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News credibility and the quest for clicks[☆]

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ABSTRACT

We examine a model of dynamic communication by a media outlet. In each period, the uninformed public can consult the outlet's report at a cost. The outlet, which is primarily driven by profit maximization, has an incentive to induce uncertainty in order to encourage future consultation and thereby generate revenue. In an intermediate cost range, the public and the outlet may be worse off with a cheaper cost of access since it leads the outlet to distort information more, by making the public's future consultation decision more responsive to the current report.

1. Introduction

In contemporary liberal societies, citizens obtain their information to a large extent via news media. Four stylized facts of media markets stand out. First, news providers generate revenue from readers (or viewers) whether directly by selling access to news or through advertising fees (Doyle, 2013; Noam, 2013). Second, the average cost of accessing news contents has decreased significantly over the years, in consequence of the rise of online news (Newman et al., 2016).[1] Third, the revenue (advertising and circulation combined) of the newspaper industry has fallen substantially.[2] Fourth, news accuracy is widely perceived to have declined in recent decades.[3]

Katherine Viner, Editor in Chief of the British newspaper the *Guardian*, summarizes the overall situation as follows: "*(T)he trouble is that the business model of most digital news organizations is based around clicks. News media around the world has reached a fever-pitch of frenzied binge-publishing, in order to scrape up digital advertising's pennies and cents. (...) The impact on journalism of the crisis in the business model is that, in chasing down cheap clicks at the expense of accuracy and veracity, news organizations undermine the very reason they exist: to find things out and tell readers the truth – to report, report, report.*"[4]

We study the relationship between the informativeness of media reports and the (exogenous) cost of access to news in a simple dynamic game of information transmission. An informed sender (a media firm) faces a representative reader, who seeks information to improve his decision-making. The state of the world is binary and follows a Markov process, and each current state entails a different (low or high) level of uncertainty about the state tomorrow. When today's state implies

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[1] While technology does allow news outlets to implement paywalls, most news media adopt either no paywall or a "soft" paywall on which key articles are still freely available. See Ingram, M. "The Guardian, Paywalls and the Death of Print Newspapers," Fortune, 27 February 2016. See also Arrese (2016) for the development of payment systems for online media.

[2] "Newspapers Fact Sheet", Pew Research Center, 9 July 2019. Cagé (2016) points to the recent decline in the number of journalists per outlet and in the amount of space for news in newspapers.

[3] "Decline in Credibility Ratings for Most News Organizations", Pew Research Center, 16 August 2012; "U.S. Adults under 30 Now Trust Information from Social Media Almost As Much As from National News Outlets", Pew Research Center, 27 October 2022.

[4] Viner, K. "How Technology Disrupted the Truth," Guardian, 12 July 2016. See also Murtha, J. "What It's Like to be Paid for Clicks," Columbia Journalism Review, 13 July 2015; and Sherman, J. "Daily Mail 'Too Unreliable' for Wikipedia," The Times, 10 February 2017. See also Rusbridger (2018) for a comprehensive description of fundamental changes that have occurred in the print media industry during the last decades. Kilgo et al. (2018) empirically study the link between social media recommendations and sensationalism in online news. Arbaoui et al. (2020) provide evidence that commercial media funded through advertising are more prone to sensationalism than publicly funded media.

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low uncertainty about tomorrow, the media firm has an incentive to lie about the current state in order to induce the rational receiver to access another report tomorrow. We find that in an intermediate cost range, cheaper access to news lowers the accuracy of the report in equilibrium since the receiver's decision whether to read a news report becomes more manipulable, which increases the sender's incentive to lie. This negative effect on the accuracy of news may be strong enough to outweigh the benefit of cheaper access for both the sender and the receiver. As a result, they may be worse off with cheaper access to news.

In our two-period model, an informed sender (S) who perfectly observes the state produces a report in each period. An uninformed receiver (R) makes two decisions in each period, namely (i) whether to access the report by incurring costs, including opportunity costs; and (ii) which action to take to match the state of the world. While S's primary source of payoffs is visits/accesses by R, he also derives a positive payoff from truth-telling, but the latter is assumed small relative to the revenue generated from visits.

We find that if the expected cost of access is in an intermediate range, equilibrium communication is partially but not fully informative.[5] S occasionally sends a false report claiming that the current state is the one that involves higher future uncertainty, with the aim of inducing a future visit. Reports are informative on average, which is why R may be willing to access them even if it is costly to do so. Equilibrium communication thus features a combination of uncertainty resolution and uncertainty generation. Our second main finding is that in the intermediate cost range, S's reports become less informative as the expected cost of access *decreases*, since R's decision to access a future report becomes more responsive to the contents of the current report, which in turn gives S a stronger incentive to misreport. Our third main finding is that this adverse effect may be strong enough that both S and R can be worse off with cheaper access to news.

We consider several extensions to our baseline model. The first extension examines the case of two news outlets, which allows R to cross-check reports from different media firms. Our main finding is that if the outlets can with some probability coordinate their reports, the accuracy of the individual reports can decrease as the cost of access decreases, as in the single outlet model. We also find that a higher level of independence in news production does not necessarily increase the accuracy of reports, especially when the cost of access is small and the reports in equilibrium are not very accurate. In other words, cross-checking may not be effective in inducing more accurate reporting when the readers' behaviour is highly manipulable.

The second extension introduces subscription pricing into our single and two-firm models, where R is required to subscribe to an outlet in order to access its reports. We find that subscription contracts do not affect equilibrium communication, while they allow a media firm to capture the readers' surplus. In the third extension, which we relegate to Appendix C, we study an infinite horizon version of our model, which is of interest both as a robustness check and as a realistic description of particular reporting contexts.[6] We find that the effects of the change in the cost of access found in the two-period model carry over to this setting.

On the face of it, our results echo some findings in the literature on planned obsolescence, in that a firm may reduce the quality of the current product for higher profits (Bulow, 1986; Waldman, 1993, 1996). In particular, Bulow (1986) showed that a durable goods monopolist may inefficiently reduce the durability of the product in the current period in order to mitigate his tendency to overproduce later (due to the lack of commitment). However, the strategic environment we examine in this paper is very different from that in the literature on planned obsolescence. In Bulow (1986) for example, lower durability/quality does not increase future demand, while in our model the purpose of reducing the quality of a report is to increase the demand in the future.[7]

The literature on strategic media has studied factors that determine the characteristics of media reports in various environments.[8] A key distinction with regards to media firms' incentives to misreport in the literature is between partisan motives and profit motives. While the partisan approach has provided important insights (e.g. Duggan and Martinelli, 2011; Stone, 2011), our model is more closely related to the other strand of the literature. In Mullainathan and Shleifer (2005) and Bernhardt et al. (2008), a profit seeking media firm biases news reports to target a subset of behavioural readers who, besides seeking information, value confirmatory news.[9] In contrast to these papers, readers in our model care only about being informed, are fully rational, and have no behavioural preferences such as a positive payoff from confirmatory news.[10]

The effects of media competition on newspaper reports have been studied extensively. Gentzkow et al. (2014) document that competitive forces in the US newspaper market made newspapers ideologically differentiate themselves from competitors and focus on serving specific tastes of consumers. Angelucci et al. (2023) show that newspapers in the US reduced the amount of local news items in response to the entry of national television news. Cagé (2020) finds that an increased number of newspapers is associated with fewer articles and less hard news provision. An et al. (2007) study theoretically how competition induces media bias as means of product differentiation, and Perego and Yuksel (2022) analyze how the differentiation of news induced by competition affects the voting behaviour of the audience. Gentzkow et al. (2015) observe that in theory, competition generally reduces distortion even if the competing firms have biases, since competition tends to align media firms' incentives with the demand of consumers especially when the consumers are unbiased. In our model the readers are also unbiased and they wish purely to be better informed, but the effect of competition (parameterized as the degree to which reports are independently produced) on the informativeness of reports is ambiguous when the cost of access is low.

The economic incentives behind the media firm's misreporting are also related to the case of credence goods, for which the buyer cannot observe the quality of treatment even after purchase.[11] The closest paper in the literature to ours is Fong (2005), where the buyer can choose whether to purchase a treatment after a "recommendation" from the seller. However, the "recommendation" in Fong (2005) is a take-it-or-leave-it offer specifying the price and type of the treatment

[5] The cost of access being in an intermediate range has a natural interpretation. If the cost is very low or zero, R will access a news report in the next period regardless of the content and informativeness of the report in the current period.

[6] While the state in the two-period model can be interpreted as the development of a specific event that has a predetermined end date, the infinite-horizon model better represents reporting on an economic or political state of affairs in perpetual evolution.

[7] In Bulow (1986), the future demand function is unaffected but the future (resale) price is negatively affected by durability through the future total supply. Moreover, in our model the media firm's incentive to misreport comes from the reader's (or buyer's) lack of commitment to a future consultation strategy, while in Bulow (1986) the driving force of low durability is time inconsistency on the seller's side.

[8] See e.g. Gentzkow et al. (2015) and Puglisi and Snyder (2015) for surveys.

[9] Empirical studies on the sources of media's ideological biases include Gentzkow and Shapiro (2006, 2010), Martin and Yurukoglu (2017), Qin et al. (2018), and Levy (2021).

[10] The empirical study by Ahern and Sosyura (2015) examines financial media reporting on speculated corporate mergers. They find that less accurate stories are more likely to feature well-known firms with broader readership appeal. This suggests that the primary source of inaccuracy is the incentive to attract a broader readership, as we argue in this paper. Angelucci and Cagé (2019) demonstrate theoretically and empirically that a reduction in advertising revenue for media firms leads to a reduction in the subscription price and the amount of journalistic contents.

[11] See Dulleck and Kerschbamer (2006) for a survey.

the expert commits to, while in our model a media firm's report is information about the state and does not affect the receiver's action space in any way.

The incentive to create future uncertainty in our paper is reminiscent of the notions of suspense and surprise in Ely et al. (2015), which refer to expected and current changes in the receiver's belief. While the receiver in their model enjoys positive payoffs directly from uncertainty as represented by suspense and surprise, the receiver in our model is strictly worse off with higher uncertainty since his action is less likely to match the state. Moreover, unlike in our model, in Ely et al. (2015) the sender commits to a disclosure policy, and the state is constant over time. In Hörner and Skrzypacz (2016) an agent endowed with commitment power can gradually reveal verifiable information concerning his ability level.

Renault et al. (2013) and Golosov et al. (2014) are among the papers that study repeated cheap talk games. In their models, the incentives to misreport stem from the misalignment of preferences with regards to the action taken by the receiver given the information provided by the sender. In our paper, the sender's payoff is primarily affected by whether the receiver acquires a report, but not the receiver's action informed by the report. The incentive to misreport stems from the drive to induce future visits, which to our knowledge has not been studied previously. A recent paper by Che et al. (2023) studies dynamic persuasion with a listening cost for the receiver as well as an information provision cost for the sender, where the receiver can stop listening and take the final action at any point. Unlike in our model, the sender cares about the receiver's action, and can commit to a disclosure policy.

We proceed as follows. Section 2 studies a single firm model with two periods. Section 3 extends the model to the case of two firms, and Section 4 discusses subscription pricing. Section 5 concludes. The proofs are relegated to Appendix A. Appendix B contains some additional discussions for the single and two-firm models. Appendix C studies an infinite horizon version of the single firm model.

2. Single media firm

There are two periods $t = 1, 2$. At each period t, the state ω_t is drawn from state space $\{A, B\}$. The prior distribution of the state in period 1 is given by $P(\omega_1 = A) = \theta$, where $\theta \in [\frac{1}{2}, 1)$. The state follows a Markov process with transition probabilities $P(\omega_{t+1} = A \mid \omega_t = A) = \gamma \in (\frac{1}{2}, 1)$ and $P(\omega_{t+1} = B \,|\omega_t = B) = \frac{1}{2}$. State B thus entails higher uncertainty about the state in the next period. There are two players, namely a sender S and a receiver R. S observes the state ω_t at the beginning of period t and chooses a message (or "report") $m_t \in \{A, B\}$ for the period. In period t, R observes neither ω_t nor ω_{t-1}.

R's decision making in each period is divided into three stages. In the first stage, R has the option to consult (or "visit") S at a cost and thereby observe m_t. The consultation cost is $w+v_t$, where $w \in [0, \frac{1}{2})$ is a fixed cost constant across both periods, and v_t is a random cost privately observed by R that is drawn anew at each period t from a uniform distribution on $(0, \frac{1}{2})$.[12] The consultation cost includes not only various material costs borne when acquiring the report but also the opportunity cost of time and cognitive effort to read the report. R observes v_t before deciding whether to consult in period t. Let d_t be a variable that takes value 1 if R consults in period t and 0 otherwise. In the second stage of period t, R chooses an action $a_t \in \{A, B\}$. The choice of a_t is based on R's posterior given m_t if it has been acquired, and otherwise on R's prior at the beginning of period t. In the last stage of period t, R observes m_t even if he chose not to visit S in period t. This captures the notion that news reports are only excludable for a short time, as information spreads fast through various channels such as word of mouth and social media.

The payoff of R for period t depends on both his consultation choice and whether his action matches the state. Besides the consultation cost already described, R obtains an action payoff of 1 in period t if $a_t = \omega_t$, and 0 otherwise.

S's payoff in period t depends on two aspects, a visit and truth-telling. S receives per visit revenue f if R makes a visit. Reporting $m_t = \omega_t$ yields a truth-telling benefit $z = fx$, where $x > 0$ is small so that $x < (1-\theta)(2\gamma - 1)$,[13] while reporting $m_t \neq \omega_t$ yields no such benefit.[14] The truth-telling benefit z captures S's intrinsic preference for reporting the truth, or unmodelled reputational concerns, which we will discuss shortly. By construction $z < f$, so that S's primary concern is to induce consultation. We assume no discounting for simplicity.

A strategy of S for the whole game is given by the combination of communication strategies for both periods. A communication strategy for period t is informative if $P(m_t = A \mid \omega_t = A) \neq P(m_t = A \mid \omega_t = B)$ and it is otherwise uninformative. We define a *simple* communication strategy for period t as one that has the following two features. First, the probability of sending $m_t \in \{A, B\}$ is only a function of the current state ω_t. Denote by τ_J^t the probability that message $m_t = J$ is sent in period t when $\omega_t = J$. The second feature is $\tau_B^t = 1$, which means that S reports truthfully whenever the state is B. Intuitively, since state B implies higher uncertainty about the future than state A, there is no reason for S to misreport if $\omega_t = B$. Thus if misreporting is to occur, it must be when $\omega_t = A$. Given $\tau_B^t = 1$, a simple communication strategy for period t is *partially informative* if $\tau_A^t \in (0, 1)$, *fully informative* if $\tau_A^t = 1$ and *uninformative* if $\tau_A^t = 0$. Thus τ_A^t measures the informativeness of the S's report in period t. A simple communication strategy for the whole game is such that S uses simple communication strategies in both periods. S is free to use any strategy, i.e. we allow S to use strategies that are not simple communication strategies.

A strategy of R for the whole game is given by the combination of strategies for both periods. R's strategy for each period specifies (i) whether he visits (i.e. acquires m_t before choosing a_t); and (ii) which action a_t he chooses, conditional on the information R has.

A strategy profile together with a set of equilibrium beliefs constitutes a perfect Bayesian equilibrium if each player's strategy is sequentially rational given his beliefs and the other's strategy, while beliefs are derived from Bayes' rule whenever possible. That is, at any point in time where a player is called upon to make a choice (e.g. choosing a message in S's case), their choice maximizes their expected utility at that point.[15]

Note that we do *not* a priori restrict our equilibrium analysis to equilibria that feature a simple communication strategy. We shall however establish that the unique equilibrium is such that S uses a simple communication strategy.

We conclude the presentation of our model with a discussion on some assumptions. The assumption that R exogenously observes m_1 at the end of period 1 captures information diffusion through free media, social networks (off- or online) or word of mouth. Technically speaking, the assumption simplifies the equilibrium analysis since R's belief at the end of $t = 1$ is independent of his consultation choice in period 1.

[12] We discuss the assumption on the support of the distribution later.

[13] This condition rules out the trivial case where the only equilibrium is truth-telling for any w.

[14] In principle, *truthful* communication (or truth-telling) and *fully informative* communication are conceptually different. S communicates truthfully if $m_t = \omega_t$ in any state ω_t. In contrast, equilibrium communication is fully informative if R can always infer the state perfectly based on the message observed, regardless of its literal meaning, using Bayes' rule and S's communication strategy. As will be clear later, unlike in standard models of cheap talk (Crawford and Sobel, 1982) fully informative equilibrium communication has to come in the form of truth-telling in our model.

[15] An alternative, which we do not explore, would be to let S commit ex ante to a communication rule with the aim of maximizing his ex ante expected payoff, as in Kamenica and Gentzkow (2011).

We interpret f as advertising revenue generated by a visit to S's website (a click), rather than as a direct transfer from R to S. Advertising revenue is typically a main source of income for many news websites that do not charge for access. This interpretation also implies that even with a subscription contract, S still has an incentive to induce clicks since they generate revenue through advertisements or valuable browsing data that can be sold to third parties. We discuss subscription contracts in Section 4.

The truth-telling benefit $z = fx$ captures journalistic commitment to truthful reporting, unmodelled reputational concerns, or potential legal costs associated with inaccurate reporting.[16] It appears natural to assume that the truth-telling benefit is proportional to the size of the market and/or per visit revenue. Alternatively we could simply assume that z is some positive constant. However, we will see later that key equilibrium quantities (in particular the truth-telling probabilities) are affected by z and f exclusively through the ratio $\frac{z}{f}$, and therefore the assumption $z = fx$ simplifies the exposition. All of our results, except those directly related to the effect of exogenous changes in f, are qualitatively unaffected even if f does not enter the truth-telling benefit.

Finally, for expositional convenience we assume that there is only one receiver up until our discussion of subscription pricing in Section 4. However, throughout the paper our representative receiver can be reinterpreted as a continuum of receivers with mass one. In this alternative interpretation, the distribution of v_t is i.i.d. across all receivers and periods.

In what follows, we present our equilibrium analysis for three adjacent ranges of w, namely low, intermediate and high. We demonstrate that the accuracy of the report in period 1 is weakly increasing in w over the three ranges, but strictly increasing in the intermediate range. Specifically, there is no information transmission in the low range, noisy transmission in the intermediate range where the report becomes more informative as w increases, and perfect transmission in the high range.

Let us note that regardless of the ranges of w, the report in period 2 must be truthful (i.e. $m_2 = \omega_2$ regardless of ω_2) in equilibrium. Since period 2 is the last period, there is no incentive to induce a future visit by misreporting. S strictly prefers to report truthfully, because of the presence of the truth-telling benefit.

2.1. Low visiting costs

We first consider the low range of w, i.e.

$$w \in \left[0, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right].$$

R's probability of visit in period 2 is increasing in the posterior probability that $\omega_1 = B$ as of the end of period 1, since the future state ω_2 is more uncertain when $\omega_1 = B$. Also, given $P(\omega_1 = B \mid m_1 = B)$, which denotes R's conditional probability that $\omega_1 = B$ when $m_1 = B$, the probability of visit in period 2 increases as the expected cost of access decreases.[17] Therefore, a lower fixed cost of access w implies that when $\omega_1 = A$, S has a higher incentive to send $m_1 = B$ in order to increase the posterior probability that R assigns to $\omega_1 = B$. Our first Proposition shows that very low w thus makes it impossible to achieve any informative communication by S in period 1, since the incentive to manipulate R's future consultation decision is too strong.

Proposition 1. *If $w \in \left[0, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$, there is a unique equilibrium. It features uninformative communication in period 1 where $m_1 = B$ regardless of ω_1, and truth-telling in period 2.*

Notice that even if $w = 0$, R still bears a uniformly distributed random cost of access $v_t \in (0, \frac{1}{2})$. Therefore, as w tends to 0 the expected cost of consultation does not tend to 0 but to $\frac{1}{4}$, which implies that R's decision whether to visit in period 2 is affected significantly by m_1. In the equilibrium, because of the random cost, $m_1 = B$ leads to a strictly higher probability of visit in period 2 than $m_1 = A$, and S never chooses $m_1 = A$. As a result m_1 is uninformative and R never visits in period 1 even when w is very small or zero.

If there was no random cost v_t, as w tends to 0, R's decision whether to visit in period 2 would become unresponsive to m_1 since he would visit for sure regardless of m_1. This in turn would support an equilibrium where m_1 is truthful and R always visits in period 1.[18] However, in the context of online news that we are concerned with in this paper, no readers would have unlimited capacity to read, so our assumption that R faces a non-negligible cost of access appears natural.

2.2. Intermediate visiting costs

Let us now assume that the fixed cost of consultation is in an intermediate range, i.e.

$$w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right). \tag{1}$$

In this range of w, R's decision whether to visit in period 2 is affected by m_1, but its influence is attenuated by the fact that a visit involves a significant cost.

2.2.1. Equilibrium

We obtain the following equilibrium characterization:

Proposition 2. *Let $w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right)$.*

(a) *There exists a unique equilibrium. In the equilibrium, S reports truthfully in period 2 and uses a partially informative simple communication strategy in period 1, where the probability that $m_1 = A$ when $\omega_1 = A$ is given by*

$$\tau_A^{1*} = \frac{2w + \theta(2\gamma - 1) + x - 1}{\theta(2\gamma + 2w + x - 2)} \in (0, 1). \tag{2}$$

R consults with probability x in period 2 if $m_1 = B$, and with probability 0 if $m_1 = A$. In both periods, R chooses $a_t = m_t$ if he consults, and $a_t = A$ if he does not consult.

(b) *τ_A^{1*} is constant in f; and it is increasing in w, γ and x.*

Recall that we allow S to use any communication strategy and thus do not restrict our analysis to equilibria that involve simple communication strategies. Proposition 2 however establishes that there is a unique equilibrium and a property of this equilibrium is that S uses a simple communication strategy. Communication in period 1 is partially informative, as S is indifferent and randomizes between $m_1 = A$ and $m_1 = B$ when $\omega_1 = A$. Note that τ_A^{1*} is the only aspect of S's communication strategy that varies with exogenous parameters.

A key aspect of Part (b) of Proposition 2 is that τ_A^{1*} is increasing in w, as illustrated in Fig. 1. This is consistent with our discussion in the Introduction, where we suggest that online news, which tends to be much less costly to access relative to traditional paper-based media, have also become less reliable.

Technically, the property that τ_A^{1*} is increasing in w is an immediate implication of the fact that S is indifferent between $m_1 = A$ and $m_1 = B$ when $\omega_1 = A$. The corresponding indifference condition is given by

$$fx = fP(d_2 = 1 \mid m_1 = B). \tag{3}$$

[16] The model features no uncertainty about S's preference or behavioural type, which is often associated with models of reputational concerns.

[17] Note that we have $P(\omega_1 = B \mid m_1 = A) = 0$ given that S uses a simple communication strategy in period 1, because S never misreports when $\omega_1 = B$.

[18] In Appendix B we provide a detailed discussion on the case where the expected random cost of access is very small. We derive a condition with respect to w and the support of the distribution of v_t such that an equilibrium with truth-telling in period 1 exists.

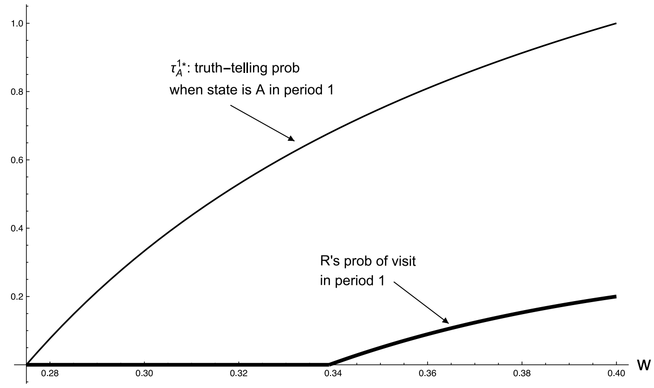


Fig. 1. Equilibrium behaviour as function of w, where $\theta = 1/2, \gamma = 3/4$, and $x = 1/5$.

The LHS corresponds to the truth-telling benefit in period 1 when sending $m_1 = A$. Recall that $m_1 = A$ induces no visit in period 2.[19] The RHS of (3) is the expected revenue from R's visit in period 2 induced by sending $m_1 = B$ (thereby misreporting), where $P(d_2 = 1 \mid m_1 = B)$ denotes R's conditional probability of visit in period 2 given $m_1 = B$. Clearly, f cancels out from both sides of (3), so that the indifference condition reduces to a requirement that the probability of visit in period 2 conditional on $m_1 = B$ equals x, the parameter for the truth-telling benefit.

It is easy to check that $P(d_2 = 1 \mid m_1 = B)$ is strictly increasing in τ_A^1. Intuitively, a higher τ_A^1 means higher uncertainty about ω_2 conditional on $m_1 = B$, since it implies that $m_1 = B$ gives a stronger indication that $\omega_1 = B$ (i.e. $m_1 = B$ is "discounted" less). Note also that as the expected cost of access increases, R's willingness to consult in period 2 after $m_1 = B$ decreases, holding fixed the informativeness of the report in period 1. Therefore, (3) implies that as the expected cost of access increases, the equilibrium truth-telling probability τ_A^{1*} must increase in order to ensure that R's visiting probability after $m_1 = B$ remains equal to x.

The relationship between the cost of access and the informativeness of the report in period 1 is a reflection of the commitment problem that R faces. A small reduction in the cost of access makes R's decision to visit more manipulable, in the sense that his probability of visit in period 2 after $m_1 = B$ increases (while the probability of visit following $m_1 = A$ remains zero). The increase in R's responsiveness to $m_1 = B$ gives rise to a larger temptation for S to misreport when $\omega_1 = A$ and, as a result, lower informativeness of the equilibrium report in period 1. A higher cost of access thus de facto acts as an imperfect but effective commitment device for R not to overreact to a message that implies higher future uncertainty.

Proposition 2 does not describe R's visiting probability in period 1, which is instead shown in Fig. 1.[20] We see that the probability of visit is zero when w is low within the intermediate range, because the low credibility of m_1 cannot justify the cost of access. As w becomes higher, the probability of visit in period 1 increases, since the improvement in the informativeness of m_1 outweighs the increase in the expected cost of access.

The intuition behind the positive effect of an increase in γ is similar to the intuition for the effect of w. For a given informativeness of m_1, an increase in γ reduces R's responsiveness to message $m_1 = B$ by reducing his uncertainty about the state in period 2 conditional on $m_1 = B$, and this lowers S's incentive to misreport.

[19] The LHS can be written as $fx + fP(d_2 = 1 \mid m_1 = A)$ where $P(d_2 = 1 \mid m_1 = A) = 0$.

[20] Since S chooses m_1 to influence R's future visit and R always observes m_1 at the end of period 1, S's equilibrium strategy in period 1 can be derived without calculating R's visiting probability in period 1.

2.2.2. Welfare

We now consider equilibrium payoffs. In order to obtain clear results, for the rest of our analysis of the intermediate range of w we assume maximal uncertainty regarding the state in period 1, i.e. $\theta = \frac{1}{2}$. This maximizes R's incentive to consult before choosing the action in period 1, all else being equal.

Proposition 3. *Let $w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right)$.*

1. *(a) R's probability of consultation in period 1 belongs to $[0,1)$ and is weakly increasing in w. If the probability of consultation in period 1 is strictly positive, it is strictly increasing in w.*

(b) For w sufficiently close to but below $\frac{1-x}{2}$, R's probability of consultation in period 1 is strictly positive and R's expected payoff in period 1 is strictly increasing in w.

2. *S's total expected payoff is weakly increasing in w.*

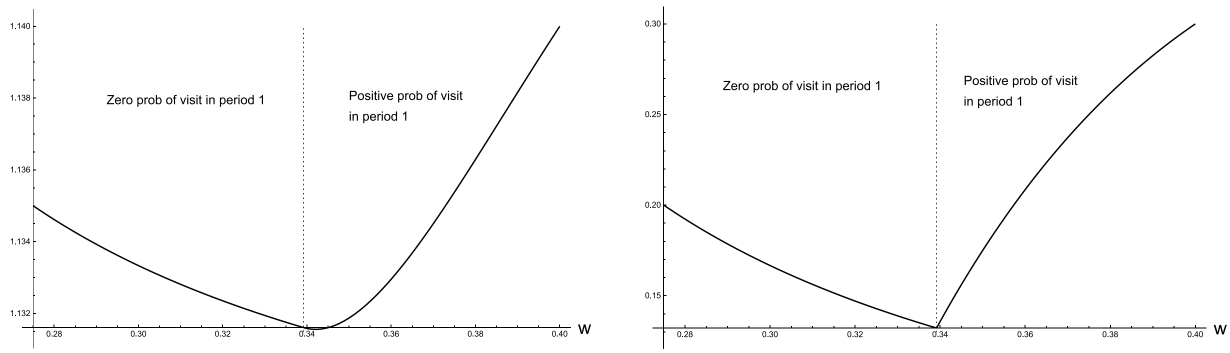
Consider Part 1. R's total expected payoff across the two periods has two components, namely the expected cost of access and the expected payoff from whether his action matches the state in each period. An increase in w generates a trade-off between increased informativeness of m_1 and an increased expected cost of access in both periods. As we saw earlier, the informativeness of m_2 is independent of w since S always reports truthfully. However, an increase in w, by improving the informativeness of m_1, helps R to better decide whether to visit in period 2. The effect of w on R's payoff in period 2 is thus ambiguous.

Meanwhile, the effect of w on R's payoff in period 1 is simpler, involving a pure trade-off between increased informativeness of m_1 and an increased cost of access in period 1. Part 1.(a) implies that R's expected cost of access incurred in period 1 weakly increases as w increases, since the probability of visit weakly increases and the average cost borne conditional on consultation strictly increases. It follows that once the probability of visit in period 1 becomes positive, an increase in w generates a clear trade-off between an adverse cost effect and a positive informativeness effect (an increase in τ_A^1). Part 1.(b) identifies a sufficient condition such that this trade-off resolves positively for R, so that an increase in w yields an increase in R's payoff in period 1.

R's expected payoff over two periods, which we derive explicitly in Appendix B, is complex and its derivative with respect to w for the intermediate range of w does not have a simple expression. Fig. 2(a) shows that the benefit of a more informative report m_1 may outweigh the increase in the expected visiting cost when w is relatively high, which is reflected by the upward sloping curve.

Part 2 of Proposition 3 provides a clear comparative statics result concerning the expected payoff of S. An increase in w has three effects. First, the higher probability of truth-telling in period 1 leads to a higher expected truth-telling payoff. Second, the expected revenue in period 1 increases if the probability of visit in the period is positive, since in this case the probability of visit in period 1 increases. Third, the expected revenue in period 2 decreases as the probability of visit in period 2 decreases. Indeed, the fact that S reports truthfully in period 2 regardless of w implies that R is less likely to visit in period 2, because the expected cost of access increases, and because $m_1 = B$ becomes less likely (since τ_A^{1*} increases). Overall, the first two positive effects weakly dominate the third.

We are interested not only in the expected payoff of S but also in the expected *accounting profit* (= total advertising revenue) of S since it captures the financial performance of a media firm, leaving out the less tangible truth-telling benefit. The expected accounting profit is calculated by simply subtracting the expected truth-telling benefit from S's expected payoff, which we derive in the proof of Proposition 3 in Appendix A. Unfortunately the sign of the derivative of the accounting profit with respect to w is non-monotonic and cannot be stated in a transparent manner. Fig. 2(b) shows that when the probability of visit in period 1 is positive, S's accounting profit may be increasing in w. As we noted earlier, an increase in w has opposing effects on the respective



(a) R's total expected payoff (b) S's total expected *accounting* profit

Fig. 2. Equilibrium payoff with respect to w, where $\theta = 1/2, \gamma = 3/4$, and $x = 1/5$.

accounting profits in periods 1 and 2 (positive in period 1 and negative in period 2). Fig. 2(b) indicates that the increase in the revenue in period 1 outweighs the decline in the revenue in period 2. Since R faces very high uncertainty in period 1 ($\theta = \frac{1}{2}$), his decision to acquire m_1 is very sensitive to the informativeness of m_1 and hence an increase in w. On the other hand, $\gamma > \frac{1}{2}$ implies that there is less overall uncertainty in period 2 than in period 1, and thus the demand for the report in period 2 is lower and less sensitive to an increase in w.

2.3. High visiting costs

Finally, let us consider the case of high w, which corresponds to

$$w \in \left[\frac{1-x}{2}, \frac{1}{2}\right).$$

When w is high, R's decision whether to visit in period 2 is hardly sensitive to the report in period 1, and this naturally lowers the incentive to misreport.

Proposition 4. *If $w \in \left[\frac{1-x}{2}, \frac{1}{2}\right)$, there exists a unique equilibrium. It features truth-telling in both periods and a strictly positive probability of consultation in period 2.*

In the equilibrium described above, S's benefit from truth-telling when $\omega_1 = A$ outweighs the benefit of inducing a (slightly) higher probability of visit by misreporting since the latter probability remains very low due to the high expected cost of access.

Note that $w < \frac{1}{2}$ ensures that in the truth-telling equilibrium, the probability of visit is strictly positive after $m_1 = B$. Truth-telling is thus induced not by the complete absence of response to m_1, but rather by the attenuated response to m_1 due to high w.

3. Two media firms

We now extend our single firm model to the case of two media firms denoted by S_1 and S_2. The possibility of observing multiple reports allows for cross-checking, which might reduce the firms' incentive to misreport. A main objective of the analysis here is to show that an increased fixed cost of access may still have a positive effect on the informativeness of reports and thus also on R's expected payoff and the firms' expected profits. We will also find that an increased level of independence in news production between the firms (i.e. a reduced possibility of coordinating reports) does not necessarily increase the equilibrium accuracy of reporting. The result stems from a credibility effect of independence, whereby two reports indicating state B are more persuasive and thus more conducive to future visits, the more likely it is that they were independently produced. This effect increases the incentive to misreport, which counteracts the disciplining effect of cross-checking. As in Section 2 we assume away pricing, which shall be discussed later in Section 4.

3.1. Model

In what follows, assumptions that we do not explicitly describe are carried over from the single-firm model (e.g. the process governing the state, the sources of payoffs, etc.). For simplicity, let the prior distribution of the state be uniform, i.e. $\theta = \frac{1}{2}$. Each firm S_i observes the state $\omega_t \in \{A, B\}$ perfectly at the beginning of period t before choosing $m_t^{S_i}$. The truth-telling benefit for S_i of sending $m_t^{S_i} = \omega_t$ is still $z = fx$.

In each period, reports are sent simultaneously with probability β and instead sequentially with probability $1 - \beta$. In the latter case, each firm is equally likely to be the first mover. The firms and R do not directly observe which scenario (simultaneous or sequential) has materialized. Unless a firm is the second mover under sequential communication, the firm is uncertain as to whether communication is simultaneous or sequential (i.e. the firm only knows that it is not the second mover). However, if communication is sequential *and* a firm is the second mover, the firm observes the first mover's message before choosing its own and correctly learns that communication is sequential. We interpret β as the degree of independence of firms in choosing their reports. In reality, reports from various outlets are often remarkably similar, reflecting the fact that journalists frequently observe other outlets' coverage of a given topic prior to writing their own report.

In each period t, after the two firms send their reports but before R chooses a_t, R can acquire $m_t^{S_1}$, $m_t^{S_2}$ or neither, but not both. Acquiring a report for period t comes at cost $w + v_t$. At the end of period 1, after choosing a_1, R observes both reports $\left\{m_1^{S_1}, m_1^{S_2}\right\}$ regardless of whether he acquired any report beforehand.

3.2. Equilibrium

Note first that as in the single firm model, both firms report truthfully in period 2 as there is no benefit from lying in this final period. As for period 1, we focus on an equilibrium that features the following strategies. The firms use a symmetric simple communication strategy represented by τ^C. If firm i is not the second mover, it chooses $m_1^{S_i} = B$ with probability 1 if $\omega_1 = B$, and $m_1^{S_i} = A$ with probability τ^C if $\omega_1 = A$. If firm j is the second mover and thereby observes $m_1^{S_i}$ before choosing its own report, firm j aligns its report with that of the first mover (firm i), i.e. $m_1^{S_i} = m_1^{S_j}$. The firms randomize independently in the simultaneous scenario. Given such strategies, the informativeness of a firm's report in period 1 is captured by τ^C. While we derive an equilibrium in symmetric simple communication strategies, the firms are allowed to deviate to any strategy.

If R consults at the beginning of period t he chooses each of the reports with equal probability, as he is indifferent between them. At the end of period 1, as R observes both reports, he cross-checks them, and thereby may be able to infer ω_1 better than if he observes a single report.

3.2.1. R's incentives

Given the firms' assumed strategies, while R does not observe whether the game is simultaneous or sequential, he correctly infers that $\omega_1 = A$ whenever at least one report indicates A. R is thus uncertain regarding ω_1 only when $m_1^{S_i} = m_1^{S_j} = B$. The conditional probability that $m_1^{S_i} = m_1^{S_j} = B$ given $\omega_1 = B$ is 1, while the conditional probability of the same message profile given $\omega_1 = A$ is

$$\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C). \tag{4}$$

The first term in (4) corresponds to the case where the firms communicate simultaneously and both misreport, whereas the second term corresponds to the case where the firms communicate sequentially, the first mover misreports and the second mover follows suit. Using (4), R's posterior given $m_1^{S_i} = m_1^{S_j} = B$ is

$$P(\omega_1 = A \mid m_1^{S_i} = m_1^{S_j} = B) \quad = \quad \frac{\frac{1}{2}\beta(1-\tau^C)^2 + \frac{1}{2}(1-\beta)(1-\tau^C)}{\frac{1}{2}+\frac{1}{2}\beta(1-\tau^C)^2 + \frac{1}{2}(1-\beta)(1-\tau^C)}. \tag{5}$$

The above posterior probability is decreasing in β and τ^C. In other words, the posterior probability assigned by R to $\omega_1 = B$ given $m_1^{S_i} = m_1^{S_j} = B$ is higher when the reports are more likely to be independently produced or are individually more informative. The conditional probability of visit in period 2 given $m_1^{S_i} = m_1^{S_j} = B$ is[21]

$$\begin{aligned} &P(d_2 = 1 \mid m_1^{S_i} = m_1^{S_j} = B) \\ &= \frac{1 - P(\omega_1 = A \mid m_1^{S_i} = m_1^{S_j} = B)\gamma - P(\omega_1 = B \mid m_1^{S_i} = m_1^{S_j} = B)\frac{1}{2} - w}{1/2}. \end{aligned} \tag{6}$$

Given the comparative statics of (5), it follows that (6) is increasing in β and τ^C.

3.2.2. Firms' incentives

Note first that $\beta = 1$ (fully independent news production) trivially supports truth-telling by both firms, assuming off-equilibrium beliefs such that any contradiction in the two reports implies $\omega_1 = A$ with probability 1. In such a case, if one firm always reports truthfully the other firm has no incentive to misreport in any state.[22]

Let us now focus on $\beta \in [0, 1)$. Suppose firm i is *not* the second mover in period 1, in which case firm i is either the first mover or the game is simultaneous. Given $\omega_1 = A$ and assuming $m_1^{S_i} = B$, firm i assigns the following probability to $m_1^{S_j} = B$ and thus to the event that R observes $m_1^{S_i} = m_1^{S_j} = B$:

$$\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}(1-\tau^C) + \frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}. \tag{7}$$

The expression above denotes the probability that firm i's lie is unexposed and cross-checking thus *fails*.[23] It is easy to see that (7) is not only decreasing in β but also decreasing in τ^C. In other words, misreporting by one firm is more likely to be exposed, the more accurate the report of the other firm is.

The degree of independence β thus overall influences a firm's incentives through two opposing effects, namely a credibility effect and a cross-checking effect. The credibility effect (already discussed above) is that for fixed $\tau^C > 0$, an increase in β leads to an increase in $P(\omega_1 = B \mid m_1^{S_i} = m_1^{S_j} = B)$, which implies that unexposed lies are more credible, since the reports are less likely to be a result of coordination between the firms. This effect *increases* an individual firm's incentive to lie when $\omega_1 = A$, since the probability of consultation in period 2 given $m_1^{S_i} = m_1^{S_j} = B$ becomes higher. The cross-checking effect is that for fixed $\tau^C > 0$, an increase in β makes it more likely that misreporting by firm i when $\omega_1 = A$ will be exposed by firm j, i.e. $m_1^{S_i} = m_1^{S_j} = B$ becomes less likely given $m_1^{S_i} = B$. This effect *reduces* an individual firm's incentive to misreport.

Recall that the second mover's equilibrium strategy is such that he always sends the same message as the first mover's message on the equilibrium path. Consider the case where $\omega_1 = A$. Intuitively, if at least one of the two reports indicates A, then R assigns probability 1 to $\omega_1 = A$ and consequently the probability of visit in period 2 is lower than if $m_1^{S_i} = m_1^{S_j} = B$. If the first mover's message is $m_1^{S_i} = A$, the second mover thus does not contradict it because otherwise the second mover would forgo the truth-telling benefit while the probability of visit in period 2 remains the same. Suppose instead that the first mover's message is $m_1^{S_i} = B$. The second mover's expected payoff from $m_1^{S_j} = B$ is at least as high as the first mover's payoff from $m_1^{S_i} = B$ because the second mover's lie is never exposed. Since the second mover's payoff from $m_1^{S_j} = A$ is the same as the first mover's payoff from $m_1^{S_i} = A$, it follows that if the first mover weakly prefers $m_1^{S_i} = B$ to $m_1^{S_i} = A$, then a fortiori the second mover prefers $m_1^{S_j} = B$ to $m_1^{S_j} = A$ as well. Therefore there is no incentive for the second mover to contradict the first mover's report.[24]

Naturally when $\omega_1 = B$, both the first mover and the second mover are better off reporting $m_1^{S_i} = m_1^{S_j} = B$ as they lead to both a higher probability of visit and the truth-telling benefit.

3.2.3. Equilibrium characterization

Let x be small enough that

$$1 - \gamma < \frac{3 - 2\gamma - 4x}{4} < \frac{1}{2} - \frac{x(1+\beta)}{1-\beta}. \tag{8}$$

If (8) holds, we have three well-defined adjacent intervals of values of w, each of which yields a qualitatively different equilibrium prediction, as in the single firm model.

First, let us consider the case where $w \leq \frac{3-2\gamma-4x}{4}$.[25]

Proposition 5 (*Low visiting costs*). *Let $w \in \left[0, \frac{3-2\gamma-4x}{4}\right]$. There exists an equilibrium that features uninformative communication in period 1 ($\tau^C = 0$) and truth-telling in period 2. There exists no equilibrium with truth-telling in both periods.*

Our complementary numerical analysis (see Appendix B) suggests that there furthermore exists no equilibrium with partially informative communication in period 1 ($\tau^C \in (0, 1)$). The general intuition for these negative findings echoes the single firm model: when w is very low, R becomes so manipulable that the incentive to misreport is too strong for informative communication. We now turn to the case of intermediate values of w.

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[21] Note that R chooses $a_2 = A$ if he does not visit in period 2, because regardless of the reports observed in period 1, R's posterior at the beginning of period 2 is such that $\omega_2 = A$ is (weakly) more likely.

[22] This type of equilibrium construction is standard in the literature on cheap talk games with multiple senders (e.g. Krishna and Morgan, 2001).

[23] If firm i learns that it is *not* the second mover, then firm i assigns probability $\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}$ to simultaneous communication and probability $\frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}$ to being the first mover under sequential communication.

[24] We give a detailed argument on the second mover's incentives in the proof of Proposition 6 in Appendix A.

[25] Regarding the left inequality in (8), the difference between $w \in [0, 1-\gamma)$ and $w \in \left[1-\gamma, \frac{3-2\gamma-4x}{4}\right]$ concerns R's probability of visit in period 2 when at least one of the messages is $m^{S_i} = A$, in which case R assigns probability 1 to $\omega_1 = A$. If $w \in [0, 1-\gamma)$, the fixed cost of access is so low that this probability of visit is strictly positive, while if $w \geq 1-\gamma$ it is zero. The difference is important in proving Proposition 5 but does not affect equilibrium communication for $w \in \left[0, \frac{3-2\gamma-4x}{4}\right]$.

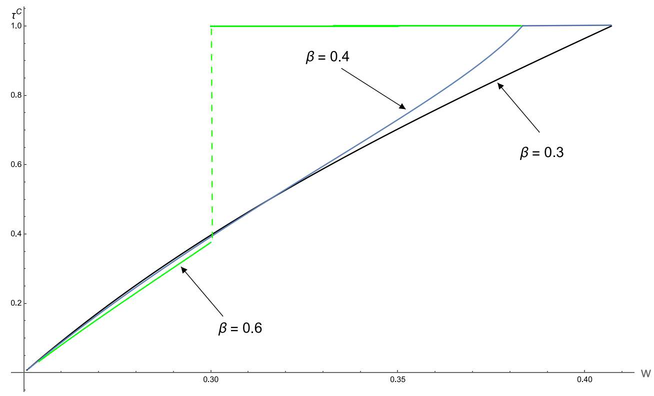


Fig. 3. Truth-telling probability in period 1 given $\omega_1 = A$, where $\gamma = 0.9$ and $x = 0.05$.

Proposition 6 (*Intermediate visiting costs*). *Let $w \in \left(\frac{3-2\gamma-4x}{4}, \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}\right)$. There exists an equilibrium that features partially informative communication in period 1 ($\tau^C \in (0,1)$) such that τ^C in the equilibrium is increasing in w. There exists no equilibrium with truth-telling or uninformative communication in period 1.*

The intuition underlying the result again echoes the single firm model, which also features noisy communication for intermediate values of w. The equilibrium randomization probability τ^C is pinned down by the requirement that firm i, if it is not the second mover, should be indifferent between $m_1^{S_i} = A$ and $m_1^{S_i} = B$ when $\omega_1 = A$. As in the single firm model, the informativeness of the reports in period 1 is increasing in w, since a higher expected cost of access makes R less likely to visit after observing $m_1^{S_i} = m_1^{S_j} = B$ and consequently misreporting less attractive.

While the expected payoff from $m_1^{S_i} = A$ is simply the truth-telling benefit fx, the expected payoff from $m_1^{S_i} = B$ is complex because the expression incorporates both the case where the game is sequential and the case where the game is simultaneous. As a result the solutions to the indifference condition with respect to τ^C are also complex.[26] We present numerical examples in Fig. 3 below to obtain economic insights. Before doing so, we lastly consider the case of high w.

Proposition 7 (*High visiting costs*). *Let $w \in \left[\frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}, \frac{1}{2}\right)$. There exists an equilibrium that features truth-telling in both periods ($\tau^C = 1$).*

As in the single firm model, the state is fully revealed when w is high, since R's decision whether to visit in period 2 becomes less sensitive to the reports. The existence of the truth-telling equilibrium is also closely associated with the degree of independence β, as we see that the condition on w above is always satisfied for $\beta \to 1$. When β is very high, misreporting is simply too likely to be exposed for it to be worthwhile.

In Fig. 3 we plot τ^C in the most informative equilibrium as a function of w, for different values of β.[27] For $\beta = 0.4$ and 0.6, the range of w that appears on the horizontal axis corresponds to the intermediate and the high intervals introduced above, whereas for $\beta = 0.3$ it only corresponds to the intermediate interval. We see that for each value of β considered, the maximum equilibrium value of τ^C is weakly increasing in w, as in the single firm model.

When w is low, not only is the accuracy of reports (as captured by τ^C) low, but a higher β is also not effective in increasing their accuracy, as evidenced by the fact that all curves are clustered. In fact, τ^C in the equilibrium can even slightly decrease as β increases. Recall that the effect of β on τ^C operates through two channels, the credibility effect and the cross-checking effect. Intuitively, when w is low, the equilibrium τ^C given β is also low, which in turn implies that both effects of β are weak. For high w, the curves are not clustered anymore, which indicates that β has a large positive effect on τ^C. The reason for this is that for high τ^C, the cross-checking effect is very strong relative to the credibility effect, since misreporting is highly likely to be exposed when communication is simultaneous. At the same time, when τ^C is high, the posterior probability that $\omega_1 = B$ given $m_1^{S_i} = m_1^{S_j} = B$ is high regardless of the value of β, which implies that the credibility effect is weak.[28]

3.3. Welfare

Fig. 4 plots R's expected payoff and a firm's ex ante expected accounting profit as functions of w in the most informative equilibrium. We consider only two values of β for clarity because the scale of the payoffs varies significantly across values of β. R's expected payoff is strictly increasing in w as long as the equilibrium reports are partially informative ($\tau^C < 1$), above which it is decreasing in w since the informativeness of the reports is constant at $\tau^C = 1$.

The expected accounting profit of the firms is increasing in w so long as the probability of visit in period 1 is positive and $\tau^C < 1$, as in the single firm model. In addition, we see that the firms are better off with a higher degree of independence β because it leads them to communicate more informatively, which in turn increases the probability of visit.

We conclude our analysis of the two-firm model with some caveats. The model embeds a very strong form of cross-checking. In particular, R exogenously observes both reports at the end of each period. This approach rules out other potential effects of w on cross-checking. For example, if R observes both reports at the end of each period only when he acquires them at a cost, cross-checking becomes endogenous, which would make the relationship between w and the informativeness of reports more complex.

The model also rules out another potentially important channel through which cross-checking affects media firms' reputation. In our model the firms' truth-telling benefit is identical and common knowledge, but if it instead was uncertain and privately observed, a firm would have an incentive to expose any misreporting by the other firm in order to signal a high truth-telling benefit. The gained reputational advantage would enable the firm to capture the full market in period 2. Adding uncertainty over the type of each firm is beyond the scope of this paper, but reputational concerns should overall strengthen the cross-checking effect and lead to more informative reporting.

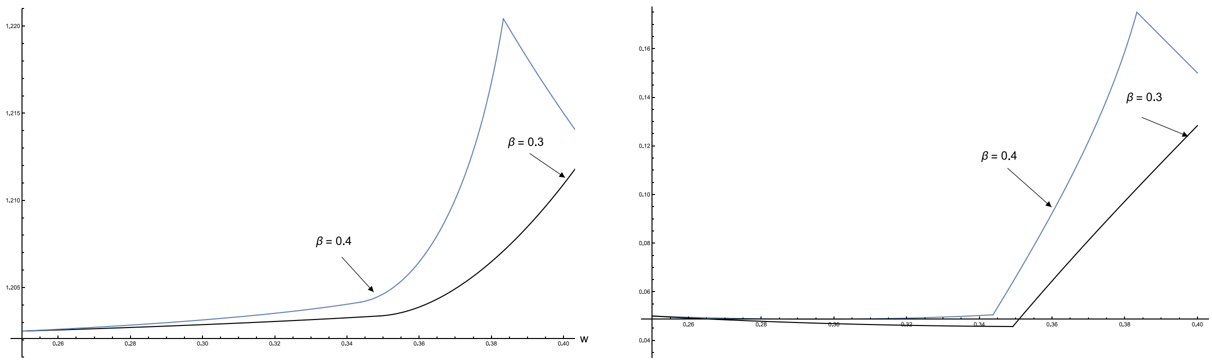
4. Remarks on pricing

So far we have assumed away pricing. In reality, accessing the full contents of a newspaper often requires payment of a fee. In the online news market, this typically takes the form of a subscription fee that grants unlimited access to news articles for a specific period of time. Practices differ substantially across outlets. Tabloid newspapers tend to offer significantly more free contents than quality newspapers (e.g. New York Times), but the latter however regularly offer drastically discounted subscription fees. Another key feature of the online news market is that subscribers generate revenue via more than just the subscription fee. Their clicks often generate not only advertising revenue but also valuable browsing data (which can be used internally or sold to third parties). Thus in many instances a media firm has an incentive to induce actual visits even after a reader has purchased a subscription.

[26] The derivation is in the proof of Proposition 6.

[27] For $\beta = 0.6$, there also exist multiple equilibria with $\tau^C \in (0,1)$ when there exists a truth-telling equilibrium.

[28] We can see from (5) that $P(\omega_1 = B \mid m_1^{S_i} = m_1^{S_j} = B) \to 1$ as $\tau^C \to 1$, regardless of β.



(a) R's total expected payoff (b) A firm's total expected *accounting* profit

Fig. 4. Equilibrium payoff with respect to w, where $\gamma = 0.9$ and $x = 0.05$.

In what follows, we discuss simple extensions of our single and multiple firm models by incorporating endogenously priced subscriptions. A subscription, once bought, allows readers to access reports without any additional payment from that point onwards. Subscribers however still incur the fixed and random costs $w + v_t$ for each visit, which are interpreted as non-monetary costs (such as opportunity cost and cognitive effort). For technical convenience, we explicitly interpret R as a continuum of receivers with mass 1, where the distribution of v_t is i.i.d. across all receivers and periods. The receivers are thus all ex ante identical. For both extensions (single and two-firm models), except for the assumptions on pricing and purchasing decisions explicitly described below, all aspects of the model are the same as in the corresponding original model without pricing.

The key observation with respect to subscription pricing is that, as long as the media firms obtain revenue from actual visits by subscribers, a paid subscription service does not fundamentally change the incentive to misreport as compared to what we saw in the previous sections. Indeed, once purchased, a subscription alters neither the marginal cost of access for the receivers nor the marginal revenue from visit for the media firm. As a result, the equilibrium behaviour of both senders and receivers is essentially the same as in the model without subscriptions (except for aspects related to subscription pricing and purchase). Furthermore, we will see that a monopolist media firm extracts all surplus from receivers via the subscription fee, while Bertrand competition pushes the subscription fee down to zero in the two-firm model.

We first extend the single firm model of Section 2. At the beginning of period 1, before S and the receivers learn their private information (ω_1 for S, and v_1's for the receivers), S offers a subscription at price p and each receiver decides whether to buy it or not. We assume for simplicity that this is the only point at which the receivers can buy a subscription. Unless a receiver purchases the subscription he cannot access m_t before choosing a_t in either period. As in the original model, every receiver observes m_t at the end of period t regardless of whether he acquired m_t before choosing a_t.[29]

Remark 1. *There exists an equilibrium in which S sets $p = p^* > 0$, every receiver buys the subscription, and the subsequent behaviour of S and receivers is the same as in the equilibrium described in Section 2. The receivers are indifferent between buying and not buying the subscription at the subscription price p^*.*

The relationship between the informativeness of reports and the (non-monetary) cost of access to news (as characterized in Propositions 1, 2 and 4) is thus unaffected by the introduction of a costly subscription.

One limitation of our argument above is that the subscription payment by assumption does not affect visiting costs. In practice, a subscription could lower the non-financial costs of accessing articles, for example through habit formation or becoming used to navigating the website. If a subscription did cause a reduction in w, this might result in lower informativeness and lower click-related revenue. While the subscription price would still be set so as to capture all the receiver surplus, the total surplus generated might as a result be lower, and a monopolist anticipating this might accordingly not consider it attractive to introduce a subscription scheme.

We now extend the two-firm model discussed in Section 3. At the beginning of period 1, before the two media firms and receivers learn their private information, the firms simultaneously offer subscriptions at respective prices p_1 and p_2 (Bertrand pricing) while each (ex ante identical) receiver decides whether to buy one subscription or none (we assume they cannot buy two, for simplicity). As earlier, this is the only point at which a receiver can buy a subscription. A receiver can only access a report from the firm that he subscribes to before choosing a_t. Each receiver observes $m_t^{S_1}$ and $m_t^{S_2}$ at the end of period t, regardless of whether he acquired a report before choosing a_t. Reports can thus be cross-checked at the end of each period.

Remark 2. *There exists a symmetric equilibrium in which, on the equilibrium path, the firms charge a subscription price of zero, every receiver buys a subscription with probability $\frac{1}{2}$ from each firm, and the subsequent behaviour of the firms and the receivers is the same as in the equilibrium described in Section 3.*

As a technical observation, note that the symmetric equilibrium that we describe above still exists if we allow readers to purchase two subscriptions, as long as they can only visit one firm in each period. While the subscriptions are free, the receivers are indifferent between holding one and two subscriptions. Allowing receivers to visit multiple outlets in period t would require the calculation of the marginal value of a second visit, which is complex and beyond the scope of the current extension.

5. Conclusion

The driving force behind information distortion in our model is a media firm's incentive to generate uncertainty about tomorrow's state in order to encourage future clicks by potential readers. We have found that for an intermediate cost level, a higher cost of access increases the informativeness of communication by dampening a reader's responsiveness to reports that generate uncertainty and thereby discouraging the media firm from distorting its private information. Moreover, in this parameter region, both the media firm and the readers might be better off with a higher cost of access. We argue that this may explain why there is a widespread concern that media reports have become less accurate in recent years. According to our model, this would be an indirect consequence of the reduced cost of access stemming from the rise of online news.

[29] The proofs of the following Remarks are in Appendix A.

The main insights of our paper appear to extend beyond the context of media. Similar incentives to misreport private information may well be present in many repeated relationships between an uninformed party and an expert who sells his advice over time (for example doctor–patient relations, or financial advisers, etc.).

Declaration of competing interest

The authors declare that they no relevant or material financial interests that relate to the research described in this paper.

Data availability

No data was used for the research described in the article.

Appendix A. Proofs

For expositional convenience, the Appendix is organized as follows: we first prove Proposition 2 for the intermediate range of w so that the structure of the equilibrium in the single firm model is clearly understood and presented. We then the give proofs of the other two Propositions for equilibrium communication, namely Proposition 1 for the low range, and Proposition 4 for the high range of w. They are followed by the proof of Proposition 3 on equilibrium payoffs for the intermediate range of w. The other proofs are presented in the order of appearance in the main text.

A.1. Proposition 2

Before proving Proposition 2, we present a series of lemmas in order to narrow down the class of strategy profiles we need to consider for the derivation of an equilibrium with informative communication in period 1.

A.1.1. Preliminary observations for Proposition 2

Lemma 1. *In any equilibrium S reports truthfully in period 2.*

Proof. In period 2, S has a strict incentive to report truthfully (i.e. to send $m_2 = \omega_2$) in any putative equilibrium. Since it is the last period, there is no incentive to induce a future visit by misreporting, and sending $m_2 = \omega_2$ yields the truth-telling benefit. □

Lemma 1 allows us to focus our equilibrium analysis on the informativeness of the report in period 1.

Lemma 2. *(a) No equilibrium features uninformative communication in period 1. (b) No equilibrium features fully informative communication in period 1.*

Proof. Step 1 The proof of part (a) is by contradiction. There are two cases of uninformative communication in period 1 to consider. The first is the case where both $m_1 = A$ and $m_1 = B$ are sent with strictly positive probability. In this case, both $m_1 = A$ and $m_1 = B$ yield the same probability of consultation in period 2 since both are uninformative. Since $R's$ consultation behaviour in period 2 is independent of m_1, S is strictly better off reporting truthfully with probability 1, as this yields the truth-telling benefit. Therefore, this case is excluded.

The second case is such that either $m_1 = A$ or $m_1 = B$ is sent with ex ante probability 1 in period 1. Assume that S always sends the same message $\widetilde{m}_1 = J \in \{A, B\}$ in period 1. Now, the out-of-equilibrium belief in period 1 that maximizes S's incentive to choose $\widetilde{m}_1$ in period 1 is that given $m'_1 \neq \widetilde{m}_1$, R assigns probability 1 to $\omega_1 = A$. Consider thus a putative equilibrium in which there is some $J \in \{A, B\}$ such that S always sends message $\widetilde{m}_1 = J$ in period 1 and if R observes $m'_1 = -J \equiv \{A, B\} \setminus J$ in period 1, he assigns probability 1 to $\omega_1 = A$. We now show that in such a putative equilibrium, S has a deviation incentive in period 1 when $\omega_1 = -J$.

Let $\omega_1 = -J$. Note that sending $m'_1 = -J$ yields the expected payoff $2fx$ across the two periods if $w > 1 - \gamma$, as implied by (1). Indeed, S obtains the truth-telling payoff in period 1, and anticipates that he will receive it also in period 2, although R will not consult in period 2. On the other hand, sending $\widetilde{m}_1 = J$ yields

$$f(1 + \theta - 2\theta\gamma - 2w) + fx.$$

Note that $(1 + \theta - 2\theta\gamma - 2w)$ is positive and denotes the probability of visit in period 2, given that R's belief about ω_1 remains unchanged (i.e. R considers that he has not learned anything concerning the state in period 1). The second term fx in the above expression is the expected truth-telling payoff in period 2. Now,

$$fx > f(1 + \theta - 2\theta\gamma - 2w)$$

rewrites as

$$w > \frac{1-x}{2} - \frac{\theta(2\gamma - 1)}{2},$$

as assumed in (1). It follows that in this putative equilibrium, S has a strict incentive to deviate and send $m'_1 = -J$ in period 1 if $\omega_1 = -J$. We conclude that there exists no equilibrium such that either $m_1 = A$ or $m_1 = B$ is sent with ex ante probability 1.

Step 2 The proof of part (b) is also by contradiction and there are again two cases to consider. Consider first the case in which S's equilibrium strategy in period 1 is such that $m_1 = \omega_1$ for any ω_1. In such an equilibrium, the assumption that w is in the intermediate range (1) implies that R consults with probability 0 in period 2 after $m_1 = A$ and with positive probability after $m_1 = B$. Let us now show that S then strictly prefers to deviate to $m_1 = B$ given $\omega_1 = A$. Let $\omega_1 = A$. Sending $m_1 = A$ leads to a visiting probability of zero in period 2, and thus yields an expected payoff of fx. In contrast, $m_1 = B$ leads to a strictly positive probability of visit in period 2, namely probability $1 - 2w$. Sending $m_1 = B$ thus yields expected payoff $f(1 - 2w)$. Now, note that $f(1 - 2w) > fx$ is equivalent to $\frac{1-x}{2} > w$, which is consistent with the assumption on w given in (1). S thus has a strict incentive to deviate to $m_1 = B$ when $\omega_1 = A$, so the equilibrium breaks down.

Consider next the case in which S's equilibrium strategy in period 1 is such that $m_1 = A$ if $\omega_1 = B$ and $m_1 = B$ if $\omega_1 = A$. In this case, the incentive to deviate given $\omega_1 = A$ is further reinforced, relative to the first case. That is, deviating to $m_1 = A$ given $\omega_1 = A$ indeed not only yields a positive probability of consultation tomorrow (as opposed to $m_1 = B$), but the deviation also yields the truth-telling benefit. Hence the putative equilibrium breaks down. Hence we conclude that there cannot be an equilibrium with fully informative communication in period 1. □

The intuition for part (a) is that in a putative equilibrium with uninformative communication in period 1, the report in period 1 should not have any influence on R's decision to visit in period 2. However, given that the report has no influence, S would strictly prefer to reveal truthfully in period 1 for the truth-telling benefit. The intuition for part (b) is that if x is small and communication is fully informative in period 1, then S must have an incentive to deviate in the period to the message indicating state B when the state is A, in order to induce a future visit. Our next two Lemmas concern R's consultation behaviour in equilibrium.

Lemma 3. *No equilibrium features zero probability of consultation in period 2.*

Proof. The proof is by contradiction. Assume first that the probability of consultation in period 2 is zero, and that both messages ($m_1 = A$ and $m_1 = B$) are sent with strictly positive probability in equilibrium. In this case, m_1 does not affect R's consultation decision on-the-equilibrium path, since the probability of visit in period 2 remains zero. This implies that S is always strictly better off sending $m_1 = \omega_1$ in period 1, as this yields the truth-telling benefit. Given truth-telling in period 1, the probability of visit in period 2 after $m_1 = B$ is $1 - 2w > 0$, which contradicts the assumption that the probability of consultation in period 2 is zero.

Second, assume that the probability of consultation in period 2 is zero, and that either $m_1 = A$ or $m_1 = B$ is sent with probability 1. However, this implies an equilibrium with uninformative communication in period 1, which we have ruled out in Lemma 2.

Thus we conclude that the probability of consultation in period 2 is strictly positive in equilibrium. □

Lemma 3 implies that, in equilibrium, some message in period 1 has to lead to consultation with positive probability in period 2. This opens up the possibility that one of the messages available to S in period 1 makes consultation in period 2 more likely than the other, thereby creating an incentive to send the former message in order to maximize the probability of consultation in period 2.

The next Lemma demonstrates that R's consultation decision in period 2 is affected significantly by m_1, if S's communication strategy for period 1 is what we call a *simple* communication strategy.

Lemma 4. *In any equilibrium featuring a simple communication strategy in period 1, R consults with probability 0 in period 2 if $m_1 = A$, and with strictly positive probability if $m_1 = B$.*

Proof. Consider an equilibrium in which S's strategy in period 1 is a simple communication strategy. From Lemma 2, any equilibrium features an informative communication strategy in period 1. Therefore, we must have $\tau_A^1 > 0$. This implies that both $m_1 = A$ and $m_1 = B$ are sent with strictly positive probability in period 1. We also know from Lemma 3 that any equilibrium must feature a positive probability of consultation in period 2. In the following we consider R's consultation behaviour in period 2 when $m_1 = A$ and $m_1 = B$, respectively.

First, if $m_1 = A$ under a simple communication strategy, R believes $P(\omega_1 = A \mid m_1 = A) = 1$. Note that if R does not consult in period 2, then he chooses action A in the period. Consulting in period 2 guarantees the correct action (as S reports truthfully) but comes at the cost $w+v_2$. Thus R consults in period 2 if and only if $1-w-v_2 \geq P(\omega_2 = A \mid m_1 = A) = \gamma$. The assumption that w is in the intermediate range (1) implies $w+\gamma > 1$, which then implies that R consults in period 2 with probability 0 after $m_1 = A$.

Let $m_1 = B$. We know from Lemma 3 that any equilibrium must feature a strictly positive probability of consultation in period 2. We furthermore know that R consults with probability 0 after $m_1 = A$. It follows that R must consult with strictly positive probability after $m_1 = B$. □

We see that R never visits in period 2 if he is sure that $\omega_1 = A$ (entailing low uncertainty about ω_2), which is the case here given $m_1 = A$.

Our last Lemma for Proposition 2 establishes that the equilibrium communication strategy in period 1 must indeed be a simple communication strategy, which implies that focusing on simple strategies in both periods is without loss of generality.

Lemma 5. *In any equilibrium that features a positive probability of consultation in period 2, the communication strategy in period 1 is a simple communication strategy.*

Proof. The proof is organized as follows. We show by contradiction that in equilibrium, $m_1 = B$ must induce a weakly higher probability of consultation in period 2 than $m_1 = A$. In turn, we show that this immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$.

Recall that the report is truthful in period 2 in any equilibrium by Lemma 1. Lemma 2 states that any equilibrium involves informative communication in period 1. If the report is informative in period 1, there is some $J \in \{A, B\}$ such that $m_1 = J$ shifts R's posterior towards $\omega_1 = B$ more than $m_1' = -J$ does, so that R is (weakly) more likely to visit in period 2 after observing $m_1 = J$ than $m_1' = -J$.

Assume that $m_1 = A$ induces a weakly higher probability of consultation in period 2 than $m_1 = B$. If this is the case, then S reports $m_1 = A$ with probability 1 when $\omega_1 = A$, since by doing so S obtains both the truth-telling benefit and induces a weakly higher probability of consultation in period 2 than by sending $m_1 = B$. However, if an equilibrium is such that S sends $m_1 = A$ for sure whenever $\omega_1 = A$, then the report $m_1 = B$ induces a strictly higher posterior probability that the state is $\omega_1 = B$ than $m_1 = A$. Indeed, note that if $\tau_A^1 = 1$, then

$$P(\omega_1 = B \mid m_1 = A) = \frac{(1-\theta)(1-\tau_B^1)}{(1-\theta)(1-\tau_B^1)+\theta} < 1$$

while

$$P(\omega_1 = B \mid m_1 = B) = \frac{(1-\theta)\tau_B^1}{(1-\theta)\tau_B^1+\theta(0)} = 1.$$

Therefore, the report $m_1 = B$ must induce a higher probability of consultation in period 2 than $m_1 = A$, but this contradicts the assumption that $m_1 = A$ induces a weakly higher probability of consultation. We have thus shown by contradiction that $m_1 = B$ must induce a strictly higher probability of consultation in period 2 than $m_1 = A$. Note furthermore that this fact immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$, since by doing so S obtains both the truth-telling benefit and induces a strictly higher probability of consultation in period 2 than by sending $m_1 = A$. □

A.1.2. Proposition 2

Proof. Part (a) We know from the previous Lemmas that if there exists an equilibrium, it satisfies the following description. It features a simple partially informative communication strategy in period 1 (Lemmas 2 and 5) and the fully informative simple communication strategy in period 2 (Lemma 1). Furthermore, R consults with strictly positive probability in period 2 if $m_1 = B$ and with probability 0 if $m_1 = A$ (Lemmas 3 and 4). In the following, we show that there exists a unique equilibrium satisfying the above description, and characterize the value of τ_A^1 as well as the consultation probability in period 2 given $m_1 = B$. We first derive the probability of visit by R in period 2 when $m_1 = B$. Given this probability, we consider S's choice between reporting truthfully ($m_1 = A$) and misreporting ($m_1 = B$) when $\omega_1 = A$. We then examine the conditions under which there exists an equilibrium of the type described above. We show that it must be unique, if it exists, and obtain the explicit solution for τ_A^1.

Assume in what follows that S uses a simple strategy in both periods, where the strategy in period 2 is truth-telling. Note that regardless of the message in period 1, if R does not visit in period 2, he chooses $a_2 = A$ in period 2. Therefore, given $m_1 = B$, a visit by R in period 2 requires

$$1 - w - v_2 > P(\omega_1 = A \mid m_1 = B)\gamma + P(\omega_1 = B \mid m_1 = B)\frac{1}{2}, \tag{9}$$

where $P(\omega_1 = B \mid m_1 = B) = \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}$. Hence, (9) is equivalent to

$$1 - w - v_2 > \left(1 - \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\right)\gamma + \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\frac{1}{2},$$

which rewrites as

$$v_2 < \frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w.$$

Thus, given $m_1 = B$, R visits in period 2 with positive probability if and only if

$$\frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w > 0. \tag{10}$$

Let

$$\varphi_B(\tau_A^1, w, \gamma) \equiv \frac{\frac{\theta-2\theta\gamma-2\theta\tau_A^1+2\theta\gamma\tau_A^1+1}{2\left(1-\theta\tau_A^1\right)} - w}{1/2} = \frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{1-\theta\tau_A^1} - 2w. \tag{11}$$

If (10) holds, then conditional on $m_1 = B$, the probability that R visits in period 2 is given by

$$P(d_2 = 1 \mid m_1 = B) = \max\left\{0, \varphi_B(\tau_A^1, w, \gamma)\right\}.$$

Note that $\varphi_B(\tau_A^1, w, \gamma) \le 1$ for any $\tau_A^1 \in [0,1]$. This is because the marginal value of visit, ignoring the cost of access, is at most $\frac{1}{2}$.

Next, let us examine the incentives of S in period 1. Recall that S reports truthfully when $\omega_1 = B$. Suppose instead that $\omega_1 = A$. Sending $m_1 = A$ yields the truth-telling benefit fx but zero probability of visit in period 2. Therefore, given $\omega_1 = A$, S is indifferent between $m_1 = A$ and $m_1 = B$ in period 1 if and only if $fx + 0 = 0 + f\varphi_B(\tau_A^1, w, \gamma)$, which is equivalent to

$$\varphi_B(\tau_A^1, w, \gamma) = x. \tag{12}$$

Note that $\varphi_B(\tau_A^1, w, \gamma)$ is strictly increasing in τ_A^1, since we have

$$\frac{\partial \varphi_B(\tau_A^1, w, \gamma)}{\partial \tau_A^1} = \frac{\theta(1-\theta)(2\gamma-1)}{\left(1-\theta\tau_A^1\right)^2} > 0. \tag{13}$$

This implies that there exists a unique solution $\tau_A^1 \in (0,1)$ to (12), if $\varphi_B(1, w, \gamma) > x$ and $\varphi_B(0, w, \gamma) < x$. Note that $\varphi_B(1, w, \gamma) > x$ rewrites as $w < \frac{1-x}{2}$, whereas $\varphi_B(0, w, \gamma) < x$ rewrites as $w > \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}$. Both of these inequalities are consistent with the assumption that w is in the intermediate range (1). Thus, there exists a unique equilibrium in which S uses a simple strategy in both periods, with the strategy in period 2 being truth-telling and the strategy in period 1 satisfying (12), if $w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right)$. Solving (12) for τ_A^1, we obtain the unique solution

$$\tau_A^{1*} = \frac{2w + \theta(2\gamma-1) + x - 1}{\theta(2\gamma + 2w + x - 2)}.$$

We have an equilibrium with $\tau_A^{1*} \in (0,1)$ for $w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right)$, as stated.

Part (b) Recall that $\gamma > \frac{1}{2}$ and that the assumption $w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right)$ implies $1 - x - 2w > 0$. We have

$$\frac{\partial \tau_A^{1*}}{\partial f} = 0,$$

$$\frac{\partial \tau_A^{1*}}{\partial w} = \frac{2(2\gamma-1)(1-\theta)}{\theta(2\gamma+2w+x-2)^2} > 0,$$

$$\frac{\partial \tau_A^{1*}}{\partial \gamma} = \frac{2(1-x-2w)(1-\theta)}{\theta(2\gamma+2w+x-2)^2} > 0,$$

$$\frac{\partial \tau_A^{1*}}{\partial x} = \frac{(2\gamma-1)(1-\theta)}{\theta(2\gamma+2w+x-2)^2} > 0. \quad \square$$

A.2. Proposition 1

Proof. Outline: Recall that throughout we assume $x < (1-\theta)(2\gamma-1)$, which is equivalent to $1 - \gamma < \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}$.

Step 1 establishes the following three useful properties of equilibrium behaviour. First, any equilibrium must feature truth-telling in period 2 regardless of the state. Second, any equilibrium must feature a positive probability of consultation in period 2. Third, any equilibrium with informative communication in period 1 must feature a *simple* communication strategy, that is, if $m_1 = B$ then $\omega_1 = B$.

Step 2 shows that if $w \in \left[0, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$ there exists no equilibrium with truth-telling in both periods.

Step 3 examines the case where $w \in [0, 1-\gamma)$ and shows that there exists no equilibrium that features an informative simple communication strategy in period 1.

Step 4 examines the case where $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$ and shows the same result as in Step 3 holds.

Step 5 shows that there exists an equilibrium that features uninformative communication such that $m_1 = B$ regardless of the state in period 1 and truth-telling in period 2.

Step 6 summarizes.

Step 1 The fact that any equilibrium must feature truth-telling in period 2 regardless of the state follows from the same argument as that of Lemma 1. The fact that any equilibrium must feature a positive probability of consultation in period 2 follows from the same argument as in the proof of Lemma 3.

We now show that any equilibrium with informative communication in period 1 must feature a simple communication strategy, that is, truth-telling in period 1 if $\omega_1 = B$. We invoke the same argument as in the proof of Lemma 5 and we repeat it for convenience in what follows. We show by contradiction that in equilibrium, $m_1 = B$ must induce a weakly higher probability of consultation in period 2 than $m_1 = A$. In turn, we show that this immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$.

Suppose that the message in period 1 is informative in period 1. Then as we have seen in the proof of Lemma 5, there is some $J \in \{A, B\}$ such that $m_1 = J$ shifts R's posterior towards $\omega_1 = B$ more than $m_1' = -J$ does, so that R is (weakly) more likely to visit in period 2 after observing $m_1 = J$ than $m_1' = -J$.

Assume that $m_1 = A$ induces a weakly higher probability of consultation in period 2 than $m_1 = B$. If this is the case, then S reports $m_1 = A$ with probability 1 when $\omega_1 = A$, since by doing so S obtains both the truth-telling benefit and induces a weakly higher probability of consultation in period 2 than by sending $m_1 = B$. However, if an equilibrium is such that S sends $m_1 = A$ for sure whenever $\omega_1 = A$, then the report $m_1 = B$ induces a strictly higher posterior that the state is $\omega_1 = B$ than $m_1 = A$. Indeed, note that if $\tau_A^1 = 1$, then

$$P(\omega_1 = B \mid m_1 = A) = \frac{(1-\theta)(1-\tau_B^1)}{(1-\theta)(1-\tau_B^1) + \theta} < 1$$

while

$$P(\omega_1 = B \mid m_1 = B) = \frac{(1-\theta)\tau_B^1}{(1-\theta)\tau_B^1 + \theta(0)} = 1.$$

Consequently, if communication in period 1 is informative, then $m_1 = B$ must induce a higher probability of consultation in period 2 than $m_1 = A$, but this contradicts the assumption that $m_1 = A$ induces a weakly higher probability of consultation. We thus conclude that if communication in period 1 is informative, then $m_1 = B$ must induce a strictly higher probability of consultation in period 2 than $m_1 = A$. Furthermore, this fact immediately implies that, if communication in period 1 is informative, then S reports $m_1 = B$ with probability 1 when $\omega_1 = B$, since by doing so S obtains both the truth-telling benefit and induces a strictly higher probability of consultation in period 2 than by sending $m_1 = A$.

Step 2 Let us show that there exists no equilibrium that features truth-telling in period 1. Assume that there is an equilibrium with truth-telling in both periods. The visiting probabilities of R in period 2 satisfy the following. We have

$$P(d_2 = 1 \mid m_1 = A) = \frac{1-\gamma-w}{1/2} > 0$$

if $w < 1-\gamma$. Meanwhile, $P(d_2 = 1 \mid m_1 = A) = 0$ if $w \ge 1-\gamma$. On the other hand,

$$P(d_2 = 1 \mid m_1 = B) = \frac{\frac{1}{2} - w}{1/2}.$$

We now consider the strategy of S in period 1. Let $\omega_1 = B$, in which case S strictly prefers to report truthfully in period 1, as this yields the truth-telling benefit and also maximizes the visiting probability in period 2. Let $\omega_1 = A$. In period 1, the expected payoff from sending $m_1 = A$ is given by fx if $w \in \left[1-\gamma, \frac{1}{2} - x - \frac{\theta(2\gamma-1)}{2}\right]$ since the probability of visit in period 2 is zero, and the corresponding expected payoff for $w < 1-\gamma$ is given by

$$fx + 2(1-\gamma-w)f.$$

Meanwhile, the expected payoff when sending $m_1 = B$ is given by $(1-2w)f$. Therefore, when $w < 1-\gamma$, S strictly prefers to report $m_1 = B$ when $\omega_1 = A$ (i.e. deviating from truth-telling) if and only if

$$fx + 2(1-\gamma-w)f < (1-2w)f,$$

which reduces to $x < \frac{2\gamma-1}{2}$. This inequality is readily implied by our stated assumption on x, namely

$$x < (1-\theta)(2\gamma-1)$$

for $\theta \in [\frac{1}{2}, 1)$.

If on the other hand $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$, S strictly prefers sending $m_1 = B$ when $\omega_1 = A$ if and only if

$$fx < (1-2w)f,$$

which is equivalent to $x < 1-2w$ and holds true given $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$.

Thus from this step we conclude that no equilibrium features truth-telling in both periods.

Step 3 Let us focus on the case where $w \in [0, 1-\gamma)$. Consider a putative equilibrium with a simple communication strategy in period 1. R's visiting probability in period 2 after $m_1 = A$ is given by

$$P(d_2 = 1 \mid m_1 = A) = \frac{1-\gamma-w}{1/2}.$$

If $m_1 = B$, a positive probability of visiting in period 2 requires

$$1 - w - v_2 > P(\omega_1 = A \mid m_1 = B)\gamma + P(\omega_1 = B \mid m_1 = B)\frac{1}{2}.$$

Note that

$$P(\omega_1 = B \,|m_1 = B) = \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}.$$

Thus given $m_1 = B$, R visits in period 2 if and only if

$$1 - w - v_2 > \left(1 - \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\right)\gamma + \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\frac{1}{2},$$

which is equivalent to $\frac{\theta-2\theta\gamma-2\theta\tau_A^1+2\theta\gamma\tau_A^1+1}{2\left(1-\theta\tau_A^1\right)} - w > v_2$. Thus we have

$$P(d_2 = 1 \mid m_1 = B) = \frac{\frac{\theta-2\theta\gamma-2\theta\tau_A^1+2\theta\gamma\tau_A^1+1}{2\left(1-\theta\tau_A^1\right)} - w}{1/2},$$

provided that the numerator in the above expression is positive (note that the numerator is never larger than $\frac{1}{2}$).

We now examine S's choice of m_1. Assume that $\omega_1 = A$. Sending $m_1 = A$ yields the truth-telling benefit fx, and R visits with probability $\frac{1-\gamma-w}{1/2}$ in period 2. Meanwhile if $m_1 = B$, S obtains no truth-telling benefit but R visits with probability $\varphi_B(\tau_A^1, w, \gamma)$ given by (11) in period 2. S is thus indifferent between $m_1 = A$ and $m_1 = B$ in period 1 if and only if

$$fx + f\frac{1-\gamma-w}{1/2} = f\varphi_B(\tau_A^1, w, \gamma). \tag{14}$$

For the rest of the current step, we use a monotonicity argument to study the existence of an equilibrium in simple strategies that features $\tau_A^1 \in (0, 1]$. We have already noted in the proof of Proposition 2 Part (a) that $\varphi_B(\tau_A^1, w, \gamma) \le 1$. Note furthermore that $\varphi_B(\tau_A^1, w, \gamma)$ is a continuous function of τ_A^1 and from (13) strictly increasing in τ_A^1. Because of the continuity and monotonicity shown in (13), there exists a unique solution to the indifference condition (14) and hence an equilibrium with informative communication in period 1 if both

$$\varphi_B(1, w, \gamma) = 1 - 2w \ge x + \frac{1-\gamma-w}{1/2} \tag{15}$$

and

$$\varphi_B(0, w, \gamma) = 1 + \theta - 2w - 2\theta\gamma < x + \frac{1-\gamma-w}{1/2} \tag{16}$$

hold, since (14) requires

$$\varphi_B(\tau_A^1, w, \gamma) = x + \frac{1-\gamma-w}{1/2} \tag{17}$$

in equilibrium. We have already seen in Step 2 that (15) holds (i.e. there is no truth-telling equilibrium). However, the assumption $x < (1-\theta)(2\gamma-1)$ implies that, contrary to (16), we have

$$\varphi_B(0, w, \gamma) = 1 + \theta - 2w - 2\theta\gamma > x + \frac{1-\gamma-w}{1/2}, \tag{18}$$

which, together with (15) and the monotonicity shown in (13), implies that (17) never holds since we have

$$\varphi_B(\tau_A^1, w, \gamma) > x + \frac{1-\gamma-w}{1/2}$$

for any $\tau_A^1 \in [0, 1]$. Thus we conclude that if $w < 1-\gamma$ there is no equilibrium that features informative communication in period 1.

Step 4 In this step, we consider $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$, in which case R never visits in period 2 if he assigns probability 1 to $\omega_1 = A$. We apply the proof of Proposition 2. The proof shows that given $w \ge 1-\gamma$, an equilibrium that features a simple communication strategy such that $\tau_A^1 \in (0, 1]$ exists only if

$$w > \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}.$$

The inequality clearly does not hold for the range of w we consider and there is thus no equilibrium that features informative communication in period 1.

Step 5 We here show that there exists an equilibrium that features uninformative communication in period 1 and truth-telling in period 2. The equilibrium is constructed as follows: S always reports $m_1 = B$ in period 1, and if R receives the off-the-equilibrium message $m_1 = A$ in period 1, he assigns probability 1 to $\omega_1 = A$. We now check that S has no incentive to deviate from $m_1 = B$ in period 1.

Let us focus on $\omega_1 = A$. Note that (18) implies that if $w < 1-\gamma$, S is strictly better off reporting $m_1 = B$ given R's off-the-equilibrium belief. In addition, (18) further implies that the same holds true for $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$, since the expected payoff from truth-telling is even lower than if $w < 1-\gamma$, now that the probability of visit given truth-telling ($m_1 = A$) is zero.

Clearly, when $\omega_1 = B$ there is no incentive to deviate from $m_1 = B$, as it yields both the truth-telling benefit and a positive probability of visit in period 2. Thus we conclude that there exists an equilibrium where S reports $m_1 = B$ regardless of the state.

Step 6 Using the observation that any equilibrium with informative communication in period 1 must feature a simple communication strategy (Step 1), we have now shown that if $w \le \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}$ and $x < (1-\theta)(2\gamma-1)$ as assumed, there exists no equilibrium with truth-telling in both periods (Step 2), and there exists no equilibrium in simple communication strategies that involve informative communication in period 1 (Steps 3 and 4) either. We then have shown that under the same assumptions, there exists an equilibrium where $m_1 = B$ regardless of the state in period 1 (Step 5). Hence we conclude that the equilibrium such that $\tau_A^1 = 0$ is the unique equilibrium. □

A.3. Proposition 4

Proof. Consider a putative equilibrium with truth-telling in both periods for $w \in \left[\frac{1-x}{2}, \frac{1}{2}\right)$. Substituting $\tau_A^1 = 1$ into $\varphi_B(\tau_A^1, w, \gamma)$, the visiting probability in period 2 given $m_1 = B$ is given by $\frac{\frac{1}{2}-w}{1/2} = 1-2w$, which is clearly positive. Given our assumptions, the visiting probability given $m_1 = A$ is on the other hand zero. We now examine the incentives of S. If $\omega_1 = A$, sending $m_1 = A$ yields the truth-telling benefit fx whereas $m_1 = B$ yields $(1-2w)f$. The truth-telling condition in this case, i.e. $fx \geq (1-2w)f$, is equivalent to $\frac{x}{2} \geq \frac{1}{2} - w$. Note that we are considering the case where $w \geq \frac{1-x}{2}$ or equivalently $\frac{x}{2} \geq \frac{1}{2} - w$. Thus we conclude that there is an equilibrium that features truth-telling in period 1 (as well as period 2) and a strictly positive probability of consultation in period 2.

The argument above implies that $\varphi_B(1, w, \gamma) \leq x$ and hence $\varphi_B(\tau_A^1, w, \gamma) < x$ for any $\tau_A^1 \in [0,1)$, since from (13) $\varphi_B(\tau_A^1, w, \gamma)$ is strictly increasing in τ_A^1. Thus the equilibrium with truth-telling in both periods is unique. □

A.4. Proposition 3

We present a proof for each part, namely Part 1.(a), Part 1.(b), and Part 2, separately below.

A.4.1. Proposition 3 : Part 1.(a)

Proof. **Step 1** Let r_1 denote the probability that R visits in period 1. We have $r_1 < 1$, since the upper bound of v_t is $\frac{1}{2}$, and furthermore the gross marginal benefit of visit is no larger than $\frac{1}{2}$.

Step 2 Let us explicitly derive r_1. Note that R, when deciding whether or not to visit in period 1, only considers his expected payoff in period 1, as he will observe m_1 at the end of period 1 whether or not he visits in the period. R's expected payoff for period 1 if he does not visit is $\theta = \frac{1}{2}$ as assumed. If R visits and $v_1 = 0$, then his expected payoff for period 1 is $\theta\tau_A^1 + (1-\theta) - w$. The increase in the payoff by visiting in period 1 is thus given by $\left(\theta\tau_A^1 + (1-\theta)\right) - w - \theta = \frac{1}{2}\tau_A^1 + (1-\frac{1}{2}) - w - \frac{1}{2}$. Therefore we have

$$r_1\left(w, x, \gamma, \frac{1}{2}\right) = P\left(v_1 \leq \frac{1}{2}\tau_A^1 + \left(1-\frac{1}{2}\right) - w - \frac{1}{2}\right) = \frac{\frac{1}{2}\tau_A^1 + (1-\frac{1}{2}) - w - \frac{1}{2}}{1/2}.$$

Substituting the equilibrium value of $\tau_A^1 = \tau_A^{1*} = \frac{2w+\theta(2\gamma-1)+x-1}{\theta(2\gamma+2w+x-2)}$ into the above, we have

$$r_1\left(w, x, \gamma, \frac{1}{2}\right) = -\frac{4w\gamma - 2x - 2\gamma - 8w + 2wx + 4w^2 + 3}{2w + x + 2\gamma - 2}.$$

Step 3 Differentiating r_1 with respect to w, we obtain

$$\frac{\partial r_1\left(w, x, \gamma, \frac{1}{2}\right)}{\partial w} = \frac{2(8w - 4w^2 - 4wx - 8w\gamma - x^2 - 4x\gamma + 4x - 4\gamma^2 + 10\gamma - 5)}{(2w + x + 2\gamma - 2)^2}.$$

Denote the expression in the parentheses in the numerator by

$$G \equiv 8w - 4w^2 - 4wx - 8w\gamma - x^2 - 4x\gamma + 4x - 4\gamma^2 + 10\gamma - 5.$$

Solving $G = 0$ for w, the unique relevant solution is given by $w = \frac{1}{2}\sqrt{2\gamma-1} - \gamma - \frac{1}{2}x + 1$. Now we have

$$\left.\frac{\partial G}{\partial w}\right|_{w=\frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1} = -4\sqrt{2\gamma-1} < 0$$

so that $\frac{\partial r_1\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w} > 0$ for $w < \frac{1}{2}\sqrt{2\gamma-1} - \gamma - \frac{1}{2}x + 1$. Note that $\frac{1-x}{2} < \frac{1}{2}\sqrt{2\gamma-1} - \gamma - \frac{1}{2}x + 1$, since

$$\left(\frac{1}{2}\sqrt{2\gamma-1} - \gamma - \frac{1}{2}x + 1\right) - \frac{1-x}{2} = \frac{1}{2}\sqrt{2\gamma-1} - \gamma + \frac{1}{2} > 0$$

as we assume $\gamma \in (\frac{1}{2}, 1)$. Therefore we have $\frac{\partial r_1\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w} > 0$ for $w < \frac{1-x}{2} < \frac{1}{2}\sqrt{2\gamma-1} - \gamma - \frac{1}{2}x + 1$. This implies that there are two possible cases. In the first case, there is $\hat{w} \in \left(\frac{1-x}{2} - \frac{(2\gamma-1)}{4}, \frac{1-x}{2}\right)$ such that $r_1 = 0$ when $w \leq \hat{w}$, and $r_1 \in (0,1)$ and r_1 is strictly increasing in w when $w > \hat{w}$. In the second case, for any $w \in \left(\frac{1-x}{2} - \frac{(2\gamma-1)}{4}, \frac{1-x}{2}\right)$, $r_1 \in (0,1)$ and r_1 is strictly increasing in w. We thus conclude that r_1 is weakly increasing in w, and if $r_1 > 0$ then r_1 is strictly increasing in w, as stated. □

A.4.2. Proposition 3 : Part 1.(b)

Proof. **Step 1** Solving $r_1\left(w, x, \gamma, \frac{1}{2}\right) = 0$ with respect to w, the unique relevant solution is given by $w = \frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4}$. Meanwhile, we have $r_1\left(\frac{1-x}{2}, x, \gamma, \frac{1}{2}\right) = x > 0$. This and $\frac{\partial r_1\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w} > 0$ for $w < \frac{1-x}{2}$, as shown above, imply $\frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4} < \frac{1-x}{2}$.

Step 2 We conclude that there exists some

$$w \in \left(\max\left\{\frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4}, 1-\gamma, \frac{1-x}{2} - \frac{(2\gamma-1)}{4}\right\}, \frac{1-x}{2}\right) \quad (19)$$

such that $r_1\left(w, x, \gamma, \frac{1}{2}\right) \in (0,1)$ in equilibrium, as stated.

Step 3 We know that R visits with positive probability in period 1 if w satisfies (19). In what follows we explicitly derive the expression for the expected payoff of R in period 1. Let K_1 denote the expected cost of access in period 1, conditional on R deciding to visit. We have

$$K_1 = w + E\left(v_2 \middle| v_2 \leq \left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}\right) = w + \frac{\left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}.$$

The expected consultation cost incurred in period 1 is

$$P(d_1 = 1)K_1 = \left(\frac{\left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{1/2}\right)\left(w + \frac{\left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}\right).$$

On the other hand, in equilibrium, the probability that R's action matches the state in period 1 is given by

$$P(\omega_1 = A)(1 - r_1) + P(\omega_1 = A)r_1[\tau_A^1(1) + (1-\tau_A^1)(0)] + P(\omega_1 = B)(1-r_1)(0) + P(\omega_1 = B)r_1(1).$$

Summarizing, R's expected payoff for period 1 is given by

$$\Pi_1^R(w, x, \gamma, \theta) = \frac{1}{2}(1 - r_1(w, x, \gamma, \theta)) + \frac{1}{2}\left(r_1(w, x, \gamma, \theta)\right)\tau_A^1(w, x, \gamma, \theta) + \frac{1}{2}r_1(w, x, \gamma, \theta) - \left(\frac{\left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{1/2}\right)\left(w + \frac{\left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}\right).$$

Step 4 Let us now consider the derivatives of this expected payoff in period 1. Substituting $\theta = \frac{1}{2}$ and $\tau_A^1 = \tau_A^{1*} = \frac{2w+\theta(2\gamma-1)+x-1}{\theta(2\gamma+2w+x-2)}$ into the above, we have the expression shown in Box I. Thus we obtain the derivative with respect to w given in Box II. It is easy to see that $(2w + x + 2\gamma - 2)^3 > 0$ under our assumptions. Let H be the expression in the large parentheses in Box II and find values of w that constitute roots of the expression ($H = 0$). The two only admissible roots are $w' \equiv \frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4}$ and $w'' \equiv \frac{1}{2}\sqrt{2\gamma-1} - \gamma - \frac{1}{2}x + 1$. As shown earlier, we have $w' < \frac{1-x}{2} < w''$. Also, we have

$$\left.\frac{\partial H}{\partial w}\right|_{w=w''} = -4(2\gamma-1)\left(x + 2\gamma - 2\sqrt{2\gamma-1}\right) < 0$$

$$\Pi_1^R\left(w,x,\gamma,\frac{1}{2}\right) = \frac{1}{4\left(2w+x+2\gamma-2\right)^2} \times \left(\begin{array}{c} 16w^4+16w^3x+32w^3\gamma-64w^3+4w^2x^2+16w^2x\gamma-48w^2x+16w^2\gamma^2-80w^2\gamma \\ +96w^2-8wx^2-24wx\gamma+52wx-16w\gamma^2+72w\gamma-64w+6x^2+16x\gamma-20x+12\gamma^2-28\gamma+17 \end{array}\right).$$

Box I.

$$\frac{\partial \Pi_1^R\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w} = \frac{1}{\left(2w+x+2\gamma-2\right)^3} \times \left(\begin{array}{c} 16w^4+24w^3x+48w^3\gamma-64w^3+12w^2x^2+48w^2x\gamma-72w^2x+48w^2\gamma^2-144w^2\gamma+96w^2 \\ +2wx^3+12wx^2\gamma-24wx^2+24wx\gamma^2-92wx\gamma+70wx+16w\gamma^3-88w\gamma^2+140w\gamma-64w \\ -2x^3-10x^2\gamma+11x^2-16x\gamma^2+40x\gamma-22x-8\gamma^3+32\gamma^2-40\gamma+15 \end{array}\right).$$

Box II.

since $\gamma \in (\frac{1}{2},1)$. Therefore, $H > 0$ for $w \in (w', w'')$, which implies $\Pi_1^R\left(w,x,\gamma,\frac{1}{2}\right)$ is increasing in w, as stated. □

A.4.3. Proposition 3 : Part 2

Proof. Let us define $E[k]$ as the expected number of times that S reports truthfully over the two periods. Given τ_A^1, we have $E[k] = \frac{1}{2}(\tau_A^1+1)+\frac{1}{2}(1+1)$. The accounting profit in period 2 is given by

$$\pi_2^S = fx\left(\frac{1}{2}(1-\tau_A^1)+\frac{1}{2}\right),$$

where $\frac{1}{2}(1-\tau_A^1)+\frac{1}{2}$ is the probability that $m_1 = B$, and x is the probability of visit in period 2 given $m_1 = B$ in equilibrium. Therefore, S's total payoff over two periods when there is no visit in period 1 is given by

$$\Pi^S = \pi_2^S + fxE[k] = fx\left(\frac{1}{2}(1-\tau_A^1)+\frac{1}{2}\right) + fx\left(\frac{1}{2}(\tau_A^1+1)+\frac{1}{2}(1+1)\right) = \frac{5fx}{2},$$

which is independent of w. In other words, when the probability of visit is zero in period 1, S's total payoff is unaffected by w.

Suppose that the probability of visit in period 1 is strictly positive. The accounting profit in period 1 in this case is given by

$$\pi_1^S = f\frac{\left(\frac{1}{2}\tau_A^1+\frac{1}{2}-w-\frac{1}{2}\right)}{1/2}.$$

Substituting the equilibrium $\tau_A^1 = \tau_A^{1*} = \frac{2w+\theta(2\gamma-1)+x-1}{\theta(2\gamma+2w+x-2)}$ into the above and differentiating it with respect to w, we have

$$\frac{\partial \pi_1^S}{\partial w} = 2f\left(\frac{2\gamma-1}{(2w+x+2\gamma-2)^2}-1\right) > 0,$$

since the expression in the large parenthesis is positive. To see this, note that $w < \frac{1-x}{2}$ and $\gamma \in (\frac{1}{2},1)$ imply $(2w+x+2\gamma-2)^2 < (2\gamma-1)^2 < 2\gamma-1 < 1$.

Thus we conclude that S's total expected payoff in equilibrium is unaffected by w when the probability of visit in period 1 is zero, and strictly increasing in w when the probability of visit in period 1 is positive. □

A.5. Preliminary observations for Propositions 5, 6, and 7

Before presenting the proofs of the Propositions, let us make two observations that we refer to repeatedly in the proofs. The first observation is that in any equilibrium the second mover follows the first mover's report in period 1. The second is the expected payoff of the sender who is not the second mover when misreporting given $\omega_1 = A$.

Lemma 6. *In any equilibrium in simple communication strategies, the second mover sends the same report as that of the first mover.*

Proof. Suppose $\omega_1 = A$, and consider first an equilibrium with $\tau^C \in (0,1)$. If the first mover S_i's report is $m_1^{S_i} = A$, then R's belief cannot be influenced by the second mover S_j's report given the (correct) belief that any contradiction in the reports implies $\omega_1 = A$, and thus the second mover is strictly better off sending the same report. If the first mover's report is $m_1^{S_i} = B$, the first mover is indifferent between the two reports, which implies that the second mover is strictly better off sending the same report $m_1^{S_j} = B$, since unlike the first mover's report, the second mover's report is never contradicted so that the probability of inducing visit in period 2 is strictly higher. If an equilibrium is such that $\tau^C = 0$, then the expected payoffs of the first mover and the second mover given $m_1^{S_i} = m_1^{S_j} = B$ are the same, because cross-checking never occurs and thus insofar as the first mover strictly or weakly prefers to report $m_1^{S_i} = B$, the second mover has no incentive to deviate. If the equilibrium is such that $\tau^C = 1$, the first mover reports $m_1^{S_i} = A$ with probability 1, and the argument above implies that the second mover is strictly better off reporting $m_1^{S_j} = A$.

Suppose $\omega_1 = B$. Both the first mover and the second mover are better off sending the same reports $m_1^{S_i} = m_1^{S_j} = B$ because of a higher probability of visit and the truth-telling benefit. □

Let us now consider the expected payoff of S_i who is not the second mover. From Lemma 6, (6) and (7), the expected payoff when $\omega_1 = A$ and S_i sends $m_1^{S_i} = B$ is given by

$$f\left(\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}(1-\tau^C)+\frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}\right)P(d_2=1 \mid m_1^{S_i}=m_1^{S_j}=B)\frac{1}{2}. \quad (20)$$

The fraction $\frac{1}{2}$ that appears at the end of the expression captures the fact that R visits one of the firms with equal probability, since both firms' reports are truthful in period 2. As in the single firm model of Section 2, $m_1^{S_i} = B$ triggers a positive probability of visit in period 2 but yields no truth-telling benefit. Meanwhile, if S_i reports truthfully then the expected payoff is given by fx, since $m_1^{S_i} = A$ induces zero probability of visit in period 2 but the truth-telling benefit.

A.6. Proposition 5

Proof. In what follows we prove that (i) there exists an equilibrium with uninformative communication in period 1; and that (ii) there does not exist an equilibrium with truth-telling in period 1. We do so separately for two cases, namely $w \in [0, 1-\gamma)$ and $w \in \left[1-\gamma, \frac{3-2\gamma-4x}{4}\right]$.
Case 1: $w \in [0, 1-\gamma)$

In this case, the conditional probability of visit in period 2 if one or both firms report A in period 1, where R believes $\omega_1 = A$, is strictly positive and given by

$$P\left(d_2 = 1 \mid \left\{m_1^{S_1}, m_1^{S_2}\right\} \in \{\{A,A\},\{A,B\},\{B,A\}\}\right) = \frac{1-\gamma-w}{1/2}.$$

Let us consider the existence of an equilibrium with uninformative communication in period 1 ($\tau^C = 0$). In what follows we compare the expected payoffs when reporting truthfully and misreporting, given $\omega_1 = A$ and assuming S_i is not the second mover. The expected payoff from S_i's viewpoint when misreporting ($m_1^{S_i} = B$) is written as

$$\begin{aligned} &f\left(\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}(1-\tau^C) + \frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}\right) P(d_2 = 1 \mid m_1^{S_i} = m_1^{S_j} = B)\frac{1}{2} + \\ &f\left(1 - \left(\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}(1-\tau^C) + \frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}\right)\right) P(d_2 = 1 \mid m_1^{S_i} = B, m_1^{S_j} = A)\frac{1}{2}, \end{aligned} \tag{21}$$

where $P(d_2 = 1 \mid m_1^{S_i} = m_1^{S_j} = B)$ is given in (6) and $P(d_2 = 1 \mid m_1^{S_i} = B, m_1^{S_j} = A) = \frac{1-\gamma-w}{1/2}$. Meanwhile, the expected payoff when reporting truthfully ($m_1^{S_i} = A$) is given by

$$f\frac{1-\gamma-w}{1/2}\frac{1}{2} + fx. \tag{22}$$

Substituting $\tau^C = 0$ into the payoff from misreporting (21) and the payoff from truth-telling (22), we see that the former is strictly larger than the latter if

$$x < \frac{2\gamma-1}{4}.$$

The above is equivalent to x being small enough that $1-\gamma < \frac{3-2\gamma-4x}{4}$, which we have assumed to guarantee that the intervals for the two cases within $w \in \left[0, \frac{3-2\gamma-4x}{4}\right]$ are well-defined. Thus we conclude that if $w \in [0, 1-\gamma)$ there exists an equilibrium that features uninformative communication in period 1.

We now study the existence of an equilibrium with truth-telling in period 1 ($\tau^C = 1$). Given $\omega_1 = A$, assuming S_i is not the second mover, consider the incentives of S_i. Substituting $\tau^C = 1$ into the payoff from misreporting (21) and comparing it to the truth-telling payoff (22), we can see that truth-telling cannot be supported in equilibrium if

$$\beta < \frac{(2\gamma-1)-2x}{(2\gamma-1)+2x},$$

which is equivalent to $1-\gamma < \frac{1}{2} - \frac{x(1+\beta)}{1-\beta}$ as already assumed in (8). Thus there is no equilibrium with truth-telling in period 1.
Case 2: $w \in \left[1-\gamma, \frac{3-2\gamma-4x}{4}\right]$

In this case, the conditional probability of visit in period 2 if one or both firms report A, where R believes $\omega_1 = A$, is zero. Let us study the existence of an equilibrium with uninformative communication in period 1 ($\tau^C = 0$). Consider the incentives of an individual firm. Given $\omega_1 = A$, assuming S_i is not the second mover, his expected payoff from $m_1^{S_i} = A$ is given by fx, since R assigns probability 1 to $\omega_1 = A$ and thus the probability of visit in period 2 is zero. On the other hand S_i's expected payoff from $m_1^{S_i} = B$ is given by (20). Substituting $\tau^C = 0$ into (20), we have $\frac{f}{4}(3-2\gamma-4w)$, which is larger than or equal to the truth-telling payoff fx, from $w \leq \frac{3-2\gamma-4x}{4}$. Thus S_i prefers to misreport, which is consistent with $\tau^C = 0$, and hence there is an equilibrium that features uninformative communication in period 1.

We now study the existence of an equilibrium with truth-telling in period 1 ($\tau^C = 1$). Given $\omega_1 = A$, assuming S_i is not the second mover, consider the incentives of S_i. Substituting $\tau^C = 1$ into the expected payoff from misreporting (20) we have $\frac{f(1-2w)(1-\beta)}{2(1+\beta)}$, which is strictly larger than the truth-telling payoff fx if $w < \frac{1}{2} - \frac{x(1+\beta)}{1-\beta}$. Since $w \leq \frac{3-2\gamma-4x}{4}$ implies $w < \frac{1}{2} - \frac{x(1+\beta)}{1-\beta}$ from (8), S_i strictly prefers to report $m_1^{S_i} = B$. Thus there is no equilibrium with truth-telling in period 1. □

A.7. Proposition 6

Proof. In an equilibrium in simple symmetric communication strategies such that $\tau^C \in (0,1)$, if S_i is not the second mover he must be indifferent between $m_1^{S_i} = A$ and $m_1^{S_i} = B$ when $\omega_1 = A$. From (20) and the truth-telling benefit fx, the indifference condition is given by

$$\left(\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}(1-\tau^C) + \frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}\right) P(d_2 = 1 \mid m_1^{S_i} = m_1^{S_j} = B)\frac{1}{2} = x \tag{23}$$

after cancelling out f from both sides. If $\tau^C \in (0,1)$ satisfies (23), then it corresponds to a mixed strategy symmetric equilibrium stated in Proposition 6. The LHS is decreasing in w. The equality is satisfied when $\tau^C = 0$ and $w = \frac{3-2\gamma-4x}{4}$, which implies that there exists an equilibrium with $\tau^C = 0$. The equality is also satisfied when $\tau^C = 1$ and $w = \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}$, in which case there exists an equilibrium with $\tau^C = 1$. Focus on $w \in \left(\frac{3-2\gamma-4x}{4}, \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}\right)$ as stated in the Proposition. The interval ensures that the LHS of (23) is strictly larger than x for $\tau^C = 1$ and strictly smaller than x for $\tau^C = 0$. Since the LHS is continuous in τ^C, by the intermediate value theorem there must be at least one interior value of $\tau^C \in (0,1)$ for which the indifference condition is satisfied.

The LHS of (23) is a complex function of β and τ^C, and as a result explicit solutions for τ^C are also complex. Thus let us proceed with a qualitative argument. We know that at the highest interior solution of (23) for $\tau^C \in (0,1)$, the LHS expression must cross x from below. Accordingly, the fact that the LHS is decreasing in w immediately implies that the highest interior solution for τ^C must be increasing in w.

Moreover, the existence of the equilibrium with $\tau^C = 0$ at $w = \frac{3-2\gamma-4x}{4}$ and the truth-telling equilibrium at $w = \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}$ together imply that for the intermediate range of $w \in \left(\frac{3-2\gamma-4x}{4}, \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}\right)$ there exists no equilibrium that features $\tau^C = 0$ or $\tau^C = 1$. □

A.8. Proposition 7

Proof. If the truth-telling benefit fx is larger than or equal to (20) given $\tau^C = 1$, then there exists a symmetric truth-telling equilibrium. Accordingly, solving the inequality with respect to w, we obtain $w \geq \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}$ as stated. □

A.9. Remark 1

Proof. In the equilibrium strategy profile in the Remark, if S deviates to $p < p^*$ off the equilibrium, then all players still subsequently behave as in Section 2. If S instead deviates to $p > p^*$, the receiver acquires no subscription and S reports truthfully. In what follows we show that the strategy profile is sequentially rational.

Step 1 Let p^* be such that an individual receiver is indifferent between playing the equilibrium described in 2 after purchasing the subscription at price p^*, and not purchasing the subscription while S reports truthfully in both periods. Note that if a receiver does not purchase the subscription, despite truthful reports in both periods, his expected payoff is the same as in the case where he would not observe a report in either period, since he chooses $a_1 = A$ in period 1 (recall $\theta \in [\frac{1}{2}, 1)$) and $a_2 = A$ in period 2 regardless of whether $m_1 = A$ or $m_1 = B$. That is, the report the receiver observes at the end of period 1 is irrelevant to his action or payoff. The following argument shows that the value of p that achieves this indifference described above, and is positive and unique.

Note that if a receiver purchases the subscription at $p = 0$ and S's strategies are the same as those in Section 2, the receiver's expected payoff is by construction strictly larger than the expected payoff when he does not purchase the subscription. This is because the subscription itself is free and R visits voluntarily with positive probability (recall that the report in period 2 is truthful), at a cost that is furthermore always strictly smaller than the marginal value of a visit. Therefore we must have $p^* > 0$ to make the receiver indifferent between purchasing the subscription and not purchasing.

Step 2 We now verify that players' strategies are sequentially rational. Note first that the subscription fee, once paid, alters neither the receiver's marginal cost of access $w + v_t$ nor S's marginal benefit from a visit. Therefore, once the subscription fee has been paid, incentives of both S and the receiver are identical to those arising in the single firm model in Section 2.

Step 3 Consider first the receiver's incentives. If $p \le p^*$, recall that a receiver purchases the subscription while his subsequent decision to visit in both periods is as described in Section 2. If the receiver does not purchase the subscription, he cannot visit in either period 1 or 2. If $p \le p^*$, the receiver's expected deviation payoff when he does not buy the subscription (in which case the accuracy of the report is as described in Section 2 since this individual deviation does not affect S's incentives) is the same as the hypothetical expected payoff that the receiver would obtain if he chose not to buy the subscription and S reveals truthfully. This is because, as noted earlier, the report the receiver observes at the end of period 1 is irrelevant to his action or payoff if he never visits in period 2. Thus if $p \le p^*$, an individual receiver always weakly prefers buying the subscription to not buying. If instead $p > p^*$, then by construction (given the definition of p^*) the receiver prefers not to purchase the subscription.

Step 4 Consider now S's incentives. Note first that if no receiver has purchased the subscription, S strictly prefers to report truthfully in both periods and thereby obtain the truth-telling benefit, since there is no incentive for S to misreport. The following argument shows that S is strictly better off setting $p \le p^*$ (where all receivers purchase the subscription) than $p > p^*$ (where no receiver purchases the subscription and S reports truthfully in both periods). If S sets $p > p^*$ and no subscription is sold, then S has no visit in either period but enjoys the truth-telling benefit in both periods. Suppose instead that S sets $p \le p^*$ and all receivers purchase the subscription. If $\omega_1 = B$, then S subsequently enjoys the truth-telling benefit in period 1 and the revenue through visits from a fraction of the receivers in period 2. If instead $\omega_1 = A$, S has to choose between the truth-telling benefit (from $m_1 = A$) and revenue from visits (from $m_1 = B$) in period 2, but S's conditional expected payoff in equilibrium must be equal to (when $\tau_A^1 \in (0,1)$ and $\tau_A^1 = 1$ in equilibrium) or larger than (when $\tau_A^1 = 0$ in equilibrium) the truth-telling benefit only. We thus conclude that S has no incentive to deviate to $p > p^*$. Finally, simply note that among any price $p \le p^*$, S strictly prefers higher p. S's optimal choice of subscription price is thus $p = p^*$. □

A.10. Remark 2

Proof. In the equilibrium strategy profile that we consider, if one firm deviates to a positive subscription price off the equilibrium, all receivers buy a subscription from the other firm and both firms communicate truthfully.

Step 1 We now verify that the assumed strategies are sequentially rational. Consider first the firms' incentives *after* firm i unilaterally deviates to $p_i > 0$ from $p_i = p_j = 0$. Suppose that given the deviation all receivers purchase a subscription from firm j, so that firm i sells no subscription. Then after the deviation, firm i has no incentive to induce future visits and strictly prefers to report truthfully in both periods to obtain the truth-telling benefit. In addition, given that firm i reports truthfully, firm j has no incentive to deviate from truth-telling in period 1 since any misreporting by firm j must be exposed through firm i's report.

Consider now firm i's incentive to unilaterally deviate to $p_i > 0$. After the deviation, firm i sells no subscription and thus receives no visits but obtains only the truth-telling benefit in both periods. Suppose instead that firm i sets $p_i = 0$ and sells its subscription to half of the receivers. If $\omega_1 = B$, then firm i subsequently enjoys the truth-telling benefit in period 1 and the revenue from visits in period 2. If instead $\omega_1 = A$, firm i has to choose between the truth-telling benefit (from $m_1^{S_i} = A$) and the revenue from visits (from $m_1^{S_i} = B$) in period 2, but firm i's conditional expected payoff in equilibrium must be equal to (when $\tau^C \in (0,1)$ and $\tau^C = 1$) or larger than (when $\tau^C = 0$) the truth-telling benefit only. We thus conclude that firm i has no incentive to deviate to $p_i > 0$, given that the receivers do not purchase firm i's subscription at that price.

Step 2 Finally, consider now each individual receiver's incentives after firm i unilaterally deviates to $p_i > 0$. Note that both firms now communicate truthfully (and thus identically), so that any receiver prefers to buy a subscription from firm j at $p_j = 0$. Consider a receiver's incentives on the equilibrium path, after he buys a subscription from one of the firms at $p_i = p_j = 0$. Clearly, the incentives regarding the decision whether to visit on this equilibrium path are the same as in the equilibrium described in Section 3. Also, each receiver is strictly better off buying a subscription at $p_i = p_j = 0$, since the subscription is free and visiting is voluntary and occurs only when it increases his expected payoff. We conclude that price competition for subscribers leads to zero price, and we can directly resort to our analysis in Section 3. □

Appendix B. Additional discussions and calculations

B.1. Small expected random cost of access in the single firm model

We have assumed that the random cost of access v_t is uniformly distributed on $(0, \frac{1}{2})$. We now allow the upper bound of the support of v_t, which we denote by c, to be smaller than $\frac{1}{2}$. As w and c become very small, the expected cost of access thus tends to 0. We will demonstrate below that if both w and c are very low, there exists a truth-telling equilibrium and it is unique. Although assuming an arbitrarily low cost of accessing media reports is not plausible with respect to our motivation and applications, the result is intuitive. If the overall cost of access is extremely low in expectation, R should access reports regardless of his belief about the current and future states. Consequently the decision whether to visit in the future becomes insensitive to the current report, and hence S does not have any incentive to misreport.

Let $v_t \sim U(0, c)$. The probability of visit in period 2 when $m_1 = B$ is denoted by

$$P(d_2 = 1 \mid m_1 = B) = \min\left\{\tilde{\varphi}_B(\tau_A^1, w, \gamma, c), 1\right\},$$

where

$$\tilde{\varphi}_B(\tau_A^1, w, \gamma, c) \equiv \frac{\frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2(1-\theta\tau_A^1)} - w}{c}.$$

We use $\tilde{\varphi}_B(\tau_A^1, w, \gamma, c)$ to prove two claims below, each of which corresponds to a separate interval of w.

In what follows we consider two cases $w \in [0, 1-\gamma)$ and $w \in \left[1-\gamma, \frac{1-2cx-\theta(2\gamma-1)}{2}\right]$, which are distinguished by R's decision to visit in period 2 when he observes $m_1 = A$. In the former case, the probability of visit is strictly positive, while in the latter case it is zero. This naturally changes the S's incentives in period 1.

Case 1: $w \in [0, 1-\gamma)$

Claim 1. *There exists a threshold $\bar{c} = \frac{1-w-\gamma}{1-x}$ such that for $c \in (\bar{c}, \frac{1}{2}]$, there is a unique equilibrium and it features $\tau_A^1 = 0$; for $c \in (0, \bar{c})$ there is a unique equilibrium and it features truth-telling ($\tau_A^1 = 1$); and for $c = \bar{c}$ any $\tau_A^1 \in [0, 1]$ can be supported in equilibrium.*

Proof. We first introduce notation and make preliminary observations, and then establish the claim for $c \in (\bar{c}, \frac{1}{2}]$, $c \in (0, \bar{c})$, and $c = \bar{c}$, respectively. Note that $\gamma \in (\frac{1}{2}, 1)$ and $x < \frac{2\gamma-1}{4}$, which follows from (8), imply $\bar{c} < \frac{1}{2}$.

The probability of visit in period 2 given $m_1 = A$, where R infers correctly that $\omega_1 = A$, is now positive and given by

$$P(d_2 = 1 \mid m_1 = A) = \min\left\{\frac{1-w-\gamma}{c}, 1\right\}.$$

We can check that, for given τ_A^1, w and γ, the derivative of $\frac{1-w-\gamma}{c}$ with respect to c is strictly negative and strictly larger (i.e. smaller in absolute value) than the derivative of $\tilde{\varphi}_B(\tau_A^1, w, \gamma, c)$, which is also strictly negative. Therefore, we have

$$x + \frac{1-w-\gamma}{c} < \tilde{\varphi}_B(\tau_A^1, w, \gamma, c) \tag{24}$$

for any $c \in (0, \frac{1}{2}]$. Given $\omega_1 = A$, S's expected payoff when reporting truthfully is

$$fx + fP(d_2 = 1 \mid m_1 = A) \tag{25}$$

and the expected payoff when misreporting ($m_1 = B$) is given by

$$fP(d_2 = 1 \mid m_1 = B). \tag{26}$$

We have seen in the proof of Proposition 2 that, if $c = \frac{1}{2}$ as assumed throughout the main text, (25) is strictly smaller than (26) for any $\tau_A^1 \in [0, 1]$, and thus the only equilibrium features $\tau_A^1 = 0$. For the comparison between (25) and (26), f cancels out, so that below we shall focus on x, $P(d_2 = 1 \mid m_1 = A)$ and $P(d_2 = 1 \mid m_1 = A)$.

First, consider $c \in (\bar{c}, \frac{1}{2}]$. Note that $\bar{c} = \frac{1-w-\gamma}{1-x}$ is equivalent to

$$x + \frac{1-w-\gamma}{\bar{c}} = 1,$$

which implies $x + P(d_2 = 1 \mid m_1 = A) = 1$ at $c = \bar{c}$. It is easy to see that at $c = \frac{1}{2}$, (25) is strictly smaller than (26). This and (24) imply that given $c \in (\bar{c}, \frac{1}{2}]$, we have $x + P(d_2 = 1 \mid m_1 = A) < P(d_2 = 1 \mid m_1 = B)$ for any $\tau_A^1 \in [0, 1]$. Therefore, the unique equilibrium features $\tau_A^1 = 0$.

Second, let us consider $c \in (0, \bar{c})$, in which case by construction we have $x + P(d_2 = 1 \mid m_1 = A) > 1$. This and (24) imply that $P(d_2 = 1 \mid m_1 = B) = 1$ for any $\tau_A^1 \in [0, 1]$. Thus (25) is strictly larger than (26) for any $\tau_A^1 \in [0, 1]$. We conclude that the unique equilibrium features truthful reporting in period 1 ($\tau_A^1 = 1$) .

Finally, if $c = \bar{c}$, then given $\omega_1 = A$, R is indifferent between $m_1 = A$ and $m_1 = B$ for any $\tau_A^1 \in [0, 1]$. Therefore any $\tau_A^1 \in [0, 1]$ can be supported in equilibrium. □

Case 2: $w \in \left[1-\gamma, \frac{1-2cx-\theta(2\gamma-1)}{2}\right]$

Claim 2. *There exists a unique equilibrium. It features $\tau_A^1 = 0$.*

Proof. Note first that $1-\gamma < \frac{1-2cx-\theta(2\gamma-1)}{2}$ for $c \le \frac{1}{2}$ so that the interval is well-defined. For this range of w, the probability of visit in period 2 given $m_1 = A$ is zero. Thus given $\omega_1 = A$, S reports truthfully if

$$fx \ge fP(d_2 = 1 \mid m_1 = B),$$

and misreports otherwise. Uninformative communication in period 1 requires $\tau_A^1 = 0$ and

$$x \le \frac{1-2w-\theta(2\gamma-1)}{2c} = \tilde{\varphi}_B(0, w, \gamma, c), \tag{27}$$

which holds by the assumptions $w \le \frac{1-2cx-\theta(2\gamma-1)}{2}$ and $c \le \frac{1}{2}$. Thus there exists an equilibrium with $\tau_A^1 = 0$.

For uniqueness, note first that from the proof of Proposition 2 any equilibrium must be in simple strategies. Now we have

$$\frac{\partial \tilde{\varphi}_B(\tau_A^1, w, \gamma, c)}{\partial \tau_A^1} > 0.$$

This implies that given $w \le \frac{1-2cx-\theta(2\gamma-1)}{2}$ and $c \le \frac{1}{2}$, we have

$$x < P(d_2 = 1 \mid m_1 = B)$$

for any $\tau_A^1 \in [0, 1]$, which in turn means that for any $\tau_A^1 \in [0, 1]$, S is strictly better off misreporting. Thus we conclude that the equilibrium with $\tau_A^1 = 0$ is unique. □

B.2. *R's ex ante total expected payoff in Fig. 2(a)*

We here derive R's total expected payoff for parameter values such that R visits with positive probability in period 1.

Step 1 Let r_1 denote the probability that R visits in period 1. Let r_J, for $J \in \{A, B\}$, denote the probability that R visits in period 2 conditional on $m_1 = J$. Note that $r_A = 0$ in equilibrium when w is in the intermediate range. Let K_1 denote the expected visiting cost incurred by R in period 1, conditional on actually visiting. Let K_J, for $J \in \{A, B\}$, denote the expected visiting cost incurred by R in period 2, conditional on $m_1 = J$. Given r_1, r_A and r_B, R's expected total payoff is given by

$$\begin{aligned}
&P(\omega_1 = A, \omega_2 = A)(1-r_1)\left[2\tau_A^1 + 2\left(1-\tau_A^1\right)\right] + \\
&P(\omega_1 = A, \omega_2 = A)r_1\left[2\tau_A^1 + \left(1-\tau_A^1\right)\right] + \\
&P(\omega_1 = A, \omega_2 = B)(1-r_1)\left[\tau_A^1 + \left(1-\tau_A^1\right)(1+r_B)\right] + \\
&P(\omega_1 = A, \omega_2 = B)r_1\left[\tau_A^1 + \left(1-\tau_A^1\right)r_B\right] + \\
&P(\omega_1 = B, \omega_2 = A)\left[(1-r_1) + 2r_1\right] + \\
&P(\omega_1 = B, \omega_2 = B)\left[(1-r_1)r_B + r_1\left(1+r_B\right)\right] \\
&\quad - \left(r_1 K_1 + \left[P(\omega_1 = A)(1-\tau_A^1) + P(\omega_1 = B)\right] r_B K_B\right),
\end{aligned}$$

where the last term $r_1 K_1 + \left[P(\omega_1 = A)(1-\tau_A^1) + P(\omega_1 = B)\right] r_B K_B$ represents the ex ante expected cost access, and the rest represents the ex ante expected action payoff. Let us provide more details below.

Step 2 The total expected cost of access incurred by R is

$$\begin{aligned}
&r_1 K_1 \\
&+ \underbrace{\left[P(\omega_1 = A)\tau_A^1\right]}_{P(m_1=A)} r_A K_A + \underbrace{\left[P(\omega_1 = A)(1-\tau_A^1) + P(\omega_1 = B)\right]}_{P(m_1=B)} r_B K_B.
\end{aligned}$$

The first term, namely the expected cost of access incurred in period 1 rewrites as

$$r_1 K_1 = \underbrace{\frac{\left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{1/2}}_{r_1} \times \underbrace{\left(w + \frac{\left(\frac{1}{2}\tau_A^1 + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}\right)}_{K_1}.$$

Let us then compute the expected cost of access incurred in period 2. We have

$$r_B = \frac{(\gamma-1)\frac{\tau_A^1-2}{\tau_A^1+2\gamma-3} - w}{1/2} = x$$

and

$$K_B = w + E\left(v_2 \,\middle|\, v_2 \le (\gamma-1)\frac{\tau_A^1 - 2}{\tau_A^1 + 2\gamma - 3} - w\right)$$
$$= w + \frac{(\gamma-1)\frac{\tau_A^1-2}{\tau_A^1+2\gamma-3} - w}{2} = w + \frac{x}{4}.$$

The probability of receiving $m_1 = B$ is given by

$$P(\omega_1 = A)(1-\tau_A^1) + P(\omega_1 = B) = \left(\frac{1}{2}(1-\tau_A^1) + \frac{1}{2}\right).$$

Recall that $r_A = 0$ in the equilibrium we consider. Thus the ex ante expected cost of access incurred in period 2 rewrites as

$$\left[P(\omega_1 = A)(1-\tau_A^1) + P(\omega_1 = B)\right] r_B K_B = \underbrace{\left(\frac{1}{2}(1-\tau_A^1) + \frac{1}{2}\right)}_{P(\omega_1=A)(1-\tau_A^1)+P(\omega_1=B)} \underbrace{x}_{r_B} \underbrace{\left(w + \frac{x}{4}\right)}_{K_B}.$$

Step 3 R's expected action payoff for both periods is given by

$$P(\omega_1 = A, \omega_2 = A)(1-r_1)\left[2\tau_A^1 + 2\left(1-\tau_A^1\right)\right] +$$
$$P(\omega_1 = A, \omega_2 = A)r_1\left[2\tau_A^1 + \left(1-\tau_A^1\right)\right] +$$
$$P(\omega_1 = A, \omega_2 = B)(1-r_1)\left[\tau_A^1 + \left(1-\tau_A^1\right)(1+r_B)\right] +$$
$$P(\omega_1 = A, \omega_2 = B)r_1\left[\tau_A^1 + \left(1-\tau_A^1\right)r_B\right] +$$
$$P(\omega_1 = B, \omega_2 = A)\left[(1-r_1) + 2r_1\right] +$$
$$P(\omega_1 = B, \omega_2 = B)\left[(1-r_1)r_B + r_1\left(1+r_B\right)\right],$$

where

$$P(\omega_1 = A, \omega_2 = A) = \frac{1}{2}\gamma,$$
$$P(\omega_1 = A, \omega_2 = B) = \frac{1}{2}(1-\gamma),$$
$$P(\omega_1 = B, \omega_2 = A) = \frac{1}{4},$$
$$P(\omega_1 = B, \omega_2 = B) = \frac{1}{4}.$$

B.3. Uninformative communication in period 1 in the two-firm model

Let us discuss the uniqueness of the equilibrium with $\tau^C = 0$ in the two-firm model stated in Proposition 5. We consider two cases in turn, one where $w \in [0, 1-\gamma)$ and the other where $w \in \left[1-\gamma, \frac{3-2\gamma-4x}{4}\right]$. As briefly mentioned in Section 3.2.3 and fully discussed in the proof of Proposition 5, the main difference between the two ranges of w concerns R's decision whether to visit in period 2 given $m_1^{S_i} = A$. While truth-telling in period 1 is trivially ruled out and the existence of an equilibrium with $\tau^C = 0$ is established in Proposition 5, we are unable to analytically prove its uniqueness in either case, due to the complexity of the indifference condition with respect to τ^C which pins down any mixed strategy equilibrium. However, we can confirm numerically that for wide ranges of parameter values there is no equilibrium with partially informative communication in period 1. We present a numerical analysis for each case below.

B.3.1. $w \in [0, 1-\gamma)$

In this case, the conditional probability of visit in period 2 if one or both firms report A in period 1, where R assigns probability 1 to $\omega_1 = A$, is strictly positive. An equilibrium with partially informative communication in period 1, if it exists, must feature $\tau^C \in (0,1)$ that equates (21) and (22) for indifference. Now the two solutions to the indifference condition, denoted by τ^{C^*}, are independent of w (as it appears in both expressions symmetrically and cancels out) and given by

$$\tau^{C^*} = \frac{\beta - 2\beta\gamma + (\beta+1)^2 x \pm \sqrt{\beta^2(1-2\gamma)^2 + ((\beta-6)\beta+1)(\beta+1)^2 x^2}}{2\beta(\beta+1)x}. \tag{28}$$

Our numerical calculations presented in Fig. 5 indicate that we have $\tau^{C^*} \notin (0,1)$ for the ranges of parameter values considered here, which cover nearly the full parameter space within the assumptions we make. Specifically, we can see that the solutions in Fig. 5 are either below 0 or above 1.

B.3.2. $w \in \left[1-\gamma, \frac{3-2\gamma-4x}{4}\right]$

In this case, the conditional probability of visit in period 2 if one or both firms report A in period 1 is zero. Fig. 6 presents three solutions to the indifference condition (23) for different values of β. We can confirm that none of the three solutions with respect to τ^C, each of which would represent a mixed strategy equilibrium if $\tau^C \in (0,1)$, falls within the interval $(0,1)$.[30]

B.4. R's ex ante total expected payoff in Fig. 4(a)

The calculation is similar to that of the ex ante total expected payoff in Fig. 2(a). R's ex ante expected payoff in the model in Section 3 is given by

$$P(\omega_1 = A, \omega_2 = A)(1-r_1)\left[2\tau^C + 2(1-\tau^C)\right] +$$
$$P(\omega_1 = A, \omega_2 = A)r_1\left[2\tau^C + (1-\tau^C)\right] +$$
$$P(\omega_1 = A, \omega_2 = B)\times$$
$$(1-r_1)\left[\left(1 - (\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C))\right)\right.$$
$$\left.+(\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C))(1+r_B)\right] +$$
$$P(\omega_1 = A, \omega_2 = B)r_1\left[\tau^C + (1-\tau^C)r_B\right] +$$
$$P(\omega_1 = B, \omega_2 = A)\left[(1-r_1) + 2r_1\right] +$$
$$P(\omega_1 = B, \omega_2 = B)\left[(1-r_1)r_B + r_1\left(1+r_B\right)\right]$$
$$-\left(r_1 K_1 + \left[\left(P(\omega_1 = A)(\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C))\right.\right.\right.$$
$$\left.\left.\left.+\ P(\omega_1 = B)\right)\right] r_B K_B\right).$$

The differences from the corresponding calculation for Fig. 2(a) are as follows. Given $\omega_1 = A$, the probability that R is still uncertain about the state at the end of period 1 (which is the case if $m_1^{S_i} = m_1^{S_j} = B$) is now $\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C)$. The probability of visit r_1 and the conditional expected cost of access K_1 are calculated in the same way as in Fig. 2(a), simply by replacing τ_A^1 with τ^C since R acquires at most one message before choosing a_1. In the last line of the above expression, the unconditional probability that R receives $m_1^{S_i} = m_1^{S_j} = B$ and thus is uncertain about the state at the end of period 1 given by

$$P(\omega_1 = A)(\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C)) + P(\omega_1 = B).$$

The probability of visit in period 2 given $m_1^{S_i} = m_1^{S_j} = B$ is denoted by r_B. The expected cost of access is denoted by K_B. These can be calculated from (6).

Appendix C. Infinite horizon

In this Appendix we extend our single firm model with two periods to an infinite horizon setup and demonstrate that our key results and intuitions do not depend on the feature that truth-telling occurs in equilibrium in the final period. In reality, the underlying state often evolves over time without a specific end date, and an infinite horizon model captures situations where in every period there is an incentive for S to induce future visits by misreporting. Technically, both players now face a recursive problem. The state ω_t in the infinite horizon model corresponds to a broad topic (politics, economy, sports, etc.) followed by readers over the long run, rather than a particular news event that develops and ends within a short period of time. An example could be

[30] The solutions close to 1 shown in Fig. 6 are all strictly decreasing in w. For example, for $\beta = 0.6$, the middle solution is $\tau^C = 1.17528$ when $w = 0.1$, and $\tau^C \approx 1.07826$ when $w = 0.25$.

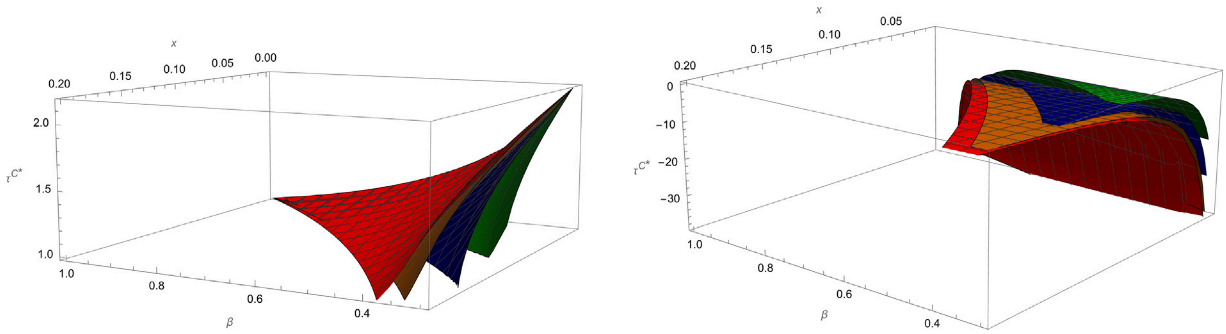


Fig. 5. The two solutions τ^{C^*} for $w \in [0, 1-\gamma)$ with respect to β and x such that $x < \frac{2\gamma-1}{4}$ and $\beta < \frac{(2\gamma-1)-2x}{(2\gamma-1)+2x}$: red, orange, blue and green surfaces correspond to $\gamma = 0.95, 0.9, 0.8$ and 0.7, respectively. Recall that the solutions are independent of w. (For the references to colour in this figure legend, the reader is referred to the web version of this article.)

that states A and B correspond to the stability of the current condition of the economy. In state A, the condition is very stable (either being good or bad) and likely to remain the same for a while, whereas in state B it is highly volatile and thus difficult to forecast.

C.1. Model

Let the time be denoted by $t = 0, 1, 2, \ldots, +\infty$. Both S and R discount their future payoffs at a common discount factor δ and they maximize the discounted future stream of payoffs in every period. The transition matrix of the state, the players, their action sets and per-period payoffs, are the same as in Section 2.

Let $\widetilde{x} \equiv x/\delta$ for convenience, since the discount factor δ affects the players' incentives only via x. Specifically, a higher discount factor means that S values revenue from future visits more, which in turn implies that the incentive to misreport becomes relatively higher, resulting in the lower effective truth-telling benefit x/δ. We focus on the interesting case of intermediate w and impose a slightly modified and simpler version of (1) as follows:

$$w \in \left(1-\gamma, \frac{1-\widetilde{x}}{2}\right). \tag{29}$$

At the beginning of each period, S observes ω_t. At the beginning of period t, R observes neither ω_t nor ω_{t-1} but instead observes $\{\omega_{t-2}, m_{t-1}\}$. The assumption that R observes ω_{t-2} captures the notion that as time goes by, a past event is more clearly recognized and understood. The assumption that m_{t-1} is observed, as in the two periods model, captures diffusion of information. S chooses a message in each period from period 1 onwards, and R chooses whether to visit and what action to take from period 2 onwards. The timing of payoff realizations is irrelevant, as long as it does not allow R to infer ω_t and ω_{t-1} during period t independently of the messages received.

A communication strategy of S for the whole game is given by a sequence of one-shot communication strategies, one for each t. A stationary communication strategy for the whole game is such that S uses the same one-shot strategy in every period. A strategy for period t is said to belong to class n if it conditions the message sent in period t on neither more nor less than $\{\omega_{t-n+1}, \ldots, \omega_{t-2}, \omega_{t-1}, \omega_t\}$. In other words, a class n strategy is conditioned on the history of the state up to $n-1$ periods ago. Note that a class n strategy is conditioned only on realized states, but not on S's own messages in the past. A*basic class n* strategy for period t is a strategy for period t such that S always sends $m_t = B$ if $\omega_t = B$.

For concreteness, let us illustrate a basic class 2 strategy for period t in some detail. First, the probability of sending $m_t \in \{A, B\}$ in period t is a function of (and only of) $\{\omega_{t-1}, \omega_t\}$. Such a strategy for period t is therefore described by four truth-telling probabilities at period t, and we denote by $\tau^t_{\omega_{t-1}\omega_t}$ the probability that $m_t = \omega_t$ given $\{\omega_{t-1}, \omega_t\}$. Second, a basic class 2 strategy for period t features $\tau^t_{\omega_{t-1}B} = 1$ for any ω_{t-1}. A basic class 2 strategy for period t is*partially informative* if neither $\tau^t_{AA} = \tau^t_{BA} = 1$ nor $\tau^t_{AA} = \tau^t_{BA} = 0$. If $\tau^t_{AA} = \tau^t_{BA} = 1$ then the basic class 2 strategy for period t is *fully informative* (and also features truth-telling). Finally, if $\tau^t_{AA} = \tau^t_{BA} = 0$ then the basic class 2 strategy for period t is *uninformative* since $m_t = B$ regardless of the state. A stationary strategy of S for the whole game that involves the infinite repetition of a basic class n strategy for each period is called a basic class n stationary strategy. In what follows, for simplicity we restrict ourselves to basic class 1 and 2 stationary strategies of S. We will see shortly that no basic class 1 strategy can form part of a stationary equilibrium, so that the simplest class of basic stationary equilibrium strategies of S is class 2.

A strategy of R for the whole game is given by a sequence of one-shot strategies, one for each t. R's strategy for each period has two components, a consultation rule and an action rule. A stationary strategy of R is such that R uses the same one-shot strategy in each period t. A strategy of R is said to belong to class n if R's choices are conditioned on neither more nor less than $\{\omega_{t-n+1}, \ldots, \omega_{t-2}, m_{t-1}, m_t\}$, where we (abusively) denote $m_t = \emptyset$ if m_t was not observed. Given our exogenous restriction to class 1 and class 2 stationary communication strategies of S, it is without loss of generality to focus on stationary strategies of R of class no larger than $n = 3$. A class 3 strategy for period t, as is the case for a strategy for period t of any class, involves a consultation rule that takes the form of a set of threshold rules. For each possible observed history (ω_{t-2}, m_{t-1}), there is a threshold value of the uniformly distributed random cost $v_t \in (0, \frac{1}{2})$, denoted by $v(\omega_{t-2}, m_{t-1})$, such that R visits in the beginning of period t if and only if $v_t \leq v(\omega_{t-2}, m_{t-1})$. We denote by $\varphi_{\omega_{t-2}m_{t-1}}$ the conditional probability that R consults in period t given (ω_{t-2}, m_{t-1}), which is simply $P(v_t \leq v(\omega_{t-2}m_{t-1}))$. A class 3 strategy of R for period t also involves an action rule which conditions R's chosen action at t on $\{\omega_{t-2}, m_{t-1}, m_t\}$.

We focus on equilibria featuring stationary strategies for both S and R, and we call such equilibria stationary. We say that a stationary equilibrium features a positive probability of consultation if there is some $(n-1)$-elements history $\{\omega_{t-n+1}, \ldots, \omega_{t-2}, m_{t-1}\}$ which has a positive stationary probability and the strategy of R specifies a positive probability of consultation in period t given this $(n-1)$-elements history. Recall that both S and R are unable to commit to future actions, so that the behaviour has to be incentive compatible at every point in time (i.e. every information set) in equilibrium.

We conclude the presentation of the model with a discussion of our focus on stationary strategy profiles. We do not consider non-stationary strategies (e.g. Grim-Trigger, Stick and Carrot) that might achieve more informative reporting through punishment for misreporting (provided a sufficiently high discount factor). Such strategies would in principle be feasible since R observes past states (though with a lag), so that R could detect misreporting and thus choose not to visit for a while after a misreport as punishment. Note that the following simple and intuitive class of such strategy profiles would not support truth-telling in equilibrium. Consider the class where punishment by R is accompanied by uninformative messages by S during the punishment phase, since otherwise

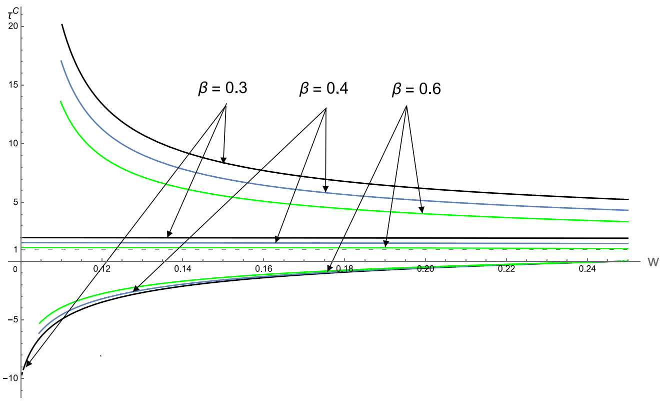


Fig. 6. Solutions to (23) assuming $\gamma = 0.9, x = 0.05$.

R might have an incentive to visit during this phase and thus fail to impose punishment. The contradiction is that if S knows that R never visits in the next period(s), S reports truthfully to obtain the truth-telling benefit, which in turn induces R to visit, making the punishment by R ineffective. Equilibrium strategies to support truth-telling would thus have to be more sophisticated, involving e.g. punishment for not punishing. Such sophisticated equilibrium constructions that feature non-stationary strategies seem implausible in the context of communication between a media firm and potentially large audience. Insofar as a report is informative concerning the state in period t, punishment (committing not to visit) at t is subject to a free-riding problem among receivers, which makes their coordinated punishment of the sender unlikely. Yet another problem with non-stationary strategies is that equilibria featuring sophisticated punishment strategies are typically not renegotiation proof, in contrast to the stationary equilibria that we study.

C.2. Equilibrium

The following Lemma outlines key features of R's equilibrium consultation.

Lemma 7. *(a) No stationary equilibrium features zero probability of consultation. (b) In any stationary equilibrium in which S uses a basic communication strategy, the probability of consultation in period t is zero if $m_{t-1} = A$, regardless of $\omega_{t-2} \in \{A, B\}$.*

The proofs of all Lemmas and Propositions in Appendix C are relegated to Appendix C.3. Lemma 7 is a consequence of the assumption that w is in the intermediate range (29): w is low enough for consultation to occur in equilibrium, but also high enough that the consultation decision is conditioned on the previous period's message. The next Lemma concerns S's equilibrium communication strategy.

Lemma 8. *(a) No stationary equilibrium features an uninformative or a fully informative communication strategy. (b) No stationary equilibrium features a basic class 1 communication strategy.*

Uninformative communication cannot be an equilibrium strategy. In such a putative equilibrium, R never consults and S deviates to truth-telling to reap the truth-telling benefit x. Equally, perfectly informative communication cannot be part of an equilibrium. In such a putative equilibrium, the visiting probability in $t+1$ is such that S deviates from truth-telling and sends $m_t = B$ even if $\omega_t = A$ (note that $m_t = A$ yields a visiting probability of 0). So if there exists a basic class 1 equilibrium, it must be such that S randomizes when $\omega_t = B$. This in turn implies that R must consult with the same probability at $t+1$ given the alternative observed histories $\omega_{t-1} = A, m_t = B$ and $\omega_{t-1} = B, m_t = B$. This, however, cannot be true.

The next Proposition presents our equilibrium characterization. In a search space containing all equilibria featuring either *basic* class 1 or basic class 2 stationary communication strategies, there exists a unique equilibrium. It features a basic class 2 strategy. The argument leading to the result builds on our Lemmas. By Lemma 7, any stationary equilibrium must feature a positive probability of consultation, as well as a zero probability of consultation at t if $m_{t-1} = A$. By Lemma 8, any stationary equilibrium features a partially informative communication strategy, which can furthermore not be a basic class 1 strategy. It follows that if we find a stationary equilibrium featuring either a basic class 1 or 2 communication strategy, then the equilibrium must feature a partially informative basic class 2 communication strategy and R must consult with positive probability at t if and only if $m_{t-1} = B$. The last part of the characterization is to prove that there exists an equilibrium satisfying the above description, and this is the focus of the proof provided in Appendix C.3.

Proposition 8. *There exists no stationary equilibrium featuring a basic class 1 communication strategy. There exists a unique stationary equilibrium featuring a basic class 2 communication strategy. In this equilibrium, R plays a class 3 strategy. Furthermore, S's communication strategy features $\tau^*_{AA}, \tau^*_{BA} \in (0,1)$ and $\tau^*_{AA} \neq \tau^*_{BA}$; and R consults with probability $\tilde{x}$ at t if $m_{t-1} = B$ and with probability 0 if $m_{t-1} = A$.*

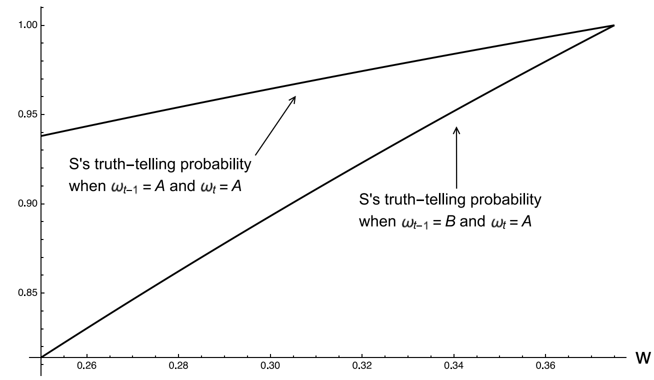
The intuition behind the equilibrium is very similar to that obtained for the analysis of the two-period model. To see that players best respond to each other, consider the following. Assume that S plays a class 2 strategy and that R plays a class 3 strategy, i.e. m_t is conditioned on both ω_t and ω_{t-1}, while R's consultation decision at $t+1$ depends only on m_t and ω_{t-1}.

Let us consider the strategy of S. The message m_t affects S's expected payoff in two ways, namely through (i) the truth-telling benefit in period t, which depends only on the current state ω_t; and (ii) the consultation probability in period $t+1$, which is induced by m_t given ω_{t-1} and fully described by $\varphi_{AA} = 0$, $\varphi_{BA} = 0$, φ_{AB} and φ_{BB}. Let us look at the second channel (ii) in more detail. When choosing m_t, S acknowledges that in period $t+1$, R will condition his visit only on ω_{t-1} and m_t, while m_{t-1} does not affect R's visiting probability in period $t+1$. It follows in turn that S conditions m_t only on $\{\omega_{t-1}, \omega_t\}$. S has no incentive to deviate and condition m_t on m_{t-1}, since m_{t-1} affects neither the truth-telling benefit in period t nor the visiting probability in period $t+1$.

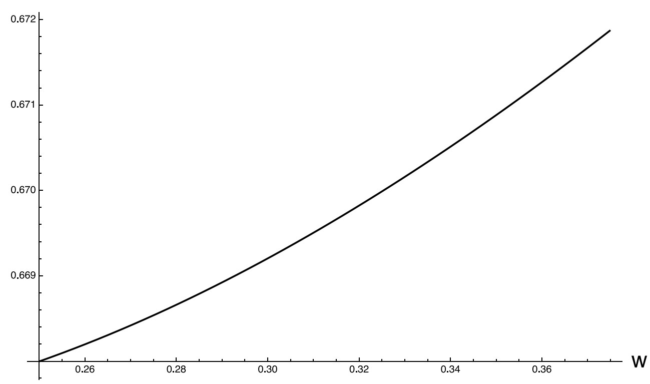
We now turn to R's equilibrium strategy. The equilibrium features $\tau^*_{AA}, \tau^*_{BA} \in (0,1)$, and note that $\varphi_{AB} = \varphi_{BB} \in (0,1)$ implies $\tau^*_{AA} \neq \tau^*_{BA}$. Indeed, if $\tau^*_{AA} = \tau^*_{BA}$, then we cannot have $\varphi_{AB} = \varphi_{BB} \in (0,1)$ since the conditional distribution of ω_t at the end of period t given $\{\omega_{t-1} = A, m_t = B\}$ and $\{\omega_{t-1} = B, m_t = B\}$ has to differ, which in turn implies that the expected payoff from visiting in period $t+1$ differs depending on ω_{t-1}. The states prior to ω_{t-1} are irrelevant to the stationary equilibrium behaviour because the assumption that the state follows a Markov process implies that only ω_{t-1} matters for the distribution of ω_t. We now present key comparative statics properties of equilibrium communication.

Proposition 9. *In the equilibrium identified in Proposition 8, τ^*_{AA} and τ^*_{BA} are constant in f, increasing in w and decreasing in $\tilde{x}(= x/\delta)$.*

The comparative statics results are qualitatively the same as in the two-period model. Clearly if τ^*_{AA} and τ^*_{BA} remained unchanged, an increase in w would lower the probability of visit at t given $m_t = B$. This implies that the equilibrium informativeness of communication must increase to keep the consultation probability equal to $\tilde{x}$. Equivalently, a higher cost of access w makes misreporting (i.e. sending $m_t = B$ when $\omega_t = A$) less attractive since R's future visit becomes less sensitive to



(a) S's equilibrium behaviour



(b) R's average per-period payoff

Fig. 7. Equilibrium behaviour and payoff with respect to w, where $\gamma = 3/4, x = 1/5$ and $\delta = 4/5$.

the current report. As S becomes more patient (δ higher and $\widetilde{x}$ lower) the incentive to misreport increases, since a future visit induced by misreporting becomes more valuable relative to the truth-telling benefit in the current period. Also, f does not enter τ^*_{AA} or τ^*_{BA} as it cancels out from the relevant indifference conditions, as in the two-period model. Proposition 9 demonstrates that the main insights obtained in the two-period setup do *not* depend on the feature that truth-telling occurs in the final period.

We now consider how R's equilibrium payoff varies with w. Let the average per-period payoff of R be the limit of his average payoff across periods 2 to T, for T tending to infinity. The realized frequency of state A over this time interval thus coincides for sure with the stationary distribution.[31]

A numerical example is presented in Fig. 7. Fig. 7(a) illustrates Proposition 9 showing that τ^*_{AA} and τ^*_{BA} are increasing in w. We see $\tau^*_{AA} \geq \tau^*_{BA}$, that is, the current report is more informative when the previous state involves less uncertainty ($\omega_{t-1} = A$). When making his consultation decision at period $t+1$ after observing $m_t = A$, R also considers the (observed) state in period $t-1$, and $\omega_{t-1} = A$ implies lower uncertainty about ω_{t+1} than $\omega_{t-1} = B$. This means that R is less manipulable when $\omega_{t-1} = A$, which leads to a lower incentive for S to misreport. Fig. 7(b) shows that R's per-period payoff may be increasing in w.[32] That is, as in the two-period model, the benefit of an increase in w (more accurate reports) may outweigh the cost (increased average cost of access) so that R may suffer overall from a cheaper cost of access to reports.

S's per-period average accounting profit on the other hand is decreasing in w. Note that an increase in w leads to a higher probability of truth-telling when $\omega_t = A$ as we saw in Proposition 9 while the probability of visit $\widetilde{x}$ after $m_t = B$ remains unchanged (Proposition 8). This implies that the average visiting probability decreases, because $m_t = A$, after which R never visits, becomes more likely. The effect of w on S's accounting profit thus contrasts with what we found in the two-period model, where the accounting profit may be increasing in w. The difference is due to the high initial uncertainty in the two-period model (we set the maximal initial uncertainty $\theta = \frac{1}{2}$ for the numerical example in Section 2.2). In Fig. 1, we can see that the probability of visit in period 1 may be *increasing* in w since the high uncertainty in that period makes the decision to visit very sensitive to the informativeness of m_1.

C.3. Proofs and calculations

C.3.1. Lemma 7

Proof. Part (a) Consider a putative stationary equilibrium with no consultation. In such an equilibrium, S would have a strict incentive to send $m_t = \omega_t$ in every period. So such an equilibrium would have to feature a fully informative basic communication strategy of S. But then, given our parameter assumptions, R would have a strict incentive to consult with positive probability after $m_t = B$.

Part (b) In an equilibrium where $m_{t-1} = A$ implies that $\omega_{t-1} = A$ for sure, a necessary condition for R to choose to consult with positive probability in period t given $m_{t-1} = A$ is $1 - w > \gamma$, or equivalently $w < 1 - \gamma$ since the gross benefit of consultation (ignoring consultation costs) is at most $1 - \gamma$. This benefit corresponds to a scenario where S's communication is fully informative at t. A fortiori, if S's report is not fully informative in period t, R never visits in period t given $m_{t-1} = A$ if $w > 1 - \gamma$. Now, simply note that states $w > 1 - \gamma$ according to the assumption (29). □

C.3.2. Lemma 8

Proof. Step 1 Part (a) Consider a putative stationary equilibrium featuring an uninformative communication strategy of S. Such an equilibrium must feature no consultation. But in a putative stationary equilibrium featuring no consultation, S would strictly favour sending $m_t = \omega_t$ for sure for any value of ω_t, thereby deviating from his assumed uninformative stationary strategy.

Step 2 Consider a putative stationary equilibrium featuring a fully informative stationary communication strategy of S. Consider first the case where S's strategy is such that S always sends $m_t = \omega_t$. Assume now that $\omega_t = A$. Message $m_t = A$ leads to a visiting probability of 0 in period $t+1$ and thus yields a period $t+1$ expected payoff over t and $t+1$ of fx. In contrast, $m_t = B$ leads to a strictly positive probability of visit of $\frac{\frac{1}{2}-w}{1/2} = 1 - 2w$ in period $t+1$. It thus yields an expected payoff over t and $t+1$ of $\delta f(1 - 2w)$. Now, note that $\delta f(1 - 2w) \leq fx$ is equivalent to $\frac{1-\widetilde{x}}{2} \leq w$, which contradicts the assumption that w is in the intermediate range (29).

Consider now the case where S's strategy is such that S always sends $m_t \neq \omega_t$. Assume now that $\omega_t = A$. Message $m_t = B$ leads to a visiting probability of 0 in period $t+1$ and thus yields an expected payoff of 0 over t and $t+1$. In contrast, $m_t = A$ yields the truth-telling

[31] We derive the exact expression for this average per-period payoff in Appendix C.3.

[32] We give the calculation of the payoff in Appendix C.3.5. The parameter values are the same as in Fig. 2 produced for the two-period model, except for the discount factor δ and the relevant range of w.

benefit fx in period t and also leads to a strictly positive probability of visit of $1-2w$ in period $t+1$. It follows trivially that in this putative equilibrium, S strictly prefers to deviate to $m_t = A$ given $\omega_t = A$.

Step 3 Part (b) Consider a putative stationary equilibrium featuring a stationary basic communication strategy of S that belongs to class 1. Consider two different cases. In the first case, the stationary strategy of S builds on a pure strategy for period t. It is thus either fully informative or uninformative. We have already shown in Steps 1 and 2 above that this cannot be true in equilibrium. The second case to consider is that the stationary strategy of S builds on a mixed strategy of S in each period (which must be identical in all periods by the stationarity assumption). By definition it must be true that S sends $m_t = B$ whenever $\omega_t = B$. So the randomization must take place when the state is A. Recall that $m_t = \omega_t$ yields the immediate truth-telling benefit fx. Note also that if $m_t = A$, then R is thus sure that $\omega_t = A$. Given our assumptions on parameter values, it follows immediately that R consults with probability 0 at $t+1$ if $m_t = A$. Note furthermore that if given ω_t, S randomizes between messages A and B at t, then it must be true that both messages yield the same expected payoff for S over t and $t+1$. Using the fact that R consults with probability 0 at $t+1$ if $m_t = A$, the corresponding indifference condition is equivalent to stating that (after cancelling out f) the following has to hold:

$$P(d_{t+1} = 1 \,|m_t = B, \omega_{t-1} = A) = P(d_{t+1} = 1 \,|m_t = B, \omega_{t-1} = B) = \widetilde{x}.$$

However, this in turn requires that

$$P(\omega_{t+1} = B \,|m_t = B, \omega_{t-1} = A) = P(\omega_{t+1} = B \,|m_t = B, \omega_{t-1} = B),$$

which guarantees that the benefit of consulting at $t+1$ after $m_t = B$ is the same, regardless of the observed value of ω_{t-1}. This equality can however not be satisfied given the Markov process and the assumed communication strategy of S. □

C.3.3. Proposition 8

Proof. **Step 1** Let us first show that there is no equilibrium that features $\tau_{AA} = 1$ or $\tau_{BA} = 1$ (a fortiori there is no truth-telling equilibrium, where $\tau_{AA} = \tau_{BA} = 1$). If there is such an equilibrium, it has to be that S is better off reporting $\omega_t = A$ truthfully for at least one of the relevant histories, namely either

$$x + \delta P(d_t = 1 \mid m_{t-1} = A) f \geq 0 + \delta P(d_t = 1 \mid \omega_{t-2} = A, m_{t-1} = B) f \quad (30)$$

or

$$x + \delta P(d_t = 1 \mid m_{t-1} = A) f \geq 0 + \delta P(d_t = 1 \mid \omega_{t-2} = B, m_{t-1} = B) f. \quad (31)$$

Part (b) of Lemma 8 states that $\varphi_{AA} = \varphi_{BA} = 0$ while Part (a) of the Lemma states that $\varphi_{AB} > 0$ or $\varphi_{BB} > 0$. Meanwhile, $\tau_{AA} = 1$ or $\tau_{BA} = 1$ implies $\widetilde{v}(A,B) = \frac{1}{2} - w$ or $\widetilde{v}(B,B) = \frac{1}{2} - w$; and hence $\varphi_{AB} = 1-2w$ or $\varphi_{BB} = 1-2w$, respectively. Recall that $\widetilde{x} \equiv x/\delta$. From (30) and (31), if $\varphi_{JB} = 1-2w$, S reports truthfully when $\omega_{t-1} = J$ and $\omega_t = A$ if

$$f\widetilde{x} \geq (1-2w) f. \quad (32)$$

However, (32) simplifies to $w \geq \frac{1-\widetilde{x}}{2}$, which contradicts the assumption (29). Therefore, neither $\tau_{AA} = 1$ nor $\tau_{BA} = 1$ can be supported in equilibrium.

Step 2 Let us prove the existence of an equilibrium that features $\tau_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$. First let us consider R's decision to consult. In order to calculate his expected payoffs when he consults and when he does not consult, we need to consider his conditional expectations about the previous state ω_{t-1} and the current state ω_t, given ω_{t-2} and m_{t-1}. Note that

$$P(\omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B) = 1 - P(\omega_t = B \mid \omega_{t-2} = A, m_{t-1} = B)$$
$$= \underbrace{\frac{1-\gamma}{\gamma(1-\tau_{AA}) + (1-\gamma)}}_{P(\omega_{t-1}=B|\omega_{t-2}=A,m_{t-1}=B)} \times \frac{1}{2} + \underbrace{\left(1 - \frac{1-\gamma}{\gamma(1-\tau_{AA}) + (1-\gamma)}\right)}_{P(\omega_{t-1}=A|\omega_{t-2}=A,m_{t-1}=B)} \times \gamma$$

and

$$P(\omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B) = 1 - P(\omega_t = B \mid \omega_{t-2} = B, m_{t-1} = B)$$
$$= \underbrace{\frac{\frac{1}{2}}{\frac{1}{2}(1-\tau_{BA}) + \frac{1}{2}}}_{P(\omega_{t-1}=B|\omega_{t-2}=B,m_{t-1}=B)} \times \frac{1}{2} + \underbrace{\left(1 - \frac{\frac{1}{2}}{\frac{1}{2}(1-\tau_{BA}) + \frac{1}{2}}\right)}_{P(\omega_{t-1}=A|\omega_{t-2}=B,m_{t-1}=B)} \times \gamma.$$

Basic sender strategies imply $P(\omega_{t-1} = A \mid \omega_{t-2}, m_{t-1} = A) = 1$ for $\omega_{t-2} \in \{A, B\}$.

The conditional joint distribution of the present and the previous state satisfies

$$P(\omega_{t-1} = A, \omega_t = A \,|\omega_{t-2}, m_{t-1}) = P(\omega_{t-1} = A \,|\omega_{t-2}, m_{t-1})\gamma$$

and

$$P(\omega_{t-1} = B, \omega_t = A \,|\omega_{t-2}, m_{t-1}) = P(\omega_{t-1} = B \,|\omega_{t-2}, m_{t-1})\frac{1}{2}.$$

Since R chooses $a_t = A$ when $m_t = A$ and $a_t = B$ when $m_t = B$, his expected payoff for period t conditional on consultation in period t (exclusive of the visiting costs) is given by

$$\begin{aligned}\pi_t^R(d_t = 1 \mid \omega_{t-2} = A, m_{t-1} = B) = {} & P(\omega_t = B \mid \omega_{t-2} = A, m_{t-1} = B) \\ & + P(\omega_{t-1} = A, \omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B)\tau_{AA} \\ & + P(\omega_{t-1} = B, \omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B)\tau_{BA},\end{aligned}$$

where $P(\omega_t = B \mid \omega_{t-2} = A, m_{t-1} = B)$ represents his payoff (of 1) multiplied by the conditional probability that $\omega_t = B$. If R does not consult at t, his expected payoff for period t is given by

$$\pi_t^R(d_t = 0 \mid \omega_{t-2} = A, m_{t-1} = B) = P(\omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B)$$

since he chooses $a_t = A$. The gross benefit of consultation given $\omega_{t-2} = A$ and $m_{t-1} = B$ is given by

$$\Delta_t^R\left(\omega_{t-2} = A, m_{t-1} = B\right) \equiv \pi_t^R(d_t = 1 \mid \omega_{t-2} = A, m_{t-1} = B) - \pi_t^R(d_t = 0 \mid \omega_{t-2} = A, m_{t-1} = B).$$

Similarly, for $\omega_{t-2} = B$ and $m_{t-1} = B$ we have

$$\begin{aligned}\pi_t^R(d_t = 1 \mid \omega_{t-2} = B, m_{t-1} = B) = {} & P(\omega_t = B \mid \omega_{t-2} = B, m_{t-1} = B) \\ & + P(\omega_{t-1} = A, \omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B)\tau_{AA} \\ & + P(\omega_{t-1} = B, \omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B)\tau_{BA},\end{aligned}$$

and

$$\pi_t^R(d_t = 0 \mid \omega_{t-2} = B, m_{t-1} = B) = P(\omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B).$$

Thus the gross benefit of consulting given $\omega_{t-2} = B$ and $m_{t-1} = B$ is given by

$$\Delta_t^R\left(\omega_{t-2} = B, m_{t-1} = B\right) \equiv \pi_t^R(d_t = 1 \,|\omega_{t-2} = B, m_{t-1} = B) - \pi_t^R(d_t = 0 \,|\omega_{t-2} = B, m_{t-1} = B).$$

The above gross benefits, together with w and the realized value of v_t, determine R's best response given τ_{AA} and τ_{BA}.

Step 3 An equilibrium featuring a basic communication strategy as well as $\tau_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$ requires two indifference conditions to hold simultaneously (one for $\omega_{t-2} = A$ and the other for $\omega_{t-2} = B$), namely

$$\underbrace{fx}_{\text{truth-telling benefit + no visit}} = \underbrace{\delta\left(\frac{\Delta_t^R\left(\omega_{t-2} = A, m_{t-1} = B\right) - w}{1/2}\right) f}_{\text{no truth-telling benefit + positive prob of visit in } t+1} \quad (33)$$

and

$$fx = \delta\left(\frac{\Delta_t^R\left(\omega_{t-2} = B, m_{t-1} = B\right) - w}{1/2}\right) f. \quad (34)$$

Solving simultaneously for τ_{AA} and τ_{BA}, we obtain three pairs of solutions.

The first pair is given by $\tau_{AA} = \frac{\tilde{x}+2w+4\gamma-2}{2\gamma}$ and $\tau_{BA} = \tilde{x} + 2w$. The second pair of solutions is given by

$$\tau_{AA} = \frac{(\tilde{x} + 2w + 6\gamma - 1)\gamma + \sqrt{8\gamma^2\left(1 - \tilde{x} - 2w - 4\gamma^2\right) + \left(6\gamma^2 + \tilde{x}\gamma - \gamma + 2w\gamma\right)^2}}{4\gamma^2},$$

and

$$\tau_{BA} = \frac{(3 + \tilde{x} + 2w - 2\gamma)\gamma + \sqrt{8\gamma^2\left(1 - \tilde{x} - 2w - 4\gamma^2\right) + \left(6\gamma^2 + \tilde{x}\gamma - \gamma + 2w\gamma\right)^2}}{4\gamma(1-\gamma)}.$$

The third pair is given by:

$$\tau^*_{AA} = \frac{(\tilde{x} + 2w + 6\gamma - 1)\gamma - \sqrt{8\gamma^2\left(1 - \tilde{x} - 2w - 4\gamma^2\right) + \left(6\gamma^2 + \tilde{x}\gamma - \gamma + 2w\gamma\right)^2}}{4\gamma^2} \tag{35}$$

and

$$\tau^*_{BA} = \frac{(3 + \tilde{x} + 2w - 2\gamma)\gamma - \sqrt{8\gamma^2\left(1 - \tilde{x} - 2w - 4\gamma^2\right) + \left(6\gamma^2 + \tilde{x}\gamma - \gamma + 2w\gamma\right)^2}}{4\gamma(1-\gamma)}. \tag{36}$$

For $\tau_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$, the first solution requires $w < 1 - \gamma$, which is at odds with the assumption that w is in the intermediate range (29). Similarly, the second solution requires $\tilde{x} < 0$ while we have assumed $\tilde{x} > 0$. The third solution gives $\tau^*_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$ for $\gamma \in (\frac{1}{2}, 1)$ and

$$w \in \left(-\frac{\tilde{x}}{2}, \frac{1-\tilde{x}}{2}\right),$$

which is satisfied under our assumptions. Thus we conclude that the third pair of solutions (35) and (36) pins down the unique equilibrium in stationary basic strategies. □

C.3.4. Proposition 9

Proof. We use the closed form expressions for τ^*_{AA} and τ^*_{BA} appearing in (35) and (36). We obtain

$$\frac{\partial \tau^*_{AA}}{\partial w} = \frac{2 - \frac{\partial}{\partial w}F(w,\tilde{x},\gamma)}{4\gamma}, \tag{37}$$

$$\frac{\partial \tau^*_{BA}}{\partial w} = \frac{2 - \frac{\partial}{\partial w}F(w,\tilde{x},\gamma)}{4(1-\gamma)}, \tag{38}$$

$$\frac{\partial \tau^*_{AA}}{\partial \tilde{x}} = \frac{1 - \frac{\partial}{\partial \tilde{x}}F(w,\tilde{x},\gamma)}{4\gamma} \tag{39}$$

$$\frac{\partial \tau^*_{BA}}{\partial \tilde{x}} = \frac{1 - \frac{\partial}{\partial \tilde{x}}F(w,\tilde{x},\gamma)}{4(1-\gamma)} \tag{40}$$

where

$$F(w,\tilde{x},\gamma) = \sqrt{8\gamma^2\left(1 - \tilde{x} - 2w - 4\gamma^2\right) + \left(6\gamma^2 + \tilde{x}\gamma - \gamma + 2w\gamma\right)^2}.$$

Partially differentiating F, we have

$$\frac{\partial F(w,\tilde{x},\gamma)}{\partial w} = \frac{2\gamma^2(\tilde{x} + 6\gamma + 2w - 5)}{\sqrt{\gamma^2\left(\tilde{x}^2 + 2\tilde{x}(6\gamma + 2w - 5) + (3-2\gamma)^2 + 4w^2 + 4(6\gamma - 5)w\right)}},$$

and

$$\frac{\partial F(w,\tilde{x},\gamma)}{\partial \tilde{x}} = \frac{\gamma^2(\tilde{x} + 6\gamma + 2w - 5)}{\sqrt{\gamma^2\left(\tilde{x}^2 + 2\tilde{x}(6\gamma + 2w - 5) + (3-2\gamma)^2 + 4w^2 + 4(6\gamma - 5)w\right)}}.$$

The partial derivatives above are all negative when $\tilde{x} + 6\gamma + 2w - 5 < 0$. This inequality holds if $\gamma \in (\frac{1}{2}, 1)$ and $w < \frac{1-\tilde{x}}{2}$, as assumed in (29). Therefore, from (37) to (40) we obtain $\frac{\partial \tau^*_{AA}}{\partial w} > 0$, $\frac{\partial \tau^*_{BA}}{\partial w} > 0$, $\frac{\partial \tau^*_{AA}}{\partial \tilde{x}} > 0$, and $\frac{\partial \tau^*_{BA}}{\partial \tilde{x}} > 0$. □

C.3.5. R's per-period average payoff

Step 1 Let $\overline{\Pi}^R$ be R's average per-period payoff in the identified equilibrium. We have

$$\begin{aligned}\overline{\Pi}^R = &\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1}) \\ &\times \Big[P(\omega_t = B\,|\omega_{t-1})\varphi_{\omega_{t-2}m_{t-1}} \\ &\quad + P(\omega_t = A\,|\omega_{t-1})(1 - \varphi_{\omega_{t-2}m_{t-1}}(1 - \tau_{\omega_{t-1}A}))\Big] \\ &- \sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1})\varphi_{\omega_{t-2}m_{t-1}}\left(\frac{v(\omega_{t-2}m_{t-1})}{2} + w\right),\end{aligned}$$

where the first term is the average action payoff and the second term is the average per-period consultation cost. In what follows we will derive each term separately.

Step 2 Let us give explicit expressions for some of the variables above. In the equilibrium we have $\varphi_{BA} = \varphi_{AA} = 0$ and $\varphi_{AB} = \varphi_{BB} = \tilde{x}$. The fact that $\varphi_{AB} = \frac{v(AB)}{1/2} = \tilde{x}$ and $\varphi_{BB} = \frac{v(BB)}{1/2} = \tilde{x}$ implies $v(AB) = v(BB) = \frac{\tilde{x}}{2}$. We also have $\tau_{AB} = \tau_{BB} = 1$, and

$$\tau_{AA}(w,\tilde{x},\gamma) = \frac{(\tilde{x} + 2w + 6\gamma - 1)\gamma - \sqrt{8\gamma^2(1 - \tilde{x} - 2w - 4\gamma^2) + (6\gamma^2 + \tilde{x}\gamma - \gamma + 2w\gamma)^2}}{4\gamma^2}$$

and

$$\tau_{BA}(w,\tilde{x},\gamma) = \frac{\left(3 + \tilde{x} + 2w - 2\gamma\right)\gamma - \sqrt{8\gamma^2(1 - \tilde{x} - 2w - 4\gamma^2) + (6\gamma^2 + \tilde{x}\gamma - \gamma + 2w\gamma)^2}}{4\gamma(1-\gamma)}.$$

Note also that

$$\begin{aligned}
P(\omega_{t-2} = A, \omega_{t-1} = A, m_{t-1} = A) &= \mu_A \gamma \tau_{AA},\\
P(\omega_{t-2} = B, \omega_{t-1} = A, m_{t-1} = A) &= \mu_B \frac{1}{2}\tau_{BA},\\
P(\omega_{t-2} = A, \omega_{t-1} = A, m_{t-1} = B) &= \mu_A \gamma(1 - \tau_{AA}),\\
P(\omega_{t-2} = B, \omega_{t-1} = A, m_{t-1} = B) &= \mu_B \frac{1}{2}(1 - \tau_{BA}),\\
P(\omega_{t-2} = A, \omega_{t-1} = B, m_{t-1} = B) &= \mu_A(1 - \gamma),\\
P(\omega_{t-2} = B, \omega_{t-1} = B, m_{t-1} = B) &= \mu_B \frac{1}{2},\\
P(\omega_{t-2} = A, \omega_{t-1} = B, m_{t-1} = A) &= \mu_A(1 - \gamma)(0) = 0,\\
P(\omega_{t-2} = B, \omega_{t-1} = B, m_{t-1} = A) &= \mu_B \frac{1}{2}(0) = 0,
\end{aligned}$$

where $\mu_A = \frac{1}{3-2\gamma}$.

Step 3 Let us derive the average per-period action payoff. Note that the average action payoff corresponds to the sum of the probabilities of the three events (denoted by Events 1, 2 and 3, respectively) in which R chooses an action that matches the state. Event 1 is that the state is B and R visits, in which case R receives message B with probability 1. Event 2 is that the state is A and R visits and receives message A. Event 3 is that the state is A and R does not consult. It follows that the expected probability of R choosing the correct action is given by

$$\begin{aligned}&\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1})\Big[P(\omega_t = B\,|\omega_{t-1})\varphi_{\omega_{t-2}m_{t-1}} \\ &\quad + P(\omega_t = A\,|\omega_{t-1})(1 - \varphi_{\omega_{t-2}m_{t-1}}(1 - \tau_{\omega_{t-1}A}))\Big].\end{aligned}$$

Using the fact that

$$P(\omega_{t-2} = A, \omega_{t-1} = B, m_{t-1} = A) = P(\omega_{t-2} = B, \omega_{t-1} = B, m_{t-1} = A) = 0,$$

we may write

$$\begin{aligned}&\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1}) \\ &\times \left[\begin{array}{c} P(\omega_t = B\,|\omega_{t-1})\varphi_{\omega_{t-2}m_{t-1}} \\ +P(\omega_t = A\,|\omega_{t-1})(1 - \varphi_{\omega_{t-2}m_{t-1}}(1 - \tau_{\omega_{t-1}A})) \end{array}\right] \\ =\;& \\ &\mu_A\gamma\tau_{AA}\left[P(\omega_t = B\,|\omega_{t-1} = A)\varphi_{AA}\right. \\ &\quad \left.+ P(\omega_t = A\,|\omega_{t-1} = A)(1 - \varphi_{AA}(1 - \tau_{AA}))\right]\end{aligned}$$

$$
\begin{aligned}
&+ \mu_B \frac{1}{2}\tau_{BA}\left[P(\omega_t = B\,|\omega_{t-1} = A)\varphi_{BA}\right.\\
&\left.+ P(\omega_t = A\,|\omega_{t-1} = A)(1-\varphi_{BA}(1-\tau_{AA}))\right]\\
&+ \mu_A\gamma(1-\tau_{AA})\left[P(\omega_t = B\,|\omega_{t-1} = A)\varphi_{AB}\right.\\
&\left.+ P(\omega_t = A\,|\omega_{t-1} = A)(1-\varphi_{AB}(1-\tau_{AA}))\right]\\
&+ \mu_B\frac{1}{2}(1-\tau_{BA})\left[P(\omega_t = B\,|\omega_{t-1} = A)\varphi_{BB}\right.\\
&\left.+ P(\omega_t = A\,|\omega_{t-1} = A)(1-\varphi_{BB}(1-\tau_{AA}))\right]\\
&+ \mu_A(1-\gamma)\left[P(\omega_t = B\,|\omega_{t-1} = B)\varphi_{AB}\right.\\
&\left.+ P(\omega_t = A\,|\omega_{t-1} = B)(1-\varphi_{AB}(1-\tau_{BA}))\right]\\
&+ \mu_B\frac{1}{2}\left[P(\omega_t = B\,|\omega_{t-1} = B)\varphi_{BB}\right.\\
&\left.+ P(\omega_t = A\,|\omega_{t-1} = B)(1-\varphi_{BB}(1-\tau_{BA}))\right].
\end{aligned}
$$

Using the observations in Step 2, the above expression in turn rewrites as

$$
\begin{aligned}
&\mu_A\gamma\tau_{AA}\left[P(\omega_t = A\,|\omega_{t-1} = A)\right]\\
&+\mu_B\frac{1}{2}\tau_{BA}\left[P(\omega_t = A\,|\omega_{t-1} = A)\right]\\
&+\mu_A\gamma(1-\tau_{AA})\left[P(\omega_t = B\,|\omega_{t-1} = A)\widetilde{x} + P(\omega_t = A\,|\omega_{t-1} = A)(1-\widetilde{x}(1-\tau_{AA}))\right]\\
&+\mu_B\frac{1}{2}(1-\tau_{BA})\left[P(\omega_t = B\,|\omega_{t-1} = A)\widetilde{x} + P(\omega_t = A\,|\omega_{t-1} = A)(1-\widetilde{x}(1-\tau_{AA}))\right]\\
&+\mu_A(1-\gamma)\left[P(\omega_t = B\,|\omega_{t-1} = B)\widetilde{x} + P(\omega_t = A\,|\omega_{t-1} = B)(1-\widetilde{x}(1-\tau_{BA}))\right]\\
&+\mu_B\frac{1}{2}\left[P(\omega_t = B\,|\omega_{t-1} = B)\widetilde{x} + P(\omega_t = A\,|\omega_{t-1} = B)(1-\widetilde{x}(1-\tau_{BA}))\right],
\end{aligned}
$$

which further simplifies to

$$
\begin{aligned}
&\left(\frac{1}{3-2\gamma}\right)\gamma\tau_{AA}\left(w,\widetilde{x},\gamma\right)\gamma + \left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}\tau_{BA}\left(w,\widetilde{x},\gamma\right)\gamma\\
&+\left(\frac{1}{3-2\gamma}\right)\gamma\left(1-\tau_{AA}\left(w,\widetilde{x},\gamma\right)\right)\left[(1-\gamma)\widetilde{x}+\gamma\left(1-\widetilde{x}\left(1-\tau_{AA}\left(w,\widetilde{x},\gamma\right)\right)\right)\right]\\
&+\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}(1-\tau_{BA}\left(w,\widetilde{x},\gamma\right))\left[(1-\gamma)\widetilde{x}+\gamma\left(1-\widetilde{x}\left(1-\tau_{AA}\left(w,\widetilde{x},\gamma\right)\right)\right)\right]\\
&+\left(\frac{1}{3-2\gamma}\right)(1-\gamma)\left[\frac{1}{2}\widetilde{x}+\frac{1}{2}\left(1-\widetilde{x}\left(1-\tau_{BA}\left(w,\widetilde{x},\gamma\right)\right)\right)\right]\\
&+\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}\left[\frac{1}{2}\widetilde{x}+\frac{1}{2}\left(1-\widetilde{x}\left(1-\tau_{BA}\left(w,\widetilde{x},\gamma\right)\right)\right)\right].
\end{aligned}
$$

Step 4 Invoking Step 1 and Step 2, we can write R's per period expected visiting cost as

$$
\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1})\varphi_{\omega_{t-2}m_{t-1}}\left(\frac{v(\omega_{t-2}m_{t-1})}{2}+w\right)
$$
$$
=\begin{bmatrix} P(\omega_{t-2}=A,\omega_{t-1}=A,m_{t-1}=B)\\ +P(\omega_{t-2}=B,\omega_{t-1}=A,m_{t-1}=B)\\ +P(\omega_{t-2}=A,\omega_{t-1}=B,m_{t-1}=B)\\ +P(\omega_{t-2}=B,\omega_{t-1}=B,m_{t-1}=B)\end{bmatrix}\times\widetilde{x}\left(\frac{\widetilde{x}}{4}+w\right),
$$

which in turn rewrites as

$$
\begin{pmatrix} \frac{1}{3-2\gamma}\gamma(1-\tau_{AA}\left(w,\widetilde{x},\gamma\right))\\ +\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}(1-\tau_{BA}\left(w,\widetilde{x},\gamma\right))\\ +\frac{1}{3-2\gamma}(1-\gamma)+\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}\end{pmatrix}\widetilde{x}\left(\frac{\widetilde{x}}{4}+w\right).
$$

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