The Shale Revolution, Geopolitical Risk, and Oil Price Volatility

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Abstract

The U.S. shale revolution, using new technologies to extract crude oil, has led to new dynamics in the supply side of the global oil market. We ask whether the shale revolution has dampened the role of geopolitical risk in oil price volatility. We extend a reduced form Structural Break Threshold Vector Autoregressive (SBT-VAR) model to a structural SBT-VAR model and identify the structural innovations by allowing conditional heteroskedasticity. Compared with the conventional reduced form VAR and TVAR models, an SBT-VAR with a constant threshold and a break in April 2014 are supported by the data. We then analyse the conditional (co)variance impulse response concerning two distinct shock scenarios, one with only a geopolitical risk shock, and the other with a simultaneous shale production shock and a geopolitical risk shock. The volatility responses are due to the identified contemporaneous relationships amongst geopolitical risk, shale production and oil prices, and are conditional on volatilities at the points in time. With the extra unit shale production shock, we find that the volatility response of oil prices to a geopolitical risk shock is higher, but the response is less correlated with the geopolitical risk factor.

JEL classification: C32, Q43

Keywords: U.S. shale oil revolution, Geopolitical risk, Oil price volatility, Structural break threshold VAR models

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> *So the economics of oil have changed. The market will still be subject to political shocks: war in the Middle East or the overdue implosion of Vladimir Putin's kleptocracy would send the price soaring. But, absent such an event, the oil price should be less vulnerable to shocks or manipulation. Even if the 3m extra b/d that the United States now pumps out is a tiny fraction of the 90m the world consumes, America's shale is a genuine rival to Saudi Arabia as the world's marginal producer. That should reduce the volatility not just of the oil price but also of the world economy. Oil and finance have proved themselves the only two industries able to tip the world into recession. At least one of them should in future be a bit more stable.*

The Economist, *Sheikhs v shale - The new economics of oil*, Dec 4th 2014

1 Introduction

Understanding the price dynamics of crude oil is crucial for policymakers, business leaders, and consumers. The variation in oil prices is susceptible to supply shocks, in particular, supply shocks by physical disruptions. Therefore, it is essential to analyse the impacts of geopolitical risk, which has been one of the most crucial security risks to the oil supply.

Due to the advances in technology in fracking shale oil, the supply condition in the global oil market has changed. It was confirmed by the U.S. Energy Information Administration on July 16, 2018, "the U.S. oil output from seven major shale formations is expected to rise to a record 7.47 million barrels per day in August 2018". As the shale revolution continues to drive oil production, we have observed low oil prices between June 2014 and February 2016. Although the shale revolution is not solely responsible for the substantial fall in oil prices since 2014, Kilian (2017) argues that the Brent price of crude oil was lower by $10 than it would have been in the absence of the fracking boom. Monge et al. (2017) investigate the relationship between the shale oil revolution in the U.S. and

WTI oil price behaviour, which suggests they are negatively correlated during 2009-2014. Shakya et al. (2022) confirm significant information spillover between drilling activities and energy prices, and the spillover effect increased since the shale boom.

The long quote above constitutes one of the hypotheses for the significant technology changes in the shale oil industry,: "(the shale revolution) should reduce the volatility of the oil price". Thus, in this chapter, one of our objectives is to investigate the impact of geopolitical uncertainties on oil price volatility under a simultaneous shale oil production shock.

Most oil price-related research focuses on the nonlinear relationship between oil prices and GDP growth, such as in Hamilton (2003). Consistent with most of the findings in the literature, Hamilton (2003) concludes that upside risks in oil prices are more of a threat than downside risks in real economic activities. The framework laid by Hamilton (2009) and Kilian (2009) led many recent studies, such as Prest (2018), to identify the oil shocks and improve our understanding of their historical causes and consequences. On the other hand, research shed light on oil price forecasting (Degiannakis and Filis, 2018), and the relationship between oil prices and the financial market (Degiannakis et al., 2018a,b). Further, oil price realised volatility is widely applied in this field (Degiannakis and Filis, 2017, 2022). The oil price driving force has been identified from both the supply and the demand sides, see Dées et al. (2007) and Hamilton (2009). Empirical evidence also supports that most of the historical oil price shocks were caused by physical disruptions of supply, see Hamilton (2009). Hamilton (2009) finds that the oil price run-up of 2007-08 was a joint effect of stagnating world production and demand. We follow a similar direction, and besides evaluating the oil price volatility responses to exogenous shocks, we focus on the nonlinear oil price dynamics and its response to geopolitical risks amidst the shale revolution.

With respect to econometric modelling, the seminal chapter by Sims (1980) proposes to use the Vector Autoregressive (VAR) to conduct macroeconomic analysis. For instance, a bivariate VAR model is employed in Kilian and Vigfusson (2011) to study how GDP responds to asymmetric oil price changes.

The baseline VAR model has been generalized in many different dimensions to capture nonlinear dynamics in macroeconomic variables. For example, a time-varying-parameter VAR (TVP-VAR) (Koop and Korobilis, 2013) and a smooth-transition VAR (Hubrich and Teräsvirta, 2013) are proposed to address different types of nonlinearities. To answer our question, of whether the response of oil price to geopolitical risk has changed under the shale revolution, we apply a generalization of the baseline VAR model - a structural break threshold VAR (SBT-VAR) model proposed by Galvão (2006).

The Structural Break Threshold Vector Autoregressive (SBT-VAR) model is a nonlinear structural model developed based on the baseline VAR model. The VAR model is applied in macroeconomic dynamics for decades. Further, nonlinear dynamic terms are developed in many ways. The structural-break estimator in baseline VAR is a useful tool to identify the relationship changes in a group of variables. Research on the dynamics between oil and other assets has been used structural break VAR in varying topics (Avalos, 2014; Bondia et al., 2016; Enders and Jones, 2016; Fasanya et al., 2018). The supply shock is one of the main reasons for oil price fluctuations, even structural breaks as well. The structural break (SB) point and the parameters in the before-SB and post-SB periods estimation are presented in a large amount of literature (Baumeister and Peersman, 2013; Cashin et al., 2014; Kilian and Zhou, 2019). To answer our research question, whether the response of oil price to geopolitical risk has changed under the shale oil production revolution, the VAR model fitted in the structural break estimators (SB-VAR) is efficient. Threshold effect in oil price drivers triggers regime shift in oil price behaviours. Geopolitical risk causes interruption in the oil supplies. Therefore, we adopted the geopolitical threat risk (GPR) which can cause oil supply side shocks, as the threshold variable in this research. The interaction between shale oil production and oil price under different regimes is examined by estimating threshold terms. Several pieces of literature shed light on the interactivities in oil supply shock and oil price by adopting threshold VAR (TVAR) (Atems et al., 2015; Balcilar et al., 2022; Barrales-Ruiz and Mohammed, 2021; Sek, 2019; Van Robays, 2016). Further, Nonlinear structural models, incorporating thresholds and breaks are proposed in Baum and Koester (2011) and Galvão and Marcellino (2013). Therefore, we use the SBT-VAR specification to investigate the new dynamic in shale oil revolution production in the U.S. oil market. It is able to provide the results of structural break time, and the threshold value estimated simultaneously. Based on the previous research, SBT-VAR is an appropriate approach to deal with the research question in this paper. Meanwhile, it fills the potential research gap in this field.

Cholesky decomposition is generally used to solve this problem, i.e., by imposing exclusion restrictions on parameter $\boldsymbol{A_0}$. However, the estimation accuracy relies on the ordering of the variables. Therefore, justifications for the ordering have to be made after consulting economic theory. The exclusion restriction on the impact effects of structural shocks has been applied in Sims (1980) and Kilian (2009). In Blanchard and Quah (1989), identification is achieved by restrictions on the long-run effects. For instance, the restriction is imposed on the aggregate demand shock, assuming the aggregate demand shock has no long-run impact on the GNP. Restrictions, such as on the signs of the responses of specific variables to a shock, were used for identification in Uhlig (2005). Interested readers can find a comprehensive review of the various identification methods in Kilian and Lütkepohl (2017). Because of the drawbacks of identifications through Cholesky decomposition and sign restriction methods, Bouakez et al. (2013), Bouakez et al. (2014), Lütkepohl and Netšunajev (2014), Lütkepohl and Netšunajev (2017) and Elder and Serletis (2010) propose to use a GARCH specification for the structural innovation to identify the SVAR model. In this chapter, we utilized the identification procedure by allowing heteroskedasticity in the structural innovations. It illustrates that using an exclusion restriction on structural model identification may arrive at very different inferences of the impulse response functions.

With respect to empirical applications in the literature, SVAR has been utilized to analyse the oil prices' responses to different measures of geopolitical risks. For instance, Coleman (2012) finds that the frequency of fatal terrorist attacks in the Middle East and the U.S. troop numbers in the Middle East explain a significant amount of variation in crude oil prices. Chen et al. (2016) find a significant and positive causal effect of OPEC political risk on Brent crude oil prices. Rather than using dummy variables and focusing on a specific geopolitical event as in Noguera-Santaella (2016), we apply a geopolitical risk index, constructed by Caldara and Iacoviello (2018), to a structural break threshold VAR (SBT-VAR) model. With the SBT-VAR model specification, the unknown break-point and threshold can be estimated using maximum likelihood. Then, the reduced form SBT-VAR model is generalized to its structural form, which is identified through a GARCH specification in the structural innovations as in Bouakez et al. (2013) and Lütkepohl and Netšunajev (2017). Finally, we analyse the impulse response functions of oil prices to geopolitical shocks. Further, we compare the volatility impulse responses of oil prices to geopolitical shocks under two distinct scenarios. In the first scenario, oil price volatilities respond to a simultaneous shale production shock and a geopolitical risk shock. In the second scenario, the oil price volatilities respond to a hypothetical shock from only one source, i.e., the geopolitical risk.

The chapter is organized as follows: Section 2 presents the model. Section 3 illustrates the empirical results. Section 4 concludes.

2 Method

2.1 Reduced Form SBT-VAR and Structural SBT-VAR

First, we apply a reduced form Structural Break Threshold VAR (SBT-VAR) model proposed by Galvão (2006) to identify the changing dynamics in the oil price. The reduced form SBT-VAR is specified as:

$$\begin{aligned}\boldsymbol{y_t} \;=\;& \left[\boldsymbol{\Gamma_1 X_t} I_{1,t-d_1}\left(r_1\right)+\boldsymbol{\Gamma_2 X_t}\left(1-I_{1,t-d_1}\left(r_1\right)\right)\right] I_t\left(\tau\right)+\\ & \left[\boldsymbol{\Gamma_3 X_t} I_{2,t-d_2}\left(r_2\right)+\boldsymbol{\Gamma_4 X_t}\left(1-I_{2,t-d_2}\left(r_2\right)\right)\right]\left(1-I_t\left(\tau\right)\right)+\boldsymbol{u_t}, \qquad (1)\end{aligned}$$

where $I\left(\cdot\right)$ is an indicator function. Denote the threshold as r_i, the delay parameter as d_i, the break-point as τ, and the transition variable as z, then, the threshold and break indicator functions are defined as $I_{i,t-d_i}\left(r_i\right)=1\left(z_{t-d_i}\leq r_i\right)$, and $I_t\left(\tau\right)=1\left(t\leq\tau\right)$. The error term (or statistical innovation), $\boldsymbol{u_t}$, follows a multivariate normal, $\boldsymbol{u_t}\overset{i.i.d}{\sim} MN\left(\mathbf{0},\boldsymbol{\Sigma_u}\right)$, and $\boldsymbol{\Sigma_u}\in\mathbb{R}^{k\times k}$.

Structural form SBT-VAR (denote SBT-SVAR hereafter) is applied to disentangle the marginal effects of exogenous shocks. The SBT-SVAR is specified as follows:

$$\begin{aligned}\boldsymbol{A_0 y_t} \;=\;& \left[\boldsymbol{A_1 X_t} I_{1,t-d_1}\left(r_1\right)+\boldsymbol{A_2 X_t}\left(1-I_{1,t-d_1}\left(r_1\right)\right)\right] I_t\left(\tau\right)+\\ & \left[\boldsymbol{A_3 X_t} I_{2,t-d_2}\left(r_2\right)+\boldsymbol{A_4 X_t}\left(1-I_{2,t-d_2}\left(r_2\right)\right)\right]\left(1-I_t\left(\tau\right)\right)+\boldsymbol{\epsilon_t}, \quad (2)\end{aligned}$$

where $\boldsymbol{u_t}=\boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}$ or $\boldsymbol{A_0 u_t}=\boldsymbol{\epsilon_t}$. The coefficient matrix $\boldsymbol{\Gamma_i}$ can be found from $\boldsymbol{\Gamma_i} = \boldsymbol{A_0}^{-1}\boldsymbol{A_i}$ with $i = 1, 2, 3, 4$.

The (co)variance of the error term of the model in the reduced form $\boldsymbol{\Sigma_u}$ in and the unconditional variance of structural shocks $\boldsymbol{\Sigma_\epsilon}$ in can be linked by $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\boldsymbol{\Sigma_\epsilon}\left(\boldsymbol{A_0}^{-1}\right)^T$. We impose normalization on the $\boldsymbol{\Sigma_\epsilon}$. Then, $\boldsymbol{\Sigma_\epsilon}$ is normalized to an identify matrix, $\boldsymbol{I_k}$ with $\boldsymbol{I_k} \in \mathbb{R}^{k\times k}$, and $\boldsymbol{\Sigma_u}$ is

$$\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\left(\boldsymbol{A_0}^{-1}\right)^T, \tag{3}$$

$\boldsymbol{A_0}$ is the upper-triangular matrix by applying a Cholesky factorization. It is a straightforward solution to identify the structural shocks in a structural SBT-VAR (SBT-SVAR) model. However, using the Cholesky decomposition implies restrictions on the direction of contemporaneous effects of structural shocks. Therefore, we apply a flexible method for STB-SVAR model identification - allowing heteroskedastic structural errors.

2.2 GARCH Structural Errors for identify STB-SVAR

GARCH (structural) innovations is used to identify SVAR has been proposed in (Lütkepohl and Netšunajev, 2017; Bouakez et al., 2013, 2014; Sentana and Fiorentini, 2001; Elder and Serletis, 2010). We identify elements in $\boldsymbol{A_0}$ by using statistical innovations $\boldsymbol{\hat{u}_t}$ from the SBT-VAR model and a GARCH specification of heteroskedasticity in $\boldsymbol{\Sigma_{\epsilon,t}}$ The statistical innovation $\boldsymbol{u_t}$ and the structural innovations $\boldsymbol{\epsilon_t}$ are linked by $\boldsymbol{A_0}$, where

$$\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}, \tag{4}$$

In the spirit of Lütkepohl and Milunovich (2016), a GARCH(1,1) process is specified for the conditional variance of the structural innovations:

$$\boldsymbol{\Sigma_{\epsilon,t}} = (\boldsymbol{I_k} - \Delta_1 - \Delta_2) + \Delta_1 \circ \left(\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}}^T\right) + \Delta_2 \circ \boldsymbol{\Sigma_{\epsilon,t-1}}, \tag{5}$$

where Δ_1 and Δ_2 are diagonal matrices, and "$\circ$" denotes the Hadamard product operator. If Δ_1 and Δ_2 are null, then $\boldsymbol{\Sigma_{\epsilon,t}}$ is constant. Whereas, if Δ_1 and Δ_2 are positive semi-definite, then $(\boldsymbol{I_k} - \Delta_1 - \Delta_2)$ is positive definite, which indicates that at least one of the structural innovations follow a GARCH(1,1) process.

Therefore, the GARCH(1,1) specification for an individual conditional structural variance is

$$\sigma^2_{m,t|t-1} = (1-\gamma_m-\delta_m) + \gamma_m \epsilon^2_{m,t-1} + \delta_m \sigma^2_{m,t-1|t-2}, \quad m = 1,\ldots,k. \tag{6}$$

We follow a two-step procedure, see Bouakez et al. (2013) and Bouakez et al. (2014), to estimate the ARCH coefficients Δ_1 and GARCH coefficients Δ_2. The first step requires extracting the estimated statistical innovations $\hat{\boldsymbol{u}}_{\boldsymbol{t}}$ from SBT-VAR specification. The second step involves estimating the structural parameters, i.e. the non-zero elements in $\boldsymbol{A_0}$, as well as Δ_1 and Δ_2.

2.3 Generalized Impulse Response Functions with SBT-SVAR

To evaluate the impact effects of structural shocks in a non-linear SBT-SVAR system, we apply the Generalized Impulse Response Functions (GIRF) proposed by (Koop et al., 1996), which requires a h-step ahead forecasting of the conditional mean of $\boldsymbol{y_{t+h}}$, in response to a one-unit structural shock, $E\left[\boldsymbol{y_{t+h}} \mid \boldsymbol{\xi_t}, \mathcal{F}_{t-1}\right]$, and a h-step ahead forecasting of $\boldsymbol{y_{t+h}}$ only conditional on the history, $E\left[\boldsymbol{y_{t+h}} \mid \mathcal{F}_{t-1}\right]$. The GIRF is then

$$GIRF\left(\boldsymbol{h}, \boldsymbol{\xi_t}, \mathcal{F}_{t-1}\right) = E\left[\boldsymbol{y_{t+h}} \mid \boldsymbol{\xi_t}, \mathcal{F}_{t-1}\right] - E\left[\boldsymbol{y_{t+h}} \mid \mathcal{F}_{t-1}\right]. \tag{7}$$

2.4 Variance Impulse Response Functions with GARCH structural errors

We compare the dynamic responses of the conditional covariance between oil prices and geopolitical risk under two distinct scenarios - one with a simultaneously perturbing shale production structural shock, and one without.

In the spirit of Koop et al. (1996), Hafner and Herwartz (2006) propose a variance impulse response function (VIRF). Denote $\boldsymbol{\Sigma_{u,t}}$ as the initial conditional variance preceding the structural shock $\boldsymbol{\xi_t}$, the general expression for VIRF is

$$V_{t+h}\left(\boldsymbol{\xi_t}\right) = E[\text{vech}\left(\boldsymbol{\Sigma_{u,t+h}}\right) \mid \boldsymbol{\xi_t}, \mathcal{F}_{t-1}] - E[\text{vech}\left(\boldsymbol{\Sigma_{u,t+h}}\right) \mid \mathcal{F}_{t-1}]. \tag{8}$$

The VIRF calculates the differences between the expectation of volatility conditional on a perturbing shock $\boldsymbol{\xi_t}$ and the history $\mathcal{F}_{t-1}$, and the expectation of volatility only conditional on the history. Hafner and Herwartz (2006) consider a vec representation of multivariate GARCH(1,1) specified as

$$\operatorname{vech}(\boldsymbol{\Sigma_{u,t}}) = \boldsymbol{W} + \tilde{\boldsymbol{A}}_{\mathbf{1}} \operatorname{vech}\left(\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^{T}\right) + \tilde{\boldsymbol{B}}_{\mathbf{1}} \operatorname{vech}(\boldsymbol{\Sigma_{u,t-1}}). \tag{9}$$

In the spirit of Hafner and Herwartz (2006) by using VARMA representation of GARCH model, the general analytic expression for VIRF is as follows :

$$V_{t+h}\left(\boldsymbol{\xi_t}\right) = \boldsymbol{\phi_h}\boldsymbol{D_k^{+}}\left(\boldsymbol{\Sigma_{u,t}}^{1/2} \otimes \boldsymbol{\Sigma_{u,t}}^{1/2}\right)\boldsymbol{D_k} \operatorname{vech}\left(\boldsymbol{\xi_t}\boldsymbol{\xi_t}^{T} - \boldsymbol{I_k}\right), \tag{10}$$

where $\boldsymbol{\phi_h} = \left(\tilde{\boldsymbol{A}}_{\mathbf{1}} + \tilde{\boldsymbol{B}}_{\mathbf{1}}\right)^{h-1} \tilde{\boldsymbol{A}}_{\mathbf{1}}$, $\boldsymbol{D_m}$ denotes the duplication matrix defined by the property $\boldsymbol{vec(Z)} = \boldsymbol{D_m} \operatorname{vech}(Z)$ for any symmetric $(m \times m)$ matrix $\boldsymbol{Z}$, and $\boldsymbol{D_m^{+}}$ denotes its Moore-Penrose inverse.

Motivated by van der Weide (2002), Bauwens et al. (2006), and (Engle and Kroner, 1995), we transform the GO-GARCH into the BEKK. Then vec GARCH representation is settled down. Finally, the VIRF can be calculated using the estimated $\widehat{\boldsymbol{A_0}}^{-1}$ from the identified SBT-SVAR model.

There are two methodologies to calculate variance impulse response function. One is the conditional moment profile framework ()Gallant1993, another one the VIRF proposed by Hafner and Herwartz (2006). The main different between them is imposing varying shocks in return when general impulse response functions. In our case, we choose fixed values of hypothetical $\boldsymbol{\xi_t}$, and investigate the conditional (co)variances respond to these hypothetical orthogonal shocks $\boldsymbol{\xi_t}$, given different points in time with a heteroskedastic $\boldsymbol{\Sigma_{u,t}}$.

We impose a hypothetical structural shock $\boldsymbol{\xi_t}$ at time t as $\boldsymbol{\xi_t} = \boldsymbol{\epsilon_t} = (1, 0, 0)^{T}$. The order of the variables indicates that there is only one unit hypothetical structural shock imposed on the geopolitical risk at time t. Let another type of unit hypothetical structural shock $\boldsymbol{\xi_t^{*}}$ be imposed on both geopolitical risk and shale production simultaneously, i.e. $\boldsymbol{\xi_t^{*}} = \boldsymbol{\epsilon_t^{*}} = (1, 1, 0)^{T}$. Denoting the VIRF with shock $\boldsymbol{\xi_t^{*}}$ for h periods ahead as V_{t+h}^{*}, and VIRF with shock $\boldsymbol{\xi_t}$ as V_{t+h}, calculates the differences in VIRF under two different circumstances given a perturbing geopolitical shock, i.e. one under a simultaneous shale shock, whereas the other without.

$$V_{t+h}^{*}\left(\boldsymbol{\epsilon_t^{*}}\right) - V_{t+h}\left(\boldsymbol{\epsilon_t}\right) = \boldsymbol{\phi_h}\boldsymbol{D_k^{+}}\left(\boldsymbol{\Sigma_{u,t}}^{1/2} \otimes \boldsymbol{\Sigma_{u,t}}^{1/2}\right)\boldsymbol{D_k} \operatorname{vech}\left(\boldsymbol{\xi_t^{*}}\boldsymbol{\xi_t^{*T}} - \boldsymbol{\xi_t}\boldsymbol{\xi_t^{T}}\right) \tag{11}$$

We now look at the differences in variance responses in the two scenarios using the conditional volatility profiles. Suppose the shock $\boldsymbol{\delta_t}$ directly inflicts on $\boldsymbol{y_t}$ takes the form $\boldsymbol{\delta_t} = \boldsymbol{\Sigma_t^{1/2}} \boldsymbol{\epsilon_t}$, and suppose a hypothetical shock $\boldsymbol{\epsilon_t} = (1, 0, 0)^T$, the corresponding conditional variance profile is denoted as $\boldsymbol{v_{t+h}}$. In another scenario, suppose the hypothetical shock consists of both a geopolitical risk shock and a shale production shock, i.e. $\boldsymbol{\epsilon_t^*} = (1, 1, 0)^T$, and shock $\boldsymbol{\delta_t^*}$ takes the form $\boldsymbol{\delta_t^*} = \boldsymbol{\Sigma_t^{1/2}} \boldsymbol{\epsilon_t^*}$, denote the corresponding conditional variance profile as $\boldsymbol{v_{t+h}^*}$. The difference between these two conditional (co)variance profiles under the two different direct perturbations is

$$v_{t+h}^* \left(\boldsymbol{\delta_t^*} \right) - v_{t+h} \left(\boldsymbol{\delta_t} \right) = \boldsymbol{\phi_{t+h}} \left(\text{vech} \left(\boldsymbol{\Sigma_{u,t}^{1/2}} \boldsymbol{\epsilon_t^*} \boldsymbol{\epsilon_t^{*T}} \boldsymbol{\Sigma_{u,t}^{1/2}} \right) - \text{vech} \left(\boldsymbol{\Sigma_{u,t}^{1/2}} \boldsymbol{\epsilon_t} \boldsymbol{\epsilon_t^T} \boldsymbol{\Sigma_{u,t}^{1/2}} \right) \right)$$
$$= V_{t+h}^* \left(\boldsymbol{\epsilon_t^*} \right) - V_{t+h} \left(\boldsymbol{\epsilon_t} \right) \tag{12}$$

Therefore, using a fixed baseline, the impact differences between the two fixed hypothetical shocks $\boldsymbol{\epsilon_t^*}$ and $\boldsymbol{\epsilon_t}$ has the same analytical expression in the conditional volatility profile as that in the VIRF framework.

In order to evaluate the impact effects of shale revolution, we analyse the differences in VIRF, $V_{t+h}^* \left(\boldsymbol{\epsilon_t^*} \right) - V_{t+h} \left(\boldsymbol{\epsilon_t} \right)$, or the differences in two conditional volatilities profiles, $v_{t+h}^* \left(\boldsymbol{\epsilon_t^*} \right) - v_{t+h} \left(\boldsymbol{\epsilon_t} \right)$, under the two types of hypothetical shocks, i.e. shock $\boldsymbol{\epsilon_t^*}$ represents that shale production is imposed with a simultaneous shock as well as geopolitical risk, whereas $\boldsymbol{\epsilon_t}$ indicates only the geopolitical risk variable is imposed with a unit shock.

3 Empirical results

In this section, we aim to analyse the changing dynamics of oil prices under the shale oil revolution. Also, we would like to pin down the impact effects of geopolitical risk on oil prices by allowing simultaneous shale production shocks.

3.1 Data

The dataset constructed of geopolitical risk, shale oil production, and oil price is in monthly frequency from January 2000 to October 2017. Regarding the measurement of geopolitical risk, we use an index constructed by Caldara and Iacoviello (2018). Caldara and Iacoviello (2018) follow the method proposed in Saiz and Simonsohn (2013) and Baker et al. (2016) and conduct an automatic text search over 11 English newspapers, including 6 from the U.S., 4 British ones, and 1 Canadian newspaper. The index is constructed by counting the frequency of articles related to geopolitical risks, i.e. $GPR = NumberofGPRrisk\ \ articles/NumberofTotal\ \ articles$. In Caldara and Iacoviello (2018), geopolitics is defined as "the practice of states and organizations to control and compete for territory", whereas geopolitical risk is defined as "risks associated with wars, terrorist acts, or tensions between states that affect the normal and peaceful course of international relations". Hence, the geopolitical threat risk index constructed by Caldara and Iacoviello (2018) relates to threats of conflicts in the world and it represents the proportion of western mainstream media coverage in English-speaking countries. In fig. (1), the top left plots the monthly geopolitical threat risk index (GPR).

Dataset transformation is applied to fit estimation. To be specific, the GPR index is standardized using $\left(GPR_t - \overline{GPR}\right)/\sigma_{GPR}$. The monthly shale oil production (ShaleP), measured by thousands of barrels per day, is collected from the U.S. Energy Information Administration (EIA). The West Texas Intermediate (WTI) crude oil spot prices, in dollars per barrel, are also collected from the U.S. EIA. Then, the WTI prices are adjusted to real prices using a monthly CPI index. The CPI index (for all urban consumers) is collected from the Bureau of Labour Statistics, [1982-84=100] [1]. The return of oil price is calculated by $R_t = 100 \times \log\left(\Delta OilP_t\right)$. Fig. (1) plots all original and transformed data series.

3.2 Estimation results

First, the data $\boldsymbol{y_t} = \left(GPR_t, 100 \times \Delta \log ShaleP_t, 100 \times \Delta \log OilP_t\right)^T$ is fitted using five competing models, which are VAR, TVAR, SBVAR, SBTVAR with changing thresholds, and SBTVAR with a constant threshold. The geopolitical threat risk Index, GPR_t, is chosen as the threshold variable z. Table (1) indicates that based on the LR, FPF and AI criterion, the optimal lag length is selected as $p = 2$ in the baseline VAR model from an Ordinary Least Square (OLS) estimation. Therefore, we pre-specify $p = 2$ in the reduced form SBT-VAR.

[1] https://www.bls.gov/cpi/tables/supplemental-files/historical-cpi-u-201805.pdf

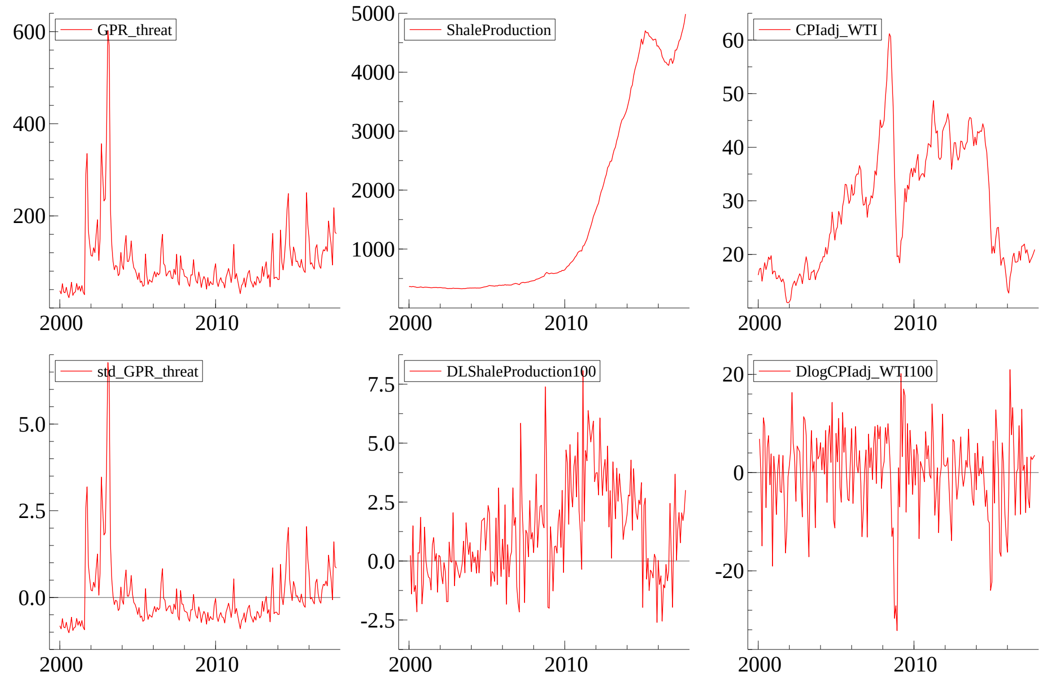

Figure 1: Plots of data series (in monthly frequency). Top panel from left to right plots: monthly GPR-threat index, monthly shale oil production (ShaleP) in thousands of barrels per day, CPI adjusted monthly WTI oil price. Bottom panel from left to right plots: $GPR - threat_t$, standardized monthly GPR-threat index, $100 \times \Delta \log ShaleP_t$, monthly percentage change in shale oil production, $100 \times \Delta \log OilP_t$, monthly return on WTI

Table 1: Optimal lag length p selection in the baseline VAR

lag	LL	LR	df	p	FPF	AIC	HQIC	SBIC
0	-1497.62				301.937	14.223	14.243	14.271
1	-1365.41	264.41	9	0.000	93.918	13.056	13.133*	13.247*
2	-1351.3	28.227*	9	0.001	89.48*	13.008*	13.142	13.341*

Table 2: Estimation results using the reduced form models

	TVAR	SBVAR	SBTVARc	SBTVAR
$\hat{d}$	12	–	1	15, 23
$\hat{r}$	0.088	–	-0.089	-0.357
	(-4.819, 4.417)		(-12.332, -1.284)	(0.043, 2.229)
	TR1[-0.4252]			0.306
	TR2[0.0333]			(-3.345, 13.099)
$\hat{\tau}$	–	2007	2014.417	2013.75
		(2006.917, 2007.667)	(2013.483, 2014.750)	(2013.482, 2014.750)
		SB1[2007.000]		
		SB2[2011.750]		
T[189]	139	59	105	68
	50	130	43	72
			9	41
			32	8

Table (2) presents the estimation results using the competing models in their reduced form. Then, we follow a "specific to general" approach in Galvão (2006), to select the model based on bounded Wald (W) and LM statistics.

Denote θ_1 under the null and θ_2 under the alternative, the W and LM statistics are

$$\begin{aligned} W(\theta_2) &= n\left(\frac{SSR\left(\hat{\theta}_1\right) - SSR(\theta_2)}{SSR(\theta_2)}\right), \\ LM(\theta_2) &= n\left(\frac{SSR\left(\hat{\theta}_1\right) - SSR(\theta_2)}{SSR\left(\hat{\theta}_1\right)}\right). \end{aligned}$$

Based on Altissimo and Corradi (2002), BWald and BLM are the maximum values of a Wald and LM statistic over a grid of possible values for the nuisance parameter. Using the asymptotic bounds $(1/2ln(ln(T)))$, the decision rule is that the model under alternative will be selected if Bounded Wald (BWald) or Bounded LM statistic (BLM) is larger than 1. The bounded Wald statistic (BWald) is

$$BWald = \left[\frac{1}{2\ln(\ln(T))}\left[\underset{\theta_2^L \leq \theta_2 \leq \theta_2^U}{sup} W(\theta_2)\right]^{1/2}\right] > 1. \tag{13}$$

Interested readers can refer to the simulation study in Galvão (2006) with respect to the ability of BWald and BLM to discriminate between different reduced form VAR specifications.

Table (3) presents the estimated BWald and BLM for the model selection procedure. Step 1 consists of two sets of model comparisons in 1A and 1B. The baseline VAR is compared with the alternative TVAR and SBVAR models. The BWald and BLM in 1B are both larger than 1. Therefore, SBVAR is selected based on the decision rule. As pointed out in Galvão (2006), only if none of the alternative hypotheses is rejected using the decision rule, the VAR shall be chosen. Otherwise, if at least one of the statistics suggests rejection of the VAR, we have to proceed to step 2.

We then proceed to step 2, i.e. 2A1, 2A2, 2B1, and 2B2, which consists of four sets of model comparison. According to the statistics in step 1 (1A and 1B) in table (3), BWald (BLM) with SBVAR under the alternative is larger than BWald (BLM) with TVAR under the alternative, we use the statistics in step 2B1 and

Table 3: BWald and LM bounds with monthly data 2000 : 1 - 2017 : 10. Selection rule: if BWald(BLM) >1, choose model under alternative.

	H_0 VS H_A	BWald	BLM
1A	VAR : TVAR	0.834	0.818
1B	**VAR : SBVAR**	**1.116**	**1.078**
2A1	TVAR : SBTVARc	1.514	1.422
2A2	TVAR : SBTVAR	0.894	0.874
2B1	**SBVAR : SBTVARc**	**1.308**	**1.248**
2B2	SBVAR : SBTVAR	0.513	0.509
X1	TVAR : 3R-TVAR	1.171	1.127
X2	SBVAR : 2-SBVAR	1.277	1.221

2B2 to verify whether the inclusion of a threshold improves the SBVAR using estimated SBT-VAR and SBT-VARc under the alternative. Because the statistic with SBT-VARc under the alternative (in 2B1) is larger than the statistic with SBT-VAR under the alternative (in 2B2), the inclusion of a constant threshold has to be considered. Therefore, from table (3), the SBT-VARc model is chosen based on the decision rule. If both statistics in 2B are smaller than 1, the SBVAR would have been chosen.

According to the estimation results in table (2), a break is detected in April/-May, 2014. The 90% confidence interval, computed using bootstrap, for the break is between April, May 2013 and September 2014.

3.3 GARCH structural innovations and a flexible $\mathbf{A_0}$

Fig. (2) plots the estimated residuals $\hat{u}_t$ from the reduced form SBT-VARc model. From the histograms and a clustering patten of $\hat{u}_t$, we are motivated to use the GO-GARCH model and allow heteroskedasticity in the conditional (co)variances. Hence, with the GO-GARCH representation, we are able to identify ${\boldsymbol{A_0}}^{-1}$ in

$$\boldsymbol{u_t} = {\boldsymbol{A_0}}^{-1}\boldsymbol{\epsilon_t},$$

as well as to identify the heteroskedastic structural shocks $\boldsymbol{\epsilon_t}$.

Table (4) presents the estimated results from the GO-GARCH model in eq. (18) and eq. (20). From table (4), the structural shocks in the standardized

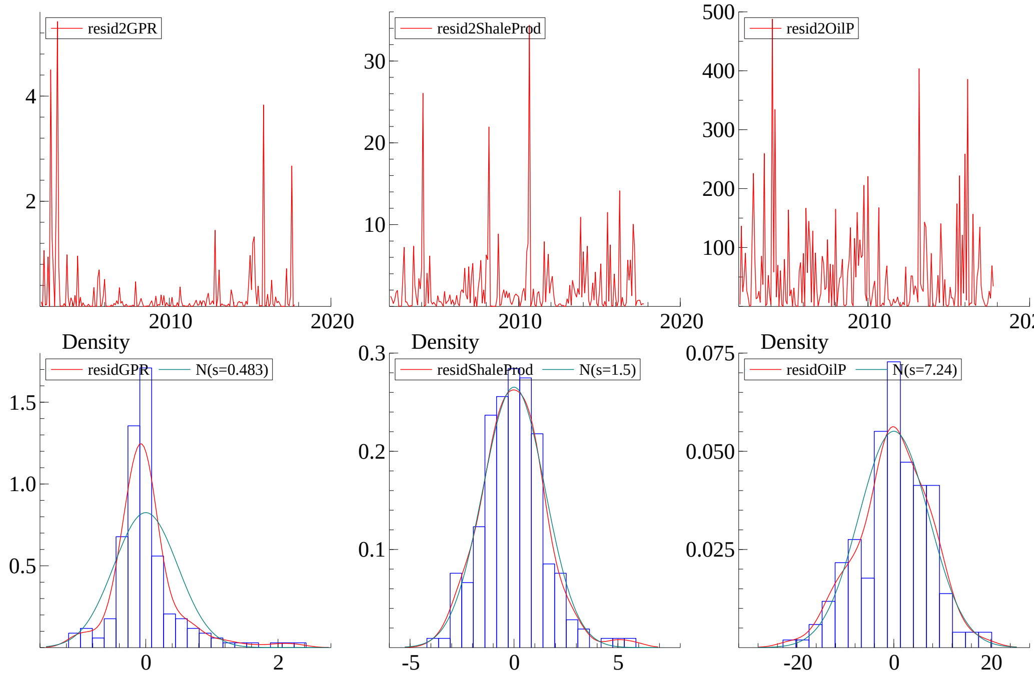

Figure 2: Heterskadsitic statistic innovation $\hat{\boldsymbol{u}}_t$ and fitted distribution

geopolitical threat risk $\boldsymbol{\epsilon_{t,GPR}}$ is estimated to follow a univariate GARCH(1,1) process and the structural shocks in the oil returns $\boldsymbol{\epsilon_{t,OilP}}$ are estimated to follow an ARCH process.

Table 4: Identify $\mathbf{A_0}^{-1}$ with GARCH structural errors

$\widehat{\boldsymbol{A_0^{-1}}}$		-0.945	0.323	-0.023
		0.129	0.126	-0.983
		-0.311	-0.947	-0.074
			coef.	t-prob
STAD GPR Threat		$\hat{\gamma}_1$	0.122	0.034
		$\hat{\delta}_1$	0.817	0.000
Δ log Shale		$\hat{\gamma}_2$	0.099	0.108
		$\hat{\delta}_2$	0.000	0.1878
Δ log OilP		$\hat{\gamma}_3$	0.320	0.011
		$\hat{\delta}_3$	-0.054	0.490

Fig. (3) plots the estimated conditional variance, covariance and correlation for $\boldsymbol{u_{t,GPR}}$, $\boldsymbol{u_{t,ShaleP}}$ and $\boldsymbol{u_{t,OilP}}$. Towards the end of 2011, there is a big spike in the conditional covariance between shale production and oil prices, largely due to the spike in the shale production variance. This result coincides with the time line of the shale revolution after the successful horizontal drilling experiment in Bakken Shale Play[2].

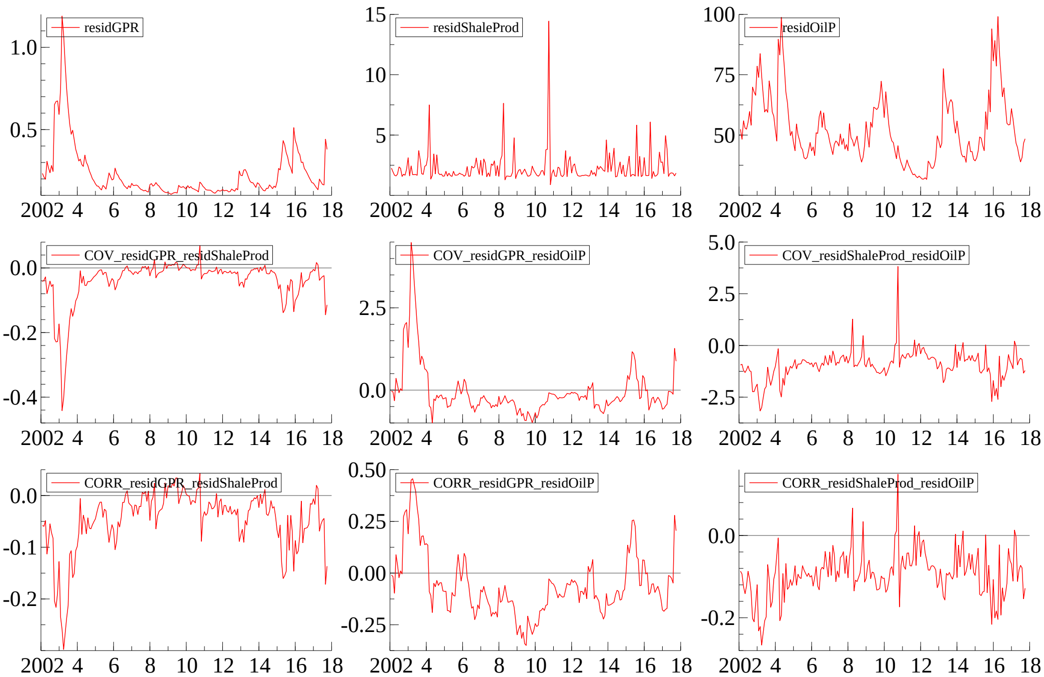

Figure 3: Estimated conditional (co)variance and correlation using GO-GARCH structural errors. Top panel plots the conditional variances, middle panel plots the conditional covariances, and bottom panel plots the conditional correlation.

3.4 Generalized impulse responses and variance impulse response functions

Using eq. (23), we analyse the responses of oil returns to one-unit structural shock in the geopolitical risk index. Because the geopolitical risk variable (GPR_t) is chosen as the threshold variable, a structural shock - in combination with the feedback effects from other endogenous variables such as $ShaleP_t$ and $OilP_t$ -

[2]For more information relating to the history of advances in the technology of shale production, through fracking oil from its rock formation, please see https://bakkenshale.com

may trigger a regime switch. Therefore, the impulse response functions in the nonlinear SBT-SVARc may not appear to be smooth decaying functions as it would be in a linear baseline VAR. Moreover, according to eq. (14), the regime switch also depends on the estimated delay variable $\hat{d}_i$. Therefore, the impulse response functions for h step ahead also depends on the history $\mathcal{F}_{t+h-1}$.

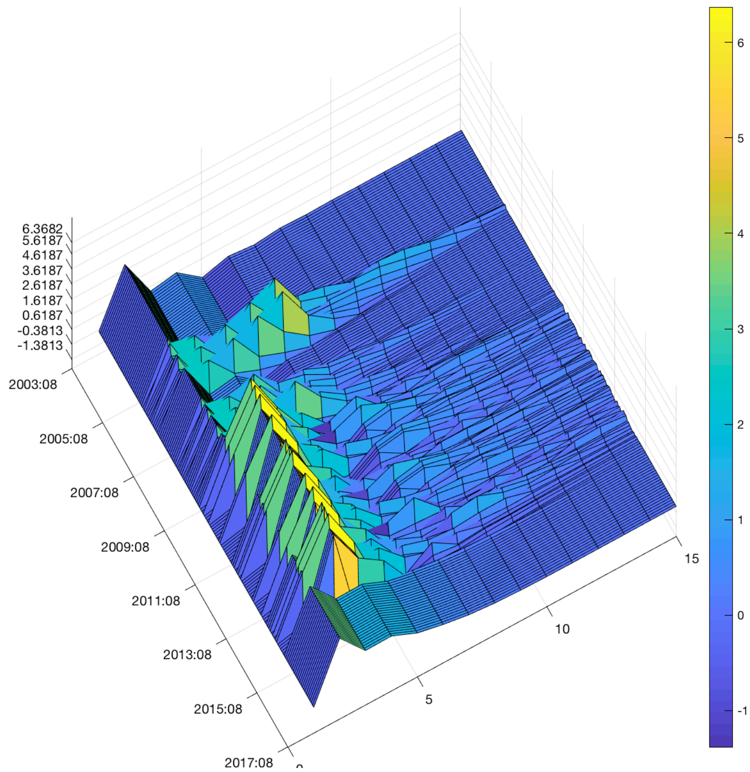

Figure 4: ΔOil price responses to shock in GPR with GO-GARCH structural errors

Using the estimated results from the reduced form SBT-VARc and $\widehat{\boldsymbol{A_0}^{-1}}$ in table (4), we impose a structural shock $\boldsymbol{\epsilon_t} = (1, 0, 0)^T$, with $t = 1, \ldots, 170$. That is imposing a one-unit shock on geopolitical risk, where $\epsilon_{t,GPR} = 1$, starting from August 2003. We would like to see how oil returns respond to the same size of geopolitical shocks over time. The same exercise is repeated over 170 periods. Fig. (4) plots these 170 generalized impulse response functions of oil price returns to one - unit shock on the geopolitical risk variable ϵ_t using eq. (23).

We pick the first series (August 2003) and the last series (October 2017) from fig. (4) and plot them together for an ad hoc comparison. Fig. (5) compares the oil price responses to a one-unit geopolitical risk shock before and after the

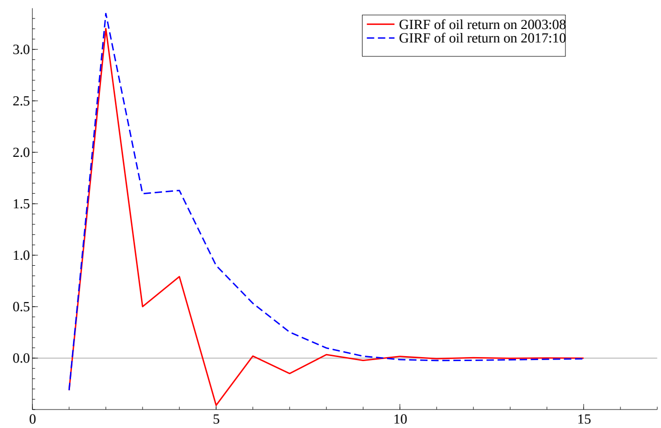

Figure 5: ΔOil price responses to shock in GPR a comparison between 2003:08 and 2017:10. Dash line plots the GIRF of ΔOil price in August 2003. Solid line plots the GIRF of ΔOil price in August 2017.

estimated break in 2014. Both fig. (4) and fig. (5) show that the generalized impulse response functions are smoother towards the end of the sample of 170.

Given that the size of the hypothetical structural shocks is fixed and the identified $\widehat{\boldsymbol{A_0}}^{-1}$ does not vary with time, the smoothness in the response functions implies that the impose structural shock $\boldsymbol{\epsilon_t}$ did not induce abrupt regime switches after the break. Given the GPR index around August 2003 and October 2017 are on a similar level, see from fig. (1), this difference in the impulse response functions must be owing to a joint effort from the feedback coefficient matrices ($\widehat{\boldsymbol{A_3}}$ and $\widehat{\boldsymbol{A_4}}$) and identified contemporaneous relationship amongst the variables $\widehat{\boldsymbol{A_0}}^{-1}$. The shock impact effects have a smoother spread over time after the break.

Appendix (4) plots the generalized impulse response functions of oil prices to a one-unit structural shock in the geopolitical risk variable using the Cholesky identification method, where $\widehat{\boldsymbol{A_0}}^{-1}$ is restricted to be the lower triangular of $\boldsymbol{\Sigma_u}$ via a Cholesky decomposition. We argue that, in line with the existing literature, the Cholesky method to identify structural models implies restrictive and unrealistic assumptions.

Rather than focusing on the effects of a single historical event as proposed in Hafner and Herwartz (2006), we aim to uncover the (co)variance response functions to different hypothetical orthogonal shocks at different points in time. The two hypothetical shocks we impose on the system are $\boldsymbol{\epsilon_t^*}$ and $\boldsymbol{\epsilon_t}$, where

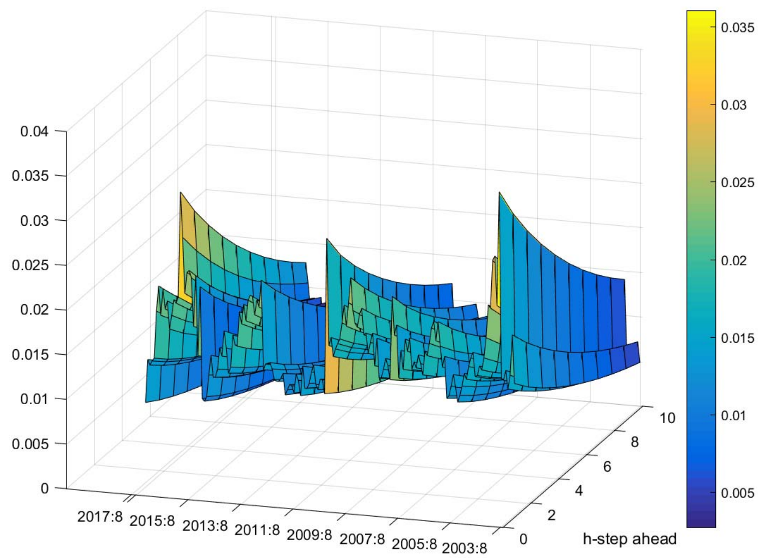

Figure 6: $V^*_{t+h}\left(\boldsymbol{\epsilon^*_t}\right) - V_{t+h}\left(\boldsymbol{\epsilon_t}\right)$ for $\sigma^2_{OilP,t}$. The imposing shock $\boldsymbol{\xi^*_t} = (1,1,0)^T$ represents a structural shock with one unit on geopolitical risk with a simultaneous unity shale production shock. Shock $\boldsymbol{\xi_t} = (1,0,0)^T$ represents only geopolitical risk variable is imposed with a unit structural shock.

$\boldsymbol{\epsilon^*_t} = (1,1,0)^T$ represents a simultaneous unity geopolitical risk shock and a unity shale production shock, whereas $\boldsymbol{\epsilon_t} = (1,0,0)^T$ represents the hypothetical unit structural shock is only imposed on the geopolitical risk variable.

The variance impulse responses $V^*_{t+h}\left(\boldsymbol{\epsilon^*_t}\right)$ and $V_{t+h}\left(\boldsymbol{\epsilon_t}\right)$ are then calculated using eq. (24). Given the discussion in section (4), the difference in VIRF, $V^*_{t+h}\left(\boldsymbol{\epsilon^*_t}\right) - V_{t+h}\left(\boldsymbol{\epsilon_t}\right)$, and the difference in two conditional volatilities profiles, $v^*_{t+h}\left(\boldsymbol{\epsilon^*_t}\right) - v_{t+h}\left(\boldsymbol{\epsilon_t}\right)$, have the same analytic expression. Therefore, using eq. (31), the function $\boldsymbol{\phi_h D^+_k}\left(\boldsymbol{\Sigma^{1/2}_{u,t}} \otimes \boldsymbol{\Sigma^{1/2}_{u,t}}\right)\boldsymbol{D_k}\,\mathrm{vech}\left(\boldsymbol{\xi^*_t \xi^{*T}_t} - \boldsymbol{\xi_t \xi_t}^T\right)$ is determined by $\boldsymbol{\phi_h}$ and the estimated time-varying (co)variance $\boldsymbol{\Sigma_{u,t}}$. Fig. (6) plots the volatility impulse response difference of oil returns and fig. (7) plots the difference of $cov\left(GPR_t, OilP_t\right)$ responses, to the two types of hypothetical shocks respectively.

From fig. (6), the largest VIRF difference is at around 2003 and 2015 (0.035). The smallest VIRF difference is at around 2010(0.01), which implies that the extra unit shale oil production shock induces small positive increases in oil price volatility. In other words, the conditional volatility of oil prices is higher under a simultaneous geopolitical risk and shale oil production shock, compared with

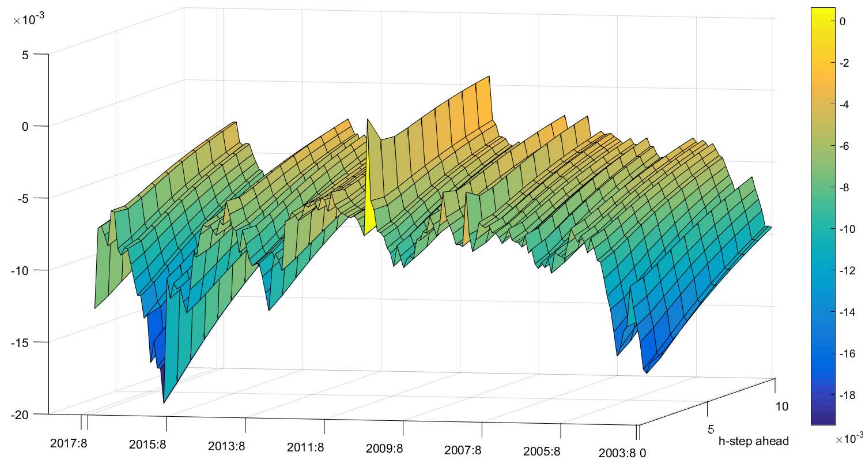

Figure 7: $V^*_{t+h}\left(\boldsymbol{\epsilon^*_t}\right) - V_{t+h}\left(\boldsymbol{\epsilon_t}\right)$ for $cov_{GPR_t,OilP_t}$. The imposing shock $\boldsymbol{\xi^*_t} = (1,1,0)^T$ represents a structural shock with one-unit on geopolitical risk with a simultaneous unity shale production shock. Shock $\boldsymbol{\xi_t} = (1,0,0)^T$

the counterpart scenario where the shock is only imposed on the geopolitical risk variable. Because the VIRF difference function, in eq. (31), mainly depends on the conditional (co)variance at that time point, by examining fig. (3), the conditional variance $\boldsymbol{\sigma_{OilP}}$ in $\boldsymbol{\Sigma_{u,t}}$ is maximized at around 2003 and 2015 periods, and minimized around 2010, it is not surprising that a positive shale production shock does not lower the conditional volatility response in the oil price to geopolitical risk shocks.

From fig. (7), the geopolitical risk and oil price covariance responses $(\sigma_{GPR_t,OilP_t})$ to the extra unit shale production shock $(\boldsymbol{\epsilon^*_t} = (1,1,0)^T)$ is decreased by $5 \sim 20 \times 10^{-3}$ compared with the covariance responses without shale production $(\boldsymbol{\epsilon_t} = (1,0,0)^T)$ in the 170 periods, apart from a small window around 2010. The differences of $\sigma_{GPR_t,OilP_t}$, which are the 4$^{\text{th}}$ elements in vech $\left(V^*_{t+h}\left(\boldsymbol{\epsilon^*_t}\right) - V_{t+h}\left(\boldsymbol{\epsilon_t}\right)\right)$, were maximized around 2015 and 2003, and minimized round 2010. The shape of the differences (co)variance response surface over these 170 sample periods corresponds to the estimated conditional GO-GARCH (co)variance, which is plotted in fig. (3).

3.5 Implications

In this section, we summarise the value of this paper in reality to benefit policy-makers, economists, traders, and so on. First, SBT-VAR specification application in the oil market is efficient in the structural break time point and threshold effect value estimation. The supply shock caused by geopolitical risk generally results in a structural break in oil price behaviour. Further, the dynamics between oil prices and market shocks have changed depending on the different regimes. Therefore, we recommend SBT-VAR in further research and macroeconomics forecasting of its practical significance mentioned above. It is favourable to measure the oil dynamics (market interactions) in different market conditions and recognise the crucial timing to policymakers, economists, and traders.

Second, two improvements to the impulse response function are applied in this article. One is the structural SBT-VAR identification disentangles the marginal effects of exogenous shocks. Another is the GARCH structural errors unrestricted the interaction order between oil price, geopolitical risk, and shale revolution production. For example, the oil price shock influenced geopolitical risk in Venezuela during the oil price plunge of 2014, while oil revenue accounted for about 95% of Venezuela's export earrings. IRF based on reduced form fails to identify the responses, and biased results diverge from economic reality. It measures the time profile of the effect of shocks at a given point in time on the expected future values of the variables. It is beneficial to policymakers, economists, traders, and other stakeholders for an economic system to react to a shock at a given moment they are interested.

Third, the variance impulse response function under two scenarios provides a straightforward approach to contrast. We could observe the oil market dynamics with and without the shale revolution. Variance estimation and forecast are helpful to optimal portfolio construction. Further, risk measurement is crucial to traders. In theory, it provides a new way to consider how the macroeconomic conditions switch affects the energy market in the big picture. Stakeholders could evaluate the impact of the financial market shock on the asset's variance. It provides practical guidance for financial participants.

4 Conclusions

In this chapter, we focus on two aspects: First, we investigate the impact of geopolitical uncertainties on oil price volatility under shale oil revolution. Second, we evaluate the oil price impulse responses under a structural geopolitical risk shock amidst the shale revolution.

We extend a reduced form structural break threshold vector autoregressive (SBT-VAR) model to its' structural form (SBT-SVAR). Then, we allow a flexible contemporaneous relationship amongst the variables and identify the structural innovations by allowing heteroskedasticity. Compared with the conventional reduced-form VAR and TVAR models, an SBT-VAR with a constant threshold is recognised. Meanwhile, the break-point in 2014 is identified. We find that the Cholesky decomposition method with the variables in fixing order may lead to misinterpretation and overestimation in terms of the responses of oil prices to the geopolitical risk shocks. Over a 170 sample period, the impulse response functions of oil prices to a unity structural geopolitical risk shock become smoother after the estimated break-point in 2014 compared with counterparts before. Then we conclude: The identified feedback coefficient matrices in a structural SBT-VAR model result in the imposed shock smoothly spread over time after 2014 when given a similar level of the threshold variable (the geopolitical risk index in this paper).

The analyses of the (co)variance impulse response concerning two distinct shock scenarios (one with only a geopolitical risk shock, the other with simultaneous shale production and geopolitical risk shocks) are used to evaluate the shale revolution shock impact on the dynamics between oil price and geopolitical risk. We find the conditional volatility of oil prices is higher with a shale production shock than without. Allowing changes in the unconditional variances could be a further extension of this research.

The covariance response between geopolitical risk and oil price is reduced by $5 \sim 20 \times 10^{-3}$ with the extra unit shale production shock. The scale of the differences in the (co)variance responses over this 170 sample period depends on the identified ${\boldsymbol{A_0}}^{-1}$ in the SBT-SVAR model, as well as the GO-GARCH specification in the statistical innovations. The differences in the (co)variance responses under the two scenarios also correspond to the estimated conditional volatilities at those points in time.

In this article, we discuss three types of further research suggestions. First, adding crude oil price drivers, such as crude oil inventory in the U.S. market, to the current topic is valuable. Second, it is interesting to discuss further whether the U.S. would become more dominant in the world crude oil market through shale oil production boosting. Finally, it probably results in different impulse responses if another structure model identification method is applied. We expect different results when using varying methods to redo the same analysis. It will be interesting to compare the difference and make a valuable conclusion.

Acknowledgement

We would like to thank all workshop participants at HSI2018. We also thank Toshiaki Watanabe, Raffaella Giacomini, Roberto Leon-Gonzalez, Yasuhiro Omori, Etsuro Shioji and Tatsuma Wada for their valuable comments.

Appendix

A: Reduced Form SBT-VAR and Structural SBT-VAR

In order to identify the changing dynamics in the oil price, first, we apply a reduced form Structural Break Threshold VAR (SBT-VAR) model proposed by Galvão (2006). The SBT-VAR is a generalization of the baseline VAR model, which is specified in eq. (34).

The reduced form SBT-VAR is specified as

$$\begin{aligned}\boldsymbol{y_t} &= \left[\boldsymbol{\Gamma_1 X_t} I_{1,t-d_1}\left(r_1\right)+\boldsymbol{\Gamma_2 X_t}\left(1-I_{1,t-d_1}\left(r_1\right)\right)\right] I_t\left(\tau\right)+\\ &\quad \left[\boldsymbol{\Gamma_3 X_t} I_{2,t-d_2}\left(r_2\right)+\boldsymbol{\Gamma_4 X_t}\left(1-I_{2,t-d_2}\left(r_2\right)\right)\right]\left(1-I_t\left(\tau\right)\right)+\boldsymbol{u_t},\end{aligned} \tag{14}$$

where $I\left(\cdot\right)$ is an indicator function. Denote the threshold as r_i, the delay parameter as d_i, the break-point as τ, and the transition variable as z, then, the threshold and break indicator functions are defined as $I_{i,t-d_i}\left(r_i\right)=1\left(z_{t-d_i}\leq r_i\right)$, and $I_t\left(\tau\right)=$

$1(t \leq \tau)$. The error term (or statistical innovation), $\boldsymbol{u_t}$, follows a multivariate normal, $\boldsymbol{u_t} \overset{i.i.d}{\sim} MN(\boldsymbol{0}, \boldsymbol{\Sigma_u})$, and $\boldsymbol{\Sigma_u} \in \mathbb{R}^{k \times k}$.

As mentioned before, a reduced form model is not sufficient for the impulse responses analysis because the correlation of the statistical innovations makes it impossible to disentangle the marginal effects of exogenous shocks. Therefore, in order to analyse how the oil prices respond to the structural shocks of geopolitical risks before and after the shale revolution, we are motivated to identify the mutually independent structural shocks in a regime-changing system.

Nonlinear structural models, incorporating thresholds and breaks are proposed in Baum and Koester (2011) and Galvão and Marcellino (2013). We extend the reduced form SBT-VAR to its structural form and denote the structural model as SBT-SVAR hereafter. The SBT-SVAR is specified as follows:

$$\begin{aligned} \boldsymbol{A_0 y_t} &= \left[\boldsymbol{A_1 X_t} I_{1,t-d_1}(r_1) + \boldsymbol{A_2 X_t}\left(1 - I_{1,t-d_1}(r_1)\right)\right] I_t(\tau) + \\ &\quad \left[\boldsymbol{A_3 X_t} I_{2,t-d_2}(r_2) + \boldsymbol{A_4 X_t}\left(1 - I_{2,t-d_2}(r_2)\right)\right]\left(1 - I_t(\tau)\right) + \boldsymbol{\epsilon_t}, \end{aligned} \tag{15}$$

where $\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}$ or $\boldsymbol{A_0 u_t} = \boldsymbol{\epsilon_t}$. The coefficient matrix $\boldsymbol{\Gamma_i}$ can be found from $\boldsymbol{\Gamma_i} = \boldsymbol{A_0}^{-1}\boldsymbol{A_i}$ with $i = 1, 2, 3, 4$.

The (co)variance of the error term of the model in the reduced form $\boldsymbol{\Sigma_u}$ in eq. (14) and the unconditional variance of structural shocks $\boldsymbol{\Sigma_\epsilon}$ in eq. (15) can be linked by $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\boldsymbol{\Sigma_\epsilon}\left(\boldsymbol{A_0}^{-1}\right)^T$. We impose normalization on the $\boldsymbol{\Sigma_\epsilon}$. Then, $\boldsymbol{\Sigma_\epsilon}$ is normalized to an identify matrix, $\boldsymbol{I_k}$ with $\boldsymbol{I_k} \in \mathbb{R}^{k \times k}$, and $\boldsymbol{\Sigma_u}$ is

$$\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\left(\boldsymbol{A_0}^{-1}\right)^T, \tag{16}$$

or in the form of the precision matrix

$$\boldsymbol{\Sigma_u}^{-1} = \boldsymbol{A_0}^T\boldsymbol{A_0}. \tag{17}$$

$\boldsymbol{A_0}$ is the upper-triangular matrix by applying a Cholesky factorization.

Similarly to the case with a simple structural VAR in eq. (35), restricting $\mathbf{A_0}$ as a lower triangular matrix and using the Cholesky decomposition offers a straightforward solution to identify the structural shocks in a structural SBT-VAR (SBT-SVAR) model in eq. (15). However, using the Cholesky decomposition implies restrictions on the direction of contemporaneous effects of structural shocks.

For instance, suppose the order of the variables is fixed and the structural error is $\boldsymbol{\epsilon_t} = (\boldsymbol{\epsilon_{t,GPR}}, \boldsymbol{\epsilon_{t,ShaleP}}, \boldsymbol{\epsilon_{t,OilP}})^T$, where $\boldsymbol{\epsilon_{t,GPR}}$, $\boldsymbol{\epsilon_{t,ShaleP}}$, and $\boldsymbol{\epsilon_{t,OilP}}$ are the orthogonal mutually independent structural shocks to geopolitical risk, to shale oil production, and to oil prices at time t, respectively. A lower triangular restriction in $\mathbf{A_0}^{-1}$ implies that a structural shock in geopolitical risk has an instantaneous impact on shale oil production, the oil price, not vice versa. Similarly, the shale oil production shocks, $\boldsymbol{\epsilon_{t,ShaleP}}$, instantaneously affect the oil price, but not vice versa. These assumptions on the contemporaneous relationship amongst the variables appear to be too restrictive and unrealistic.

Next section demonstrates another flexible method for identification - allowing heteroskedastic structural errors. The identification method is applied to the STB-SVAR model in our empirical applications.

B: GARCH Structural Errors

Rather than using the Cholesky decomposition method to identify eq. (15), we can exploit the conditional heteroskedasticity, i.e. $\boldsymbol{\Sigma_{\epsilon,t}}$, in the structural shocks $\boldsymbol{\epsilon_t}$, to identify more unrestricted elements in $\boldsymbol{A_0}$. The SVAR with GARCH (structural) innovations has been proposed in Lütkepohl and Netšunajev (2017), Bouakez et al. (2013), Bouakez et al. (2014), and Sentana and Fiorentini (2001). A SVAR with GARCH-m type innovations is proposed in Elder and Serletis (2010).

In the context of analysing the responses of oil prices and oil price volatilities to geopolitical risk shocks under the shale production, a flexible $\boldsymbol{A_0}$ is needed. A flexible $\boldsymbol{A_0}$ requires a relaxation of restrictions on the contemporaneous relationship amongst the variables. In other words, a flexible $\boldsymbol{A_0}$ allows impact effects of an oil price shock ($\boldsymbol{\epsilon_{t,OilP}}$) on shale oil production and geopolitical risk. For instance, the low oil prices since 2014 did not help with the recent Venezuela crisis[3] and it is reasonable to believe that the oil price shock has had impact effects on geopolitical risk in Venezuela.

[3]"Its oil revenues account for about 95% of its export earnings. But when the oil price plummeted in 2014, Venezuela was faced with a shortfall of foreign currency." in *How Venezuela's crisis developed and drove out millions of people, BBC, Aug 22, 2018* https://www.bbc.co.uk/news/world-latin-america-36319877

By utilizing the estimated statistical innovations $\hat{\boldsymbol{u}}_{\boldsymbol{t}}$ from the SBT-VAR model in eq. (14) and a GARCH specification of heteroskedasticity in $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}}$, we can identify more unrestricted elements in $\boldsymbol{A}_{\mathbf{0}}$ in a more realistic setting. Recall that the statistical innovation $\boldsymbol{u}_{\boldsymbol{t}}$ and the structural innovations $\boldsymbol{\epsilon}_{\boldsymbol{t}}$ are linked by $\boldsymbol{A}_{\mathbf{0}}$, where

$$\boldsymbol{u}_{\boldsymbol{t}} = \boldsymbol{A}_{\mathbf{0}}{}^{-1}\boldsymbol{\epsilon}_{\boldsymbol{t}}, \tag{18}$$

or $\boldsymbol{A}_{\mathbf{0}}\boldsymbol{u}_{\boldsymbol{t}} = \boldsymbol{\epsilon}_{\boldsymbol{t}}$, and the unconditional statistical innovation is $\boldsymbol{\Sigma}_{\boldsymbol{u}} = \left(\boldsymbol{A}_{\mathbf{0}}{}^{-1}\right)\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}\left(\boldsymbol{A}_{\mathbf{0}}{}^{-1}\right)^{T}$. For convenience, the unconditional variance of the structural innovations are normalized to unity, i.e. $E\left(\boldsymbol{\epsilon}_{\boldsymbol{t}}\boldsymbol{\epsilon}_{\boldsymbol{t}}{}^{T}\right) = \boldsymbol{I}_{\boldsymbol{k}}$ with $\boldsymbol{I}_{\boldsymbol{k}} \in \mathbb{R}^{k\times k}$. Denote the information set up to t is $\mathcal{F}_t$, $E_{t-1}(\cdot) \equiv E(\cdot \mid \mathcal{F}_{t-1})$, the heteroskadastic (co)variance of the statistical innovation and structural innovation conditional on the historical information are $\boldsymbol{\Sigma}_{\boldsymbol{u},\boldsymbol{t}} = E_{t-1}\left(\boldsymbol{u}_{\boldsymbol{t}}\boldsymbol{u}_{\boldsymbol{t}}{}^{T}\right)$ and $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}} = E_{t-1}\left(\boldsymbol{\epsilon}_{\boldsymbol{t}}\boldsymbol{\epsilon}_{\boldsymbol{t}}{}^{T}\right)$.

In Lütkepohl and Milunovich (2016), a GARCH(1,1) process is specified for the conditional variance of the structural innovations:

$$\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}} = (\boldsymbol{I}_{\boldsymbol{k}} - \Delta_1 - \Delta_2) + \Delta_1 \circ \left(\boldsymbol{\epsilon}_{\boldsymbol{t-1}}\boldsymbol{\epsilon}_{\boldsymbol{t-1}}{}^{T}\right) + \Delta_2 \circ \boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t-1}}, \tag{19}$$

where Δ_1 and Δ_2 are diagonal matrices, and "$\circ$" denotes the Hadamard product operator. If Δ_1 and Δ_2 are null, then $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}}$ is constant. Whereas, if Δ_1 and Δ_2 are positive semi-definite, then $(\boldsymbol{I}_{\boldsymbol{k}} - \Delta_1 - \Delta_2)$ is positive definite, which indicates that at least one of the structural innovations follow a GARCH(1,1) process. Therefore, the GARCH(1,1) specification for an individual conditional structural variance is

$$\sigma^2_{m,t|t-1} = (1 - \gamma_m - \delta_m) + \gamma_m \epsilon^2_{m,t-1} + \delta_m \sigma^2_{m,t-1|t-2}, \quad m = 1, \ldots, k. \tag{20}$$

We follow a two-step procedure, see Bouakez et al. (2013) and Bouakez et al. (2014), to estimate the ARCH coefficients Δ_1 and GARCH coefficients Δ_2 by maximizing the likelihood function as follows:

$$\log L \approx -T \log\left|\det\left(\mathbf{A_0}\right)\right| - \frac{1}{2}\sum_{t=1}^{T} \log\left|\det\left(\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}}\right)\right| - \frac{1}{2}\sum_{t=1}^{T} \boldsymbol{\epsilon}_{\boldsymbol{t}}{}^{T}\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}}{}^{-1}\boldsymbol{\epsilon}_{\boldsymbol{t}}, \tag{21}$$

where the initialization $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\mathbf{0}} = \left(\boldsymbol{\epsilon}_{\mathbf{0}}\boldsymbol{\epsilon}_{\mathbf{0}}{}^{T}\right) = \boldsymbol{I}_{\boldsymbol{k}}$. This initialization is consistent with the intercept term $\boldsymbol{I}_{\boldsymbol{k}}$ in eq. (19).

The first step requires extracting the estimated statistical innovations $\hat{\boldsymbol{u}}_{\boldsymbol{t}}$ from eq. (14). The reduced form SBT-VAR(p) in eq. (14) is estimated using maximum likelihood (ML)[4], where the order of p is pre-selected by estimating the baseline VAR in eq. (33) using AIC and BIC. Please refer to Galvão (2006) for discussions regarding the estimation procedure with the reduced form SBT-VAR model. Then, the $T \times k$ matrix of statistical innovation $\hat{u}_t$ can be treated as observables in the second step.

The second step involves estimating the structural parameters, i.e. the non-zero elements in $\boldsymbol{A_0}$, as well as Δ_1 and Δ_2. As pointed in Lütkepohl and Netšunajev (2017), this GARCH type structural error resembles the Generalized Orthogonal GARCH (GO-GARCH) model proposed in van der Weide (2002). As pointed out in Lanne and Saikkonen (2007), the GO-GARCH is a special case of factor-GARCH model. This GO-GARCH representation in the statistical innovation not only helps us to identify $\boldsymbol{A_0}$ but also offers us a convenient form for the (co)variance impulse response analysis in the next step.

C: Generalized Impulse Response Functions with SBT-SVAR

In (Hamilton, 1994, p.92, p.327), the impulse response functions reflect how the perturbing shock spreads across time. Evaluation of the dynamic consequence of structural shocks is a particular interest of policy makers. Denote h as the periods succeeding one unit structural shock $\boldsymbol{\xi_{j,t}}$ on variable j at time t, given the information available up to t as $\mathcal{F}_{t-1}$, the impulse response functions, IR in a linear covariance stationary VAR(p) system can be calculated by

$$IR\left(\boldsymbol{h}, \boldsymbol{\xi_{j,t}}, \mathcal{F}_{t-1}\right) = \frac{\partial \boldsymbol{y_{t+h}}}{\partial \boldsymbol{\xi_{j,t}}} \tag{22}$$

using the Wold representation of a VAR(p).

However, in a nonlinear system, such as the SBT-SVAR model specified in eq. (15), eq. (22) can no longer be used to calculate the impulse response functions. In the nonlinear SBT-SVAR model, the variable responses to a shock not only depend on the estimated delay variable d_i, but also depend on the history preceding the

[4] The ML estimation is achieved by $\hat{r}_1, \hat{r}_2, \hat{\tau} = \min\limits_{\substack{L \leq r_1 \leq r_U \\ r_L \leq r_2 \leq r_U \\ \tau_L \leq \tau \leq \tau_U}} \log\left(det\left(\hat{\boldsymbol{\Sigma}}\left(r_1, r_2, \tau\right)\right)\right)$

shock. Moreover, the perturbing shock might trigger a regime switch if the threshold variable z goes above the threshold r_i.

To evaluate the impact effects of structural shocks in a non-linear SBT-SVAR system, we apply the Generalized Impulse Response Functions (GIRF) proposed by (Koop et al., 1996), which requires a h-step ahead forecasting of the conditional mean of $\boldsymbol{y_{t+h}}$, in response to a one-unit structural shock, $E\left[\boldsymbol{y_{t+h}} \mid \boldsymbol{\xi_t}, \mathcal{F}_{t-1}\right]$, and a h-step ahead forecasting of $\boldsymbol{y_{t+h}}$ only conditional on the history, $E\left[\boldsymbol{y_{t+h}} \mid \mathcal{F}_{t-1}\right]$. The GIRF is then

$$GIRF\left(\boldsymbol{h}, \boldsymbol{\xi_t}, \mathcal{F}_{t-1}\right) = E\left[\boldsymbol{y_{t+h}} \mid \boldsymbol{\xi_t}, \mathcal{F}_{t-1}\right] - E\left[\boldsymbol{y_{t+h}} \mid \mathcal{F}_{t-1}\right]. \tag{23}$$

D: Variance Impulse Response Functions with GARCH structural errors

Besides evaluating the impact effects of structural shocks on the conditional means by using the GIRF suggested by Koop et al. (1996), we are also interested in tracing the dynamic responses of the conditional (co)variances $\boldsymbol{\Sigma_{u,t}}$ to perturbing structural shocks. In particular, we want to compare the dynamic responses of the conditional covariance between oil prices and geopolitical risk under two distinct scenarios - one with a simultaneously perturbing shale production structural shock and one without.

In the spirit of Koop et al. (1996), Hafner and Herwartz (2006) propose a variance impulse response function (VIRF). Denote $\boldsymbol{\Sigma_{u,t}}$ as the initial conditional variance preceding the strucutral shock $\boldsymbol{\xi_t}$, the general expression for VIRF is

$$V_{t+h}\left(\boldsymbol{\xi_t}\right) = E[\text{vech}\left(\boldsymbol{\Sigma_{u,t+h}}\right) \mid \boldsymbol{\xi_t}, \mathcal{F}_{t-1}] - E[\text{vech}\left(\boldsymbol{\Sigma_{u,t+h}}\right) \mid \mathcal{F}_{t-1}]. \tag{24}$$

The VIRF calculates the differences between the expectation of volatility conditional on a perturbing shock $\boldsymbol{\xi_t}$ and the history $\mathcal{F}_{t-1}$, and the expectation of volatility only conditional on the history. Hafner and Herwartz (2006) consider a vec representation of multivariate GARCH(1,1) specified as

$$\text{vech}(\boldsymbol{\Sigma_{u,t}}) = \boldsymbol{W} + \boldsymbol{\tilde{A}_1}\,\text{vech}\left(\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T\right) + \boldsymbol{\tilde{B}_1}\,\text{vech}(\boldsymbol{\Sigma_{u,t-1}}). \tag{25}$$

Using eq. (24) and using VARMA representation of GARCH model, the general analytic expression for VIRF in Hafner and Herwartz (2006) is as follows:

$$V_{t+h}\left(\boldsymbol{\xi_t}\right) = \boldsymbol{\phi_h}\boldsymbol{D_k^+}\left(\boldsymbol{\Sigma_{u,t}}^{1/2} \otimes \boldsymbol{\Sigma_{u,t}}^{1/2}\right)\boldsymbol{D_k}\,\text{vech}\left(\boldsymbol{\xi_t}\boldsymbol{\xi_t}^T - \boldsymbol{I_k}\right), \tag{26}$$

where $\boldsymbol{\phi_h} = \left(\tilde{\boldsymbol{A}}_{\boldsymbol{1}} + \tilde{\boldsymbol{B}}_{\boldsymbol{1}}\right)^{h-1} \tilde{\boldsymbol{A}}_{\boldsymbol{1}}$, $\boldsymbol{D_m}$ denotes the duplication matrix defined by the property $\boldsymbol{vec(Z)} = \boldsymbol{D_m}\,\mathrm{vech}(Z)$ for any symmetric $(m \times m)$ matrix $\boldsymbol{Z}$, and $\boldsymbol{D}^{+}_{\boldsymbol{m}}$ denotes its Moore-Penrose inverse.

In the previous section, we propose to parameterize the heteroskedastic statistical innovations $\boldsymbol{u_t}$ with a GO-GARCH model. The analytic expression of VIRF in eq. (26) offers an obvious solution to analyse the dynamic impact effects of a structural shock on the conditional (co)variances.

In order to apply eq. (26), we have to make a connection between the vec GARCH and the GO-GARCH(1,1) representations. In van der Weide (2002) and Bauwens et al. (2006), the GO-GARCH model is a generalization of the Orthogonal-GARCH model, which is also a special case of the factor GARCH models. Thus, the GO-GARCH model is nested in the general BEKK model (Engle and Kroner, 1995), where its properties follow from those of the BEKK model. Hence, we first transform the GO-GARCH into the BEKK, and then into its vec GARCH representation. See Appendix (4) for the transformation from a GO-GARCH model to a vec GARCH. Finally, the VIRF in eq. (26) can be calculated using the estimated $\widehat{\boldsymbol{A_0}}^{-1}$ from the identified SBT-SVAR model.

The conditional moment profile framework proposed in Gallant et al. (1993) is similar to the VIRF proposed in Hafner and Herwartz (2006). A comparison of the conditional moment profile of volatility to the baseline profile, we refer to it as conditional volatility profile hereafter, is analogous to VIRF. In Gallant et al. (1993), the shocks are interpreted as a direct perturbation on $\boldsymbol{y_t}$. Therefore, the statistical innovation $\boldsymbol{u_t}$ can be viewed as an impulse or shock adding on the contemporaneous $\boldsymbol{y_t}$.

The analytic expressions of the conditional volatility profile are also given in Hafner and Herwartz (2006) based on the types of shock and baseline. Suppose the baseline $\boldsymbol{u_0}$ is fixed to $\mathbf{0}$, a shock is fixed at $\boldsymbol{\delta_t}$, the conditional volatility profile is denoted as $\boldsymbol{v_{t+h}}\left(\boldsymbol{\delta_t}\right)$,

$$v_{t+h}\left(\boldsymbol{\delta_t}\right) = E[\mathrm{vech}\left(\boldsymbol{\Sigma_{u,t+h}}\right) \mid \boldsymbol{u_t} = \boldsymbol{u_0} + \boldsymbol{\delta_t}, \mathcal{F}_{t-1}] - E[\mathrm{vech}\left(\boldsymbol{\Sigma_{u,t+h}}\right) \mid \boldsymbol{u_t} = \boldsymbol{u_0}, \mathcal{F}_{t-1}]. \tag{27}$$

We skip the derivations in Hafner and Herwartz (2006), but only demonstrate the analytic expression of VIRF from Hafner and Herwartz (2006) achieved using the VMA representation,

$$\begin{aligned} v_{t+h}\left(\boldsymbol{\delta_t}\right) &= \boldsymbol{\phi_h}\left[\operatorname{vech}\left((\boldsymbol{u_0}+\boldsymbol{\delta_t})(\boldsymbol{u_0}+\boldsymbol{\delta_t})^T\right)-\operatorname{vech}\left(\boldsymbol{u_0}\boldsymbol{u_0}^T\right)\right] \\ &= \boldsymbol{\phi_h}\boldsymbol{D_k^+}\operatorname{vec}\left(\boldsymbol{\delta_t}\boldsymbol{\delta_t}^T+2\boldsymbol{\delta_t}\boldsymbol{u_0}^T\right). \end{aligned} \tag{28}$$

Given the baseline $\boldsymbol{u_0}$ is fixed to $\mathbf{0}$, the conditional volatility profile simplifies to

$$v_{t+h}\left(\boldsymbol{\delta_t}\right)=\boldsymbol{\phi_h}\operatorname{vech}\left(\boldsymbol{\delta_t}\boldsymbol{\delta_t}^T\right). \tag{29}$$

Suppose a fixed shock with size $\boldsymbol{\delta_t}=\boldsymbol{\Sigma_{u,t}}^{1/2}\boldsymbol{\xi_t}$, the conditional volatility profile is then

$$\begin{aligned} v_{t+h}\left(\boldsymbol{\Sigma_{u,t}}^{1/2}\boldsymbol{\xi_t}\right) &= \phi_h\operatorname{vech}\left(\boldsymbol{\Sigma_{u,t}}^{1/2}\boldsymbol{\xi_t}\boldsymbol{\xi_t}^T\boldsymbol{\Sigma_{u,t}}^{1/2}\right) \\ &= V_{t+h}\left(\boldsymbol{\xi_t}\right)+\boldsymbol{\phi_h}\operatorname{vech}\left(\boldsymbol{\Sigma_{u,t}}^{1/2}\right). \end{aligned} \tag{30}$$

Comparing eq. (24) and eq. (30), there is a clear connection between the VIRF and the conditional volatility profile. However, the interpretations of the perturbing shocks and the initial conditions are different. Gallant et al. (1993) argue for a representative impulse response sequence, that is either using an “average” initial condition or taking the average of many impulse-response sequences conditional on many different initial conditions draws drawn from their marginal density. Regarding the perturbing shocks, Gallant et al. (1993) experiment with different sizes of shocks in a bivariate system. Hafner and Herwartz (2006) argue that the choices of innitial condition (baseline) and shock could be arbitary. Hafner and Herwartz (2006) give a comprehensive discussion of the four different resulting $v_{t+h}\left(\boldsymbol{\delta_t}\right)$ due to the permutations of (either fixed or random) baselines and shocks.

Hafner and Herwartz (2006) compare the conditional volatility profile in eq. (27) with VIRF in eq. (24), focusing on the impact effects of two specific events on the conditional volatilities.

To calculate the conditional volatility profile $v_{t+h}(\boldsymbol{\delta_t})$ in (27), Hafner and Herwartz (2006) set the baseline as zero, i.e. $\boldsymbol{u_0} = \mathbf{0}$. The estimated residual $\boldsymbol{\hat{u}_t}$ on the event day t is considered to be the perturbing shock $\boldsymbol{\delta_t}$, i.e. $\boldsymbol{\delta_t} = \boldsymbol{\hat{u}_t}$. Given the interpretation of shocks is different in the VIRF framework than in the conditional volatility profile, to calculate $V_{t+h}(\boldsymbol{\xi_t})$ in eq. (24), a standardized estimated residual is taken as the shock, i.e. $\boldsymbol{\xi_t} = \boldsymbol{\hat{\epsilon}_t}$. Hence, using eq. (18) we can estimate the shock $\boldsymbol{\hat{\epsilon}_t}$ on day t using the estimated residual $\boldsymbol{\hat{u}_t}$ and the estimated volatility state $\boldsymbol{\Sigma_{u,t}}$. In this way, the identified structural shock $\boldsymbol{\hat{\epsilon}_t}$ in Hafner and Herwartz (2006) is interpreted as a materialised shock, which reflects the information in independent news.

In our case, we neither focus on the impact effects of a historical shock $\boldsymbol{\xi_t}$ on a particular day, treating $\boldsymbol{\xi_t} = \boldsymbol{\hat{\epsilon}_t}$ as proposed in Hafner and Herwartz (2006), nor try to directly inflict different contemporaneously related $\boldsymbol{u_t}$ on $\boldsymbol{y_t}$, treating $\boldsymbol{\delta_t} = \boldsymbol{\hat{u}_t}$ as proposed in Gallant et al. (1993), we arbitrarily choose fixed values of hypothetical $\boldsymbol{\xi_t}$, and investigate the conditional (co)variances respond to these hypothetical orthogonal shocks $\boldsymbol{\xi_t}$, given different points in time with a heteroskedastic $\boldsymbol{\Sigma_{u,t}}$.

Suppose the dependent variable $\boldsymbol{y_t}$ is ordered as $\boldsymbol{y_t} = (GPR_t, 100\times\Delta\log ShaleP_t, 100 \times \Delta \log OilP_t)^T$, where $100 \times \Delta \log ShaleP_t$ calculates the percentage change in the shale production and $100 \times \Delta \log OilP_t$ calculates the oil returns. Let a hypothetical structural shock $\boldsymbol{\xi_t}$ at time t as $\boldsymbol{\xi_t} = \boldsymbol{\epsilon_t} = (1,0,0)^T$, the order of the variables indicates that there is only one unit hypothetical structural shock imposed on the geopolitical risk at time t. Similarly, let another type of unit hypothetical structural shock $\boldsymbol{\xi_t^*}$ be imposed on both geopolitical risk and shale production simultaneously, i.e. $\boldsymbol{\xi_t^*} = \boldsymbol{\epsilon_t^*} = (1,1,0)^T$. Denoting the VIRF with shock $\boldsymbol{\xi_t^*}$ for h periods ahead as V^*_{t+h}, and VIRF with shock $\boldsymbol{\xi_t}$ as V_{t+h}, using eq. (26),

$$V^*_{t+h}\left(\boldsymbol{\epsilon_t^*}\right) - V_{t+h}\left(\boldsymbol{\epsilon_t}\right) = \boldsymbol{\phi_h D_k^+}\left(\boldsymbol{\Sigma_{u,t}}^{1/2} \otimes \boldsymbol{\Sigma_{u,t}}^{1/2}\right)\boldsymbol{D_k}\,\mathrm{vech}\left(\boldsymbol{\xi_t^* \xi_t^{*T}} - \boldsymbol{\xi_t \xi_t^T}\right) \quad (31)$$

calculates the differences in VIRF under two different circumstances given a perturbing geopolitical shock, i.e. one under a simultaneous shale shock, whereas the other without.

We now look at the differences in variance responses in the two scenarios using the conditional volatility profiles. Suppose the shock $\boldsymbol{\delta_t}$ directly inflicts on $\boldsymbol{y_t}$ takes the form $\boldsymbol{\delta_t} = \boldsymbol{\Sigma_t^{1/2}\epsilon_t}$, and suppose a hypothetical shock $\boldsymbol{\epsilon_t} = (1,0,0)^T$, the corresponding conditional variance profile is denoted as $\boldsymbol{v_{t+h}}$. In another scenario, suppose the hypothetical shock consists of both a geopolitical risk shock and a shale production shock, i.e. $\boldsymbol{\epsilon_t^*} = (1,1,0)^T$, and shock $\boldsymbol{\delta_t^*}$ takes the form $\boldsymbol{\delta_t^*} = \boldsymbol{\Sigma_t^{1/2}} \boldsymbol{\epsilon_t^*}$, denote the corresponding conditional variance profile as $\boldsymbol{v_{t+h}^*}$. Using eq. (27), the difference between these two conditional (co)variance profiles under the two different direct perturbations is

$$\begin{aligned} v_{t+h}^* \left(\boldsymbol{\delta_t^*}\right) - v_{t+h} \left(\boldsymbol{\delta_t}\right) &= \boldsymbol{\phi_{t+h}} \left(\operatorname{vech} \left(\boldsymbol{\Sigma_{u,t}^{1/2}} \boldsymbol{\epsilon_t^*} \boldsymbol{\epsilon_t^{*T}} \boldsymbol{\Sigma_{u,t}^{1/2}} \right) - \operatorname{vech} \left(\boldsymbol{\Sigma_{u,t}^{1/2}} \boldsymbol{\epsilon_t} \boldsymbol{\epsilon_t^T} \boldsymbol{\Sigma_{u,t}^{1/2}} \right) \right) \\ &= V_{t+h}^* \left(\boldsymbol{\epsilon_t^*}\right) - V_{t+h} \left(\boldsymbol{\epsilon_t}\right) \end{aligned} \tag{32}$$

Therefore, using a fixed baseline, the impact differences between the two fixed hypothetical shocks $\boldsymbol{\epsilon_t^*}$ and $\boldsymbol{\epsilon_t}$ has the same analytical expression in the conditional volatility profile as that in the VIRF framework.

As seen from eq. (26), the three functions, $V_{t+h}^* \left(\boldsymbol{\epsilon_t^*}\right) - V_{t+h} \left(\boldsymbol{\epsilon_t}\right)$, $V_{t+h}^* \left(\boldsymbol{\epsilon_t^*}\right)$ and $V_{t+h} \left(\boldsymbol{\epsilon_t}\right)$, share the same rate of decay, which is determined by $\boldsymbol{\phi_h}$. Also, because we fix the size of shocks in $\boldsymbol{\epsilon_t^*}$ and $\boldsymbol{\epsilon_t}$, $\operatorname{vech}\left(\boldsymbol{\epsilon_t^*} \boldsymbol{\epsilon_t^{*T}}\right) - \operatorname{vech}\left(\boldsymbol{\epsilon_t} \boldsymbol{\epsilon_t^T}\right)$ is also fixed. Therefore, the volatility impulse responses difference, $V_{t+h}^* \left(\boldsymbol{\epsilon_t^*}\right) - V_{t+h} \left(\boldsymbol{\epsilon_t}\right)$, will mainly depend on the conditional variance $\boldsymbol{\Sigma_{u,t}}$ at that point in time. It also depends on $\widehat{\boldsymbol{A_0}^{-1}}$, which is also reflected in the decay parameter $\boldsymbol{\phi_h}$.

In order to evaluate the impact effects of shale revolution, we analyse the differences in VIRF, $V_{t+h}^* \left(\boldsymbol{\epsilon_t^*}\right) - V_{t+h} \left(\boldsymbol{\epsilon_t}\right)$, or the differences in two conditional volatilities profiles, $v_{t+h}^* \left(\boldsymbol{\epsilon_t^*}\right) - v_{t+h} \left(\boldsymbol{\epsilon_t}\right)$, under the two types of hypothetical shocks, i.e. shock $\boldsymbol{\epsilon_t^*}$ represents that shale production is imposed with a simultaneous shock as well as geopolitical risk, whereas $\boldsymbol{\epsilon_t}$ indicates only the geopolitical risk variable is imposed with a unit shock.

E: VAR and Cholesky decomposition

The baseline VAR model is specified as follows:

$$\boldsymbol{y_t} = \boldsymbol{\Gamma_0} + \boldsymbol{\Gamma_1} \boldsymbol{y_{t-1}} + \ldots + \boldsymbol{\Gamma_p} \boldsymbol{y_{t-p}} + \boldsymbol{u_t}, \tag{33}$$

where $\boldsymbol{y_t} = (y_{1t}, \ldots, y_{kt})^T$ and $\boldsymbol{y_t} \in \mathbb{R}^{k\times 1}$. The vector of intercepts $\boldsymbol{\Gamma_0} \in \mathbb{R}^{k\times 1}$ and the coefficients are squared matrix $\boldsymbol{\Gamma_i} \in \mathbb{R}^{k\times k}$ with $i = 1, \ldots, p$. Therefore, eq.

(33) can be summarized as

$$\boldsymbol{y_t} = \boldsymbol{\Gamma} \boldsymbol{X_t} + \boldsymbol{u_t}, \tag{34}$$

denoting $\boldsymbol{\Gamma} = (\boldsymbol{\Gamma_0}, \boldsymbol{\Gamma_1}, \ldots, \boldsymbol{\Gamma_p})$, and $\boldsymbol{X_t} = (1, \boldsymbol{y_{t-1}}, \boldsymbol{y_{t-2}}, \ldots, \boldsymbol{y_{t-p}})^T$ where $\boldsymbol{\Gamma} \in \mathbb{R}^{k \times (kp+1)}$, and $\boldsymbol{X_t} \in \mathbb{R}^{(kp+1) \times 1}$. The error term $\boldsymbol{u_t} \overset{i.i.d}{\sim} MN\left(\boldsymbol{0}, \boldsymbol{\Sigma_u}\right)$, where $\boldsymbol{\Sigma_u} \in \mathbb{R}^{k \times k}$ and $MN\left(\boldsymbol{0}, \boldsymbol{\Sigma}\right)$ denotes a multivariate normal distribution with mean $\boldsymbol{0}$ and a constant covariance matrix $\boldsymbol{\Sigma}$.

In order to quantify the dependent variable $\boldsymbol{y_t}$'s response to a specific exogenous shock, mutually independent structural shocks have to be identified. Because of the covariance $\boldsymbol{\Sigma}_u$ specification in a reduced form VAR system, a hypothetical shock, therefore, cannot be isolated from other error terms. Hence, it is impossible to provide a clear interpretation of the impulse response function using the reduced form VAR. Therefore, the reduced form VAR has been extended to a structural VAR (SVAR) in the literature. There are many ways to identify the structural shocks in SVAR, such as by imposing different identification restrictions. An SVAR model is specified as follows:

$$\boldsymbol{A_0} \boldsymbol{y_t} = \boldsymbol{A_1} \boldsymbol{X_t} + \boldsymbol{\epsilon_t}, \tag{35}$$

where $\boldsymbol{\epsilon_t}$ are serially and mutually uncorrelated structural innovations. The reduced form VAR in eq. (34) and SVAR in eq. (35) can be linked with $\boldsymbol{u_t} = \boldsymbol{A_0}^{-1} \boldsymbol{\epsilon_t}$ and $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right) \boldsymbol{\Sigma_\epsilon} \left(\boldsymbol{A_0}^{-1}\right)^T$, where $\boldsymbol{u_t}$ and $\boldsymbol{\Sigma_u}$ can be achieved by estimation and treated as observables. From eq. (34) and eq. (35), $\boldsymbol{\Gamma}$ in VAR has the form $\boldsymbol{\Gamma} = \boldsymbol{A_0}^{-1} \boldsymbol{A_1}$, where $\boldsymbol{A_1}$ is a reflection of the feedback dynamics in SVAR.

Identifying the structural shocks has been a focus in the literature. One obvious solution to identification is to use the Cholesky decomposition, i.e. $\boldsymbol{\Sigma_\epsilon}$ is normalized to unity, then $\boldsymbol{\Sigma_u}$

$$\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right) \left(\boldsymbol{A_0}^{-1}\right)^T. \tag{36}$$

By Cholesky decomposition, $\boldsymbol{A_0}^{-1}$ is the lower triangular. Thus, the statistical innovation $\boldsymbol{u}_t$ depends recursively on the mutually uncorrelated structural innovation $\boldsymbol{\epsilon}_t$.

F: GO-GARCH in BEKK representation

Denote information set up to t is $\mathcal{F}_t$, $E_{t-1}(\cdot) \equiv E(\cdot \mid \mathcal{F}_{t-1})$ and the statistical innovation is

$$\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}, \tag{37}$$

where $\boldsymbol{\Sigma_{\epsilon,t}} = E_{t-1}\left(\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}^T\right)$ and $\boldsymbol{\Sigma_{u,t}} = E_{t-1}\left(\boldsymbol{u_t}\boldsymbol{u_t}^T\right)$. The unconditional variance of the structural innovations are normalized to unity, i.e. $E\left(\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}^T\right) = \boldsymbol{I}$. The unconditional variance of the statistical innovation is $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\boldsymbol{\Sigma_\epsilon}\left(\boldsymbol{A_0}^{-1}\right)^T$. The heteroskedasticity in the structural innovations can be specified with a GARCH(1,1) process as follows

$$\boldsymbol{\Sigma_{\epsilon,t}} = (\boldsymbol{I} - \Delta_1 - \Delta_2) + \Delta_1 \circ \left(\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}}^T\right) + \Delta_2 \circ \boldsymbol{\Sigma_{\epsilon,t-1}}, \tag{38}$$

where Δ_1 and Δ_2 are diagonal matrices, and "$\circ$" denotes the Hadamard product operator. If Δ_1 and Δ_2 are null, then $\boldsymbol{\Sigma_{\epsilon,t}}$ is constant. Δ_1 and Δ_2 are positive semi-definite, and $(\boldsymbol{I} - \Delta_1 - \Delta_2)$ is positive definite, which indicate that at least one structural innovation is GARCH(1,1). Therefore, the GARCH(1,1) for an individual conditional structural variance is

$$\sigma^2_{m,t|t-1} = (1 - \gamma_m - \delta_m) + \gamma_m \epsilon^2_{m,t-1} + \delta_m \sigma^2_{m,t-1|t-2}, \quad m = 1, \ldots, k. \tag{39}$$

Therefore, the linear combination of $\boldsymbol{A_0^{-1}}$ and $\boldsymbol{\epsilon_t}$ fit in a GO-GARCH representation proposed in van der Weide (2002).

$$\boldsymbol{A_0}^{-1} = \boldsymbol{P}\boldsymbol{\Lambda}^{1/2}\boldsymbol{U}^T, \tag{40}$$

where $\boldsymbol{P}$ and $\boldsymbol{\Lambda}$ denote the matrices with the orthogonal eigenvectors and the eigenvalues of $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\boldsymbol{\Sigma_\epsilon}\left(\boldsymbol{A_0}^{-1}\right)^T$, respectively. $\boldsymbol{U}$ is the orthogonal matrix of eigenvectors of $\boldsymbol{A_0}^{-1}\boldsymbol{A_0}^{-1T}$. In van der Weide (2002), the matrices $\boldsymbol{P}$ and $\boldsymbol{\Lambda}$ are estimated directly by means of unconditional information, e.g. from the sample covariance matrix $\boldsymbol{\Sigma_u}$. Therefore, in the GO-GARCH model, to identify $\boldsymbol{A_0}^{-1}$, we have to identify the orthogonal matrix $\boldsymbol{U}$. Considering the GO-GARCH is nested in the more general BEKK model, we fit the GO-GARCH in the BEKK representation.

Consider the BEKK model proposed by Baba, Engle, Kraft, and Kroener in Baba et al. (1990),

$$\boldsymbol{\Sigma_{u,t}} = \boldsymbol{C} + \sum_{i=1}^{k} \boldsymbol{A_i} \boldsymbol{u_{t-1}} \boldsymbol{u_{t-1}}^T \boldsymbol{A_i}^T + \sum_{j=1}^{k} B_j \Sigma_{u,t-1} B_j^T, \tag{41}$$

matrices $\{\boldsymbol{A_{i=1}^k}\}$ and $\{\boldsymbol{B_{j=1}^k}\}$ are restricted to have identical eigenvector matrix $\boldsymbol{A_0}^{-1}$, where the eigenvalues of A_i and B_j are all zero except for the i-th and j-th one, respectively. Assume C can be decomposed as $\boldsymbol{A_0}^{-1}\boldsymbol{D_c}\boldsymbol{A_0}^{-1T}$, where $\boldsymbol{D_c}$ is a positive definite diagonal matrix. Then the associate BEKK parameterization is equivalent to a GO-GARCH(1,1).

Proof. matrices $\{\boldsymbol{A_{i=1}^k}\}$ and $\{\boldsymbol{B_{j=1}^k}\}$ are restricted to have identical eigenvector matrix $\boldsymbol{A_0}^{-1}$ so that they can be diagonalized as

$$\boldsymbol{A_i} = \boldsymbol{A_0}^{-1}\boldsymbol{D_{A_i}}\boldsymbol{A_0}^{-1T} \quad \boldsymbol{B_j} = \boldsymbol{A_0}^{-1}\boldsymbol{D_{B_j}}\boldsymbol{A_0}^{-1T}, \tag{42}$$

where $\boldsymbol{D_{A_i}}$ and $\boldsymbol{D_{B_j}}$ denote diagonal eigenvalue matrices. Note all elements of $\boldsymbol{D_{A_i}}$ and $\boldsymbol{D_{B_j}}$ are zero except for the i-th and j-th elements. Therefore, denoting the only non-zero elements a_i in $\boldsymbol{D_{A_i}}$ and b_j in $\boldsymbol{D_{B_j}}$, where a_i and b_j represent the only non-zero eigenvalue of $\boldsymbol{A_i}$. By substitution we have

$$\begin{aligned}\boldsymbol{\Sigma_{u,t}} =& \boldsymbol{A_0}^{-1}\boldsymbol{D_c}\boldsymbol{A_0}^{-1T} + \sum_{i=1}^{k} \boldsymbol{A_0}^{-1}\boldsymbol{D_{A_i}}\boldsymbol{A_0}\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T\boldsymbol{A_0}^T\boldsymbol{D_{A_i}}\boldsymbol{A_0}^{-1T} \\ &+ \sum_{j=1}^{k} \boldsymbol{A_0}^{-1}\boldsymbol{D_{B_j}}\boldsymbol{A_0}\boldsymbol{\Sigma_{u,t-1}}\boldsymbol{A_0}^T\boldsymbol{D_{B_j}}\boldsymbol{A_0}^{-1T}\end{aligned}, \tag{43}$$

which can be simplified to

$$\boldsymbol{\Sigma_{u,t}} = \boldsymbol{A_0}^{-1}\left[\boldsymbol{D_c} + \sum_{i=1}^{k}\boldsymbol{D_{A_i}}\boldsymbol{A_0}\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T\boldsymbol{A_0}^T\boldsymbol{D_{A_i}} + \sum_{j=1}^{k}\boldsymbol{D_{B_j}}\boldsymbol{A_0}\boldsymbol{\Sigma_{u,t-1}}\boldsymbol{A_0}^T\boldsymbol{D_{B_j}}\right]\mathbf{A_0}^{-1T}. \tag{44}$$

According to eq. (37), $\boldsymbol{u_t}$, where $\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}$ or $\boldsymbol{\epsilon_t} = \boldsymbol{A_0}\boldsymbol{u_t}$, represents the unobserved components in the GO-GARCH model. Denoting $\boldsymbol{\Sigma_{\epsilon,t}} = \boldsymbol{A_0}\boldsymbol{\Sigma_{u,t}}\boldsymbol{A_0}$ as the conditional covariance of $\boldsymbol{\epsilon_t}$, by arranging eq. (43), we find

$$\boldsymbol{\Sigma_{\epsilon,t}} = \boldsymbol{D_c} + \sum_{i=1}^{k}\boldsymbol{D_{A_i}}\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}}^T\boldsymbol{D_{A_i}} + \sum_{j=1}^{k}\boldsymbol{D_{B_j}}\boldsymbol{\Sigma_{\epsilon,t-1}}\boldsymbol{D_{B_j}}. \tag{45}$$

By the properties of matrices $\boldsymbol{D_{A_i}}$ and $\boldsymbol{D_{B_j}}$, it follows that the sum can be re-written using Hadamard product as

$$\sum_{i=1}^{k} \boldsymbol{D_{A_i} \epsilon_{t-1} \epsilon_{t-1}}^T \boldsymbol{D_{A_i}} = \boldsymbol{D_A} \circ \boldsymbol{\epsilon_{t-1} \epsilon_{t-1}}^T, \quad \sum_{j=1}^{k} \boldsymbol{D_{B_j} \Sigma_{\epsilon,t-1} D_{B_j}} = \boldsymbol{D_B} \circ \boldsymbol{\Sigma_{\epsilon,t-1}}, \tag{46}$$

where $\boldsymbol{D_A} = diag\,(a_1, \ldots, a_k)$ and $\boldsymbol{D_B} = diag\,(b_1, \ldots, b_k)$. Then $\boldsymbol{D_c}$, $\boldsymbol{D_A} \circ \boldsymbol{u_{t-1} u_{t-1}}^T$, and $\boldsymbol{D_B} \circ \boldsymbol{\Sigma_{\epsilon,t-1}}$, are all diagonal, and $\boldsymbol{\Sigma_{\epsilon,t}}$, the conditional covariance matrix of $\boldsymbol{\epsilon_t}$, is also diagonal. Therefore, eq. (46) implies a univariate GARCH(1,1) specification for $\boldsymbol{\epsilon_t}$ as it is assumed by the GO-GARCH model. Therefore, using our GARCH(1,1) specification in eq. (38),

$$\begin{aligned} \boldsymbol{C} &= \boldsymbol{A_0}^{-1} \boldsymbol{D_c A_0}^{-1T} \\ \boldsymbol{A_i} &= \boldsymbol{A_0}^{-1} \boldsymbol{D_{A_i} A_0}^{-1T} \\ \boldsymbol{B_j} &= \boldsymbol{A_0}^{-1} \boldsymbol{D_{B_j} A_0}^{-1T}. \end{aligned}$$

With the estimates of $\boldsymbol{D_A}$ and $\boldsymbol{D_B}$, we can find matrices $\{\boldsymbol{D_{A_i}}\}$ with $i = 1, \ldots, k$ and $\{\boldsymbol{D_{B_j}}\}$ with $j = 1, \ldots, k$. After formalizing the GO-GARCH into the BEKK form, eq. (41) can be transformed to the corresponding vec form.

Vectorizing eq. (41),

$$\text{vech}\,(\boldsymbol{\Sigma_{u,t}}) = \text{vech}\,(\boldsymbol{C}) + \sum_{i=1}^{k} \text{vech}\left(\boldsymbol{A_i u_{t-1} u_{t-1}}^T \boldsymbol{A_i}^T\right) + \sum_{i=1}^{k} \text{vech}\left(\boldsymbol{B_j \Sigma_{u,t-1} B_j}^T\right), \tag{47}$$

Denote $\otimes$ as the Kronecker product operator, recognizing vec $\left(xy^T\right) = y \otimes x$ and using the product rule [5] with Kronecker product, eq. (47) becomes

$$\text{vec}\,(\boldsymbol{\Sigma_{u,t}}) = \text{vec}\,(\boldsymbol{C}) + \sum_{i=1}^{k} (\boldsymbol{A_i} \otimes \boldsymbol{A_i})\,\text{vec}\left(\boldsymbol{u_{t-1} u_{t-1}}^T\right) + \sum_{i=1}^{k} (\boldsymbol{B_j} \otimes \boldsymbol{B_j})\,\text{vec}\,(\boldsymbol{\Sigma_{u,t-1}})\,. \tag{48}$$

Denoting $\boldsymbol{D_k}$ as the duplication matrix, given vec($\boldsymbol{A}$) = $\boldsymbol{D_k}$ vech($\boldsymbol{A}$), eq. (48) is

[5] The product rule is $(A \otimes B)(C \otimes D) = AC \otimes BD$, and $\text{vec}[(Ax)\left(y^T B\right)] = \left(B^T y\right) \otimes (Ax) = \left(B^T \otimes A\right)(y \otimes x) = \left(B^T \otimes A\right) \text{vec}\left(xy^T\right)$. $\text{vec}\,(ABC) = \left(C^T \otimes A\right) \text{vec}\, B$

$$\boldsymbol{D_k}\,\text{vech}\left(\boldsymbol{\Sigma_{u,t}}\right) = \boldsymbol{D_k}\,\text{vech}(\boldsymbol{C}) + \sum_{i=1}^{k}\left(\boldsymbol{A_i} \otimes \boldsymbol{A_i}\right)\boldsymbol{D_k}\,\text{vech}\left(\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T\right) + \sum_{i=1}^{k}\left(\boldsymbol{B_j} \otimes \boldsymbol{B_j}\right)\boldsymbol{D_k}\,\text{vech}\left(\boldsymbol{\Sigma_{u,t-1}}\right). \quad (49)$$

Define the generalized inverse of $\boldsymbol{D_k}$ as $\boldsymbol{D_k^+} = \left(\boldsymbol{D_k}^T\boldsymbol{D_k}\right)^{-1}\boldsymbol{D_k}^T$, that is a $(k \times (k+1)/2) \times (k^2)$ matrix, where $\boldsymbol{D_k}$ where $\boldsymbol{D_k^+}\boldsymbol{D_k} = \boldsymbol{I_k}$ Then, we can have a unique transformation from BEKK to vech as follows:

$$\text{vech}(\Sigma_{u,t}) = \text{vech}(C) + D_k^+\left(\sum_{i=1}^{k}(A_i \otimes A_i)\right)D_k\,\text{vech}\left(u_{t-1}u_{t-1}^T\right) + D_k^+\left(\sum_{i=1}^{k}(B_j \otimes B_j)\right)D_k\,\text{vech}\left(\Sigma_{u,t-1}\right).$$

Given the vech model

$$\text{vech}(\boldsymbol{\Sigma_{u,t}}) = \boldsymbol{W} + \tilde{\boldsymbol{A}}\,\text{vech}\left(\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T\right) + \tilde{\boldsymbol{B}}\,\text{vech}(\boldsymbol{\Sigma_{u,t-1}}), \quad (50)$$

We have the following relations

$$\begin{aligned}\tilde{\boldsymbol{A}} &= \boldsymbol{D_k^+}\left(\sum_{i=1}^{k}(\boldsymbol{A_i} \otimes \boldsymbol{A_i})\right)\boldsymbol{D_k}, \\ \tilde{\boldsymbol{B}} &= \boldsymbol{D_k^+}\left(\sum_{i=1}^{k}(\boldsymbol{B_j} \otimes \boldsymbol{B_j})\right)\boldsymbol{D_k}.\end{aligned}$$

After fitting the GO-GARCH in a vech GARCH form, we can apply the results in Hafner and Herwartz (2006) and calculate VIRF.

G: GIRF with a Cholesky decomposition

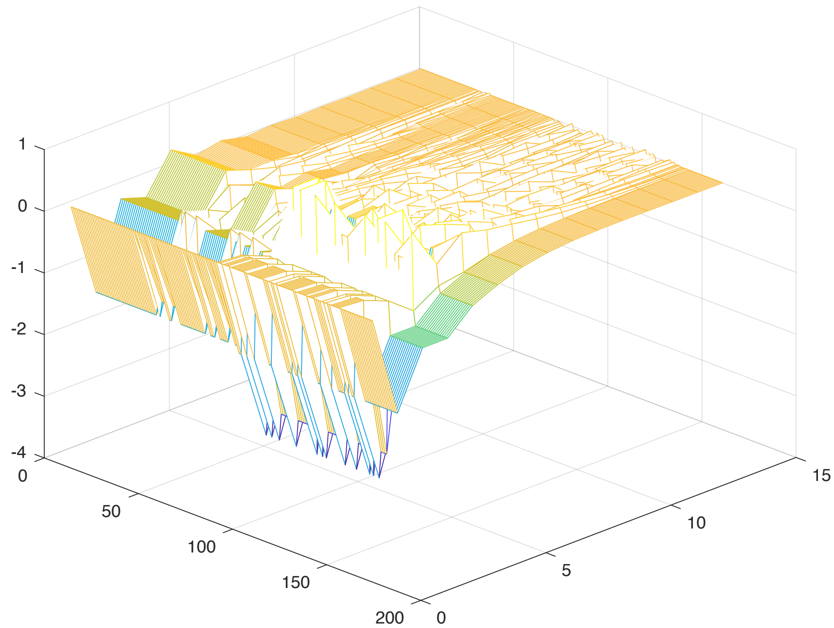

Figure 8: ΔOil price responses to a structural shock in GPR by a Cholesky decomposition.

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