MATHTASK: DESIGNING AND IMPLEMENTING SITUATION SPECIFIC ACTIVITIES FOR MATHEMATICS TEACHER EDUCATION AND PROFESSIONAL DEVELOPMENT

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Abstract – MathTASK is a research and development programme that engages mathematics teachers with challenging and highly contextualised classroom situations in the form of tasks (mathtasks). Teacher responses to these tasks reveal their mathematical and pedagogical discourses and provide opportunities to articulate, reflect and shift said discourses. These tasks have been used as instruments for research as well as teacher education and professional development in the UK, Greece and Brazil. In this overview presentation of the MathTASK programme[1], we first introduce the MathTASK programme and its principles- and we exemplify these principles with one mathtask (*simplification* task). We then present a summary of theoretical constructs that have emerged in the course of analysis of MathTASK data before concluding with observations on the benefits of using mathtasks as a means to trigger and facilitate mathematics teachers' reflection on their practice.

Keywords : MathTASK, classroom situations, mathematical discourse, pedagogical discourse, mathematics teaching

I. THE MathTASK PROGRAMME : AN OVERVIEW

Mathematics teachers have high aspirations when they enter the classroom. They want their students to understand, appreciate and enjoy mathematics. Often though, what they face in the classroom is nowhere near these aspirations: students' responses may not make sense, addressing individual needs is difficult, the class does not cooperate, technology is confusing and the resources not exactly what is needed[2]. MathTASK[3], a research and development programme that brings together researchers, mathematics teacher educators[4] (thereafter teacher educators) and teachers from the UK, Greece and Brazil, aims to help teachers deal with the challenging situations they often face in the classroom - and, ultimately, to help mathematics teachers transform their aspirations into effective classroom strategies. To this aim, we design situation-specific tasks for mathematics teachers and then invite teachers to engage with these tasks. We call these tasks *mathtasks*. Tasks are presented to teachers as short narratives that comprise a classroom situation where a teacher and students deal with a mathematical problem and a conundrum that may arise from the different responses to the problem put forward by different students. The mathematical problem, the student responses and the teacher reactions are all inspired by the vast array of issues that typically emerge in the complexity of the mathematics classroom and that prior research highlights as seminal. MathTASK so far has focused on four sets of these issues: different or potentially flawed approaches to the mathematical problem taken by different class members; classroom management issues triggered by the exchanges during the lesson and interfering with students' mathematical learning; creative, or not, tensions emerging from the use of digital resources in mathematical problem solving; and, inclusion in mathematical activity of typically under-included learners, such as learners with some disability. Teachers are invited to engage with these tasks through

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[1] This overview is based on the presentation at the Association Tunisienne de Didactique des Mathématiques (ATDM) conference, March 8-11, 2017 and recent developments of the MathTASK programme discussed at Biza et al. (2021).

[2] See a brief animation that describes MathTASK at: https://youtu.be/gt0HZBfBBGI

[3] We use MathTASK (https://www.uea.ac.uk/groups-and-centres/a-z/mathtask) when we refer to the overall programme and its principles, whereas we use mathtask to refer to specific tasks designed with the principles of the MathTASK.

[4] Mathematics teacher educators are those who engage with the education of pre- or in- service teachers.

reflecting, responding in writing and discussing. At the heart of MathTASK is the claim that, through setting out from – and sharpening the focus on – particular elements of mathematics embedded in classroom situations that are likely to occur in actual practice, consistent, specific and research-informed mathematics pedagogies can emerge (Biza, Kayali, Moustapha-Corrêa, Nardi & Thoma, 2021).

II. PRINCIPLES AND RESEARCH DESIGN

The focus of our work is the exploration of teachers' pedagogical and mathematical discourses in their preparation for teaching and in the reflection on their own teaching practices, especially in relation to their interaction with their educators (e.g. in undergraduate or postgraduate course for teachers) or with their colleagues (e.g. when they discuss their teaching during their daily routine or during an in-service professional development course). Teacher education courses expect teachers to transform the theoretical input of these courses into what they do in their everyday work in the classroom. Research has reported the overt discrepancy between theoretically and out of context expressed teacher views about mathematics and pedagogy and actual practice (e.g. Speer, 2005; Thompson, 1992). Speer (2005) claims, for example, that, instead of discussing about teaching practices in the abstract, a discussion of these in a concrete context can provide shared understanding between researchers and participating teachers of the beliefs that are attributed by researchers to teachers. With this observation in mind, in our work we start from specific classroom situations that can provide a trigger for exchanges and build shared insights between researchers and teachers. Specifically, we invite pre- and in- service teachers to reflect on fictional but realistic and research grounded classroom situations (mathtasks) that include a mathematical problem and a reaction by one or more students (and a teacher) to this problem (Biza & Nardi, 2019; Biza, Nardi & Joel, 2015; Biza, Nardi & Zachariades 2007, 2009, 2018; Nardi, Biza & Zachariades 2012).

In MathTASK, a classroom situation is a classroom event or an instance of when teachers have to take a decision about how they would react. The choice of the incident is grounded on issues that research and experience have identified as seminal; it is focused enough to promote teachers' structured reflections; and, it is broad enough to open a meta-discussion on more general issues related to the teaching of mathematics. In this respect, in designing these tasks we bear in mind the following principles:

- The mathematical content of the task concerns a topic or an issue that is known for its subtlety or for causing difficulty to students, this information is drawn from the literature and/or teaching experience – Mathematically Significant Pedagogical Opportunities to build on Student Thinking (MOSTs): *student mathematical thinking*, *mathematically significant* (Leatham, Peterson, Stockero and Van Zoest, 2015).
- The student's response reflects this subtlety (or lack of) or difficulty and provides an opportunity for the teacher to reflect on and demonstrate the ways in which s/he would help the student achieve subtlety or overcome difficulty - Mathematically Significant Pedagogical Opportunities to build on Student Thinking (MOSTs): *pedagogical opportunity* (Leatham, et al., 2015).
- The teacher's pedagogical approach concerns mathematical, pedagogical and epistemological issues that are known for their subtlety or for being challenging to teachers – Practical Rationality of Teaching (Herbst & Chazan 2003) and Spectrum of Warrants (Nardi, Biza & Zachariades, 2012).

- Mathematical content and student/teacher responses provide a context in which teachers' discourses are evidenced – Mathematical Discourse for Teaching (Cooper, 2014) – also in relation to teachers' knowledge, beliefs and intended practices (mathematical, pedagogical and epistemological) that are allowed to surface – Mathematical Knowledge for Teaching (Ball, Thames & Phelps, 2008) and Knowledge Quartet (Turner & Rowland 2011).
- Mathematical content and students'/teachers' actions and interactions are contextualised to the curriculum and the educational context teachers are familiar (e.g. contextual information about the class and students level allows the teachers to situate themselves as teachers of that class)

1. Exemplification of the MathTASK design principles: The "Simplification Task"

The principles we discussed earlier are demonstrated in the "Simplification Task" (Biza et al., 2015). In Figure 1, the mathtask is with comments on the side that explain its design.

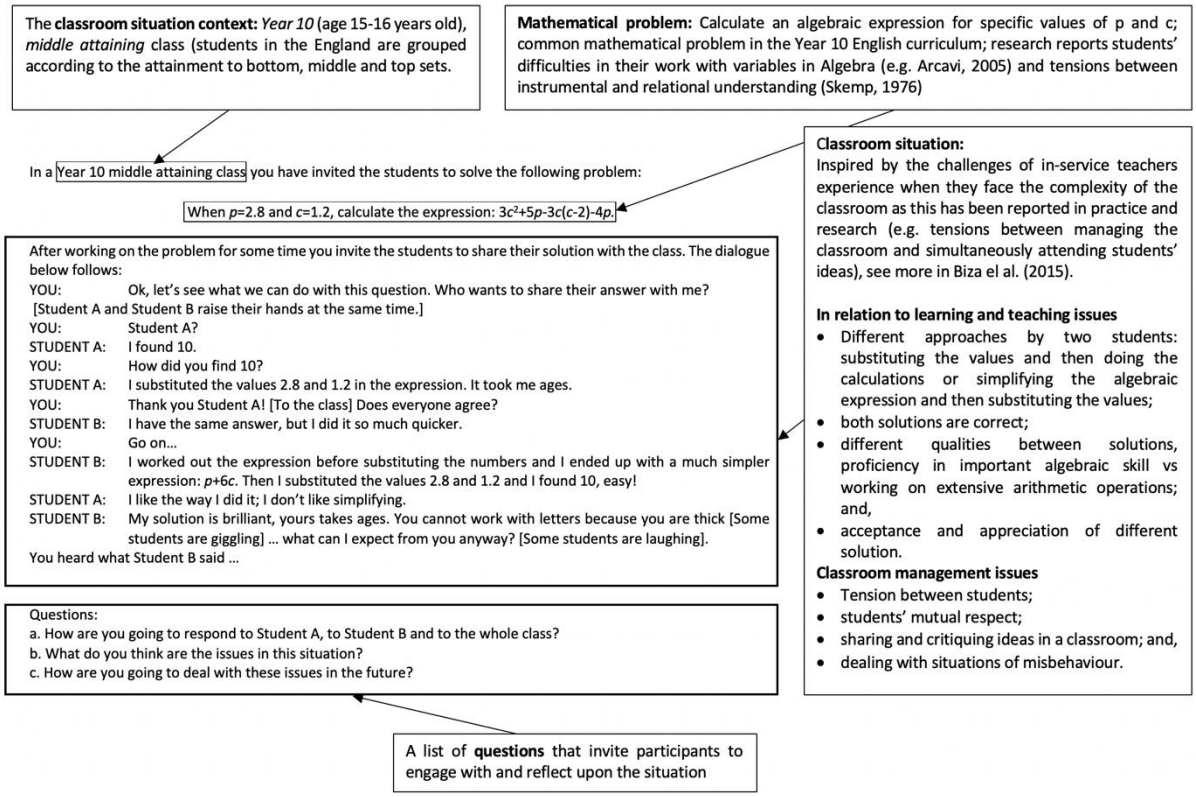

Figure 1 – The Simplification Task (Biza et al., 2015, p. 188) annotated

THEORETICAL CONSTRUCTS PROPOSED BY THE USE OF MATHTASKS

Findings from the use of mathtasks in research have revealed the complex set of considerations mathematics teachers take into account when they make decisions or reflect upon their teaching. To give an example, when we asked a mathematics teacher if they would "accept a graph-based argument as proof", he replied:

> Mathematically, in the classroom, I would welcome it at lesson-level and I would analyse it and praise it, but not in a test". Asked to elaborate, he said: "Through [the graph-based argument] I would try to lead the discussion towards a normal proof...with the definition, the slope, the derivative, etc.". Asked to justify he said: "This is what we, mathematicians, have learnt so far. To ask for precision. ... we have

> this axiomatic principle in our minds. ... And this is what is required in the exams. And we are supposed to prepare the students for the exams. (Biza et al. 2009, p. 34)

The teacher above seems to approach visual argumentation from three different and interconnected perspectives: the restrictions of the current educational setting, in this case the university entrance examinations; the epistemological constraints with regard to what makes an argument a proof within the mathematical community; and, finally, the pedagogical role of visual argumentation as a means towards the construction of formal mathematical knowledge. These three perspectives reflect three roles that a mathematics teacher needs to balance: educator (responsible for facilitating students' mathematical learning), mathematician (accountable for introducing the normal practices of the mathematical community) and professional (responsible for preparing candidates for one of the most important examinations of their student career).

This observation led us to the analysis of the arguments put forward by secondary mathematics teachers in their written responses to a classroom situation described in one mathtask and the follow up interviews. Our analysis aimed to discern, differentiate and discuss the range of influences (epistemological, pedagogical, curricular, professional and personal) on the arguments teachers put forward in their scripts and interviews. We focused particularly on the warrants of these arguments, in the light of Toulmin's (1958) model of informal arguments and Freeman's (2005) classification of warrants, and we proposed the following classification:

- an a priori warrant is, for example, resorting to a mathematical theorem or definition (a priori–epistemological) or resorting to a pedagogical principle (a priori–pedagogical);
- an institutional warrant is, for example, a justification of a pedagogical choice on the grounds of it being recommended or required in a textbook (institutional–curricular) or on the grounds that it reflects the standard practices of the mathematics community (institutional–epistemological);
- an empirical warrant is, for example, the citation of a frequent occurrence in the classroom (according to the arguer's teaching experiences, empirical–professional) or resorting to personal learning experiences in mathematics (empirical–personal);
- an evaluative warrant is a justification of a pedagogical choice on the grounds of a personally held view, value or belief. (Nardi et al. 2012, pp. 160-161).

In a different study, we analysed teachers' responses to mathtasks in relation to their competencies in diagnosing issues in students' responses and to respond to these issues. The analysis suggested a typology of four interrelated characteristics of teachers' responses:

- *Consistency:* how consistent a response is in the way it conveys the link between the respondent's stated beliefs and their intended practice,
- *Specificity:* how contextualised and specific a response is to the teaching situation in the task,
- *Reification of pedagogical discourse:* how reified the pedagogical discourse of the response is in order to describe the pedagogical and didactical issues of the classroom situations and the intended practice presented in the script, and
- *Reification of mathematical discourse:* how reified the mathematical discourse of the response is in relation to the identification of the underpinning mathematical content of the classroom situations and the transformation of this mathematical content into the intended practice presented in the script. (Biza et al. 2018, p.64)

Towards the analysis of the student data we collect during the delivery of the courses – and sample in this chapter – we deploy a typology of four interrelated characteristics that emerged from themes identified as pertinent for mathematics teacher education and professional development in our prior research (see detailed rationale, definitions and examples in Biza et al., 2018; Biza & Nardi, 2019, pp. 46-47) and is tailored to the commognitive underpinnings of our work.

- *Consistency:* how consistent a response to a mathtask is, namely how well-linked the respondent's utterances on stated pedagogical priorities are with their utterances on intended reaction in the teaching situation under consideration.
- *Specificity:* how contextualised and specific a response to a mathtask is, namely how explicitly relevant the respondent's utterances are to the teaching situation under consideration.
- *Reification of RME discourse:* how reified the use of theories and findings from research into the teaching and learning of mathematics – that students are becoming familiar with during the course – appear in a response to a mathtask.
- *Reification of mathematical discourse*: how reified mathematical discourse – that students are familiar with, through prior mathematical studies – appears in a response to a mathtask.

III. CONCLUSION

In this overview presentation of the MathTASK programme we summarised the principles of the MathTASK programme and we exemplify those principles through one example, the simplification task. We presented also theoretical constructs proposed through the use of mathtask for research purposes. In recent years mathtasks have been used for research and teacher professional development purposes. Moustapha-Corrêa and colleagues, in a master's course for in-service teachers in Brazil, designed and applied mathtasks with a twofold purpose: to educate in-service teachers and to conduct research on those teachers' discursive shifts on what is mathematics and what mathematical truth is (Moustapha-Corrêa, 2020; Moustapha-Corrêa et al., 2019; Moustapha-Corrêa et al., 2021). Also, mathtask design was influenced by research observation from a secondary mathematics classroom (Kayali, 2019; Kayali & Biza, 2017, 2021). Also, mathtasks were used in the CAPTeaM (Challenging Ableist Perspectives on the Teaching of Mathematics) project. In this project mathtasks were used to engage teachers across educational levels and in different national and institutional contexts with reflection on the inclusion of disabled learners in mathematics (Nardi, Healy, Biza & Fernandes, 2017). Mathtask are also used in the introduction of Education and Mathematics undergraduate students to mathematics education research and practices (Biza & Nardi, in press; Nardi & Biza, in press).

In all examples, the mathematical content is central and always intertwined with the pedagogy of mathematics teaching. Teachers very often act at the boundaries of the teaching discourses (grounded on their experiences as students or as teachers), the mathematical discourses (grounded on the mathematical component of their education) and the pedagogical discourses (grounded on the pedagogical component of their education. MathTASK programme aims to bring these discourses together.

Overall, we see the situation-specific task design we propose and the theoretical findings from the use of mathtasks in research – classification of warrants (Nardi et al., 2012) and typology of four characteristics (Biza et al., 2018) – as potent research tools and components of formative and summative assessment in teacher education programmes. By accentuating the

specificity of the classroom situation, we invite teachers to reflect upon students' (and another teacher's) approaches and imagine their own intended practice. We thus gain insight into teachers' views and, crucially, challenging aspects of these views.

Teachers who participated in MathTASK workshops said that: "[t]hese activities made me reflect on my teaching practice" or "[m]y engagement with these tasks helped me deepen my own mathematical knowledge" or "[m]y engagement with these tasks helped me anticipate students' answers and their mistakes as well as their different ways of solving or approaching mathematical concepts". This balance between mathematics and pedagogy in teachers' reflections is exactly at the heart of MathTASK.

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