-1} \times Q_{i,t-1}$ | -0.0611*** |
| | (-2.71) |
| $Q_{i,t-1} \times \sigma_{it}^{FEV}$ | 0.132*** |
| | (4.95) |
| $\sigma_{it}^{FEV} \times \sigma_{it}^{FEV}$ | -0.0763*** |
| | (-3.75) |
| $cf_{i,t-1} \times cf_{i,t-1}$ | 0.0124*** |
| | (4.23) |
| $N$ | 14444 |
| $J$ | 60 |
| $\chi^2$ | 23.70 |
| p-value | 0.479 |
| $A - Bond$ | 1.186 |
| p-value | 0.236 |

Table 3.A.1: Significant Interaction Terms between Variables

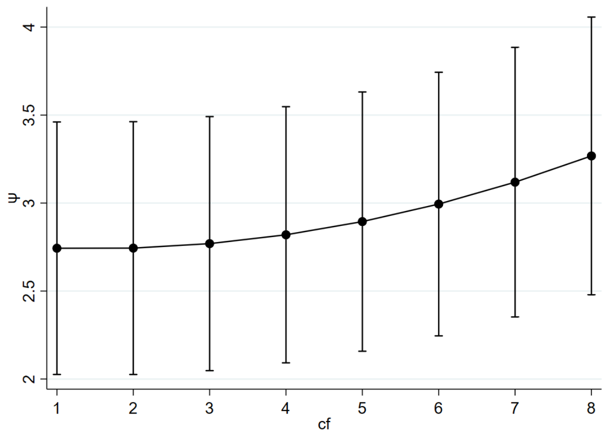
### 3.A.2 Quadratic Forms




(a) Predicted Investment Rate as a Function of Q



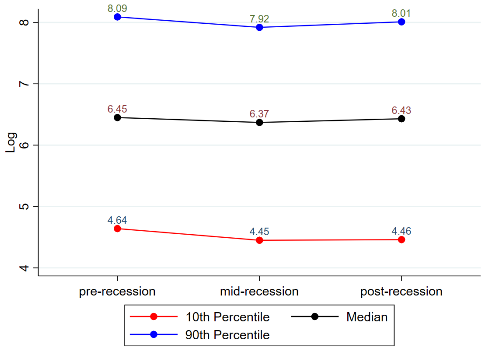

(b) Predicted Investment Rate as a Function of Uncertainty



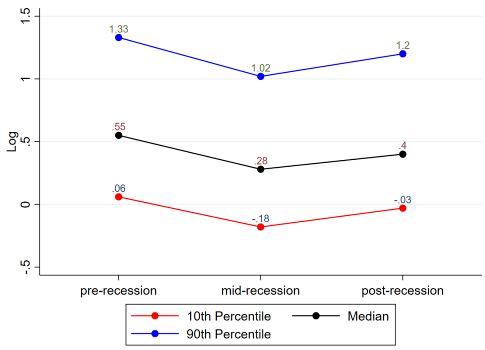

(c) Predicted Investment Rate as a Function of Cash Flows

Figure 3.A.1: Quadratic Forms of Variables inside a 95% Confidence Interval

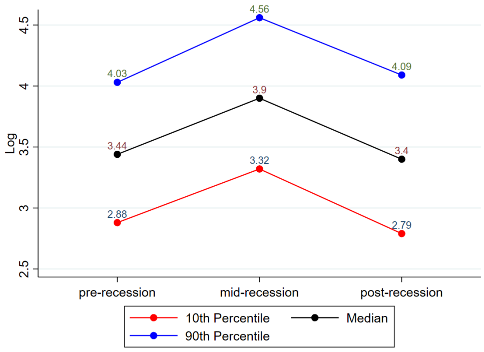
### 3.A.3 Median Values of the Variables around the Great Recession



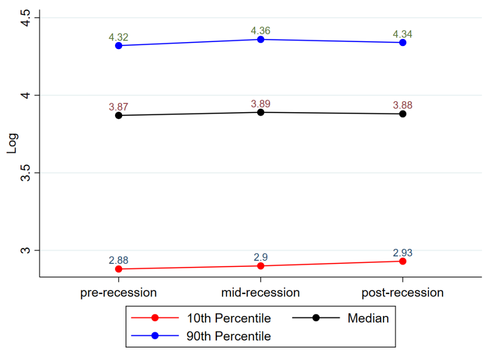

(a) Net Sales



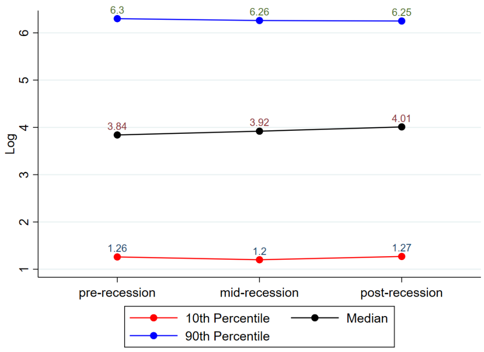

(b) Q



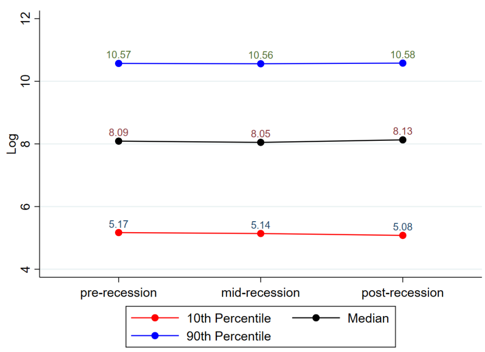

(c) Uncertainty




(d) Leverage




(e) Cash Flows




(f) Size

Figure 3.A.2: Changes in Variables around the Great Recession

# Chapter 4

# Identifying Uncertainty and Stock Market Volatility Shocks with an Instrumental Variable SVAR model

## 4.1 Introduction

The real options theory introduced in the previous chapters makes testable predictions about the effect of uncertainty on real economic activity. As the value of waiting for more information increases, firms delay investment and hiring decisions until the uncertainty is resolved, which decreases economic activity. Testing these hypotheses is difficult because uncertainty is not observable; economists must construct measures of uncertainty based on available data. One common approach is to use the volatility of stock market returns, on the justification that periods where returns substantially deviate from their average value suggests that it is harder to predict the future value of investments. According to the efficient markets hypothesis (EMH), changes in firms' stock prices should reflect changes in investors' beliefs about the future value of firms.

However, stock market volatility (SMV) can vary independently of uncertainty. As a result, any estimates of the effect of uncertainty on the macroeconomy will be tainted by variation in the uncertainty measure which is actually caused by movements in SMV independently of uncertainty. While previous researchers have appreciated this problem, none have shown the differential effects of shocks to SMV and uncertainty on the macroeconomy. Furthermore, the causal relationship between uncertainty and SMV has been overlooked by previous literature, which tends to focus on the response of the *level* of stock market returns to uncertainty shocks[1]. Having a clear picture of these relationships is crucial from a policy making perspective as well as an academic perspective, as central banks must have exact knowledge of what type of shock they are facing before targeting them with policy instruments.

This chapter uses a set-identification approach to estimate the macroeconomic effects of structural shocks to uncertainty and stock market volatility in an instrumental variable structural vector auto-regression (SVAR-IV) model. The variation in the price of gold around events which make future innovations in economic variables harder to predict is used to construct a proxy for uncertainty shocks. Exogenous shocks to the credit spread between Baa-rated corporate bonds and the 10-year treasury bond rate proxy for shocks to risk preferences which affect SMV. I show that uncertainty and SMV, despite being treated as interchangeable in previous studies, produce different impulse response functions for key macroeconomic variables. Specifically, uncertainty shocks account for more of the variation in real economic variables and have a relatively smaller impact on prices. The opposite is true for the identified shocks to SMV, which have a weaker effect on real economic activity but make up a greater proportion of the variation in prices. Furthermore, changes in SMV originating from shocks to risk preferences do not cause a significant contemporaneous movement in uncertainty, and as the shock is resolved uncertainty will actually decrease as agents update their forecasting modes. These results validate the predictions of several authors who suggest that stock market based measures of economic uncertainty will lead to erroneous conclusions.

Alongside these primary findings, I address some shortcomings in the previous literature this chapter builds upon. Specifically, I ensure the results are robust to potential measurement error in the gold price instrument for uncertainty shocks and improve on the methodology used by Stock & Watson (2012) to isolate exogenous innovations in a series when constructing the proxy for structural shocks to SMV. Additionally, I use a moving block bootstrap (MBB) to construct confidence intervals for the impulse response functions (IRFs) in section 4.5. This is an improvement on the

[1] One exception is Baker et al. (2016), who show that their measure of Economic Policy Uncertainty is a significant predictor of the 30-day implied volatility of options traded on the S&P 500 (the VIX index introduced later in the chapter). However, they do not try to establish the direction of causal relationships between the two.

wild bootstrapping approach of Piffer & Podstawski (2018) who also identify uncertainty shocks with an instrument based on the variation in gold prices around events which increase uncertainty.

I use the measure of uncertainty constructed by Jurado, Lugdvigson, and Ng (2015). Their paper is widely cited in related literature and has the advantage of being independent of any theoretical assumptions. In section 4.5.4, I compare the results generated from this measure to those generated by using the Economic Policy Uncertainty (EPU) index of Baker et al. (2016) as the measure of uncertainty. SMV is measured by the standard deviation of logarithmic returns on the S&P500 as in Aït-Sahalia et al. (2021). Both of these variables are used at the monthly frequency. When choosing the other macroeconomic variables in the VAR model, I follow Bloom (2009), who was the first to examine the impact of uncertainty shocks on the macroeconomy and remains one of the most influential papers in the field. Piffer & Podstawski (2018) also follow their specification.

In this framework, set-identification of the contemporaneous effects from structural shocks to uncertainty and SMV can be achieved in three steps. First, run the reduced-form VAR model and estimate its residuals and covariance matrix. Second, perform a linear regression of the reduced-form residuals from each equation in the VAR on the instruments for the structural shocks. The estimated coefficients from these regressions represent the contemporaneous effects of shocks to uncertainty and SMV but are only correct up to a constant scale factor determined by the relationship between the instruments and the *structural* shocks. The third step combines the estimated coefficients from step two and the information in the reduced-form covariance matrix to obtain the desired contemporaneous responses of variables to structural uncertainty and SMV shocks.

After an overview of the related literature, I provide a comprehensive overview of the methodological approach in this paper in section 4.3, expanding on the steps outlined above. Section 4.4 then presents the variables and instruments used in the chapter. The results are presented and discussed in section 4.5, followed by a brief conclusion.

## 4.2 Related Literature

### 4.2.1 Uncertainty and Volatility

Jurado et al. (2015) define uncertainty as the conditional volatility of a disturbance that is *unforecastable* from the perspective of economic agents. The h-step ahead uncertainty of a variable $x_{it} \in \boldsymbol{x}_t = (x_{1t}, \ldots, x_{Nt})$, where $\Omega_t$ is the information available to agents at time t, is

$$\mathcal{U}^x_{it}(h) \equiv \sqrt{\mathbb{E}\left[(x_{i,t+h} - \mathbb{E}[x_{i,t+h}|\Omega_t])^2 \,|\Omega_t\right]}. \tag{4.2.1}$$

There must be some forecast error to prevent this expression begin equal to zero, so $\Omega_t$ does not allow agents to perfectly predict the future. Uncertainty is a forward-looking concept and any future variation that can be predicted based on the information set available to agents in the current period is not uncertain. Using this definition, Jurado et al. construct a measure of macroeconomic uncertainty by taking a weighted average of the conditional volatility of forecast errors from over 130 economic time series.

$$\mathcal{U}^x_t(h) \equiv \frac{1}{N}\sum_{i=1}^{N}\mathcal{U}^x_{it}(h). \tag{4.2.2}$$

This represents a gold-standard in uncertainty measurement but is difficult to construct and does not have an analogue for low-frequency firm-level data, which may be the researcher's primary interest. This fact explains the popularity of measures of uncertainty based on SMV. In the macroeconomic literature, these can be found in Bloom (2009) and Chuliá et al. (2017) while Leahy & Whited (1996), Bloom et al. (2007), Bulan (2005), and Panousi & Papanikolaou (2012) provide examples using firm-level data, as does chapter three of this thesis. Gilchrist et al. (2014) examines SMV-derived uncertainty on both the firm-level and aggregate level.

In theory, a firm's stock price reflects the expected present discounted value of all its future profit flows, taking into account all relevant information available at the time (Malkiel, 2003). A representative sample indexing the value of U.S. equities should, therefore, reveal investors' best estimate of the expected future performance of the U.S. economy. The average deviation of changes in stock prices around the mean gives an indication of how difficult it is for investors to predict the future profitability of firms, taking into account both idiosyncratic information about the firm and information about the macroeconomy. Future movements in stock prices are also not forecastable under the EMH, as they are driven by random shocks which cause investors to update their expectations about the future profitability of firms. From this theoretical view of the stock market, the literature cited above justifies the use of SMV as a measure of economic uncertainty.

However, as was discussed briefly in chapter three, there are problems with this argument. Fundamentally, volatility measures the spread of a variable's distribution while uncertainty is concerned with the difficulty estimating the parameters of that distribution (Aït-Sahalia et al., 2021). Variables with a higher volatility tend to be harder to predict because the range of possible future values tends to be wider, but it is the deviation of an observed value from an agent's best estimate in the previous period which underlines the concept of uncertainty (Jurado et al., 2015).

Stock returns are also partly predictable based on an asset's loading on risk factors such as those identified by Fama & French (1992) or Carhart (1997). Bulan (2005) and Gilchrist et al. (2014) used asset pricing models to try to remove this predictable information but the evidence presented in chapter three casts doubt as to whether this will lead to actual differences in the estimated effect of SMV-measured uncertainty on real economic variables compared to using the standard deviation of returns without conditioning on any risk factors.

More importantly, SMV can vary even when there is no change in the difficulty forecasting economic variables. This can be due to factors such as the influence of noise traders who invest in equities based on speculation not related to firm fundamentals (De Long et al., 1990, 1991), changes in the willingness to supply liquid capital which leads to a burst of equity trading (Allen & Gale, 1994), or changes in risk preferences which cause large movements to or from risky investments such as equities (Drees & Eckwert, 1997; Campbell & Cochrane, 1999; Brandt & Wang, 2003). There is also the observed tendency for stock returns to become more volatile after decreases in price. One common argument suggests this occurs because firms' debt-to-equity ratios increase following a decline in asset prices, which makes stocks more risky and more volatile (Aït-Sahalia et al., 2013)[2]. Risk is another concept which can be conflated with volatility but the two are actually distinct,

[2]But see the evidence against this hypothesis in Hasanhodzic & Lo (2011). The hypothesis, known as the 'leverage effect', is often attributed to Black (1976), however, it is difficult to find the original source.

an investment could display a very large average deviation from its mean but if the downside risk remains low it does not represent significant risk to the investor. Changes in any of these factors can trigger variation in SMV independently of uncertainty.

Previously, Schwert (1989) described a 'volatility paradox' by showing that SMV is unrelated to other forms of macroeconomic volatility. In contrast, Ahn & Lee (2006) found evidence of a bidirectional relationship between SMV and the volatility of real output in several countries. Arnold & Vrugt (2008) appreciate that ex post measures of volatility can smooth over significant ex ante variation. They construct a measure of uncertainty based on the dispersion of forecasts in the Survey of Professional Forecasters and demonstrate that this measure is significantly related to SMV. Furthermore, they perform Granger causality tests between SMV and the forecast dispersion of several macroeconomic variables. They find that uncertainty in four of the variables is significantly linked with SMV but overall there is little evidence of bidirectional relationships. They do not attempt to estimate contemporaneous relationships in their VAR model. For a large country like the U.S., global asset markets can also be affected by increases in domestic uncertainty, Su et al. (2019) find that higher U.S. uncertainty causes increases in SMV in both industrialised and emerging foreign economies.

Joëts et al. (2017) examined the disconnect between uncertainty and the volatility of returns for 19 commodities. Using a structural threshold vector autoregressive model (TVAR), they show that agricultural and industrial price volatility is highly sensitive to uncertainty, while volatility in precious metals shows a less aggressive response. The largest disconnect is found between uncertainty and oil price volatility, meaning that shocks to the latter often did not cause any increase in uncertainty. Similarly, this chapter finds that, on average, shocks to SMV do not have a significant impact on macroeconomic uncertainty. However, Van Robays (2016) found that higher uncertainty tends to cause increases in oil price volatility because it heightens the sensitivity of oil prices to supply and demands shocks. Notably, uncertainty in her study is equated with the volatility of industrial production rather than on expectations or forecast errors.

Jurado et al. (2015) and Baker et al. (2016) construct alternative measures of macroeconomic uncertainty. The former is based on the aggregation of forecast errors described in equations 4.2.1 and 4.2.2 while the latter is based on the number of times keywords related to economic uncertainty appear in newspapers. Aït-Sahalia et al. (2021) demonstrates that the news-based measure by Baker et al. (2016) is frequently in a state of disconnect with stock market volatility, as measured by the standard deviation of returns on the S&P 500. The authors show that equity returns are positively related to uncertainty but negatively related to SMV. They suggest that uncertainty causes a capital flight to safe assets which lowers their required return while driving up the required return on risky assets. Higher volatility is associated with lower returns because lower returns increase debt to equity ratios and thereby make stocks more risky as explained in Aït-Sahalia et al. (2013). Their model thereby presents a solution as to why the sign of the relationship between returns and volatility appears to be sample dependent; it may be confounded by the effects of uncertainty.

### 4.2.2 Macroeconomic Effects

There is a large body of literature discussing the macroeconomic effects of uncertainty but no studies so far have focused on separating shocks to uncertainty from those affecting SMV independently of uncertainty. Bloom (2009) measures uncertainty shocks based on large spikes in the Chicago Board of Options Exchange VXO index and shows that uncertainty causes declines in economic activity followed by quick recoveries and a period of 'overshoot' in a SVAR model. The VXO is calculated from the implied volatility of a hypothetical option on the S&P 100 index with 30 days until expiry and is commonly used as a measure of economic uncertainty. Its derivation from financial options makes it a forward looking measure but Jurado et al. (2015) notes that its variation will still be contaminated by factors affecting SMV independently of uncertainty. Their uncertainty measure based on the conditional volatility of forecast errors shows that large uncertainty shocks are less common than Bloom's analysis would suggest, but they lead to more persistent declines in economic activity when they do occur.

Both these studies used a Cholesky decomposition of the reduced form covariance matrix to identify structural shocks. This imposes a recursive ordering between variables which is difficult to justify on theoretical grounds. Kilian et al. (2022) notes that the common practice of changing the ordering of variables in the VAR and checking whether the IRFs differ is not a valid check for of the correct ordering of variables. Stock & Watson (2012) and Mertens & Ravn (2013) developed an approach to identifying one or more structural shocks based on instrumental variables. If a variable can be found which is both correlated with the targeted structural shock and independent from the other shocks in the model, then the relationships between the variables in the VAR and the shock of interest can be recovered from data on this variable and the reduced-form covariance matrix.

Finding instruments which are only correlated with the variable of interest is a formidable task given the endogeneity of macroeconomic variables over the business cycle (Ramey, 2016). Stock & Watson (2012) also find that uncertainty shocks cause declines in economic activity, but they are unable to separate uncertainty shocks from shocks to financial market liquidity or risk. However, their instrument for uncertainty shocks is based on exogenous shocks to the VIX[3], specifically, the residuals from an AR(2) regression. Because their instrument comes from financial markets, it will naturally pick up shocks to factors such as risk preferences which may affect volatility independently of uncertainty. However, they also find correlation between the innovations in the uncertainty measure of Baker et al. (2016) and the credit spread measure in Gilchrist & Zakrajšek (2012). Piffer & Podstawski (2018) instead use the variation in gold prices around specific events which should make the economic future harder to predict as a proxy for uncertainty shocks. They are able to separate uncertainty shocks affecting uncertainty from news shocks affecting the level of the S&P 500 using Mertens & Ravn's (2013) approach for identifying multiple shocks.

Gilchrist et al. (2014) distinguished shocks to uncertainty from shocks to the credit spread in a recursively identified SVAR model, finding that uncertainty shocks affect the real economy primarily through increases in the credit spread. In their view, uncertainty interferes with firms' supply of credit and causes delays in their planned investment expenditures which are already attenuated through the real option effects of uncertainty (Bernanke, 1983; Dixit & Pindyck, 1994; Abel & Eberly, 1996). Merton (1974) demonstrated that because limited liability provides equity holders with limited downside risk, the payoff of levered equity is equivalent to holding a European call option if the value of the firm is modelled as a geometric Brownian motion. Likewise, holders of risky debt like corporate bonds face a payoff structure identical to the writer of a put option, they

[3]The VIX is based on a the implied volatility of a hypothetical option on the S&P 500 with 30 days to maturity. It thus has a broader market coverage compared to the VXO which is based on the S&P 100.

have a limited upside gain but stand to lose all of their initial investment if the firm defaults. Higher volatility of the firm's assets therefore benefits equity holders at the expense of bond holders, which means the interest paid on debt has to rise to compensate the increase in investor risk. Hence, the increase in uncertainty causes increases in the credit spread.

## 4.3 Instrumental Variable SVAR Models

### 4.3.1 SVAR Models

SVAR models estimate dynamic relationships between time series variables. By identifying the structural shocks to the system, they also reveal the contemporaneous effects between the variables. Consider a $k \times 1$ vector $\boldsymbol{y}_t$ which follows a VAR process with $k$ variables and $p$ lags. The structural shocks are given by then $k \times 1$ vector $\boldsymbol{\epsilon}_t \sim (0, \boldsymbol{I}_k)$ and with $\mathbb{E}(\epsilon_{i,t}, \epsilon_{j,t} = 0)$ so that the various shocks are uncorrelated. Then, equation 4.3.1 describes the dynamics of $\boldsymbol{y}_t$ where the $k \times k$ matrix $\boldsymbol{A}$ contains information about the contemporaneous relationships between the variables and $\boldsymbol{C}_p$ is another $k \times k$ matrix containing information of relationships $p$ periods ago.

$$\boldsymbol{A}\boldsymbol{y}_t = \boldsymbol{\alpha} + \boldsymbol{C}_1\boldsymbol{y}_{t-1} + \cdots + \boldsymbol{C}_p\boldsymbol{y}_{t-p} + \boldsymbol{\epsilon}_t \tag{4.3.1}$$

The contemporaneous relationships in $\boldsymbol{A}$ creates a simultaneity problem meaning the effects of innovations in the structural shocks on one of the variables cannot be identified because all other variables move in the same period. Pre-multiplying by $\boldsymbol{A}^{-1} = \boldsymbol{B}$ and defining $\boldsymbol{\alpha}^* = \boldsymbol{B}\boldsymbol{\alpha}$, $\boldsymbol{C}_p^* = \boldsymbol{B}\boldsymbol{C}_p$, and $\boldsymbol{u}_t = \boldsymbol{B}\boldsymbol{\epsilon}_t$ gives the reduced form model for which a conventional linear estimator like OLS can recover the parameters.

$$\boldsymbol{y}_t = \boldsymbol{\alpha}^* + \boldsymbol{C}_1^*\boldsymbol{y}_{t-1} + \cdots + \boldsymbol{C}_p^*\boldsymbol{y}_{t-p} + \boldsymbol{u}_t \tag{4.3.2}$$

Because all information about the contemporaneous relationships between the variables is now in $\boldsymbol{B}$, the researcher needs to find a way to recover the elements of this matrix. A common method is to perform a Cholesky decomposition of the covariance matrix $\boldsymbol{\Sigma}$ such that $\boldsymbol{\Sigma} = \boldsymbol{B}\boldsymbol{B}'$ where $\boldsymbol{B}$

is a lower triangular matrix. Given $\mathbb{E}(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\Sigma}$ if the errors are homoskedastic and the covariance matrix for $\boldsymbol{\epsilon}_t$ is normalised to $\boldsymbol{I}_k$ then it follows that

$$\boldsymbol{B}\,\mathbb{E}(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t')\boldsymbol{B}' = \boldsymbol{B}\boldsymbol{B}'.$$

This is commonly referred to as imposing a recursive ordering among variables because all elements of $\boldsymbol{B}$ above the lead diagonal are equal to zero, meaning shocks to variables ordered lower in the $\boldsymbol{y}_t$ vector do not contemporaneously affect variables placed higher in the ordering. Despite its simplicity, Cholesky identification is not appropriate in a macroeconomic model where it is not reasonable to assume that the variables only influence each other after a lag, which is certainly the case for fast-moving variables like uncertainty and SMV.

### 4.3.2 Identification by External Proxy

Another possible way to recover the elements in $\boldsymbol{B}$ is to use instrumental variables contained in a vector $\boldsymbol{z}_t$ as a proxy for the structural shocks in $\boldsymbol{\epsilon}_t$ (Stock & Watson, 2012; Mertens & Ravn, 2013). This chapter examines the dynamic macroeconomic effects of shocks to economic uncertainty and equity market volatility, hence, $\boldsymbol{z}_t$ is a $2 \times 1$ vector containing instruments for structural shocks to uncertainty and SMV. Decomposing the residuals from equation 4.3.2 ($\hat{\boldsymbol{u}}_t$) into

$$\hat{\boldsymbol{u}}_t = \boldsymbol{B}^*\boldsymbol{\epsilon}_t^* + \tilde{\boldsymbol{B}}\tilde{\boldsymbol{\epsilon}}_t \tag{4.3.3}$$

where $\boldsymbol{\epsilon}_t^*$ is a $2 \times 1$ vector of the structural shocks of interest (the ones which will be identified using external instruments), $\boldsymbol{B}^*$ is a $k \times 2$ matrix of the contemporaneous effects of those shocks, $\tilde{\boldsymbol{\epsilon}}_t$ is a $(k-2) \times 1$ vector collecting all other structural shocks which will not be identified, and $\tilde{\boldsymbol{B}}$ is a $k \times (k-2)$ matrix of the contemporaneous effects of $\tilde{\boldsymbol{\epsilon}}_t$.

If two instruments can be found for the structural shocks of interest which are both relevant such that $\mathbb{E}\left(\boldsymbol{\epsilon}_t^*\boldsymbol{z}_t'\right) = \boldsymbol{\Phi}$, where $\boldsymbol{\Phi}$ is a $2 \times 2$ non-zero and full column-rank matrix, and exogenous such that $\mathbb{E}\left(\tilde{\boldsymbol{\epsilon}}_t\boldsymbol{z}_t'\right) = 0$, then the $2 \times 1$ vector $\boldsymbol{z}_t$ can be used to estimate the parameters in the two columns of $\boldsymbol{B}^*$. Although $\boldsymbol{\epsilon}_t^*$ is not observed, the reduced form errors $\hat{\boldsymbol{u}}_t$ provide a way of obtaining the parameters in $\boldsymbol{B}^*$ up to the sign convention $\boldsymbol{\Phi}$ because of the relation

$$\mathbb{E}\left(\hat{\boldsymbol{u}}_t \boldsymbol{z}_t'\right) = \boldsymbol{B}^* \, \mathbb{E}\left(\boldsymbol{\epsilon}_t^* \boldsymbol{z}_t'\right) + \tilde{\boldsymbol{B}} \, \mathbb{E}\left(\tilde{\boldsymbol{\epsilon}}_t \boldsymbol{z}_t'\right) = \boldsymbol{B}^* \boldsymbol{\Phi}. \tag{4.3.4}$$

Now run $2 \times k$ linear regressions of the variables in $\hat{\boldsymbol{u}}_t$ on the variables in $\boldsymbol{z}_t$ where $i = 1, \ldots, k$ indexes the rows of a matrix, $j = 1, 2$ indexes the columns of a matrix, and $\gamma_{ij}$ is an element of the $k \times 2$ matrix $\boldsymbol{\Gamma}$,

$$\hat{u}_{it} = \delta_{ij} + \gamma_{ij} z_{jt} + \eta_{ij,t}, \tag{4.3.5}$$

where, if the instruments in $\boldsymbol{z}_t$ are normalised to have unit variance

$$\gamma_{ij} = \mathbb{E}\left(\hat{u}_{it} z_{jt}\right) = \sum_{h=1}^{2} b_{ih}^* \phi_{hj}. \tag{4.3.6}$$

The matrix $\boldsymbol{\Gamma}$ thus contains the contemporaneous effects of the structural shocks of interest up to the constant matrix of unknown weights $\boldsymbol{\Phi}$. Some algebra (given in appendix 4.A.1) yields $\boldsymbol{B}^*$ from $\boldsymbol{\Gamma}$, thus recovering two columns of the $B$ matrix which can be used for impulse response analysis and forecast error decompositions.

The caveat is that this method produces many candidate estimates for $\boldsymbol{B}^*$ dependent on a draw from a distribution of random orthogonal matrices (see the appendix for details). Drawing 1000 orthogonal matrices, $\boldsymbol{Q}$, generates a sample of 1000 $\boldsymbol{B}^*$ matrices from the identified set. Note that equations 4.3.4 and 4.3.6 imply that $\boldsymbol{\Gamma} = \boldsymbol{B}^* \boldsymbol{\Phi}$ which in turn implies that $\left(\boldsymbol{B}^*\right)^{-1} \boldsymbol{\Gamma} = \boldsymbol{\Phi}$. Remembering that $\boldsymbol{\Phi} = \mathbb{E}\left(\boldsymbol{\epsilon}_t^* \boldsymbol{z}_t'\right)$, there are restrictions which can be placed on $\boldsymbol{\Phi}$ to restrict the set of $\boldsymbol{B}^*$ to those which are economically intuitive. Piffer & Podstawski (2018) let

$$\begin{bmatrix} \phi_{11} > 0 & \phi_{22} - \phi_{12} > \psi \\ \phi_{11} - \phi_{21} > \psi & \phi_{22} > 0 \end{bmatrix}. \tag{4.3.7}$$

These conditions imply that increases in the proxy for structural shocks to uncertainty must be associated with increases in reduced-form shocks to uncertainty, increases in the risk aversion proxy must be associated with increases in reduced-form shocks to SMV, and that the instruments must both be more correlated with the shock they target compared to the other shock by a positive quantity $\psi$. A higher $\psi$ demands the instruments are more correlated with the shocks they target relative to the other shock, so will tend to decrease the size of the identified set which passes the restrictions. As in Piffer & Podstawski (2018), the results presented in this paper are robust to variations in $\psi$ between 0 and 0.2. I use a baseline of $\psi = 0.12$.

### 4.3.3 Reporting IRFs and FEVD

The main tool for interpreting the results of SVAR models is the IRF. Impulse responses are the difference between forecasts of $\boldsymbol{y}_t$ produced from a model in which the system is affected by a shock and one in which no shock occurs (Hamilton, 1994, pp.318-319). From (Lütkepohl, 2005, pp.46, 58), equation 4.3.2 can be written in moving average form

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \sum_{i=0}^{\infty} \boldsymbol{\Lambda}_i \boldsymbol{u}_{t-i} = \boldsymbol{\mu} + \sum_{i=0}^{\infty} \boldsymbol{\theta}_i \boldsymbol{\epsilon}_{t-i} \tag{4.3.8}$$

where, given $L^i$ is an operator producing the $i^{th}$ lag, $\boldsymbol{\Lambda}(L) = \sum_{i=0}^{\infty} \boldsymbol{\Lambda}_i L^i$ such that $\boldsymbol{\Lambda}(L)\boldsymbol{C}(L) = I_k$, $\boldsymbol{\mu} = \boldsymbol{\Lambda}(L)\boldsymbol{\alpha}^*$, and $\boldsymbol{\theta}(L) = \sum_{i=0}^{\infty} \boldsymbol{\Lambda}_i \boldsymbol{B}$. Hence, the $s$ step ahead response to a shock $\varepsilon$ at time $t$ is

$$\mathbb{E}\left[\boldsymbol{y}_{t+s} \mid \boldsymbol{\epsilon}_t = \varepsilon\right] - \mathbb{E}\left[\boldsymbol{y}_{t+s} \mid \boldsymbol{\epsilon}_t = 0\right] = \boldsymbol{\theta}_s \varepsilon \tag{4.3.9}$$

because $\varepsilon$ is propagated through the system by the autoregressive coefficients contained in $\boldsymbol{\Lambda}_s$. I calibrate the IRFs so that the shock of interest (uncertainty or volatility) causes a one standard deviation increase in the variable it targets on impact. For example, if $\sigma_{y_{it}}$ is the standard deviation of variable of interest $y_{it}$, then the shock is calibrated to increase $y_{it}$ by $\sigma_{y_{it}}$ on impact.

If $jl$ represents the $jl^{th}$ element of the matrix $\boldsymbol{\theta}_i$, then dividing $\sum_{i=0}^{s-1} \boldsymbol{\theta}_{jl,i}^2$ by the total forecast variance of variable $l$ in the VAR system, which is the $l^{th}$ element on the leading diagonal of

$$\sum_{i=0}^{s-1} \boldsymbol{\Lambda}_i \boldsymbol{\Sigma} \boldsymbol{\Lambda}_i',$$

gives the $s$ step-ahead forecast error decomposition of variable $j$. This is the total contribution of the shock to the variance of variable $l$. A shock could cause a persistent increase or decrease in a variable, however, this response might only account for a small proportion of the total variation, which implies the shock is less influential in explaining that variable's dynamics. This is why forecast error decompositions are a useful tool to examine the relationships between macroeconomic variables.

As discussed, the method outlined in section 4.3.2 will only recover two columns of the $\boldsymbol{B}$ matrix. Furthermore, there will be as many estimates as there are $\boldsymbol{B}^*$ satisfying the identification restrictions in equation 4.3.7. One way to summarise the IRFs generated from this method is to take the median of all the estimates. However, Fry & Pagan (2011) point out that doing so mixes information from different $\boldsymbol{B}^*$ matrices and renders forecast error decompositions meaningless as they can take values greater than one. They develop a method of finding a Median Target (MT) $\boldsymbol{\theta}_i^{MT}$ matrix which is as close as possible to median of the estimated $\boldsymbol{\theta}_i$. The elements of all the $\boldsymbol{\theta}_i$ are standardised by subtracting their median and dividing by their standard deviation and stacked in a column vector $\boldsymbol{\omega}$. If there are $s$ steps and $k$ variables in the VAR, then with two identified shocks this vector will have dimensions $2ks \times 1$. The chosen $\boldsymbol{\theta}_i^{MT}$ is whichever $\boldsymbol{\theta}_i$ minimises the value of $\boldsymbol{\omega}'\boldsymbol{\omega}$. The reported IRFs and the forecast error decomposition is based on $\boldsymbol{\theta}_i^{MT}$.

I use a Moving Block Bootstrap of Jentsch & Lunsford (2022) to generate the 68% confidence bands for the IRF estimates. The quantiles of the bootstrapped distribution are based on the standard percentile intervals discussed in (Lütkepohl, 2005, p.710). Piffer & Podstawski (2018) use a form of Wild Bootstrap to generate their confidence bands, which, unlike the MBB, does not take into account heteroskedasticity such as GARCH effects in the residuals of the VAR and leads to asymptotically invalid estimates of confidence bands (Bruns & Lütkepohl, 2022). I compare the bands from the MBB to those from the Proxy Residual Based Bootstrap (PRBB) of Bruns & Lütkepohl (2022), which may attain higher accuracy in small samples. The method of constructing the bootstrapped estimates is discussed in appendix 4.A.2.

Ludvigson et al. (2021), while also adopting a bootstrap procedure for statistical inference, note that the lack of a consistent point estimate makes inference especially challenging in set-identified SVAR-IV models. Giacomini et al. (2022) and Braun & Brüggemann (2022) both use Bayesian inference. The former build on the robust Bayesian approach of Giacomini & Kitagawa (2021) allowing the researcher to relax controversial point-identifying restrictions without introducing an unrevisable prior into the model. In the latter, the influence of the prior does not vanish asymptotically but the authors show how to calculate Bayes factors to test the validity of external instruments. The moment-inequality framework of Granziera et al. (2018) and the delta method inference of Gafarov et al. (2018) can only place restrictions on a single shock.

## 4.4 Data

### 4.4.1 Macroeconomic Data

I use the uncertainty index constructed by Jurado et al.'s (2015) as the measure of uncertainty; it is based on the one-month-ahead forecast error produced from the principle component of over 130 macroeconomic series and is hereafter called the JLN measure. I deviate from Bloom (2009) and Piffer & Podstawski (2018) in not using the VIX as a measure of uncertainty because, as Jurado et al. (2015) point out, it is constructed from options dependent on the performance of the S&P 500 and will therefore pick up changes in volatility which are not related to uncertainty. SMV is measured by the monthly standard deviation of returns on the S&P 500 index, calculated as the volatility of daily returns multiplied by the square root of the number of days in the month. As in Bloom (2009), I examine the impact of volatility while holding the level of returns constant by including the monthly returns of the S&P 500 in the model, the mean of daily returns multiplied by the number of days in the month.

The macroeconomic variables are available from the Federal Reserve Bank of St. Louis website and are selected and transformed based on the models of Bloom (2009) and Piffer & Podstawski (2018). Their monthly time series in the sample period between January 1969 and March 2022 are given in figure 4.4.1. Three of the variables are prices. The Federal Funds Rate (FFR) is the rate at which the Federal Reserve loans to corporate banks, wage growth is the log-difference in average hourly earnings for non-supervisory employees in the manufacturing sector, and inflation is the log-difference in the Consumer Price Index (CPI). The other three variables capture real activity. Hours worked is the average weekly hours worked in the manufacturing sector, labour growth is the log-difference in total number of employees in the manufacturing sector, and production growth is measured by the log-difference in the Federal Reserve Board of Governors production index. The very large falls in production and labour growth at the end of the sample are caused by the fallout of the Coronavirus pandemic.

In light of the real options literature in Dixit & Pindyck (1994), it makes sense to include both hours worked and the total number of employees because a negative uncertainty shock may incentivise firms to temporarily reduce the hours worked by their labour stock rather than making the partially irreversible decision of firing workers. The optimal lag length of 3 for the reduced-form VAR was chosen by the Akaike information criterion. The conclusions of this paper do not change if the lag order is increased to 5, which is the initial lag order Piffer & Podstawski (2018) provide in their codes available on Michele Piffer's website.

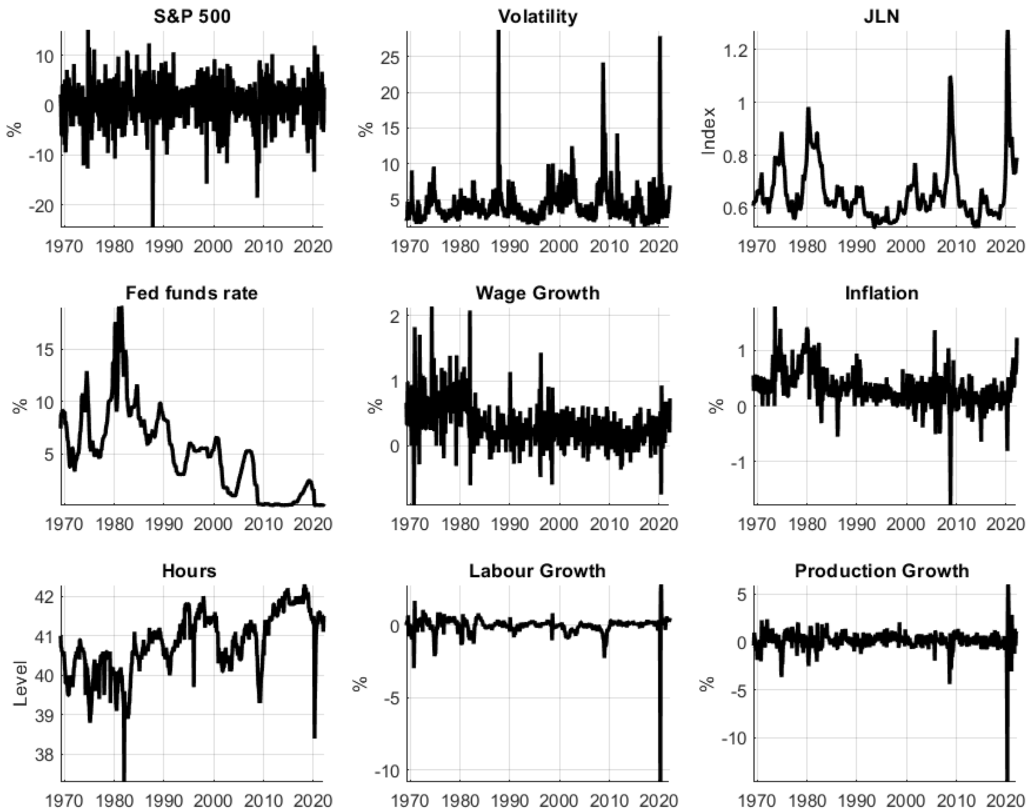



Figure 4.4.1: Macroeconomic Variables

### 4.4.2 Uncertainty Proxy Construction

Following Piffer & Podstawski (2018), I use the intraday variation in the price of gold on the London Bullion Market around historical events expected to cause an increase in uncertainty as a proxy measure for uncertainty shocks. In theory, gold prices should be correlated with uncertainty shocks because gold is considered a safe-haven asset whose demand rises following an increase in uncertainty. By calculating the intraday variation around specific events, it is easier to justify that these changes are truly caused by uncertainty shocks and not by changes in risk preferences, the other structural shock identified in the model, or indeed by the other structural shocks which are not identified. The official price of gold is quoted on weekdays at 10:30 and 15:00 GMT.

I extend the events database used by Piffer & Podstawski (2018) so that it covers a range from January 1969 to March 2022. Piffer & Podawski use the publication times of news articles (primarily from Bloomberg) to determine the exact time when news of an uncertainty event reached the market. Like them, I collect events which might change how easy it is for agents to forecast economic variables and that are not the direct result of another economic shock, such as a change in interest rates. Typical examples are wars, terrorist attacks, natural distastes, and election results. The final list of the events added to the Piffer & Podstawski (2018) database and used in the construction of the proxy is given in appendix 4.A.8. For the sample period of 1969-2022, roughly 10% of months contain uncertainty shocks, which is comparable to the ratio used in Piffer & Podstawski (2018) and Mertens & Ravn (2013).

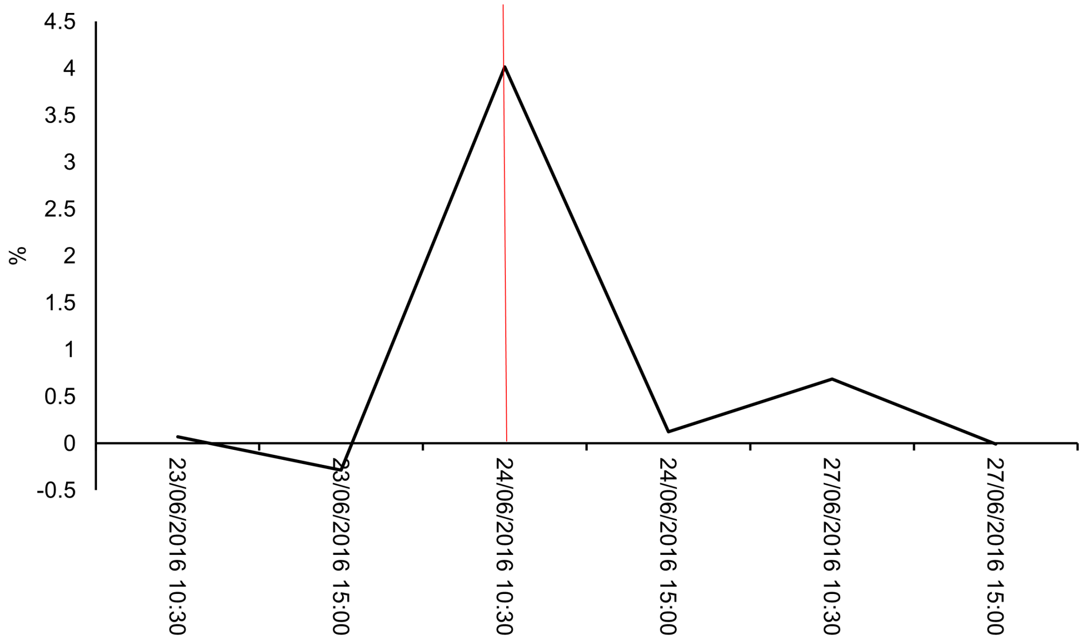
Figure 4.4.2 gives two examples of the variation of gold prices around an event; panel 4.4.2a shows the spike after the result of the Brexit referendum was announced and panel 4.4.2b shows the fall in gold prices after the election of President Clinton. The unexpected Brexit vote not only caused short-term fluctuations in global asset markets but also created uncertainty about future trade agreements and potential disruptions in the flow of goods from the European Union's export sector (Graziano et al., 2020). The announcement came in the evening of the 23rd of June, so the jump in gold price came the morning after. In contrast, the poor state of the economy and perception that it was being mismanaged under President George H.W. Bush gave Clinton an edge in the 1992 election Doherty & Gimpel (1997), and the announcement of the result resolves some of the uncertainty about government policy in the coming four years, because many of the candidate's plans will be revealed during the election campaign[4]. Again, the result was announced on the night of the 3rd of November, so the gold price drops the morning after.

For the period before 1980, I mostly use the New York Times (NYT) to determine when the news of an uncertainty event reached the market. Baker et al. (2016) also used the NYT as a trusted news source to construct their Economic Policy Uncertainty Index. Events generally appear in the NYT the morning after they occurred, for example, the OPEC siege on 21st of December 1975, where armed terrorists held 60 hostages at the semi-annual meeting of OPEC leaders in Vienna, is reported in the NYT the day after. Some events, such as the resignation of president Nixon on August 8th 1974 were broadcast on television, so it is easy to pinpoint when they occurred. I mostly use the times quoted on articles written by Reuters news agency for the period from 2015 (the end of Piffer & Podawski's sample) to 2022. I label the uncertainty instrument based on the exact times events occurred $gold_E$, where the $E$ subscript stands for 'exact'.

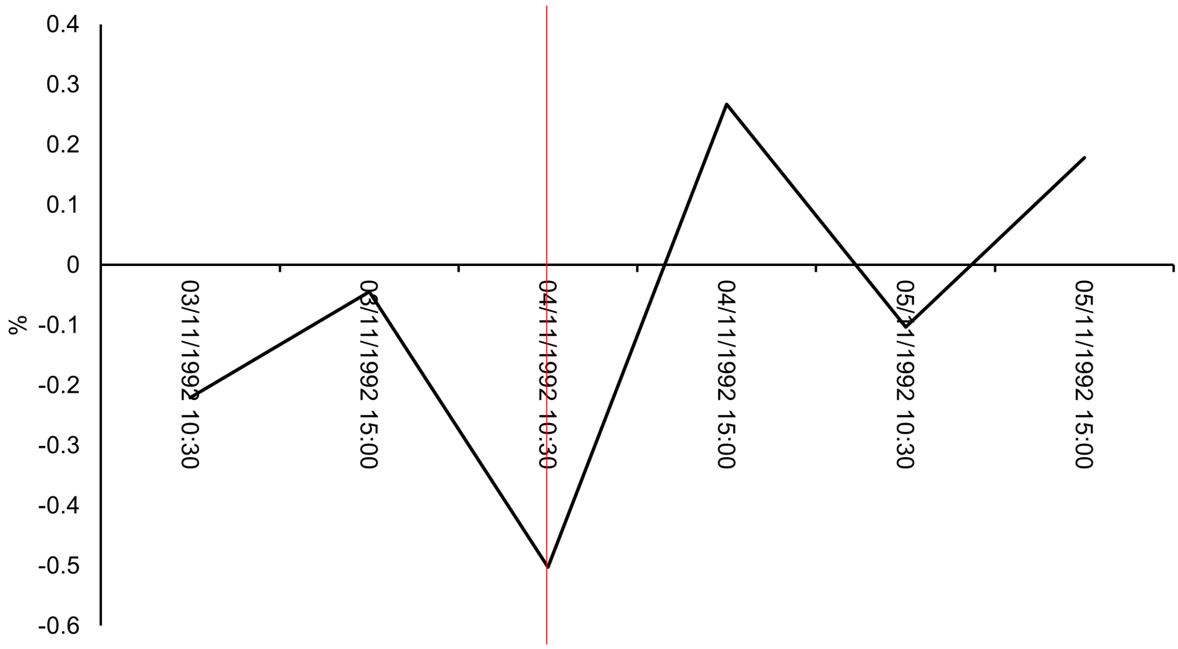
However, information on the exact time of events is often not available. This occurs in around 17% of cases in the original Piffer & Podawski database, which rises to 35% in the first 10 years, reflecting that fact that it is harder to pinpoint the exact times events occur early in the sample. In such cases, the exact time news of the event reached the market must be guessed. It is also harder to guess whether news may have reached informed investors before newspapers were able to publish the story in the early years of the sample.

To see how much guessing might impact results, I construct another proxy which identifies when news of the event reached the market through a simple search algorithm that chooses the largest absolute change in the price of gold in each 24 period on the date the event occurred. Given the spot price of gold is quoted twice every day, at 10:30 and 15:00, there are three possible intervals in which the event can affect the quote in a 24 hour period. Consider an even occurring on the 2nd of January and let $P_{time}^{date}$ denote the price of gold at a specific time on the date given by the superscript. If news of the event reached the market between 00:00 and 10:30, its effect on the gold

[4]Election results have quite an unpredictable effect on the gold price. On the one hand, they should give a signal about what policy decisions will be made over the next four years. On the other hand, if the presidential candidate displays erratic behaviour, or the election divides executive and legislature power, it also seems possible that uncertainty will increase. In light of this, I have not attempted to assign an expected sign to these events in the manner explained later in the text.




(a) Increase in Gold Price after Brexit




(b) Fall in Gold Price after Clinton is Elected

Figure 4.4.2: Examples of Changes in Gold Price after Uncertainty Shocks

price will show up between $P^{2nd}_{10:30} - P^{1st}_{15:00}$. Applying the same reasoning to other intervals between the price quotes, the event could also affect the gold price between $P^{2nd}_{15:00} - P^{2nd}_{10:30}$ and $P^{3rd}_{10:30} - P^{2nd}_{15:00}$. The algorithm chooses whichever of these changes is largest in absolute value.

It is possible that the candidate uncertainty shock actually had no effect on the gold price, so contrary to the researcher's belief, markets did not believe the event made the future easier or harder to predict and justified a rebalancing of portfolios. But the algorithm would incorrectly attribute the largest jump in the gold price on that day to the candidate uncertainty shock. To mitigate the risk of incorrectly identifying candidate uncertainty events which actually had no affect on gold prices, I assign an expected sign to each event and demand that the sign of the largest absolute change during the day matches the expected sign of the uncertainty shock. For example, on the evening of the 17th of October 1973, Americans were made aware that Arab states were issuing an oil embargo against countries supporting Israel. This event will disrupt supply chains and make the economic future harder to predict, so the expected sign of the change in the gold price is positive. However, the largest movement in gold prices that day was a $-1.46\%$ decline. The mismatch in expected and actual signs of the gold price means the event is not selected by the algorithm, or used in the instrument based on exact times. I label the gold instrument based on the algorithm $gold_A$.

As an additional check on the robustness of the instrument, I compare it to another which calculates the mean gold price change across the 24-hour period when the shock occurred (labelled $gold_M$). Given the search algorithm always chooses the largest variation in the 24-hour period, it will likely overestimate the response of gold prices to uncertainty shocks. The proxy based on the 24-hour mean will likely underestimate the response because the random variations in gold price will mitigate the jump caused by the uncertainty event[5]. I expect the results generated by the proxy based on exact times to fall between those generated by the search algorithm and the intraday mean.

Once events and the corresponding changes in the gold price of are identified, a value of zero is assigned to days without a shock and the percentage change in the price of gold on that day is used for those with a shock. These daily values are then summed every month to produce the proxy of uncertainty shocks. Naturally, months with no shocks will have a value of zero. Like Piffer & Podstawski (2018), I winsorise the final instrument at the one percent level to mitigate the effect of outliers. Figure 4.4.3 presents the instrument for uncertainty shocks based on exact times.

[5]It is worth noting that there is little evidence that the daily variation in the price of gold exhibits strong autocorrelation, based on AR(3) models run for each year in the sample.

### 4.4.3 Financial Market Shocks Proxy Construction

Some shocks are likely to be very closely related to uncertainty shocks and could be a potential confounding influence. Piffer & Podstawski (2018) note that their uncertainty measure might be correlated with events that provide markets with information about future productivity. Barsky & Sims (2011) note that these 'news shocks', are significant predictors for the future dynamics of consumption, hours worked, and output. To ensure their identified uncertainty shocks are separated from news shocks Piffer & Podstawski use a set-identification approach to estimate the effects of the two structural shocks in the same model. They use the Principle Component from several measures of news shocks in the literature as an instrument to identify shocks to the *level* of the S&P 500[6].

Building on the literature surrounding the disconnect between uncertainty and volatility, I use the same methodology to disentangle uncertainty shocks from shocks to credit and risk conditions by creating an instrument for that latter from the exogenous shocks to the credit spread between Baa-rated corporate bonds and the interest rate on a 10-year treasury bond. Using credit spreads to proxy shocks in financial conditions has precedent in SVAR literature via Stock & Watson (2012) who use the measure of credit spreads constructed by Gilchrist & Zakrajšek (2011; 2012) from corporate bond premiums. While Stock & Watson (2012) do not find evidence to suggest that their uncertainty and financial tightness proxies identify different shocks, this may be explained by their use of the VIX as a proxy for uncertainty, which Jurado et al. (2015) points out also varies independently of uncertainty due to changes in stock market volatility.

Although credit spreads show a strong association with economic uncertainty (Gilchrist et al., 2014), they should also contain information relevant to SMV which is orthogonal to uncertainty. For example, changes in risk appetite (Campbell & Cochrane, 1999; Brandt & Wang, 2003) or leverage (Bollerslev et al., 2006) can affect stock market volatility even if there is no change in underlying uncertainty. Credit spreads will certainly reflect investors' risk appetite because the required interest rate on corporate bonds will be higher when investors are more risk averse. Indeed, Tang & Yan (2010) find that investor sentiment is the most important determinant of credit spreads at

[6] I do not identify news shocks explicitly in the model, so it is possible that the identified uncertainty shocks are contaminated by news shocks. However, looking at the list of events used in appendix 4.A.8, it is often difficult to see how they would give a clear signal about future productivity, and much clearer to see how they would make it harder to accurately forecast the economic future.

the aggregate level. As the value of debt will tend to increase as the risk premium on corporate bonds increases, higher credit spreads will also lead to a stronger leverage effect under the interpretation that a fall in share prices increases the debt-to-equity ratio, implying a greater default risk. Hence, innovations in the credit spread can affect financial market volatility with no change in macroeconomic uncertainty. For simplicity, I will refer to these as shocks to risk preferences. Table 4.4.1 shows the credit spreads has a stronger relationship, both higher t-statistics and $R^2$, with the volatility of the S&P 500 index compared with JLN and EPU macroeconomic uncertainty indicators.

| | JLN | EPU | S&P500 |
|---|---|---|---|
| $\beta_{cs}$ | 0.0608*** | 34.4419*** | 0.3193*** |
| | (13.7964) | (9.6989) | (16.3992) |
| $\beta_0$ | 0.5241*** | 41.6357*** | 0.2382*** |
| | (54.5313) | (4.8431) | (5.8620) |
| $R^2$ | 0.2048 | 0.1745 | 0.2456 |
| T | 741 | 447 | 828 |

Table 4.4.1: Correlation of Credit Spreads, Uncertainty, and Stock Market Volatility

To minimise the risk that the proxy for financial market shocks is tainted by other macroeconomic shocks, I use the method of Stock & Watson (2012) and construct the proxy variable from the residuals of the best-fitting autoregressive model according to the Akaike information criterion. This is a good first step to extract the exogenous component of the series, however, it does not take into account that other variables could cause changes in the credit spread. To deal with this issue, I also include changes in the one-month treasury bill and the oil price in the regression to control for other shocks identified in Piffer & Podstawski (2018) as potential sources of contamination. Theoretically, this method leaves only the purely exogenous part of the series, which I label $cs$.

Phillips-Perron and Dicky-Fuller tests reject the presence of a unit root at the 1% significance level in the credit spread series. Oil prices and the T-bill rate enter the regression in first differences because the same tests suggest these series are non-stationary. From table 4.4.2, the best fitting autoregressive model uses three lags of the credit spread. Increases in the oil price have a statistically significant negative effect on credit spreads, suggesting that higher oil prices decrease the required premium on corporate bonds necessary to attract external funds. Jiang et al. (2021) find that oil

| | Credit Spread |
|---|---|
| $\Delta oil\ price$ | -0.0012*** |
| | (-3.6364) |
| $\Delta Tbill$ | -0.6871 |
| | (-0.6883) |
| $\beta_0$ | 2.0433*** |
| | (9.4179) |
| Lags | |
| $L1$ | 1.2816*** |
| | (50.5768) |
| $L2$ | -0.4598*** |
| | (-10.7128) |
| $L3$ | 0.1450*** |
| | (5.0658) |
| AIC | -386 |
| N | 704 |

Table 4.4.2: Best-Fitting AR Model for Credit Spreads

demand shocks tend to decrease the credit spread and put the association down to higher economic activity in the short run because of the shock. After controlling for oil prices and the lags of the credit spread, changes in the short-term interest rate do not have a significant impact. The proxy for financial market shocks is constructed as the residuals from this regression and is presented in figure 4.4.3.

### 4.4.4 Tests of Instrument Suitability

Olea et al. (2021) argue that a strong instrument has a heteroskedasticity robust F-statistic greater than 10 in the first stage regression (equation 4.3.5). The F-statistic from a comparison of a restricted model including only a constant term and an unrestricted model including the variable of interest is equivalent to the square of the variable of interest's t-statistic in the unrestricted model. The row $F$ in table 4.4.3 shows the F-statistic for the uncertainty and financial market volatility instruments in the first stage regressions. The first two columns assume the error variance is constant, which is what Piffer & Podstawski (2018) report in their first stage regressions. The last two columns show the results when heteroskedasticity-robust standard errors are used. All of the instruments have been normalised so the magnitude of the coefficients in each column are comparable.

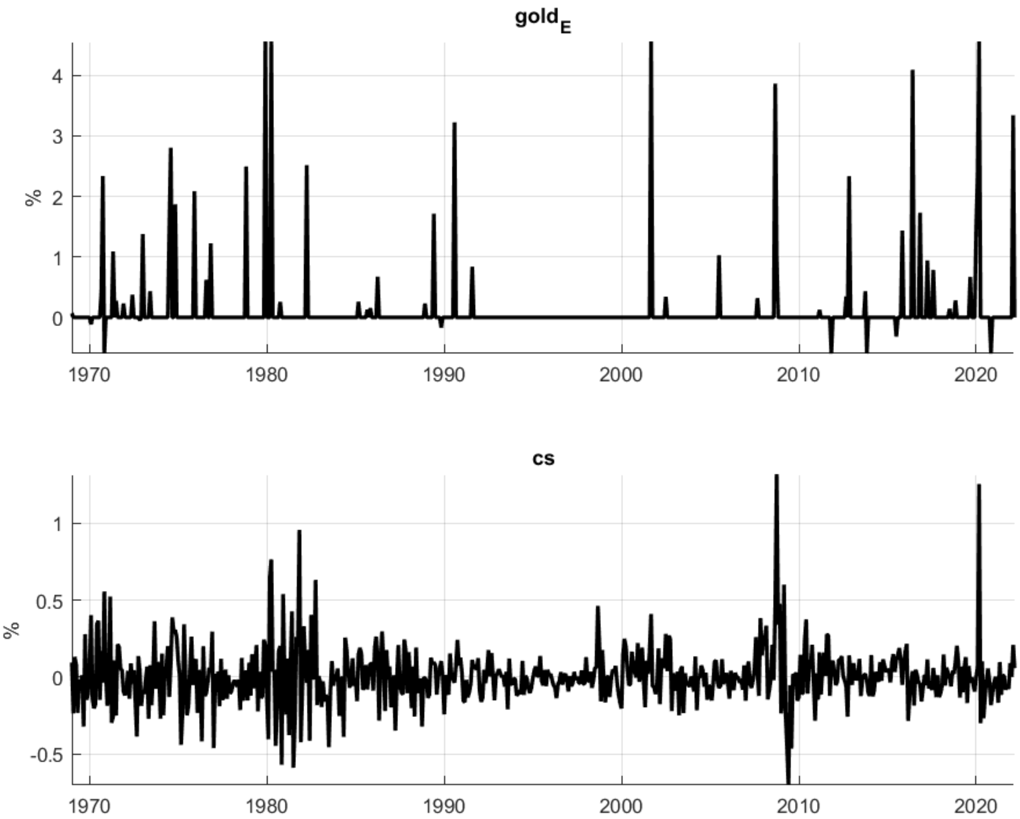



Figure 4.4.3: Instruments for Uncertainty and Financial SMV Shocks

Both instruments are highly correlated with the reduced-form residuals of the variables they target. Importantly, they are more correlated with the variables they target compared to the one they do not. The F-statistics in the first two columns are very high which would suggest the instruments are very strong. However, after controlling for heteroskedasticity the F-statistics decline substantially, in the $gold_E$ and $cs$ cases, they fall to just below 10. Thus, while there is good evidence to suggest the instruments are relevant, they are close to the threshold of what Olea et al. (2021) classify as a strong instrument when taking into account potential bias in the standard errors. In contrast to the findings of Stock & Watson (2012), however, it appears that using the variation of gold prices around events that change how easy it is for agents to predict the future can separate uncertainty shocks from shocks to SMV.

Table 4.4.4 shows the relationships between the instruments and reduced-form shocks to the other variables in the model. It reports the t-statistics from both a model which assumes the error

| | Homoskedastic Standard Errors | | Robust Standard Errors | |
|---|---|---|---|---|
| | $\hat{\boldsymbol{u}}_{JLN}$ | $\hat{\boldsymbol{u}}_{volatility}$ | $\hat{\boldsymbol{u}}_{JLN}$ | $\hat{\boldsymbol{u}}_{volatility}$ |
| $gold_E$ | 0.0042*** | 0.3979*** | 0.0042*** | 0.3979* |
| F | 74.70 | 28.10 | 9.58 | 3.10 |
| $R^2$ | 0.1050 | 0.0423 | 0.1050 | 0.0423 |
| $gold_A$ | 0.0042*** | 0.3347*** | 0.0042*** | 0.3347 |
| F | 74.82 | 19.62 | 10.17 | 2.31 |
| $R^2$ | 0.1051 | 0.0299 | 0.1051 | 0.0299 |
| $gold_M$ | 0.0040*** | 0.2868*** | 0.0040*** | 0.2868 |
| F | 69.95 | 14.30 | 10.12 | 1.64 |
| $R^2$ | 0.0990 | 0.0220 | 0.0990 | 0.0220 |
| $cs$ | 0.0012** | 0.4884*** | 0.0012 | 0.4884*** |
| $F$ | 5.44 | 44.76 | 1.45 | 9.61 |
| $R^2$ | 0.0085 | 0.0657 | 0.0085 | 0.0657 |
| N | 639 | 639 | 639 | 639 |

Table 4.4.3: Relevance of Instruments

variance is constant (the first number in parentheses) and using heteroskedasticity-robust standard errors (the second number in parentheses). The stars next to the coefficients indicate statistical significance in the constant error variance case.

The uncertainty shock instrument is negatively correlated with the level of the S&P 500, and there is weaker evidence to suggest it is correlated with wage and production growth (these coefficients become insignificant when using robust standard errors). The credit spread instrument only shows a statistically significant correlation with shocks to wage growth at the 1% level, so there is some evidence that shocks to the credit spread pick up these kinds of shocks. However, the associated F-statistic is still only $-2.8^2 = 7.84$ when using heteroskedasticity-robust standard errors, which is the largest robust F-statistic observed in the table. The lack of significant relationship between $cs$ and $\hat{\boldsymbol{u}}_{S\&P500}$ implies that when controlling for shocks to the monthly volatility of the S&P 500, shocks to risk preferences do not significantly correlate with shocks to the level of the S&P 500. In general, the instruments are strongly correlated with the variables they target and only weakly correlated with the other variables in the model, suggesting the instruments are only predicting variation in the variables they target. There may be some concern that the uncertainty proxy also picks up shocks to the level of the S&P 500 but the correlations are still much weaker than they are for JLN uncertainty.

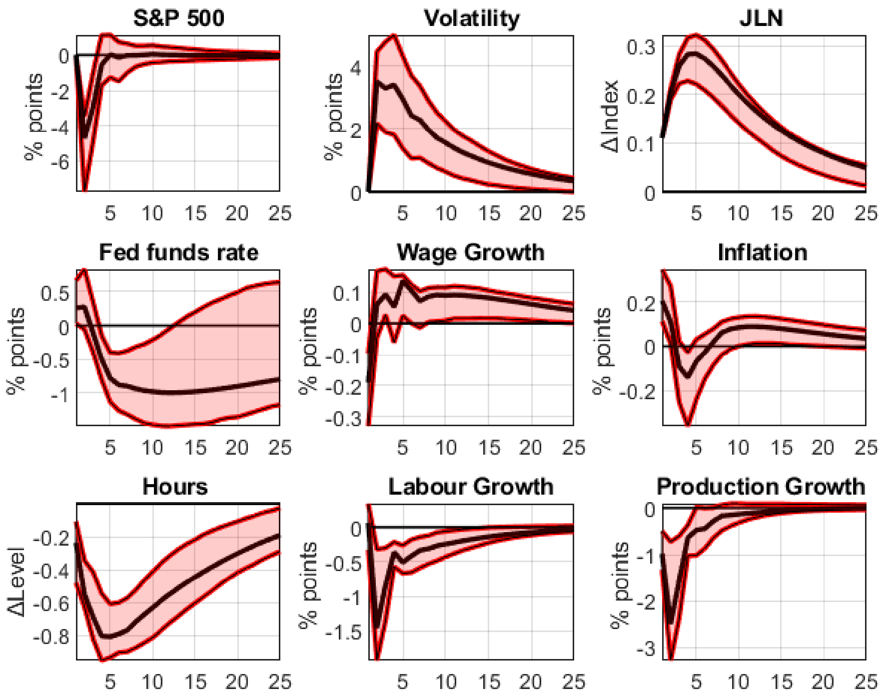
|  | $\hat{u}_{S\&P500}$ | $\hat{u}_{FFR}$ | $\hat{u}_{Wages}$ | $\hat{u}_{Inflation}$ | $\hat{u}_{Hours}$ | $\hat{u}_{Labour}$ | $\hat{u}_{Production}$ |
|---|---|---|---|---|---|---|---|
| $gold_E$ | -0.6218*** | 0.0005 | -0.0227** | 0.0039 | 0.0062 | -0.0072 | -0.0633** |
|  | (-3.7096) | (0.0284) | (-2.0271) | (0.4440) | (0.7037) | (-0.3790) | (-2.0015) |
|  | (-2.7479) | (0.0204) | (-1.8185) | (0.4176) | (0.9873) | (-0.2786) | (-1.5055) |
| $R^2$ | 0.0211 | 0.0000 | 0.0064 | 0.0003 | 0.0008 | 0.0002 | 0.0062 |
| $gold_A$ | -0.5896*** | 0.0051 | -0.0218* | 0.0074 | 0.0066 | -0.0054 | -0.0568* |
|  | (-3.5141) | (0.2836) | (-1.9442) | (0.8374) | (0.7526) | (-0.2842) | (-1.7966) |
|  | (-2.7149) | (0.2113) | (-1.7912) | (0.8189) | (1.0989) | (-0.2146) | (-1.4058) |
| $R^2$ | 0.0190 | 0.0001 | 0.0059 | 0.0011 | 0.0009 | 0.0001 | 0.0050 |
| $gold_M$ | -0.4825*** | 0.0051 | -0.0274** | 0.0050 | 0.0017 | -0.0195 | -0.0863*** |
|  | (-2.8662) | (0.2834) | (-2.4557) | (0.5651) | (0.1907) | (-1.0298) | (-2.7352) |
|  | (-2.4532) | (0.2235) | (-1.9007) | (0.6476) | (0.3083) | (-0.6090) | (-2.1179) |
| $R^2$ | 0.0127 | 0.0001 | 0.0094 | 0.0005 | 0.0001 | 0.0017 | 0.0116 |
| $cs$ | -0.3136* | -0.0329* | -0.0322*** | -0.0146* | 0.0144* | 0.0260 | 0.0128 |
|  | (-1.8849) | (-1.8544) | (-2.9300) | (-1.6796) | (1.6739) | (1.3956) | (0.4093) |
|  | (-1.3276) | (-1.1193) | (-2.7960) | (-1.3272) | (1.4718) | (0.9829) | (0.2805) |
| $R^2$ | 0.0055 | 0.0054 | 0.0133 | 0.0044 | 0.0044 | 0.0030 | 0.0003 |
| $N$ | 639 | 639 | 639 | 639 | 639 | 639 | 639 |

Table 4.4.4: Correlation between Instruments and Shocks to Other Variables

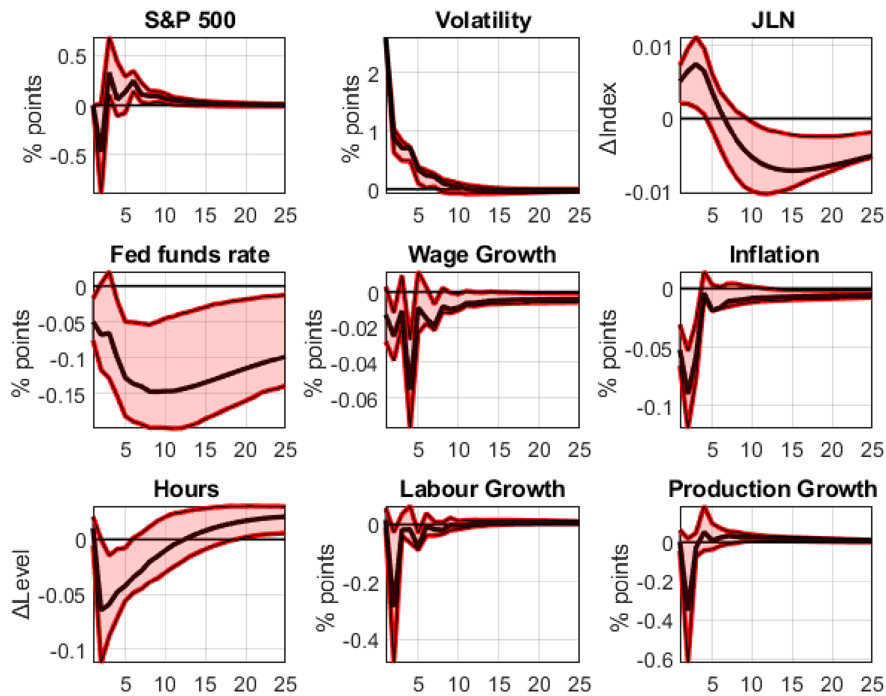
## 4.5 Results

### 4.5.1 Cholesky-Identified Impulse Responses

The shortcomings of using a Cholesky decomposition to identify structural shocks were discussed in 4.3.1, however, it is used by Bloom (2009) and Jurado et al. (2015) as their main identification strategy and by Piffer & Podstawski (2018) as a method of comparison. It thus offers a useful reference case from which to discuss the results from the SVAR-IV model. Therefore, I first present the IRFs generated by a structural shock causing a one standard deviation increase in the variables of interest, identified by a Cholesky decomposition of the covariance matrix. The black line in 4.5.1a and 4.5.1b show the median target estimate generated from the data and the red region contains the 68% confidence interval from a MBB. The ordering of the variables is the same as the ordering in figure 4.4.1, when read from left to right. It is motivated from Bloom (2009) and Piffer & Podstawski (2018) and implies that shocks to the level of the S&P 500 can affect all variables contemporaneously while shocks to any variable lower in the order affects those above it only after one month. As previously discussed, it is difficult to justify ruling out contemporaneous relationships among some of the variables.




(a) Uncertainty Shock Increasing JLN by One Standard Deviation




(b) Financial Market Shock Increasing SMV by One Standard Deviation

Figure 4.5.1: Cholesky Identified Impluse Responses

Figure 4.5.1a shows that uncertainty shocks generate an immediate economic downturn; output growth, the growth in employment, and the number of hours worked all fall and the hourly wage declines. This is consistent with the real-options interpretation of uncertainty shocks; firms cut back on hiring, reduce the activity of their labour force, and cut production as they hold onto their options to wait for more information before expanding their operations. There is also a contemporaneous increase in inflation, which subsequently falls after 3 months. Leduc & Liu (2016) predict that uncertainty shocks will act as an aggregate demand shock and should therefore cause decreases in inflation. While their DSGE model allows for monopolistic competition in the market for the output good, if firms have sufficiently strong market power they could increase prices to compensate for the decrease in profits after the uncertainty shock. The FFR is cut to combat the stagnant economy in accordance with the U.S. monetary rule, but it takes some time before the FED reacts. Notice the uncertainty shock is persistent, which corroborates with the results in Jurado et al. (2015), and that the uncertainty shock also causes a persistent increase in the volatility of the S&P 500 after one month, while the level of the stock market is only briefly affected.

In figure 4.5.1b, shocks to SMV cause an immediate increase in the volatility of the S&P 500 but the effect is notably less persistent than was the case for uncertainty shocks. The effect of the shock on the level of the JLN index is not much above zero and the change in the index only remains positive for about 6 months, compared to over two years in response to the uncertainty shock. There is also a period of decreasing uncertainty from about six months after the shock to financial market volatility. Hence, according to the IRFs generated from a Cholesky decomposition, the initial shock to SMV causes a modest increase in uncertainty but agents soon update their forecasting models in response to the shock, so the increase in uncertainty is undone.

The macroeconomic effects are again indicative of an economic downturn, but there is much lower confidence in the IRFs as the bootstrapped confidence intervals include zero for the three real economic variables in the last row. The FFR is cut, wages decrease, but in contrast to the effects of the uncertainty shock inflation now also decreases. The shock triggering the increase in SMV thus generates the type of downturn more consistent with the aggregate demand shocks in Leduc & Liu (2016). Note that if uncertainty is ordered before SMV in the VAR, the SMV shock produces no increase in the JLN uncertainty index which instead shows a minor but persistent decline (see the appendix 4.A.3). This finding already suggests that increases in SMV are not necessarily accompanied by increases in uncertainty but the strong restrictions which the Cholesky identification strategy puts on the data means these results should be interpreted with caution.

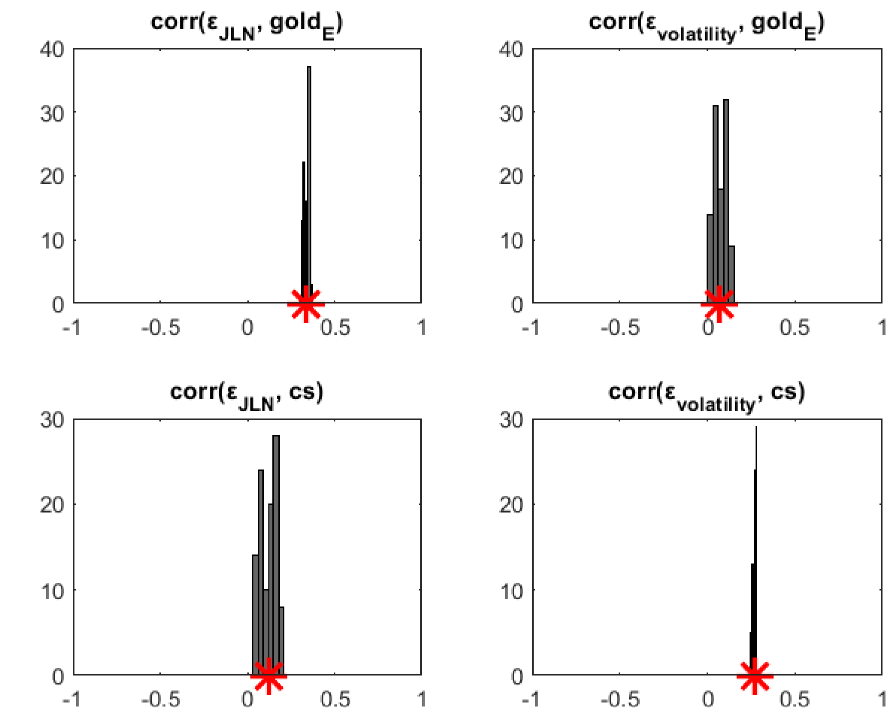
### 4.5.2 Estimated Structural Shocks

As explained in section 4.3.2, I set identify uncertainty and financial market volatility shocks by imposing restrictions on the correlation structure between the structural shocks and the instruments in the matrix $\mathbf{\Phi}$. As section 4.3.2 and appendix 4.A.1 explained, multiple matrices of contemporaneous effects, $\boldsymbol{B}^*$ are obtained from draws of random orthogonal matrices. Because $\mathbf{\Phi} = (\boldsymbol{B}^*)^{-1}\mathbf{\Gamma}$, there are many $\mathbf{\Phi}$ (I generate 3000 to explore the identified set but performing more draws would have no effect on estimation accuracy). The restrictions in the matrix 4.3.7 reduces the set of $\mathbf{\Phi}$ to those for which increases in the gold price instrument are associated positive structural shocks to uncertainty, shocks to risk preferences are associated with positive structural shocks to SMV, and those for which the instruments are more correlated with the shocks they target by a constant factor $\psi$.

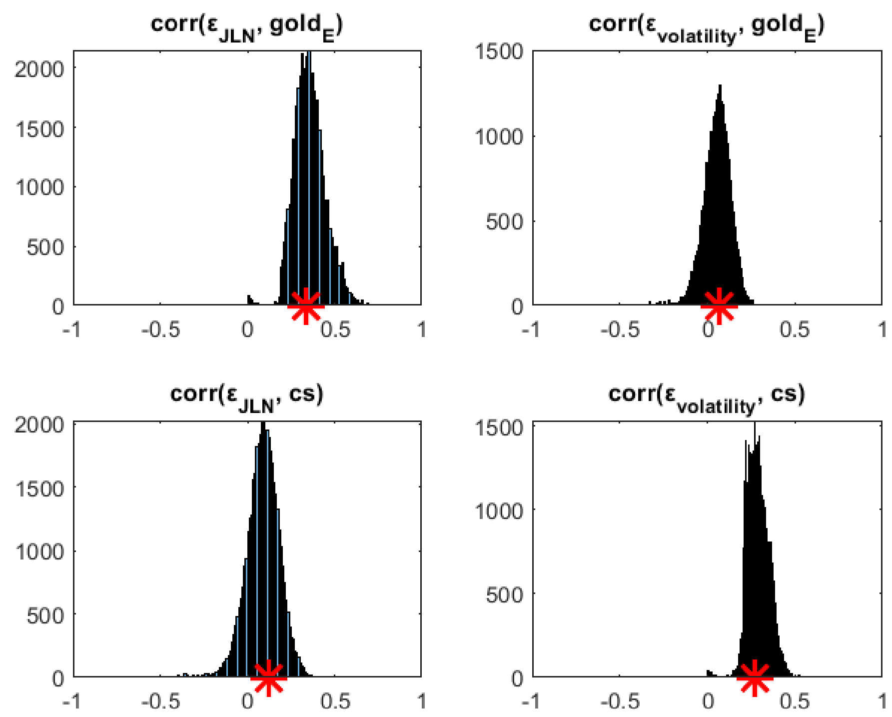
Figure 4.5.2a shows a histogram of estimated correlations from the matrices $\mathbf{\Phi}$ which satisfy the restrictions in 4.3.7 from the original sample. The red star shows the correlations in $\mathbf{\Phi}$ associated with the $\boldsymbol{B}^*$ matrix which was used to produce the median target IRFs in the next section. I will call this matrix $\mathbf{\Phi}^*$. Figure 4.5.2b does the same for the correlations generated from the bootstrapping. Bootstrapping produces many more observations because it repeats the process for 1000 newly generated samples. Regardless of the differences in observations, the results are the same. The histograms on the lead diagonal show that the correlation between structural shocks and the instruments that target them is positive. Furthermore, the off-diagonal plots show that the instruments are less correlated with the structural shocks which they do not target, as expected from the reduced form regressions in table 4.4.3.

The structural shocks generated from the sample correlations in $\mathbf{\Phi}^*$ are presented in figure 4.5.3[7]. Some of the largest spikes in the series coincide around the Great Recession and the Coronavirus

[7]The scale of the proxies on the vertical axis is slightly different in the figure compared to 4.4.3 because they have been normalised.




(a) Estimated Instrument and Structural Shock Correlation from Sample




(b) Estimated Instrument and Structural Shock Correlation from Bootstrapping

Figure 4.5.2: Estimated Correlation between Structural Shocks and Instruments

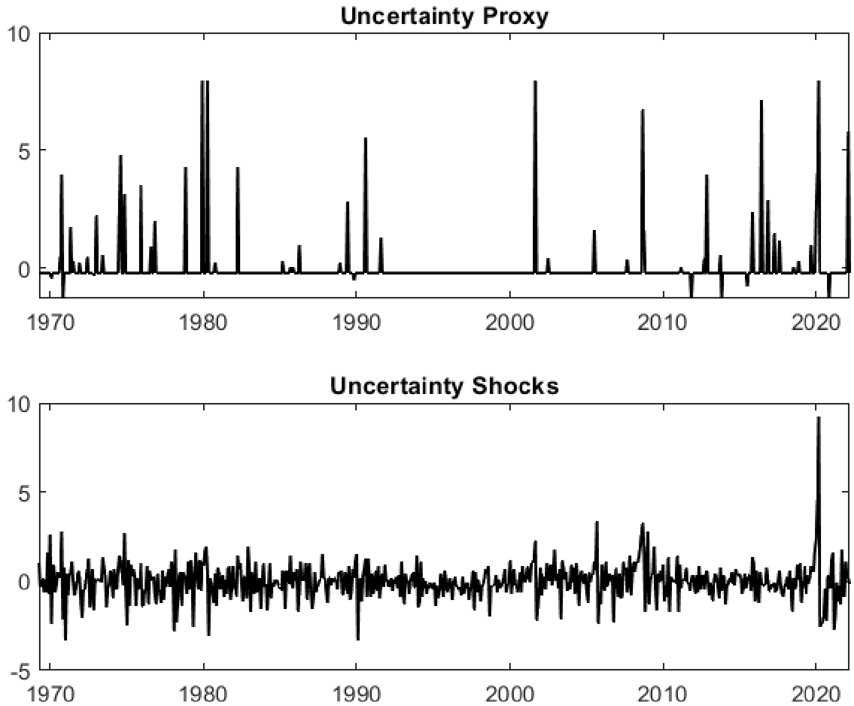
pandemic, however, corroborating the results of (Aït-Sahalia et al., 2021), there are several events which affected SMV independently on economic uncertainty and vice-versa. The most notable is the Black Monday event in October 1987, which caused a very large shock in SMV but had a relatively minor effect on economic uncertainty. In contrast, the 9/11 terrorist attacks caused a large shock to economic uncertainty but had a relatively mild effect on SMV. Other major shocks to economic uncertainty not accounted for in the database appear in September 2005 (perhaps the aftermath of Hurricane Katrina) and January 1970.

There are 16 months during the 1970s which display a structural uncertainty shock greater than one standard deviation from the mean (which is normalised to one). In contrast, only 6 months feature a SMV shock greater than one standard deviation from the mean over the same period. This highlights that some periods can display several large shocks to uncertainty without corresponding shocks to SMV. Aït-Sahalia et al. (2021) and Jurado et al. (2015) previously gave cursory evidence of this fact, and was a major motivation for constructing uncertainty measures not based solely on stock markets. The notable difference in uncertainty shocks in the early 1970s compared with SMV shocks possibly reflects a more politically volatile period of American history due to the Cold War and tensions with Middle Eastern oil-producing states. The decade with the highest frequency of large (those greater than one standard deviation from the mean) uncertainty shocks occurred in the 2000s (24), while the 1980s saw the highest frequency of large SMV shocks (15).

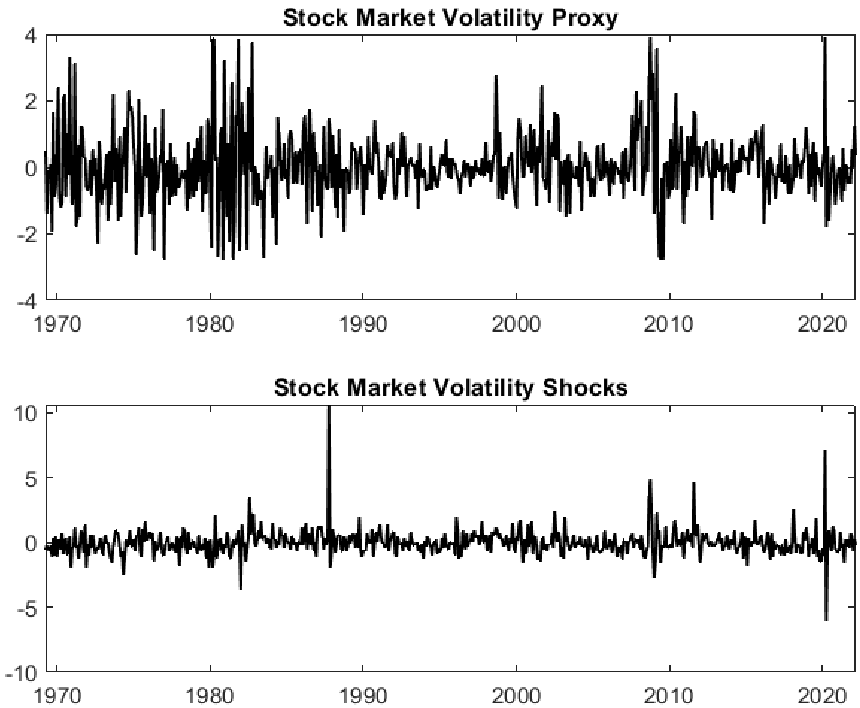
### 4.5.3 Set Identified SVAR-IV Impulse Responses

While the IRFs display some differences to the Cholesky case presented above, the essential results from section 4.5.1 regarding the impact of an uncertainty shock hold for the set identified SVAR-IV model[8]. Uncertainty is persistently higher for 24 months after an uncertainty shock causing a one standard-deviation increase in JLN uncertainty. SMV increases contemporaneously by around 5 percentage points, though the 68% confidence interval for this relationship covers quite wide range of values, while the level of returns on the stock market falls by around 10 percentage points. The increase in SMV is less persistent than was suggested by the Cholesky decomposition. However, the

[8]When comparing the estimates and confidence bands, it is important to note that the Cholesky decomposition used point identification while the SVAR-IV model used set-identification, so they were estimated differently.




(a) Estimated Structural Shocks to Uncertainty




(b) Estimated Structural Shocks to SMV

Figure 4.5.3: Estimated Structural Shocks

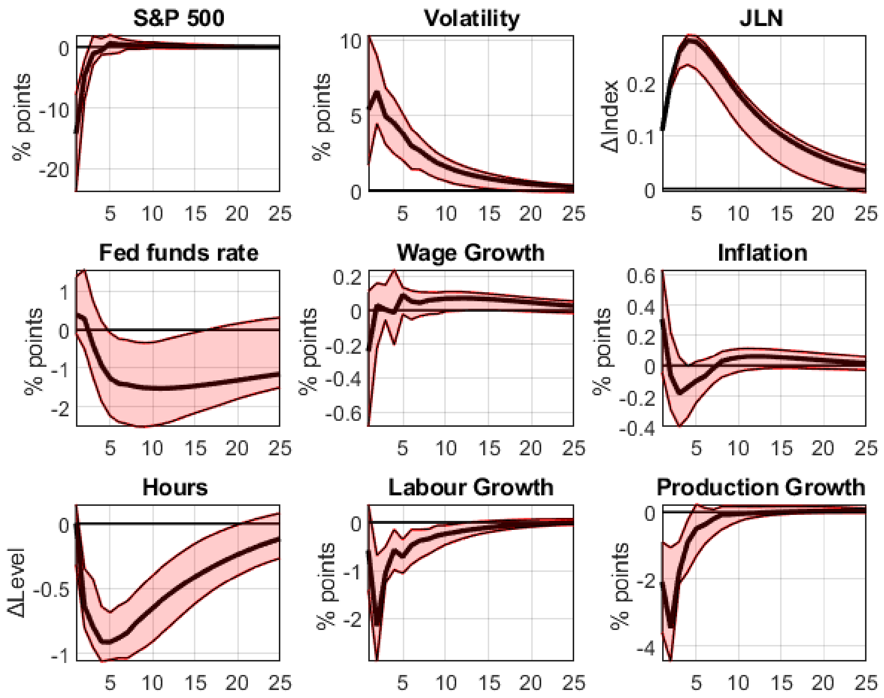
contemporaneous effect of uncertainty on the real economy is more severe, suggesting recursively identified models of uncertainty like Bloom (2009) will underestimate the impact of uncertainty shocks on the real economy. The recovery in industrial production is faster in the set-identified model, but growth in employment and hours worked remains depressed well over a year after the original shock. As in the Cholesky decomposition, inflation increases after the uncertainty shock, which challenges the notion in Leduc & Liu (2016) that uncertainty shocks behave like aggregate demand shocks.

One key result from figure 4.5.4b is that shocks to risk appetite causing a one standard deviation increase in SMV results in no significant contemporaneous change in JLN uncertainty. Studies using SMV as a measure of economic uncertainty will therefore capture variation from other sources (such as shocks to risk preferences) and falsely attribute this to variation in uncertainty. Furthermore, the macroeconomic effects of the two shocks are quite different. Like in the Cholesky case, and still in contrast to the uncertainty shock, the inflation rate falls following the shock to risk preferences. The other price variables in the model, the FFR and wages, also show persistent decreases, with the Fed responding contemporaneously to the shock.

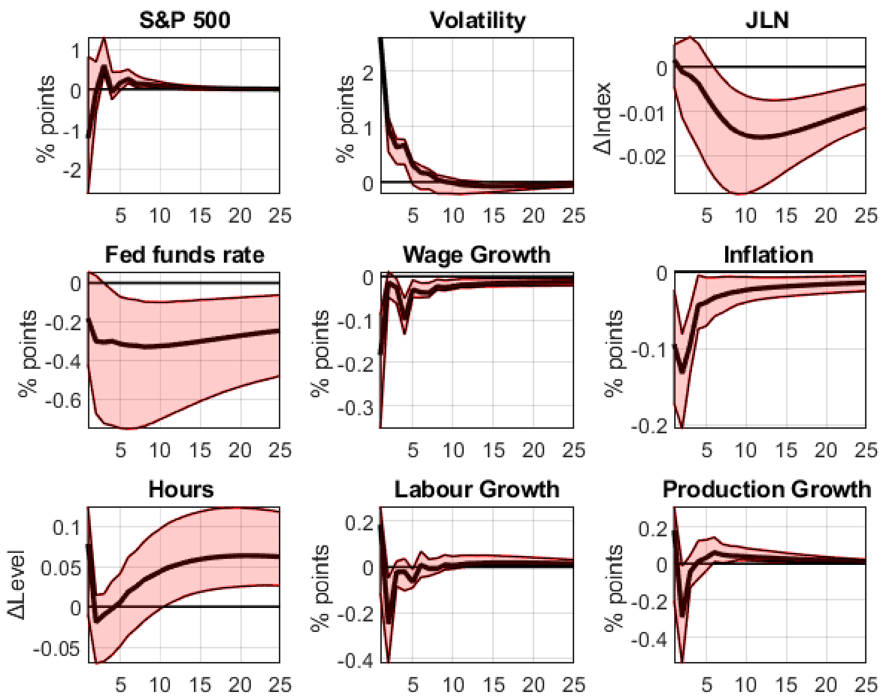
The contemporaneous effect of the identified SMV shock on production, hours worked, and labour growth is positive according to the median target IRF. Once a shock arrives, there is an initial burst of activity before a period where the availability of credit is lower, as financial institutions and governments act to mitigate the perceived risk in the market. Unlike uncertainty shocks, shocks to financial market volatility do not trigger an option value of waiting for more information before making economic decisions. If firm's profits are convex functions of their equity returns, an increase in volatility will cause a short-run burst in profit and activity which is then eroded as tighter borrowing constraints bite[9]. However, the confidence bands are wide, so caution should be taken when interpreting this result.

Figure 4.5.5 shows the forecast error variance decomposition of the variables in the model to an uncertainty shock (red line) and a shock to SMV (blue line). The solid lines represent the estimates from the set-identified model while the dashed lines represent those from a recursively-identified

[9]See the related discussion of the 'volatility overshoot' in Bloom (2009) and the positive relationship between volatility and activty in the model of capital accumulation by Abel (1983).




(a) Uncertainty Shock Increasing JLN by One Standard Deviation




(b) Financial Market Shock Increasing SMV by One Standard Deviation

Figure 4.5.4: Set Identified IRFs Using Instrumental Variables

model. The 68% confidence bands for these estimates are shown in appendix 4.A.4. A greater proportion of the variation in nominal variables can be explained through shocks to SMV rather than uncertainty. Around 18% of the variation in inflation and 8% of the variation in the FFR can be explained by SMV shocks after one year. As shown by the IRFs, the Fed will cut the FFR to a lower rate in response to a shock causing a one standard deviation increase in JLN uncertainty in an attempt to stimulate activity, however, the contemporaneous response is small in magnitude, explaining why SMV shocks make up a much higher proportion of the FFR's forecast error variance for the first months. The Fed works faster in easing borrowing costs after a shock to SMV compared to an uncertainty shock.

However, uncertainty shocks account for around 25% of the variation in production growth after just three months, and after a year make up around 30% of the variation in hours worked and 22% of the variation in employment growth. This backs up the evidence from the IRFs in 4.5.4 that uncertainty shocks are a highly influential driver of real economic activity. Note also the very small amount of the variation in JLN uncertainty that is caused by shocks to SMV. Uncertainty shocks do make up a more substantial part of the variation in SMV, but again the evidence confirms that the two concepts are distinct and that a significant part of the variation in SMV is driven by shocks to risk preferences and the willingness to supply credit independently of uncertainty.

### 4.5.4 Extensions and Robustness

In appendix 4.A.5 I show that the 68% confidence bands generated from the PRBB method of Bruns & Lütkepohl (2022) are narrower than those generated by the MBB method. The improvement is most obvious for the real variables; growth in employment and growth in production. However, the bands remain quite wide for some variables when the shock first hits, indicating that it is difficult to predict the exact contemporaneous effect of the shocks. It is especially difficult to predict the exact response of the FFR to both uncertainty and SMV shocks, though the confidence interval does lie entirely below zero in the latter case.

Changing the method of calculating the uncertainty proxy does not have a significant effect on the results. Choosing the largest jump in gold price on the day of the uncertainty shock or using

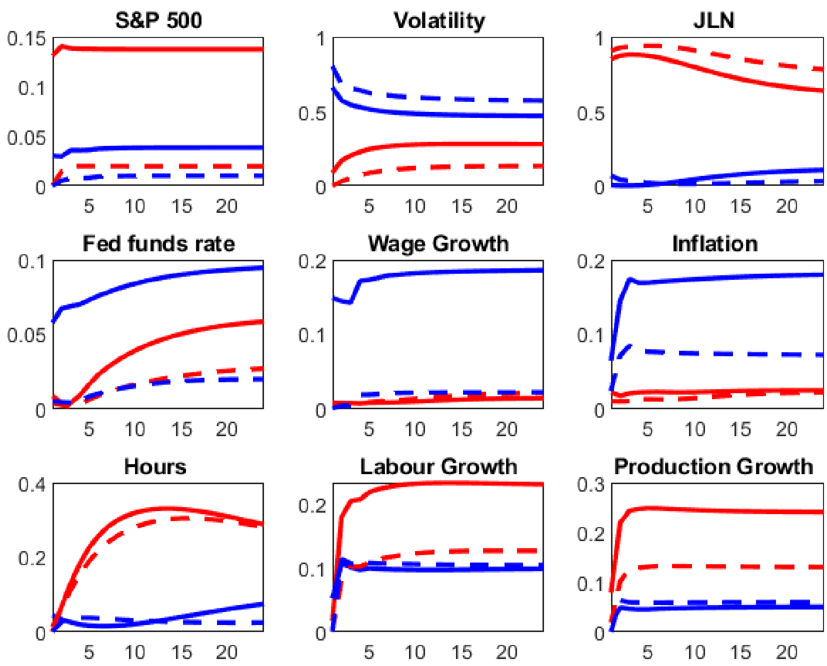



Figure 4.5.5: Forecast Error Variance Decomposition

the intra-day mean produces results that are virtually indistinguishable from using the exact time in the day. This finding is useful for future work because it is much easier to calculate the intra-day mean around uncertainty shocks than it is to identify exactly when news of the shock reached the market. The forecast error decompositions in all three models are also very similar.

It is possible that the unprecedented macroeconomic effects of the Coronavirus Pandemic are distorting the IRFs. Removing the years after 2019 from the sample does reveal some differences. In response to uncertainty shocks, JLN increases slightly less in the months after the shocks but the real economic variables in the last row of figure 4.A.7a show a larger contemporaneous decline. Inflation still increases but uncertainty shocks still do not seem to create large fluctuations in prices. Uncertainty shocks also cause a slightly smaller increase in SMV with the years after 2019 removed from the sample, suggesting the uncertainty created by the pandemic transferred into higher SMV more than would be expected given previous experience. The most notable changes to the estimated impulse responses from SMV shocks in the pre-Coronavirus sample is that the estimated contemporaneous effect on production growth and labour growth is now slightly negative, which is more in line with expectations. This suggests the earlier positive contemporaneous relationships between SMV shocks and the real economic variables are chance findings.

It is interesting to see how the results of the previous analysis change when using a different measure of economic uncertainty. One of the most widely cited is the the EPU index of Baker et al. (2016), which is based on the frequency of keywords related specifically to economic uncertainty about future economic policy in newspapers (see references in Aït-Sahalia et al. (2021), Chuliá et al. (2017), and Gulen & Ion (2016)). I reran the analysis but with this index used in place of JLN as the measure of uncertainty[10].

| | Homoskedastic Standard Errors | | Robust Standard Errors | |
|---|---|---|---|---|
| | $\hat{\boldsymbol{u}}_{EPU}$ | $\hat{\boldsymbol{u}}_{volatility}$ | $\hat{\boldsymbol{u}}_{EPU}$ | $\hat{\boldsymbol{u}}_{volatility}$ |
| $gold_E$ | 0.1350*** | 0.8953*** | 0.1350*** | 0.8953** |
| F | 79.98 | 67.73 | 18.66 | 6.41 |
| $R^2$ | 0.1532 | 0.1329 | 0.1532 | 0.1329 |
| $cs$ | 0.0727*** | 1.1057*** | 0.0727** | 1.1057*** |
| F | 16.5 | 86.73 | 4.60 | 13.23 |
| $R^2$ | 0.0360 | 0.1640 | 0.0360 | 0.1640 |
| N | 444 | 444 | 444 | 444 |

Table 4.5.1: Relevance of Instruments with EPU as the Uncertainty Measure

From table 4.5.1, the gold price instrument based on exact times is correlated with both shocks to the reduced-form residuals of EPU uncertainty and SMV, however, as required for set-identification, the F-statistic is much larger for EPU uncertainty. Indeed, the F-statistic is now larger than 10 even after adjusting the covariance matrix to account for heteroskedasticity. Several of the events in the uncertainty proxy are related to changes in the U.S. political landscape or emergency changes in government policy (such as the response to the Coronavirus pandemic or the 2008 recession). Hence, it makes sense that the instrument is able to proxy for shocks to this uncertainty measure as well. Naturally, newspapers are also likely to make references to government policy changes and uncertainty after the other shocks identified in the proxy. With the EPU index included in the model instead of the JLN, the robust F-statistic of the credit spread instrument is also above 10. Overall, both instruments remain relevant after the change in uncertainty index.

Compared to the JLN case, table 4.5.2 reveals that using EPU as the uncertainty measure leads to a greater concern that the instruments also pick up other kinds of shocks. Even after control-

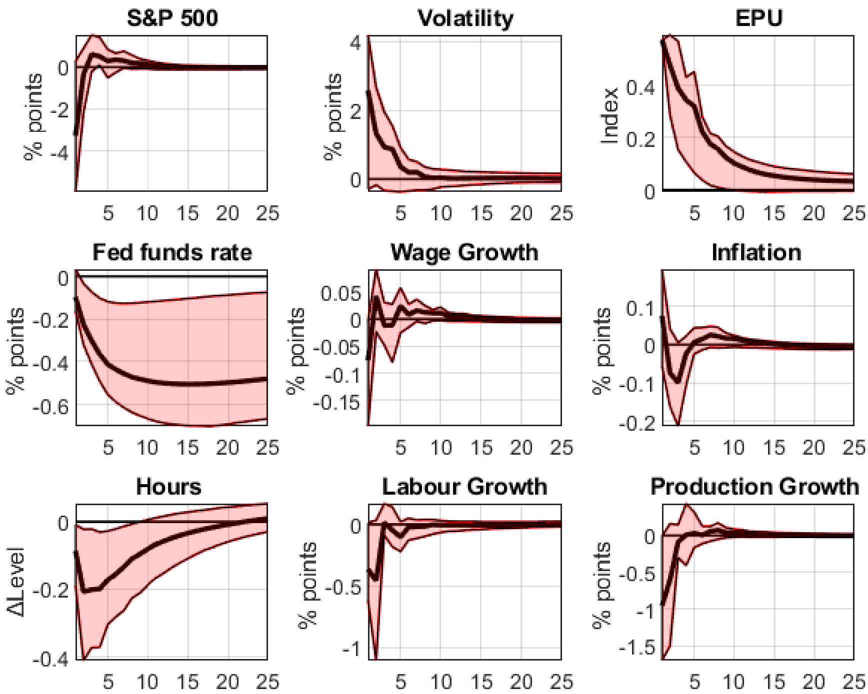
[10] The Akaike information criterion again selected three months as the optimal lag length and the VAR was stable.

| | $\hat{\boldsymbol{u}}_{S\&P500}$ | $\hat{\boldsymbol{u}}_{FFR}$ | $\hat{\boldsymbol{u}}_{Wages}$ | $\hat{\boldsymbol{u}}_{Inflation}$ | $\hat{\boldsymbol{u}}_{Hours}$ | $\hat{\boldsymbol{u}}_{Labour}$ | $\hat{\boldsymbol{u}}_{Production}$ |
|---|---|---|---|---|---|---|---|
| $gold_E$ | -1.1278*** | -0.0341*** | -0.0151 | 0.0046 | -0.0094 | -0.0323 | -0.1582*** |
| | (-5.3648) | (-4.3873) | (-1.4312) | (0.4312) | (-0.9315) | (-1.3151) | (-3.6345) |
| | (-4.6516) | (-2.3050) | (-1.8898) | (0.3513) | (-1.2345) | (-2.7444) | (-1.8658) |
| $R^2$ | 0.0611 | 0.0417 | 0.0046 | 0.0004 | 0.0020 | 0.0039 | 0.0290 |
| $cs$ | -1.3468*** | -0.0428*** | -0.0009 | -0.0273** | 0.0173 | 0.0930*** | 0.0572 |
| | (-5.7925) | (-4.9892) | (-0.0797) | (-2.3004) | (1.5383) | (3.4467) | (1.1675) |
| | (-3.9794) | (-3.1282) | (-0.0933) | (-1.6568) | (1.0872) | (1.8893) | (0.5574) |
| $R^2$ | 0.0706 | 0.0533 | 0.0000 | 0.0118 | 0.0053 | 0.0262 | 0.0031 |
| $N$ | 444 | 444 | 444 | 444 | 444 | 444 | 444 |

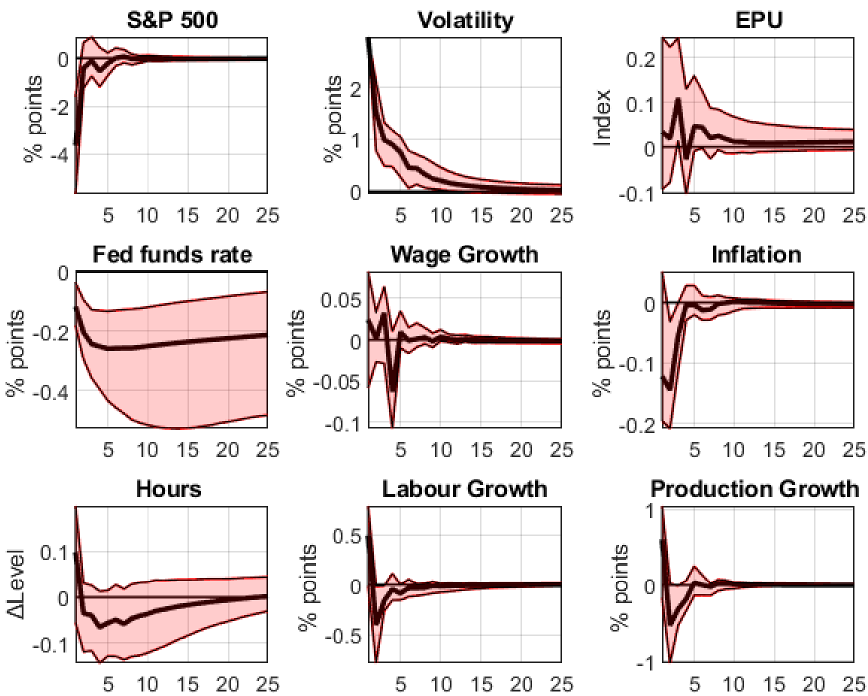
Table 4.5.2: Correlation between Instruments and Shocks to Other Variables with EPU as the Measure of Uncertainty

ling for heteroskedasticity, as seen from the second t-statistic in parentheses below the estimated coefficient, both instruments are highly correlated with shocks to the level of stock market returns and to the FFR. There is also weaker evidence of a correlation between the uncertainty proxy and shocks to production growth (as was the case for the JLN measure of uncertainty) and between the risk preference proxy and shocks to employment growth. These correlations are again much weaker than those for the variable the instruments target, but the IRFs should be interpreted bearing in mind the fact that the proxies are more likely to be picking up other types of shocks compared to the JLN case.

An uncertainty shock causing a one standard deviation increase in the EPU index produces qualitatively similar impulse responses to the JLN case, however, the magnitude and persistence of the changes are generally lower and the confidence bands (generated by the MBB) contain zero even for the real economic variables in the last row. The Fed cuts the FFR to tackle the negative economic shock and inflation increases on impact. The uncertainty shock also causes a smaller and less persistent increase in SMV compared to the JLN case, and the confidence bands span zero so there is only weak evidence for the relationship. Panel 4.5.6b shows that shocks to risk preferences causing a one standard deviation increase in SMV do not lead to a large increase in EPU uncertainty on impact. This further confirms that shocks to SMV do not necessarily imply increases in economic uncertainty. The median target IRF still suggests that there is a contemporaneous increase in real activity following the shock to credit spreads but the evidence for this result is weak given the span of the confidence bands. Consistent with section 4.5.3, inflation decreases in response to this shock.




(a) Uncertainty Shock




(b) Financial Market Volatility Shock

Figure 4.5.6: Set-identified IRFs with EPU Uncertainty

However, the FFR now appears to be cut more aggressively in response to the uncertainty shock rather than the risk preference shock.

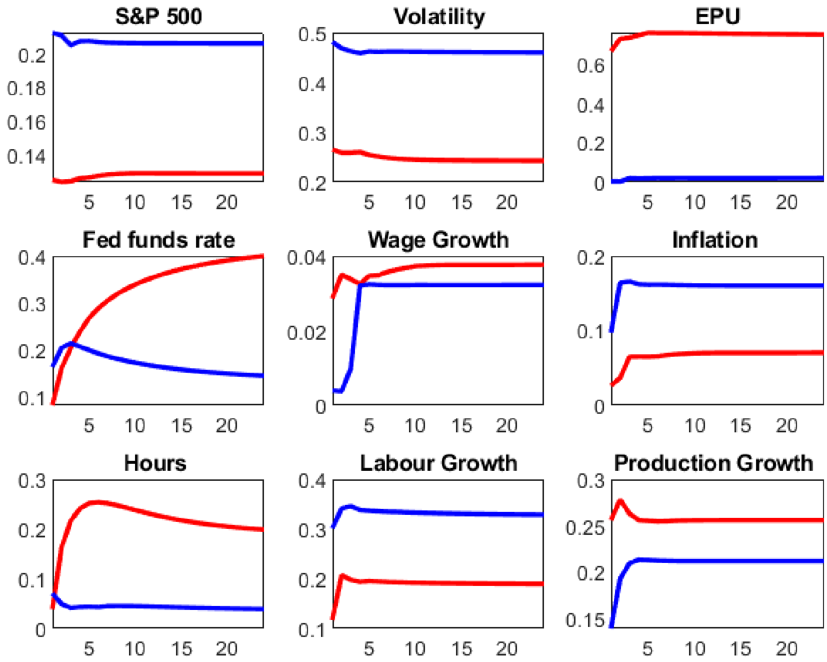



Figure 4.5.7: Forecast Error Variance Decomposition Using EPU as the Measure of Uncertainty

There are some notable changes in the forecast error decomposition compared to the JLN case[11]. First, it appears that credit spread shocks now make up a larger proportion of the variance in the level of the S&P 500 and labour growth compared to uncertainty shocks. It is understandable why uncertainty shocks would be a more important driver of the number of hours worked compared to employment growth, because reducing the hours worked by the current labour force rather than making the partially irreversible decision to fire workers is optimal when the economic climate is more uncertain. Meanwhile, firms facing credit supply issues may find labour stock adjustments necessary to reduce operating costs. The uncertainty shock also makes up a much larger proportion of the total variance in the FFR compared to figure 4.5.5. Table 4.5.2 showed that the uncertainty proxy was more correlated with shocks to the level of the S&P 500 and industrial production when using EPU as the uncertainty measure. This could explain why the identified uncertainty shocks appear to capture more of the variation in U.S. monetary policy.

[11] Again, the confidence bands for these forecast error decompositions are shown in appendix 4.A.4.

## 4.6 Conclusion

This chapter confirms the hypothesis that SMV can vary without any substantial change in economic uncertainty. Using an instrumental variable SVAR model, the identified uncertainty shocks cause a persistent increase in SMV, but shocks to volatility coming from increases in the credit spread do not lead to a corresponding increase in economic uncertainty. Some of the variation in SMV comes from sources other than uncertainty and measures of economic uncertainty based solely on stock markets may come to erroneous conclusions about the relationship between uncertainty and the real economy. Shocks to uncertainty and the credit spread also cause different IRFs for key macroeconomic indicators. Using the JLN index as the measure of uncertainty, these shocks make up a larger proportion of the variance in production and labour markets (especially the number of hours worked), while shocks to the credit spread make up a larger proportion of the variation in prices.

Aside from the implications for economists designing measures of economic uncertainty, this chapter also suggests some avenues of future research. First, it would be interesting to examine whether the volatility of portfolios from different economic sectors are affected differently by uncertainty shocks. It may be that some industries are more affected by changes in economic uncertainty than others which may in turn be reflected in the volatility of their stock returns. Second, this chapter only considered one instrument to identify shocks to SMV, however, future studies could examine other measures of time-varying risk or liquidity preferences. An instrument based on the variation in risk or liquidity preferences around specific events would mitigate concerns that the instrument may still contain some endogenous variation. Finally, there is currently no widely-available measure of business fixed investment at the monthly frequency. This is unfortunate given the strength of irreversibilities in capital stock adjustments and the importance of investment in short-run and long-run economic growth. Introducing a monthly index of business investment would therefore be valuable for the field.

# Appendix 4.A

## 4.A.1 Finding the Elements of the B Matrix

After obtaining the $k \times 2$ matrix $\mathbf{\Gamma}$ from the regression of the reduced form residuals on the proxies for the structural shocks, partition this matrix such that $\mathbf{\Gamma}_{11}$ is a $2 \times 2$ matrix collecting the responses of the uncertainty and equity market volatility reduced form shocks to the proxies and $\mathbf{\Gamma}_{21}$ is a $(k-2) \times 2$ matrix containing the responses of the other variables. Now perform the same decomposition for the $k \times k$ matrices $\boldsymbol{B}$ and $\mathbf{\Sigma}$ such that, for example,

$$B = \begin{bmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} \\ \boldsymbol{B}_{21} & \boldsymbol{B}_{22} \end{bmatrix} \text{ with dimensions } \begin{bmatrix} (2 \times 2) & (2 \times (k-2)) \\ ((k-2) \times 2) & ((k-2) \times (k-2)) \end{bmatrix}.$$

Piffer & Podstawski (2018) show in their technical appendix that with $\boldsymbol{G} = \mathbf{\Gamma}_{21}\mathbf{\Gamma}_{11}^{-1}$ and the covariance matrix $\mathbf{\Sigma}$ can be used to obtain

$$\hat{\boldsymbol{B}}^{*} = \begin{bmatrix} \hat{\boldsymbol{B}_{11}} \\ \boldsymbol{G}\hat{\boldsymbol{B}_{11}} \end{bmatrix} \tag{4.A.1}$$

which is a consistent estimator of the true relationship between the structural and reduced form shocks. In 4.A.1, $\hat{\boldsymbol{B}_{11}} = \hat{\boldsymbol{B}_{11}^{c}}\boldsymbol{Q}$ where $\boldsymbol{Q}$ is any random $2 \times 2$ orthogonal matrix and $\hat{\boldsymbol{B}_{11}^{c}}$ is the lower Cholesky decomposition of $\mathbf{\Sigma}_{11} - \widehat{\boldsymbol{B}_{12}\boldsymbol{B}_{12}'}$ where

$$\widehat{\boldsymbol{B}_{12}\boldsymbol{B}_{12}'} = (\mathbf{\Sigma}_{21} - \boldsymbol{G}\mathbf{\Sigma}_{11})' \mathbf{\Pi} (\mathbf{\Sigma}_{21} - \boldsymbol{G}\mathbf{\Sigma}_{11})$$

and

$$\mathbf{\Pi} = \mathbf{\Sigma}_{22} + \boldsymbol{G}\mathbf{\Sigma}_{11}\boldsymbol{G}' - \mathbf{\Sigma}_{21}\boldsymbol{G}' - \boldsymbol{G}\mathbf{\Sigma}_{21}'.$$

## 4.A.2 Bootstrapped Confidence Intervals

To implement the MBB, choose a block length $\ell$ by rounding $5.03T^{0.25}$ to the next highest integer, as suggested by Jentsch & Lunsford (2022), and organise the reduced form residuals and the instruments in the following matrix

$$\begin{bmatrix} \begin{pmatrix} \hat{\boldsymbol{u}}_1 \\ \boldsymbol{z}_1 \end{pmatrix} & \begin{pmatrix} \hat{\boldsymbol{u}}_2 \\ \boldsymbol{z}_2 \end{pmatrix} & \cdots & \begin{pmatrix} \hat{\boldsymbol{u}}_\ell \\ \boldsymbol{z}_\ell \end{pmatrix} \\ \begin{pmatrix} \hat{\boldsymbol{u}}_2 \\ \boldsymbol{z}_2 \end{pmatrix} & \begin{pmatrix} \hat{\boldsymbol{u}}_3 \\ \boldsymbol{z}_3 \end{pmatrix} & \cdots & \begin{pmatrix} \hat{\boldsymbol{u}}_{\ell+1} \\ \boldsymbol{z}_{\ell+1} \end{pmatrix} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \begin{pmatrix} \hat{\boldsymbol{u}}_{T-\ell+1} \\ \boldsymbol{z}_{T-\ell+1} \end{pmatrix} & \begin{pmatrix} \hat{\boldsymbol{u}}_{T-\ell+2} \\ \boldsymbol{z}_{T-\ell+2} \end{pmatrix} & \cdots & \begin{pmatrix} \hat{\boldsymbol{u}}_T \\ \boldsymbol{z}_T \end{pmatrix} \end{bmatrix}.$$

Select $s = [T/\ell]$ random rows from this matrix, where $[\cdot]$ chooses the smallest integer such that $\ell s \geq T$, and concatenate them horizontally. Then, keep the first $T$ observations, demean the residuals and multiply them by $\sqrt{T/(T - kp - 1)}$ to ensure the bootstrapped sample of $\boldsymbol{y}_t^{boot}$ is generated using a mean-zero error as in equation 4.A.2. These residuals are used to generate a new series $1, \ldots, T$ using $p$ random consecutive values from the original sample to initiate the series,

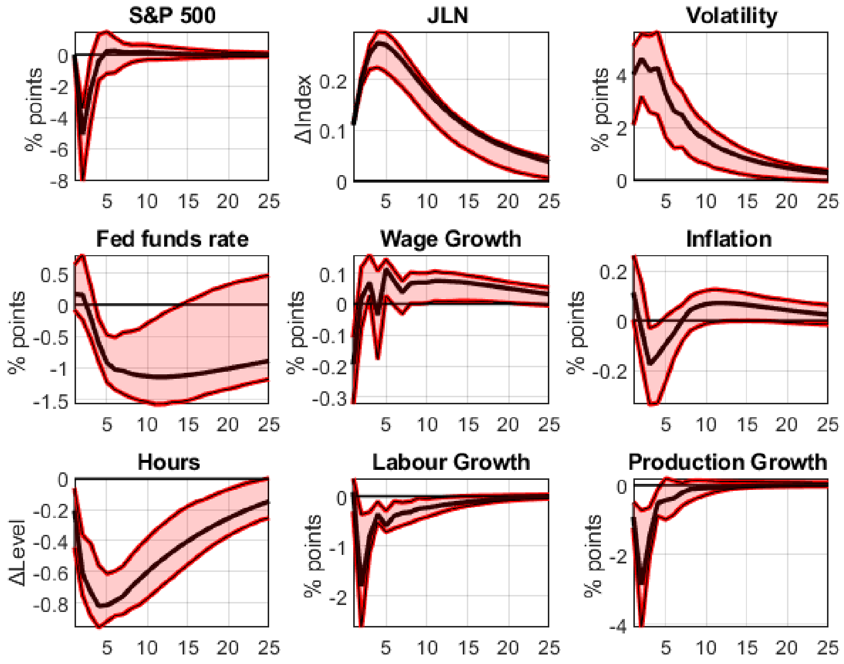
$$\boldsymbol{y}_t^{boot} = \hat{\boldsymbol{\alpha}}^* + \hat{\boldsymbol{C}}_1^* \boldsymbol{y}_{t-1}^{boot} + \cdots + \hat{\boldsymbol{C}}_p^* \boldsymbol{y}_{t-p}^{boot} + \hat{\boldsymbol{u}}_t.$$

For the PRBB, after obtaining the reduced form residuals and recovering the estimated structural shocks, perform the regression

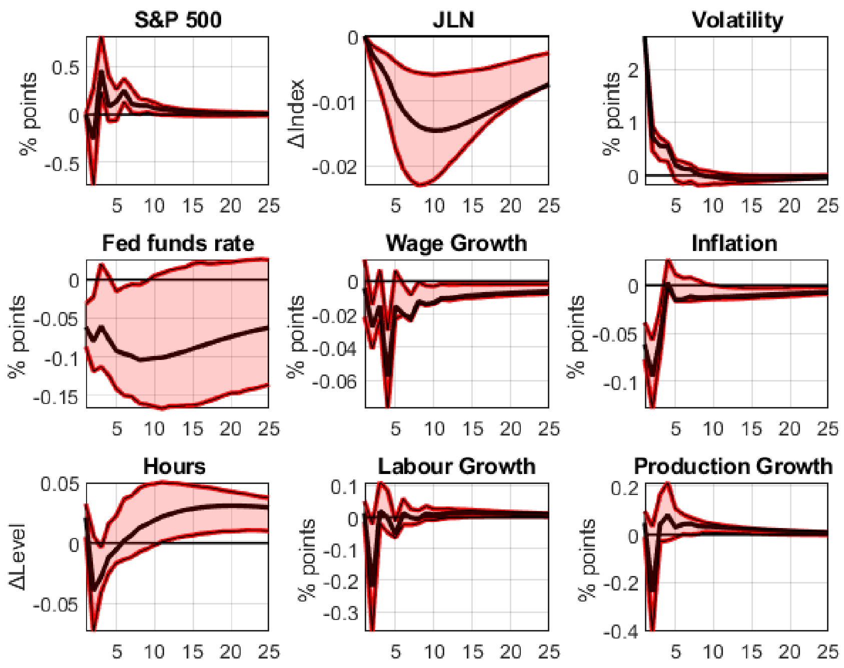
$$\boldsymbol{z}_t = \boldsymbol{\mu} + \beta \hat{\boldsymbol{\epsilon}}_t + \boldsymbol{v}_t$$

then for each $t \in T$ draw a random column of data $[\hat{\boldsymbol{u}}_t \; \hat{\boldsymbol{v}}_t \; \hat{\boldsymbol{\epsilon}}_t]'$ with replacement, where $\hat{\boldsymbol{v}}_t$ are the residuals from the regression above. A new series of $\boldsymbol{y}_t$ is then generated in much the same way as for the MBB case but a new series of $\boldsymbol{z}_t$ is also generated based on the previous regression equation multiplied by $D_t \in \{0, 1\}$, which follows a Bernoulli distribution with the probability of $D_t = 0$ equal to the proportion of zeros the instrument displayed in the original sample. Instruments used to estimate structural shocks frequently take a value of zero at a given date because there was no observed shock, see figure 4.4.3 as an example.

### 4.A.3 Cholesky Identification with JLN ordered before SMV




(a) Uncertainty Shock Increasing JLN by One Standard Deviation




(b) Financial Market Shock Increasing SMV by One Standard Deviation

Figure 4.A.1: Cholesky Identified Impluse Responses with JLN Ordered Before SMV

### 4.A.4 Confidence Intervals for Forecast Error Decompositions

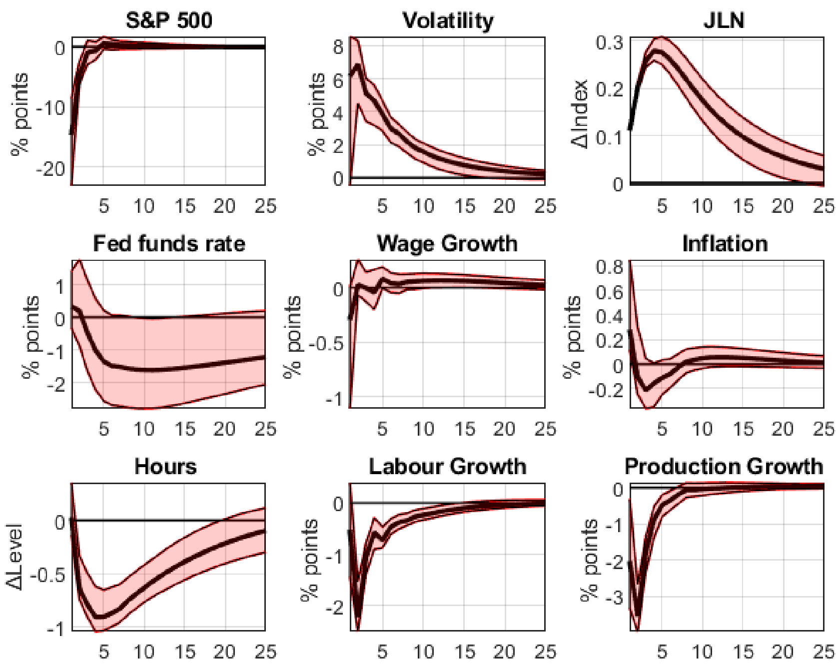
| | Months after the Shock | | | | |
|---|---|---|---|---|---|
| Variable | 0 months | 6 months | 12 months | 18 months | 24 months |
| **Cholesky Decomposition - Uncertainty Shock** | | | | | |
| S&P 500 | 0, 0 | 0.012, 0.047 | 0.013, 0.047 | 0.013, 0.047 | 0.013, 0.047 |
| Volatility | 0, 0 | 0.041, 0.172 | 0.048, 0.198 | 0.052, 0.204 | 0.052, 0.205 |
| JLN | 0.835, 0.973 | 0.875, 0.939 | 0.774, 0.898 | 0.681, 0.852 | 0.617, 0.826 |
| FFR | 0.001, 0.018 | 0.006, 0.032 | 0.006, 0.054 | 0.007, 0.07 | 0.008, 0.081 |
| Wages | 0.001, 0.017 | 0.007, 0.024 | 0.009, 0.03 | 0.011, 0.035 | 0.012, 0.038 |
| Inflation | 0.004, 0.026 | 0.013, 0.051 | 0.018, 0.055 | 0.021, 0.061 | 0.022, 0.063 |
| Hours | 0.003, 0.048 | 0.157, 0.308 | 0.197, 0.375 | 0.192, 0.377 | 0.173, 0.348 |
| Labour | 0, 0.015 | 0.073, 0.168 | 0.085, 0.187 | 0.086, 0.188 | 0.085, 0.188 |
| Production | 0.005, 0.044 | 0.085, 0.184 | 0.085, 0.186 | 0.084, 0.186 | 0.084, 0.186 |
| **Cholesky Decomposition - Volatility Shock** | | | | | |
| S&P 500 | 0, 0 | 0.008, 0.027 | 0.008, 0.028 | 0.008, 0.028 | 0.008, 0.028 |
| Volatility | 0.75, 0.857 | 0.51, 0.665 | 0.484, 0.647 | 0.473, 0.638 | 0.469, 0.634 |
| JLN | 0.015, 0.122 | 0.005, 0.028 | 0.008, 0.039 | 0.01, 0.051 | 0.013, 0.056 |
| FFR | 0.001, 0.023 | 0.003, 0.037 | 0.004, 0.039 | 0.004, 0.042 | 0.004, 0.043 |
| Wages | 0, 0.005 | 0.012, 0.039 | 0.012, 0.039 | 0.012, 0.04 | 0.012, 0.039 |
| Inflation | 0.008, 0.039 | 0.034, 0.113 | 0.034, 0.111 | 0.034, 0.109 | 0.034, 0.107 |
| Hours | 0, 0.005 | 0.005, 0.077 | 0.007, 0.07 | 0.01, 0.066 | 0.012, 0.063 |
| Labour | 0, 0.007 | 0.022, 0.216 | 0.023, 0.214 | 0.023, 0.213 | 0.024, 0.213 |
| Production | 0, 0.005 | 0.009, 0.153 | 0.01, 0.153 | 0.012, 0.153 | 0.012, 0.153 |
| **SVAR-IV - Uncertainty Shock (JLN)** | | | | | |
| S&P 500 | 0.043, 0.235 | 0.06, 0.226 | 0.06, 0.225 | 0.06, 0.225 | 0.06, 0.225 |
| Volatility | 0.029, 0.231 | 0.128, 0.405 | 0.138, 0.408 | 0.141, 0.403 | 0.141, 0.4 |
| JLN | 0.598, 0.869 | 0.583, 0.808 | 0.5, 0.721 | 0.433, 0.655 | 0.394, 0.619 |
| FFR | 0.001, 0.055 | 0.012, 0.07 | 0.013, 0.097 | 0.013, 0.105 | 0.013, 0.109 |

| | | | | | |
|---|---|---|---|---|---|
| Wages | 0.001, 0.033 | 0.007, 0.039 | 0.008, 0.041 | 0.01, 0.044 | 0.01, 0.045 |
| Inflation | 0.001, 0.06 | 0.014, 0.068 | 0.018, 0.069 | 0.02, 0.07 | 0.02, 0.07 |
| Hours | 0, 0.018 | 0.136, 0.364 | 0.178, 0.397 | 0.172, 0.38 | 0.151, 0.344 |
| Labour | 0.001, 0.083 | 0.168, 0.301 | 0.181, 0.309 | 0.18, 0.306 | 0.177, 0.302 |
| Production | 0.016, 0.193 | 0.178, 0.314 | 0.178, 0.31 | 0.176, 0.307 | 0.176, 0.307 |
| **SVAR-IV - Volatility Shock (JLN)** | | | | | |
| S&P 500 | 0.005, 0.146 | 0.022, 0.149 | 0.023, 0.149 | 0.024, 0.149 | 0.024, 0.149 |
| Volatility | 0.402, 0.69 | 0.281, 0.511 | 0.272, 0.495 | 0.268, 0.488 | 0.267, 0.484 |
| JLN | 0.001, 0.03 | 0.008, 0.068 | 0.022, 0.132 | 0.036, 0.168 | 0.043, 0.186 |
| FFR | 0.007, 0.319 | 0.013, 0.332 | 0.014, 0.32 | 0.015, 0.313 | 0.016, 0.308 |
| Wages | 0.026, 0.254 | 0.055, 0.256 | 0.057, 0.256 | 0.059, 0.256 | 0.061, 0.255 |
| Inflation | 0.006, 0.15 | 0.066, 0.252 | 0.07, 0.251 | 0.07, 0.254 | 0.072, 0.257 |
| Hours | 0.003, 0.083 | 0.011, 0.05 | 0.017, 0.074 | 0.024, 0.112 | 0.029, 0.153 |
| Labour | 0.005, 0.112 | 0.042, 0.165 | 0.045, 0.165 | 0.051, 0.166 | 0.056, 0.166 |
| Production | 0.002, 0.059 | 0.023, 0.108 | 0.029, 0.11 | 0.032, 0.11 | 0.033, 0.111 |
| **SVAR-IV - Uncertainty Shock (EPU)** | | | | | |
| S&P 500 | 0.027, 0.167 | 0.046, 0.168 | 0.046, 0.169 | 0.046, 0.169 | 0.046, 0.169 |
| Volatility | 0.034, 0.343 | 0.041, 0.318 | 0.043, 0.314 | 0.044, 0.314 | 0.044, 0.314 |
| EPU | 0.497, 0.807 | 0.572, 0.749 | 0.561, 0.736 | 0.554, 0.726 | 0.548, 0.717 |
| FFR | 0.012, 0.178 | 0.119, 0.361 | 0.128, 0.386 | 0.125, 0.403 | 0.125, 0.41 |
| Wages | 0.003, 0.063 | 0.017, 0.073 | 0.018, 0.075 | 0.019, 0.075 | 0.019, 0.075 |
| Inflation | 0.002, 0.084 | 0.041, 0.131 | 0.041, 0.133 | 0.042, 0.133 | 0.043, 0.133 |
| Hours | 0.011, 0.094 | 0.074, 0.387 | 0.067, 0.362 | 0.062, 0.339 | 0.059, 0.331 |
| Labour | 0.013, 0.15 | 0.039, 0.282 | 0.037, 0.28 | 0.037, 0.281 | 0.038, 0.28 |
| Production | 0.075, 0.389 | 0.086, 0.371 | 0.086, 0.371 | 0.086, 0.371 | 0.086, 0.371 |
| **SVAR-IV - Volatility Shock (EPU)** | | | | | |
| S&P 500 | 0.125, 0.421 | 0.126, 0.382 | 0.126, 0.381 | 0.126, 0.381 | 0.126, 0.381 |
| Volatility | 0.418, 0.685 | 0.373, 0.597 | 0.369, 0.59 | 0.368, 0.587 | 0.366, 0.585 |

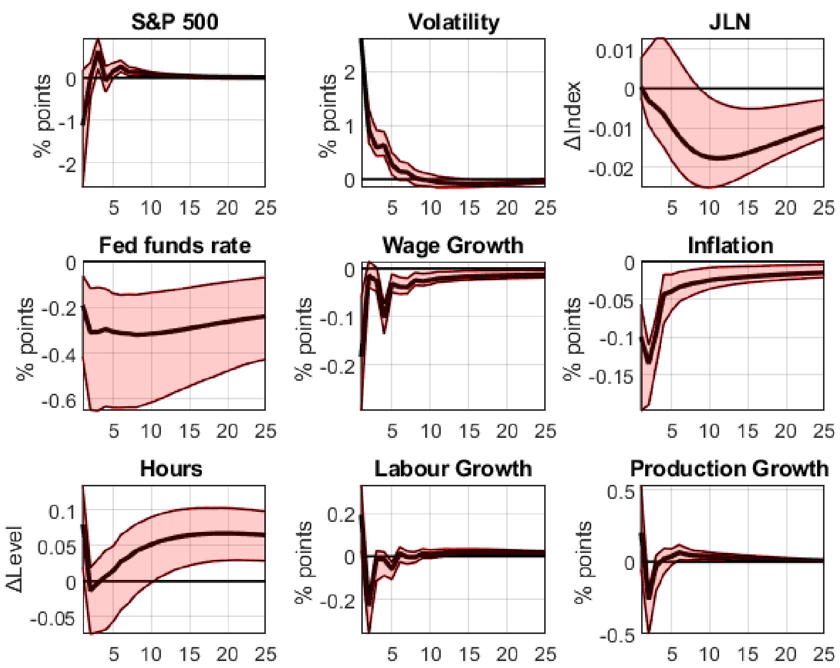
| | | | | | |
|---|---|---|---|---|---|
| EPU | 0.003, 0.07 | 0.024, 0.12 | 0.027, 0.122 | 0.028, 0.123 | 0.029, 0.125 |
| FFR | 0.066, 0.24 | 0.125, 0.31 | 0.11, 0.299 | 0.097, 0.293 | 0.089, 0.294 |
| Wages | 0, 0.023 | 0.013, 0.073 | 0.014, 0.074 | 0.014, 0.074 | 0.014, 0.074 |
| Inflation | 0.007, 0.161 | 0.046, 0.251 | 0.046, 0.25 | 0.047, 0.25 | 0.047, 0.25 |
| Hours | 0.006, 0.165 | 0.033, 0.119 | 0.033, 0.135 | 0.033, 0.139 | 0.035, 0.139 |
| Labour | 0.017, 0.373 | 0.148, 0.362 | 0.158, 0.36 | 0.156, 0.359 | 0.157, 0.359 |
| Production | 0.005, 0.196 | 0.07, 0.262 | 0.072, 0.261 | 0.072, 0.261 | 0.073, 0.261 |

Table 4.A.1: Boostrapped 68% Confidence Intervals for the Forecast Error Decompositions at Six-Month Steps

### 4.A.5 Proxy Residual Based Bootstrap



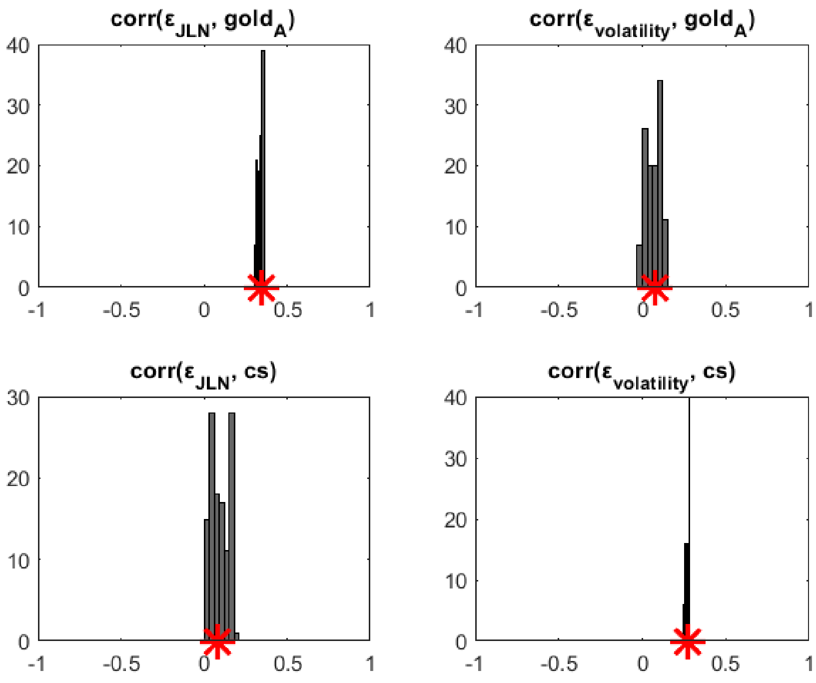

(a) Confidence Bands for an Uncertainty Shock Generated by the PRBB Method



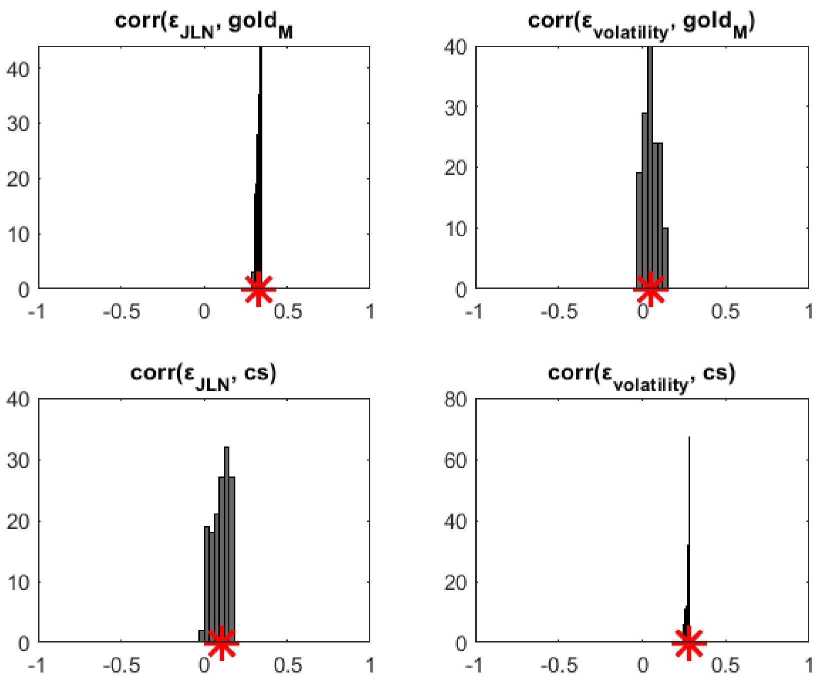

(b) Confidence Bands for a Volatility Shock Generated by the PRBB Method

Figure 4.A.2: IRFs with PRBB

## 4.A.6 Other Gold Instruments

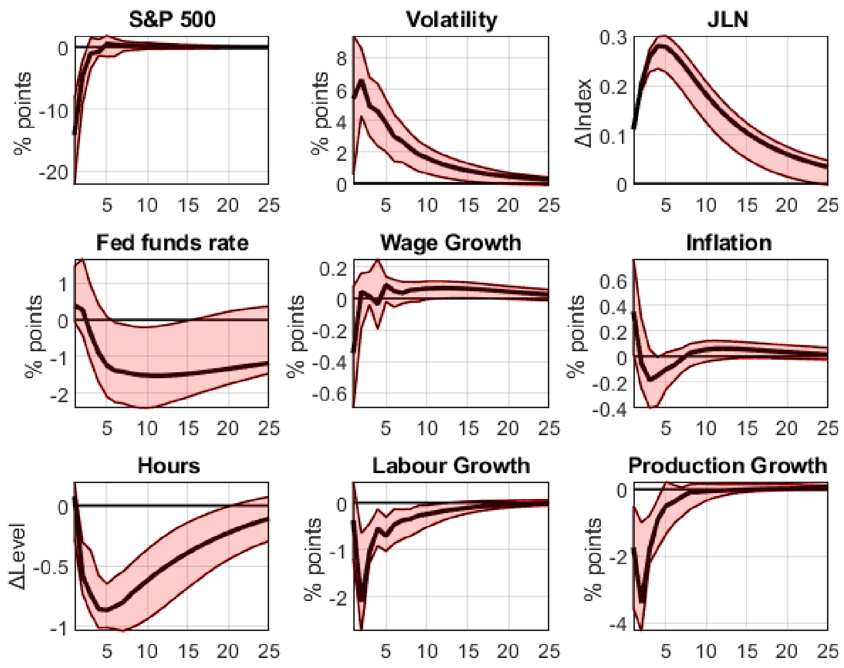



(a) Estimated Instrument and Structural Shock Correlation from Sample Using $g_A$ as Proxy for Uncertainty Shocks

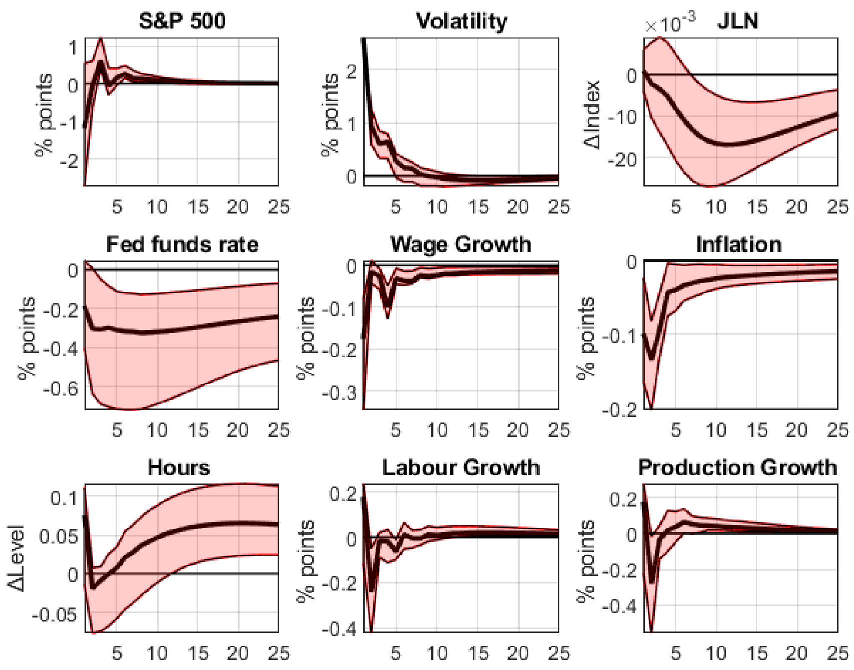



(b) Estimated Instrument and Structural Shock Correlation from Sample Using $g_M$ as Proxy for Uncertainty Shocks

Figure 4.A.3: Correlations between Estimated Structural Shocks and Alternative Gold Instruments

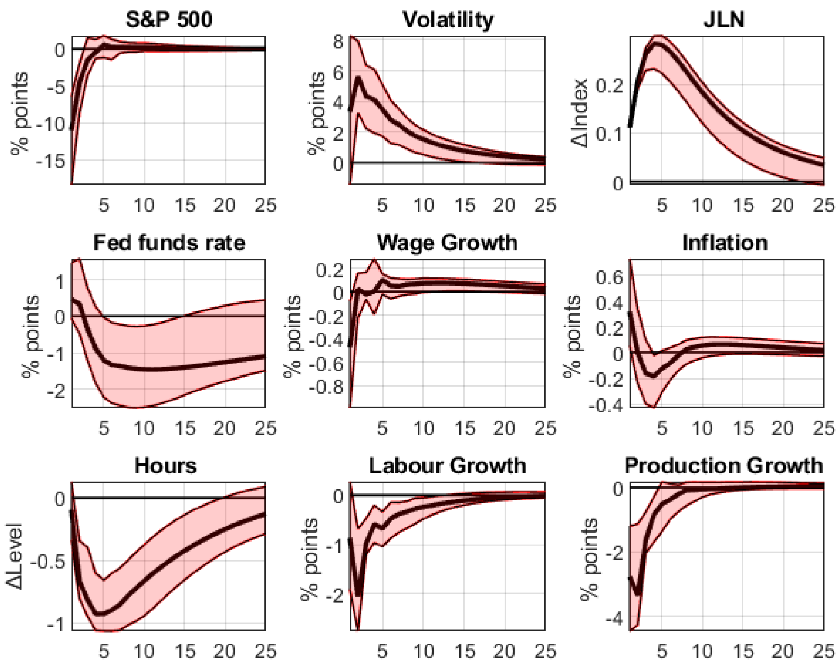



(a) Responses to an Uncertainty Shock Using $g_A$ as Proxy for Uncertainty Shocks

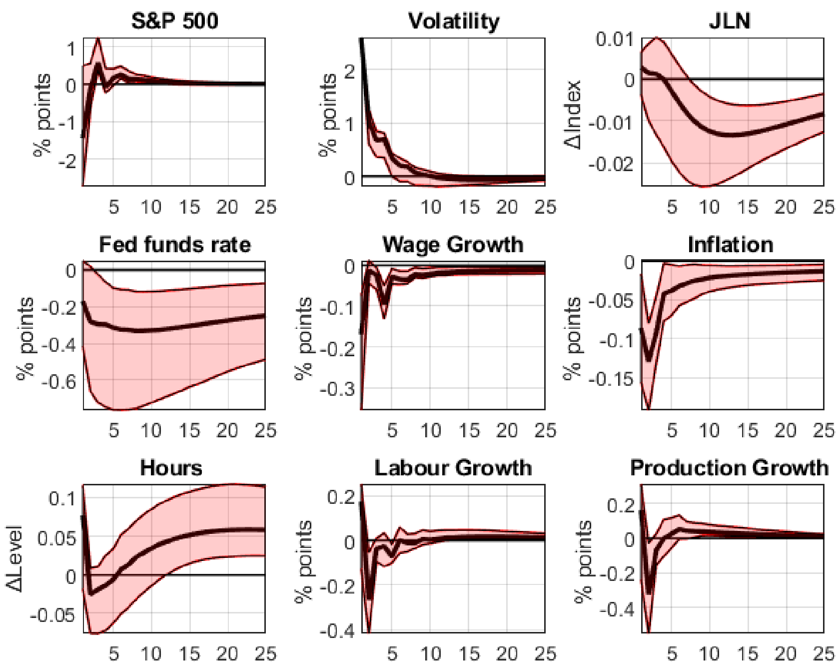



(b) Responses to a Volatility Shock Using $g_A$ as Proxy for Uncertainty Shocks

Figure 4.A.4: IRFs when Using $gold_A$ as Instrument for Uncertainty Shocks

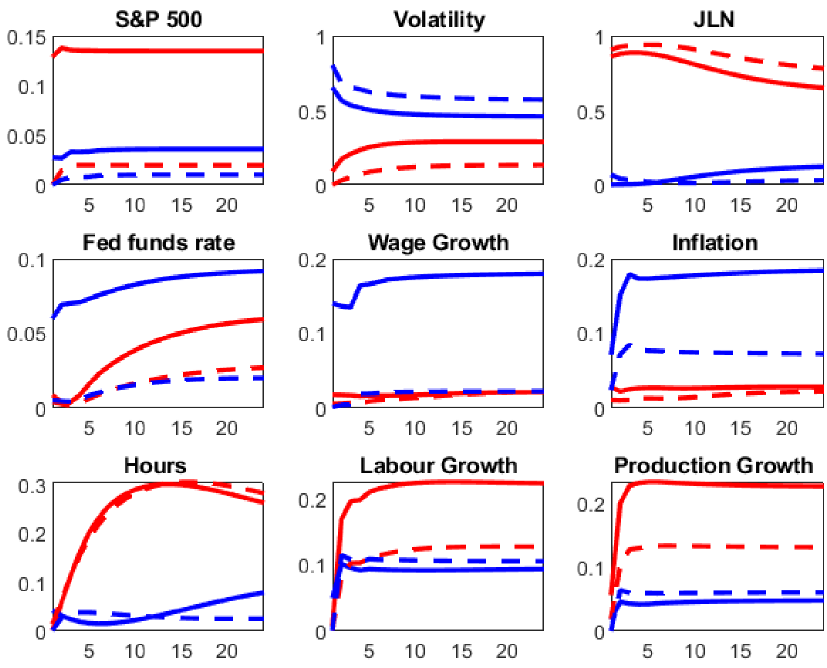



(a) Responses to an Uncertainty Shock Using $g_M$ as Instrument for Uncertainty Shocks

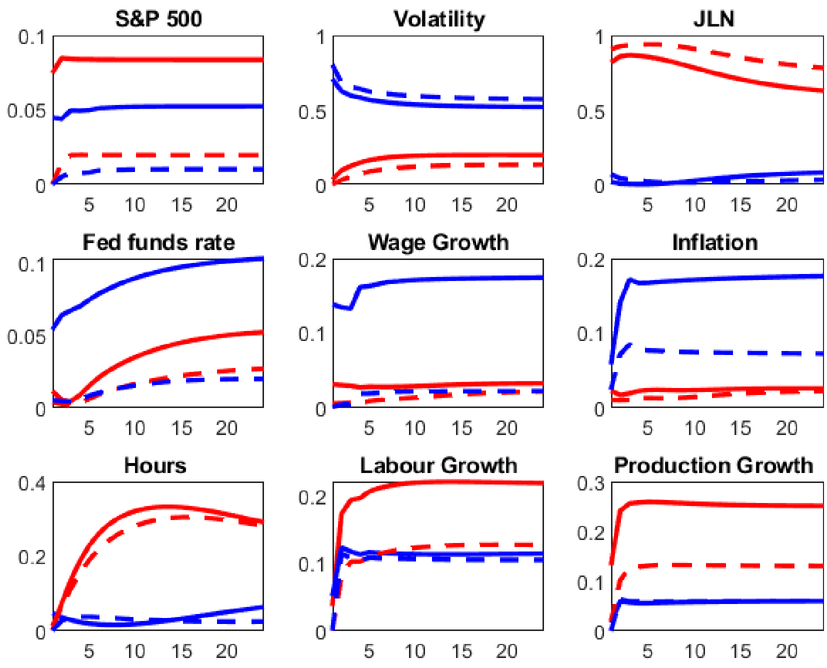



(b) Responses to a Volatility Shock Using $gold_M$ as Proxy for Uncertainty Shocks

Figure 4.A.5: IRFs when Using $gold_M$ as Instrument for Uncertainty Shocks



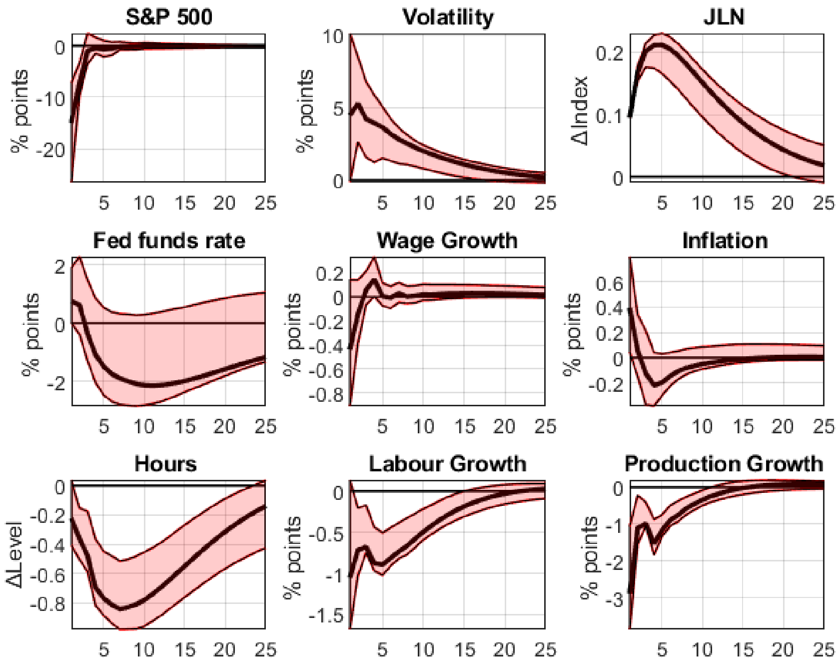

(a) Forecast Error Variance Decomposition Using $g_A$ as Proxy for Uncertainty Shocks



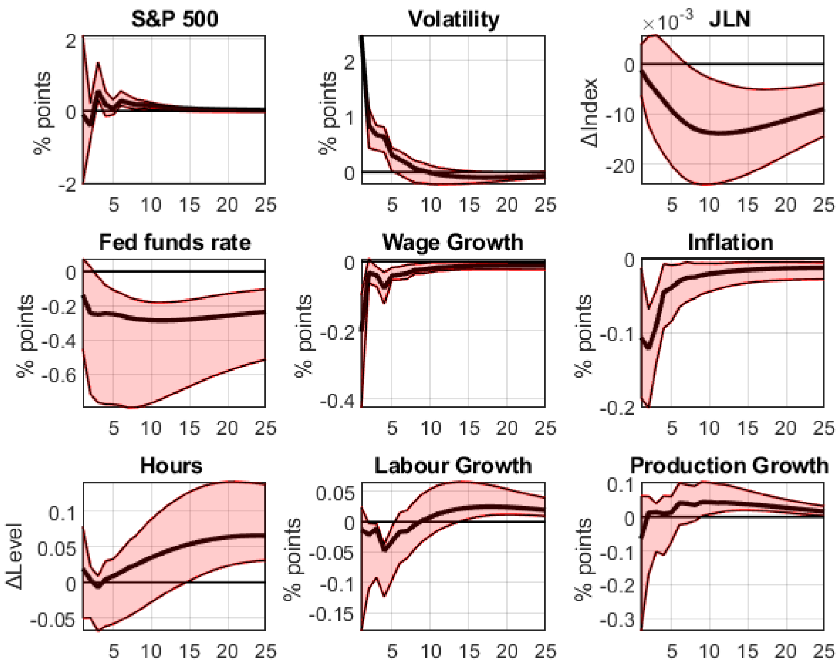

(b) Forecast Error Variance Decomposition Using $g_M$ as Proxy for Uncertainty Shocks

Figure 4.A.6: FEVD for Alternative Gold Instruments

## 4.A.7 Pre-Coronavirus Sample




(a) Responses to an Uncertainty Shock for a Sample Excluding the Years After the Coronavirus Pandemic where Uncertainty in the Model is Measured by JLN




(b) Responses to a Volatility Shock for a Sample Excluding the Years After the Coronavirus Pandemic where Uncertainty in the Model is Measured by JLN

Figure 4.A.7: IRFs for Sample Excluding the Years After the Coronavirus Pandemic

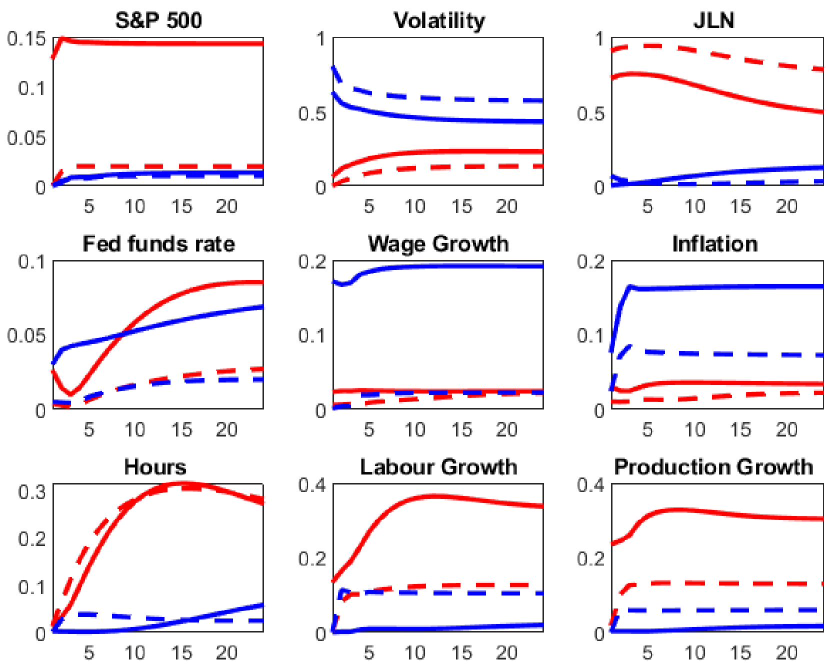



Figure 4.A.8: Forecast Error Variance Decomposition for a Sample Excluding the Years After the Coronavirus Pandemic

## 4.A.8 List of Events Used in the Proxy for Uncertainty Shocks

The following events are those which appear in conjunction with those in Piffer & Podstawski (2018) to create the proxy for uncertainty shocks. Their sample covers the period between 1979 and most of 2015, so these events cover the years between 1969 and 1978 as well as between late 2015 and 2022. A star next to the digital time in the third column indicates the day before or after the date given in the second column. Whether it refers to the day before or after is clear from the ordering of the times. For example, for the first entry in the table, 10:30* means 10:30 on the 29th of January 1969. Likewise, for the eighth entry, 15:00* indicates the 3rd of May 1970. There is no gold quote on the weekend, so if an event occurred on a Saturday or Sunday the effect in the gold price will not show up until the succeeding Monday. Sources are provided in the last column, mostly from the NYT before 1979 but also from Reuters (R), The Guardian (G), and the British Broadcasting Company (BBC).

It is worth mentioning that the outbreak of the Yom-Kippur war on the 6th of October 1973 (a Saturday) is associated with a large spike on the gold price on the AM fix for October the 8th

(3.81%). In hindsight, this event was important because it also triggered the first oil crisis of the 1970s. Much like the Russian invasion of Ukraine, including this event in the uncertainty proxy actually leads to a lower correlation with the JLN uncertainty residuals, implying the residuals were not particularly high in October 1973. Including this event does not change the results but does weaken the robust F-statistic in the first stage regression. Likewise, taking the Russian invasion of Ukraine out of the sample increases the robust F-statistic. Because these events are associated with large increases in the gold price, if they are not correlated with the residuals then they will have a large impact on the variance of the regression error, so it makes sense that the robust F-statistic will be sensitive to their inclusion.

| Event | Date | Time | Gold Price | Source |
|---|---|---|---|---|
| Santa Barbara Oil Spill. | 28/01/1969 | 15:00-10:30* | 0.71% | (Clarke & Hemphill, 2002) |
| Arthur Burns becomes head of the Federal Reserve. | 31/02/1970 | 15:00-10:30* | -0.11% | NYT |
| Attempted assassination of Hussein-bin-Talal of Jordan and threat of war from Iraq. | 01/09/1970 | 15:00-10:30* | 0.41% | NYT |
| Quebec Liberation Front Kidnap a British delegate. | 05/10/1970 | 15:00-10:30* | 0.52% | NYT |
| Salvador Allende elected president of Chile. | 25/10/1970 | 15:00-10:30* | 1.82% | NYT |
| Democrats hold the House and Senate during rising tensions over Vietnam War. | 04/11/1970 | 15:00-10:30* | -1.65% | NYT |
| 500000 die in the Bhola cyclone. | 12/11/1970 | 15:00-10:30* | 0.11% | NYT |

| | | | | |
|---|---|---|---|---|
| Dollar floods European Markets and trading is stopped in a handful of European countries. | 04/05/1970 | 15:00*-10:30 | 1.09% | NYT |
| Chile nationalises copper mines. | 11/07/1971 | 15:00-10:30* | 0.27% | (Gedicks, 1973) |
| India-Pakistan war. | 05/06/1971 | 15:00-10:30* | 0.22% | NYT |
| Hurricane Anges. | 23/06/1972 | 15:00*-10:30 | 3.64% | NYT |
| Nixon Re-elected. | 07/11/1972 | 15:00-10:30* | -0.047% | NYT |
| Nixon announces 60-day retail price freeze to combat inflation. | 14/06/1973 | 15:00*-10:30 | 0.43% | NYT |
| Cyprus coup d'etat. | 15/07/1973 | 15:00-10:30* | 1.43% | NYT |
| Nixon announces resignation. | 08/08/1974 | 15:00-10:30* | 0.16% | NYT |
| Turko-Cypriat relations break down and fighting resumes. | 13/08/1974 | 15:00-10:30* | 2.64% | NYT |
| Democrats make large gains in midterm elections. | 05/11/1974 | 15:00-10:30* | -0.047% | NYT |
| OPEC siege. | 21/12/1975 | 15:00-10:30* | 2.08% | NYT |
| Hurricane Belle. | 10/08/1976 | 15:00*-10:30 | 0.62% | NYT |
| Jimmy Carter wins U.S. Presidential election. | 02/11/1976 | 15:00-10:30* | 1.22% | NYT |
| Libyan-Egyptian war. | 22/07/1977 | 15:00*-10:30 | 0.21% | NYT |
| Democrats hold majority in house and senate in midterm elections. | 08/11/1978 | 15:00-10:30* | 2.5% | NYT |
| France strikes on Syria after multiple ISIL attacks in Paris. | 13/11/2015 | 15:00*-10:30 | 1.20% | R |
| Turkey shoot down Russian plane. | 24/11/2015 | 15:00*-10:30 | 0.23% | BBC |

| | | | | |
|---|---|---|---|---|
| Result from the Brexit vote. | 23/06/2016 | 15:00-10:30* | 4.10% | BBC |
| Donald Trump elected president. | 08/11/2016 | 15:00-10:30* | 1.37% | R |
| America bombs air base in Syria. | 06/04/2017 | 15:00-10:30* | 0.94% | NYT |
| Barcelona attacks. | 17/08/2017 | 15:00-10:30* | 0.79% | R |
| American tariffs on Chinese imports spark fears of a trade war. | 06/07/2016 | 15:00*-10:30 | 0.14% | R |
| Democrats win control of the house in U.S. midterms. | 06/11/2018 | 15:00-10:30* | 0.28% | R |
| Pelosi announces the plan to impeach Donald Trump. | 24/09/2019 | 15:00-10:30* | 0.67% | G |
| Death of Qasem Soleimani. | 02/01/2020 | 15:00-10:30* | 1.33% | R |
| WHO declares a global health emergency. | 30/01/2020 | 15:00-10:30* | 0.16% | G |
| Markets crash due to worsening situation in China over weekend. | 24/02/2020 | 15:00*-10:30 | 2.38% | G |
| UK enters lock down. | 23/03/2020 | 15:00-10:30* | 4.86% | BBC |
| Pfizer-Biontech announce COVID-19 Vaccine. | 09/11/2020 | 10:30-15:00 | -4.61% | BBC |
| Russia invades Ukraine. | 24/02/2022 | 15:00*-10:30 | 3.34% | R |

Table 4.A.2: List of Events Used in the Proxy for Uncertainty Shocks

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